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NEUTRON SELF-SHIELDING OF A PURELY ABSORBING CYLINDRICAL ROD

Joel Bengston

Since the general solution of the problem of the neutron self-shielding of a cylindrical absorbing rod embedded in an infinite medium has not been obtained in closed form, the solution to a very special case treated here is useful for rough estimates. If we consider a medium with cylindrical symmetry in which the total neutron cross section is independent of position, the integral equation for the flux may be written¹

$$\rho(r) = \int_0^{\infty} G(r, r') \left[\sum_{sc} \rho(r') + S(r') \right] r' dr' \quad (1a)$$

where

$$G(r, r') = \sum_{tot} \int_1^{\infty} K_0 \left(\sum_{tot} y r_+ \right) I_0 \left(\sum_{tot} y r_- \right) dy$$

In principle, this equation could be solved by iteration of the source term. The lowest order of approximation, however, consists of putting $\sum_{sc} = 0$ in the righthand side of Eq. (1a). If we consider two regions such that the source term is

$$\begin{aligned} S(r) &= 0, \quad r < R \\ &= S_0, \quad r > R, \end{aligned} \quad (2)$$

then the contribution due to the source term alone in Eq. (1a) is:

$$\begin{aligned} \rho_s(r \leq R) &= \sum S_0 \int_R^{\infty} r' dr' \int_1^{\infty} K_0 \left(\sum y r' \right) I_0 \left(\sum y r \right) dy \\ &= S_0 R \int_1^{\infty} \frac{dy}{y} I_0 \left(\sum r y \right) K_1 \left(\sum R y \right) \end{aligned} \quad (3)$$

1. A. M. Weinberg and L. C. Noderer, Theory of Neutron Chain Reactions, ORNL CF-51-5-98, Vol. I (May 15, 1951).

The above is the fundamental equation which will now be evaluated for some special cases. The flux at the center of the rod is

$$\rho_s(0) = SR \int_1^{\infty} \frac{dy}{y} K_1(\sum Ry) \quad (4a)$$

To evaluate this integral, use the integral representation in Watson's Bessel functions, p. 185:-

$$K_1(Z) = Z \int_1^{\infty} e^{-Zt} (t^2 - 1)^{1/2} dt.$$

Then

$$\begin{aligned} \rho_s(0) &= SR \int_{\sum R}^{\infty} dz \int_1^{\infty} e^{-Zt} (t^2 - 1)^{1/2} dt \\ &= SR \int_1^{\infty} e^{-\sum Rt} \left(1 - \frac{1}{t^2}\right)^{1/2} dt \quad (4b) \\ &= SR \left[E_0(Z) - \frac{1}{2} E_2(Z) - \frac{1}{8} E_4(Z) - \frac{1}{16} E_6(Z) - \frac{5}{128} E_8(Z) \dots \right]_{Z = \sum R} \\ &= \frac{S}{\sum} \left[Z \left\{ E_0(Z) - \frac{1}{2} E_2(Z) - \frac{1}{8} E_4(Z) - \frac{1}{16} E_6(Z) - \frac{5}{128} E_8(Z) \right. \right. \\ &\quad \left. \left. - \frac{5}{128} \times \frac{7}{10} E_{10}(Z) - \frac{5}{128} \times \frac{7}{10} \times \frac{9}{12} E_{12}(Z) \dots \right\} \right]_{Z = \sum R} \end{aligned}$$

This expansion is unfortunately very slowly convergent, since asymptotic expansions give

$$\frac{E_{n+2}(Z)}{E_n(Z)} \sim \frac{Z + n - 1}{Z + n + 1} \quad (5)$$

Since the series in Eq. (4b) has all terms but the first of the same sign, it is useful to have an upper bound for the remainder after N terms. Let

us write this as

$$R = - \sum_{m=N+1}^{\infty} C_m E_{2m}(Z) \quad (6a)$$

where

$$C_1 = \frac{1}{2}; C_m = \frac{1 \cdot 1 \cdot 3 \cdot 5 \cdots (2m-3)}{2 \cdot 4 \cdot 6 \cdot 8 \cdots (2m)}, m \geq 2. \quad (6b)$$

But these coefficients are just those in the expansion

$$\lim_{x \rightarrow 1} (1-x)^{1/2} = 1 - \sum_{m=1}^{\infty} C_m = 0. \quad (6c)$$

The remainder is then,

$$\begin{aligned} -R &\leq \sum_{m=N+1}^{\infty} C_m E_{2N+2} \\ &\leq (1 - \sum_{m=1}^N C_m) E_{2N+2} \end{aligned} \quad (6d)$$

where the $\sum_1^N C_m$ is just the sum of the coefficients already used in the evaluation, and may be readily found numerically.

Another useful quantity is the flux at the surface of the rod:

$$\rho_s(R) = SR \int_1^{\infty} \frac{dy}{y} I_0(\sum Ry) K_1(\sum Ry). \quad (7a)$$

It may be verified by differentiation that

$$\int I_0(Z) K_1(Z) \frac{dZ}{Z} = -Z (I_1 K_1 + I_0 K_0)_Z - I_0 K_1. \quad (7b)$$

It then turns out that

$$\rho_s(R) = \frac{S}{\sum R} \left[Z^2 (I_1 K_1 + I_0 K_0)_Z + Z I_0(Z) K_1(Z) - Z \right]_{Z=\sum R}. \quad (7c)$$

We can also get the average value of the flux in the rod by integration,

$$\begin{aligned}
 \langle \rho_s \rangle_{\text{rod}} &= \frac{2}{R^2} \int_0^R \rho(r) r dr \\
 &= \frac{2S}{R} \int_1^\infty \frac{dy}{y} K_1(\sum R y) \int_0^R I_0(\sum r y) r dr \\
 &= \frac{2S}{\sum} \int_1^\infty \frac{dy}{y^2} K_1(\sum R y) I_1(\sum R y) .
 \end{aligned} \tag{8a}$$

A useful approximation is²

$$K_v(Z) I_v(Z) \approx \frac{1}{2 \sqrt{v^2 + Z^2}} \tag{8b}$$

which is valid to 5% or better every where for $v = 1$. Substituting this result in Eq. (8a) we obtain,

$$\begin{aligned}
 \langle \rho_s \rangle_{\text{rod}} &= \frac{S}{\sum} \int_1^\infty \frac{dy}{y^2 (1 + \sum^2 R^2 y^2)^{1/2}} \\
 &= \frac{S}{\sum} \left[\sqrt{1 + \sum^2 R^2} - \sum R \right] .
 \end{aligned} \tag{8c}$$

This approximation can be improved somewhat by a correction factor. It is evident that most of the contribution to the integral evaluated in Eq. (8c) is near the beginning of the range. Use this fact to multiply the above result by the actual value of $2K_1(Z) I_1(Z) \sqrt{1 + Z^2}$ at $Z = \sum R$.

$$\therefore \langle \rho_s \rangle_1^{\text{rod}} = \langle \rho_s \rangle_0^{\text{rod}} \times 2K_1(\sum R) I_1(\sum R) \sqrt{1 + \sum^2 R^2} \tag{8d}$$

The above formulas represent the exact solution when there is no scattering either inside or outside the rod, the absorption cross sections are the same in both regions, and there is a spatially uniform source of

2. Yost Wheeler and Breit, Phys. Rev. 49, 181 (1936).

neutrons outside. However, it has been found in the case of plane geometry that the ratio

$$\frac{\langle \rho \rangle_{\text{foil}}}{\rho(\text{edge})} \quad (9)$$

is relatively independent of the properties of the outer medium. It is to be expected that the same will be true of the ratio of corresponding quantities in the cylindrical case and that the flux distribution within a purely absorbing rod is given reasonably well by the above treatment, independent of the properties of the outer medium.

It has been found by R. R. Bate that the ratio Eq. (9) is a linear function of $\sum T$ for small foil thickness ($\lesssim 1$ mean free path). It turns out that the corresponding quantity in the cylindrical case, as determined from Eqs. (8d) and (7c) can also be approximated as a linear function (of $\sum R$) for small rod size

$$\frac{\langle \rho_s \rangle_{\text{rod}}}{\rho(R)} \simeq 1 - 0.37 \sum R, \text{ for } \sum R \lesssim 1. \quad (10)$$

For greater rod radii, this formula will overestimate this self-shielding effect by a progressively larger amount.