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REACTORS-RESEARCH AND TESTING

DANGER COEFFICIENT MEASUREMENTS
USING A WATER BOILER REACTOR

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ABSTRACT

The use of a water boiler reactor as a danger coefficient test instrument has been investigated, with emphasis on the testing of reactor materials for neutron absorbing impurities and the testing of uranium for enrichment variations. After calibration of the WBNS coarse control rod, tests were made on a variety of materials including beryllium, boron trifluoride, iron and steel, natural uranium metal and oxide, and uranium metal of various enrichments. Several experiments were conducted to evaluate the sources and magnitudes of errors in danger coefficient testing. On the basis of the data obtained from sample testing and auxiliary experiments, the sensitivity of the WBNS for absorption and enrichment testing was determined, as well as other constants relating to danger coefficient measurements.

The Water Boiler Neutron Source has been found to be well suited to danger coefficient testing. Small absorption differences between samples of similar geometry and nuclear properties can be determined to a standard deviation of 0.002 cm^2 . The danger coefficient technique using the WBNS compares favorably with other methods of absorption detecting methods, such as the shotgun test or chemical analysis. Enrichment differences between natural uranium samples of only 500 grams can be detected to ± 0.0008 per cent U^{235} by weight, and between 3 per cent enriched samples to ± 0.0028 per cent U^{235} . In enrichment testing relative values can be found to an accuracy equivalent to those obtained from spectrographic analyses. With improvements in the WBNS core arrangement so that larger samples could be handled, and relocation of the glory hole to the flux center of the core, greater sensitivity could be obtained. The WBNS compares favorably to other reactors used for danger coefficient work.

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I. INTRODUCTION

Over the past three years an intermittent series of danger coefficient measurements have been made using the NAA Water Boiler Neutron Source. Considerable data and information have been gathered concerning the procedures, the sensitivity and accuracy, and the advantages and disadvantages of certain techniques with particular materials. The program has demonstrated the utility of the danger coefficient method for the measurement of impurities in uranium metal and oxide, for the determination of the U^{235} enrichment in uranium metal and for the testing of several miscellaneous materials.

This report is intended as a final summary of the measurement program. An attempt has been made to arrange the results into two parts, the first representing the general theory, the experimental technique, and the calibration of the coarse control rod, the second representing actual data obtained in the testing of materials in the WBNS. Many of the routine measurements have been made with the object of proving the utility of this method as compared with other testing methods such as chemical or spectrographic analyses and the "shotgun" method. When possible, therefore, results obtained with the WBNS are compared with independent data such as measurements made elsewhere or computed values.

II. THEORY

In general, the testing program at NAA using the Water Boiler Neutron Source has involved either the determination of the difference between the total absorption cross sections of two or more samples, or the determination of the difference in U^{235} enrichment of two or more uranium samples. For each sample comparison the change in cross section, $\Delta \Sigma_a$, or the change in enrichment, ΔE , is obtained from a measured change in reactivity, Δk .

Suppose a "standard" sample of pure material is inserted into a reproducible position within a reactor, and the nuclear reaction is brought to criticality. A sample identical in every respect to the standard, except for the presence of a small absorptive impurity and a small amount of fissionable material, is then substituted, and the change in reactivity observed. Theory predicts the following

for small reactivity changes¹:

$$\Delta k = \frac{\int \left[-nv \Delta \Sigma_a(v) F(r, v) + \nu nv \Delta \Sigma_f(v) F_H(r) - \Delta D(r, v) \left| \nabla(nv) \right| \left| \nabla F(r, v) \right| \right] dv dV}{\int nv \Sigma_f(r, v) dv dV} \quad \dots (1)$$

where $\Delta \Sigma_a(v)$ = macroscopic absorption cross section of impurity and fissile material evaluated at neutron velocity, v .

$\Delta \Sigma_f(v)$ = macroscopic fission cross section of fissile material evaluated at neutron velocity, v .

ΔD = change in diffusion coefficient due to impurity and fissile material.

$\Sigma_f(r, v)$ = macroscopic fission cross section of reactor.

$F(r, v)$ = effectiveness* of a neutron of velocity, v , added to the assembly at r .

$F_H(r) = F(r, v)$ averaged over the fission neutron spectrum.

Assuming homogeneity of the impurity and the fissile material in the sample volume, and ignoring second order effects as well as the contribution of ΔD , one can write

$$\Delta k = \frac{\overline{nv}_a}{\overline{nv}_o \overline{\Sigma}_f} \left(-\overline{\Delta \Sigma}_a F_L + \nu \overline{\Delta \Sigma}_f F_H \right) = \frac{\nu \Delta f F_H - \Delta a F_L}{R_f} \quad \dots (2)$$

where $\overline{nv}_a$ = average neutron flux over sample.

$\overline{nv}_o$ = average neutron flux in reactor core.

$\overline{\Delta \Sigma}_a$ = total absorption cross section (in cm^2) of impurity and fissile material, averaged with respect to energy using the neutron flux as a weighting factor.

$\overline{\Delta \Sigma}_f$ = total fission cross section of fissile material, averaged as above.

$\overline{\Sigma}_f$ = total fission cross section of reactor, averaged as above.

$F_L = F(r, v)$ averaged over sample volume and neutron velocities in absorption energy spectrum.

$F_H = F_H(r)$ averaged over sample volume.

* $F(r, v)$ is actually a probability. It differs from $P(r, v)$, the probability that a neutron introduced at r with velocity v will produce a fission, in that $F(r, v)$ includes all successive generations. $F(r, v)$ can be called the "infinite order iteration" of $P(r, v)$.

Δa = rate of neutron absorption in impurity and fissile material.

Δf = fission rate in fissile material.

R_f = total fission rate in reactor core.

In practice, where one is performing either absorption impurity testing or enrichment testing, the effects upon Δk of $\Delta \Sigma_a$ or $\Delta \Sigma_f$ are considered separately. For the case of absorption testing, therefore,

$$\Delta k = - \frac{\bar{n} \bar{v}_a \Delta \Sigma_a}{\bar{n} \bar{v}_o \Sigma_f} F_L = - \frac{\Delta a}{R_f} F_L \quad \dots (3)$$

For the case of samples differing only in the amount of fissile material,

$$\Delta \Sigma_a = (1 + \alpha) \Delta \Sigma_f \quad \dots (4)$$

where α is the capture-to-fission ratio in the fissionable material. Therefore,

$$\Delta k = \frac{\bar{n} \bar{v}_a \Delta \Sigma_f}{\bar{n} \bar{v}_o \Sigma_f} \left[\nu F_H - (1 + \alpha) F_L \right] = \frac{\Delta f}{R_f} \left[\nu F_H - (1 + \alpha) F_L \right] \quad \dots (5)$$

Consequently, for small Δf the observed value of Δk is directly proportional to the change in fission cross section between the two samples.

Returning to the effect of ΔD upon Δk it will be noted that the factor $|\nabla(nv)|$ is involved. In the ideal danger coefficient test reactor, the sample position lies at the flux center of the assembly, where the flux gradient is zero. Changes in the diffusion coefficient caused by differences in the samples will have no effect upon Δk in such a case. Unfortunately, the test hole in the WBNS is displaced 3 inches below core center, resulting in a non-negligible value of $|\nabla(nv)|$ in the vertical direction through the sample. Now for isotropic scattering

$$D = \frac{1}{3(\Sigma_s + 0.2 \Sigma_a)} \quad \dots (6)$$

so that an increase in either Σ_a or Σ_s causes a decrease in D . From Eq. (1) it is seen that $\Delta \Sigma_a$ has a negative effect upon Δk , but this $\Delta \Sigma_a$ gives rise to a negative value of ΔD , which in turn affects Δk positively. In short, the existence of a flux gradient at the sample position somewhat decreases the sensitivity of the reactor in measurements of $\Delta \Sigma_a$.

Effects due to differences in inelastic scattering cross section have not been discussed. In routine sample testing these effects are generally negligible, and actual evaluation of their magnitude would be inconveniently difficult.

Although Δk can be found from the resulting reactor periods by means of the inhour equation², it is sometimes convenient to relate the change in reactivity to a change in control rod position. If the control rod is readjusted to provide reactor criticality after the addition of the absorptive impurity, the difference in rod position Δx corresponds to the Δk calculated from the period. In most practical situations the reactivity effect of the control rod per unit change of position, $\Delta k/\Delta x$, is a non-linear function of position. A curve of $\Delta \Sigma_a/\Delta x$ versus x (which is equivalent to $\Delta k/\Delta x$ versus x) can be obtained by means of alternate measurements with two samples whose cross sectional difference is already known. In order to advance the critical position so as to cover the entire useful range of x , an auxiliary shim rod can be employed. Validity of this type of control rod calibration depends upon the absence of shadowing between sample, control rod, and shim rod, as well as the constancy of the ratio

$$\frac{\Delta k}{\Delta \Sigma_a} = - \frac{\bar{n} v_a F_L}{\bar{n} v_o \bar{\Sigma}_f} \quad \dots (7)$$

for all absorbers tested.

III. APPARATUS

The NAA Water Boiler Neutron Source has been described in detail in another report,³ and only the particular items relevant to danger coefficient testing need be discussed here. Figure 1 presents a cutaway view from the top of the reactor, showing the arrangement currently in use. Passing completely through the central exposure facility is a magnesium tube 12 feet in length with a 7/8-inch ID and 1-inch OD. Four lucite push rods, each 3 feet long by 0.85 inch diameter, are used as fillers. By means of these rods, samples can be pushed into position from one side of the reactor in a semi-continuous fashion, obviating shutdown of the chain reaction during sample changes. The through tube itself is securely clamped at both ends, so that the end into which samples are loaded can be used as a reference point which is exactly 5 feet from core center.

Fig. 1. Cutaway View of WBNS and Associated Apparatus

Measurements are made using the coarse control rod, whose position can be determined to 0.001 inch by means of a 3-foot steel scale with a vernier, attached to the control rod assembly. A shim rod, consisting of a cadmium-filled aluminum tube sliding into one of the stringers in the reflector, is used to adjust the WBNS reactivity so that criticality will occur at a suitable control rod position. Safety rods, the fine control rod, and the radium-beryllium source are withdrawn during testing.

Small changes in reactor power level are measured by the system shown in Fig. 2. It consists of a boron trifluoride-filled ion chamber 10 inches in diameter by 5 feet in length, a bucking current source in the form of a volt box and series resistor, and a vibrating reed electrometer driving a strip chart recorder. The overall sensitivity of the system is adjustable; a value of 1.3 cm on the recorder chart per milliwatt of reactor power is currently in use. The noise level, based on the recorded signal with the reactor shut down, is about ± 0.1 cm RMS consisting of relatively high-frequency components (~ 1 cps); at 200 mw this figure is ± 0.3 cm. The chart is operated at a speed of two inches per minute.

IV. CONTROL ROD CALIBRATION

A. SAMPLES

The samples employed in the control rod calibration were wafers of Bakelite impregnated with boric acid. A cylindrical graphite sample holder was constructed of 0.85 inch diameter and 4.0 inch length. The remaining length of the through tube was filled, as usual, with lucite. To make the wafers, reagent grade boric acid and black Bakelite metallurgical mounting powder were carefully weighed and milled. The material was pressed into a block which was then cut into four rectangular plates 1.0 by 0.75 by 0.06 inches. Knowing the weight ratio of boron to Bakelite in the original mixture, and assuming homogeneity, one could ascertain the weight of boron in each wafer from the total weight of the wafer. Four wafers of pure Bakelite were prepared in the same manner.

B. MEASUREMENTS

The control rod travel is from 34.664 inches (completely in) to 3.930 inches (completely out). The region most useful for danger coefficient measurements is, however, under 24 inches. With the critical position farther in than this value,

Fig. 2. Schematic of Power Level Measuring and Recording System

the reactivity is so sensitive to rod position that the amount of movement of the control rod necessary to compensate for the absorption effects of the samples is too small to provide the desired accuracy in the results. The region between 24 inches and 9 inches has been calibrated.

Because of small slow variations in the reactivity of the WBNS, probably resulting from gradual drifts in the core temperature or from possible formation of bubbles in the fuel solution, it was desirable to make the measurements in a cyclic manner. Thus, if sample A were to be compared with sample B, the critical position for A would be determined, then that for B, and finally for A again. The critical position for B should then be compared with the average critical position for A. It was assumed here that the drift in reactivity was constant during the period required for the three measurements. As demonstrated in later experiments, this assumption is valid.

Values of $\Delta\Sigma_a/\Delta x$ were obtained at approximately 1-inch steps along the length of the control rod in the region of interest. Each value required three critical position determinations.

1. Two pure Bakelite samples
2. Two boron-impregnated Bakelite samples
3. Pure Bakelite samples repeated.

The shim rod was moved between each set of three measurements so that the average of the critical positions of blank and boron samples occurred at the 1-inch steps of control rod travel.

C. DETERMINATION OF CRITICAL POSITIONS

To determine each critical control rod position, three power level traces were made with the control rod set at slightly different positions in the vicinity of criticality. These control rod settings were such that the power level increased slowly for the first trace, decreased slowly for the second, and increased again for the third. The rod setting was made close enough to the critical position so that the power did not change by more than 5 per cent during any of the 8-1/2 minute traces.

The power of a reactor on a stable, finite period is given by

$$P = P_0 e^{\frac{t - t_0}{T}} = P_0 \left[1 + \frac{t - t_0}{T} + \frac{(t - t_0)^2}{2T^2} + \dots \right] \quad \dots (8)$$

where P_o = the power at time t_o
 P = the power at a later time t
 T = the reactor period.

For time intervals $t - t_o = \Delta t$ such that $\Delta t \ll T$, the power change $P - P_o = \Delta P$ during Δt is approximately

$$\Delta P = \frac{P_o}{T} \Delta t \quad \dots (9)$$

The time allotted to the power trace, 8-1/2 minutes, is smaller than the reactor periods permitted by a factor of at least 20. Hence, the trace observed on the recorder chart is normally a straight line upon which are superimposed random fluctuations. Figure 3 is a photograph of a typical power trace. From the best straight line fit, knowledge of the bucking voltage, and previous calibration of the electrometer, $P_{av} = (P + P_o)/2$ and ΔP for the 8-1/2 minute interval are determined. The reciprocal period, given by

$$\frac{1}{T} = \frac{1}{P_{av}} \frac{\Delta P}{\Delta t} \quad \dots (10)$$

is then plotted against rod position x , and the best straight line drawn through the resulting three points. The rod position at the $1/T = \text{zero}$ intercept is then the desired position, x_c .

D. CONSTRUCTION OF THE CALIBRATION CURVES

If the critical positions for the two determinations on the blanks are $x_{co}(1)$ and $x_{co}(2)$ respectively, and that the boron sample is x_{ca} , then the change in control rod setting, Δx_c , is given by

$$\Delta x_c = \frac{x_{co}(1) + x_{co}(2)}{2} - x_{ca} \quad \dots (11)$$

Also

$$x = x_{ca} + \frac{\Delta x_c}{2} \quad \dots (12)$$

The value of $\Delta \Sigma_a$ in cm^2 can be calculated from the known weight of boron and its assumed 2200 m/sec microscopic cross section (750 ± 10 barns). $\Delta \Sigma_a$ was a constant equal to about 0.15 cm^2 throughout the measurements. Figure 4 shows the "differential calibration curve" for the WBNS coarse control rod, that is, a plot of $\Delta x_c / \Delta \Sigma_a$ versus x . This differential curve is linear within other experimental

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Fig. 3. Photograph of Actual WBNS
Power Trace

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Fig. 4. Coarse Control Rod Differential Calibration Curve

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errors over a range of about one inch in control rod position. Hence, if the rod positions when performing routine testing differ by less than one inch, the following equation may be safely used:

$$\Delta \Sigma_a = (R)_{av} \Delta x \quad \dots (13)$$

where $(R)_{av}$ is the average value of $R(x) = \frac{\Delta \Sigma_a}{\Delta x}$ between the two critical rod positions.

If, however, Δx is large, it is necessary to integrate $R(x)$ over the range Δx . For this purpose an "integral calibration curve" has been plotted based on the relation

$$\Sigma_a(x) = \int_{x_1}^x \frac{dI_a}{dx} dx \cong \int_{x_1}^x \frac{\Delta \Sigma_a}{\Delta x} dx \quad \dots (14)$$

Figure 5 shows $\Sigma_a(x)$ plotted versus x , where $\Sigma_a(x)$ has been normalized to zero at $x = x_1 = 9$ inches. The absorption difference between two samples A and B is then

$$\Delta \Sigma_a = \Sigma_a(x_A) - \Sigma_a(x_B) \quad \dots (15)$$

E. CALIBRATION ERRORS

Errors in the calibration result from uncertainty in the boron standards and limitations and errors in the experimental technique.

The uncertainty in the boron standards arises from the uncertainty in the absorption cross section of boron and from the uncertainty in the weight of boron in the standards. The cross section of boron was assumed to be 750 ± 10 barns, an apparent error of ± 1.3 per cent. However, since the cross sections for other materials are often referred to the boron cross section, this error is unimportant. It is necessary to remember only that the coarse control rod has been calibrated relative to boron whose cross section was assumed to be 750 barns.

The weight uncertainty is not as clear cut. In fabricating the standards, great care was taken to insure knowledge of the boron content to 0.1 per cent. Because it was suspected that some boron had been lost in the milling and pressing operations, several corroborative measurements were made. These included

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Fig. 5. Coarse Control Rod Integral Calibration Curve

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chemical analyses of the samples, the use of borated D_2O and of gold foils as a check on the control rod calibration, and danger coefficient measurements inter-comparing the boron samples. The results obtained permitted only an estimate of the maximum possible error in the boron weights, namely -10 to +2 per cent. However, since all of the calibration measurements were made with the same two samples, the relative shape of the calibration curve should be quite accurate.

The experimental technique was designed to minimize existing uncertainties. Cycling of the samples made negligible the errors resulting from slow reactivity drifts. To prevent unintentional changes in reactivity, the shim rod was securely fixed in position once it had been set for a desired critical control rod position. Samples were always put into the same position with the same orientation. Electrometer drift was frequently checked, and was found to be entirely negligible over the period necessary to make an 8-1/2 minute trace. Finally, before initiating runs in the morning, and again at the end of the day, the sensitivity of the entire detecting system was determined by calibrating the electrometer in terms of centimeters deflection on the strip chart versus volts input signal. The two calibrations always agreed within a negligible error.

The analysis of the data is straightforward and offers little uncertainty other than that in drawing the straight line through the strip chart trace and reading the graphs of reciprocal period versus rod position. These uncertainties are estimated to contribute an error equivalent to less than 0.7 per cent uncertainty in the cross section of the boron standard.

V. ROUTINE SAMPLE TESTING

A. EXPERIMENTAL PROCEDURE

In some respects, the routine of danger coefficient measurements differs from the procedure followed in calibrating the coarse control rod. One of the principal differences is the way in which the critical positions of the control rod are found. After positioning of a given sample, the rod is adjusted to provide as nearly as possible a critical chain reaction. When the power trace, as observed on the recorder at the reactor console, appears steady (under 0.5 per cent change per minute), the 8-1/2 minute trace on the electrometer-recorder is made. During this time, the actual rod position x is read, not the rod position x_c which

would provide exact criticality. As described before, from the straight line drawn through the trace P_{av} and ΔP are found. From the ratio $\Delta P/P_{av}$ the correction $\delta x = x_c - x$ is calculated from a standard "correction curve". This curve is based on the constancy of $\Delta k/\Delta \Sigma_a$ within the limitations implied by Eq. (7). Noting that the reactivity in inhours equals the inverse of the period in hours for periods much longer than the average neutron generation time, one can write

$$\frac{\Delta k}{\Delta \Sigma_a} = \frac{1}{T \Delta \Sigma_a} \equiv \alpha \quad \dots(16)$$

Combining Eq. (10) and (16) gives

$$\frac{\Delta P}{P_{av}} = \alpha \Delta \Sigma_a \Delta t \quad \dots(17)$$

for $\Delta t \ll T$. Since the control rod has been calibrated in terms of a known amount of boron, it forms a convenient known variation of $\Delta \Sigma_a$. A small displacement δx from the critical rod position will introduce a change of $R(x)\delta x$ in absorption resulting in a power change

$$\frac{\Delta P}{P_{av}} = \alpha R(x) \delta x \Delta t \quad \dots(18)$$

In the description of the control rod calibration it was mentioned that three values of $\frac{1}{T}$ were plotted versus x to find x_c for a given sample loading. Therefore, any value of $\frac{1}{T}$ can be related to a value of $\delta x = x_c - x$. From Eq. (1) and (18), $R(x)$ is found for the x_c ascertained above. This procedure results in a value of $\alpha = -23 \text{ ih/cm}^2$, a figure confirmed by repeating the above steps for several critical positions. The correction curve may be obtained utilizing Eq. (18), which is rewritten below.

$$\delta x = \frac{1}{\alpha R(x) \Delta t} \frac{\Delta P}{P_{av}} = C(x) \frac{\Delta P}{P_{av}} \quad \dots(19)$$

Since Δt is constant (8-1/2 minutes) for all runs, and α has a negligible variation for long periods, a curve of $C(x)$ as drawn in Fig. 6, gives the corrections to be applied to the observed control rod position.

Under the discussion of the control rod calibration, it was noted that the cyclic system of sample testing permits a correction for the gradual reactivity drift of the WBNS. When several samples are to be compared, one is chosen as

A1

Fig. 6. Coarse Control Rod Correction Curve

a "standard", and denoted by "A"; the testing then proceeds in the following fashion: A B A C A D A . . . At the conclusion of the entire set of measurements a "drift curve" is plotted relating the values of x_c for the standard to the time of day, whence exact values of x_c are read corresponding the times at which B, C, D, . . . were tested. When it is felt that accuracy can be sacrificed in favor of speed, an order such as A B C A D E A . . . is often employed. More will be said about the drift of the WBNS in Section VII.

B. BERYLLIUM

Three groups of pressed beryllium cylinders were supplied to NAA in late 1953 by the Brush Beryllium Company of Cleveland, Ohio. One set of cylinders was pure QMV beryllium, while the two others contained 495 ppm cobalt and 526 ppm silver, respectively. All the cylinders were a nominal 1/2 inch diameter, but of lengths of 1, 2, and 4 inches. In order to center the samples in the through tube and fill the annular void remaining, graphite sleeves were fabricated of 0.5-inch ID and about 0.85-inch OD in the three lengths required.

Table I presents the data obtained in the entire series of measurements, which were performed over a period of about two weeks of intermittent testing.

The value of the mean standard deviation of the measured incremental cross sections implies a precision for danger coefficient measurements using the WBNS of about 0.0043 cm^2 (a 95 per cent confidence limit). The largest sample supplied was 0.5 inch in diameter and 4 inches in length, resulting in a beryllium absorption cross section of 0.015 cm^2 . Therefore, the minimum equivalent shotgun value* obtained with this size sample and confidence of 95 per cent is 29. A sample of dimensions 8 x 0.85 inch diameter would decrease this figure to 5.0.

*The calculated "danger coefficient" of an absorptive impurity in any material is given by

$$K_d = \frac{\sigma_{ai}}{\sigma_{ao}} \frac{A_o}{A_i} \quad \dots (20)$$

where σ_a is microscopic absorption cross section, A is atomic weight, and the subscripts i and o refer to the impurity and to the material containing the impurity, respectively. If the absorption Σ_{ai} of the impurity is small compared to the absorption Σ_{ao} of the sample (or major constituent), the fractional absorption due to the impurity is

$$\frac{\Sigma_{ai}}{\Sigma_{ao}} = \frac{M_i}{M_o} K_d \quad \dots (21)$$

When expressed in per cent, the quantity Σ_{ai}/Σ_{ao} is often referred to as a "shotgun value".

TABLE I
RESULTS OF TESTS ON BERYLLIUM SAMPLES

Sample	Dimensions and Weights	$\Delta\Sigma_a$ (calculated) cm ²	$\Delta\Sigma_a$ (measured) cm ² 4/7/54	$\Delta\Sigma_a$ (measured) cm ² 4/13/54	$\Delta\Sigma_a$ (measured) cm ² 4/23/55	$\Delta\Sigma_a$ (measured) cm ²	ϵ (measured) cm ²	$\Delta\Sigma_a$ (measured) $-\Delta\Sigma_a$ (calculated) cm ²
A1	1 x 0.5 in diam	(standard)	(standard)	(standard)	(standard)	(standard)	(standard)	(standard)
A2		0.000	0.002	0.004	0.000	0.002	0.002	0.002
C1		0.001	0.002	0.005	-0.001	0.002	0.003½	0.001
C2		0.001	0.006	0.006	0.000	0.004	0.003½	0.003
S1		0.001	0.001	0.008	-0.002	0.002	0.005	0.001
S2		0.001	0.002	0.005	-0.001	0.002	0.003	0.001
A3	2 x 0.5 in. diam	(standard)	(standard)	(standard)		(standard)	(standard)	(standard)
A4		0.000	0.000	0.000		0.000	0.000	0.000
C3		0.002	0.000	0.000		0.000	0.000	-0.002
C4		0.002	-0.001	0.004		0.001½	0.003½	-0.000½
S3		0.002	-0.003	0.004		0.000½	0.005	-0.001½
S4		0.002	-0.003	0.002		-0.000½	0.003½	-0.002½
A5	4 x 0.5 in. diam	(standard)	(standard)	(standard)		(standard)	(standard)	(standard)
A6		0.000	0.000	0.001		0.000½	0.000½	0.000½
C5		0.005	0.007	0.007		0.007	0.000	0.002
C6		0.005	0.005	0.004		0.004½	0.000½	-0.000½
S5		0.004	0.006	0.005		0.005½	0.000½	0.001½
S6		0.004	0.010	0.007		0.008½	0.002	0.004½
							$\bar{\epsilon} = 0.0022$	$av = 0.0005$

where: $\Delta\Sigma_a$ (calculated) = the calculated value of Σ_a (sample) - Σ_a (standard) based on thermal neutron cross sections.

$\Delta\Sigma_a$ (measured) = the measured values of $\Delta\Sigma_a$ in cm² as obtained on the dates mentioned.

ϵ (measured) = the standard deviation, based on two or three measurements of the same sample.

There is a tendency for the values of $\Delta\Sigma_a$ (measured) to exceed those of $\Delta\Sigma_a$ (calculated). In order to understand the reason for this discrepancy, one needs only to remember that the calibration of the control rod is completely valid only if $\Delta k/\Delta\Sigma_a$ as given in Eq. (9) remains constant. In practical testing, however, this ratio undergoes considerable variation. According to the definition, F_L is a measure of the effectiveness of a thermal neutron introduced at any point in the impurity or sample. For samples of low cross section such as beryllium or graphite, the thermal neutron is considered to diffuse immediately into the active region of the core. Owing to the high absorption cross section of the fuel, the neutron in question will probably cause a fission near the sample. The fast neutrons produced by this fission have an effectiveness dependent in part upon the moderating power of the sample. In the case at hand, the moderating power of beryllium is about twice that of graphite; hence, it follows that F_L is higher for beryllium than for graphite. Remembering that the Bakelite wafers used in the control rod calibration were enclosed in a graphite cylinder, one sees that beryllium samples cause an increase in $\Delta k/\Delta\Sigma_a$. Furthermore, because of its higher moderating power, beryllium will also give rise to a greater value of the thermal flux ratio $\bar{n}_v/\bar{n}_{v_0}$, resulting in a further increase in $\Delta k/\Delta\Sigma_a$.

C. BORON-TRIFLUORIDE FILLED CYLINDERS

During 1954 a number of danger coefficient measurements were conducted on $B^{10}F_3$ -filled aluminum capsules in conjunction with the Reactor Safety Program. The capsules themselves were 4 inches long by about 0.75 inch diameter, with thin walls and negligible absorption cross section compared to that of the B^{10} enclosed. A description of the techniques employed in filling the capsules with known amounts of gas is given in a previous report⁴; it is sufficient to state here that the uncertainties in the B^{10} content were kept to a negligibly low magnitude. Table II presents a summary of the data obtained on the WBNS over a period of several weeks of intermittent measurements. A graph of the measured versus the calculated total cross sections is shown in Fig. 7.

The decrease in the ratio $\Delta k/\Delta\Sigma_a$ is very evident from the curves shown. The asymptote of the "ideal" case would be obtained if Bakelite wafers impregnated with varying amounts of boron and enclosed in a cylindrical graphite holder were tested. Actually, there was no moderator associated with the samples. The

Fig. 7. Cross Section Measurements on B¹⁰F₃ Capsules

TABLE II

MEASUREMENTS ON BORON-TRIFLUORIDE FILLED CAPSULES

Capsule No.	B ¹⁰ Content mg	Σ_a (calculated) cm ²	Σ_a (measured)* cm ²
4	12.65	3.05	2.05
5	15.88	3.82	2.45
6	8.01	1.93	1.38
7	7.56	1.82	1.30
8	11.35	2.74	1.85
9	4.22	1.02	0.75
10	7.31	1.76	1.24
11	3.62	0.87	0.63
13	6.10	1.47	1.07
14	5.79	1.395	1.00
15	5.20	1.25	0.89
16	5.21	1.26	0.93

*normalized to Σ_a (measured) = 0 for an empty capsule

result is a decrease in F_L and $\bar{n}\bar{v}_a/\bar{n}\bar{v}_o$ such that $\frac{F'_L \bar{n}\bar{v}'_a/\bar{n}\bar{v}'_o}{F_L \bar{n}\bar{v}_a/\bar{n}\bar{v}_o} = 0.76$ where the unprimed quantities represent those existing during calibration of the control rod, and the primed factors apply to the B¹⁰F₃ capsule testing. Because samples are frequently tested which have no moderating qualities themselves, nor are they enclosed in holders made of moderating material, the factor of 0.76 should be applied even when the absorption cross section introduced is negligible. As can be seen from Fig. 6, there is a further decrease in F_L and $\bar{n}\bar{v}_a/\bar{n}\bar{v}_o$ as the total cross section of the sample increased, and the above ratio drops because of self-shielding effects.

D. IRON AND STEEL

In conjunction with the construction of the Sodium Reactor Experiment, several measurements were made on iron and steel in various alloys and geometries. It developed that determination of absolute values of $\Delta\Sigma_a$ was difficult in view of the low values of F_L and $\bar{n}\bar{v}_a/\bar{n}\bar{v}_o$ encountered. For example, five samples of iron

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and steel were compared by the danger coefficient method with results as shown in Table III. Each sample was a cylinder 2.000 inches long by 0.850 inch diameter with dimensional tolerances of ± 0.002 inches.

TABLE III
RESULT OF IRON AND STEEL MEASUREMENTS

Sample	Weight g	Calculated Σ_a cm^2	Calculated $\Delta\Sigma_a$ cm^2	Measured $\Delta\Sigma_a$ cm^2	$\frac{\text{Measured } \Delta\Sigma_a}{\text{Calculated } \Delta\Sigma_a}$
Armco	145.60	3.78	(standard)	(standard)	--
304	146.49	4.44	0.66	0.340	0.52
304 ELC	146.62	4.44	0.66	0.385	0.58
347	147.43	5.21	1.43	0.430	0.30
1020	145.75	3.96	0.18	0.078	0.43

In an effort to obtain a better idea of the "self-shielding" in stainless steel samples, several wires of Type 304 steel were cut, each 2.0 inches long by 0.091 inches in diameter. Measurements were made with a variable number of wires; the voids left by the omitted wires were not filled with other material. The wires were enclosed in a graphite sleeve 2.0 inches long by 0.5-inch ID and 0.85-inch OD. The results are shown in Table IV.

TABLE IV
COMPARISONS OF STEEL WIRES TO INVESTIGATE SELF-SHIELDING

Sample wires	Weight g	Calculated Σ_a cm^2	Calculated $\Delta\Sigma_a$ cm^2	Measured $\Delta\Sigma_a$ cm^2	$\frac{\text{Measured } \Delta\Sigma_a}{\text{Calculated } \Delta\Sigma_a}$
20	33.80	1.022	(standard)	(standard)	} 0.65
19	32.11	0.971	-0.051	-0.033	
10	16.90	0.511	-0.511	-0.363	} 0.78
9	15.21	0.460	-0.562	-0.403

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Another measurement, involving two Type 304 stainless steel samples of rectangular cross section and of mean weight equal to 83.7 grams, gave a measured $\Delta\Sigma_a$ of 0.128 cm^2 as compared with the calculated value of 0.233 cm^2 , the ratio here being 0.55. One concludes that in using the WBNS for comparison of Type 304 or similar types of stainless steel in samples of 50 to 150 grams, observed values of $\Delta\Sigma_a$ should be multiplied by roughly a factor of 2 to convert to actual absorption cross section differences. Small sample weight will serve to increase F_L and $\bar{n}\bar{v}_a/\bar{n}\bar{v}_o$, but the sensitivity for detection of impurities in the steel will be lowered as the total cross section is decreased.

VI. URANIUM TESTING

In dealing with fissionable material (or any material of variable isotopic content) it is not possible by the danger coefficient technique to distinguish between changes in enrichment (or isotopic content) and the presence of impurities. One or the other must be a priori known or assumed, unless only values of Δk are to be related to the samples. An extensive series of measurements was made with natural uranium to determine the utility of the system for purity testing. Additional measurements were conducted on slightly enriched uranium samples to ascertain the merits of the WBNS for enrichment testing. The first series of tests was done with both uranium and uranium oxide; the second employed only the metal.

A. PURITY TESTING

1. Uranium metal - In May, 1955, a series of 15 natural uranium slugs supplied by Hanford were compared by the danger coefficient method. The purpose of these tests was to ascertain whether the WBNS produces data comparable to results previously obtained at the W305 pile at Hanford.

The original slugs, of dimensions of 8 inches in length and 1.36 inches diameter, were machined to a length of 3.875 inches and a diameter of 0.750 inch. A comparison of samples from different locations in one slug indicated homogeneity of absorption cross section throughout the cylinder. Later results suggested, however, that at least one slug was inhomogeneous, so that this point is not yet resolved.

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The first series of tests produced results which were in disagreement with the Hanford data. In September 1955 the measurements were repeated with several refinements. A comparison of natural uranium cylinders with large weight differences revealed that weight corrections for the Hanford samples were not negligible, as had previously been thought. When natural uranium is compared to a void in the WBNS core, the reactivity difference observed is small, but not zero; the negative contribution from Σ_a nearly balances the positive contribution from Σ_f and Σ_s . The actual weight correction amounts to $+0.00021 \text{ cm}^2/\text{gm}$.

Another effect which proved troublesome was the variation in length among the cylinders, the value of which was a maximum of 4 mils. A negative increment of length is replaced by a corresponding length of lucite rod. Four mils of lucite, compared to air, has an apparent reactivity effect of at least -0.001 cm^2 . To eliminate this difficulty, a graphite sleeve of 0.755-inches ID, 0.850-inches OD, and 3.9 inches in length was fabricated, so that the distance between the ends of the lucite rods adjacent to the sample was held constant at 3.9 inches. It was also found that in order to detect the small cross sectional differences involved, the lucite rods had to be kept in the same order and orientation. Evidently there are small variations in diameter among the four rods, as well as differences in a given rod. Therefore, the two ends adjacent to the sample must be the same throughout a given series of runs.

In Table V are shown the results of these measurements, along with the Hanford data for purposes of comparison. Shotgun values were also computed from chemical analyses performed on the slugs by Mallinckrodt Chemical Works. It should be noted that sample 946-1 was undersize, the procedure being to compare it to an undersized sample from slug M1779D-4 and extrapolating the result to the larger size.

The older measured values of $\Delta\Sigma_a$ indicated that slug No. 3609-1 possessed the most favorable nuclear characteristics; therefore, this sample was chosen as the "standard". In the table, $\delta(\Delta\Sigma_a)$ indicates the correction for the weight differences. From the results of enrichment testing, self-shielding factors $(F'_L \bar{n}v'_a/\bar{n}v'_o)/(F_L \bar{n}v_a/\bar{n}v_o)$ were available for the cylinder sizes used here: values of 0.57 for the 0.6 inch diameter and 0.525 for the 0.75 inch diameter were used.

The cylinder cut from slug No. M1779D-2 contained visible voids and pits, which fact accounts for its low weight. Even after a weight correction the value

TABLE V
DANGER COEFFICIENT MEASUREMENTS ON HANFORD URANIUM METAL

Slug No.	Weight g	ΔWt g	No. of Runs	$\overline{\Delta \Sigma}_a$ (measured) cm^2	$\delta(\Delta \Sigma_a)$ cm^2	$\overline{\Delta \Sigma}_a$ (corrected for wt) cm^2
3609-1	536.65	standard	--	standard	standard	standard
3609-2	535.7	-0.95	1	0.0004	-0.0002	0.0002
3609-3	535.0	-1.65	1	0.0003	-0.0003	0
3609-4	536.7	+0.05	2	0.0007	0	0.0007
3609-5	534.7	-1.95	2	0.0013	-0.0004	0.0009
M1779D-1	533.8	-2.85	1	0.0018	-0.0006	0.0012
M1779D-2	521.15	-15.5	1	0.0107	-0.0033	0.0074
M1779D-3	532.3	-4.35	2	0.0034	-0.0009	0.0025
M1779D-4	533.8	-2.85	4	0.0018	-0.0006	0.0012
M1779D-5	533.2	-3.45	1	0.0031	-0.0007	0.0024
946-1	--	--	--	--	--	0.0079
946-2	533.55	-3.1	1	0.0085	-0.0007	0.0078
946-3	531.6	-5.05	1	0.0152	-0.0011	0.0141
946-4	531.9	-4.75	1	0.0122	-0.0010	0.0112
946-5	530.0	-6.65	1	0.0069	-0.0014	0.0055
946-1	339.7	standard	--	standard	standard	standard
M1779D-4	341.5	+1.2	1	-0.0049	+0.0003	-0.0046

Slug Set No.	Average $\overline{\Delta \Sigma}_a$ cm^2	Normal av $\overline{\Delta \Sigma}_a$ cm^2	Normal av $\overline{\Delta \Sigma}_a$ (Corrected for self shielding) cm^2	Equivalent NAA shotgun	Hanford values shotgun	Shotgun from chemical analysis
3609	0.004	0	0	0.000	0.00	0.000
M1779D	{ 0.0029 0.0018*	{ 0.0025 0.0014*	{ 0.0048 0.0027*	{ 0.046 0.026* }	0.016	0.000
946	0.0093	0.0089	0.0169	0.162	0.20	0.110

for $\Delta\Sigma_a$ of 0.0074 cm^2 looks peculiarly high. Therefore, the 0.0074 cm^2 is not averaged into the data marked by an asterisk (*).

The precision of the NAA shotgun values should be on the order of ± 0.004 . The observed standard deviations of $\bar{\Delta\Sigma}_a$ (corrected for weight) for the slugs in each set are: 0.0004 cm^2 for No. 3609, $0.0026 (0.0007*) \text{ cm}^2$ for No. M1779D, and 0.0033 cm^2 for No. 946. These figures in terms of shotgun values become 0.007, 0.047 (0.013*), and 0.060. One can surmise from these data that heterogeneity of the original slugs might account for some of the disagreement between NAA and Hanford.

An advantage of the danger coefficient method as used with the WBNS is high sensitivity. The total weight of a group of five slugs as tested in the W305 pile was 18,000 grams for a reported standard deviation in the shotgun value of 0.006, whereas 2650 grams of material in the WBNS should yield the figure of 0.004. Furthermore, since the nuclear characteristics of the slugs from a single rod may have a considerable variation, individual measurements on the slugs are valuable when the time element is not critical. It is noteworthy that this type of testing procedure can reveal a trend such as noticed in No. 946, namely that the central portion of the rod seems to contain more impurities than the ends. Such data could provide a clue to improvements in the fabrication technique or in reactor loading patterns.

2. Uranium Oxides - A relatively large part of the danger coefficient work at NAA has involved uranium oxides. As in the case of uranium metal testing, the chief object of the oxide program has been to prove the utility of a water boiler reactor in routine production quality control, in comparison with such methods as chemical analysis and shotgun testing.

Soon after the receipt in 1953 of six UO_3 samples from Mallinckrodt Chemical Works, a preliminary set of danger coefficient measurements were performed. The inconsistent results obtained indicated that the oxides as received were unsuitable for danger coefficient testing. An investigation by the Chemistry Group revealed an actual composition of the oxides described by $\text{UO}_3 \cdot 0.4 \text{ H}_2\text{O}$, with a considerable variation in the $0.4 \text{ H}_2\text{O}$ among the samples. Other data have shown that 0.07 grams of water in the center of the WBNS core produces a positive reactivity change equivalent to a $\Delta\Sigma_a$ of -0.002 cm^2 . For the weight of oxide used in each sample ($\sim 200\text{g}$), this value of $\Delta\Sigma_a$ corresponds to a variation in $0.4 \text{ H}_2\text{O}$ of about

+0.006 H₂O. It can be seen, therefore, that the presence of varying quantities of water in the oxides serves to mask completely the effects of any absorptive impurities.

A procedure was developed by the Chemistry Group which involves igniting the UO₃ oxide in a dry atmosphere at 850° C for at least two hours, thus reducing the compound to the more stable and water-free U₃O₈. Subsequent measurements with this oxide proved to be satisfactory.

Magnesium containers were used with dimensions of 0.8 inches long by 0.85 inches in diameter. One end was supplied with a removable magnesium cap, the other end welded closed. Calibrations of the containers were initially performed with the containers empty. However, it was felt necessary to check these results for the effect of the presence of oxide. In spite of the fact that U₃O₈ has about the same reactivity effect on the WBNS as does air, the oxide probably causes a hardening of the neutron spectrum near the sample position. In other words, the absorption of thermal neutrons in the uranium is balanced reactivity-wise by the emission of fission neutrons and by the moderation due to the scattering properties of oxygen and uranium. If the net effect is to raise the neutron temperature, the assumed $1/v$ absorption cross section of magnesium will be effectively lowered.

In order to check this possibility, a thin-walled magnesium container was constructed whose outside dimensions permitted a sliding fit into the standard containers; this container was loaded with 200 grams of a typical U₃O₈ sample. A series of tests using this oxide-filled container in conjunction with the large containers revealed that on the average $\Delta\Sigma_a$ (filled) = 0.8 $\Delta\Sigma_a$ (empty). Data on the empty containers are shown in Table VI.

Loading of the containers presented some problems. A ± 5 per cent variation in the oxide densities (under similar conditions of ignition and packing) meant that when the oxides were loaded and tamped, a weight variation of equal magnitude would exist (± 10 grams for the size containers used). It was then necessary to empty sufficient oxide out of all the containers except the one containing the lowest weight of oxide, until all the oxide weights agreed within 10 milligrams. The containers were then agitated in order to spread the contents as evenly as possible throughout the 8-inch length.

TABLE VI
CALIBRATION OF MAGNESIUM CONTAINERS

Container	Mass g	$\Delta\Sigma_a$ 2/22/54 cm ²	$\Delta\Sigma_a$ 5/26/54 cm ²	$\Delta\Sigma_a$ 6/3/55 cm ²	$\overline{\Delta\Sigma}_a$ empty cm ²	$0.8\overline{\Delta\Sigma}_a$ (empty) = $\Delta\Sigma_a$ (filled) cm ²
A	32.50	(standard)	(standard)	(standard)	(standard)	(standard)
B	32.39	0.004	0.001	0.003	0.0027	0.002
C	32.23	0.004	0.003	0.005	0.004	0.003
D	32.93	--	0.008	0.007	0.0075	0.006

Table VII shows the results of the measurements on the six oxides, which are converted to shotgun values for comparison with the Mallinckrodt data in Table VIII.

TABLE VII
MEASUREMENTS OF MALLINCKRODT URANIUM OXIDE

Sample Number	Container	Oxide net wt g	$\Delta\Sigma_a$ (measured) cm ²	Container Correction cm ²	net $\Delta\Sigma_a$ cm ²
63600	C	180.86	(standard)	(standard)	(standard)
57619	B	180.86	0.0085	0.001	0.0095
63600	C	215.79	(standard)	(standard)	(standard)
63627	A	215.79	0.0025	0.003	0.0055
63506	D	215.79	0.012	-0.003	0.009
63600	A	205.92	(standard)	(standard)	(standard)
63627	B	205.92	0.0075	-0.002	0.0055
63636	C	205.92	0.008	-0.003	0.005
63638	D	205.92	0.009	-0.006	0.003

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TABLE VIII
COMPARISON OF NAA AND MALLINCKRODT SHOTGUN VALUES

Oxide Number	NAA Shotgun	Mallinckrodt Shotgun
63600	0.00	0.00
63638	0.09	0.01
63636	0.15	0.00
63627	0.16	0.00
63506	0.25	0.03
57619	0.32	0.08

It will be noted that, while the figures for the "NAA Shotgun" do not numerically agree with those obtained by Mallinckrodt, there exists a definite qualitative correlation. It is quite likely that shotgun values as obtained at Mallinckrodt do not really represent the contamination in the oxide material; a discussion of the shotgun test procedure is contained in Ref. 5. Spectrographic analyses of the same samples indicate discrepancies between the calculated and the shotgun measured absorptive impurity cross sections. For instance, the total $\sum_a$ of the impurities in 200 grams of No. 57619 minus the total $\sum_a$ in 200 grams of No. 63600 is calculated to be 0.007 cm^2 , based upon spectrographic analyses (which, incidentally, omitted data on cadmium content). The danger coefficient measurements gave a value of $\Delta\sum_a = 0.010 \pm 0.002 \text{ cm}^2$, which is in fairly good agreement with 0.007 cm^2 . It would appear, therefore, that in this particular case the Mallinckrodt shotgun value is low by a factor of three.

A spectrographic analysis of the pellet formed from sample No. 57619 for the shotgun test revealed that the weight ratios of impurities were not retained during the chemical process. For instance, 20 ppm of Cu in the oxide was concentrated to a value of 1000 ppm in the pellet, whereas Mg, which was 1000 ppm in the oxide, became only 250 ppm in the pellet.

It was suspected that some of the discrepancies in the danger coefficient measurements could be due to differences in particle size among the samples. As a check on this possibility, two uranium oxide samples, No. 63506 and No. 57076,

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were ignited and then ground to 100 mesh particle size. The samples were subsequently compared by the danger coefficient method, with the results shown in Table IX. Previous measurements are shown for comparison.

Equalization of the particle size decreased the shotgun value of No. 63506, relative to No. 75076, by a factor of 2. This change is not in the direction expected, for if No. 63506 had contained impurities of high microscopic absorption cross section in large-sized particles, grinding to 100 mesh would have decreased and self-shielding of the impurities and increased the apparent shotgun value. There is some possibility that the two oxides absorbed different amounts of water during the grinding and sieving process, or that the impurities were preferentially absorbed on the laboratory apparatus used. Investigation into the effects of particle size is continuing.

Tentative conclusions regarding uranium oxide testing are that preparation and loading of the samples for danger coefficient measurements are crucial factors, and that shotgun tests do not reflect the actual impurity content in the oxide.

B. ENRICHMENT TESTING

1. Enrichment Calibration - Uranium samples of several enrichments and sizes were compared by the danger coefficient method, with the object of graphing the relation between change in enrichment versus apparent $\Delta\Sigma_a$. Since the value of $\Delta\Sigma_a/\Delta E$ is relatively large for the WBNS, this reactor should be very useful for routine enrichment testing on uranium metal samples.

Three sizes of uranium cylinders were employed, all 3.875 inches in length, but with diameters of 0.500, 0.600, and 0.750 inches. All dimensions were kept to a tolerance of ± 0.002 inches. Of the five enrichments used, the lower three, 0.491, 0.711, and 0.902 per cent U^{235} by weight, were obtained from uranium utilized at NAA in lattice measurements. A special quota of material from Westinghouse Atomic Power Division provided an enrichment of 1.299 per cent. Since this metal was available only in a 0.600 inch diameter, the 0.750 inch diameter cylinder for the danger coefficient tests was formed from four carefully machined segments obtained from the 0.600 inch stock. Finally, the 2.778 per cent material represents the fuel for the Sodium Reactor Experiment; this figure is the mean enrichment as determined from isotopic analyses of the 26 ingots used in manufacturing the SRE slugs. The consistency of the isotopic analyses was

TABLE IX

DATA SHOWING EFFECTS OF VARYING U_3O_8 PARTICLE SIZE

100 Mesh Particle Size	Sample	Cont.	Oxide wt g	Measured $\Delta\Sigma_a$ cm ²			$\Delta\Sigma_a$ corrected cm ²	Shotgun
				1st run	2nd run	mean		
	75076	N	215.8	standard	standard	standard	standard	0.00
	63506	L	215.7	0.0075	0.0083	0.0079	0.0074	0.21
Mixed Particle Sizes	75076	M	200.55	standard	standard	standard	standard	0.00
	63600	A	201.15	-0.0026	-0.0008	-0.0017	0.0071	0.21
	63600	C	215.79	standard	standard	standard	standard	0.21
	63506	D	215.79	0.0103	0.0133	0.0118	0.0088	0.46

checked by the danger coefficient method as described later in this report. Graphite sleeves were machined of 3.875 inch lengths and 0.850-inch OD, but of varying inside diameters to accommodate the three cylinder sizes used. As usual, the lucite push rods occupied the remainder of the through tube during testing.

Table X lists the data obtained in this series of tests. All the figures quoted for 1.000 inch long samples were obtained in a preliminary experiment⁶, and are included only for comparison. The measured values of $\Delta\Sigma_a \equiv \Sigma_a(\text{sample}) - \Sigma_a(\text{natural uranium})$ have a standard deviation of about 0.005 cm^2 , but a larger systematic error may exist, resulting from an error in the coarse rod integral calibration curve. However, this systematic error will be cancelled when these curves are subsequently used in conjunction with the water boiler.

To obtain $\Delta\sigma_a(\text{eff})$ in barns, $\Delta\Sigma_a$ in cm^2 is divided by N , the number of atoms in the sample. This is equivalent to applying a weight correction to each sample (since the sample weights differ from that of the standard), and then calculating $\Delta\sigma_a(\text{eff})$ using the N corresponding to the standard.

A set of values of $\Delta\sigma_a(\text{eff})$ result which are based on 0.711 per cent enrichment, but it is desirable to change the reference point to 0.000 per cent enrichment for more convenient presentation and analysis of the data. To obtain zero enrichment values for a given sample size, $\Delta\sigma_a(\text{eff})$ was first plotted versus enrichment, the derivative of this curve for various values of enrichment was then plotted against enrichment on semi-log paper, and a straight line drawn through the points. The derivative for zero enrichment was found by extrapolation and used to extrapolate the first curve to zero enrichment. The value of $\Delta\sigma_a(\text{eff})$ thus found was then used to normalize the data to $\sigma_a(\text{eff}) = 0$ at zero enrichment. The " Δ " is eliminated to avoid confusion with a differential curve described below.

Since the derivative of the $\sigma_a(\text{eff})$ curve is fairly well fitted by an exponential, the normalized $\sigma_a(\text{eff})$ curve itself can be approximated by

$$\sigma_a(\text{eff}) = C(1 - e^{-mE}) \quad \dots (22)$$

where E is the enrichment in per cent. The values of C and m for each size cylinder were found by a trail and error procedure and are shown in Fig. 8.

In practical testing, one compares a sample of slightly different enrichment with a cylinder of known enrichment, impurities assumed equal. From the

TABLE X
RESULTS OF URANIUM ENRICHMENT TESTS

Dimensions (inches)	Enrichment (wt % U ²³⁵)	Weight (grams)	Total No. of Atoms $\times 10^{-24}$	Measured $\Sigma_a(\text{sample}) - \Sigma_a(\text{natural U})$ (cm ²)	$\Delta\sigma_a(\text{eff})$ (barns)	Normalized $\sigma_a(\text{eff})$ (barns)
1.000 x 0.750	0.491	137.10	0.3467	0.197	0.568	(-1.46)
	0.711	--	--	--	--	(-2.03)
	0.902	137.24	0.3470	-0.158	-0.455	(-2.485)
3.875 x 0.500	0.491	235.78	0.5964	0.325	0.545	-1.355
	0.711	236.29	0.5977	--	--	-1.90
	0.902	237.59	0.6010	-0.247	-0.412	-2.31
	1.299	237.79	0.6015	-0.710	-1.180	-3.08
	2.778	237.30	0.6009	-1.885	-3.13	-5.03
3.875 x 0.600	0.491	339.48	0.8588	0.430	0.501	-1.34
	0.711	339.81	0.8597	--	--	-1.84
	0.902	341.21	0.8632	-0.350	-0.405	-2.245
	1.299	339.82	0.8597	-0.920	-1.070	-2.91
	2.778	341.45	0.8642	-2.420	-2.80	-4.64
3.875 x 0.750	0.491	529.86	1.340	0.640	0.478	-1.27
	0.711	530.43	1.342	--	--	-1.75
	0.902	533.31	1.349	-0.480	-0.356	-2.11
	1.299	530.69	1.342	-1.295	-0.965	-2.715
	2.778	533.44	1.320	-3.265	-2.41	-4.16

Fig. 8. Effective σ_a for Uranium of Different Enrichments and Geometries

observed value of $\Delta\Sigma_a$ in cm^2 one wishes to obtain ΔE , the difference in weight per cent U^{235} enrichment. A graph of $\Delta E/\Delta\Sigma_a$ versus E is shown in Fig. 9. This was obtained by noting that

$$N\sigma_a = \Sigma_a = C(1 - e^{-mE}) \quad \dots(23)$$

so that

$$\frac{\Delta E}{\Delta\Sigma_a} = \frac{e^{mE}}{NCm} \quad \dots(24)$$

As an example of the use of this graph, suppose two 3.875 by 0.600 inch diameter uranium cylinders of equal impurities are compared by the danger coefficient method, the "standard" being of known 0.7110 per cent enrichment. It is found that the unknown possesses an effective cross section 0.010 cm^2 greater than the standard (i. e., $\Delta\Sigma_a = 0.010 \text{ cm}^2$). One obtains from Fig. 9 a value of $\Delta E/\Delta\Sigma_a$ of $-0.54\%/\text{cm}^2$ for 0.71 per cent enrichment and this size cylinder. Therefore, the unknown has an enrichment of $0.7110 - 0.54 \times 0.010 = 0.7056$ per cent.

The standard deviation of a measurement of $\Delta\Sigma_a$ using the WBNS is 0.002 cm^2 . From Fig. 9 it is seen that for the 0.750 inch diameter size this corresponds to values of E of 0.0009, 0.0015, and 0.0027 per cent U^{235} by weight for mean enrichments of 1, 2 and 3 per cent, respectively.

2. Routine Measurements of SRE Slugs - Fifty-two slugs selected from the SRE fuel were recently compared by the danger coefficient method. These slugs are 6.0 inches long by 0.75 inch in diameter and are of nominal 2.778 per cent by weight U^{235} enrichment. The purpose of these tests was to vindicate the accuracy of the isotopic analyses performed on the original ingots, and to make sure no gross absorption impurities are present in the slugs.

To represent the 26 ingots, two slugs were chosen from each, and the pair tested as a unit. It is fairly certain that the two slugs in a pair were identical in enrichment and impurities, as they were chosen from the same rod. Thus; two slugs in a pair numbered 8212-3 and 8212-6 are slugs number 3 and 6 from rod number 2 from the bottom third of ingot number 82.

In the interests of speed, an abnormal testing procedure was employed: 12 pairs of slugs were initially lined up in the twelve-foot through tube, such that one pair entirely occupied the WBNS core. The reactor was brought up to and

Fig. 9. "Sensitivity" of the WBNS for Enrichment Testing

leveled off at 200 mw. A five minute trace of the power drift was made with the BF_3 chamber-electrometer-recorder setup, after which the next pair was pushed into the core by inserting two more slugs at one end of the through tube. Ordinarily, the change in reactivity was small enough to allow the next tract to proceed without adjustment of the coarse rod.

The reactivity drift during the five hours needed for the tests was estimated by repeating the measurement on the first slug pair at the end of the testing, with the coarse rod adjusted to its initial position. This is a crude method of ascertaining the drift, as compared with running the standard sample several times during the testing, but justifiable for the accuracy desired here.

Because of the considerable increase in reactivity caused by the presence of the SRE material in the glory hole, the coarse rod was operating at a position only about 1-1/2 inches from entirely "in". The calibration curves for danger coefficient measurements do not encompass this point, but a rough value of $\Delta\Sigma_a/\Delta x$ for this rod position was obtained by observing the reactor period for a few different settings of the rod around critical (for a given loading).

A standard deviation of 0.005 cm^2 , instead of the usual 0.002 cm^2 , has been assigned to the measurements of $\Delta\Sigma_a$ because of the abbreviated testing procedure used. The standard deviation of the mass spectrographic analyses is 0.0035 weight per cent U^{235} , as reported by Oak Ridge.

The data are plotted in Fig. 10. A fairly consistent relationship is indicated between the relative enrichment values obtained by the danger coefficient tests and the mass spectrographic analyses. Results obtained using the two techniques agree within the standard deviations quoted for the measurements. The straight line shown is not based upon a least squares analysis, but is sufficiently accurate to provide a value of the WBNS "sensitivity" for the case of filling the whole through tube with SRE fuel. A value for $\Delta E/\Delta\Sigma_a$ of $-0.33\%/\text{cm}^2$ is obtained from the slope of the line, as compared with $-1.28\%/\text{cm}^2$ for a cylinder only 3.875 inches in length. If the entire five-foot string of SRE slugs in reflector and core were as effective as a single sample in the core, a value of $-0.0825\%/\text{cm}^2$ would have been expected. The weighting factor of $0.0825/0.33 = 0.25$ should correspond to:

$$\frac{\overline{nv}^2 \text{ of 3.875 in. of glory hole}}{\overline{nv}^2 \text{ of entire glory hole}}$$

Fig. 10. Comparison of Measured $\Delta \Sigma_a$ of SRE Slugs with Enrichment as Found by Isotopic Analysis

From flux distribution measurements³ in the glory hole this ratio can be evaluated. The result is a factor of 0.266, which agrees favorably with the above figure.

VII. PRECISION OF THE DANGER COEFFICIENT METHOD

Considerable work has been done at NAA on the analysis of the errors inherent in the danger coefficient technique, their causes, and possible remedies. As pointed out previously, the error of an individual measurement of Δk is strongly dependent upon reactivity drift, frequency of retesting the standard, and accuracy of the control rod calibration. The correctness of $\Delta \Sigma_a$ depends upon the above as well as knowledge of the neutron effectiveness and the disadvantage factor in the case at hand.

A. DRIFT CURVES

Three danger coefficient runs designed specifically to reveal sources of errors in the method were conducted in 1954. The purpose and results of each are enumerated below.

1. A full eight-hour run was made during which no samples were inserted, nor was the control rod moved, except for negligible amounts in the order of mils for the purpose of keeping the WBNS critical at 200 mw. The object of this experiment was a rough determination of the "natural" reactivity drift with other variables eliminated or held constant.

During the eight hours about 40 traces were taken; the control rod was moved only three times, a distance of about 10 mils each time. The rod was moved by hand, taking advantage of play in the motor gears--this done to guarantee no side motion of the rod.

A plot of critical rod position versus time as shown in Fig. 11 revealed a smooth drift with a maximum value of +18 mils/hr or $-0.003 \text{ cm}^2/\text{hr}$ (at the average rod position of 15.360 inches). Standard deviation of an individual measurement was calculated to be 4.2 mils or 0.0008 cm^2 at the control rod operating point.

2. An experiment was next conducted to discover the combined effect of control rod motion and power transients (if any) upon the apparent

Fig. 11. Natural Reactivity Drift of WBNS

reactivity drift. The following procedure was employed: measurements were made for a period of about six hours with no sample in the through tube; after each measurement the control rod was moved sufficiently to give a drop in power to 150 mw followed by a rise to 250 mw, and finally a drop back to 200 mw--the variation lasting for a period of two minutes. This procedure simulates the normal transients encountered in danger coefficient measurements with the through tube.

A plot of the critical rod position versus time, Fig. 12, revealed that the natural reactivity drift was not changed by the control rod motion or the power transients. The maximum rate of drift was, in fact, less than that found in experiment 1, a value of -4 mils/hr or $+0.001 \text{ cm}^2/\text{hr}$. The random fluctuations gave a standard deviation per measurement of 3.9 mils or 0.0009 cm^2 at the average rod position, also about the same as before.

3. The final item to be investigated was the effect of an error in sample positioning upon the value of $\Delta\Sigma_a$. In order to simulate normal conditions, the one inch pure Be sample was chosen since the one inch series of Be samples had given trouble in past measurements. Measurements were made giving the cross section of the sample at various off-center positions as compared with the standard value at the core center (see Fig. 13). It appears that an error $\pm 1/4$ inch would have no effect on the measurements, and in practice, an error that large would be extremely unlikely.

B. EMPIRICAL ERROR ANALYSIS

All the danger coefficient data obtained at NAA has been carefully examined in order to obtain an overall value of standard deviation applicable to a single measurement of $\Delta\Sigma_a$. The results of this empirical analysis are given in Table XI, where

- ϵ = standard deviation of a measurement of $\Delta\Sigma_a$,
 $\epsilon_{95\%}$ = error for 95 per cent confidence limits,
 and a Gaussian distribution is assumed.

Fig. 12. Reactivity Drift of WBNS with Power Transients

$\Delta \Sigma_a$ OF SAMPLE, NORMALIZED TO ZERO AT CORE CENTER

Fig. 13. Traverse of Beryllium Sample Through WBNS Core Showing Variation of Measured Σ_a with Position

TABLE XI
STATISTICAL ERRORS IN DANGER COEFFICIENT TESTING

Case	Experimental Conditions	Materials Involved	Mean ϵ cm ²	Mean $\epsilon_{95\%}$ cm ²
1	Repeat measurements separated by one or more days in time	Beryllium Magnesium containers Uranium slugs	0.0028	0.0055
2	Repeat measurements made on same day	Beryllium Magnesium containers Uranium slugs Uranium oxide	0.0017	0.0034
3	Fluctuations above and below smooth drift curve	None	0.0008	0.0015

It is important to emphasize the experimental conditions common to the measurements which underwent this error analysis:

1. The measured values of $\Delta \Sigma_a$ were rarely over 0.020 cm²; for values of $\Delta \Sigma_a$ over 0.1 cm² one can expect ϵ to increase because the integral calibration curve can be read only to about ± 0.008 cm².
2. The order of testing samples has generally been A B A C A D A . . . where A is the "standard". Most measurements were done during a total period of four to eight hours, with the result that fairly accurate WBNS drift curves were obtained by the repeated runs on the standard. The value of ϵ will increase for 1) shorter total time for a set of measurements, and 2) fewer runs on the standard (such as an A B C A D E A . . . scheme).
3. Roughly 15 minutes was consumed by each measurement (sample or standard), half of this time being employed for the actual WBNS power trace.
4. Only one sample at a time was inside the WBNS proper, with the geometry and filling of the remaining spaces kept constant throughout a given set of measurements.

With the stipulations above, it would be safe to assume a standard deviation of about 0.002 cm^2 . The value of 0.0028 cm^2 under case 1 is almost surely due to physical changes in samples and standards over extended periods of time. With a few refinements in sample handling technique, this value could be decreased to that listed under case 2. Empirical results indicate, however, that the standard deviation of 0.0008 cm^2 listed under case 3 is unrealistically low.

C. ERRORS IN OBTAINING $\Delta\Sigma_a$ FROM Δk

As discussed before, the coarse rod calibration curve applied strictly only to samples of the same nuclear properties as the Bakelite-boron standards with graphite holders. In the ideal case the rod would be recalibrated for each type of sample measured; in the testing of uranium oxides, for example, a calibration would be conducted using two samples composed of the same type and geometry of oxide, one impregnated with a small known amount of boron. Even this procedure, however, fails to eliminate all the difficulties, since future measurements of $\Delta\Sigma_a$ would be referred to boron, a strict $1/v$ absorber. Suppose the impurity were a resonance absorber, the effect upon the WBNS reactivity would then depend upon $F(r, v)$ at resonance, the value of which would lie between F_L and F_H . The measured $\Delta\Sigma_a$ would indicate a boron impurity equivalent to the resonance impurity in the WBNS core. However, because the thermal flux in this type of reactor is relatively low compared to the resonance flux, the $\Delta\Sigma_a$ obtained would be an overestimate of the effect of the resonance impurity in a reactor with a high cadmium ratio (such as a graphite-uranium reactor). Conversely, should the impurity consist of cadmium, the observed $\Delta\Sigma_a$ would be an underestimate when referred to a graphite-uranium reactor. Therefore, values of $\Delta\Sigma_a$ obtained in routine sample testing using the WBNS should be treated as somewhat arbitrary unless some knowledge of the elements constituting the impurity exists independently.

D. WBNS CONSTANTS

The sensitivity for the WBNS has already been quoted as -23 inhours per cm^2 . For a water boiler type reactor, 360 inhours is equivalent to 1 per cent in Δk . Therefore, $\Delta\Sigma_a/\Delta k$ is found to equal $-15.7 \text{ cm}^2/\% \Delta k$. From flux traverses with an aluminum foil holder inserted into the core, a value of $nv_a/\bar{n}v_0 = 1.4$ was obtained (where nv_a = thermal flux at center of central exposure facility). From

the WBNS critical mass of 638 grams, Σ_f is calculated to be 950 cm^2 . Substitution of these values into Eq. (2) (where it is assumed that $\bar{\Sigma}_f \cong \Sigma_f$) gives $F_L = 0.43$.

In the absence of U^{235} foil measurements in the WBNS core, determination of F_H is less straightforward. It is known that comparison of various sizes of natural uranium cylinders results in negligible values of Δk . If scattering effects are disregarded, the condition required for this situation is

$$\nu \sigma_f F_H = \sigma_a F_L \quad \dots (25)$$

for a given volume of sample. Insertion of numerical values ($\sigma_f = 4.14$ barns, $\sigma_a = 7.65$ barns, $\nu = 2.48$ neutrons per fission, and $F_L = 0.43$) gives $F_H = 0.32$.

On the other hand, from enrichment testing data for completely depleted uranium, the values shown on Table XII are obtained where Eq. (2) was used to calculate F_H .

TABLE XII
VALUES OF F_H CALCULATED FROM ENRICHMENT TESTS

Cylinder Size (inches)	Enrichment %	Measured $\frac{\Delta \Sigma_a}{\Delta E}$ in $\frac{\text{cm}^2}{\%}$	Measured $\frac{\Delta k}{\Delta E}$ in $\frac{\%}{\%}$	Calculated $\frac{\Delta k}{\Delta E}$ in $\frac{\%}{\%}$	Calculated $\frac{\Delta \Sigma_a}{\Delta E}$ in $\frac{\text{cm}^2}{\%}$	Calculated $\frac{\Delta \Sigma_f}{\Delta E}$ in $\frac{\text{cm}^2}{\%}$	Calculated F_H
3.875 x 0.500	0.00	-1.82	0.172	0.165	4.14	3.46	0.34
3.875 x 0.600	0.00	-2.63	0.248	0.238	5.96	4.99	0.34
3.875 x 0.750	0.00	-4.00	0.377	0.371	9.29	7.76	0.34

In Table XII, "calculated $\frac{\Delta k}{\Delta E}$ " = $\frac{\Delta M^{235} \Delta k}{\Delta E \Delta m}$, and the quantity $\Delta k / \Delta m$ designates the WBNS mass coefficient of reactivity referred to the center of the central exposure facility or 0.050%/g of U^{235} . The good agreement between $\Delta k / \Delta E$ (measured) and $\Delta k / \Delta E$ (calculated) indicates that for zero enrichment samples, $\bar{\nu} \nu_a / \bar{\nu} \nu_o$ and F_L , which are used to calculate F_H , are constants (but both drop rapidly as the enrichment increases).

Finally, it is to be pointed out that k_{∞} for the WBNS is about 1.56. If the reasonable assumption is made that fast neutron leakage from the WBNS is very much greater than slow neutron leakage, then the fraction of fast neutrons lost equals $k_{\infty} - 1/k_{\infty}$ or 36 per cent. Therefore, as a rough approximation $F_H = 0.64 F_L$ averaged over the entire reactor. Assuming this relation applies as well to the sample position, $F_H = 0.64 \times 0.43 = 0.28$. If this value of F_H is averaged with the figures of 0.32 and 0.34 noted above, an approximate evaluation of $F_H = 0.31$ is obtained.

A table of the WBNS constants is presented below, with figures for the KAPL Thermal Test Reactor⁷, the Reactivity Measurement Facility at Arco⁸, and the old Argonne Pile (CP-2)⁹ included for comparison.

The quoted sensitivities of the WBNS and the TTR imply that the WBNS is only half as sensitive as the TTR. However, the sensitivity ratio can also be formulated as follows:

$$\frac{\frac{\Delta k}{\Delta \Sigma_a} \text{ (WBNS)}}{\frac{\Delta k}{\Delta \Sigma_a} \text{ (TTR)}} = \frac{\frac{nv_a F_L}{\bar{nv}_o \Sigma_f} \text{ (WBNS)}}{\frac{nv_a F_L}{\bar{nv}_o \Sigma_f} \text{ (TTR)}} = \frac{1.4 \times 0.43 \times 1/950}{2.3 \times 0.45 \times 1/4140} = 2.5 \quad \dots (26)$$

where it has been assumed that the effective microscopic fission cross section is the same in both reactors. The above ratio implies that if a $\Delta \Sigma_a$ of 0.001 cm^2 can be detected using the TTR, a $\Delta \Sigma_a$ of 0.0004 cm^2 should be observable with the WBNS, instead of 0.002 cm^2 as quoted. Although the instrumentation and technique used at KAPL may account for part of the discrepancy, it is felt that most of the difference is due to the nature of a water boiler type reactor. Bubble formation and temperature drifts (both long and short term) are disadvantages characteristic of a water boiler. Routine power traces have occasionally revealed step changes in power or reactivity whose cause is suspected to be bubble formation and release. These transients often make analysis of the traces somewhat arbitrary, although the errors resulting are small. It should be remembered that with a central glory hole the WBNS sensitivity would be improved by a factor of approximately 1.3.

TABLE XIII

REACTOR CONSTANTS FOR THE WBNS, TTR, RMF, AND CP-2

	WBNS	TTR	RMF	Argonne CP-2
Critical mass	634 g	2700 g	4990 g	--
$\Delta k/\Delta m$	0.050%/g	0.006%/g	0.0445%/g	--
$nv_a/\bar{n}v_o$	1.4	2.3	--	--
Cadmium ratio at sample	3.7 for 95 mg/cm ² indium foils	5.9	--	--
$\Delta k/\Delta T$	-0.0245%/° C	-0.01%/° C	-0.01%/° C	--
$\Delta k/\Delta P$	--	--	--	-0.0083%/cm ² Hg
$\Delta \Sigma_a/\Delta k$ at sample	-15.7 cm ² /%	-20 cm ² /%	-250 cm ² /%	-670 cm ² /%
Σ_a	1440 cm ²	--	--	--
Σ_f	950 cm ²	4140 cm ²	7640 cm ²	--
F_L at sample	0.43	0.45	--	--
F_H at sample	0.31	0.42	--	--
$\Delta \Sigma_a/\Delta x$	0.15 cm ² at x = 15 inches	--	--	--
Quoted sensitivity	0.002 cm ²	0.001 cm ²	0.03 cm ²	0.03 cm ²

VIII. CONCLUSION

Comparison between actual danger coefficient measurements using the WBNS and data independently obtained has demonstrated the utility of a water boiler type reactor for production testing. The difference in the absorption cross sections of two samples can be measured to an accuracy of 0.002 cm^2 , this figure representing a standard deviation. Such an error implies a precision of 0.06 in the shotgun values of uranium oxide samples of weights of 200 grams. The standard deviation of shotgun values for solid natural uranium cylinders of 500 to 600 grams in weight is 0.04. Considerable improvement can be gained by centering the glory hole and by increasing its inside diameter. With a two inch glory hole passing through the maximum flux region and the entire 12 inches through the core occupied by the sample, the shotgun errors quoted above would be reduced to roughly 0.01 for the oxides and 0.003 for the metal.

In the field of enrichment testing, the WBNS has been found capable of detecting an enrichment difference between two 4.00 inch long by 0.750 inch diameter natural uranium samples of 0.0008 per cent U^{235} by weight, or a difference between samples of 3 per cent enrichment of 0.0028 per cent U^{235} by weight, where again standard deviations are quoted. As before, improvement results from increasing the sample size, although for enrichments much over 1 per cent the increased self-shielding limits the advantages of increased size.

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