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CF- 56-6- 175

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COPY NO. 20

DATE: June 27, 1956

SUBJECT: THE H.R.R. THIMBLE DESIGN PROBLEM

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THE H.R.R. THIMBLE DESIGN PROBLEM

Design of cylindrical thimbles to house the Homogeneous Research Reactor experiments is concerned with the following principal considerations:

- (a) Thimble material must be transparent to neutrons (low absorption); zircaloy II has been investigated herein.
- (b) The thimbles being immersed in the fuel solution are subjected to intense heating which is the effect of gamma radiation produced by fission of fuel nuclei, decay of fission products and neutron capture within the housing wall. The intensity of the energy release is estimated to be as high as 4 watts per gram of zircaloy (a 6 inch I.D., 3/4 inch thick cylinder is a heat source of approximately 80 kilowatts per foot length).
- (c) The thimbles are subjected externally to a reactor operating pressure of 1400 psia; in this study a shell has been selected sufficiently strong to withstand this full pressure load - no reliance has been placed on internal coolant pressure for shell support.
- (d) Space requirements for some experiments indicate a minimum internal diameter of 6 inches.
- (e) Corrosion of thimble wall by reactor fuel solution.
- (f) Fabrication problem of joining zircaloy thimbles to reactor steel shell (discussed elsewhere).

After investigation, conclusions reached are:

- (1) A 6 inch I.D. 3/4 inch thick zircaloy II cylindrical thimble can successfully meet the H.R.R. design requirements for strength against buckling and yielding.
- (2) Additional shell thickness is required to counter thimble corrosion by fuel solution; up to 1/4 inch may be added to the basic thickness (0.75 inch) before thermal stress becomes prohibitive.
- (3) It is recommended that the ratio of internal diameter to shell thickness for tubes less than 6 inches I.D. be held to a basic 8 to 1 plus a correction for corrosion.
- (4) A 6 inch internal diameter is near maximum thimble size without a requirement of internal pressurization of the shell.

The bases for these conclusions are contained in the ensuing analysis:

SYMBOL LISTING

3.

a	inside radius cylindrical shell	ft
b	outside radius cylindrical shell	ft
D_o	external diameter cylindrical shell	ft
E	modulus of elasticity of shell	psi
f	β for maximum shell temperature	---
k	shell conductivity	Btu/ $^{\circ}$ F ft hr
l	shell thickness	ft
h_{ca}	film coefficient, coolant side	Btu/ $^{\circ}$ F ft ² hr
h_{cb}	film coefficient, fuel solution side	Btu/ $^{\circ}$ F ft ² hr
P	external pressure load	psia
Q	heat generation per unit volume	Btu/ft ³ hr
r	cylindrical radial coordinate	ft
S	tangential shell force per unit length	lb/ft
T	shell temperature	$^{\circ}$ F
T_a	inside surface shell temperature	$^{\circ}$ F
T_b	outside surface shell temperature	$^{\circ}$ F
T_{ca}	internal coolant temperature	$^{\circ}$ F
T_{cb}	fuel solution temperature	$^{\circ}$ F
α	shell expansion coefficient	1/ $^{\circ}$ F
β	ratio of r to a	---
ϵ	ratio of b to a	---
ν	poisson's ratio	---
σ_e	shell tangential stress	psi
σ_r	shell radial stress	psi
σ_z	shell axial stress	psi

A cylindrical shell under external pressure loading can fail either by buckling or yielding. In the former case, an elastic instability develops between the loading and shell reaction when the loading approaches a critical value; the resulting unbalance of forces causes progressive deformation of the shell. In the latter case, combined stresses exceed yield strength causing plastic flow failure.* Thus shell thickness has been selected by the more critical of two considerations; namely,

- a) External pressure shall not exceed 1/2 the theoretical buckling pressure corresponding to the maximum temperature of the shell**.
- b) Maximum combined stress (pressure plus thermal) shall not exceed the material elastic limit.

A. Buckling Instability

An external pressure loading of the shell is supported by a tangential force per unit length designated S . The theoretical value of S at which the shell buckles is given by:

$$(1) \quad S_{cr} = \frac{E l^3}{4(1-\nu^2)b^2}$$

The critical external pressure loading may be related to the critical tangential force by integrating the stress distribution across the thickness of the shell. Thermal stresses induced in the shell because of temperature gradients integrate to zero; there remains the pressure stress distribution:

$$S_{cr} = \int_a^b \sigma_{pe} dr = P_{cr} \int_a^b \frac{b^2}{b^2 - a^2} \left[\frac{a^2}{r^2} + 1 \right] dr = P_{cr} b$$

$$(2) \quad P_{cr} = \frac{E l^3}{4(1-\nu^2)b^3}$$

E is the tangent elasticity modulus of the shell material which is dependent on shell temperature and shell compressive stress.

The buckling pressure for a zircaloy II cylindrical shell versus ratio of external diameter to thickness for temperatures equal 100, 300, 600 and 800°F is presented in Figure 1.

* These design criteria were chosen after consultations with Dr. R. G. Sturm (See inter-company correspondence dated June 18, 1956 - R. F. Hughes to W. R. Gall).

** This requirement, for the shells investigated, gives a buckling safety factor based on mean shell temperature of greater than 2.5.

*** Timoshenko's "Theory of Elastic Stability", Chapter 4.

B. Shell Stress

The theory of constant energy of distortion was used to determine shell strength against yield. According to this criteria, the shell will fail when the stresses in the three cylindrical coordinate directions are of such magnitude that,

$$(3) \quad (\sigma_{\theta} - \sigma_r)^2 + (\sigma_{\theta} - \sigma_z)^2 + (\sigma_z - \sigma_r)^2 = 2(\text{YIELD STRENGTH})^2$$

The yield strength of zircaloy II versus temperature is shown in Figure 2.

A study of stress-strain data for zircaloy II indicated the elastic limit to be approximately 1/2 yield strength corresponding to 0.20% permanent strain; therefore, a factor of safety of 2 based on yield strength was demanded for all points of the shell.

The pressure stress distribution in the shell is given readily by:

$$(4) \quad \begin{aligned} \sigma_{\theta} &= P \frac{\epsilon^2(B^2+1)}{(\epsilon^2-1)B^2} \\ \sigma_{rz} &= 1/2 \sigma_{\theta} \\ \sigma_{rr} &= P \frac{\epsilon^2(B^2-1)}{(\epsilon^2-1)B^2} \end{aligned}$$

where $\epsilon = b/a$, $B = r/a$

It remains to consider the thermal stress distribution in the shell which is dependent on the following reactor design variables:

- (a) Reactor Fuel Solution Temperature (fixed by design at 500°F)
- (b) Thimble Internal Coolant Temperature
- (c) Thimble Internal Coolant Velocity
- (d) Gamma-Ray Heating of Thimble Wall (taken to be 4 watts per gram of zircaloy)
- (e) Thimble Thickness, l
- (f) Ratio of External to Internal Thimble Radii, ϵ
- (g) Thimble Material Properties (elasticity modulus E , thermal expansion coefficient α , thermal conductivity k , and poisson's ratio ν).

The temperature distribution must satisfy the differential equation:

$$(5) \quad \frac{d^2 T}{dr^2} + \frac{1}{r} \frac{dT}{dr} + \frac{Q}{k} = 0$$

where Q is the heat released per unit volume of shell due to gamma radiation; its magnitude is taken as 26 watts per cm^3 (4 watts per gram).

It is convenient to define a parameter " f " which is the particular value of $\beta = r/a$ that locates the maximum temperature in the wall. We may express the solution of (5), also the temperature drop to coolant and temperature drop to fuel solution in terms of " f ".

$$(6) \quad T - T_a = \frac{Q l^2}{2k(\epsilon-1)^2} \left[f^2 \ln \beta - \frac{\beta^2-1}{2} \right] \quad *$$

$$(7) \quad T_a - T_{ca} = \frac{Q l}{2h_{ca}} \left[\frac{f^2-1}{\epsilon-1} \right]$$

$$(8) \quad T_b - T_{cb} = \frac{Q l}{2h_{cb}} \left[\frac{\epsilon^2-f^2}{\epsilon(\epsilon-1)} \right]$$

For maximum shell temperature greater than fuel solution temperature, " f " varies between the value 1 and ϵ as indicated in the following table:

Remarks

$f = 1$ all heat transferred to fuel solution, $r = a$ insulated

$f = \sqrt{(\epsilon^2-1)/2 \ln \epsilon}$ minimum thermal stress, $T_a = T_b$

$f = \epsilon$ all heat transferred to coolant, $r = b$ insulated

" f " is defined by the intensity of heat generation in the wall and the difference between fuel solution temperature and coolant temperature according to the implicit relation.

$$(9) \quad T_{cb} - T_{ca} = \frac{Q l}{2(\epsilon-1)} \left[\frac{f^2 \ln \epsilon}{(\epsilon-1)k/l} - \frac{\epsilon+1}{2k/l} + \frac{f^2-1}{h_{ca}} - \frac{\epsilon^2-f^2}{\epsilon h_{cb}} \right]$$

The thermal stress distribution in the wall is determined from the temperature distribution given by equation (6)**:

$$(10) \quad \sigma_{to} = \frac{E \alpha Q l^2}{16k(1-\nu)(\epsilon^2-1)(\epsilon-1)^2} \left[4f^2(\epsilon^2 + \frac{\epsilon^2}{\beta^2}) \ln \epsilon - 4f^2(\epsilon^2-1)(\ln \beta + 1) + 1 - \epsilon^4 + (3\beta^2 - \frac{\epsilon^2}{\beta^2})(\epsilon^2-1) \right]$$

$$(11) \quad \sigma_{tr} = \frac{E \alpha Q l^2}{16k(1-\nu)(\epsilon^2-1)(\epsilon-1)^2} \left[4f^2(\epsilon^2 - \frac{\epsilon^2}{\beta^2}) \ln \epsilon - 4f^2(\epsilon^2-1) \ln \beta + 1 - \epsilon^4 + (\beta^2 + \frac{\epsilon^2}{\beta^2})(\epsilon^2-1) \right]$$

$$(12) \quad \sigma_{tz} = \frac{E \alpha Q l^2}{8k(1-\nu)(\epsilon^2-1)(\epsilon-1)^2} \left[4f^2 \epsilon^2 \ln \epsilon - 2f^2(\epsilon^2-1)(2 \ln \beta + 1) + 1 - \epsilon^4 + 2\beta^2(\epsilon^2-1) \right]$$

*

See Appendix

**

Timoshenko's "Theory of Elasticity", Chapter 14.

1. Thimble Design Selected

Figure 1 indicates that the thinnest shell possible (factor of safety of 2 against buckling)* for a temperature of 600°F is one with a ratio of external diameter to thickness of 10. Considering a minimum internal diameter of 6 inches dictated by experimental space requirements, the minimum wall thickness is 3/4 inches. This cylinder was analyzed for yield failure.

The capacity of the internal coolant (water) to remove heat generated in the tube wall is dependent on its temperature and velocity. The temperature distribution in the cylindrical shell is given in Figure 3 for a coolant temperature of 200°F and a coolant velocity of 5 feet/sec. The capacity of the fuel solution to remove heat was estimated from an average velocity of 1 ft/sec over the tubes. The thimble thermal stress and pressure stress distributions are presented in Figure 4. Critical stress occurs at the maximum temperature point of the shell; the yield strength is a minimum there and both thermal and pressure stress contribute to compression of wall.

The data of Table I shows that a 6 inch I.D.-3/4 inch thick thimble meets both the buckling and yield strength design criteria.

2. Effect of Tube Size on Stress

Tube size was varied at constant ratio of diameter to thickness (safety factor against buckling maintained at 2) to determine the largest shell within the yield criteria. Table I indicates that the maximum diameter allowable is only slightly greater than 6 inches I.D.

3. Effect of Tube Thickness on Stress

Increasing shell thickness beyond 3/4 inches does not strengthen the shell against yield; lower pressure stress is more than offset by higher thermal stress and reduced yield strength. The results are in Table II.

4. Effect of Internal Coolant Temperature on Stress

Table III presents the variation of shell critical stress with coolant temperature. An increase in coolant temperature increases the design safety; the decrease in yield strength of the cylinder is more than offset by reduction of thermal stress.

5. Coolant Required

The coolant (water) required for a 6 inch diameter five foot long thimble is estimated to be from 75 to 100 gallons per minute.

6. Corrosion

Corrosion of the thimble by fuel solution requires additional wall thickness above the requirement for strength against yielding and buckling. Table II indicates that the maximum wall thickness (limited by thermal stress) for a 6 inch I.D. tube is approximately 1 inch. Thus, up to 1/4 inches may be added to thimble basic thickness to counter corrosion.

* Based on maximum shell temperature.

APPENDIX

HEAT GENERATION IN CYLINDRICAL SHELLS

A SOLUTION OF $\frac{d^2T}{d\pi^2} + \frac{1}{\pi} \frac{dT}{d\pi} + \frac{Q}{k} = 0$ IS:

$$\begin{aligned} T - T_a &= C_0 + C_1 \ln \pi/a - \frac{1}{4} \frac{Qa^2}{k} \frac{\pi^2}{a^2} \\ &= C_0 + C_1 \ln \beta - \frac{1}{4} \frac{Ql^2}{k(\epsilon-1)^2} \beta^2 \end{aligned}$$

@ $\pi = a$, $T = T_a$, $\therefore C_0 = \frac{1}{4} \frac{Ql^2}{k(\epsilon-1)^2}$ SINCE $\beta = 1$

@ $\pi = \pi^*$, $T = T_{\text{MAX}}$, $\beta = f$, $\frac{dT}{d\pi} = 0$

$$\frac{dT}{d\pi} = C_1 / \beta a - \frac{1}{4} \frac{Ql^2}{k(\epsilon-1)^2} \frac{2\beta}{a} = 0$$

$$C_1 = \frac{f^2}{2} \frac{Ql^2}{k(\epsilon-1)^2} \quad \text{SINCE } \beta = f$$

$$\rightarrow \therefore T - T_a = \frac{Ql^2}{2k(\epsilon-1)^2} \left[f^2 \ln \beta - \frac{\beta^2 - 1}{2} \right]$$

$$\rightarrow (T_a - T_{ca}) h_{ca} = \frac{\pi(\pi^{*2} - a^2) Q}{2\pi a} = \frac{f^2 - 1}{\epsilon - 1} \frac{Ql}{2}$$

$$\rightarrow (T_b - T_{cb}) h_{cb} = \frac{\pi(b^2 - \pi^{*2}) Q}{2\pi b} = \frac{\epsilon^2 - f^2}{\epsilon(\epsilon-1)} \frac{Ql}{2}$$

TABLE I

9.

DIAMETER VS STRESS

	BASIC SHELL	MAXIMUM DIAMETER SHELL	
→ Internal Dia.	6	6.4	in.
Thickness	3/4	4/5	in.
External Pressure	1400	1400	psia
Internal Pressure	0	0	psia
Heat Generation	4	4	watts/gram
Heat Flux Inward to Coolant	138,000	141,000	Btu/hr ft ²
Heat Flux Outward to Fuel Solution	31,000	38,000	Btu/hr ft ²
Temperatures			
Coolant	201	204	°F
Fuel Solution	500	500	°F
Inside Surface	278	282	°F
Outside Surface	566	581	°F
Critical Stress Point (Radius)	3.60	3.82	in.
Maximum Shell Temperature	585	603	°F
Thermal Stress (Critical Point)			
Tangential	-2912	-3100	psia
Radial	+ 110	+ 117	psia
Axial	-2803	-2990	psia
Pressure Stress (Critical Point)			
Tangential	-6590	-6590	psia
Radial	-1190	-1190	psia
Axial	-3295	-3295	psia
Combined Stress (Equation 3)	-7390	-7530	psia
Yield Strength	15700	15200*	psia
→ Safety Factor - Yield	2.13	2.02*	
D _o /l	10	10	
Safety Factor - Buckling (max. shell temperature)	2	2	

* For zircaloy II, elastic limit equal approximately 1/2 yield strength.

TABLE II

10.

THICKNESS VS STRESS

	BASIC SHELL	SHELL A	SHELL B	
Internal Dia.	6	6	6	in.
→ Thickness in.	3/4	7/8	1	in.
External Pressure	1400	1400	1400	psia
Internal Pressure	0	0	0	psia
Heat Generation	4	4	4	watts/gram
Temperatures				
Coolant	201	200	198	°F
Fuel Solution	500	500	500	°F
Inside Surface	278	282*	285*	°F
Outside Surface	566	590*	590*	°F
Critical Stress Point (Radius)	3.60	3.64	3.67	in.
Maximum Shell Temperature	585	625	672	°F
Thermal Stress (Critical Point)				
Tangential	-2912	-3110	-3468	psia
Radial	+ 110	+ 135	+ 161	
Axial	-2803	-3084	-3307	
Pressure Stress (Critical Point)				
Tangential	-6590	-5880	-5340	psia
Radial	-1190	-1120	-1060	
Axial	-3295	-2940	-2670	
Combined Stress (Equation 3)	-7390	-7000	-6950	psia
Yield Strength	15700	14800	14200	psia
→ Safety Factor-Yield	2.13	2.11	2.04	
D_o/l	10	8.86	8	
Safety Factor-Buckling (max. shell temperature)	2.0	2.4	2.6	

* Boiling heat transfer to fuel solution

TABLE III

COOLANT TEMPERATURE VS STRESS

	SHELL C	BASIC SHELL	SHELL D	
Internal Dia.	6	6	6	in.
Thickness	3/4	3/4	3/4	
External Pressure	1400	1400	1400	psia
Internal Pressure	0	0	0	
Heat Generation	4	4	4	watts/gram
Temperatures				
→ Coolant	113	201	294	°F
Fuel Solution	500	500	500	
Inside Surface	197	278	364	
Outside Surface	545	566	587	
Critical Stress Point (Radius)	3.65	3.60	3.56	in.
Maximum Shell Temperature	555	585	618	°F
Thermal Stress (Critical Point)				
Tangential	-3638	-2912	-2320	psia
Radial	+ 115	+ 110	+ 107	
Axial	-3551	-2803	-2211	
Pressure Stress (Critical Point)				
Tangential	-6590	-6590	-6590	psia
Radial	-1190	-1190	-1190	
Axial	-3295	-3295	-3295	
Combined Stress (Equation 3)	-7990	-7390	-6800	psia
Yield Strength	16500	15700	15000	psia
→ Safety Factor Yield	2.07	2.13	2.21	
D _o /l	10	10	10	
Safety Factor Buckling (max. shell temperature)	2	2	2	

12 fig 1

BUCKLING PRESSURE VERSUS RATIO OF EXTERNAL
DIAMETER TO THICKNESS
(ZIRCALOY-2 CYLINDRICAL SHELLS)

023 012

FIGURE 1

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TEMPERATURE DISTRIBUTION IN 6 in. I.D. - 3/4 in. THICK CYLINDRICAL ZIRCALOY SHELL

FIGURE 3

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023 015

THERMAL STRESS AND PRESSURE STRESS DISTRIBUTIONS
IN 6 in. I.D.-3/4 in. CYLINDRICAL ZIRCALOY SHELL

Figure 4