TABLES OF ELECTRON WAVEFUNCTIONS AND ASSOCIATED DATA

E.L. CHURCH

December 1966

BROOKHAVEN NATIONAL LABORATORY

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December 1966

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ABSTRACT

Part I reviews the role of penetration effect in internal conversion, and provides a collection of formulae for the analysis of such effects in the K and L shells. Part II provides tables of the bound and free electron wavefunctions at the nucleus, which appear as normalization constants in part I. These results, which include the effects of the finite nuclear size and atomic screening, are derived from the tabulations of Band, Guman, and Sogomonova, which represent intermediate steps in the internal-conversion-coefficient calculations of Sliv and Band.

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INTRODUCTION

This report is divided into two parts: Part I gives a collection of expressions useful for the analysis and interpretation of static and dynamic penetration effects in internal conversion, including detailed recipes for monopole, dipole, and quadrupole transitions converting in the K and L shells. Part II contains numerical tables of the bound and free electron wavefunctions at the center of the nucleus, which appear as normalization constants in the discussions of Part I. These wavefunction data include the static effects of the finite nuclear size, and atomic screening.

The essential ingredients of this report were prepared several years ago and privately circulated. Those in Part I are based on a review article on finite-nuclear-size effects,¹ and those in Part II were prepared immediately thereafter as the basis for a systematic analysis of experimental conversion anomalies. To date, only one aspect of this study has been published,² since such an analysis requires, in general, the radial matrix elements for internal conversion, and tables of these have only recently become available.³ The numerical data in Part II are based on the intermediate steps in the Sliv-Band calculations of internal coefficients, which have appeared in the book by Band, Guman, and Sogomonova.^{4,5} Similar data have already been used by the Leningrad group for the interpretation of electron capture⁶ and monopole conversion results.⁷

There are two comprehensive tabulations of internal-conversion coefficients in the literature; those of Rose⁸ and of Sliv-Band.^{9,9A} They are both based on the relativistic calculation of electron wavefunctions in a Fermi-Thomas Dirac potential, and include the effects of the finite nuclear size in a necessarily model-dependent way.^{3,8,9,9A} It has long been known that the tabulated conversion coefficients of Rose and Sliv-Band frequently differ by amounts greater than their

assigned limits of significance. Many of these differences are attributable to typographical and interpolation errors, and to their different handling of finite-nuclear-size effects; although there are residual differences yet to be explained in a systematic way.^{10,11} There have recently been recalculations of a number of specific internal conversion coefficients, which differ principally from the earlier calculations in their treatment of screening.¹¹⁻¹³ It is hoped that there will soon be systematic recalculations to resolve the remaining differences, and to provide a comprehensive frame of reference for the analysis of penetration and other physical effects.

Changes in the calculated values of the usual conversion contribution may seriously affect the values of the penetration contributions deduced from experiment, since the two contributions add in the conversion process. However, the present work is only formally connected with this "atomic" aspect of the problem; it is principally concerned with the subsequent separation of the penetration contribution, C_λ , into its electron and nuclear factors, C and λ . The electron factor, C , involves the product of the bound and free electron wavefunctions at the nucleus, such as the results tabulated in Part II. The finite-size and screening effects included in such results are generally only moderate corrections to the Coulomb wavefunctions in heavier nuclei. Possible changes in these wavefunctions resulting from recalculation, using different screening or static nuclear models, are expected to be small^{14,1}; especially on the scale of significance of nuclear models which are available for the interpretation of the deduced values of the dynamic nuclear-matrix element parameter, λ .

The results in this report are stated in terms of finite-size effects in internal conversion coefficients, although they also have immediate application

to other conversion processes, such as directional and polarization correlations.¹⁵⁻¹⁷

The numerical data may also be useful for the tabulation of derived quantities and in fields which are not discussed explicitly, such as beta-decay processes, hyperfine effects, and atomic screening.

PART I

I.1 Electron Wavefunctions in a Central Potential¹⁸

The Dirac electron wavefunction in a central potential, $\Psi(\vec{r})$, may be written:

$$\Psi(\vec{r}) = \begin{pmatrix} -i\chi_{-\kappa}(\hat{r})f(r)_{\kappa} \\ \chi_{\kappa}(\hat{r})g(r)_{\kappa} \end{pmatrix}, \quad (1)$$

where χ represents the spin-angular factors, f and g are the radial wavefunctions, and κ is the usual relativistic quantum number:¹⁹

$$\kappa = \begin{cases} -(\ell + 1) & \text{for } j = \ell + \frac{1}{2} \\ \ell & \text{for } j = \ell - \frac{1}{2} \end{cases}. \quad (2)$$

The radial functions satisfy the coupled equations:

$$\begin{aligned} + \frac{dg}{dr} + \frac{\kappa + 1}{r} g - (W - V + 1) f &= 0, \\ - \frac{df}{dr} + \frac{\kappa - 1}{r} f - (W - V - 1) g &= 0, \end{aligned} \quad (3)$$

where W is the total energy and V is the central potential felt by the electrons. For the particular case of the Coulomb potential, $V(r) = -\alpha Z/r$, the bound and free solutions are well known.^{20,21} In the present context $V(r)$ is the electrostatic potential for a finite spherical nuclear charge distribution, and $V(0)$ is finite.

The regular solutions of (3) are conveniently classified in terms of "large" and "small" components, although these classifications depend on

the radial distance. At atomic distances the magnitude of $g(r)$ generally dominates that of $f(r)$, and g is termed "large" in that region. However, at small distances, of the order of nuclear dimensions, the situation is reversed for positive values of κ . The classifications are then:

	"nuclear" distances	"atomic" distances
"large" components	$g(r)_{- \kappa }, f(r)_{+ \kappa }$	$g(r)_{\pm \kappa }$
"small" components	$f(r)_{- \kappa }, g(r)_{+ \kappa }$	$f(r)_{\pm \kappa }$

(4)

The indicial behavior of the solutions of (3) may be exhibited by expanding the solutions about the origin. The large and small components at the nucleus are easily seen to begin as:

$$\begin{aligned}
 g(r)_{-|\kappa|} &= g(0)_{-|\kappa|} r^{|\kappa|-1} + \dots, & f(r)_{-|\kappa|} &= + \frac{V(0)-W+1}{2(j+1)} g(0)_{-|\kappa|} r^{|\kappa|} + \dots, \\
 f(r)_{+|\kappa|} &= f(0)_{+|\kappa|} r^{|\kappa|-1} + \dots, & g(r)_{+|\kappa|} &= - \frac{V(0)-W-1}{2(j+1)} f(0)_{+|\kappa|} r^{|\kappa|} + \dots,
 \end{aligned}
 \tag{5}$$

where $g(0)_{-|\kappa|}$ and $f(0)_{+|\kappa|}$ are phase and normalization constants, yet to be specified.

The phase and normalization of the solutions of (3) are fixed by convention. There are two common phase conventions, which may be expressed:

$$\begin{aligned}
 \text{(A):} & \quad \begin{cases} g(0)_{-|\kappa|} \\ f(0)_{+|\kappa|} \end{cases} \quad \text{are positive} \\
 \text{(B):} & \quad \begin{cases} g(0)_{-|\kappa|} \\ f(0)_{+|\kappa|} \end{cases} \quad \begin{array}{l} \text{is positive} \\ \text{is negative.} \end{array}
 \end{aligned}
 \tag{6}$$

In short, (A) states that the large components at nuclear distances are positive, while (B) states that the large components at atomic distances are (initially) positive.

The normalization of the boundstate wavefunction is generally:

$$\int_0^{\infty} r^2 dr (f^2 + g^2) = 1. \quad (7)$$

There are two common conventions for the normalization of the free wavefunctions:

(a): Normalization per unit energy interval.

(b): Normalization to one particle in a large sphere
of radius Ω . (8)

These may be expressed analytically as:^{20,21}

$$(a): \int_{W-}^{W+} dW' \int_0^{\infty} r^2 dr (f_W f_{W'} + g_W g_{W'}) = 1, \quad (9)$$

and:

$$(b): \int_0^{\Omega} r^2 dr (f_W^2 + g_W^2) = 1. \quad (10)$$

Free functions normalized according to (b) are converted to normalization (a) by multiplying by the factor:

$$\sqrt{\frac{W\Omega}{\pi P}} \quad ; \quad P \equiv \sqrt{W^2 - 1}. \quad (11)$$

Phase and normalization conventions need only be stated in terms of one radial component since the ratio of the large and small components is fixed by the original equations, (3). For example, the sign of f/g at small distances is the sign of κ .

The remainder of this report makes use of the classification in terms of large and small components at "nuclear" distances, given in (4); the phase convention (A) in (6); and the normalization convention (a) in (8). Although these conventions are not used universally in the literature, they recommend themselves naturally to the present discussions.

Complete solutions of (3) may be easily derived in the form of series expansions about the origin.^{22,23} Matumoto and Yamada have given particularly useful form of such an expansion:²⁴

$$g(r)_{-|\kappa|} = g(0)_{-|\kappa|} r^{|\kappa|-1} \left[J_{|\kappa|} - \frac{W+1}{2|\kappa|-1} r^2 \varphi_{|\kappa|} + 2 \frac{2|\kappa|W+1}{4|\kappa|^2-1} K_{|\kappa|}^{(2)} + \mathcal{O}(WR)^2 + \dots \right], \quad (12)$$

$$f(r)_{+|\kappa|} = f(0)_{+|\kappa|} r^{|\kappa|-1} \left[J_{|\kappa|} - \frac{W-1}{2|\kappa|-1} r^2 \varphi_{|\kappa|} + 2 \frac{2|\kappa|W-1}{4|\kappa|^2-1} K_{|\kappa|}^{(2)} + \mathcal{O}(WR)^2 + \dots \right],$$

and;

$$f(r)_{-|\kappa|} = g(0)_{-|\kappa|} r^{|\kappa|} \left[\varphi_{|\kappa|} - \frac{W-1}{2|\kappa|+1} J_{|\kappa|} - 2 \frac{2|\kappa|W+1}{4|\kappa|^2-1} K_{|\kappa|}^{(1)} + \mathcal{O}(WR)^2 + \dots \right], \quad (13)$$

$$g(r)_{+|\kappa|} = -f(0)_{+|\kappa|} r^{|\kappa|} \left[\varphi_{|\kappa|} - \frac{W+1}{2|\kappa|+1} J_{|\kappa|} - 2 \frac{2|\kappa|W-1}{4|\kappa|^2-1} K_{|\kappa|}^{(1)} + \mathcal{O}(WR)^2 + \dots \right],$$

for the large and small components, respectively. The Matumoto-Yamada functions J , φ , and K are only functions of r and j , but not the energy, W ; and are related to the potential, $V(r)$, according to:

$$J_{|\kappa|}(r) = 1 - \int_0^r dr_1 r_1 V(r_1) \varphi_{|\kappa|}(r_1), \quad (14)$$

$$\varphi_{|\kappa|}(r) = r^{-2|\kappa|-1} \int_0^r dr_1 r_1^{2|\kappa|} V(r_1) J_{|\kappa|}(r_1),$$

and:

$$\begin{aligned} K_{|\kappa|}^{(2)}(r) &= \int_0^r dr_1 r_1 V(r_1) \{ J_{|\kappa|}(r_1) + K_{|\kappa|}^{(1)}(r_1) \}, \\ K_{|\kappa|}^{(1)}(r) &= r^{-2|\kappa|-1} \int_0^r dr_1 r_1^{2|\kappa|} V(r_1) \{ r_1^2 \varphi_{|\kappa|}(r_1) - K_{|\kappa|}^{(2)}(r_1) \}. \end{aligned} \quad (15)$$

Explicit expressions for these functions are easily obtained by inserting a particular form for $V(r)$. For example, for a uniformly charged sphere of radius R :

$$V(r) = -\frac{3}{2} \frac{\alpha Z}{R} \left[1 - \frac{1}{3} \left(\frac{r}{R} \right)^2 \right]; \quad r \leq R. \quad (16)$$

The Matumoto-Yamada functions then appear as expansions in the parameter:

$$\xi^2 \equiv (\alpha Z/2)^2,$$

which is of the order of 10^{-1} or less in cases of practical interest.

The leading terms of the Matumoto-Yamada functions are given in Table I.1, (p. 23), for a uniformly charged sphere for electrons having an arbitrary value of $j = |\kappa| - \frac{1}{2}$. Much more complete expansions for the particular cases of $j = 1/2$ and $j = 3/2$ are given in Tables I.2 - I.7 (pp. 24-29). The wavefunctions represented by these cases are the most useful ones, since they describe electrons which are weighted most importantly at the nucleus.

The remaining sections of Part I make use of the expansions of the electron wavefunctions described above to derive practical expressions for various quantities of physical interest in terms of the phase and normalization constants, $g(0)_{-|\kappa|}$ and $f(0)_{+|\kappa|}$. Tables of numerical values of these constants are then given in Part II.

I.2 EO Internal Conversion^{1,25}

The rate of EO conversion-electron ejection from an initial state i to the final state f is:¹⁸

$$\mathcal{W}_e(\text{EO}) = 2\pi\alpha^2 (2j_i+1) \left| \langle f | \left| \frac{1}{e} Q H(r) \right| | i \rangle \right|^2, \quad (17)$$

where Q is the nuclear charge-density operator, and $H(r)$ is the radial function:

$$H(r) = \int_0^r r_1^2 dr_1 \left[\frac{1}{r} - \frac{1}{r_1} \right] \left[f_f(r_1) f_i(r_1) + g_f(r_1) g_i(r_1) \right]. \quad (18)$$

Substituting the previous expansions of the electron wavefunctions, f and g , into (18) allows the rate of electron ejection to be written in the standard form:

$$\mathcal{W}_e(\text{EO}) = \Omega(Z, k) \left| \rho_\mu \right|^2 \quad (19)$$

where Ω is a purely electronic factor, and ρ_μ is the nuclear "strength parameter":

$$\rho_\mu \equiv \langle f | \left| \frac{1}{e} Q \left[\left(\frac{r}{R} \right)^\mu - \sigma \left(\frac{r}{R} \right)^{\mu+2} + \dots \right] | i \rangle \right|. \quad (20)$$

Explicit expressions for Ω and σ are given in Table I.8 (p. 30) for EO conversion in the K , L_1 , L_{11} , and L_{111} shells. The values in the Table are approximate in that small terms, of order WR , have been deliberately omitted.

For the K , L_1 , and L_{11} shells, $\mu = 2$, while for the L_{111} shell, $\mu = 4$. Conversion ratios involving K , L_1 , and L_{11} are then essentially independent of nuclear properties, while conversion in the L_{111} shell is generally negligible.

I.3 Penetration Effects in Internal Conversion for $L > 0$ ^{1,2,26-28}

Conversion-electron ejection for multipoles of order $L > 0$ proceeds via the transverse electromagnetic field, which is the same field responsible for gamma-ray emission. There is then an intimate connection between these two competing processes, and the ratio of their rates is defined to be the internal-conversion-coefficient; the ICC. In the limit of a point nucleus the ICC is essentially independent of dynamical nuclear properties. The purpose of this section is to quote expressions which may be used for the analysis of those dynamical contributions to the ICC arising from the fact that the nucleus does have a finite size. Because of the smallness of the nucleus on the scale of atomic dimensions, this "nuclear" aspect of the problem is essentially separable from the "atomic" problem; that is, the actual computation of ICC's. In the discussion of penetration contributions to internal conversion given below, the atomic aspect only enters through the appearance of the electron phase and normalization constants, $g(0)_{-|\kappa|}$ and $f(0)_{+|\kappa|}$.

The rate of internal-conversion electron ejection from an initial state i to the final states f , may be written:¹⁸

$$\begin{aligned} \mathcal{W}_e^{(EL)} = & \frac{8\pi^2 \alpha^2}{L(L+1)(2L+1)} k^2 \frac{2I_f+1}{2I_i+1} \sum_f C_{fi} |G_{fi} \langle f | \left| \frac{1}{e} \vec{j}_n \cdot \vec{A}(EL) \right| | i \rangle \\ & + \langle f | \left| \frac{1}{e} \vec{j}_n \cdot \vec{\nabla} \Phi(r) Y_L \right| | i \rangle + \langle f | \left| \frac{1}{e} \vec{j}_n \cdot \frac{\hat{r}}{r} \Theta(r) Y_L \right| | i \rangle|^2 \\ & + \mathcal{O}(\alpha^3) + \dots \end{aligned} \quad (21)$$

and:

$$\begin{aligned} \mathcal{W}_e^{(ML)} = & \frac{8\pi^2 \alpha^2}{L(L+1)(2L+1)} k^2 \frac{2I_f+1}{2I_i+1} \sum_f B_{fi} |R_{fi} \langle f | \left| \frac{1}{e} \vec{j}_n \cdot \vec{A}(ML) \right| | i \rangle \\ & + \langle f | \left| \frac{1}{e} \vec{j}_n \cdot \vec{A}(ML) \Psi(r) \right| | i \rangle|^2 + \mathcal{O}(\alpha^3) + \dots \end{aligned} \quad (22)$$

The corresponding rates of gamma-ray emission are:

$$W_{\gamma}^{(FI)} = 8\pi\alpha k \frac{2I_f+1}{2I_i+1} |\langle i || \frac{1}{e} \vec{j}_n \cdot \vec{A} \left(\frac{EL}{ML} \right) || i \rangle|^2 + \mathcal{O}(\alpha^2) + \dots \quad (23)$$

In these expressions, I , $\vec{j}_n$, and k are the nuclear spin, current, and transition energy, respectively. The reduced nuclear matrix elements appearing on the right of the above expressions are defined by:

$$\langle f || O_L || i \rangle \equiv \left[\begin{matrix} I_i & L & I_f \\ C_{M_i} & M & M_f \end{matrix} \right]^{-1} \langle I_f M_f || O_{LM} || I_i M_i \rangle, \quad (24)$$

where C is the usual Clebsch-Gordan coefficient.¹⁹

The vector multipole potentials appearing in the leading matrix elements in (21), (22), and (23) are explicitly:

$$\begin{aligned} \vec{A}(EL) &= - \frac{1}{k \sqrt{L(L+1)}} \vec{\nabla} \times (\vec{r} \times \vec{\nabla}) j_L(kr) Y_L(\hat{r}) \\ &= + \frac{1}{k \sqrt{L(L+1)}} \left\{ \vec{\nabla} \frac{d}{dr} r + k^2 \vec{r} \right\} j_L(kr) Y_L(\hat{r}), \end{aligned} \quad (25a)$$

and:

$$\vec{A}(ML) = \frac{1}{\sqrt{L(L+1)}} (\vec{r} \times \vec{\nabla}) j_L(kr) Y_L(\hat{r}), \quad (25b)$$

where j_L is the spherical Bessel function. The quantities C_{fi} and B_{fi} appearing in (21) and (22) are the angular factors given by Rose^{8,39}, and G_{fi} and R_{fi} are the radial electron integrals:

$$\begin{aligned} G_{fi}(EL) &= \int_0^\infty r^2 dr \left\{ [g_f(r) f_i(r) - f_f(r) g_i(r)] k \right. \\ &\quad \left. - [f_f(r) f_i(r) + g_f(r) g_i(r)] \frac{d}{dr} \right\} r h_L^{(1)}(kr), \end{aligned} \quad (26)$$

and:

$$R_{fi}(ML) = \int_0^{\infty} r^2 dr \left[g_f(r) f_i(r) + f_f(r) g_i(r) \right] h_L^{(1)}(kr), \quad (27)$$

where $h_L^{(1)}$ is a spherical Hankel function. Φ , Θ , and Ψ are electron penetration functions. Explicit expressions for these are given in the literature;^{1,2} and their long-wavelength expansions, which are sufficient for practical purposes, are given in Table I.9 (p. 31).

Two approximations are customarily made in the calculations of ICC's.^{8,9,11-13} First, only the leading terms in (21)-(23) are retained, and terms of higher order in the fine-structure constant, α , are dropped. Second, the radial integral for electric transitions, (26), is replaced by the form:

$$\begin{aligned} R_{fi}(EL) = & (\kappa_i - \kappa_f) \int_0^{\infty} r^2 dr \left[g_f(r) f_i(r) + f_f(r) g_i(r) \right] h_{L-1}^{(1)}(kr) \\ & + L \int_0^{\infty} r^2 dr \left[g_f(r) f_i(r) - f_f(r) g_i(r) \right] h_{L-1}^{(1)}(kr) \\ & + L \int_0^{\infty} r^2 dr \left[f_f(r) f_i(r) + g_f(r) g_i(r) \right] h_L^{(1)}(kr), \end{aligned} \quad (28)$$

which contains no radial derivative. It is easily seen that the two forms, (26) and (28), are equal when the central potential, $V(r)$, felt by the electron is the same in its initial and final states. In all screened calculations of ICC's to date, the bound and free electron wavefunctions are computed for different potentials; the difference being the potential of the converting electron in its initial state. The resulting approximation in the calculated conversion coefficient, however, is only one of a variety introduced by the use of non-orthogonal wavefunctions, and approximate screening potentials in general.

The dimensionality of the approximations introduced by the neglect of higher-order corrections and screening approximations are of relative order α and $1/Z$, respectively, but the magnitudes of the various multiplying factors have only been partially investigated.^{1,11,29} Such effects give rise to significant corrections to existing conversion calculations. For example, Pietenpol has recently shown that in certain cases the higher-order electrodynamic effects in gamma-ray emission, (23), are of the order of 10% of the leading term.³⁰ In internal conversion, higher-order effects also introduce dependences on detailed nuclear properties, such as static nuclear moments and polarizability; and the details of screening ultimately introduce dependences on the chemical and physical state of the converting atom.

Finite-nuclear-size effects, higher-order electrodynamic corrections, and atomic screening, place physical limitations on the immediate significance of any simple tabulation of internal-conversion properties.

I.4 Long-Wavelength Expansions

The vector potentials, A , and the penetration functions, Φ , Θ , and Ψ , enter into the arguments of nuclear matrix elements and are relevant only in the region of the order of nuclear dimensions. Since $kR \ll 1$ for all energies of interest in conversion-electron spectroscopy, it is convenient to expand these quantities in powers of kr , and to keep only the leading terms of significance. This is the usual long-wavelength expansion. The two leading terms in the expansion of the electric and magnetic vector multipole potentials are:^{2,31}

$$\begin{aligned} \vec{A}_{(EL)} = & \sqrt{\frac{L+1}{L}} \frac{1}{k} \frac{(kR)^L}{(2L+1)!!} \left[\vec{\nabla} \left(\frac{r}{R} \right)^L Y_L(\hat{r}) \right. \\ & - (kR)^2 \left\{ \frac{(L+3)}{2(L+1)(2L+3)} \vec{\nabla} \left(\frac{r}{R} \right)^{L+2} Y_L(\hat{r}) - \frac{1}{L+1} \frac{r}{r} \left(\frac{r}{R} \right)^{L+2} Y_L(\hat{r}) \right\} \\ & \left. + \mathcal{O}(kR)^4 + \dots \right], \end{aligned} \quad (29)$$

and:

$$\begin{aligned} \vec{A}_{(ML)} = & \frac{1}{\sqrt{L(L+1)}} \frac{(kR)^L}{(2L+1)!!} (\vec{r} \times \vec{\nabla}) \left(\frac{r}{R} \right)^L Y_L(\hat{r}) \left[1 \right. \\ & \left. - \frac{(kR)^2}{2(2L+3)} \left(\frac{r}{R} \right)^2 + \mathcal{O}(kR)^4 + \dots \right]. \end{aligned} \quad (30)$$

The leading terms in the above expansions give rise to the well-known "textbook" forms of the nuclear gamma-ray transition probabilities and matrix elements:^{32,33}

$$\mathcal{W}_\gamma \begin{pmatrix} EL \\ ML \end{pmatrix} = 8\pi Q \frac{L+1}{L} \frac{k^{2L+1}}{[(2L+1)!!]^2} B \begin{pmatrix} EL \\ ML \end{pmatrix}, \quad (31)$$

where:

$$B(EL) \approx \frac{2I_f+1}{2I_i+1} \left| \langle f || \frac{1}{e} Q r^L Y_L || i \rangle \right|^2, \quad (32)$$

$$B(ML) \approx \frac{2I_f+1}{2I_i+1} \left| \langle f || \frac{1}{L+1} \frac{1}{e} \vec{j}_n \cdot (\vec{r} \times \vec{\nabla}) r^L Y_L || i \rangle \right|^2.$$

The higher terms in (29) and (30) give rise to additional nuclear matrix elements in the expression for the rate of gamma-ray emission; but these terms are weighted by the factor of $(kR)^2$, and are negligible relative to the leading term when the leading matrix element itself is not inhibited. For very forbidden gamma-ray transitions the higher terms in the long-wavelength expansion may make significant contributions to the transition rates, since they need not be simultaneously inhibited. The nuclear-model-dependent selection rules which the leading and higher matrix elements obey have been discussed at length in the literature.^{1,2,26,33-38}

The various forms of the nuclear matrix elements which arise from the long-wavelength expansion of the electron penetration functions have the same form as the matrix elements arising in the expansion of the vector potentials indicated above. However, in the electron case these higher matrix elements are weighted much more importantly relative to the leading term than in the gamma-ray case; and it is this situation which is exploited in the study of penetration effects in internal conversion. For example, Emery and Perlman have recently used this correspondence to show that the penetration matrix elements necessary to explain the conversion anomalies in certain very highly hindered E1 transitions are sufficient to account for approximately 10% of the rate of the competing gamma-ray transitions.³⁸ In extreme cases the penetration terms may dominate in the conversion process.² Electric-monopole conversion, on the other hand, proceeds

entirely via penetration effects, since there is no corresponding gamma-ray matrix element.

The long-wavelength expansions of the electron penetration functions, $\bar{\phi}$, Θ , and Ψ , are given in Table I.9, (p. 31).² These results are expressed in terms of the radial electron wavefunctions, f and g , of the initial and final electron states. If the expansions of these wavefunctions, given in section I.1, are substituted into the results in the Table, the penetration functions may be expressed in the form of a series in the quantity $(r/R)^2$. The expansion forms the basis for the explicit expressions for the ICC's given in the following section.

I.5 Final Formulation and Approximations

Substituting the expansions of the electron wavefunctions I.1 into the long-wavelength expansions of the penetration functions given in Table I.9 allows the approximate ICC's to be written in particularly simple forms. For electric transitions:

$$\begin{aligned} \text{ICC}(S; \text{EL}) \approx \sum_{n_f} & |K_{fi}(\text{EL}) R_{fi}(\text{EL}) \\ & + i C_{j \cdot \nabla}^{(n)} \lambda_{j \cdot \nabla}^{(n)} \\ & + i C_{j \cdot r}^{(m)} \lambda_{j \cdot r}^{(m)}|^2, \end{aligned} \quad (33)$$

where:

$$\begin{aligned} \lambda_{j \cdot \nabla}^{(n)} = \frac{N}{1 + \sigma_{j \cdot \nabla}^{(n)} + \dots} & \left[\frac{\langle f | \vec{j}_n \cdot \vec{\nabla} \left(\frac{r}{R} \right)^n r^{L_{Y_L}} | i \rangle}{\langle f | \vec{j}_n \cdot \vec{A}(\text{EL}) | i \rangle} \right. \\ & \left. + \sigma_{j \cdot \nabla}^{(n)} \frac{\langle f | \vec{j}_n \cdot \vec{\nabla} \left(\frac{r}{R} \right)^{n+2} r^{L_{Y_L}} | i \rangle}{\langle f | \vec{j}_n \cdot \vec{A}(\text{EL}) | i \rangle} + \dots \right], \end{aligned} \quad (34)$$

and:

$$\begin{aligned} \lambda_{j \cdot r}^{(m)} = \frac{N}{1 + \sigma_{j \cdot r}^{(m)} + \dots} & \left[\frac{\langle f | \vec{j}_n \cdot \frac{\hat{r}}{r} \left(\frac{r}{R} \right)^m r^{L_{Y_L}} | i \rangle}{\langle f | \vec{j}_n \cdot \vec{A}(\text{EL}) | i \rangle} \right. \\ & \left. + \sigma_{j \cdot r}^{(m)} \frac{\langle f | \vec{j}_n \cdot \frac{\hat{r}}{r} \left(\frac{r}{R} \right)^{m+2} r^{L_{Y_L}} | i \rangle}{\langle f | \vec{j}_n \cdot \vec{A}(\text{EL}) | i \rangle} + \dots \right]. \end{aligned} \quad (35)$$

The corresponding expressions for magnetic transitions are:

$$\begin{aligned} ICC(S; ML) \approx \sum_{\kappa_f} & |K_{fi}(ML) R_{fi}(ML) \\ & + i C_{j \cdot A}^{(\ell)} \lambda_{j \cdot A}^{(\ell)}|^2, \end{aligned} \quad (36)$$

and:

$$\begin{aligned} \lambda_{j \cdot A}^{(\ell)} = \frac{1}{1 + \sigma_{j \cdot A}^{(\ell)} + \dots} & \left[\frac{\langle f | \vec{j}_n \cdot \vec{A}(ML) \left(\frac{r}{R} \right)^\ell | i \rangle}{\langle f | \vec{j}_n \cdot \vec{A}(ML) | i \rangle} \right. \\ & \left. + \sigma_{j \cdot A}^{(\ell)} \frac{\langle f | \vec{j}_n \cdot \vec{A}(ML) \left(\frac{r}{R} \right)^{\ell+2} | i \rangle}{\langle f | \vec{j}_n \cdot \vec{A}(ML) | i \rangle} + \dots \right]. \end{aligned} \quad (37)$$

These expressions are obvious generalizations of those defined in the literature.¹ Their structure is the following: S stands for the initial, the bound, electron state, and κ_f is the quantum number of the final electron states. K_{fi} and R_{fi} are the angular and radial integrals which comprise the leading term in the conversion process. The C_λ terms, which add to the leading terms, are the penetration contributions. The C's are penetration coefficients, which depend on the product of the bound and free electron wavefunctions at the nucleus, and the λ 's are essentially nuclear factors, involving the ratios of a series of penetration matrix elements to the corresponding gamma-ray matrix element. The coefficients σ , which weight the second successively higher-order term in the λ 's, are generally small, and only the first two terms in the series are usually retained.

There are two characteristically different λ 's for electric transitions and one for magnetic transitions; designated by the subscripts $j \cdot \nabla$, $j \cdot r$, and $j \cdot A$, respectively. The factor:

$$N = \frac{1}{k} \frac{k^L}{(2L+1)!!}, \quad (38)$$

is inserted into the numerators of the two electric λ 's so that they are expressed simply if the gamma-ray matrix element in their denominators is approximated by the first term in its long-wavelength expansion (cf. eq. (42),(43)).

The quantities λ describe the dynamic contributions of the finite nuclear size to the conversion process. To prepare a specific table of ICC's these penetration contributions must be specified in some reasonable way, although the choice is arbitrary within rather wide limits. There are two different conventions in use in the literature: Rose has chosen the "no-penetration" model,^{8,28} and Sliv-Band have chosen the "surface-current" model.⁹ These conventions correspond to the following values of λ :

Model	$\lambda_{j \cdot \nabla}$	$\lambda_{j \cdot r}$	$\lambda_{j \cdot A}$
"no-penetration"	0	0	0
"surface-current"	$\sqrt{\frac{L}{L+1}}$	0	.1

(39)

For $\lambda \sim 1$, it is found that the penetration contributions are at most a few per cent of the leading conversion terms in (33) and (36), KR, so that the difference between the ICC's calculated for these models should generally be small; although they may reach 10% or more in certain cases.

Explicit expressions for the quantities, K, C, σ , etc., are given in Tables I.10 - I.12 for E1 and E2 transitions, and in Tables I.13 and I.14 for M1 and M2 transitions (pp. 32-36).^{8,39} The results for E1 transitions include the effects of the well-known cancellations which occur in $C_{j \cdot \nabla}$ for certain

electron transitions.^{2,27} The results in these Tables are approximate in that small terms, of the order WR , have been deliberately omitted. For example, for the particularly important case of $M1$ conversion via spin $1/2 \rightarrow$ spin $1/2$ electron transitions, the first-order corrections to the tabulated values of $C_{j.A}$ are given by the factors:

$$1 + \frac{1}{3} \frac{R}{\alpha Z} (W_i + W_f \mp 2), \quad (40)$$

where the double sign refers to $\kappa = -1$ and $\kappa = +1$ electron transitions, respectively.

The electron radial matrix elements, R_{fi} , appearing in (33) and (36) have been tabulated by Rose for K-shell conversion in a Coulomb potential by a point nucleus.^{8,40} On the other hand, Sliv and Band have tabulated the quantities:

$$M_{fi}(EL) = K_{fi} R_{fi} + i \sqrt{\frac{L}{L+1}} C_{j.A}, \quad (41)$$

$$M_{fi}(ML) = K_{fi} R_{fi} + i C_{j.A},$$

for K- and L-shell conversion, including atomic screening and the effects of the finite nuclear size.³ The static effects of the finite nuclear size were taken into account by taking the nucleus to be a uniformly charged sphere (as done in the present discussion) and the dynamic effects of the finite nuclear size were included on the basis of the "surface-current" model, which gives rise to the two terms proportional to C appearing on the right of (41). Both the tabulated values of R and M are based on the phase convention of (A) given in (6).^{3,4,8,9,9A,41}

Because of the approximate nature of the nuclear models which are available to interpret the values of λ deduced from experiment, it may be convenient to simplify the expressions for λ given by (34), (35), and (37) by making two additional approximations: First, by replacing the gamma-ray matrix elements in the denominators by their long-wavelength limits, and second, by taking all penetrat...

matrix elements in the numerators in a given λ to be equal. This latter approximation is reasonable since the cases of practical interest are usually those for which the penetration matrix elements in the numerator are "allowed". Then:

$$\lambda_{j \cdot \nabla}^{(n)} \approx \sqrt{\frac{L}{L+1}} \frac{\langle f | | \vec{j}_n \cdot \vec{\nabla} \left(\frac{r}{R} \right)^n r^{L_{Y_L}} | | i \rangle}{\langle f | | \vec{j}_n \cdot \vec{\nabla} r^{L_{Y_L}} | | i \rangle} = \sqrt{\frac{L}{L+1}} \frac{\langle f | | Q \left(\frac{r}{R} \right)^n r^{L_{Y_L}} | | i \rangle}{\langle f | | Q r^{L_{Y_L}} | | i \rangle}, \quad (42)$$

$$\lambda_{j \cdot r}^{(m)} \approx \sqrt{\frac{L}{L+1}} \frac{\langle f | | \vec{j}_n \cdot \hat{r} \left(\frac{r}{R} \right)^m r^{L_{Y_L}} | | i \rangle}{\langle f | | \vec{j}_n \cdot \vec{\nabla} r^{L_{Y_L}} | | i \rangle} = \frac{i}{k} \sqrt{\frac{L}{L+1}} \frac{\langle f | | \vec{j}_n \cdot \hat{r} \left(\frac{r}{R} \right)^m r^{L_{Y_L}} | | i \rangle}{\langle f | | Q r^{L_{Y_L}} | | i \rangle}, \quad (43)$$

and:

$$\lambda_{j \cdot A}^{(\ell)} \approx \frac{\langle f | | \vec{j}_n \cdot (\vec{r} \times \vec{\nabla}) \left(\frac{r}{R} \right)^\ell r^{L_{Y_L}} | | i \rangle}{\langle f | | \vec{j}_n \cdot (\vec{r} \times \vec{\nabla}) r^{L_{Y_L}} | | i \rangle}. \quad (44)$$

The cases of greatest practical interest are those for which the exponents n , m , and ℓ have their minimum value of 2.

The expressions for the ICC's themselves may also be expressed in a variety of approximate forms. For M1 conversion, for example, (36) may be written:

$$ICC(M1)_\lambda \approx \left\{ |1 - (\lambda - 1)C'| \chi + [1 - \chi] \right\}^2 ICC(M1)_{\lambda=1}, \quad (45)$$

where $\lambda \equiv \lambda_{j \cdot A}^{(2)}$ and:

$$C' \equiv C_{j \cdot A}^{(2)} / i M_{j_f = \frac{1}{2}}, \quad (46)$$

$$\chi \equiv |M_{j_f = \frac{1}{2}}|^2 / ICC(M1).$$

The only approximation involved here is the neglect of the penetration contributions coming from electron transitions to states with $j_f > 1/2$. The above

expression is valid for the K shell and all three L shells. However, since $\chi \approx$ for the K, L_1 , and L_{11} shells, and $\chi \approx 0$ for L_{111} , (45) may be further approximated by the classical forms:

$$ICC(M1)_{\lambda} \approx [1 - (\lambda - 1)C_S]^2 ICC(M1)_{\lambda=1}, \quad (47)$$

for the K, L_1 , and L_{11} shells, and:

$$ICC(M1)_{\lambda} \approx ICC(M1)_{\lambda=1}, \quad (48)$$

for the L_{111} shell, where:

$$C_S \approx \chi C' \approx |C_{j.A}^{(2)}| / [ICC(M1)]^{1/2}. \quad (49)$$

The values of M and the ICC's appearing in (46) and (49) are those corresponding to the "surface-current" model, $\lambda_{j.A}=1$; although the expressions could easily be written in terms of the "no-penetration" model, $\lambda_{j.A}=0$.^{1,28}

In general, penetration effects for ML conversion are greatest for electron transitions from the state of spin j_i to the final state of spin $j_f = L - j_i$; and when present, such transitions dominate the conversion process. The particular recipes given for M1 transitions, therefore, may be easily generalized to higher magnetic transitions.

Approximations such as the above, and others, have appeared in the literature.^{1,26,42,43} For example, earlier studies frequently made use of the additional simplification, $\sigma = 0$. Approximations such as the above may be more convenient than the precise forms, and may represent them quite well in certain cases. However, the detailed analysis of accurate experimental data recommends comparison with the general forms, (33) and (36), in conjunction with the radial electron integrals which are now available.

Table I.1

Indicial terms of the Matumoto-Yamada functions for a uniformly charged sphere of radius R. In this and in the following tables, $\xi \equiv \alpha Z/2$ and $X \equiv r/R$.

$$J_{|\kappa|}(X) = 1 - \xi^2 \left[\frac{9}{2(2|\kappa|+1)} X^2 - \frac{3}{4} \left(\frac{1}{2|\kappa|+1} + \frac{1}{2|\kappa|+3} \right) X^4 \right. \\ \left. + \frac{1}{6(2|\kappa|+3)} X^6 \right] + \mathcal{O}(\xi^4) + \dots$$

$$\varphi_{|\kappa|}(X) = - \frac{\xi}{R} \left\{ \left[\frac{3}{2|\kappa|+1} - \frac{1}{2|\kappa|+3} X^2 \right] + \mathcal{O}(\xi^2) + \dots \right\}$$

$$K_{|\kappa|}^{(2)}(X) = - \xi R \left\{ \left[\frac{3}{2} X^2 - \frac{1}{4} X^4 \right] + \mathcal{O}(\xi^2) + \dots \right\}$$

$$K_{|\kappa|}^{(1)}(X) = 0 - \mathcal{O}(\xi^2) + \dots$$

Table I.2

Expansion of J_1 for a uniformly charged sphere.

$$\begin{aligned}
 J_1(x) = & 1 - \xi^2 \left[\frac{3}{2} x^2 - \frac{2}{5} x^4 + \frac{1}{30} x^6 \right] \\
 & + \xi^4 \left[\frac{27}{40} x^4 - \frac{12}{35} x^6 + \frac{29}{420} x^8 - \frac{16}{2475} x^{10} + \frac{1}{3960} x^{12} \right] \\
 & - \xi^6 \left[\frac{81}{560} x^6 - \frac{3}{28} x^8 + \frac{19}{560} x^{10} - \frac{881}{150150} x^{12} \right. \\
 & \quad \left. + \frac{74303}{126126000} x^{14} - \frac{83}{2524500} x^{16} + \frac{1}{1211760} x^{18} \right] \\
 & + \mathcal{O}(\xi^8) + \dots
 \end{aligned}$$

$$J_1(0) = 1$$

$$J_1(1) = 1 - \xi^2(1.133333) + \xi^4(0.394978) - \xi^6(0.066118) + \dots$$

Table I.3

Expansion of J_2 for a uniformly charged sphere.

$$\begin{aligned}
 J_2(X) = & 1 - \xi^2 \left[\frac{9}{10} X^2 - \frac{9}{35} X^4 + \frac{1}{42} X^6 \right] \\
 & + \xi^4 \left[\frac{81}{280} X^4 - \frac{11}{70} X^6 + \frac{53}{1540} X^8 - \frac{177}{50050} X^{10} + \frac{1}{6552} X^{12} \right] \\
 & - \xi^6 \left[\frac{27}{560} X^6 - \frac{117}{3080} X^8 + \frac{207}{16016} X^{10} - \frac{173}{71500} X^{12} \right. \\
 & \quad \left. + \frac{6701}{20420400} X^{14} - \frac{1047}{64664600} X^{16} + \frac{1}{2240784} X^{18} \right] \\
 & + \mathcal{O}(\xi^8) + \dots
 \end{aligned}$$

$$J_2(0) = 1$$

$$J_2(1) = 1 - \xi^2(0.666667) + \xi^4(0.163175) - \xi^6(0.021045) + \dots$$

Table I.4

Expansion of φ_1 for a uniformly charged sphere.

$$\begin{aligned}\varphi_1(X) = & -\frac{\xi}{R} \left\{ \left[1 - \frac{1}{5} X^2 \right] \right. \\ & - \xi^2 \left[\frac{9}{10} X^2 - \frac{27}{70} X^4 + \frac{1}{18} X^6 - \frac{1}{330} X^8 \right] \\ & + \xi^4 \left[\frac{81}{280} X^4 - \frac{53}{280} X^6 + \frac{1}{20} X^8 - \frac{681}{100100} X^{10} \right. \\ & \left. \left. + \frac{13}{27000} X^{12} - \frac{1}{67320} X^{14} \right] \right. \\ & \left. - \mathcal{O}(\xi^6) + \dots \right\}\end{aligned}$$

$$\varphi_1(0) = -\frac{\xi}{R} \{1\}$$

$$\varphi_1(1) = -\frac{\xi}{R} \left\{ \frac{4}{5} - \xi^2(0.566811) + \xi^4(0.143663) - \dots \right\}$$

Table I.5

Expansion of φ_2 for a uniformly charged sphere.

$$\begin{aligned} \varphi_2(X) = & - \frac{\xi}{R} \left\{ \left[\frac{3}{5} - \frac{1}{7} X^2 \right] \right. \\ & - \xi^2 \left[\frac{27}{70} X^2 - \frac{13}{70} X^4 + \frac{23}{770} X^6 - \frac{1}{546} X^8 \right] \\ & + \xi^4 \left[\frac{27}{280} X^4 - \frac{213}{3080} X^6 + \frac{401}{20020} X^8 \right. \\ & \left. \left. - \frac{4507}{1501500} X^{10} + \frac{2399}{10210200} X^{12} - \frac{1}{124488} X^{14} \right] \right. \\ & \left. - \mathcal{O}(\xi^6) + \dots \right\} \end{aligned}$$

$$\varphi_2(0) = - \frac{\xi}{R} \left\{ \frac{3}{5} \right\}$$

$$\varphi_2(1) = - \frac{\xi}{R} \left\{ 0.457143 - \xi^2(0.228039) + \xi^4(0.044528) - \dots \right\}$$

Table I.6

Expansions of $K_1^{(2)}$ and $K_1^{(1)}$ for a uniformly charged sphere.

$$K_1^{(2)}(X) = - \xi R \left\{ \left[\frac{3}{2} X^2 - \frac{1}{4} X^4 \right] - \xi^2 \left[\frac{27}{20} X^4 - \frac{153}{280} X^6 + \frac{8}{105} X^8 - \frac{7}{1800} X^{10} \right] + \mathcal{O}(\xi^4) + \dots \right\}$$

$$K_1^{(2)}(0) = 0$$

$$K_1^{(2)}(1) = - \xi R \left\{ \frac{5}{4} - \xi^2 (0.875873) + \dots \right\}$$

$$K_1^{(1)}(X) = - \xi^2 \left[\frac{3}{10} X^2 - \frac{13}{140} X^4 + \frac{1}{180} X^6 \right] + \mathcal{O}(\xi^4) + \dots$$

$$K_1^{(1)}(0) = 0$$

$$K_1^{(1)}(1) = - \xi^2 (0.212698) + \dots$$

Table I.7

Expansions of $K_2^{(2)}$ and $K_2^{(1)}$ for a uniformly charged sphere.

$$K_2^{(2)}(x) = -\xi R \left\{ \left[\frac{3}{2} x^2 - \frac{1}{4} x^4 \right] - \xi^2 \left[\frac{27}{28} x^4 - \frac{23}{56} x^6 + \frac{19}{308} x^8 - \frac{31}{9240} x^{10} \right] + \mathcal{O}(\xi^4) + \dots \right\}$$

$$K_2^{(2)}(0) = 0$$

$$K_2^{(2)}(1) = -\xi R \left\{ \frac{5}{4} - \xi^2(0.611905) + \dots \right\}$$

$$K_2^{(1)}(x) = -\xi^2 \left[\frac{27}{70} x^2 - \frac{19}{140} x^4 + \frac{3}{308} x^6 \right] + \mathcal{O}(\xi^4) + \dots$$

$$K_2^{(1)}(0) = 0$$

$$K_2^{(1)}(1) = -\xi^2(0.259740) + \dots$$

Table I.8

The quantities μ , Ω , and σ , for E0 conversion in the K and L shells. Static finite nuclear-size effects are included for the case of a uniformly charged sphere of radius R.

Shell	μ	Ω	σ
K	2	$\frac{\pi\alpha^2}{9} R^4 [g_{-1}(0)g_{-1}(0)]^2$	$+ \frac{3}{5} \left(\frac{\alpha Z}{2}\right)^2$
L ₁	2	$\frac{\pi\alpha^2}{9} R^4 [g_{-1}(0)g_{-1}(0)]^2$	$+ \frac{3}{5} \left(\frac{\alpha Z}{2}\right)^2$
L ₁₁	2	$\frac{\pi\alpha^2}{9} R^4 [f_{+1}(0)f_{+1}(0)]^2$	$+ \frac{3}{5} \left(\frac{\alpha Z}{2}\right)^2$
L ₁₁₁	4	$\frac{\pi\alpha^2}{50} R^8 [g_{-2}(0)g_{-2}(0)]^2$	$+ \frac{24}{35} \left(\frac{\alpha Z}{2}\right)^2$

Table I.9

Long-wavelength expansions of the electron penetration functions Φ , Θ , and Ψ .

$$\begin{aligned}\Phi(r) = & i \frac{\sqrt{L(L+1)}}{2L+1} \left(\frac{r}{k}\right)^2 \left\{ \int_0^r dr_1 \left[\frac{r_1^{L-2}}{r_1^{L-1}} - \frac{r_1^{L+2}}{r_1^{L+3}} \right] [f_f(r_1)f_i(r_1) + g_f(r_1)g_i(r_1)] \right. \\ & + (kr) \int_0^r dr_1 \left[\frac{1}{L} \frac{r_1^{L-3}}{r_1^{L-2}} + \frac{1}{L+1} \frac{r_1^{L+3}}{r_1^{L+4}} \right] [g_f(r_1)f_i(r_1) - f_f(r_1)g_i(r_1)] \\ & \left. + \mathcal{O}(kr)^2 + \dots \right\}.\end{aligned}$$

$$\begin{aligned}\Theta(r) = & - \frac{i}{\sqrt{L(L+1)}} \frac{r^3}{k} \left\{ [g_f(r)f_i(r) - f_f(r)g_i(r)] \right. \\ & - \frac{(kr)}{2L+1} \int_0^r dr_1 \left[(L+1) \frac{r_1^{L+2}}{r_1^{L+3}} + L \frac{r_1^{L-2}}{r_1^{L-1}} \right] [f_f(r_1)f_i(r_1) + g_f(r_1)g_i(r_1)] \\ & \left. + \mathcal{O}(kr)^2 + \dots \right\}.\end{aligned}$$

$$\begin{aligned}\Psi(r) = & i \frac{(2L-1)!!}{k^{L+1}} \left\{ \int_0^r dr_1 \left[\frac{1}{r_1^{L-1}} - \frac{r_1^{L+2}}{r_1^{2L+1}} \right] [g_f(r_1)f_i(r_1) + f_f(r_1)g_i(r_1)] \right. \\ & \left. + \mathcal{O}(kr)^2 + \dots \right\}.\end{aligned}$$

Table I.10

The factor K_{fi} for E1 and E2 transitions converting in the K and L shells.

EL	S	n_i	n_f	$K_{fi}(EL)$
E1	K, L_1	-1	+1	$\sqrt{\frac{1}{3}} \sqrt{\pi \alpha k}$
			-2	$\sqrt{\frac{2}{3}} \sqrt{\pi \alpha k}$
	L_{11}	+1	-1	$\sqrt{\frac{1}{3}} \sqrt{\pi \alpha k}$
			+2	$\sqrt{\frac{2}{3}} \sqrt{\pi \alpha k}$
	L_{111}	-2	-1	$\sqrt{\frac{2}{3}} \sqrt{\pi \alpha k}$
			+2	$\sqrt{\frac{2}{15}} \sqrt{\pi \alpha k}$
			-3	$\sqrt{\frac{6}{5}} \sqrt{\pi \alpha k}$

E2	K, L_1	-1	+2	$\sqrt{\frac{2}{15}} \sqrt{\pi \alpha k}$
			-3	$\sqrt{\frac{1}{5}} \sqrt{\pi \alpha k}$
	L_{11}	+1	-2	$\sqrt{\frac{2}{15}} \sqrt{\pi \alpha k}$
			+3	$\sqrt{\frac{1}{5}} \sqrt{\pi \alpha k}$
	L_{111}	-2	+1	$\sqrt{\frac{2}{15}} \sqrt{\pi \alpha k}$
			-2	$\sqrt{\frac{2}{15}} \sqrt{\pi \alpha k}$
			+3	$\sqrt{\frac{2}{35}} \sqrt{\pi \alpha k}$
			-4	$\sqrt{\frac{12}{35}} \sqrt{\pi \alpha k}$

Table I.11

The quantities n , $C_{j,\nabla}$ and $\sigma_{j,\nabla}$ for E1 and E2 transitions converting in the K and L shells. Static finite-nuclear size effects are included for the case of a uniformly charged sphere of radius R.

EL	S	n_f	n	$C_{j,\nabla}^{(n)} / (1 + \sigma_{j,\nabla}^{(n)} + \dots)$	$\sigma_{j,\nabla}^{(n)}$
E1	K, L_1	+1	2	$+ f_{+1}(0) g_{-1}(0) \sqrt{\frac{\pi Q}{k}} \frac{1}{10} \sqrt{\frac{2}{3}} (2-5k) R^2$	$- \frac{6}{7} \frac{2-7k}{2-5k} \left(\frac{QZ}{2} \right)^2$
		-2	2	$+ g_{-2}(0) g_{-1}(0) \sqrt{\frac{\pi Q}{k}} \frac{\sqrt{3}}{5} R^2$	$- \frac{9}{14} \left(\frac{QZ}{2} \right)^2$
	L_{11}	-1	2	$+ g_{-1}(0) f_{+1}(0) \sqrt{\frac{\pi Q}{k}} \frac{1}{10} \sqrt{\frac{2}{3}} (2+5k) R^2$	$- \frac{6}{7} \frac{2+7k}{2+5k} \left(\frac{QZ}{2} \right)^2$
		+2	2	$+ f_{+2}(0) f_{+1}(0) \sqrt{\frac{\pi Q}{k}} \frac{\sqrt{3}}{5} R^2$	$- \frac{9}{14} \left(\frac{QZ}{2} \right)^2$
	L_{111}	-1	2	$+ g_{-1}(0) g_{-2}(0) \sqrt{\frac{\pi Q}{k}} \frac{\sqrt{3}}{5} R^2$	$- \frac{9}{14} \left(\frac{QZ}{2} \right)^2$
		+2	4	$+ f_{+2}(0) g_{-2}(0) \sqrt{\frac{\pi Q}{k}} \frac{3}{35\sqrt{15}} (1-7k) R^4$	$- \frac{4}{5} \frac{1-9k}{1-7k} \left(\frac{QZ}{2} \right)^2$
		-3	4	$+ g_{-3}(0) g_{-2}(0) \sqrt{\frac{\pi Q}{k}} \frac{3}{14} \sqrt{\frac{3}{5}} R^4$	$- \frac{2}{3} \left(\frac{QZ}{2} \right)^2$
E2	K, L_1	+2	2	$- f_{+2}(0) g_{-1}(0) \sqrt{\frac{\pi Q}{k}} \frac{3}{7\sqrt{5}} QZ R^1$	$- \frac{1}{18} - \frac{1}{2} \left(\frac{QZ}{2} \right)^2$
		-3	2	$+ g_{-3}(0) g_{-1}(0) \sqrt{\frac{\pi Q}{k}} \frac{15}{14} \sqrt{\frac{6}{5}} R^2$	$- \frac{2}{3} \left(\frac{QZ}{2} \right)^2$
	L_{11}	-2	2	$+ g_{-2}(0) f_{+1}(0) \sqrt{\frac{\pi Q}{k}} \frac{3}{7\sqrt{5}} QZ R^1$	$- \frac{1}{18} - \frac{1}{2} \left(\frac{QZ}{2} \right)^2$
		+3	2	$+ f_{+3}(0) f_{+1}(0) \sqrt{\frac{\pi Q}{k}} \frac{15}{14} \sqrt{\frac{6}{5}} R^2$	$- \frac{2}{3} \left(\frac{QZ}{2} \right)^2$
	L_{111}	+1	2	$+ f_{+1}(0) g_{-2}(0) \sqrt{\frac{\pi Q}{k}} \frac{3}{7\sqrt{5}} QZ R^1$	$- \frac{1}{18} - \frac{1}{2} \left(\frac{QZ}{2} \right)^2$
		-2	2	$+ g_{-2}(0) g_{-2}(0) \sqrt{\frac{\pi Q}{k}} \frac{3}{7\sqrt{5}} R^2$	$- \frac{2}{3} \left(\frac{QZ}{2} \right)^2$
		+3	4	$- f_{+3}(0) g_{-2}(0) \sqrt{\frac{\pi Q}{k}} \frac{1}{14} \sqrt{\frac{3}{35}} QZ R^3$	$- \frac{10}{99} - \frac{6}{11} \left(\frac{QZ}{2} \right)^2$
		-4	4	$+ g_{-4}(0) g_{-2}(0) \sqrt{\frac{\pi Q}{k}} \frac{\sqrt{5}}{14} R^4$	$- \frac{36}{55} \left(\frac{QZ}{2} \right)^2$

Table I.12

The quantities m , $C_{j \cdot r}$, and $\sigma_{j \cdot r}$ for E1 and E2 transitions converting in the K and L shells. Static finite-nuclear-size effects are included for the case of a uniformly charged sphere of radius R .

EL	S	n_f	m	$C_{j \cdot r}^{(m)} / (1 + \sigma_{j \cdot r}^{(m)} + \dots)$	$\sigma_{j \cdot r}^{(m)}$
E1	K, L_1	+1	2	$+ f_{+1}(0) g_{-1}(0) \sqrt{\frac{\pi Q}{k}} \sqrt{\frac{3}{2}} R^2$	$-2 \left(\frac{\alpha Z}{2} \right)^2$
		-2	4	$+ g_{-2}(0) g_{-1}(0) \sqrt{\frac{\pi Q}{k}} \frac{\sqrt{3}}{5} \alpha Z R^3$	$-\frac{1}{7} - \frac{9}{7} \left(\frac{\alpha Z}{2} \right)^2$
	L_{11}	-1	2	$- g_{-1}(0) f_{+1}(0) \sqrt{\frac{\pi Q}{k}} \sqrt{\frac{3}{2}} R^2$	$-2 \left(\frac{\alpha Z}{2} \right)^2$
		+2	4	$+ f_{+2}(0) f_{+1}(0) \sqrt{\frac{\pi Q}{k}} \frac{\sqrt{3}}{5} \alpha Z R^3$	$-\frac{1}{7} - \frac{9}{7} \left(\frac{\alpha Z}{2} \right)^2$
	L_{111}	-1	4	$- g_{-1}(0) g_{-2}(0) \sqrt{\frac{\pi Q}{k}} \frac{\sqrt{3}}{5} \alpha Z R^3$	$-\frac{1}{7} - \frac{9}{7} \left(\frac{\alpha Z}{2} \right)^2$
		+2	4	$+ f_{+2}(0) g_{-2}(0) \sqrt{\frac{\pi Q}{k}} \sqrt{\frac{3}{5}} R^4$	$-\frac{36}{25} \left(\frac{\alpha Z}{2} \right)^2$
		-3	6	$+ g_{-3}(0) g_{-2}(0) \sqrt{\frac{\pi Q}{k}} \frac{9}{35} \sqrt{\frac{3}{5}} \alpha Z R^5$	$-\frac{5}{27} - \left(\frac{\alpha Z}{2} \right)^2$
E2	K, L_1	+2	2	$+ f_{+2}(0) g_{-1}(0) \sqrt{\frac{\pi Q}{k}} \frac{1}{3} \sqrt{5} R^2$	$-\frac{9}{5} \left(\frac{\alpha Z}{2} \right)^2$
		-3	4	$+ g_{-3}(0) g_{-1}(0) \sqrt{\frac{\pi Q}{k}} \frac{1}{7} \sqrt{30} \alpha Z R^3$	$-\frac{7}{45} - \frac{6}{5} \left(\frac{\alpha Z}{2} \right)^2$
	L_{11}	-2	2	$- g_{-2}(0) f_{+1}(0) \sqrt{\frac{\pi Q}{k}} \frac{1}{3} \sqrt{5} R^2$	$-\frac{9}{5} \left(\frac{\alpha Z}{2} \right)^2$
		+3	4	$+ f_{+3}(0) f_{+1}(0) \sqrt{\frac{\pi Q}{k}} \frac{1}{7} \sqrt{30} \alpha Z R^3$	$-\frac{7}{45} - \frac{6}{5} \left(\frac{\alpha Z}{2} \right)^2$
	L_{111}	+1	2	$+ f_{+1}(0) g_{-2}(0) \sqrt{\frac{\pi Q}{k}} \frac{1}{3} \sqrt{5} R^2$	$-\frac{9}{5} \left(\frac{\alpha Z}{2} \right)^2$
		-2	4	$- g_{-2}(0) g_{-2}(0) \sqrt{\frac{\pi Q}{k}} \frac{1}{12} \sqrt{5} k R^4$	$-\frac{108}{25} \left(\frac{\alpha Z}{2} \right)^2$
		+3	4	$+ f_{+3}(0) g_{-2}(0) \sqrt{\frac{\pi Q}{k}} \sqrt{\frac{15}{7}} R^4$	$-\frac{9}{7} \left(\frac{\alpha Z}{2} \right)^2$
		-4	6	$+ g_{-4}(0) g_{-2}(0) \sqrt{\frac{\pi Q}{k}} \frac{2}{35} \sqrt{\frac{2}{5}} \alpha Z R^5$	$-\frac{15}{77} - \frac{72}{77} \left(\frac{\alpha Z}{2} \right)^2$

Table I.13

The factor K_{fi} for M1 and M2 transitions converting in the K and L shells.

ML	S	κ_i	κ_f	$K_{fi}(ML)$
M1	K, L_1	-1	-1	$2\sqrt{\frac{1}{3}}\sqrt{\pi\alpha k}$
			+2	$\sqrt{\frac{2}{3}}\sqrt{\pi\alpha k}$
	L_{11}	+1	+1	$2\sqrt{\frac{1}{3}}\sqrt{\pi\alpha k}$
			-2	$\sqrt{\frac{2}{3}}\sqrt{\pi\alpha k}$
	L_{111}	-2	+1	$\sqrt{\frac{2}{3}}\sqrt{\pi\alpha k}$
			-2	$4\sqrt{\frac{2}{15}}\sqrt{\pi\alpha k}$
			+3	$\sqrt{\frac{6}{5}}\sqrt{\pi\alpha k}$

M2	$K_1 L_1$	-1	-2	$\sqrt{\frac{6}{5}}\sqrt{\pi\alpha k}$
			+3	$2\sqrt{\frac{1}{5}}\sqrt{\pi\alpha k}$
	L_{11}	+1	+2	$\sqrt{\frac{6}{5}}\sqrt{\pi\alpha k}$
			-3	$2\sqrt{\frac{1}{5}}\sqrt{\pi\alpha k}$
	L_{111}	-2	-1	$\sqrt{\frac{6}{5}}\sqrt{\pi\alpha k}$
			+2	0
			-3	$\sqrt{\frac{10}{7}}\sqrt{\pi\alpha k}$
			+4	$4\sqrt{\frac{3}{35}}\sqrt{\pi\alpha k}$

Table I.14

The quantities ℓ , $C_{j \cdot A}^{(\ell)}$, and $\sigma_{j \cdot A}^{(\ell)}$ for M1 and M2 transitions converting in the K and L shells. Static finite-nuclear-size effects are included for the case of a uniformly charged sphere of radius R.

ML	S	n_f	ℓ	$C_{j \cdot A}^{(\ell)} / (1 + \sigma_{j \cdot A}^{(\ell)} + \dots)$	$\sigma_{j \cdot A}^{(\ell)}$
M1	K, L_1	-1	2	$- g_{-1}(0) g_{-1}(0) \sqrt{\frac{\pi Q}{k^3}} \frac{\sqrt{3}}{5} \alpha Z R^1$	$-\frac{1}{14} - \frac{6}{7} \left(\frac{\alpha Z}{2} \right)^2$
		+2	2	$+ f_{+2}(0) g_{-1}(0) \sqrt{\frac{\pi Q}{k^3}} \frac{1}{5} \sqrt{\frac{3}{2}} R^2$	$-\frac{15}{14} \left(\frac{\alpha Z}{2} \right)^2$
	L_{11}	+1	2	$+ f_{+1}(0) f_{+1}(0) \sqrt{\frac{\pi Q}{k^3}} \frac{\sqrt{3}}{5} \alpha Z R^1$	$-\frac{1}{14} - \frac{6}{7} \left(\frac{\alpha Z}{2} \right)^2$
		-2	2	$+ g_{-2}(0) f_{+1}(0) \sqrt{\frac{\pi Q}{k^3}} \frac{1}{5} \sqrt{\frac{3}{2}} R^2$	$-\frac{15}{14} \left(\frac{\alpha Z}{2} \right)^2$
	L_{111}	+1	2	$+ f_{+1}(0) g_{-2}(0) \sqrt{\frac{\pi Q}{k^3}} \frac{1}{5} \sqrt{\frac{3}{2}} R^2$	$-\frac{15}{14} \left(\frac{\alpha Z}{2} \right)^2$
		-2	4	$- g_{-2}(0) g_{-2}(0) \sqrt{\frac{\pi Q}{k^3}} \frac{3}{35} \sqrt{\frac{6}{5}} \alpha Z R^3$	$-\frac{10}{81} - \frac{4}{5} \left(\frac{\alpha Z}{2} \right)^2$
		+3	4	$+ f_{+3}(0) g_{-2}(0) \sqrt{\frac{\pi Q}{k^3}} \frac{3}{28} \sqrt{\frac{6}{5}} R^4$	$-\frac{14}{15} \left(\frac{\alpha Z}{2} \right)^2$

M2	K, L_1	-2	2	$- g_{-2}(0) g_{-1}(0) \sqrt{\frac{\pi Q}{k^5}} \frac{6}{7} \sqrt{\frac{6}{5}} \alpha Z R^1$	$-\frac{1}{12} - \frac{3}{4} \left(\frac{\alpha Z}{2} \right)^2$
		+3	2	$+ f_{+3}(0) g_{-1}(0) \sqrt{\frac{\pi Q}{k^5}} \frac{3}{7} \sqrt{5} R^2$	$-\left(\frac{\alpha Z}{2} \right)^2$
	L_{11}	+2	2	$+ f_{+2}(0) f_{+1}(0) \sqrt{\frac{\pi Q}{k^5}} \frac{6}{7} \sqrt{\frac{6}{5}} \alpha Z R^1$	$-\frac{1}{12} - \frac{3}{4} \left(\frac{\alpha Z}{2} \right)^2$
		-3	2	$+ g_{-3}(0) f_{+1}(0) \sqrt{\frac{\pi Q}{k^5}} \frac{3}{7} \sqrt{5} R^2$	$-\left(\frac{\alpha Z}{2} \right)^2$
	L_{111}	-1	2	$- g_{-1}(0) g_{-2}(0) \sqrt{\frac{\pi Q}{k^5}} \frac{6}{7} \sqrt{\frac{6}{5}} \alpha Z R^1$	$-\frac{1}{12} - \frac{3}{4} \left(\frac{\alpha Z}{2} \right)^2$
		-3	4	$- g_{-3}(0) g_{-2}(0) \sqrt{\frac{\pi Q}{k^5}} \frac{3}{14} \sqrt{\frac{10}{7}} \alpha Z R^3$	$-\frac{40}{297} - \frac{8}{11} \left(\frac{\alpha Z}{2} \right)^2$
		+4	4	$+ f_{+4}(0) g_{-2}(0) \sqrt{\frac{\pi Q}{k^5}} \sqrt{\frac{5}{21}} R^4$	$-\frac{48}{55} \left(\frac{\alpha Z}{2} \right)^2$

PART II

II.1 Electron Wavefunctions at the Center of the Nucleus

Band, Guman, and Sogomonova^{4,5} have published tables of the bound and free (continuum) electron wavefunctions which appeared as intermediate steps in the calculation of the Sliv-Band tables of internal-conversion coefficients.^{3,9,16} These wavefunctions are calculated including the static effects of the finite-nuclear size by taking it to be a uniformly charged sphere, and the effects of atomic screening based on the Fermi-Thomas-Dirac statistical model. Details of these calculations are given in the literature.^{3,5,9,9A} The present tables of electron wavefunctions are derived from those of Band et al. in the following way:

- (1) The free function normalization is changed to the "energy" normalization, (a) in (8).
- (2) The values of the wavefunctions, which are given at the nuclear surface in the original reference, are transformed into the values at the center of the nucleus, using the expansions of the electron wavefunctions in a uniformly charged sphere inside the nucleus discussed in Section I.1; specifically, the first three terms of (12) plus Tables I.2-I.7.
- (3) The wavefunctions derived in this manner are interpolated to cover in detail the ranges of Z and k of practical interest.

The normalization change is introduced to facilitate comparison with earlier discussions and data in the literature.^{14,19-21,23,44,45} Values are tabulated at the origin rather than at the nuclear surface as a convenience in analysis, as illustrated in Part I. Interpolation is a necessary evil arising from the relatively small number of atomic numbers (and energies) for which data are given in the original reference.

The present tables give wavefunctions at $r = 0$ for four bound functions, the K, L_1 , L_{11} , and L_{111} shells, and for the four free (continuum) functions with $j = 1/2$ and $3/2$ ($\kappa = \pm 1$ and ± 2), since these are weighted most strongly at the nucleus.⁴⁶ Only the "large" components, $g(0)_{-\kappa}$ and $f(0)_{+\kappa}$, are tabulated since the expressions given in Part I are stated most simply in terms of these components. If desired, the values of the "small" components may be easily calculated from the large ones using (5) or (13).

Values of the bound-state eigenvalues, nuclear radii, etc., are also given, including the values of the atomic number, A , for which the original data were computed. Variations of A by up to ten percent are expected to change the tabulated electron wavefunctions by less than one percent.^{47,48}

Table II.1 gives the wavefunctions for each of the 16 values of Z and for each energy for which data are given in the original reference. These results form the basis for the interpolated results given in Table II.2. The interpolated results cover the practical range: $Z = 55(1)100$ for 7-9 nuclear transition energies between $k = \sim 0.05$ and 1.0, converting in both the K and L shells. Together, the tables contain approximately 4,300 wavefunction entries.

II.2 Accuracy and Interpolation

Band, Guman, and Sogomonova quote no measure of significance of their tabulated results; although their bound functions are generally given to four significant figures and the free functions to three figures. The Sliv-Band tables of internal-conversion-coefficients which are based on these wavefunctions, have an assigned significance of $\pm (1-2)\%$; and the discussion of these calculations in reference 9A indicates that the free-function normalization has been determined to within 0.5%. The number of terms used in the expansion of the electron wavefunctions inside the nucleus as given in Part I is sufficient to insure that the transformation from the surface to center is accurate to $\sim 0.1\%$. The data in Table II.1, therefore, have essentially the same accuracy as the data in the original reference.

The data in Table II.2 were obtained from those in Table II.1 by interpolation and extrapolation. The bound functions were fitted by least-squares polynomials of varying degrees, and the results calculated from these polynomials were smoothed by comparison with the corresponding calculations for a pure Coulomb potential. With two exceptions ($Z = 61$, L_{11} and L_{111}), the final interpolations agree with the input data to within 0.2%.

The free functions were also fitted by least-squares polynomials; first in energy and then in atomic number. The energy interpolations included all the input energies indicated in Table II.2, and the interpolation in atomic number included the 12 input values of Z between 57 and 98. The present listing of the free functions in terms of the energies of gamma rays converting in the K and L shells is purely a matter of convenience. In fact, they are each functions of a single variable: the kinetic energy of the free functions, T . In

the double asterisked cases free functions are given for the L_{11} and L_{111} shells as well as for the K and L_1 shells. But since the subshell differences are small, only the K and L_1 results are cross interpolated to intermediate values of Z.

The degrees of the polynomials used in interpolating the free functions were chosen individually and arbitrarily by taking the lowest degree which gave results generally agreeing with the input data to appreciably better than 1%. This procedure is believed to be more realistic than an exact fit through the input points, since it allows for random uncertainties in the input data, and the fact that they may not be truly represented by a simple polynomial. The interpolation uncertainty in the free functions is undoubtedly greater than that in the bound functions. The consistency of the free-function interpolation was observed by comparing the results obtained by fitting polynomials of varying degrees to a given set of input data. As expected, the "goodness" estimated in this way is very variable, and no meaningful overall estimate can be assigned. A figure of several percent, for example, would be an overestimate in most cases, and an underestimate in others. Values at the ends of the interpolation interval are particularly susceptible to uncertainties, and the few extrapolated values may only be representative.

Taken all in all, the data in Table II.2 are believed to be a reasonable representation of the data in the original reference, and to form a practical basis for the analysis of experimental data as illustrated in Part I of this report.

Table II.1

Bound and free electron wavefunctions at the center of the nucleus and other data, based on the work of Band, Guman, and Sogomonova.

Z	= atomic number.
A	= atomic weight deduced from the nuclear radius.
$1/R$	= reciprocal nuclear radius given in the original reference.
$1 - W_B$	= binding energies determined from the computed eigenvalues.
k	= nuclear transition energy. The corresponding kinetic energy of the free functions is: $T = k - (1 - W_B)_{L_1}$.
$\left. \begin{array}{l} g_{- n }(0) \\ f_{+ n }(0) \end{array} \right\}$	= values of the electron wavefunctions at the origin.

The units are: $m_e = \hbar = c = 1$.⁴⁷

$$Z = 25 \quad Mm$$

$$A = 55.55 \quad 1/R = 84.3313$$

BOUND

	$1 - W_B$	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
K	0.01277	0.1702			
L _I	0.00150	0.05191			
L _{II}	0.00128		0.003642		
L _{III}	0.00126			0.002262	

CONTINUUM

L _I	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
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CONTINUUM VALUES NOT GIVEN
IN ORIGINAL REFERENCE

$$Z = 33 \quad A_{\Delta}$$

$$A = 71.18 \quad V_R = 77.6390$$

BOUND

	$1 - W_B$	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
K	0.02329*	0.2784*			
L _I	0.00308	0.08884			
L _{II}	0.00273		0.008575		
L _{III}	0.00269			0.005088	

CONTINUUM

L _I	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.05008	1.23 ₁	0.226 ₈	0.142 ₇	0.0216 ₃
	0.10008	1.28 ₀	0.306 ₆	0.197 ₄	0.0430 ₁
	0.15008	1.34 ₇	0.376 ₁	0.250 ₀	0.0656 ₈
	0.20008	1.38 ₁	0.441 ₆	0.301 ₃	0.0901 ₇
	0.30008	1.48 ₇	0.552 ₀	0.400 ₂	0.144 ₅
	0.40008	1.59 ₅	0.674 ₂	0.504 ₈	0.205 ₆
	0.50008	1.65 ₈	0.781 ₁	0.609 ₇	0.268 ₄
	0.70008	1.89 ₁	0.985 ₂	0.834 ₇	0.424 ₅

* THESE TWO QUANTITIES ARE NOT GIVEN IN THE ORIGINAL REFERENCE, 4. THE VALUES INDICATED HAVE BEEN DERIVED FROM THE RESULTS GIVEN IN REFERENCE 6.

$$Z = 41 \quad N_b$$

$$A = 92.86 \quad 1/R = 71.0561$$

BOUND

	$1 - W_B$	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
K	0.03729	0.4214			
L _I	0.00536	0.1394			
L _{II}	0.00493		0.01723		
L _{III}	0.004742			0.009671	

CONTINUUM

L _I	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.01536	1.480	0.2232	0.1301	0.01037
	0.05036	1.495	0.2998	0.1821	0.02739
	0.10036	1.555	0.3919	0.2414	0.05196
	0.15036	1.610	0.4701	0.2998	0.07866
	0.20036	1.663	0.5410	0.3573	0.1071
	0.30036	1.791	0.6844	0.4711	0.1689
	0.40036	1.897	0.8117	0.5858	0.2390
	0.50036	2.009	0.9348	0.7038	0.3171
	0.70036	2.248	1.170	0.9572	0.4876
	1.00036	2.658	1.510	1.387	0.7984
	1.50036	3.101	2.044	2.215	1.451
	2.00036	3.561	2.568	3.189	2.260

$$Z = 49 \quad I_m$$

$$A = 114.70 \quad 1/R = 66.2248$$

BOUND

	$1 - W_B$	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
K	0.05498	0.6099			
L _I	0.0084177	0.2086			
L _{II}	0.00783		0.03163		
L _{III}	0.007455			0.01661	

CONTINUUM

L _I	* _k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.01842	1.79 ₁	0.323 ₂	0.177 ₆	0.0155 ₃
	0.05042	1.83 ₁	0.399 ₀	0.226 ₄	0.0341 ₈
	0.10042	1.89 ₈	0.505 ₂	0.296 ₂	0.0637 ₅
	0.15042	1.93 ₅	0.598 ₂	0.364 ₂	0.0946 ₉
	0.20042	2.00 ₃	0.681 ₆	0.427 ₂	0.128 ₄
	0.30042	2.13 ₉	0.842 ₈	0.554 ₂	0.200 ₁
	0.40042	2.26 ₀	1.00 ₅	0.685 ₅	0.278 ₁
	0.50042	2.38 ₁	1.13 ₇	0.821 ₂	0.366 ₀
	0.70042	2.62 ₈	1.40 ₆	1.11 ₀	0.566 ₇
	1.00042	2.98 ₇	1.79 ₇	1.59 ₁	0.914 ₁
	1.50042	3.58 ₄	2.40 ₉	2.52 ₁	1.65 ₈
	2.00042	4.13 ₈	3.00 ₅	3.62 ₆	2.58 ₈

* THE VALUE OF $1 - W_B$ FOR THE L_I SHELL HAS BEEN ROUNDED TO 0.00842 FOR THE CALCULATION OF THESE K VALUES.

$$Z = 57 \quad \text{La}$$

$$A = 138.84 \quad 1/R = 62.1392$$

BOUND

	$1 - W_B$	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
K	0.07665	0.8598			
L _I	0.0124	0.3036			
L _{II}	0.0117		0.05490		
L _{III}	0.0109			0.02663	

CONTINUUM

L_1	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.0224	2.20 ₄	0.462 ₃	0.239 ₁	0.0228 ₄
	0.0504	2.24 ₂	0.535 ₉	0.286 ₁	0.0429 ₀
	0.1004	2.30 ₅	0.657 ₉	0.363 ₂	0.0776 ₁
	0.1504	2.37 ₅	0.764 ₆	0.438 ₄	0.114 ₀
	0.2004	2.42 ₈	0.869 ₀	0.509 ₇	0.152 ₃
	0.3004	2.58 ₇	1.05 ₂	0.656 ₄	0.235 ₄
	0.4004	2.73 ₀	1.23 ₃	0.807 ₈	0.329 ₂
	0.5004	2.85 ₈	1.39 ₉	0.957 ₀	0.429 ₆
	0.7004	3.14 ₅	1.72 ₇	1.29 ₄	0.659 ₈
	1.0004	3.62 ₀	2.15 ₉	1.84 ₄	1.06 ₄
	1.5004	4.21 ₁	2.87 ₇	2.89 ₇	1.91 ₄
	2.0004	4.85 ₄	3.55 ₅	4.15 ₁	2.96 ₀

$$Z = 61 \quad P_m$$

$$A = 146.92 \quad 1/R = 60.9788$$

BOUND

	$1 - W_B$	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
K	0.08909	1.014			
L _I	0.01469	0.3644			
L _{II}	0.0140		0.07096		
L _{III}	0.0128			0.03299	

CONTINUUM

L _I	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
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CONTINUUM VALUES NOT GIVEN
IN ORIGINAL REFERENCE

$$Z = 65 \quad Tl$$

$$A = 159.10 \quad 1/R = 59.3800$$

BOUND

	$1 - W_B$	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
K	0.1027	1.193			
L _I	0.0173	0.4357			
L _{II}	0.0165		0.09215		
L _{III}	0.0150			0.04080	

CONTINUUM

L _I	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.0273	2.71 ₃	0.661 ₁	0.312 ₅	0.0330 ₂
	0.0503	2.78 ₁	0.728 ₃	0.358 ₂	0.0528 ₉
	0.1003	2.82 ₇	0.867 ₂	0.442 ₃	0.0948 ₈
	0.1503	2.90 ₄	0.992 ₀	0.528 ₁	0.138 ₁
	0.2003	3.01 ₇	1.11 ₇	0.612 ₅	0.182 ₉
	0.3003	3.14 ₄	1.33 ₉	0.776 ₇	0.281 ₃
	0.4003	3.32 ₄	1.54 ₃	0.948 ₈	0.389 ₉
	0.5003	3.47 ₅	1.74 ₂	1.13 ₂	0.507 ₇
	0.7003	3.79 ₁	2.11 ₀	1.53 ₀	0.770 ₄
	1.0003	4.25 ₃	2.64 ₇	2.13 ₁	1.23 ₅
	1.5003	5.00 ₉	3.47 ₃	3.35 ₂	2.20 ₂
	2.0003	5.75 ₂	4.25 ₃	4.80 ₂	3.43 ₆

$$Z = 69 \quad T_m$$

$$A = 169.31 \quad 1/R = 58.1630$$

BOUND

	$1 - W_B$	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
K	0.1174	1.401			
L _I	0.0202	0.5219			
L _{II}	0.0193		0.1183		
L _{III}	0.0174			0.04985	

CONTINUUM

L _I	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
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CONTINUUM VALUES NOT GIVEN

IN ORIGINAL REFERENCE

$$Z = 73 \quad Ta$$

$$A = 180.79 \quad 1/R = 56.9053$$

BOUND

	$1 - W_B$	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
K	0.1335	1.641			
L _I	0.0234	0.6208			
L _{II}	0.0224		0.1517		
L _{III}	0.0199			0.06057	

CONTINUUM

L _I	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.0334	3.36 ₆	0.935 ₁	0.409 ₈	0.0476 ₉
	0.0504	3.44 ₄	0.989 ₂	0.443 ₀	0.0650 ₃
	0.1004	3.51 ₂	1.15 ₉	0.543 ₉	0.115 ₉
	0.1504	3.58 ₇	1.31 ₉	0.638 ₉	0.168 ₀
	0.2004	3.70 ₃	1.45 ₀	0.733 ₈	0.220 ₇
	0.3004	3.84 ₂	1.71 ₁	0.929 ₃	0.335 ₃
	0.4004	4.07 ₆	1.96 ₄	1.13 ₄	0.460 ₇
	0.5004	4.26 ₆	2.21 ₄	1.34 ₁	0.601 ₀
	0.7004	4.61 ₄	2.63 ₇	1.77 ₃	0.912 ₄
	1.0004	5.15 ₂	3.27 ₁	2.48 ₁	1.44 ₉
	1.5004	6.02 ₀	4.23 ₄	3.92 ₆	2.57 ₀
	2.0004	6.84 ₉	5.20 ₃	5.59 ₄	3.95 ₂

$$Z = 77 \quad I_n$$

$$A = 193.00 \quad 1/R = 55.6785$$

BOUND

	$1 - W_B$	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
K	0.1509	1.922			
L _I	0.0259	0.7426			
L _{II}	0.0248		0.1939		
L _{III}	0.0216			0.07325	

CONTINUUM

L _I	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
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CONTINUUM VALUES NOT GIVEN

IN ORIGINAL REFERENCE

$$Z = 81 \quad Tl$$

$$A = 204.19 \quad 1/R = 54.6427$$

BOUND

	$1 - W_B$	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
K	0.1697	2.250			
L _I	0.0308	0.8839			
L _{II}	0.0297		0.2473		
L _{III}	0.0256			0.08774	

CONTINUUM

L _I	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.0408	4.23 ₉	1.33 ₃	0.534 ₆	0.0679 ₄
	0.0498	4.27 ₀	1.38 ₄	0.553 ₆	0.0801 ₃
	0.0998	4.37 ₄	1.58 ₂	0.668 ₁	0.142 ₀
	0.1498	4.46 ₆	1.74 ₉	0.779 ₃	0.204 ₀
	0.1998	4.60 ₂	1.93 ₆	0.893 ₁	0.267 ₈
	0.2998	4.82 ₃	2.24 ₁	1.11 ₅	0.405 ₈
	0.3998	5.03 ₃	2.53 ₇	1.34 ₇	0.556 ₀
	0.4998	5.26 ₇	2.82 ₅	1.58 ₉	0.710 ₇
	0.6998	5.67 ₀	3.36 ₁	2.09 ₆	1.07 ₄
	0.9998	6.31 ₀	4.09 ₀	2.91 ₆	1.71 ₄
	1.4998	7.28 ₀	5.22 ₂	4.54 ₅	3.01 ₂
	1.9998	8.27 ₂	6.27 ₅	6.45 ₅	4.60 ₈

$$Z = 84 \quad P_0$$

$$A = 213.88 \quad 1/R = 53.8042$$

BOUND

	$1 - W_B$	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
K	0.1849	2.531			
L _I	0.0340	1.009			
L _{II}	0.0329		0.2967		
L _{III}	0.0279			0.1003	

CONTINUUM

L _I	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.0440	4.63 ₁	1.52 ₇	0.589 ₄	0.0780 ₄
	0.0500	4.63 ₈	1.55 ₄	0.602 ₃	0.0859 ₇
	0.1000	4.77 ₃	1.76 ₃	0.721 ₅	0.152 ₁
	0.1500	4.88 ₉	1.96 ₀	0.840 ₉	0.218 ₈
	0.2000	5.01 ₈	2.13 ₄	0.956 ₃	0.286 ₆
	0.3000	5.26 ₁	2.48 ₁	1.18 ₈	0.433 ₆
	0.4000	5.48 ₂	2.80 ₀	1.43 ₇	0.589 ₃
	0.5000	5.69 ₃	3.10 ₂	1.69 ₇	0.761 ₆
	0.7000	6.17 ₁	3.69 ₁	2.21 ₆	1.14 ₆
	1.0000	6.80 ₀	4.44 ₅	3.10 ₅	1.83 ₃
	1.5000	7.82 ₃	5.71 ₀	4.81 ₈	3.20 ₆
	2.0000	8.87 ₄	6.77 ₇	6.84 ₁	4.89 ₆

$$Z=88 \quad Ra$$

$$A=225.93 \quad 1/R=52.8302$$

BOUND

	$1-W_B$	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
K	0.2066	2.963			
L _I	0.0387	1.206			
L _{II}	0.0375		0.3792		
L _{III}	0.0312			0.1195	

CONTINUUM

L _I	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.0487	5.215	1.83 ₂	0.668 ₈	0.0920 ₅
	0.0497	5.235	1.88 ₃	0.675 ₆	0.0946 ₁
	0.0997	5.35 ₃	2.08 ₁	0.797 ₂	0.167 ₇
	0.1497	5.48 ₄	2.30 ₂	0.927 ₂	0.241 ₈
	0.1997	5.66 ₄	2.47 ₆	1.05 ₄	0.317 ₉
	0.2997	5.89 ₅	2.84 ₂	1.29 ₈	0.470 ₈
	0.3997	6.12 ₉	3.20 ₁	1.57 ₅	0.644 ₅
	0.4997	6.37 ₂	3.54 ₆	1.84 ₂	0.831 ₁
	0.6997	6.85 ₇	4.17 ₄	2.42 ₅	1.25 ₁
	0.9997	7.56 ₉	5.03 ₇	3.37 ₈	1.97 ₆
	1.4997	8.70 ₀	6.33 ₁	5.23 ₁	3.45 ₃
	1.9997	9.89 ₆	7.56 ₅	7.41 ₄	5.28 ₆

$$Z = 92 \quad U$$

$$A = 237.76 \quad 1/R = 51.9391$$

BOUND

	$1 - W_B$	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
K	0.2301	3.473			
L _I	0.0438	1.445			
L _{II}	0.0425		0.4835		
L _{III}	0.0348			0.1419	

CONTINUUM

L _I	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.0498	5.986	2.213	0.7560	0.1049
	0.0998	6.024	2.438	0.8947	0.1859
	0.1498	6.165	2.661	1.026	0.2631
	0.1998	6.402	2.870	1.162	0.3477
	0.2998	6.611	3.310	1.434	0.5167
	0.3998	6.897	3.692	1.726	0.7039
	0.4998	7.115	4.045	2.009	0.9142
	0.6998	7.677	4.752	2.638	1.367
	0.9998	8.459	5.686	3.674	2.150
	1.4998	9.611	7.174	5.668	3.760
	1.9998	10.69	8.405	8.014	5.736

$$Z = 95 \quad A_m$$

$$A = 246.87 \quad 1/R = 51.2926$$

BOUND

	$1 - W_B$	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
K	0.2490	3.916			
L _I	0.0480	1.655			
L _{II}	0.0467		0.5816		
L _{III}	0.0375			0.1612	

CONTINUUM

L _I	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
*	0.0510	6.61 ₂	2.56 ₄	0.840 ₃	0.114 ₆
	0.0580	6.48 ₄	2.53 ₈	0.841 ₄	0.124 ₂
	0.1000	6.66 ₆	2.78 ₄	0.969 ₅	0.199 ₅
	0.1500	6.73 ₀	3.02 ₁	1.10 ₆	0.284 ₁
	0.2000	6.91 ₅	3.27 ₉	1.24 ₅	0.378 ₅
	0.3000	7.22 ₈	3.73 ₁	1.53 ₅	0.555 ₇
	0.4000	7.53 ₈	4.12 ₈	1.84 ₄	0.756 ₆
	0.5000	7.78 ₉	4.49 ₁	2.14 ₅	0.973 ₃
	0.7000	8.37 ₉	5.22 ₆	2.81 ₉	1.46 ₃
	1.0000	9.10 ₄	6.23 ₄	3.91 ₁	2.29 ₄
	1.5000	10.34	7.73 ₂	5.99 ₃	4.01 ₀
	2.0000	11.5 ₇	9.11 ₅	8.49 ₂	6.12 ₀

* A k OF 0.0500 MIGHT BE EXPECTED HERE, IMPLYING A TYPOGRAPHICAL ERROR IN THE ORIGINAL REFERENCE. IN THAT CASE, THIS ROW WOULD READ:

0.0500 7.31₅ 2.83₆ 0.929₆ 0.126₈

$$Z = 98 \quad Cf$$

$$A = 255.86 \quad 1/R = 50.6845$$

BOUND

	$1 - W_B$	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
K	0.2691	4.418			
L _I	0.0526	1.893			
L _{II}	0.0514		0.7079		
L _{III}	0.0405			0.1842	

CONTINUUM

L _I	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.0621	7.13 ₄	2.97 ₃	0.922 ₈	0.141 ₁
	0.0626	7.09 ₄	2.93 ₆	0.925 ₂	0.141 ₉
	0.0996	7.28 ₄	3.18 ₆	1.04 ₉	0.214 ₉
	0.1496	7.40 ₈	3.43 ₃	1.19 ₇	0.306 ₄
	0.1996	7.57 ₆	3.68 ₀	1.36 ₁	0.403 ₁
	0.2996	7.87 ₉	4.20 ₃	1.66 ₂	0.595 ₇
	0.3996	8.18 ₇	4.61 ₁	1.97 ₆	0.815 ₈
	0.4996	8.51 ₅	4.99 ₂	2.27 ₃	1.04 ₉
	0.6996	9.09 ₈	5.78 ₆	3.03 ₄	1.56 ₁
	0.9996	9.96 ₈	6.86 ₅	4.16 ₂	2.45 ₉
	1.4996	11.3 ₃	8.47 ₀	6.39 ₅	4.27 ₂
	1.9996	12.4 ₇	9.92 ₈	9.01 ₃	6.50 ₀

Table II.2

Bound and free electron wavefunctions at the center of the nucleus, interpolated and extrapolated from the data in Table II.1.

Z	= atomic number. Asterisks indicate listings in Table II.1.*
A	= atomic weight. These values are chosen to interpolate between the values given in Table II.1.
1/R	= reciprocal nuclear radius calculated from the atomic weight, A.
1 - γ_1	= the K-shell Coulomb binding energy: $\gamma_1 \equiv \sqrt{1 - (\alpha Z)^2}$. ⁴⁷
1 - W_B	= the <u>empirical</u> binding energies. ⁴⁹
k	= nuclear transition energy. The corresponding kinetic energy of the free functions is: $T = k - (1 - W_B)$.

$\left. \begin{array}{l} g(0)_{-|n|} \\ f(0)_{+|n|} \end{array} \right\}$ = values of the electron wavefunctions at the origin.

* A double asterisk indicates that both bound and free functions are given in the original reference; a single asterisk indicates that only bound functions are listed.

The units are: $m_e = \hbar = c = 1$.⁴⁷

$$Z = 55 \quad C.A.$$

$$A = 133$$

$$1/R = 63.04$$

$$1 - \gamma_1 = 0.084075$$

BOUND

	$1 - W_B$	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
K	0.07042	0.7897			
L_1	0.01118	0.2770			
L_{II}	0.01049		0.04799		
L_{III}	0.00981			0.02376	

CONTINUUM

K	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.15	2.18	0.589	0.330	0.0663
	0.20	2.24	0.703	0.404	0.103
	0.25	2.29	0.797	0.475	0.139
	0.30	2.37	0.877	0.545	0.176
	0.35				
	0.40				
	0.50	2.66	1.21	0.828	0.354
	0.70	2.90	1.54	1.15	0.570
	1.00	3.40	2.00	1.66	0.906
L_1	0.05	2.10	0.499	0.270	0.0407
	0.10	2.18	0.596	0.344	0.0729
	0.15	2.25	0.705	0.417	0.109
	0.20	2.32	0.803	0.487	0.146
	0.30	2.47	0.982	0.626	0.223
	0.40	2.60	1.15	0.770	0.312
	0.50	2.72	1.31	0.922	0.416
	0.70	3.00	1.64	1.24	0.632
	1.00	3.48	2.05	1.78	1.02

$$Z = 56 \quad Ba$$

$$A = 136$$

$$1/R = 62.57$$

$$1-\gamma_1 = 0.087307$$

BOUND

	$1-W_B$	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
K	0.07327	0.8242			
L_1	0.01172	0.2901			
L_{II}	0.01101		0.05133		
L_{III}	0.01027			0.02516	

CONTINUUM

K	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.15	2.23	0.604	0.335	0.0664
	0.20	2.29	0.718	0.410	0.103
	0.25	2.36	0.818	0.482	0.140
	0.30	2.43	0.905	0.553	0.179
	0.35				
	0.40				
	0.50	2.72	1.25	0.842	0.359
	0.70	2.98	1.58	1.16	0.578
	1.00	3.46	2.04	1.69	0.919
L_1	0.05	2.17	0.518	0.278	0.0418
	0.10	2.24	0.622	0.353	0.0749
	0.15	2.31	0.733	0.427	0.112
	0.20	2.38	0.834	0.498	0.150
	0.30	2.53	1.02	0.640	0.228
	0.40	2.66	1.19	0.787	0.319
	0.50	2.79	1.35	0.941	0.425
	0.70	3.07	1.68	1.27	0.645
	1.00	3.54	2.10	1.81	1.04

$$** \quad \bar{z} = 57 \quad La$$

$$A = 139$$

$$1/R = 62.12$$

$$1 - \gamma_1 = 0.090609$$

BOUND

	$1 - W_B$	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
K	0.07618	0.8597			
L ₁	0.01226	0.3038			
L _{II}	0.01153		0.05486		
L _{III}	0.01073			0.02663	

CONTINUUM

K	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.15	2.28	0.621	0.341	0.0665
	0.20	2.35	0.734	0.416	0.104
	0.25	2.42	0.839	0.490	0.142
	0.30	2.49	0.938	0.562	0.181
	0.35				
	0.40				
	0.50	2.77	1.29	0.859	0.363
	0.70	3.05	1.62	1.18	0.588
	1.00	3.50	2.09	1.72	0.933
L ₁	0.05	2.24	0.537	0.286	0.0429
	0.10	2.30	0.654	0.363	0.0771
	0.15	2.37	0.765	0.438	0.115
	0.20	2.44	0.869	0.511	0.153
	0.30	2.58	1.06	0.656	0.234
	0.40	2.72	1.23	0.805	0.327
	0.50	2.86	1.40	0.960	0.434
	0.70	3.14	1.73	1.29	0.658
	1.00	3.62	2.16	1.84	1.06

* * $z = 57$ La

CONTINUUM

L_{II}	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.05	2.24	0.538	0.287	0.0431
	0.10	2.30	0.655	0.363	0.0773
	0.15	2.37	0.766	0.438	0.115
	0.20	2.44	0.870	0.512	0.153
	0.30	2.58	1.06	0.657	0.234
	0.40	2.72	1.23	0.805	0.327
	0.50	2.86	1.40	0.961	0.435
	0.70	3.14	1.73	1.29	0.658
	1.00	3.62	2.16	1.84	1.06
L_{III}	0.05	2.24	0.540	0.288	0.0436
	0.10	2.30	0.657	0.365	0.0779
	0.15	2.37	0.768	0.440	0.116
	0.20	2.44	0.871	0.513	0.154
	0.30	2.58	1.06	0.658	0.235
	0.40	2.73	1.23	0.806	0.328
	0.50	2.87	1.40	0.962	0.435
	0.70	3.14	1.73	1.29	0.659
	1.00	3.62	2.16	1.85	1.07

$$Z = 58 \quad \text{Ce}$$

$$A = 141$$

$$1/R = 61.82$$

$$1 - \gamma_1 = 0.093983$$

BOUND

	$1 - W_B$	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
K	0.07915	0.8965			
L_1	0.01282	0.3180			
L_{II}	0.01207		0.05862		
L_{III}	0.01120			0.02814	

CONTINUUM

K	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.15	2.34	0.637	0.346	0.0665
	0.20	2.40	0.751	0.422	0.104
	0.25	2.48	0.861	0.497	0.143
	0.30	2.55	0.960	0.571	0.183
	0.35				
	0.40				
	0.50	2.84	1.32	0.872	0.368
	0.70	3.12	1.66	1.20	0.587
	1.00	3.58	2.13	1.75	0.946
L_1	0.05	2.31	0.559	0.295	0.0441
	0.10	2.37	0.676	0.372	0.0789
	0.15	2.43	0.789	0.448	0.117
	0.20	2.50	0.895	0.522	0.157
	0.30	2.65	1.09	0.669	0.239
	0.40	2.79	1.26	0.821	0.334
	0.50	2.93	1.43	0.980	0.443
	0.70	3.22	1.77	1.32	0.670
	1.00	3.69	2.21	1.88	1.08

$$Z = 59 \quad Pr$$

$$A = 143$$

$$1/R = 61.53$$

$$1 - \gamma_1 = 0.097427$$

BOUND

	$1 - W_B$	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
K	0.08218	0.9344			
L _I	0.01338	0.3328			
L _{II}	0.01261		0.06262		
L _{III}	0.01167			0.02973	

CONTINUUM

K	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.15	2.40	0.655	0.351	0.0664
	0.20	2.46	0.769	0.428	0.104
	0.25	2.54	0.884	0.505	0.144
	0.30	2.61	0.988	0.580	0.185
	0.35				
	0.40				
	0.50	2.90	1.36	0.886	0.373
	0.70	3.19	1.70	1.22	0.606
	1.00	3.65	2.18	1.78	0.960
L _I	0.05	2.38	0.580	0.303	0.0453
	0.10	2.43	0.703	0.382	0.0810
	0.15	2.50	0.818	0.459	0.120
	0.20	2.57	0.926	0.534	0.160
	0.30	2.72	1.12	0.684	0.244
	0.40	2.86	1.30	0.838	0.341
	0.50	3.01	1.48	1.00	0.452
	0.70	3.29	1.81	1.35	0.684
	1.00	3.76	2.27	1.91	1.10

$$Z = 60 \quad Nd$$

$$A = 145$$

$$1/R = 61.25$$

$$1 - \gamma_1 = 0.100945$$

BOUND

	$1 - W_B$	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
K	0.08527	0.9737			
L _I	0.01395	0.3483			
L _{II}	0.01316		0.06686		
L _{III}	0.01215			0.03139	

CONTINUUM

K	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.15	2.46	0.672	0.356	0.0662
	0.20	2.52	0.788	0.435	0.105
	0.25	2.60	0.906	0.512	0.146
	0.30	2.68	1.02	0.589	0.187
	0.35				
	0.40				
	0.50	2.97	1.39	0.901	0.378
	0.70	3.27	1.74	1.24	0.615
	1.00	3.72	2.22	1.81	0.975
L _I	0.05	2.44	0.602	0.312	0.0465
	0.10	2.49	0.731	0.391	0.0831
	0.15	2.56	0.847	0.470	0.123
	0.20	2.63	0.958	0.547	0.164
	0.30	2.78	1.16	0.700	0.250
	0.40	2.93	1.34	0.856	0.348
	0.50	3.08	1.52	1.02	0.462
	0.70	3.37	1.86	1.38	0.698
	1.00	3.84	2.33	1.94	1.12

$$* \quad Z = 61 \quad P_m$$

$$A = 147$$

$$1/R = 60.97$$

$$1 - \gamma_1 = 0.104535$$

BOUND

	$1 - W_B$	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
K	0.08843	1.014			
L ₁	0.01454	0.3645			
L ₁₁	0.01372		0.07135		
L ₁₁₁	0.01264			0.03313	

CONTINUUM

K	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.15	2.52	0.691	0.362	0.0660
	0.20	2.59	0.809	0.441	0.105
	0.25	2.67	0.930	0.520	0.147
	0.30	2.74	1.04	0.598	0.189
	0.35				
	0.40				
	0.50	3.04	1.43	0.916	0.383
	0.70	3.34	1.78	1.26	0.624
	1.00	3.79	2.27	1.84	0.989
L ₁	0.05	2.51	0.625	0.320	0.0478
	0.10	2.56	0.758	0.401	0.0853
	0.15	2.63	0.877	0.481	0.126
	0.20	2.70	0.990	0.560	0.168
	0.30	2.85	1.20	0.715	0.256
	0.40	3.00	1.38	0.874	0.356
	0.50	3.16	1.56	1.04	0.471
	0.70	3.45	1.91	1.41	0.712
	1.00	3.92	2.39	1.98	1.14

$$Z = 62 \quad S_m$$

$$A = 150$$

$$1/R = 60.56$$

$$1 - \gamma_1 = 0.108200$$

BOUND

	$1 - W_B$	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
K	0.09166	1.057			
L _I	0.01514	0.3812			
L _{II}	0.01431		0.07611		
L _{III}	0.01315			0.03493	

CONTINUUM

K	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.15	2.59	0.709	0.367	0.0656
	0.20	2.65	0.830	0.447	0.105
	0.25	2.73	0.954	0.528	0.148
	0.30	2.81	1.07	0.607	0.191
	0.35				
	0.40				
	0.50	3.11	1.47	0.932	0.388
	0.70	3.42	1.82	1.28	0.633
	1.00	3.87	2.32	1.87	1.00
L _I	0.05	2.58	0.649	0.329	0.0490
	0.10	2.63	0.787	0.412	0.0875
	0.15	2.69	0.908	0.492	0.129
	0.20	2.76	1.02	0.572	0.172
	0.30	2.92	1.23	0.731	0.262
	0.40	3.08	1.42	0.893	0.364
	0.50	3.24	1.60	1.06	0.481
	0.70	3.53	1.96	1.44	0.726
	1.00	4.00	2.45	2.01	1.17

$$Z = 63 \quad Eu$$

$$A = 153$$

$$1/R = 60.16$$

$$1 - \gamma_1 = 0.111939$$

BOUND

	$1 - W_B$	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
K	0.09495	1.100			
L _I	0.01576	0.3986			
L _{II}	0.01491		0.08114		
L _{III}	0.01365			0.03681	

CONTINUUM

K	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.15	2.65	0.729	0.372	0.0652
	0.20	2.72	0.852	0.454	0.106
	0.25	2.80	0.979	0.536	0.150
	0.30	2.88	1.10	0.617	0.194
	0.35				
	0.40				
	0.50	3.18	1.50	0.947	0.393
	0.70	3.50	1.87	1.31	0.643
	1.00	3.95	2.38	1.90	1.02
L _I	0.05	2.65	0.673	0.338	0.0504
	0.10	2.70	0.816	0.422	0.0897
	0.15	2.76	0.939	0.504	0.132
	0.20	2.84	1.06	0.586	0.176
	0.30	2.99	1.27	0.747	0.268
	0.40	3.16	1.47	0.912	0.372
	0.50	3.32	1.65	1.08	0.491
	0.70	3.62	2.01	1.46	0.740
	1.00	4.09	2.52	2.05	1.19

$$Z = 64 \quad Gd$$

$$A = 156$$

$$1/R = 59.77$$

$$1 - \eta_1 = 0.115755$$

BOUND

	$1 - W_B$	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
K	0.09832	1.146			
L _I	0.01639	0.4168			
L _{II}	0.01552		0.08647		
L _{III}	0.01417			0.03876	

CONTINUUM

K	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.15	2.72	0.748	0.377	0.0646
	0.20	2.78	0.876	0.460	0.106
	0.25	2.87	1.00	0.544	0.151
	0.30	2.94	1.13	0.626	0.196
	0.35				
	0.40				
	0.50	3.25	1.54	0.963	0.398
	0.70	3.58	1.92	1.33	0.652
	1.00	4.03	2.43	1.93	1.04
L _I	0.05	2.72	0.699	0.348	0.0517
	0.10	2.77	0.846	0.433	0.0920
	0.15	2.84	0.971	0.516	0.135
	0.20	2.91	1.09	0.599	0.180
	0.30	3.07	1.31	0.764	0.274
	0.40	3.23	1.51	0.932	0.380
	0.50	3.40	1.70	1.11	0.502
	0.70	3.70	2.06	1.49	0.755
	1.00	4.18	2.58	2.09	1.21

$$** \frac{1}{2} = 65 \quad T\&$$

$$A = 159$$

$$1/R = 59.39$$

$$1 - \gamma_1 = 0.119648$$

BOUND

	$1 - W_B$	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
K	0.10176	1.193			
L _I	0.01704	0.4358			
L _{II}	0.01615		0.09211		
L _{III}	0.01471			0.04080	

CONTINUUM

K	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.15	2.79	0.768	0.382	0.0640
	0.20	2.85	0.902	0.467	0.106
	0.25	2.93	1.03	0.552	0.152
	0.30	3.02	1.15	0.635	0.198
	0.35				
	0.40				
	0.50	3.34	1.57	0.974	0.405
	0.70	3.65	1.95	1.35	0.661
	1.00	4.14	2.50	1.98	1.05
	0.05	2.78	0.729	0.358	0.0531
	0.10	2.83	0.865	0.443	0.0938
	0.15	2.91	0.994	0.528	0.139
	0.20	2.99	1.12	0.612	0.184
	0.30	3.16	1.34	0.778	0.280
	0.40	3.32	1.55	0.949	0.388
	0.50	3.47	1.74	1.13	0.511
	0.70	3.79	2.11	1.53	0.769
	1.00	4.25	2.65	2.13	1.24

** $z=65$ T&

CONTINUUM

L_{II}	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.05	2.78	0.731	0.359	0.0535
	0.10	2.83	0.866	0.444	0.0943
	0.15	2.91	0.995	0.528	0.139
	0.20	3.00	1.12	0.612	0.185
	0.30	3.16	1.34	0.779	0.280
	0.40	3.32	1.55	0.950	0.388
	0.50	3.47	1.74	1.13	0.512
	0.70	3.79	2.11	1.53	0.770
	1.00	4.25	2.65	2.13	1.24

L_{III}	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.05	2.78	0.735	0.361	0.0546
	0.10	2.84	0.870	0.446	0.0956
	0.15	2.91	0.999	0.531	0.140
	0.20	3.00	1.12	0.615	0.186
	0.30	3.17	1.34	0.782	0.282
	0.40	3.32	1.55	0.953	0.390
	0.50	3.47	1.74	1.13	0.514
	0.70	3.80	2.12	1.53	0.771
	1.00	4.25	2.65	2.13	1.24

$$Z = 66 \quad Dy$$

$$A = 161$$

$$1/R = 59.15$$

$$1 - \gamma_1 = 0.123619$$

BOUND

	$1 - W_B$	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
K	0.10527	1.242			
L_1	0.01771	0.4557			
L_{11}	0.01679		0.09815		
L_{111}	0.01525			0.04294	

CONTINUUM

K	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.15	2.86	0.789	0.387	0.0633
	0.20	2.93	0.925	0.474	0.106
	0.25	3.01	1.06	0.560	0.153
	0.30	3.09	1.19	0.646	0.200
	0.35				
	0.40				
	0.50	3.41	1.62	0.994	0.409
	0.70	3.75	2.01	1.37	0.672
	1.00	4.21	2.55	1.99	1.07
L_1	0.05	2.86	0.754	0.367	0.0545
	0.10	2.91	0.907	0.455	0.0967
	0.15	2.99	1.04	0.541	0.142
	0.20	3.06	1.16	0.627	0.189
	0.30	3.22	1.39	0.798	0.286
	0.40	3.40	1.60	0.972	0.397
	0.50	3.57	1.80	1.16	0.523
	0.70	3.88	2.17	1.55	0.786
	1.00	4.36	2.72	2.17	1.26

$$Z = 67 \quad H_0$$

$$A = 165$$

$$1/R = 58.67$$

$$1 - \gamma_i = 0.127669$$

BOUND

	$1 - W_B$	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
K	0.10885	1.293			
L_1	0.01839	0.4765			
L_{II}	0.01745		0.1045		
L_{III}	0.01580			0.04517	

CONTINUUM

K	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.15	2.94	0.810	0.392	0.0625
	0.20	3.00	0.952	0.481	0.106
	0.25	3.08	1.09	0.568	0.154
	0.30	3.16	1.22	0.655	0.202
	0.35				
	0.40				
	0.50	3.49	1.66	1.01	0.414
	0.70	3.84	2.06	1.39	0.682
	1.00	4.30	2.61	2.02	1.08
L_1	0.05	2.94	0.783	0.377	0.0559
	0.10	2.99	0.940	0.466	0.0991
	0.15	3.06	1.07	0.554	0.146
	0.20	3.14	1.20	0.642	0.193
	0.30	3.31	1.43	0.816	0.293
	0.40	3.48	1.65	0.993	0.405
	0.50	3.66	1.85	1.18	0.534
	0.70	3.98	2.23	1.58	0.802
	1.00	4.46	2.79	2.21	1.29

$$Z = 68 \quad E_n$$

$$A = 167$$

$$1/R = 58.43$$

$$1 - \gamma_1 = 0.131799$$

BOUND

	$1 - W_B$	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
K	0.11250	1.346			
L_1	0.01908	0.4983			
L_{II}	0.01813		0.1113		
L_{III}	0.01636			0.04750	

CONTINUUM

K	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.15	3.02	0.832	0.397	0.0616
	0.20	3.08	0.979	0.487	0.106
	0.25	3.16	1.12	0.577	0.154
	0.30	3.24	1.25	0.665	0.204
	0.35				
	0.40				
	0.50	3.58	1.71	1.03	0.419
	0.70	3.93	2.12	1.42	0.692
	1.00	4.40	2.67	2.05	1.10
L_1	0.05	3.01	0.814	0.388	0.0574
	0.10	3.07	0.973	0.478	0.102
	0.15	3.15	1.11	0.568	0.149
	0.20	3.22	1.24	0.657	0.198
	0.30	3.39	1.48	0.834	0.299
	0.40	3.57	1.70	1.01	0.414
	0.50	3.75	1.90	1.20	0.545
	0.70	4.08	2.30	1.62	0.819
	1.00	4.57	2.86	2.25	1.31

$$* Z = 69 \quad T_m$$

$$A = 169$$

$$1/R = 58.20$$

$$1 - \gamma_1 = 0.136011$$

BOUND

	$1 - W_B$	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
K	0.11623	1.400			
L ₁	0.01980	0.5210			
L _{II}	0.01882		0.1185		
L _{III}	0.01692			0.04991	

CONTINUUM

K	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.15	3.09	0.854	0.402	0.0607
	0.20	3.16	1.01	0.494	0.106
	0.25	3.24	1.15	0.585	0.155
	0.30	3.32	1.29	0.675	0.205
	0.35				
	0.40				
	0.50	3.66	1.75	1.04	0.425
	0.70	4.02	2.17	1.44	0.702
	1.00	4.50	2.74	2.09	1.12
L ₁	0.05	3.09	0.846	0.398	0.0589
	0.10	3.15	1.01	0.490	0.104
	0.15	3.23	1.15	0.581	0.153
	0.20	3.31	1.28	0.672	0.203
	0.30	3.48	1.52	0.852	0.306
	0.40	3.66	1.74	1.04	0.423
	0.50	3.84	1.96	1.23	0.556
	0.70	4.18	2.36	1.65	0.836
	1.00	4.67	2.94	2.29	1.34

$$Z = 70 \quad Yb$$

$$A = 172$$

$$1/R = 57.86$$

$$1 - \gamma_1 = 0.140305$$

BOUND

	$1 - W_B$	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
K	0.12003	1.457			
L _I	0.02053	0.5447			
L _{II}	0.01953		0.1260		
L _{III}	0.01750			0.05242	

CONTINUUM

K	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.15	3.18	0.877	0.407	0.0596
	0.20	3.24	1.04	0.501	0.106
	0.25	3.32	1.18	0.594	0.156
	0.30	3.40	1.32	0.685	0.207
	0.35				
	0.40				
	0.50	3.75	1.80	1.06	0.430
	0.70	4.11	2.23	1.46	0.712
	1.00	4.61	2.81	2.12	1.14
L _I	0.05	3.17	0.879	0.409	0.0605
	0.10	3.23	1.04	0.503	0.107
	0.15	3.31	1.18	0.595	0.157
	0.20	3.40	1.32	0.687	0.208
	0.30	3.58	1.57	0.871	0.313
	0.40	3.76	1.80	1.06	0.432
	0.50	3.94	2.02	1.26	0.568
	0.70	4.28	2.43	1.68	0.853
	1.00	4.78	3.02	2.34	1.37

$$Z = 71 \quad Lu$$

$$A = 175$$

$$1/R = 57.53$$

$$1-\gamma_1 = 0.144683$$

BOUND

	$1-W_B$	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
K	0.12391	1.516			
L ₁	0.02127	0.5694			
L _{II}	0.02025		0.1341		
L _{III}	0.01809			0.05503	

CONTINUUM

K	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.15	3.26	0.900	0.412	0.0585
	0.20	3.32	1.07	0.508	0.105
	0.25	3.40	1.21	0.602	0.156
	0.30	3.49	1.35	0.695	0.209
	0.35				
	0.40				
	0.50	3.85	1.84	1.08	0.436
	0.70	4.21	2.28	1.49	0.722
	1.00	4.71	2.88	2.15	1.16
L ₁	0.05	3.25	0.915	0.420	0.0621
	0.10	3.32	1.08	0.516	0.109
	0.15	3.40	1.23	0.610	0.160
	0.20	3.49	1.36	0.703	0.212
	0.30	3.67	1.62	0.891	0.320
	0.40	3.86	1.85	1.08	0.441
	0.50	4.04	2.08	1.28	0.580
	0.70	4.39	2.50	1.71	0.870
	1.00	4.90	3.10	2.38	1.40

$$Z = 72 \quad Hf$$

$$A = 178$$

$$1/R = 57.20$$

$$1 - \gamma_1 = 0.149146$$

BOUND

	$1 - W_B$	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
K	0.12789	1.578			
L _I	0.02206	0.5952			
L _{II}	0.02102		0.1426		
L _{III}	0.01871			0.05775	

CONTINUUM

K	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.15	3.34	0.924	0.417	0.0572
	0.20	3.41	1.10	0.515	0.105
	0.25	3.48	1.24	0.611	0.157
	0.30	3.57	1.39	0.706	0.210
	0.35				
	0.40				
	0.50	3.94	1.89	1.09	0.441
	0.70	4.31	2.34	1.51	0.732
	1.00	4.83	2.95	2.19	1.17
L _I	0.05	3.34	0.952	0.432	0.0637
	0.10	3.41	1.12	0.529	0.112
	0.15	3.50	1.27	0.624	0.164
	0.20	3.58	1.41	0.720	0.218
	0.30	3.77	1.66	0.910	0.327
	0.40	3.96	1.91	1.10	0.451
	0.50	4.14	2.14	1.31	0.592
	0.70	4.50	2.57	1.75	0.889
	1.00	5.02	3.18	2.43	1.42

$\ast \ast \eta = 73 \quad Ta$

$A = 181$

$1/R = 56.88$

$1-\gamma_1 = 0.153695$

BOUND

	$1-W_B$	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
K	0.13193	1.641			
L_1	0.02286	0.6221			
L_{II}	0.02179		0.1517		
L_{III}	0.01934			0.06058	

CONTINUUM

K	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.15	3.43	0.948	0.422	0.0559
	0.20	3.50	1.13	0.522	0.104
	0.25	3.57	1.28	0.619	0.157
	0.30	3.66	1.42	0.715	0.211
	0.35				
	0.40				
	0.50	4.04	1.94	1.11	0.445
	0.70	4.42	2.42	1.54	0.744
	1.00	4.95	2.99	2.19	1.19
L_1	0.05	3.44	0.988	0.443	0.0652
	0.10	3.51	1.16	0.543	0.115
	0.15	3.59	1.31	0.639	0.169
	0.20	3.68	1.45	0.735	0.222
	0.30	3.87	1.71	0.929	0.333
	0.40	4.07	1.96	1.13	0.459
	0.50	4.26	2.21	1.34	0.605
	0.70	4.62	2.64	1.77	0.911
	1.00	5.15	3.27	2.48	1.45

* * $Z = 73$ Ta

CONTINUUM

L_{II}	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.05	3.45	0.990	0.444	0.0657
	0.10	3.51	1.16	0.544	0.116
	0.15	3.59	1.32	0.641	0.169
	0.20	3.68	1.45	0.736	0.223
	0.30	3.87	1.71	0.930	0.334
	0.40	4.07	1.97	1.13	0.460
	0.50	4.26	2.21	1.35	0.606
	0.70	4.62	2.64	1.77	0.912
	1.00	5.15	3.27	2.48	1.45
L_{III}	0.05	3.45	1.00	0.450	0.0680
	0.10	3.52	1.17	0.549	0.118
	0.15	3.60	1.32	0.645	0.172
	0.20	3.69	1.46	0.741	0.226
	0.30	3.88	1.72	0.935	0.337
	0.40	4.07	1.97	1.14	0.463
	0.50	4.26	2.22	1.35	0.610
	0.70	4.62	2.64	1.78	0.915
	1.00	5.16	3.28	2.49	1.46

$$Z = 74 \quad W$$

$$A = 184$$

$$1/R = 56.57$$

$$1 - \gamma_1 = 0.158333$$

BOUND

	$1 - W_B$	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
K	0.13606	1.708			
L_1	0.02368	0.6502			
L_{II}	0.02259		0.1613		
L_{III}	0.01997			0.06352	

CONTINUUM

K	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.15				
	0.20	3.59	1.16	0.529	0.104
	0.25	3.66	1.32	0.628	0.157
	0.30	3.76	1.47	0.726	0.213
	0.35				
	0.40				
	0.50	4.15	1.99	1.13	0.452
	0.70	4.52	2.47	1.56	0.752
	1.00	5.06	3.11	2.26	1.21
L_1	0.05	3.52	1.03	0.456	0.0671
	0.10	3.60	1.20	0.556	0.118
	0.15	3.69	1.36	0.655	0.173
	0.20	3.78	1.50	0.754	0.228
	0.30	3.98	1.77	0.952	0.342
	0.40	4.17	2.02	1.15	0.471
	0.50	4.36	2.27	1.37	0.617
	0.70	4.73	2.72	1.82	0.926
	1.00	5.27	3.36	2.53	1.48

$$z = 75 \quad \text{Re}$$

$$A = 187$$

$$1/R = 56.27$$

$$1 - \gamma_1 = 0.163059$$

BOUND

	$1 - W_B$	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
K	0.14027	1.776			
L_1	0.02451	0.6794			
L_{II}	0.02340		0.1715		
L_{III}	0.02062			0.06659	

CONTINUUM

K	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.15				
	0.20	3.68	1.20	0.536	0.103
	0.25	3.75	1.35	0.637	0.157
	0.30	3.85	1.51	0.737	0.214
	0.35				
	0.40				
	0.50	4.25	2.04	1.14	0.458
	0.70	4.63	2.54	1.58	0.763
	1.00	5.19	3.19	2.29	1.23
L_1	0.05	3.61	1.07	0.469	0.0688
	0.10	3.70	1.25	0.570	0.120
	0.15	3.79	1.41	0.671	0.177
	0.20	3.89	1.55	0.772	0.234
	0.30	4.09	1.83	0.973	0.350
	0.40	4.28	2.09	1.18	0.481
	0.50	4.48	2.34	1.40	0.630
	0.70	4.85	2.80	1.85	0.946
	1.00	5.40	3.45	2.58	1.51

$$Z = 76 \quad O_{\Delta}$$

$$A = 190$$

$$1/R = 55.97$$

$$1 - \gamma_1 = 0.167877$$

BOUND

	$1 - W_B$	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
K	0.14457	1.848			
L_1	0.02537	0.7100			
L_{II}	0.02424		0.1823		
L_{III}	0.02127			0.06979	

CONTINUUM

K	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.15				
	0.20	3.77	1.23	0.544	0.102
	0.25	3.85	1.39	0.646	0.157
	0.30	3.95	1.55	0.748	0.216
	0.35				
	0.40				
	0.50	4.36	2.10	1.16	0.463
	0.70	4.75	2.60	1.61	0.773
	1.00	5.32	3.27	2.33	1.25
L_1	0.05	3.71	1.12	0.482	0.0706
	0.10	3.80	1.30	0.585	0.123
	0.15	3.90	1.46	0.688	0.181
	0.20	4.06	1.61	0.790	0.239
	0.30	4.20	1.89	0.995	0.358
	0.40	4.40	2.15	1.20	0.492
	0.50	4.59	2.41	1.42	0.644
	0.70	4.98	2.88	1.89	0.966
	1.00	5.54	3.55	2.63	1.54

$$* \quad Z = 77 \quad \text{Ir}$$

$$A = 193$$

$$1/R = 55.68$$

$$1-\gamma_1 = 0.172787$$

BOUND

	$1-W_B$	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
K	0.14895	1.922			
L _I	0.02627	0.7418			
L _{II}	0.02510		0.1938		
L _{III}	0.02195			0.07311	

CONTINUUM

K	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.15				
	0.20	3.87	1.27	0.551	0.100
	0.25	3.95	1.44	0.655	0.157
	0.30	4.06	1.59	0.758	0.216
	0.35				
	0.40				
	0.50	4.47	2.16	1.18	0.469
	0.70	4.87	2.68	1.63	0.784
	1.00	5.45	3.36	2.36	1.27
L _I	0.05	3.81	1.17	0.496	0.0724
	0.10	3.90	1.34	0.600	0.126
	0.15	4.01	1.51	0.705	0.186
	0.20	4.11	1.66	0.808	0.245
	0.30	4.32	1.95	1.02	0.366
	0.40	4.52	2.22	1.23	0.502
	0.50	4.72	2.49	1.46	0.658
	0.70	5.11	2.97	1.93	0.986
	1.00	5.68	3.65	2.69	1.58

$$Z = 78 \quad Pt$$

$$A = 196$$

$$1/R = 55.39$$

$$1-\gamma_1 = 0.177791$$

BOUND

	$1-W_B$	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
K	0.15342	1.999			
L _i	0.02719	0.7750			
L _{ii}	0.02598		0.2060		
L _{iii}	0.02263			0.07658	

CONTINUUM

K	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.15				
	0.20	3.97	1.31	0.558	0.0989
	0.25	4.06	1.48	0.664	0.156
	0.30	4.16	1.64	0.769	0.217
	0.35				
	0.40				
	0.50	4.59	2.22	1.20	0.474
	0.70	4.99	2.75	1.66	0.794
	1.00	5.59	3.45	2.40	1.29
L _i	0.05	3.91	1.22	0.510	0.0743
	0.10	4.01	1.40	0.616	0.130
	0.15	4.12	1.56	0.722	0.190
	0.20	4.23	1.72	0.828	0.251
	0.30	4.44	2.01	1.04	0.375
	0.40	4.64	2.29	1.26	0.513
	0.50	4.84	2.56	1.49	0.672
	0.70	5.24	3.06	1.97	1.01
	1.00	5.82	3.75	2.74	1.61

$Z = 79$ Au

$A = 199$

$1/R = 55.11$

$1-\gamma_1 = 0.182890$

BOUND

	$1-W_B$	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
K	0.15798	2.079			
L_1	0.02809	0.8098			
L_{II}	0.02688		0.2189		
L_{III}	0.02332			0.08018	

CONTINUUM

K	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.15				
	0.20	4.08	1.35	0.565	0.0972
	0.25	4.16	1.52	0.673	0.156
	0.30	4.27	1.68	0.780	0.218
	0.35				
	0.40				
	0.50	4.71	2.28	1.22	0.480
	0.70	5.11	2.82	1.68	0.805
	1.00	5.73	3.55	2.44	1.31
L_1	0.05	4.02	1.27	0.524	0.0762
	0.10	4.13	1.45	0.632	0.133
	0.15	4.24	1.62	0.740	0.195
	0.20	4.35	1.78	0.847	0.257
	0.30	4.57	2.03	1.06	0.384
	0.40	4.77	2.37	1.29	0.525
	0.50	4.97	2.65	1.52	0.686
	0.70	5.38	3.16	2.01	1.03
	1.00	5.97	3.86	2.80	1.64

$$Z = 80 \quad \text{Hg}$$

$$A = 202$$

$$1/R = 54.84$$

$$1-\gamma_1 = 0.188088$$

BOUND

	$1-W_B$	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
K	0.16263	2.163			
L_1	0.02905	0.8461			
L_{II}	0.02781		0.2326		
L_{III}	0.02404			0.08392	

CONTINUUM

K	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.15				
	0.20	4.18	1.38	0.572	0.0953
	0.25	4.27	1.57	0.682	0.155
	0.30	4.39	1.73	0.791	0.218
	0.35				
	0.40				
	0.50	4.84	2.34	1.23	0.485
	0.70	5.24	2.90	1.71	0.815
	1.00	5.88	3.64	2.48	1.34
L_1	0.05	4.14	1.32	0.539	0.0781
	0.10	4.25	1.51	0.649	0.136
	0.15	4.36	1.68	0.758	0.200
	0.20	4.47	1.85	0.868	0.263
	0.30	4.70	2.15	1.09	0.392
	0.40	4.90	2.45	1.32	0.536
	0.50	5.11	2.73	1.55	0.701
	0.70	5.53	3.25	2.05	1.05
	1.00	6.13	3.97	2.86	1.68

$$** z = 81 \quad Tl$$

$$A = 204$$

$$1/R = 54.66$$

$$1 - \eta = 0.193385$$

BOUND

	$1 - W_B$	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
K	0.16738	2.249			
L _i	0.03003	0.8840			
L _{ii}	0.02876		0.2474		
L _{iii}	0.02477			0.08781	

CONTINUUM

K	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.15				
	0.20	4.29	1.43	0.580	0.0931
	0.25	4.39	1.62	0.692	0.155
	0.30	4.50	1.80	0.805	0.220
	0.35	4.62	1.97	0.918	0.285
	0.40				
	0.50	4.96	2.43	1.26	0.492
	0.70	5.39	2.99	1.73	0.825
	1.00	6.00	3.77	2.53	1.36
L _i	0.05	4.27	1.38	0.554	0.0805
	0.10	4.37	1.58	0.667	0.140
	0.15	4.48	1.76	0.780	0.205
	0.20	4.59	1.93	0.892	0.270
	0.30	4.82	2.24	1.12	0.404
	0.40	5.04	2.54	1.35	0.551
	0.50	5.26	2.82	1.58	0.717
	0.70	5.67	3.36	2.09	1.07
	1.00	6.31	4.09	2.91	1.71

* * | $z = 81$ T2

CONTINUUM

L_{II}	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.05	4.27	1.39	0.557	0.0819
	0.10	4.37	1.58	0.670	0.142
	0.15	4.48	1.76	0.782	0.207
	0.20	4.59	1.93	0.895	0.272
	0.30	4.82	2.25	1.12	0.406
	0.40	5.05	2.54	1.35	0.553
	0.50	5.26	2.82	1.59	0.719
	0.70	5.67	3.37	2.10	1.07
	1.00	6.31	4.09	2.92	1.72
L_{III}	0.05	4.28	1.41	0.566	0.0865
	0.10	4.38	1.60	0.679	0.147
	0.15	4.49	1.78	0.792	0.212
	0.20	4.60	1.95	0.904	0.278
	0.30	4.83	2.26	1.13	0.412
	0.40	5.06	2.55	1.36	0.560
	0.50	5.27	2.83	1.60	0.727
	0.70	5.68	3.38	2.11	1.08
	1.00	6.32	4.10	2.93	1.73

$Z = 82$ Pb

$A = 208$

$1/R = 54.31$

$1 - \gamma_1 = 0.198783$

BOUND

	$1 - W_B$	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
K	0.17223	2.340			
L _I	0.03104	0.9238			
L _{II}	0.02974		0.2628		
L _{III}	0.02551			0.09184	

CONTINUUM

K	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.15				
	0.20	4.40	1.47	0.587	0.0908
	0.25	4.51	1.66	0.700	0.153
	0.30	4.62	1.84	0.813	0.219
	0.35	4.75	2.02	0.930	0.286
	0.40				
	0.50	5.10	2.48	1.27	0.495
	0.70	5.51	3.07	1.76	0.837
	1.00	6.19	3.85	2.56	1.38
L _I	0.05	4.38	1.44	0.570	0.0822
	0.10	4.50	1.63	0.683	0.143
	0.15	4.61	1.81	0.797	0.210
	0.20	4.74	1.98	0.910	0.276
	0.30	4.97	2.30	1.14	0.411
	0.40	5.18	2.61	1.38	0.560
	0.50	5.39	2.91	1.62	0.732
	0.70	5.83	3.46	2.13	1.10
	1.00	6.46	4.20	2.98	1.75

$Z = 83$ Bi

$A = 211$

$1/R = 54.05$

$1-\gamma_1 = 0.204285$

BOUND

	$1-W_B$	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
K	0.17716	2.433			
L_1	0.03207	0.9655			
L_{II}	0.03074		0.2793		
L_{III}	0.02626			0.09604	

CONTINUUM

K	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.15				
	0.20	4.52	1.51	0.594	0.0881
	0.25	4.63	1.72	0.709	0.151
	0.30	4.75	1.89	0.824	0.219
	0.35	4.87	2.08	0.942	0.288
	0.40				
	0.50	5.23	2.55	1.29	0.500
	0.70	5.66	3.16	1.79	0.847
	1.00	6.35	3.96	2.60	1.40
L_1	0.05	4.51	1.50	0.586	0.0842
	0.10	4.63	1.69	0.702	0.147
	0.15	4.75	1.88	0.817	0.215
	0.20	4.87	2.06	0.932	0.283
	0.30	5.11	2.38	1.16	0.420
	0.40	5.33	2.70	1.40	0.573
	0.50	5.54	3.00	1.66	0.748
	0.70	5.99	3.57	2.18	1.12
	1.00	6.63	4.33	3.04	1.78

$$** \quad z = 84 \quad P_0$$

$$A = 214$$

$$1/R = 53.79$$

$$1 - \gamma_1 = 0.209893$$

BOUND

	$1 - W_B$	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
K	0.18221	2.531			
L _I	0.03313	1.009			
L _{II}	0.03178		0.2968		
L _{III}	0.02702			0.1004	

CONTINUUM

K	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.15				
	0.20	4.63	1.55	0.601	0.0854
	0.25	4.77	1.76	0.718	0.149
	0.30	4.89	1.95	0.836	0.218
	0.35	5.01	2.14	0.954	0.288
	0.40				
	0.50	5.36	2.64	1.31	0.506
	0.70	5.82	3.25	1.81	0.857
	1.00	6.49	4.10	2.65	1.43
L _I	0.05	4.64	1.56	0.603	0.0864
	0.10	4.77	1.76	0.720	0.150
	0.15	4.90	1.96	0.838	0.219
	0.20	5.02	2.14	0.957	0.289
	0.30	5.25	2.48	1.19	0.431
	0.40	5.48	2.80	1.44	0.587
	0.50	5.71	3.10	1.69	0.764
	0.70	6.16	3.69	2.22	1.15
	1.00	6.80	4.44	3.10	1.83

* * $\bar{z} = 84$ P_0

CONTINUUM

L_{II}	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.05	4.64	1.56	0.606	0.0877
	0.10	4.77	1.77	0.723	0.152
	0.15	4.90	1.96	0.841	0.221
	0.20	5.02	2.14	0.959	0.291
	0.30	5.25	2.48	1.20	0.433
	0.40	5.48	2.80	1.44	0.589
	0.50	5.71	3.11	1.69	0.766
	0.70	6.17	3.69	2.23	1.15
	1.00	6.80	4.45	3.11	1.84

L_{III}	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.05	4.65	1.58	0.618	0.0936
	0.10	4.79	1.79	0.735	0.158
	0.15	4.91	1.98	0.853	0.228
	0.20	5.03	2.16	0.971	0.298
	0.30	5.26	2.50	1.21	0.440
	0.40	5.49	2.82	1.45	0.598
	0.50	5.72	3.12	1.70	0.775
	0.70	6.18	3.71	2.24	1.16
	1.00	6.81	4.46	3.12	1.86

$$Z = 85 \quad \text{At}$$

$$A = 217$$

$$1/R = 53.55$$

$$1 - \gamma_1 = 0.215608$$

BOUND

	$1 - W_B$	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
K	0.18736	2.633			
L_1	0.03422	1.055			
L_{II}	0.03284		0.3153		
L_{III}	0.02780			0.1049	

CONTINUUM

K	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.15				
	0.20	4.75	1.60	0.608	0.0821
	0.25	4.90	1.83	0.728	0.148
	0.30	5.02	2.01	0.847	0.218
	0.35	5.14	2.20	0.968	0.290
	0.40				
	0.50	5.52	2.71	1.33	0.510
	0.70	5.96	3.34	1.85	0.869
	1.00	6.68	4.19	2.69	1.45
L_1	0.05	4.79	1.64	0.620	0.0886
	0.10	4.91	1.84	0.740	0.154
	0.15	5.03	2.03	0.859	0.225
	0.20	5.16	2.22	0.978	0.296
	0.30	5.41	2.56	1.22	0.440
	0.40	5.64	2.89	1.47	0.600
	0.50	5.85	3.21	1.73	0.782
	0.70	6.32	3.80	2.27	1.17
	1.00	6.99	4.59	3.17	1.86

$$Z = 86 \quad E_m$$

$$A = 220$$

$$1/R = 53.30$$

$$1 - \gamma_1 = 0.221434$$

BOUND

	$1 - W_B$	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
K	0.19260	2.739			
L ₁	0.03533	1.103			
L _{II}	0.03392		0.3353		
L _{III}	0.02860			0.1096	

CONTINUUM

K	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.15				
	0.20				
	0.25	5.03	1.88	0.737	0.145
	0.30	5.16	2.07	0.859	0.217
	0.35	5.28	2.26	0.981	0.291
	0.40				
	0.50	5.67	2.79	1.35	0.515
	0.70	6.11	3.44	1.88	0.880
	1.00	6.86	4.31	2.73	1.47
L ₁	0.05	4.94	1.71	0.638	0.0908
	0.10	5.06	1.91	0.759	0.158
	0.15	5.18	2.11	0.881	0.231
	0.20	5.31	2.30	1.00	0.304
	0.30	5.57	2.65	1.25	0.450
	0.40	5.80	2.99	1.50	0.613
	0.50	6.02	3.32	1.77	0.799
	0.70	6.50	3.92	2.32	1.20
	1.00	7.18	4.73	3.24	1.90

$$Z = 87 \quad Fx$$

$$A = 223$$

$$1/R = 53.06$$

$$1 - \gamma_1 = 0.227373$$

BOUND

	$1 - W_B$	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
K	0.19795	2.849			
L _I	0.03648	1.153			
L _{II}	0.03503		0.3564		
L _{III}	0.02940			0.1145	

CONTINUUM

K	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.15				
	0.20				
	0.25	5.18	1.94	0.747	0.143
	0.30	5.30	2.14	0.870	0.216
	0.35	5.43	2.33	0.994	0.291
	0.40				
	0.50	5.82	2.88	1.37	0.519
	0.70	6.28	3.54	1.90	0.890
	1.00	7.04	4.44	2.78	1.49
L _I	0.05	5.09	1.79	0.656	0.0930
	0.10	5.21	1.99	0.780	0.162
	0.15	5.33	2.20	0.903	0.236
	0.20	5.47	2.39	1.03	0.311
	0.30	5.73	2.75	1.28	0.461
	0.40	5.97	3.10	1.54	0.627
	0.50	6.19	3.43	1.80	0.817
	0.70	6.68	4.05	2.37	1.22
	1.00	7.37	4.88	3.30	1.94

$$** z = 88 \quad Ra$$

$$A = 226$$

$$1/R = 52.82$$

$$1 - \gamma_1 = 0.233427$$

BOUND

	$1 - W_B$	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
K	0.20339	2.964			
L _I	0.03765	1.206			
L _{II}	0.03617		0.3788		
L _{III}	0.03022			0.1195	

CONTINUUM

K	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.15				
	0.20				
	0.25	5.31	2.02	0.755	0.140
	0.30	5.45	2.22	0.880	0.216
	0.35	5.59	2.41	1.01	0.292
	0.40	5.73	2.60	1.13	0.367
	0.50	5.98	2.96	1.39	0.523
	0.70	6.44	3.64	1.93	0.902
	1.00	7.21	4.57	2.84	1.51
L _I	0.05	5.23	1.88	0.675	0.0949
	0.10	5.35	2.09	0.799	0.166
	0.15	5.50	2.29	0.925	0.243
	0.20	5.64	2.48	1.05	0.319
	0.30	5.90	2.85	1.31	0.471
	0.40	6.14	3.20	1.57	0.641
	0.50	6.36	3.54	1.84	0.835
	0.70	6.86	4.18	2.43	1.25
	1.00	7.57	5.04	3.38	1.98

* * $Z = 88$ Ra

CONTINUUM

L_{II}	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.05	5.23	1.89	0.679	0.0968
	0.10	5.36	2.09	0.803	0.169
	0.15	5.50	2.29	0.928	0.245
	0.20	5.64	2.49	1.05	0.321
	0.30	5.91	2.85	1.31	0.473
	0.40	6.14	3.21	1.57	0.643
	0.50	6.36	3.54	1.85	0.838
	0.70	6.86	4.18	2.43	1.25
	1.00	7.57	5.04	3.38	1.98
L_{III}	0.05	5.25	1.91	0.694	0.105
	0.10	5.38	2.12	0.819	0.178
	0.15	5.52	2.32	0.944	0.255
	0.20	5.66	2.51	1.07	0.330
	0.30	5.92	2.88	1.33	0.483
	0.40	6.15	3.23	1.59	0.655
	0.50	6.38	3.56	1.86	0.851
	0.70	6.88	4.20	2.45	1.26
	1.00	7.58	5.06	3.40	2.01

$$Z = 89 \quad A_c$$

$$A = 229$$

$$1/R = 52.59$$

$$1 - \gamma_1 = 0.239600$$

BOUND

	$1 - W_B$	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
K	0.20894	3.083			
L_1	0.03881	1.262			
L_{II}	0.03731		0.4028		
L_{III}	0.03105			0.1247	

CONTINUUM

K	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.15				
	0.20				
	0.25	5.48	2.07	0.765	0.137
	0.30	5.60	2.28	0.894	0.213
	0.35	5.73	2.48	1.02	0.291
	0.40	5.91	2.67	1.15	0.368
	0.50	6.15	3.06	1.41	0.528
	0.70	6.62	3.76	1.96	0.912
	1.00	7.42	4.70	2.87	1.54
L_1	0.05	5.42	1.95	0.695	0.0978
	0.10	5.53	2.16	0.822	0.170
	0.15	5.66	2.38	0.950	0.248
	0.20	5.80	2.58	1.08	0.326
	0.30	6.07	2.96	1.34	0.483
	0.40	6.32	3.32	1.61	0.657
	0.50	6.55	3.66	1.88	0.854
	0.70	7.06	4.31	2.47	1.28
	1.00	7.78	5.18	3.45	2.02

$$Z = 90 \quad Th$$

$$A = 232$$

$$1/R = 52.37$$

$$1 - \gamma_1 = 0.245893$$

BOUND

	$1 - W_B$	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
K	0.21459	3.208			
L_1	0.04006	1.320			
L_{II}	0.03854		0.4280		
L_{III}	0.03190			0.1302	

CONTINUUM

K	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.15				
	0.20				
	0.25	5.64	2.14	0.775	0.134
	0.30	5.76	2.36	0.906	0.212
	0.35	5.89	2.56	1.03	0.291
	0.40	6.07	2.75	1.17	0.369
	0.50	6.31	3.16	1.43	0.533
	0.70	6.80	3.87	1.99	0.923
	1.00	7.61	4.84	2.92	1.57
L_1	0.05	5.59	2.03	0.715	0.100
	0.10	5.70	2.26	0.845	0.175
	0.15	5.83	2.47	0.974	0.254
	0.20	5.97	2.68	1.10	0.334
	0.30	6.25	3.07	1.37	0.494
	0.40	6.50	3.44	1.64	0.672
	0.50	6.74	3.79	1.92	0.873
	0.70	7.26	4.45	2.52	1.31
	1.00	7.99	5.34	3.52	2.07

$$Z = 91 \quad Pa$$

$$A = 235$$

$$1/R = 52.14$$

$$1-\gamma_1 = 0.252311$$

BOUND

	$1-W_B$	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
K	0.22036	3.338			
L _I	0.04130	1.381			
L _{II}	0.03975		0.4548		
L _{III}	0.03275			0.1359	

CONTINUUM

K	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.15				
	0.20				
	0.25	5.80	2.21	0.784	0.130
	0.30	5.93	2.44	0.918	0.210
	0.35	6.06	2.65	1.05	0.290
	0.40	6.23	2.84	1.19	0.370
	0.50	6.48	3.26	1.45	0.537
	0.70	6.98	3.99	2.02	0.933
	1.00	7.82	4.98	2.98	1.59
L _I	0.05	5.77	2.12	0.736	0.103
	0.10	5.87	2.35	0.868	0.179
	0.15	6.00	2.57	1.00	0.260
	0.20	6.15	2.79	1.13	0.342
	0.30	6.44	3.19	1.40	0.506
	0.40	6.69	3.57	1.68	0.688
	0.50	6.93	3.92	1.97	0.893
	0.70	7.47	4.60	2.58	1.34
	1.00	8.21	5.51	3.59	2.11

$$* * z = 92 \quad U$$

$$A = 238$$

$$1/R = 51.92$$

$$1 - \gamma_1 = 0.258856$$

BOUND

	$1 - W_B$	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
K	0.22624	3.473			
L _I	0.04258	1.445			
L _{II}	0.04099		0.4834		
L _{III}	0.03360			0.1418	

CONTINUUM

K	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.15				
	0.20				
	0.25	5.98	2.28	0.794	0.126
	0.30	6.07	2.50	0.929	0.206
	0.35	6.23	2.72	1.06	0.289
	0.40	6.39	2.94	1.20	0.371
	0.50	6.68	3.35	1.48	0.540
	0.70	7.14	4.11	2.05	0.947
	1.00	8.07	5.11	3.02	1.61
L _I	0.05	5.98	2.21	0.757	0.105
	0.10	6.04	2.44	0.891	0.183
	0.15	6.18	2.66	1.03	0.266
	0.20	6.35	2.88	1.16	0.348
	0.30	6.65	3.30	1.44	0.516
	0.40	6.89	3.69	1.72	0.702
	0.50	7.11	4.06	2.01	0.916
	0.70	7.68	4.74	2.64	1.37
	1.00	8.46	5.69	3.67	2.15

* * $z = 92$ U

CONTINUUM

L_{II}	R	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.05	5.98	2.22	0.761	0.108
	0.10	6.04	2.44	0.895	0.186
	0.15	6.19	2.67	1.03	0.268
	0.20	6.35	2.88	1.17	0.351
	0.30	6.65	3.30	1.44	0.518
	0.40	6.89	3.69	1.72	0.705
	0.50	7.11	4.07	2.02	0.919
	0.70	7.68	4.75	2.64	1.37
	1.00	8.46	5.69	3.68	2.16

L_{III}					
	0.05	5.98	2.25	0.782	0.119
	0.10	6.06	2.48	0.916	0.198
	0.15	6.21	2.70	1.05	0.281
	0.20	6.38	2.92	1.19	0.363
	0.30	6.67	3.33	1.46	0.532
	0.40	6.91	3.72	1.75	0.721
	0.50	7.13	4.09	2.04	0.937
	0.70	7.71	4.77	2.67	1.39
	1.00	8.45	5.71	3.71	2.19

$$Z = 93 \quad N_p$$

$$A = 241$$

$$1/R = 51.71$$

$$1-\gamma_1 = 0.265533$$

BOUND

	$1-W_B$	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
K	0.23222	3.615			
L _I	0.04386	1.512			
L _{II}	0.04226		0.5140		
L _{III}	0.03446			0.1480	

CONTINUUM

K	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.15				
	0.20				
	0.25				
	0.30	6.27	2.61	0.942	0.204
	0.35	6.40	2.83	1.08	0.288
	0.40	6.55	3.04	1.21	0.372
	0.50	6.85	3.48	1.50	0.546
	0.70	7.37	4.23	2.08	0.955
	1.00	8.23	5.28	3.08	1.64
L _I	0.05				
	0.10	6.24	2.56	0.916	0.189
	0.15	6.37	2.79	1.05	0.273
	0.20	6.52	3.02	1.19	0.358
	0.30	6.82	3.44	1.47	0.531
	0.40	7.09	3.83	1.76	0.721
	0.50	7.34	4.20	2.05	0.935
	0.70	7.90	4.90	2.70	1.40
	1.00	8.67	5.86	3.75	2.20

$$Z = 94 \quad Pu$$

$$A = 244$$

$$1/R = 51.49$$

$$1-\gamma_1 = 0.272343$$

BOUND

	$1-W_B$	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
K	0.23829	3.762			
L _I	0.04520	1.582			
L _{II}	0.04357		0.5468		
L _{III}	0.03533			0.1545	

CONTINUUM

K	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.15				
	0.20				
	0.25				
	0.30	6.46	2.70	0.954	0.201
	0.35	6.58	2.92	1.09	0.286
	0.40	6.72	3.16	1.23	0.373
	0.50	7.03	3.60	1.52	0.550
	0.70	7.57	4.36	2.11	0.966
	1.00	8.45	5.44	3.14	1.67
L _I	0.05				
	0.10	6.44	2.67	0.941	0.194
	0.15	6.56	2.91	1.08	0.279
	0.20	6.72	3.14	1.22	0.366
	0.30	7.02	3.58	1.51	0.543
	0.40	7.30	3.98	1.80	0.733
	0.50	7.56	4.34	2.10	0.956
	0.70	8.13	5.07	2.76	1.43
	1.00	8.91	6.05	3.83	2.25

* * $z = 95$ A_m

$A = 247$

$1/R = 51.28$

$1 - \gamma_1 = 0.279292$

BOUND

	$1 - W_B$	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
K	0.24449	3.916			
L_1	0.04656	1.655			
L_{II}	0.04490		0.5821		
L_{III}	0.03621			0.1613	

CONTINUUM

K	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.15				
	0.20				
	0.25				
	0.30	6.66	2.77	0.967	0.197
	0.35	6.74	3.03	1.10	0.284
	0.40	6.89	3.27	1.24	0.374
	0.50	7.24	3.72	1.53	0.556
	0.70	7.78	4.50	2.15	0.972
	1.00	8.72	5.56	3.16	1.71
L_1	0.05				
	0.10	6.66	2.78	0.970	0.199
	0.15	6.74	3.03	1.11	0.286
	0.20	6.89	3.28	1.24	0.375
	0.30	7.24	3.72	1.54	0.558
	0.40	7.53	4.13	1.84	0.755
	0.50	7.78	4.50	2.15	0.974
	0.70	8.38	5.22	2.81	1.46
	1.00	9.10	6.23	3.91	2.29

* * $z = 95$ Am

CONTINUUM

L_{II}	R	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
0.05					
0.10	6.66	2.79	0.973	0.201	
0.15	6.74	3.04	1.11	0.288	
0.20	6.90	3.28	1.25	0.378	
0.30	7.24	3.73	1.54	0.560	
0.40	7.53	4.13	1.84	0.757	
0.50	7.79	4.51	2.16	0.977	
0.70	8.38	5.22	2.82	1.47	
1.00	9.10	6.24	3.92	2.30	

L_{III}					
0.05					
0.10	6.67	2.83	0.998	0.217	
0.15	6.77	3.09	1.13	0.305	
0.20	6.93	3.32	1.27	0.394	
0.30	7.27	3.76	1.57	0.578	
0.40	7.56	4.17	1.87	0.777	
0.50	7.81	4.54	2.19	0.999	
0.70	8.41	5.25	2.85	1.49	
1.00	9.09	6.27	3.95	2.33	

$$Z = 96 \quad C_m$$

$$A = 250$$

$$1/R = 51.08$$

$$1 - \gamma_1 = 0.286383$$

BOUND

	$1 - W_B$	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
K	0.25079	4.076			
L _I	0.04795	1.731			
L _{II}	0.04626		0.6204		
L _{III}	0.03711			0.1685	

CONTINUUM

K	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.15				
	0.20				
	0.25				
	0.30	6.84	2.89	0.979	0.194
	0.35	6.96	3.12	1.12	0.282
	0.40	7.08	3.39	1.26	0.374
	0.50	7.43	3.84	1.56	0.558
	0.70	7.99	4.64	2.17	0.987
	1.00	8.91	5.77	3.26	1.72
L _I	0.05				
	0.10	6.85	2.91	0.994	0.204
	0.15	6.97	3.15	1.14	0.293
	0.20	7.12	3.40	1.28	0.384
	0.30	7.44	3.87	1.58	0.570
	0.40	7.74	4.28	1.88	0.774
	0.50	8.01	4.66	2.19	1.00
	0.70	8.60	5.41	2.89	1.49
	1.00	9.40	6.44	3.99	2.35

$Z = 97$ Bk

$A = 253$

$1/R = 50.87$

$1 - \gamma_1 = 0.293620$

BOUND

	$1 - W_B$	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
K	0.25723	4.244			
L_1	0.04938	1.810			
L_{II}	0.04765		0.6625		
L_{III}	0.03804			0.1761	

CONTINUUM

K	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.15				
	0.20				
	0.25				
	0.30	7.03	3.00	0.991	0.190
	0.35	7.15	3.23	1.13	0.280
	0.40	7.29	3.50	1.28	0.373
	0.50	7.63	3.97	1.59	0.562
	0.70	8.21	4.78	2.20	0.998
	1.00	9.14	5.95	3.32	1.75
L_1	0.05				
	0.10	7.06	3.04	1.02	0.209
	0.15	7.18	3.28	1.17	0.300
	0.20	7.33	3.54	1.32	0.392
	0.30	7.66	4.02	1.62	0.584
	0.40	7.96	4.44	1.92	0.792
	0.50	8.25	4.82	2.24	1.02
	0.70	8.85	5.59	2.96	1.53
	1.00	9.66	6.65	4.08	2.40

* * $z = 98$ Cf

$A = 256$

$1/R = 50.68$

$1 - \gamma_1 = 0.301009$

BOUND

	$1 - W_B$	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
K	0.26375	4.418			
L_I	0.05082	1.893			
L_{II}	0.04908		0.7078		
L_{III}	0.03896			0.1842	

CONTINUUM

K	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.15				
	0.20				
	0.25				
	0.30	7.24	3.12	1.00	0.187
	0.35	7.37	3.34	1.15	0.277
	0.40	7.51	3.60	1.31	0.371
	0.50	7.82	4.12	1.62	0.566
	0.70	8.45	4.93	2.23	1.01
	1.00	9.34	6.16	3.41	1.78
L_I	0.05				
	0.10	7.28	3.19	1.05	0.215
	0.15	7.42	3.43	1.20	0.307
	0.20	7.56	3.69	1.36	0.402
	0.30	7.88	4.19	1.67	0.598
	0.40	8.19	4.62	1.97	0.813
	0.50	8.50	4.99	2.28	1.05
	0.70	9.10	5.78	3.03	1.56
	1.00	9.96	6.86	4.16	2.46

* * $z = 98$ Cf

CONTINUUM

L_{II}	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.05				
	0.10	7.28	3.19	1.05	0.218
	0.15	7.42	3.43	1.20	0.310
	0.20	7.57	3.70	1.36	0.405
	0.30	7.88	4.20	1.67	0.601
	0.40	8.20	4.62	1.97	0.816
	0.50	8.51	4.99	2.28	1.05
	0.70	9.10	5.79	3.04	1.56
	1.00	9.97	6.86	4.16	2.47

L_{III}					
	0.05				
	0.10	7.31	3.24	1.09	0.237
	0.15	7.45	3.49	1.24	0.330
	0.20	7.60	3.76	1.40	0.426
	0.30	7.92	4.25	1.71	0.624
	0.40	8.23	4.67	2.01	0.841
	0.50	8.54	5.03	2.32	1.08
	0.70	9.13	5.84	3.09	1.59
	1.00	10.00	6.87	4.18	2.51

$$Z = 99 \quad E_A$$

$$A = 258$$

$$1/R = 50.54$$

$$1 - \gamma_1 = 0.308553$$

BOUND

	$1 - W_B$	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
K	0.27042	4.600			
L _I	0.05407	1.978			
L _{II}	0.05053		0.7573		
L _{III}	0.03992			0.1928	

CONTINUUM

K	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.15				
	0.20				
	0.25				
	0.30	7.45	3.22	1.02	0.181
	0.35	7.56	3.46	1.16	0.274
	0.40	7.77	3.70	1.34	0.368
	0.50	8.05	4.25	1.64	0.570
	0.70	8.68	5.08	2.26	1.02
	1.00	9.63	6.31	3.45	1.81
L _I	0.05				
	0.10	7.51	3.31	1.08	0.220
	0.15	7.62	3.57	1.23	0.314
	0.20	7.78	3.84	1.39	0.411
	0.30	8.12	4.35	1.70	0.612
	0.40	8.44	4.79	2.01	0.831
	0.50	8.74	5.17	2.33	1.07
	0.70	9.36	5.96	3.10	1.60
	1.00	10.2	7.08	4.25	2.50

$$Z = 100 \text{ Fm}$$

$$A = 261$$

$$1/R = 50.35$$

$$1 - \gamma_1 = 0.316259$$

BOUND

	$1 - W_B$	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
K	0.27719	4.790			
L _I	0.05382	2.066			
L _{II}	0.05202		0.8116		
L _{III}	0.04088			0.2019	

CONTINUUM

K	k	$g_{-1}(0)$	$f_{+1}(0)$	$g_{-2}(0)$	$f_{+2}(0)$
	0.15				
	0.20				
	0.25				
	0.30	7.67	3.33	1.03	0.176
	0.35	7.77	3.58	1.18	0.270
	0.40	8.05	3.79	1.37	0.364
	0.50	8.27	4.40	1.66	0.574
	0.70	8.92	5.24	2.30	1.03
	1.00	9.88	6.50	3.52	1.84
L _I	0.05				
	0.10	7.75	3.46	1.11	0.226
	0.15	7.86	3.72	1.26	0.322
	0.20	8.01	4.00	1.42	0.420
	0.30	8.36	4.53	1.74	0.627
	0.40	8.69	4.97	2.06	0.852
	0.50	9.00	5.35	2.38	1.10
	0.70	9.63	6.16	3.17	1.63
	1.00	10.5	7.31	4.34	2.56

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$$(\gamma + i \frac{\alpha Z W}{P}) e^{i\eta} = \zeta \left\{ \left[\frac{(n-\gamma)(nW-\gamma)}{2(W+1)} \right]^{1/2} - i \frac{n}{|n|} \left[\frac{(n+\gamma)(nW-\gamma)}{2(W-1)} \right]^{1/2} \right\},$$

where $\zeta = +1$ and $-n/|n|$, respectively.

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$$X \equiv \sqrt{\frac{2\pi Q}{3}} R^2 (1 + \sigma_j \cdot \nabla^+ \dots) (\lambda_j \cdot \nabla^- \frac{1}{\sqrt{2}}), \quad Y \equiv \sqrt{\frac{3\pi Q}{2}} k R^2 (1 + \sigma_j \cdot r^+ \dots) \lambda_{j \cdot r} \quad .$$

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