ON THRESHOLDS OF PARAMETRIC
INSTABILITIES NEAR THE
LOWER HYBRID FREQUENCY

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On Thresholds of Parametric Instabilities Near the
Lower Hybrid Frequency

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ABSTRACT

Resonant decay instabilities of a pump wave with frequency ω_0 near the lower-hybrid frequency ω_{LH} are analyzed with respect to the wavenumber k of the decay waves and the ratio ω_0/ω_{LH} to determine the decay process with the minimum threshold. It was found that the lowest thresholds are for decay into an electron plasma (lower hybrid) wave plus either a backward ion-cyclotron wave, an ion Bernstein wave, or a low frequency sound wave. For $\omega_0 < (2\omega_{LH})^{1/2}$, it was found that these decay processes can occur and have faster growth than ion quasimodes provided the drift velocity (ce_0/B_0) is much less than the sound speed. In many cases of interest, electromagnetic corrections to the lower-hybrid wave rule out decay into all but short wavelength ($k\rho_i > 1$) waves. The experimental results are consistent with the linear theory of parametric instabilities in a homogeneous plasma.

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The threshold of decay of a pump wave of frequency ω_0 near the lower-hybrid frequency^{1,2} ω_{LH} into an electron plasma (lower-hybrid) wave, and a low frequency ion wave is calculated for a range of ω_0 between ω_{LH} and $3\omega_{LH}$. These thresholds are analyzed as a function of wavenumber k and angle θ of the wavevector k to a uniform magnetic field B_0 . It is shown that short wavelength ion-cyclotron waves, low frequency sound waves ($|\omega| < \omega_{ci}$), and ion Bernstein waves can be excited for angles θ such that $(m_i/m_e) \cos^2 \theta < 1$ and have lower minimum thresholds than the previously calculated ion quasimode.³ For low pump powers the growth rate of these resonant decay instabilities is greater than for the nonresonant quasimode.

When $m_i \cos^2 \theta / m_e \ll 1$ and $k_\perp \rho \approx 1$ where ρ is the ion Larmor radius, the parallel phase velocity of the lower-hybrid waves becomes comparable to the speed of light when the ion temperature exceeds a kilovolt, and the lower-hybrid dispersion relation is no longer satisfied. In many cases of interest to lower-hybrid heating of tokamaks, this consideration rules out all decay instabilities except short-wavelength ($k\rho > 1$) waves and lower-hybrid waves.

The dispersion relation governing the decay of a pump wave near the lower-hybrid frequency has been derived previously in the dipole approximation.^{1,4} The orientation of the electric and uniform magnetic fields, and the associated wavevectors are shown in Fig. 1. The pump wave could be either an electron-plasma wave or a whistler wave⁵ and by assumption $|k_0| < |k_1|, |k_2|$. The dispersion relation governing the decay is given by

$$\epsilon(\omega) \epsilon(\omega - \omega_0) + \frac{\mu^2}{4} \chi_i(\omega) \chi_e(\omega) = 0 \quad (1)$$

where $\chi_i(\omega)$ [$\chi_e(\omega)$] is the ion (electron) susceptibility at frequency $|\omega| \ll \omega_0$, $\epsilon(\omega) = 1 + \chi_e(\omega) + \chi_i(\omega)$, and $\mu = (cE_0/B_0)(k_y/\omega_0)$. The threshold for decay into weakly damped waves is $\gamma_0^2 = \gamma_i \gamma_e$ where γ_i (γ_e) is the damping rate of the ion (electron) wave in the absence of the pump and γ_0 is the growth rate of the decay waves in the absence of damping. This condition is equivalent to the equation

$$\frac{u_0^2}{C_s^2} = \frac{4\omega_0^2}{\omega_{pi}^2} k^2 \lambda_{De}^2 \epsilon_I(\omega_e) \epsilon_I(\omega_i), \quad (2)$$

provided $|\omega_i| \ll k_{\parallel} v_e$. In Eq. (2), we define $u_0 = cE_0/B_0$, the $E_0 \times B_0$ drift velocity, C_s is the sound speed, λ_{De} is the Debye length, $\epsilon_I(\omega)$ is the imaginary part of the dielectric at frequency ω , and $\omega_i(\omega_e)$ is the real part of the frequency of an ion (electron) wave. The dispersion relation for the electron plasma wave is

$$\omega_e = \omega_{LH} \left\{ 1 + \frac{m_i}{m_e} \cos^2 \theta \left(1 + 3 \frac{k_{\parallel}^2 v_e^2}{\omega_e^2} \right) + \frac{3k^2 v_i^2}{\omega_e^2} + k^2 \lambda_{De}^2 \epsilon \left[\left(1 + \frac{m_i}{m_e} \cos^2 \theta \right) \frac{\epsilon}{4(1+\epsilon)} - \frac{m_i}{m_e} \cos^2 \theta \right] \right\}^{1/2}, \quad (3)$$

where $\omega_{LH} = \omega_{pi}/(1 + \omega_{pe}^2/\omega_{ce}^2)^{1/2}$, $\epsilon = \omega_{pe}^2/\omega_{ce}^2$, and the damping rate is the sum of collisional damping γ_e^{coll} given by the equation

$$\gamma_e^{coll} = \frac{1}{2} \frac{m_i}{m_e} v_e \frac{\omega_{LH}^2}{\omega_e^2} \left(\cos^2 \theta + \frac{\omega_e^2}{\omega_{ce}^2} \right),$$

and electron and ion Landau damping given by the equation

$$\gamma_C^L = \frac{\omega_e^2 \sqrt{\pi}}{(1+\omega_{pe}^2/\omega_{ce}^2) 2k^3 \lambda_D^3 \omega_{pe} \cos \theta} \left[\exp \left(- \frac{\omega_e^2}{k^2 \lambda_{De}^2 \omega_{pe}^2 \cos^2 \theta} \right) + \left(\frac{T_e}{T_i} \right)^{3/2} \left(\frac{m_i}{m_e} \right)^{1/2} \exp \left(- \frac{\omega_e^2 T_e}{\omega_{pi}^2 T_i k^2 \lambda_{De}^2} \right) \right], \quad (4)$$

where ν_e is the electron-ion collision frequency, ω_{pe} (ω_{pi}) the electron (ion) plasma frequency, and $\omega_{ce} = |eB_0/m_e c|$ the electron-cyclotron frequency. The damping rate is related to the imaginary part of the dielectric by $\gamma_e = |\epsilon_I / \partial \epsilon / \partial \omega_e|$ where $|\partial \epsilon / \partial \omega_e| = 2(1+\omega_{pe}^2/\omega_{ce}^2)/\omega_e$.

The dipole approximation for the pump wave has the consequence that the wavenumbers of the high and low frequency waves are equal. The dispersion character of the low frequency waves depends on the product of the wavenumber k and the ion gyroradius ρ . The electrostatic approximation was assumed in deriving Eq.(3) for the lower hybrid wave, that is, we assumed $n_{||} = ck_{||}/\omega_{LH} \gg 1$. Assuming that $(k_{||}/k) \sim (m_e/m_i)^{1/2}$ and that $k_0 \sim 1$, we find that the parallel index of refraction $n_{||} \approx (m_e c^2/T_i)^{1/2} (m_e/m_i)^{1/2}$. For hot plasmas ($T_i \approx \text{keV}$) the parallel index of refraction can only be large if $k\rho > 1$; however, there is no such restriction on small laboratory experiments where typically $T_i < T_e \sim 10\text{eV}$. For reference the lower hybrid wave frequency including electromagnetic corrections is given by

$$\omega^2 = \omega_{LH}^2 \frac{\left[1 + \frac{m_i}{m_e} \cos^2 \theta \left(1 + \frac{\omega_{pi}^2}{c^2 k_{||}^2} \right) \right]}{\left[1 + \frac{\omega_{pe}^2}{c^2 k^2} \left(1 + \epsilon \frac{\omega_{pe}^2}{c^2 k^2} \right) \right]}$$

where $\epsilon = \omega_{pe}^2 / \omega_{ce}^2$. Terms of order $\omega_{pe} / ck_{\perp}$ are important also because $k_{\perp} \approx (m_i / m_e)^{1/2} k_{\parallel}$.

Another important parameter involved in coupling of high and low frequency waves is the ratio of the ion gyroradius to the electron Debye length, that is, $(\rho_i / \lambda_{De}) = (T_i / T_e)^{1/2} (m_i / m_e)^{1/2} \omega_{ce} / \omega_{pe}$. Often in laboratory scale machines this ratio is not large because $T_i \ll T_e$ and decay into short wavelength waves.

$(k\rho_i > 1)$ has high thresholds because the electron wave will be Landau damped.

There are many possible low frequency decay waves in general. For $|\omega| \ll \omega_{ci}$ and $\lambda = k_{\perp}^2 \rho_i^2 < 0.269^5$, a low frequency sound wave propagates with dispersion given by

$$\omega_s = k_{\parallel} C_s = k_{\parallel} C_s \left(\frac{I_o(\lambda) \exp(\lambda)}{[1 + (1 - I_o) \exp(-\lambda) \frac{T_e}{T_i}]} \right)^{1/2} \approx k_{\parallel} C_s$$

Near the harmonics of the ion cyclotron frequency ω_{ci} , both the electrostatic ion cyclotron wave and the ion Bernstein wave propagate. The parallel phase velocity, $(\omega / k_{\parallel})$ of the ion cyclotron wave is less than the electron thermal velocity V_e but the ion Bernstein wave phase velocity obeys the opposite inequality, i.e., $\omega / k_{\parallel} < V_e$ for the ion cyclotron wave but $\omega / k_{\parallel} > V_e$ for the ion Bernstein wave. The ion cyclotron wave frequency is above the ion cyclotron frequency harmonics and has the approximate dispersion given by

$$\omega_c = n\omega_{ci} \left[1 + \frac{T_e}{2T_i} \left(\frac{k_{\perp}\rho_i}{2} \right)^{2n} \right]$$

for the forward wave defined for $\lambda \ll 1$. For $\lambda > n^2 + 1/2$ the forward wave evolves into the backward wave with dispersion given by

$$\omega_c = n\omega_{ci} \left(1 + \frac{(k_{\perp}\rho_i)^{-1}}{\sqrt{2\pi} \left(1 + \frac{T_i}{T_e} \right)} \right)$$

The ion Bernstein wave frequency lies just below the n^{th} harmonic for $k_{\perp}\rho_i \ll 1$ for $n \geq 2$ and asymptotically approaches the $(n-1)^{\text{th}}$ harmonics as $k_{\perp}\rho \rightarrow \infty$. Near the second harmonic $\omega = 2\omega_{ci} (1 - k_{\perp}^2 \rho^2 / 2)$. For $|\omega| < \omega_{ci}$ and $k_{\perp}\rho_i < \left(\frac{T_i}{T_e} \right)^{1/2}$. The cold plasma ion Bernstein wave of frequency

$$\omega = \frac{\omega_{ci} \left(\frac{m_i}{m_e} \right)^{1/2} \cos \theta}{\left(1 + \frac{m_i \cos^2 \theta}{m_e} \right)^{1/2}}$$

can propagate. Away from the cyclotron harmonics, and for $kC_s \gg \omega_{ei}$ a high frequency sound wave with dispersion $\omega = kC_s (1 + 3T_i/T_e)^{1/2}$, propagates but is highly damped unless $T_e \gg T_i$.

A competing process is nonresonant decay via nonlinear Landau damping into lower-hybrid waves and ion quasimodes.³ The threshold for this process is always higher when the selection rules for resonant decay, i.e., $\omega_o = \omega_e + \omega_i$ and $k_o = k_e + k_i \approx 0$ can be satisfied. It has been pointed out previously⁴ that the selection rules cannot be satisfied for forward ion-cyclotron waves or high frequency sound waves when $\cos^2 \theta < m_e/m_i$. However, low frequency sound waves, backward ion-cyclotron waves, and ion Bernstein waves can be excited. For sound waves the most important damping processes are electron and ion Landau damping which gives

$$\epsilon_I(\omega_s) = \frac{(\pi)^{1/2}}{k^2 \lambda_{De}^2} \left[\left(\frac{m_e}{m_i} \right)^{1/2} + \left(\frac{T_e}{T_i} \right)^{3/2} \exp \left(-\frac{T_e}{T_i} \right) \right] \quad (5)$$

For backward ion-cyclotron waves, electron Landau damping and ion-cyclotron damping predominate so that $\epsilon_I(\omega_i)$ is given by

$$\epsilon_I(\omega_i) = \frac{(\pi)^{1/2}}{k^2 \lambda_{De}^2} \frac{n}{k \rho \cos \theta} \left(\frac{m_e T_i}{m_i T_e} \right)^{1/2} \left[1 + \left(\frac{m_i}{m_e} \right)^{1/2} \left(\frac{T_e}{T_i} \right)^{3/2} \frac{\exp \left(-\frac{C}{k^4 \rho_i^4} \right)}{\sqrt{2\pi}^{1/2} k \rho_i} \right] \quad (6)$$

where $C = n^2 / \left[(1 + T_i/T_e)^2 (2\pi) \cos^2 \theta \right]$. Because of the small angles to the normal to B_0 and since $k \lambda_{De} = k \rho_i (T_e/T_i)^{1/2} \omega_{ci}/\omega_{pi} \sim (m_e/m_i)^{1/2} k \rho_i$ is much less than one, Landau damping of the electron wave is small compared to collisional damping. If the electron wave is collisionally damped, then $\epsilon_I(\omega_e)$ is only weakly dependent on k so that the minimum threshold obtains when $k^2 \lambda_{De}^2 \epsilon_I(\omega_i)$ is a minimum subject to the previously mentioned constraints. The minimum thresholds are obtained following the procedure outlined in Appendix A. The frequency selection rules are assumed to be satisfied. The minimum threshold for low frequency ($|\omega| < \omega_{ci}$) sound waves is given by

$$\Gamma_s = \frac{U_o^2}{C_s^2} = 4\pi^{1/2} \frac{v_e}{\omega_o} \left[\left(\frac{m_e}{m_i} \right)^{1/2} + \left(\frac{T_e}{T_i} \right)^{3/2} \exp \left(-\frac{T_e}{T_i} \right) \right] \cdot \left(\frac{\omega_o^2}{\omega_{LH}^2} - 1 + \frac{\omega_o^2}{\omega_{ce} \omega_{ci}} \right) \quad (7)$$

where we used the frequency selection rule to obtain $m_i \cos^2 \theta / m_e \approx -1 + \omega_o^2 / \omega_{LH}^2$. The minimum threshold Γ_c for backward ion cyclotron waves is given by:

$$\Gamma_c = 4\sqrt{\pi}g \frac{v_e}{\omega_o} n^{1/2} \left(\frac{m_e}{m_i}\right)^{1/4} \left(\frac{\omega_o^2}{\omega_{LH}^2} - 1\right)^{-1/4} \left(\frac{\omega_o^2}{\omega_{LH}^2} - 1 + \frac{\omega_o^2}{\omega_{ce}\omega_{ci}}\right) \quad (8)$$

where

$$g \equiv (2\pi \ln 4b)^{1/4} \left(\frac{T_i}{T_e}\right)^{1/2} \left(1 + \frac{T_i}{T_e}\right)^{1/2},$$

and ,

$$b \equiv \left(\frac{m_i}{m_e}\right)^{1/4} \frac{T_e}{T_i} \left(\frac{T_e}{T_i} + 1\right)^{1/2} (2\pi n^2)^{-1/4} \left(\frac{\omega_o^2}{\omega_{LH}^2} - 1\right)^{1/4},$$

and b is assumed to be greater than one. The wavenumber for which the threshold is a minimum is given approximately by

$$\frac{k^2 \rho^2}{n} = \left[2\pi \frac{m_e}{m_i} \left(\frac{\omega_o^2}{\omega_{LH}^2} - 1\right) \left(1 + \frac{T_i}{T_e}\right)^2 \ln 4b \right]^{-1/2}$$

and has the approximate value $k\rho \approx (m_i/m_e)^{1/4}$. In deriving the thresholds for these waves, we have neglected coupling to a high frequency wave at frequency $\omega_o + \omega$ which is justified so long as $\omega_i \gg \gamma_e$. For the low frequency sound waves, this inequality sets lower limit to the value of k. Of course, if $T_e \sim T_i$, then sound waves are highly damped.

Resonant decay into lower hybrid waves and ion Bernstein waves is also possible. The threshold formula, Eq. (2), no longer applies because $\omega_i > k_{\parallel} V_e$ in this case. The threshold is now determined by the equation

$$\frac{U_o^2}{C_s^2} = \frac{4\omega_o^2}{k^2 C_s^2} \frac{\epsilon_I(\omega_i) \epsilon_I(\omega_e)}{(\chi_e(\omega_i))^2},$$

where $\chi_e = -\omega_{pe}^2 \cos^2 \theta / \omega^2$. Cold plasma ion Bernstein waves are damped only by electron Landau damping and collisions. Landau damping is negligible if $k\rho \ll (T_i/T_e)^{1/2}$ so the threshold for decay is very low. Hot plasma ion Bernstein waves for a minimum threshold when electron Landau damping and ion cyclotron damping balance. This balance can only occur for $k\rho < 1$ because both damping rates are increasing functions of $k\rho$ for short wavelengths. The minimum threshold for the lowest frequency hot plasma ion Bernstein wave is given by:

$$\Gamma_B = \frac{U_o^2}{C_s^2} = 128\sqrt{\pi} \frac{\nu_e}{\omega_o} \frac{\omega_o^2}{\omega_{ce}^2} \left(\frac{T_i}{T_e}\right)^5 \frac{\left(\frac{m_i}{m_e}\right)^{8/3}}{\Delta^5} \exp - \frac{\left(\frac{m_i}{m_e}\right)^{2/3}}{\Delta^2} \quad (9)$$

where

$$\Delta \equiv \left[\left(\frac{\omega_o - 2\omega_{ci}}{\omega_{LH}} \right)^2 - 1 \right]^{1/2}$$

The wavenumber at the threshold minimum is $k\rho = (m_e/m_i)^{1/3} (T_e/T_i)^{1/2}$. For wavenumbers above this value electron Landau damping exceeds ion cyclotron damping.

All these thresholds should be compared with the quasimode threshold Γ_{qm}^3 given by:

$$\Gamma_{qm} = \frac{U_o^2}{C_s^2} = \frac{4}{0.76} \frac{\nu_e}{\omega_o} \left(\frac{\omega_o^2}{\omega_{LH}^2} - 1 + \frac{m_i}{m_e} \frac{\omega_o^2}{\omega_{ce}^2} \right) \quad (10)$$

which is the threshold for nonlinear Landau damping on electrons.

This process is possible in a magnetic field when the parallel phase velocity $\omega/k_{\parallel}$ of the beatwave is less than the electron thermal velocity and when the phase velocity ω/k is less than the sound speed.

In a hot plasma when $\omega_0 < \sqrt{2} \omega_{LH}$ only decay into quasimodes and backward ion cyclotron waves is possible. If $(\omega_0/\omega_{LH})^2 - 1 \gg m_e/m_i$ then the threshold for decay into ion cyclotron waves is lower than the threshold for quasimode decay but not much lower if $\omega_0 < \sqrt{2} \omega_{LH}$.

If the electron temperature is high enough or the density is low enough so that the electron wave is Landau damped rather than collisionally damped, the conclusions must be altered because $\epsilon_I(\omega_e)$ becomes a strongly increasing function of $k\lambda_{De}$ (see Eq. 14). It remains true that for $\omega_0 < \sqrt{2} \omega_{LH}$ only backward ion-cyclotron waves or low frequency sound waves can be resonantly excited.

However, for ion-cyclotron waves, the minimum value of the threshold occurs for the smallest value of $k\rho$ consistent with small electron Landau damping and with $e^{-\lambda} I_n(\lambda) \ll 1$. Thus, $k\rho$ must be greater than $k_{\min}\rho$ where $k_{\min}\rho = n(\pi T_i/T_e)^{1/2} (m_i \cos^2 \theta / m_e)^{-1/2}$. Low frequency ion sound waves in this case have a minimum threshold when the collisional and Landau dampings of the high frequency wave are approximately equal, i.e., when the wavenumber of the decay wave is so large that Landau damping equals collisional damping. The wavenumber for which this occurs is given by

k_v where

$$k_v \lambda_{De} = \frac{\left(\frac{T_e}{T_i} \right)^{1/2} \frac{\omega_0}{\omega_{pi}}}{\left[\ln \left(\frac{\omega_{LH}}{v_e} \right) \right]^{1/2}}$$

If $\omega_o > \sqrt{2}\omega_{LH}$, then high frequency ion sound waves and forward ion-cyclotron waves can be excited as well and the analysis is more complicated. For the case that the electron wave is collisionally damped, i.e., $k_v \gg k$, then the lowest thresholds are for the backward ion-cyclotron and the low frequency ion sound wave. For reference, we include the threshold for decay into high frequency ion acoustic waves ($\omega = kC_s$) given by

$$\Gamma_A = \frac{U_o^2}{C_s^2} = 4 \sqrt{\frac{\pi}{2}} \frac{v_e}{\omega_o} \left(\frac{\omega_o^2}{\omega_{LH}^2} - 1 \right)^{1/2} \left(1 + \frac{3T_i}{T_e} \right)^{1/2} \quad (11)$$

including ion temperature corrections to the sound speed and keeping only electron Landau damping terms which is valid provided $T_e \gg T_i$ and $\omega_o \gg \omega_{LH}$. This formula should be compared to Eq. (8) for the threshold for decay into ion cyclotron waves. The ion cyclotron decay threshold is lower if $\omega_o/\omega_{LH} \gg 1$ provided $k\rho < (m_i/m_e)^{1/4} (\omega_{LH}/\omega_o)^{1/4}$.

All the decay processes considered so far have very low thresholds because the lower hybrid wave is very weakly damped. In general, the growth rate, γ , of a resonant decay instability when the frequency selection rules are satisfied exactly can be found from the formula

$$\gamma = - \frac{\gamma_i + \gamma_e}{2} + \frac{1}{2} \left[(\gamma_i - \gamma_e)^2 + 4\gamma_o^2 \right]^{1/2}$$

where γ_i (γ_e) is the damping rate of the low (high) frequency decay wave and γ_o is the growth rate in the absence of damping. Since $\gamma_i \gg \gamma_e$ for cases considered in this paper, the growth

rate near threshold is $\gamma = \frac{\gamma_o^2}{\gamma_i} - \gamma_e$. If the pump electric field E_o is strong enough so that $\gamma_o > \gamma_i$, then $\gamma = \gamma_o - (\gamma_i + \gamma_e)/2$ provided that $\gamma_o \ll \omega_i, |\omega_i - \omega_{ci}|$. The growth rate γ_o and the ion wave damping rate γ_i are given below for each instability considered for the wavenumber that minimizes the threshold (if such a wavenumber corresponds to nonzero growth). The damping rate of the lower hybrid wave in each case is given by

$$\gamma_e = \left(\frac{\gamma_e}{2} \right) \left(1 - \frac{\omega_{LH}^2}{\omega_o^2} + \frac{\omega_{LH}^2}{\omega_{ci} \omega_{ce}} \right).$$

1. Cold Plasma Ion Bernstein Wave

Growth rate:

$$\gamma_o = \frac{U_o}{4C_s} (\omega_{LH} \omega_{ci})^{1/2} \left(1 - \frac{\omega_{LH}^2}{\omega_o^2} \right)^{1/4} \left(\frac{T_e}{T_i} \right)^{1/2} k\rho \quad (12)$$

Damping rate:

$$\frac{\gamma_i}{\omega_{ci}} = \frac{(\pi)^{1/2}}{2} \left(\frac{\omega_o^2}{\omega_{LH}^2} - 1 \right)^{1/2} \left(\frac{\omega_{LH}}{\omega_o} \right)^6 \left(\frac{T_i}{T_e} \right)^{3/2} (k\rho)^{-3} \exp \left(- \frac{T_i}{T_e} \frac{\omega_{LH}^2}{\omega_o^2} \frac{1}{k_p^2} \right)$$

$$\text{where } k\rho < \left(\frac{T_i}{T_e} \right)^{1/2} \frac{\omega_{LH}}{\omega_o}$$

2. Low Frequency Sound wave

Growth rate:

$$\gamma_o = \frac{3}{16} \frac{U_o}{C_s} \left(1 - \frac{\omega_{LH}^2}{\omega_o^2} \right)^{1/2} \left(\frac{m_e}{3m_i} \right)^{1/4} (\omega_o \omega_{ci})^{1/2} \quad (13)$$

Damping rate:

$$\frac{\gamma_i}{\omega_{ci}} = \frac{1}{2} \left(\frac{\pi}{3} \right)^{1/2} \sqrt{\frac{m_e}{m_i}} \left(\frac{\omega_o^2}{\omega_{LH}^2} - 1 \right)^{1/2} \left[\left(\frac{m_e}{m_i} \right)^{1/2} + \left(\frac{T_e}{T_i} \right)^{3/2} \exp \left(- \frac{T_e}{T_i} \right) \right]$$

$$\text{for } k\rho = \left(\frac{T_i}{3T_e} \right)^{1/2} < 0.5$$

3. Backward ion cyclotron waves

Growth rate:

$$\gamma_o = \frac{1}{2\sqrt{2}} \frac{U_o}{C_s} (\omega_o \omega_{ci})^{1/2} \left(\frac{\omega_o^2}{\omega_{LH}^2} - 1 \right)^{1/8} \left(\frac{m_e}{m_i} \right)^{1/8} \frac{\omega_{LH}}{\omega_o} \left(\frac{n}{2\pi} \right)^{1/4} g^{1/2} \quad (14)$$

Damping rate:

$$\frac{\gamma_i^c}{n\omega_{ci}} = \sqrt{\pi} \sqrt{\frac{m_e}{m_i}} (\ln 4b)^{1/2} \left(\frac{T_i}{T_e} \right)^{1/2} \left(1 + \frac{T_e}{T_i} \right)$$

4. Hot Plasma Ion Bernstein wave ($\omega_{ci} < \omega \lesssim 2\omega_{ci}$)

Growth rate:

$$\gamma_o^B = \frac{1}{8} \frac{U_o}{C_s} \sqrt{\omega_o \omega_{ci}} \frac{\omega_{LH}}{\omega_o} \left[\left(\frac{\omega_o - 2\omega_{ci}}{\omega_{LH}} \right)^2 - 1 \right]^{1/2} \left(\frac{m_e}{m_i} \right)^{2/3} \left(\frac{T_e}{T_i} \right)^{3/2} \quad (15)$$

Damping rate:

$$\frac{\gamma_i}{\omega_{ci}} = 8\sqrt{\pi} \frac{T_i}{T_e} d \exp(-d)$$

where $d \equiv (m_i/m_e)^{2/3} / (-1 + \omega_o^2/\omega_{LH}^2)$.

5. High Frequency sound waves ($\omega = kC_s$)

Growth rate:

$$\gamma_o = \frac{1}{4} \frac{U_o}{C_s} (\omega_o \omega_{ci})^{1/2} \frac{\omega_{LH}}{\omega_o} \left(3 + \frac{T_e}{T_i} \right)^{3/4} \left(\frac{T_e}{T_i} \right)^{1/2} \sqrt{k\rho} \quad (16)$$

Damping rate:

$$\frac{\gamma_i}{\omega_{ci}} = \frac{\left(1 + \frac{3T_i}{T_e} \right)^{5/2}}{2} \sqrt{\pi} \sqrt{\frac{T_e}{T_i}} k\rho \left[\frac{1}{\left(\frac{\omega_o^2}{\omega_{LH}^2} - 1 \right)^{1/2}} + \left(\frac{T_e}{T_i} \right)^{3/2} \exp\left(-\frac{3}{2} - \frac{T_e}{2T_i} \right) \right]$$

For nonresonant ion quasimodes the growth rate is given by

$$\gamma_{qm} = \frac{0.76\sqrt{\pi}}{8} \frac{U_o^2}{C_s^2} \frac{\omega_{LH}^2}{\omega_o^2} \omega_o - \gamma_e, \quad (17)$$

provided that $|\gamma_{qm}| \ll k_{||} v_e$ and $\omega_o < \sqrt{2} \omega_{LH}$.

As mentioned previously not all these waves can be excited in a given plasma. For a fusion reactor plasma with $T_i \sim \text{keV}$ only backward ion cyclotron, high frequency sound waves, and quasimodes are possible decay waves. High frequency sound waves are heavily damped unless both $\omega_o \gg \sqrt{2} \omega_{LH}$ and $T_e \gg T_i$ which is unlikely in a fusion reactor. The most likely low frequency decay wave will be either a backward ion cyclotron wave or a quasimode if $\omega_o < \sqrt{2} \omega_{LH}$ or only an ion cyclotron wave if $\omega_o > \sqrt{2} \omega_{LH}$. Above the threshold for both processes but for $\gamma_o^C < \gamma_i^C$, the growth rates for both instabilities are proportional to the square of the electric pump field.

In a plasma with $\omega_{ce} = 4\omega_{pe}$, $T_e = 3T_i$, $\omega_o = \sqrt{2}\omega_{LH}$, the growth rates are equal when $m_i/m_p = 2$ and $U_o/C_s = 0.5$. The variation of the minimum threshold Γ_c for ion-cyclotron waves as a function of $(\omega_o - \omega_{LH})$ is shown in Fig. 1 together with the growth rate for the wavenumber that determines the minimum threshold. This wavenumber is given in Fig. 2 as a function of $\omega_o - \omega_{LH}$. This threshold, Γ_c , has a minimum for $\omega_o^2/\omega_{LH}^2 = 1 + \omega_{pe}^2/3(\omega_{pe}^2 + \omega_{ce}^2)$.

The results of the previous sections assumed that the wavenumber of the pump wave was much smaller than the decay wave wavenumbers. For the case that the pump wave is electrostatic, this restriction can be removed so that the generalization of Eq. (1)

to $\underline{k}_0 \neq 0$ electrostatic pumps is the equation

$$\epsilon(\underline{k}, \omega) \epsilon(\underline{k}-\underline{k}_0, \omega-\omega_0) - \frac{\tilde{\mu}^2}{4} \chi_e^2(k, \omega) \frac{k^2}{(\underline{k}-\underline{k}_0)^2},$$

where $\tilde{\mu} = |(\underline{k} \times \underline{E}_0) \cdot \hat{e}_z| c/\omega_0 B_0$ and $|\chi_e(k, \omega)| \gg |\chi_e(k-k_0, \omega-\omega_0)|$ by assumption. In general the satisfaction of the frequency and wavenumber selection rules requires three dimensions in wavevector space especially when coupled with the threshold minimization procedure. If the high frequency wave is collisionally damped, the problem is considerably simplified. In particular, the resonant decay into ion cyclotron waves is possible. Let (k_0, ψ_0, θ_0) , (k_1, ψ_1, θ_1) and (k_2, ψ_2, θ_2) be the spherical coordinates in phase space of the wavenumbers of the electrostatic pump, lower hybrid decay wave, and ion cyclotron decay wave respectively. Frequency relation determines θ_1 in terms of k_2 and θ_0 . Wavenumber selection determines k_1 , ψ_1 , and θ_2 . The remaining variables are k_2 and ψ_2 which can be chosen so that k_2 minimizes the damping of the cyclotron wave and ψ_2 to maximize the cross product in $\tilde{\mu}$. As long as the lower hybrid decay wave is collisionally damped, finite pump wavevectors will only raise the threshold by the ratio $(1 + \frac{k_0^2}{k^2})$.

The results of a number of laboratory scale experiments have been reported^{1,7}. It is interesting to compare the predictions of the homogeneous theory for these plasmas and large scale Tokamaks. It was shown that only short wavelength ($k\rho > 1$) waves can be excited in Tokamaks. However, in the laboratory experiments, decay into long wavelengths is allowed. The relevant parameters of the experiment reported in Ref. 1 by Chu et al, are $T_i \gtrsim T_e = 0.2\text{ev}$, $\omega_0/\omega_{LH} \approx 8$, $\omega_{pe} \ll \omega_{ce}$, $\frac{m_i}{m_p} = 133$ and $ck_{||}/\omega_{pi} \gg 1$ even for

$k\rho \ll 1$. The spectrum observed was consistent with decay into ion cyclotron waves as is expected from homogenous theory. The experiments described in Ref. 7 have the parameters $\omega_o/\omega_{LH} \approx 1.2$, $m_i/m_p = 1$, $T_e/T_i > 30-70$, $T_e \sim 3-7\text{ev}$, $\omega_{pe} \sim \omega_{ce}$, $n_o \approx 1-5 \times 10^{10} \text{cm}^{-3}$. The threshold for decay into ion cyclotron waves is dominated by the Landau damping of the lower hybrid wave. Thus, the lowest thresholds are for ion Bernstein waves, low frequency sound waves, and quasimodes. In fact, the results are consistent with quasimode decay though the measured wavelengths perpendicular to the magnetic field are in the range of the minimum threshold for resonant decay into ion Bernstein waves.

Hooke and Bernabei¹ observed decay into high frequency acoustic waves. In this experiment, $m_i/m_p = 4$, $T_e \sim 10\text{ev} \approx 10T_i$, $\omega_o \gg \omega_{LH}$, $U_o/C_s < 1$, $B_o = 10^3 \text{G}$, $n_o = 10^{10} \text{cm}^{-3}$. Because $T_e \gg T_i$ and $\omega_o \gg \omega_{LH}$, decay into high frequency sound waves is competitive with ion cyclotron waves. In fact, decay into high frequency sound waves has larger growth rates far above threshold than decay into ion cyclotron waves.

In none of these experiments were the ions hot enough to make the electrostatic approximation invalid for the lower hybrid wave in marked contrast to the case for Tokamaks.

We have examined the various parametric instabilities that can occur in a homogeneous plasma with a pump wave near the lower-hybrid resonance. We found that the lowest threshold correspond to decay into lower-hybrid waves and backward ion-cyclotron waves, and ion-acoustic waves in the dipole approximation for the pump wave. For low pump powers, these waves also have the largest growth rates. Naturally, for larger pump powers the non-resonant decay processes,

i.e., quasimodes, ³ can have larger growth rates. However, electromagnetic corrections to the lower-hybrid dispersion were shown to have a profound effect on the decay which limits the possible low frequency decay waves to the backward ion-cyclotron wave, ion quasimodes, or ion Bernstein waves with $k_{\perp}\rho > 1$ for all waves for hot ion plasmas ($T_i \sim \text{keV}$). Experimental results were shown to be consistent with the predictions of parametric theory for homogeneous plasmas. The modification of the results for non-dipole pumps ($k_0 \neq 0$) was shown to have little effect on the thresholds for decay into backward ion cyclotron waves.

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APPENDIX

The minimum thresholds are found in a straightforward manner. For each pair of decay waves, one of which is always a lower hybrid wave, there is a value of wavenumber k for which the threshold is a minimum. We assume $k^2 \lambda_{Di}^2 \ll m_i/m_e \cos^2 \theta$ and $|\omega_i| \ll \omega_{LH}$ so that the frequency selection rules determine $m_i/m_e \cos^2 \theta \approx \omega_o^2/\omega_{LH}^2 - 1$. If the lower hybrid wave is collisionally damped, then the only wavenumber dependent quantity in the threshold formula, Eq. (2) is $k^2 \lambda_{De}^2 \epsilon_I(\omega_1)$. The threshold $\hat{\Gamma}_c$ for decay into lower hybrid waves and backward ion cyclotron waves ($k_i \rho_i > 1$) is shown in Fig. 4 versus $k \rho_i$ for a hydrogen plasma with $\omega_o^2 = 1.5 \omega_{LH}^2$, $T_e = 500 \text{ ev}$, and $\omega_{pe} = .8 \omega_{ce}$. It is apparent that this threshold has a minimum for $k \rho_i \approx 3$. This result can be obtained readily by differentiation of the analytic expression for Γ_c using Eqs. (2) and (6). Minimum thresholds for other decays are obtained similarly.

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Fig. 1. Orientation of the electric fields of pump E_0 and decay waves E_1 and E_2 , and the corresponding wavenumbers k_0 , k_1 , and k_2 . A uniform magnetic field B_0 is aligned along the Z axis. The wavenumber of the pump $|k_0| \ll |k_1|, |k_2|$, the wavenumbers of the decay waves. The dominant coupling arises from the electron $E_0 \times B_0$ drift velocity. The decay waves are in the yz plane; the pump field is in the xz plane.

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Fig. 2. The plot of the threshold Γ_c normalized to Γ_0 versus the difference of the ratio ω_0/ω_{LH} from one (upper curve). A plot of the growth rate γ_c normalized by γ_0 versus $(\omega_0/\omega_{LH}) - 1$ provided the threshold for decay is greatly exceeded and for the wavenumber that minimizes the threshold. Here $\Gamma_0 = (2\pi^3 m_e/m_i)^{1/4} (T_e/T_i)^{1/2} (\ln 4b)^{1/4} v_e/\omega_{LH}$ and $\gamma_0 = 0.113 \omega_{ci} (\omega_{pe}/\omega_{ce})^{1/2} (1 + \omega_{pe}^2/\omega_{ce}^2)^{-1/4} (\ln 4b)^{1/4} u_0/c_s$.

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Fig. 3. A plot of the product of wavenumber k_c and ion gyroradius that minimizes the threshold Γ_c versus $(\omega_o/\omega_{LH}) - 1$. $2\omega_{pe}^2 = \omega_{ce}^2$, $3T_i = T_e = 2\text{keV}$ and $n = 10^{13} \text{ cm}^{-3}$ were assumed.

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Fig. 4. The threshold for decay of a dipole pump into a lower hybrid wave and a backward ion cyclotron wave versus wavenumber. The parameters $m_i = m_p$, $T_e = 3T_i$, $\omega_{pe} = 0.8\omega_{ce}$, $\omega_o = 1.2\omega_{LH}$, $n_i = 10^{14} \text{ cm}^{-3}$, $v_e = 3.5 \times 10^{-6} \omega_o^2$ were assumed and $\hat{\Gamma}_c = 10^6 U_o^2 / C_s^2$.

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