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**THE CALCULATION OF STEAM FLOW AND HYDRAULIC FRACTURING
IN A POROUS MEDIUM WITH THE KRAK CODE**

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Carl E. Keller, Alvin H. Davis, and John N. Stewart, Jr.

ABSTRACT

A computer program named KRAK has been developed for calculating the flow of steam and water in a porous medium and also the growth of a crack produced by the high pressure fluid. The program is written for a two-dimensional cylindrically symmetric geometry. The model treats the thermodynamics of the two-phase steam-water system accurately. It also takes into account heat conduction through the porous medium and between the medium and the fluid. Crack growth is treated in a quasi-static approximation, but within this limitation the detailed crack shape is obtained. Darcy flow is assumed within the porous medium, but fluid flow in the crack itself is calculated accurately from the equations for flow between parallel plates for all ranges of the Reynold's number. A suite of illustrative calculations is given showing the dependence of crack growth rates on the main parameters of the problem: source pressure, permeability, porosity, medium saturation, tensile strength, etc.

I. INTRODUCTION

The KRAK code is a computer program for calculating the flow and condensation of steam in a porous medium in two dimensions. The code also has the capability of calculating the growth of a steam-driven hydro-fracture in the medium. The original purpose of the code development was to calculate the rate of crack growth from a high-pressure steam-filled cavity generated underground by a nuclear explosion.

The chief advantage of the KRAK code is the detailed physical model used for the steam flow calculation. Conversely, the disadvantage is the computer time requirements. However, the KRAK code has proved very useful in determining the significance of the major physical parameters and the validity of major assumptions used for faster calculations.

This report describes the physical model used to calculate steam flow, condensation, heat conduction and convection, and also the dimensions of a propagating crack if it is allowed to develop. In the appendices are the details of the difference equations, the crack calculation, and the equation of state.

II. METHOD

The KRAK code is based upon the equations expressing conservation of energy and mass together with an appropriate flow equation. For the high-velocity flow in the crack momentum conservation applies, but in the porous medium Darcy flow is assumed. Heat flow is calculated including conduction through the rock, conduction from the fluid in the pore space into the rock, and convection of the fluid through the pore space.

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Basically, the problem is to calculate the pressure field as a function of time, since all flow depends upon the pressure gradients and if a crack develops, its dimensions are a function of the pressure inside the crack. Given a pressure field, the calculational procedure is:

1. Calculate the flow velocities in the problem.
2. Use the flow velocities to calculate mass and energy fluxes.
3. Calculate energy transfer due to heat conduction.
4. Calculate new mass and energy densities throughout the mesh.
5. Use equation of state relationships to calculate a new pressure field.
6. Calculate crack dimensions (if a crack is propagating).
7. Start at (1.) again.

We apply the conservation equations and thermodynamic relationships as finite difference equations to a finite spatial grid and use an explicit technique for advancing in time. The coordinate system is a two-dimensional, cylindrical (r, z) system with an axis of symmetry. Parts of the calculation use an implicit scheme for advancing in time.

III. CONSERVATION EQUATIONS

Conservation of mass requires that the rate of mass accumulation in a volume element dV be equal to the negative divergence of the mass flux through the element. Simply:

$$\frac{\partial \rho}{\partial t} dV = -\nabla \cdot \vec{\phi} dV = -\nabla \cdot \rho \vec{v} dV$$

where ρ is the density, ϕ the mass flux, v the velocity of the fluid. Since our porous material has a porosity of ϵ and the fluid only flows through that pore space, the above equation for a volume element dV containing porous material and fluid is:

$$\epsilon \frac{\partial \rho}{\partial t} = -\nabla \cdot \rho \vec{v} \quad (1)$$

where v is now defined as the volumetric flow velocity ($m^3/s \cdot \text{area}$).

A basic assumption is that the flow velocity through the porous material obeys Darcy's law.

Darcy's law says simply that the velocity is proportional to the pressure gradient:

$$\vec{v} = \frac{-K}{\mu} \nabla P \quad (2)$$

where K is the permeability, μ the viscosity, and P the pressure.

Energy transport assumptions result in the relationship for the fluid:

Net heat gain = heat gained - heat lost

or

$$\frac{dU}{dt} = -\nabla \cdot \rho \vec{v} C_p T - \beta \Delta T / \epsilon \quad (3)$$

where the first term on the right is the heat convection due to fluid flow, and the second term is the loss due to conduction into the medium. C_p is the specific heat of the fluid at constant pressure; T is the temperature of the fluid; ΔT is the temperature difference between the fluid and medium, $(T_f - T_m)$; β is the volumetric conductivity of the medium ($J/^\circ C \cdot m^3 \cdot s$); and U is the fluid energy per unit volume in the volume element.

We found it helpful to look at the energy flow from a different point of view. Energy conservation can also be written in the form used in Ref. 1 (Chap. 15) called by the authors "Macroscopic Energy Balance" (in one dimension for simplicity and without the heat conduction terms):

$$\begin{aligned} \frac{d}{dt} (U_{tot} + K_{tot} + \phi_{tot}) dV = & \rho_1 v_1 U_1 A_1 - \rho_2 v_2 U_2 A_2 \quad (4) \\ & + \frac{1}{2} \rho_1 v_1^3 A_1 - \frac{1}{2} \rho_2 v_2^3 A_2 \\ & + \rho_1 v_1 \phi_1 A_1 - \rho_2 v_2 \phi_2 A_2 \\ & + Q - W \\ & + P_1 v_1 A_1 - P_2 v_2 A_2 \end{aligned}$$

where U_{tot} is the total internal energy density, K_{tot} is the total kinetic energy density and ϕ_{tot} is the total potential energy density. ρ_1 , v_1 , P_1 and A_1 are the density, velocity, pressure, and area, respectively, at the one side of the element dV , and the subscript 2 just applies to the other side. Note U_1 and U_2 are energies per unit mass. Q is the heat added to the system, and W is the work done by the system (excluding flow work). Since we are using Eulerian

coordinates, W is zero. Q is usually zero unless we have a source term. On the right-hand side of Eq.

(4):

Line 1 is the flux of internal energy;

Line 2 is the flux of kinetic energy;

Line 3 is the flux of potential energy;

Line 5 is the power applied through the surfaces A_1 and A_2 (i.e., a work component not included in W).

We will ignore the kinetic and potential energy terms and need to show later that they are negligible. Hence 4 becomes

$$\frac{dU_{\text{tot}}}{dt} dV = \rho_1 v_1 U_1 A_1 - \rho_2 v_2 U_2 A_2 + P_1 v_1 A_1 - P_2 v_2 A_2 \quad (5)$$

Dividing by the volume ($dV = A dx$) where $A \sim A_1 \sim A_2$ for very small dx and taking the limit as $dx \rightarrow 0$:

$$\frac{dU_{\text{tot}}}{dt} = -\vec{v} \cdot (\rho U + P) \vec{v} \quad (6)$$

When one considers only the fluid filling the pore space in the volume dV and the possibility for heat exchange between the fluid and the porous medium, the second term on the right side of Eq. (3) must be added to Eq. (6) as a source term:

$$\frac{dU_{\text{tot}}}{dt} = -\vec{v} \cdot (\rho U + P) \vec{v} - \beta \Delta T / \epsilon \quad (7)$$

Note that comparison of Eqs. (3) and (7), with the realization that $U_{\text{tot}} = U$ (as defined in Eq. (3)), implies that $\rho v C_p T = (\rho U + P) v$ or $C_p T = U + P/\rho$. But these are both common definitions of specific enthalpy (per unit mass) and therefore equal (see Ref. 2, p. 180). The term $(\rho U + P) v$ might be called an enthalpy flux.

The energy conduction into the medium from the fluid is described by:

$$\frac{\partial U_m}{\partial t} = \rho_m C_m \frac{\partial T_m}{\partial t} = \beta \Delta T \quad (8)$$

where ρ_m , C_m , and T_m are the medium density, specific heat, and temperature, respectively.

Usually the energy conducted through the rock is very small compared to the energy transferred by fluid flow. However, under special conditions of very low permeability outside the crack and high steam temperatures inside the crack, the major loss of energy through the crack walls is by conduction. For that reason conduction from cell to cell is included:

$$\frac{dU_m}{dt} dV \text{ (due to conduction)} = -\nabla \cdot \kappa \nabla T dV \quad (9)$$

where κ is the rock conductivity.

Adding Eqs. (8) and (9) yields the total energy accumulation rate due to conduction from the fluid and from adjacent rock:

$$\rho_m C_m \frac{\partial T_m}{\partial t} = \beta \Delta T - \nabla \cdot \kappa \nabla T \quad (10)$$

Equations (1), (2), (7), and (10) are adequate to describe the model used for flow through the porous medium. However, some equation-of-state relationship is required to relate the energy and mass density of the steam and water to a pressure for the mixture.

IV. EQUATIONS OF STATE

The pore space of a given spatial zone can be occupied by water alone, water and steam, or steam alone. The first is the fully saturated case and will be discussed later. The second case (water and steam) is also called the saturated situation but refers to the condition of steam coexisting with water. For this condition any one of the state variables (steam or water density, temperature, or pressure) is sufficient to determine the distribution of the energy and mass in a zone between steam and liquid water. KRAK calculates and keeps the total fluid mass for a zone and the average energy per unit volume. By use of an iteration scheme the density of the steam in a mixed zone (i.e., one filled with steam and water) is determined. Once the steam density is known, the pressure and the temperature are easily obtained from the saturation curves.

A zone is defined as mixed if the internal energy is less than 2512 J/g (600 cal/g). Figure B-4 (Appendix B) shows the actual internal energy of steam on the saturation curve as a function of temperature.

Only at pressures above 14.5 MPa (145 bars) is the value of 2512 J/g wrong by more than 84 J/g.

The total energy density of the steam-water mixture is:

$$E_f = \bar{C}_w \left[\rho_w (1-F) + \rho_s F \right] T + \rho_s H_v F \quad (11)$$

where H_v is the heat of vaporization, F is the steam volume fraction, and ρ_w and ρ_s are the water and steam densities, respectively. $\bar{C}_w$ is defined as

$$\bar{C}_w \equiv \frac{1}{T_s} \int_0^{T_s} C_w(T) dT \quad (12)$$

where T_s is just the saturation temperature and C_w is the specific heat of water as a function of T .

Fortunately $\bar{C}_w T + H_v$ on the saturation curve is within 84 J/g of 2512 J/g over the range of interest (i.e., 0°C to 320°C). At ~370°C (the critical temperature) the error is ~390 J/g. Therefore a reasonably good approximation is:

$$E_f^{\text{tot}} = \bar{C}_w \rho_w T (1-F) + (2512 \text{ J/g}) \rho_s F \quad (13)$$

The temperature as a function of steam density on the saturation curve is fit reasonably well by:

$$\begin{aligned} T &= 47.264 \ln \rho_s + 450.6^\circ\text{C} \text{ for } \rho_s \geq 6 \times 10^{-4} \text{ Mg/m}^3 \\ \text{and} \\ T &= 24.16 \ln \rho_s + 279.31^\circ\text{C} \text{ for } \rho_s < 6 \times 10^{-4} \text{ Mg/m}^3 \end{aligned} \quad (14)$$

Figure B-2 (Appendix B) shows these fits versus experimental data.

The density of water as a function of temperature is described reasonably well by,

$$\frac{1}{\rho_w} = 1 + (T/475)^2; (T \text{ in } ^\circ\text{C}) \quad (15)$$

See Figure B-3 (Appendix B).

If the internal energy is greater than 2512 J/g, the pore space is assumed to be filled with superheated steam. Superheated steam is assumed to be adequately described by the perfect gas equation except that the perfect gas relationship is normalized

to provide a smooth transition from the saturated region. This "normalization" is done by calculating a temperature difference from the saturation temperature at the known steam density using the excess internal energy above 2512 J/g and a constant specific heat. Then the slope $(\frac{\partial P}{\partial T})$ of the perfect gas equation is used to calculate the pressure change from the saturation pressure at the known steam density. This approach provides a smooth transition in temperature and pressure between the saturated region and the superheated region. Table B-II (Appendix B) shows a few examples of the comparison of this equation of state's results with the steam tables. The results are well within the general approximation level of the code. The coupling with the saturation curve is appropriate from less than 10 to ~20,000 kPa. See Appendix B for more details of the equation-of-state treatment.

Water and steam viscosities are calculated from the following fits:

$$\nu_w = 0.136 T^{-.89} \text{ (Pa}\cdot\text{s)} \quad (16)$$

$$\nu_s = (T/65)^{.552} \times 10^{-5} \text{ (Pa}\cdot\text{s)} \quad (17)$$

These fits are compared to the data in Figs. 1 and 2.

V. TWO-PHASE FLOW

The first model used for the two-phase flow was one which assumed that the phases were mixed homogeneously (e.g., water droplets in steam or steam bubbles in the water). Consequently, there was a need for only one flow velocity as implied by Eq. (1). Energy transport was as described by Eq. (7). However, some major problems became evident. Examination of the difference equations shows that in order to calculate energy and mass flux across cell boundaries, one must average the various parameters at the boundaries. An example is the enthalpy flux portion of Eq. (7). The flow of internal energy through area A_1 minus the flow through area A_2 is:

$$\rho_1 v_1 U_1 A_1 - \rho_2 v_2 U_2 A_2$$

The subscript 1 parameters are the average between cells a and b:

Fig. 1. Viscosity of H₂O (liquid) vs. temperature (T) on saturation curve.

• - SATURATION CURVE DATA
x - SUPERHEATED STEAM

Fig. 2. Viscosity of H₂O (steam) vs. temperature (T) on saturation curve.

$$U_1 \equiv (U_a + U_b) / 2 \quad (18)$$

where $U_a = U'_a / \rho_a$ and $U_b = U'_b / \rho_b$ (primed U 's are energy per unit volume);

$$\rho_1 \equiv (\rho_a + \rho_b) / 2 \quad (19)$$

hence

$$\rho_1 U_1 \equiv \frac{1}{2} (U'_a / \rho_a + U'_b / \rho_b) (\rho_a + \rho_b) \quad (20)$$

However, another reasonable average for $\rho_1 U_1$ develops as follows:

$$\rho_1 U_1 \equiv U'_1 = (U'_a + U'_b) / 2 \quad (21)$$

For slowly varying parameters, the two schemes give reasonably close results. But if $\rho_a \gg \rho_b$ as is commonly the case for the flow of steam into water, the two expressions may be different by a factor of 50 or more. In the simple case where U'_a or $U'_b = 0$, the two averages are different by a factor of 2. Ideally, one would like to pick the scheme which averages over the smallest range. Both methods were tried and the results were not very satisfactory.

Another two-phase flow model is that which is described in Refs. 4, 8, and 9. This model allows the separate but simultaneous flow of steam and water and avoids some of the interpolation problems just described. Each phase has its own velocity for a common pressure gradient.

The velocity is based upon Darcy flow using the viscosity of the separate phase and an effective permeability for that phase. We prefer to think of the effective permeability as the normal medium permeability multiplied by the fraction of the flow area effectively open to flow of that phase. Hence,

$$v_1 = - \frac{K f_1}{\mu_1} \nabla P \quad (22)$$

Morrison (Ref. 9) uses an empirically derived formulation for f_1 such that $f_1 = a_1^3$ where a_1 is the volume fraction of the original unfilled pore space now occupied by the 1th phase.

The net result is that one applies the Eqs. (1), (2), and (7) to each phase. The equations are coupled by the fact that a given volume element contains the mass and energy of both phases. The advantage of the model is that one averages properties between steam

and steam, or water and water, and not between different phases.

VI. TREATMENT OF FULLY SATURATED ZONES

Since the flow equations are not limited to gas flow, one can in principle calculate the flow of liquid water as well as steam. In fact, the compressibility of water is so low that the time step required for water flow under large pressure gradients makes the computer time requirements prohibitive. Consequently, the KRAK code runs well as long as none of the zones become fully saturated. When such a situation develops it is treated as a special case.

Liquid water is assumed to be incompressible so that when a mass change in a zone occurs which would cause the zone to exceed full saturation, the excess mass above that required to reach full saturation is passed on to the adjacent cell through the top or right-hand boundaries (because the mesh sweep is such that the left and bottom adjacent zones have already been calculated). The saturated zone's pressure is raised artificially until the excess mass is lost.

The new pressure in the saturated zone is determined by interpolation from the surrounding zones, rather than via the equation-of-state relations, unless the pressure from the equation-of-state calculation exceeds the interpolated pressure. The larger of the two pressures is used. If the mesh were swept in alternating directions the bias introduced by forcing the excess mass into uncalculated zones might be negated, but currently the mesh sweep is only from left to right and bottom to top. For a typical crack propagation calculation where the crack starts in the lower left-hand corner, the bias is probably not unrealistic. Whereas the saturated zone is not treated rigorously, the effect upon the overall steam flow is reasonable; namely, saturated zones are regions of very low permeability and the saturated regions can be pushed about by the pressure field.

VII. FLOW IN THE CRACK

Steam and/or liquid water flow in the crack is usually at much higher velocities than the flow through a porous region. Consequently the flow is based upon a different model. The classic model most appropriate seems to be for flow between rough parallel plates. The flow equation called the Fanning

equation, ascribes the pressure drop to the inertia of the fluid plus friction. Hence,

$$\frac{\partial P}{\partial r} = -\rho \frac{dv}{dt} - \frac{2fv^2\rho}{D_e} \quad (23)$$

where P is the pressure in the crack, ρ the fluid density, v the fluid velocity in the r -direction (i.e., along the crack), f the Fanning friction factor, and D_e the "equivalent diameter" of the crack. The Fanning friction factor is a function of the Reynolds' number of the flow. f and D_e are described in Appendix C.

Rewriting the above equation in Eulerian form and solving for $\frac{\partial v}{\partial t}$,

$$\frac{\partial v}{\partial t} = -\frac{1}{\rho} \frac{\partial P}{\partial r} - v \frac{\partial v}{\partial r} - 2 \frac{fv^2}{D_e} \quad (24)$$

where we have put $\frac{dv}{dt} = \frac{\partial v}{\partial t} + v \frac{\partial v}{\partial r}$.

This equation is used to calculate the change in the fluid velocity for each time step. A Crank-Nicholson type of implicit solution is used for the nonlinear set of simultaneous equations for the zones along the crack. Appendix C gives more of the details. The solution is not fully implicit because the pressure gradient is assumed constant during each time step.

VIII. CRACK DIMENSIONS

The crack is assumed to be "penny-shaped"; that is, very flat, with axial symmetry and an approximately elliptical cross section. The crack propagates in the r -direction along the line $z = 0$. By symmetry, only one quadrant need be calculated. In general, the crack does not begin at $r = 0$; there can be an offset, r_c , representing, for example, the radius of a large cavity. To describe the opening of the crack, the nodes of the mesh representing the porous medium are moved. Since our primary concern is not in the detailed dynamics of this motion but only in the resulting crack size, we adopt a quasi-static approach in which the crack is allowed to reach its equilibrium shape at each time step. This assumption will be valid when the speed of crack

propagation is much less than the elastic wave speeds in the medium. We assume further that the only significant motion resulting from the opening of the crack is in the z-direction.

The crack is allowed to propagate to a point at which the pressure at the tip just exceeds some critical value, which is typically given by the tensile strength of the medium plus the stress across the crack (e.g., the horizontal stress in the medium in the case of a crack propagating vertically). For the assumptions stated, the shape of the crack can be computed from an integral expression due to Sneddon (Ref. 7) involving the pressure along the crack, the confining stress, and the Young's modulus and Poisson's ratio of the medium. The details are given in Appendix D. This calculation determines the displacement of each node of the mesh along the crack. At the present time, the nodes outside the crack are all displaced similarly to preserve their volume. In the real situation they would undergo some compression, but this is difficult to calculate exactly.

IX. ZONING

The output of any finite difference program will be dependent to some extent on the sizes of the zones used, since these will affect the determinations of the gradients. If the gradients are not too steep, so that the change across a single zone is small, the accuracy of the results will not be greatly affected. In the present case this seems to be true for the radial zones extending along the crack. Since flow along the crack is relatively rapid, steep pressure gradients cannot be maintained for any appreciable period of time, except perhaps near the crack tip.

Figure 3 shows crack length vs. time for a typical run (the parameters of this are the same as for the "standard" run given in the next section except that the medium permeability is 0.001 darcy) with three different values for the length of the initial radial zone. For the largest zone size (150 mm) the crack, which starts with a length of one-half the first zone, initially advances a little more rapidly than in the other two cases. The coarser zoning also produces a less smooth curve. Successive cuts by a factor of three in zone size, however, show that the crack length becomes less and less affected. Furthermore, the slopes of all the curves approach the

Fig. 3. Effect of radial zone size on crack growth.

same value, indicating that the crack velocity is independent of zone size. Although the 15-mm zone presumably gives the most accurate results, we used the 50-mm zone in subsequent runs since the error was small and the computing time was reduced.

The situation for the axial zoning (perpendicular to the faces of the crack) is not so simple. In this case the flow is governed by the Darcy equation instead of the Fanning equation so that the velocity is directly proportional to the pressure gradient. This gradient is generally quite steep, especially when the crack has just opened; then, high pressure steam comes quickly into contact with the crack face, which is customarily at ambient (approximately one atmosphere) pressure. The calculated pressure gradient, and consequently the flow velocity, is then inversely proportional to the thickness of the axial zone just outside the crack. Since the crack propagation rate is dependent on a balance between the flow along the crack and the Darcy flow through the faces of the crack, it is to be expected that this zoning will be critical.

Two opposing effects occur, however, as the axial zone size becomes smaller. The smaller zone

results in a larger calculated gradient and a greater flow through the crack face. But this high flow velocity into a smaller zone causes the zone to fill rapidly and develop a back pressure which inhibits further flow. Figure 4 shows the results of this same case for a series of runs, with the indicated widths for the first axial zone. With a 10-mm-wide zone, the crack propagates rapidly. Reducing the zone size to 1 mm and then to 0.3 mm causes a pronounced reduction not only in the initial growth but in the final velocity attained. Further decreases in zone size actually cause a slight speedup in crack growth because of the rapid development of back pressure, but the final velocity is relatively unaffected. Apparently, to insure that the axial zoning does not substantially affect the results, it is necessary to make the zones small enough so that the "dead time" required for the flow to build up an appreciable back pressure is small compared to the time

required for the crack to propagate through a single radial zone.

Unfortunately, the extremely small axial zone width required in the present example — approximately 0.1 mm or less — leads to practical problems. Small zones require a much smaller time step to maintain stability, and consequently a great increase in computing time. To avoid these difficulties, we developed a so-called "penetration depth approximation" to allow the use of a larger axial zone size without introducing a correspondingly inaccurate pressure gradient. The idea is that the pressure gradient between the crack and the first axial zone, instead of being calculated in the usual way between zone centers, is calculated only across a distance corresponding to the degree to which the zone has been "filled". The quantity used to fill the zone can either be actual fluid from the crack, or energy. Both methods have been tried and the energy method has given better results — that is, results closer to the finely zoned case.

In the energy controlled penetration depth approximation, the idea is that the energy flowing into the first axial zone is used first to heat the water immediately adjacent to the crack instead of being spread uniformly through the zone. As soon as the water next to the crack has reached some critical temperature — say 100° C, where back pressure might begin to be apparent — the next layer of water is heated, and so on. In practice, the code modification is quite simple. Initially, the pressure gradient at the face of a newly cracked zone is calculated using only the half-width of the crack itself (equivalent to a zero width of the adjacent axial zone). Then all the flows are calculated in the normal way, and the state of the fluid in the first axial zone is computed from the equation of state assuming equilibrium throughout the zone, as is done in the rest of the program. This leads to a temperature T for the fluid in the zone which is higher than the initial temperature T_0 . Then the penetration depth is calculated as

$$L = \Delta z \left(\frac{T - T_0}{T_c - T_0} \right) \quad (25)$$

where T_c is the chosen critical temperature and Δz is the thickness of the first axial zone. For a

Fig. 4. Effect of axial zone size on crack growth.

constant specific heat, this equation merely corresponds to putting all the additional energy of the zone into a layer of thickness L . In the next time step, the pressure gradient is calculated using L plus the half-width of the crack. As soon as L becomes equal to $(\Delta z/2)$, the calculation is returned to the normal procedure.

In Fig. 5 we see the effect of using this penetration depth method with a value of $T_c = 100^\circ\text{C}$ and with various axial zone sizes. Although the effect of zone size is still evident, the 1-mm curve now lies very close to the very finely zoned conventional runs (the dashed curve is the 0.1-mm case from the previous graph). Our subsequent parameter study was done using this energy controlled penetration depth approximation with a first axial zone of essentially 1-mm thickness and $T_c = 100^\circ\text{C}$.

Fig. 5. Effect of axial zone size on crack growth with penetration depth approximation. (The dashed curve is from the previous figure for comparison).

The choice of critical temperature, T_c , will also affect the problem somewhat, though not too seriously, as shown in Fig. 6. Here the three solid curves are repeated from the previous graph, and the corresponding dashed curves show the same cases with $T_c = 170^\circ\text{C}$. Since more energy is now required to fill the zone, the crack does not propagate quite as far in a given time, but the crack velocity is approximately the same.

It should be noted that even though the penetration depth method allowed the use of a zone size of 1 mm instead of 0.1 mm, the zoning is still much finer than might have been considered necessary a priori. Careful attention to zone size is required in any problem involving Darcy flow and steep pressure gradients. The aspect ratio of the zones in this problem is quite large since the smallest radial zone is 50 mm in length while the crack itself starts at

Fig. 6. Effect of T_c on crack growth with penetration depth approximation.

a width of 0.02 mm. Because of the disparity in radial and axial flow rates in the crack, a large aspect ratio is required, and with care instabilities can be avoided. As the problem advances, the crack widens considerably at its base, but the conditions near the tip of the crack remain about the same.

X. RESULTS

A. Comparison with Other Calculations

Ideally one would like to test the KRAK code against experimental results. We do not have experimental results available yet, but we have compared some simple KRAK calculations with appropriate analytical results.

By injecting high-temperature dry steam into a porous medium of the same temperature, one can reasonably approximate the conditions of isothermal perfect gas flow in a porous medium. This problem has been treated analytically by Morrison (Ref. 3) for one-dimensional flow in a pipe as a function of time. By allowing the flow in a pipe to reach a steady state we can also compare the pressure distribution with the very simple one-dimensional steady state solution

$$P(x) = \left[P_0^2 - \frac{x}{L} (P_0^2 - P_L^2) \right]^{1/2}$$

where P_0 is the inlet pressure, P_L is the outlet pressure, and L is the length of the porous column.

Table I lists the initial conditions used for the comparison calculations. Figure 7 is a comparison of the KRAK pressure distribution (at 13 s) with the analytic results for steady state flow. Steady state flow was achieved after about 3 seconds.

The results from Morrison's transient calculations give the time required for the complete expulsion of the original gas in the pore space, for our

TABLE I

INITIAL CONDITIONS FOR KRAK COMPARISON CALCULATION

Source Pressure = 3.44 MPa
Pipe Length = 0.37 m
Permeability = 0.1 darcy
Porosity = 0.3
Temperature of Source and Medium = 600° C
Fluid - Dry Steam

Fig. 7. Comparison of KRAK with analytic results for steady state pressure distribution.

geometry, as ~2.08 s. The KRAK code calculates the complete expulsion as occurring at ~2.3 s.

Figure 8 compares Morrison's pressure distribution at 0.8 s before steady state is achieved, with the KRAK results.

In light of the fact that the KRAK calculation is not exactly isothermal, the comparisons are considered quite good. Because the KRAK code does allow the gas to cool upon expansion, the pressure front should lag the isothermal front as it does.

B. Parameter Study

As an illustration of the uses of the KRAK program, and as a preliminary investigation of the significance of various source parameters and medium characteristics on the rate of crack propagation, we have performed a limited parameter study. Clearly a complete investigation of the parameter space is not feasible. Hence we have picked a single representative case and then varied individual parameters around the value chosen for that case. Table II gives

Fig. 8. Comparison of the KRAK and analytic transient results.

TABLE II
INPUT VALUES FOR THE STANDARD CASE

<u>Medium</u>	<u>Crack</u>
1. Porosity — 30%	1. Conductivity:
2. Saturation — 20%	Volumetric — $7.0 \text{ MJm}^{-3}\text{C}^{-1}\text{s}^{-1}$
3. Density — 2.0 Mg/m^3	Rock — $8.4 \text{ J}^\circ\text{C}^{-1}\text{m}^{-1}\text{s}^{-1}$
4. Permeability — 0.002 darcy	2. "Density" — 1.0 Mg/m^3
5. Young's modulus — 3000 MPa	3. Friction factor — VARIABLE
6. Poisson's ratio — 0.3	4. Initial permeability — 0.002 darcy
7. Conductivity:	<u>Source</u>
Volumetric — $7.0 \text{ MJm}^{-3}\text{C}^{-1}\text{s}^{-1}$	1. Temperature — 1000°C
Rock — $8.4 \text{ J}^\circ\text{C}^{-1}\text{m}^{-1}\text{s}^{-1}$	2. Density — 0.16 Mg/m^3
8. Temperature — 30°C	3. Pressure — 8275 kPa
9. Tensile strength — 0 kPa	4. Steam fraction — 1.0
	5. Cavity radius — 20 m

the main input parameters for this "standard" case, which is intended to bear a slight resemblance to NTS tuff.

A very detailed physical description of conditions at all points in the mesh as a function of time is available, if desired, as output from the KRAK code. Generally, for this parameter study we present only curves of crack length vs. time. However, for the standard case we show more details, both to give the reader a better idea of the working of the program and to illustrate some of the output options available. Figure 9 shows the region of the mesh near the crack at a time of 0.05 sec after the initiation

of the problem. Note that the axial or z-coordinate is not on the same scale as the r-coordinate; the crack is very long and narrow. Figure 10 gives the pressure distribution within the crack at the same time. Figure 11 gives the width at the base of the crack as a function of time. The slight waviness in the curve is an effect of zoning. The net amount of mass flowing into the crack as a function of time is given in Fig. 12. The energy flow into the crack is also monitored and is quite similar in form. Finally, a typical pressure history is shown in Fig. 13

Fig. 9. Shape of crack at 0.05 s — standard run.

Fig. 10. Pressure distribution inside crack at 0.05 s — standard run.

Fig. 11. Crack width at base vs. time.

for a zone in the crack centered at 80 mm past the cavity boundary. The pressure rises steeply when the crack first breaks into the zone. The waviness again reflects zoning farther up the crack, since the breaking of a new zone will send a small rarefaction reverberating along the crack. Time histories of other variables, such as temperature, saturation, etc. are also available for any desired zone in the mesh.

Fig. 12. Total steam injected into crack.

Fig. 13. Pressure history in crack 80 mm from base of crack.

The parameters which were varied in the calculations were the permeability, saturation, porosity and tensile strength of the medium, the source pressure, and the depth of burial of the source. Figure 14 shows the effect of permeability, which is perhaps

Fig. 14. Effect of permeability on crack growth.

the most sensitive single parameter. The curve marked 0.002 darcy is our standard run. An increase in permeability, which increases the leakage rate through the crack faces, sharply reduces the crack velocity. For the present case, the crack will not propagate at permeabilities much greater than 5 millidarcys.

The effect of saturation is shown in Fig. 15, where the curve labeled 20% is our standard. Here the behavior is a little more difficult to explain. A study of the details of the calculation suggests that for the case of 50% saturation the crack propagates more slowly because the heat capacity of the additional water in the medium is quite effective in quenching the steam passing through the crack faces. This delays the buildup of back pressure in the zone next to the crack. However, in the 95% case only a small amount of steam needs to be quenched to produce full saturation in the zone next to the crack, and a fully saturated zone has a smaller effective permeability. If we had run to later times, we would expect a saturated region to develop for all cases. However, one must remember that it is the situation

near the crack tip which most strongly influences the growth rate.

Figure 16 shows the effect of decreasing the porosity from the 30% value of the standard run. Note that the permeability is the same for each case. The crack velocity increases as the volume of pore space decreases, since the back pressure develops more rapidly and thus the axial leakage rate is decreased.

The effect of source pressure is quite strong, as shown in Fig. 17. The pressure was varied from 6895 kPa to 9653 kPa (1000 to 1400 psi.) with the standard run at 8274 kPa (1200 psi.). The extremely large changes in crack velocity illustrate the strong positive feedback which is operative in the problem. A higher source pressure not only is able to crack the medium more easily by overcoming the compressive stresses or material strength resisting fracture, but by opening the crack wider allows greater flow along the crack. Thus the high-pressure steam is transmitted much more rapidly to the crack tip promoting further fracture. There is at the same time somewhat greater leakage through the crack faces, but this effect is simply proportional to the pressure

Fig. 15. Effects of saturation on crack growth.

Fig. 16. Effect of porosity on crack growth.

Fig. 17. Effect of source pressure on crack growth.

Fig. 18. Effect of depth of burial on crack growth.

increase and does not counterbalance the other processes.

A similar effect is seen when the depth of burial is varied, as shown in Fig. 18, where the standard run is labeled 304.8 m. In this problem, a change in depth of burial simply produces a proportional change in the compressive stresses resisting fracture.

The effect of varying the tensile strength of the medium is also similar but not quite identical. Figure 19, which shows the results, does not contain the standard run. Instead, all these cases were run at a higher source pressure of 9653 kPa (1400 psi.), and the curves are labeled with values of tensile strength in kPa (note 100 kPa = 1 bar). The curve marked 0 is the same as the high pressure curve in Fig. 17. An increased tensile strength requires a higher pressure to initiate cracking. However, once cracking has occurred in any radial zone, the tensile strength plays no further role and the width of the crack (which helps control the radial flow rate) is restricted only by the compressive stresses due to overburden. Thus the effect is somewhat different from an increase in depth of burial. Small tensile

Fig. 19. Effect of tensile strength on crack growth.

strengths make comparatively little changes in propagation velocity, but a large enough value can, of course, prevent crack propagation entirely.

APPENDIX A

KRAK CODE DIFFERENCE EQUATIONS

The four equations of mass, momentum and energy conservation (equations 1, 2, 7, and 10) are:

$$c \frac{\partial \rho}{\partial t} = -\nabla \cdot \rho \vec{v} \quad (A-1)$$

$$\vec{v} = -\frac{K}{\mu} \nabla P \quad (A-2)$$

$$\frac{\partial U}{\partial t} = \nabla \cdot (\rho U \vec{v}) - \beta \Delta T / c \quad (\text{for the fluid}) \quad (A-3)$$

$$\rho_m C_m \frac{\partial T_m}{\partial t} = \beta \Delta T - \nabla \cdot \kappa \nabla T \quad (\text{for the porous medium}) \quad (A-4)$$

The difference approximations used are based upon the mesh definition sketched in Fig. A-1.

$\vec{v}$ is reduced to its components in the r and z directions:

$$v_r = \frac{-K_{ij}^r (P_{i,j-1} - P_{i,j+1})}{\mu_{ij}^r (r_{i,j-1} - r_{i,j+1}) / 2} \quad (A-5)$$

Fig. A-1. The KRAK mesh.

$$v_{r+dr} = \frac{-K_{i,j+1}^r (P_{i,j} - P_{i,j+2})}{\mu_{i,j+1}^r (r_{i,j} - r_{i,j+2}) / 2} \quad (A-6)$$

$$v_z = \frac{-K_{i,j}^z (P_{i-1,j} - P_{i+1,j})}{\mu_{i,j}^z (z_{i-1,j} - z_{i+1,j}) / 2} \quad (A-7)$$

$$v_{z+dz} = \frac{-K_{i+1,j}^z (P_{i,j} - P_{i+2,j})}{\mu_{i+1,j}^z (z_{i,j} - z_{i+2,j}) / 2} \quad (A-8)$$

where

$$K_{ij}^z = \frac{z_{i+1,j} - z_{i-1,j}}{\left(\frac{z_{i+1,j} - z_{i,j}}{K_{ij}} \right) + \left(\frac{z_{i,j} - z_{i-1,j}}{K_{i-1,j}} \right)} \quad (A-9)$$

$$K_{ij}^r = \frac{r_{i,j+1} - r_{i,j-1}}{\left(\frac{r_{i,j+1} - r_{i,j}}{K_{ij}} \right) + \left(\frac{r_{i,j} - r_{i,j-1}}{K_{i,j-1}} \right)} \quad (A-10)$$

and K_{ij} is the permeability of the ij^{th} cell.

The densities at the four interfaces are:

$$\rho_r = (\rho_{i,j-1} + \rho_{i,j}) / 2$$

$$\rho_{r+dr} = (\rho_{i,j} + \rho_{i,j+1}) / 2$$

$$\rho_z = (\rho_{i-1,j} + \rho_{i,j}) / 2$$

$$\rho_{z+dz} = (\rho_{i,j} + \rho_{i+1,j}) / 2$$

The temperatures at the interfaces are:

$$T_r = (T_{i,j-1} + T_{i,j}) / 2$$

and similarly for T_{r+dr} (i.e., simple linear averaging). Note that temperatures are needed only at the interfaces to calculate temperature-dependent properties such as viscosity.

Currently, different approximations are used for the mixed flow calculation and the two-velocity flow calculation. For the mixed flow calculation

$$\rho_r U_r \equiv (U_{i,j-1} + U_{ij}) (\rho_{i,j-1} + \rho_{ij}) / 4$$

and a corresponding expression at $r+dr$ (i.e., linear averaging of each factor).

For the two-velocity (duoflo) model we have a small preference for

$$\rho_r U_r \equiv (\rho_{i,j-1} U_{i,j-1} + \rho_{ij} U_{ij}) / 2$$

For both cases,

$$P_r \equiv (P_{i,j-1} + P_{ij}) / 2$$

Equation (A-1) can be written as

$$\begin{aligned} c \frac{\partial \rho}{\partial t} \Delta V = & \rho_r v_r A_r - \rho_{r+dr} v_{r+dr} A_{r+dr} \\ & + \rho_z v_z A_z - \rho_{z+dz} v_{z+dz} A_{z+dz} \end{aligned}$$

where, $A_r = 2\pi r_{ij} (z_{i+1,j} - z_{ij})$

$$A_{r+dr} = 2\pi r_{i,j+1} (z_{i+1,j+1} - z_{i,j+1})$$

$$A_z = \pi (r_{i,j+1}^2 - r_{ij}^2)$$

$$A_{z+dz} = \pi (r_{i+1,j+1}^2 - r_{i+1,j}^2)$$

and $\Delta V \equiv (A_z + A_{z+dz}) (z_{i+1,j} - z_{ij}) / 2$

The first term on the right side of Eq. (A-3) is just an enthalpy flux similar to the mass flux above. Therefore

$$\begin{aligned} \frac{\partial U}{\partial t} \Delta V = & (\rho_r U_r + P_r) v_r A_r - (\rho_{r+dr} U_{r+dr} + P_{r+dr}) \\ & \times (v_{r+dr} A_{r+dr}) + (\text{the } z\text{-components}) \\ & - \beta (T_f - T_m) / \epsilon \end{aligned}$$

where U is the internal energy of the fluid. Note, the above equation, with the exception of the conduction term, is applied to each phase for the duoflo model. The total energy change is then the sum of the changes for each phase.

The energy change in the medium is

$$\rho_m C_m \frac{\partial T_m}{\partial t} = \beta \Delta T - \nabla \cdot \kappa \nabla T$$

Here we express the divergence of the heat flux in difference form as follows:

$$\begin{aligned} \nabla \cdot \kappa \nabla T = & \frac{1}{\Delta V} \left[\left(\kappa_{r+dr} \frac{\Delta T}{\Delta r} \right)_{r+dr} A_{r+dr} - \kappa_r \frac{\Delta T}{\Delta r} \right]_r A_r \\ & + \left(\kappa_{z+dz} \frac{\Delta T}{\Delta z} \right)_{z+dz} A_{z+dz} - \kappa_z \frac{\Delta T}{\Delta z} \right]_z A_z \end{aligned}$$

$$\text{where } \frac{\Delta T}{\Delta r} \Big|_{r+dr} = 2 (T_{i,j+1} - T_{ij}) / (r_{i,j+1} - r_{ij})$$

$$\frac{\Delta T}{\Delta r} \Big|_r = 2 (T_{ij} - T_{i,j-1}) / (r_{ij} - r_{i,j-1})$$

$$\frac{\Delta T}{\Delta z} \Big|_{z+dz} = 2 (T_{i+1,j} - T_{ij}) / (z_{i+1,j} - z_{ij})$$

$$\frac{\Delta T}{\Delta z} \Big|_z = 2 (T_{ij} - T_{i-1,j}) / (z_{ij} - z_{i-1,j})$$

and

$$\kappa_r = (\kappa_{i,j-1} + \kappa_{ij}) / 2$$

$$\kappa_{r+dr} = (\kappa_{ij} + \kappa_{i,j+1}) / 2$$

$$\kappa_z = (\kappa_{i-1,j} + \kappa_{ij}) / 2$$

$$\kappa_{z+dz} = (\kappa_{ij} + \kappa_{i+1,j}) / 2$$

Therefore

$$\begin{aligned} \Delta T_m = & \frac{\partial T_m}{\partial t} \Delta t \\ = & \frac{\beta (T_f - T_m) \Delta t}{\rho_m C_m} - \frac{2}{\rho_m C_m} \\ & \times \left[\left(\kappa_{r+dr} \frac{\Delta T}{\Delta r} \right)_{r+dr} A_{r+dr} - \kappa_r \frac{\Delta T}{\Delta r} \right]_r A_r + (z\text{-terms}) \Delta t. \end{aligned}$$

Likewise, the fluid mass change, ΔM , is

$$\Delta M = c \frac{\partial \rho}{\partial t} \Delta V \Delta t$$

and the fluid energy change is

$$\Delta U = \frac{\partial U}{\partial t} \Delta t$$

APPENDIX B

EQUATION-OF-STATE DETAILS

The difficulty of treating two-phase fluid flow becomes evident when one tries to calculate the pressure of a two-phase mixture of water and steam. The following definitions apply to the notation used for describing the method used in the KRAK code:

ρ_s $\equiv$ steam density

ρ_w $\equiv$ liquid water density

M $\equiv$ total fluid mass

V $\equiv$ fluid volume = cdV

$\bar{\rho}$ $\equiv$ average fluid density

F $\equiv$ the volume fraction of the pore space occupied by the steam.

Hence

$$\bar{\rho} = M/V = \rho_s F + \rho_w (1-F) \quad (B-1)$$

The most common situation is that in which the pore space of a cell is occupied by both phases (i.e., the steam is saturated). Under that condition it is only necessary to know one of the state variables (e.g., pressure, temperature, steam density, etc...) to determine the others. However, it is not so easy to calculate one of the state variables because of the transcendental equations relating them. The KRAK code uses an iteration scheme to solve for the temperature of the mixture, using the energy density in the pore space and the total fluid mass, as follows:

First a steam density is guessed (usually the one from the previous time step). That steam density is used to calculate a temperature for the mixture based upon the energy density:

$$T_E = (E_f - F \cdot H \cdot \rho_s) / [\bar{C}_w \cdot \rho_w \cdot (1-F)] \quad (B-2)$$

where E_f is the energy per unit volume in the pore space, $\bar{C}_w$ is the average specific heat of liquid

water, and H is the internal energy of saturated steam, ~ 2512 J/g (600 cal/g). This relationship is derived from the simple statement that the total energy in the pore space is the sum of the energy in the steam and in the water.

Another temperature, T_s , is determined from the relationship of temperature to saturated steam density:

$$T_s = 47.264 \ln \rho_s + 450.8^\circ\text{C for } \rho_s \geq 6 \times 10^{-4} \text{ Mg/m}^3$$

and

(B-3)

$$T_s = 24.16 \ln \rho_s + 279.31^\circ\text{C for } \rho_s \leq 6 \times 10^{-4} \text{ Mg/m}^3.$$

T_E and T_s are compared. If they are significantly different, the steam density guess is improved, and a new pair of temperatures is calculated until a temperature is found which satisfies the energy conservation condition and the saturation condition. That temperature is used to calculate the pressure and other necessary state variables.

During each cycle of the above iteration the values for ρ_w and $\bar{C}_w$ must be determined for the new temperature. $\bar{C}_w$ is read from a table (see Table B-1) and ρ_w is determined from

$$\frac{1}{\rho_w} = 1 + (T/475)^2 ; (T \text{ in } ^\circ\text{C}) \quad (B-4)$$

The steam fraction F is given by

$$F = (\bar{\rho} - \rho_w) / (\rho_s - \rho_w) \quad (B-5)$$

which is just an expression of conservation of mass.

The pressure of the steam as a function of saturated steam density is given by

TABLE B-1

INTERNAL ENERGY CONTENT OF WATER VS. TEMPERATURE
(Data from Ref. 6)

Temperature (°C)	$\bar{C}_w$ (J/g)
10	4.186
100	4.186
200	4.261
250	4.345
300	4.483
330	4.626
350	4.776
360	4.893
370	5.115
380	5.115

$$P \text{ (in MPa)} = 169.5 \rho_s \quad (\text{B-6})$$

The quality of the fits of the saturation parameters of pressure, temperature, and water density as a function of the steam density or temperature is seen in Figures B-1, B-2, and B-3. At pressures above 10^7 Pa (100 atmospheres) the fit is poor but that does not affect most of our problems. Better fits can and will be obtained.

The assumption that the internal energy of steam on the saturation curve is a constant 2512 J/g (600 cal/g) is shown to be reasonable, for our purposes, by Fig. B-4, which is a plot of the actual internal energy of saturated steam versus temperature.

If the saturated steam pressure is less than the pore pressure, the pressure in the cell is defined as equal to the pore pressure. Above 600 cal/g the fluid is a superheated steam with a temperature

Fig. B-1. Steam pressure vs. density at saturation. (Data points from Ref. 5).

Fig. B-2. Steam density vs. temperature at saturation. (Data points from Ref. 5).

calculated by assuming the steam was generated along a path which is reasonably easy to calculate (since the choice of path makes no difference in the final relationship of pressure, temperature and internal energy). The path chosen is along the saturation curve to the steam density ρ_s which corresponds to a saturation temperature T_s ; then along a constant volume path to the T , P , and E of the cell's fluid. The temperature is defined as

$$T = T_s + (E/\rho_s - 2512 \text{ J/g}) / C_v \quad (\text{B-7})$$

where C_v is the specific heat at constant volume. In other words, the right-hand term is the temperature increment above the saturation temperature for a perfect gas. Comparisons of the above results to the steam table data are shown in Table B-II.

The superheated steam pressure is calculated in a like manner, with the pressure on the saturation curve at density ρ_s added to the pressure increment for a perfect gas of the above temperature T :

$$P = P_{sat} + \rho_s R (T - T_s) / MW \quad (\text{B-8})$$

Fig. B-3. Water density vs. temperature at saturation. (Data points from Ref. 5).

where P_{sat} is the pressure on the saturation curve at ρ_g and R/MW is the gas constant over the average molecular weight of H_2O (i.e., 18).

A major advantage of this equation of state for superheated steam is that it is continuous with the formulation in the saturated region. Considerable effort might be spent on refinement of the equation-of-state formulation, but other approximations of the KRAK code suggest that a steam-like material is good enough.

Fig. B-4. Internal energy of steam vs. temperature at saturation. (Data points from Ref. 5).

TABLE B-II

COMPARISON OF SUPERHEATED STEAM TEMPERATURE AND PRESSURE CALCULATED WITH KRAK EOS TO STEAM TABLE DATA (Data from Ref. 6)

ρ (Mg/m ³)	Enthalpy (J/kg)	(MPa) (Calc./Steam Tables)	T (°C)
8.40×10^{-3}	2.82×10^6	1.86/1.72	268/215
5.96×10^{-3}	3.19×10^6	1.7/1.7	391/371
9.42×10^{-3}	3.53×10^6	3.25/3.44	541/538
28.9×10^{-3}	2.98×10^6	7.2/6.9	386/338
17.9×10^{-3}	3.63×10^6	6.5/6.9	609/593

APPENDIX C

FLOW IN THE CRACK

I. The Flow Equations

The flow equation described in the main body of the report was as follows:

$$\frac{\partial v}{\partial t} = -\frac{1}{\rho} \frac{\partial P}{\partial r} - v \frac{\partial v}{\partial r} - 2 \frac{f' v^2}{D_e} \quad (C-1)$$

where ρ is the average density of the steam-water mixture in the crack, v the velocity, f' the Fanning friction factor, and D_e the "equivalent diameter" of the crack. D_e is defined in Ref. 10 as

$$D_e = 2\sqrt{2} \Delta z$$

where Δz is the half-width of the crack, so that from Eq. (C-1)

$$\frac{2f' v^2}{D_e} = \frac{f v^2}{\Delta z} \quad (C-2)$$

with $f \equiv f'/\sqrt{2}$.

In our system of cylindrical coordinates, the crack is limited to growth in the radial direction only. Because of azimuthal symmetry the flow velocity determined by Eq. (C-1) is purely radial. Note that there is Darcy flow in the axial direction through the crack walls into the porous medium, but this is usually at much smaller velocity than flow down the crack and is not coupled with it. Figure 9 of the text shows the crack geometry. The calculation of the crack shape is described in Appendix D.

Originally the crack flow velocities at each cell boundary were calculated explicitly, but the velocity distribution was subject to unstable oscillations under certain conditions. A much more satisfactory approach has been to use a semi-implicit method which is described here.

Using the old velocity, v_j , at time t^n and at radial mesh coordinate r_j , a new velocity is calculated at t^{n+1} from the relation

$$v_j^{n+1} = v_j^n + \frac{\partial v_j}{\partial t} \left(t^{n+1} - t^n \right) \quad (C-3)$$

The derivative $\partial v_j / \partial t$ should be the appropriate average over the time interval, Δt . We use the weighted average as follows:

$$\frac{\partial v_j}{\partial t} = \alpha \frac{\partial v_j}{\partial t} \Big|_n + (1-\alpha) \frac{\partial v_j}{\partial t} \Big|_{n+1} \quad (C-4)$$

where α is the fractional weight applied to the derivative at time t^n . We use an α of 0.05.

Combining Eqs. (C-1 thru C-4), we obtain

$$v_j^{n+1} = v_j^n - \left[\alpha \left[\frac{1}{\rho} \frac{\partial P}{\partial r} \Big|_n + v_j^n \frac{\partial v_j}{\partial r} \Big|_n + f \frac{v_j^2}{\Delta z} \Big|_n \right] + (1-\alpha) \left[\frac{1}{\rho} \frac{\partial P}{\partial r} \Big|_{n+1} + v_j^{n+1} \frac{\partial v_j}{\partial r} \Big|_{n+1} + f \frac{v_j^2}{\Delta z} \Big|_{n+1} \right] \right] \Delta t \quad (C-5)$$

Notice that we assume the pressure gradient does not change significantly from t^n to t^{n+1} . Therefore, the solution is not fully implicit. The form of the difference equations for each partial derivative is

$$\frac{1}{\rho} \frac{\partial P}{\partial r} \equiv 4 \frac{(P_{1j} - P_{i,j-1})}{(\rho_{1j} + \rho_{i,j-1})(r_{i,j+1} - r_{i,j-1})} \quad (C-6)$$

$$v_j \frac{\partial v_j}{\partial r} \equiv v_j \frac{(v_{j+1} - v_{j-1})}{(r_{i,j+1} - r_{i,j-1})} \quad (C-7)$$

$$f \frac{v_j^2}{\Delta z} \equiv f \frac{v_j |v_j|}{(z_{i+1,j} - z_{i,j})} \quad (C-8)$$

Equation (C-5) can be written as

$$v_j^{n+1} = v_j^n + a + \alpha G_j^n + (1-\alpha) G_j^{n+1} \quad (C-9)$$

Here we have defined

$$G_j^n \equiv b v_j^n v_{j+1}^n + c v_j^n v_{j-1}^n + d v_j^n \left| v_j^n \right| \quad (C-10)$$

and similarly for G_j^{n+1} ; also

$$a \equiv 4 \frac{-\Delta t (P_j^n - P_{j-1}^n)}{(\rho_j + \rho_{j+1})(r_{j+1} - r_{j-1})}$$

$$b \equiv -\Delta t / (r_{j+1} - r_{j-1})$$

$$c \equiv -b$$

$$d \equiv -\Delta t f / (z_{i+1,j} - z_{i,j})$$

We have dropped the i subscript since it is just the index of the crack zone (i.e., $i = 1$).

At the current time t^n the v^n 's are known so that we can calculate an F_j^n such that

$$F_j^n = v_j^n + a + \alpha G_j^n \quad (C-11)$$

If we now collect all the terms in Eq. (C-9) with superscript $n+1$ to the left-hand side and then divide by v_j^{n+1} , we can write the result in the form

$$Dv_j^{n+1} + Bv_{j+1}^{n+1} + Cv_{j-1}^{n+1} = F_j^n/v_j^{n+1} \quad (C-12)$$

where

$$D \equiv \frac{1}{v_j^{n+1}} - (1-\alpha)d \frac{v_j^{n+1}}{v_j^{n+1}}$$

$$B \equiv -(1-\alpha)b$$

$$C \equiv -(1-\alpha)c$$

Using the factor $|v_j^{n+1}|/v_j^{n+1}$ assures the correct direction for the frictional force.

The reason for all of the manipulation to get Eq. (C-12) is to develop a form for a set of simultaneous equations which can be solved to get the velocity distribution in the crack at time t^{n+1} . Equation (C-12) is one of a set of equations which can be described in matrix form as a skew symmetric tridiagonal matrix. However, the equations are nonlinear unless one can make a very good guess at v_j^{n+1} where it appears in the denominator. That is what we do to get a set of linear equations. We guess a v_j^{n+1} , calculate the matrix of coefficients, solve the matrix for the velocities, correct our guess of v_j^{n+1} , and repeat the process until successive values of the velocities are nearly equal.

In order to solve for the v 's we also need boundary conditions. At the base of the crack we define a constant velocity from the boundary to the center of the first zone. At the first zone beyond the tip, we define the velocity to be zero.

After many other attempts at formulating the problem, the above scheme was found to converge very quickly. It also was the only scheme which converged.

11. The Fanning Friction Factor

A major influence upon the calculated flow in the crack is the friction factor, f . The value of f can change by a factor of ten or more depending upon the crack roughness and the Reynolds number of the flow. We have approximated f by a fit to the experimental data assembled by Schlichting (Ref. 11). Figure C-1 is a plot of the friction factor used by the KRAK code for the existing flow and relative roughness conditions. The roughness is defined as $\sqrt{2}\Delta z/s$, where s is the particle size protruding from the crack wall. If s is greater than the crack width, $2\Delta s$, the roughness is defined to be $\sqrt{2}/2$. The code calculates the Reynolds number and then a friction factor according to the appropriate roughness (Fig. C-1). However, the allowable friction factors are bounded from below by the diagonal line at the bottom. If the Reynolds number is less than 2500, only the single curve on the left is used.

Fig. C-1. Fanning Friction Factor vs. Reynolds Number for Different Values of Roughness.

APPENDIX D

CRACK DIMENSIONS

At the beginning of the crack calculation the pressure is known at the center of each cell along the crack. The crack length is determined first by finding where the given pressure distribution just reaches a critical value P_{cr} given by

$$P_{cr} = \sigma_h + T_0 \quad (D-1)$$

where T_0 = tensile strength of the medium

σ_h = horizontal stress in the medium.

This applies, of course, to the case of a vertically propagating crack. σ_h will vary with r and may be preassigned to each mesh point, as may T_0 . For the case of tuff T_0 may often be neglected. In this special case also σ_h is usually the least principal stress and is due solely to the weight of overburden; it is then given by

$$\sigma_h = \frac{\nu}{1-\nu} \rho_m gH \quad (D-2)$$

where ρ_m = density of the rock medium

ν = Poisson's ratio of the medium

g = acceleration of gravity

H = distance to the surface.

The exact crack length is then determined by the following procedure. First, the farthest zone with a pressure greater than P_{cr} is determined. If the location of the crack tip as determined by a linear extrapolation of the last two pressure points lies within the zone, that location is used. If it would lie in the next zone (where the central pressure is below P_{cr}), the crack tip position is found instead by interpolation between the central pressure in that zone and the boundary pressure at the previous zone as determined by the extrapolation. The crack length c thus depends solely on the current pressure

distribution, and not on its length in the previous time cycle. The crack is allowed to close as well as open.

Given the crack length c , the crack shape is determined using the static solution given by Sneddon (Ref. 7). The displacement Δz in the z -direction of the crack surface (originally very close to $z = 0$) is given by (for $\rho < 1$)

$$\Delta z = \frac{4(1-\nu^2)c}{\pi E} \int_{\rho}^1 \frac{\mu d\mu}{(\mu^2 - \rho^2)^{1/2}} \int_0^1 \frac{x P_e(x\mu c) dx}{(1-x^2)^{1/2}} \quad (D-3)$$

where $P_e(r) = P(r) - \sigma_h(r)$

E = Young's modulus for the medium

$\rho \equiv \frac{r'}{c}$; $r' = r - r_c$

r_c = offset (or cavity) radius.

Here μ and x are integration variables (essentially dummy variables for ρ). Since c and $P(r)$ are known, Δz can be calculated as a function of ρ (i.e., of r). This integral expression is exact for a purely elastic medium and is used since the exact crack size is critical in determining flow down the crack. Plastic effects near the tip of the crack are not taken into account. This probably results in the crack being less blunt in the tip region than the above formula indicates; however, this should not affect the crack flow drastically. The entire mesh is moved by the same amount in the z -direction for each r value; thus, strains in the mesh due to crack expansion are not considered in the current version of the code. Displacements in the r -direction are usually small and in any case do not affect the flow pattern critically; they are neglected here.

The evaluation of the double integral in Eq. (D-3) is straightforward except where the integrands become singular. These end points are treated by using the analytic form of the integral with a constant pressure function over a small region near the singularity.

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