MASTER

TITLE
OPERATING INSTRUCTIONS FOR TWO I B M PROGRAMS
FOR
TRI-AXIAL ELASTIC THERMAL STRESSES

WANL-TMI-243

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SUMMARY

This report describes the procedure for using two I B M programs which were set up to solve for the thermal stress in a long unrestrained member having a constant cross section provided that the temperature is known at all points in the cross section. The I B M program cards are available from C. P. Saalbach, Mathematics Department, and I. N. Kallin, Reactor Mechanical Design Section.

BACKGROUND

A common structural element in a nuclear reactor is a long solid block of material with or without a pattern of cooling holes running the full length. Cooling holes, where used are positioned so as to reduce temperature gradients, however in any case, a certain non-symmetrical non-uniform temperature profile usually results. As a result of thermal analysis, these temperatures are available as a temperature map or grid for a particular cross section. It was desired to compute the maximum thermal stress that occurred at a section to predict any possible failure.

The equations were readily available. (See ref. 1, 2, and Appendix). The actual calculation of stress was quite lengthy due to the complexity of the temperature profile. The Mathematics Department programmed the equations for the I B M machine and the programs were subsequently checked out on test cases.

The first program gives only the stress in the axial direction. The second program gives the radial and tangential stresses around a hole or hot spot. The three stresses thus computed are the principle stresses at a point and should be sufficient to show whether any failure would occur.

USE OF THE PROGRAMS

For the first program it is first necessary to find the neutral axis about which the section will rotate. This can be done by eye or by balancing squares of a grid. The neutral axis is the centroid of the cross-sectional area. The direction of temperature gradient must be normal to the neutral axis. The area is broken up into squares and with each square there is associated a temperature, (t) an area, (a) and a distance (y) from the neutral axis. The "y" distance is taken as positive on the warmer side of the neutral axis and negative on the cooler side. These three columns of values are listed on a standard I B M form (#33406, available from the supply room) along with a constant value for Young's Modulus and a constant value for coefficient of expansion. The positioning of values on the I B M form is very important and should follow the example (page 7) explicitly.

The output sheets print all the input information and a stress (σ) for each station or square. Positive values of stress are tension and negative values are compression. At the top of the page are printed the total area "A", section inertia, "I", a value called "Sumtay" and two values called "Negsig" and "Possig".

"Negsig" is the summation of compressive stress times area. "Possig" is the summation of tensile stress times area. Since there is no external load on the part, these two values should be equal and this serves as a check on the program. If we divide "Sumtay" by the area

moment of inertia "I", the units are degrees per inch and this quotient is the effective linear gradient of the section. We can solve for the average temperature by taking the values for any station:

$$\sigma_{T_n} = E \alpha \left[-T_n + T_{avg.} + Y_n \left(\frac{\sum T_{ay}}{I} \right) \right]$$

The only unknown in this equation is $T_{avg.}$. Knowing $\left(\frac{\sum T_{ay}}{I} \right)$ and $T_{avg.}$ we can then compute the axial stress at any point inside of one of the squares which may have an unusually high or low temperature.

The stresses printed out in the first program are correct for a thin member, but as the thickness in the width direction becomes large, perhaps fifty per cent of the depth or greater, the effect of Poisson's ratio must be included. This is done by multiplying the computed stress by $\frac{1}{1 - \mu}$.

The second program evaluates the local stresses in the plane of the section at hot or cold spots. The most dangerous spot will probably have a cooling hole at its center. At the cool walls of the hole, bi-axial tensile stresses will result (the radial stress is zero at the wall) and this is where cracking is most likely to occur. Isothermal lines must be drawn around the hole and then these egg shaped isotherms must be approximated by circles concentric with the hole. The area considered should be as large as possible, consistent with a pattern of circular isotherms. An effective area which has a diameter four times the hole diameter will probably be accurate enough for most purposes.

The temperature of the rings should be plotted against radius. This curve can then be used for interpolation to assign a value of temperature to each of a number of thinner rings. To each thin ring is assigned a temperature "t", a mean radius "r", and a thickness "H". These values are listed on an I B M form as shown on Page 8. Other input information is Young's Modulus "E", coefficient of expansion, α , and Poisson's ratio, μ .

The print-out will give three stresses in each ring; radial stress, tangential stress, and axial stress (Sig Z). The value of axial stress will be less accurate than that computed by the first program, and the first program value should be used, since it represents a more realistic restraint condition. Also printed out are the input information and some values for checking purposes. $V +$ and $V -$ are the summations of tensile and compressive tangential stresses times ring thickness and they should have the same magnitude since we assumed no external pressures on the "hot spot" cylinder. $H +$ and $H -$ are the summations of axial stresses times thickness times radius at the hot spot and these two values should have the same magnitude.

POTENTIAL AND LIMITATIONS

Once the temperature is known at all points of a member these programs give a quick means for finding the tri-axial state of stress at any point in the member. The first program also indirectly computes the net change in dimensions of the whole member. That is, it computes the average temperature at the section in question and also the effective linear temperature gradient across the section. The average temperature gives the net growth of the member and the effective linear temperature gradient permits computation of the bowing of the member. Of course, if temperatures vary in magnitude along the length of the member also, then a number of sections must be analyzed and the effects of each section added to compute the total growth and amount of bow.

These programs apply only to thermal stresses in the elastic region. If any yielding takes place, the stresses will be drastically redistributed, the computed peak values being lowered. In this respect, the values are conservative. Constant values of "E" and " α " are used at each section. This will introduce an error, however for many materials these values are not precisely known as a function of temperature. Also, temperature differences of much more than 150° in a short distance will cause yielding in many materials anyway. Variations of "E" and " α " are generally less than 10 per cent over this range.

The precision of the computed stresses printed out by the I B M machine depends on how fine a grid was used in selecting areas and temperatures; however, the program values can be used in a hand calculation, as described on Page 4 under "Use of the Programs", to precisely compute the stress at a point for which the temperature is known exactly.

The second program which treats the circular "hot spots" assumes that the radial and tangential stresses are zero at the outside of the circular area considered. This will introduce an error which is the reason for taking as large circular area as is practical.

CODING FORM FOR IBM TYPE 7090 COMPUTER
FORTRAN - FAP

TITLE		ANALYST		PHONE		DATE		PROGRAM NO.		DECK NO.		L.S. NO.		SHEET	
		C.P. SAALBACH										131		1 OF 1	
FORTRAN		STAT. NO.		#		FORTRAN STATEMENT						DECK NO.		SEQ. NO.	
FAP		SYMBOL				OPERATION		ADDRESS, TAG, DECREMENT		REMARKS					
REF.		1 2 3 4 5 6		7		8 9 10 11 12 13 14		15 16 17 18 19 20 21 22 23 24 25 26 27 28 29		30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49		50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72		73 74 75 76 77 78 79 80	
A						T		TEST CASE		INGMAR KALLIN		PROGRAM 1			
22.0								44.0E+6		5.2E-6					
1.0						100.		-2.5							
1.0						110.		-1.5							
1.0						120.		-0.5							
1.0						130.		0.5							
1.0						140.		1.5							
1.0						150.		2.5							
1.0						150.		2.5							
2.0						160.		1.5							
1.0						130.		0.5							
1.0						120.		-1.5							
1.0						110.		-1.5							
2.0						90.		-2.5							
1.0						150.		2.5							
1.0						130.		0.5							
1.0						120.		-0.5							
1.0						110.		-1.5							
1.0						150.		2.5							
1.0						140.		1.5							
1.0						130.		0.5							
1.0						120.		-0.5							
1.0						110.		-1.5							
1.0						100.		-2.5							

TEST CASE INGMAR KALLIN

PROGRAM 1.

E 0.440000E 08 ALPHA 0.520000E-05 A 0.240000E 02 I 0.700000E 02
 SUMTAY 0.810000E 03 NEGSIG -0.105030E 05 POSSIG 0.105030E 05

N	A	T	Y	SIGMA
1	0.100000E 01	0.100000E 03	-0.250000E 01	-0.708190E 03
2	0.100000E 01	0.110000E 03	-0.150000E 01	-0.348647E 03
3	0.100000E 01	0.120000E 03	-0.500000E 00	0.108953E 02
4	0.100000E 01	0.130000E 03	0.500000E 00	0.370438E 03
5	0.100000E 01	0.140000E 03	0.150000E 01	0.729981E 03
6	0.100000E 01	0.150000E 03	0.250000E 01	0.108952E 04
7	0.100000E 01	0.150000E 03	0.250000E 01	0.108952E 04
8	0.200000E 01	0.160000E 03	0.150000E 01	-0.384602E 04
9	0.100000E 01	0.130000E 03	0.500000E 00	0.370438E 03
10	0.100000E 01	0.120000E 03	-0.500000E 00	0.108953E 02
11	0.100000E 01	0.110000E 03	-0.150000E 01	-0.348647E 03
12	0.200000E 01	0.900000E 02	-0.250000E 01	0.157981E 04
13	0.100000E 01	0.150000E 03	0.250000E 01	0.108952E 04
14	0.100000E 01	0.130000E 03	0.500000E 00	0.370438E 03
15	0.100000E 01	0.120000E 03	-0.500000E 00	0.108953E 02
16	0.100000E 01	0.110000E 03	-0.150000E 01	-0.348647E 03
17	0.100000E 01	0.150000E 03	0.250000E 01	0.108952E 04
18	0.100000E 01	0.140000E 03	0.150000E 01	0.729981E 03
19	0.100000E 01	0.130000E 03	0.500000E 00	0.370438E 03
20	0.100000E 01	0.120000E 03	-0.500000E 00	0.108953E 02
21	0.100000E 01	0.110000E 03	-0.150000E 01	-0.348647E 03
22	0.100000E 01	0.100000E 03	-0.250000E 01	-0.708190E 03

CODING FORM FOR IBM TYPE 7090 COMPUTER
FORTRAN - FAP

TITLE		ANALYST		PHONE		DATE		PROGRAM NO.		DECK NO.		L.S. NO.		SHEET	
		C.P. SAALBACH										33025		1 OF 1	
FORTRAN		STAT. NO.		#		FORTRAN STATEMENT						DECK NO.		SEQ. NO.	
FAP		SYMBOL				OPERATION		ADDRESS, TAG, DECREMENT		REMARKS					
REF.		1 2 3 4 5 6		7		8 9 10 11 12 13 14		15 16 17 18 19 20 21 22 23 24 25 26 27 28 29		30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49		50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72		73 74 75 76 77 78 79 80	
T						R		H		TEST CASE		PROGRAM 2		INGMAR KALLIN	
14.0						.0000052		44000000.		0.3		0.1875		0.78	
1.7						.195		.0125							
3.3						.213		.025							
6.1						.238		.025							
9.7						.275		.05							
13.2						.325		.05							
15.9						.375		.05							
17.8						.425		.05							
19.2						.475		.05							
20.3						.525		.05							
21.0						.575		.05							
21.5						.625		.05							
21.9						.675		.05							
22.0						.725		.05							
22.0						.765		.03							

TEST CASE PROGRAM 2 INGMAR KALLIN

ALPHA 5.200000E-06 V^E 4.400000E 07 MU 3.000000E-01 A 1.875000E-01 B 7.800000E-01
 SUMTH 5.388286E 00 V⁺ 1.606417E 02 V⁻ -1.603922E 02 H⁺ 3.879430E 02 H⁻ -3.840056E 02

J	T	R	H	SIGR	SIGT	SIGZ
1	1.700000E 00	1.950000E-01	1.250000E-02	2.139794E 02	5.375043E 03	5.589023E 03
2	3.300000E 00	2.130000E-01	2.500000E-02	5.984434E 02	4.467608E 03	5.066051E 03
3	6.100000E 00	2.380000E-01	2.500000E-02	9.354552E 02	3.215396E 03	4.150852E 03
4	9.700000E 00	2.750000E-01	5.000000E-02	1.105125E 03	1.869041E 03	2.974166E 03
5	1.320000E 01	3.250000E-01	5.000000E-02	1.125610E 03	7.045557E 02	1.830166E 03
6	1.590000E 01	3.750000E-01	5.000000E-02	1.014378E 03	-6.672632E 01	9.476517E 02
7	1.780000E 01	4.250000E-01	5.000000E-02	8.581398E 02	-5.315167E 02	3.266231E 02
8	1.920000E 01	4.750000E-01	5.000000E-02	6.954750E 02	-8.264518E 02	-1.309768E 02
9	2.030000E 01	5.250000E-01	5.000000E-02	5.403107E 02	-1.030830E 03	-4.905197E 02
10	2.100000E 01	5.750000E-01	5.000000E-02	3.996819E 02	-1.119002E 03	-7.193197E 02
11	2.150000E 01	6.250000E-01	5.000000E-02	2.765100E 02	-1.159258E 03	-8.827483E 02
12	2.190000E 01	6.750000E-01	5.000000E-02	1.692534E 02	-1.182744E 03	-1.013491E 03
13	2.200000E 01	7.250000E-01	5.000000E-02	7.810031E 01	-1.124277E 03	-1.046177E 03
14	2.200000E 01	7.650000E-01	3.000000E-02	1.933203E 01	-1.065509E 03	-1.046177E 03

SET UP OF A "HOT SPOT" FOR PROGRAM TWO. ALSO SHOWING
TANGENTIAL STRESS RESULTS

Set-Up of a "Hot Spot" for Program Two
Also Showing Tangential Stress Results

References: Timoshenko, S. : "Theory of Elasticity" 2nd Edition.
McGraw Hill Book Co.

Roark, R. J. : "Formulas for Stress and Strain"
McGraw Hill Book Co.

Stallone, M. J. : Course Notes for "Mechanical Design
Theory" General Electric Company, Evendale, Ohio

APPENDIX

Program 1

Axial thermal stresses in a long plate with a non-linear temperature gradient. Input:

$$\alpha = \underline{\hspace{1cm}} . \underline{\hspace{1cm}} \times 10^{-6}$$

$$E = \underline{\hspace{1cm}} . \underline{\hspace{1cm}} \times 10^6$$

<u>Station</u>	<u>Area</u>	<u>Temperature</u>	<u>Lever Arm</u>
(n)	(An)	(Tn)	(y)
1	-	-	-
2	-	-	-
3	-	-	-
1	-	-	-
1	-	-	-
n _f	-	-	-

COMPUTATIONS

$$\sigma_n = E \alpha \left[-T_n + \frac{\sum T_n A_n}{\sum A_n} + Y_n \frac{\sum T_n A_n Y_n}{\sum A_n Y_n} \right]$$

The three terms in the bracket represent:

T_n - the temperature at the point in question.

$\frac{\sum T_n A_n}{\sum A_n}$ = the average temperature at the section.

$Y_n \frac{\sum T_n A_n Y_n}{\sum A_n Y_n^2}$ = the effective linear gradient times the distance of the point in question from the neutral axis.

$$I = \sum A_n Y_n^2$$

$$\text{SUMTAY} = \sum T_n A_n Y_n$$

$$\text{NEGSIG} = \sum A_n (\sigma_n) \text{ minus}$$

$$\text{POSSIG} = \sum A_n (\sigma_n) \text{ plus}$$

$$\text{TAVG.} = \frac{\sum T_n A_n}{\sum A_n}$$

Program Two

Thermal stress in a long cylinder with a central hole and a symmetrical temperature gradient.

Input:

$\alpha = \underline{\hspace{1cm}} \cdot \underline{\hspace{1cm}} \times 10^{-6}$ Hole radius a = $\underline{\hspace{1cm}} \cdot \underline{\hspace{1cm}}$
E = $\underline{\hspace{1cm}} \cdot \underline{\hspace{1cm}} \times 10^6$ Outer radius b = $\underline{\hspace{1cm}} \cdot \underline{\hspace{1cm}}$
 $\mu = \underline{\hspace{1cm}} \cdot \underline{\hspace{1cm}}$

<u>Station</u>	<u>Temperature</u>	<u>Mean Radius</u>	<u>Ring Thickness</u>
(n)	(t)	(r)	(h)
1	-	-	-
2	-	-	-
3	-	-	-
-	-	-	-
-	-	-	-
n _f	-	-	-

COMPUTATION

$$B_n = \sum_{n=1}^{n-1} T_n r_n h_n + 1/2 T_n r_n h_n$$

$$\sigma_r = \frac{\alpha E}{1-\mu} \left(\frac{1}{r_m^2} \right) \left[\frac{r_n^2 - a^2}{b^2 - a^2} \sum_{n=1}^{n=n_f} T_n r_n h_n - B_n \right]$$

$$\sigma_\theta = \frac{\alpha E}{1-\mu} \left(\frac{-1}{r_n^2} \right) \left[\left(\frac{r_n^2 + a^2}{b^2 - a^2} \sum_{n=1}^{n=n_f} T_n r_n h_n \right) + B_n - T_n r_n^2 \right]$$

$$\sigma_z = \frac{\alpha E}{1-\mu} \left[\left(\frac{2}{b^2 - a^2} \right) \left(\sum_{n=1}^{n=n_f} T_n r_n h_n \right) - T_n \right]$$

$$V_+ = \sum (\sigma_z)_+ \quad hr$$

$$V_- = \sum (\sigma_z)_- \quad hr$$

$$H_+ = \sum (\sigma_\theta)_+ \quad h$$

$$H_- = \sum (\sigma_\theta)_- \quad h$$

PRINT OUT

All Input Data,

$$\left. \begin{array}{l} \sigma_r \\ \sigma_\theta \\ \sigma_z \end{array} \right\} \text{At each station}$$

V_+

V_-

H_+

H_-