ROLE OF THE PION MASS IN TRIPLE-REGGE PHYSICS*

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Abstract

This paper is an extension of the investigations of Abarbanel et al., who examined asymptotic total cross sections in a multiperipheral model and obtained the surprising result that the scale for the cross sections is provided not by μ , the mass of the exchanged pion (as anticipated on geometrical grounds) but by m_V , the central mass of the dominant low energy π - π resonance entering the kernel. In this paper the role of the pion mass in triple-Regge physics is clarified by examining the pion pole dominance model for the triple-Regge couplings g_{ijk} . It is found that m_V provides the scale for the inclusive link and that for this reason the couplings g_{ijk} depend very little on the intercept α_k of the exchanged reggeon. In the exclusive links if $i = j = \text{pomeron}$, m_V once again is the only active energy scale, whereas if $i = j = R$, the reggeons of intercept 0.5 or less, the pion mass enters the couplings g_{RRk} in an essential way. It is shown that the smallness of μ^2/m_V^2 is responsible for the largeness of the ratios g_{RRk}/g_{PPk} . These features of the model, which are in qualitative agreement with experiment, are put to a quantitative test.

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I. Introduction

The theorist's view of the role of the pion mass μ in hadronic processes has an interesting history. Following Yukawa's discovery that exchanging a particle of mass m produces a force of range $1/m$, there has existed the belief, based on geometrical reasoning, that hadron-hadron cross sections would be controlled by the lightest hadron, the pion. The corresponding cross section $\pi\pi^{-2} = 60 \text{ mb}$, is in fact of the order of magnitude of observed high energy total cross sections. The geometrical view was nevertheless challenged by the investigation of Abarbanel, Chew, Goldberger, and Saunders [1] who calculated meson-meson asymptotic total cross section within a multiperipheral model involving an N -dimensional multiplet of pions obeying an $SU(n)$ symmetry. They obtained the surprising result that as the pion mass μ was reduced to zero, the total cross section approached a smooth limit of order

$$\sigma^t(\infty) \sim \frac{16\pi^3}{Nm_V^2} \quad (1)$$

where m_V is the central mass of the dominant low-energy resonance multiplet in elastic $\pi\pi$ scattering. For $m_V = 900 \text{ MeV}$ and $N=8$, they obtained a cross section of about 30 mb, acceptable in magnitude but totally non-geometric in character -- the scale of the cross section being provided by the direct channel mass m_V rather than the t -channel mass μ .

Since m_V is the only mass left in the problem, it also sets the scale for the Regge expansion. The authors of ref. [1] obtain for the asymptotic form of the absorptive part of the elastic amplitude, the expression of the form

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$$A(S, 0) \underset{S \rightarrow \infty}{\sim} 16\pi^3 \left(\frac{S}{m_V^2} \right)^\alpha \quad (2)$$

due to the leading pole α . Clearly the same mass m_V will set the scale for the Regge expansions in meson-baryon and baryon-baryon amplitudes calculated within this model.

It has been known phenomenologically that a universal scale factor $S_0 = 1\text{GeV}^2$ ($\approx m_V^2$) is the natural one for Regge expansions[†], in the sense that if one expands the absorptive part of the forward elastic amplitude for a typical process as

$$A(S, 0) \underset{S \rightarrow \infty}{\sim} \beta_P \left(\frac{S}{S_0} \right)^{\alpha_P} + \sum_R \beta_R \left(\frac{S}{S_0} \right)^{\alpha_R} + \dots \quad (3)$$

the residue β_P of the pomeron is commensurate in magnitude with the β_R corresponding to the lower (intercept ≈ 0.5) trajectories. [2, 3]

Thus the largeness of the variable (S/S_0) is a direct and reliable measure of the convergence of the expansion. In contrast, if one uses μ^2 as the scale factor, one obtains for the same amplitude an expansion:

$$A(S, 0) \underset{S \rightarrow \infty}{\sim} \beta_P \left(\frac{\mu^2}{S_0} \right)^{\alpha_P} \left(\frac{S}{\mu^2} \right)^{\alpha_P} + \sum_R \beta_R \left(\frac{\mu^2}{S_0} \right)^{\alpha_R} \left(\frac{S}{\mu^2} \right)^{\alpha_R} + \dots \quad (4)$$

$$\cdot \beta'_P \left(\frac{S}{\mu^2} \right)^{\alpha_P} + \sum_R \beta'_R \left(\frac{S}{\mu^2} \right)^{\alpha_R} + \dots$$

In this expansion the largeness of (S/μ^2) is not a reliable measure of the convergence of the series, since the relatively large residues accompanying the lower poles delay the convergence in this variable.

[†] In this paper the scale factors S_0 and m_V^2 (both set equal to 1GeV^2) will be used interchangeably. In some theoretical contexts the scale factor will be denoted by m_V^2 to emphasize its origin within the model, while in a phenomenological context the symbol S_0 will be preferentially used.

Are we to infer from the above that the only role of the pion mass is to make plausible (due to its smallness) the hypothesis of pion-pole dominance in each link of the multiperipheral chain? What about the geometrical connection between the pion mass and hadronic cross sections?

The answer to this question is implicit in ref. [1]. We find there that only the part of the total cross section arising from Regge poles substantially above zero in the t -channel angular momentum plane (and hence important at high energies) is μ^2 -independent, whereas that associated with lower poles (and hence dominant at low energies) is sensitive to μ^2 . To see how this comes about let us examine the trace of the kernel[†] used in ref. [1]:

$$T_r K_\lambda = \frac{1}{16 - i(\lambda + 1)} \int_{-\infty}^0 \frac{dt}{(\mu^2 - t)^2} \left[\frac{-t}{m_\pi^2 - 2t} \right]^{\lambda + 1} \quad (5)$$

where λ is the angular momentum in the t -channel. For poles substantially above $\lambda = 0$ ($\lambda \gg 1$) the dependence on μ^2 is feeble and to a good approximation one may set $\mu^2 = 0$. [1] For poles around $\lambda \approx 0$ and below, the dependence on μ^2 is crucial and in fact for $\lambda \leq 0$, setting $\mu^2 = 0$ will cause the divergence of the integral. For poles in this region the physical value of μ^2 will enter the description introducing an additional energy scale.[‡] Herein lies the possibility of a reconciliation with the geometric ideas. If we consider a process like $\pi\pi \rightarrow VV$ (where V is the $\pi\pi$ resonance) due to one pion exchange, the cross section will indeed diverge if μ^2 is set equal to zero (in accordance with geo-

[†] This is the only place where μ enters as the mass of the exchanged object. Its occurrence as the mass of the external particles elsewhere does not interest us.

[‡] The fact that such low-lying poles are sensitive to μ^2 is only of academic interest since they do not feature in phenomenological Regge fits.

metrical ideas), and we must use the physical value of μ^2 . This dependence on μ^2 does not, however, conflict with the results of Abarbanel et al., since the energy dependence of this cross section corresponds to a low lying pole at $\lambda = 2\alpha_\pi - 1 = -1$. It is only when an infinite number of exclusive processes, involving an infinite number of pion links add up inclusively to produce a high lying Regge pole that the dependence on μ^2 drops out.

After this lengthy prelude let us turn to the question at hand, the role of the pion mass in triple-Regge physics. While all reactions of the type $ab \rightarrow cX$ fall under the latter category, we will confine ourselves to a reaction $p(p_1) + p(p_2) \rightarrow p(p_3) + X$ which alone has been investigated in detail. In the limit $M^2 = (p_1 + p_2 - p_3)^2 \rightarrow \infty$, $(S/M^2) \rightarrow \infty$ and $t = (p_3 - p_1)^2$ fixed, let us write the inclusive cross section as

$$\frac{d\sigma}{dt d(M^2/S)} = \left(\frac{S_0}{S}\right) \sum_{i,j,k} G_{ijk}(t) \left(\frac{S}{M^2}\right)^{\alpha_i(t) + \alpha_j(t)} \left(\frac{M^2}{S_0}\right)^{\alpha_k(0)} \quad (6)$$

where the coefficients G_{ijk} are measured in $\text{mb} \cdot \text{GeV}^{-2}$. Experimentalists presently use two trajectories, the pomeron (P) of intercept unity and a lower trajectory (R) of intercept around 0.5 or below. Only diagonal terms ($i=j$ in G_{ijk}) are employed. The results from a variety of sources are summarized in Table I. We shall discuss this table in greater detail later. For the present let us note two conspicuous features:

(i) The coefficients G_{iik} have only a feeble dependence on the inclusive reggeon k , that is G_{iiP} and G_{iiR} are commensurate in magnitude (reflecting the suitability of $S_0 = 1 \text{ GeV}^2$ as the scale factor in this link).

Some readers may object to the above generalization on the

grounds that there are fits such as refs. [5, 6] in which G_{RRP} is substantial while G_{RRR} is absent. I would like to draw the attention of such readers to ref. [5] where it is pointed out that even a substantial coefficient G_{RRR} (of the same order as G_{RRP}) could easily be omitted in a fit since its presence makes little difference to the inclusive cross section and the χ^2 values. Notice, however, that fits in which G_{RRR} does occur (refs. [4, 7]), it does so with a magnitude similar to G_{RRP} .

(ii) The coefficients G_{iik} have a marked dependence on the reggeon i : the coefficients G_{RRk} are an order of magnitude larger than the coefficients G_{ppk} . Consequently the variable (S/M^2) does not provide a reliable index of the convergence of the expansion in the two exclusive links. From our earlier discussion it would seem that a new energy scale has made its appearance and is discriminating between P and R .

In this paper both these features will be related to the role of the pion mass in the triple-Regge region. The analysis will be based on the pion pole dominance model for triple-Regge couplings. While the model formulas for these couplings have appeared in the literature [8] and have been numerically evaluated by Sørensen [9] my purpose here is to focus attention on the following features of the model which have not been emphasized in the past:

(i) The inclusive link carrying reggeon k (Fig. 2) has a smooth behavior as $\mu^2 \rightarrow 0$. In this limit the only energy scale is S_0 , a circumstance which will be seen to be responsible for the weak dependence of G_{ijk} on the reggeon k .

(ii) The μ^2 dependence in the exclusive links is similar to that encountered in ref. [1]: If the links carry high spin reggeons i and j , μ^2 may be set equal to zero and $m_V^2 = S_0$ provides the scale, while if

i and j are low spin reggeons the physical value of μ^2 enters in an essential way. The crucial difference here is that even the trajectories R of intercept > 0.5 (which are very much a part of the triple-Regge fits to the data) are classified as low spin reggeons. The entry of the small pion mass into the coefficients G_{RRk} is seen to be the cause of the large ratios G_{RRk}/G_{PPk} and the resultant delay in the convergence of the triple-Regge series in the variable (S/M^2) .

The paper is organized as follows. The notations and conventions are established in Section II. A brief discussion of the model, leading to the formulas for the triple-Regge couplings, is presented in Section III. These formulas are analyzed in Section IV to display the role of the pion mass. The quantitative predictions of the model are compared with experiment in Section V.

II. Notations and Conventions

From a theoretical standpoint the following expansion of the inclusive cross section is more appropriate than eq. (6):

$$\begin{aligned} \frac{d\sigma}{dt d(M^2/S)} &= \left(\frac{S_0}{S}\right) \cdot \frac{1}{16\pi S_0} \sum_{i,j,k} \beta_{ppi}(t) \xi_i(t) \beta_{ppj}(t) \xi_j^*(t) \\ &\times \left(\frac{S}{M^2}\right)^{\alpha_i(t)+\alpha_j(t)} \cdot g_{ijk}(t) \operatorname{Im} \xi_k(0) \cdot \left(\frac{M^2}{S_0}\right)^{\alpha_k} \beta_{ppk}(0) \text{ mb} \cdot \text{GeV} \end{aligned} \quad (7)$$

In this expansion $\beta_{ppi}(t)$ is the coupling of reggeon i to protons, $\alpha_i(t)$ its trajectory ($\alpha_i(0) = \alpha_i$) and $\xi_i(t)$ its signature factor given by $\{1 - \exp(i\pi\alpha_i(t))\}$ for even and $\{-i - \tan(\frac{1}{2}\pi\alpha_i(t))\}$ for odd signatures.

The triple-Regge coupling $g_{ijk}(t)$ has dimensions GeV^{-2} and will be measured in mb ($1 \text{ mb} = 2.5 \text{ GeV}^{-2}$). The normalization of the β 's is

such that a single pole i contributes to the total cross section an amount

$$\sigma_{pp,i}^t(S) \stackrel{S \rightarrow \infty}{\sim} \frac{1}{S} \cdot [\text{Im } \xi_i(0)] \beta_{ppi}^2(0) \left(\frac{S}{S_0}\right)^{\alpha_i} \text{ GeV}^{-2} \quad (8)$$

With the present choice of signature factors $\beta_{pp\omega}^2(0)$ and $\beta_{ppP}^2(0)$ will both be positive. Since only $\beta_{ppi}^2(0)$ is defined by eq. (8) we will agree that $\beta_{ppi}(0)$ is the positive square root of $\beta_{ppi}^2(0)$. This defines the sign of g_{ijk} . A comparison of eqs. (7 and 8) provides the connection between the triple-Regge coefficients G_{ijk} and the triple-Regge couplings g_{ijk} :

$$G_{ijk}(t) = \frac{1}{16\pi S_0} \beta_{ppi}(t) \xi_i(t) \beta_{PFj}(t) \xi_j^*(t) \beta_{ppk}(0) [\text{Im } \xi_k(0)] g_{ijk} \quad (9)$$

mb · GeV⁻²

For the off-diagonal coefficients ($i \neq j$) let us define the quantity

$$G_{ijk} = G_{ijk} + G_{jik} = 2 \text{ Re } G_{ijk} \quad (10)$$

In the fits carried out so far, the Regge poles used are the pomeron (P) and the next family of poles -- referred to collectively as R. The effect of pion exchange is included either directly by means of a $\pi\pi P$ term or indirectly, by using $\alpha_R(t) = 0.2 \cdot t$ instead of the conventional $\alpha_R(t) = 0.5 \cdot t$ in the RRP term. In some cases $\alpha_R = 0.2 \cdot t$ is used uniformly. In all the fits carried out so far, only diagonal coefficients G_{iik} are employed. The results from the analyses of NAL [4, 5], ISR [6] and global [7] data are presented in Table I. The trajectories employed in the different fits are indicated there. While both refs. [6] and [7] give analytic expressions for $G_{iik}(t)$, the values at specific t -values are presented here so that they may be compared with other measurements. The regions of small $|t|$ (defined arbitrarily by $|t| <$

0.16 GeV^2) is avoided since it seems controversial -- coefficients which turn over in this region according to some fits (e.g., G_{RRP} of ref. [7] do not turn over according to others (ref. [6]).

As pointed out in the introduction, our object here is to understand why the coefficients G_{iik} have a feeble dependence on reggeon k and a strong dependence on reggeon i . If we recall that β_{PPi} have about the same magnitude for P and R and that $|\xi_P(t)|^2$ and $|\xi_R(t)|^2$ are also of similar magnitude, we deduce from eq. (9) that g_{iik} will exhibit a similar dependence on the indices i and k at least for small $|t|$.^{*} In the next two sections we will therefore examine g_{iik} within the pion pole dominance model and understand how the pion mass μ produces the above mentioned dependence on i and k . We will finally return to G_{iik} in Section V when the model is compared with experiment quantitatively.

III. The Pion Pole Dominance Model for g_{ijk}

Since this model has been discussed at length in refs. [8] and [9] only a brief description will be provided here, emphasizing those aspects which are germane to the subsequent discussions. Among all the exclusive events contributing to the inclusive cross section, consider those in which the particle from X closest to the proton in rapidity (labelled 4 in Fig. 1) is a pion, $\pi(p_4)$. When $u = (p_3 + p_4 - p_1)^2 = \mu^2$, the amplitude factorizes;

^{*} We know from total cross section measurements that $\beta_{ppP}(0) \approx \beta_{ppR}(0)$. For $\alpha_R = 0.5$ and $\alpha_P = 1$, $|\xi_R|^2 = 2|\xi_P|^2$ at $t=0$. We are assuming that this commensurability will persist for modest values of $|t|$.

$$M_{pp \rightarrow p\pi X'}(u) \xrightarrow{u \rightarrow \mu^2} \frac{M_{\pi p \rightarrow \pi p} M_{\pi p \rightarrow X'}}{u - \mu^2} \quad (11)$$

The key assumption in the model (based on the smallness of μ^2) is that in the physical region ($u < 0$) the amplitude is given by the factorized form of eq. (11) modified by form factors which account for the off-shell nature of the exchanged pion. This assumption has been tested against experiment for the case $X' = \pi p$ and found to be reliable. [11]

Calculations of the inclusive cross section in the triple-Regge region from the model amplitude and the identification of g_{ijk} are schematically represented in Fig. 2. The details may be found in refs. [8,9]. The following is the result

$$g_{ijk}(t) = \frac{\Gamma(1+\alpha_k) \times 3}{16\pi^3} \left(\frac{1}{2.5}\right) \cdot \int_{-\infty}^0 du \left[2\sqrt{\frac{ut}{m_V^2}} \sinh q \right]^{\alpha_i(t)+\alpha_j(t)} \\ \left(\frac{u}{t}\right)^{\frac{1+\alpha_k}{2}} \frac{1}{(\mu^2 - u)^2} \cdot P_{\alpha_i(t)+\alpha_j(t)}^{-1-\alpha_k} (\coth q) \beta_{\pi\pi i}(t, u, \mu^2) \beta_{\pi\pi j}(t, u, \mu^2) \\ \times \beta_{\pi\pi k}(0, u, u) \quad \text{mb.} \quad (12)$$

One considers in this model just the vacuum trajectories P and P' , since the ω is forbidden by G-parity, while the ρ and A_2 couple weakly to the external protons. Thus the three types of pions that can be exchanged are accounted for by a factor 3 in eq. (12). The scale factor $m_V^2 = S_0 = 1 \text{ MeV}^2$ implicit in refs. [8,9] is explicitly displayed here and $\cosh q = \frac{\mu^2 - t - u}{2\sqrt{ut}}$. The residue $\beta_{\pi\pi i}(t, y^2, z^2)$ is the coupling of a reggeon i of mass $\sqrt{t}$ to pions of mass y and z . Only $\beta_{\pi\pi i}(t, \mu^2, \mu^2) \equiv \beta_{\pi\pi i}(t)$ is measurable[†] and we shall use the ABFST [12] form

[†] The coupling $\beta_{\pi\pi i}(t)$ that obtained from Regge fits using the standard scale factor of $1/\sqrt{V^2}$

factors to go off-shell:[†]

$$\beta_{\pi\pi i}(t, y^2, z^2) = \beta_{\pi\pi i}(t) \left[\frac{m_V^2 + t/4 - \mu^2}{m_V^2 - 1/2(y^2 + z^2 - t/2)} \right]^{1+\alpha_i(t)} \quad (13)$$

For future reference, let us note that for t , y^2 and z^2 much smaller than m_V^2 , we can use the approximation

$$\beta_{\pi\pi i}(t, y^2, z^2) \simeq \beta_{\pi\pi i}(t) e^{\frac{1/2(y^2 + z^2)}{m_V^2} (1+\alpha_i)} \quad (14)$$

IV. The Role of the Pion Mass

As given by eq. (12) the coupling $g_{ijk}(t)$ defies any simple analysis. However, the formula simplifies greatly at $t=0$:

$$g_{ijk}(0) = \frac{3}{16\pi^3} \frac{1}{1+\alpha_k} \cdot \frac{1}{(2.5)} \int_{-\infty}^0 du \frac{1}{(\mu^2 - u)^2} \left(\frac{\mu^2 - u}{m_V^2} \right)^{\alpha_i + \alpha_j} \\ \left(\frac{-u}{\mu^2 - u} \right)^{1+\alpha_k} \beta_{\pi\pi i}(0, \mu^2, u) \beta_{\pi\pi j}(0, \mu^2, u) \beta_{\pi\pi k}(0, u, u) \quad \text{mb} \quad (15)$$

and using eq. (14)

$$g_{ijk}(0) = \frac{3}{16\pi^3} \frac{1}{1+\alpha_k} \cdot \frac{1}{(2.5)} \beta_{\pi\pi i}(0) \beta_{\pi\pi j}(0) \beta_{\pi\pi k}(0) \times \int_{-\infty}^0 du \\ \frac{1}{(\mu^2 - u)^2} \cdot \left(\frac{\mu^2 - u}{m_V^2} \right)^{\alpha_i + \alpha_j} \left(\frac{-u}{\mu^2 - u} \right)^{1+\alpha_k} e^{\left[\omega_{ijk} \cdot \left(\frac{u}{m_V^2} \right) \right]} \quad \text{mb} \quad (16)$$

[†] Sorensen uses the form factor $\left[\frac{m_V^2}{m_V^2 - 1/2(y^2 + z^2 - t/2)} \right]^{1+\alpha_i(t)}$ which

doesn't reduce to unity on shell. For the range of small t he considers, this causes little error.

where $\omega_{ijk} = \frac{\alpha_i + \alpha_j + 2}{2} + 1 + \alpha_k$ varies in the limited range 3-4 for conventional P and R trajectories. Let us begin by examining the dependence of $g_{ijk}(0)$ on reggeon k. We see that $(-u/(\mu^2 - u))^{1+\alpha_k}$ smoothly approaches unity as $\mu^2 \rightarrow 0$ and may be evaluated in that limit. The dependence of $g_{iik}(0)$ on k is then due to the factors $(1+\alpha_k)^{-1}$ and $\beta_{\pi\pi k}(0)$ in front of the integral in eq. (16) and the form factor $e^{\omega_{ijk} u/m_V^2}$ within. The dependence of these quantities on the reggeon k is weak. That the coupling $\beta_{\pi\pi k}(0)$ could become sensitive to μ^2 (and possibly be very large) for $\alpha_k \sim 0$ is of academic interest, since such low-lying Regge poles do not occur in triple-Regge fits.

Let us now turn to the diagonal couplings and consider the dependence of g_{iik} on reggeon i.

$$g_{iik}(0) = \frac{3}{16\pi^3} \frac{1}{1+\alpha_k} \cdot \frac{1}{2.5} \beta_{\pi\pi i}^2(0) \beta_{\pi\pi k}(0) \times \int_{-\infty}^0 \frac{du}{(\mu^2 - u)^2} \\ \times \left(\frac{\mu^2 - u}{m_V^2} \right)^{2\alpha_i} e^{\omega_{iik} u/m_V^2} \quad \text{mb} \quad (17)$$

where the $\mu^2 \rightarrow 0$ limit of $(-u/(\mu^2 - u))^{1+\alpha_k}$ has been taken. If $i=P$ (with $\alpha_P=1$), the integral is independent of μ^2 and the scale is provided by m_V^2 :

$$g_{PPk}(0) = \frac{3}{16\pi^3} \frac{1}{2.5} \cdot \frac{1}{1+\alpha_k} \beta_{\pi\pi P}^2(0) \beta_{\pi\pi k}(0) \left(\frac{1}{m_V^2} \right) \frac{1}{\omega_{PPk}} \quad \text{mb} \quad (18)$$

If $i=R$, with $\alpha_R=0.5$ we obtain

$$g_{RRk}(0) = \frac{3}{16\pi^3} \frac{1}{2.5} \frac{1}{1+\alpha_k} \cdot \beta_{\pi\pi R}^2 \beta_{\pi\pi k}(0) \left(\frac{1}{m_V^2} \right) \log \left(\frac{m_V^2}{\omega_{RRk} \mu^2} \right) \quad \text{mb} \quad (19)$$

Notice how the physical value of μ^2 has entered in an essential way and how setting $\mu^2 = 0$ causes $g_{RRk}(0)$ to diverge (as anticipated by geometrical reasoning). The crucial difference between the inclusive cross sections discussed here and the total cross sections discussed in ref. [1] is that for the latter, μ^2 was expected to enter only for trajectories with $\alpha \lesssim 0$ (which do not feature in phenomenological Regge fits) while in the present case even the trajectories of intercept $\simeq 0.5$ (which are very much a part of triple-Regge fits) are μ^2 -dependent.[†]

For $R=P'$, if we recall that $\beta_{\pi\pi P}(0) \simeq \beta_{\pi\pi P'}(0)$ [2, 3] we obtain from eqs. (18, 19) the rough estimate:

$$\frac{g_{RRk}(0)}{g_{PPk}(0)} \sim \omega_{PPk} \left(\ln \frac{m_V^2}{\omega_{RRk} \mu^2} \right) \simeq 10 \quad (20)$$

for an average ω of 3.5 and $m_V^2/\mu^2 \simeq 50$.

Whether or not this ratio will be observed experimentally is decided by the corrections that must be applied to the model. The two key approximations made in the model were that:

(i) particle 4 in Fig. 1 is a pion, and that

(ii) granted (i), the amplitude is dominated by the pion pole in u .

It is not clear how approximation (ii) affects the ratio g_{RRk}/g_{PPk} . On the other hand the effect of the corrections to assumption (i) are easier to analyse, since even in which particle 4 is not a pion make additive corrections to the inclusive cross section and to the triple-Regge

[†] Whereas singular behavior of $g_{RRk}(0)$ in the $\mu^2 \rightarrow 0$ limit obtains only for $\alpha_R \leq 0.5$, a strong dependence on μ^2 is expected even if α_R were slightly above 0.5. We can see from eq. (17) that the dependence on μ^2 decreases smoothly with increasing α_i and ultimately disappears for $\alpha_i = \alpha_P = 1$.

couplings g_{ijk} calculated in the pion model. Let us consider for definiteness the impact of events in which particle 4 is a kaon, on the ratio g_{RRk}/g_{PPk} . If we assume for simplicity kaon pole dominance, the above calculations can be repeated* with $\mu^2 \rightarrow m_K^2$ and $\beta_{\pi\pi i}(0) \rightarrow \beta_{KKi}(0)$. The contributions to $g_{PPk}(0)$ will be of the same order as in the pion case since empirically $\beta_{\pi\pi i}(0) \simeq \beta_{KKi}(0)$, [2] and the meson mass drops out for such couplings. By contrast, the contributions to $g_{RRk}(0)$ will be much smaller than in the pion case due to the dependence of these couplings on the meson mass (eq. 20). A more detailed analysis suggests that the corrections to $g_{RRk}(0)$ will be of the same order as the corrections to $g_{PPk}(0)$. The net effect of the kaon events then, will be to lower the ratios $g_{RRk}(0)/g_{PPk}(0)$ calculated in the pion model.

That m_V and not the meson masses (μ or m_K) controls $g_{PPP}(0)$ is of theoretical interest for two reasons. First, the above circumstance lends credibility to the estimate of $g_{PPP}(0)$ by Abarbanel et al., [13] who assumed that an SU(3) octet of mesons contribute equally to $g_{PPP}(0)$. Had $g_{PPP}(0)$ a dependence on meson mass expected by geometrical reasoning, their assumption would have been grossly violated by the sizeable mass difference between the pions and the kaons within the octet. Secondly, and more importantly, a non-vanishing $g_{PPP}(0)$ of a scale decided by μ^2 would have led to an embarrassingly large value for the dimensionless parameter $\eta_P = \frac{S_0}{32\pi\alpha_1} g_{PPP}^2(0)$, which, according to these authors measures $1-\alpha_1$, the deviation of the pomeron intercept from unity. These features are not accidental, for consis-

* We are assuming that the form factors are the same in both cases for want of a more realistic alternative.

tency of the model requires that if the asymptotic total cross sections have a smooth limit as the meson mass vanishes, so must the triple-pomeron coupling $g_{PPP}(0)$.

Let us pause now to understand the physical origin of the factor $(\mu^2 - u)^{2\alpha_i}$ (eq. 17) which played a crucial role in the subsequent analysis.[†] Consider the schematic form of the amplitude for $pp \rightarrow p\pi X'$ in Fig. 2 along the t-channel. We see there a reggeon i of mass $\sqrt{t}$ coupling to a π - π system consisting of a real pion of mass μ and a virtual pion of mass $\sqrt{u}$. We will discover that the factor $(\mu^2 - u)^{\alpha_i}$ corresponds to the usual threshold factor[‡] which inhibits the coupling of the π - π system to high spin reggeons i near the π - π threshold. Since μ is small and $\sqrt{u}$ tends to be small (due to the pole factor $(\mu^2 - u)^{-2}$), the CM energy of the π - π system $\sqrt{t}$, is close to threshold if t is zero (as in our analysis) or small. The threshold factors are thus very effective and the coupling to the pomeron, which has the highest spin, is suppressed the most.

How did the threshold factor enter the coupling g_{iik} ? We know in the usual Regge analysis of $ab \rightarrow cd$ that the question of whether or not the residues exhibit threshold behavior is decided by the choice of the asymptotic variable. At high energies in the S-channel, if we expand the amplitude in terms of $\cos\theta_t = S/2pq$ (where p and q are the CM momenta of $a\bar{c}$ and $\bar{b}d$ respectively, in the t-channel), the contribution of a single Regge pole i is the of the form

[†] Whereas the factor $((\mu^2 - u)/m_V^2)^{2\alpha_i}$ enters eq. (17), we will not go into the origin of $(m_V^2)^{-2\alpha_i}$ here, since the latter turns out to be a matter of simple algebra. The curious reader will be informed of its entry by means of a footnote.

[‡] This factor gets squared when we calculate E_{iik} .

$$M_i(S, t) \underset{t \text{ fixed}}{\overset{\cos\theta_t \rightarrow \infty}{\sim}} \xi_i(t) \gamma_{aci}(t) \left(\frac{S}{2pq}\right)^{\alpha_i(t)} \gamma_{bdi} \quad (21)$$

where $\gamma_{aci}(t) \underset{p \rightarrow 0}{\sim} (p)^{\alpha_i(t)}$ and $\gamma_{bdi}(t) \underset{q \rightarrow 0}{\sim} (q)^{\alpha_i(t)}$. If however, we expand in terms of (S/S_0) , as

$$M_i(S, t) \underset{t \text{ fixed}}{\overset{S \rightarrow \infty}{\sim}} \xi_i(t) \beta_{aci}(t) \left(\frac{S}{S_0}\right)^{\alpha_i(t)} \beta_{bdi}(t) \quad (22)$$

the "reduced" residues β will not exhibit strong threshold behavior.

Consider now the Regge expansion in the exclusive link carrying reggeon i (Fig. 2) which leads to the triple-Regge expansion of eq. (7). To which of the two possibilities eqs. (21, 22) does it correspond? Something inbetween, is the answer. To see why, note that for large M^2 ,

$$\sqrt{\frac{t-4m^2}{2}} \cos\theta_t = \sqrt{2p} \cos\theta_t \simeq \left(\frac{S}{M^2}\right) \times \sqrt{2t} \quad (23)$$

where m is the proton mass and p the CM momentum of the protons 1 and 3 (Fig. 2) in the t -channel. It follows that an expansion in the variable (S/M^2) corresponds to removing the threshold behavior only from the proton end and introducing an additional factor of $\sqrt{2t}$ into the missing mass end.[†] We may therefore anticipate in the coupling of reggeon i to X (that is, to the π - π system in our model) a factor $(\sqrt{2t} \cdot \sqrt{2}q)^{\alpha_i(t)}$ and in $g_{iik}(0)$ a factor $(\sqrt{2t} \cdot \sqrt{2}q)^{2\alpha_i}$, where q is the CM momentum of the π - π system in the t -channel. If we now recall

[†] Notice that in expanding in terms of (S/M^2) one also omits the Regge scale factor S_0 (compare with eq. (22)), which then gets absorbed into the missing mass end. The overall factor attached to this end is then $\sqrt{2t}/S_0 \cdot \sqrt{2t}/m_{\pi}^2$. In the interest of clarity the factor $1/m_{\pi}^2$ is suppressed in these discussions.

that

$$2\sqrt{t}q = \lambda^{1/2}(t, u, \mu^2) = (\mu^2 - u) \quad \text{at } t=0, \quad (24)$$

we understand the origin of the factor $(\mu^2 - u)^{2\alpha_i}$ in $g_{iik}(0)$.[†]

We have restricted our discussions to the point $t=0$ so as to exploit the simple formula for $g_{ijk}(0)$. The numerical estimates for $g_{ijk}(t)$ given by Sorensen [9] indicate that the major features encountered at $t=0$ persist for modest values of $|t|$ (up to $\approx .25 \text{ GeV}^2$). As we move away from zero, the following considerations control the ratios $g_{RRR}(t)/g_{PPk}(t)$ and $G_{RRk}(t)/G_{PPk}(t)$:

(i) The threshold effects which discriminated between P and R will get weaker as we move in the negative t direction, since this takes us away from the $\pi\text{-}\pi$ threshold. This will tend to lower the ratio $g_{RRk}(t)/g_{PPk}(t)$.

(ii) Due to the small slope of P, the difference $\alpha_P(t) - \alpha_R(t)$ increases with $|t|$ -- which in turn boosts the ratio $g_{RRk}(t)/g_{PPk}(t)$.

(iii) The residues $\beta_{\pi\pi P}(t)$ have a sharper t -falloff than $\beta_{\pi\pi R}(t)$, which enhances $g_{RRk}(t)/g_{PPk}(t)$. A similar consideration applies to $\beta_{ppP}(t)$ and $\beta_{ppR}(t)$ which tends to boost the ratio $G_{RRk}(t)/G_{PPk}(t)$ (see eq. (9)).

We will take these considerations into account when we put the model to a quantitative test in Section V. Let us now summarize our findings in somewhat more general terms. Consider $g_{ijk}(t)$ as the coupling of two reggeons i and j to the ends of a ladder k (Fig. 3). The ladder can contain any species of particles. Due to the choice of (S/M^2) as the asymptotic variable in eqs. (6, 7), the couplings of

[†] The triangle function $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2zx$.

reggeons i and j to the edges of the ladder (that is, to the particles ab and ac respectively) contain threshold factors. The masses of a , b , and c insinuate themselves into $g_{ijk}(t)$ through these threshold factors and the propagators of band c . If the intercept of the reggeons i and j are substantially above 0.5 (that is if $i=j=\text{pomeron}$) these masses drop out. For trajectories R , of intercept ≈ 0.5 these masses enter the couplings and can play an important role. If these masses squared are commensurate with the scale factor of 1 GeV^2 (e.g., for kaons) their effect will be minimal. If, on the other hand, a , b , and c are pions (as in the model) the entry of the small pion mass μ into the couplings g_{RRk} will boost them up way above g_{PPk} . The consequent delay in the convergence of the series in the variable (S/M^2) may therefore be attributed to the new energy scale brought in by the pion mass, over and above S_0 .

V. Quantitative Comparison with Experiment

The object of this section is to compare the ratios of triple-Regge coefficients calculated within the model with experiment. The existing analyses omit off-diagonal coefficients G_{ijk} ($i \neq j$) in their fits -- either arbitrarily or on the basis of certain exchange degeneracy arguments.[7] In ref. [14] these exchange degeneracy arguments are criticized as being inapplicable in the triple-Regge region. It is pointed out there that according to the pion pole dominance model one of the off-diagonal terms (PRP) is expected to make a significant contribution (typically 30%) to the inclusive cross section. The fact that this possibly important term has been omitted in the data analysis makes a term by term comparison of the model with experiment pointless. We will therefore perform a comparison of average quantities, the sole purpose

of which will be to demonstrate that the ratios G_{RRk}/G_{PPk} given by the model are of the same order as the measured ones. Since there exists no unique prescription for the kind of average that must be employed, the following average ratio is chosen arbitrarily:

$$\frac{\langle G_{RRk} \rangle}{\langle G_{PPk} \rangle} = \frac{G_{RRR} + G_{RRP}}{G_{PPR} + G_{PPP}} \quad (25)$$

Since no measurement has been performed at $t=0$, the comparison will be made[†] at $t = -0.16 \text{ GeV}^2$. The comparison will be made only with fits that use the conventional trajectory $\alpha_R = 0.5+t$, since we can identify the latter with the P' and use its known residues and signature factor. The corresponding operation for the effective trajectory $\alpha_R = 0.2+t$ is ambiguous. To obtain $G_{iik}(t)$, eqs. (9) and (12) were combined, the residues of ref. [3] were used and the value of the complicated integral in eq. (12) extracted from Sorensen's paper. The results are given in Table II. It is encouraging to note that the difference between the model prediction for $\langle G_{RRk} \rangle / \langle G_{PPk} \rangle$ and the measured ones is no greater than the differences among the latter.

VI. Conclusions

We started with the surprising result of ref. [1] that in a multi-peripheral model the scale for the asymptotic cross sections is provided not by the mass μ of the exchanged pions but by m_V , the central mass of the low energy $\pi\pi$ resonance that entered the kernel -- a non-

[†] We wish to remain as close as possible to the point $t=0$, to which much of our discussion was confined. The choice of $t = -0.16 \text{ GeV}^2$ permits a comparison with refs. [5, 6, and 7].

geometric feature. Nevertheless, reconciliation with geometrical ideas was possible, since according to ref. [1], the μ^2 -independence was true for only the higher singularities ($\lambda \geq 1/2$) while lower singularities were allowed to exhibit a dependence on μ^2 expected on geometrical grounds.

In this paper we tried to understand the role played by μ^2 in triple-Regge physics by considering the pion pole dominance model for triple-Regge couplings. We saw that of the three links carrying reggeons i, j and k (Fig. 2) the inclusive link (k) was controlled by m_V^2 and not μ^2 and for this reason had only a feeble dependence on α_k (provided α_k was well above zero). The situation in the exclusive links resembled in part that encountered in ref. [1] -- the higher reggeons were controlled just by m_V^2 , while the lower ones were controlled by μ^2 as well. The crucial feature here was that even the poles R of intercept 0.5, which play a prominent role in triple-Regge fits, were classified as low. The entry of the small pion mass into the couplings g_{RRk} was seen to boost them by a factor of about ten over the couplings g_{ppk} . The new mass scale introduced by the pion into g_{RRk} (and hence G_{RRk}) may then be viewed as the cause of the delayed convergence of the triple-Regge expansion in the variable (S, M^2) describing the exclusive links.

The tendency of the pion mass μ to enter the couplings via the pole factor $(\mu^2 - u)^{-2}$ and to boost them in magnitude due to its smallness, were offset either wholly (in $g_{ppk}(0)$) or in part (in $g_{RRk}(0)$) by the angular momentum barrier factors $(\mu^2 - u)^{2\alpha_i}$. It was pointed out that, had the model generated a non-vanishing $g_{pp-p}(0)$ with a scale set by μ^2 rather than m_V^2 , an embarrassingly large η_p would have resulted.

The quantitative predictions of the model were compared with experiment. It was found that the ratio of averaged couplings,

$\langle G_{RRk} \rangle / \langle G_{PPk} \rangle$ given by the model was of the same order as the measured ones.

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NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Atomic Energy Commission, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

Table 1. Data in triple-Regge couplings $G_{ijk}(t)$ in mb GeV⁻².

Source	Trajectories	$\frac{1}{t}(\text{GeV}^2)$	G_{PPP}	G_{PPR}	G_{RRP}	G_{RRR}
Ref. [4] (NAL)	$\sigma_P = 1$	.33	.21	.87	33.7	30.4
	$\sigma_R = .5+t$	.45	.14	.56	27.7	31.5
Ref. [5] (NAL)	$\sigma_P = 1+.25t$	.16	1.3	3.8	108	--
	$\sigma_R = .2+t$ in RRP	.20	1.2	3.3	91	--
	$= .5+t$ in others	.25	1.0	2.3	78	--
	(fit III)	.33	.7	1.8	67	--
	$\sigma_P = 1+.25t$	.16	.92	3.7	26	--
	$\sigma_R = .5+t$	.20	.84	3.6	24	--
	$\pi\pi P$ included	.25	.75	2.3	23	--
	as per Ref. [10]	.33	.52	1.8	21	--
	(fit IV)					
Ref. [6] (ISR) (The G's given in Ref. [4] have been multiplied by π to get the G's used by others.)	$\sigma_P = 1$	.16	.83	--	15.7	--
	$\sigma_R = .5+t$	.20	.70	--	15.7	--
		.25	.57	--	15.7	--
		.33	.41	--	15.7	--
	$\sigma_P = 1+.15t$	.16	1.16	--	15.7	--
	$\sigma_R = .5+t$	.20	1.0	--	15.7	--
		.25	.84	--	15.7	--
		.33	.65	--	15.7	--
Ref. [7]	$\sigma_P = 1+.25t$	.16	2.6	1.6	96.3	86.2
	$\sigma_R = .2+t$	.20	1.25	1.6	86.5	65.8
		.25	1.0	1.4	72.8	51.6
		.33	.75	1.2	54	37.8

Table II. Comparison of the pion-pole dominance model predictions for $G_{iik}(t)$ (in $\text{mb} \cdot \text{GeV}^{-2}$) with experiment, at $t = -0.16 \text{ GeV}^2$.

Source	G_{PPP}	G_{PPR}	G_{RRP}	G_{RRR}	$\frac{\langle G_{RRk} \rangle}{\langle G_{PPk} \rangle}$
Ref. [5] fit IV	.92	3.7	26	--	11.3
Ref. [6] $\alpha_P(t) = 1$	.83	--	15.7	--	18.9
Ref. [6] $\alpha_P(t) = 1 + \frac{1}{4}t$	1.16	--	15.7	--	13.5
Model	.38	.81	5.8	11	14.1

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Figure Captions

- Fig. 1. Rapidity plot of an exclusive event contributing to the inclusive cross section in the pion pole dominance model.
- Fig. 2. Schematic derivation of $g_{ijk}(t)$ in the pion pole dominance model.
- Fig. 3. The ladder description of the triple-Regge couplings. The dotted lines remind us that we are considering the absorptive part. The couplings γ have threshold factors while the couplings β do not.

Fig. 1

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$$\frac{d\sigma}{dt d(M^2/S)} \approx \sum_{X', p_4, u} \left| \sum_i \right|$$

The top diagram illustrates a particle interaction. It features external lines labeled 1, 2, 3, and 4 = π . A vertex labeled X' is shown with a hatched area. Internal lines are labeled i and $u = -v$. The bottom diagram shows a more complex interaction with multiple vertices and internal lines labeled i, j, k, u, μ, π . The sum $\sum_{i,j,k,u}$ is indicated to the left of the diagram.

Fig. 2

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Fig. 3