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STRESS ANALYSIS OF CYLINDRICAL SHELLS

F. J. Stanek

OAK RIDGE NATIONAL LABORATORY
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for the
U.S. ATOMIC ENERGY COMMISSION

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PREFACE

This report presents in a convenient form the principal formulas and tables of numerical values for the stress analysis of cylindrical shells loaded axisymmetrically. The analysis is based on the classical small-deflection theory. Parts I and II give the formulas and the numerical values of all necessary functions and are sufficient for a complete analysis of a specific cylindrical shell. Parts III and IV contain, respectively, several illustrative examples and the derivation of the cylindrical-shell formulas.

The author is indebted to the personnel of the Numerical Analysis Department (headed by J. E. Rowe) at the Oak Ridge Gaseous Diffusion Plant for evaluating and tabulating the various functions included in this report. The programing and computing were done under the supervision of I. R. Parsley.

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CONTENTS

Part I. Cylindrical-Shell Formulas and Their Application	1
Introduction	1
Symbol and Sign Convention for Cylindrical Shells	1
Cylindrical-Shell Formulas	2
Procedure	3
Part II. Values of All Functions	5
Introduction	5
Table 1. The Four M_n Functions.....	6
Table 2. The Four Q_n Functions.....	8
Table 3. The Four N_n Functions.....	10
Table 4. The Four W'_n Functions.....	12
Summary of Functions Evaluated and Tabulated in Part II.....	14
Part III. Illustrative Examples	15
Example 1.....	15
Example 2.....	17
Example 3.....	18
The Cylindrical-Container Problem	20
The Standpipe Problem.....	22
Part IV. Derivation of the Cylindrical-Shell Formulas	25

Part I

CYLINDRICAL-SHELL FORMULAS AND THEIR APPLICATION
INTRODUCTION

Almost any thin cylindrical shell, loaded with any combination of the axisymmetrical loads shown in Fig. 1, may be analyzed by the method and procedure presented. The maximum radial displacement should be small relative to the thickness of the shell.

The stress analysis of cylindrical shells by the classical small-deflection theory stems from the solution of a fourth-order differential equation whose independent dimensionless coordinate y is defined by the simple coordinate transformation

$$y = \beta x,$$

where x is the axial coordinate along the surface of the cylinder and β is a shell parameter. Hence, the cylindrical-shell formulas are expressed in terms of functions of y . The numerical values of these functions are provided.

To analyze a cylindrical shell, a set of simultaneous equations is solved to determine the constants of integration. When they have been evaluated, the desired results are obtained by substitution in the appropriate formula. For long cylindrical shells two of the constants of integration are taken equal to zero.

SYMBOL AND SIGN CONVENTION FOR CYLINDRICAL SHELLS*

x = axial coordinate of shell (in.)

a = radius of middle surface of shell (in.)

b = thickness of shell (in.)

L = length of shell (in.)

$$\beta = \left[\frac{3(1 - \mu^2)}{a^2 b^2} \right]^{1/4} = \text{shell parameter (in.}^{-1}\text{)}$$

$y = \beta x$ = dimensionless coordinate of shell

w = radial displacement of middle surface of shell, positive inward (in.)

u = axial displacement of middle surface of shell, positive in positive x direction (in.)

p = uniform internal pressure (psi)

Q = shearing force in shell (lb/in.) (see Fig. 1 for positive direction)

N = normal membrane force in shell, tension positive (lb/in.)

M = bending moment in shell, positive when tensile stresses are in outside surface (lb·in./in. = lb)

E = Young's modulus of elasticity for the material of the shell (psi)

μ = Poisson's ratio for the material of the shell (dimensionless)

C_n = the four constants of integration C_1 , C_2 , C_3 , and C_4 (lb/in.)

C_5 = another constant of integration (lb/in.)

Fig. 1

*The units designated are the most common set used in the USA; however, any consistent set may be used, as the numerical values in the tables are dimensionless.

CYLINDRICAL-SHELL FORMULAS

In each of the formulas 1, 3, 5, 6, 7, and 8 below,

$$\sum C_n (\dots)_n = C_1 (\dots)_1 + C_2 (\dots)_2 + C_3 (\dots)_3 + C_4 (\dots)_4,$$

where $(\dots)_n$ are functions of y whose values are tabulated in Part II.

Shell Parameter

$$\beta^4 = \frac{3(1 - \mu^2)}{a^2 b^2} \quad (i)$$

$$\left. \begin{aligned} &= \frac{2.73}{a^2 b^2} \\ \beta^2 &= \frac{1.6523}{ab} \end{aligned} \right\} \quad (\text{for } \mu = 0.3) \quad (ii)$$

Coordinate Transformation

$$y = \beta x \quad (iii)$$

Internal Bending Moments

$$M_x = \frac{1}{4a\beta^2} \sum C_n M_n \quad (1)$$

$$M_\theta = \mu M_x \quad (2)$$

Internal Membrane Forces

$$Q = \frac{1}{4a\beta} \sum C_n Q_n \quad (3)$$

$$N_x = \text{constant} = \frac{\text{axial load}}{2\pi a} \quad (4)$$

$$N_\theta = -\sum C_n N_n + ap \quad (5)$$

Displacement of Middle Surface

$$w = \frac{a}{Eb} (\sum C_n N_n + \mu N_x - ap) \quad (6)$$

$$\frac{dw}{dx} = \frac{a\beta}{Eb} \sum C_n w'_n \quad (7)$$

$$u = \frac{1}{Eb\beta} \left[\frac{\mu}{4} \sum C_n Q_n + (N_x - \mu ap) y + C_5 \right] \quad (8)$$

Internal Bending Stresses (+ for outside surface, - for inside surface)

$$\sigma_{Bx} = \pm \frac{6M_x}{b^2} \quad (9)$$

$$\sigma_{B\theta} = \pm \frac{6M_{\theta}}{b^2} \quad (10)$$

Internal Membrane Stresses

$$\sigma_{mx} = \frac{N_x}{b} \quad (11)$$

$$\sigma_{m\theta} = \frac{N_{\theta}}{b} \quad (12)$$

Total or Principal Stresses (+ for outside surface, - for inside surface)

$$\sigma_x = \frac{N_x}{b} \pm \frac{6M_x}{b^2} \quad (13)$$

$$\sigma_{\theta} = \frac{N_{\theta}}{b} \pm \frac{6M_{\theta}}{b^2} \quad (14)$$

PROCEDURE

Numerical values of the sub- n functions are tabulated in Part II for the range of y values from 0 through 10 (in increments of 0.1).

It is usually more convenient to place the origin of the x coordinate at the loaded or restrained end of the cylindrical shell. Then the y values are in the range from 0 through βL . The exact value of $y_L = \beta L$ of the cylinder may not be one of the y values listed in the tables, in which case the nearest value is selected and the value of L is adjusted slightly.

Four simultaneous equations are solved to evaluate the four constants of integration C_n when a short cylinder ($y_L < 10$) is analyzed. Each equation in this set is a mathematical statement of a "boundary condition" or "edge condition" which is specified for the cylinder. The four boundary conditions are usually obtained by specifying two of the four quantities M_x , Q , w , and dw/dx at each end of the cylinder.

For long cylinders ($y_L > 10$) the constants of integration C_3 and C_4 are taken equal to zero, because the end effects decay very rapidly as x increases. Hence, only two boundary conditions are necessary, at the "origin" end of the cylinder. If loads are applied at both ends of a long cylinder, two "origin" ends may be established and treated as if there were two separate long cylinders with loads at one end of each.

The constant of integration C_5 appears in the formula for the axial displacement u only. Hence its value is obtained by specifying u at some y value (usually at y_0 or y_L), which generally stipulates merely an axial displacement of the cylinder. Hence, C_5 may usually be taken equal to zero.

Some of the above points and others are illustrated in several examples in Part III. The additional necessary relations to analyze a cylindrical container (cylinder closed with a flat circular plate at one or both ends) are also given. The standpipe is also treated.

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Part II VALUES OF ALL FUNCTIONS

INTRODUCTION

The values of all functions which appear in the cylindrical-shell formulas are tabulated for the range of y values from 0 through 10.0, in increments of 0.1.

The value of the function is the eight-digit number (preceded by a minus sign if negative) multiplied by ten raised to the power (digit with sign) following the eight-digit number. For example, the values of the four M_n functions at $y = 1.8$ from Table 1 are

$$M_1 = 7.5112456 -2 = 7.5112456 \times 10^{-2} = 0.075112456 ,$$

$$M_2 = 3.2195190 -1 = 3.2195190 \times 10^{-1} = 0.32195190 ,$$

$$M_3 = -2.7489828 \quad 0 = -2.7489828 ,$$

$$M_4 = -1.1782869 \quad 1 = -1.1782869 \times 10 = -11.782869 .$$

TABLE 1
M FUNCTIONS

Y	1	2	3	4
0.0	-2.0000000 0	0	2.0000000 0	0
0.1	-1.8006340 0	1.8066598 -1	2.1992993 0	-2.2066592 -1
0.2	-1.6048213 0	3.2531331 -1	2.3941120 0	-4.8531041 -1
0.3	-1.4154614 0	4.3785347 -1	2.5791388 0	-7.9782103 -1
0.4	-1.2348113 0	5.2206979 -1	2.7481230 0	-1.1618877 0
0.5	-1.0645615 0	5.8157256 -1	2.8937781 0	-1.5808781 0
0.6	-9.0590763 -1	6.1976469 -1	3.0077192 0	-2.0576911 0
0.7	-7.5961886 -1	6.3981803 -1	3.0804061 0	-2.5945899 0
0.8	-6.2610108 -1	6.4465773 -1	3.1010987 0	-3.1930104 0
0.9	-5.0545559 -1	6.3695389 -1	3.0578279 0	-3.8533462 0
1.0	-3.9753232 -1	6.1911973 -1	2.9373883 0	-4.5747100 0
1.1	-3.0197814 -1	5.9331433 -1	2.7253565 0	-5.3546692 0
1.2	-2.1828020 -1	5.6144956 -1	2.4061409 0	-6.1889568 0
1.3	-1.4580393 -1	5.2520049 -1	1.9630657 0	-7.0711610 0
1.4	-8.3826832 -2	4.8601785 -1	1.3785024 0	-7.9923906 0
1.5	-3.1567276 -2	4.4514245 -1	6.3404560 -1	-8.9409237 0
1.6	1.1790512 -2	4.0362089 -1	-2.8925104 -1	-9.9018400 0
1.7	4.7075480 -2	3.6232168 -1	-1.4105742 0	-1.0856641 1
1.8	7.5112456 -2	3.2195190 -1	-2.7489828 0	-1.1782869 1
1.9	9.6707914 -2	2.8307363 -1	-4.3229574 0	-1.2653724 1
2.0	1.1263867 -1	2.4612008 -1	-6.1498616 0	-1.3437698 1
2.1	1.2364327 -1	2.1141111 -1	-8.2453141 0	-1.4098228 1
2.2	1.3041555 -1	1.7916795 -1	-1.0622457 1	-1.4593381 1
2.3	1.3360011 -1	1.4952713 -1	-1.3291112 1	-1.4875600 1
2.4	1.3378969 -1	1.2255331 -1	-1.6256836 1	-1.4891500 1
2.5	1.3152374 -1	9.8251209 -2	-1.9519848 1	-1.4581768 1
2.6	1.2728839 -1	7.6576294 -2	-2.3073845 1	-1.3881152 1
2.7	1.2151727 -1	5.7444599 -2	-2.6904694 1	-1.2718599 1
2.8	1.1459321 -1	4.0741332 -2	-3.0989019 1	-1.1017528 1
2.9	1.0685050 -1	2.6328562 -2	-3.5292661 1	-8.6963092 0
3.0	9.8577660 -2	1.4051926 -2	-3.9769052 1	-5.6689497 0
3.1	9.0020497 -2	3.7463733 -3	-4.4357497 1	-1.8460212 0
3.2	8.1385405 -2	-4.7589009 -3	-4.8981383 1	2.8641198 0
3.3	7.2842780 -2	-1.1636306 -2	-5.3546357 1	8.5537895 0
3.4	6.4530451 -2	-1.7056471 -2	-5.7938467 1	1.5314100 1
3.5	5.6557101 -2	-2.1185462 -2	-6.2022365 1	2.3232670 1
3.6	4.9005573 -2	-2.4182604 -2	-6.5639542 1	3.2390909 1
3.7	4.1936064 -2	-2.6198829 -2	-6.8605716 1	4.2860857 1
3.8	3.5389132 -2	-2.7375463 -2	-7.0714386 1	5.4701514 1
3.9	2.9388530 -2	-2.7843401 -2	-7.1725671 1	6.7954628 1
4.0	2.3943815 -2	-2.7722642 -2	-7.1375471 1	8.2639990 1
4.1	1.9052755 -2	-2.7122082 -2	-6.9370096 1	9.8750100 1
4.2	1.4703503 -2	-2.6139566 -2	-6.5387415 1	1.1624432 2
4.3	1.0876548 -2	-2.4862108 -2	-5.9077667 1	1.3504241 2
4.4	7.5464733 -3	-2.3366290 -2	-5.0065106 1	1.5501755 2
4.5	4.6834726 -3	-2.1718764 -2	-3.7950539 1	1.7598882 2
4.6	2.2546892 -3	-1.9976845 -2	-2.2314929 1	1.9771323 2
4.7	2.2536685 -4	-1.8189167 -2	-2.7243176 0	2.1987736 2
4.8	-1.4401802 -3	-1.6396375 -2	2.1263924 1	2.4208863 2
4.9	-2.7777517 -3	-1.4631840 -2	5.0093210 1	2.6386656 2
5.0	-3.8225951 -3	-1.2922372 -2	8.4198159 1	2.8463383 2

TABLE 1

M FUNCTIONS

Y	1	2	3	4
5.1	-4.6088640 -3	-1.1288925 -2	1.2399297 2	3.0370766 2
5.2	-5.1692007 -3	-9.7472790 -3	1.6985777 2	3.2029151 2
5.3	-5.5344204 -3	-8.3086920 -3	2.2212275 2	3.3346753 2
5.4	-5.7332820 -3	-6.9805164 -3	2.8104965 2	3.4219000 2
5.5	-5.7923423 -3	-5.7667732 -3	3.4681098 2	3.4528005 2
5.6	-5.7358742 -3	-4.6686845 -3	4.1946633 2	3.4142241 2
5.7	-5.5858414 -3	-3.6851578 -3	4.9893613 2	3.2916409 2
5.8	-5.3619195 -3	-2.8132246 -3	5.8497258 2	3.0691606 2
5.9	-5.0815544 -3	-2.0484357 -3	6.7712784 2	2.7295838 2
6.0	-4.7600514 -3	-1.3852096 -3	7.7471969 2	2.2544906 2
6.1	-4.4106855 -3	-8.1713825 -4	8.7679468 2	1.6243790 2
6.2	-4.0448297 -3	-3.3725307 -4	9.8208919 2	8.1885424 1
6.3	-3.6720943 -3	6.1746894 -5	1.0889887 3	-1.8311531 1
6.4	-3.3004712 -3	3.8730289 -4	1.1954856 3	-1.4028756 2
6.5	-2.9364843 -3	6.4683685 -4	1.2991370 3	-2.8616863 2
6.6	-2.5853362 -3	8.4761963 -4	1.3970217 3	-4.5802286 2
6.7	-2.2510542 -3	9.9666754 -4	1.4856994 3	-6.5780221 2
6.8	-1.9366305 -3	1.1006611 -3	1.5611716 3	-8.8727350 2
6.9	-1.6441553 -3	1.1658856 -3	1.6188462 3	-1.1479387 3
7.0	-1.3749433 -3	1.1981882 -3	1.6535083 3	-1.4409424 3
7.1	-1.1296489 -3	1.2029519 -3	1.6592962 3	-1.7669681 3
7.2	-9.0837599 -4	1.1850826 -3	1.6296899 3	-2.1261209 3
7.3	-7.1077426 -4	1.1490064 -3	1.5575066 3	-2.5177966 3
7.4	-5.3612905 -4	1.0986764 -3	1.4349158 3	-2.9405384 3
7.5	-3.8343934 -4	1.0375872 -3	1.2534661 3	-3.3918804 3
7.6	-2.5148870 -4	9.6879464 -4	1.0041377 3	-3.8681787 3
7.7	-1.3890541 -4	8.9494004 -4	6.7741188 2	-4.3644306 3
7.8	-4.4216329 -5	8.1827748 -4	2.6337540 2	-4.8740852 3
7.9	3.4108774 -5	7.4070336 -4	-2.4815184 2	-5.3888450 3
8.0	9.7618598 -5	6.6378669 -4	-8.6744676 2	-5.8984622 3
8.1	1.4784948 -4	5.8880033 -4	-1.6046816 3	-6.3905336 3
8.2	1.8629949 -4	5.1675132 -4	-2.4696735 3	-6.8502982 3
8.3	2.1440827 -4	4.4841107 -4	-3.4715885 3	-7.2604414 3
8.4	2.3354174 -4	3.8434402 -4	-4.6185963 3	-7.6009105 3
8.5	2.4498053 -4	3.2493497 -4	-5.9174676 3	-7.8487551 3
8.6	2.4991250 -4	2.7041470 -4	-7.3731167 3	-7.9779892 3
8.7	2.4942785 -4	2.2088364 -4	-8.9880791 3	-7.9594948 3
8.8	2.4451723 -4	1.7633378 -4	-1.0761931 4	-7.7609746 3
8.9	2.3607178 -4	1.3666835 -4	-1.2690644 4	-7.3469574 3
9.0	2.2488512 -4	1.0171980 -4	-1.4765872 4	-6.6788836 3
9.1	2.1165656 -4	7.1265532 -5	-1.6974185 4	-5.7152697 3
9.2	1.9699565 -4	4.5042022 -5	-1.9296238 4	-4.4119836 3
9.3	1.8142733 -4	2.2756903 -5	-2.1705887 4	-2.7226259 3
9.4	1.6539783 -4	4.0995688 -6	-2.4169265 4	-5.9906207 2
9.5	1.4928082 -4	-1.1249956 -5	-2.6643818 4	2.0079055 3
9.6	1.3338383 -4	-2.3613505 -5	-2.9077325 4	5.1476822 3
9.7	1.1795467 -4	-3.3308592 -5	-3.1406910 4	8.8688306 3
9.8	1.0318774 -4	-4.0643489 -5	-3.3558072 4	1.3217821 4
9.9	8.9230243 -5	-4.5913277 -5	-3.5443760 4	1.8237529 4
10.0	7.6188031 -5	-4.9396939 -5	-3.6963516 4	2.3965504 4

TABLE 2

Q FUNCTIONS

Y	1	2	3	4
0.0	-2.0000000 0	-2.0000000 0	-2.0000000 0	2.0000000 0
0.1	-1.9813000 0	-1.6199680 0	-1.9786334 0	2.4199552 0
0.2	-1.9301346 0	-1.2795080 0	-1.9088016 0	2.8794224 0
0.3	-1.8533148 0	-9.7760792 -1	-1.7813177 0	3.3769598 0
0.4	-1.7568811 0	-7.1274152 -1	-1.5862354 0	3.9100107 0
0.5	-1.6461340 0	-4.8298893 -1	-1.3129000 0	4.4746561 0
0.6	-1.5256723 0	-2.8614295 -1	-9.5002801 -1	5.0654103 0
0.7	-1.3994369 0	-1.1980083 -1	-4.8581627 -1	5.6749960 0
0.8	-1.2707588 0	1.8556650 -2	9.1911681 -2	6.2941090 0
0.9	-1.1424095 0	1.3149830 -1	7.9551830 -1	6.9111740 0
1.0	-1.0166520 0	2.2158741 -1	1.6373217 0	7.5120983 0
1.1	-8.9529246 -1	2.9133618 -1	2.6293127 0	8.0800257 0
1.2	-7.7972976 -1	3.4316936 -1	3.7828160 0	8.5950977 0
1.3	-6.7100442 -1	3.7939656 -1	5.1080952 0	9.0342267 0
1.4	-5.6984468 -1	4.0219101 -1	6.6138881 0	9.3708930 0
1.5	-4.7670973 -1	4.1357518 -1	8.3068781 0	9.5749693 0
1.6	-3.9183038 -1	4.1541140 -1	1.0191091 1	9.6125890 0
1.7	-3.1524620 -1	4.0939716 -1	1.2267215 1	9.4460670 0
1.8	-2.4683944 -1	3.9706435 -1	1.4531851 1	9.0338862 0
1.9	-1.8636572 -1	3.7978154 -1	1.6976681 1	8.3307668 0
2.0	-1.3348142 -1	3.5875875 -1	1.9587560 1	7.2878370 0
2.1	-8.7767839 -2	3.3505439 -1	2.2343542 1	5.8529137 0
2.2	-4.8752403 -2	3.0958350 -1	2.5215838 1	3.9709245 0
2.3	-1.5927015 -2	2.8312724 -1	2.8166712 1	1.5844878 0
2.4	1.1236373 -2	2.5634300 -1	3.1148336 1	-1.3653360 0
2.5	3.3272530 -2	2.2977495 -1	3.4101615 1	-4.9380798 0
2.6	5.0712094 -2	2.0386468 -1	3.6954997 1	-9.1926920 0
2.7	6.4072666 -2	1.7896187 -1	3.9623293 1	-1.4186095 1
2.8	7.3851876 -2	1.5533454 -1	4.2006546 1	-1.9971491 1
2.9	8.0521939 -2	1.3317906 -1	4.3988969 1	-2.6596351 1
3.0	8.4525734 -2	1.1262959 -1	4.5438001 1	-3.4100102 1
3.1	8.6274124 -2	9.3766869 -2	4.6203518 1	-4.2511476 1
3.2	8.6144305 -2	7.6626504 -2	4.6117263 1	-5.1845503 1
3.3	8.4479085 -2	6.1206474 -2	4.4992567 1	-6.2100146 1
3.4	8.1586921 -2	4.7473980 -2	4.2624367 1	-7.3252566 1
3.5	7.7742563 -2	3.5371639 -2	3.8789695 1	-8.5255035 1
3.6	7.3188176 -2	2.4822969 -2	3.3248633 1	-9.8030452 1
3.7	6.8134893 -2	1.5737235 -2	2.5745859 1	-1.1146757 2
3.8	6.2764595 -2	8.0136684 -3	1.6012872 1	-1.2541590 2
3.9	5.7231931 -2	1.5451292 -3	3.7710437 0	-1.3968030 2
4.0	5.1666455 -2	-3.7788267 -3	-1.1264518 1	-1.5401546 2
4.1	4.6174837 -2	-8.0693272 -3	-2.9380004 1	-1.6812019 2
4.2	4.0843068 -2	-1.1436063 -2	-5.0856904 1	-1.8163173 2
4.3	3.5738656 -2	-1.3985559 -2	-7.5964743 1	-1.9412007 2
4.4	3.0912763 -2	-1.5819817 -2	-1.0495244 2	-2.0508265 2
4.5	2.6402237 -2	-1.7035292 -2	-1.3803828 2	-2.1393936 2
4.6	2.2231534 -2	-1.7722156 -2	-1.7539830 2	-2.2002816 2
4.7	1.8414534 -2	-1.7963800 -2	-2.1715304 2	-2.2260167 2
4.8	1.4956194 -2	-1.7836555 -2	-2.6335256 2	-2.2082471 2
4.9	1.1854088 -2	-1.7409592 -2	-3.1395977 2	-2.1377335 2
5.0	9.0997766 -3	-1.6744967 -2	-3.6883199 2	-2.0043567 2

TABLE 2

Q FUNCTIONS

Y	1	2	3	4
5.1	6.6800611 -3	-1.5897789 -2	-4.2770063 2	-1.7971470 2
5.2	4.5780783 -3	-1.4916480 -2	-4.9014928 2	-1.5043374 2
5.3	2.7742716 -3	-1.3843113 -2	-5.5559027 2	-1.1134478 2
5.4	1.2472344 -3	-1.2713798 -2	-6.2323965 2	-6.1140339 1
5.5	-2.5569111 -5	-1.1559115 -2	-6.9209104 2	1.5309262 0
5.6	-1.0671896 -3	-1.0404559 -2	-7.6088874 2	7.8043920 1
5.7	-1.9006836 -3	-9.2709992 -3	-8.2810023 2	1.6977205 2
5.8	-2.5486950 -3	-8.1751441 -3	-8.9188864 2	2.7805652 2
5.9	-3.0331187 -3	-7.1299902 -3	-9.5008621 2	4.0416946 2
6.0	-3.3748419 -3	-6.1452610 -3	-1.0001687 3	5.4927064 2
6.1	-3.5935472 -3	-5.2278237 -3	-1.0392326 3	7.1435678 2
6.2	-3.7075767 -3	-4.3820828 -3	-1.0639746 3	9.0020378 2
6.3	-3.7338412 -3	-3.6103474 -3	-1.0706772 3	1.1073002 3
6.4	-3.6877741 -3	-2.9131684 -3	-1.0551981 3	1.3357732 3
6.5	-3.5833211 -3	-2.2896475 -3	-1.0129684 3	1.5853056 3
6.6	-3.4329558 -3	-1.7377165 -3	-9.3899889 2	1.8550446 3
6.7	-3.2477217 -3	-1.2543866 -3	-8.2789721 2	2.1435016 3
6.8	-3.0372916 -3	-8.3596934 -4	-6.7389810 2	2.4484451 3
6.9	-2.8100410 -3	-4.7826974 -4	-4.7090755 2	2.7667849 3
7.0	-2.5731314 -3	-1.7675510 -4	-2.1256589 2	3.0944507 3
7.1	-2.3326007 -3	7.3303003 -5	1.0767187 2	3.4262644 3
7.2	-2.0934586 -3	2.7670662 -4	4.9643100 2	3.7558108 3
7.3	-1.8597806 -3	4.3823211 -4	9.6028998 2	4.0753031 3
7.4	-1.6348054 -3	5.6254733 -4	1.5056226 3	4.3754542 3
7.5	-1.4210265 -3	6.5414784 -4	2.1384143 3	4.6453465 3
7.6	-1.2202833 -3	7.1730593 -4	2.8640410 3	4.8723164 3
7.7	-1.0338454 -3	7.5603463 -4	3.6870187 3	5.0418425 3
7.8	-8.6249381 -4	7.7406116 -4	4.6107099 3	5.1374606 3
7.9	-7.0659459 -4	7.7481212 -4	5.6369968 3	5.1406932 3
8.0	-5.6616810 -4	7.6140529 -4	6.7659089 3	5.0310154 3
8.1	-4.4095086 -4	7.3664982 -4	7.9952152 3	4.7858520 3
8.2	-3.3045183 -4	7.0305080 -4	9.3199717 3	4.3806247 3
8.3	-2.3400280 -4	6.6281934 -4	1.0732030 4	3.7888529 3
8.4	-1.5080228 -4	6.1788575 -4	1.2219507 4	2.9823143 3
8.5	-7.9954443 -5	5.6991551 -4	1.3766222 4	1.9312875 3
8.6	-2.0502206 -5	5.2032720 -4	1.5351106 4	6.0487236 2
8.7	2.8544203 -5	4.7031148 -4	1.6947574 4	-1.0285842 3
8.8	6.8183448 -5	4.2085100 -4	1.8522906 4	-3.0009566 3
8.9	9.9403435 -5	3.7274013 -4	2.0037601 4	-5.3436865 3
9.0	1.2316532 -4	3.2660491 -4	2.1444755 4	-8.0869384 3
9.1	1.4039103 -4	2.8292210 -4	2.2689455 4	-1.1258916 4
9.2	1.5195363 -4	2.4203767 -4	2.3708221 4	-1.4884254 4
9.3	1.5867043 -4	2.0418423 -4	2.4428512 4	-1.8983261 4
9.4	1.6129826 -4	1.6949739 -4	2.4768326 4	-2.3570203 4
9.5	1.6053077 -4	1.3803086 -4	2.4635912 4	-2.8651723 4
9.6	1.5699733 -4	1.0977033 -4	2.3929643 4	-3.4225007 4
9.7	1.5126326 -4	8.4646076 -5	2.2538080 4	-4.0275741 4
9.8	1.4383123 -4	6.2544253 -5	2.0340251 4	-4.6775893 4
9.9	1.3514352 -4	4.3316966 -5	1.7206230 4	-5.3681288 4
10.0	1.2558497 -4	2.6791093 -5	1.2998012 4	-6.0929019 4

TABLE 3

N FUNCTIONS

Y	1	2	3	4
0.0	0	1.0000000 0	0	1.0000000 0
0.1	9.0332989 -2	9.0031699 -1	1.1033296 -1	1.0996496 0
0.2	1.6265665 -1	8.0241065 -1	2.4265521 -1	1.1970560 0
0.3	2.1892674 -1	7.0773069 -1	3.9891051 -1	1.2895694 0
0.4	2.6103489 -1	6.1740566 -1	5.8094383 -1	1.3740615 0
0.5	2.9078628 -1	5.3228075 -1	7.9043904 -1	1.4468890 0
0.6	3.0988234 -1	4.5295382 -1	1.0288456 0	1.5038596 0
0.7	3.1990902 -1	3.7980943 -1	1.2972949 0	1.5402031 0
0.8	3.2232887 -1	3.1305054 -1	1.5965052 0	1.5505493 0
0.9	3.1847695 -1	2.5272779 -1	1.9266731 0	1.5289139 0
1.0	3.0955987 -1	1.9876616 -1	2.2873550 0	1.4686942 0
1.1	2.9665716 -1	1.5098907 -1	2.6773346 0	1.3626783 0
1.2	2.8072478 -1	1.0914010 -1	3.0944784 0	1.2030704 0
1.3	2.6260025 -1	7.2901966 -2	3.5355805 0	9.8153287 -1
1.4	2.4300892 -1	4.1913416 -2	3.9961953 0	6.8925121 -1
1.5	2.2257123 -1	1.5783638 -2	4.4704618 0	3.1702280 -1
1.6	2.0181045 -1	-5.8952559 -3	4.9509200 0	-1.4462552 -1
1.7	1.8116084 -1	-2.3537740 -2	5.4283205 0	-7.0528708 -1
1.8	1.6097595 -1	-3.7556228 -2	5.8914344 0	-1.3744914 0
1.9	1.4153682 -1	-4.8353957 -2	6.3268621 0	-2.1614787 0
2.0	1.2306004 -1	-5.6319333 -2	6.7188492 0	-3.0749308 0
2.1	1.0570556 -1	-6.1821637 -2	7.0491139 0	-4.1226571 0
2.2	8.9583976 -2	-6.5207774 -2	7.2966908 0	-5.3112285 0
2.3	7.4763564 -2	-6.6800056 -2	7.4377999 0	-6.6455560 0
2.4	6.1276656 -2	-6.6894842 -2	7.4457500 0	-8.1284180 0
2.5	4.9125605 -2	-6.5761869 -2	7.2908840 0	-9.7599238 0
2.6	3.8288147 -2	-6.3644193 -2	6.9405763 0	-1.1536922 1
2.7	2.8722300 -2	-6.0758633 -2	6.3592995 0	-1.3452347 1
2.8	2.0370666 -2	-5.7296604 -2	5.5087641 0	-1.5494509 1
2.9	1.3164281 -2	-5.3425250 -2	4.3481546 0	-1.7646330 1
3.0	7.0259631 -3	-4.9288830 -2	2.8344748 0	-1.9884526 1
3.1	1.8731866 -3	-4.5010248 -2	9.2301059 -1	-2.2178749 1
3.2	-2.3794505 -3	-4.0692703 -2	-1.4320599 0	-2.4490692 1
3.3	-5.8181531 -3	-3.6421390 -2	-4.2768947 0	-2.6773178 1
3.4	-8.5282353 -3	-3.2265225 -2	-7.6570499 0	-2.8969233 1
3.5	-1.0592731 -2	-2.8278551 -2	-1.1616335 1	-3.1011183 1
3.6	-1.2091302 -2	-2.4502786 -2	-1.6195455 1	-3.2819771 1
3.7	-1.3099415 -2	-2.0968032 -2	-2.1430429 1	-3.4303358 1
3.8	-1.3687732 -2	-1.7694566 -2	-2.7350757 1	-3.5357193 1
3.9	-1.3921701 -2	-1.4694265 -2	-3.3977314 1	-3.5862836 1
4.0	-1.3861321 -2	-1.1971907 -2	-4.1319995 1	-3.5687735 1
4.1	-1.3561041 -2	-9.5263776 -3	-4.9375050 1	-3.4685048 1
4.2	-1.3069783 -2	-7.3517512 -3	-5.8122159 1	-3.2693707 1
4.3	-1.2431054 -2	-5.4382742 -3	-6.7521205 1	-2.9538833 1
4.4	-1.1683145 -2	-3.7732366 -3	-7.7508774 1	-2.5032553 1
4.5	-1.0859382 -2	-2.3417363 -3	-8.7994410 1	-1.8975269 1
4.6	-9.9884225 -3	-1.1273446 -3	-9.8856616 1	-1.1157465 1
4.7	-9.0945834 -3	-1.1268342 -4	-1.0993868 2	-1.3621588 0
4.8	-8.1981874 -3	7.2009012 -4	-1.2104432 2	1.0631962 1
4.9	-7.3159199 -3	1.3888759 -3	-1.3193328 2	2.5046605 1
5.0	-6.4611859 -3	1.9112976 -3	-1.4231692 2	4.2099079 1

TABLE 3

N FUNCTIONS

Y	1	2	3	4
5.1	-5.6444625 -3	2.3044320 -3	-1.5185383 2	6.1996484 1
5.2	-4.8736395 -3	2.5846004 -3	-1.6014575 2	8.4928887 1
5.3	-4.1543460 -3	2.7672102 -3	-1.6673376 2	1.1106137 2
5.4	-3.4902582 -3	2.8666410 -3	-1.7109500 2	1.4052483 2
5.5	-2.8833866 -3	2.8961712 -3	-1.7264003 2	1.7340549 2
5.6	-2.3343423 -3	2.8679371 -3	-1.7071120 2	2.0973316 2
5.7	-1.8425789 -3	2.7929207 -3	-1.6458204 2	2.4946807 2
5.8	-1.4066123 -3	2.6809598 -3	-1.5345803 2	2.9248629 2
5.9	-1.0242179 -3	2.5407772 -3	-1.3647919 2	3.3856392 2
6.0	-6.9260478 -4	2.3800257 -3	-1.1272453 2	3.8735985 2
6.1	-4.0856913 -4	2.2053427 -3	-8.1218951 1	4.3839734 2
6.2	-1.6862654 -4	2.0224149 -3	-4.0942712 1	4.9104459 2
6.3	3.0873447 -5	1.8360471 -3	9.1557657 0	5.4449435 2
6.4	1.9365145 -4	1.6502356 -3	7.0143778 1	5.9774282 2
6.5	3.2341842 -4	1.4682421 -3	1.4308432 2	6.4956851 2
6.6	4.2380981 -4	1.2926681 -3	2.2901143 2	6.9851087 2
6.7	4.9833377 -4	1.1255271 -3	3.2890110 2	7.4284970 2
6.8	5.5033056 -4	9.6831524 -4	4.4363675 2	7.8058580 2
6.9	5.8294280 -4	8.2207767 -4	5.7396933 2	8.0942311 2
7.0	5.9909407 -4	6.8747163 -4	7.2047119 2	8.2675414 2
7.1	6.0147594 -4	5.6482444 -4	8.8348407 2	8.2964813 2
7.2	5.9254131 -4	4.5418800 -4	1.0630605 3	8.1484497 2
7.3	5.7450319 -4	3.5538713 -4	1.2588983 3	7.7875330 2
7.4	5.4933818 -4	2.6806452 -4	1.4702692 3	7.1745788 2
7.5	5.1879359 -4	1.9171967 -4	1.6959402 3	6.2673306 2
7.6	4.8439732 -4	1.2574435 -4	1.9340893 3	5.0206886 2
7.7	4.4747002 -4	6.9452704 -5	2.1822153 3	3.3870594 2
7.8	4.0913874 -4	2.2108165 -5	2.4370426 3	1.3168770 2
7.9	3.7035168 -4	-1.7054387 -5	2.6944225 3	-1.2407592 2
8.0	3.3189335 -4	-4.8809299 -5	2.9492311 3	-4.3372338 2
8.1	2.9440016 -4	-7.3924740 -5	3.1952668 3	-8.0234081 2
8.2	2.5837566 -4	-9.3149745 -5	3.4251491 3	-1.2348368 3
8.3	2.2420554 -4	-1.0720414 -4	3.6302207 3	-1.7357942 3
8.4	1.9217201 -4	-1.1677087 -4	3.8004553 3	-2.3092982 3
8.5	1.6246749 -4	-1.2249027 -4	3.9243775 3	-2.9587338 3
8.6	1.3520735 -4	-1.2495625 -4	3.9889946 3	-3.6865583 3
8.7	1.1044182 -4	-1.2471392 -4	3.9797474 3	-4.4940395 3
8.8	8.8166888 -5	-1.2225861 -4	3.8804873 3	-5.3809656 3
8.9	6.8334174 -5	-1.1803589 -4	3.6734787 3	-6.3453219 3
9.0	5.0859898 -5	-1.1244256 -4	3.3394418 3	-7.3829361 3
9.1	3.5632766 -5	-1.0582828 -4	2.8576349 3	-8.4870926 3
9.2	2.2521011 -5	-9.8497823 -5	2.2059918 3	-9.6481190 3
9.3	1.1378451 -5	-9.0713665 -5	1.3613130 3	-1.0852943 4
9.4	2.0497844 -6	-8.2698913 -5	2.9953103 2	-1.2084632 4
9.5	-5.6249778 -6	-7.4640408 -5	-1.0039527 3	-1.3321909 4
9.6	-1.1806753 -5	-6.6691916 -5	-2.5738411 3	-1.4538663 4
9.7	-1.6654296 -5	-5.8977333 -5	-4.4344153 3	-1.5703455 4
9.8	-2.0321744 -5	-5.1593871 -5	-6.6089106 3	-1.6779036 4
9.9	-2.2956639 -5	-4.4615121 -5	-9.1187645 3	-1.7721880 4
10.0	-2.4698470 -5	-3.8094016 -5	-1.1982752 4	-1.8481758 4

TABLE 4

W PRIME FUNCTIONS

Y	1	2	3	4
0.0	1.0000000 0	-1.0000000 0	1.0000000 0	1.0000000 0
0.1	8.0998400 -1	-9.9064997 -1	1.2099826 0	9.8931668 -1
0.2	6.3975400 -1	-9.6506730 -1	1.4397112 0	9.5440079 -1
0.3	4.8880396 -1	-9.2665741 -1	1.6884799 0	8.9065887 -1
0.4	3.5637076 -1	-8.7844055 -1	1.9550054 0	7.9311770 -1
0.5	2.4149446 -1	-8.2306702 -1	2.2373281 0	6.5645000 -1
0.6	1.4307147 -1	-7.6283616 -1	2.5327052 0	4.7501401 -1
0.7	5.9900413 -2	-6.9971845 -1	2.8374980 0	2.4290814 -1
0.8	-9.2783250 -3	-6.3537939 -1	3.1470545 0	-4.5955840 -2
0.9	-6.5749151 -2	-5.7120473 -1	3.4555870 0	-3.9775915 -1
1.0	-1.1079371 -1	-5.0832602 -1	3.7560492 0	-8.1866083 -1
1.1	-1.4566809 -1	-4.4764623 -1	4.0400128 0	-1.3146563 0
1.2	-1.7158468 -1	-3.8986488 -1	4.2975488 0	-1.8914080 0
1.3	-1.8969828 -1	-3.3550221 -1	4.5171133 0	-2.5540476 0
1.4	-2.0109551 -1	-2.8492234 -1	4.6854465 0	-3.3069441 0
1.5	-2.0678759 -1	-2.3835486 -1	4.7874846 0	-4.1534390 0
1.6	-2.0770570 -1	-1.9591519 -1	4.8062945 0	-5.0955455 0
1.7	-2.0469858 -1	-1.5762310 -1	4.7230335 0	-6.1336076 0
1.8	-1.9853218 -1	-1.2341972 -1	4.5169431 0	-7.2659258 0
1.9	-1.8989077 -1	-9.3182862 -2	4.1653834 0	-8.4883407 0
2.0	-1.7937938 -1	-6.6740708 -2	3.6439185 0	-9.7937800 0
2.1	-1.6752719 -1	-4.3883920 -2	2.9264568 0	-1.1171771 1
2.2	-1.5479175 -1	-2.4376202 -2	1.9854623 0	-1.2607919 1
2.3	-1.4156362 -1	-7.9635077 -3	7.9224390 -1	-1.4083356 1
2.4	-1.2817150 -1	5.6181867 -3	-6.8266801 -1	-1.5574168 1
2.5	-1.1488747 -1	1.6636265 -2	-2.4690399 0	-1.7050808 1
2.6	-1.0193234 -1	2.5356047 -2	-4.5963460 0	-1.8477498 1
2.7	-8.9480933 -2	3.2036333 -2	-7.0930475 0	-1.9811646 1
2.8	-7.7667270 -2	3.6925938 -2	-9.9857454 0	-2.1003273 1
2.9	-6.6589530 -2	4.0260969 -2	-1.3298176 1	-2.1994485 1
3.0	-5.6314793 -2	4.2262867 -2	-1.7050051 1	-2.2719001 1
3.1	-4.6883435 -2	4.3137062 -2	-2.1255738 1	-2.3101759 1
3.2	-3.8313252 -2	4.3072153 -2	-2.5922751 1	-2.3058632 1
3.3	-3.0603237 -2	4.2239543 -2	-3.1050073 1	-2.2496283 1
3.4	-2.3736990 -2	4.0793461 -2	-3.6626283 1	-2.1312183 1
3.5	-1.7685819 -2	3.8871281 -2	-4.2627518 1	-1.9394847 1
3.6	-1.2411484 -2	3.6594088 -2	-4.9015226 1	-1.6624317 1
3.7	-7.8686172 -3	3.4067447 -2	-5.5733737 1	-1.2872929 1
3.8	-4.0068342 -3	3.1382297 -2	-6.2707949 1	-8.0064361 0
3.9	-7.7256460 -4	2.8615966 -2	-6.9840149 1	-1.8855218 0
4.0	1.8894134 -3	2.5833228 -2	-7.7007730 1	5.6322592 0
4.1	4.0346636 -3	2.3087419 -2	-8.4060098 1	1.4690002 1
4.2	5.7180316 -3	2.0421534 -2	-9.0815866 1	2.5428452 1
4.3	6.9927797 -3	1.7869328 -2	-9.7060038 1	3.7982372 1
4.4	7.9099084 -3	1.5456382 -2	-1.0254133 2	5.2476221 1
4.5	8.5176457 -3	1.3201118 -2	-1.0696968 2	6.9019140 1
4.6	8.8610780 -3	1.1115767 -2	-1.1001408 2	8.7699151 1
4.7	8.9819000 -3	9.2072667 -3	-1.1130084 2	1.0857652 2
4.8	8.9182774 -3	7.4780972 -3	-1.1041235 2	1.3167628 2
4.9	8.7047957 -3	5.9270440 -3	-1.0688668 2	1.5697989 2
5.0	8.3724834 -3	4.5498883 -3	-1.0021784 2	1.8441600 2

TABLE 4

W PRIME FUNCTIONS

Y	1	2	3	4
5.1	7.9488945 -3	3.3400306 -3	-8.9857349 1	2.1385031 2
5.2	7.4582399 -3	2.2890391 -3	-7.5216868 1	2.4507464 2
5.3	6.9215563 -3	1.3871358 -3	-5.5672390 1	2.7779514 2
5.4	6.3568991 -3	6.2361721 -4	-3.0570170 1	3.1161982 2
5.5	5.7795577 -3	-1.2784556 -5	7.6546311 -1	3.4604552 2
5.6	5.2022793 -3	-5.3359479 -4	3.9021960 1	3.8044437 2
5.7	4.6354996 -3	-9.5034182 -4	8.4886025 1	4.1405011 2
5.8	4.0875720 -3	-1.2743475 -3	1.3902826 2	4.4594432 2
5.9	3.5649951 -3	-1.5165593 -3	2.0208473 2	4.7504311 2
6.0	3.0726305 -3	-1.6874209 -3	2.7463532 2	5.0008438 2
6.1	2.6139119 -3	-1.7967736 -3	3.5717839 2	5.1961629 2
6.2	2.1910414 -3	-1.8537884 -3	4.5010189 2	5.3198730 2
6.3	1.8051737 -3	-1.8669206 -3	5.5365011 2	5.3533858 2
6.4	1.4565842 -3	-1.8438871 -3	6.6788659 2	5.2759904 2
6.5	1.1448237 -3	-1.7916605 -3	7.9265282 2	5.0648420 2
6.6	8.6885827 -4	-1.7164779 -3	9.2752230 2	4.6949944 2
6.7	6.2719330 -4	-1.6238609 -3	1.0717508 3	4.1394861 2
6.8	4.1798467 -4	-1.5186458 -3	1.2242225 3	3.3694905 2
6.9	2.3913487 -4	-1.4050205 -3	1.3833924 3	2.3545378 2
7.0	8.8377552 -5	-1.2865657 -3	1.5472253 3	1.0628294 2
7.1	-3.6651502 -5	-1.1663004 -3	1.7131322 3	-5.3835933 1
7.2	-1.3835331 -4	-1.0467293 -3	1.8779054 3	-2.4821550 2
7.3	-2.1911606 -4	-9.2989031 -4	2.0376516 3	-4.8014499 2
7.4	-2.8127367 -4	-8.1740271 -4	2.1877271 3	-7.5281131 2
7.5	-3.2707392 -4	-7.1051326 -4	2.3226732 3	-1.0692071 3
7.6	-3.5865297 -4	-6.1014167 -4	2.4361582 3	-1.4320205 3
7.7	-3.7801732 -4	-5.1692272 -4	2.5209212 3	-1.8435094 3
7.8	-3.8703058 -4	-4.3124691 -4	2.5687303 3	-2.3053550 3
7.9	-3.8740606 -4	-3.5329729 -4	2.5703466 3	-2.8184984 3
8.0	-3.8070264 -4	-2.8308405 -4	2.5155077 3	-3.3829544 3
8.1	-3.6832491 -4	-2.2047543 -4	2.3929260 3	-3.9976076 3
8.2	-3.5152540 -4	-1.6522591 -4	2.1903123 3	-4.6599858 3
8.3	-3.3140967 -4	-1.1700140 -4	1.8944265 3	-5.3660149 3
8.4	-3.0894288 -4	-7.5401139 -5	1.4911571 3	-6.1097534 3
8.5	-2.8495775 -4	-3.9977221 -5	9.6564373 2	-6.8831112 3
8.6	-2.6016360 -4	-1.0251103 -5	3.0243618 2	-7.6755529 3
8.7	-2.3515574 -4	1.4272101 -5	-5.1429212 2	-8.4737870 3
8.8	-2.1042550 -4	3.4091724 -5	-1.5004783 3	-9.2614530 3
8.9	-1.8637006 -4	4.9701717 -5	-2.6718432 3	-1.0018800 4
9.0	-1.6330245 -4	6.1582658 -5	-4.0434942 3	-1.0722378 4
9.1	-1.4146105 -4	7.0195517 -5	-5.6294580 3	-1.1344727 4
9.2	-1.2101883 -4	7.5976815 -5	-7.4421272 3	-1.1854111 4
9.3	-1.0209212 -4	7.9335214 -5	-9.4916306 3	-1.2214256 4
9.4	-8.4748696 -5	8.0649128 -5	-1.1785101 4	-1.2384163 4
9.5	-6.9015430 -5	8.0265385 -5	-1.4325861 4	-1.2317956 4
9.6	-5.4885163 -5	7.8498667 -5	-1.7112504 4	-1.1964822 4
9.7	-4.2323038 -5	7.5631629 -5	-2.0137870 4	-1.1269040 4
9.8	-3.1272127 -5	7.1915616 -5	-2.3387946 4	-1.0170125 4
9.9	-2.1658483 -5	6.7571760 -5	-2.6840644 4	-8.6031152 3
10.0	-1.3395546 -5	6.2792485 -5	-3.0464510 4	-6.4990063 3

SUMMARY OF FUNCTIONS EVALUATED AND TABULATED IN PART II

Table 1

$$M_1 = -2e^{-y} \cos y$$

$$M_2 = 2e^{-y} \sin y$$

$$M_3 = 2e^y \cos y$$

$$M_4 = -2e^y \sin y$$

Table 2

$$Q_1 = -2e^{-y} (\cos y + \sin y)$$

$$Q_2 = 2e^{-y} (\sin y - \cos y)$$

$$Q_3 = 2e^y (\sin y - \cos y)$$

$$Q_4 = 2e^y (\cos y + \sin y)$$

Table 3

$$N_1 = e^{-y} \sin y$$

$$N_2 = e^{-y} \cos y$$

$$N_3 = e^y \sin y$$

$$N_4 = e^y \cos y$$

Table 4

$$W'_1 = e^{-y} (\cos y - \sin y)$$

$$W'_2 = -e^{-y} (\cos y + \sin y)$$

$$W'_3 = e^y (\cos y + \sin y)$$

$$W'_4 = e^y (\cos y - \sin y)$$

Part III
ILLUSTRATIVE EXAMPLES

EXAMPLE 1

A cylindrical shell with a uniform thickness of $\frac{1}{4}$ in. has a radius of its middle surface of 4 in. A radial outward shearing force of Q_0 lb/in. is applied at one end of the cylinder, while the other end is rigidly fixed. The length of the cylinder is 2 in. and Poisson's ratio is equal to 0.3. Determine the circumferential membrane stress at the end where the shearing force is applied and the axial bending stress at the fixed end.

Solution

Since $ab = 1$, formula ii yields

$$\beta = 1.2854 .$$

Then, the exact value of y_L is $(1.2854)2 = 2.5708$. The nearest value appearing in the tables is 2.6, and hence the adjusted value of L is 2.0227.

When the origin of x is at the end of the cylinder with the shearing force, the boundary conditions are

$$\left. \begin{array}{l} M_x = 0 \\ Q = Q_0 \end{array} \right\} \quad \text{at } y = y_0 = 0 ,$$

$$\left. \begin{array}{l} w = 0 \\ \frac{dw}{dx} = 0 \end{array} \right\} \quad \text{at } y = y_L = 2.6 .$$

Substitution in formulas 1, 3, 6, and 7 of these values and the proper values from the tables yields the following set of simultaneous equations:

C_1	C_2	C_3	C_4	Q_0
-2	0	2	0	= 0
-2	-2	-2	2	= 20.566
0.038288	-0.063644	6.9406	-11.537	= 0
-0.10193	0.025356	-4.5963	-18.477	= 0

This set may be solved very readily by first noting that $C_1 = C_3$ (first equation). The solution is

$$\begin{aligned} C_1 &= -0.081092Q_0 , & C_2 &= -10.114Q_0 , \\ C_3 &= -0.081092Q_0 , & C_4 &= 0.0067408Q_0 . \end{aligned}$$

Then, by formula 5 and the values in Table 3 for $y = 0$,

$$N_\theta = -(C_2 + C_4) = 10.107Q_0 ;$$

and, by formula 12,

$$\sigma_{m\theta} = 10.107Q_0/b = 40.428Q_0 .$$

Formula 1 and the values in Table 1 for $y = 2.6$ yield

$$M_x = 0.037828(0.12729C_1 + 0.076576C_2 - 23.074C_3 - 13.881C_4) = 0.037552Q_0 .$$

Then formula 9 yields

$$\sigma_{Bx} = 6(0.037552)Q_0/b^2 = 3.6050Q_0 ,$$

a tensile stress at the outside surface.

For this example it is equally convenient to place the origin of x at the fixed end. The boundary conditions are then

$$\left. \begin{array}{l} M_x = 0 \\ Q = -Q_0 \end{array} \right\} \quad \text{at } y = y_L = 2.6 ,$$

$$\left. \begin{array}{l} w = 0 \\ \frac{dw}{dx} = 0 \end{array} \right\} \quad \text{at } y = y_0 = 0 .$$

The negative sign must now appear in the second condition because an *outward* shearing force is negative when applied at the "terminal" end.

The set of simultaneous equations corresponding to this set of boundary conditions is

C_1	C_2	C_3	C_4		Q_0
0.12729	0.076576	-23.074	-13.881	=	0
0.050712	0.20386	36.955	-9.1927	=	-20.566
0	1	0	1	=	0
1	-1	1	1	=	0

This set is readily solved by noting that $C_2 = -C_4$ (third equation). The solution is

$$\begin{aligned} C_1 &= -0.88881Q_0 , & C_2 &= -0.64061Q_0 , \\ C_3 &= -0.39241Q_0 , & C_4 &= 0.64061Q_0 . \end{aligned}$$

Then

$$N_\theta \text{ (for } y = 2.6) = (-0.034031C_1 + 0.040771C_2 - 2.7236C_3 - 7.3907C_4) = 10.107Q_0$$

and

$$M_x \text{ (for } y = 0) = 0.037828(-2C_1 + 2C_3) = 0.037552Q_0 ,$$

which agree with the previous results.

EXAMPLE 2

A cylindrical shell with a diameter of 2.88 in. and a uniform thickness of 0.1 in. has a length of 3 in. A uniform bending moment of M_0 lb.-in./in. is applied along one end. Determine the circumferential membrane force, the radial displacement, and the slope of the radial displacement at the loaded end. ($\mu = 0.3$.)

Solution

Since $a = 1.44$ in. and $b = 0.1$ in., $\beta = 3.3874$. The cylinder in this example is considered as a long cylinder ($y_L = 10.162 > 10$), so $C_3 = C_4 = 0$ and hence only two boundary conditions are required. With the origin of x at the loaded end, these are

$$\left. \begin{array}{l} M_x = M_0 \\ Q = 0 \end{array} \right\} \quad \text{at } y = y_0 = 0 .$$

The corresponding set of simultaneous equations is

$$-2C_1 + 0 = 4M_0 a \beta^2 ,$$

$$-2C_1 - 2C_2 = 0 .$$

Hence

$$C_1 = -2M_0 a \beta^2 ,$$

$$C_2 = 2M_0 a \beta^2 .$$

Then formulas 5, 6, and 7 yield, respectively,

$$N_\theta = -C_2 = -2M_0 a \beta^2 = -33.045M_0 ,$$

$$Ew = aC_2/b = 2M_0 a^2 \beta^2/b = 47.585M_0 ,$$

$$E(dw/dx) = a\beta(C_1 - C_2)/b = -4M_0 a^2 \beta^3/b = -3223.8M_0 .$$

EXAMPLE 3

A radial line load of P lb/in. is applied around the circumference of a long cylinder at a distance of c in. from an end (Fig. 2a). Determine the internal axial bending moment (M_x) at the load when (1) $\beta c > 10$, (2) $\beta c = 2$.

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Fig. 2

Solution

(1) When the load is at a sufficient distance from the ends the analysis is simplified because of symmetry conditions. Hence, only two boundary conditions are required:

$$\begin{aligned} Q &= P/2 & \text{at } y = 0, \\ \frac{dw}{dx} &= 0 & \text{at } y = 0. \end{aligned}$$

The set of simultaneous equations is

$$\begin{aligned} -2C_1 - 2C_2 &= 2a\beta P, \\ C_1 - C_2 &= 0. \end{aligned}$$

Accordingly

$$\begin{aligned} C_1 &= C_2 = -a\beta P/2, \\ M_x &= -2C_1/4a\beta^2 = P/4\beta. \end{aligned}$$

(2) When $\beta c = 2$, six constants of integration must be determined, four for portion 1 and two for portion 2 (Fig. 2b). The six required boundary conditions are

$$\begin{aligned} \left. \begin{aligned} Q_1 &= 0 \\ M_{x1} &= 0 \end{aligned} \right\} & \text{at } y_1 = \beta c = 2, \\ Q_1 &= P_1 & \text{at } y_1 = 0, \end{aligned}$$

$$\left. \begin{aligned} M_{x1} &= M_{x2} \\ w_1 &= w_2 \\ \left(\frac{dw}{dx}\right)_1 &= -\left(\frac{dw}{dx}\right)_2 \end{aligned} \right\} \text{ at } y_1 = y_2 = 0$$

The last three are known as "compatibility conditions." Note that $(dw/dx)_1$ is opposite in sign to $(dw/dx)_2$ because the positive directions of x_1 and x_2 are opposite. The two sets of constants of integration are distinguished by prefixing their subscript with 1 or 2 in the following set of simultaneous equations:

C_{11}	C_{12}	C_{13}	C_{14}		P_1	C_{21}	C_{22}
-0.13348	0.35876	19.587	7.2878	=	0	0	0
0.11264	0.24612	-6.1499	-13.438	=	0	0	0
-2	-2	-2	2	=	$4a\beta$	0	0
-2	0	2	0	=	0	-2	0
0	1	0	1	=	0	0	1
1	-1	1	1	=	0	-1	1

The solution is

$$C_{11} = C_{12} = -1.0332a\beta P_1,$$

$$C_{13} = 0.026693a\beta P_1,$$

$$C_{14} = -0.039801a\beta P_1,$$

$$C_{21} = -1.0599a\beta P_1,$$

$$C_{22} = -1.0730a\beta P_1.$$

Next, P_1 is determined by the compatibility condition

$$Q_2 = P - P_1 \quad \text{at } y_2 = 0.$$

Hence

$$P - P_1 = (-2C_{21} - 2C_{22})/4a\beta = 1.0659P_1,$$

$$P_1 = 0.48404P.$$

Then the internal axial bending moment at the load is

$$M_x = M_{x2} = -2C_{21}/4a\beta^2 = 0.25652P/\beta.$$

THE CYLINDRICAL-CONTAINER PROBLEM

This problem is the analysis of a cylindrical shell which is closed with a flat circular plate at one or both ends (Fig. 3) and subjected to an internal pressure and/or a concentrated axial force at the center of the plate. Initially, the constants of integration must be determined with the aid of the compatibility conditions at the junction. The necessary equations when the "origin" end is closed are given below. When the "terminal" end is closed, one side of Eqs. 2 and 3 must be multiplied by -1 and the subscript 0 replaced with L . It is always much more convenient to designate the closed end as the "origin" end. Hence, the equations for a closed "terminal" end are required primarily for simultaneous use with the closed "origin" end when the cylindrical container is extremely short and closed at both ends.

The first equation is an equation of equilibrium; that is, the sum of axial forces on the plate must be zero. The second equation is based on the assumption that the original right angle between plate and cylinder is maintained; that is, at the junction the slope of the lateral displacement of the plate is equal to the slope of the radial displacement of the cylinder. The third equation equates the radial displacement of the cylinder and the radial displacement of the plate at the junction (it is assumed that the radial displacement of the plate is influenced only by the radial membrane force Q_0 of the plate). In Eqs. 2 and 3, Poisson's ratio = 0.3 for the material of the plate and it is assumed that the thicknesses t and b are small relative to the other dimensions.

The following symbols are used in addition to those established in Part I.

F = axial tensile force on cylinder applied at center of plate (lb)

$T = F/2\pi$ = load parameter due to force F (lb)

t = thickness of plate (in.)

$r = a - (b/2)$ = inside radius of cylinder (in.)

E_p = Young's modulus of elasticity of plate (psi)

Fig. 3

Compatibility Equations

$$N_x = (T + 0.5r^2 p)/a, \quad (1)$$

$$\frac{dw}{dx_0} = \frac{2.1a}{E_p t^3} [4M_0 + 2T + 0.5(2r^2 - a^2)p], \quad (2)$$

$$w_0 = \frac{0.7Q_0 a}{E_p t}. \quad (3)$$

Example

Determine the principal stresses σ_x at a plate-cylinder junction when the container is subjected to an internal pressure of p psi. The geometrical constants of the container ($E_p = E$) are

$$t = 0.3 \text{ in.}, \quad b = 0.2 \text{ in.}, \quad a = 3 \text{ in.}$$

Solution

When the cylinder is long two boundary conditions are sufficient:

$$\left. \begin{array}{l} \text{Compatibility Eq. 2} \\ \text{Compatibility Eq. 3} \end{array} \right\} \quad \text{at } y = 0$$

For this cylindrical shell

$$N_x = 0.5r^2 p/a = 1.4017p,$$

$$E \frac{dw}{dx_0} = \frac{a\beta}{b} (C_1 - C_2) = 24.893(C_1 - C_2),$$

$$M_0 = \frac{1}{4a\beta^2} (-2C_1) = -0.060522C_1,$$

$$Ew_0 = \frac{a}{b} (C_2 + 0.3N_x - ap) = 15C_2 - 38.692p,$$

$$Q_0 = \frac{1}{4a\beta} (-2C_1 - 2C_2) = -0.10043(C_1 + C_2).$$

These equations and the two boundary conditions yield the following set of simultaneous equations:

$$81.380C_1 - 24.893C_2 = 912.33p,$$

$$0.70301C_1 + 15.703C_2 = 38.692p,$$

whose solution is

$$C_1 = 11.803p, \quad C_2 = 1.936p.$$

Therefore

$$M_0 = (-0.060522)(11.803p) = -0.71434p.$$

Formula 13 yields:

$$\sigma_x = 100.14p \text{ at the outside surface (a compressive stress),}$$

$$\sigma_x = 114.16p \text{ at the inside surface (a tensile stress).}$$

THE STANDPIPE PROBLEM

A cylindrical container is frequently used as a standpipe to retain a vertical column of liquid, which causes the internal pressure to vary linearly in the axial direction. The internal pressure can be expressed by the following relationship:

$$p = A + B\beta x = A + By, \quad (a)$$

where A and B are constants and β is the shell parameter defined in Part I.

To analyze a standpipe the following modifications must be made:

1. In formulas 5 and 6 the internal pressure p must be replaced with $A + By$, and in formula 8 it must be replaced with $A + By/2$.
2. The right side of formula 7 is augmented by adding

$$\frac{-a^2 \beta B}{Eb}.$$

3. If the standpipe has a flat plate resting on a smooth horizontal surface so that the same uniform pressure acts on both sides of the plate, $N_x = 0$ and the terms $2T + 0.5(2r^2 - a^2)p$ must be omitted from compatibility Eq. 2 of the previous problem.

Example

A standpipe 8 ft long has a diameter of 6 ft and a thickness of $\frac{1}{4}$ in. The bottom end is fixed. Determine the axial bending stresses at the bottom end when the standpipe is filled with a liquid whose weight per unit volume is γ lb/in.³.

Solution

For this cylinder

$$a = 36 \text{ in.},$$

$$\beta = \frac{1.6523}{36(1/4)} = 0.42847.$$

With the origin of the x axis at the bottom end the expression for the internal pressure is

$$p = \gamma(96 - x) = \gamma\left(96 - \frac{y}{\beta}\right),$$

and therefore in Eq. a

$$A = 96\gamma,$$

$$B = -\gamma/\beta = -2.3339\gamma.$$

Since $\beta x_L > 10$, only two boundary conditions are necessary:

$$\left. \begin{array}{l} w = 0 \\ \frac{dw}{dx} = 0 \end{array} \right\} \quad \text{at } y = 0,$$

which, with the modified form of formulas 6 and 7, yield the following set of simultaneous equations:

$$C_2 - 3456.0\gamma = 0 ,$$

$$C_1 - C_2 - 84.020\gamma = 0 ,$$

whose solution is

$$C_1 = 3372.0\gamma ,$$

$$C_2 = 3456.0\gamma .$$

Then formula 9 at $y = 0$ yields

$$\begin{aligned}\sigma_{Bx} &= \frac{6(-2C_1)}{(1/4)^2 (4) (36) (0.42847)^2} \\ &= -24,489.0\gamma \\ &= -884.05 \text{ psi} ,\end{aligned}$$

when $\gamma = 0.0361$ (the weight per unit volume of water, in lb/in.³). The negative sign denotes a compressive stress at the outside surface.

Part IV

DERIVATION OF THE CYLINDRICAL-SHELL FORMULAS

The symbol and sign convention established in Part I is used in Part IV. Formulas 1-8 of Part I are derived below. The basic fourth-order differential equation for cylindrical shells originates from the following sets of equations.¹

Equations of Equilibrium

$$\frac{dN_x}{dx} = 0, \quad (1)$$

$$N_\theta + \frac{dQ}{dx} a = ap, \quad (2)$$

$$Q + \frac{dM_x}{dx} = 0. \quad (3)$$

Force-Displacement Relations

$$N_x = \frac{Eb}{1 - \mu^2} \left(\frac{du}{dx} - \mu \frac{w}{a} \right), \quad (4)$$

$$N_\theta = \frac{Eb}{1 - \mu^2} \left(-\frac{w}{a} + \mu \frac{du}{dx} \right), \quad (5)$$

$$M_x = D \frac{d^2 w}{dx^2}, \quad (6)$$

$$M_\theta = \mu M_x, \quad (7)$$

where $D = Eb^3/12(1 - \mu^2)$, the flexural rigidity of the shell.

The basic fourth-order differential equation is derived in the following manner. Integration of Eq. 1 yields formula 4, which is

$$N_x = \text{constant}.$$

Elimination of du/dx from Eqs. 4 and 5 and of Q from Eqs. 2 and 3 yields, respectively,

$$N_\theta - \mu N_x = -\frac{Eb}{a} w, \quad (8)$$

$$N_\theta - a \frac{d^2 M_x}{dx^2} = ap, \quad (9)$$

which are combined by eliminating N_θ :

$$\frac{d^2 M_x}{dx^2} + \frac{Eb}{a^2} w = \frac{\mu}{a} N_x - p.$$

¹S. Timoshenko, *Theory of Plates and Shells*, 1st ed., chap. XI, McGraw-Hill, New York, 1940. (In this report the positive directions for the bending moments are changed and Z is replaced with $-p$.)

Substitution of Eq. 6 for M_x yields the fourth-order differential equation

$$\frac{d^4 w}{dx^4} + 4\beta^4 w = \frac{\mu N_x}{aD} - \frac{p}{D} , \quad (10)$$

where

$$\beta^4 = \frac{Eb}{4a^2 D} = \frac{3(1 - \mu^2)}{a^2 b^2} .$$

Introduction of the dimensionless coordinate $y = \beta x$ yields

$$\frac{d^4 w}{dy^4} + 4w = \frac{4a}{Eb} (\mu N_x - ap) . \quad (10a)$$

The complete general solution of Eq. 10a is

$$w = \frac{a}{Eb} [e^{-y} (\sin y C_1 + \cos y C_2) + e^y (\sin y C_3 + \cos y C_4) + \mu N_x - ap] , \quad (11)$$

which is formula 6 with N_n defined as follows:

$$N_1 = e^{-y} \sin y ,$$

$$N_2 = e^{-y} \cos y ,$$

$$N_3 = e^y \sin y ,$$

$$N_4 = e^y \cos y .$$

The successive derivatives of Eq. 11 with respect to x are readily obtained by the coordinate transformation relation

$$\frac{d(\dots)}{dx} = \beta \frac{d(\dots)}{dy} .$$

Hence, formula 7 is derived by differentiating Eq. 11 (N_x and p are constant) and defining

$$W'_n = \frac{dN_n}{dy} \quad (n = 1, 2, 3, 4) .$$

Formula 1 is derived by differentiating Eq. 11 twice, substituting in Eq. 6, and defining

$$M_n = \frac{d^2 N_n}{dy^2} = \frac{dW'_n}{dy} .$$

Equation 7 is formula 2.

Formula 3 is derived by differentiating formula 1, substituting in Eq. 3, and defining

$$Q_n = \frac{-dM_n}{dy} .$$

Formula 5 is derived by substituting formula 6 (Eq. 11, above) in Eq. 8.

Formula 8 is derived in the following manner: Elimination of N_θ from Eqs. 2 and 5, substitution of Eq. 10 for w , and solving for $Eb \, du/dx$ yields

$$Eb \frac{du}{dx} = -\frac{a}{\mu} \left[(1 - \mu^2) \frac{dQ}{dx} + D \frac{d^4 w}{dx^4} \right] + N_x - \mu a p ;$$

Integration with respect to x and substitution of $-Q$ for $D(d^3 w/dx^3)$ (see Eqs. 3 and 6) yields

$$u = \frac{1}{Eb} \left[a\mu Q + (N_x - \mu a p) x + \frac{C_5}{\beta} \right] ,$$

where the constant of integration C_5 is divided by β for later simplification. Then substitution of formula 3 for Q and of y/β for x yields formula 8.

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