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ABSTRACT

This contract has concentrated on nonlinear and linear theory of instabilities in finite plasma for purposes of comparison with actual experiment whenever possible. Emphasis has been on nonlinear threshold saturation calculations, in order to establish the mechanisms responsible for saturation of instabilities and to identify the instability in one of its most commonly observed states. A number of fluid instabilities, collisionless, collisional or weakly ionized, have previously been treated this way. We have now succeeded in doing the equivalent analysis for an instability governed by the Vlasov equation; the collisionless drift. The saturation mechanism appears to be ion heating. As a preliminary to this work, it was necessary to extend linear theory of the collisionless drift in sheared fields to include both finite boundaries and radial variation of the potential. A set of useful "universal" stability boundaries is obtained. We have also obtained a better calculation of the stability criteria for sheared plasma with localized modes. Finally, a general theory of linear stability boundaries for hot plasma in mirror machines, in the presence of both density and temperature gradients, was obtained.

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Introduction

A principal goal of this project, during the past academic year, was to extend the methods of threshold nonlinear saturation analysis⁽¹⁾ to collisionless plasma governed by the Vlasov equation. Despite a number of difficult roadblocks and novel problems, the effort has been entirely successful. We have been able to calculate saturation amplitudes and nonlinear frequency shifts for the collisionless drift instability and have identified the stabilization mechanism. Further details are given below in section 2.

The linear treatment of the drift wave in sheared magnetic fields was improved in two ways. One was to establish stability limits for a plasma with finite boundaries with radial variation of the potential included. This is discussed in section 1 below and is mainly applicable to experiments in Q-machines and hydrogen arcs. The second improvement is a calculation of the shear stability limit in plasmas with localized modes. This method retains the full nonresonant and resonant electron effects and solves the appropriate differential equation for the eigenvalue in two relevant regions and matching. The result agrees extremely well with previous perturbation theory methods and is discussed in section 3. It is appropriate to experiments in Tokamak devices.

In addition to this work, a study of stability boundaries for temperature and density gradient flute modes in mirror machines was completed and has been accepted for publication in the Physics of Fluids. Details are also given in section 4 below.

1. Collisionless Drift Mode (Linear Theory in Finite Plasma)

The starting point for any threshold non-linear saturation analysis

must be a well defined point of marginal stability for a plasma. Previous linear theories⁽²⁻⁴⁾ of this instability are not easily used for this purpose because they do not include the combined effects of finite boundaries and radial variation of the perturbed potential. We have recalculated stability boundaries for a nearly Maxwellian cylindrical plasma with an experimental radial density variation of characteristic length L_N . The perturbed potential is represented by the Galerkin trial function $\sin(\pi x/\ell)$ where ℓ is the distance to the radial boundary. The resulting dispersion relation is a function of four parameters, $\Theta = -\omega/|k_z|v_+\sqrt{2}$, $\Gamma = -k_y a_i/2|k_z|L_N$, $q = 2L_N k_z/a_i L_s |k_z|$ and $\beta_s = e^s I_0(s)$ with $s = [k_y^2 + (\frac{\pi}{\ell})^2]a_i^2$. Here L_s is the magnetic field shear length, a_i is the ion larmor radius, v_+ the ion thermal speed and k_y and k_z the wave numbers of the perturbation. For marginal stability, Θ is real and can be eliminated between the real and imaginary parts of the dispersion equation, thus yielding a "universal" stability curve in Γ, β_s space for any given shear parameter q .

Using this diagram, one then plots all possible modes that can occur in any finite experimental system and sees whether the envelope of the mode region overlaps the unstable region below the marginal stability curve.

Details are given in a report by Simon, Gross and Catto⁽⁵⁾ [COO-3497-6] which is attached to this annual report, as well as in a paper which has been submitted to the Physics of Fluids. Using the universal diagrams, one can easily calculate the parameters necessary to make any system marginally stable. It also shows that there is a value of the shear parameter (≈ 0.72) which stabilizes all modes. Of course these numerical

results are for a given ion species (hydrogen) and do not apply to experimental situations in which the marginally stable mode is expected to be localized and entirely unlike the trial function used above. There are already experiments in which the observed eigenmodes are of the form used above⁽⁶⁻⁷⁾, although both of these were in the unstable saturated regime and did not actually reach the marginal stability point.

2. Collisionless Drift Mode (Non-linear Theory)

Assume now that we have prepared a plasma experiment in a marginally stable state. That is, there is a single mode of particular wave numbers k_y, k_z with a real eigenvalue $\bar{\omega}$, all other allowable modes having damped eigenvalues. All this for a particular value of L_N, L_S, L_Z, l and a_1 . Here all symbols have the same meanings as in the previous section, and L_Z is the finite length of the system in the z-direction. Suppose now that the magnetic field is increased slightly by the fractional amount Δ ($\Delta \ll 1$). The marginally stable mode will begin to grow in amplitude and shift in frequency. After a time, the system will have reached a steadily oscillating state in which there is a mode of amplitude Γ and real frequency ω , where $\omega = \bar{\omega} + \delta\omega$. Here we expect $|\Gamma|^2$ and $\delta\omega$ to be of order Δ .

The actual values of $|\Gamma|^2$ and $\delta\omega$ are found by the methods of non-linear analysis pioneered by Bogoliubov and coworkers. We systematically expand the final eigenmodes and potentials in powers of $\sqrt{\Delta}$ and also study the particular fourier frequencies corresponding to $\bar{\omega}, 2\bar{\omega}$, and 0. At every stage, one demands the absence of secular terms. The analysis is similar to that described in ref. (1) and in the numerous applications of this method to plasmas governed by fluid equations. However, the added complications of

working in 6-space $(\vec{x}, \vec{v})$ rather than 3-space $(\vec{x})$ were hardly trivial especially in regard to the proper treatment of multiple Landau poles.

We have obtained the second order corrections to the equilibrium distribution functions and potential. It was found necessary to calculate these corrections to first order in a_i/λ , for consistency. We also obtained the adjoint solution to the system of equations consisting of the two Vlasov equations (one for each species) and Poisson's equation. Finally, the condition that the third order equation in $\sqrt{\lambda}$ be non-secular yielded an equation determining $|\Gamma|^2$ and $\delta\omega$.

Numerical answers have already been obtained for the case of a plasma in a magnetic field without shear (linearly stabilized by finite length in the z-direction). We found that there is indeed a proper nonlinear saturation of the system and with a positive frequency shift. The mechanism of nonlinear saturation appears to be heating of the ions. This increases their Landau damping and reestablishes balance. There is also a readjustment of the density variation so as to increase the slope close to the origin and to decrease it in the outer regions. However this effect is equally stabilizing and destabilizing and on average does not produce saturation.

Further study of the saturation mechanism will be carried out. We also plan to include mode coupling ($2\bar{\omega}$ frequency) and to do numerical calculations for plasmas in sheared magnetic fields. A preliminary abstract by Gross and Simon⁽⁸⁾ has been submitted for the Albuquerque Meeting and is attached to this report.

3. Resonant and Non-resonant Effects on the Stability of Drift Waves in a Sheared Magnetic Field

In a sheared magnetic field, $\underline{B} = B(\hat{z} + y\hat{x}/L_s)$, the parallel wave number becomes a function of x , $k_{||} = k_{||}(x)$. As a result, a drift wave is able to extract energy via the resonant electrons in only a localized region about a rational surface ($x = 0$). Because $k_{||} = k_{||}(x)$, however, the resonant and non-resonant electron interaction is also affected, and in fact depends on x in a rather complicated manner. Consequently, previous solutions of this eigenvalue problem have not been completely self-consistent in the treatment of the electrons. By solving the appropriate differential equation in the two relevant regions and matching, a self-consistent stability criterion has been found for the least stable mode. The result shows that the perturbation theory method, which treats the electrons in the low phase velocity (fluid) limit and retains the resonant electrons, agrees extremely well with the self-consistent solution of the differential equation, which retains the full non-resonant (low through high phase velocities) and resonant electron effects.

This work was carried out by P. J. Catto at the University of Rochester in collaboration with M. N. Rosenbluth of the Institute for Advanced Study. An abstract by Catto and Rosenbluth⁽⁹⁾ for the Albuquerque meeting is attached to this report.

4. Stability Boundaries for Temperature and Density Gradient Flute Modes in Mirror Machines

This work, which was in progress at the time of our last annual report, was completed and has been accepted for publication in the Physics of Fluids [see attached report, by Abhyankar and Simon, C00-3497-7]. It

was also part of the Ph.D. thesis of V. G. Abhyankar. The results can be summarized in the following fashion.

(a) We studied flute-like linear perturbations on a bounded cylindrical plasma in a magnetic field. The governing equations are a set of collisionless fluid equations known as the Varma equations. The equilibrium magnetic field, temperature and density all vary radially.

(b) We derived expressions for the stability surface in δ' , λ' , P space where δ' is related to the ratio of the density gradient to the magnetic field gradient; λ' is related to the ratio of the temperature gradient to the magnetic field gradient, and P is simply related to the ratio of the particle density to the square of the wave number. The nature of this surface is sketched in the several regions of behavior in this space. One can identify density gradient instability onset and temperature gradient instability onset points.

(c) Some of the temperature gradient onset conditions are predicted to occur for very short wavelengths in contradiction to predictions using the Vlasov equation. This was resolved by corrections to the Varma equations.

(d) The Varma equations were derived directly from the Vlasov equation, showing that a delta-function distribution is necessary and sufficient for their validity. A modification of this distribution yields a modified Varma equation which has reasonable stability onset conditions under all circumstances.

(e) A qualitative explanation was obtained for the explosive instability behavior which was seen experimentally⁽¹¹⁾ and noted in the nonlinear analysis of Simon and Weng⁽¹²⁾.

5. Personnel:

(a) Principal Investigator -- The principal investigator has devoted 25% of his time to this project during the 1973-1974 academic year. He also spent two months on it during the summer of 1974. He will be on sabbatical leave for the remainder of the calendar year, at the Institute for Advanced Study, Princeton, New Jersey, and will continue to work on problems related to plasma instability as well as other topics specifically related to magnetic confinement fusion. Of course, there will be no salary recovery by the University of Rochester during the academic year leave period. His graduate student (L. Gross) should complete his thesis at Rochester during this period.

(b) Faculty Colleague (P. J. Catto) -- Dr. Catto has devoted 20% of his time to this project during the year. He will supervise one graduate student (T. Speziale) beginning in Sept., 1974.

(c) Graduate Research Assistants --

Mr. V. G. Abhyankar (MAS Dept.) completed his thesis work in the spring and received his Ph.D. degree in May. He has been offered a position at the Physical Research Lab in Ahmedabad, India. Mr. Leonard Gross is expected to complete a thesis on nonlinear saturation of the collisionless drift mode by January, 1975.

Three new graduate students will begin plasma theory theses. Mr. Tom Speziale (MAS Dept.) will begin thesis research on delayed reradiation due to resonant absorption of electromagnetic waves in plasma, under the direction of Dr. Catto. Mr. Stephano Migliuolo (Physics Dept.) will continue course work in Physics and will begin a thesis under Dr. Simon's direction in Sept. 1975. Mr. Ahmed Metwalli will work on topics related to

laser fusion until Dr. Simon's return in September, 1975.

(d) Other Federal Support -- None of the personnel working on this project have other federal support.

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9. P. J. Catto and M. N. Rosenbluth, "Resonant and Non-Resonant Electron Effects on the Stability of Drift Waves in a Sheared Magnetic Field", C00-3497-9, Aug. 9, 1974.
10. V. G. Abhyankar and A. Simon, "Stability Boundaries for Temperature and Density Gradient Flute Modes in Mirror Machines", C00-3497-7,

May 1974 (PPG-3). To be published in Phys. Fluids, Nov. 1974.

11. R. J. Colchin, J. L. Dunlap and H. Postma, "Evolution and Stabilization of Flutes in a Magnetic Mirror", Phys. Fluids 13, 501 (1970).
12. A. Simon and C. I. Weng, Phys. Fluids 15, 2341 (1972).

PREVIOUS PUBLICATIONS UNDER THIS CONTRACT

1. A. Simon, "Nonlinear Transport in Weakly Ionized Plasma", Phys. Fluids 11, 1181 (1968), UR-3942-1.
2. A. Simon, "Convection of Weakly Ionized Plasma in a Non-uniform Magnetic Field", Phys. Fluids 11, 1186 (1968), UR-3942-2.
3. J. J. Kim and A. Simon, "Nonlinear Theory of the ExB Instability", Phys. Fluids 12, 895 (1969), UR-3942-3.
4. A. Simon and J. N. Shiau, "Quasilinear Theory of the Positive Column Spiral Instability", Phys. Fluids 12, 2630 (1969), UR-3942-5.
5. J. N. Shiau and A. Simon, "Nonlinear Theory of the Positive Column Spiral Instability", Phys. Fluids 13, 2569 (1970), UR-3942-7.
6. A. M. Sleeper and A. Simon, "Nonlinear Theory of the Drift-Dissipative Instability", Phys. Fluids 13, 2757 (1970), UR-3942-8.
7. Albert Simon and C. I. Weng, "Quasilinear Theory of the Mirror Flute Instability", Phys. Fluids 15, 2341 (1972), C00-3497-1.
8. Donald A. Monticello and Albert Simon, "Nonlinear Theory of the Collisional Drift-Wave Instability", C00-3497-4. To appear in Physics of Fluids, May 1974.
9. V. G. Abhyankar and A. Simon, "Linear Threshold of the Buneman-Farley Instability", Phys. Fluids 17, 261 (1974), C00-3497-3.

10. V. G. Abhyankar and A. Simon, "Stability Boundaries for Temperature and Density Gradient Flute Modes in Mirror Machines", C00-3497-7.
Accepted for publication by Phys. Fluids.

THESES COMPLETED UNDER THIS CONTRACT

1. John J. Kim, "The Second Order Shifts in Frequency and Amplitude in the Non-Linear Theory of ExB Instability", Physics Dept., 1969.
2. Jeng-Nan Shiau, "Small Amplitude Nonlinear Theory of the Positive Column Spiral Instability", MAS Dept., 1969.
3. Arthur M. Sleeper, "The Drift Dissipative Instability", Physics Dept., 1970.
4. Cheng-I Weng, "Nonlinear Theory of the Mirror Flute Instability", MAS Dept., 1972.
5. Donald A. Monticello, "Nonlinear Theory of the Collisional Drift Wave Instability", Phys. Dept., 1973.
6. Vishwas Govind Abhyankar, "Density and Temperature Gradient Flute-Mode Instabilities", MAS Dept., 1974.

List of Publications

<u>Title</u>	<u>Authors</u>	<u>Journal/Status</u>	<u>Percentage AEC Support</u>
Nonlinear Theory of the Collisional Drift-Wave Instability	D. A. Monticello and A. Simon	Physics of Fluids (to appear, May 1974 issue)	100%
Linear Threshold of the Buneman-Farley Instability	V. G. Abhyankar and A. Simon	Physics of Fluids Jan., 1974	100%
Stability Boundaries for Temperature and Density Gradient Flute Modes in Mirror Machines	V. G. Abhyankar and A. Simon	Physics of Fluids (accepted for publication)	100%
Shear Stability Boundaries for Collisionless Drift Modes in Finite Geometry	A. Simon, L. Gross and P. J. Catto	Physics of Fluids (submitted)	75%

Table I