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SOME PROBLEMS IN THE SAFETY OF FAST REACTORS

by

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2

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TABLE OF CONTENTS

	Page
INTRODUCTION	4
KINETICS STUDIES	5
Methods Adopted	5
Some Results Obtained	8
1. Very Fast System	8
2. Fast Systems	12
3. Below Prompt Critical	14
PROBLEMS CONNECTED WITH EBR-I BEHAVIOR	17
THERMAL INERTIA	20
APPENDICES	
A. Solution of Kinetics Equation for n , Given k_{ex}	21
B. Solution of the Kinetics Equations for k_{ex} , Given n	24
C. Analytical Solution for k_{ex} , Given n	26
D. Analytical Solution for n , Given k_{ex}	29
E. Temperature Distribution in Two-Region Cylindrical Geometry with Experimentally Rising Heat Source in Central Region	32
F. Time Variation of Stored Heat in Two-Region Fuel Cell During Transient	35

4

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INTRODUCTION

The characteristics of fast reactors have been studied at Argonne National Laboratory (ANL) to determine those unique features which must be considered in addition to features held in common with thermal reactors when looking at the problem of reactor safety and environmental hazards.

Fast reactors differ from less energetic systems in two principal respects. The short lifetime of prompt neutrons represents one, the nature of the reactivity coefficients the other. The lifetime of prompt neutrons in a fast system differs from other characteristic time constants, such as those of delayed neutron emitters, of mechanical motion, and of thermal transport, by several orders of magnitude. The inherent reactivity coefficients are of considerably different character, with competition between positive and negative components resulting in net coefficients which are small and vary in sign with the rate of the changes. The net coefficients are strongly dependent on detail of core design and fabrication.

These unique characteristics of fast reactors pose special problems among which are: mechanical and thermal inertia may be significant; difficulties in handling the solutions of the kinetics equations arise; the possibility of operational instability is strong; detailed temperature-dependent physical behavior of core materials and components becomes important; potentially very high rates of energy release alter the containment problem; it becomes technically more difficult to examine experimentally the kinetic behavior in areas of abnormal operation.

An attempt is made in this report to set down the procedures and results of looking at some fast reactor problems at ANL. The first accounts show the methods adopted for handling the kinetic equations, and the results of some general studies. This is followed by some information on stability with reference to EBR-I, including experiments. Finally some procedures and a study are presented which deal with the problem of thermal inertia.

5

KINETICS STUDIES

A safety survey of fast reactor systems was begun at Argonne National Laboratory by studying the characteristic kinetic behavior of such systems. To make this study possible, the problem of programming and coding the solution to the equations describing this behavior for the available digital computer (AVIDAC) was undertaken.

Methods Adopted

The one-group, space-independent model for neutron density seems adequate and easiest to handle. Since the time dependence is being studied for a large range of problems, it is necessary to be specific regarding the delayed neutrons, so that six such groups of precursors with their characteristic times and relative yields are considered. The seven rate equations obtained completely describe the system providing the variation of the reactivity with time and neutron density is known.

The characteristics of the neutron system are dependent on the following parameters: prompt neutron lifetime, the ratio delayed neutron precursor yield per prompt neutron, the delayed neutron precursor decay constants, and the source strength. Thus, the equations become

$$\dot{n} = \frac{k_{ex}(1-\beta) - \beta}{\ell} n + \sum_{i=1}^6 \lambda_i c_i + S \quad \left(\beta = \sum_{i=1}^6 \beta_i \right)$$

(or alternately

$$\dot{n} = \frac{k_{ex}}{\ell} n - \sum_{i=1}^6 \dot{c}_i + S)$$

$$\dot{c}_i = \frac{(1 + k_{ex}) \beta a_i}{\ell} n - \lambda_i c_i \quad (i = 1, 2, \dots, 6) \quad a_i = \frac{\beta_i}{\beta}$$

$$k_{ex} = f(n, t),$$

wherein

n = neutron density

c_i = concentration of i 'th delayed neutron precursor

β = fraction of neutrons that are delayed

a_i = fraction of delayed neutrons of the i 'th group ($\sum a_i = 1$)

λ_i = decay constant for the i 'th group

S = constant source term

k_{ex} = excess reactivity = $k_{effective} - 1$

t = time variable

ℓ = effective prompt lifetime.

The solution has been coded (designation: RE-13) as a step-by-step scheme using a fourth-order Runge-Kutta method without other approximation. Several integration schemes have been investigated, but the one chosen seems optimum from the standpoint of compatibility of machine time and accuracy.

The coding was arranged to provide a 130-word gap in the memory at the end of the problem code. This accommodates any form of $f(n,t)$ and additional punch-out orders and sub-routines can be included without breaking into the main body of the problem code. For each reactor or class of problems $f(n,t)$ has a different form. Mechanical and thermal inertia can thus be accounted for. To solve kinetic problems it is only necessary to prescribe the form of k_{ex} . If the sub-routine for this form is not already in the library, a new one is developed and inserted.

Unfortunately, the presence of the prompt neutron lifetime in the problem, which becomes very small for fast reactors, and the necessity of performing subtractions yielding small differences between large numbers limits the interval size to about $10^3 \ell$. Thus, for classes of problems in which the variables change slowly with time, the number of points which must be computed can become enormous.

Because of the severe limitation on interval size encountered in use of RE-13 for some solutions on AVIDAC, an attempt was made to manage the computations with an interval size which need be only small enough that n , k_{ex} and c_i may be reasonably assumed to vary linearly from point to point. The general approach used is to integrate the usual kinetics equations over the interval size to obtain the change in the variables. By working with only the changes in the variables many of the steps involving small differences between large numbers are eliminated. The resulting code is designated RE-29. The procedure is detailed in Appendix A.

Again in this scheme computation of Δk is divorced from the main body of the code so that any number of forms for expressing k_{ex} may be utilized with little additional coding time.

It is of interest to note that these solutions may be obtained by hand computation at the cost of about 15 to 20 minutes/point. The interval size is restricted to that interval in which k_{ex} , n , and c_i can reasonably be considered linear and which will not cause the numerator of the Δn expression to go to zero for $k_{ex} > \beta$.

This latter restriction seems severe, but means only that when the excess reactivity is in the neighborhood of prompt critical, this code has no advantage over RE-13. It is generally true also that solutions in this region find ρ varying greatly, so that linearity and definition require smaller interval sizes anyhow. In many problems, however, especially operational ones, k_{ex} is below prompt critical with dk_{ex}/dt of the order of 10^{-4} /second, so that the elapsed true time for the problem is of the order of minutes. In such problems RE-13 would require many days of computing time, whereas RE-29, using intervals of the order of 10^4 larger would yield the solution in about the true elapsed time.

In addition to the problem of obtaining the time variation of neutron flux for a prescribed mode of variation in reactivity, it was decided that the converse problem of obtaining the time variation of reactivity consistent with a given time variation in neutron flux would be a useful tool. Attempts to arrange a program which would accomplish this were stimulated by a need for analyzing experimental data. The solution by H. J. Wheeler was programmed and coded as RE-22 for AVIDAC and is outlined in Appendix B. Here, use of a trapezoidal integration scheme has imposed limits on the interval sizes. An improved Runge-Kutta method of integration has greatly relieved the limit and has been adopted in AVIDAC code RE-31. This latter scheme will be published in the near future. It is interesting to note that the calculation of RE-22 can be performed with a desk computer at the rate of five to ten intervals per hour.

Analytical solutions of the kinetics equation have been obtained by Harold Greenspan of ANL. These solutions parallel the synthesis scheme of RE-29 and the analysis scheme of RE-22 outlined above, but are limited to cases in which the flux or reactivity can be represented by functions whose Laplace transforms exist. The solution which yields k_{ex} has been obtained in general and specifically for the case wherein ρ is assumed to be sinusoidally varying with time. The analytical solution is outlined in Appendix C. The solution which gives ρ has been obtained for the case when the reactivity is a periodically varying function of time and a functional of the neutron density involving a delay time. It is presented in Appendix D.

Adoption of methods suitable for digital machines was not based on a comparison of these machines with analog types, but rather on the fact that the available analog machine (REAC, a general type) did not at the time have adequate components to handle the problem. Nor were the usual components adequate to define the solution when used for fast reactors in which the prompt lifetime is several orders of magnitude smaller than the delay group time characteristics and other characteristic times occurring in the problems such as thermal relaxation times.

8

As a matter of fact, the digital machine methods are inferior to analog machine methods for a great many of the problems encountered, especially those in the range of normal operation of the reactor. ANL has since acquired more components, and a very fast integrator has been developed here so that the REAC will now accept fast reactor problems.

Many parametric studies can and will be carried out on REAC at much less expense in time and money than similar studies on AVIDAC. It is visualized that a few ranging problems will be run with AVIDAC in the future, but that the details within these ranges will be left to REAC. Only for that class of problems dealing with behavior above prompt critical, where the solution covers many decades in a short time, does AVIDAC have a distinct advantage over REAC.

The fast memory of AVIDAC is limited to 512 storages, so that only rather simpler reactivity functions can be utilized, and crutches must be used when past-history problems are run. Of course, GEORGE will eliminate this difficulty in the future.

Problems involving the kinetic behavior of the whole plant will certainly be run on the analog machine (investigations of operational stability and studies of the transfer function, for example). Problems of the excursion type on the other hand are best suited to the digital machine.

Both machine types should and will be jointly used in studying reactor kinetics and safety problems. Both have their regions of best operation, and these regions will become more apparent through experience.

Some Results Obtained

1. Very Fast System

For the first investigations of kinetic behavior it seemed advantageous to simulate the burst experiments on Godiva at Los Alamos, since the model was rather simpler, more condensed, and with a lifetime considerably shorter than a practical fast reactor. Then, too, experimental data were available to compare with computed results.

These experiments¹ on a condensed assembly represent the case of instantaneous addition of reactivity in excess of prompt critical to a system with very low initial neutron density and very short lifetime ($<10^{-8}$ sec). The system shuts itself down (i.e., reduces reactivity below prompt critical) by thermal expansion without transfer of heat during the

¹Orndoff, J.D., APDA-105 (Nov., 1954)

excursion. Thus, one could take an expression for reactivity variation that depended only on temperature. k_{ex} had some initial value which was reduced by thermal expansion during the excursion and hence the $f(n,t)$ chosen could be taken as

$$\dot{k}_{ex} = B (n - n_0)$$

or

$$\dot{k}_{ex} = A + Bn$$

The A represents change which would be brought about if the heat due to initial power level were not assumed to be carried away. This was always negligible in considering the Godiva experiments because n_0 is so low. The B represents the reactivity coefficient and

$$Bn \propto \left(\frac{1}{V} \frac{dV}{d\theta} \right) \left(\frac{dk}{k} / \frac{dV}{V} \right) \frac{n\bar{v}\bar{\Sigma}_f}{\rho s_H}$$

for a one-energy group, space-independent model. The volume temperature coefficient of expansion $\frac{1}{V} \frac{dV}{d\theta}$ was chosen to be three times a linear coefficient of $14 \times 10^{-6}/^{\circ}\text{C}$. The per cent change in reactivity dk/k was taken to be about $3/4$ of the per cent change in volume, $\frac{dV}{V}$. The density, ρ , was assumed to be 19 g/cc and the specific heat s_H as $0.0287 \text{ cal/g-}^{\circ}\text{C}$. Values of $\bar{v}\bar{\Sigma}_f$ were chosen to characterize the energy group in such a way that

$$\bar{v}\bar{\Sigma}_f = \frac{1}{\ell \nu}$$

The lifetime ℓ was obtained from Los Alamos as $7 \times 10^{-9} \text{ sec}$ and ν was taken as 2.5. The energy was converted by assuming $3 \times 10^{10} \text{ fissions/watt-second}$.

The six delay group data of Hughes² were used so that:

i	λ_i	β_i
1	0.01246	0.00025
2	0.0315	0.00166
3	0.1535	0.00213
4	0.456	0.00241
5	1.612	0.00085
6	14.3	0.00025
sum = β		0.00755

²Hughes, Dabbs, Kahn, and Hall, Phys. Rev., 73, 111 (1948)

10

To simulate the experimental condition in the computations, the reactor was assumed initially operating at delayed critical with a very low neutron density (n_0). It was further assumed that the delayed neutron precursors were at an equilibrium and their derivatives ($\dot{c}_{i0}$) zero. The variations with time of n , $\int n dt$, $n/\dot{n}$, and k_{ex} were then obtained using the RE-13 program on AVIDAC for the values of k_{ex0} above prompt critical as shown in the table below. The values of maximum fission rate, total fissions in the burst, temperature rise, period, and ratio of peak width at half height to period are also given.

Prob. No.	Δk , ϕ Above Prompt Crit.	k_{ex0}	Peak Fiss. Rate, Fiss./Sec. $\times 10^{16}$	Total Fiss. $\times 10^{16}$	E, Mega-Watt -Sec.	$\Delta\theta$, $^{\circ}\text{C}$	Peak Location, μs	Width at 1/2 Height, μs	Period, μs	Width Period
G-1	30	0.009815	218000	2.78	.927	144	59.4	11.3	3.19	3.52
G-2	20	0.00906	94300	1.83	.610	95	84.5	17.0	4.86	3.50
G-3	15	0.008683	51700	1.36	.452	70	109	23.0	6.57	3.50
G-4	12	0.008456	32500	1.07	.357	55	133	29.0	8.32	3.49
G-5	10	0.008305	21500	0.881	.293	46	155	35.7	10.1	3.52
G-6	7	0.008079	9850	0.596	.198	31	216	53	15.0	3.52
G-7	4	0.007852	2680	0.311	.1036	16	352	102	29.1	3.51
G-8	2	0.007701	397	0.125	.0413	6.4	702	265	75.5	3.53
G-11	1.5	0.007663	148	0.082	.0273	4.2	1000	442	125	3.54
G-9	1	0.007626	22	0.037	.0123	1.9	1720	1330	350	3.8

Variation of fission rate with t is plotted in Fig. 1 for some values of Δk above prompt critical.

Some of the relations are shown graphically, with experimental data superimposed, in Figs. 2 to 5. The experimental values of the peak fission rates are higher than the calculated rates for periods less than about $20 \mu\text{s}$. Higher peaks would be expected if the mechanical inertia caused the expansion to lag, resulting in a slower reduction of reactivity. The spike widths differ for both the very short periods and the long periods. For the shorter periods the expansion may carry further than indicated in calculations where inertia is neglected. This overshoot of the expanding system to a radius larger than could be obtained for a temperature rise of this magnitude slowly attained would result in a more rapid shutdown from the burst peak and, hence, a narrower spike. In the bursts which took more time, the peaks were broadened at half height by neutrons scattered back into the assembly from the room. There exists a more pronounced tail in the experiments than in the calculations. Later experiments made at Los Alamos outdoors showed tails like those of the calculations.

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In the case of the temperature rise, the 10 per cent discrepancy between estimated and measured values might be attributed to having 15 Mev/fission escaping the core without being converted to heat. There does not seem to be anything wrong with the temperature measurements or the assumed values of the specific heat. It does not seem likely that significant heat would be transferred from the core during the experiment. The temperature rise necessary to cause an expansion which will result in reduction in reactivity sufficient to reflect about prompt critical is given by

$$\Delta\theta = \frac{2\Delta k(\phi \text{ above prompt critical}) \bar{\rho}}{100 dk/d\theta} = 5.92 \Delta k(\phi \text{ above prompt critical})$$

for $dk/d\theta = (Vdk/kdV)(dV/Vd\theta) = (0.73)(42 \times 10^{-6}) = 30.7 \times 10^{-6}$. The values given for temperature rise by this expression are compared here with the values from AVIDAC.

Δk , Cents Above Prompt Critical	$\Delta\theta$, °C for Reflection	$\Delta\theta$, °C from Integral Under First Peak (AVIDAC)
30	148	144
20	98	95
15	74	70
12	59	55
10	49	46
7	35	31
4	20	16
2	9.8	6.4
1.5	7.4	4.2
1	4.9	1.9

From these calculations one can obtain the following relationships, wherein α is the reciprocal period corresponding to the initial excess reactivity:

$$\text{Total number of Fissions in burst, } F = 9 \times 10^{10} \alpha$$

$$\text{Maximum Fission Rate } (dF/dt)_{\max.} = 2.22 \times 10^{10} \alpha^2 = 1/4 F \alpha$$

$$\text{Temperature Rise } \Delta\theta = 4.6 \times 10^{-4} \alpha$$

$$\text{Spike Width} = 3.5/\alpha$$

$$\text{Half Width} \times \text{Peak Height} = 7/8 \text{ of total fissions in burst.}$$

12

2. Fast Systems

Following the success in calculating a condensed system like Godiva, an attempt was made to look at calculations for a more dilute system having a longer lifetime and other characterizations more nearly like a fast power reactor. In such a reactor the possible mechanism for reducing reactivity is different in that it is brought about principally by linear expansion of fuel rods rather than by volumetric expansion. This and the change in lifetime (hence $\nabla \bar{\Sigma} f$) constitute the principal differences between the two systems. Another difference lies in the relation between the per cent change in volume and per cent change in k , since the new system is cylindrical and effectively experiences a change in core height only.

The expansion can be taken care of by using the linear rather than the volumetric coefficient so that one-third the volumetric coefficient is chosen for δ_T . If such dilute reactors are characterized by a lifetime of 10^{-7} sec, the reactivity coefficient is decreased by a factor of $100/7$ from that of the condensed system. $(dk/k)/(dV/V)$ might be as small as half its former value. There may be a 10 per cent uncertainty in density, a possible variation of the specific heat from 1 to 1.6 times, and of the coefficient of expansion from 1 to 1.4 times, the value chosen for Godiva. For the ranges suggested here it is found that the coefficient B for the fast reactor lies between 0.16 and 0.005 times that used for Godiva. A most probable value was felt to be 0.04 times that of Godiva and so a value of $B = -10^{-9}$ was chosen to represent reactors of this character. The effect of using a more pessimistic value of B was studied also. The range of reactivity temperature coefficients covered was $(5 \times 10^{-6} < dk/d\theta < 10 \times 10^{-6})/^{\circ}\text{C}$.

The solution for $n(t)$ was obtained first for a step input of reactivity, using $\beta = 10^{-7}$ and the same delayed neutron data as before, so that $\dot{k}_{\text{ex}} = 10^{-9}n_0 - 10^{-9}n$ and $\dot{k}_{\text{ex}}(0) = 0$ with $k_{\text{ex}}(0) = 0.008$. The results for this and other values of the coefficient B , both more optimistic and pessimistic, are shown in Fig. 6. Of more interest were cases in which k_{ex} was initially zero and $\dot{k}_{\text{ex}}(0)$ not zero. Then $\dot{k}_{\text{ex}} = (R + 10^{-9}n_0) - 10^{-9}n$. This allowed study of the problem wherein reactivity was inserted at a constant rate and reduced by prompt coefficient in proportion to the temperature, i.e., $\int n dt$. Four cases were tried starting at low power ($n_0 = 1$) in which R was chosen to be about \$1000/sec, \$100/sec, \$10/sec, and \$1/sec. For these cases n , $\int n dt$, and k_{ex} are presented in Figs. 7 through 10. Results are summarized below.

R	Max k_{ex}	Max $\log_{10} n$	$\log_{10} \int n dt$ First Peak	Min Period ($n/\dot{n}$)
8/sec ($\sim$ \$1000/sec)	0.0132	11.28	7.10	18.8 μs
0.8/sec ($\sim$ \$100/sec)	0.0094	10.22	6.55	65.4 μs
0.08/sec ($\sim$ \$10/sec)	0.0080	9.10	6.02	234 μs
0.008/sec ($\sim$ \$1/sec)	0.0077	7.85	5.47	1110 μs

The relation between R and $k_{\text{ex,max}}$ is plotted in Fig. 11, while that between the R and τ_{min} is shown in Fig. 12. The relation between n and R is plotted in Fig. 13, showing values of n for each peak and valley. Variation of $\int n dt$ with R is shown in Fig. 14.

Several studies were made to show the effect on n and $\int n dt$ of starting at operating power levels instead of very low power. To do this a value of $n_0 = 10^7$ was chosen instead of $n_0 = 1$. The effect of this is typified by the dotted curves shown on Fig. 8. It is seen that the value of the integral is independent of n_0 after the oscillations damp.

Since there was uncertainty in the coefficient B , a most pessimistic case was computed wherein $B = -2 \times 10^{-10}$ and $k_{\text{ex}} (R + 2 \times 10^{-10} n_0) - 2 \times 10^{-10} n$. The results are plotted as dotted curves in Fig. 7. Using limiting values of B in this way yields a region in which the correct results must lie when exact information is lacking.

The calculations show that the total energy release is about proportional to $\sqrt{R}$ and is represented by

$$\int n dt = 4 \times 10^6 R^{0.55}$$

The maximum excess reactivity reached in the first burst is also about proportional to $\sqrt{R}$:

$$(k_{\text{ex}} - \beta)_{\text{max}} = 1.86 \times 10^{-3} R^{0.53}$$

or

$$\Delta k, \text{ cents above prompt critical} = 25 R^{0.53}$$

The height of the first peak is about proportional to R :

$$n_{\text{max}} = 2 \times 10^{10} R^{1.1}$$

Hence,

$$\int n dt = 20 \sqrt{2 n_{\text{max}}}$$

Also, the minimum period attained is nearly inversely proportional to the $\sqrt{R_0}$:

$$\tau_{\text{min}} = 60 \times 10^{-6} / R^{0.54}$$

For a given excess reactivity or a given rate, n_{max} is nearly inversely proportional to the reactivity coefficient B .

14

The temperatures attained in the fuel would be for a value of $dk/d\theta = 6 \times 10^{-6}$:

R	$\Delta\theta, ^\circ\text{C}$		$d\theta/dt$ after Oscillations
	After 1st Burst	After Oscillations	
8/sec	1890	4450	1.3×10^6 degrees/second
0.8/sec	540	780	1.3×10^5 degrees/second
0.08/sec	160	160	1.3×10^4 degrees/second
0.008/sec	50	50	1.3×10^3 degrees/second

This implies that for a rate greater than 100 per cent per second one could expect the uranium to melt during the first burst. However, one must remember that the reactivity remains at prompt critical for as long as one continues the rate. Presumably one would not have more than a certain per cent available. Heating would continue at a rate of about 1.7×10^5 R degrees per second until available reactivity were compensated for by temperature rise and expansion.

3. Below Prompt Critical

An attempt was made to organize information about the kinetic behavior of fast reactors in the operational region below prompt critical so that control and safety system requirements could be set up. Such problems as determining the rates of change of reactivity which can be handled without damage to the core are coupled by the control system response characteristics through detectors to the reactor kinetic behavior. Detector signals which operate such systems might be placed in two categories, those which work on flux level and those which work on the time rate of change of flux. Information about these are combined in the ratio of flux to the time derivative of the flux or $n/\dot{n}$.

In the one-group model used here,

$$n/\dot{n} = \frac{\ell}{k_{\text{ex}} (1 - \beta) - \beta + \frac{\ell}{n} \sum_i \lambda_i c_i}.$$

The rates of rise are characterized by this quantity, $n/\dot{n}$, which becomes the asymptotic period or e-folding time after k_{ex} has remained constant for some time. For k_{ex} not equal to zero one can consider this quantity as the "instantaneous" period. It is exactly this quantity which is the "pile period meter" output.

15

In addition, one observes that the inverse quantity is

$$\dot{n}/n = (1/n) (dn/dt) = F(n)$$

or

$$dn/n = F(n) dt$$

Now, integration yields

$$\int dn/n = \log_e n + \text{const} = \int F(n) dt$$

Thus, n at any time t can be obtained (if $n/\dot{n}$ vs t is known) by a simple integration. This gives the information necessary to work with the flux level detector signal.

Thus, the principal kinetic operative characteristics can be presented in a simple picture by a plot of $n/\dot{n}$ vs k_{ex} for various rates of change of reactivity $\dot{k}_{ex}$. By requiring the digital computer to punch out $n/\dot{n}$, sufficient information was obtained to yield Fig. 15 for a fast reactor with a characteristic lifetime of 10^{-7} sec.

The behavior of the function $n/\dot{n}$ for $\dot{k}_{ex} = \text{constant}$ appears to be as follows:

There are two limiting curves, one for $\dot{k}_{ex} = \infty$, which is defined by $n/\dot{n} = \frac{\ell}{k_{ex}(1-\beta)}$; the other for $\dot{k}_{ex} = 0$, which is defined by the "inhour" formula such that

$$n/\dot{n} = \frac{\ell}{k_{ex} \left(1 - \sum_{i=1}^6 \frac{\beta_i}{1 + \lambda_i n/\dot{n}} \right) - \sum_{i=1}^6 \frac{\beta_i}{1 + \lambda_i n/\dot{n}}}$$

At very small k_{ex} , $n/\dot{n}$ becomes $\frac{\sum_{i=1}^6 \beta_i}{k_{ex}}$, which is parallel to $k_{ex} \frac{\ell}{(1-\beta)}$.

For $k_{ex} > \beta$, $n/\dot{n}$ becomes $\frac{\ell}{k_{ex}(1-\beta) - \beta}$, which in turn approaches $\frac{\ell}{k_{ex}(1-\beta)}$ as an asymptote.

For each rate $\dot{k}_{ex}$ between 0 and ∞ , the value of $n/\dot{n}$ first follows the curve $n/\dot{n} = \frac{\ell}{k_{ex}(1-\beta)}$ after $t = 0$. As $n/\dot{n}$ approaches the value $\frac{\beta}{k_{ex}(1-\beta)}$, the function has a transition to the nearly constant value as $\frac{\ell}{k_{ex}(1-\beta)}$ becomes small in comparison. In this region, and until the limiting curve for

16

$\dot{k}_{ex} = 0$ is approached, the function behaves about like $\frac{\beta}{\dot{k}_{ex}(1-\beta)} - t$. Further transition ends when $n/\dot{n}$ behaves finally like the asymptotic period.

A further study, in which $\dot{k}_{ex}$ was made a linear function of time, resulted in a behavior of the function $n/\dot{n}$ illustrated by the curve marked $\dot{k}_{ex} = 100 t$. This curve can also be obtained from the data for $\dot{k}_{ex} = \text{constant}$ at corresponding values of k_{ex} ; that is, calculate $k_{ex} = \frac{100 t^2}{2}$ and $\dot{k}_{ex} = 100 t$. Select the point on the corresponding $\dot{k}_{ex} = \text{constant}$ curve where it reaches this value of k_{ex} . For example, when $t = 10^{-3}$ sec, $\dot{k}_{ex} = 0.1$ and $k_{ex} = 5 \times 10^{-5}$. Select the curve for $\dot{k}_{ex} = 0.1$ and find the proper value of $n/\dot{n}$ where this curve reaches the value $k_{ex} = 5 \times 10^{-5}$.

17
PROBLEMS CONNECTED WITH EBR-I BEHAVIOR

Operational experience with EBR-I indicated an instability which became more pronounced as coolant flow rates were reduced. Such a condition can be characteristic of a reactor system in which the reactivity coefficient is sometimes positive and/or when a time delay related by heat capacity/heat removal is postulated. To define the problem the reactor transfer function was obtained by use of an oscillator technique. Two groups of experiments were run, one in March of 1955 and one in November 1955 (both at NRTS, Idaho). The first yielded power amplitude response and phase shifts, primarily. The second group was aimed at taking a close look at temperature variations. Analysis of the second group is as yet incomplete and is not reported here, except for some observations.

The amplitude response of the system to impressed frequencies over the spectrum from 0.05 to 50 radians/sec at zero power (2 kw), half power (550 kw), and full power (1100 kw) at several flow rates was obtained from the data. The results are summarized in Fig. 16.

The possibility of an external feedback was postulated on the basis of temperature variation of the coolant at the core inlet after first coming down through the blanket whose temperature near the core-blanket interface also varied. This model was discarded when measurements in the November tests showed that variations in the coolant temperature at the core inlet did not vary measurably (less than one degree) at any time.

The model must then evidently be an internal feedback system, which may have a prompt and delayed temperature coefficient, the latter being dependent on heat capacity and rate of heat removal. One surely gets instability if both of these are positive or if the prompt is positive and the delayed negative. Stability is conditional on the relative magnitudes if the prompt is negative and the delayed either positive or negative. This model is presently being studied.

Operationally, the net coefficient is negative for normal flow rates. However, it had been observed that at zero flow the net coefficient is positive.³ The long-time constant of the optical galvanometer used to follow the decrease in $P/\dot{P}$ (shortening of period) made the calculation of reactivity change suspect since the period was allowed to decrease from an initial low power value of 32 seconds to a high power value of 6 seconds. The decrease began when sensible heating first occurred. It was felt important to repeat this experiment with adequate measuring instrumentation. Measurements were to include fuel slug temperature and power level.

³H. V. Lichtenberger, et al., ANL-5023 (Feb. 1953)

The reactor was placed above critical on a period of ~ 60 seconds ($k_{ex} \sim 0.001$) at 20 watts. Sometime after the power level had risen to the point where sensible heating began, the value of $P/\dot{P}$ began to decrease. It was planned to shut the reactor off when the period got to about one second or when the power approached 1500 kw. Both circumstances occurred nearly simultaneously, so that the scram order occurred just below 1500 kw. The rods were scrammed but their worth was insufficient to reduce the reactivity adequately and before the slow-moving blanket control could come into play, the power had risen above 1500 kw and the period was about one-half second. The power trace was not recorded above 2000 kw but came back on scale two seconds later. Apparently, the thermocouple on the low-time constant recorder had failed and was recording NaK temperature rather than slug temperature. The slower recorders were unable to follow the fuel temperature excursion. The heat generation had proceeded to the point where the fuel element temperature was at the melting point of either the uranium or the eutectic formed with the stainless steel jacket. Normal coolant supply was restored, but a flow restriction became evident after a short time. This indicated the possibility that the jackets took some time to melt through. The core is presently being removed.

In spite of the damage to the reactor, loss of power trace, and thermocouple failure, it was possible to realize the aims of the experiment. The power-time trace yielded the time variation of reactivity. A calorimetric scheme allowed division of energy between NaK and the fuel elements in the faster portion of the excursion after thermocouple failure.

The power, energy release, temperatures, and reactivity for this experiment are shown as a function of time in Figs. 17 to 25. The reactivity-energy release and reactivity coefficient-uranium temperature variations are shown in Figs. 26 and 27.

It appears that experiments and analysis to date will yield information about the size and magnitude of the reactivity coefficients, but will not reveal the physical cause. It has been established only that the feedback is internal, and not external. For the present, the least likely candidate for a mechanism is a Doppler effect. Competition between radial and axial motion of the core materials seems most probable. (Motions of only a few thousandths of an inch in the radial direction can produce significant reactivity changes.) Changes in leakage at the core-blanket interface and density changes in the sodium may also be significant contributors to the net coefficient. Evidence that the tendency towards instability increased greatly when the spacer fins were left off in the second core loading at present seems to indicate that the radial motion of the fuel is the major factor. Most of these are temperature dependent so that the relation between power, heat capacity, and cooling rate would seem to yield the dominant parameters.

19

The studies of the behavior of EBR-I have required further solutions of the kinetics equations wherein the reactivity is periodically varying. A first step was a correlation check. To accomplish this, the response of the flux to a flux-independent input of reactivity was determined for a system with a lifetime of 4×10^{-8} sec. This corresponded to experimentally determined responses on EBR-I at very low power.

A sub-routine was coded for AVIDAC to be used with RE-29 (see Appendix A) in which

$$k_{ex} = \delta k \sin \omega t.$$

Using a value of $\delta k = 0.0002$ and $n_0 = 1$, calculations were performed for various values of ω , covering the range from 0.05 to 10 radians/sec. The response of n for each ω is given in Fig. 28. From these response curves, the values $\frac{n_{max} - n_{min}}{2} = \Delta n$ were obtained, and the quantity $n_0 \delta k / \Delta n$ was calculated. In addition, the phase shift, ψ (represented by the angular displacement of peaks on the n curve from peaks on the k_{ex} curve), was measured. These quantities represent the "transfer function" for the model and are shown in Fig. 29.

The experimentally determined (at 2 kw) values, obtained with the use of RE-22 (see Appendix B) properly normalized, are superimposed, along with the linearized analytical solution obtained by use of H. Greenspan's solution (see Appendix D). The agreement gives some assurance that linearization has resulted in no practical loss in accuracy for variation in flux of up to about 20%, so that except for undamped resonances or resonances at very low frequency the analytical solution should give reliable results if the proper model for $k_{ex} = f(n, t)$ is adopted.

THERMAL INERTIA

In investigations of rapid excursions, great simplification is possible if it can be assumed that there is no transfer of heat out of the source material. It was recognized that for some characteristic excursion time (period or $n/\dot{n}$) this assumption no longer yields a sufficiently accurate picture. If the period is long enough, the energy released should be divided between fuel element and coolant in determining the changes in reactivity of the system.

To determine when the thermal inertia becomes sufficiently important the time variation of stored heat in a sodium-cooled uranium rod during a transient was studied. Use was made of a solution by H. Greenspan of ANL to the problem of temperature distribution in a two-region cylindrical geometry wherein the central region had an exponentially rising heat source. For convenience, this solution by Greenspan has been summarized in Appendix E. It has been previously reported in classified literature.⁴ It complements similar solutions by Greenspan for slab and spherical geometry.⁵

Specifically, the temperature distribution obtained by Greenspan could be integrated to obtain the heat content of the central region or its derivative integrated to obtain the flow between the central and outer regions, following which the ratio of heat in the uranium rod to total heat generated (sodium stagnant), could be obtained. Development of the analytical expression for this fraction is given in Appendix F.

The computational work is rather large, so that only one specific problem has been solved. The material and geometry of the regions is a close approximation of the small pin element proposed for EBR-II. The results are shown in Fig. 30, where the fraction of heat remaining in the fuel has been plotted against the reciprocal period for various ratios of time to period. The fraction indicates the size of the correction which must be made to the reactivity coefficients in each excursion investigated. It appears that such corrections can certainly be ignored for problems in which the period is less than a millisecond, but should be considered for excursions slower than this. Estimates for other sizes of rods can be made by determining the asymptotic value for uniform temperature and intuitively adjusting the curves; or, with greater accuracy, the curve for large values of time/period can be calculated without too much work and the adjustments for smaller values of the ratio can then be made.

⁴Reactor Engineering Division Quarterly Report ANL-5511, p. 174 (January, 1956)

⁵Reactor Engineering Division Quarterly Report ANL-4951, p. 13 (December 15, 1952)

APPENDIX A

Solution of Kinetics Equation for n, Given k_{ex}
(AVIDAC Code RE-29) --due to R. Brittan

The kinetics equations considered are the usual set for a one-group, space-independent model:

$$\left. \begin{aligned} \dot{n} &= \frac{k_{ex}}{\ell} n - \sum_i \dot{c}_i + S \\ \dot{c}_i &= (1 + k_{ex}) \frac{\beta a_i}{\ell} n - \lambda_i c_i \end{aligned} \right\} \quad i = 1, 2, \dots, 6$$

$$k_{ex} = f(n, t) \quad ,$$

using the notations and definitions given previously.

The change in n between two points j and j+1 can be obtained by integrating the first equation:

$$n_{j+1} - n_j = \int_{t_j}^{t_{j+1}} dn = \frac{1}{\ell} \int_{t_j}^{t_{j+1}} k_{ex} n dt - \sum_i (c_{i,j+1} - c_{i,j}) + S (t_{j+1} - t_j) \quad .$$

Using the notation $n_{j+1} = n_j + \Delta n$; $k_{ex,j+1} = k_{ex,j} + \Delta k$;
 $c_{i,j+1} = c_{i,j} + \Delta c_i$ and $t_{j+1} = t_j + \Delta t$, this becomes

$$\Delta n = \frac{1}{\ell} \int_{t_j}^{t_{j+1}} k_{ex} n dt - \sum_i \Delta c_i + S \Delta t \quad .$$

If k_{ex} and n are assumed to vary linearly over the interval Δt , then

$$k_{ex} = k_{ex,j} + \Delta k \left(\frac{t-t_j}{\Delta t} \right) \quad \text{and} \quad n = n_j + \Delta n \left(\frac{t-t_j}{\Delta t} \right) \quad \text{for} \quad t_j < t < t_{j+1} \quad .$$

The product $k_{ex}n$ is formed from these and integrated to yield

$$\Delta n = \frac{1}{\ell} \left[n_j \left(k_{ex,j} + \frac{\Delta k}{2} \right) + \frac{\Delta n}{2} \left(k_{ex,j} + \frac{2\Delta k}{3} \right) \right] \Delta t - \sum_i \Delta c_i + S \Delta t \quad .$$

Similarly, the change in c_i between two points j and j+1 is obtained by integrating the second equation:

$$c_{i,j+1} - c_{i,j} = \int_{t_j}^{t_{j+1}} dc_i = \frac{\beta a_i}{\ell} \int_{t_j}^{t_{j+1}} n dt + \frac{\beta a_i}{\ell} \int_{t_j}^{t_{j+1}} k_{ex} n dt - \lambda_i \int_{t_j}^{t_{j+1}} c_i dt .$$

Again changing the notation, replacing k_{ex} by $k_{ex} + \Delta k \frac{(t-t_j)}{\Delta t}$,
 n by $n_j + \Delta n \frac{(t-t_j)}{\Delta t}$ and c_i by $c_{i,j} + \Delta c_i \frac{(t-t_j)}{\Delta t}$, performing the integrations
 and replacing

$$\left[\frac{\beta a_i}{\ell} (1 + k_{ex,j}) n_j - \lambda_i c_i \right] \text{ by } \left(\frac{dc_i}{dt} \right)_j ,$$

there is obtained

$$\Delta c_i = \frac{\frac{\beta a_i}{\ell} \left[n_j \Delta k + \Delta n \left(1 + k_{ex,j} + \frac{2\Delta k}{3} \right) \right] \frac{\Delta t}{2} + \left(\frac{dc_i}{dt} \right)_j \Delta t}{\left(1 + \lambda_i \frac{\Delta t}{2} \right)} .$$

These Δc_i can be summed over all i 's and inserted in the expression for Δn to finally yield

$$\Delta n = \frac{\frac{\Delta t}{\ell} n_j \left[k_{ex,j} + \frac{\Delta k}{2} \left(1 - \beta \sum_i \frac{a_i}{1 + \lambda_i \frac{\Delta t}{2}} \right) \right] - \sum_i \left[\frac{\left(\frac{dc_i}{dt} \right)_j \Delta t}{\left(1 + \lambda_i \frac{\Delta t}{2} \right)} \right] + S \Delta t}{1 - \frac{\Delta t}{2\ell} \left[\left(k_{ex,j} + \frac{2\Delta k}{3} \right) \left(1 - \beta \sum_i \frac{a_i}{1 + \lambda_i \frac{\Delta t}{2}} \right) - \beta \sum_i \frac{a_i}{1 + \lambda_i \frac{\Delta t}{2}} \right]} .$$

The values for Δk can be obtained directly from the third equation if k_{ex} is dependent on t only, and by iteration between Δk and Δn if k_{ex} is also dependent on n . In the latter case, the order of procedure should be

Compute $\Delta k^{(0)}$ assuming Δn has the value between $j-1$ and j points

Compute $\Delta n^{(1)}$ using this $\Delta k^{(0)}$

Compute $\Delta k^{(1)}$ using $\Delta n^{(1)}$

Compute $\Delta n^{(2)}$ using $\Delta k^{(1)}$

Continue to convergence

Compute Δc_i

Compute $\left(\frac{dc_i}{dt} \right)_{j+1} = \frac{2 \Delta c_i}{\Delta t} - \left(\frac{dc_i}{dt} \right)_j$.

23

In the program for AVIDAC, computation of Δk is divorced from the main body of the code so that any number of forms for expressing k_{ex} may be utilized with little additional coding time.

A number of forms of $k_{ex} = f(n,t)$ have been coded to date. Representative of these are:

$$\dot{k}_{ex} = A + Bn$$

$$\dot{k}_{ex} = At$$

$$\dot{k}_{ex} = \frac{\Omega}{\omega} \cos \omega t$$

Others are available in the AVIDAC library.

APPENDIX B

Solution of the Kinetics Equations for k_{ex} , Given n .
(AVIDAC Code RE-22) --due to H. J. Wheeler

There has been increasing demand for means of analyzing time-dependent neutron flux data obtained on reactors with a view to determining the time variation of reactivity required to produce such flux data. The means has been provided by arranging for a solution of the reactor kinetics equations on AVIDAC for a given input of flux-time data. This is the converse of the solution outlined in Appendix A for a given excess reactivity variation. The procedure is as follows:

Consider again the first two kinetics equations, which are rewritten here as

$$\frac{dn}{dt} = \frac{k_{ex}(t)(1 - \beta) - \beta}{\ell} n(t) + \sum_{i=1}^6 \lambda_i c_i(t) \quad (1)$$

$$\frac{dc_i}{dt} = \frac{[1 + k_{ex}(t)]\beta a_i}{\ell} n(t) - \lambda_i c_i(t) \quad (i = 1, 2 \dots 6) \quad (2i)$$

(symbols have same meaning as above)

The neutron density data for which the associated k_{ex} variation is desired is available for points in time at h second intervals. Equations (2i) can be integrated to give

$$c_i(t) = \frac{\beta a_i}{\ell} e^{-\lambda_i t} \int_{t_0}^t [k_{ex}(y) + 1] n(y) e^{\lambda_i y} dy + e^{-\lambda_i(t - t_0)} c_i(t_0)$$

and at time h later,

$$c_i(t + h) = e^{-\lambda_i h} c_i(t) + \frac{\beta a_i}{\ell} e^{-\lambda_i(t + h)} \int_t^{t+h} [k_{ex}(y) + 1] n(y) e^{\lambda_i y} dy.$$

This equation can be solved for $k_{ex}(t + h)$ in terms of $c_i(t + h)$, $c_i(t)$, $n(t)$ and $k_{ex}(t)$ if the integral is evaluated by trapezoidal integration. The result of doing this is to yield six simultaneous equations in seven unknowns where it is assumed that $c_i(t)$, $n(t)$ and $k_{ex}(t)$ are known. This is the case when $t = t_0$. Equation (1) provides the seventh equation if we replace $\frac{dn}{dt}$ by $\frac{n(t + h) - n(t - h)}{2h}$ (which is the three-point derivative of n at t).

There are now seven simultaneous equations in seven unknowns if we set $t = t_0$. These can be solved for $c_i(t_0 + h)$ and $k_{ex}(t_0 + h)$. Now set $t = t_0 + h$ and solve for $c_i(t_0 + 2h)$ and $k_{ex}(t_0 + 2h)$, and so on.

The order of solution used here is

$$k_{ex}(t+h) = \frac{1}{1 - \beta \left(1 - \frac{h}{2} \sum \lambda_i a_i\right)} \left\{ \frac{\frac{\ell}{2h} [n(t+2h) - n(t)] - \sum \lambda_i e^{-\lambda_i h} \left[\ell c_i(t) + \frac{k_{ex}(t) + 1}{2} \beta a_i h n(t) \right]}{n(t+h)} + 1 \right\} - 1$$

$$c_i(t+h) = e^{-\lambda_i h} \left\{ c_i(t) + \frac{h}{2} \frac{k_{ex}(t) + 1}{\ell} \beta a_i n(t) \right\} + \frac{h}{2} \frac{k_{ex}(t+h) + 1}{\ell} \beta a_i n(t+h) .$$

The initial conditions are that k_{ex} has been constant (k_{ex_c}) long enough so that the concentration of delayed emitters have the asymptotic form; that is, it is assumed that the flux has the form $n(t) = n_0 e^{t/\tau}$ (where τ is the asymptotic period) and that t is large enough so that the expression for the delayed emitters

$$c_i(t) = \frac{\beta a_i n_0}{\ell} \left[\frac{(k_{ex_c} + 1) \lambda_i e^{t/\tau} + (1/\tau - \lambda_i k_{ex_c}) e^{-\lambda_i t}}{\lambda_i (\lambda_i + 1/\tau)} \right]$$

can be replaced by

$$c_i(t_0) = \frac{\beta a_i (k_{ex_c} + 1) n(t_0)}{\lambda_i + 1/\tau} .$$

Because of using trapezoidal rule for integrating the expression

$$\int_{t_0}^t [k_{ex}(y) + 1] n(y) e^{\lambda_i y} dy ,$$

errors have been introduced which depend on the interval size h , independently of how the flux and reactivity change over this interval. These errors are of the order of magnitude of

$$-\frac{\beta h^3}{12} \sum a_i \lambda_i^3$$

and for Hughes' data, when $h \approx 0.1$, this is roughly -4×10^{-5} . For $h \leq 0.01$ the only limit imposed on interval size arises from the assumption that n is linear over the interval, essentially.

APPENDIX C

Analytical Solution for k_{ex} , given n .
--due to H. Greenspan

If the flux can be represented by a function whose Laplace transform exists, then the one-group, space-independent kinetics equations can be solved analytically for excess reactivity. The essential point in the solution is to work with $\mathcal{L}\{k_{ex}n\}$ rather than with $\mathcal{L}\{k_{ex}\}$. The solution is developed as follows:

Consider again the basic kinetic equations

$$\frac{dn}{dt} = \frac{1-\beta}{\ell} k_{ex} n - \frac{\beta}{\ell} n + \sum_i \lambda_i c_i \quad (1)$$

and

$$\frac{dc_i}{dt} = \frac{\beta_i}{\ell} k_{ex} n + \frac{\beta_i}{\ell} n - \lambda_i c_i \quad (2)$$

Now let

$$n(t) = n_0 + n_1(t); \quad c_i(t) = c_{i_1}(t) + c_{i_0};$$

$$\mathcal{L}\{n_1(t)\} = N(s); \quad \mathcal{L}\{c_i(t)\} = C_i(s); \quad \mathcal{L}\{k_{ex}(t)n(t)\} = G(s) \quad .$$

$$\text{At } t = 0 \text{ assume } n_1(0) = c_{i_1}(0) = k_{ex}(0) = 0 \quad ,$$

so that

$$c_{i_0} = \frac{\beta_i n_0}{\lambda_i \ell} \quad (3)$$

The transformed equations for dn/dt and dc_i/dt are then

$$sN(s) = \frac{1-\beta}{\ell} G(s) - \frac{\beta}{\ell} N(s) + \sum_i \lambda_i C_i(s)$$

$$sC_i(s) = \frac{\beta_i}{\ell} G(s) + \frac{\beta_i}{\ell} N(s) - \lambda_i C_i(s) \quad .$$

Combining these

$$sN(s) = \frac{1-\beta}{\ell} G(s) - \frac{\beta}{\ell} N(s) + \frac{1}{\ell} [N(s) + G(s)] \sum_i \frac{\lambda_i \beta_i}{s + \lambda_i} \quad .$$

Solving for $G(s)$, the final result is

$$G(s) = N(s) \left[\frac{s \left(\ell + \sum_i \frac{\beta_i}{s + \lambda_i} \right)}{1 - s \sum_i \frac{\beta_i}{s + \lambda_i}} \right] \quad (4)$$

The expression

$$\frac{s \left(\ell + \sum_i \frac{\beta_i}{s + \lambda_i} \right)}{1 - s \sum_i \frac{\beta_i}{s + \lambda_i}} = Q(s)$$

appearing here and in the solution treated in Appendix D is of the same form as the inhour equation if the asymptotic pile period is substituted for s . It can be written as the ratio of two polynomials in s :

$$Q(s) = \frac{P_1(s)}{P_2(s)}$$

The coefficients of $P_1(s)$ depend only on the lifetime of the pile and on the β_i 's and λ_i 's, and the coefficients of P_2 depend only on the β_i 's and λ_i 's, where

$$P_1(s) = [a_7 \ell s^7 + a_6 s^6 + a_5 s^5 + a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s]$$

and

$$P_2(s) = (s - \mu_1) (s - \mu_2) (s - \mu_3) (s - \mu_4) (s - \mu_5) (s - \mu_6).$$

The problem has been worked for the case where $n_1(t) = c \sin \omega t$.

Then

$$k_{\text{ex}}^n = c\omega \left\{ \frac{e^{j\omega t} P_1(j\omega)}{2j\omega P_2(j\omega)} + \frac{e^{-j\omega t} P_1(-j\omega)}{-2j\omega P_2(-j\omega)} + \sum_i \frac{e^{\mu_i t} P_1(\mu_i)}{(\mu_i^2 + \omega^2) \frac{d}{ds} P_2(s)} \Big|_{s=\mu_i} \right\}$$

or, more conveniently,

$$k_{\text{ex}}^n = cR \sin(\omega t + \psi) + c\omega \sum_i \frac{e^{\mu_i t} P_1(\mu_i)}{(\mu_i^2 + \omega^2) \frac{d}{ds} P_2(s)} \Big|_{s=\mu_i}$$

where

$$R = \sqrt{\frac{(\text{Real } P_1)^2 + (\text{Im } P_1)^2}{(\text{Real } P_2)^2 + (\text{Im } P_2)^2}}$$

$$\psi = \arctan \frac{\text{Im } P_1}{\text{Real } P_1} - \arctan \frac{\text{Im } P_2}{\text{Real } P_2}$$

Since $\mu_i < 0$, the summation drops out for large enough t . The asymptotic form for $k_{\text{ex}} n$ then is

$$k_{\text{ex}} n = cR \sin(\omega t + \psi)$$

Since

$$n = n_0 + n_1(t) = n_0 + c \sin \omega t ,$$

one finds that

$$k_{\text{ex}} = \frac{R \sin(\omega t + \psi)}{\frac{n_0}{c} + \sin \omega t}$$

For the Hughes' six-delay group data, the values of the a 's and μ 's in the polynomials above are:

$a_1 =$	0.58078×10^{-4}	$\mu_1 =$	-0.012506
$a_2 =$	49.1212×10^{-4}	$\mu_2 =$	-0.031546
$a_3 =$	667.7001×10^{-4}	$\mu_3 =$	-0.154319
$a_4 =$	1981.3818×10^{-4}	$\mu_4 =$	-0.457148
$a_5 =$	1173.4164×10^{-4}	$\mu_5 =$	-1.61315
$a_6 =$	76.0744×10^{-4}	$\mu_6 =$	-14.005
$a_7 =$	1.0076		

APPENDIX D

Analytical Solution for n , Given k_{ex} --due to H. Greenspan

An analytical solution for the variation of neutron density with time in a one-group, space-independent model has been obtained when the reactivity is a function of time and a functional of the neutron density involving a delay time, Γ . Specifically

$$k = 1 + a [n(t) - n_0] + bH(t - \Gamma)[n(t - \Gamma) - n_0] + c \sin \omega t.$$

The linearized solution is obtained as follows:

Letting

$$n = n_0 + n_1(t) \quad n_1 \ll n_0$$

and

$$c_i = c_{i_0} + c_{i_1}(t) \quad c_{i_0} = \frac{\beta_i n_0}{\lambda_i \ell},$$

with zero (0) subscripts denoting steady state values,

$$\begin{aligned} H(t - \Gamma) &= 0 & t < \Gamma \\ H(t - \Gamma) &= 1 & t \geq \Gamma \end{aligned}$$

and

$$n_1(t - \Gamma) = \bar{n}_1;$$

also

$$k = 1 + k_{ex}.$$

The basic kinetics equations set forth previously can be written (ignoring all squared terms in n_1):

$$\begin{aligned} \ell \frac{dn}{dt} = \ell \frac{dn_1}{dt} &= -\beta n_1 + a n_0 n_1 (1 - \beta) + b n_0 H \bar{n}_1 (1 - \beta) + \\ &+ n_0 (1 - \beta) c \sin \omega t + \ell \sum_i \lambda_i c_i \end{aligned}$$

and

$$\frac{dc_i}{dt} = \frac{dc_{i_1}}{dt} = -\lambda_i c_i + \frac{\beta_i}{\ell} [n_1 + a n_0 n_1 + b n_0 H \bar{n}_1 + n_0 c \sin \omega t].$$

30

Using the information that $n_1(0) = 0$ and $c_{i_1}(0) = 0$, one writes the Laplace transforms of these equations, where

$$\mathcal{L}\{n_1(t)\} = N(s) \text{ and } \mathcal{L}\{c_{i_1}(t)\} = C_i(s),$$

as follows:

$$\ell s N(s) = -\beta N(s) + a n_0 N(s)(1 - \beta) + b n_0 e^{-s\Gamma} (1 - \beta) N(s) +$$

$$\frac{n_0 c (1 - \beta) \omega}{s^2 + \omega^2} + \ell \sum_i \lambda_i C_i(s)$$

and

$$(s + \lambda_i) C_i(s) = \frac{\beta_i}{\ell} \left[N(s) + a n_0 N(s) + b n_0 e^{-s\Gamma} N(s) + n_0 c \frac{\omega}{s^2 + \omega^2} \right].$$

From the latter,

$$\ell \sum_i \lambda_i C_i(s) = \sum_i \beta_i \left(1 - \frac{s}{s + \lambda_i} \right) \left[N(s) + a n_0 N(s) + b n_0 e^{-s\Gamma} N(s) + n_0 c \frac{\omega}{s^2 + \omega^2} \right].$$

This may be substituted into the $\ell s N(s)$ equation above, and by proper algebraic manipulations it is found that

$$N(s) = \frac{n_0 c \omega}{(s^2 + \omega^2) W(s)},$$

wherein

$$W(s) = Q(s) - n_0(a + b e^{-s\Gamma})$$

$$Q(s) = \left[\frac{s \left(\ell + \sum_i \frac{\beta_i}{s + \lambda_i} \right)}{\left(1 - s \sum_i \frac{\beta_i}{s + \lambda_i} \right)} \right]$$

as before.

Then $n_1(t)$ is obtained from the transform as

$$n_1(t) = n_0 c \omega \text{ times sum of residues of } \frac{1}{(s^2 + \omega^2) W(s)}$$

and is

$$n_1(t) = n_0 c \omega \left[\frac{e^{j\omega t}}{2j\omega W(j\omega)} + \frac{e^{-j\omega t}}{-2j\omega W(-j\omega)} \right] +$$

$$n_0 c \omega \sum_i \frac{e^{\mu_i t}}{(\mu_i^2 + \omega^2) \left. \frac{d}{ds} W(s) \right|_{s=\mu_i}} .$$

This may be put in a more convenient form:

$$n_1(t) = \frac{n_0 c}{|W(j\omega)|} \sin(\omega t + \psi) + n_0 c \omega \sum_i \frac{e^{\mu_i t}}{(\mu_i^2 + \omega^2) \left. \frac{d}{ds} W(s) \right|_{s=\mu_i}}$$

where $\psi = \arctan \frac{\text{Im. } W(j\omega)}{\text{Real } W(j\omega)} .$

Depending on the combination of $n_0 a$, $n_0 b$ and Γ , the roots of $W(s)$ are determined and these determine the character of $n_1(t)$. In many cases $n_0 a$, $n_0 b$ and Γ are such that $W(s)$ has no roots with positive real parts, but $W(s)$ may have roots which are pure imaginary, have a negative real part, or both. The formula above for $n_1(t)$ does not apply for all such values of t , $n_0 a$, $n_0 b$, Γ , and ω which would make $n_1(t) > n_0$, but even with such combinations it can be used to investigate the development of an unstable situation.

APPENDIX E

Temperature Distribution in Two-Region Cylindrical Geometry with Experimentally Rising Heat Source in Central Region

--due to H. Greenspan

Solutions to the time-dependent heat conduction and temperature distribution problems in fuel elements have been previously reported for the cases of plates and spheres.⁶ Since then there has been a demand for a solution for the case of cylinders in some safety studies. The problem has been solved in two-region cylindrical geometry for an exponentially rising heat source in the central region, as follows:

Consider a composite cylinder made up of a central cylinder of heat source material surrounded by a cylinder of non-source material. The source material occupies the region 1 ($0 \leq r \leq r_1$). Here the density is ρ_1 , specific heat c_1 , conductivity k_1 , and source strength $Q_1 e^{\alpha t}$. Here α is the inverse of the e-folding time or period of the source whose strength increases exponentially with time. The non-source material occupies the region $r_1 \leq r \leq r_2$ and has the properties ρ_2 , c_2 , k_2 with $Q_2 = 0$.

The space-time-dependent equations defining the temperatures $T_i(r, t)$ are, as usual,

$$\frac{\partial T_i}{\partial t} = \kappa_i^2 \left[\frac{\partial^2 T_i}{\partial r^2} + \frac{1}{r} \frac{\partial T_i}{\partial r} \right] + q_i e^{\alpha t} \quad , \quad (i = 1, 2)$$

where

$$\kappa_i^2 = \frac{k_i}{\rho_i c_i} \quad \text{and} \quad q_i = \frac{Q_i}{\rho_i c_i}$$

and

$$\left. \begin{array}{l} q_2 = 0 ; \quad \frac{\partial T_2}{\partial r} = 0 \quad \text{at} \quad r = r_2 ; \quad k_1 \frac{\partial T_1}{\partial r} = k_2 \frac{\partial T_2}{\partial r} \end{array} \right\} \text{at } r = r_1$$

$$T_1 \text{ is finite at } r = 0 ; \quad T_i(r, 0) = 0$$

(Such a condition would exist, for example, if this composite cylinder were formed as a representative cell in an array of cells. This would correspond closely to a group of uranium rods in sodium.)

⁶Reactor Engineering Division Quarterly Report ANL-4951,
(December 15, 1952)

33

The method of Laplace transformations is applied to the equation above to obtain the transform of T_i , thus:

$$V_i(r, s) = A_i(s) I_0(\mu_i r) + B_i(s) K_0(\mu_i r) + \frac{q_i}{s(s-\alpha)} \quad \left(\mu_i^2 = \frac{s}{\kappa_i^2} \right).$$

Applying the conditions above, one obtains

$$V_1(r, s) = \frac{q_1 \rho [I_1(\mu_2 r_1) K_1(\mu_2 r_2) - I_1(\mu_2 r_2) K_1(\mu_2 r_1)]}{s(s-\alpha) D(s)} I_0(\mu_1 r) + \frac{q_1}{s(s-\alpha)}$$

and

$$V_2(r, s) = \frac{q_1 I_1(\mu_1 r_1)}{s(s-\alpha) D(s)} [I_0(\mu_2 r) K_1(\mu_2 r_2) + I_1(\mu_2 r_2) K_0(\mu_2 r)] ,$$

where

$$\rho = \frac{K_2 \kappa_1}{K_1 \kappa_2}$$

and

$$D(s) = I_1(\mu_1 r_1) [I_0(\mu_2 r_1) K_1(\mu_2 r_2) + I_1(\mu_2 r_2) K_0(\mu_2 r_1)] \\ - \rho I_0(\mu_1 r_1) [I_1(\mu_2 r_1) K_1(\mu_2 r_2) - I_1(\mu_2 r_2) K_1(\mu_2 r_1)] .$$

The inverse transformation is applied to these to obtain the temperature distribution in the two regions:

$$T_1(r, t) = \frac{q_1}{\alpha} (e^{\alpha t} - 1) + \frac{\rho q_1 e^{\alpha t} I_0\left(\frac{\sqrt{\alpha}}{\kappa_1} r\right) \left[I_1\left(\frac{\sqrt{\alpha}}{\kappa_2} r_1\right) K_1\left(\frac{\sqrt{\alpha}}{\kappa_2} r_2\right) - I_1\left(\frac{\sqrt{\alpha}}{\kappa_2} r_2\right) K_1\left(\frac{\sqrt{\alpha}}{\kappa_2} r_1\right) \right]}{\alpha D(\alpha)} \\ - \rho q_1 \pi \sum_{j=1}^{\infty} \frac{J_0\left(\frac{\lambda_j}{\kappa_1} r\right) \left[J_1\left(\frac{\lambda_j}{\kappa_2} r_2\right) Y_1\left(\frac{\lambda_j}{\kappa_2} r_1\right) - J_1\left(\frac{\lambda_j}{\kappa_2} r_1\right) Y_1\left(\frac{\lambda_j}{\kappa_2} r_2\right) \right] e^{-\lambda_j^2 t}}{\lambda_j (\lambda_j^2 + \alpha) \frac{d}{d\lambda} D(-\lambda^2) \Big|_{\lambda = \lambda_j}} \\ - \frac{\rho q_1}{\alpha} \left[\frac{\kappa_1}{\kappa_2 \left(\frac{r_1^2}{r_1^2 - r_2^2} \right) - \rho \kappa_1} \right]$$

54

$$\begin{aligned}
T_2(r, t) = & \frac{q_1 e^{\alpha t} I_1\left(\frac{\sqrt{\alpha}}{\kappa_1} r_1\right) \left[I_0\left(\frac{\sqrt{\alpha}}{\kappa_2} r\right) K_1\left(\frac{\sqrt{\alpha}}{\kappa_2} r_2\right) + I_1\left(\frac{\sqrt{\alpha}}{\kappa_2} r_2\right) K_0\left(\frac{\sqrt{\alpha}}{\kappa_2} r\right) \right]}{\alpha D(\alpha)} \\
& - q_1 \pi \sum_{j=1}^{\infty} \frac{J_1\left(\frac{\lambda_j}{\kappa_1} r_1\right) \left[J_1\left(\frac{\lambda_j}{\kappa_2} r_2\right) Y_0\left(\frac{\lambda_j}{\kappa_2} r\right) - J_0\left(\frac{\lambda_j}{\kappa_2} r\right) Y_1\left(\frac{\lambda_j}{\kappa_2} r_2\right) \right] e^{-\lambda_j^2 t}}{\lambda_j (\lambda_j^2 + \alpha) \frac{d}{d\lambda} D(-\lambda^2) \Big|_{\lambda=\lambda_j}} \\
& - \frac{q_1}{\alpha} \left[\frac{\kappa_2 \left(\frac{r_1^2}{r_1^2 - r_2^2} \right)}{\kappa_2 \left(\frac{r_1^2}{r_1^2 - r_2^2} \right) - \rho \kappa_1} \right] .
\end{aligned}$$

In these equations λ_j are the roots of $D(-\lambda^2) = 0$. For large values of αt , i.e., after several periods (say $\alpha t = 6$) the last two terms in each equation above can be neglected. For investigation of the problem near the beginning of the excursion, these terms are important.

APPENDIX F

Time Variation of Stored Heat in 2-Region Fuel Cell
During Transient --due to R. O. Brittan

The study of thermal inertia effects was meant to determine the maximum reactor periods for which it could be reasonably assumed that heat transfer out of the element could be neglected. The solution of the temperature distribution for an exponentially rising source in a two-region cylindrical cell provides a means of determining the fraction of total heat produced remaining in the element for various αt and $\alpha(\frac{1}{\alpha} = \text{e-folding time})$. One procedure used is to calculate the ratio of heat content in the element (region 1) to total heat in the cell as

$$R_H = \frac{H_1}{H} = \frac{\rho_1 c_1 \int_0^{r_1} T_1 2\pi r dr}{Q_1 \pi r_1^2 \int_0^{t_1} e^{\alpha t} dt}$$

by substituting Greenspan's expression for T_1 (Appendix E). The calculation involved evaluation of the summation terms, except for large αt . In addition, a second scheme can be used to obtain the ratio R_H by calculating the heat leaving the region 1, i.e., H_2 . Then

$$R_H = \frac{H - H_2}{H} = \frac{Q_1 \pi r_1^2 \int_0^{t_1} e^{\alpha t} dt - \left[-2\pi r_1 K_1 \int_0^{t_1} \frac{\partial T_1}{\partial r} \bigg|_{r=r_1} dt \right]}{Q_1 \pi r_1^2 \int_0^{t_1} e^{\alpha t} dt} .$$

Both schemes lead to identical results, the latter less circuitously. The procedure of the second scheme is in some detail as follows. (The notation and definitions used are the same as in Appendix E.)

From basic heat flow considerations, the heat leaving region 1 is

$$H_{1\text{out}} = H_{2\text{in}} = -2\pi r_1 K_1 \int_0^{t_1} \frac{\partial T_1}{\partial r} \bigg|_{r=r_1} dt .$$

36

Partial differentiation of the expression for $T_1(r,t)$ written previously yields

$$\left. \frac{\partial T_1}{\partial r} \right|_{r=r_1} = \frac{\rho q_1 \frac{\sqrt{\alpha}}{\kappa_1}}{\alpha} M e^{\alpha t} + \frac{\rho q_1 \pi}{\kappa_1} \sum_{j=1}^{\infty} N e^{-\lambda_j^2 t},$$

where

$$M = \frac{I_1\left(\frac{\sqrt{\alpha}}{\kappa_1} r_1\right) \left[I_1\left(\frac{\sqrt{\alpha}}{\kappa_2} r_1\right) K_1\left(\frac{\sqrt{\alpha}}{\kappa_2} r_2\right) - I_1\left(\frac{\sqrt{\alpha}}{\kappa_2} r_2\right) K_1\left(\frac{\sqrt{\alpha}}{\kappa_2} r_1\right) \right]}{D(\alpha)}$$

and

$$N = \frac{J_1\left(\frac{\lambda_j}{\kappa_1} r_1\right) \left[J_1\left(\frac{\lambda_j}{\kappa_2} r_2\right) Y_1\left(\frac{\lambda_j}{\kappa_2} r_1\right) - J_1\left(\frac{\lambda_j}{\kappa_2} r_1\right) Y_1\left(\frac{\lambda_j}{\kappa_2} r_2\right) \right]}{(\lambda_j^2 + \alpha) \frac{d}{d\lambda} D(-\lambda^2) \Big|_{\lambda=\lambda_j}}.$$

The total amount of heat produced (see above) is

$$H = Q_1 \pi r_1^2 \int_0^{t_1} e^{\alpha t} dt = Q_1 \pi r_1^2 \frac{(e^{\alpha t} - 1)}{\alpha}.$$

From these relations the ratio is found to be

$$R_H = \frac{H - H_2}{H} = 1 + \frac{2 \rho \kappa_1}{r_1 \sqrt{\alpha}} M + \frac{2 \rho \kappa_1 \pi \alpha}{(e^{\alpha t} - 1)} \sum_{j=1}^{\infty} \frac{N}{\lambda_j^2} (1 - e^{-\lambda_j^2 t}).$$

The calculations involve evaluating $D(\alpha)$ for each α investigated. One must also make sufficient evaluations of $D(-\lambda^2)$ to determine the first several values of λ_j , at which $D(-\lambda^2)$ is zero and the slopes $\frac{d}{d\lambda} D(-\lambda^2)$ at each λ_j . Next, N is evaluated for each λ_j and α and the summation obtained for each α . For large αt the summation term may be neglected.

The R_H has been obtained over the range $1 < \alpha < 10^5$ and $0 < \alpha t < 6$ and $\alpha t < 6$ for the following specific problem:

37

Region 1

Material: Uranium

$$\rho_1 = 19 \text{ gm/cc}$$

$$c_1 = 0.0432 \text{ cal/gm } ^\circ\text{C}$$

$$k_1 = 0.0816 \text{ cal/cm sec } ^\circ\text{C}$$

$$r_1 = 0.20828 \text{ cm}$$

Region 2

Material: Sodium

$$\rho_2 = 0.834 \text{ gm/cc}$$

$$c_2 = 0.3041 \text{ cal/gm } ^\circ\text{C}$$

$$k_2 = 0.1617 \text{ cal/gm sec } ^\circ\text{C}$$

$$r_2 = 0.33731 \text{ cm}$$

The results are shown in Fig. 30.

Fig. 1
 Fission Rate vs. Time for Δk
 Values Above Prompt Critical

Fig. 2
Peak Fission Rate vs. Reciprocal Period

ANL-FWT-15377-A

Fig. 3
Total Fission Yield vs. Excess Reactivity

$$\begin{aligned} \ell^* &= 7 \times 10^{-9} \\ \beta &= 0.00755 \\ \dot{k}_{ex} &= -2.60 \times 10^{-6} (n - n_0) \end{aligned}$$

Fig. 4
Ratio of Peak Width at
Half-Height to Period

Fig. 5
Relation Between Temperature Rise
and Total Number of Fissions

Fig. 6
 Variation of n and $\int n dt$ with Shutdown
 Effectiveness with $k_{ex_0} = 0.008$

44

Fig. 7

n and $\int n dt$ for Rate of 8/sec ($\sim 1000/\text{sec}$)

RE-7-15394-B

45

Fig. 8
 n and $\int n dt$ for Rate of 0.8/sec ($\sim \$100/\text{sec}$)

RE-7-15392-B

Fig. 9

n and $\int ndt$ for Rate of 0.08/sec ($\sim 10/\text{sec}$)

RE-7-15393-B

47

Fig. 10
 n and $\int n dt$ for Rate of 0.008/sec ($\sim \$1/\text{sec}$)

RE-7-15386-B

RE-7-15378-A

Fig. 11
Maximum k_{ex} for Various Rates

Fig. 12
Variation of Minimum Period with Rate

Fig. 13
Variation of Damping Characteristics with Rate

RE-7-15387-B

Fig. 14
 $\int ndt$ vs. Rate for First Peak

RE-7-15384-A

Fig. 15

Kinetic Behavior

Fig. 16
EBR-I Oscillation Tests
Amplitude Response

Fig. 17

Fig. 18

Fig. 19

Fig. 20

Fig. 21

Fig. 22

Fig. 23

Fig. 24

Fig. 25
Measured & Estimated Temperatures

Fig. 26
Reactivity - Energy Relation

Fig. 27

Uranium Temperature Reactivity Coefficient

Fig. 28

Oscillatory Response of Neutron Population
To Sinusoidal Reactivity Input at $\sim$ Zero Power

$$A = 0.0002$$

$$\ell = 4 \times 10^{-8} \text{ seconds}$$

(AVIDAC Results From RE-29 Program)

Fig. 29

Response to an Impressed Periodic Variation in
 Reactivity on a Reactor at Zero Power Having
 an Effective Lifetime of 4×10^{-8} seconds

RE-7-17071-A

67