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26 6p. N715

BROOKHAVEN NATIONAL LABORATORY
Associated Universities, Inc.
Upton, New York

ACCELERATOR DEPARTMENT
Informal Report

INCLUSIVE EXPERIMENTS IN THE TRIPLE-REGGE REGION

Frank E. Paige

May 29, 1975

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ABSTRACT

The triple-Regge model is reviewed. Inclusive experiments in the triple-Regge region are suggested and discussed.

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1. INTRODUCTION

In this report we describe the theoretical interest in measuring at ISABELLE inclusive cross sections in which one of the particles has $x = q_{\parallel}/q_{\parallel\text{max}}$ close to one. The principle motivation for discussing such experiments now is that they require the detection and analysis of high-momentum particles at small angles. Thus they seem likely to impose constraints on the design of any large general-purpose spectrometer and on the length of the straight sections of the machine.

In Sec. 2 we give a brief review of the triple-Regge formula and related theoretical ideas. In Sec. 3 we propose a number of inclusive experiments and discuss them using the triple-Regge model as a guide. It should be stressed, however, that the motivation for these experiments is more general than the theoretical model. No matter what the Pomeron is, we will want to explore how Pomerons interact with each other and how such interactions depend on energy. Measuring inclusive cross sections $x=1$ at ISABELLE will give information on these questions.

2. THEORETICAL REVIEW

2.1 Kinematics

We consider the inclusive reaction $a+b \rightarrow c+X$, where X denotes anything. The invariant cross section $E \frac{d\sigma}{d^3q}$ is a function of three variables, which may be taken to be

$$s = (p_a + p_b)^2, \quad x = \frac{q_{\parallel}}{q_{\parallel\text{max}}} \approx \frac{2q_{\parallel}}{(s)^{\frac{1}{2}}}, \quad \text{and } q_{\perp}^2. \quad (1)$$

Here q is the momentum of c in the center-of-mass frame, and $q_{\parallel}$ and $q_{\perp}$ are its components parallel and transverse to the incident momenta. Mueller¹ has shown that if the Pomeron is taken to be a simple pole with $\alpha(0) = 1$, then

$$E \frac{d\sigma}{d^3q} \sim f(x, q_{\perp}^2), \quad s \rightarrow \infty. \quad (2)$$

We are interested in the end-of-spectrum region, $x \approx 1$. In this region it is convenient to express the inclusive cross section in terms of s , $M^2 = (p_a + p_b - p_c)^2$, and $t = (p_a - p_c)^2$. For large s and $x > 0$ these variables are related to the previous ones by

$$M^2 = (1-x)s + O(1),$$

$$t = -\frac{1}{x} q_{\perp}^2 + (1-x)m_a^2 - \frac{(1-x)}{x} m_c^2 + O\left(\frac{1}{s}\right) \quad (3)$$

Thus $s \gg M^2$ for $x \approx 1$. These formulas remain valid for colliding beams of unequal energy provided that $1-x$ is interpreted as the fractional energy loss of the scattered particle. We also have

$$\frac{s}{\pi} \frac{d\sigma}{dtdM^2} = E \frac{d\sigma}{d^3q} \quad (4)$$

2.2 Triple-Regge Formula

The derivation of the triple-Regge formula² is summarized by the graphs shown in Fig. 1. We first assume that for $s \gg M^2$ the reaction $a+b \rightarrow c+X$ can be treated as a quasi-two-body one, and, in particular, that the amplitude is dominated by one or more Regge poles $\alpha_i(t)$. Then

$$\begin{aligned} \frac{d\sigma}{dtdM^2} &= \frac{1}{16\pi s^2} \sum_X \left| \sum_i \beta_{a\bar{c},i}(t) \gamma_{X,i} \xi_i(t) (\cos\theta_t)^{\alpha_i(t)} \right|^2 \\ &= \frac{1}{16\pi s^2} \sum_{ij} \beta_{a\bar{c},i}(t) \beta_{a\bar{c},j}(t) \xi_i(t) \xi_j^*(t) (\cos\theta_t)^{\alpha_i(t)+\alpha_j(t)} \\ &\quad \times \sum_X \gamma_{X,j}^* \gamma_{X,i} \end{aligned} \quad (5)$$

where $\beta_{a\bar{c},i}(t)$ and $\xi_i(t)$ are the usual Regge residue functions and signature factors, and where the sum on X includes integration over all internal variables. By a unitarity this sum gives the imaginary part of the forward Reggeon-particle amplitude³ $A_{\alpha_j b; \alpha_i b}(M^2, t)$. Thus

$$\begin{aligned} \frac{d\sigma}{dtdM^2} &= \frac{1}{16\pi s^2} \sum_{ij} \beta_{a\bar{c},i}(t) \beta_{a\bar{c},j}(t) \xi_i(t) \xi_j^*(t) (\cos\theta_t)^{\alpha_i(t)+\alpha_j(t)} \\ &\quad \times \text{Im } A_{\alpha_j b; \alpha_i b}(M^2, t) \end{aligned} \quad (6)$$

We further assume that for large M^2 the Reggeon-particle amplitude

itself has Regge behavior:

$$\text{Im } A_{\alpha_j b; \alpha_i b}(M^2, t) \sim \sum_k g_{ij,k}(t) \beta_{b\bar{b},k}(0) (M^2)^{\alpha_k(0)}. \quad (7)$$

The $g_{ij,k}(t)$ are the triple-Regge couplings. We also note that for M^2 large and $s \gg M^2$

$$\cos \theta_t \sim \frac{s}{M^2}. \quad (8)$$

Thus

$$\begin{aligned} \frac{d\sigma}{dt dM^2} &= \frac{1}{16\pi s^2} \sum_{ijk} \beta_{a\bar{c},i}(t) \beta_{a\bar{c},j}(t) \xi_i(t) \xi_j^*(t) \beta_{b\bar{b},k}(0) g_{ij,k}(t) \\ &\times \left(\frac{s}{M^2}\right)^{\alpha_i(t)+\alpha_j(t)} (M^2)^{\alpha_k(0)}. \end{aligned} \quad (9)$$

This is the triple-Regge formula. Using Eqs. (3) and (4), we can rewrite it as

$$\begin{aligned} E \frac{d\sigma}{d^3q} &= \frac{1}{16\pi^2} \sum_{ijk} \beta_{a\bar{c},i}(t) \beta_{a\bar{c},j}(t) \xi_i(t) \xi_j^*(t) \beta_{b\bar{b},k}(0) g_{ij,k}(t) \\ &\times \left(\frac{1}{1-x}\right)^{\alpha_i(t)+\alpha_j(t)-\alpha_k(0)} (s)^{\alpha_k(0)-1}. \end{aligned} \quad (10)$$

It should be noted that the trajectory α_k enters in a different way than α_i and α_j . In labeling triple-Regge terms, therefore, we will always write the indices in the order (ijk).

2.3 Phenomenology

Several authors have used the triple-Regge formula to analyze data on inclusive reactions at large x . In a detailed study Field and Fox⁴ found that the available data on $pp \rightarrow pX$ for $0.8 \lesssim x < 1$ and $|t| \lesssim 1 \text{ GeV}^2$ are generally consistent with the triple-Regge formula. These authors include six triple-Regge terms, namely (ijk) = PPP, MMP, PPM, MMM, $\pi\pi P$, and $\pi\pi M$, where P denotes the Pomeron, M denotes the ω , f , ρ , and A_2 trajectories, and π denotes the pion trajectory. Setting $\alpha_P = 1$, $\alpha_M = \frac{1}{2}$, and $\alpha_\pi = 0$ gives the nominal behaviors for these terms shown in Table I. Evidently the triple-Pomeron term dominates for s

very large and x very close to one.

Table I: Triple-Regge terms for $\alpha_p = 1$, $\alpha_M = \frac{1}{2}$, $\alpha_\pi = 0$.

(ijk)	$E \frac{d\sigma}{d^3q}$	$\frac{d\sigma}{dtdM^2}$
PPP	$(1-x)^{-1}$	$(M^2)^{-1}$
MMP	1	$(s)^{-1}$
PPM	$(1-x)^{-3/2}(s)^{-1/2}$	$(M^2)^{-3/2}$
MMM	$(1-x)^{-1/2}(s)^{-1/2}$	$(M^2)^{-1/2}(s)^{-1}$
$\pi\pi P$	$(1-x)$	$(M^2)(s)^{-2}$
$\pi\pi M$	$(1-x)^{1/2}(s)^{-1/2}$	$(M^2)^{1/2}(s)^{-2}$

The $pp \rightarrow nX$ cross section has been measured for several energies at the ISR.⁵ It can be fit⁵ by a single triple-Regge term (ijk) = $\pi\pi P$. There is no evidence for an $\pi\pi M$ term as might have been expected.

Other non-diffractive reactions have also been analyzed, and substantial disagreement with the behavior expected from the triple-Regge model has been found.⁶ However, the data are at rather low energy and do not extend to x near one. It would be useful to have an analysis of data in the FNAL energy range.

2.4 Correlations

We consider here reactions $a+b \rightarrow c+d+X$ in which particle c is near $x=1$ and particle d is in the missing mass. In the triple-Regge model such reactions can be viewed as inclusive reactions for Reggeon-particle scattering. The inclusive cross sections normalized to the Reggeon-particle cross section are expected to be similar to those for particle-particle scattering.⁷

For definiteness let us assume that $\alpha_k = \alpha_p$ and that $\alpha_i = \alpha_j = \alpha$. Then for $a+b \rightarrow c+X$ we have

$$E_c \frac{d\sigma}{d^3q_c} = \frac{1}{16\pi^2} \beta_{ac,\alpha}^2(t) |\xi_\alpha(t)|^2 \beta_{bb,P}(0) g_{\alpha\alpha,P}(t) \left(\frac{1}{1-x_c}\right)^{2\alpha(t)-1} . \quad (11)$$

The corresponding graph for $a+b \rightarrow c+d+X$ with d in the central region of the missing mass is shown in Fig. 2a. Using factorization of the Pomeron, we see that this graph gives

$$E_c E_d \frac{d\sigma}{d^3q_c d^3q_d} = \frac{1}{16\pi^2} \beta_{ac,\alpha}^2(t) |\xi_\alpha(t)|^2 \beta_{bb,P}(0) g_{\alpha\alpha,P}(t) \gamma_d(q_{\alpha 1}^2) \left(\frac{1}{1-x_c}\right)^{2\alpha(t)-1} , \quad (12)$$

where $\gamma_d(q_{\alpha 1}^2)$ is the same central-region vertex¹ as occurs in particle-particle scattering; see Fig. 2b. This formula should apply for

$$-\frac{1}{2} \log s + \Delta < y_d < \frac{1}{2} \log s - \log \frac{1}{1-x_c} - \Delta \quad (13)$$

where Δ is some suitable rapidity gap, say $\Delta \approx 2$.

From Eq. (11)-(13) it follows immediately that the average multiplicity of particles in the missing mass is⁷

$$\bar{n}(M^2) = c \log M^2 + O(1) , \quad (14)$$

with the same constant c as in the formula for the total multiplicity,

$$\bar{n}(s) = c \log s + O(1) . \quad (15)$$

Obviously one can also consider reactions in which several particles are detected in the missing mass.

2.5 Theoretical Problems

There is a theoretical difficulty with the triple-Regge formula. If the triple-Pomeron coupling does not vanish at $t=0$, then the contribution of the triple-Pomeron term to the total cross section grows like $\log \log s$ as $s \rightarrow \infty$.^{8,9} However, if this coupling does vanish at $t=0$, then by a chain of arguments the Pomeron is found to decouple at $t=0$ in all reactions including the total cross section.^{9,10} In either case the total cross section cannot be dominated by a simple Regge pole with

$\alpha(0) = 1$ as $s \rightarrow \infty$. Regge cuts which vanish as an inverse power of $\log s$ relative to the pole at $t=0$ do not change the conclusion.

A possible resolution of this problem can be given within the framework of the Reggeon calculus.^{11,12} One assumes that there is a bare Pomeron which is a simple pole with $\alpha(0) = 1$ and which has a small but nonzero triple-Pomeron coupling. At accessible energies this bare Pomeron gives a zeroth order approximation, and corrections to it can be calculated by perturbation theory. At extremely high energies, say $\log s \gg 10$, perturbation theory fails and one must sum all Reggeon graphs. Progress in doing this by using the renormalization group has been made recently.^{13,14}

The main conclusion from all of this is that one probably should not expect the experimental triple-Pomeron coupling to vanish at $t=0$.

3. SUGGESTED EXPERIMENTS

We suggest here a number of inclusive experiments in which one of the detected particles has a value of x near one. These are the reactions to which the triple-Regge model can be applied. While this model is certainly oversimplified, we expect such reactions to give information on the interactions of the Pomeron with itself and with other Reggeons.

3.1 $p+p \rightarrow p+X$

This reaction has been studied extensively at lower energies, and Field and Fox⁴ have made a triple-Regge fit to the existing data. Their preferred fit (Solution I) extrapolated to infinite energy is shown in Fig. 3.

At ISABELLE we will be particularly interested in studying the triple-Pomeron term and therefore the peak at $x=1$. We will therefore want to measure the cross section for as large values of $\frac{s}{M^2}$, or for values of x as close to one, as possible. Obviously the attainable values of $1-x$ are limited by the momentum resolution of the spectrometer; perhaps $x = 0.99$ is a reasonable goal. This corresponds to $M^2 = 1600$ for $s = 160,000$. For a triple-Regge analysis it would be best to present the data as a function of x for fixed t .

There does not seem to be any compelling reason to try to reach very small values of $|t|$. A reasonable minimum would be $|t| = 0.25 \text{ GeV}^2$. This corresponds to $q_{\perp} = 0.5 \text{ GeV}$ and thus to $\theta = 2.5 \text{ mr.}$ for $q_{\parallel} = 200 \text{ GeV}$. Since $t \propto q_{\perp}^2$, one does not gain very rapidly in the minimum angle

required by increasing the minimum $|t|$. Also, at larger values of $|t|$ the peak at $x = 1$ becomes less pronounced, as can be seen from Fig. 3.

The dynamics of Pomeron interactions are likely to be reflected in logarithmic deviations from the triple-Regge formula. It is probably impossible to study logarithmic variations in $(1-x)$. From Fig. 3 it is apparent that the MFP term is still significant at $1-x = 0.04$, and the resolution of any spectrometer will probably limit $(1-x)$ to $(1-x) \gtrsim 0.01$. It should, however, be possible to look for a logarithmic dependence on s at fixed x and t . For x fixed at a value sufficiently close to one this dependence could be ascribed to the large M^2 behavior of the Pomeron-particle amplitude.

Obviously a measurement of the energy dependence requires good relative normalization at different energies. This might be achieved by keeping the momentum p_a fixed and varying p_b ; then a fixed spectrometer setting corresponds to a fixed x and t . Suppose that $p_a = 200$ GeV and that $x = 0.98$. Then if p_b is varied from 30 GeV to 200 GeV, M^2 varies from 480 GeV^2 to 3200 GeV^2 . This is comparable to the ISR range for s , over which the proton-proton cross section increases by about 10%.

3.2 $p+p \rightarrow c+X, c \neq p$

There is as yet little evidence that the triple-Regge model works for non-diffractive reactions,^{5,6} but we expect it to do so. Then measuring these cross sections at ISABELLE would give information on the large M^2 behavior of Reggeon-particle amplitudes for various Reggeons. Since fewer triple-Regge terms can contribute in non-diffractive reactions, it would be less important to measure the cross section at very small values of $(1-x)$.

3.3 $p+p \rightarrow p+d+X$

We are interested here in reactions in which the proton has x near one and particle d is one of the particles in the missing mass. The large values of M^2 attainable at ISABELLE should simplify the interpretation of such reactions and thus aid the understanding of diffractive processes. For example, multiplicity data for presently available values of M^2 are consistent with being proportional either to $\log M^2$, as in the triple-Regge model, or to M , as in a fireball model.¹⁵

In this experiment one should trigger on a proton with x near one and then measure the rapidity distribution of particles in the missing mass. For $s = 160,000 \text{ GeV}^2$ and $x = 0.98$ we have $\log M^2 \approx 8$. This is

comparable with the rapidity range at the ISR, so if the triple-Regge model is correct, we should see a clear central plateau. See Sec. 2.4 above. This prediction actually holds for any fixed range of x and t , but it is most interesting for x near one, where the triple-Pomeron term dominates.

It would also be interesting to measure the azimuthal correlation between the detected proton and the particle d . The cross section is independent of the azimuthal angle if the Pomeron is a simple pole,¹⁶ so the observation of such a correlation should give some information about the nature of the Pomeron.

Ideally one would like to identify the particles in the missing mass. However, if this is difficult it would still be interesting to measure the inclusive cross section for all charged particles.

One could also imagine studying more complicated reactions, such as rapidity or azimuthal correlations between two particles in the missing mass.

3.4 $p+p \rightarrow p+p+X$

The interest here is in measuring one proton near $x=1$ and the other near $x=-1$. Then the double-triple-Regge graph, Fig. 4, is relevant, and one can study the four-Reggeon amplitude.

At $s = 160,000 \text{ GeV}^2$, $x_1 = 0.95$, and $x_2 = -0.95$, we have $s' = (p_a + p_b - p_c - p_d)^2 = 400 \text{ GeV}^2$, so we can assume that the Pomeron dominates in all channels. Using factorization we obtain for the cross section

$$\frac{d\sigma}{dx_1 dt_1 dx_2 dt_2} = \frac{1}{256} \beta_p^2(t_1) |\xi_p(t_1)|^2 \beta_p^2(t_2) |\xi_p(t_2)|^2 g_{PPP}(t_1) g_{PPP}(t_2) \\ \times \left(\frac{1}{1-x_1}\right)^{2\alpha_p(t_1)-1} \left(\frac{1}{1-x_2}\right)^{2\alpha_p(t_2)-1} \quad (16)$$

To get an idea of the size of the cross section we integrate this expression over

$$-\infty < t_1 < 0, \quad 0.9 < x_1 < 1 - \frac{s_0}{s}, \quad s_0 = 4 \text{ GeV}^2. \quad (17)$$

Then with $\beta_p^2(0) = 40 \text{ mb}$ we find using Solution I of Field and Fox⁴ that

$$\sigma = 0.11 \text{ mb} \quad (18)$$

The cross section should be independent of s for fixed x_i and t_i .

The graph shown in Fig. 4 has no dependence on the angle between p_{c1} and p_{d1} . In general, however, there can be a dependence on this angle whenever the middle Pomeron is replaced by anything other than a simple Regge pole.¹⁶ Such a dependence can occur even if one has simple poles in the t_1 and t_2 channels. That is, the four-Pomeron amplitude can depend on this angle as well as on its other variables.

4. SUMMARY

The $p+p \rightarrow p+X$ experiment is relatively straightforward and will certainly be done. The $p+p \rightarrow p+d+X$ experiment seems particularly interesting since it sheds light directly on the nature of high-mass diffractive processes. The other experiments are probably of lower priority although still worthwhile.

ACKNOWLEDGMENTS

I wish to thank Satoshi Ozaki, Kenneth Foley and Alan M. Thorndike for useful discussions about possible experiments.

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FIGURE CAPTIONS

- Fig. 1 Derivation of the triple-Regge formula.
- Fig. 2 (a) Graph corresponding to Eq. (12). (b) Graph involving the same central vertex $\gamma_d(q_{d1}^2)$.
- Fig. 3 Solid curves: Triple-Regge fit to $pp \rightarrow pX$ extrapolated to infinite energy. Dashed curves: Triple-Pomeron term. [From Ref. 4, Solution I.]
- Fig. 4 The double-triple-Regge graph.

Fig. 1

FIG. 3

Fig. 4