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NUCLEAR DIVISION

RN-S-0542

To AEC-NASA Space Nuclear Propulsion Office

SEISMIC-RESPONSE ANALYSIS OF
ENGINE TEST STAND NO. 1

NERVA Program

Contract SNP-1

November 1969

NUCLEAR ROCKET OPERATIONS

MASTER

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SEISMIC-RESPONSE ANALYSIS OF
ENGINE TEST STAND NO. 1

D. Tow

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NERVA Program Office
Nuclear Rocket Operations

TABLE OF CONTENTS

	<u>Page</u>
I. Introduction	1
II. Summary	3
III. Technical Discussion	5
A. Description of Engine Test Stand (ETS-1)	5
B. Formulation of Structural Dynamic Model	7
C. Modal Analysis	12
1. Method of Analysis	12
2. Eigenvalue Equation Solution	18
3. Coupling of X, Y, and θ Motions	19
4. Computer Input Data	19
D. Natural Frequencies and Mode Shapes	26
E. Seismic-Response Analysis	47
1. General Nature of Problem	47
2. Modal Damping	47
3. Seismic Ground-Motion Data	49
4. Outline of Seismic-Response Analysis Method	51
5. Response Spectrum Analysis as Applied to Seismic Ground Motion	56
F. Seismic-Response Results	61
IV. Conclusions and Recommendations	70
References	71

TABLES

<u>No.</u>	<u>Title</u>	<u>Page</u>
1	Degrees of Freedom	11
2	Coupling of X, Y, and θ Motions	20
3	Beam Properties	24
4	Weight, Mass, and Mass Moment of Inertia	25
5	ETS-1 Natural Frequencies	27
6	ETS-1 Modal Analysis for Seismic Response; Mode Shape, $f_1 = 1.27$ Hz	29
7	ETS-1 Modal Analysis for Seismic Response; Mode Shape, $f_2 = 1.96$ Hz	30
8	ETS-1 Modal Analysis for Seismic Response; Mode Shape, $f_3 = 2.08$ Hz	31
9	ETS-1 Modal Analysis for Seismic Response; Mode Shape, $f_4 = 2.65$ Hz	32
10	ETS-1 Modal Analysis for Seismic Response; Mode Shape, $f_5 = 3.96$ Hz	33
11	ETS-1 Modal Analysis for Seismic Response; Mode Shape, $f_6 = 5.31$ Hz	34
12	ETS-1 Modal Analysis for Seismic Response; Mode Shape, $f_7 = 6.63$ Hz	35
13	ETS-1 Modal Analysis for Seismic Response; Mode Shape, $f_8 = 6.78$ Hz	36
14	ETS-1 Modal Analysis for Seismic Response; Mode Shape, $f_9 = 7.10$ Hz	37
15	ETS-1 Modal Analysis for Seismic Response; Mode Shape, $f_{10} = 7.40$ Hz	38
16	ETS-1 Modal Analysis for Seismic Response; Mode Shape, $f_{11} = 8.48$ Hz	39
17	ETS-1 Modal Analysis for Seismic Response; Mode Shape, $f_{12} = 9.57$ Hz	40
18	ETS-1 Modal Analysis for Seismic Response; Mode Shape, $f_{13} = 10.1$ Hz	41
19	ETS-1 Modal Analysis for Seismic Response; Mode Shape, $f_{14} = 12.5$ Hz	42
20	ETS-1 Modal Analysis for Seismic Response; Mode Shape, $f_{15} = 14.4$ Hz	43
21	ETS-1 Modal Analysis for Seismic Response; Mode Shape, $f_{16} = 15.8$ Hz	44

TABLES (cont.)

<u>No.</u>	<u>Title</u>	<u>Page</u>
22	ETS-1 Modal Analysis for Seismic Response; Mode Shape, $f_{17} = 16.0$ Hz	45
23	ETS-1 Modal Analysis for Seismic Response; Mode Shape, $f_{18} = 19.6$ Hz	46
24	Maximum Response for Benham Event	62
25	Maximum Response for Design Event	66

FIGURES

<u>No.</u>	<u>Title</u>	<u>Page</u>
1	ETS-1 Structure	6
2	ETS-1 Structural-Dynamic-Model Discrete-Mass-Point Location	9
3	ETS-1 Structural-Dynamic-Model Beam Members	10
4	Typical Model Section	13
5	Forces Acting on Typical Beam	14
6	Free Body of i'th Element	16
7	ETS-1 Natural Frequencies	28
8	Seismic Ground Motion at ETS-1, Benham Event	50
9	Seismic Ground Motion for Design Event	52
10	Maximum Response for Benham Event	63
11	Maximum Column Response for Benham Event	64
12	Maximum Response for Design Event	67
13	Maximum Column Response for Design Event	68
14	Response Spectrum Comparison for Design Event	69

I. INTRODUCTION

Some years ago, the Atomic Energy Commission (AEC) began exploding thermonuclear devices of considerable size and yield in its program for underground testing of nuclear weapons at the Nevada Test Site. Structural-response analyses of the Nuclear Rocket Development Station (NRDS) facilities at Jackass Flats, Nevada were performed by several organizations prior to each test series. The purpose was to ensure the structural integrity of the facility structures, which are within 50 mi of the nuclear detonation center, where they would be subjected to high ground shock and seismic-type disturbances. Structures and facilities were modified where necessary. One set of analyses was performed on Engine Test Stand No. 1 (ETS-1), which is in the northwest corner of NRDS. During the seismic ground-motion disturbance, ETS-1 is stimulated into oscillations, the magnitudes of which determine dynamic loads and stresses.

The seismic response of the ETS-1 structure is primarily governed by the magnitudes and distributions of the mass and stiffness elements and by the degree of damping present. A knowledge of the response of the ETS-1 test-stand structure to such an excitation is important, from the standpoint of design to ensure its safe life and to protect the structure, equipment, and personnel from excessive vibration damage or from injury.

A detailed seismic-response analysis was undertaken for ETS-1, objectives of which can be grouped into two main categories: (1) to provide structural vibration characteristics of ETS-1 and (2) to evaluate the seismic response of the ETS-1 structural components for a given seismic disturbance, characterized by both horizontal and vertical acceleration-time histories.

The seismic-response analysis of ETS-1 was subdivided into three phases. The objective of Phase I of the analysis was to formulate an ideal structural dynamic model that was free of major mathematical complexity but which would structurally and dynamically simulate the actual structure under consideration. Considerable time and thought were given to this phase; the

final structural dynamic model consists of a system of discrete masses, interconnected by equivalent beams and linear or rotational springs, or both, to form a 70-degrees-of-freedom system.

Phase II of the analysis consisted of conducting the modal analysis of the test-stand structure. Modes were determined by using an eigenvalue-eigenvector computer program on the IBM 360/65 computer, and the 70 resonant frequencies and mode shapes were defined.

Phase III is the seismic-response analysis. Results of the modal analysis and a predetermined time-history of seismic ground motion, as well as a carefully chosen critical damping ratio, were used as input to a seismic-response computer program, which gave the dynamic response of the structures in terms of accelerations, velocities, and displacements. Critical stresses can be determined from design drawings, and the structure can be checked for its ability to withstand the seismic loads impressed upon it. Structural modifications to ETS-1 were made as required.

The effects of two seismic events have been analyzed to date. In the first case, the actual measured ground-motion data obtained from the Benham seismic event were used as input to the seismic-response computer program to obtain response solutions, and these results were then compared with the measured structural responses. This verification of calculated responses with measured data ensures the completeness and accuracy of the structural dynamic model. For the second analytical case, one of the largest ground acceleration levels (0.27-g zero-to-peak) predicted for future tests was used as input for the seismic-response calculations. Data used for the ground acceleration-time history were generated by Environmental Research Corporation (ERC) by applying appropriate scaling factors to existing data relative to source energy, local geological and soil composition, propagation path, and transmission loss.

For aluminum and steel structures, 2% is a conservative value for viscous damping and was therefore retained in the seismic-response analysis.

II. SUMMARY

A detailed seismic-response analysis of ETS-1 has been completed, and the two main objectives of the analysis were met: (1) to establish a valid structural dynamic model for the test stand and (2) to predict test-stand dynamics under the influence of future nuclear events and initiate appropriate strengthening measures.

A structural dynamic model of ETS-1 was developed as the basis for this investigation; the model shows good correlation between predicted and actual test-stand frequency and responses. The measured fundamental test-stand frequency shows a mean value of 2.08 Hz for motion perpendicular to the shield wall as compared with an analytical value of 1.96 Hz. In addition, results of the modal analysis show that ETS-1 exhibits many other theoretical resonant frequencies. There are 37 such frequencies below 50 Hz, of which 19 are below 20 Hz.

For the Benham seismic-response analysis, the input ground motion used was obtained from the acceleration-time history recorded on a concrete pad located on the ground near the ETS-1 facility during that event. For this motion, the maximum calculated accelerations in the y-direction (perpendicular to the shield wall) at the top of the test stand (elev 3916.0 ft) were 0.465 and 0.487 g (Stations 22 and 23, respectively) as compared with the measured value of 0.45 g determined from a playback of the original unfiltered data. Responses calculated at other story levels of the test stand in the same direction show fair correlations. The maximum seismic magnification factor (ratio of maximum response acceleration to maximum input acceleration) was slightly greater than 4.

For the maximum-g design event considered, the time history of ground accelerations (generated by ERC) was used as input to the same structural dynamic model; and maximum absolute accelerations at the top of the test stand

were calculated as 0.78 and 0.83 g, respectively. In addition, 28 time-history responses, in the form of accelerations, velocities, and displacements on the test stand and engine, were calculated. The maximum seismic magnification factor for the design event was about 3.

As a check on the consistency of Aerojet's interpretation of the ERC-furnished time history of ground acceleration, a computer program was developed to give the responses of simple single-degree-of-freedom systems to such an input. The response spectra generated by Aerojet agreed excellently with currently furnished ERC shock spectra.

Aerojet computer capabilities were further developed to handle responses of multiple-degree-of-freedom systems to arbitrary-input ground accelerations in time-history form. The above two computer programs, together with the existing eigenvalue-eigenvector computer program, can be used to solve any future seismic-response-type problems of a completely general nature.

Other maximum-g design events are currently being evaluated.

III. TECHNICAL DISCUSSION

A. DESCRIPTION OF ENGINE TEST STAND (ETS-1)

The principal structural components of the XE engine are the nozzle, reactor, pressure vessel (including pump-discharge, diluent, turbine-inlet, and turbine-exhaust lines), lower- and upper-thrust-structure modules, an external shield, the turbopump assembly, and the test-stand adapter.

The ETS-1 test stand supports the test engine, the 77,000-gal liquid-hydrogen run tank (considered empty during the analysis), and the miscellaneous piping systems required for engine operation. The test stand is a three-dimensional structure made of 6061-T6 aluminum and comprises four main vertical columns (hollow cylinders 40 in. in diameter and 1 in. thick) that are connected by a network of cross braces and trusses. The engine load is transmitted to the main support structure through the four thrust columns (located 90 deg apart) and a network of box girders.

At the test stand, the test hardware (XE engine) is mated to the facility thrust ring to provide an integral test system. The fluid-transfer systems connect to the test stand to provide engine coolant, postoperational cooldown fluids, pneumatic-actuation supply fluids, inlet gas purge, water for cooling facility components, and shielding.

The test-stand structure is shown in Figure 1. The 3-ft-thick, 95-ft-long reinforced concrete shield wall, running parallel to the X-Z plane adds to the complexity of the structure under investigation. There are three movable weather doors on the three unprotected sides of the test stand. The top story consists primarily of the roof structure on the elevator shaft, flare-stack supports, and miscellaneous structural items. The total dry weight of the test stand was calculated to be 600,000 lb, and the XE engine weighs an additional 39,500 lb.

STORY ELEVATION

5 3916.00

4 3901.00

3 3886.00

2 3866.0

1 3848.67

3810.00

FIGURE 1 - ETS-1 STRUCTURE

B. FORMULATION OF STRUCTURAL DYNAMIC MODEL

A complex structure such as ETS-1 cannot be dynamically represented by a simple model because of its structural arrangement and configuration. Considerable time and effort were spent, both on the analytical studies and the structural configuration, to derive a mathematical model that would dynamically and structurally simulate the test stand. Another basic task in formulating such a model is to understand the basic characteristics of the structure and the corresponding component load-transfer paths. By knowing these different characteristics, a mathematical model can be derived that is sufficiently similar to the actual structure in dynamic characteristics.

Because of the structural symmetry with respect to the main center line of the test stand, only half of the ETS-1 three-dimensional structure is considered in the basic analysis. The advantages of using symmetry for a structure of this kind are as follows: (1) the complex structure is reduced to a simpler one by using only a portion of the structure, but the solution provided is as good as when the whole structure is analyzed; (2) for a given matrix size, greater detail can be incorporated into the model; and (3) both the preparation of computer input data and the amount of computer time are reduced to a minimum.

Past engineering experience has shown that, with judicious selection, 70 degrees of freedom is sufficient to define the dynamic characteristics of a structure such as the ETS-1.

The basic model of the test stand includes the main vertical columns and their associated connecting trusses, the thrust-measuring columns and thrust supporting system, and the network of box girders and cross braces. The various structural members atop the main column above the fifth story have little influence on the overall motion of the structure and can therefore be treated as a mass appendage to the top mass points (mass points 22 and 23

in Figure 2). Additional masses from the liquid-hydrogen run tank, cross braces, platforms, floor grating, piping, and miscellaneous structural connectors were included and distributed into various node points.

The ETS-1 test-stand structure was modeled as shown in Figure 2, where the numbers indicate the locations of the equivalent concentrated masses, and in Figure 3, where the numbers refer to the structural members. The model consists of 26 discrete mass points, each of which is free to translate and rotate.

Each point in the model has, at most, three degrees of freedom: translation in the X and Y directions and rotation in the X-Y (θ) plane. It is not necessary that each mass point selected be allowed all three degrees of freedom, because those motions that are minimal or insignificant can be neglected. The complete model contains 70 degrees of freedom. The final selections are listed in Table 1.

The lack of full moment connections at the locations where the vertical columns join the concrete pad at the base of the test stand was considered in the analysis. Calculations of structural fixity at these joints were made, and equivalent springs were found. An on-site inspection of the 3-ft reinforced concrete shield wall was made in order to accurately determine the degree of coupling between the wall and the aluminum stand members, with allowance also made for the wall bracing members; an idealized spring was calculated by using the influence-coefficient technique, standard as a structural-analysis procedure.

All analytical calculations were based on the test stand with the modifications made prior to the Benham seismic event.

FIGURE 3 - ETS-1 STRUCTURAL-DYNAMIC-MODEL BEAM MEMBERS

TABLE 1
DEGREES OF FREEDOM*

<u>Mass</u>	<u>X</u>	<u>Y</u>	<u>θ</u>
1	x	x	x
2	x	x	-
3	x	x	x
4	x	x	x
5	x	x	x
6	x	x	-
7	x	x	x
8	x	x	x
9	x	x	x
10	x	x	x
11	x	x	-
13	x	x	x
14	x	x	x
15	x	x	x
16	x	x	x
17	x	x	-
18	x	x	x
19	x	x	x
20	x	x	-
21	x	x	x
22	x	x	x
23	x	x	x
24	x	x	-
25	x	x	-
26	x	x	x
27	<u>-</u>	<u>x</u>	<u>x</u>
Subtotals:	25	26	19
Total: 70			

*An x indicates applicability of a given degree of freedom.

C. MODAL ANALYSIS

The matrix-type input to the computer program for modal analysis is too voluminous to be included in its entirety in this report; however, major elements of the analysis are outlined below.

1. Method of Analysis

The ETS-1 is dynamically simulated by dividing it into a series of discrete masses connected by beams or idealized springs, or both. Calculation of the free-vibration modes requires sectional representation of the system mass and stiffness characteristics. The mass matrix representation allows for uniform mass distributions, for concentrated masses, and for the mass moment of inertia of each section. Mass points are assumed at the ends of all idealized beams, with half of the mass properties of each section distributed at each end of the section. The stiffness matrix representation of the system is in the form of a set of influence coefficients at each station. The stiffness influence coefficients, as well as the equations of motion, are formulated as follows:

A typical section of the model is shown in Figure 4, where $i - 1$, i , and $i + 1$ are respectively mass-point locations.

For a typical beam between mass points (Figure 5), the following matrix equation can be written to relate the deformations to the forces and moments:

$$\begin{bmatrix} \Delta_i \\ \theta_i \end{bmatrix} = \begin{bmatrix} a_{fi} & a_{mi} \\ b_{fi} & b_{mi} \end{bmatrix} \begin{bmatrix} F_{i+1} \\ M_{i+1} \end{bmatrix}$$

This matrix equation can be inverted to give:

FIGURE 4 - TYPICAL MODEL SECTION

FIGURE 5 - FORCES ACTING ON TYPICAL BEAM

$$\begin{bmatrix} F_{i+1} \\ M_{i+1} \end{bmatrix} = \begin{bmatrix} K_{Fi} & K_{FTi} \\ K_{TFi} & K_{Ti} \end{bmatrix} \begin{bmatrix} \Delta_i \\ \theta_i \end{bmatrix},$$

where

- a_{fi} = relative deflection due to unit force,
- a_{mi} = relative deflection due to unit moment,
- b_{fi} = relative rotation due to unit force,
- b_{mi} = relative rotation due to unit moment,
- K_{Fi} = force required to produce a unit relative deflection,
- K_{FTi} = force required to produce a unit relative rotation,
- K_{TFi} = moment required to produce a unit relative deflection,
- K_{Ti} = moment required to produce a unit relative rotation.

For a uniform beam, these influence coefficients become:

$$\begin{aligned} a_{fi} &= \frac{L_i^3}{3EI_i}, & K_{Fi} &= \frac{12EI_i}{L_i^3}, \\ a_{mi} &= \frac{L_i^2}{2EI_i}, & K_{FTi} &= \frac{-6EI_i}{L_i^2}, \\ b_{fi} &= \frac{L_i^2}{2EI_i}, & K_{TFi} &= \frac{-6EI_i}{L_i^2}, \\ b_{mi} &= \frac{L_i}{EI_i}, & K_{Ti} &= \frac{4EI_i}{L_i}. \end{aligned}$$

By applying the above-derived force and moment equations to the typical section of the model, the forces and moments acting on the i th element are found to be those shown in Figure 6. It follows that the equations of motion are:

FIGURE 6 - FREE BODY OF ITH ELEMENT

$$\begin{aligned}
& M_i \ddot{y}_i + y_{i-1}(-K_{Fi-1}) + y_i(K_{Fi-1} + K_{Fi}) + y_{i+1}(-K_{Fi}) \\
& + \theta_{i-1}(-K_{FTi-1} - K_{Fi-1}L_{i-1}) \\
& + \theta_i(K_{FTi-1} + K_{FTi} + K_{Fi}L_i) - \theta_{i+1}(K_{FTi}) = 0. \quad (1)
\end{aligned}$$

$$\begin{aligned}
& I_i \ddot{\theta}_i + y_{i-1}(-K_{TFi-1}) + y_i(K_{TFi-1} + K_{TFi} + K_{Fi}L_i) + y_{i+1}(-K_{TFi} - K_{Fi}L_i) \\
& + \theta_{i-1}(-K_{Ti-1} - K_{TFi-1}L_{i-1}) + \theta_i(K_{Ti-1} + K_{Ti} + K_{FTi}L_i + K_{TFi}L_i + K_iL_i^2) \\
& + \theta_{i+1}(-K_{Ti} - K_{FTi}L_i) = 0. \quad (2)
\end{aligned}$$

For a uniform beam, the following relationships hold:

$$\begin{aligned}
K_{FTi} &= K_{TFi}, \\
K_{FTi} + K_{Fi}L_i &= -K_{FTi}, \\
K_{Ti} + K_{FTi}L_i &= \frac{-K_{Ti}}{2}, \\
2K_{FTi}L_i + K_{Fi}L_i^2 &= 0.
\end{aligned}$$

Therefore, Equations 1 and 2 may be simplified to:

$$\begin{aligned}
& M_i \ddot{y}_i + y_{i-1}(-K_{Fi-1}) + y_i(K_{Fi-1} + K_{Fi}) + y_{i+1}(-K_{Fi}) \\
& + \theta_{i-1}(K_{FTi-1}) + \theta_i(K_{FTi-1} - K_{FTi}) + \theta_{i+1}(-K_{FTi}) = 0; \quad (3)
\end{aligned}$$

$$\begin{aligned}
I_i \ddot{\theta}_i + y_{i-1} (-K_{FTi-1}) + y_i (K_{FTi-1} - K_{FTi}) + y_{i+1} (K_{FTi}) \\
+ \theta_{i-1} \left(\frac{K_{Ti-1}}{2} \right) + \theta_i (K_{Ti-1} + K_{Ti}) + \theta_{i+1} \left(\frac{K_{Ti}}{2} \right) = 0. \quad (4)
\end{aligned}$$

Equations 3 and 4 are the general equations of motion for the i th element of a continuous elastic beam, with each element assumed to be uniform in itself.

2. Eigenvalue Equation Solution

The undamped equation of motion (the type shown by Equations 3 and 4) for each particular degree of freedom has been formulated in the previous section. These equations are then arrayed into matrix form:

$$[M] \{\ddot{y}\} + [K] \{y\} = 0, \quad (5)$$

where $[M]$ stands for a general mass and inertial matrix and $[K]$ for a general stiffness matrix.

The natural modes of vibration may be obtained by assuming the displacements $\{y\}$ to be sinusoidal functions of time. Therefore, by assuming harmonic motion, one finds

$$\{\ddot{y}\} = -\omega^2 \{y\},$$

with $\omega = 2\pi f$ where f is the natural frequency in Hz; and Equation 5 becomes

$$-\omega^2 [M] \{y\} + [K] \{y\} = 0. \quad (6)$$

By mathematical methods, the $[M]$ and $[K]$ matrices are used to form a new matrix $[A]$ such that

$$\omega^2 \{y\} = [A] \{y\}. \quad (7)$$

Equation 7 is an eigenvalue equation that is routinely solved on the IBM 360/65 computer for the eigenvalues (ω_n^2) and the eigenvector (y or mode shape) by using the Givens Method (Reference 1). In essence, the method is as follows:

The matrix A is reduced to a triple-diagonal (Jacobi) matrix J by using a series of plane rotations. A recursion formula is then used to locate the eigenvalues. Next the eigenvector corresponding to a given eigenvalue is obtained by rotating the matrix $J - \omega_n^2 I$ (where I is the unit identity matrix) to triangular form. The eigenvector of the original matrix A is obtained by using the same plane rotations as used to obtain matrix J. The eigenvectors are then normalized.

The matrices [M] and [K] must be real, symmetrical, and of the same size; these conditions are met by the physical nature of the dynamic problems. Establishing such large sets of equations requires considerable data preparation and consumes a lot of time.

3. Coupling of X, Y, and θ Motions

Table 2 shows the coordinates that enter into the equations describing the motion at the various mass points. Because of the structural complexity of the test stand, no motion can be a pure X, Y, or θ motion.

4. Computer Input Data

The computer input data can be derived from Table 3, which gives beam properties, and Table 4, which gives weight, mass, and mass moment of inertia.

TABLE 2
COUPLING OF X, Y, AND θ MOTIONS

Degree of Freedom	Coupled Degrees							
X_1	.	θ_1	X_2	X_3	θ_3	X_5	X_{24}	
Y_1	.	θ_1	Y_2	Y_3	Y_5	θ_5	Y_{24}	
θ_1	X_1	Y_1	.	X_3	θ_3	Y_5	θ_5	Y_{24}
X_2	X_1	.	Y_2	X_4	X_5	Y_5	X_7	Y_7
Y_2	Y_1	X_2	.	Y_4	X_5	Y_5	X_7	Y_7
X_3	X_1	θ_1	.	θ_3	X_4	θ_4	X_{26}	
Y_3	Y_1	.	Y_4	Y_{26}				
θ_3	X_1	θ_1	X_3	.	X_4	θ_4	θ_{26}	
X_4	X_2	X_3	θ_3	.	θ_4	X_7	X_{25}	
Y_4	Y_2	Y_3	.	θ_4	Y_7	θ_7	Y_{25}	
θ_4	X_3	θ_3	X_4	Y_4	.	Y_7	θ_7	Y_{25}
X_5	X_1	X_2	Y_2	.	Y_5	X_6	X_8	
Y_5	Y_1	θ_1	X_2	Y_2	X_5	.	θ_5	Y_6 Y_8 θ_8
θ_5	Y_1	θ_1	Y_5	.	Y_8	θ_8		
X_6	X_5	.	Y_6	X_7	X_{16}	Y_{16}	X_{18}	Y_{18}
Y_6	Y_5	X_6	.	Y_7	X_{16}	Y_{16}	X_{18}	Y_{18}
X_7	X_2	Y_2	X_4	X_6	.	Y_7	X_{15}	
Y_7	X_2	Y_2	Y_4	θ_4	Y_6	X_7	.	θ_7 Y_{15} θ_{15}
θ_7	Y_4	θ_4	Y_7	.	Y_{15}	θ_{15}		
X_8	X_5	.	θ_8	X_9	θ_9	X_{16}		
Y_8	Y_5	θ_5	.	θ_8	Y_9	Y_{16}	θ_{16}	

TABLE 2 (cont.)

Degree of Freedom	Coupled Degrees									
θ_8	Y_5	θ_5	X_8	Y_8	.	X_9	θ_9	Y_{16}	θ_{16}	
X_9	X_8	θ_8	.	θ_9	X_{10}					
Y_9	Y_8	.	θ_9	Y_{10}	θ_{10}					
θ_9	X_8	θ_8	X_9	Y_9	.	Y_{10}	θ_{10}			
X_{10}	X_9	.	Y_{10}	X_{11}	Y_{11}	X_{19}				
Y_{10}	Y_9	θ_9	X_{10}	.	θ_{10}	X_{11}	Y_{11}	Y_{13}	Y_{19}	
θ_{10}	Y_9	θ_9	Y_{10}	.	θ_{19}					
X_{11}	X_{10}	Y_{10}	.	Y_{11}	X_{13}	Y_{13}				
Y_{11}	X_{10}	Y_{10}	X_{11}	.	X_{13}	Y_{13}				
X_{13}	X_{11}	Y_{11}	.	Y_{13}	X_{14}	X_{21}				
Y_{13}	Y_{10}	X_{11}	Y_{11}	X_{13}	.	θ_{13}	Y_{14}	θ_{14}	Y_{21}	
θ_{13}	Y_{13}	.	Y_{14}	θ_{14}	θ_{21}					
X_{14}	X_{13}	.	θ_{14}	X_{15}	θ_{15}					
Y_{14}	Y_{13}	θ_{13}	.	θ_{14}	Y_{15}					
θ_{14}	Y_{13}	θ_{13}	X_{14}	Y_{14}	.	X_{15}	θ_{15}			
X_{15}	X_7	X_{14}	θ_{14}	.	θ_{15}	X_{18}				
Y_{15}	Y_7	θ_7	Y_{14}	.	θ_{15}	Y_{18}	θ_{18}			
θ_{15}	Y_7	θ_7	X_{14}	θ_{14}	X_{15}	Y_{15}	.	Y_{18}	θ_{18}	
X_{16}	X_6	Y_6	X_8	.	Y_{16}	X_{17}	X_{19}			
Y_{16}	X_6	Y_6	Y_8	θ_8	X_{16}	.	θ_{16}	Y_{17}	Y_{19}	θ_{19}
θ_{16}	Y_8	θ_8	Y_{16}	.	Y_{19}	θ_{19}				
X_{17}	X_{16}	.	Y_{17}	X_{18}	X_{19}	Y_{19}	X_{21}	Y_{21}		

TABLE 2 (cont.)

Degree of Freedom	Coupled Degrees							
Y_{17}	Y_{16}	X_{17}	.	Y_{18}	X_{19}	Y_{19}	X_{21}	Y_{21}
X_{18}	X_6	Y_6	X_{15}	X_{17}	.	Y_{18}	X_{21}	
Y_{18}	X_6	Y_6	Y_{15}	θ_{15}	Y_{17}	X_{18}	.	θ_{18} Y_{21} θ_{21}
θ_{18}	Y_{15}	θ_{15}	Y_{18}	.	Y_{21}	θ_{21}		
X_{19}	X_{10}	X_{16}	X_{17}	Y_{17}	.	Y_{19}	X_{20}	X_{22}
Y_{19}	Y_{10}	Y_{16}	θ_{16}	X_{17}	Y_{17}	X_{19}	.	θ_{19} Y_{20} Y_{22} θ_{22}
θ_{19}	θ_{10}	Y_{16}	θ_{16}	Y_{19}	.	Y_{22}	θ_{22}	
X_{20}	X_{19}	.	Y_{20}	X_{21}	X_{22}	Y_{22}	X_{23}	Y_{23}
Y_{20}	Y_{19}	X_{20}	.	Y_{21}	X_{22}	Y_{22}	X_{23}	Y_{23}
X_{21}	X_{13}	X_{17}	Y_{17}	X_{18}	X_{20}	.	Y_{21}	X_{23}
Y_{21}	Y_{13}	X_{17}	Y_{17}	Y_{18}	θ_{18}	Y_{20}	X_{21}	.
θ_{21}	θ_{13}	Y_{18}	θ_{18}	Y_{21}	.	Y_{23}	θ_{23}	θ_{21} Y_{23} θ_{23}
X_{22}	X_{19}	X_{20}	Y_{20}	.	Y_{22}	θ_{22}	X_{23}	θ_{23}
Y_{22}	Y_{19}	θ_{19}	X_{20}	Y_{20}	X_{22}	.	θ_{22}	Y_{23}
θ_{22}	Y_{19}	θ_{19}	X_{22}	Y_{22}	.	X_{23}	θ_{23}	
X_{23}	X_{20}	Y_{20}	X_{21}	X_{22}	θ_{22}	.	Y_{23}	θ_{23}
Y_{23}	X_{20}	Y_{20}	Y_{21}	θ_{21}	Y_{22}	X_{23}	.	θ_{23}
θ_{23}	Y_{21}	θ_{21}	X_{22}	θ_{22}	X_{23}	Y_{23}	.	
X_{24}	X_1	.						
Y_{24}	Y_1	θ_1	.					
X_{25}	X_4	.						

TABLE 2 (cont.)

<u>Degree of Freedom</u>	<u>Coupled Degrees</u>					
Y_{25}	Y_4	θ_4	.			
X_{26}	X_3	.				
Y_{26}	Y_3	.	θ_{26}	Y_{27}	θ_{27}	
θ_{26}	θ_3	Y_{26}	.	Y_{27}	θ_{27}	
Y_{27}	Y_{26}	θ_{26}	.	θ_{27}		
θ_{27}	Y_{26}	θ_{26}	Y_{27}	.		

TABLE 3
BEAM PROPERTIES

<u>Beam</u>	<u>L, in.</u>	<u>A, in.²</u>	<u>I, in.⁴</u>
1 and 6	464	122.5	23,310
2 and 5	180	9.56	80.08
3 and 4	120	81.0	3,600
7 and 10	208	122.5	23,310
8 and 9	275	17.64	324.16
11 and 12	180	9.56	80.08
13 and 16	120	122.5	23,310
14 and 15	300	11.02	87.92
24 and 25	120	122.5	23,310
26 and 27	180	9.56	80.08
28 and 31	180	9.12	108.5
29 and 30	255	6.68	88.12
32 and 33	180	5.02	56.73
34 and 37	180	9.12	108.5
35 and 36	255	6.08	88.12
38	360	9.35	218.13

*For beam numbering, refer to Figure 3.

TABLE 4
WEIGHT, MASS AND MASS MOMENT OF INERTIA

<u>Point*</u>	<u>Weight, lb</u>	<u>Mass, lb-sec²/in.</u>	<u>I , lb-sec²-in.</u>
1	24,760	64.079	346,597
2	29,750	76.993	237,822
3	5,803	15.018	36,788
4	60,702	157.096	2,395,987
5	6,107	15.805	65,767
6	2,470	6.392	56,488
7	20,145	52.135	720,680
8	2,310	5.978	22,155
9	19,759	51.137	1,108,060
10	5,048	13.065	1,065,290
11	1,510	3.908	20,797
13	5,048	13.065	1,065,290
14	19,759	51.137	1,108,060
15	2,310	5.978	22,155
16	3,290	8.514	23,176
17	2,310	5.978	56,488
18	17,281	44.723	673,820
19	2,480	6.419	16,872
20	2,372	6.139	56,488
21	16,312	42.216	653,687
22	7,372	19.077	218,454
23	27,774	71.878	220,475
24	4,715	12.202	245,850
25	4,715	12.202	245,850
26	10,008	25.900	72,538
27	35,394	91.600	277,000

*For mass point locations, refer to Figure 2.

D. NATURAL FREQUENCIES AND MODE SHAPES

ETS-1 exhibits many theoretical resonant frequencies, some of which are important test-stand modes, whereas others are associated only with localized vibrations, as can be seen by studying the mode shapes. The number of mode shapes corresponds to the number of natural frequencies and to the total number of degrees of freedom used in the analysis. Only the lower frequencies can be expected to be accurate to a high degree of certainty. There are 37 indicated resonant frequencies below 50 Hz; these frequencies are listed in Table 5 and provide a general representative view of the range of frequencies under consideration. The lowest natural frequency in Table 5 is 1.27 Hz, and the associated mode shape (Table 6) identifies this frequency as a localized engine mode. For higher modes than those given in Table 5, reference should be made to the modal-analysis computer output.

For motion perpendicular to the shield wall, the mean, measured, test-stand, fundamental, natural frequency, as interpolated from the Benham-seismic-event response data, ranged from 2.00 Hz ($T = 0.5$ sec) to 2.17 Hz ($T = 0.46$ sec). This range compares favorably with the calculated first test-stand mode of 1.96 Hz.

The first 18 natural frequencies of ETS-1 are plotted in Figure 7. The corresponding sets of mode shapes, which are used later in the response analysis, are summarized in detail in Tables 6 through 23.

TABLE 5
ETS-1 NATURAL FREQUENCIES

<u>Mode No.</u>	<u>f_n, Hz</u>	
1	1.27	(Engine Mode)
2	1.96	(Fundamental Test-Stand Mode)
3	2.08	
4	2.65	
5	3.96	
6	5.31	
7	6.63	
8	6.78	
9	7.10	
10	7.40	
11	8.48	
12	9.57	
13	10.1	
14	12.5	
15	14.4	
16	15.8	
17	16.0	
18	19.6	
19	19.9	
20	20.8	
20	22.8	
22	24.2	
23	24.5	
24	26.7	
25	28.9	
26	29.6	
27	31.3	
28	32.2	
29	34.7	
30	35.5	
31	35.7	
32	37.6	
33	39.9	
34	43.1	
35	43.7	
36	46.1	
37	49.0

FIGURE 7 - ETS-1 NATURAL FREQUENCIES

TABLE 6

ETS-1 MODAL ANALYSIS FOR SEISMIC RESPONSE;
MODE SHAPE, $f_1 = 1.27$ Hz

(Primarily an Engine Mode)

<u>Mass Point</u>	<u>X</u>	<u>Y</u>	<u>θ</u>
1	105	3,976	10
2	-3	4,271	-
3	-16	3,971	-50
4	-105	3,880	11
5	54	4,832	2
6	-11	4,920	-
7	-54	4,837	2
8	43	5,028	1
9	93	5,119	1
10	91	5,470	1
11	0	5,326	-
13	-91	5,471	1
14	-92	5,142	1
15	-44	5,055	1
16	37	5,194	1
17	-8	5,265	-
18	-37	5,222	1
19	24	5,457	1
20	-18	5,528	-
21	-24	5,473	1
22	-9	5,645	3
23	9	5,681	2
24	3	17	-
25	-3	14	-
26	-16	4,096	-572
27	-	97,080	-587

TABLE 7

ETS-1 MODAL ANALYSIS FOR SEISMIC RESPONSE;
MODE SHAPE, $f_2 = 1.96$ Hz

(Fundamental Test-Stand Mode)

<u>Mass Point</u>	<u>X</u>	<u>Y</u>	<u>θ</u>
1	1,765	-17,576	-44
2	66	-20,103	-
3	113	-17,368	-12
4	-1,804	-17,164	-49
5	2,432	-27,697	-55
6	147	-29,658	-
7	-2,484	-27,907	-58
8	2,674	-34,476	-50
9	729	-35,992	-58
10	774	-50,413	-58
11	-39	-51,766	-
13	-845	-50,423	-57
14	-799	-36,225	-57
15	-2,732	-34,822	-51
16	2,800	-40,549	-48
17	111	-42,137	-
18	-2,861	-41,022	-50
19	2,751	-50,109	-57
20	385	-51,830	-
21	-2,860	-50,441	-64
22	3,604	-58,178	-64
23	-3,773	-59,118	-41
24	51	-75	-
25	-53	-58	-
26	114	-17,353	-168
27	-	9,995	-173

TABLE 8

ETS-1 MODAL ANALYSIS FOR SEISMIC RESPONSE;
MODE SHAPE, $f_3 = 2.08$ Hz

<u>Mass Point</u>	<u>X</u>	<u>Y</u>	<u>θ</u>
1	7	-52	0
2	-86	-59	-
3	-13	-51	0
4	-7	-51	0
5	10	-183	-1
6	-51	-98	-
7	-10	13	1
8	12	-286	0
9	38	-639	2
10	38	-254	2
11	37	-255	-
13	53	-281	-3
14	54	460	-3
15	-11	55	-1
16	13	-235	1
17	377	-182	-
18	-11	-95	-2
19	30	96	26
20	-556	-247	-
21	20	-606	-32
22	21	-803	-1,496
23	30	357	1,521
24	0	0	-
25	0	-1	-
26	-13	-51	0
27	-	19	0

TABLE 9

ETS-1 MODAL ANALYSIS FOR SEISMIC RESPONSE;
MODE SHAPE, $f_4 = 2.65$ Hz

<u>Mass Point</u>	<u>X</u>	<u>Y</u>	<u>θ</u>
1	58	1,400	2
2	-10	1,597	-
3	-21	1,386	-1
4	-76	1,370	3
5	89	1,832	2
6	-31	2,066	-
7	-113	1,825	2
8	118	2,221	4
9	365	1,906	6
10	372	3,512	6
11	-23	2,881	-
13	-419	3,514	7
14	-411	1,765	7
15	-146	2,204	5
16	129	2,795	5
17	-52	3,059	-
18	-159	2,838	5
19	680	3,290	30
20	-46	2,231	-
21	-753	3,320	38
22	1,840	-1,956	-1,526
23	-1,971	-1,995	-1,485
24	2	10	-
25	-2	8	-
26	-21	1,386	10
27	-	-310	11

TABLE 10

ETS-1 MODAL ANALYSIS FOR SEISMIC RESPONSE;
MODE SHAPE, $f_5 = 3.96$ Hz

<u>Mass Point</u>	<u>X</u>	<u>Y</u>	<u>θ</u>
1	1,907	28,601	21
2	-1,637	30,347	-
3	-974	28,404	-25
4	-2,242	28,100	42
5	2,806	26,565	-36
6	-606	25,392	-
7	-3,245	29,598	-21
8	3,376	19,977	-75
9	-1,304	25,750	-188
10	-1,175	-21,390	-189
11	123	-19,373	-
13	1,450	-21,441	-225
14	1,595	34,614	-224
15	-3,861	24,052	-82
16	3,618	8,580	-99
17	64	4,316	-
18	-4,110	9,530	-140
19	4,557	-19,450	-190
20	990	-24,160	-
21	-4,991	-20,012	-447
22	7,087	-39,353	23
23	-8,064	-42,286	14
24	56	270	-
25	-65	203	-
26	-1,001	28,502	174
27	-	-82	183

TABLE 11

ETS-1 MODAL ANALYSIS FOR SEISMIC RESPONSE;
MODE SHAPE, $f_6 = 5.31$ Hz

<u>Mass Point</u>	<u>X</u>	<u>Y</u>	<u>θ</u>
1	1,491	19,438	8
2	4,177	18,644	-
3	695	19,433	-10
4	-2,179	19,277	-20
5	2,129	16,158	-25
6	1,561	10,132	-
7	-3,078	-6,526	-112
8	2,350	12,229	50
9	-228	20,961	-92
10	-174	-2,028	-92
11	-2,744	-6,087	-
13	-5,293	-1,834	106
14	-5,252	-27,733	102
15	-3,476	-4,158	-26
16	2,434	3,803	-77
17	-2,308	774	-
18	-3,736	-2,042	40
19	2,283	-4,216	-95
20	-1,699	-8,685	-
21	-6,851	-4,276	1,005
22	4,858	-24,423	31
23	-11,007	-27,266	95
24	-43	203	-
25	-63	281	-
26	731	19,584	111
27	-	1,299	118

TABLE 12

ETS-1 MODAL ANALYSIS FOR SEISMIC RESPONSE;
MODE SHAPE, $f_7 = 6.63$ Hz

<u>Mass Point</u>	<u>X</u>	<u>Y</u>	<u>θ</u>
1	2,151	-1,581	96
2	21,687	-2,642	-
3	29,390	-1,397	55
4	2,513	-1,204	-323
5	2,257	13,493	89
6	5,697	-2,692	-
7	3,004	-21,463	-3
8	2,116	20,487	-33
9	-3,625	60,762	-263
10	-3,576	-4,841	-263
11	-2,093	-2,345	-
13	-551	-4,670	126
14	-600	-36,328	127
15	3,181	-19,176	73
16	1,880	7,536	-139
17	354	2,110	-
18	3,232	-495	220
19	-1,158	-2,198	-253
20	-302	587	-
21	1,843	-2,178	-283
22	-2,879	9,061	-20
23	4,488	11,252	-21
24	63	-297	-
25	73	925	-
26	31,826	-1,402	-9
27	-	246	-11

TABLE 13

ETS-1 MODAL ANALYSIS FOR SEISMIC RESPONSE;
MODE SHAPE, $f_8 = 6.78$ Hz

<u>Mass Point</u>	<u>X</u>	<u>Y</u>	<u>θ</u>
1	4,045	27,476	-12
2	1,413	21,498	-
3	19,683	27,839	49
4	-1,636	27,817	-288
5	5,111	449	-214
6	-550	-1,657	-
7	-2,539	-120	-80
8	5,174	-20,200	-40
9	10,098	-65,594	294
10	10,028	7,828	295
11	3,338	-3,191	-
13	-3,535	7,677	91
14	-3,582	-15,175	92
15	-2,517	-7,658	-31
16	5,321	-14,914	83
17	2,367	-8,388	-
18	-2,578	-9,817	-12
19	6,223	4,132	289
20	2,721	4,083	-
21	-1,488	4,322	-182
22	6,842	-633	16
23	-2,128	-1,132	-1
24	118	354	-
25	-48	1,161	-
26	21,397	28,239	153
27	-	3,088	165

TABLE 14

ETS-1 MODAL ANALYSIS FOR SEISMIC RESPONSE;
MODE SHAPE, $f_9 = 7.10$ Hz

<u>Mass Point</u>	<u>X</u>	<u>Y</u>	<u>θ</u>
1	2,778	-9,057	73
2	2,772	-7,793	-
3	30,283	-8,996	48
4	5,708	-8,807	-217
5	3,044	-4,401	-9
6	4,253	-2,161	-
7	7,212	2,386	241
8	3,307	-5,685	-5
9	3,062	-9,365	-19
10	3,104	-14,231	-20
11	9,391	-4,858	-
13	14,900	-14,351	-297
14	14,850	59,936	-299
15	7,869	20,084	-25
16	3,429	-5,747	2
17	11,137	-6,272	-
18	8,241	-1,218	-292
19	4,130	-10,778	-16
20	13,432	-8,686	-
21	15,865	-12,078	433
22	2,898	7,474	-15
23	24,708	9,454	-2
24	81	-318	-
25	166	530	-
26	33,197	-9,123	-51
27	-	-642	-56

TABLE 15

ETS-1 MODAL ANALYSIS FOR SEISMIC RESPONSE;
MODE SHAPE, $f_{10} = 7.40$ Hz

<u>Mass Point</u>	<u>X</u>	<u>Y</u>	<u>θ</u>
1	1,274	15,363	-55
2	1,494	11,289	-
3	-9,282	15,491	-4
4	1,726	15,367	111
5	2,074	-1,074	-105
6	9,255	-8,805	-
7	2,546	1,595	-210
8	2,173	-16,076	-143
9	-3,671	-7,836	-137
10	-3,708	-41,889	-137
11	2,970	-31,775	-
13	9,382	-41,934	-38
14	9,171	-32,362	-39
15	3,343	-22,995	-194
16	2,405	-35,993	-168
17	8,578	-45,306	-
18	3,496	-52,113	-307
19	-5,775	-34,203	-110
20	5,586	-18,494	-
21	18,484	-37,841	4
22	-17,060	42,135	-53
23	41,346	55,819	-56
24	37	340	-
25	50	-144	-
26	-10,259	15,752	85
27	-	1,626	93

TABLE 16

ETS-1 MODAL ANALYSIS FOR SEISMIC RESPONSE;
MODE SHAPE, $f_{11} = 8.48$ Hz

<u>Mass Point</u>	<u>X</u>	<u>Y</u>	<u>θ</u>
1	-8,510	-194	-123
2	-20,690	-1,667	-
3	-39,139	-81	66
4	-15,189	24	-319
5	-10,697	-8,489	16
6	-17,140	-7,846	-
7	-18,820	-9,016	123
8	-11,794	-6,952	-14
9	-14,362	4,728	-59
10	-14,524	-9,842	-58
11	-18,963	-15,254	-
13	-20,930	-9,863	-88
14	-20,676	12,008	-87
15	-20,393	701	16
16	-12,252	-9,304	-21
17	-25,449	-8,134	-
18	-20,985	-9,670	-173
19	-19,183	-6,769	-51
20	-42,287	3,520	-
21	-28,860	-7,154	54
22	-28,826	14,919	-7
23	-40,195	24,930	-8
24	-248	357	-
25	-442	935	-
26	-44,743	-67	-1
27	-	264	-3

TABLE 17

ETS-1 MODAL ANALYSIS FOR SEISMIC RESPONSE;
MODE SHAPE, $f_{12} = 9.57$ Hz

<u>Mass Point</u>	<u>X</u>	<u>Y</u>	<u>θ</u>
1	-1,461	-4,968	128
2	-18,703	132	-
3	37,130	-5,021	-98
4	-5,920	-4,915	154
5	-3,550	561	-10
6	-227	2,414	-
7	-9,783	11,992	50
8	-4,203	-1,275	-38
9	-5,968	10,657	-1
10	-6,136	10,478	-1
11	-11,079	4,084	-
13	-14,292	10,641	90
14	-13,988	-11,877	91
15	-11,931	11,366	-82
16	-4,309	-7,850	-60
17	-21,160	-12,658	-
18	-13,021	-30,412	-658
19	-11,697	6,870	3
20	-39,726	12,586	-
21	-25,403	8,676	-53
22	-17,406	2,770	5
23	-49,657	8,519	3
24	-43	-436	-
25	-172	-511	-
26	44,181	-5,193	-27
27	-	-935	-28

TABLE 18

ETS-1 MODAL ANALYSIS FOR SEISMIC RESPONSE;
MODE SHAPE, $f_{13} = 10.1$ Hz

<u>Mass Point</u>	<u>X</u>	<u>Y</u>	<u>θ</u>
1	-1,847	2,086	70
2	-8,571	1,281	-
3	23,013	2,231	-77
4	-6,676	2,319	126
5	-3,597	-2,817	-75
6	-20,968	-1,573	-
7	-10,148	-7,180	-204
8	-4,556	-10,114	-26
9	-6,770	-26,924	-48
10	-6,874	-38,841	-48
11	-10,185	-43,675	-
13	-11,455	-39,138	-188
14	-11,121	2,704	-187
15	-11,783	-18,945	56
16	-5,102	-7,326	47
17	-15,627	1,437	-
18	-12,311	22,401	699
19	-10,428	-27,221	-44
20	-42,137	-12,045	-
21	-23,024	-32,220	82
22	-25,485	13,957	-10
23	-32,558	32,421	-13
24	-54	-182	-
25	-194	-345	-
26	27,932	2,287	12
27	-	61	16

TABLE 19

ETS-1 MODAL ANALYSIS FOR SEISMIC RESPONSE;
MODE SHAPE, $f_{14} = 12.5$ Hz

<u>Mass Point</u>	<u>X</u>	<u>Y</u>	<u>θ</u>
1	-11,643	-1,222	-66
2	-91,001	-1,477	-
3	8,004	-490	61
4	-6,418	263	-45
5	-16,600	-19,950	-2
6	-20,359	3,574	-
7	-8,719	24,950	105
8	-17,261	-11,011	103
9	-16,922	4,334	-30
10	-16,956	-3,159	-30
11	-13,669	5,424	-
13	-6,578	-3,075	-67
14	-6,843	-19,614	66
15	-7,659	21,341	-65
16	-17,246	5,469	148
17	-1,911	8,758	-
18	-7,038	22,856	104
19	-13,013	-2,991	-37
20	27,703	-13,143	-
21	8,912	-3,872	-14
22	-9,168	1,003	-6
23	44,380	-11,303	-4
24	-339	184	-
25	-187	137	-
26	11,043	-508	-2
27	-	47	-5

TABLE 20

ETS-1 MODAL ANALYSIS FOR SEISMIC RESPONSE;
MODE SHAPE, $f_{15} = 14.4$ Hz

<u>Mass Point</u>	<u>X</u>	<u>Y</u>	<u>θ</u>
1	-3,851	1,850	-123
2	-17,507	-10,215	-
3	3,244	3,164	26
4	-5,535	4,267	-14
5	-5,669	-20,199	-19
6	8,016	-30,676	-
7	-7,071	-29,295	-649
8	-6,555	-19,188	-34
9	-7,992	27,959	40
10	-8,200	38,064	41
11	-6,829	44,441	-
13	-2,937	38,144	-46
14	-3,114	49,528	-47
15	-6,709	-66,149	-104
16	-6,544	-30,143	-116
17	-13,186	-35,736	-
18	-6,639	-66,764	153
19	-12,048	23,832	60
20	15,789	18,809	-
21	5,470	38,712	5
22	-4,870	6,284	0
23	8,927	-15,218	4
24	-112	395	-
25	-161	95	-
26	5,190	3,394	17
27	-	521	22

TABLE 21

ETS-1 MODAL ANALYSIS FOR SEISMIC RESPONSE;
MODE SHAPE, $f_{16} = 15.8$ Hz

<u>Mass Point</u>	<u>X</u>	<u>Y</u>	<u>θ</u>
1	-33,312	3,164	401
2	25,634	-4,390	-
3	125	3,125	175
4	8,832	2,794	7
5	-43,769	36,932	-62
6	-30,659	6,670	-
7	10,169	261	34
8	-50,485	15,078	-283
9	-65,044	-11,777	57
10	-65,954	3,502	65
11	-36,888	-67,485	-
13	9,002	3,226	12
14	9,153	282	12
15	9,742	4,159	24
16	-51,346	9,811	-161
17	-45,325	-10,954	-
18	9,189	4,810	-24
19	-75,729	-2,202	83
20	-49,516	-23,592	-
21	9,260	1,694	-1
22	-107,578	1,653	-5
23	12,928	-16,818	-3
24	-971	-1,187	-
25	257	12	-
26	222	3,452	19
27	-	692	20

TABLE 22

ETS-1 MODAL ANALYSIS FOR SEISMIC RESPONSE;
 MODE SHAPE, $f_{17} = 16.0$ Hz

<u>Mass Point</u>	<u>X</u>	<u>Y</u>	<u>θ</u>
1	-3,137	3,266	-264
2	-31,846	-13,742	-
3	-2,368	4,590	342
4	34,467	5,444	-58
5	-3,954	-22,773	19
6	27,026	-10,312	-
7	40,710	15,223	-102
8	-3,652	-13,112	105
9	-2,065	2,948	-32
10	-2,000	-4,957	-32
11	32,044	40,268	-
13	51,108	-4,597	6
14	51,172	-5,735	3
15	44,007	-640	-141
16	-3,391	1,569	127
17	11,306	-5,262	-
18	44,611	-5,705	59
19	-7,632	-653	-46
20	-60,254	1,734	-
21	23,378	-2,265	0
22	-21,168	-16,876	5
23	-47,591	6,704	3
24	-91	848	-
25	1,005	244	-
26	-4,447	5,121	29
27	-	1,095	29

TABLE 23

ETS-1 MODAL ANALYSIS FOR SEISMIC RESPONSE;
MODE SHAPE; $f_{18} = 19.6$ Hz

<u>Mass Point</u>	<u>X</u>	<u>Y</u>	<u>θ</u>
1	-996	-18,379	1,022
2	-9,252	11,151	-
3	-3,277	-17,624	-261
4	1,156	-14,330	-109
5	745	46,079	-95
6	12,072	40,912	-
7	2,294	36,759	-610
8	3,206	34,254	-43
9	5,155	-7,858	15
10	5,542	-4,392	14
11	5,721	-8,008	-
13	2,112	-4,706	-2
14	2,075	-4,178	-2
15	2,402	9,408	41
16	4,254	30,903	-20
17	20,756	20,096	-
18	2,237	13,990	-120
19	17,229	-54	-5
20	-12,945	-2,065	-
21	672	-9,340	0
22	34,257	-38,353	3
23	-972	11,676	0
24	-29	-3,464	-
25	34	165	-
26	-10,000	-20,466	-87
27	-	-3,597	-159

E. SEISMIC-RESPONSE ANALYSIS

1. General Nature of Problem

The dynamic model of the ETS-1, by using matrix methods of linear-elastic structural analysis, may be represented by a set or sets of equations of motion of the type

$$[M] \{\ddot{q}\} + [C] \{\dot{q}\} + [K] \{q\} = \{F(t)\}, \quad (8)$$

where $[M]$, $[C]$, and $[K]$ represent the mass inertia, viscous damping, and stiffness matrices in the q coordinates, respectively; and $\{\ddot{q}\}$, $\{\dot{q}\}$, and $\{q\}$ are response vectors in acceleration, velocity, and displacement, respectively. The forcing-function vector, $\{F(t)\}$, represents the time-history of the ground acceleration.

The mass and stiffness matrices were formulated in the modal-analysis section by using lumped-mass-parameter and influence-coefficient techniques. The damping matrix, which represents the linear viscous damping force in the structure, is an important parameter to be considered when dynamic-response solutions are needed. The transient external excitation force induced at the base of the stand is the seismic-disturbance ground motion. Because descriptions of modal damping, of seismic ground-motion information, and of methods for solving Equation 8 are pertinent to the response analysis such descriptions are discussed briefly below.

2. Modal Damping

Both viscous and structural damping are extremely important in calculating the dynamic response of structures subjected to an earthquake or similar-type seismic disturbance. Structural damping is a very complicated phenomenon, even for an otherwise linear structural system, and is still a subject of much research. With current technology, simulation of structural damping in dynamic analysis is limited almost exclusively to use of the

Voigt-type biparameter model (linear viscosity or dashpot damping). The damped structural dynamic model is thus approximated by Equation 8, where $[C]$ is the viscous damping matrix.

The basic assumption in computing the modal damping matrix $[C]$ is that the normal modes are not coupled by damping. Thus, each normal mode has its own modal damping. This assumption results in a close approximation of the true solution when the system is lightly damped, a condition that may be defined as meaning the system has a critical damping ratio of no greater than 5% (from definition of viscous damping).

Determining the damping coefficient to be used in the dynamic-response analysis is one of the most important and difficult steps in the entire analysis. Most available test results on damping are based on very small amplitude variations and probably do not accurately reflect the true damping characteristics of the structure for the large-amplitude motions associated with a severe seismic event. Although a small reduction in the assumed damping coefficient may more than proportionately change the calculated dynamic response of a structural system, any calculated increase in dynamic response could justify the use of a larger damping coefficient, particularly if component parts enter the structural yielding range.

The damping values currently used in the dynamic analysis of structural systems generally give only approximate values for the total energy loss in the vibrating system. Damping values currently used in the seismic field, and considered to be conservative, are summarized as follows (Reference 2):

<u>Item</u>	<u>Percent Critical Damping</u>
Containment structure and all internal concrete structures (see Structures - Rocking)	2.0
Conventional reinforced-concrete structures above ground; i.e., shear walls and rigid frames	5.0

<u>Item</u>	<u>Percent Critical Damping</u>
Steel structures	
Welded	1.0
Bolted or riveted	2.0
Piping systems (welded)	0.5
Structures founded on base material (rocking)	
On rock, $c > 6000$ fps	2.0
On firm soil, $c \geq 2000$ fps	5.0
On soft soil, $c < 2000$ fps	7.0

This summary shows that damping must be estimated on its own merits for every type of structure. The list gives values on which a reasonably reliable analysis can be based.

For structural calculations at NRDS, a critical-damping value of 2% is currently used in evaluating aluminum structures of bolted or welded construction, or both, as recommended by J. A. Blume and Associates. A 2% value ($\xi = C/C_{cr} = 0.02$) was therefore used for all modes in Aerojet's seismic-response analysis of ETS-1.

3. Seismic Ground-Motion Data

To provide for a direct comparison of analytical results with records of actual motion during the Benham event, more than ten accelerometers were installed at various levels of the test stand prior to the Benham event. The two acceleration records seen in Figure 8 show the actual ground acceleration-time histories recorded in two mutually perpendicular directions on a concrete pad located on the ground near the ETS-1 facility. Track 1 of Figure 8 shows the ground acceleration perpendicular to the shield wall and was used for the theoretical Benham-event response analysis. Track 7 shows the motion parallel to the shield wall.

FIGURE 8 - SEISMIC GROUND MOTION AT ETS-1, BENHAM EVENT

The ERC-generated acceleration-time history of ground motion perpendicular to the shield wall for the design event (0.27 g, zero to peak) is given in Figure 9. This history was used for the design-event response analysis.

4. Outline of Seismic-Response Analysis Method

For a complex structure like that of ETS-1, dynamic response is best analyzed by using a generalized coordinate representation of the system. This approach, in effect, reduces the complex elastic system to a simple spring-mass system involving rigid-body coordinates. The modal analysis yields a transformation matrix that uncouples the equations of motion so they may be solved individually for any known excitation force. Solutions to the uncoupled equations of motion together with the transformation matrix provide sufficient information for computing structural dynamic responses.

In the classic normal-mode method, the generalized coordinate representation of the structural system requires that the forces be represented as generalized forces. Likewise, the masses, damping, and stiffness coefficients are represented in generalized forms. The generalized forces are defined as the excitation forces acting in the direction of motion of the system and are proportional to the modal displacements of the system. The generalized force on each mode is the summation of the products of the individual forces and their individual displacements in the mode. The analytical solution to determine the dynamic response of the system by using the normal-mode method is straightforward and well known. The solution is outlined here for clarity and completeness (also see Reference 3).

The differential equations of motion of a system of n degrees of freedom in the q coordinate system take the following matrix form:

$$[M] \{\ddot{q}\} + [C] \{\dot{q}\} + [K] \{q\} = \{F(t)\} \quad (8)$$

A coordinate transformation

$$\{q\} = [\phi] \{z\} \quad (9a)$$

is applied in which each column of ϕ is a modal column of the system (obtained from the eigenvalue-eigenvector computer program) and $\{z\}$ represents the normal coordinates. From Equation (9a),

$$\{\dot{q}\} = [\phi] \{\dot{z}\} \quad (9b)$$

and

$$\{\ddot{q}\} = [\phi] \{\ddot{z}\}. \quad (9c)$$

By substituting Equation 9 into Equation 8 and premultiplying by the transpose of $[\phi]$, Equation 8 becomes

$$[\phi]^T [M] [\phi] \{\ddot{z}\} + [\phi]^T [C] [\phi] \{\dot{z}\} + [\phi]^T [K] [\phi] \{z\} = [\phi]^T \{F(t)\}. \quad (10)$$

From the orthogonality relations, it follows that

$$[\phi]^T [M] [\phi] = [M_n], \quad (11)$$

$$[\phi]^T [K] [\phi] = [\omega_n^2] [M_n] = [K_n], \quad (12)$$

where $[M_n]$ and $[K_n]$ are the generalized mass and generalized stiffness for the n th mode.

By comparing the triple matrix product

$$[\phi]^T [C] [\phi] \quad (13)$$

With Equations 11 and 12, it becomes apparent that this triple matrix product will result in a diagonal matrix only when the damping matrix $[C]$ is proportional to the mass matrix $[M]$; that is, when $[C] = 2 \xi \omega [M]$. (14)

By using Equation 14 in Equation 13 and comparing with Equation 11, it follows that

$$[\phi]^T [C] [\phi] = 2 \xi \omega [M_n]. \quad (15)$$

Substituting Equations 11, 12, and 14 into Equation 10 results in a set of n decoupled differential equations of motion

$$[M_n] \{\ddot{z}\} + 2 \xi \omega [M_n] \{\dot{z}\} + [\omega_n^2] [M_n] \{z\} = [\phi]^T \{F(t)\}. \quad (16)$$

The n th mode of Equation (16) has the form

$$M_n \ddot{z}_n + 2 \xi_n \omega_n M_n \dot{z}_n + \omega_n^2 M_n z_n = \sum_i \phi_{in} F_i(t), \quad (17)$$

where ϕ_{in} is the modal amplitude at the i th station for the n th mode and $F_i(t)$ is the sectional force acting at the i th station.

Dividing through by M_n in Equation 17 (and noting that the mode shapes can be normalized to produce unit generalized masses in all modes from the eigenvalue-eigenvector computer program) gives the following for the n th mode:

$$\ddot{z}_n + 2 \xi_n \omega_n \dot{z}_n + \omega_n^2 z_n = \sum_i \phi_{in} F_i(t). \quad (18)$$

In the present seismic-response analysis, the only external forces that act on the structure are those produced by the seismic ground acceleration, $a_o(t)$, in the lateral direction perpendicular to the shield wall. Thus, the inertial force, $F_i(t)$, in Equation 18 becomes

$$F_i(t) = -m_i a_o(t). \quad (19)$$

Substituting Equation 19 into Equation 18 gives

$$\ddot{z}_n + 2 \xi_n \omega_n \dot{z}_n + \omega_n^2 z_n = - \sum_i \phi_{in} m_i a_o(t),$$

which can be written as

$$\ddot{z}_n + 2 \xi_n \omega_n \dot{z}_n + \omega_n^2 z_n = - m_n a_o(t), \quad (20)$$

where $m_n = \sum_i \phi_{in} m_i$ is the modal participation factor for the nth mode.

Equation 20 can be solved to determine the relative displacements, velocities, and accelerations of the structural system in generalized coordinates by subtracting out the motion at the base of the test stand. To calculate the response of the mass points, Equation 9a is used to transform from generalized coordinates back to the mass-point coordinates; thus, in matrix form

$$\{q_i\} = [\phi_{in}] \{z_n\}, \quad (21a)$$

$$\{\dot{q}_i\} = [\phi_{in}] \{\dot{z}_n\}, \quad (21b)$$

$$\{\ddot{q}_i\} = [\phi_{in}] \{\ddot{z}_n\}. \quad (21c)$$

The absolute motion of the test stand is of interest to designers and analysts for evaluating the structural integrity of components. The absolute acceleration is calculated from Equation 21c by adding in the input seismic ground acceleration $a_o(t)$ at the base. Thus,

$$\{\ddot{q}_i\}_{abs} = [\phi_{in}] \{\ddot{z}_n\} + \delta \{a_o(t)\}, \quad (22)$$

where $\delta = 1$ for all motions in the y direction and
 $\delta = 0$ for all motions in the x direction.

A digital-computer program using the normal-mode method was written to facilitate the response solutions. The necessary computer input data are the ground acceleration-time history, damping coefficient, and the modal data obtained from the eigenvalue-eigenvector computer program. The response values (displacement and velocity relative to ground and absolute acceleration) are printed out at each time increment of the computer output to give a complete time history of the response of the system.

For a structure as complex as ETS-1, many modes must be included to obtain satisfactory convergence of responses; that is, to be able to discard the remaining higher modes as insignificant contributors to the overall seismic response. This effort has been evaluated, and it was found that eighteen modes would give excellent results and good convergence. These eighteen modes were therefore used in the seismic-response analysis. The results for the ETS-1 test stand are given in Section F.

5. Response Spectrum Analysis as Applied to Seismic Ground Motion

In addition to supplying Aerojet with an acceleration-time history for ground motion for the 0.27-g zero-to-peak design condition, SNPO, through Environmental Research Corporation, furnished a response spectrum representative of the seismic event. This response spectrum represents the maximum response of a single-degree-of-freedom system with a given damping coefficient as a function of the period of the system. Use of the response spectrum to uncover significant characteristics of ground motion is important in establishing coefficients for estimating the seismic loads on simple structural systems. Though seismic-response spectra can be used to determine the maximum responses of a structure having one degree of freedom, they provide only an approximate indication of the maximum response to be expected in multiple-degree-of-freedom systems. Nevertheless, the basic characteristics of the response spectrum and its uses will be covered in this section. References 4 through 7 give a complete description on this subject.

Consider a mechanical system comprising a single concentrated mass M , a linear spring of spring constant K , and a viscous damping element with damping coefficient C . Assume motion of the system to be possible only in the direction of the spring, so the system has but one degree of freedom. The absolute displacement of the mass will be denoted by x , and the absolute displacement of the ground will be denoted by y . The relative velocity and acceleration between the mass and the ground are $(\dot{x} - \dot{y})$ and $(\ddot{x} - \ddot{y})$, respectively.

The response of this single-degree-of-freedom system to a given seismic excitation may be calculated by means of the Duhamel integral equation:

$$(x - y) = \frac{1}{\omega \sqrt{1 - \xi^2}} \int_0^t \ddot{y}(\tau) e^{-\xi\omega(t-\tau)} \sin \omega \sqrt{1 - \xi^2} (t - \tau) d\tau, \quad (23)$$

in which

ω = circular frequency of the undamped system.

ξ = percent of critical damping = $C/2\omega M$,

$\ddot{y}(\tau)$ = seismic ground-acceleration history;

τ is a time parameter that disappears in the course of the integration. For small damping of the magnitude encountered in most structures ($\xi < 0.2$), this expression can be simplified by recognizing that $\sqrt{1 - \xi^2} \approx 1$. Thus, Equation 23 may be written as:

$$(x - y) = \frac{1}{\omega} \int_0^t \ddot{y}(\tau) e^{-\xi\omega(t-\tau)} \sin \omega(t - \tau) d\tau. \quad (24)$$

Differentiating Equation 24 (again using the approximation for small damping) yields

$$(\dot{x} - \dot{y}) = \int_0^t \ddot{y}(\tau) e^{-\xi\omega(t-\tau)} \sin \omega(t-\tau) d\tau; \quad (25)$$

$$\ddot{x} = \omega \int_0^t \ddot{y}(\tau) e^{-\xi\omega(t-\tau)} \sin \omega(t-\tau) d\tau. \quad (26)$$

Examination of Equations 24 through 26 shows that the response of the mechanical system depends directly on the magnitude of the integral. This integral has the dimensions of velocity; its maximum value is called the spectral velocity and is designated S_v ; i.e.

$$S_v = \left[\int_0^t \ddot{y}(\tau) e^{-\xi\omega(t-\tau)} \sin \omega(t-\tau) d\tau \right]_{\max}. \quad (27a)$$

The form of the integral clearly shows that, for a given seismic acceleration history, $\ddot{y}(\tau)$, the spectral velocity depends only on the undamped circular frequency of vibration, ω (or its inversely related quantity, period of vibration, T), and on the critical damping ratio, ξ . Two other often used spectral response quantities that are closely related to the spectral velocity are the spectral acceleration, S_a , and the spectral displacement, S_d . These quantities represent the maximum acceleration and displacement, and are defined as follows:

$$S_a = \omega S_v, \quad (27b)$$

$$S_d = \frac{1}{\omega} S_v. \quad (27c)$$

Using the above notation gives

$$(x - y)_{\max} = \frac{1}{\omega} S_v, \quad (28)$$

$$(\dot{x} - \dot{y})_{\max} = S_v, \quad (29)$$

$$\ddot{x}_{\max} = \omega S_v. \quad (30)$$

Thus, the response spectrum can be expressed in terms of displacements, velocities, or accelerations, depending on the response quantities being sought. The relative-displacement spectrum (Equation 28) is important in design problems because the stresses and strains in the structure are directly proportional to these relative displacements. The absolute-acceleration response spectrum (Equation 30) is important in any situation in which experimental measurements are to be obtained as a check on analytical calculations, because the structural response that is easiest to measure directly is the absolute acceleration. The second important reason for using the absolute-acceleration response spectrum is the fact that, in the structural design of buildings, dams, and other heavy reinforced-concrete structures, the so-called seismic coefficient or lateral force coefficient is directly proportional to the absolute-acceleration response spectrum. In engineering seismology, perhaps the most often used characteristic is the velocity response spectrum, which directly involves the equating of energy input from a given seismic event and energy dissipations in a structure. S_v thus becomes a significant parameter in evaluating the dynamic response of structures.

For structures having only a single degree of freedom, the response spectrum analysis leads to the maximum relative displacements and forces in the structure. However, in multiple-degree-of-freedom structures, such as ETS-1, the maximum response cannot be determined directly from the modal maximum, as was noted above, because the various modes do not achieve

their maxima simultaneously. Therefore, the solution can only be obtained by approximation. Various techniques for combining the modal maxima to approximate the total response of the structure have been suggested by many investigators. The root-mean-square (rms) method is probably one of the more widely used methods of calculating the maximum response for multiple-degree-of-freedom systems. In this method, the total maximum response is obtained by taking the square root of the sum of the squares of the individual modal maxima. A second method that is frequently used is direct superposition of the absolute values of the modal maxima; however, because the modal maxima actually occur at different times, this approximation may overestimate the response considerably. Because the first mode generally contributes the greater part of the total response and the higher modes essentially provide a correction to the first-mode effect, a third method to consider for obtaining the total response is to take the first-mode effect plus some fraction of the higher-mode terms.

Of the various methods mentioned above for approximating the total response of a structure, no one method is particularly suited for every structural system. One method may overestimate the displacement but may underestimate the moment in the structure. For this reason, the ground-motion acceleration-time history is always used as input to obtain the most reliable response results; and the response spectrum analysis is used primarily to verify the ERC-generated spectrum curve.

F. SEISMIC-RESPONSE RESULTS

The analysis involved two principal investigations. (1) confirmation of the Benham-event measured responses and (2) calculation of the response of the test stand to a design event for which input data were furnished by ERC.

Complete details of the Benham-event findings are given in Reference 3, in which the responses for 30 coordinates for the Benham input (Track 1, Figure 8) were calculated and plotted. The calculated maximum absolute accelerations in the y direction at the top of the test stand (elev 3916.0) were 0.465 and 0.487 g for Stations 22 and 23, respectively. The measured response near these locations for the Benham event was 0.45 g from the original unfiltered data playback. The calculated response results for the other story levels of the test stand showed a fair correlation with the measured responses. Some of the differences may be due to cross-coupling effects of the ground-motion input parallel to the wall which were not considered in the present analysis. In addition to verifying the response results, the calculated first-mode resonant frequency of ETS-1 was found to agree very closely with the measured test-stand frequency (see Section D). The results of the response analysis for the Benham event are presented in Table 24 and Figures 10 and 11.

In addition to the Benham response analysis, Reference 3 gives the response results obtained by using an ERC-furnished input design tape, ERC Tape No. D41242-D3-14-36, for ground-motion velocity-time history. The predicted maximum absolute accelerations in the y direction at the top of the test stand are rather large, 1.50 and 1.58 g for Stations 22 and 23, respectively.

TABLE 24

MAXIMUM RESPONSE FOR BENHAM EVENT

(Using Time-History Ground-Motion Input - Perpendicular to Shield Wall; 2% Damping; Track 1)

Response Location	Response		Response		Absolute Max Value, g
	Max Pos Value, g (Time, sec)		Max Neg Value, g (Time, sec)		
x ₁	0.02	(1.467)	-0.02	(1.340)	0.02
y ₁	0.23	(1.218)	-0.25	(1.348)	0.25
x ₂₂	0.06	(1.467)	-0.06	(1.347)	0.06
y ₂₂	0.37	(1.336)	-0.465	(1.476)	0.465
x ₂₄	0.001	(1.467)	-0.001	(1.340)	0.001
y ₂₄	0.115	(1.184)	-0.115	(1.304)	0.115
x ₂₅	0.001	(1.337)	-0.001	(1.473)	0.001
y ₂₅	0.115	(1.184)	-0.115	(1.304)	0.115
y ₂₆	0.23	(1.218)	-0.25	(1.348)	0.25
y ₂₇	0.09	(12.378)	-0.10	(11.983)	0.10
y ₃	0.24	(1.733)	-0.23	(1.351)	0.24
y ₄	0.23	(1.217)	-0.24	(1.348)	0.24
y ₅	0.24	(1.734)	-0.19	(1.865)	0.24
y ₈	0.22	(2.982)	-0.16	(1.867)	0.22
y ₉	0.27	(2.983)	-0.22	(1.866)	0.27
y ₁₀	0.22	(1.330)	-0.31	(1.478)	0.31
y ₁₁	0.22	(1.329)	-0.30	(1.475)	0.30
y ₁₆	0.20	(2.988)	-0.13	(2.811)	0.20
y ₁₉	0.21	(1.328)	-0.30	(1.477)	0.30
y ₂₃	0.39	(1.337)	-0.487	(1.477)	0.487
y ₂	0.25	(1.733)	-0.25	(1.349)	0.25
y ₆	0.24	(1.734)	-0.19	(1.865)	0.24
y ₇	0.27	(1.734)	-0.21	(1.864)	0.27
y ₁₃	0.22	(1.330)	-0.31	(1.478)	0.31
y ₁₄	0.33	(1.737)	-0.27	(1.865)	0.33
y ₁₅	0.25	(2.981)	-0.19	(1.866)	0.25
y ₁₇	0.19	(2.991)	-0.13	(2.792)	0.19
y ₁₈	0.21	(2.987)	-0.13	(2.819)	0.21
y ₂₀	0.25	(1.332)	-0.33	(1.477)	0.33
y ₂₁	0.22	(1.329)	-0.30	(1.477)	0.30

x Direction: Parallel (in the vertical direction) to concrete shield wall

y Direction: Perpendicular to concrete shield wall

DIRECTION PERPENDICULAR TO CONCRETE SHIELD WALL
2% DAMPING, TRACK 1

UNIT IN g's

FIGURE 10 - MAXIMUM RESPONSE FOR BENHAM EVENT

2% Damping

Using Time History Ground Motion Input - Perpendicular to Shield Wall (Track 1)

- Calculated Results, Column 1
- △-----△ Calculated Results, Column 2
- Actual Measured Unfiltered Data

FIGURE 11 - MAXIMUM COLUMN RESPONSE FOR BENHAM EVENT

The results of the response analysis for the Figure 9 design event (0.27-g zero-to-peak maximum input) are shown in Table 25 and Figures 12 and 13, where maximum-response results for a total of 30 coordinates are shown. The maximum calculated absolute accelerations in the y direction at the top of the test stand were 0.78 and 0.83 g for Stations 22 and 23, respectively. These results should be considered as superseding those in Reference 3, which were based on an earlier ground-motion velocity tape furnished by ERC.

For the design event, both ERC and Aerojet calculated the response spectra for a single-degree-of-freedom system. Figure 14 shows excellent agreement between the response spectra.

TABLE 25
 MAXIMUM RESPONSE FOR DESIGN EVENT
 (2% Damping, ERC Tape No. D45359-A4-3)

Response Coordinate	Response		Response		Absolute Max Value, g
	Max Pos Value, g	(Time, sec)	Max Neg Value, g	(Time, sec)	
x ₁	0.037	(3.360)	-0.031	(3.240)	0.04
y ₁	0.483	(4.120)	-0.498	(3.240)	0.50
x ₂₂	0.119	(3.880)	-0.119	(3.240)	0.12
y ₂₂	0.699	(3.520)	-0.782	(3.360)	0.78
x ₂₄	0.001	(3.360)	-0.001	(3.240)	0.001
y ₂₄	0.272	(2.560)	-0.197	(2.680)	0.272
x ₂₅	0.001	(3.240)	-0.001	(3.360)	0.001
y ₂₅	0.273	(2.560)	-0.198	(2.680)	0.273
y ₂₆	0.482	(4.120)	-0.497	(3.240)	0.50
y ₂₇	0.110	(10.280)	-0.098	(8.960)	0.11
y ₃	0.480	(4.120)	-0.495	(3.240)	0.50
y ₄	0.474	(4.120)	-0.491	(3.240)	0.49
y ₅	0.470	(4.120)	-0.457	(3.760)	0.47
y ₈	0.388	(3.600)	-0.358	(3.280)	0.39
y ₉	0.501	(3.600)	-0.450	(4.240)	0.50
y ₁₀	0.424	(9.480)	-0.497	(3.360)	0.50
y ₁₁	0.433	(9.480)	-0.477	(3.360)	0.48
y ₁₆	0.308	(9.480)	-0.337	(9.240)	0.34
y ₁₉	0.419	(9.480)	-0.468	(3.360)	0.47
y ₂₃	0.754	(3.520)	-0.830	(3.360)	0.83
y ₂	0.515	(4.120)	-0.522	(3.240)	0.52
y ₆	0.453	(4.120)	-0.446	(3.760)	0.45
y ₇	0.512	(3.640)	-0.510	(3.760)	0.51
y ₁₃	0.424	(9.480)	-0.498	(3.360)	0.50
y ₁₄	0.694	(3.640)	-0.620	(3.760)	0.69
y ₁₅	0.469	(3.640)	-0.442	(3.760)	0.47
y ₁₇	0.326	(9.480)	-0.355	(9.240)	0.36
y ₁₈	0.311	(9.480)	-0.350	(9.240)	0.35
y ₂₀	0.448	(4.000)	-0.542	(3.360)	0.54
y ₂₁	0.423	(9.480)	-0.478	(3.360)	0.48

2% DAMPING: ERC TAPE NO. D45359-A4-3

UNIT IN g's

FIGURE 12 - MAXIMUM RESPONSE FOR DESIGN EVENT

2% DAMPING ERC TAPE NO. D45359-A4-3

FIGURE 13 - MAXIMUM COLUMN RESPONSE FOR DESIGN EVENT

FIGURE 14 - RESPONSE SPECTRUM COMPARISONS FOR DESIGN EVENT

IV. CONCLUSIONS AND RECOMMENDATIONS

The prime objective of developing analytical and modeling techniques to solve for the seismic response of the ETS-1 test stand has been achieved with good results. Analytical solutions for both the resonant frequency and seismic response show good correlation with measured data obtained from the Benham seismic event. A response spectrum for the 0.27-g design event was determined by using the Aerojet single-degree-of-freedom model computer program, and the results checked very closely with the shock spectrum furnished by ERC. As a result of this correlation, a high level of confidence has now been achieved in the overall analytical structural dynamic program. In addition, time-history maximum responses for the 0.27-g design event were predicted for various stations of the test stand and the engine.

Two new digital-computer programs have been developed to solve the differential equations of motion for single- (response-spectrum program) and multiple-degree-of-freedom (time-history program) systems subjected to seismic-event or earthquake-type disturbances.

For future seismic-event analysis, it is recommended that the structural dynamic model of ETS-1 be revised to reflect up-to-date information on structural modifications proposed for, or being made to, the test-stand structure.

For other structural systems, it is recommended that the degree of sophistication of analytical procedures be adjusted for consistency with the complexity of the structure being considered. For complex structures such as ETS-1, the ground-motion time history should be used in all cases as input to calculate responses. For relatively simple structures, the calculated single-degree-of-freedom response spectrum may be used in the response analysis.

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