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**EFFECT OF RANDOM TEMPERATURE FLUCTUATIONS
ON CREEP AND CREEP RUPTURE
OF CYLINDRICAL TUBES**

by

W. N. Huang and R. A. Valentin

BASE TECHNOLOGY

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ARGONNE NATIONAL LABORATORY, ARGONNE, ILLINOIS

Prepared for the U.S. ATOMIC ENERGY COMMISSION

Division of Reactor Research and Development

under Contract W-31-109-Eng-38

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Printed in the United States of America
Available from
National Technical Information Service
U.S. Department of Commerce
5285 Port Royal Road
Springfield, Virginia 22151
Price: Printed Copy \$4.00; Microfiche \$1.45

ANL-8006

LMFBR Structural Materials
and Design Engineering
(UC-79h)

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Components Technology Division

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NOMENCLATURE

a, b	Inner and outer radii, respectively	T_a, T_b	Inner- and outer-surface temperatures, respectively
A	Cross-sectional area of rod	$\bar{T}_a, \bar{T}_b$	Means of T_a and T_b
A_r	Fraction of load carrying area	u	Radial creep velocity
$\tilde{A}, \tilde{B}, \tilde{C}$	Temperature-dependent constants	$U()$	Unit step function (Heaviside function)
b	Tube thickness	α	Radius ratio
C, ν	Material constants	$\bar{\alpha}, \bar{\beta}$	Means of $\hat{\alpha}$ and $\hat{\beta}$
d_1, d_2	Functions of temperature (Eq. 75)	α_R	Value of α corresponding to time to fracture t_{R0}^*
d_3	Flux-dependent parameter (Eq. 76)	β	Parameter (Eq. 77b)
$C()$	Autocovariance of indicated quantity	δ	Parameter (Eq. 22)
d	Correlation interval	δ_{ij}	Kronecker delta
D	Damage factor	ϵ	Nondimensional creep-viscosity parameter
$\text{erf}()$	Error function	ϵ_e	Effective strain
E	Modulus of elasticity	$\dot{\epsilon}_c$	Strain-rate parameter
$f()$	Probability density function	$\dot{\epsilon}_{c0}$	Value of $\hat{\epsilon}_c$ at $\hat{T} = T_0$
$F(T)$	Function of temperature (Eq. 4)	$\dot{\epsilon}_{ij}$	Strain-rate tensor
H	Statistical constant (Eq. 70)	η	Argument for autocovariance of stationary random process ($= t_1 - t_2$)
K	Temperature-independent constant (Eq. 76)	γ, ξ	Experimentally determined functions
l	Proportionality constant	$\rho, \rho_{\alpha\beta}$	Correlation coefficients
$\text{MAX}\{ \}$	Most probable value of indicated quantity	ω, λ	Deterministic constants
n	Creep exponent	Ω	Normalization factor
p	Internal pressure	ϕ	Integrated neutron flux
Q	Constant load	σ_a	Stress acting over undamaged area
r	Radial coordinate	σ_c	Reference stress (chosen as $R_1 p b_1^{-1}$)
R	Tube radius	σ_e	Effective stress
R_i, b_i	Initial values of R and b	σ_K	Constant stress (Eq. 42)
s_{ij}	Stress deviator	$\sigma_r, \sigma_\theta, \sigma_z$	Stresses
t	Time	σ_0	Initial stress
t_K	Time to rupture for which $\sigma = \sigma_K$	$\sigma()$	Standard deviation of indicated quantity
t_R	Time to brittle fracture	τ	Nondimensional temperature
t^*	Time to rupture	$< >$	Expectation value operator
t_D^*	Time to ductile fracture	$()_0$	Indicates nominal value, i.e., value when temperature equals mean value
t_H^*	Hoff's ductile-fracture time	$(\hat{ })$	Designates random quantity
t_K^*	Kachanov's brittle-fracture time		
t_s^*	Söderquist's brittle-fracture time		
T	Temperature		
T_0 or $\bar{T}$	Mean value of $\hat{T}$		

EFFECT OF RANDOM TEMPERATURE FLUCTUATIONS ON CREEP AND CREEP RUPTURE OF CYLINDRICAL TUBES

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ABSTRACT

The design of LMFBR plants requires extensive creep analysis, since many in-core structures and primary system components operate at elevated temperatures. In the design process, exact information on the temperature field is often lacking, and this, together with the temperature sensitivity of creep phenomena, leads to uncertainties in derived quantities such as strain and total deformation.

This study considers the qualitative and quantitative implications of random temperature data by deriving expressions for the expectation value and variance of quantities such as displacement and stress in creep of thin, pressurized tubes. Also, a nondeterministic analog of an existing theory of creep rupture is used to study the impact of random temperature fluctuations on time to brittle and ductile rupture. Data for stainless steel are applied in various numerical examples, and these, together with analytical considerations, are used to formulate qualitative generalizations of importance to the design process.

I. INTRODUCTION

The vast majority of components designed for service in commercial and research reactors operate in temperature ranges below those for which creep phenomena are significant. As the emphasis in reactor design shifts to more advanced systems, however, operating temperatures increase and a careful consideration of creep becomes a mandatory aspect of design. In particular, with the advent of large liquid-metal-cooled or high-temperature gas-cooled reactors designed for long service lives, accumulation of appreciable creep strains in core structures and primary piping becomes inevitable. For example, liquid-metal fast breeder reactors (LMFBR's) such as the FTR and the first demonstration plants will operate at temperatures in excess of 800°F. This has required extensive elaboration of the applicable codes and standards so that the creep response of the stainless steels that form the major structural elements can be adequately factored into the design process.

Since creep rates, creep rupture properties, plastic response, thermal ratcheting, etc., are all temperature-sensitive phenomena, exact knowledge of distributions and variations in temperature is required in most design studies. In early design iterations, however, the exact geometry and operating conditions may not be well defined, and hence, temperature data may be lacking or inexact. Also, even after a design has been made final, the exact operating conditions may vary about nominal values and result in temperature fields that vary in some ill-defined manner. Since creep response is temperature-sensitive, lack of precise knowledge of temperature will be reflected in uncertainties in derived quantities such as displacement rates. For this reason, it is desirable to investigate the effect of random temperature variations on creep response. It is of great potential value to know what derived quantities are affected by such variations so that given, say, the mean temperature and its standard deviation, the mean and standard deviation of the resulting creep rates can be estimated. Of equal importance is the need to make qualitative statements about such problems. A knowledge of what quantities are relatively unaffected by random temperature perturbations immediately clarifies design by uncoupling certain aspects of the design process.

The creep of real materials cannot be treated as a strictly deterministic process, even in the absence of variations in temperature. The random distribution of material imperfections on a microscopic level and macroscopic spatial variations in average material properties produces data scatter in even the most carefully controlled experiments. This aspect of creep will not be treated in this report. For current literature in this area, see Cozzarelli and Huang;¹ their basic approach in uncoupling the "random temperature problem" from the "random imperfection problem" will be used. Justification for this uncoupling is the relative insensitivity of the creep exponent to temperature, together with its strong dependence on imperfection structure. The converse holds for the reference strain rate used in the constitutive law.

Section II concentrates on the problem of random temperature in the steady creep of thin circular tubes, since this geometry is probably of greatest concern in both piping and design of core internals. Section III considers the problem of creep rupture in the same geometry. Section IV summarizes the qualitative aspects of the study and indicates areas in need of further attention.

II. EFFECT OF RANDOM TEMPERATURE VARIATIONS ON THE CREEP OF PRESSURIZED TUBES

In considering the effect of random temperature fluctuations on the stress and deformation state in a pressurized tube at elevated temperature, it is assumed that the material has entered a stage of secondary creep. Thus, the general form of the constitutive equations for the material is

$$\dot{\epsilon}_{ij} = \frac{3}{2} \dot{\epsilon}_c \sigma_c^{-n} \sigma_e^{n-1} s_{ij} \quad (1)$$

It is also assumed that Eq. 1 will hold when the temperature is varying slowly; thus temperature will be considered a random process in time. Since experimental evidence indicates that temperature has only a slight effect on the creep exponent, but a marked effect on the reference creep strain rate, only $\dot{\epsilon}_c$ will be treated as a random material parameter. Thus Eq. 1 is written

$$\hat{\epsilon}_{ij} = \frac{3}{2} \hat{\epsilon}_c \sigma_c^{-n} \hat{\sigma}_e^{n-1} \hat{s}_{ij}, \quad (2)$$

where

$$\hat{\epsilon}_c = F(\hat{T}); \quad (3)$$

with $F(\cdot)$ a known monotonically increasing function.

On physical grounds, it is reasonable to assume that

$$F(\hat{T}) = \omega \exp(-\lambda \hat{T}^{-1}). \quad (4)$$

However, it is more convenient to approximate the temperature dependence, Eq. 3, by

$$\hat{\epsilon}_c(t) = \dot{\epsilon}_{c0} \exp[\hat{\tau}(t)], \quad (5)$$

where

$$\hat{\tau}(t) = \lambda T_0^{-2} [\hat{T}(t) - T_0]. \quad (6)$$

Equations 5 and 6 are a good approximation to the exponential expression in Eq. 4 and result in a more tractable statistical treatment of the problem.

Assuming that $\hat{T}$ is a stationary normal process with mean T_0 implies $\hat{\tau}$ is stationary normal with zero mean and the first-order density function

$$f(\tau) = (2\pi)^{-1/2} \sigma_\tau^{-1} \exp[-\tau^2 (2\sigma_\tau^2)^{-1}]. \quad (7)$$

Using the definition

$$\hat{\epsilon}(t) \equiv \dot{\epsilon}_{c0}^{-1} \hat{\epsilon}_c(t) = \exp[\hat{\tau}(t)], \quad (8)$$

and the results of Cozzarelli and Huang,¹ it may be shown that $\hat{\epsilon}(t)$ is a stationary random process with the first-order density function

$$f(\epsilon) = [(2\pi)^{1/2} \sigma_\tau \epsilon]^{-1} \exp[-(\ln \epsilon)^2 (2\sigma_\tau^2)^{-1}] U(\epsilon). \quad (9)$$

Equation 9 is the density function for the lognormal distribution, and its properties are well known; for example, see Ref. 2. The major statistical properties of this distribution that will be used in the development to follow are

$$\langle \hat{\epsilon} \rangle = \exp(\sigma_\tau^2/2) > 1, \quad (10)$$

$$\sigma_e^2 = \exp(\sigma_\tau^2)[\exp(\sigma_\tau^2) - 1] > \sigma_\tau^2, \quad (11)$$

and

$$C_e(\eta) = \exp(\sigma_\tau^2)\{\exp[C_\tau(\eta)] - 1\}, \quad (12)$$

where $C_\tau(\eta)$ is the autocovariance of $\hat{\tau}$, and $\eta = t_1 - t_2$.

Consider a thin cylindrical tube of radius $\hat{R}(t)$ and thickness $\hat{b}(t)$ subjected to a constant internal pressure p . The various stress components are

$$\hat{\sigma}_\theta = 2\hat{\sigma}_z = p\hat{R}\hat{b}^{-1}, \quad \hat{\sigma}_r \equiv 0, \quad (13)$$

$$\hat{\sigma}_e = \frac{\sqrt{3}}{2} \frac{p\hat{R}}{\hat{b}}, \quad (14)$$

and

$$\hat{s}_\theta = -\hat{s}_r = \frac{\hat{\sigma}_\theta}{2}, \quad \hat{s}_z \equiv 0; \quad (15)$$

and, from Eq. 2, the strain-rate expressions are

$$\hat{\epsilon}_\theta = -\hat{\epsilon}_r = \hat{\epsilon}_c \left(\frac{\sqrt{3}}{2}\right)^{n+1} \left(\frac{\hat{\sigma}_\theta}{\sigma_c}\right)^n \text{ and } \hat{\epsilon}_z = 0. \quad (16)$$

Assuming Eq. 2 holds for logarithmic strain,

$$\hat{\epsilon}_\theta = \ln \frac{\hat{R}(t)}{R_i}, \quad (17)$$

and noting that incompressibility implies

$$R_i b_i = \hat{R}(t)\hat{b}(t), \quad (18)$$

it follows that

$$\hat{\sigma}_\theta(t) = \frac{R_i p}{b_i} \exp[2\hat{\epsilon}_\theta(t)]. \quad (19)$$

Hence, from Eq. 16,

$$\hat{\epsilon}_\theta = B\hat{\epsilon}_c(t) \exp[2n\hat{\epsilon}_\theta(t)], \quad (20)$$

where, for convenience, $\sigma_c = R_i p b_i^{-1}$, and thus $B = (\sqrt{3}/2)^{n+1}$. The solution to Eq. 20 considered as a differential equation in $\hat{e}_\theta$ is

$$\hat{e}_\theta(t) = \ln \left[1 - 2nB \int_0^t \hat{e}_c(\xi) d\xi \right]^{-1/2n} \quad (21)$$

Let

$$\hat{R}(t) = R_i + \delta \Delta \hat{R}(t), \quad (22)$$

where δ is defined so that $\delta \Delta \hat{R} \ll R_i$. Thus, to a first approximation,

$$\left[\frac{R(t)}{R_i} \right]^{-2n} \cong 1 - 2n\delta \Delta \hat{R}(t) R_i^{-1}.$$

From this approximation and Eqs. 17 and 21, it follows that

$$\delta \Delta \hat{R}(t) = BR_i \hat{e}_{c0} \int_0^t \hat{e}(\xi) d\xi. \quad (23)$$

Defining the nominal value $\Delta R_0(t)$ as that resulting when $\hat{T} = T_0$, it also follows that $\delta \Delta R_0(t) = BR_i \hat{e}_{c0} t$; thus,

$$\frac{\Delta \hat{R}(t)}{\Delta R_0(t)} = \frac{1}{t} \int_0^t \hat{e}(\xi) d\xi. \quad (24)$$

In the special case when $\hat{e}(\xi)$ is assumed to be a random variable $\hat{e}$, Eq. 24 gives $\Delta \hat{R}(t)/\Delta R_0(t) = \hat{e}$, and the density function of $\Delta \hat{R}(t)/\Delta R_0(t)$ becomes the lognormal density function $f(e)$. In the general case of $\hat{e}(\xi)$, determination of the density function is extremely complicated and shall not be attempted here; however, its mean and variance will be determined as follows.

From Eqs. 10 and 24 and the stationarity of $\hat{e}(\xi)$, we obtain the mean

$$\left\langle \frac{\Delta \hat{R}(t)}{\Delta R_0(t)} \right\rangle = \exp\left(\frac{\sigma_T^2}{2}\right), \quad (25)$$

in which the mean diameter change always exceeds the nominal value based on the mean temperature.

We now examine the second-order statistics of the problem. From Eq. 24, it follows that

$$\left\langle \frac{\Delta \hat{R}(t_1)}{\Delta R_0(t_1)} \frac{\Delta \hat{R}(t_2)}{\Delta R_0(t_2)} \right\rangle = \frac{1}{t_1 t_2} \int_0^{t_1} \int_0^{t_2} \langle \hat{e}(\xi_1) \hat{e}(\xi_2) \rangle d\xi_1 d\xi_2, \quad (26)$$

where

$$\langle \hat{\epsilon}(\xi_1) \hat{\epsilon}(\xi_2) \rangle = \exp(\sigma_\tau^2) \exp[C_\tau(\eta)], \quad \eta = \xi_1 - \xi_2. \quad (27)$$

It is physically reasonable to choose the autocovariance

$$C_\tau(\eta) = \sigma_\tau^2 \exp\left(-\frac{|\eta|}{d}\right), \quad (28)$$

where d is some assigned correlation interval modeling the qualitative aspects of the characteristic times associated with the assumed temperature field. In the present case, d will be considered a parameter to define the time scale. From Eq. 28, it can be shown that $\hat{\epsilon}_c(t)$, and thus $\hat{\epsilon}(t)$, are ergodic in the mean. From Eqs. 26-28, the autocorrelation function is

$$\left\langle \frac{\Delta \hat{R}(t_1)}{\Delta R_0(t_1)} \frac{\Delta \hat{R}(t_2)}{\Delta R_0(t_2)} \right\rangle = \frac{\exp(\sigma_\tau^2)}{t_1 t_2} \int_0^{t_1} \int_0^{t_2} \exp\left[\sigma_\tau^2 \exp\left(-\frac{|\eta|}{d}\right)\right] d\xi_1 d\xi_2. \quad (29)$$

From Eq. 24, the variance at time t is given by

$$\sigma_{\Delta R / \Delta R_0}^2 = \frac{1}{t} \int_0^{2t} \left(1 - \frac{\eta}{2t}\right) C_\epsilon(\eta) d\eta, \quad (30)$$

where

$$C_\epsilon(\eta) = \exp(\sigma_\tau^2) \left\{ \exp\left[\sigma_\tau^2 \exp\left(-\frac{|\eta|}{d}\right)\right] - 1 \right\}.$$

The integral in Eq. 30 is evaluated to give

$$\sigma_{\Delta R / \Delta R_0}^2 \sigma_\epsilon^{-2} = [\exp(\sigma_\tau^2) - 1]^{-1} \frac{d}{t} \sum_{k=1}^{\infty} \frac{\sigma_\tau^{2k}}{k(k!)} \left\{ 1 - \frac{d}{2kt} \left[1 - \exp\left(-\frac{2kt}{d}\right) \right] \right\}, \quad (31)$$

which is used to develop the limits

$$\lim_{d/t \rightarrow 0} \left(\frac{\sigma_{\Delta R / \Delta R_0}^2}{\sigma_\epsilon^2} \right) = 0, \quad \lim_{d/t \rightarrow \infty} \left(\frac{\sigma_{\Delta R / \Delta R_0}^2}{\sigma_\epsilon^2} \right) = 1. \quad (32)$$

Although the expression for the mean diameter change, Eq. 25, is time-independent, the variance, Eq. 31, is an explicit function of time; thus $\Delta \hat{R} / \Delta R_0$ is only stationary in the mean. Figure 1 shows the variance ratio, Eq. 31, plotted as a function of d/t for several values of σ_τ . The diameter change is clearly sensitive to perturbations in the temperature field.

Fig. 1. Variance of Radius Change as a Function of Dimensionless Correlation Time for Several Values of Dimensionless Temperature Variance

As a specific example, consider a Type 304 stainless steel tube operating at $T_0 = 1560^\circ\text{R}$. Using data from Ref. 3, assuming $\sigma_T = 10^\circ, 20^\circ$, and 25° , and taking $d/t = 1$, evaluation of Eqs. 10, 11, 25, and 31 yields the results given in Table I.

TABLE I. Mean and Standard Deviation of $\hat{\Delta R}/\Delta R_0$

σ	σ_T	$\hat{\Delta R}/\Delta R_0$	σ_{ϵ}	$\sigma_{\Delta R/\Delta R_0}$
10	0.3463	1.0618	0.3792	0.2740
20	0.6926	1.2712	0.9978	0.7210
25	0.8658	1.4550	1.5378	1.1110

In addition to the fact that $\langle \hat{\Delta R}/\Delta R_0 \rangle > 1$, the variance in the random temperature is not related in any simple manner with the resulting variance in the radius change, other than an apparent monotonic behavior showing that increases in σ_T will increase $\sigma_{\Delta R/\Delta R_0}$.

To determine the stress response of the cylinder, note that

$$\hat{\sigma}_{\theta}(t) = \frac{p[\hat{R}(t)]^2}{R_i b_i} \quad (33)$$

and, since Eqs. 17 and 21 imply

$$\hat{R}(t) = R_i \left[1 - 2nB \int_0^t \hat{\epsilon}_c(\xi) d\xi \right]^{-1/2n} \quad (34)$$

it follows that

$$\hat{\sigma}_{\theta}(t) = \frac{pR_i}{b_i} \left[1 - 2nB\epsilon_{c0} \int_0^t \hat{\epsilon}(\xi) d\xi \right]^{-1/n} \quad (35)$$

Therefore,

$$\frac{\hat{\sigma}_\theta(t)}{\sigma_{\theta_0}(t)} = \left[\frac{1 - 2nB\dot{\epsilon}_{c_0}t}{1 - 2nB\dot{\epsilon}_{c_0} \int_0^t \hat{\epsilon}(\xi) d\xi} \right]^{1/n} \quad (36)$$

To determine the density function, mean, and variance of $\sigma_\theta(t)/\sigma_{\theta_0}(t)$ either explicitly or by numerical methods would be a prohibitive task. It is true, however, that when Eq. 28 is valid, $\hat{\epsilon}(t)$ is ergodic in the mean. Thus for a sufficiently long time,

$$\frac{1}{t} \int_0^t \hat{\epsilon}(\xi) d\xi \cong E\{\hat{\epsilon}(\xi)\} = \exp(\sigma_\tau^2/2);$$

and

$$\frac{\hat{\sigma}_\theta(t)}{\sigma_{\theta_0}(t)} \cong \left[\frac{1 - 2nB\dot{\epsilon}_{c_0}t}{1 - 2nB\dot{\epsilon}_{c_0}t \exp(\sigma_\tau^2/2)} \right]^{1/n}$$

For times of the order of $t = 10^5$ hr (for Type 304 stainless steel),

$$\left. \begin{aligned} 2nB\dot{\epsilon}_{c_0}t &\ll 1 \\ \frac{\hat{\sigma}_\theta(t)}{\sigma_{\theta_0}(t)} &\cong 1 \end{aligned} \right\} \quad (37)$$

implying that the stress state is almost deterministic.*

III. RANDOM TEMPERATURE EFFECTS IN CREEP RUPTURE

To assess the effects of uncertainty in temperature on the ability to predict creep rupture, it is necessary to work within the framework provided by a well-defined theory. Since an extensive literature on random temperature considerations in this area does not exist, it is first necessary to study the qualitative aspects of the problem. For this purpose, there is no need to have a detailed model or to incorporate comprehensive materials data. Such refinement can follow after the general nature of the problem has been studied, and after there is some indication that the effects of random temperature are of sufficient interest to justify production of a design-oriented formulation.

*It is not true that as $t \rightarrow \infty$, $\hat{\sigma}_\theta(t)/\sigma_{\theta_0}(t) \rightarrow 1$. Rather, Eq. 37 holds for physically realistic values of t (e.g., 10^5 hr).

Thus, in the development that follows, emphasis is placed on relatively elementary phenomenological theories exhibiting the basic aspects of ductile and brittle creep rupture.

For many structural materials, the data on creep rupture have the same general form. On a log-log plot of initial stress σ_{10} versus time to failure t^* , there are two distinctive regions. For small values of t^* and large stresses, the rupture is ductile, and the curves have a relatively shallow slope with increasing t^* . For low stress and long times, the rupture is brittle and the slope much greater. The ductile part of the curve can be treated readily, using the phenomenological theory of Hoff.⁴ By assuming a power law for secondary creep and working with logarithmic strain, Hoff showed that the time to rupture was given by

$$t_H^* = \frac{1}{n} \left(\frac{\sigma_c}{\sigma_{10}} \right)^n, \quad (38)$$

where n and σ_c are material constants in the creep law. In this simplified theory, t_H^* represents the time required for the original section to reduce to zero area.

To account for the long-term material deterioration, which, in some sense, is the essence of brittle creep rupture, Kachanov⁵ developed a phenomenological cumulative-damage theory. As presented by Odqvist,⁶ this theory assumes that deterioration is a direct result of tensile stresses. In its one-dimensional formulation, a prismatic rod of cross-sectional area A is subjected to a constant tensile load P . It is assumed that, at any given time, only some fraction of the area A_r is able to carry the load; thus a damage factor is defined as

$$D = \frac{A - A_r}{A} \quad (39)$$

The actual stress on the supporting area σ_r increases above its nominal value $\sigma = P/A$ as A_r decreases. It is assumed that the rate of increase of the damage is

$$\frac{dD}{dt} = f(\sigma_r) = f\left(\frac{\sigma}{1 - D}\right), \quad (40)$$

with the initial condition $D = 0$ at $t = 0$. Integration of this equation then yields the time to rupture t_R for which $D = 1$. If f is of the simple form

$$f(x) = Cx^v,$$

where C and v are material constants, it follows that

$$\int_0^1 (1-D)^v dD = \frac{1}{1+v} = C \int_0^{t_R} \sigma^v dt. \quad (41)$$

By assuming that the various material constants are determined such that t_K corresponds to a time to rupture for which the nominal stress $\sigma = \sigma_K =$ constant, the above criterion may be written

$$\int_0^{t_R} \left(\frac{\sigma}{\sigma_K} \right)^v dt = t_K. \quad (42)$$

This is essentially the form presented by Kachanov⁵ and is equivalent to the linear cumulative creep-damage law given in Refs. 7 and 8.

In considering the effect of random temperature, emphasis will be placed on use of the criterion given in Eq. 42 or in slight generalizations of this result. The treatment of random temperature effects in ductile creep rupture will be handled independently of the Hoff theory, since the Kachanov formulation can be suitably modified to describe both ductile and brittle creep rupture.

Before considering the detailed random-temperature case, the possible generalizations of the basic failure criteria should be noted and their implications explored in a deterministic sense. In particular, the multidimensional extension of Eq. 42 will be taken as

$$\int_0^{t_R} \left| \frac{\sigma_{\max}}{\sigma_K} \right|^v dt = t_K, \quad (43)$$

where $\sigma_{\max}$ is the maximum principal stress and is assumed to be tensile. It is possible that the effective stress σ_e should replace $\sigma_{\max}$ in Eq. 43; however, use of $\sigma_{\max}$ results in a simpler analysis, which is generally in qualitative agreement with experiment.

A. The Deterministic Formulation

Following Odqvist,⁶ the creep rupture problem for a thin pressurized tube may be formulated by introducing the notation

$$\sigma_{\theta 0} = p R_i b_i^{-1}, \quad \alpha = R R_i^{-1}, \quad (44)$$

and using logarithmic strains, to obtain

$$\epsilon_{\theta} = \ln \alpha, \quad \dot{\epsilon}_{\theta} = \dot{\alpha} \alpha^{-1}. \quad (45)$$

Incompressibility then implies

$$\sigma_{\theta} = \sigma_{\theta 0} \alpha^2 \quad (46)$$

and, using the creep law in power form with σ_{θ_0} as the reference stress,

$$\dot{\alpha} = B \dot{\epsilon}_{c0} \alpha^{2n+1}, \quad (47)$$

where

$$B = \left(\frac{\sqrt{3}}{2} \right)^{n+1}$$

Noting that $\alpha = 1$ at $t = 0$, Eq. 47 is integrated to give

$$1 - \alpha^{-2n} = 2nB\dot{\epsilon}_{c0}t. \quad (48)$$

For ductile rupture, $\alpha \rightarrow \infty$, and thus the time to ductile failure is given by

$$t_{D0}^* = (2nB\dot{\epsilon}_{c0})^{-1}. \quad (49)$$

To estimate the time to brittle failure t_{K0}^* , it is first noted that

$$\sigma_{\max} = \sigma_{\theta} = \sigma_{\theta_0} \alpha^2, \quad (50)$$

and the integral criteria, Eq. 43, then implies

$$t_K = \int_0^{t_{K0}^*} \left| \frac{\sigma_{\theta_0} \alpha^2}{\sigma_K} \right|^v dt = \left(\frac{\sigma_{\theta_0}}{\sigma_K} \right)^v \int_1^{\alpha_R} \alpha^{2v} \frac{dt}{d\alpha} d\alpha, \quad (51)$$

where α_R is that value of α corresponding to the time to failure t_{K0}^* .

Computing $dt/d\alpha$ from Eq. 48 and substituting into Eq. 51 gives

$$t_K = \left(\frac{\sigma_{\theta_0}}{\sigma_K} \right)^v (B\dot{\epsilon}_{c0})^{-1} \int_1^{\alpha_R} \alpha^{2v-2n-1} d\alpha. \quad (52)$$

Examination of existing data shows $v < n$, and evaluation of the integral in Eq. 52 yields

$$\alpha_R^{-2n} = \left[1 - 2(n-v) B\dot{\epsilon}_{c0} \left(\frac{\sigma_K}{\sigma_{\theta_0}} \right)^v t_K \right]^{n/(n-v)} \quad (53)$$

Since Eq. 48 implies

$$1 - \alpha_R^{-2n} = 2nB\dot{\epsilon}_{c0}t_{K0}^*, \quad (54)$$

this and Eq. 53 give for the time to brittle failure

$$t_{K0}^* = (2nB\dot{\epsilon}_{c0})^{-1} \left\{ 1 - \left[1 - 2(n-v) B\dot{\epsilon}_{c0} \left(\frac{\sigma_K}{\sigma_{\theta 0}} \right)^v t_K \right]^{n/(n-v)} \right\}. \quad (55)$$

In the nondeterministic development to follow, the treatment parallels that given above. Consideration was given to the possibility of using a more comprehensive model of the brittle failure mechanism based on the work of Söderquist;⁹ however, in the deterministic cases evaluated, the differences proved minor (see appendix). For this reason and for simplicity, Kachanov's formulation was adjudged adequate.

B. The Random-temperature Case

If the assumptions stated in Sec. II are applied to the case in which temperature is a random process in t , Eq. 47 becomes

$$\hat{\alpha} = B\dot{\epsilon}_{c0} \hat{\epsilon} \hat{\alpha}^{2n+1}, \quad (56)$$

and, since $\hat{\alpha} = 1$ at $t = 0$,

$$1 - \hat{\alpha}^{-2n} = 2nB\dot{\epsilon}_{c0} \int_0^t \hat{\epsilon}(\xi) d\xi. \quad (57)$$

From the properties of the lognormal density function, the expectation value of Eq. 57 is

$$1 - \langle \hat{\alpha}^{-2n} \rangle = 2nB\dot{\epsilon}_{c0} t \exp(\sigma_T^2/2), \quad (58)$$

and, since ductile failure will correspond to $\langle \hat{\alpha}^{-2n} \rangle = 0$,

$$t_D^* = [2nB\dot{\epsilon}_{c0} \exp(\sigma_T^2/2)]^{-1}. \quad (59)$$

Comparison of Eq. 59 with the deterministic result, Eq. 49, yields

$$\frac{t_D^*}{t_{D0}^*} = \exp(-\sigma_T^2/2). \quad (60)$$

Equation 60 is not dependent on the internal pressure.

Table II lists the expectation values of t_D^*/t_{D0}^* for typical variations in the temperature.

TABLE II. Expectation Value of t_D^*/t_{D0}^*

$\sigma_T, ^\circ R$	σ_T	t_D^*/t_{D0}^*
10	0.3463	0.9418
20	0.6926	0.7866
25	0.8658	0.6873

It is apparent that temperature variations have a significant effect on lifetime. For example, if the temperature fluctuates randomly between 1500 and 1620°R with normal distribution, the expected lifetime is approximately one-fourth of that computed on a deterministic basis using the mean temperature. This is in qualitative agreement with the effects of random temperature on lifetime of viscoelastic structures.¹⁰

Evaluation of analogous results for brittle fracture is somewhat more complicated. Starting with the nondeterministic result comparable to that given in Eq. 52,

$$t_K = \left(\frac{\sigma_{\theta_0}}{\sigma_K} \right)^v (B \dot{\epsilon}_{c_0} \hat{\epsilon})^{-1} \int_1^{\hat{\alpha}_R} \alpha^{2v-2n-1} d\alpha \quad (61)$$

or

$$t_K = \left(\frac{\sigma_{\theta_0}}{\sigma_K} \right)^v [2(n-v) B \dot{\epsilon}_{c_0} \hat{\epsilon}]^{-1} [1 - \hat{\alpha}_R^{-2(n-v)}]; \quad (62)$$

thus,

$$\hat{\alpha}_R^{-2(n-v)} = 1 - 2(n-v) B \dot{\epsilon}_{c_0} \left(\frac{\sigma_K}{\sigma_{\theta_0}} \right)^v t_K \hat{\epsilon}. \quad (63)$$

Rewriting Eq. 63 as

$$\hat{\alpha}_R^{-2n} = (1 - g \hat{\epsilon})^{n/(n-v)}, \quad (64a)$$

where

$$g = 2(n-v) B \dot{\epsilon}_{c_0} \left(\frac{\sigma_K}{\sigma_{\theta_0}} \right)^v t_K, \quad (64b)$$

and noting Eq. 57 implies that, at failure,

$$1 - \hat{\alpha}_K^{-2n} = 2n B \dot{\epsilon}_{c_0} \int_0^{t_K^*} \hat{\epsilon}(\xi) d\xi, \quad (65)$$

Eqs. 64 and 65 may be combined to give

$$1 - (1 - g \hat{\epsilon})^{n/(n-v)} = 2n B \dot{\epsilon}_{c_0} \int_0^{t_K^*} \hat{\epsilon}(\xi) d\xi, \quad (66)$$

which yields the expectation value

$$\left. \begin{aligned} & 1 - \langle (1 - g\hat{\epsilon})^{n/(n-v)} \rangle = 2nB\dot{\epsilon}_{c0} \langle \hat{\epsilon} \rangle t_K^* \\ \text{or} \quad & t_K^* = \frac{1 - \langle (1 - g\hat{\epsilon})^{n/(n-v)} \rangle}{2nB\dot{\epsilon}_{c0} \langle \hat{\epsilon} \rangle} \end{aligned} \right\} \quad (67)$$

Finally, on dividing Eq. 67 by the deterministic result given by Eq. 55,

$$\frac{t_K^*}{t_{K0}^*} = \frac{1 - \langle (1 - g\hat{\epsilon})^{n/(n-v)} \rangle}{[1 - (1 - g)^{n/(n-v)}] \langle \hat{\epsilon} \rangle} \quad (68)$$

Equation 68 for brittle failure differs from the ductile case in two respects: (1) It is dependent on the pressure p , and (2) since there is zero probability that $\hat{\epsilon} > g^{-1}$, the lognormal density function for $\hat{\epsilon}$ is replaced with a truncated lognormal density function. Thus,

$$f(\epsilon) = (\Omega \sqrt{2\pi} \sigma_\tau \epsilon)^{-1} \exp[-(\ln \epsilon)^2 (2\sigma_\tau^2)^{-1}] [U(\epsilon) - U(\epsilon - g^{-1})], \quad (69)$$

where the normalization function is

$$\Omega = \frac{1}{2} + \operatorname{erf}\{[\ln(g^{-1})]\sigma_\tau^{-1}\},$$

and the error function $\operatorname{erf}(x)$ is defined as

$$\operatorname{erf}(x) \equiv \frac{1}{\sqrt{2\pi}} \int_0^x e^{-y^2/2} dy.$$

The expectation value of $\hat{\epsilon}$ is given by

$$\langle \hat{\epsilon} \rangle = H\Omega^{-1} \exp(\sigma_\tau^2/2), \quad (70)$$

where

$$H = \frac{1}{2} + \operatorname{erf}\left[\frac{\ln(g^{-1}) - \sigma_\tau^2}{\sigma_\tau}\right].$$

Again, as a specific example, consider a Type 304 stainless steel tube. Let $\sigma_K = 12,300$ psi, the stress causing creep rupture in a uniaxial creep rupture test in 10^5 hr (see ASME Code Case 1331-5), and let $v/n = 0.7$ as suggested by Kachanov.¹¹ Under these conditions, Eq. 68 may be evaluated in a straightforward manner for particular values of p and σ_τ . Typical results are listed in Table III.

TABLE III. Expectation Value of t_K^*/t_{K0}^*

p , psi	$\sigma_{\theta 0}$, psi	σ_T	t_K^*/t_{K0}^*
200	2,133	0.3463	0.99996
		0.6926	0.99975
		0.8658	0.99948
1153	12,300	0.3463	0.99792
		0.6926	0.98897
		0.8658	0.97856
2000	21,333	0.3463	0.99318
		0.6926	0.96472
		0.8658	0.93372

Examination of the foregoing results reveals that the time to brittle failure exhibits a relatively minor sensitivity to variations in temperature. The sensitivity increases slightly with pressure, but is not particularly significant with respect to the uncertainty involved in the physical model assumptions. This observation is in qualitative agreement with experiment. For example, Oding¹² gives results on rupture strength of a particular alloy for cylindrical specimens tested both at constant temperatures of 800 and 850°C, and cycled over the ranges 700-800-700 and 700-850-700°C. The rupture strength in the cyclic tests was equal to or slightly greater than that observed in the constant-temperature tests conducted at the maximum cycle temperature.

IV. EFFECTS OF RANDOM SPATIAL VARIATIONS IN TEMPERATURE ON CREEP PREDICTIONS

In previous sections, considering plane strain problems for thin tubes made it possible to treat temperature as a function of time only and thus simplify the analysis. However, in many reactor components it is necessary to determine the creep behavior of thick sections; therefore, the assumption of random temperature variations requires a study of the temperature process as a function of both space and time.

To assess some of the difficulties associated with such spatial variations, several sample problems were considered. Due to the mathematical complexities involved in adequately treating end conditions, finite cylinders were excluded. Instead, the analysis of an infinite, thick-walled tube was outlined. Initially, it was hoped that a further simplification would result if only irradiation-induced creep was considered, since, based on current theoretical and experimental evidence, it is reasonable to assume that strain is a linear function of stress for this creep mechanism. Unfortunately, this linear stress dependence is of minor importance when compared with the additional difficulties resulting from inclusion of the description of volume change associated with irradiation-induced creep.

In the uniaxial case and at temperatures below those for which thermal creep is significant, the following equation is typical of the form used in describing irradiation-induced creep strain:^{13,14}

$$\epsilon_{irr} = \tilde{A} \frac{\sigma}{E} \left[1 - \exp\left(-\frac{\phi t}{\tilde{B}}\right) \right] + \tilde{C} \sigma^n \phi t, \quad (71)$$

where ϕ is the integrated fast neutron flux (>0.1 MeV), t is irradiation time, and $\tilde{A}$, $\tilde{B}$, and $\tilde{C}$ are temperature-dependent constants. Based on current theoretical models and available experimental results, the irradiation creep exponent n is near unity for low stress levels. In what follows, it will be assumed that $n \approx 1$. Also, since the transient aspects of irradiation-induced creep in stainless steels saturate at relatively low fluence ($\phi t \sim 10^{21}$ n cm⁻²), the first term in Eq. 71 will be neglected. The remaining steady-state term has been attributed to vacancy cascade nucleation and subsequent collapse into dislocation loops. This cannot be considered a constant volume process; hence, generalization of Eq. 71 to multiaxial stress states cannot follow the conventional formulation in terms of deviatoric components of the stress tensor. Some attempts have been made to convert equations such as Eq. 71 into expressions in terms of effective stress and effective strain; however, these are based on rather limited experimental evidence and involve further assumptions concerning the stress dependence of irradiation-induced isotropic swelling. For example, Gilbert and Straalsund¹⁵ established a simple proportionality between the ratio of irradiation-induced effective strain to effective stress and the ratio of strain to stress in uniaxial data (where the data also contain effects of swelling). This allowed the steady-state irradiation-induced creep to be expressed in the form

$$\epsilon_e = d_1 \sigma_e \phi t, \quad (72)$$

where d_1 is some suitable temperature-dependent constant.

Since irradiation-induced creep is not a constant-volume process, it is reasonable to assume (as in Ref. 15) that

$$\dot{\epsilon}_{ij} = \gamma \sigma_{ij} + \xi s \delta_{ij}, \quad (73)$$

where γ and ξ are experimentally determined functions independent of the stress state

$$s = \frac{1}{3}(\sigma_r + \sigma_\theta + \sigma_z),$$

and δ_{ij} is the Kronecker delta. Here, it will be assumed that $\xi = \ell \gamma$, where ℓ is some proportionality constant.*

*Gilbert and Straalsund used limited data for Type 304 stainless steel to fix $\ell = 9$.

Consistency between Eqs. 72 and 73 requires that

$$d_1 = \gamma \left(1 + \frac{\ell}{3}\right); \quad (74)$$

thus,

$$\dot{\epsilon}_{ij} = \frac{d_1 \varphi}{(1 + \ell/3)} (\sigma_{ij} + \ell s \delta_{ij}) \equiv d_2 (\sigma_{ij} + \ell s \delta_{ij}). \quad (75)$$

McElroy et al.¹⁴ assumed that the function analogous to d_1 was a function of the temperature T and the neutron spectrum. Since exact results for the spectral dependence are not available, only the temperature dependence will be included. Data presented by McElroy show that the temperature dependence of irradiation-induced creep can be fitted with a simple exponential. From this, the final form considered for the constitutive equation becomes

$$\dot{\epsilon}_{ij} = d_3 \exp(-KT) (\sigma_{ij} + \ell s \delta_{ij}), \quad (76)$$

where d_3 is an appropriate flux-dependent parameter, and K is a known constant.

In the deterministic case, the temperature field in a thick cylinder having inner and outer radii a and b , respectively, is

$$T = \ell n r + \beta, \quad (77a)$$

where

$$\alpha = \frac{T_b - T_a}{\ell n(b/a)}, \quad \beta = \frac{T_a \ell n b - T_b \ell n a}{\ell n(b/a)}, \quad (77b)$$

and T_a and T_b are the inner- and outer-surface temperatures. If it is assumed that T_a and T_b are normal random variables $\hat{T}_a$ and $\hat{T}_b$, with means $\bar{T}_a$ and $\bar{T}_b$ and variances $\sigma_{T_a}^2$ and $\sigma_{T_b}^2$, the corresponding first-order density functions are then

$$\left. \begin{aligned} f(T_a) &= [\sqrt{2\pi} \sigma_{T_a}]^{-1} \exp[-(T_a - \bar{T}_a)^2 (2\sigma_{T_a}^2)^{-1}] \\ \text{and} \\ f(T_b) &= [\sqrt{2\pi} \sigma_{T_b}]^{-1} \exp[-(T_b - \bar{T}_b)^2 (2\sigma_{T_b}^2)^{-1}] \end{aligned} \right\} \quad (78)$$

It is further assumed that the joint density function of $\hat{T}_a$ and $\hat{T}_b$ is bivariate normal and is given by

$$f(T_a, T_b) = \left[2\pi\sigma_{T_a}\sigma_{T_b}(1-\rho^2)^{1/2} \right]^{-1} \exp \left\{ -[2(1-\rho^2)]^{-1} \left[\frac{(T_a - \bar{T}_a)^2}{\sigma_{T_a}^2} - \frac{2\rho(T_a - \bar{T}_a)(T_b - \bar{T}_b)}{\sigma_{T_a}\sigma_{T_b}} + \frac{(T_b - \bar{T}_b)^2}{\sigma_{T_b}^2} \right] \right\}, \quad (79)$$

where ρ is the correlation coefficient. Linear functions of normal random variables remain normally distributed, and if two random variables are jointly normal, then two functions of these two random variables are also jointly normal. The form of α and β ensures that $\hat{\alpha}$ and $\hat{\beta}$ are jointly normal. It is easily shown that, for $\hat{\alpha}$,

$$\bar{\alpha} = \langle \hat{\alpha} \rangle = \frac{\bar{T}_a - \bar{T}_b}{\ln(b/a)} \quad \left. \begin{array}{l} \text{and} \\ \sigma_{\hat{\alpha}}^2 = \langle \hat{\alpha}^2 \rangle - \langle \hat{\alpha} \rangle^2 = [\ln(b/a)]^{-2} (\sigma_{T_b}^2 - 2\rho\sigma_{T_a}\sigma_{T_b} + \sigma_{T_a}^2) \end{array} \right\}; \quad (80)$$

and, for $\hat{\beta}$,

$$\bar{\beta} = \frac{\bar{T}_a \ln b - \bar{T}_b \ln a}{\ln(b/a)} \quad \left. \begin{array}{l} \text{and} \\ \sigma_{\hat{\beta}}^2 = [\ln(b/a)]^{-2} [\sigma_{T_a}^2 (\ln b)^2 - 2\rho\sigma_{T_a}\sigma_{T_b} \ln b \ln a + \sigma_{T_b}^2 (\ln a)^2] \end{array} \right\}; \quad (81)$$

that is, $\hat{\alpha}$ and $\hat{\beta}$ are normal with means and variances as above. The joint density function is

$$f(\alpha, \beta) = \left[2\pi\sigma_{\alpha}\sigma_{\beta}(1-\rho_{\alpha\beta}^2)^{1/2} \right]^{-1} \exp \left\{ -[2(1-\rho_{\alpha\beta}^2)]^{-1} \left[\frac{(\alpha - \bar{\alpha})^2}{\sigma_{\alpha}^2} - \frac{2\rho_{\alpha\beta}(\alpha - \bar{\alpha})(\beta - \bar{\beta})}{\sigma_{\alpha}\sigma_{\beta}} + \frac{(\beta - \bar{\beta})^2}{\sigma_{\beta}^2} \right] \right\}, \quad (82)$$

where the correlation coefficient is

$$\rho_{\alpha\beta} = \frac{\langle (\hat{\alpha} - \bar{\alpha}) (\hat{\beta} - \bar{\beta}) \rangle}{\sigma_{\alpha}\sigma_{\beta}}$$

$$= \frac{\rho\sigma_{T_a}\sigma_{T_b} (\ln a + \ln b) - (\sigma_{T_b}^2 \ln a + \sigma_{T_a}^2 \ln b)}{\left\{ (\sigma_{T_b}^2 - 2\rho\sigma_{T_a}\sigma_{T_b} + \sigma_{T_a}^2) [\sigma_{T_a}^2 (\ln b)^2 - 2\rho\sigma_{T_a}\sigma_{T_b} \ln a \ln b + \sigma_{T_b}^2 (\ln a)^2] \right\}^{1/2}} \quad (83)$$

Since

$$\hat{T} = \hat{\alpha} \ln r + \hat{\beta}, \quad (84)$$

$\hat{T}(r)$ is a normal process, and its first-order density function, mean, and variance may be computed from the above expressions:

$$\left. \begin{aligned} f(T; r) &= (2\pi\sigma_T)^{-1} \exp \left[-\frac{(T - \bar{T})^2}{2\sigma_T^2} \right] \\ \bar{T} &= \bar{\alpha} \ln r + \bar{\beta} \\ \sigma_T^2 &= \sigma_{\beta}^2 + 2\rho_{\alpha\beta}\sigma_{\alpha}\sigma_{\beta} \ln r + \sigma_{\alpha}^2 (\ln r)^2 \end{aligned} \right\} \quad (85)$$

and

In the special case where $\hat{\alpha} \equiv 0$, i.e., where the temperature is a random variable, $\hat{T} = \hat{\beta}$, it is possible to show that the stresses are deterministic. The radial creep velocity u can be shown to be given by

$$\frac{\hat{u}}{u_0} = \exp[K(\bar{\beta} - \hat{\beta})], \quad (86)$$

where u_0 is the nominal value (the value at $\hat{T} = \bar{T}$). Define

$$\hat{\tau} \equiv K(\bar{\beta} - \hat{\beta});$$

thus,

$$\langle \hat{\tau} \rangle = 0,$$

$$\sigma_{\tau}^2 = K^2 \sigma_{\beta}^2,$$

and $\hat{\tau}$ is normal with mean and variance above. Since, from Ref. 1,

$$\frac{\hat{u}}{u_0} = e^{\hat{\tau}}$$

has a lognormal density function, it immediately follows that

$$\left\langle \left(\frac{\hat{u}}{u_0} \right)^n \right\rangle = \exp(n^2 K^2 \sigma_\beta^2 / 2),$$

$$\left\langle \frac{\hat{u}}{u_0} \right\rangle = \exp\left(\frac{K^2 \sigma_\beta^2}{2}\right) > 1,$$

$$\sigma_{u/u_0}^2 = \exp(K^2 \sigma_\beta^2) [\exp(K^2 \sigma_\beta^2) - 1] > K^2 \sigma_\beta^2,$$

and

$$\text{MAX} \left\{ \frac{\hat{u}}{u_0} \right\} = \exp(-K^2 \sigma_\beta^2) \ll 1.$$

In the general case, when $\hat{\alpha} \neq 0$, the statistical properties of the stresses and other quantities of interest can be obtained only by numerical means. Introducing a suitable stress-function formulation makes it possible to obtain expressions for the stresses in a form somewhat similar to that occurring in the deterministic case; however, this formalism is of use only in numerical studies, since it involves exponents which are themselves random functions. Since the constitutive equation, Eq. 76, on which this development rests is based on scant experimental evidence, it seems unreasonable to pursue the solution of this class of problems to the point where it would be necessary to treat such difficult numerical evaluations. Some indication of the uncertainty surrounding the formulation of constitutive descriptions of irradiation-induced creep can be gained by examining the various expressions in Ref. 16. It is immediately obvious that even the general trend of temperature dependence has not been established, nor has it been established that such a dependence exists.

For the above reasons, it is concluded that a study of the effects of spatial randomness in temperature on creep deformation cannot be profitably based on a "case study" using irradiation-induced creep. Simplifications that result from the linear stress dependence are more than outweighed by a lack of adequate theoretical and experimental descriptions of the physical process. In particular, the lack of conservation of volume during irradiation-induced creep and the lack of extensive experimental evidence concerning this phenomenon preclude a rational three-dimensional formulation of appropriate constitutive equations.

V. SUMMARY AND CONCLUSIONS

The problems considered in the previous sections have concentrated on the effect of random temperature on various aspects of creep of cylindrical shells. Although quantitative results are strictly applicable only to this

geometry and hold only for specific parameters used in the examples, certain qualitative features form a basis for more general conclusions.

For steady creep of a thin tube, a random temperature field was shown to result in an almost deterministic stress state. At the same time, predictions of the deformation rate were sensitive to temperature variations, increases in the variance of the temperature producing an increase in the variance of the deformation rate. If the latter is true in a general sense, it could have considerable impact on computations needed to satisfy design criteria. For example, if a design satisfies both the deterministic stress criteria and the strain limits that exist in elevated-temperature rules, and if both are near their maximum allowable values, any actual temperature fluctuations could result in violation of the strain limits while the stresses remain within the allowable range. Based on this consideration, designers should not approach the limit of allowable strains unless they have reliable temperature information. On the other hand, they should be able to approach the allowed stress levels, even when unsure of temperature variations about given nominal values.

For the most part, general conclusions on the effect of random temperatures on creep rupture are somewhat obscured by uncertainties relating to the mathematical model. However, based on the simple model used in Sec. III, it is reasonable to conclude that any randomness in the temperature can be reflected only in a prediction of a decrease in the time to failure. The results indicate that this decrease is negligible for brittle fracture, even for a very large variance of temperature. Qualitatively, this holds over a wide range of stress levels in the case considered.

Attempts to gain insight into the effect of random spatial variations in temperature, using the special case of irradiation-induced creep, were not successful. The simplicity of the relationship between strain and stress was more than offset by the lack of data and uncertainty about the correct form of the constitutive relationship. Although the analytical problem proved to be tractable, it was not possible to use this to obtain any conclusive numerical results.

From the above remarks, it appears that the greatest potential for extending this initial study lies in the area of creep deformation of specific structures, rather than in pursuit of more specialized topics such as irradiation-induced creep. Of particular interest is the effect of random temperature variations on creep-buckling predictions, since design restrictions and existing standards associated with this mode of failure are severe and involve large factors of safety. Any study that explores the sensitivity or predicted times to failure would aid the early stages of design by allowing a rational assessment of the applicability of such factors to cases in which definite information on temperature fields is unavailable.

APPENDIX

Deterministic Calculations Using Söderquist's Creep-damage Model

Söderquist⁹ has modified the Rabotnov and Kachanov creep and damage relations⁶ through the use of a simple phenomenological model. Assuming the temperature is deterministic and constant, the basic equations are

$$\dot{\epsilon} = \dot{\epsilon}_{c0} \sigma_c^{-n} \left(\frac{Q}{A} \right)^n (1 - D)^{-n} \quad (\text{A.1})$$

and

$$\dot{D} = C \left(\frac{Q}{A} \right)^v (1 - D)^{-v}, \quad (\text{A.2})$$

where D is a measure of damage due to creep ($0 \leq D \leq 1$), Q is a constant load, A is the specimen cross-sectional area, and C and v are constants in the damage law

$$\dot{D} = C \sigma_a^v, \quad (\text{A.3})$$

where σ_a is the stress acting over the undamaged area. This is essentially a simplification of the Rabotnov law in that the number of empirical constants are reduced.

For the thin-tube problem, Eqs. A.1 and A.2 become

$$\dot{\epsilon}_\theta = \dot{\epsilon}_{c0} \left(\frac{\sqrt{3}}{2} \right)^{n+1} \left(\frac{\sigma_\theta}{\sigma_c} \right)^n (1 - D_\theta)^{-n} \quad (\text{A.4})$$

and

$$\dot{D}_\theta = C \sigma_\theta^v (1 - D_\theta)^{-v}. \quad (\text{A.5})$$

Substitution of Eqs. 45 and 46 into Eq. A.4 gives

$$\alpha^{-(2n+1)} \dot{\alpha} = B \dot{\epsilon}_{c0} (1 - D_\theta)^{-n}, \quad (\text{A.6})$$

which is analogous to Eq. 47 used in the simplified treatment. Dividing Eq. A.6 by D_θ , as given in Eq. A.5, and using Eq. 46 then gives

$$\frac{1}{2(v-n)} \frac{d}{dt} [\alpha^{2(v-n)}] = \dot{\epsilon}_{c0} B C^{-1} \sigma_{\theta 0}^{-v} \left[\frac{-1}{1 - (n-v)} \right] \frac{d}{dt} (1 - D_\theta)^{1 - (n-v)} \quad (\text{A.7})$$

Equation A.7, together with the initial condition $\alpha = 1$ for $D_\theta = 0$, then yields

$$(1 - D_\theta)^{-n} = \{1 - g[1 - \alpha^{-2(n-v)}]\}^{-n/[1-(n-v)]}, \quad (\text{A.8})$$

where

$$g = \frac{1 - (n - v)}{2(n - v)} \frac{C\sigma_{\theta_0}^v}{\dot{\epsilon}_{c0}B}.$$

Substitution of Eq. A.8 into Eq. A.6 reduces the equation for α to the form

$$\dot{\alpha} = a\alpha^m(b + g\alpha^{-k})^q, \quad (\text{A.9})$$

where

$$a = B\dot{\epsilon}_{c0}, \quad m = 2n + 1, \quad b = 1 - g,$$

$$q = \frac{-n}{1 - (n - v)}, \quad \text{and} \quad k = 2(n - v).$$

Denoting by α_R the value of α at the time of brittle failure, use of the failure criteria Eq. 51, together with Eq. A.9, gives

$$t_K = \frac{1}{a} \left(\frac{\sigma_{\theta_0}}{\sigma_K} \right)^v \int_1^{\alpha_R} \frac{\alpha^{2v} d\alpha}{\alpha^{m(b + g\alpha^{-k})^q}}. \quad (\text{A.10})$$

Also, since t_K is defined as that particular value of the time to rupture for which the stress remains constant, it can be shown that

$$C\sigma_K^v t_K = \frac{1}{v + 1}; \quad \text{hence,}$$

$$g = \frac{1 + v - n}{2(n - v)(1 + v)} \frac{(\sigma_{\theta_0}/\sigma_K)^v}{\dot{\epsilon}_{c0} t_K B}.$$

From direct integration of Eq. A.9,

$$t_{s0}^* = \frac{1}{a} \int_1^{\alpha_R} \frac{d\alpha}{\alpha^{m(b + g\alpha^{-k})^q}}. \quad (\text{A.11})$$

Equations A.10 and A.11 together form a parametric representation of the time to brittle rupture.

To compare Eqs. A.10 and A.11 to the simpler result given by Eq. 55, a specific numerical example is considered. Let σ_K equal the stress causing rupture in 10^5 hr in uniaxial creep rupture test. For Type 304 stainless steel at 1100°F , this would give $\sigma_K = 12,300$ psi (see ASME Code Case 1331-5).

Kachanov¹¹ gives, as the limits on v , the expression $0.62 < v/n < 1$, and suggests 0.7 as a representative mean value. Table IV compares the numerical results obtained from Eqs. A.10, A.11, and 55 for various values of internal pressures.

TABLE IV. Comparison between Calculated Values
of $t_{K_0}^*$ and $t_{s_0}^*$

p, psi	σ_{θ_0} , psi	Time to Rupture, hr	
		$t_{K_0}^*$: Eq. 55	$t_{s_0}^*$: Eqs. A.10 and A.11
200	2,133	6.907×10^8	6.905×10^8
1153	12,300	9.895×10^4	9.743×10^4
2000	21,333	5.998×10^3	5.709×10^3

From these results it is noted that the modified theory shows a reduction in the time to rupture; however, the difference is too slight to justify the increase in computational complexity that would result from this theory.

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