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The Inevitability of
Gravitational Collapse

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ABSTRACT

We prove that a sufficient accumulation of cold, uncharged, nonrotating matter cannot be stable against gravitational collapse. The highlights of our proof are:

- (1) We make fewer assumptions than previous arguments for collapse. In particular, we refrain from making the plausible, but unproved, "causality" assumption.
- (2) The amount of matter is measured by its baryon number, not its mass, since mass can be radiated away.
- (3) The precise analog of the argument that the Schwarzschild radius is proportional to the radius cubed does not suffice as a proof of our theorem for a very simple reason. As a result, it may be that an incompressible fluid is not the most resistant to collapse.
- (4) A crucial step in the proof appears to result from the details of the equations, and we are unable to provide obvious physical insight into this mechanism.

It is commonly accepted that general relativity predicts that gravitational collapse is inevitable. If enough nonrotating cold, uncharged matter is accumulated in a small region it cannot remain in static equilibrium, according to this belief.

This belief is widespread¹, and yet the arguments in its favor have not been entirely compelling. It is the purpose of this paper to examine this hypothesis, to formulate a rather precise theorem, and to attempt to prove or disprove the theorem using rigorous arguments.

The question to which we are addressing ourselves is intermediate between two extremes. On one hand, inevitability of gravitational collapse in our hypothetical situation does not mean that "black holes" - which are the result of collapse - necessarily actually exist. It is a problem of astronomy to discover whether enough matter ever is present in a star at a sufficiently late stage of its evolution, and whether enough angular momentum is lost from the star in order to allow collapse. We are concerned with a gedanken experiment, which may not have astronomical parallels.

On the other hand, if any gedanken experiment is allowed, it is simple to construct situations which lead to collapse through the Schwarzschild radius. One has only to make an appropriate hollow region in the center of enough matter in the right sized region. Rather than this, we attempt to duplicate, to the extent possible, the situation in a star. We only ask if there is a maximum to the amount of matter that can be assembled into a configuration of stable static equilibrium. If such a

maximum exists, then the addition of more matter must presumably cause a collapse.

There have been many arguments in the past supporting the existence of a maximum to the amount of material. We wish to review a few of these arguments in order to point out the differences between those arguments and ours, and where our argument goes beyond them.

A common argument for collapse concerns the relation between the Schwarzschild radius and the actual radius of the star. The Schwarzschild radius is given in terms of the star's mass by

$$R_S = 2G M$$

where G is the gravitational constant, and here as throughout this paper we choose units in which

$$c = 1$$

The actual radius can also be expressed in terms of the star's mass as

$$R = \left(\frac{M}{\frac{4}{3} \pi \rho} \right)^{1/3}$$

where ρ is the average density. Thus the ratio of these radii is

$$R_S/R \propto M^{2/3}$$

As M is increased, eventually this ratio surpasses unity, and collapse must occur then if not before.

One difficulty with the above argument is that it assumes that the average density of the star does not decrease as its mass increases. Although this is straightforward to prove in nonrelativistic gravity, it could possibly be violated in general relativity.

Another plausibility argument can be made. In nonrelativistic gravity mass attracts mass, but in order to be consistent with relativity pressure must also attract pressure. Therefore the gravitational attraction is proportional to the pressure squared or larger, and will eventually be able to overcome the repulsive force, given by the gradient of the pressure which is only proportional to the first power of the pressure.

This argument has the same sort of difficulty as the first. It may be that the average pressure decreases as more matter is added to the star, or it may be that the pressure never gets sufficiently large in order to have this argument apply.

Another reason to believe that collapse is inevitable is that all model equations of state which have been studied exhibit collapse. Most notably, both constant density matter and matter made of a degenerate free fermi gas of neutrons both show collapse. It is plausible that constant density is the equation of state most resistant to collapse. If that is the case, the original argument that $R_g/R \propto M^{2/3}$ holds, and collapse is inevitable. We shall return to this question, and in fact we shall argue that it appears that non-constant density equations of state may actually be more resistant to collapse.

The remainder of the model calculations suffer from another defect. It is generally arranged that these model equations of state obey the so-called "causality" property. In fact, it has been established¹ that this "causality" property is sufficient to make collapse inevitable. It is therefore important to examine this property, to see whether it might be violated, and whether a violation might be likely.

The basic argument for this property is that sound waves cannot travel faster than the speed of light in vacuum. The velocity (group velocity) of sound at very low frequencies is $\sqrt{dP/d\rho}$. Therefore, the argument goes, $dP/d\rho < 1$, which can be integrated to give $P < \rho$. This is the "causality" property. The step of this argument which is not rigorous is that the group velocity of waves may exceed C . In fact, examples of this phenomenon are known². The velocities which are required to be less than C are the signal and energy transport velocities (and in addition, the infinite frequency limit of the phase velocity). These velocities are, in usual situations, close in value to the group velocity, and may be replaced by the group velocity for most practical applications. However, if superdense matter is unusual (in this sense), then the causality property $P < \rho$ cannot be established for such matter³.

Is it likely that superdense matter is unusual? This question cannot be answered without more knowledge about the properties of such matter than we have at present. The group velocity of low frequency sound waves is unlikely to be significantly different

from the signal and energy transport velocities unless there is strong absorption of low frequency sound. Thus low frequency excitations different from sound may provide a mechanism for the breakdown of the "causality" property of the equation of state. The question posed at the beginning of this paragraph reduces to the question of whether such excitations exist.

In superdense matter the nucleons are touching or even overlapping. It is known that there is a strong attraction between pi mesons and nucleons. Thus the effective mass of the pi mesons will be greatly reduced by the presence of the nucleons. It is possible that pi mesons might be able, under such conditions, to exist at very low frequencies, and provide an excitation into which sound waves can decay. Therefore, until more is known about the equation of state, it is not safe to rule out a possible violation of the "causality" property.

For the benefit of those readers who are somewhat familiar with this line of reasoning, we point out that the condition of stability, i.e., that the matter under consideration be in its ground state, puts further conditions on the relationship between the strong absorption of sound and the violation of $p < \rho$. In order to be consistent with such stability, we require the strong absorption to occur at a frequency at which the signal and energy transport velocities, and probably the group velocity are in the opposite direction to the phase velocity. Otherwise a dispersion relation essentially equivalent to the Kramers Kronig relation for the index of refraction of light could be used to establish $p < \rho$.

Almost all of the arguments which we have so far discussed have a further difficulty. These arguments assume that the amount of matter is measured by its mass. However, as one imagines matter being added to a star, hydrogen atom by hydrogen atom, an amount of radiation, in the form of photons and neutrinos, will be emitted from the star. The mass of this radiation may be almost exactly as much mass as was added. Thus it is possible that the total mass of a star is bounded although an unlimited amount of matter is added.

A better measure of the amount of matter is provided by the baryon number of the star. According to current knowledge, baryon number and charge are the only absolutely conserved quantities which cannot be carried by massless particles. All other quantities which might be used to measure the amount of matter can be radiated away.

Since charge couples to the long range coulomb force it is to be expected that a charged star will attract charged matter of the opposite sign charge. We may reasonably expect, therefore, that a superdense star will be nearly uncharged. Thus baryon number remains the only good measure of the amount of matter.

One further well known comment on the equation of state is needed. The temperature imagined for neutron stars is well below the Fermi energy of the nucleons. Thus the equation of state is essentially independent of the temperature in this region. It seems safe to assume that we may treat the star as cold, i.e., $T = 0$.

We are now in a position to state the theorem which we wish to prove or disprove. It is:

Theorem There exists a number $N_{\max}$ such that every possible uncollapsed cold superdense star in a condition of stable static equilibrium has a baryon number less than $N_{\max}$, under the following assumptions:

- (1) General relativity
- (2) The density and pressure of matter in stable equilibrium are given uniquely in terms of the baryon number density.
- (3) The pressure and density tend to infinity together. In particular the pressure cannot become infinite while the density remains finite (This is probably a true result of causality.)
- (4) There is no massless particle which carries baryon number. The lightest thing which carries baryon number (e.g., iron metal) has a mass/baryon number $= \mu_0 > 0$, and at zero pressure it has a finite density $\rho_0 > 0$.
- (5) The pressure is allowed to become slightly negative. However $-P < \rho/3$.
- (6) The star is spherically symmetric. In particular it is not rotating.

Assumption (2) has already been discussed. All quantities not given in terms of baryon number density are assumed to have been radiated away. For example neutrinos or antineutrinos were emitted in order to adjust the lepton number density to a value appropriate to the baryon number density. Assumption (3) is

much weaker than the "causality" property $P < \rho$. We allow equations of state in which P can exceed ρ by any amount, just so P is finite if ρ is finite. Assumption (4) is made in order to know the conditions on the outside of the star. It can easily be replaced by considerably weaker assumptions, such as that the star have finite radius (although not necessarily a radius bounded independently of the baryon number of the star). However, since this assumption is almost certainly valid, we will not discuss the weaker forms. Assumption (5) is a statement about matter under ordinary conditions. At pressures near zero solid iron is the stable form of matter. For iron, or any other element or combination of elements which happen to be in a region of low pressure, the theoretical maximum tensile strength can be calculated. This number is determined by the chemical energy per atom which is about a factor 10^8 smaller than the mass of the atom. Thus the assumption $P + \rho/3 > 0$ is safe. The pressure cannot become negative at superhigh densities because of assumption (3). Assumption (6) is made in order to exclude rotation. It is expected that all other deviations from spherical symmetry are damped out, with the possible exception of the magnetic field of the star. Assumption (6) also rules out magnetic fields.

There are two conditions on the equation of state that can be derived⁴. The first of these is that stability requires the pressure and density to vary monotonically. We write this as

$$\frac{d\rho}{dP} > 0, \quad (1)$$

with the understanding, however, that phase transitions may be included. (These correspond to $d\rho/dP = a \delta(P - P_0)$ with $a > 0$). The second condition follows from conservation of energy. The volume per baryon is $1/n$ where n is the baryon number density. The work done per nucleon in compressing matter is equal to the nucleon's gain in energy. Thus $-P d(1/n) = d(\rho/n)$, from which $\rho/n^2 dn = d\rho/n - \rho/n^2 dn$. The "chemical potential μ is defined as

$$\mu \cdot dn = d\rho \quad (2)$$

The energy condition then becomes

$$\mu n = P + \rho \quad (3)$$

We differentiate this, and use (2) to obtain a useful relation

$$nd\mu = dP \quad (4)$$

The remainder of the equations we need are consequences of general relativity. The 3 dimensional space metric is given by

$$d\ell^2 + r^2(d\theta^2 + \sin^2 \theta d\varphi^2).$$

(i.e., a circle at distance ℓ from the center of the star has a circumference $2\pi r$.) The two radial coordinates ℓ and r are related: $r = r(\ell)$. We define S by $S = \frac{dr}{d\ell}$. Denoting the number of baryons inside the radius r by $N = N(r)$, and derivatives with respect to r by a prime, we have

$$\frac{dN}{4\pi r^2 d\ell} = n$$

or

$$N' = 4\pi r^2 n/S \quad (5)$$

The derivative of the mass m of material inside r is given by a similar formula. However it must be corrected for the gravitational binding energy

$$m' = 4\pi r^2 \rho / S - \text{gravitational binding}$$

The result of general relativity is

$$m' = 4\pi r^2 \rho \tag{6}$$

Actually, since there is no operational meaning to the mass, Eq.(6) can be taken as a definition of $m(r)$. It agrees with the externally observed mass at the surface. Note that Eq.(6) only differs from its non-gravitational part by the absence of the denominator S . We shall show that $0 < S \leq 1$, so that the gravitational binding decreases m as expected.

It turns out that the quantity S is given by the mass m internal to the radius r , and by r , but by no other quantity. The value of S is

$$S = \sqrt{1 - 2Gm/r} \tag{7}$$

This is precisely the same as its form external to the star. Only m has a slightly more general meaning.

The remaining equation is the generalization of the nonrelativistic equation of hydrostatic equilibrium

$$P' = - \frac{G m \rho}{r^2} + \text{relativistic corrections.}$$

This equation becomes the Tolman-Oppenheimer-Volkoff equation

$$P' = - \frac{G(m + 4\pi r^3 P)(\rho + P)}{r^2 S^2} \tag{8}$$

The proof consists in manipulating these equations. The equations we have given are standard, and appear for example in Ref. 1.

Equation (8) can be divided by n , and using equations (3) and (4) can be written as

$$\mu' = - \frac{G(m + 4\pi r^3 P) \mu}{r^2 S^2} \quad (9)$$

This shows $\mu' < 0$. By using Equations (1), (2), and (4), and $\mu_0 = \mu$ on the surface > 0 , we immediately obtain $P' < 0$, $\rho' < 0$, $n' < 0$. The equation $\rho' < 0$ allows us to bound the mass m :

$$\begin{aligned} m(r) &= \int_0^r \rho(r') 4\pi r'^2 dr' > \rho(r) \int_0^r 4\pi r'^2 dr' \\ &= \frac{4}{3} \pi r^3 \rho(r) \end{aligned} \quad (10)$$

Therefore, in particular, $m > 0$ except at $r = 0$, and by Eq. (7), $S < 1$.

In the case of a constant density equation of state, S can become negative. This can be understood by analogy with the equator on the surface of the earth. In the Southern hemisphere, as the distance ℓ from the north pole increases, the circumference $2\pi r$ of a circle of latitude becomes smaller. Therefore $S = dr/d\ell$ is negative. In that case r is a misleading coordinate to use since two values of ℓ - one in the northern hemisphere and one in the southern - have the same value of r . We will prove that, under the assumptions we have made, there cannot be an "equator" for a superdense star. We will assume that there is an equator, and show a violation of our assumptions. At the "equator"

$$s_e = 0,$$

and it follows that

$$2Gm_e = r_e$$

where we put the subscript e to denote quantities evaluated at the "equator." Since the mass is finite, the density ρ_e , which is less than $m_e/(4/3\pi r_e^3)$ is also finite. From Equation (7) we calculate

$$\begin{aligned} \left| (s^2)_e' \right| &= \left| -\frac{2Gm_e}{r_e} + \frac{2Gm_e}{r_e^2} \right| \\ &= \left| -8\pi G r_e^2 \rho_e + \frac{2G m_e}{r_e^2} \right| < \infty \end{aligned}$$

Thus s^2 vanishes linearly in $(r-r_e)$ (or faster), i.e., $s^2 \leq \text{const.}(r-r_e)$.

We can write Equation (8) near $r = r_e$ as

$$\begin{aligned} |P'| &\approx \left| \frac{\text{const.} \left(P + \frac{m_e}{4\pi r_e^3} \right) (P + \rho_e)}{s^2} \right| \\ &\geq \left| \text{const.} \left(P + \frac{m_e}{4\pi r_e^3} \right) (P + \rho_e) \frac{1}{r-r_e} \right| \end{aligned}$$

This can be integrated to give

$$\left| \ln \frac{P + \rho_e}{P + \frac{m_e}{4\pi r_e^3}} \right| \geq \text{const.} \ln |r-r_e| + \text{const.}$$

The right side of this equation is infinite at the equator, so the left side must be also. This requires either $P + \rho_e = 0$ or $P + \frac{m_e}{4\pi r_e^3} = 0$. However, we have made the reasonable assumption that $P + \rho/3 > 0$. Therefore

$$P + \rho_e = (P_e + \rho_e/3) + \frac{2\rho_e}{3} > 0$$

and

$$P + \frac{m_e}{4\pi r_e^3} = (P_e + \rho_e/3) + \frac{m_e - \frac{4}{3}\pi r_e^3 \rho_e}{4\pi r_e^3 \rho_e} > 0.$$

Therefore no equator is possible. It follows that r and ℓ are uniquely related, and r can be used as the radial coordinate with no complications. Moreover, at $r = 0$, $S = 1$ and since S cannot ever be zero we have $S > 0$.

At the surface of the star $P = 0$, $\rho = \rho_0 > 0$, $\mu = \mu_0 > 0$, $n = n_0 > 0$. Since P' , ρ' , μ' and n' are all negative we have $P > 0$, $\rho > \rho_0$, $\mu > \mu_0$, $n > n_0$ everywhere in the star.

We now turn to the main part of our proof. We want to bound the baryon number N . The plausability argument for a bound on the amount of matter in a superdense star uses the mass instead of the baryon number. This suggests multiplying the numerator and denominator of Eq.(5) by μ which is the mass per baryon number. We obtain

$$\begin{aligned} N' &= 4\pi r^2 n\mu/\mu S \\ &= 4\pi r^2 (\rho+P) / \mu S \end{aligned} \quad (11)$$

We treat the numerator and denominator separately:

The numerator is expected to be about $3m/r$, and if $2Gm/r \approx 1$, it will be roughly $3/2G$. Thus we should attempt to bound it by a constant. That can be done as follows:

$$\begin{aligned} -(\mu n)' &= \mu(-n') + n(-\mu') > n(-\mu') \\ &= \frac{G(m + 4\pi r^3 \rho) \cdot n\mu}{r^2 S^2} \end{aligned}$$

m is larger than $4/3\pi r^3 \rho$ and $4\pi r^3 \rho$ is larger than $4/3\pi r^3 P$. Therefore their sum is larger than $4/3\pi r^3 (\rho + P)$

$$\frac{4}{3}\pi r^3 (\rho + P) = \frac{4}{3}\pi r^3 n\mu.$$

Moreover $S < 1$ so $1/S > 1$. Thus

$$\begin{aligned} -(\mu n)' &> \frac{G\left(\frac{4}{3}\pi r^3 n\mu\right)(n\mu)}{r^2} \\ &= \frac{4}{3}\pi G r (n\mu)^2 \end{aligned}$$

Dividing by $(\mu n)^2$ we get

$$\left(\frac{1}{\mu n}\right)' > \frac{4}{3}\pi G r.$$

Integrating:

$$\frac{1}{\mu n} > \frac{2}{3}\pi G r^2 + \left(\frac{1}{\mu n}\right)_{r=0} > \frac{2}{3}\pi G r^2.$$

So that

$$4\pi r^2 n\mu < \frac{6}{G} \tag{12}$$

This rigorous bound is 4 times the rough estimate.

We now turn to the denominator of Equation (11). Using Equations (7), (9), and (6) we obtain

$$(\mu S)' = -4\pi r G n \frac{\mu^2}{S} \quad (13)$$

which is negative. Therefore, if $r_1 < r$

$$\mu(r_1)S(r_1) > \mu(r)S(r) \quad (14)$$

Using Equations (11), (12), and (14), we find

$$N'(r_1) < \frac{6}{G \mu(r) S(r)} \quad \text{if } r_1 < r.$$

Integrating this from $r_1 = 0$ to $r_1 = r$, we get

$$N(r) < \frac{6r}{G \mu(r) S(r)}$$

Choosing R to be the radius of the star we get

$$N_{\text{tot}} < \frac{6R}{G \mu_0 S(R)} \quad (15)$$

It is possible to bound R . However it is not possible to bound $1/S$. S must be greater than zero, but may be arbitrarily close to zero.

A physical interpretation of this result can be made. The baryon number is roughly proportional to the volume of the star. However the volume is roughly a factor $1/S$ larger than the flat space volume $4/3\pi R^3$. Thus if S is very close to zero, the volume can be made extremely large and many baryons can be put in the star.

Therefore, it may well be that compressible matter can cause more baryons to be allowed in a superdense star than constant

density matter. If the density is very high near the center, S will quickly get very small. Then if the density is moderate away from the center many baryons can be put in the large volume where $S \approx 0$ without increasing the star's mass much. The plausibility arguments for collapse based on the constant density model, as well as the upper limit for the baryon number of a nonrotating neutron star derived from such arguments may be very misleading. We must allow the star's volume to be large.

Can the bound (15) be improved? Or does there exist an equation of state for which N_{tot} is unbounded? If the former is true the theorem is proved, while if the latter is true the theorem is disproved. We shall show that, in fact, the bound can be improved and that the theorem is true.

From Equation (13) we see that $(\mu S)'$ is proportional to $\mu^2/S = \mu^3/(\mu S)$. The exponent 3 in the numerator of this expression turns out to be equal to the number of dimensions of space. This mathematical fact allows the proof to be completed. We find that

$$\left(\frac{\mu^3}{\mu S}\right)' = -\frac{3G\mu^2}{r^2 S^3} \left[\left(m - \frac{4}{3}\pi r^3 \rho\right) + \frac{8}{3}\pi r^3 p \right] \quad (16)$$

The three in the coefficient of ρ came from the exponent 3 of μ . Because space has three dimensions, we were able to show that

$$m - \frac{4}{3}\pi r^3 \rho > 0.$$

Therefore

$$\left(\frac{\mu^3}{\mu S}\right)' < 0.$$

If $r_1 > r$,

$$\frac{\mu^2(r_1)}{S(r_1)} > \frac{\mu^2(r)}{S(r)}$$

and $n(r_1) > n(r)$.

Therefore, Equation (13) gives

$$-\frac{d}{dr_1} (\mu(r_1)S(r_1)) > 4\pi r_1 G n(r) \mu^2(r) / S(r) \quad (17)$$

which integrates to

$$\mu(r_2) S(r_2) - \mu(r) S(r) > 2\pi G(r^2 - r_2^2) n(r) \mu^2(r) / S(r) \quad (18)$$

where r_2 is any radius smaller than r . Using the Schwarz inequality

$$\begin{aligned} \mu(r_2)S(r_2) &> \mu(r)S(r) + 2\pi G(r^2 - r_2^2) n(r) \mu^2(r) / S(r) \\ &\geq \sqrt{8\pi G(r^2 - r_2^2) n(r) \mu^3(r)} \end{aligned} \quad (19)$$

The right hand side is independent of $S(r)$, which represents an improvement over the bound $\mu(r_2)S(r_2) > \mu(r)S(r)$. The price paid for this improvement is the factor $\sqrt{r^2 - r_2^2}$ which now appears.

We now insert bounds (12) and (19) into (11)

$$N' < \frac{6}{\sqrt{8\pi G^3(r^2 - r_2^2) n(r) \mu^3(r)}} \quad (20)$$

and integrate from $r_2 = 0$ to $r_2 = r$

$$\begin{aligned}
 N(r) &< \frac{6}{\sqrt{8\pi G^3 n(r) \mu^3(r)}} \int_0^r \frac{dr_2}{\sqrt{r^2 - r_2^2}} \\
 &= \frac{3\pi}{\sqrt{8\pi G^3 n(r) \mu^3(r)}} \\
 &= \sqrt{\frac{9\pi}{8 G^3 \mu^2(r) (P(r) + \rho(r))}} \quad (21)
 \end{aligned}$$

Evaluating at the surface $r = R$

$$N_{\text{tot}} < \sqrt{\frac{9\pi}{8G^3 \mu_o^2 \rho_o}} \quad (22)$$

which proves the theorem.

The reader will notice that the proof from Equation (16) on has been entirely mathematical. I do not have a physical interpretation of this part of the proof.

It is of interest to compare bound (22) with (15). We bound R by

$$\frac{4}{3} \pi R^3 < \frac{m}{\rho_o} < \frac{R}{2G \rho_o}$$

or

$$R < \sqrt{\frac{3}{8\pi G \rho_o}}$$

Thus (15) becomes

$$N_{\text{tot}} < \sqrt{\frac{108}{8\pi G^3 \mu_o^2 \rho_o S^2}}$$

So even if S is replaced by 1, (22) is a slightly stronger bound

(by a factor of $\sqrt{12/\pi}$).

The numerical value of the bound (22) is very poor. We have

$$N_{\text{tot}} < \sqrt{\frac{9\pi}{8 G^3 \rho_0}} \quad (23)$$

If we put in the density of iron, 8 gm/cc, we get $N_{\text{tot}}/\mu_0 < 5 \times 10^8$ solar masses. However Equation (21) can be used at as high a density as the equation of state is known and the nucleon number of the remainder of the star can be found by integration. Very little matter will be at densities less than nuclear density. Thus if we put $\rho_0 =$ nuclear density in Eq.(22) we get the much better bound ~ 100 solar masses. This limit is still about two orders of magnitude above limits estimated on the basis of model equations of state. The rigorous limit can probably be improved somewhat. However it does indicate that a sufficiently hard equation of state may give a surprisingly large limit to the amount of matter in a neutron star. Such a situation might have important consequences to astrophysics.

To summarize the main points of this paper:

- (1) The so-called "causality" condition on the equation of state may be invalid.
- (2) Baryon number should be used to measure the amount of matter in a superdense star.
- (3) There does exist an upper limit to the baryon number of a cold nonrotating "neutron" star.
- (4) However, because of the possibility that the ratio of true volume to flat space volume can be large, the rigorous proof of the inevitability of collapse depends on

subtle mathematical details.

(5) The maximum baryon number may be larger than usual estimates.

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