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MATRIX INVERSION ON IBM 650 -PROGRAM 40

by

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PROGRAM 40: MATRIX INVERSION ON IBM 650

1.

General Description

This program can be used to solve equations of the form

$$|M| |x| = |b| \quad (1.)$$

where M is the matrix given by

$$M = \begin{vmatrix} e_{11} & e_{12} & \dots & e_{1n} \\ \vdots & & & \vdots \\ e_{n1} & \dots & \dots & e_{nn} \end{vmatrix} \quad (1.2)$$

b is the column vector given by

$$b = \begin{vmatrix} b_{11} \\ b_{21} \\ \vdots \\ b_{n1} \end{vmatrix} \quad (1.3)$$

and x's are the solutions of the equations. The program uses floating point arithmetic. The inverse matrix is also available when the calculation is completed. Any number of b vectors within the limits of storage capacity can be used at the same time as the matrix inversion is being performed, yielding a corresponding number of solutions to the sets of equations.

Preparation of Data

A special blank form is available for the preparation of data for this program. The sheet is arranged so that four cards may be keypunched from each sheet, using 80 columns per card. The first word (of 10 digits) is to the left and is used as a control word for the data loading. Digit 1 (low order digit) is used to designate the number of words of data to be loaded from the card. In general 7 words should be loaded per card except for the last card which will contain only as many words as are needed. Digits 5 through 8 are used to designate the address at which the first word of data is to be located. Subsequent words of data are loaded into successive positions in the memory. The address on the first card is 0000, and successive cards will be 0007, 0014, 0021, etc. The last word of data must have an address of less than 1764.

Subsequent data is used to store the elements of the matrix and the b vectors. The elements of the M matrix and of the b vectors have the following locations in storage.

$$\begin{array}{l}
 0000; 0001; 0002; \dots (n+b)-1 \\
 n+b; \dots z(n+b) \\
 \vdots \\
 \vdots \\
 \vdots \\
 \vdots \\
 (n-1)(n+b); \dots (n-1)(n+b)+(n+b)-1 \\
 n(n+b) \dots n(n+b)+(n+b)-1
 \end{array}$$

(2.1)

Thus it is seen that data is entered in the following order: First row of M matrix, first row of b vectors, second row of M matrix, second row of b vectors, etc.

An element e_{nn} is written as

$$e_{nn} = A \times 10^a, \quad A = x.xxx \text{ xxxx} \quad (2.2)$$

The exponent plus 50 is entered in the 2 high order digits of the entry word. The mantissa is the 8 low order digits of the word. The sign of e_{nn} is entered in the space following the word.

Results

The results are obtained in two forms. First, the inverted matrix and x columns are punched out from drum storage, using 7 words per card. The inverted matrix and the x columns have the following arrangement in storage, occupying the same locations on the drum as the original M matrix and b vectors.

$$\begin{array}{c} \left| \begin{array}{ccccccc} X_{11} & X_{12} & \cdots & X_{1b} & e'_{11} & e'_{12} & \cdots & e'_{1n} \\ X_{21} & & & & & & & \vdots \\ \vdots & & & & & & & \vdots \\ \vdots & & & & & & & \vdots \\ \vdots & & & & & & & \vdots \\ \vdots & & & & & & & \vdots \\ \vdots & & & & & & & \vdots \\ X_{n1} & \cdots & X_{nb} & e'_{n1} & \cdots & \cdots & e'_{nn} \end{array} \right| \end{array}$$

The second set of output gives the x columns in column form, up to a maximum of 7 columns per card. Successive cards are used for successive rows. If more than 7 column vectors are used, additional sets of cards are punched. The results from this second set will appear as follows:

01	x_{11}	x_{12}	.	.	.	.	x_{17}
02	x_{21}	x_{22}	.	.	.	.	x_{27}
.							.
.							.
.							.
.							.
.							.
.							.
.							.
.							.
.							.
.							.
n	x_{n1}	x_{n2}	.	.	.	.	x_{n7}

Keypunching Instructions

Data should be keypunched from the special data sheets using 80 columns per card. The first word of a card is the 10 digits to the left. The sign of words 2 through 8 must be entered in Columns 20, 30, 40, 50, 60, 70 and 80. A 12-punch should be used for a positive number and a 11 punch for a negative number. In addition Columns 1 and 10 must have a 12 punch. The key punch data need not be printed. Any blank space to the right of each field should be punched as zeros. The last card may have blank fields, which should be punched as zero. All columns must have a digit punch (0-9), and Cols. 20, 30, 40, 50, 60, 70 and 80 must have an 11 or 12-punch.

Special Cards

Two special control cards are needed. Special forms are available for this purpose.

Card 1. The first word of data is a control word written as follows: (n) 000000 (n / b) where n is the order of the matrix and b is the number of b vectors. If, for example, we have a 14th order matrix with 4 b vectors, the control word is 1400000018. If only the inverse of the matrix is desired the control word is 1400000014. This is not a load card. Two cards are required.

Card 2. This card contains the following information:

Columns 1 - 10 69 0000 8003

" 11 - 20 69 (LA) 8003

where LA is the address of the last element of the matrix. This can be computed from the formula

$$LA = (n) (n / b) - 1$$

This card is a load card, and must have 12 punches in Columns 1, 10 and 20.

Control Panel Wiring

Load cards are identified by a 12-punch in Column 1. Non-load cards must be wired to read 8 - 10 digits words. Punching is 8 - 10 digits words.

Program Operation

The Console is set at 70 1951 1951. Keypunched matrix data is inserted behind block A, followed by control card 1. Block B is followed by control card 2. Block C is followed by control card 1.

Time Requirements

The approximate time required for the calculation is $0.072 n^3$ seconds, where n is the order of the matrix.