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**A TECHNIQUE
FOR GENERATING PRESSURE-VOLUME RELATIONSHIPS
AND FAILURE ENVELOPES FOR ROCKS**

T. R. Butkovich

November 1, 1973

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Abstract

A technique has been devised for generating pressure-specific volume (P - V) and maximum shear stress-mean stress (τ - $\bar{P}$) relationships for rocks. These relationships are necessary input for making underground nuclear explosion code calculations. With the new technique, it is possible to obtain good estimates of the P - V relationship for various types of rocks. It is also possible to obtain good estimates of the τ - $\bar{P}$ relationship for rocks with weak intergranular bonds; such rocks are characterized by tuffs, alluviums, and saturated rocks in which pore pressure controls the strength of the rock. The results produced by this technique should prove useful in doing preliminary calculations and parameter studies without undertaking an extensive rock property measurement program. The necessary input parameters for generating these relationships are bulk density, grain density, weight fraction water, and elastic bulk modulus.

A TECHNIQUE FOR GENERATING PRESSURE-VOLUME RELATIONSHIPS AND FAILURE ENVELOPES FOR ROCKS

To study the effects and application of underground nuclear and high explosive detonations, a rather complete knowledge of the behavior of surrounding rocks under shock wave induced stress conditions is needed. Extensive experimental evidence shows that effects of the explosion; the attenuation of the shock wave generated by the explosion, the extent of ground motion (that is, surface acceleration and displacement), cavity size and shape, and the volume of rock broken; are all strong functions of the properties of the rock medium in which the detonation occurs.

Calculations made to predict the effects of the explosion require knowledge of the pressure-specific volume (P-V) relationship for the rock medium. Tests have been devised to measure the Hugoniot equation-of-state under shock conditions. Techniques are also available to measure hydrostatic compressibility to 40 kbar, and shear-stress vs mean-stress failure envelopes from triaxial shear-strength tests. These experiments are generally costly and time consuming.

Many geological materials that are of interest may differ only slightly in mineral constituents. They can have, however, a variety of porosities and water saturations that affect the P-V relationship and the strength curves. In a measurement program it is customary to in-

vestigate each composite separately. The samples are normally prepared from cores taken from different depths at the proposed explosion site.

Other investigators have recognized the need for developing P-V relationships based on easily determinable rock properties (such as bulk density, porosity, and water content) measured in situ or from samples recovered from the explosion site. These relationships would then be used in making predictions or for parameter studies using computer codes designed for underground explosion phenomenology calculations. Obviously, measurements on rock samples provide the best results, but because of time and expense it is not always possible to obtain such measurements.

Dennen¹ has taken the mineral constituents of rock masses and, with a limited knowledge of the compressibility and fraction of each constituent, has constructed Hugoniot equations-of-state for some rock types. More recently Schuster and Isenberg² have designed mathematical models for calculating P-V relationships for geological materials subjected to shock loading. Results from their models compare well with P-V data. Results are also given by Schuster and Isenberg along adiabatic release paths. They indicate that their model must be used cautiously below 50 kt, because variation in

density, porosity, and water content can dramatically affect the low pressure properties. Pressure-Volume curves have been constructed for a 40% average dry porosity tuff, comparing calculations for four different water saturations: dry, 50%, 90% and 100% (Ref. 3). These results showed that for the same energy yield and depth of burst, vastly different attenuations of the ground shock and different motions are produced in the same rock having different water contents. Crowley,⁴ in a continuation of this work was also interested in investigating the attenuation of the shock front as it propagates through rock masses of various porosities and saturations, obtained similar results by calculation.

Besides the multiminerale nature of rock, there occurs in many rocks, porosities that vary from a fraction of 1% to as much as 50% or more by volume. In most cases at least some of the porosity is water filled, depending on local conditions and the presence of a water table. Therefore, in constructing P-V relationships, three things have to be considered: the compressibility of the solid matrix of the rock, that is, the portion that contains no pores; the compressibility of the water component of the rock; and the irreversible removal of the air filled porosity on compression, namely the compaction of the rock. The relationship between the specific volume of the solid matrix rock, water and air component all at the same pressure can be expressed by

$$V_S = \frac{V_R + Z(V_W - V_R)}{1 - \psi}, \quad (1)$$

where V is specific volume, and $V \equiv 1/\rho$; ρ is the density in g/cm^3 . The subscripts S, W, and R refer to the rock-air-water system, water, and the solid matrix of the rock, respectively. The symbol Z is the weight fraction water and ψ is the air filled porosity. Both V_R and V_W are functions of pressure, and Z is constant with pressure. With increasing pressure and compaction, ψ decreases to near zero.

A typical P-V curve for a rock is shown in Fig. 1. This will help illustrate some of the assumptions that were made. The value of V_0 is $1/\rho_0$, the measured bulk density of the rock-air-water system. The rock is assumed to be elastic to some threshold pressure P_T , where the

Fig. 1. Typical pressure-volume relationship for rock with some air-filled porosity.

air filled porosity begins to collapse. As pressure is increased above P_T , the air filled porosity is removed irreversibly. At the merge pressure P_m , all the air filled porosity has been removed. If P is

released from P_m or above, the release path will go to V where $\psi = 0$. Release from maximum pressures between P_T and P_m will return to between V_0 and $V(t = 0)$.

Compressibility of Grain Density Rock

To obtain a relationship between V_R and pressure, Hugoniot data for rocks of very low to zero porosity were assembled. Granite, granodiorite, quartz, basalt, anorthosite, and andesite tend to have low porosity where the bulk density and grain density are approximately the same. Grain density ρ_G refers to the density of the solid matrix rock component at ambient pressure. Hugoniot data are obtained from strong shocks commonly generated by high explosive charges placed near a specimen of rock. The transit time of the shock front through the specimen is measured to obtain the shock velocity U_s . Various methods are available to measure the free surface velocity of the specimen from which particle velocity U_p is determined.⁵ From the conservation of mass, momentum, and energy, the so-called Rankine-Hugoniot condition can be derived which relates U_s and U_p with pressure P , specific volume V , and specific internal energy E of the specimen.

$$P - P_0 = \rho_0 U_s U_p \quad (2)$$

$$V = \left(\frac{U_s - U_p}{U_s} \right) V_0 \quad (3)$$

$$E - E_0 = \frac{(P - P_0)(V_0 - V)}{2} \quad (4)$$

The subscript 0 refers to the pre-shocked condition. The symbol P_0 is the ambient pressure; it is assumed to be negligibly small compared to P .

The references from which the Hugoniot data were taken for near grain density rock are given in Table 1.

The Hugoniot data is plotted in the $U_s - U_p$ plane (Fig. 2). For a large number of materials, the experimentally determined variables of U_s and U_p associated with a single shock are linearly related. At present there is no theoretical justification for the linear relationship. However, the Hugoniot can be

Table 1. Hugoniot data on a variety of near grain density rock used in analysis.

Material ^a	Average density (g/cm ³)	Ref. No.
Granite	2.65	6
Granite	2.675	7
Granite	2.676	8
Granodiorite	2.68	9
Granodiorite	2.68	10
Andesite	2.621	8
Quartz	2.65	11
Basalt	2.86	10
Basalt	2.82	12
Anorthosite	2.734	13

^aThe water content was assumed to be zero for all these materials.

Fig. 2. Hugoniot equation of state data for near grain density rock used in analysis.

determined by the preshocked density and an experimentally determined U_s vs U_p relationship.

In solid materials (such as rocks) in the lower stress states, the measured value of U_s may be less than the sonic velocity in the preshocked material. This phenomenon is attributed to an elastic precursor. Furthermore, rock minerals undergo polymorphic phase transitions. If the higher pressure phase has a lower shock velocity than the lower pressure phase, a double wave structure will develop. However, this disappears at significantly high pressures. The data shown in Fig. 2 for $U_p < 0.21$ cm/μsec shows the traditional scatter due to elastic precursors or double wave structure. At $U_p > 0.21$ cm/μsec, the disturbance

traveling through the specimen reverts to a single shock.

At $U_p < 0.21$ cm/μsec, the Hugoniot data is not useful to obtain a U_s vs U_p relationship. The P-V relationship for the low pressure region is obtained from hydrostatic compressibility measurement. Hydrostatic compressibility has been measured on many rock types including a near-grain-density granite¹⁴ (Fig. 24). A grain density relationship was also derived from hydrostatic data for Paintbrush Tuff¹⁴ with $\rho_G = 2.37$ g/cm³ (Fig. 12). This curve is based on the assumption that all the porosity was removed on compression to 50 kbar and that the release to $P = 0$ was $V = 1/\rho_G = 1/2.37$. The linear relationships shown by the broken lines in Fig. 2 were obtained in this manner. The upper line shown is for $\rho_G = 2.8$ g/cm³, and the lower line is for $\rho_G = 2.3$ g/cm³. The linear U_s vs U_p relationship is of the form

$$U_s = a_1 + b_1 U_p, \quad (5)$$

where

$$\begin{aligned} a_1 &= 0.3427 \rho_G - 0.432 \\ b_1 &= 4.598 - 1.568 \rho_G \end{aligned} \quad (6)$$

for $U_p < 0.21$ cm/μsec.

For $U_p > 0.21$ cm/μsec, the U_s vs U_p relationship was assumed to be the same for all grain densities in the range used. The best fit straight line to the data yields:

$$\begin{aligned} a_1 &= 0.235 \\ b_1 &= 1.60. \end{aligned}$$

Using Eqs. (2) and (3), U_p can be determined at any pressure:

$$U_p = \frac{\sqrt{a_1^2 + 4b_1 P V_G} - a_1}{2b_1} \quad (7)$$

and

$$V_R = \left(\frac{U_s - U_p}{U_s} \right) V_G \quad (8)$$

It is interesting to note that the intercepts at $U_p = 0$ are in the range corresponding to shock velocities between 0.38 to 0.53 cm/ μ sec (12,500 to 17,500 ft/sec) for densities $\rho_G = 2.37$ and 2.80 g/cm³ respectively. This is within the range of sonic velocities measured at many places at depths where rocks are dense and air filled porosity is small or insignificant.

From Eqs. (5), (6), (7), and (8) V_R as a function of P was calculated; it is shown in Fig. 3 for ρ_G from 2.3 to 2.8 g/cm³ along with the hydrostats used to derive the relationship. The basic assumption displayed is that the compressibility is a

function of grain density only. There is some experimental evidence that this assumption is reasonable.

Fig. 3. Pressure-volume relationships for different grain density rocks obtained from analysis.

Compressibility of Water

Hugoniot data has been measured for water⁵ and is plotted in the $U_s - U_p$ plane in Fig. 4. Here the U_s vs U_p plot appears to be linear for $U_p < 0.2$ cm/ μ sec. Above this value the data better fits a curved line. The linear portion of the data was fitted to

$$U_s = 0.176 + 1.67 U_p \quad \text{for } U_p < 0.20 \text{ cm}/\mu\text{sec.}$$

However, in this region, phase transformations of water are quite apparent in the hydrostatic measurements. Whether or not the phase transformation occurs and the amount of volume change at the transition depend on whether water is present and how much water is present respectively. However, it has been reported¹⁵ volume changes are larger for some rocks than others with similar water contents. This is believed to be

Fig. 4. Hugoniot equation of state data for water showing correction for phase transitions at 9.6 and 24.0 kbar.

due to the chemical state of the water in the samples. Therefore it is assumed that the water content refers to the free water that would undergo phase transformation. Similar phase transformations are observed in low pressure shock Hugoniot measurements; however, they seem to be time dependent. The 10 kbar transition in hydrostatic measurements is caused by the solidification of water to ice VI which is a pseudo-orthorhombic structure. Another transition is observed at about 24 kbar from the pseudo-orthorhombic to body-centered cubic (ice VII).¹⁵

The $U_s - U_p$ curve modified for these phase transitions are shown in Fig. 4. There are three options for calculating the curve: (1) leave out the phase transformation; (2) calculate discontinuous

P-V curves at points corresponding to 10 and 24 kbar; or (3) smooth through the transformations as shown by the broken line. The third option was used in calculations given later in this report.

For the smoothed through phase transformations shown by the broken line in Fig. 4,

$$U_s = a_2 + b_2 U_p.$$

For $U_p < 0.128$ cm/μsec,

$$a_2 = 0.176$$

and

$$b_2 = 1.156.$$

For $0.128 < U_p < 0.2$ cm/μsec,

$$a_2 = 0$$

and

$$b_2 = 2.595.$$

Since $\rho_0 \approx V_0 = 1.0$ g/cm³ was assumed,

$$U_p = \frac{\sqrt{a_2^2 + 4b_2 P}}{2b_2} - a_2 \quad (9)$$

for $U_p < 0.20$ cm/μsec.

The curved portion of the line was fitted to

$$U_s = a_2 U_p^{b_2}$$

where

$$a_2 = 1.368$$

$$b_2 = 0.6132$$

Then

$$U_p = \left(\frac{P}{a_2} \right)^{\frac{1}{b_2+1}}$$

and

$$V_w = \left(\frac{U_s - U_p}{U_s} \right), \quad (10)$$

Air Filled Porosity

The total porosity contained in a rock is defined by

$$\phi = \frac{\rho_G - \rho_O(1 - Z)}{\rho_G},$$

and the initial air filled porosity is defined by

$$\psi_i = \phi - \rho_O Z.$$

Compression of the material leads to densification or compaction upon removal of the air filled porosity. There is a large body of evidence¹⁵ in hydrostatic compression that materials are at a higher density after unloading because of the reduction of air filled porosity. Loading and unloading data indicate that the air filled porosity is reduced exponentially. The compression of Paintbrush tuff (Fig. 12) can be used as an example. This rock, a high porosity (~40%) dry rock, when loaded hydrostatically to 3 kbar and unloaded shows a large amount of porosity removed. When loaded to 24 kbar and unloaded, a considerably smaller amount of additional air filled porosity is removed.

An examination of the hydrostatic data for a number of different rocks with different water contents and air filled porosities indicates that the pressure where all the air filled porosity is removed (P_m) is a function of the weight fraction of water (Z) present in the rock. The

functional relationship is difficult to ascertain directly. A relationship was developed by a cut and try method to best agree with data from rocks with a number of different water contents. This is

$$P_m = 0.00065 Z^{-0.944}, \quad (11)$$

where P_m is in mbar. It is also assumed that $P_m = 0.05$ mbar for $Z \leq 0.01$.

In many porous, unsaturated rocks, a so-called threshold pressure P_T is observed (Fig. 30). Below this threshold, apparently the intergranular strength is great enough to keep the bonds between

Fig. 5. Relationship used to obtain threshold pressure P_T from weight fraction water and air-filled porosity.

	Paintbrush tuff	Schooner tuff	Diamond dist. tuff
ρ_o (g/cm ³)	1.40	1.763	1.884
ρ_G (g/cm ³)	2.37	2.683	2.403
Z	0.01	0.084	0.15
ψ	0.4012	0.2500	0.0516
P_T (mbar)	0.000069	0.00009	0.000123
P_m (mbar)	0.05	0.00674	0.0039

Fig. 6. Some examples of air-filled porosity as a function of pressure obtained from calculational model.

Comparison of Data with Calculations

A wide variety of materials was chosen for making comparisons of calculations with measurements. The ability for making comparisons was limited by the lack of knowledge of two or more of

grains intact, and a pressure greater than P_T is required before collapse of the air filled pores is initiated. Examination of the hydrostatic compressibility for a number of rocks indicates that the value of this threshold pressure is dependent on the initial air filled porosity and also the water content. Stephens,¹⁶ in an analysis of a large body of hydrostatic data taken on tuffs and rhyolite from the Nevada Test Site, suggests the relationships that were used here. Analytic expressions for these relationships are:

$$0 < Z < 0.05$$

$$P_T = \exp(-3.579\psi_i - 6.908)$$

$$0.10 > Z \geq 0.05$$

$$P_T = \exp(-4.859\psi_i - 8.112)$$

$$Z > 0.10$$

$$P_T = \exp(-3.869\psi_i - 8.805)$$

where P_T is in mbar. These equations are plotted in Fig. 5.

The exponential relationship relating air filled porosity to pressure on loading is

$$\frac{\ln P - \ln P_m}{\ln P_T - \ln P_m} \psi_i = \psi.$$

As $P \rightarrow P_m$, ψ goes to 0 irreversibly.

Figure 6 shows some examples of ψ as a function of increasing P .

the following variables: bulk density, grain density, water content, and air filled porosity. For high pressure Hugoniot measurements, usually only bulk density is reported so that only data for

known dry materials can be used. Materials chosen for comparison of Hugoniot data with calculations are shown in Table 2.

Comparison of calculation with hydrostatic data was made on materials that included a wide spread of densities, water contents, and air filled porosities. In

Table 2. Materials chosen for comparison between calculation and shock-Hugoniot data.

Materials	Fig. No.	ρ_o (g/cm ³)	ρ_G (g/cm ³)	ψ_i	Z	ϕ	Ref. No.
Granite	7	2.67 ^a	2.70	0.0110	0	0.0110	13,6,7,8,9,10
Coconino sandstone	8	1.98	2.65	0.2528	0	0.2528	8
Pillow lava	9	2.61	2.81	0.0006	0.042	0.1102	8
Playa sediments	10	1.95	2.635	0.2748	0	0.2748	17
Paintbrush tuff	11	1.40	2.37	0.4035	0.01	0.4091	7,8

^aValues ranged from 2.64-2.68 g/cm³.

Fig. 7. Pressure-volume relationship for a granite to 1 Mbar, calculated from model, compared with Hugoniot data.

Fig. 8. Pressure-volume relationship for Coconino sandstone to 1 Mbar, calculated from model, compared with Hugoniot data.

Fig. 9. Pressure-volume relationship for a Pillow Lava to 1 Mbar, calculated from model, compared with Hugoniot data.

many cases the authors state a minimum air filled porosity. These values were used along with measured bulk density and water content to determine the grain density used. Table 3 lists the pertinent information regarding materials used for these comparisons. As mentioned previ-

Fig. 10. Pressure-volume relationship for a Playa to 1 Mbar, calculated from model, compared with Hugoniot data.

ously, all the calculations were made to smooth through the water phase transitions observed in the measurements.

Fig. 11. Pressure-volume relationship for Paintbrush tuff to 1 Mbar, calculated from model, compared with Hugoniot data.

Table 3. Materials chosen for comparison between calculation and hydrostatic compression—P-V curves.

Material	Fig. No.	ρ_0 (g/cm ³)	ρ_G (g/cm ³)	v_i	Z	ϕ	Ref. No.
Paintbrush tuff	12	1.406	2.37	0.4035	0.004	0.4091	15
Schooner tuff	13	1.604	2.656	0.1201	0.276	0.5628	18
Hudson Seal tuff	14	1.679	2.462	0.0189	0.30	0.5226	18
Schooner tuff	15	1.766	2.693	0.2497	0.084	0.3971	18
Hudson Seal tuff	16	1.858	2.465	0.0199	0.205	0.4008	18
Diamond Dust tuff	17	1.884	2.403	0.0510	0.15	0.3336	19
Diamond Mine tuff	18	1.895	2.382	0.048	0.144	0.3190	20
Jorum rhyolite	19	1.905	2.495	0.015	0.194	0.3846	16
Diamond Dust tuff	20	1.925	2.461	0.046	0.15	0.3351	19
Hudson Seal tuff	21	1.977	2.459	0.0271	0.144	0.3118	18
Jorum rhyolite	22	2.051	2.685	0.047	0.147	0.3484	16
Benham tuff	23	2.108	2.58	0.001	0.134	0.2924	18
Hardhat granite	24	2.648	2.70	0.0025	0.005	0.0291	15

Fig. 12. Pressure-volume relationship for Paintbrush tuff to 40 kbar, calculated from model, compared with hydrostatic compressibility data.

Fig. 13. Pressure-volume relationship for Schooner tuff to 40 kbar, calculated from model, compared with hydrostatic compressibility data.

Fig. 14. Pressure-volume relationship for Hudson Seal tuff to 40 kbar, calculated from model, compared with hydrostatic compressibility data.

Fig. 15. Pressure-volume relationship for Schooner tuff to 40 kbar, calculated from model, compared with hydrostatic compressibility data.

Fig. 16. Pressure-volume relationship for Hudson Seal tuff to 40 kbar, calculated from model, compared with hydrostatic compressibility data.

Fig. 17. Pressure-volume relationship for Diamond Dust tuff to 40 kbar, calculated from model, compared with hydrostatic compressibility data.

Fig. 18. Pressure-volume relationship for Diamond Mine tuff to 40 kbar, calculated from model, compared with hydrostatic compressibility data.

Fig. 19. Pressure-volume relationship for Jorum rhyolite to 40 kbar, calculated from model, compared with hydrostatic compressibility data.

Fig. 20. Pressure-volume relationship for Diamond Dust tuff to 40 kbar, calculated from model, compared with hydrostatic compressibility data.

Fig. 21. Pressure-volume relationship for Hudson Seal tuff to 40 kbar, calculated from model, compared with hydrostatic compressibility data.

Fig. 22. Pressure-volume relationship for Jorum rhyolite to 40 kbar, calculated from model, compared with hydrostatic compressibility data.

Fig. 23. Pressure-volume relationship for Benham tuff to 40 kbar, calculated from model, compared with hydrostatic compressibility data.

Fig. 24. Pressure-volume relationship for Hardhat granite to 40 kbar, calculated from model, compared with hydrostatic compressibility data.

P-V Unloading Curves

Unloading P-V curves from P_m , the pressure at which it is assumed that all the air filled porosity is removed irreversibly on compression, to $P = 0$ are shown in Figs. 25 and 26. The Paintbrush tuff (Fig. 25) is a dry, high porosity medium that requires a relatively high pressure to remove all the air filled porosity. The Diamond Dust tuff (Fig. 26)

is a partially saturated, relatively high water content rock. This model allows no reintroduction of porosity due to bulk-ing in unloading, which seems to be suggested by some hydrostatic compressibility unloading path measurements.

Fig. 25. Example of calculated loading-unloading P-V curves for a high air filled porosity rock.

Fig. 26. Example of calculated loading-unloading P-V curves for a low air filled porosity rock.

Failure Envelopes for Rocks

Shear stress (τ) vs mean pressure (P_{av}) failure envelopes have been measured on a large variety of rock materials. The failure envelope is an important input parameter to the underground explo-

sion codes at LLL. Cavity size, rock displacement, and the amount of rock broken by the explosion are all strongly influenced by the strength of the rock.³ Failure envelopes are usually determined

by triaxial compression tests at varying confining pressures, uniaxial compression and Brazil Tests.²¹ Typically, the $\tau - P_{av}$ curves show an increasing τ with P_{av} , with the slope of the curve decreasing, until the material approaches a Von Mises solid at some P_{av} .

Some materials behave somewhat differently. At the low pressure end of the $\tau - P_{av}$ curve, the usual shape occurs, and τ approaches a constant value with increasing P_{av} . However, as P_{av} continues to increase, τ again increases until it reaches a new higher plateau. The first plateau is thought to be caused by the progressive collapse of the matrix framework into the existing pore volume at more or less constant shear stress. With increasing volume compaction at higher pressures, all of these materials increase in strength until the pore fluid pressure limits the value of τ .²⁰ Therefore, this behavior would be characteristic of partially saturated rock with significant air filled porosity and high bond strength between the grains. Some calculations were made that indicate an insignificant difference in results of calculations with the second plateau and those without it.²²

Bonner *et al.*²³ have observed that the shear strength of alluvium is directly influenced by initial water content. Examinations of failure envelopes for other rock types with weak intergranular bonds, such as the Nevada Test Site tuffs, indicate that for these types of rock a general statement that initial water content directly influences the shear strength can be made. Data exist that indicate that partially saturated rocks with high intergranular bond strength can exhibit as high

a shear strength as the dry rock below P_m where the air filled porosity becomes zero and pore pressures are considerably below the mean stress.

Figure 27 shows a group of selected data for different water content rocks. Mostly the data is for alluvium²³ and tuffs.²⁴ Data for Hardhat granodiorite²⁴ is also presented. Not all data for rocks will fit within the range of maximum measured shear strength in this group, since this group is more representative of the weak or fully saturated materials. For example, Schock^{25,26} has measured shear strength as high as 13 kbar at high mean stress for very dry granites and sandstones. The data reported here for

Fig. 27. Maximum shear stress vs mean stress failure envelopes measured for rocks with different water content.

granodiorite is for a saturated material. However, since the porosity is of the order of 1%, the weight fraction water is only about 0.5%. Estimates of the water content for the so-called dry alluvium and lithic tuff were 0.5% and 2%, respectively.

Data in the form shown in Fig. 27 can be fitted to a hyperbolic equation²⁷;

$$\tau = \frac{P}{a_3 + b_3 P}, \quad (12)$$

where $1/a_3$ is the initial slope and $1/b_3$ the asymptotic value that τ approaches as P increases. Values for a_3 and b_3 can be obtained from the data by plotting P/τ vs P , which yields a straight line in slope-intercept form:

$$a_3 + b_3 P = P/\tau.$$

Data were plotted in a form acceptable to the SOC and TENSOR codes that use τ vs $\bar{P}$, where $\bar{P} = P_{av} + \tau/3$. The fit to the data shown in Fig. 28 is $\exp(14.6Z + 0.3)$.

For $Z \leq 0.02$,

$$b_3 = 25000 Z + 200.$$

For $Z > 0.02$,

$$b_3 = 47500 Z - 250.$$

The shear stress failure envelopes produced by Eq. (12) above are modified to include the unconfined compressive strength. The threshold pressure (P_T), defined earlier in this report as the mean stress (P_{av}) at which the porosity begins to be removed irreversibly, was assumed to be a measure of the unconfined compressive strength. For an unconfined compressive test, the shear stress at

failure is half the applied stress or $\tau = \sigma_1/2$, and the mean stress $P_{av} = \sigma_1/3$. $\bar{P} = P_{av} + \tau/3 = \sigma_1/3 + \sigma_1/6 = \sigma_1/2 = \tau$.

The unconfined compressive strength will lie on a 45° line from $\bar{P} = 0$, so that

Fig. 28. Modification to hyperbolic fit to strength data for rock with densities of 2.3 to 2.8 g/cm³, zero water content and different total porosities. This example demonstrates the maximum modification to the fit.

Fig. 29. Calculated maximum shear strength vs $\bar{P}$ failure envelopes compared with measurements for $Z = 0, 0.01, 0.02$, and 0.05 .

$\bar{P} = \tau$. The unconfined compressive strength point was calculated from

$$\tau_1 = \frac{\bar{P}_1}{a_3 + b_3 \bar{P}_1},$$

where $\bar{P}_1 = P_T + \tau_1/3$; and

$$\tau_i = \bar{P}_1$$

$$= \frac{\sqrt{[b_3 P_T (b_3 a_3 + 3a_3 b_3 P_T + 2) + a_3 (3a_3 - 2) + 1]}}{2b_3}$$

$$- (3a_3 + 3b_3 P_T - 1).$$

The next point ($\tau_2, \bar{P}_2$) calculated was on the fitted curve, such that a straight line passing through ($\tau_1, \bar{P}_1$) was tangent to the curve at $\bar{P}_2$ so that

Fig. 30. Calculated maximum shear strength vs $\bar{P}$ failure envelopes compared with measurements for $Z = 0.05, 0.10, 0.20$, and 0.30 .

$$\bar{P}_2 = \frac{-\sqrt{a_3 b_3 \tau_1 (b_3 \tau_1 + a_3 - 1)}}{b_3 (b_3 \tau_1 - 1)} - a_3 b_3 \tau_1.$$

Figure 29 shows the modification of the hyperbolic curve for $Z = 0$, for different total porosities ϕ . For $Z = 0$, the air filled porosity $\psi = \phi$. Figures 30 and 31 show calculated results compared with measurements without the unconfined compressive strength modification.

Fig. 31. Pressure-volume relationships for central Nevada tuff to 40 kbar calculated from model assuming $P_T = 0.0007$ and 0.0001 Mbar, compared with hydrostatic compressibility data.

Results and Discussions

A technique has been devised for generating pressure-volume and maximum shear stress-mean stress relationships for rocks. These relationships are necessary input for making underground nuclear explosion code calculations. The technique makes possible good estimates of the P-V relationship for different types of rocks, and good τ - $\bar{P}$ estimates for rocks with weak intergranular bonds that are characterized by tuffs, alluvium, and saturated rocks where pore pressure controls the strength of the rock. The results produced by the technique discussed in this report should prove useful for making preliminary calculations and parameter studies without undertaking an extensive rock property measurement program. The necessary input parameters for generating these relationships are bulk density, grain density, weight fraction water, and elastic bulk modulus.

The technique is not intended to replace a rock property measurement program. Measurements are better than the estimates produced by the technique. An on-going measurement program should verify some of the assumptions or suggest how the technique might be modified and improved. Measurements such as the compressibility of the solid matrix of the rock would be very useful to check a basic assumption made here that compressibility of the solid matrix is a function of density alone and that the function is linear (Fig. 3).

A study of the comparison between measurements and calculations as indicated here shows that in most cases the

agreement is excellent. However, there are a few cases in which the behavior of the material is atypical. In Mt. Helen tuff,²⁸ which is dry and highly porous, an unusually high P_T was measured. Figure 5 would suggest that $P_T \approx 100$ bars. However, the data indicate that $P_T \approx 700$ bars. Figure 7 shows the difference using either value compared with data. Paintbrush tuff (Fig. 12) has slightly higher porosity and similar water content, yet it has a $P_T \approx 70$ bars.

For a Jorum rhyolite with a bulk density of 2.188 g/cm^3 and a measured water content of $Z = 0.025$, Stephens¹⁶ reports a minimum air filled porosity $\psi = 0.076$.

Fig. 32. Pressure-volume relationships for Jorum rhyolite to 40 kbar calculated from model assuming initial air-filled porosities $\psi_i = 0.076, 0.10$, and 0.12 compared with hydrostatic compressibility data.

Using these values, $\rho_g = 2.454 \text{ g/cm}^3$. However, calculation of P-V using these values do not agree well with measurements. Higher air filled porosities which result in higher grain densities, were tried, and the calculations were run again. Figure 32 shows that an air filled porosity of 0.12 is required for excellent agreement between calculation and measurement. At this point it should be emphasized that for many of the other materials the grain density was calculated from a minimum air filled porosity reported by the experimenters.

The technique described here is to be included in the generator for the SOC and TENSOR codes. Appendix A lists the Fortran program for generating the P-V and τ - $\bar{P}$ curves. Appendix A also shows the format and explains other quantities

calculated by the code. For a complete P-V loading curve to 2 Mbar, the unloading curve from P_m , the merge pressure, and a τ vs $\bar{P}$ curve to $\bar{P} \approx 0.1$ Mbar, the user need only specify the bulk density, grain density of the solid matrix of the rock, weight fraction of water, and elastic bulk modulus for the rock conglomerate. Unless a different P_T is specified, P_T is calculated from the model as a default value.

The water-ice phase changes may be included as discontinuities in the P-V curves at 9.6 and 24 kbar, or omitted as specified. A third option is to fair through the phase transitions to produce a smooth curve as was done for all the comparisons between P-V calculations and measurements illustrated in the figures of this report.

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Appendix A

PMUGEN Code

This program was written using the language specifications employed by LRLTRAN, the Livermore extended Fortran language. It is, therefore, probably incompatible with Fortran compilers external to the Livermore facility.

OPERATING INSTRUCTIONS

Required Input Parameters

RHO $\emptyset$ — bulk density (g/cm^3)
 RHOG — grain density (g/cm^3)
 Z — water content wt fraction

Optional Input Parameters

T — tensile strength (Mbar) (default value is $\emptyset$)
 K — bulk modulus (Mbar)
 PT — threshold pressure (Mbar)
 AA-AE — various constants used in various steps throughout the code.
 BA-BE —
 OUTP — sends the output to various output media.
 OUTP = $\emptyset$; default value if no value for OUTP given, sends output to the online printer.
 OUTP = 1; sends output to the teletype.
 OUTP = 2; sends output to a family of HSP disk files beginning with HSP $\emptyset$.
 MODE — controls the source of input.
 MODE = $\emptyset$; default value, expects input to originate from teletype.
 MODE = 1; input read from a disk file called PMU. Note, the disk file should be in monitor mode and the first card in the file must correspond to the following format restrictions:
 Col. 1 11 through 40
 BOX $\wedge$ NNN $\wedge$ ^^ID INFORMATION
 ENDD — controls the flow of the code
 ENDD = $\emptyset$; continuous input expected. If running from teletype, the code will be in a wait status after each problem cycle. The execution time is relatively quick, so the user should not become unnerved if he receives "RCV TE" when he interrogates the program,

ENDD = 1; will terminate execution at the completion of current problem cycle. This is the default value.

ENDD = 2; will terminate the execution of the program immediately.

PFLG — controls the punch card output.

PFLG = 0; default value, no cards will be punched for this set of problem parameters.

PFLG = 1; punch card output will be generated for P-V curves.

PFLG = 2; punch card output will be generated for P-Mu curves.

OPT — controls the type of curve generated.

OPT = 1; smooth curve generated ignoring phase changes (default value).

OPT = 2; the curve generated will contain shifts owing to phase changes.

OPT = 3; the curve generated will fair through the phase shifts.

EXAMPLES OF USAGE

The PMUGEN file uses the following technique under the LLL operating system:

```
EXE^L107125^PMUGEN^DR.^/T^V (LF)
```

This will create a file called PMUGEN under the user's ID number.

Example A PMUGEN^/T^V (LF)
 RHO0=1.905^RHOG=2.495^Z=.194 (LF)
 OUTP=1^PFLG=1^GO^(LF)

At this point the problem will execute, send the output to teletype and punch a set of cards for a P-V loading curve that disregards phase changes, a P-V unloading curve, and a K-P curve, then terminate.

Example B PMUGEN^/T^V (LF)
 RHO0=1.905^Z=.194^PFLG=2^OUTP=2 (LF)
 RHOG=2.495^OPT=2^GO

This problem will execute, send output to a HSP disk file called HSP0, punch a set of cards for a "P-Mu" loading curve which contains shifts due to phase changes, a "P-Mu" unloading curve and a K-P curve, then terminate.

Example C

PMUGEN MODE=1 GO T V (LF)

PMUGEN will immediately go to disk looking for a monitor-formatted file called "PMU". This file may look like the following:

```

Card 1 — BOX NNN THIS IS A TEST SET A
Card 2 — RHOG=2.495 OUTP= PFLG=1 ENDD=
Card 3 — RHO=1.905 Z=.194 K=.12041
Card 4 — GO1
          RHOG=2.495 RHO=1.905 PFLG=2 Z=.194 OPT=3
          GO2
          .
          .
          .
          .
Card M — RHO=1.905 RHOG=2.495 PFLG= Z=.194
Card N — OUTP=1 Z=.194 ENDD=1
          GON

```

In this instance, the first card will become the identification page of the output. Problem one will then be processed, output will go to the on-line printer and P-V cards will be punched for curves disregarding phase changes. The second problem will be executed but this time cards for a P-Mu curve will be generated. Since the value of OUTP was not changed, the output will continue going to the on-line printer, and PMUGEN will continue expecting input from the disc file PMU. When cards M and N are read the code will execute according to this set of data, discontinue punch card output, send output to the user's teletype. Since the last value of OPT was 3 (from the second problem), the execution path will generate output data for a curve that has faired through the phase shifts. PMUGEN will then terminate since ENDD=1.

NOTE: The NAMELIST statement, which is used to process input variables, does not begin processing until it encounters a variable name which does not appear in its input list. Thus when NAMELIST encounters GO1, GO2, etc., it immediately stops reading input parameters and commences processing the variables it has read up to that point. Exercise great care, therefore, not to misspell any input variable to PMUGEN since NAMELIST will interpret that as an end flag and stop reading input.

```

*      FORTKAN          PMUGEN
C      *****
C      *
C      *              VERSION OF 11/13/73
C      *
C      *          PUBLISHED IN  UCCN 51441
C      *
C      *  OUTP=0  OUTPLT TO PRINTER
C      *  OUTP=1  OUTPLT TO TELETYPE.
C      *  OUTP=2  OUTPLT TO HSP FILE CALLED HSPU
C      *
C      *  MODE=0  INPUT FROM TELETYPE
C      *  MODE=1  INPUT FROM DISC FILE CALLED PMU
C      *
C      *  ENDD=0  CONTINUOUS INPUT
C      *  ENDD=1  TERMINATE AT THE END OF THIS CYCLE
C      *  ENDD=2  TERMINATE IMMEDIATELY
C      *
C      *  PFLG=0  NO PUNCH CARD OUTPLT REQUESTED
C      *  PFLG=1  DATA CARDS FOR P-V CURVES PLACHED
C      *  PFLG=2  DATA CARDS FOR P-MU CURVES PUNCHED
C      *
C      *  DEFAULT VALLES: MODE=0, OUTP=0, ENDD=1, PFLG=0
C      *****
C      CODE ANALYSIS
C      PROGRAM MUP(PRINT,PUNCH,TAPE2,,HSP)
C      COMMON /GOBCCM/ IKNOW(1)
C      CALL ASSIGN (3,15)
C      CLICHE SEND (6,3)
C      IF ( 1 - 1 ) , 4 , 5
C      PRINT 2 3
C      GO TO 6
4      WOT 59, 2 , 3
C      GO TO 6
5      WOT 3, 2 3
6      CONTINUE
C      ENDCCLICHE
C      CALL OCCDROP(6RPMUGEN,4R+TED,0)
C      INTEGER OPT, ENDD, OUTP, PFLG
C      REAL MUL, MUL, K
C      DIMENSION P(40),VR(40),Vh(40),PSI(40),VS(40),VSU(25),PR(40),PR(17)
C      DIMENSION MUL(40),MUU(25),PBAR(17),TAU(17),CPML(40),CPMU(25)
C      DATA (PR(1)=0.,.0002,.0004,.0006,.0008,.001,.002,.003)
C      DATA (PR(9)=.004,.005,.006,.007,.008,.009,.0096,.01)
C      DATA (PR(17)=.015,.02,.024,.03,.035,.04,.05,.07,.1)
C      DATA (PR(26)=.15,.2,.25,.3,.35,.4,.5,.7,.9,1.,1.5,2.)
C      DATA (PBAR(1)=-1.,0.,.0005,.001,.0015,.002,.003,.004)
C      DATA (PBAR(9)=.006,.008,.01,.02,1.), (PB(1)=-1.), (I=0.)
C      DATA (E=2.71828183),(DENCM=.00004186), (MODE=0), (ENDD=1)
C      DATA (TADD=0.), (OUTP=C), (PT=0.), (CPT=1), (PFLG=0), (PFLG=0)
C      DATA (MASK = 77777777777777777700B)
C      NAMELIST /TEDDY/ RHOO,RHCG,Z,K,AA,AB,AC,AD,AE,BA,BB,EC,EC,EE
C      NAMELIST /TEDDY/ TADD,ENDD,OUTP,MODE,PT,CPT, PFLG, T
C      AA=.3427 $ AB=.432 $ AC=.235 $ AD=.146 $ AE=1.368
C      BA=4.598 $ BB=1.568 $ BC=1.6 $ BC=1.86 $ BE=.6132

```

```

ZERO      WOT 59,ZOT
          INPUT DATA TEDDY 59, 3
          IF(MODE - 1)ZEROB, ,ZERCA
          MUDE = 2
          CALL ASSIGN (2,0,3RPM,0)
          RIT 2, ZOTY, (IKNOW(1),I=36,38)
ZEROA     INPUT DATA TEDDY 2,3
ZERUB     IF(OUTP - 1)ZEROC, ,ZERCC
          CALL ASSIGN (61,3)
ZEROC     IF(ENCD .GE. 2)ZOTZ,
          IF(K .EQ. C.) ,ZEROF
          IF((RHOC .LT. 1.3) .OR. (RHCG .LE. 2.2)) ,ZERCE
          SEND OUTP ZOTM
          GO TO ZERCK
ZEROE     C = (.318 * RHGO) - .2875
          GNC = (2.8147 - LOGF(C)) / 13.044
          K = (((C**2) * RHOC) * (1. + GNC)) / (3. * (1. - GNC))
ZEROF     VC = 1./RHOO
          PHI = 1. - ((RHGO * (1.-2)) / RHCG)
          PSII = PHI - (RHOC * Z)
          IF(PT .EQ. 0.) ,THREE
          PT1 = E**((-6.579 * PSII) - 6.908)
          PT2 = E**((-3.869 * PSII) - 8.805)
          IF(Z .LE. .05) ,ONE
          PT = PT1
          GO TO THREE
ONE        IF(Z .LT. .1) ,TWC
          P(1) = (Z - .05)/.05
          PT2 = (PT1/PT2) ** P(1)
          PT = PT1/PT2
          GO TO THREE
TWO        PT = PT2
THREE     IF(Z ~ .02) , ,FCUR
          PM = .05
          GO TO CINQ
FOUR      PM = .00065 * (Z**(-.944))
C          * * * * * INSERT PT AND PM INTO PRESSURE ARRAY * * * *
C
CINQ      LPT = LPM = 0
          IDA = 37
          PT = PT .INT. MASK
          PP=PT
          PM = PM .INT. MASK
          L = 1
          DO CINQA I = 1,IDA
CINQA     P(I) = PR(I) .INT. MASK
FIVE      DO SEVEN I = 1,IDA
          IF(P(I) - PP)SEVEN, ,FIVEA
          IF(PP .EQ. PT) ,EIGHT
          LPT = I
          PP = PM
          GO TO SEVEN
FIVEA     IDA = IDA + 1
          DO SIX J = IDA,I+1,-1

```



```

SIX      P(J) = P(J-1)
          P(1) = PP
          IF(PM .EQ. PP)EIGHT,
          LPT = 1
          PP = PM
          GO TO FIVE
SEVEN    CONTINUE
          SEND OUTP ZOTJ
          GO TO ZERCK
EIGHT    LPM = 1
C        * * * * * LPM LPT ARE INSERTION FLINT INDICIES * * *
          PPT = LOGF(PT)
          PPM = LOGF(PM)
          PPP = PPT - PPM
C
C        * * * * * FIND VR FOR EACH PRESSURE * * * * *
C
          CALL CSGR
C
C        * * * * * FIND V FOR LOADING CURVE * * * * *
C
          VS(1) = VC
          DO ELFN M = 2,10A
            IF(M - LPT) , ,ELFN1
            VS(M) = (K * VC) / (K + P(M))
            GO TO ELFN7
          ELFN1 VS(M) = (VR(M) + 2 * (Vh(M) - VR(M))) / (1. - PSII(M))
            IF(M .EQ. (LPT+1)) ,ELFN7
            W1 = VS(M) * (P(M) - P(LPT))
            W2 = VS(LPT) - VS(M)
            C1=-1.
            IF(W2 .NE. C.C) C1 = W1/W2
            B1 = K * 1.C5
            IF((C1 .GT. B1) .OR. (C1 .LE. C.O)) ,ELFN7
C
C        * * * * * RECALCLATE AIR-FILLED POROSITY * * * * *
C
          IF(PSII .GT. .005) ,ELFN3
          C1 = (VS(LPT) - VR(LPT) - Z * (Vh(LPT) - VR(LPT))) / VS(LPT)
          IF(C1 .GT. .005) ,ELFN3
          W1 = (ABS(1. - (C1/PSII))) * 100.
          PSII = C1
          SEND OUTP ZOTB, PSII,W1
          GO TO ELFN2
          ELFN3 PSII = C.
          SEND OUTP ZOTA
          ELFN4 W1 = (VS(LPT) - Z*Vh(LPT)) / (1. - Z)
            W2 = 100. * (ABS((VR(LPT)-W1)/VR(LPT)))
            IF(W2 .GT. 10.1) ,ELFN5
            SEND OUTP ZOTQ
            GO TO ZERCK
          ELFN5 RHOG = RHOG * VR(LPT) / W1
            SEND OUTP ZOTR, RHOG,W2
            W1 = PHI
            PHI = 1. - ((RHOG*(1.-Z))/RHCG)

```

```

SEND CULP ZCIV, W1,PHI
ELFN2 CALL USCR
ELFN6 VS(M) = (VR(M) + Z * (VW(M) - VR(M))) / (L - PSI(M))
ELFN7 MUL(M) = (VC - VS(M)) / VS(M)
      DPML(M) = (P(M) - P(M-1)) / (MUL(M) - MUL(M-1))
ELFN  CONTINUE
C
C      * * * * * FIND EF AND EV * * * * *
C
      C1 = 420.
      SIGMA2 = .5 * (VC - VS(2)) * P(2)
      W1 = SIGMA1 = B1 = 0.
      DO EINS I = 3,IDA
        A1 = P(I) * (VC - VS(I)) * .5
        SIGMA1 = (P(I) + P(I-1)) * (VS(I-1) - VS(I)) * .5
        SIGMA2 = SIGMA2 + SIGMA1
        W2 = (A1 - SIGMA2) / DENCM
        IF (W2 - C1) ,DCUS,UNS
        W1 = W2
        GO TO EINS
      UNS PE = P(I-1) + (((C1-W1) * (P(I)-P(I-1))) / (W2-W1))
        VE = VS(I-1) - (((PE-P(I-1)) * (VS(I-1)-VS(I))) / (P(I)-P(I-1)))
        GO TO TROIS
      DCUS PE = P(I)
        VE = VS(I)
      TROIS EF = (PE * (VO - VE)) / (Z * VC)
        IF (B1 .EQ. C.) ,ZWEI
        B1 = 1.
        PEF = PE
        A2 = EF
        C1 = 2800.
        GO TO EINS
      ZWEI PEV = PE
        EV = EF
        GO TO DREI
      EINS CONTINUE
C
C      * * * * * UNLOADING CURVE * * * * *
C
      DREI IF (PM .EQ. .C5) ,VIER
        DO VIER J = 1,IDA
          IF (PIJ) .EQ. .C5) ,VIER
          LPM = J
          GO TO FUMF
      VIER CONTINUE
      FUMF VSU(1) = VR(1) + Z * (VW(1) - VR(1))
        MUU(1) = (VO - VSU(1)) / VSU(1)
        DO SECHS I = 2,LPM
          VSU(I) = VR(I) + Z * (VW(I) - VR(I))
          MUU(I) = (VC - VSU(I)) / VSU(I)
          OPMU(I) = (P(I) - P(I-1)) / (MUL(I) - MUU(I-1))
      SECHS CONTINUE
        A3 = E ** ((14.6*Z) + .3)
        IF (Z - .02) ,QUATRE,CLATRE
        B3 = 200. + 25000. * Z

```

```

      GL TO SEP
      B3 = 47500. * Z - 250.
      SGP IF((T .LT. 0.) .OR. (T .GT. .CC011)) ,SEPT
      SEND OUTP ZOTL
      GO TO ZERCK
      SGP1 W1 = ((6. * A3) + (3. * B3 * PT) + 2.) * (B3 * PT)
      W2 = A3 * (3. * A3 - 2.)
      C1 = SQRTF(3. * (W1 + W2) + 1.)
      W1 = (C1 - 3.*A3 - 3.*B3*PT + 1.) / (2.*B3)
      C1 = SQRTF((A3*B3*W1) * (B3*W1 + (A3 - 1.))) + (A3*B3*W1)
      W2 = C1/(B3 - (B3 * B3 * W1))

C
C      * * * * * K - PEAR STRENGTH ENVELOPE * * * * *
C
      IF(T) ,SEPTF,SEPTA
      SEND OUTP ZOTS
      GO TO ZERCK
      SEPTA IF (T .GT. W1) ,SEPTB $$$ T 0
      SEND OUTP ZOTT
      SEPTB PB(2) = -T
      PB(3) = 0.
      PB(4) = W1
      PB(5) = W2
      TAU(3) = (W1*T) / (W1 + T)
      TAU(4) = W1
      TAU(5) = PB(5) / (A3 + (B3 * PB(5)))
      I = 6
      SEPTC L = I
      DO SEPTD J = 1,17
      IF(PBAR(J) .GT. PB(I-1)) ,SEPTC
      PB(L) = PBAR(J)
      L = L + 1
      SEPTD CONTINUE
      L = L - 1
      DC SEPTD J = 1,L
      SEPTD TAU(J) = PB(J) / (A3 + (B3 * PB(J)))
      PB(L) = 1.
      GO TO SEPTG
      SEPTF PB(2) = 0 $$$ T = 0
      PB(3) = W1
      PB(4) = W2
      TAU(3) = W1
      TAU(4) = PB(4) / (A3 + (B3 * PB(4)))
      I = 5
      GO TO SEPTC
      SEPTG TAU(L) = TAU(L-1)
      SEND OUTP ZOTC
      SEND OUTP ZOTO, RHQC, RHCG, Z, K, PEF, PEV
      SEND OUTP ZOTE
      SEND OUTP ZOTO, PHI, PSII, PT, PM, A2, EV, T
      SEND OUTP ZOTF
      SEND OUTP ZOTG
      SEND OUTP ZOTH, (P(I), VS(I), MUL(I), DPMU(I), I = 1,ICA)
      SEND OUTP ZOTI
      SEND OUTP ZOTH, (P(I), VSG(I), MUU(I), DPMU(I), I=1,LPM)

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SEND OUTP ZCTK
SEND OUTP ZOTL, (TAL(I),PE(I),I=1,L)
IF (PFLG - 1) ZEROL, ZERCG
PUNCH ZOTN, (P(I),VSL(I),I=1,ICA)
PUNCH ZOTN, (P(I),VSL(I),I=1,LPM)
GO TO ZERCH
ZEROG PUNCH ZOTN, (P(I),MUL(I),I=1,ICA)
PUNCH ZOTN, (P(I),MUL(I),I=1,LPM)
ZEROH PUNCH ZOTN, (TAL(I),PE(I),I=1,L)
PUNCH ZOTN, (TAU(I),PE(I),I=1,L)
ZEROI IF (JFLG .EQ. C) ,ZERCK
JFLG = 1
ITEMP = IKNOW(36) .INT. 7H
IF (ITEMP .EQ. 7HBOX (69) ,ZERCK
IF (OUTP .EQ. 1) ZERCK,
WUT 59, ZCTO
RIT 59, ZOTY, ITEMP
ITEMP = (ITEMP .SHR. 24) .INT. 7H
IKNOW(36) = (IKNOW(36) .INT. 1CR ) .UN. ITEMP
IF (MODE .EQ. 1) ZERCJ,
IKNOW(37) = 1CH*OUTPLT FR
IKNOW(38) = 1CHOM PMLGEN*
ZERQJ WUT 59, ZCTP, (IKNOW(I),I=36,38)
ZEROK IF (ENDC .EQ. C.) ,ZCTZ
AL = .146
BL = 1.86
PI = T = K = C.
IF (OUTP .EQ. 2) CALL EMPTY(3)
IF (MODE - 1) ZERO, ZERCA, ZERQA
ZOT FORMAT( INPUT DATA )
ZOTA FORMAT( RECALCULATED AIR-FILLED POROSITY (PSII) IS LESS THAN ZERO.
* / INPUT VALUES OF BULK MODULUS AND PT SHOULD BE CHECKED. CODE /
* WILL SET AIR-FILLED POROSITY TO ZERC AND RECALCULATE THE GRAIN
* / DENSITY (RHOG). )
ZOTB FORMAT( AIR-FILLED POROSITY WAS RECALCULATED TO BE ,F8.5,/ WHICH
* IS ,F7.3, PERCENT CHANGE FROM ORIGINAL VALUE. )
ZOTC FORMAT(//3X, RHOC ,7X, RHOG ,9X, Z ,11X, K ,8X, PF ,9X, PV )
ZOTD FORMAT(//F9.5,2X)//)
ZOTE FORMAT(4X, PHI ,8X, PSI ,8X, PT ,9X, PM ,9X, EF ,9X, EV ,9X, T )
ZOTF FORMAT(//16X, LOADING PATH //)
ZOTG FORMAT(5X, P ,10X, V ,1CX, ML ,8X, CP/DML )
ZOTH FORMAT(4(F10.6,1X))
ZOTI FORMAT(///15X, UNLOADING PATH //)
ZOTJ FORMAT(//5X, NO INSERTION POINT FOUND )
ZOTK FORMAT(///4X, TAL ,7X, PBAR )
ZOTL FORMAT(2(F10.6,1X))
ZOTM FORMAT(2X///, GRAIN DENSITY IN ERROR, OR RHOC LESS THAN 1.3 RUN A
* BORTED. )
ZOTN FORMAT(1C(8F7.5//))
ZOTO FORMAT( TYPE IN 3 CHARACTER BOX AC. )
ZOTP FORMAT( OUTPUT WILL GC TO ,3A10)
ZOTQ FORMAT( RECALCULATED GRAIN DENSITY HAS CHANGED MORE THAN 10% OF
* / ITS ORIGINAL VALUE. CODE WILL NOT CONTINUE THIS PROBLEM. /
* CHECK INPUT VALUES OF BULK MODULUS AND PT. )
ZOTR FORMAT( GRAIN DENSITY HAS BEEN RECALCULATED. NEW RHOG IS ,F8.5,/

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      ' WHICH IS ,F7.3, PERCENT CHANGE FROM INPUT VALUE. )
ZOTN FORMAT( NEGATIVE TENSILE STRENGTH INPUT, CODE WILL NOT CONTINUE )
ZOTF FORMAT( INPUT TENSILE STRENGTH IS GREATER THAN UNCCNFINEC COMPRESS
      'IVE / DETERMINED FROM CALCULATED OR INPUT THRESHOLD POINT )
ZOTG FORMAT(// INPUT TENSILE STRENGTH EQUALS OR EXCEEDS BCUNES / CODE
      ' WILL NOT CONTINUE THIS PROBLEM. )
ZOTV FORMAT( INITIAL TOTAL POROSITY CHANGE FROM ,F7.5, TO ,F7.5,
      ' . )
ZOTY FORMAT(3A10)
C
C      * * * * * ENTRY USCR * * * * *
C
      ENTRY USCR
      DO NINE I = 1,IDA
        A1 = (AA * RHCG) - AB
        B1 = BA - (BB * RHCG)
        GO TO NINEB
      NINEA
        A1 = AC
        B1 = BC
      NINEB
        C1 = (4. * B1 * P(I)) / RHCG
        C2 = SQRTF((A1**2) + C1) - A1
        LPR = C2 / (2. * B1)
        IF(A1 .EQ. AC)NINEC,
        IF(LPR .GE. .21)NINEA,
      NINEC
        USR = A1 + (B1 * LPR)
        VR(I) = (USR - LPR) / (USR * RHCG)
      NINE
      CONTINUE
C
C      * * * * * FIND VV FOR EACH PRESSURE * * * * *
C
      DO TEN I = 1,IDA
        GO TO (TEN7,TEN1,TEN5) OPT
      TEN1
        IF (P(I) .LE. .0056)TEN7,
        IF (P(I) .LE. .024) ,TEN2
        AD = .106
        BD = 1.86
        GO TO TEN7
      TEN2
        IF(P(I) .LE. .04) ,TEN3
        AD = .09
        BD = 1.86
        GO TO TEN7
      TEN3
        IF(P(I) .LE. .1) ,TEN4
        AD = 0.
        BU = 2.595
        GO TO TEN7
      TEN4
        AD = .146
        BL = 1.86
        GO TO TEN7
      TEN5
        IF(P(I) .LE. .006)TEN4,
        IF(P(I) .LE. .04) ,TEN3
        AD = .1632
        BD = 1.28
      TEN7
        A1 = SQRTF((4.*BD) * P(I) + (AC**2)) - AD
        IF(P(I) .EQ. 0.) A1 = 0.
        IF(P(I) .LE. .1) ,TEN8

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      UPW = A1 / (2 * UC)
      USW = AC + (A1/2.)
      GO TO TEN9
TEN8  UPW = (P(I)/AE)**(1./(BE + 1.))
      USW = AE * (UPW**BE)
TEN9  IF(USW .EQ. C.) TEN,
      VW(I) = (USW - UPW) / LSW
      IF(I .LE. LPT) TEN,
      PSI(I) = PSII * ((LEGF(P(I)) - PPM)/PPP)
      IF(PSI(I) .LT. C.) PSI(I)=C.
TEN   CONTINUE
      RETURN USCK
ZOTZ  CALL EXIT
      END

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