BNL 19008
EP&S 74-3

MASTER

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Upton, New York

ACCELERATOR DEPARTMENT
Informal Report

INITIATION OF THERMONUCLEAR DETONATIONS

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June 5, 1974

ABSTRACT

In the following note we derive some simple relations describing a thermonuclear detonation in a deuterium-tritium mixture. We propose an ignition system which consists of a storage ring, (or set of them) in which a bunched beam is stacked up to the transverse space charge limit. The particles are then extracted and focused to an appropriate spot. It is this sudden heating which then initiates the thermonuclear shock wave in the fuel. The practical problems of storing and focusing of the beam and containment of the detonation are not treated in this note, but will be dealt with in a later paper.

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Initiation of Thermonuclear Detonations

I. We consider a cold plasma, of density ρ_0 , and a detonation wave moving at velocity U . Behind the detonation wave we have a density ρ_1 , a velocity μ , pressure P , and internal specific energy ΔE . The conservation of mass, momentum, and energy give rise to the three Rankine-Hugoniot shock relations.¹

$$\begin{aligned}\rho_0 U &= \rho_1 (U - \mu) \\ P &= \rho_0 U \mu \\ \Delta E &= \frac{1}{2} \mu^2\end{aligned}\quad (1)$$

If we take the plasma to obey the perfect gas law, we have the equation of state.

$$P = \frac{2}{3} n_1 T = \rho_1 V_{th}^2 \quad (2)$$

where $n_1 = \rho_1/M$, and $T \equiv 3/2 M V_{th}^2$, and M is the mass of the ions. We can now solve these equations as follows:

$$\begin{aligned}\rho_1 / \rho_0 &= 4 \\ U &= \frac{4}{\sqrt{3}} V_{th}\end{aligned}\quad (3)$$

We need one more equation to solve for V_{th} , or what is the same thing, T . We obtain this relation from the following considerations. A wave is moving with velocity U through a medium of density ρ_0 , and heating it to temperature T . Therefore, there must be a power source which provides power at the following rate:

$$P_{in} = U n_0 T \text{ watts/meter}^2$$

Using our previous relations we can rewrite this as follows:

$$P_{in} = \frac{4}{3} \sqrt{\frac{2}{M}} n_0 T^{3/2}$$

Now the power source is the fusion reaction, and the shock wave is propagated by the 3.5 MeV α -particles losing energy in the plasma. The rate of production of α particles/unit volume is $R = n^2 \langle \sigma v \rangle$, where $\langle \sigma v \rangle$ is a function

of temperature. Since the α particles have a range, λ_α , only the α particles within this distance will be able to supply heat to the shock front.

Taking 1/6 for the geometry factor, we obtain:

$$P_{out} = \frac{E_{\alpha} n^2}{24} \langle \sigma v \rangle \lambda_{\alpha}$$

where E_{α} is the energy of the α particle.

The range of α particles in a plasma is a function of temperature,² and the relation is given as: $\lambda_{\alpha} = .78 \times 10^{10} T^{3/2} n_1^{-1}$. Equating P_{in} to P_{out} , we have our final relation:

$$\frac{4}{3} \sqrt{\frac{2}{M}} = .78 \times 10^{10} \frac{E_{\alpha}}{6} \langle \sigma v \rangle \quad (4)$$

This determines a value of $\langle \sigma v \rangle$ which is required in order to support a detonation in a DT mixture. Note that the density does not come into this relation. We obtain $\langle \sigma v \rangle \approx 3.8 \times 10^{-23}$.³ This corresponds to a temperature of about 7 KeV. If our estimates were wrong by a factor of two, the required temperature would rise to 9 KeV. Because we did not take radiation losses into account in equation 4, it is likely that the higher estimate is more appropriate. We also did not take the heating due to the 14 Mev neutrons into account. This is reasonable for considering the conditions for initiating the detonation. After the detonation has proceeded for a distance like the neutron mean free path, we have a large additional source of power. Therefore temperatures in the core would be expected to rise, and other fuels could be burned.

II. We have seen in the previous section that a temperature of about 10 KeV is required to propagate a detonation wave in a D-T mixture. In this section we will estimate the amount of energy input required to initiate a detonation. The critical mass is on the order of a sphere of radius λ_{α} , the range of the 3.5 Mev α -particle. Therefore we have:

$$M_{crit} \approx \frac{4\pi}{3} \rho \lambda_{\alpha}^3$$

We define a compression factor α , by the relation $\rho = \rho_0 \alpha$. We will take ρ_0 to be the density of a liquid D-T mixture, or more explicitly, corresponding to an electron density of 5×10^{28} meter⁻³. At the ignition temperature we have $\lambda_{\alpha} = 10^{-2} \alpha^{-1}$, so the critical mass becomes $M_{crit} = (4\pi/3) 10^{-6} \rho_0 \alpha^{-2}$. The energy necessary to bring this mass up to the critical temperature then is:

$$E_{\text{crit}} \approx 600/\alpha^2 \text{ MJoules} \quad (5)$$

Several points are worthy of note here. One is the strong dependence upon α . The laser fusion scheme make use of this in the extreme, with $\alpha \approx 10^4$. The other point relates to our assumption about the α particle range. In the case that we have high magnetic fields present, the range might be substantially shorter. Since this factor enters cubically into the equation, one might indeed not expect equation 5 to provide more than a rough estimate.

III. The importance of initiating a detonation in order to achieve high energy gains can be seen from the following considerations. We can define a reaction time $t_{\text{reaction}}^{-1} = n \langle \sigma v \rangle$. For an inertially confined plasma, the expansion time for a critical mass is $t_{\text{exp}} = \lambda_{\alpha} / v_{\text{th}}$. If we put in 10 Kev to heat the plasma, and obtain 17000 Kev from each fusion reaction, then the net energy yield is given by:

$$Y_{\text{crit}} = 1700 \langle \sigma v \rangle \cdot \frac{n \lambda_{\alpha}}{v_{\text{th}}} \approx 100$$

Note that this is independent of α . Such a small yield ratio makes it difficult to achieve a net useful energy gain. In order to break even one must consider the heating efficiency, τ_h , the boiler/turbine efficiency, τ_{th} , which is usually around .35, and the beam energy/input energy for the trigger device, τ_b . If a detonation is started, however, the yield is virtually unlimited. The problem becomes one of containment for the explosion.

IV. So far we have only discussed the energy requirements for initiating a thermonuclear reaction. In this section we will examine the power requirements. In order to get some feeling for the times that are involved we write down the following time constants:

$$\begin{aligned} t_{\text{Bremsstrahlung}} &= \frac{2 \times 10^{-8}}{\alpha} \\ t_{\text{reaction}} &= \frac{29 \times 10^{-8}}{\alpha} \\ t_{\text{conduction}} &\approx \frac{1.33 \times 10^{-8}}{\alpha} \\ t_{\text{expansion}} &= \frac{1.8 \times 10^{-8}}{\alpha} \end{aligned}$$

A word of caution is required for the time constant for thermal conductivity. This depends upon the electrical conductivity, and anomalous resistivity due to turbulence effects may increase this substantially. On the other hand, turbulent mass transport may also tend to enhance the heat conduction. Experimental results are needed here.

Using equation 5 for the critical energy, we obtain an approximate value of the power requirement to be $P_c \approx 6 \times 10^{16} \alpha^{-1}$ watts.

V. Now we will consider energy deposition in the plasma by relativistic charged particles. A particle with charge Z will lose $.6 Z^2$ Mev in transversing the diameter of a critical mass of D-T. Since the particles we will be considering will have much higher energies than this, they will generally be capable of substantial penetration. In what follows, we will always assume that we are dealing with a cylindrically symmetrical distribution, with the energy deposition along the axis. The number of charge particles required for ignition is given by:

$$N_c = \frac{6 \times 10^{21}}{Z^2 \alpha^2} \quad (6)$$

Two extremes are of interest in this regard. Electrons are attractive because of their low mass. This gives rise to radiation damping in a storage ring, and to pair production and Bremstrahlung in its interactions. By "showering" the electrons before entering the DT region, substantial particle multiplication can be achieved. On the other hand, high Z ions, like Uranium, are required in much fewer numbers. Equation 6 exhibits the basic design problem. It is to devise a means of achieving high compressions, and to obtain a large number of particles in a short time.

The instantaneous current required in the charged particle beam is given by:

$$i_c = \frac{zeNc}{t} = \frac{z}{Z^2 \alpha} \times 10^{11} \quad (7)$$

where z refers to the charge of the particle before it is stripped in the energy loss process. For the case of electrons, this must be corrected by the particle multiplication factor K , which could be as high as 100.

VI. So far we have specified the instantaneous current, bunch width, and particle numbers needed for ignition of an "unconfined" system. These all critically depend on α . In this section we will derive a relation independent of α which will tell us more about the properties of the charged particle beams. We consider the transverse space charge limit for an unneutralized beam in a circular strong focusing storage device. In the limit of tight bunches and large apertures, one can neglect the effects of image forces. In this case the space charge limiting current can be written:⁴

$$i_{\max} = .8 \times 10^7 e \frac{\gamma^3}{R} \frac{A}{z} \quad (8)$$

Here we have taken ϵ to be the transverse emittance, and assumed a tune shift of .25. R is the average radius of the storage ring. Clearly, we would like ϵ to be large. However, since we must focus this beam onto a spot, there is a limit to this. Since the radius of the spot must be no larger than $10^{-2}/\alpha$, and if we assume .1 radian for the greatest angular divergence which is practical, we obtain $\epsilon \approx 10^{-2}/\alpha$. Inserting this into 8, and equating with 7, we obtain the following result, which is independent of α :

$$\frac{A \gamma^3}{R} = 1.25 \times 10^7 \left(\frac{z}{2}\right)^2 K^{-1} \quad (9)$$

For electrons, this comes out to be $\gamma^3/R = 2.3 \times 10^{10} K^{-1}$. If we take R to be 100 meters, and $K \approx 100$ we obtain $\gamma \approx 2800$, corresponding to about 1.5 Gev. For U ions, we use the relation $BR = P/ze$, and express γ as a function of the average magnetic field. We obtain:

$$\gamma = 66 \sqrt{\frac{z}{B}}$$

$$BR = \frac{4.76 \times 10^4}{\sqrt{Bz}}$$

Since BR is a quantity one wishes to keep small, one wants high B and high z . Taking $B \approx 3$, and $z = 92$, we obtain $R = 939$ meters, $\gamma = 366$. This is equivalent to a 900 Gev proton storage ring.

VII. It must be emphasized that most of the foregoing relations are based upon an assumption that magnetic confinement did not effect our estimate of α . Furthermore, the magnetic fields will act to retard the expansion

the plasma across the field lines. The magnetic rigidity of a 3.5 Mev α particle is .26 T-m. The field associated with a line current is given by:

$$B = 2 \times 10^{-7} \frac{1}{r}$$

The condition that we have enough current to bend the α particle is then:

$$i = 5 \times 10^8 \text{ amps} \quad (10)$$

This condition is essentially independent of the size of the current source. Hence it does not depend upon α . Comparing this result with that of equation 7, we see that in fact the magnetic fields may play a large role indeed. The establishment of large magnetic fields in a dense hot plasma is a subject of considerable experimental and theoretical interest at the present time.

Distribution: B2

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