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Survey of Recent Applications of Reactor Noise Techniques
at Argonne National Laboratory*

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Abstract

Recent applications of reactor noise techniques to the improvement of subcritical reactivity and β_{eff} measurements in fast critical assemblies and to fast power reactor surveillance by neutron/acoustic noise cross correlation are reviewed.

The Inverse Kinetics Rod Drop (IKRD) technique has been the primary subcriticality measurement technique applied at ANL because of its relative insensitivity to counter efficiency and prompt lifetime changes and the fact that a near-critical reference is not needed. Estimates of the random errors in IKRD reactivity measurements resulting from noise effects have been developed by simulation studies and theoretical analysis. Relative standard deviations of 1 to 20% are indicated by these methods for initial Δk from -0.0005 to -0.035, respectively, under conditions typical of the Fast Test Reactor (FTR) mockup experiments in the ZPR-9 Fast Critical Facility. Space-dependent kinetics calculations have been applied to the IKRD technique to study systematic errors that are functions of detector location as well as changes in kinetics parameters and effective source strength in the course of a rod drop. In various situations, systematic errors from these causes may be as low as several parts per hundred for detectors located outside the axial reflector due to cancellation of errors. For other detector locations, the principal error components might be in the same direction, thereby amplifying the resultant error.

Noise measurements have been used as cross checks on IKRD results. The break frequency derived from cross power spectral density or polarity correlation has been found not to be a good reactivity estimator in fast power reactor critical assemblies presumably because wide disparities between neu-

tron lifetimes in the core and reflector produce deviations from point kinetics in the range of the break frequency. Various two-detector mid-frequency polarity coherence function methods have produced good reactivity estimates, although these methods depend on near-critical reference measurements and are perturbed by differences in detector efficiency between the reference and measured systems. Correction methods to account for detector efficiency and β_{eff} changes have been developed.

The accuracy of fast critical experiment measurements of $\Delta k/k$ are directly affected by the accuracy with which β_{eff} is known. Recent conflicting experimental data on β_i , λ_i has been the incentive for use of a noise-based integral β_{eff} measurement technique. The technique is a modified detector noise variance-to-mean method using sampling intervals from 8 to 200 sec; β_{eff} has been measured to a precision of $\pm 2.9\%$ in the FTR mockup critical assembly.

Exploratory work on the use of neutron/acoustic noise cross correlation as a tool for surveillance of fast power reactor systems has been carried out using the Experimental Breeder Reactor II. Cross correlation of neutron signals from in-core detectors with acoustic signals from accelerometers mounted on various mechanical components connected into the core structure has been studied up to several thousand hertz. Flow-induced subassembly vibrations at about 10 hertz have been observed.

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I. Studies of Random and Systematic Errors in Subcriticality Measurements by the Inverse Kinetics Rod Drop Technique

Measurements of the reactivity of subcritical plutonium fast reactor assemblies are of key importance for physics measurements as well as for operational safety. Emphasis in recent work at ANL has been placed on the refinement of inverse kinetics rod drop (IKRD), cross power spectral density (CPSD), and polarity correlation methods for subcritical reactivity measurements.¹

IKRD has been the primary subcriticality measurement technique applied in the fast critical assembly program at ANL because of its relative insensitivity to counter efficiency and prompt lifetime changes and because a near critical reference is not needed. The fact that a critical reference is not needed is of particular significance for assemblies that never approach critical. Data are taken by banks of high-efficiency ionization chambers mechanically isolated from the reactor structure to minimize microphonic perturbations during the drop. The data are analyzed by on-line computers using various algorithms²⁻⁴ that involve inverse-kinetics analysis of the flux record coupled with determination of the optimum estimate of source strength.

IKRD reactivity and source estimates are subject to random errors arising from neutron noise and systematic errors arising from detector location and changes in kinetics parameters and effective source strength in the course of a rod drop. Although the algorithms used for IKRD data analysis are not completely amenable to direct analysis of random and systematic errors, error estimates have been developed through simulation studies,^{5,6,7} a semi-empirical approach,⁸ and by direct theoretical analysis of a simplified

algorithm that assumes instantaneous rod drops. These error estimates are discussed in the following sections.

A. Precision Estimates by Simulation Studies

Cohn⁵ has carried out simulation studies of subcriticality measurements by IKRD to investigate the statistical error properties of this model. The studies were performed within the framework of the point-reactor model since the statistical error is independent of the systematic errors due to deviations from the point model and uncertainties in the basic kinetics parameters. Studies of systematic error sources are discussed in Section I-C below. The method and results of the Cohn studies are summarized below.

Method

The simulation process consisted, first, of the generation of a flux profile typical of the rod drop being modeled. This profile was then perturbed by noise in the form of pseudorandom Gaussian variates applied to each flux point. The noisy profile was then applied as input to the inverse-kinetics algorithm under test, and the results of that algorithm were recorded as estimates of initial and final reactivity. The process was replicated 500 times for each test. Two versions of the simulation program were written. One simulates detection noise only, while the other includes the reactor-noise component.

Noise-free flux profiles were generated by the direct-kinetics algorithm of Kaganove.⁹ Pseudorandom Gaussian variates were developed by use of the polar algorithm (Ref. 10, p. 104) whose input was uniform random numbers developed by a mixed-congruential generator.

The kinetics parameters were chosen to be typical of a plutonium-fueled fast critical assembly, and are listed in Table I. The source strength was

Table I. Kinetic Parameters for Simulations

Delayed Neutrons:

<u>Group</u>	<u>Decay Constant λ_i, sec⁻¹</u>	<u>Fraction β_i</u>
1	0.0130	0.0000820
2	0.0314	0.0006716
3	0.136	0.0005852
4	0.340	0.0010885
5	1.32	0.0004748
6	3.50	0.0001579
<hr/> TOTAL		$\beta = 0.00306$

Prompt Lifetime $\ell = 4 \times 10^{-7}$ sec.

chosen sufficient to yield 200,000 detected neutrons per second at 1% sub-critical. This is typical of the detection rate that has been observed in the FTR and FFTF-EMC assemblies in ZPR-9. The detection efficiency was taken as 0.00025, which is characteristic of the SP-11 bank of four ion chambers on ZPR-9, and thus is typical of the maximum efficiency that might be obtained with conventional instrumentation. The drop is assumed to be linear in k over a time interval of 0.3 sec. A time step of 0.15 sec was used, with initial conditions maintained for 20 sec before the drop. The flux was followed for 180 sec after the drop.

The main simulation program calls upon a subroutine to perform the direct-kinetics calculation and also calls upon a subroutine that embodies the inverse-kinetics algorithm under test. The Cohn inverse-kinetics algorithm of Ref. 2 and the Carpenter inverse-kinetics algorithm of Ref. 3 were tested and compared in these studies.

For the simulations without reactor noise, the direct kinetics was run once and the resulting noise-free flux record stored in memory throughout the run. A noisy flux record was then constructed by perturbation of each datum with a Gaussian random variate with standard deviation equal to the number of detected neutrons in that time interval. The inverse-kinetics algorithm was then invoked, with the noisy flux record as its input, to yield estimates of initial and final Δk .

The above operations were replicated 501 times. The first time, the inverse kinetics operated on the noise-free flux. This served as a check on the accuracy of the entire calculation. For the remaining 500 replications, sums were accumulated to obtain the average and standard deviation of the initial and final Δk .

The importance of the noise-free-flux test cannot be overemphasized. Results have agreed with the input initial and final subcriticalities to four or five significant figures. Such agreement is not only a cross-check on the direct kinetics, but is a necessary condition for the validity of an inverse-kinetics algorithm. If an algorithm does not return the correct result on a noise-free run, then an unbiased result from an ensemble of noisy runs must be regarded solely as a fortuitous cancellation of errors.

Reactor noise was simulated by means of a noise-equivalent-source model previously developed.¹¹ Here, the direct-kinetics calculation must be executed anew for each noisy run. At each time step, a Gaussian random variate was superimposed on the source term to simulate the reactor-noise component. After the direct-kinetics algorithm was executed, another Gaussian random variate was superimposed on the output flux to simulate the detection-noise component as before.

The variance of the random variate to be added to the source term was taken as the variance of the noise-equivalent source integrated over one time step. This latter was calculated from the formula¹²

$$\sigma^2 = \frac{2}{(\Delta t)^2} \int_0^\infty \frac{1 - \cos \omega \Delta t}{\omega^2} \Psi(\omega) d\omega$$

where $\Psi(\omega)$ is the spectral density of the noise-equivalent source in units of angular frequency, and Δt is the duration of one time step. Since

$$\int_0^\infty \frac{1 - \cos ax}{x^2} dx = \frac{a\pi}{2}$$

we get

$$\sigma^2 = \pi \Psi / \Delta t$$

because $\Psi(\omega)$ is white. (If the Schottky formula as given in Eqs. 1 or 2 of Ref. 11 is transformed to units of angular frequency and substituted in the above, the result agrees with the predictions of Poisson statistics.)

The theory of Ref. 11 was derived for a critical reactor. For a subcritical system, a few minor changes are needed. First, on the r.h.s. of Eq. 3 of Ref. 11, unity is replaced by k . Secondly, an additional entry is added to Table I of Ref. 11 to account for the neutrons introduced by the source. For this entry, the average rate of occurrence is equal to the source strength S_0 and the net number of neutrons produced is one. We have

$$n = \frac{\ell S_0}{1 - k} ,$$

so

$$S_0 = \frac{r(1 - k)}{\ell} .$$

The noise-equivalent source then becomes

$$\Psi(\omega) = \frac{n}{\pi \ell} \left(\frac{\overline{v^2 k} - \overline{v}(3k-2)}{\overline{v}} \right)$$

and the variance becomes

$$\sigma^2 = \frac{n}{\ell \Delta t} \left(\frac{\overline{v^2 k} - \overline{v}(3k-2)}{\overline{v}} \right)$$

Introduce the Diven parameter

$$D = (\overline{v^2} - \overline{v}) / \overline{v^2} .$$

We then have

$$\sigma^2 = \frac{n}{\lambda \Delta t} \left(k D \bar{v} + 2(1-k) \right).$$

Let Q be the desired detection rate (here $2 \times 10^5/\text{sec}$ at 1% subcritical as previously mentioned) and ϵ be the detector efficiency in number of detections per fission. Since the fission rate is $n k / \lambda v$, we have at 1% subcritical

$$Q = n k \epsilon / \lambda \bar{v}$$

and the source strength becomes

$$S_0 = \frac{Q \bar{v} (1 - k)}{k \epsilon} \cong 2000 \bar{v} / \epsilon.$$

(It would undoubtedly be more straightforward if the detector efficiency were expressed in terms of total neutron absorptions instead of in terms of fissions. In any case, the detection rate in these simulations was taken as being proportional to the neutron population.)

The neutron count integrated over each time step, as calculated by the direct kinetics algorithm, was multiplied by $\epsilon / \bar{v}$ to obtain the detector count. That quantity was also used as the variance for the Gaussian random variate to be added to the count.

Results

The results are shown in Table II. It is apparent that the reactor-noise component made an appreciable addition to the total error only for initial Δk smaller in magnitude than 1%. (Since the reactor noise places the lower limit on attainable error, worthwhile gains in precision could be had further subcritical if detector efficiency could be increased.) With or without the reactor noise, the Carpenter algorithm gave slightly smaller error than the

Table II. Results of Rod-Drop Simulations

		Fractional Standard Deviation of Estimate of Initial Subcriticality by a Single Drop			
		Reactor Noise Absent		Reactor Noise Present	
Initial Δk	Final Δk	Algorithm of:		Ref. 1	Ref. 2
		Ref. 1	Ref. 2		
-0.0005	-0.001	0.0035	0.00099	0.016	0.0071
-0.001	-0.002	0.0030	0.0014	0.013	0.0065
-0.002	-0.004	0.0037	0.0024	0.012	0.0072
-0.005	-0.01	0.0076	0.0071	0.014	0.012
-0.01	-0.02	0.018	0.018	0.022	0.021
-0.015	-0.025	0.037	0.035		
-0.02	-0.04		0.045	0.050	0.049
-0.025	-0.035	0.094 ^a			
-0.035	-0.025	0.079			
-0.035	-0.045	0.219	0.209		
-0.035	-0.055	0.132	0.127		
-0.045	-0.035	0.20 ^a			

^aApplies to final subcriticality.

Cohn algorithm, the difference becoming quite marked for initial Δk smaller in magnitude than 0.5%. Apparently, the fitting process in the Cohn algorithm is suboptimal for these.

Rod drops are normally performed by insertion of boron rods. That is, the configuration to be measured is the initial configuration in the experiment and is a clean configuration with no extraneous rods present. The drop is made by insertion of boron rods to remove reactivity.

One can conceive of an alternative approach in which fuel rods rather than boron rods are used, with the configuration being measured being the less-reactive rather than the more-reactive configuration. Such fuel rods could run in channels similar to those for the present boron rods, and could be of quite simple configuration. Since they would not be present in the configuration being measured, their detailed neutronics would be of no consequence, and their only function would be to carry reactivity. In particular, they could be made of uranium instead of plutonium. The only concern would be to avoid significant changes in shape function. It turns out that such a mode of operation could offer significant gains in precision.

Suppose, for example, that one wishes to measure a subcriticality of 3.5% by making a rapid reactivity change of 1%. There are four ways of doing this, viz:

1. The present method--start at 3.5% and rapidly insert boron rods to reach 4.5%.
2. Start at 4.5% and rapidly remove boron rods to reach 3.5%.
3. Start at 2.5% and rapidly remove fuel rods to reach 3.5%.
4. Start at 3.5% and rapidly insert fuel rods to reach 2.5%.

(It is recognized that methods 2 and 4 are at variance with safety requirements. However, it is worthwhile examining them if only to find out what, if anything, is being forgone--not very much, as it turns out.)

These four protocols were simulated with the Cohn algorithm and detection noise only. The results are included in Table II, which shows the standard deviation obtained for the subcriticality of interest, i.e., the 3.5% configuration. It can be seen that the use of fuel rods instead of boron rods would gain more than a factor of two in precision in this situation.

Very little in precision is gained in going from a negative to a positive insertion. However, where a given drop is replicated a number of times, a sequence of positive and negative drops could make for more efficient use of reactor time than the present method in which the rods are reset at their slow withdrawal rate after each drop.

Results of a statistical analysis of the simulation results are shown in Table III for the largest and smallest initial subcriticalities studied. All moments through the fourth were calculated from the 500 noisy runs for each case and, from these, the coefficients of skewness and kurtosis were calculated. The former is equal to $(m_3/m_2)^{3/2}$ and has expectation value of 0 ± 0.109 for a sample of 500 items. The latter is equal to $(m_4/m_2)^2 - 3$ and has expectation -0.012 ± 0.216 for a similar sample. For the largest subcriticality, the Δk estimates show significant skewness and kurtosis, and a just significant bias. The negative values of skewness indicate that the distributions are skewed toward more negative (i.e., numerically larger) subcriticality values, in agreement with the observed bias. The positive values of kurtosis indicate that the distributions are more sharply peaked than the Gaussian.

Table III. Distribution Parameters of Results

Case			Initial Δk Estimates				Initial Multiplication Estimates			
Initial Δk	Final Δk	Algorithm	Mean		Skewness	Kurtosis	1/Mean		Skewness	Kurtosis
-0.035	-0.055	Cohn	-0.0354	± 0.0002	-1.031	2.739	-0.0349	± 0.0002	0.047	0.086
		Carpenter	-0.0355	± 0.0002	-0.977	2.859	-0.350	± 0.0002	0.000	0.073
-0.0005	-0.001	Cohn	-0.0004765	± 0.0000004	0.162	-0.033	-0.0004999	± 0.0000004	-0.254	-0.006
(With Reactor Noise)		Carpenter	-0.0005002	± 0.0000002	0.045	-0.229	-0.00050012	± 0.00000005	-0.083	-0.216

On the other hand, the same data expressed as multiplication show no significant skewness, bias or kurtosis, nor can these be seen at all for the smallest subcriticality.

What are the implications of these results? First, it appears that multiplication is unskewed and unbiased. (This also applies to the quantity b of Ref. 3, which is linearly related to final multiplication.) If so, then Δk cannot help being skewed and biased just on account of the reciprocal relationship between the two. Thus, to present statistically nice numbers, results would have to be presented in terms of multiplication. If, on the other hand, one is constrained to present Δk , an unavoidable amount of skewness, kurtosis, and bias must be accepted. Second, replications of a given measurement should be combined through an average of $1/\Delta k$ instead of an average of Δk directly as suggested by Carpenter.

B. Precision Estimates by Theoretical Analysis

Bennett¹ has reported a theoretical analysis of the random errors in IKRD measurements based on a nonstationary calculation of the chain-associated conditional probabilities of neutron detection. The analysis is based on a simplified three-point algorithm in order to make the analysis tractable. A summary of Bennett's analysis and results are summarized here. The resulting calculated random errors support Cohn's simulation results described in Section I-A.

The Three-Point Method

The text-book method for deducing initial subcriticality from a rapid reduction in multiplication is by observing flux levels just prior to and just following a rod drop as well as to observe the asymptotic level reached upon decay of all transients to negligible values. The resulting "three-point

method" where ϕ_1 , ϕ_2 and ϕ_3 refer to the three relevant fluxes may use, as the basis for analysis, the two ratios $(\phi_2 - \phi_3)/\phi_1$ and ϕ_3/ϕ_1 defined as

$$\frac{\phi_2 - \phi_3}{\phi_1} = \frac{\Delta k_2 - \Delta k_1}{\Delta k_2 + \beta(1 - \Delta k_2)} \frac{\beta}{\Delta k_2} \quad \text{and} \quad (1)$$

$$\phi_3/\phi_1 = \Delta k_1/\Delta k_2 \quad . \quad (2)$$

Δk_1 and Δk_2 are the initial and final values of $1-k$ and β is the effective delayed neutron fraction. The assumptions in arriving at Eqs. (1) and (2) are those inherent in the simple model of prompt-jump point-reactor kinetics with a neutron source present. In order to focus more precisely upon the effects of noise, no allowance is made for spatial or energy-dependent phenomena and the transition from Δk_1 to Δk_2 is assumed instantaneous. This assumption of an instantaneous reactivity change is very good for drops made routinely on fast-reactor mockups.

Experimental values for flux levels during a rod drop are derived by integrating the output of a neutron-monitor detector over finite periods of time. The experimental result then consists of a record of several thousand numbers representing average fluxes over a basic sampling interval δ , which is usually a few tenths of a second. The record must contain an adequate sample of the stationary pre-drop time region as well as of the stationary post-drop time region. Due to the presence of fluctuations in average flux values, it is not prudent to use only three points averaged over the basic sampling interval δ , as implied in Eq. (1) (with ϕ_2 defined just after the drop is complete). Instead, one wants to use as much of the profile as possible to extract the maximum information and to minimize the effects of ran-

dom statistics. This leads naturally to a redefinition of the three pertinent fluxes as shown in Fig. 1. Extended time intervals t and τ are chosen with the objective of minimizing random effects; the extended interval for definition of ϕ_2 commences at a time t safely after the drop is complete and continues to a time $t + \tau$ during which the profile is decaying and is approaching its ultimate stationary post-drop value ϕ_3 . ϕ_2 should be an average over the transient region with the upper time limit $t + \tau$ chosen to maximize the ratio of transient integral to noise over the same interval.

Regardless of the choice of t and τ , a unique functional relationship exists expressing initial subcriticality in the form

$$\Delta k_1 = \text{REACT} [(\phi_2 - \phi_3)/\phi_1, \phi_3/\phi_1]. \quad (3)$$

Δk_1 is the initial $1 - k$, $(\phi_2 - \phi_3)/\phi_1$ and ϕ_3/ϕ_1 involve only integrals of the profile over appropriate time intervals divided by the interval. REACT is a function which returns a precise value for Δk_1 given precise values of the two arguments and the interval t to $t + \tau$ over which the average transient ϕ_2 is derived. When ϕ_2 is estimated just after the drop and over a very short time interval, Δk_1 can be written explicitly as a function of $(\phi_2 - \phi_3)/\phi_1$ and ϕ_3/ϕ_1 by solving Eqs. (1) and (2). When ϕ_2 is estimated over the extended interval t and $t + \tau$ (Fig. 1), no simple expression exists and the full delayed-neutron kinetics must be used but a unique functional relationship exists although recourse to a computer is necessary for its evaluation.

The average flux ϕ_2 over the transient interval is not, especially for drops far from critical, much in excess of the asymptotic flux ϕ_3 . The difference $\phi_2 - \phi_3$ will be small and will be the source of essentially all of the random error in the determination of Δk_1 . Errors in ϕ_1 and ϕ_3 may be

Fig. 1 - A Schematic of a Rod Drop Showing the Fluxes and Time Intervals Used in the Generalized Three-Point Method. ϕ_2 is Determined as an Average Over the Transient Decay.

maintained at negligible values simply by extending the pre- and post-drop time periods sufficiently.

Noise may be calculated from knowledge of detection rate and efficiency and the pile noise component is readily incorporated. The effect of noise upon Δk_1 may be predicted by determining three values of the argument $(\phi_2 - \phi_3)/\phi_1$ --the value derived from the profile and that value increased and decreased by the r.m.s. statistical error in $(\phi_2 - \phi_3)/\phi_1$. By use of Eq. (3) this results in three values for reactivity--the value from the measured result (the most probable value) and the reactivity error bounds which are skewed since the functional relationship is not linear, but which retain meaning in that the frequency of additional (ensemble) analyses will appear within the limits with the usual r.m.s. (68%) probability.

Calculation of Random Errors

The calculation of fluctuations in the pre- and post-drop asymptotic stationary averages ϕ_1 and ϕ_3 is straightforward and, in fact, is almost the original variance analysis for the Los Alamos "Water Boiler" reported in Ref. 13 (see also Ref. 12). Stationarity implies that the reactor was leveled off at some arbitrary power and operated for a long time compared to delayed neutron precursor lifetimes thus allowing all transient effects to disappear. The reactor will always exhibit a dynamic behavior due to random noise effects, but if the system is stationary these effects, in the sense of mean values or ensemble-averaged values, are time independent.

Obviously, however, a reactor is not stationary just after a rod drop - in fact, it is just the measurement of the transient effect which makes a sub-criticality determination possible. Since the contribution to the error analysis from the stationary fluxes ϕ_1 and ϕ_3 is trivial (or can be made so),

the transient average ϕ_2 dominates the error analysis. Statistics in ϕ_2 can only be derived allowing for the non-stationary effects associated with a sudden change in multiplication. In many cases of interest the transient constitutes a relatively small perturbation on the final state flux and we might reasonably expect that it would exert little influence upon post-drop fluctuations except near critical.

The solution for the non-stationary variance in integrated flux following a rod drop makes use of the method of derivation used originally¹³ for the stationary case. The significant differences are that the ensemble-averaged fission rate are permitted to vary according to the familiar prompt-jump solution to the kinetics equations. In addition to permitting a time variation of the mean profile, the chain behavior will be modified to account for the "bending" on crossing the discontinuity in subcriticality occurring at drop time of those chains initiated prior to the drop. Pile noise can be related to the time behavior of fission-initiated neutron chains and the "bending" at drop time is a necessary complication to the exact analysis of the post-drop variance.

Define the detector variance over the post drop interval σ^2 , according to

$$\sigma^2 = \frac{\langle (C - \langle C \rangle)^2 \rangle}{\langle C \rangle^2} \quad (4)$$

where C denotes the integral of detector output over the interval τ

$$C = \int_t^{t+\tau} \phi(t') dt'. \quad (5)$$

Brackets $\langle \rangle$ indicate an ensemble average.

Bennett has shown that σ^2 may be expressed as

$$\sigma^2 = \frac{1}{\langle C \rangle} + \frac{2\epsilon^2 D}{\langle C \rangle^2} \int_t^{t+\tau} dt_2 \int_t^{t_2} dt_1 \int_{-\infty}^{t_1} F(t_0) \chi(t_0, t_2) \chi(t_0, t_1) dt_0. \quad (6)$$

where:

ϵ = detector efficiency, neutron detection rate/pile fission rate.

$F(t)$ = pile fission rate.

$D = \langle \nu(\nu-1) \rangle / \langle \nu \rangle^2$.

$$\chi(t, t_0) = \sum_p A_p e^{s_p(t-t_0)} \quad 0 < t_0. \quad (7)$$

$$\chi(t, t_0) = \sum_p A_p G_p(t_0) e^{s_p t} \quad t_0 < 0. \quad (8)$$

$\chi(t, t_0)$ in Eq. (7) describes the decay of neutron chains originating at time t_0 after the drop. $\chi(t, t_0) dt$ is the probability that a neutron will be lost from the system, in any way, in the interval dt as a consequence of a single neutron introduced at t_0 . s_p and A_p are poles and residues of the transfer function $T(s)$ evaluated for post-drop kinetics and subcriticality. When t_0 is less than zero, the flux and precursor densities at drop time are boundary values for a solution of the chain decay in the post-drop period; the result is given in Eq. (8). The amplitudes $G_p(t_0)$ are functions of both pre- and post-drop kinetics and subcriticalities as well as of t_0 .

The first term on the RHS of Eq. (7) represents the detection noise contribution; the second term represents the pile noise contribution. In Table IV the variance contribution from random detection alone is listed together with the total variance (detector plus pile) for some typical rod-drop

Table IV. Comparison of Total Variance with Detector Variance Only

Start time = 0.5 sec, Stop Time = 25.0 sec $\epsilon = 2.5 \times 10^{-4}$,
Fission Rate = 2.4×10^9 at $1-k = 0.01$

Initial $1-k$	Final $1-k$	Detector Variance Only	Total Variance
0.0005	0.001	0.469 E-08	0.105 E-05
0.001	0.002	0.106 E-07	0.619 E-06
0.002	0.004	0.234 E-07	0.352 E-06
0.005	0.01	0.630 E-07	0.209 E-06
0.01	0.02	0.131 E-06	0.209 E-06
0.02	0.03	0.202 E-06	0.255 E-06

situations. Notice the strong impact that chain-related events have on precision in subcriticality measurements where a high detector efficiency exists. The total variance decreases by about a factor of 5 over the range of final $1 - k$ from 0.001 to 0.030. This of course reflects the domination of random processes by the pile and not the detector. Despite the fact that power drops as subcriticality increases the variance is actually decreasing. If, on the other hand, one chooses to ignore the pile and retain only detector noise, the variance will increase by about a factor of 40 over the range 0.001 to 0.030.

C. Accuracy Estimates by Simulation Studies

Space-dependent kinetics calculations⁶ using the QX-1 Code¹⁴ have been applied to the IKRD technique to study systematic errors⁷ that are functions of detector location as well as changes in kinetics parameters and effective source strength in the course of a rod drop. It appears that the resultant error may be represented as arising from a change in detector efficiency. In various situations, systematic errors of several parts per hundred may arise from these causes for detectors located outside the axial reflector; studies of FTR-3-like configurations have indicated that some error components tend to cancel others when the detector is located there. Under other conditions, the principal error components might be in the same direction, thereby amplifying the resultant error.

Meneley and Fuller⁶ have reported a study of systematic errors in rod drop measurements for the FTR-3 illustrative of the possible systematic effects. The FTR-3 assembly was a two-zone system in which the ^{240}Pu spontaneous fission rate was more than double in the outer zone, 28.2 fis/cm³-sec compared with 13.5 fis/cm³-sec in the inner zone. The radial dimensions of the system are shown in Table V; the extrapolated height was taken to be 134.76 cm.

Table V. FTR-3 Radial Dimensions
for Rod-Drop Simulation

Region	Radius, cm
Central rod zone	3.12
Inner boundary, rod zone 2	9.36
Outer boundary, rod zone 2	11.25
Outer boundary, core zone 1	37.58
Outer boundary, core zone 2	60.16
Outer boundary, control ring	62.13
Outer boundary, reflector	85.11
Outer boundary, shield	97.09

A rod drop from $k = 0.980$ to $k = 0.9735$ involving five control rods in the central zone was simulated. The QX-1 code was run to a reactor time of 30 sec; the 29-group fluxes were used to compute ^{10}B detector reaction rates as a function of radius within the system. This code produced values of "apparent reactivity" as would be measured by a detector at radius r , in the absence of statistical fluctuations. (The initial subcritical reactivity was known exactly from the effective source calculated by QX-1.) The difference between final and initial reactivity was taken as the "apparent rod worth" measured at radius r . Figure 2 shows the ratio of this apparent rod worth to the true rod worth calculated by QX-1. The usual placement of detectors is outside the shield. From Figure 2 it can be seen that the rod worth measured in this way would be $\sim 8\%$ low. This is inferred from the fact that the Fig. 2 ratio is almost constant through the reflector and shield, a range in which the neutron spectrum is softened considerably. The measured value of $(1 - k)$, again in the absence of statistical variations, would be approximately 2% low in magnitude.

Fig. 2. Ratio of Apparent Rod Worth as Measured by a ^{10}B Detector at Position r , to True Rod Worth Calculated by QX-1

The work of Shaftman, Ref. 7, discusses the components of the systematic errors arising from spatial effects and kinetic parameter changes during a rod drop. The work is based on QX-1 studies of the FTR-3 Assembly. There is evidence of considerable fortuitous cancellation of systematic errors in the FTR-3 Assembly leading to the conclusion that some analytical space-dependent kinetics checks are needed to validate the accuracy of rod-drop measurements in any complex assembly.

II. Errors in Subcriticality Measurements by Noise Techniques

A. Experience with Cross Power Spectral Density and Polarity Coherence Subcriticality Measurements

Cohn¹⁵ has reported ANL experience in comparing IKRD and noise-based reactivity measurements. Noise methods have been applied to the measurement of subcriticality in the FTR Engineering Mockup critical experiment program as a cross-check on subcriticality measurements by rod drop.² Initial efforts used the two-detector cross power spectral density (CPSD) method.¹⁶

Equipment for these measurements included, as high-efficiency neutron detectors, two NE-905 glass scintillators, each 1-1/2-in. diam and 1-in. thick containing about 4.7 g of ⁶Li. Each photomultiplier output enters a General Radio multifilter, which allows selection of one of thirty 1/3-octave bands with center frequencies from 100 to 80,000 cycles. Both filters are set to the same frequency. Their outputs enter the two inputs of an analog multiplier. The multiplier output is integrated by a voltage-to-frequency converter, having two pulse outputs, one active on positive excursions of the multiplier output and the other active on negative excursions. These outputs are counted in two separate scalers, and the difference between the scaler readings indicates the CPSD.

The system is calibrated by exposure of one scintillator to white noise in the form of a ^{60}Co source. Both multifilter inputs are connected to the one detector output, and measurements are taken as described above. For each band, the reading taken on a reactor run is divided by the white-noise reading, calibrating out equipment frequency response.

With this equipment, Cohn found it possible to measure roll-off frequencies to 1 or 2% in 1 h, from near critical down to 2% subcritical. However, the CPSD method gave unsatisfactory results apparently because of departures from point kinetics in the reactor behavior. Near critical, observed roll-off frequency was up to 10% greater with the detectors in the center of the core than with the detectors in the reflector. Furthermore, measured subcriticalities disagreed sharply with rod-drop results, being low by as much as 30% of the latter.

Another method has been developed by Cohn and has given results agreeing with rod-drop measurements within 0.1% Δk . Like the methods of Seifritz¹⁷ and Aibrecht¹⁸ it measures coherence between signals from two neutron detectors, but unlike their methods it uses the same equipment as for the CPSD measurements. While the CPSD at a given frequency is indicated by the difference between the two above-mentioned scaler readings, the coherence is indicated by their ratio.

This ratio R is given by

$$R = \frac{\int_0^\infty \int_0^\infty xy P(x,y) dx dy + \int_{-\infty}^0 \int_{-\infty}^0 xy P(x,y) dx dy}{\int_0^\infty \int_{-\infty}^0 xy P(x,y) dx dy + \int_{-\infty}^0 \int_0^\infty xy P(x,y) dx dy} ,$$

where $P(x,y)$ is the bivariate Gaussian probability density function

$$P(x,y) = \frac{1}{2\pi\sqrt{1-\gamma^2}} \exp \left[\frac{1}{2(1-\gamma^2)} (x^2 - 2\gamma xy + y^2) \right]$$

(in standard measure). Here γ is the coherence or correlation, and ranges from zero with no correlation to unity with complete correlation. We obtain

$$R = \frac{2/\pi(\gamma \sin^{-1} \gamma + \sqrt{1-\gamma^2}) + \gamma}{2/\pi(\gamma \sin^{-1} \gamma + \sqrt{1-\gamma^2}) - \gamma}.$$

The value of γ corresponding to a measured R is determined from this formula.

With coherence determined, subcriticality in dollars is given by

$$\$ = (1 + \$_1) \sqrt{\left(\frac{\gamma_1}{1-\gamma_1} \right) \left(\frac{1-\gamma_2}{\gamma_2} \right)} - 1,$$

where γ_2 is the coherence observed in the subcritical state, γ_1 is the coherence observed at the near-critical reference, and $\$_1$ is the subcriticality of the reference. (This is a variant of a previous formula¹⁷ that was restricted to a critical reference.) Results are not sensitive to detector location; good results have been obtained with the detectors either in the center of the core or in the axial reflector. The coherence measurements are made at the lowest multifilter frequency, e.g., 100 Hz.

Danofsky,¹⁹ et al., have reported a series of subcritical reactivity measurements, from approximately 1 to 22\$, performed in the Zero-Power Plutonium Reactor (ZPPR), during the control rod group experiments of Assembly 3 Phase 1A, using the polarity coherence function technique.¹⁷ The noise measurement system, which was a modified version of equipment

previously designed at ANL²⁰ utilized ⁶Li-loaded glass scintillation detectors with a data collection and analysis system consisting of scalars and a VARIAN 620/i computer. Coherence function reactivities, ρ_2 , were calculated from the relationship

$$\rho_2 = 1 - (1 - \rho_1) \left(\frac{\beta_1}{\beta_2} \right) \left(\frac{W_2}{W_1} \right)^{1/2} \left[\frac{\rho_1}{1 - \rho_1} \frac{1 - \rho_2}{\rho_2} \right]^{1/2}$$

which accounts for a nonconstant detector efficiency W and delayed-neutron fraction β . For these measurements, the near critical reference reactivity ρ_1 was -0.017% with a corresponding coherence function ρ_1 of 0.9257 ± 0.0005 . Noise reactivity values were compared with source multiplication and inverse kinetics results. The detector efficiency and β ratios, which were used to correct the noise data and the multiplication results, were calculated with the fast reactor code QX-1.¹⁴ The three methods generally agreed down to about -2% . Source multiplication and noise reactivities generally agreed over the range of the experiments, down to -9% , although the multiplication results were systematically higher and the difference increased with decreasing reactivity; IKRD results deviate below -2% and were 13% lower than the other results at -4.64% .

B. Corrections to the Two-Detector Polarity Coherence Method

The multiplication results discussed in Section II-A were generally larger than the coherence results, even after applying the required calculated corrections. This difference between the two methods increased as the degree of subcriticality became greater. Since the multiplication results were expected to be the most accurate, a study was conducted to identify systematic errors in the noise values. Sources of errors in the experimental

data were investigated and found to be insignificant. This included looking at noise pickup, improper discriminator settings, non-Gaussian count distributions, background effects, signal distortion, unequal detector efficiencies, and improper bandwidth settings. The formula derived by Seifritz¹⁷ for the measurement of reactivity by the polarity coherence method is

$$\rho_2 = 1 - \left(\frac{1 - \rho_2}{\rho_2} - \frac{\rho_c}{1 - \rho_c} \right)^{1/2} \quad (\text{in dollars}), \quad (1)$$

where ρ_c and ρ_2 are the plateau values of the coherence function of the delayed-critical and the subcritical reactor, respectively. This formula assumes detectors of equal efficiency. Equation (1) may be modified to allow the calibration to be at any reactivity, ρ_1 , and to include calculated ratios of both beta effective, β_1/β_2 , and efficiency of the detectors, ϵ_2/ϵ_1 . The resulting degree of subcriticality is given by

$$\rho_2 = 1 - (1 - \rho_1) \left(\frac{\beta_1^2}{\beta_2^2} \frac{\epsilon_2}{\epsilon_1} \frac{1 - \rho_2}{\rho_2} - \frac{\rho_1}{1 - \rho_1} \right)^{1/2}. \quad (2)$$

A new equation (also based on point kinetics) has been derived by Carpenter²¹ that reduces the systematic error in noise measurements using Eq. (2). For a reactor near critical, most of the prompt-fission chains are started by a delayed neutron causing a fission in ^{239}Pu (or ^{238}U which has nearly the same $\bar{\nu}$); when the reactor is far from critical, most of the chains are started by spontaneous fission of ^{240}Pu ($\bar{\nu}' = 2.25$). No mechanism exists in Eq. (2) to account for the change in correlation caused by a change in the number of neutrons initiating decay of the chain. In addition, there will be a similar spatial effect since the distribution of

the start of the chains will change both the average number of neutrons in a chain and the fluctuation in the number although this latter effect has not been considered in the present work.

Carpenter has shown that if the changes in correlation due to the variation with subcriticality of the types of fission events is accounted for Eq. (2) becomes

$$\beta_2 = 1 - (1 - \beta_1) \left[\frac{\beta_1^2}{\beta_1^2} \frac{\epsilon_2}{\epsilon_1} \frac{1 - \rho_2}{\rho_2} \frac{\rho_1}{1 - \rho_1} \right]^{1/2} \left[\frac{\frac{\bar{v}^2 - \bar{v}}{\bar{v}^2} + \frac{1}{k_{p_2}} - 1}{\frac{\bar{v}^2 - \bar{v}}{\bar{v}^2} + \frac{1}{k_{p_1}} - 1} \right]. \quad (3)$$

Equation (3) is the same as Eq. (2), except for the terms in the last bracket. If $(1 - k_p)/k_p$ is small compared to one, Eqs. (3) and (2) are identical. It has been found, however, that at 30 \$ subcritical, β_2 is 2 \$ more negative when Eq. (3) is used, compared to using Eq. (2) for a calibration at critical.

For other cases, the chains are not predominantly started by only ^{240}Pu or only delayed neutrons, and a more complicated, but straightforwardly derived, expression must be used. At 20 \$ the "correction" is about -1.4 \$, and at 10 \$ about -0.35 \$. These changes produce better agreement between the far-subcritical multiplication and noise measurements.

III. Improved Variance-to-Mean β_{eff} Measurement

Recently, the value of β_{eff} in fast breeder reactors has received further attention; new experimental data²² on delayed-neutron parameters lead to results that differ by as much as 12% from earlier data.²³ The impact of such changes on reactor design is significant because the interpretation

of all reactivity measurements in critical assemblies would be affected in the same proportion. Bennett and Travelli²⁴ have recently described an experimental technique by which β_{eff} is measured and its application to a fast critical assembly studied as part of the Fast Flux Test Reactor program.

The technique used is based on a measurement of the fluctuations in the output current of a neutron monitor with the reactor in a stationary state of known subcriticality (in dollar units). The subcriticality in dollars may be measured by an inverse-kinetics or a rod-drop flux profile analysis. The technique is similar to a variation, reported in Ref. 12, on the familiar variance-to-mean measurement. An alternative definition of variance was made use of in this experiment due to a tendency of the reactor flux to drift over a long time period. Instead of the fluctuation about the mean, the fractional fluctuation about a local flux average was found and compared against a theoretical result based on stationary fission decay-chain statistics. $\sigma^2(\tau)$ is defined here as the fractional mean square difference of flux at a point from the average of the flux at points just before and just after.

Bennett has shown that the product of absolute fission rate F by sampling time τ by variance σ^2 can be written as

$$F\tau\sigma^2(\tau) = W + \frac{2D [1 + \$\beta_{\text{eff}}]^2}{\beta_{\text{eff}}^2 [1 + \$]^2} \sum_k f_k(\tau) \quad . \quad (1)$$

The k τ -dependent functions $f_k(\tau)$ correspond to singularities of the zero-power transfer function; these singularities number one greater than the number of delayed-neutron groups. The $f_k(\tau)$ depend on the subcriticality in dollars denoted by $\$$. W is a constant referred to as detector background.

W is small compared to the second term on the RHS of Eq. (1) for the slightly subcritical state (-\$.368) of the basic measurement. W is evaluated in a supplementary far-subcritical measurement (-\$.8.2) where W dominates the RHS of Eq. (1). D is the "Diven" factor and is a measure of the neutron dispersion in fission.

A measurement of $\sigma^2(\tau)$ for τ in multiples of 8 sec was made on an assembly for which a careful evaluation of the total fission rate had been made. A fission rate, $F = 2.05 \pm 0.082 \times 10^{10}$ total fissions/sec-at the flux level of the variance measurement, was found by combining detailed ^{239}Pu foil irradiation data with 29-group two-dimensional diffusion theory calculations. Figure 3 contains a plot of the measured values for the product $F\sigma^2(\tau)$ versus τ . By adjusting β_{eff} via Eq. (1), a best-fit theoretical curve through the experimental data was found for $\beta_{\text{eff}} = 0.00298$. The error in the derived value of β_{eff} was estimated to be $\pm 2.9\%$ from estimated errors in F , D , σ^2 , and $\$$. The sensitivity of the calculated variance to changes in β is illustrated in Fig. 4.

The value of β_{eff} for the core as calculated from the older²³ delayed-neutron data is 0.00287. Its value as calculated from the newer²² data is 0.00310. The experimental value is intermediate between the two calculated results and has an uncertainty about half that attributed to uncertainties in the basic delayed-neutron data.

IV. Neutron Noise, Acoustic Noise Cross Correlation

Measurements in EBR-II

In 1970 a modest signature analysis effort was initiated as part of the surveillance effort for the EBR-II reactor. Since its initiation, this effort has evolved along three principal lines. One line has developed into a continuous signature surveillance procedure for monitoring major reactor

Fig. 3 . Comparison of Measured and Calculated Values of $F\sigma^2(\tau)$. Beta has been Adjusted for a Weighted Best Fit and Has the Value 0.00298.

Fig. 4 . Sensitivity of the Calculated Local Variance to Changes in Beta. Beta has been Increased and Decreased by 5% Around Its Best Value to Produce the Two Curves Shown.

components (i.e., mechanical signatures of the primary sodium pumps and intermediate heat exchanger, and signatures of the reactor core via the neutron flux). The other lines have been to use noise analysis techniques for trouble shooting and investigation of specific phenomena.²⁵

Of particular interest has been a small but persistent nominal 10 Hz oscillation, in the EBR-II neutron flux. Several investigations have indicated that mechanical motion of the reactor-core subassemblies, induced by sodium flow, are responsible for this 10 Hz oscillation.^{25,26} Pursuing these postulations, mathematical modeling of the reactor subassemblies has determined that their natural frequency does lie near 10 Hz.

Investigation into means of experimentally measuring subassembly vibrations in the reactor core during operation indicated that the reactor control rods were the best transmitters of core vibrations available for monitoring purposes. To make the required mechanical vibration measurements, accelerometers, affixed to magnetic mounts, were attached to the reactor control rods in a planned test sequence. The magnetic mounts allowed the accelerometer fixture to decouple from the control rod in the event of a reactor trip. Vibrational data from the accelerometers together with the oscillatory portion of the neutron flux were recorded on parallel channels of magnetic tape for a later comprehensive analysis.

Reduction of the data obtained by Price,²⁷ et. al., by frequency spectral analysis and correlation techniques shows that vibrations on several control rods corresponds directly to oscillations displayed in the neutron flux. Figure 5 clearly shows this correspondence between the 10.5 Hz radial vibrations on control rod 5 and the 10.5 Hz oscillation in the neutron flux. No axial vibration is present at 10.5 Hz on control rod 5. As can be seen in the neutron flux oscillation, a second frequency peak is present

CONDITIONS: 500 KWT, CR 5 AT 11.00"
 FILTER B.P. 5 TO 20 Hz

Fig. 5. Comparison of Frequency Spectra for Control Rod Number 5 and Neutron Flux in EBR-II

at 10.75 Hz and a third peak, with substantially larger amplitude, is present at 12 Hz. The 10.75 Hz oscillation is attributed to another control rod that has a slightly different fuel loading, while the 12 Hz oscillation is attributed to vibration of reactor subassemblies other than the control rods.

Evidence that the 12 Hz flux oscillation is due to driver type subassemblies is inferred from the relationship for natural frequencies of beams in lateral vibration. This relationship shows that for beams of the same length, composition, and attachment the natural frequency is proportional to the square root of the mass per unit length. To a reasonable approximation, the typical EBR-II driver and control rod subassemblies satisfy these conditions. The value computed for the ratio of the natural frequencies of a driver and a control rod subassembly using this approximation is 1.1. This value corresponds quite well with the ratio of the 12 and 10.5 Hz flux oscillation frequencies (1.14).

Recent measurements by Cohn and Anderson have extended the range of neutron/acoustic noise correlation studies to several thousand hertz to explore the potential of high frequencies for reactor diagnostics.

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