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Preliminary Estimates of the Thermal
and Hydraulic Performance
of Advanced Reactor Cores

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Advanced Reactor Project

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Preliminary Estimates of the Thermal and Hydraulic Performance of Advanced Reactor Cores

INTRODUCTION

For preliminary core studies in which no design parameters have been fixed it is desirable to have generalized core diameter and pressure drop equations and a rapid method of solution for these equations. This report presents the equations used in Advanced Reactor core calculations and a semi-graphical method for their solution. The following fuel elements are analyzed: continuous plates, parallel-flow rods, cross-flow rods, tubular flow channels in a solid matrix of fuel, and pellets.

EQUATIONS FOR IDEAL CORE DIAMETER

The following paragraphs give the equations for ideal (no structural material or control rods) core diameter for each type of fuel element and the heat transfer correlations on which they are based. The design criterion used in the derivation of the equations is that the cores will operate with a prescribed maximum metal surface temperature. The detailed derivations of the equations and a discussion of the assumed flux distribution are presented in Appendix A.

An artificial parameter ψ is used in all the following core diameter equations; it is defined by the equation

$$\psi = \frac{T_{sm} - T_{avg}}{\Delta T_{hc}} - 0.5 \frac{F_{\Delta T} - 1}{F_{\Delta T}}$$

where

$$\Delta T_{hc} = \frac{F_{\Delta T} Q}{c_p W \times 10^6}$$

Continuous Plates. The ideal diameter of a continuous plate core is given by

$$D_c = \left[4/5 \frac{P_r^{2/3}}{\mu^{0.2}} \cdot \frac{F_0}{F_{\Delta T}} \cdot \frac{t^{1/2}}{L/D_c} \cdot \frac{(1 + M/W)^{0.2}}{(M/W)^{1/2}} \cdot \frac{W^{0.2}}{(\psi^2 - 0.252)^{1/2}} \right]^{1/4}$$

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and is based on the Colburn equation

$$\frac{h D_c}{k} = 0.023 R_c^{0.8} Pr^{1/3}$$

Parallel-Flow Rods. The ideal diameter of a parallel-flow rod core is given by

$$D_c = \left[181 \frac{Pr^{2/3}}{\mu^{0.2}} \cdot \frac{F_\theta}{F_{\Delta T}} \cdot \frac{D_r^{1.2}}{L/D_c} \cdot \frac{(1+M/W)^{0.2}}{(M/W)^{1.2}} \cdot \frac{W^{0.2}}{(\psi^2 - 0.252)^{1/2}} \right]^{1/1.4}$$

and is also based on the Colburn equation.

Cross-Flow Rods. The ideal diameter of a staggered cross-flow rod core with M/W greater than 0.369 is given by

$$D_c = \left[171 \frac{Pr^{2/3}}{\mu^{0.2}} \cdot \frac{F_\theta}{F_{\Delta T}} \cdot \frac{D_r^{1.2}}{L/D_c} \cdot \frac{(1+M/W)^{0.6}}{M/W} \left[(1+M/W)^{1/2} - 1.129(M/W)^{1/2} \right]^{0.8} \cdot \frac{W^{0.2}}{(\psi^2 - 0.252)^{1/2}} \right]^{1/1.4}$$

and is based on the equation

$$\frac{h D_r}{k} = 0.0322 \left(\frac{G_{max} D_r}{\mu} \right)^{0.8} Pr^{1/3}$$

obtained by generalizing the results found at Brookhaven National Laboratory (1)* for one rod size and temperature.

Tubes. The ideal diameter of a solid core with tubular flow channels is given by

$$D_c = \left[181 \frac{Pr^{2/3}}{\mu^{0.2}} \cdot \frac{F_\theta}{F_{\Delta T}} \cdot \frac{D_r^{1.2}}{L/D_c} \cdot (1+M/W)^{0.2} \cdot \frac{W^{0.2}}{(\psi^2 - 0.252)^{1/2}} \right]^{1/1.4}$$

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and is based on the Colburn equation.

Pellets. The ideal diameter of a pellet core with 38 percent voids (2) is given by

$$D_c = \left[31.5 \frac{P_r^{0.7}}{\mu^{0.3}} \cdot \frac{F_\theta}{F_{\Delta T}} \cdot \frac{D_p^{1.3}}{L/D_c} \cdot \frac{W^{0.3}}{(\psi^2 - 0.252)^{1/2}} \right]^{1/1.6}$$

* Numbers in parentheses refer to list of references.

and is based on Denton's correlations (3)

$$\frac{h}{C_p G} = 0.58 \left(\frac{G D_p}{\mu} \right)^{-0.3} P_r^{-0.7}$$

in which G is calculated using the total core cross-section as the flow area.

To facilitate solution of these equations multiplication factors have been calculated by dividing the various property, $F_\theta / F_{\Delta T}$, element size, L/D_c , M/W , W , and ψ functions by reference values. The geometric factors and the thermal and hydraulic factors for plates, rods, and tubes are plotted in figures 1 and 2, respectively. Similar factors for pellets are plotted in figures 3 and 4. Figure 5 gives C_p vs T_{avg} for various system pressures and is included as an aid in the calculation of ΔT_{hc} .

To find a core diameter it is necessary to multiply the appropriate multiplication factors and a reference core diameter. The reference core diameters are as follows:

<u>Fuel Element</u>	<u>Reference D_c, ft.</u>
Plates	6.43
Parallel-flow rods	11.02
Cross-flow rods	6.01
Tubes	13.81
Pellets	8.45

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Figure No. 1

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Figure No. 2

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Figure No. 3

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Figure No. 4

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Figure No. 5

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PRESSURE DROP EQUATIONS

The following paragraphs give the pressure drop equations, based on the ideal core diameter, for each type of fuel element and the friction factors and shock losses assumed in their derivation. Detailed derivations of the equations are presented in Appendix B.

Continuous Plates and Tubes. The pressure drop in a continuous plate or tube core is given by

$$\Delta P = \frac{20.2}{\rho} \cdot \frac{W^2}{D_c^4} \cdot (1 + M/W)^2 \cdot \left(1 + \frac{4}{3} \beta\right)$$

where

$$\beta = \frac{F_\theta}{F_{AT}} \cdot Pr^{2/3} \cdot (\psi^2 - 0.252)^{-1/2}$$

This equation includes 1.5 velocity heads for the combined entrance and exit shock loss and is based on a friction factor for clean, smooth pipes

$$f = 0.046 / Re^{0.2}$$

Parallel-Flow Rods. The pressure drop in a parallel-flow rod core is given by

$$\Delta P = \frac{4.72}{\rho} \cdot \frac{W^2}{D_c^4} \cdot (1 + M/W)^2 \cdot [1 + 1.2 (L/D_c) D_c + 5.71 \beta]$$

where β is as defined above for plates and tubes. This equation includes 0.35 velocity heads each for entrance, exit, and rod junction shock losses. It assumes a 10 inch rod length and is also based on the smooth pipe friction factor

$$f = 0.046 / Re^{0.2}$$

Cross-Flow Rods. The pressure drop in a staggered cross-flow rod core with M/W greater than 0.369 is given by

$$\Delta P = 1.27 \frac{\mu^{0.15}}{\rho} \cdot \frac{W^{1.85}}{D_c^{2.70}} \cdot \frac{L/D_c}{D_r^{1.15}} \cdot \frac{(1 + M/W)^{0.425} (M/W)^{1/2}}{[(1 + M/W)^{1/2} - 1.129 (M/W)^{1/2}]^{1.85}} \left[1 + \frac{0.478 (M/W)^{0.54}}{[1.252 (1 + M/W)^{1/2} - (M/W)^{1/2}]^{1.08}} \right]$$

and is based on Jakob's friction factor for staggered tubes (4)

$$f = \left[0.23 + \frac{0.11}{(x_t - 1)^{1.08}} \right] \left(\frac{G_{max} D_r}{\mu} \right)^{-0.15}$$

Pellets. The pressure drop in a pellet core is given by

$$\Delta P = 126 \frac{\mu^{0.2}}{\rho} \cdot \frac{W^{1.8}}{D_c^{2.6}} \cdot \frac{L/D_c}{D_p^{1.2}}$$

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and is based on the work of Brown (2), which for 38 percent voids gives

$$\Delta P = \frac{1750}{144} \cdot f \cdot \frac{L}{D_p} \left(\frac{G^2}{29\rho} \right)$$

where

$$f = 0.19 / \left(46 \frac{G D_p}{\mu} \right)^{0.2}$$

and G is calculated using the total core cross-section as the flow area.

Multiplication factors similar to those used in the determination of core diameter have been calculated for the above pressure drop equations. The factors for plates and tubes, cross-flow rods, and pellets are plotted in figures 6, 8, and 9, respectively. For parallel-flow rods use the T_{avg} , W , M/W , and D_c factors given in figure 6 for plates and tubes and the nomograph in figure 7. Figure 10 gives curves of $Pr^{2/3}$ vs T_{avg} and $(\psi^{0.252})^{-1/2}$ vs ψ to facilitate determination of β for plate, tube, and parallel-flow rod pressure drop calculations. The reference pressure drops are as follows:

<u>Fuel Element</u>	<u>Reference ΔP, Psi</u>
Plates and Tubes	381
Parallel-flow rods	389
Cross-flow rods	2810
Pellets	4240

Figure No. 6

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NOMOGRAPH FOR USE IN PARALLEL-FLOW ROD PRESSURE DROP CALCULATIONS

KEY:

CONNECT $\left\{ \begin{array}{l} L/D_c \text{ AND } D_c \\ \beta \text{ AND MULT. FACTOR} \end{array} \right.$
BY PARALLEL LINES

$$\beta = \frac{F_{\theta}}{F_{\Delta T}} \times Pr^{2/3} \times \frac{1}{\sqrt{\psi^2 - 0.252}}$$

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Figure No. 7

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Figure No. 8

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Figure No. 9

Figure No. 10

NOMENCLATURE

A_f = flow area, ft^2
 A_s = surface area, ft^2
 b = effective core length, ft
 C_p = specific heat of the coolant, Btu/lb $^{\circ}F$
 D_c = ideal core diameter, ft
 D_e = $4 A_f$ /wetted perimeter = equivalent diameter, ft
 D_p = pellet diameter, ft
 D_r = rod diameter, ft
 D_t = tube diameter, ft
 f = friction factor
 $F_{\Delta T}$ = maximum to average factor for coolant temperature rise
 F_y = maximum to average factor for axial heat flux having a cosine distribution
 F_e = maximum to average factor for film temperature difference
 g = acceleration of gravity, ft/hr 2
 G = $W \times 10^6 / A_f$ = coolant mass flow rate, lb/hr ft^2
 h = heat transfer coefficient, Btu/hr ft^2 $^{\circ}F$
 k = thermal conductivity of the coolant, Btu/hr ft^2 $^{\circ}F$ /ft
 L = core length, ft
 M/W = ideal metal to water ratio
 N = number of rows of cross-flow rods
 p = rod pitch, ft
 ΔP = pressure drop, psi
 Pr = $c_p \mu / k$ = Prandtl number
 Q = core heat load, But/hr
 Re = GD_e / μ = Reynolds number
 S/V = surface to volume ratio, ft $^{-1}$
 t = plate thickness, ft
 ΔT_{hc} = coolant temperature rise in the hot channel, $^{\circ}F$
 T_{avg} = average coolant temperature, $^{\circ}F$
 $(= T_{in} + \Delta T_{hc} / 2 F_{\Delta T} , \text{ regardless of temperature distribution})$
 T_{in} = inlet coolant temperature, $^{\circ}F$
 T_s = metal surface temperature $^{\circ}F$
 T_{sm} = maximum metal surface temperature, $^{\circ}F$
 W = coolant flow rate, lbs/hr $\times 10^{-6}$

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NOMENCLATURE (CONT'D)

x_t = ratio of transverse pitch to rod diameter
 y = axial distance from the center of the core, ft
 β = artificial dimensionless parameter
 θ_{av} = average film temperature difference, °F
 θ_{max} = maximum film temperature difference, °F
 μ = viscosity of the coolant, lb/ft hr
 ρ = density of the coolant, lb/ft³
 ψ = artificial dimensionless parameter

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APPENDIX A

Derivation of the Equations for Ideal Core Diameter

The derivation of the equations for the ideal core diameter of all types of cores is based on the heat balance

$$Q = h A_s \theta_{av}$$

in which A_s is a function of core diameter,

$$A_s = \frac{\pi}{4} D_c^3 \frac{L}{D_c} \cdot \frac{S}{V}$$

and h is also a function of core diameter through the Reynolds number.

The design criterion that the core will operate with a prescribed maximum metal surface temperature is used to determine θ_{av} . To find θ_{av} as a function of T_{sm} the axial heat flux distribution must be known.

For chemical control a cosine distribution has been specified for preliminary Advanced Reactor calculations (WAPD-ReL(S)-7). For preliminary blanket mechanical control calculations two cosine distributions are of interest. The first, representative of normal operation, is in the lower half of the core and produces 90 percent of the power output (WAPD-ReL(S)-11). The second, representative of maximum xenon override, is in the upper half of the core and also produces 90 percent of the power output.

For a given T_{avg} and ΔT_{hc} the first cosine sees coolant initially at T_{in} and with a temperature rise of $0.9 \Delta T_{hc}$ in the hot channel, while the second cosine sees coolant initially at $T_{in} + 0.1 \Delta T_{hc}$ with a temperature rise of $0.9 \Delta T_{hc}$. Assuming film temperature difference to be directly proportional to heat flux and using the same T_{sm} and F_y for the above distributions, θ_{max} will be larger and core diameter smaller for the first cosine.

An intermediate coolant temperature distribution, initially at T_{in} with a temperature rise of $1.0 \Delta T_{hc}$, and thus an intermediate core diameter would result from a hypothetical cosine distribution over the entire length of the core. A factor of 1.8 to account for the concentration of power in one-half of the core must be used in F_θ for this hypothetical distribution as well as for the two actual distributions.

It is expected that the Advanced Reactor core will not be required to override maximum xenon. The flux distribution assumed above for normal operation is only a preliminary estimate. Therefore, since the hypothetical cosine distribution over the entire length of the core provides a margin of safety and can possibly be adapted to other flux distributions merely by changing the 1.8 factor in F_θ , it is used below in the derivation of θ_{av} as a function of T_{sm} .

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Amorosi and McCullough (5) give the following equation for the metal surface temperature in the hot channel based on a cosine axial flux distribution:

$$\frac{T_s - T_{in}}{\Delta T_{hc}} = \frac{\theta_{max}}{\Delta T_{hc}} \cos \pi \frac{L}{b} \cdot \frac{y}{L} + \frac{F_y}{\pi} \cdot \frac{b}{L} \left[\sin \pi \frac{L}{b} \cdot \frac{y}{L} + \frac{\pi}{2 F_y} \cdot \frac{L}{b} \right]$$

At the point of maximum metal surface temperature

$$0 = - \frac{\theta_{max}}{\Delta T_{hc}} \sin \pi \frac{L}{b} \cdot \frac{y}{L} + \frac{F_y}{\pi} \cdot \frac{b}{L} \cos \pi \frac{L}{b} \cdot \frac{y}{L}$$

or

$$\tan \pi \frac{L}{b} \cdot \frac{y}{L} = \frac{F_y}{\pi} \cdot \frac{b}{L} / \frac{\theta_{max}}{\Delta T_{hc}}$$

and

$$\sin \pi \frac{L}{b} \cdot \frac{y}{L} = \frac{\frac{F_y}{\pi} \cdot \frac{b}{L}}{\left[\left(\frac{\theta_{max}}{\Delta T_{hc}} \right)^2 + \left(\frac{F_y}{\pi} \cdot \frac{b}{L} \right)^2 \right]^{1/2}} ; \cos \pi \frac{L}{b} \cdot \frac{y}{L} = \frac{\frac{\theta_{max}}{\Delta T_{hc}}}{\left[\left(\frac{\theta_{max}}{\Delta T_{hc}} \right)^2 + \left(\frac{F_y}{\pi} \cdot \frac{b}{L} \right)^2 \right]^{1/2}}$$

Substituting these values in the equation for T_s gives

$$\frac{T_{sm} - T_{in}}{\Delta T_{hc}} = \left[\left(\frac{\theta_{max}}{\Delta T_{hc}} \right)^2 + \left(\frac{F_y}{\pi} \cdot \frac{b}{L} \right)^2 \right]^{1/2} + 0.5$$

Assuming a conservative value of F_y , say 1.5, $\frac{L}{b} = 0.95$ and

$$\frac{T_{sm} - T_{in}}{\Delta T_{hc}} = \left[\left(\frac{\theta_{max}}{\Delta T_{hc}} \right)^2 + 0.252 \right]^{1/2} + 0.5$$

Solving for θ_{av}

$$\theta_{av} = \frac{F_{\Delta T} Q}{F_{\theta} C_P W \times 10^6} \left[\left(\frac{T_{sm} - T_{in}}{\Delta T_{hc}} - 0.5 \right)^2 - 0.252 \right]^{1/2}$$

or since

$$T_{in} = T_{avg} - \Delta T_{hc} / 2 F_{\Delta T}$$

$$\theta_{av} = \frac{F_{\Delta T} Q}{F_{\theta} C_P W \times 10^6} \left[\left(\frac{T_{sm} - T_{avg}}{\Delta T_{hc}} - 0.5 \frac{F_{\Delta T} - 1}{F_{\Delta T}} \right)^2 - 0.252 \right]^{1/2}$$

Defining

$$\frac{T_{sm} - T_{avg}}{\Delta T_{hc}} - 0.5 \frac{F_{\Delta T} - 1}{F_{\Delta T}}$$

as an artificial parameter ψ

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$$\theta_{av} = \frac{F_{\Delta T} Q}{F_{\theta} c_p W \times 10^6} (\psi^2 - 0.252)^{1/2}$$

Substituting for A_s and θ_{av} in the heat balance

$$Q = \frac{\pi}{4} h D_c^3 \frac{L}{D_c} \cdot \frac{S}{V} \cdot \frac{F_{\Delta T} Q}{F_{\theta} c_p W \times 10^6} (\psi^2 - 0.252)^{1/2}$$

or

$$D_c^3 = \frac{F_{\theta} c_p W \times 10^6}{\frac{\pi}{4} h F_{\Delta T} \frac{L}{D_c} \cdot \frac{S}{V} (\psi^2 - 0.252)^{1/2}}$$

Substitution for h and S/V , and the solution for D_c follows for each type of core.

CONTINUOUS PLATES

$$S/V = \frac{Z}{t} \cdot \frac{M/W}{1 + M/W} ; \quad A_f = \frac{\pi}{4} D_c^2 \frac{1}{1 + M/W}$$

$$h = 0.023 \frac{K}{D_e} \left(\frac{G D_e}{\mu} \right)^{0.8} Pr^{1/3}$$

$$D_e = \frac{4 A_f}{\frac{\pi}{4} D_c^2 \frac{S}{V}} = \frac{2t}{M/W}$$

$$D_c^3 = \frac{F_{\theta} c_p W \times 10^6 \left(\frac{2t}{M/W} \right)^{0.2}}{\frac{\pi}{4} F_{\Delta T} \frac{L}{D_c} \cdot \frac{Z}{t} \cdot \frac{M/W}{1 + M/W} (0.023) K Pr^{1/3} \left[\frac{W(1 + M/W) \times 10^6}{\frac{\pi}{4} D_c^2 \mu} \right]^{0.8} (\psi^2 - 0.252)^{1/2}}$$

$$D_c = \left[415 \frac{Pr^{2/3}}{\mu^{0.2}} \cdot \frac{F_{\theta}}{F_{\Delta T}} \cdot \frac{t^{1.2}}{L/D_c} \cdot \frac{(1 + M/W)^{0.2}}{(M/W)^{1/2}} \cdot \frac{W^{0.2}}{(\psi^2 - 0.252)^{1/2}} \right]^{1/4}$$

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PARALLEL - FLOW RODS

$$S/V = \frac{4}{D_r} \cdot \frac{M/W}{1 + M/W} ; \quad A_f = \frac{\pi}{4} D_c^2 \cdot \frac{1}{1 + M/W}$$

$$h = 0.023 \frac{k}{D_c} \left(\frac{G D_c}{\mu} \right)^{0.8} Pr^{1/3}$$

$$D_c = \frac{4 A_f}{\frac{\pi}{4} D_c^2 \frac{3}{V}} = \frac{D_r}{M/W}$$

$$D_c^3 = \frac{F_0 C_p W \times 10^6 \left(\frac{D_r}{M/W} \right)^{0.2}}{\frac{\pi}{4} F_{DT} \frac{L}{D_c} \cdot \frac{4}{D_r} \cdot \frac{M/W}{1 + M/W} (0.023) k Pr^{1/3} \left[\frac{W(1 + M/W) \times 10^6}{\frac{\pi}{4} D_c^2 \mu} \right]^{0.8} (\psi^2 - 0.252)^{1/2}}$$

$$D_c = \left[181 \frac{Pr^{2/3}}{\mu^{0.2}} \cdot \frac{F_0}{F_{DT}} \cdot \frac{D_r^{1.2}}{L/D_c} \cdot \frac{(1 + M/W)^{0.2}}{(M/W)^{1.2}} \cdot \frac{W^{0.2}}{(\psi^2 - 0.252)^{1/2}} \right]^{1/4}$$

CROSS-FLOW RODS

$$S/V = \frac{4}{D_r} \cdot \frac{M/W}{1 + M/W}$$

$$h = 0.0322 \frac{k}{D_r} \left(\frac{G_{max} D_r}{\mu} \right)^{0.8} Pr^{1/3}$$

For staggered rods on a tilted square pitch the minimum flow area is

$$A_f = \frac{\pi}{4} D_c^2 \cdot \frac{2(p - D_r)}{\sqrt{2} p} \quad \text{for } \frac{D_r}{p} > 0.586$$

$$A_f = \frac{\pi}{4} D_c^2 \cdot \frac{\sqrt{2} p - D_r}{\sqrt{2} p} \quad \text{for } \frac{D_r}{p} < 0.586$$

Considering only $\frac{D_r}{p} > 0.586$ ($M/W > 0.369$)

$$A_f = \frac{\pi}{4} D_c^2 \sqrt{2} \left(1 - \frac{D_r}{p} \right)$$

Since

$$M/W = \frac{\frac{\pi}{4} D_r^2}{p^2 - \frac{\pi}{4} D_r^2}$$

$$\frac{D_r}{p} = 1.129 \left(\frac{M/W}{1 + M/W} \right)^{1/2}$$

and

$$A_f = \frac{\pi}{4} D_c^2 \sqrt{2} \left[1 - 1.129 \left(\frac{M/W}{1 + M/W} \right)^{1/2} \right]$$

$$D_c^3 = \frac{F_\theta C_P W \times 10^6 D_r^{0.2}}{\frac{\pi}{4} F_{\Delta T} \frac{L}{D_c} \cdot \frac{4}{D_r} \cdot \frac{M/W}{1 + M/W} (0.0322) K P_r^{1/3} \left[\frac{W \times 10^6}{\frac{\pi}{4} D_c^2 \sqrt{2} \mu \left[1 - 1.129 \left(\frac{M/W}{1 + M/W} \right)^{1/2} \right]} \right]^{0.8} (\psi^2 - 0.252)^{1/2}}$$

$$D_c = \left[171 \frac{P_r^{2/3}}{\mu^{0.2}} \cdot \frac{F_\theta}{F_{\Delta T}} \cdot \frac{D_r^{1.2}}{L/D_c} \cdot \frac{(1 + M/W)^{0.6}}{M/W} \left[(1 + M/W)^{1/2} - 1.129 (M/W)^{1/2} \right]^{0.8} \cdot \frac{W^{0.2}}{(\psi^2 - 0.252)^{1/2}} \right]^{1/4}$$

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TUBES

$$S/V = \frac{4}{D_t} \cdot \frac{1}{1+M/W} ; \quad A_f = \frac{\pi}{4} D_c^2 \frac{1}{1+M/W}$$

$$h = 0.023 \frac{k}{D_t} \left(\frac{G D_t}{\mu} \right)^{0.8} Pr^{1/3}$$

$$D_c^3 = \frac{F_\theta c_p W \times 10^6 D_t^{0.2}}{\frac{\pi}{4} F_{\Delta T} \frac{L}{D_c} \cdot \frac{4}{D_t} \cdot \frac{1}{1+M/W} (0.023) k Pr^{1/3} \left[\frac{W(1+M/W) \times 10^6}{\frac{\pi}{4} D_c^2 \mu} \right]^{0.8} (\psi^2 - 0.252)^{1/2}}$$

$$D_c = \left[181 \frac{Pr^{2/3}}{\mu^{0.2}} \cdot \frac{F_\theta}{F_{\Delta T}} \cdot \frac{D_t^{1.2}}{L/D_c} \cdot (1+M/W)^{0.2} \cdot \frac{W^{0.2}}{(\psi^2 - 0.252)^{1/2}} \right]^{1/4}$$

PELLETS

Assuming 38 percent voids (2)

$$S/V = \frac{6}{D_p} (0.62) = \frac{3.72}{D_p} ; \quad A_f = \frac{\pi}{4} D_c^2$$

$$h = 0.58 c_p G \left(\frac{G D_p}{\mu} \right)^{-0.3} Pr^{-0.7}$$

$$D_c^3 = \frac{F_\theta c_p W \times 10^6 D_p^{0.3} Pr^{0.7}}{\frac{\pi}{4} F_{\Delta T} \frac{L}{D_c} \cdot \frac{3.72}{D_p} (0.58) c_p \mu^{0.3} \left[\frac{W \times 10^6}{\frac{\pi}{4} D_c^2} \right]^{0.7} (\psi^2 - 0.252)^{1/2}}$$

$$D_c = \left[31.5 \frac{Pr^{0.7}}{\mu^{0.3}} \cdot \frac{F_\theta}{F_{\Delta T}} \cdot \frac{D_p^{1.3}}{L/D_c} \cdot \frac{W^{0.3}}{(\psi^2 - 0.252)^{1/2}} \right]^{1/6}$$

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APPENDIX B

Derivation of Pressure Drop EquationsCONTINUOUS PLATES

Assuming 1.5 velocity heads for the combined entrance and exit shock loss,

$$\Delta P = \left(1.5 + 4f \frac{L}{D_c} \right) \frac{G^2}{288 \rho}$$

From the core diameter derivation

$$D_c = \frac{2t}{M/W} ; A_f = \frac{\pi}{4} D_c^2 \frac{1}{1 + M/W}$$

Assuming a friction factor for clean, smooth pipes

$$f = 0.046 / Re^{0.2}$$

and

$$\begin{aligned} \Delta P &= \frac{W^2 \times 10^{12} (1 + M/W)^2}{\left(\frac{\pi}{4}\right)^2 D_c^4 (32.2)(3600)^2 288 \rho} \left[1 + \frac{4(0.046) L/D_c D_c \mu^{0.2}}{1.5 \left[\frac{W(1 + M/W) \times 10^6}{\frac{\pi}{4} D_c^2} \right]^{0.2} \left(\frac{2t}{M/W}\right)^{1.2}} \right] \\ &= \frac{20.2}{\rho} \cdot \frac{W^2}{D_c^4} \cdot (1 + M/W)^2 \left[1 + 0.00321 \frac{L/D_c D_c^{1.4} \mu^{0.2} (M/W)^{1.2}}{W^{0.2} (1 + M/W)^{0.2} t^{1.2}} \right] \end{aligned}$$

From the core diameter equation

$$\frac{L/D_c D_c^{1.4} \mu^{0.2} (M/W)^{1.2}}{W^{0.2} (1 + M/W)^{0.2} t^{1.2}} = 415 \frac{F_{\theta}}{F_{\Delta T}} \cdot Pr^{2/3} \cdot \frac{1}{(\psi^2 - 0.252)^{1/2}}$$

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Defining

$$\frac{F_0}{F_{AT}} \cdot P_r^{2/3} \cdot \frac{1}{(\psi^2 - 0.252)^{1/2}}$$

as an artificial parameter β

$$\Delta P = \frac{20.2}{\rho} \cdot \frac{W^2}{D_c^4} \cdot (1 + M/W)^2 \cdot \left(1 + \frac{4}{3} \beta\right)$$

PARALLEL-FLOW RODS

Assuming 0.35 velocity heads each for entrance, exit, and rod junction shock losses and a rod length of 10 inches,

$$\Delta P = \left[0.35(1 + 1.2L) + 4f \frac{L}{D_e} \right] \frac{G^2}{288 g \rho}$$

From the core diameter derivation

$$D_c = \frac{D_r}{M/W} ; \quad A_f = \frac{\pi}{4} D_c^2 \frac{1}{1 + M/W}$$

Assuming $f = 0.046 / \text{Re}^{0.2}$ as for plates and tubes

$$\Delta P = \frac{4.72}{\rho} \cdot \frac{W^2}{D_c^4} \cdot (1 + M/W)^2 \left[1 + 1.2 (L/D_c) D_c + 0.0316 \frac{L/D_c D_c^{1.4} \mu^{0.2} (M/W)^{1.2}}{W^{0.2} (1 + M/W)^{0.2} D_r^{1.2}} \right]$$

From the core diameter equation

$$\frac{L/D_c D_c^{1.4} \mu^{0.2} (M/W)^{1.2}}{W^{0.2} (1 + M/W)^{0.2} D_r^{1.2}} = 181 \frac{F_0}{F_{AT}} \cdot P_r^{2/3} \cdot \frac{1}{(\psi^2 - 0.252)^{1/2}} = 181 \beta$$

Therefore

$$\Delta P = \frac{4.72}{\rho} \cdot \frac{W^2}{D_c^4} \cdot (1 + M/W)^2 \cdot \left[1 + 1.2 (L/D_c) D_c + 5.71 \beta \right]$$

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CROSS-FLOW RODS

$$\Delta P = 4fN \frac{G_{\max}^2}{2889\rho}$$

$$N = \frac{2L}{\sqrt{2}\rho}$$

From the core diameter derivation

$$A_f = \frac{\pi}{4} D_c^2 \sqrt{2} \left(1 - \frac{D_r}{P}\right)$$

for $M/W > 0.369$. The friction factor f for staggered tubes given by Jakob (4) is

$$f = \left[0.23 + \frac{0.11}{(x_t - 1)^{1.08}} \right] \left(\frac{G_{\max} D_r}{\mu} \right)^{-0.15}$$

$$x_t = \frac{\sqrt{2}\rho}{D_r}$$

$$\Delta P = \frac{4\mu^{0.15}}{288(32.2)(3600)^2\rho} \left[0.23 + \frac{0.11}{\left(\frac{\sqrt{2}\rho}{D_r} - 1\right)^{1.08}} \right] \frac{2L}{\sqrt{2}\rho} \cdot \frac{D_r}{D_r^{1.15}} \cdot \frac{(W \times 10^6)^{1.85}}{\left[\frac{\pi}{4} D_c^2 \sqrt{2} \left(1 - \frac{D_r}{P}\right) \right]^{1.85}}$$

Substituting $\frac{D_r}{P} = 1.129 \left(\frac{M/W}{1 + M/W} \right)^{1/2}$

$$\Delta P = 1.27 \frac{\mu^{0.15}}{\rho} \cdot \frac{W^{1.85}}{D_c^{2.70}} \cdot \frac{L/D_c}{D_r^{1.15}} \cdot \frac{(1 + M/W)^{0.425} (M/W)^{1/2}}{\left[(1 + M/W)^{1/2} - 1.129 (M/W)^{1/2} \right]^{1.85}} \left[1 + \frac{0.478 (M/W)^{0.54}}{\left[1.252 (1 + M/W)^{1/2} - (M/W)^{1/2} \right]^{1.08}} \right]$$

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TUBES

Assuming 1.5 velocity heads for the combined entrance and exit shock loss,

$$\Delta P = \left(1.5 + 4f \frac{L}{D_t} \right) \frac{G^2}{288 g \rho}$$

Assuming a friction factor for clean, smooth pipes

$$f = 0.046 / Re^{0.2}$$

and

$$\Delta P = \frac{20.2}{\rho} \cdot \frac{W^2}{D_c^4} \cdot (1 + M/W)^2 \left[1 + 0.007375 \frac{L/D_c D_c^{1.4} \mu^{0.2}}{W^{0.2} (1 + M/W)^{0.2} D_t^{1.2}} \right]$$

From the core diameter equation

$$\frac{L/D_c D_c^{1.4} \mu^{0.2}}{W^{0.2} (1 + M/W)^{0.2} D_t^{1.2}} = 181 \beta$$

Therefore

$$\Delta P = \frac{20.2}{\rho} \cdot \frac{W^2}{D_c^4} \cdot (1 + M/W)^2 \cdot \left(1 + \frac{4}{3} \beta \right)$$

PELLETS

From the work of Brown (2), assuming 38 percent voids,

$$\Delta P = \frac{1750}{144} \cdot f \cdot \frac{L}{D_p} \left(\frac{G^2}{2g\rho} \right)$$

where

$$f = 0.19 / \left(46 \frac{G D_p}{\mu} \right)^{0.2}$$

Therefore

$$\Delta P = \frac{1750 (0.19) \mu^{0.2}}{288 (32.2) (3600)^2 (46)^{0.2}} \cdot \frac{L/D_c}{D_p^{1.2}} \cdot \frac{D_c}{\rho} \cdot \left(\frac{W \times 10^6}{\frac{\pi}{4} D_c^2} \right)^{1.8}$$

$$\Delta P = 126 \frac{\mu^{0.2}}{\rho} \cdot \frac{W^{1.8}}{D_c^{2.6}} \cdot \frac{L/D_c}{D_p^{1.2}}$$

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