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DERIVATION OF LINEARIZED, DIFFERENCE-DIFFERENTIAL
EQUATIONS FOR ANALOG COMPUTER STUDIES
OF GAS-FLOW OSCILLATIONS

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ABSTRACT

The basic heat transfer and flow equations applicable to studies of flow oscillations in NERVA engines are first linearized, then expressed as difference-differential approximations, and finally rewritten in a form suitable for analog computer solution. Block diagrams are presented which illustrate an analog procedure for solving the linearized, difference-differential equations.

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NOMENCLATURE

x	=	Distance along the flow path, ft.
S	=	Inner circumference of the assumed coolant flow tube, ft.
S_m, S_n	=	Portions of S corresponding to the two axially-split regions of the tube wall, ft.
A	=	Cross sectional flow area of the tube, ft^2 .
t	=	Time, sec.
ρ	=	Hydrogen density, lb/ft^3 . (Function of x and t .)
V	=	Hydrogen velocity, ft/sec .
P	=	Hydrogen pressure, lb/ft^2 absolute. (Function of x and t .)
H	=	Hydrogen enthalpy, BTU/lb . (Function of x and t .)
T	=	Hydrogen temperature, $^{\circ}\text{R}$. (Function of x and t .)
T_m, T_n	=	Tube wall temperatures in the two axially-split regions designated by subscripts m and n , respectively, $^{\circ}\text{R}$. (Functions of x and t .)
Q_m, Q_n	=	Heat generation rates in the tube wall regions, $\text{BTU}/\text{ft}^3/\text{sec}$. (Functions of x and t .)
q_m, q_n	=	Heat transfer rates to the hydrogen flow from the two wall regions, $\text{BTU}/\text{ft}^2/\text{sec}$. (Functions of x and t .)
T'_m, T'_n	=	Assumed external boundary temperatures of the two wall regions, $^{\circ}\text{R}$.
ρ_m, ρ_n	=	Densities of the two wall regions, lb/ft^3 .
c_m, c_n	=	Specific heat capacities of the two wall regions, $\text{BTU}/\text{lb } ^{\circ}\text{R}$. (Function of the wall temperatures. . assumed constant in the linearized equations.)
A_m, A_n	=	Cross sectional areas of the two tube wall regions, ft^2 .

NOMENCLATURE (Continued)

h_m, h_n = Hydrogen film coefficients associated with the two wall regions, BTU/sec ft² °R.
(Internal heat transfer resistance of the wall is neglected and the heat transfer from the mid-thickness of the wall to the hydrogen is assumed to be dependent only on the film coefficient.)

ξ_m, ξ_n, γ = Constants in the expressions for the film coefficients h_m, h_n .
(Equations 7d and 7e)

U'_m, U'_n = Heat transfer coefficients,* mid-thickness to external temperature boundary of the two wall regions, BTU/sec ft² °R.

f = Friction factor (nondimensional).

g = Acceleration due to gravity = 32.2 ft/sec².

J = Conversion factor = 778 ft-lb/BTU.

a, b, R = Constants in the equation of state.

C_p, β = Constants in the enthalpy relationship (Equation 5).

o = Subscript indicating steady state value of a variable quantity.

x_i = i^{th} discrete position along the flow path, ft.

i = Subscript denoting the value of any variable at the position x_i .

Δx = $x_{i+1} - x_i$

$x_{i+1/2}$ = $x_i + \Delta x/2$

* The hot-side heat transfer coefficient of the nozzle cooling tubes, U'_n , may be assumed to be a function of the nozzle exit gas flow rate and the temperatures of the gas and the tube walls. The other heat transfer coefficient, U'_m , may be assumed constant for each reactor system component being represented.

1.0 INTRODUCTION

The following is a derivation of a set of linearized heat transfer and flow equations suitable for analog computer studies of flow oscillations in NERVA reactors and their feedsystems. Using these equations it should be possible to construct an analog representation of a reactor and feedsystem having roughly twenty nodes.

Single-phase, gas flow is assumed in deriving the linearized equations. This assumption provides a good starting point for analysis. Two-phase effects can be added later when an understanding of single-phase oscillations has been attained.

The derivation starts with general equations of heat transfer and flow for gaseous hydrogen flow through a hollow, cylindrical tube. This tube is considered to have its heat capacity lumped radially and is assumed to be divided along its length into two regions which are thermally insulated from each other. These two wall regions are assumed to have different thermal properties and are subject to two different outside temperature boundary conditions. Constants and variables associated with the two wall regions are labeled with subscripts, m and n. It should be possible to apply these generalized equations, and their linearized, difference-differential approximations, to describe heat transfer and flow conditions in any portion of the reactor, deleting terms in the equations where these terms do not apply.

The two-wall-region representation is actually required only in the nozzle cooling passages, where it is necessary to distinguish between hot and cold sides of the cooling tubes. Here, the boundary temperature on the hot side, T_m , would be the nozzle exit gas temperature. The cold side (subscript, n) would include the heat-storage effects of the supporting structure outside the tubes. The boundary temperature for this region, T'_n , would depend on whether all or only a portion of the supporting structure was included.

One-region representations would be employed for flow channels in the core, reflector and thermal shield. In these three cases the single material region (m) would represent only a thin slice of material adjacent to the coolant and the boundary temperature T'_m would be the temperature of the bulk of the material, which could be assumed constant for the expected frequencies of flow oscillations.

As a first step in the flow oscillation study it is suggested that all heat transfer to coolant channel walls be neglected. Then, the effects of heat transfer in various reactor regions, starting with the nozzle cooling passages, should be investigated. The amount of wall material included in the variable-temperature region (m) may be varied to study the effects of wall heat capacity on the flow oscillations.

The derivation presented below first linearizes the general partial differential equations of heat transfer and flow, then approximates spatial derivatives in these equations by finite differences, and finally changes the resulting equations into a form suitable for analog computation.

2.0 Basic Equations

Continuity Equation:

$$\frac{\partial \rho}{\partial t} + \frac{\partial G}{\partial x} = 0. \quad (1)$$

Momentum Equation:

$$\frac{\partial P}{\partial x} + f \frac{\rho V^2}{2g} \frac{S}{A} = \frac{1}{g} \left[\rho \frac{\partial V}{\partial t} + G \frac{\partial V}{\partial x} \right] \quad (2a)$$

or

$$\frac{\partial}{\partial x} \left[P - \frac{GV}{g} \right] = -\frac{Sf}{2Ag} \rho V^2 + \frac{1}{g} \frac{\partial G}{\partial t} \quad (2b)$$

or

$$\frac{\partial}{\partial x} \left[P - \frac{1}{g} \frac{G^2}{\rho} \right] = -\frac{Sf}{2Ag} \frac{G^2}{\rho} + \frac{1}{g} \frac{\partial G}{\partial t} \quad (2c)$$

Energy Equation:

$$\frac{\partial (GH)}{\partial x} + \frac{\partial (\rho H)}{\partial t} - \frac{1}{J} \frac{\partial P}{\partial t} = q \frac{S}{A} \quad (3)$$

Equation of State:

$$(P + a\rho^2) \left(\frac{1}{\rho} - b \right) = RT \quad (4)$$

Enthalpy Relationship:

$$H = c_p T + \beta \quad (5)$$

Heat Transfer in the Material:*

$$\frac{\partial T_m}{\partial t} = \frac{Q_m}{\rho_m c_m} - \frac{q_m S_m}{\rho_m c_m A_m} - \frac{S_m U'_m (T_m - T'_m)}{\rho_m c_m A_m} \quad (6a)$$

$$\frac{\partial T_n}{\partial t} = \frac{Q_n}{\rho_n c_n} - \frac{S_n U'_n (T_n - T'_n)}{\rho_n c_n A_n} - \frac{q_n S_n}{\rho_n c_n A_n} \quad (6b)$$

$$Q_m = \psi_m(x) n(t) \quad (6c)$$

$$Q_n = \psi_n(x) n(t) \quad (6d)$$

* In the interest of generality, two separate material regions are assumed to be in contact with the fluid. These regions, represented by lumped-parameter approximations, are denoted by the subscripts m and n. The two-region representation is actually required only in the nozzle cooling passages, where it is necessary to distinguish between hot and cold sides of the cooling tubes. The boundary temperature on the hot side, T'_m , would be the nozzle exit gas temperature. The cold side region (subscript, n) would include the heat-storage effects of the supporting structure outside the tubes. The boundary temperature for this region, T'_n , would depend on whether all or only a portion of the supporting structure was included. One-region representations would be employed for the core, reflector, and thermal shield. In these three cases the single material region (m) would represent only a thin slice of material adjacent to the coolant and the boundary temperature T'_m would be the temperature of the bulk of the material, which could be assumed constant for the expected frequencies of flow oscillations.

Material-to-Coolant Heat Transfer:

$$q = q_m \frac{S_m}{S} + q_n \frac{S_n}{S} \quad (7a)$$

$$q_m = h_m (T_m - T) \quad (7b)$$

$$q_n = h_n (T_n - T) \quad (7c)$$

$$h_m = \xi_m G^{0.8} \left(\frac{T_m}{T} \right)^{-\gamma} \quad (7d)$$

$$h_n = \xi_n G^{0.8} \left(\frac{T_n}{T} \right)^{-\gamma} \quad (7e)$$

3.0 Linearizing Assumptions

Any variable Z , that is a function of x and t (or possibly of t only) shall be assumed to be the sum of an initial value,

$$Z(x, 0) \equiv Z_o \quad (8)$$

and an increment,

$$\Delta Z \equiv Z(x, t) - Z_o \quad (9)$$

The linearized forms of the partial differential equations will be expressed in terms of normalized increments (i.e., $\frac{\Delta Z}{Z_o}$).

4.0 Linearized Forms

Continuity Equation:

$$\rho_o \frac{\partial \left(\frac{\Delta \rho}{\rho_o} \right)}{\partial t} + G_o \frac{\partial \left(\frac{\Delta G}{G_o} \right)}{\partial x} = 0 \quad (10)$$

Momentum Equation (based on 2c):

$$\begin{aligned} \frac{\partial}{\partial x} \left[P_o \frac{\Delta P}{P_o} - \frac{G_o}{g \rho_o} \left(\frac{2\Delta G}{G_o} - \frac{\Delta \rho}{\rho_o} \right) \right] \\ = \frac{Sf G_o^2}{2A g \rho_o} \left[\frac{2\Delta G}{G_o} - \frac{\Delta \rho}{\rho_o} \right] + \frac{1}{g} \frac{\partial \left(\frac{\Delta G}{G_o} \right)}{\partial t} \end{aligned} \quad (11)$$

Energy Equation:

$$\begin{aligned} G_o H_o \left[\frac{\partial \left(\frac{\Delta G}{G_o} \right)}{\partial x} + \frac{\partial \left(\frac{\Delta H}{H_o} \right)}{\partial x} \right] + \rho_o H_o \left[\frac{\partial \left(\frac{\Delta \rho}{\rho_o} \right)}{\partial t} + \frac{\partial \left(\frac{\Delta H}{H_o} \right)}{\partial t} \right] \\ - \frac{P_o}{J} \frac{\partial \left(\frac{\Delta P}{P_o} \right)}{\partial t} = q_o \frac{S}{A} \left(\frac{\Delta q}{q_o} \right) \end{aligned} \quad (12a)$$

or, using Equation 10,

$$\begin{aligned} \rho_o H_o \left[\frac{\partial \left(\frac{\Delta H}{H_o} \right)}{\partial t} + \frac{G_o}{\rho_o} \frac{\partial \left(\frac{\Delta H}{H_o} \right)}{\partial x} \right] \\ - \frac{P_o}{J} \frac{\partial \left(\frac{\Delta P}{P_o} \right)}{\partial t} = q_o \frac{S}{A} \left(\frac{\Delta q}{q_o} \right) \end{aligned} \quad (12b)$$

Equation of State:

$$\begin{aligned} \frac{\Delta P}{P_o} \left[\frac{P_o}{\rho_o} - b P_o \right] + \frac{\Delta \rho}{\rho_o} \left[- \frac{P_o}{\rho_o} + a \rho_o - 2ab \rho_o \right] \\ = RT_o \left(\frac{\Delta T}{T_o} \right) \end{aligned} \quad (13)$$

Enthalpy Relationship:

$$\frac{\Delta H}{H_o} = \frac{\Delta T}{T_o} \quad (14)$$

Heat Transfer in the Material:*

$$\begin{aligned} T_{mo} \frac{\partial \left(\frac{\Delta T_m}{T_{mo}} \right)}{\partial t} = & \left(\frac{Q_{mo}}{\rho_m c_{mo}} \right) \frac{\Delta n}{n_o} - \left(\frac{q_{mo} S_m}{\rho_m c_{mo} A_m} \right) \frac{\Delta q_m}{q_{mo}} \\ & - \frac{S_m U'_m T_{mo}}{\rho_m c_{mo} A_m} \left[\frac{\Delta T_m}{T_{mo}} - \frac{\Delta T'_m}{T_{mo}} \right] \end{aligned} \quad (15a)$$

$$\begin{aligned} T_{no} \frac{\partial \left(\frac{\Delta T_{no}}{T_{no}} \right)}{\partial t} = & \left(\frac{Q_{no}}{\rho_n c_{no}} \right) \frac{\Delta n}{n_o} - \left(\frac{q_{no} S_n}{\rho_n c_{no} A_n} \right) \frac{\Delta q_n}{q_{no}} \\ & - \frac{S_n U'_{no} (T_{no} - T'_{no})}{\rho_n c_n A_n} \frac{\Delta U'_{no}}{U'_{no}} - \frac{S_n U'_{no} T_{mo}}{\rho_m c_{mo} A_m} \left[\frac{\Delta T_m}{T_{mo}} - \frac{\Delta T'_m}{T_{mo}} \right] \end{aligned} \quad (15b)$$

Material-to-Coolant Heat Transfer

$$q_o \frac{\Delta q}{q_o} = \frac{S_m}{S} q_{mo} \frac{\Delta q_m}{q_{mo}} + \frac{S_n}{S} q_{no} \frac{\Delta q_n}{q_{no}} \quad (16a)$$

* The material heat capacities c_m and c_n are functions of the material temperatures. The incremental variations of these capacities are neglected in the linearized equations, and the capacities were assumed constant at their initial values, c_{mo} and c_{no} . The nozzle hot-side heat transfer coefficient, U'_n , is assumed variable (depending on the hot-side gas flow rate and the ratio of the hot gas temperature to the nozzle wall temperature). The equation determining $\frac{\Delta U'_n}{U'_{no}}$ is not shown here.

$$\begin{aligned} \frac{\Delta q_m}{q_{mo}} &= \frac{h_{mo}}{q_{mo}} \left[T_{mo} \frac{\Delta T_m}{T_{mo}} - T_o \frac{\Delta T}{T_o} \right] + 0.8 \frac{\Delta G}{G_o} \\ &+ \gamma \frac{\Delta T}{T_o} - \gamma \frac{\Delta T_m}{T_{mo}} \end{aligned} \quad (16b)$$

$$\begin{aligned} \frac{\Delta q_n}{q_{no}} &= \frac{h_{no}}{q_{no}} \left[T_{no} \frac{\Delta T_n}{T_{no}} - T_o \frac{\Delta T}{T_o} \right] + 0.8 \frac{\Delta G}{G_o} \\ &+ \gamma \frac{\Delta T}{T_o} - \gamma \frac{\Delta T_m}{T_{mo}} \end{aligned} \quad (16c)$$

5.0 Finite Difference Approximations

$$\frac{\partial \Delta Z}{\partial x} \approx \frac{\Delta Z_{i+1} - \Delta Z_i}{\Delta x} \quad (17)$$

$$\frac{\partial \Delta Z}{\partial t} \approx \frac{d \Delta Z_{i+1/2}}{dt} \quad (18)$$

$$\Delta Z_{i+1/2} \approx 1/2 (\Delta Z_i + \Delta Z_{i+1}) \quad (19)$$

$$\Delta Z_i = \Delta Z(x_i, t) \quad (20)$$

$$\Delta Z_{i+1} = \Delta Z(x_{i+1}, t) \quad (21)$$

$$x_{i+1} = x_i + \Delta x \quad (22)$$

where Z may represent P, ρ , G, T, T_m , q, or Q.

The steady state reference values in the linearized equations, $P_o, \rho_o, G_o, T_o, T_{mo}, q_o$, and Q_o shall be taken as the values of $P, \rho, \dots, Q_o$, respectively at $x = x_i + \frac{\Delta x}{2}$ in the interval $x_i < x < x_i + \Delta x$, and they shall be denoted by the addition of an i subscript. That is,

$$Z_{oi} \equiv Z(x_{i+1/2}, 0) = Z(x_i + \frac{\Delta x}{2}, 0) \quad (23)$$

6.0 Difference-Differential Equations

Continuity Equation:

$$\frac{d\left(\frac{\Delta \rho}{\rho_{oi}}\right)}{dt} = \frac{G_{oi}}{\rho_{oi}} \left[\frac{\frac{\Delta G_i}{G_o} - \frac{\Delta G_{i+1}}{G_o}}{\Delta x} \right] \quad (24)$$

Momentum Equation:

$$\begin{aligned} \frac{d\left(\frac{\Delta G_{i+1/2}}{G_{oi}}\right)}{dt} &= \frac{S_f G_{oi}}{2A \rho_{oi}} \left[\frac{2 \Delta G_{i+1/2}}{G_{oi}} - \frac{\Delta \rho_{i+1/2}}{\rho_{oi}} \right] \\ &+ g \frac{P_{oi}}{G_{oi}} \left(\frac{\Delta P_{i+1}}{P_{oi}} \right) + \frac{G_{oi}}{\rho_{oi}} \left(\frac{\Delta \rho_{i+1}}{\rho_{oi}} - \frac{2 \Delta G_{i+1}}{G_{oi}} \right) \\ &- g \frac{P_{oi}}{G_{oi}} \left(\frac{\Delta P_i}{P_{oi}} \right) - \frac{G_{oi}}{\rho_{oi}} \left(\frac{\Delta \rho_i}{\rho_{oi}} - \frac{2 \Delta G_i}{G_{oi}} \right) \end{aligned} \quad (25)$$

Energy Equation:

$$\begin{aligned} \frac{d\left(\frac{\Delta H_{i+1/2}}{H_{oi}}\right)}{dt} &= \frac{2 G_{oi}}{\rho_{oi}} \left[\frac{\frac{\Delta H_{i+1/2}}{H_{oi}} - \frac{\Delta H_i}{H_{oi}}}{\Delta x} \right] \\ &+ \frac{P_{oi}}{J \rho_{oi} H_{oi}} \frac{d\left(\frac{\Delta P_{i+1/2}}{P_o}\right)}{dt} + \frac{q_o S}{\rho_o H_o A} \left(\frac{\Delta q_{i+1/2}}{q_{oi}} \right) \end{aligned} \quad (26)$$

Equation of State:

$$\begin{aligned} \frac{\Delta P_{i+1/2}}{P_{oi}} \left[\frac{P_{oi}}{\rho_{oi}} - b P_{oi} \right] + \frac{\Delta \rho_{i+1/2}}{\rho_{oi}} \left[-\frac{P_{oi}}{\rho_{oi}} + a \rho_{oi} - 2ab \rho_{oi} \right] \\ = RT_{oi} \left(\frac{\Delta T_{i+1/2}}{T_{oi}} \right) \end{aligned} \quad (27a)$$

$$\begin{aligned} \frac{\Delta P_i}{P_{oi}} \left[\frac{P_{oi}}{\rho_{oi}} - b P_{oi} \right] + \frac{\Delta \rho_i}{\rho_{oi}} \left[-\frac{P_{oi}}{\rho_{oi}} + a \rho_{oi} - 2ab \rho_{oi} \right] \\ = RT_{oi} \left(\frac{\Delta T_i}{T_{oi}} \right) \end{aligned} \quad (27b)$$

$$\begin{aligned} \frac{\Delta P_{i+1}}{P_o} \left[\frac{P_{oi}}{\rho_{oi}} - b P_{oi} \right] + \frac{\Delta \rho_{i+1}}{\rho_o} \left[-\frac{P_{oi}}{\rho_{oi}} + a \rho_{oi} - 2ab \rho_{oi} \right] \\ = RT_{oi} \left(\frac{\Delta T_{i+1}}{T_{oi}} \right) \end{aligned} \quad (27c)$$

Enthalpy Relationship:

$$\frac{\Delta H_{i+1/2}}{H_{oi}} = \frac{\Delta T_{i+1/2}}{T_{oi}} \quad (28a)$$

$$\frac{\Delta H_i}{H_{oi}} = \frac{\Delta T_i}{T_{oi}} \quad (28b)$$

$$\frac{\Delta H_{i+1}}{H_{oi}} = \frac{\Delta T_{i+1}}{T_{oi}} \quad (28c)$$

Heat Transfer in the Material:

$$T_{moi} \frac{d\left(\frac{\Delta T_{mi+1/2}}{T_{moi}}\right)}{dt} = \left(\frac{Q_{mo,i}}{\rho_{m c_{m,oi}}}\right) \frac{\Delta n}{n_o} - \left(\frac{q_{moi} S_m}{\rho_{m c_{m,oi}} A_m}\right) \frac{\Delta q_{m,i+1/2}}{q_{moi}} - \frac{S_m U'_m T_{moi}}{\rho_{m c_{m,oi}} A_m} \left[\frac{\Delta T_{mi}}{T_{moi}} - \frac{\Delta T'_{mi}}{T_{moi}}\right] \quad (29a)$$

$$T_{noi} \frac{d\left(\frac{\Delta T_{n,i+1/2}}{T_{noi}}\right)}{dt} = \left(\frac{Q_{noi}}{\rho_{n c_{noi}}}\right) \frac{\Delta n}{n_o} - \left(\frac{q_{noi} S_n}{\rho_{n c_{noi}} A_n}\right) \frac{\Delta q_{n,i+1/2}}{q_{noi}} - \frac{S_n U'_n T_{noi}}{\rho_{n c_{noi}} A_n} (T_{noi} - T'_{noi}) \frac{\Delta U'_{ni}}{U'_{noi}} - \frac{S_n U'_n T_{noi}}{\rho_{n c_{noi}} A_n} \left[\frac{\Delta T_{n,i+1/2}}{T_{noi}} - \frac{\Delta T'_{n,i+1/2}}{T_{noi}}\right] \quad (29b)$$

Material-to-Coolant Heat Transfer:

$$q_o \frac{\Delta q_{i+1/2}}{q_{oi}} = \frac{S_m}{S} q_{moi} \frac{\Delta q_{m,i+1/2}}{q_{moi}} + \frac{S_n}{S} q_{noi} \frac{\Delta q_{n,i+1/2}}{q_{noi}} \quad (30a)$$

$$\frac{\Delta q_{m,i+1/2}}{q_{mo}} = \frac{h_{moi}}{q_{moi}} \left[T_{moi} \frac{\Delta T_{m,i+1/2}}{T_{moi}} - T_{oi} \frac{\Delta T_{i+1/2}}{T_{oi}} \right] + 0.8 \frac{\Delta G_{i+1/2}}{G_{oi}} + \gamma \frac{\Delta T_{i+1/2}}{T_{oi}} - \gamma \frac{\Delta T_{i+1/2}}{T_{moi}} \quad (30b)$$

$$\begin{aligned}
\frac{\Delta q_{n,i+1/2}}{q_{noi}} &= \frac{h_{noi}}{q_{noi}} \left[T_{noi} \frac{\Delta T_{n,i+1/2}}{T_{noi}} - T_{oi} \frac{\Delta T_{i+1/2}}{T_{oi}} \right] \\
&+ 0.8 \frac{\Delta G_{i+1/2}}{G_{oi}} + \gamma \frac{\Delta T_{i+1/2}}{T_{oi}} - \gamma \frac{\Delta T_{i+1/2}}{T_{noi}}
\end{aligned} \tag{30c}$$

7.0 A Method for Solving the Linearized, Difference-Differential Equations

Figures 1, 2, and 3 illustrate a method for the analog computer solution of the difference-differential equations for a spatial increment Δx . Figure 3 gives an overall picture of the analog approach. Here, the variables $\frac{\Delta G_{i+1}}{G_{oi}}$, $\frac{\Delta H_i}{H_{oi}}$, and $\frac{\Delta P_i}{P_{oi}}$ are inputs received from similar analog programs for $x_{i+1} < x < x_{i+2}$ and $x_{i-1} < x < x_i$. The outputs are $\frac{\Delta G_i}{G_{oi}}$, $\frac{\Delta H_{i+1}}{H_{oi}}$, and $\frac{\Delta P_{i+1}}{P_{oi}}$, which feed into analog simulations for other x increments. The $\Delta n/n_o$ input is received from a neutron kinetics simulation. The material boundary temperatures, $\frac{\Delta T'_{mi}}{T'_{moi}}$ and $\frac{\Delta T'_{ni}}{T'_{noi}}$ depend on the particular portion of the reactor being described by the equations. Relationships determining these temperatures are omitted here.

The analog program is shown as divided into two parts in Figure 3. One of these parts, whose relationships are shown in detail in Figure 1, determines the pressure, density, and flow variables; the other part, which is detailed in Figure 2, determines the output enthalpy, $\frac{\Delta H_{i+1}}{H_{oi}}$. Equations for the analog program are listed below.

Pressure, Density, and Flow Equations (See Figure 1):

$$\begin{aligned} \frac{\Delta \rho_i}{\rho_{oi}} = & \left[\frac{\frac{P_{oi}}{\rho_{oi}} - bP_{oi}}{a\rho_{oi} - \frac{P_{oi}}{\rho_{oi}} - 2ab\rho_{oi}} \right] \frac{\Delta P_i}{P_{oi}} \\ & - \left[\frac{RT_{oi}}{a\rho_{oi} - \frac{P}{\rho_{oi}} - 2ab\rho_{oi}} \right] \frac{\Delta H_i}{H_{oi}} \end{aligned} \quad (31)$$

$$\frac{d\left(\frac{\Delta P_{i+1/2}}{\rho_{oi}}\right)}{dt} = \frac{2G_{oi}}{\rho_{oi}} \left[\frac{\frac{\Delta G_i}{G_{oi}} - \frac{\Delta G_{i+1/2}}{G_{oi}}}{\Delta x} \right] \quad (32)$$

$$\frac{\Delta \rho_{i+1/2}}{\rho_{oi}} = \int_{-\infty}^t \left[\frac{d\left(\frac{\Delta \rho_{i+1/2}}{\rho_{oi}}\right)}{dt} \right] dt \quad (33)$$

$$\frac{\Delta \rho_{i+1}}{P_{oi}} = 2 \left(\frac{\Delta \rho_{i+1/2}}{\rho_{oi}} \right) - \frac{\Delta \rho_i}{\rho_{oi}} \quad (34)$$

$$\begin{aligned} \frac{\Delta P_{i+1}}{P_{oi}} &= \left[\frac{aP_{oi} - \frac{P_{oi}}{\rho_{oi}} - 2ab\rho_{oi}}{\frac{P_{oi}}{\rho_{oi}} - bP_{oi}} \right] \frac{\Delta \rho_{i+1}}{\rho_{oi}} \\ &+ \left[\frac{RT_{oi}}{\frac{P_{oi}}{\rho_{oi}} - bP_{oi}} \right] \frac{\Delta H_{i+1}}{H_{oi}} \end{aligned} \quad (35)$$

$$\begin{aligned}
\frac{d\left(\frac{\Delta G_{i+1/2}}{G_o}\right)}{dt} &= \frac{Sf G_{oi}}{2A \rho_{oi}} \left[\frac{2\Delta G_{i+1/2}}{G_{oi}} - \frac{\Delta \rho_{i+1/2}}{\rho_{oi}} \right] \\
&+ \frac{g P_{oi}}{G_{oi}} \left(\frac{\Delta P_{i+1}}{P_{oi}} \right) + \frac{G_{oi}}{\rho_{oi}} \left[\frac{\Delta \rho_{i+1}}{\rho_{oi}} - \frac{2\Delta G_{i+1}}{G_{oi}} \right] \\
&- \frac{g P_{oi}}{G_{oi}} \left(\frac{\Delta P_i}{P_{oi}} \right) - \frac{G_{oi}}{\rho_{oi}} \left[\frac{\Delta \rho_i}{\rho_{oi}} - \frac{2\Delta G_i}{G_{oi}} \right] \quad (36)
\end{aligned}$$

$$\frac{\Delta G_{i+1/2}}{G_{oi}} = \int_{-\infty}^t \left[\frac{d\left(\frac{\Delta G_{i+1/2}}{G_{oi}}\right)}{dt} \right] dt \quad (37)$$

$$\frac{\Delta G_i}{G_{oi}} = 2 \left(\frac{\Delta G_{i+1/2}}{G_{oi}} \right) - \frac{\Delta G_{i+1}}{G_{oi}} \quad (38)$$

Enthalpy Equations (See Figure 2):

$$\begin{aligned}
\frac{d\left(\frac{\Delta P_{i+1/2}}{P_{oi}}\right)}{dt} &= \left[\frac{a \rho_{oi} - \frac{P_{oi}}{\rho_{oi}} - 2ab \rho_{oi}}{\frac{P_{oi}}{\rho_{oi}} - b P_{oi}} \right] \frac{d\left(\frac{\Delta \rho_{i+1/2}}{\rho_{oi}}\right)}{dt} \\
&+ \left[\frac{RT_{oi}}{\frac{P_{oi}}{\rho_{oi}} - b P_{oi}} \right] \frac{d\left(\frac{\Delta H_{i+1/2}}{H_{oi}}\right)}{dt} \quad (39)
\end{aligned}$$

$$\begin{aligned}
\frac{d}{dt} \left(\frac{\Delta H_{i+1/2}}{H_{oi}} \right) &= 2 \frac{G_{oi}}{\rho_{oi}} \left[\frac{\frac{\Delta H_i}{H_{oi}} - \frac{\Delta H_{i+1/2}}{H_{oi}}}{\Delta x} \right] \\
&+ \frac{q_{oi} S}{\rho_{oi} H_{oi} A} \left(\frac{\Delta q_{i+1/2}}{q_{oi}} \right) + \frac{P_{oi}}{J \rho_{oi} H_{oi}} \frac{d}{dt} \left(\frac{\Delta P_{i+1/2}}{P_{oi}} \right) \quad (40)
\end{aligned}$$

$$\frac{\Delta H_{i+1/2}}{H_{oi}} = 2 \left(\frac{\Delta H_{i+1/2}}{H_{oi}} \right) - \frac{\Delta H_i}{H_{oi}} \quad (41)$$

$$q_o \frac{\Delta q_{i+1/2}}{q_{oi}} = \frac{S_m}{S} q_{moi} \frac{\Delta q_{m,i+1/2}}{q_{moi}} + \frac{S_n}{S} q_{noi} \frac{\Delta q_{n,i+1/2}}{q_{noi}} \quad (42)$$

$$\begin{aligned}
\frac{\Delta q_{m,i+1/2}}{q_{moi}} &= \frac{h_{moi}}{q_{moi}} \left[T_{moi} \frac{\Delta T_{m,i+1/2}}{T_{moi}} - T_{oi} \frac{\Delta H_{i+1/2}}{H_{oi}} \right] \\
&+ 0.8 \frac{\Delta G_{i+1/2}}{G_{oi}} + \gamma \frac{\Delta H_{i+1/2}}{H_{oi}} - \gamma \frac{\Delta T_{m,i+1/2}}{T_{moi}} \quad (43)
\end{aligned}$$

$$\begin{aligned}
\frac{\Delta q_{n,i+1/2}}{q_{noi}} &= \frac{h_{noi}}{q_{noi}} \left[T_{noi} \frac{\Delta T_{n,i+1/2}}{T_{noi}} - T_{oi} \frac{\Delta H_{i+1/2}}{H_{oi}} \right] \\
&+ 0.8 \frac{\Delta G_{i+1/2}}{G_{oi}} + \gamma \frac{\Delta H_{i+1/2}}{H_{oi}} - \gamma \frac{\Delta T_{n,i+1/2}}{T_{noi}} \quad (44)
\end{aligned}$$

$$\begin{aligned}
 T_{\text{moi}} \frac{d \left(\frac{\Delta T_{\text{mi}+1/2}}{T_{\text{moi}}} \right)}{dt} &= \left(\rho_{\text{m c moi}} \right) \frac{\Delta n}{n_o} - \left(\rho_{\text{m c moi} A_m} \right) \frac{\Delta q_{\text{m},i+1/2}}{q_{\text{moi}}} \\
 &\quad - \frac{S_m U'_m T_{\text{moi}}}{\rho_{\text{m c moi} A_m}} \left[\frac{\Delta T_{\text{mi}}}{T_{\text{moi}}} - \frac{\Delta T'_{\text{mi}}}{T_{\text{moi}}} \right]
 \end{aligned} \tag{45}$$

$$\frac{\Delta T_{\text{m},i+1/2}}{T_{\text{moi}}} = \int_{-\infty}^t \left[\frac{d \left(\frac{\Delta T_{\text{m},i+1/2}}{T_{\text{moi}}} \right)}{dt} \right] dt \tag{46}$$

$$T_{\text{noi}} \frac{d \left(\frac{\Delta T_{\text{n},i+1/2}}{T_{\text{noi}}} \right)}{dt} = \left(\rho_{\text{n c noi}} \right) \frac{\Delta n}{n_o} - \left(\rho_{\text{n c noi} A_n} \right) \frac{\Delta q_{\text{n},i+1/2}}{q_{\text{noi}}}$$

$$- \frac{S_n U'_n}{\rho_{\text{n c noi} A_n}} (T_{\text{noi}} - T'_{\text{noi}}) \frac{\Delta U'_{\text{ni}}}{U_{\text{noi}}} - \frac{S_n U'_n}{\rho_{\text{n c noi} A_n}} \left[\frac{\Delta T_{\text{n},i+1/2}}{T_{\text{noi}}} - \frac{\Delta T'_{\text{n},i+1/2}}{T_{\text{noi}}} \right] \tag{47}$$

$$\frac{\Delta T_{\text{n},i+1/2}}{T_{\text{noi}}} = \int_{-\infty}^t \left[\frac{d \left(\frac{\Delta T_{\text{n},i+1/2}}{T_{\text{noi}}} \right)}{dt} \right] dt \tag{48}$$

ANALOG SOLUTION FOR PRESSURE, FLOW, AND DENSITY

$$\frac{\Delta H_{i+1}}{H_{oi}}$$

FIGURE 2
ANALOG SOLUTION
FOR ENTHALPY

FIGURE 3
ANALOG SOLUTION OF THE LINEARIZED, DIFFERENCE-
DIFFERENTIAL APPROXIMATIONS OF THE
HEAT TRANSFER AND FLOW EQUATIONS