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University of California/Livermore, California

Current-Stabilization of the Curvature-driven $m = 1$ mode

G. Logan

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CURRENT-STABILIZATION OF THE
CURVATURE-DRIVEN $m = 1$ MODE

G. Logan

March 1974

Introduction

This report presents a short review of the $m = 1$ mode instability of long, thin, high-beta plasma columns in axisymmetric mirror fields, followed by a δW calculation of the magnitude of longitudinal plasma current required to stabilize the curvature-driven instability by the image currents returning in a conducting wall. The parameter regimes explored are those useful for proposed two-component mirror fusion reactors¹ and multiple-mirror reactors². The possible excitation of current-driven helical modes and their suppression by alternating the current is then discussed.

Curvature-Driven Instability of the Equilibrium

Typical parameters assumed for the equilibrium are indicated in Fig. 1. Only half of the distance L ($L = 12$ meters) between two mirrors is considered due to symmetry. The effect of end regions outside the mirrors (possibly stabilizing) is neglected by doing the calculation for an infinitely long periodic system. The plasma pressure P is assumed to be constant with z and $r < R_p$, with currents flowing only in the surface of a sharp boundary. For real systems with a diffuse radial pressure gradient, the radially-averaged beta is appropriate to use in the sharp boundary model. The plasma radius R_{po} and beta β_o are specified at the midplane, with $R_p(z)$ and $\beta(z)$ determined elsewhere by zero order pressure balance and flux conservation:

$$R_p(z) = R_{po} [(1 - \beta_o) / ((B_e^2(z) / B_{eo}^2) - \beta_o)]^{1/4}, \quad (1)$$

$$\beta(z) = P / (B_e^2(z) / 2\mu_o) = \beta_o (B_e(z) / B_{eo})^2, \quad (2)$$

where $B_e(z)$ is the external field. The external mirror field is assumed to be created by the field of current rings of radius $R_m \gg R_p$ spaced a distance $L \gg R_m$ apart, superimposed on the uniform field of a current solenoid. The ratio ϵ of plasma radius R_p to the characteristic distance L over which the

equilibrium field varies is small compared to unity, so that the zero-order field along the plasma surface is not appreciably different from the vacuum field in the absence of plasma. Perturbations of the plasma equilibrium are described in terms of the amplitude of the normal component of displacement of the plasma surface

$$\xi(z) \equiv \xi_r(z),$$

where the normal component is taken to vary as $\xi_r(z)\exp(im\theta)$, with m the azimuthal mode number. The worst displacements are taken to be incompressible ($\nabla \cdot \vec{\xi} = 0$) to minimize increases in the plasma internal energy. It will be shown that the most unstable displacements have parallel wavelengths of the order or longer than L , so that motion is almost purely transverse ($\xi_z \approx 0$).

Although the plasma betas considered are sufficient to significantly reduce the internal field, an evaluation of the stability properties based on low beta theory is still instructive. Consider the plasma surface to have some average radius of unfavorable curvature $R_c \sim L^2/2R_p$ due to the mirror field, giving rise to an outward acceleration $g \sim 2\bar{V}_i^2/R_c$ to the ions, where $\bar{V}_i$ is the ion thermal velocity. The outward acceleration leads to Rayleigh-Taylor growth rates ($k_{||} = 0$)

$$\gamma(\text{Low}\beta) \approx \sqrt{gm/R_p} \approx 2\sqrt{m}(\bar{V}_i/L) \quad (4)$$

which increase with mode number m . Eq. (4) applies when the ion Larmor radius ρ_i is sufficiently small compared to the azimuthal wavelength $2\pi R_p/m$. Rosenbluth, et. al.³ found that for a perpendicular density gradient scale length $\Delta \gg \rho_i$, all mode numbers

$$m > 5(R_p/\rho_i)\sqrt{\Delta/R_c} \quad (5)$$

are stable due to finite Larmor radius (FLR). Assuming $\Delta \sim R_p$ in a real system, the parameters assumed for the system of Fig. 1 satisfy inequality (5) for $m > 1$ by more than an order of magnitude. Eq. (5) based on a slab model breaks down for the $m = 1$ mode of a cylindrical plasma, which is not expected to be FLR stabilized³. As for the effects of finite beta on the unstable mode numbers, Schmidt⁴ has shown that the low-beta FLR stabilization criterion Eq. (5) holds for finite betas where $\Delta \gg \rho_i$ still, and provided that

$$\left(\Delta + \frac{R_c}{2}\right)^{-2} \Delta + \left(\frac{1}{R_c} - \frac{1}{\Delta}\right) > 0. \quad (6)$$

Eq. (6) is satisfied for R_c sufficiently large compared to Δ , as in our model.

Several experiments^{5, 6, 7} with plasma parameters similar to those assumed in the model have confirmed the prediction of long wavelength $m = 1$ instability with the absence of higher m modes. Therefore in the calculations that follow only the $m = 1$ mode will be considered.

Haas and Wesson⁸ have investigated the stability of high-beta mirror equilibria such as in Fig. 1 by means of the hydromagnetic energy principle⁹, for plasmas carrying no longitudinal current. Their results (Eq. 3.13 of Ref. 8) for the potential energy change δW_m of an $m = 1$ displacement $\xi(z)$ of a sharp-boundary $\beta < 1$ plasma enclosed by conducting walls of radius R_w can be written

$$\begin{aligned} \delta W_m = & \frac{1}{2\mu_0} \int_0^{L/2} \xi^2 B_e^2 \beta \frac{d^2 R_p}{dz^2} R_p dz + \frac{1}{2\mu_0} \int_0^{L/2} B_e^2 G \left[\frac{d}{dz} (R_p \xi) - 2\xi \frac{dR_p}{dz} (1-\beta) \right]^2 dz \\ & + \frac{1}{2\mu_0} \int_0^{L/2} B_e^2 (1-\beta) \left[R_p \frac{d\xi}{dz} - \xi \frac{dR_p}{dz} \right]^2 dz, \end{aligned} \quad (7)$$

where $G = (1 + (R_p/R_w)^2)/(1 - (R_p/R_w)^2)$.

The first term can be identified as the change of potential energy of the plasma column due to field curvature (the so-called "surface energy" term of Bernstein, et.al.⁹). The second term is the change in vacuum field energy between the plasma and the conducting wall (the "vacuum energy"), and the third term is due to changes in the plasma internal field energy (the "fluid energy"). No contribution to plasma energy change appears in the third term due to $\vec{\nabla} \cdot \vec{\xi}$ set equal to zero. The second and third terms give the energy change due to field line bending external and internal to the plasma, and these contributions are always positive and stabilizing. The first term can be negative due to unfavorable curvature ($d^2 R_p / dz^2 < 0$, and instability arises when this contribution exceeds the line bending contributions. For the limit $\beta \rightarrow 1$, consider a displacement $\xi = \text{constant}$ corresponding to a rigid, transverse motion of the plasma column. The second derivative of the plasma radius in the first integral of Eq.(7) is related to the normal component of field pressure gradient at the plasma surface $r = R_p(z)$:

$$\frac{d^2 R_p}{dz^2} = \frac{1}{R_c(z)} = \frac{\vec{n} \cdot \nabla (B_e^2 / 2\mu_0)}{2B_e^2 / 2\mu_0} \quad (8)$$

Thus, in the midplane region of unfavorable curvature, a radial displacement ξ results in a net unbalanced force F per unit length in the direction of ξ , assuming constant plasma pressure across the plasma column. In the mirror region where $d^2 R_p / dz^2 > 0$, the force is restoring and the energy change $\delta W = -\frac{1}{2} \int F \cdot \xi dz$ due to $\xi = \text{const.}$ in a mirror field is therefore positive in the mirror regions and negative in the midplane regions. However, since the plasma surface area ($\sim R_p$) is smaller in the mirror than in the midplane, the negative contribution of the regions of unfavorable curvature always dominate in the first term of Eq. (7).

The presence of conducting walls has a stabilizing effect which appears in the factor G of the second term in Eq. (7), but this factor is appreciable only when $R_w \approx R_p$, which is usually not practical. Since fusion α - particles make excursions of 5 cm from the plasma surface, a wall radius $R_w = 5 R_p$ is taken in the model here, for which $G \approx 1$. Although the first term of Eq. (7) goes to zero as $\beta \rightarrow 0$ while the line-bending terms generally do not, there can be displacements satisfying

$$\xi \frac{dR_p}{dz} = R_p \frac{d\xi}{dz},$$

$$\text{or } \xi(z) = \xi_0 \exp \int_{z_0}^z \frac{1}{R_p} \frac{dR_p}{dz} dz, \quad (9)$$

which make the line-bending terms go to zero. Thus, Eq. (9) gives the most unstable $\xi(z)$ for low beta.

At high beta, the most unstable $\xi(z)$ is found by maximizing the dominant first term in Eq. (7) rather than by minimizing the line-bending terms:

$$\xi(z) = \xi_0 \exp(-AB_e^2 R_p / R_c), \quad (10)$$

where A is an adjustable constant. As a check a trial function satisfying $R_p \xi = \text{const.}$ which makes the vacuum energy term zero as $\beta \rightarrow 1$ is found to make $\delta W_m > 0$, while Eq. (10) gave the most negative values of δW_m . Results for $\xi(z)$ which minimizes δW_m for the model of Fig. 1 are shown in Fig. 2, along with the radius of curvature. Note that the most unstable ξ for the high beta considered ($\beta_0 = .9$) is nearly constant with z , with a slight peak in the region of maximum unfavorable curvature.

For an infinite periodic system with an equilibrium plasma radius given by $R_p = R_{av} (1 + \delta \cos kz)$, Haas and Wesson⁸ find a maximum $m = 1$ growth rate

$$\gamma = \frac{1}{\sqrt{2}} k \delta \bar{v}_A \left\{ \left[\frac{\beta(3 - 2\beta)}{(1 - \beta + G)} \right] [1 + (1 - 2\beta)G] \right\}^{\frac{1}{2}}, \quad (11)$$

where $\bar{V}_A$ is the Alfven velocity $\sqrt{B_e^2/\mu_0 \rho_m}$, and the mass density ρ_m , B_e , β and G are averaged over z . With $G \approx 1$, an average beta $\bar{\beta} \approx .5$, $\bar{V}_A \approx 1.5 \bar{V}_i$, $\delta \approx .5$, and $k \approx 2\pi / L$, Eq. (11) gives

$$\gamma (\text{high } \beta) \approx 3(\bar{V}_i/L), \quad (12)$$

which is nearly the same as the low beta growth rate for $m = 1$ given by Eq. (4).

Haas and Wesson⁸ also investigated the stability of a plasma with a diffuse radial pressure profile, for peak betas on the axis $\beta_A > .75$ everywhere, and $\xi(z) = \text{const}$. For the same value of β_A , they found diffuse profiles slightly more unstable than for sharp boundaries, which they attribute to lower radially-averaged $\langle \beta \rangle_r$ for the diffuse plasma. Therefore, use of betas in a sharp boundary model equal to the $\langle \beta \rangle_r$ rather than β_A of a diffuse profile should give adequate results for δW_m .

Image Current Stabilization of the $m = 1$ Mode

The expression Eq. (7) for the energy change δW_m will now be extended to include the effect of a longitudinal current I_z flowing down the plasma and returning via a conducting wall in a closed circuit. The work done by a displacement of the form given by Eq. (3) against the repulsive force between the plasma and image current will appear as an additional stabilizing term in Eq. (7). Only lateral displacements $\xi_r(z) \cos \theta$ will be considered as the object is to find the minimum current required to stabilize the most unstable curvature-driven modes. Helical displacements of the form $\xi_r \cos(\theta - \omega z)$ are most unstable in the presence of a D.C. current which causes the field lines to twist, but for these current-driven modes there is an additional means of stabilization available - namely, alternating the current - which will be discussed later.

The restoring force F per unit length on a perfectly conducting plasma initially carrying a current I_z and making a lateral $m = 1$ displacement $\xi(z)$ is given by

$$F(z) = - \frac{\mu_0 I_z'^2}{2\pi} \left(\frac{\xi}{R_w^2 - \xi^2} \right), \quad (13)$$

where

$$I_z' = \frac{I_z}{(L/2)} \int_0^{L/2} \frac{\ln(R_w/R_p) dz}{\ln\left(\frac{R_w^2 - \xi^2 - R_p \xi}{R_w R_p}\right)} \quad (14)$$

is the current after making displacement $\xi(z)$ toward a conducting wall of radius R_w carrying $-I_z'$. For $\xi > 0$, $I_z' > I_z$ due to conservation of azimuthal flux between the plasma and wall and the fact that the inductance of the circuit decreases. As $\xi \rightarrow (R_w - R_p)$, the restoring force for any current becomes arbitrarily large, and therefore the instability amplitude will be limited within the conducting wall radius for non zero currents. To prevent any motion, however, a minimum current is required. In the limit of small displacements $\xi \ll (R_w - R_p)$, $I_z' \rightarrow I_z$ and F reduces to

$$F(z) = - \frac{\mu_0 I_z^2 \xi}{2\pi R_w^2} \quad (15)$$

Since $F(z)$ is linear with ξ in this limit, the corresponding work done by a small displacement is given by

$$\delta W_I = - \frac{1}{2} \int_0^{L/2} F \xi dz. \quad (16)$$

The total energy change due to a displacement $\xi(z)$ given by Eq. (3) for a current-carrying plasma is given by adding δW_I given by Eq. (16) to δW_m given by Eq. (7):

$$\delta W = \delta W_m + \delta W_I. \quad (17)$$

Eq. (17), normalized to $\int_0^{L/2} \xi^2 dz$, is numerically minimized with respect to the parameter A in Eq. (3), for increasing currents I_z . The value of I_z for which the stationary value, or minimum value, of the normalized δW just becomes positive is the minimum current to stabilize the $m = 1$ curvature mode. The results of such a minimization for the model parameters of Fig. 1 are shown in Fig. 3. Note that the minima of δW occur at the same ξ -variation parameter A for different currents. This $A \approx 45$ corresponds to $\xi(z)$ as given in Fig. 2. Note also that the plasma is stable with no current for $A \rightarrow 0$, corresponding to a rigid displacement $\xi = \text{constant}$. Fig. 3 shows that a current slightly above 4.5 kA is required for stability for any A.

The dependence of the minimum current for stabilization on the vacuum mirror ratio M, the ratio α of the mirror coil radius R_m to the plasma radius R_{po} , the ratio η of wall radius R_w to plasma radius R_{po} , and on β_0 was investigated by varying $B_{\max}$, R_m , R_p , and plasma pressure P, respectively, about the nominal values assumed in Fig. 1. The results are presented in Tables A, B, C, and D. Table A shows that $I_z \rightarrow 0$ as $M \rightarrow 1$, corresponding to the case of a uniform field. I_z increases slowly with $M > 2$ due to decreasing effects on the field curvature. Table B indicates that localized mirrors (small R_m) are more destabilizing than gradual mirror fields for the same M. Increasing L for fixed R_m increases δW_I relative to δW_m , and in the limit $L \gg R_m$, $I_z \sim 1/\sqrt{L}$. Table C illustrates the stringent requirement $(R_w - R_{po}) \approx 0$ for stability without current by placing conducting walls very close to the plasma. For large η , reducing R_p keeping R_w fixed reduces I_z proportionately. In Table D, Eq. (9) is used for $\xi(z)$ for the lower betas .1, .2 and .3, and Eq. (10) for the higher betas. For low betas, I_z increases as $\sqrt{\beta}$ as expected, but for betas $> .9$, I_z decreases slightly due to increasing line-bending energy with beta. To summarize, for $R_{po} \ll R_m \ll L$, $R_w \gg R_{po}$, $\beta_0 \leq .9$, the stabilizing current scales approximately as

$$I_z \sim B_{eo} (M-1)^{.25} (\beta_0/\alpha L)^{.5/\eta} \quad (17a)$$

Having determined the required stabilizing currents I_z to be between 2 and 10 kA depending on the reactor parameters M , α , η and β_0 , questions arise as to whether such currents will excite turbulence in the plasma and how such currents can be fed to the plasma without incurring large heat loss by electron conduction. Assuming a plasma temperature

$$kT_i = kT_e = 5 \text{ keV}, \beta_0 = .9, \text{ and } B_{e0} = 100 \text{ kG},$$

a plasma density $n \approx 2 \times 10^{16} \text{ cm}^{-3}$ obtains. The electron drift velocity V_{ed} associated with a current of 5 kA carried within a minimum skin depth c/ω_{pe} around a 3 cm diameter plasma is $V_{ed} \approx 3 \times 10^7 \text{ cm/sec}$. Since this drift velocity is less than the ion sound speed, no excitation of ion acoustic turbulence can be expected. Since the component currents associated with normal ion and electron particle loss through ambipolar sheaths at the ends are an order of magnitude larger than I_z , slightly biasing the end walls or electrode potentials asymmetrically can provide a net current of I_z out of electrons and ions leaving the plasma rather than by having the electrodes thermionically emitting. The heat insulation provided by ambipolar sheaths can therefore be maintained.

Current-Driven Kink Modes and Their Dynamic Stabilization

The growth rate for helical $m = 1$ displacements of the form $\xi_r \cos(\theta - h_z)$, (where $\xi_r = \text{const.}$) excited by the current I_z will now be estimated. In the limit $h R_w \ll 1$ and $\xi_r \ll (R_w - R_p)$, Ribe and Riesenfeld¹⁰ derive the transverse helical force per unit length of a current-carrying plasma column (Eq. (16) of Ref. 10), which includes the helical wall-image force given by Eq. (15):

$$F(\text{helical}) \approx - \left[\frac{\mu_0 I_z^2}{2\pi R_w^2} + (2 - \beta)(h R_p) \left(\frac{\pi}{\mu_0} \right) B_e^2 - h B_e I_z \right] \xi_r. \quad (18)$$

The third term in Eq. (18) is the destabilizing term which can be interpreted as a $\vec{J} \times \vec{B}_e$ force in the direction of ξ_r . This force increases with the pitch angle $\psi \propto h$ between the current column and the z-directed mirror field B_e . The second term represents the stabilizing effect of field-line bending, since the helical displacement must warp the zero-order mirror field B_e lines. The field-line bending scales as h^2 , exceeding the destabilizing term for sufficiently large h . Setting the derivative of Eq. (18) with respect to h to zero obtains the most unstable perturbation wave number:

$$h = \frac{g}{(2 - \beta)}, \quad (19)$$

$$\text{where } g = \frac{\mu_o I_z^2}{2 \pi R_p^2 B_e} \quad (20)$$

is the field line pitch wave number ($B_\theta / (R_p B_e)$). Substituting Eq. (19) with Eq. (20) into Eq. (18) obtains

$$F(\text{helical}) \approx - \left[\frac{\mu_o I_z^2}{2 \pi R_w^2} + \frac{\mu_o I_z^2}{4 \pi R_p^2 (2 - \beta)} - \frac{\mu_o I_z^2}{2 \pi R_p^2 (2 - \beta)} \right] \xi_r$$

or $F(\text{helical}) \approx - \left[\frac{\mu_o I_z^2}{2 \pi R_w^2} - \frac{\mu_o I_z^2}{4 \pi R_p^2 (2 - \beta)} \right] \xi_r. \quad (21)$

Since $(R_w/R_p)^2/4 \approx 6$ with our model parameters, the destabilizing force exceeds the wall image force for small ξ_r , and the kink-mode grows.

Using Rayleigh's principle¹¹, the growth rate of the kink-mode is determined by

$$\omega^2 = - \frac{\frac{1}{2} \int_0^{L/2} F(\text{helical}) \xi_r dz}{\frac{1}{2} \int_0^{L/2} \rho_m \xi_r^2 \pi R_p^2 dz} \quad (22)$$

Using Eq. (21) neglecting the wall-image force, and $\xi_r = \text{const}$, mass density $\rho_m = \text{const}$, one obtains

$$\gamma(\text{helical}) \approx \left[\frac{\mu_0 I_z^2}{(2\pi)^2 \rho_m (2 - \bar{\beta}) R_{av}^4} \right]^{\frac{1}{2}}, \quad (23)$$

where $\bar{\beta}$ and R_{av} are appropriate average beta and plasma radius. Using Eq. (20), Eq. (23) can be written

$$\gamma(\text{helical}) \approx g \bar{v}_A / \sqrt{2 - \bar{\beta}}, \quad (24)$$

where $\bar{v}_A$ is an average Alfven velocity $\sqrt{\bar{B}_e^2 / \mu_0 \rho_m}$. With $\bar{\beta} \approx .5$, $\bar{v}_A \approx 1.5 \bar{v}_i$, Eq. (24) becomes

$$\gamma(\text{helical}) \approx 9 \bar{v}_i / \lambda_B, \quad (25)$$

where $\lambda_B = 2\pi/g$ is the field line pitch wavelength. With $I_z = 5$ kA, $R_{av} \approx 1.2$ cm, $\bar{B}_e \approx 150$ kG, $\lambda_B \approx 12$ meters. Since $\lambda_B \approx L$ in this case, the current-driven kink-mode growth rate is about 3 times the curvature-driven growth rate in the absence of current as given by Eq. (11). Thus, by passing current through the plasma one replaces the curvature-driven instability with a faster current-driven one. However, the amplitude of the kink-mode can be suppressed by alternating the current in a time less than the kink-mode growth rate. The required frequency $f \geq \gamma(\text{helical})/2\pi$ is of the order of 100 kHz for our parameters, easily within the range of power oscillators. Considering the plasma column within a conducting wall as a coaxial transmission line, one finds a wave velocity of 10^8 m/sec , assuming most of the annular space between the current-carrying plasma and conducting wall to be occupied by a relatively high dielectric constant α -particle plasma. At a frequency of 100 kHz , the current-amplitude wavelength is $\lambda_c \approx 10^3$ meters, much longer than the L 's considered. Since $I_z \approx \text{const.}$ with z then, the impedance of the system is low. As an example

of a simple pulsed circuit that might be employed, a 10kV, $1\mu\text{F}$ capacitor discharged into the 12 meter, $3\mu\text{H}$ load of the system considered in Fig. 1 supplies the required 100 kHz , 5 kA current.

As a final note, there are two effects not considered here which may reduce the magnitude or frequency of oscillating current required for stabilization. One effect investigated by Ribe and Riesenfeld¹⁰ is the reduction of current required to stabilize the forces of unfavorable curvature owing to the oscillating nature of the current itself. They find that dynamic stabilization may occur for oscillating currents roughly R_w/R_p times smaller than the required DC image currents. Thus an oscillating current as low as 1 kA may be sufficient for our model. Another effect which may be important is damping of the kink-mode amplitudes at the end electrodes, and additional favorable curvature regions outside the end mirrors, providing a current threshold for kink-modes. The lack of kink-modes in the DCX arc¹², which carried 300 amps DC in a 10 kG mirror field, may be due to end effects. In conclusion, the low image currents ($\leq 5\text{kA}$) required for stabilization of long, thin plasma columns^{1, 2} should provide a particularly simple method of confining such plasmas in axisymmetric mirror fields.

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Fig. 1 Equilibrium (Typical)

Fig. 2 Curvature Driven Unstable Motion

Fig. 3 Normalized Energy δW as a function of variation parameter A for $I_z = 0, 3, 4.5, 6$ kA.

Minimum Stabilizing Currents

TABLE A

M	I _z (kA)
1.0	0.0
1.1	1.6
1.3	3.2
1.7	4.1
2.0	4.6
2.3	5.2
3.0	5.7
4.0	6.0

$$M \equiv \frac{B_e(\max)}{B_{eo}}$$

$$B_{eo} = 120 \text{ kG}$$

$$\alpha = 66.7$$

$$\eta = 5$$

$$\beta_o = .9$$

TABLE B

α	I _z (kA)
16.7	10.0
33.3	6.6
66.7	4.6
133	2.8
267	2.4

$$\alpha \equiv \frac{R_m}{R_{po}}$$

$$R_{po} = 1.5 \text{ cm}$$

$$M = 2$$

$$\eta = 5$$

$$\beta_o = .9$$

TABLE C

η	I _z (kA)
7.5	3.4
5.0	4.6
2.5	9.3
1.5	14.0
1.2	13.0
1.1	6.0
1.05	0.0

$$\eta \equiv \frac{R_w}{R_{po}}$$

$$R_w = 7.5 \text{ cm}$$

$$M = 2$$

$$\alpha = 66.7$$

$$\beta_o = .9$$

TABLE C

β_o	I _z (kA)
.1	1.8
.2	2.4
.3	3.0
.6	3.4
.8	4.2
.9	4.6
.95	4.2
.975	3.6

$$\beta_o \equiv \frac{P}{(B_{eo}^2/2\mu_o)}$$

$$B_{eo} = 120 \text{ kG}$$

$$M = 2$$

$$\alpha = 66.7$$

$$\eta = 5$$

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