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A PULSED EDDY CURRENT TECHNIQUE
FOR MEASURING CLAD THICKNESS

by

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D. L. Waidelich, J. A. DeShong and W. J. McGonnagle

ABSTRACT

A pulsed eddy current technique for measuring the cladding thickness of reactor fuel elements is described. This is a progress report. Further work is continuing.

INTRODUCTION

Fuel elements for nuclear reactors consist of a core of fissionable material encased in a corrosion-resistant jacket or clad. The cladding stops fission products from escaping, eliminates any reaction or solution between the fuel and coolant, restrains the fuel to a limited extent from warping and surface roughening, and reduces the danger of toxic materials from being dispersed in the coolant system. It is necessary to measure the cladding thickness on each fuel element before it is put into a reactor to determine if thin spots developed during the coating or cladding process.

A pulsed eddy current technique for measuring the cladding thicknesses of reactor fuel elements is described here. Special emphasis has been put on the measurement of zirconium cladding.

In this technique an electromagnetic field is applied at the surface of the clad fuel element by means of a probe coil. Whenever the waves encounter a sudden discontinuity, such as an interface, "echoes" are produced. The same probe coil is used for sending the electromagnetic radiation and receiving the echoes. The echo received from the front surface of the specimen is balanced out by means of a bridge circuit. The echo from the clad core interface contains information concerning the thickness of the clad. The echo has a positive peak followed by a small negative peak. The "crossing point" between the positive and negative peak or the amplitude of the echo can be used to determine clad thickness.

One of the difficult problems with this technique is the extreme sensitivity to variations in probe-to-metal spacing. Methods for eliminating or minimizing this effect have been tried, with some success.

PRELIMINARY THEORETICAL WORK

To set up the electromagnetic field it was envisaged that a small, single-layer probe coil would be placed next to the metal such that the axis of the coil would be perpendicular to the surface of the metal. This would allow an investigation of a small area and thus facilitate point-by-point thickness measurements. The field of such a coil and the mathematical determination of the echoes are quite difficult to solve. It was decided to simplify the problem by assuming that a plane wave with its front parallel to the metal surface was set up. This would indicate what kind of echo might result, but would ignore the spreading of the waves from the source, which is almost a point source.

Another approximation used was that of assuming the input pulse to be an impulse of zero time duration and of infinitely large amplitude. Actually, the input pulse will last a finite time, but if the system is linear, the response to any finite pulse may be approximated by the superposition of a number of the responses to impulses.

The cladding material is assumed to have a thickness as shown in Figure 1. The details of the analysis are carried out in Appendix A. The analysis indicates that there would be a succession of echoes from the metal, but that the echoed pulses would not resemble the input impulse very closely because dispersion would occur as the waves travel back and forth in the metal. The phenomenon resembles the heat waves traveling in a heat conductor more than it resembles electromagnetic waves in an electrical insulator or ultrasonic waves in a solid or liquid. The reflection coefficient at the air-to-metal boundary is large (almost unity), whereas the reflection coefficient at the metal-to-metal boundary is small (0.111 for zirconium clad on uranium). The first reflected wave as shown in Figure 2 would be that reflected from the air-to-metal boundary; it would approach the input pulse in size, but it would also have a long tail which decreases with time.

Figure 1

Geometry of Clad and Base Metals

Figure 2

Diagram of Reflected Waves

This tail must be balanced out if the others echoes are to be observed. Some of the energy will be transmitted into the metal and be reflected back from the metal-to-metal boundary, through the air to the probe. This echo from the interface generates in the probe coil the pulse voltage shown in Figure 3. Probably the point of this wave which can be observed most easily experimentally would be the crossing point occurring at $t = 0.367d^2\mu_1\sigma_1$, where t is the time in seconds and μ_1 and σ_1 are the permeability and conductivity, respectively, of the cladding metal. Even though the input wave is a pulse of finite length rather than a mathematical impulse of zero length, this crossing point should be proportional to the square of the thickness d of the cladding. The maximum point of Figure 3 at $t = 0.181d^2\mu_1\sigma_1$ and the minimum point at $t = 0.560d^2\mu_1\sigma_1$ might also be useful in determining d . The third wave emerging from the metal would be that reflected from the metal-to-metal boundary twice. Since the reflection coefficient for zirconium clad on a base of uranium is 0.111, the third wave should be at most about one-tenth of the second wave and probably will be much smaller than that. Hence it was decided to neglect all the waves above the second.

Figure 3

Echo from Metal-to-Metal Boundary

In order to view the second reflected wave, i.e., the wave reflected once from the metal-to-metal boundary, it is necessary to reduce the first reflected wave (that reflected from the air-to-metal boundary) by some means. It was estimated in Appendix A that this first wave would have to be reduced to approximately 1/10,000 of its original value in order to make the second reflected wave usable. Also, if 100 volts were applied to the probe coil, the detector should be sensitive enough to detect 1/100 volt. For a 15-mil coating of zirconium the crossing point will occur 0.1487 microsecond after the input impulse is applied. The detector thus should be able to exhibit waves of the order of 0.1 microsecond long.

Figure 4

Voltage Ratio of the Reflection from Metal-to Metal
Compared to Reflection from Air-to-Metal Boundary

PRELIMINARY EXPERIMENTAL WORK

The probe coil used was a single layer coil wound on a 1/4 -inch diameter ferrite (Croloy) rod. Approximately thirty turns of wire were used, spaced over about one inch of length. The metal specimens were placed at one end of the rod. It was intended later on to construct the

probe coils as shown in Figure 5, so that the magnetic flux would be concentrated at the one end of the center core. The metal specimens would be placed at this end of the probe. Two of the simple probes were constructed and then measured by using a Q-Meter. Turns were subtracted from the larger of the two probes until the inductances were nearly the same for both probes.

Fig. 5. Probe Coil Design

These two matched probes were connected in various bridge circuits in such a manner that a pulse of current could be passed through each probe simultaneously. The difference of the voltages across the two probes was then viewed on the screen of a cathode-ray oscilloscope. A number of different circuits were found to perform more or less satisfactorily and these circuits, together with some explanatory remarks, are given in Appendix B. The most promising of the five circuits presented are those of Figures B-5 and B-3. The better of the two is believed to be that of Figure B-5. The major problem was the development or procurement of a satisfactory driving transformer. This transformer should have very little capacitance between windings so that the waveform applied to the one probe is just the negative of the waveform for the other probe. The coupling coefficient of the transformer, on the other hand, should be as high as possible so that the power transferred to the probes is high. The band width of the transformer should be at least 50 megacycles per second wide. Both circuits would require a pair of probes matched electrically as closely as possible. The possibilities of balancing the bridge of Figure B-5 appear to be better than that of Figure B-3. Figure B-3 has the capacitance of the 884 tube cathode on one side. This circuit, however, has the big advantage that most of the pulse power appears in the probes. The circuit of Figure B-5 would be more easily adapted to higher voltage (and also higher power) pulse tubes than would that of Figure B-3. The insulation of the probes will probably limit the voltages that may be used in the pulses.

The purpose of the bridge circuit is to cancel out as completely as possible the voltages appearing across the probes so that the difference voltage may be observed. When a metal specimen is placed against a probe the difference voltage now contains a component caused by reflection of the input wave from the air-to-metal boundary. To balance this voltage out an identical specimen should be placed on the other probe. Then, if one

of these specimens had a metal-to-metal layer below the surface, the difference voltage would then indicate the wave reflected from this metal-to-metal surface. It is to bring out this last wave that the balancing of the bridge and the matching of the probes are done. From the theory it appears as if this balancing and matching must be good enough so that the difference voltage with two identical specimens (no metal-to-metal layers) on the probes must be about $1/10,000$ of the voltage across one probe.

As an example of some experimental results obtained from the use of the circuit of Figure B-3 the oscillograms of Figures 6 and 7 are shown. The 50-mmfd condenser of Figure B-3 was replaced by a 200-mmfd condenser and the length of the oscillogram traces are about 0.8 microsecond. The top two traces of Figure 6 are the voltages across the two probes, indicating that they are nearly alike and hence when added should very nearly cancel one another. The current pulse flowing in the probe resembles one sine-wave loop. Since the voltage across the probe should be nearly the derivative of the current, it should have both a positive and a negative part, as shown in the two top traces of Figure 6. This sum voltage (same as the difference voltage of the preceding paragraphs), when amplified about 30 times, is the third trace of Figure 6. The fourth trace was obtained by putting a 3.5-mil thick copper sheet on one probe, while the fifth trace was that obtained by putting the same copper sheet on the other probe. In the first trace of Figure 7 is shown the sum voltage when one copper sheet is placed on each probe. The second trace of Figure 7

Fig. 6. Voltages across test and standard probes.

Fig. 7. Difference voltage across test and standard probes.

was obtained with one 3-mil stainless steel sheet on the No. 1 probe and with one 3.5-mil copper sheet on the No. 2 probe. The third trace was obtained with both a steel and copper sheet (copper is back of steel) on No. 1 probe and with one copper sheet on the No. 2 probe. The fourth trace is a superposition of both the second and third traces of Figure 7. The difference between the second and third traces of Figure 7 is the result of the reflection from the steel to copper boundary below the steel to air surface. This reflected wave has a maximum, a crossing point and a minimum similar to the theoretical wave of Figure 3. The position of the crossing point in Figure 7 agrees fairly well with the value calculated from $t = 0.367d^2 \mu_1 \sigma_1$ when average values of μ_1 and σ_1 for stainless steel were used.

A number of pulse circuits involving high-vacuum tube amplifiers were tried to combine the two voltages of the probes out of phase so that the difference voltage would be obtained. Unfortunately, the interelectrode capacitance of the tubes slowed the pulse response sufficiently so that these attempts were unsuccessful. Probably any further attempts along this line should be done with distributed amplifiers.

THEORETICAL ANALYSIS

A preliminary analysis of the action of the pulsed eddy currents in metal has appeared in a previous paragraph. This analysis assumed that the input pulse had the form of a mathematical impulse, an assumption that reduced considerably the amount of computational labor. It was shown experimentally, however, that the actual pulses used in the equipment had a shape resembling the loop of a half-period of a sine wave, as shown in Figure 8. Hence it appeared that an analysis using pulses of this shape would be more useful, and such an analysis is given in Appendix C. One result indicated by this analysis is that the difference between the points at which the oscilloscope trace crosses the time axis should be approxi-

Figure 8

Approximate Shape of Actual Pulses.

mately proportional to the reciprocal of the square root of the conductance of the cladding material, if the cladding material is infinitely thick (or at least several times thicker than the skin depth at the frequency $f = 1/T$, where T is the length of the pulse shown in Figure 8). This result has checked pretty well with experimental results and indicates that, in measuring the thickness of the cladding, the conductivity of the cladding should be as nearly the same as is possible. Similar results should also be true for the base metal, although a change in the conductivity of the clad should normally be of more significance than the same change in the base metal.

The first part of Appendix C deals with the reflection from the air-to-metal boundary while the second part considers the reflection from the metal-to-metal boundary. The reflection from the air-to-metal boundary has a crossing point (amplitude is zero) which depends upon the conductivity of the clad material. Almost all of the reflection of the air-to-metal boundary is balanced out by using a bridge with two similar probes, a standard probe and a test probe. The small amount of unbalanced voltage left, however, will have a crossing point (and an amplitude) which depends upon the clad conductivity. Some experimental tests using various different metals on the test probe have verified this conclusion. The crossing point shifted in one direction with the increase of conductivity and in the opposite direction as the conductivity decreased. The amount of shift was very nearly inversely proportional to the square root of the cladding conductivity, as predicted in Appendix C.

The reflection from the metal-to-metal boundary (clad-to-base metal boundary) as analyzed in Part II of Appendix C has a crossing point (and amplitude) which varies with $a_2 = (T_2/T)$ as shown in Figure 9; here $T_2 = d^2\mu_1\sigma_1$ where d is the depth of the cladding material, μ_1 is the permeability and σ_1 the conductivity of the cladding material. Thus the position of the crossing point depends upon the thickness of the cladding material and, hence, the thickness may be determined from the crossing point. It is to be noted that a_2 depends also on the conductivity of the cladding material; thus for accurate measurements of thickness the conductivity variations must be held to a minimum. This presupposes that the cladding material is as uniform as possible. From Figure 9 for maximum sensitivity, i.e., the greatest change in the crossing point for a given variation in a_2 , the value of a_2 should be small.

Fig. 9. Crossing-Point Variation as Function of T_2/T

For a 15-mil thickness of zirconium a value of a_2 of 0.15 indicates that the length of the input pulse should be approximately three microseconds. This has been borne out by experimental evidence in that pulses of the order of 0.3 microsecond have a markedly lower sensitivity. Pulses considerably longer than 3 microseconds would improve the sensitivity slightly but lead to other difficulties, such as a lower repetition rate and consequent less brilliance on the oscilloscope trace. Another probable difficulty is that reflections from deeper metal-to-metal boundaries might become of considerable effect for longer pulse lengths. Hence for the thickness of 15 mils of zirconium, a pulse length of three microseconds seems to be a reasonable compromise.

EXPERIMENTAL WORK

The block diagram of the circuit finally adopted is that shown in Figure 10. The rate generator of the Tektronix Model 517 oscilloscope is used to trigger the pulse circuit, which consists of a thyatron discharging a capacitor through the pulse-shaping network. The pulse then is transmitted through a pulse transformer which has a secondary carefully balanced with respect to ground. The secondary then drives a bridge circuit with the standard probe in one arm and the test probe in another arm. The output of the bridge is amplified by a wide-band amplifier, and then the pulse is displayed on the oscilloscope screen. A length of coaxial cable is used to delay the sweep triggering of the oscilloscope so that any part of the pulse may be examined minutely. Figure 11 shows a schematic diagram of the circuit, Figure 12 shows an approximate equivalent circuit for capacitive discharge, and Figure 13 shows the approximate equivalent circuit of bridge. These figures are discussed in detail in Appendices D and E. Figure 14 shows the experimental set-up.

Fig. 10. Block Diagram of Circuit.

Fig. 11. Schematic Diagram of Circuit

Fig. 12. Approximate Equivalent Circuit for Capacitive Discharge.

Fig. 13. Approximate Equivalent Circuit of Bridge.

Fig. 14. Experimental Set-Up.

Evidently the probe acts somewhat like an autotransformer in that the first few turns of wire closest to the metal are closely coupled to the metal, which in turn acts very much like a single short-circuited turn of wire. The voltage induced across the first few turns produces a current which flows in the whole probe winding. This current then has the effect of producing a much larger voltage across the whole probe winding.

The cross-sectional view of the probes finally adopted is shown in Figure 15. The core material used was the ferromagnetic ceramic Croloy 20. Perhaps for shorter pulse lengths Croloy 70 would be more useful, although the sensitivity would be lower because the permeability is less. Resolution of the probes was found to be roughly proportional to the probe diameter for this particular probe configuration. Slightly smaller diameter probes were built in an attempt to improve resolution, but no significant increase in resolution was observed. Somewhat smaller deflection sensitivity did result, however. Figure 16 is a photograph of one of the probes mounted in a plastic enclosure.

Fig. 15. Cross-Sectional View of a Probe.

Fig. 16. Close-Up View of Probe.

The output of the bridge circuit may be passed directly to an oscilloscope or sent through amplifiers before being passed on to the oscilloscope. The more usual type of wide-band or pulse oscilloscope has a band width in the neighborhood of ten megacycles per second. These oscilloscopes will show the output pulse, although some of the detail usually will be lacking. For nearly complete details of the pulse an extremely wide band-width is needed and this may be obtained by the use of oscilloscopes such as the Tektronix Model 517. For further amplification beyond that contained in the Tektronix oscilloscope, wide-band distributed amplifiers, such as the Hewlett Packard Model 460A or the Spencer Kennedy Laboratory Model 202C, are needed.

Interpretation of the oscilloscope trace to give thickness of the cladding material may be done in many ways. One of the major problems, however, is to make the indication of thickness independent of the spacing between the probe and the surface of the metal. Two successful methods of overcoming the spacing problem have been developed. The first method to be described here is the "constant slope" method. The bridge circuit is balanced as nearly as possible with nearly identical clad plates on both the standard and the test probe. If possible, both plates should be spaced the same distance, say two or three mils, from the probe face. The balance is obtained by varying the 50-ohm potentiometer of the circuit shown in Figure 11, the double 5-micromicrofarad capacitor of the same circuit, and the position of the core of the test probe. Next a slight unbalance is introduced into the test probe by increasing its inductance (moving the core toward the coil). A somewhat idealized and exaggerated wave form of the voltage across the standard probe is shown as the solid line in Figure 17(a), while a similar wave form for the test probe is shown as the dotted line in Figure 17 (a). The difference voltage when amplified has the approximate shape shown as the solid line in Figure 17 (b). The effect of changing the spacing of the test probe from the clad plate is to change the slope of the difference voltage. Thus an increase in the spacing will give the dotted line of Figure 17(b). With the probe-to-plate spacing held as constant as possible, the effect of increasing cladding thickness is shown as the displacement, to the left, of the dotted line of Figure 17(c). The horizontal distance between the two crossing points of Figure 17(c) would then be almost directly proportional to the cladding thickness. The bridge circuit containing the probes amplifies to some extent the shift of the crossing point. This amplification is explained briefly in a following paragraph. Two plates having known cladding thicknesses may be used to calibrate the instrument, and a typical pair of such calibration points is shown as circled points in Figure 18. A clad plate was then measured by the instrument and the plate was then sectioned and the actual cladding thicknesses determined by optical means. The results are indicated by the crosses of Figure 18; a good amount of agreement by the two methods is apparent.

The bridge circuit itself is used in such a manner as to contribute an additional amplification of approximately thirty times. The explanation may be had by reference to Figure 19. Curve A would represent the voltage on both the standard and the test probes when exactly balanced. If now a slight inductive unbalance is given to the test probe, by moving the core of the probe, curve B results, and the difference voltage between the two probes is shown as curve D. The ordinates of curve D have been amplified somewhat. When a change in the cladding thickness occurs the major change is in the resistance of the test probe, and this results in curve C as the voltage across the test probe. The difference voltage between the standard and the test probes is now shown as curve E of

Figure 19. Notice that the shift of the crossing point for the test probe is t_1 seconds while the observed shift for the difference voltage is t_2 seconds. The amplification contributed by the bridge circuit then is the ratio of t_2 to t_1 .

Figure 17

Voltages Across Probe.

Figure 18

Calibration Curve and Test Measurements.

Figure 19

Explanation of Additional Amplification Contributed by Bridge Circuit.

The method described above will measure the thickness of one metal clad on a base metal while reducing the effect of the probe-to-metal spacing variation to a minimum. It suffers from two disadvantages:

1. The spacing between the test probe and the surface of the metal to be tested must be adjusted by the operator so that the slope of the oscilloscope trace has the required value. This is time-consuming and tedious.
2. The thickness of the cladding is observed as a shift of the crossing point on an oscilloscope screen. This does not lend itself to continuous recording.

The second method to be described gives promise of overcoming these disadvantages.

The second method of overcoming the spacing problem involves the use of points on the difference voltage curve which are not affected by the probe-to-metal spacing but are affected by the thickness of the cladding. This difference voltage is shown in the photograph of Figure 20 as a

Fig. 20. Difference Voltage as Function of Metal-to-Probe Spacing.

series of waves, each taken at a different spacing between the probe and the surface of the metal. Notice that there are at least two points at which all of the curves intersect, indicating that these points are independent of the probe-to-metal spacing. An approximate analysis of the probe is carried out in Appendix F to determine where such points should appear. The probe is assumed to have the approximate equivalent circuit shown in Figure 21. The result of the analysis is shown as the intersections A, B, C of the curves of Figure 23 which indicate the times at which points of intersection should occur. This seems to agree quite well with the observed values as shown in Figure 20. The analysis indicates that there should be more points of intersection, but the damping of

the pulse of Figure 20 prevented the other points from being observed. More importantly, the analysis also shows that the slopes of the waves of Figure 20 at the points of intersection vary with the probe-to-metal spacing, while the heights of the points above a zero line should vary with the composition or type of metal tested. Hence, the thickness of one metal clad on another should be measurable by determining the variation of the height of a point of intersection above a zero line.

Fig. 21. Approximate Equivalent Circuit of Probe.

To test this last conclusion, plates of identical base and clad metals but with different thicknesses of cladding were obtained and tested. The results are shown in Figure 22 where the top wave is for 12.6 mils average thickness, the middle one 6 mils and the bottom 23 mils. The variation of the vertical height of the point of intersection is easily discernible in this figure.

Fig. 22 Vertical Shift of Intersection Points With Clad Thickness.

Fig. 23. Theoretical Points of Intersection.

Next, attempts to move the points horizontally were tried in an effort to see if there were positions of greater sensitivity to change in cladding thickness than the points that were used originally. It was found that a capacitor across the primary of the transformer driving the probes would shift the points of intersection somewhat as shown in Figure 24.

Fig. 24. Horizontal Shift of Crossing Points With Capacitance.

This horizontal shift is not nearly as prominent as the vertical shift of Figure 22. For the shift shown, a 40-mmfd capacitor was used. This also appears to be another confirmation of the theory given in Appendix F in that changing the capacitance C of Figure 21 should change the b of equation (F-8) and thus change the position of the points of intersection of Figure 24. The actual shift observed, however, was small and was just sufficient to indicate that the sensitivity was increased slightly by this change. Other possible parameters that could have been varied to produce a horizontal shift are the inductance (L) and resistance (R). These were more difficult to vary satisfactorily and were not tried. If the parameters of the primary or secondary of the driving transformer could

be varied, some horizontal shift of the intersection points should occur.

It was next decided to investigate the fine structure of the first intersection point by increasing the vertical gain approximately ten times and the horizontal sweep rate about the same amount. This led to the photographs of Figure 25, which show at the top the sweep expanded five times and at the bottom the sweep expanded ten times. Notice now that the intersection is no longer one point but is a series of points. The total variation of probe-to-metal spacing was 23 mils for this photograph. It was observed that, if the spacing variation were kept down to 10 mils, the intersection points would effectively coincide for the purpose at hand. It was suspected that the variation of gain and phase shift with frequency of the wide-band amplifiers was probably causing much of the trouble. Several tests seemed to confirm this. Another cause might be amplitude limiting occurring in the wide-band amplifier.

It was found that vertical movement of the intersection points was possible by varying the amount of initial unbalance in the probes. Figure 26 shows the results as the initial unbalance was varied. In the lower exposure, the initial unbalance gave pulse waves in the same direction as that caused by increasing the distance between probe and metal. The top exposure had an initial unbalance in the opposite direction. The slopes of the top exposure may be either positive or negative, whereas for the other two exposures they are always positive. The top exposure then would have the advantage that with its amplitude limiting in the amplifier would be a minimum and the troubles produced by variation of gain and phase shift would be decreased. No horizontal shift occurred.

Fig. 25. Intersection Points - Expanded Sweep.

Fig. 26. Intersection Points With Varying Initial Unbalance.

The measurements were now becoming sufficiently sensitive that a more accurately reproducible positioning of the metal plates was required. A device to accomplish this was built and helped eliminate some of the uncertainties in the above measurements. Also, it was decided to try additional amplification so that the vertical movement of the intersection

points would be large enough for convenient measurement. A second wide-band amplifier (Hewlett-Packard Model 460A) was inserted and it appeared as if the gain was all that was needed. The effects of saturation were magnified, however, and it was evident that the major portion of the saturation was occurring in the final amplifiers of the oscilloscope. Also, noise and hum began to be important and probably would limit the amount of additional amplification possible.

To minimize the effects of saturation, the intersection points were located in the middle of the linear range of the vertical deflection amplifiers. This adjustment was accomplished by varying the balancing resistance of Figure 11. The slope of the trace may best be adjusted to a horizontal position by varying the air gap of the test probe. The resulting wave form is shown in Figure 27. The upper and lower saturation positions are shown as well to indicate how the final amplifiers decreased the available dynamic range. The hum showed up as a ripple on the waveform, while the effect of the noise was to broaden the trace somewhat as compared to the waveform of the preceding figures, which were taken with a great deal less amplification.

Fig. 27. Intersection Points Showing Saturation.

For calibrating the oscilloscope deflection, carefully chosen standard plate was used on the test probe. This plate had a rather thin cladding of zirconium (approximately 7 mils) on one side of the base metal (uranium with 6% niobium) and a rather thick cladding (18 mils) on the other side. Hence, it was ideal for calibration, and the results are shown as the calibration curve of Figure 28. The crosses are the usable points measured near the center of the plate. The extreme end points are those with double

Fig. 28. Calibration Curve.

circles about them and show that the measurements become unreliable when the probe is placed half on the cladding material and half on the solid material, i.e., where the base metal is no longer present. The points circled once are one-quarter of an inch farther in toward the center and indicate that at this point the measurements are all right. An average straight line is drawn through the points and this is the calibration curve relating the cladding thickness to the oscilloscope reading.

A third plate with the same composition of cladding and base metal was measured on the oscilloscope and then sectioned and measured by optical methods. Figure 29 shows the optical measurements as crosses and the oscilloscope measurements as circles. The abscissa is the distance from an arbitrary straight line drawn on the sample. Notice that, with the exception of the extreme ends, agreement is very good. Figure 30 is similar to Figure 29 except that it is for the other side of the plate. Again, agreement is good except for the extreme end points which are off the graph.

The information used in obtaining the above results are contained in a small part of the pulse, about 10 millicroseconds long. It might be

Fig. 29. Profile of Thick Side of Plate.

Fig. 30. Profile of Thin Side of Plate.

possible to construct an electronic sampling circuit to isolate this part of the pulse and have its height measured by an electronic voltmeter designed for pulse measurements. The results would yield a device which could measure the depth of the cladding directly and be almost completely free of the effect of probe-to-metal spacing. In addition the device would be directly reading and could be used to operate a recorder.

APPENDIX A

In a metal, if $\bar{E}$ and $\bar{H}$ are the Laplacian transforms of the electric and magnetic field intensities, respectively,

$$\begin{aligned}\nabla \cdot \bar{E} &= 0 \\ \nabla \cdot \bar{H} &= 0 \\ \nabla \times \bar{H} &= \sigma \bar{E} \\ \nabla \times \bar{E} &= -\mu s \bar{H} \quad ,\end{aligned}\tag{A-1}$$

where σ and μ are the conductivity and permeability of the metal and s is the complex variable of the Laplacian transform. In Figure 1, assume that the plane wave is traveling in the x direction and that the E field is in the y direction while the H field is in the z direction. It is possible to show from (A-1) that

$$\frac{d^2 H_z}{dx^2} = s \mu \sigma H_z \quad .\tag{A-2}$$

Then

$$H_z = A e^{-\tau x} + B e^{+\tau x} \quad ,\tag{A-3}$$

where

$$\tau = \sqrt{s \mu \sigma} \quad .$$

Also

$$E_y = -\frac{1}{\tau} \frac{dH_z}{dx} = \eta [A e^{-\tau x} - B e^{+\tau x}] \quad ,\tag{A-4}$$

where

$$\eta = \frac{\sqrt{s \mu}}{\sigma} \quad .$$

Similarly, in air

$$H_z = A e^{-\frac{sx}{v}} + B e^{+\frac{sx}{v}} \quad ,\tag{A-5}$$

where $v = \frac{1}{\sqrt{\mu \epsilon}}$ and μ and ϵ are the permeability and permittivity of the air.

$$E_y = -\frac{1}{s \epsilon} \frac{dH_z}{dx} = Z_0 \left[A e^{-\frac{sx}{v}} - B e^{+\frac{sx}{v}} \right] ,$$

where

$$Z_0 = \sqrt{\frac{\mu}{\epsilon}} \quad ,$$

In the base metal, assume its depth in the x direction as infinite. Then there is no returning wave so that

$$\begin{aligned} H_z &= Ae^{-\tau_2 x} \\ E_y &= \eta_2 Ae^{-\tau_2 x} \end{aligned} \quad (A-7)$$

If the air-metal surface is taken as $x = 0$, then at $x = d$:

$$\begin{aligned} Ae^{-\tau_2 d} &= A_1 e^{-\tau_1 d} + B_1 e^{+\tau_1 d} \\ \eta_2 Ae^{-\tau_2 d} &= \eta_1 (A_1 e^{-\tau_1 d} - B_1 e^{+\tau_1 d}) \end{aligned}$$

or

$$\begin{aligned} A_1 e^{-\tau_1 d} &= \left(\frac{\eta_1 + \eta_2}{2\eta_1} \right) Ae^{-\tau_2 d} \\ B_1 e^{-\tau_1 d} &= \left(\frac{\eta_1 - \eta_2}{2\eta_1} \right) Ae^{-\tau_2 d} \end{aligned}$$

In the clad metal,

$$\begin{aligned} H_z &= \left[\left(\frac{\eta_1 + \eta_2}{2\eta_1} \right) e^{-\tau_1(x-d)} + \left(\frac{\eta_1 - \eta_2}{2\eta_1} \right) e^{\tau_1(x-d)} \right] Ae^{-\tau_2 d} \\ E_y &= \left[\left(\frac{\eta_1 + \eta_2}{2} \right) e^{-\tau_1(x-d)} - \left(\frac{\eta_1 - \eta_2}{2} \right) e^{\tau_1(x-d)} \right] Ae^{-\tau_2 d} \end{aligned} \quad (A-8)$$

In the air, let the magnetic impulse wave traveling in the $(+x)$ direction have the strength H_0 . Then, from (A-5) and (A-8) at $x = 0$,

$$\begin{aligned} H_0 + B_0 &= [(\eta_1 + \eta_2)e^{\tau_1 d} + (\eta_1 - \eta_2)e^{-\tau_1 d}] \frac{Ae^{-\tau_2 d}}{2\eta_1} \\ Z_0[H_0 - B_0] &= [(\eta_1 + \eta_2)e^{\tau_1 d} - (\eta_1 - \eta_2)e^{-\tau_1 d}] \frac{Ae^{-\tau_2 d}}{2} ; \end{aligned}$$

then

$$\begin{aligned} Ae^{-\tau_2 d} &= \frac{4H_0\eta_1 Z_0}{(\eta_1 + \eta_2)(Z_0 + \eta_1)e^{\tau_1 d} + (\eta_1 - \eta_2)(Z_0 - \eta_1)e^{-\tau_1 d}} \\ B_0 &= \frac{Ae^{-\tau_2 d}}{4\eta_1 Z_0} [(\eta_1 + \eta_2)(Z_0 - \eta_1)e^{\tau_1 d} + (\eta_1 - \eta_2)(Z_0 + \eta_1)e^{-\tau_1 d}] \end{aligned} \quad (A-9)$$

From (A-9) the reflected wave in air from the surface of the metal is

$$B_0 = H_0 \left[\frac{(\eta_1 + \eta_2)(Z_0 - \eta_1)e^{\tau_1 d} + (\eta_1 - \eta_2)(Z_0 + \eta_1)e^{-\tau_1 d}}{(\eta_1 + \eta_2)(Z_0 + \eta_1)e^{\tau_1 d} + (\eta_1 - \eta_2)(Z_0 - \eta_1)e^{-\tau_1 d}} \right]$$

Now let $R_{01} = \frac{Z_0 - \eta_1}{Z_0 + \eta_1}$ = operational reflection factor at the air-to-metal boundary and $R_{12} = \frac{\eta_1 - \eta_2}{\eta_1 + \eta_2}$ = operational reflection factor at the metal-to-metal boundary. Then from (A-10)

$$\begin{aligned} B_0 &= H_0 \left[\frac{R_{01} + R_{12} e^{-2\tau_1 d}}{1 + R_{01} R_{12} e^{-2\tau_1 d}} \right] \\ &= H_0 \left[R_{01} + R_{12}(1 - R_{01}^2) e^{-2\tau_1 d} - R_{01} R_{12}^2 (1 - R_{01}^2) e^{-4\tau_1 d} + \dots \right]. \quad (A-11) \end{aligned}$$

The first term of (A-11) represents the wave reflected from the air-to-metal boundary, as shown in Fig. 2, while the second term represents the wave reflected once from the metal-to-metal boundary. The third term of (A-11) represents the wave reflected twice from the metal-to-metal boundary, and so on. Since the reflection factor R_{12} will normally be small, each term of the series of (A-11) will be small compared to the preceding terms. Hence it is necessary only to consider the first two terms.

The first term is

$$H_0 R_{01} = H_0 \left(\frac{b - \sqrt{s}}{b + \sqrt{s}} \right) \quad (A-12)$$

where

$$b = \sqrt{\frac{\mu \sigma_1}{\epsilon \mu_1}},$$

Using a transform table, the reflected wave from the air-to-metal surface is

$$H_1 = 4 H_0 b^2 \left[\frac{1}{\sqrt{\pi b^2 t}} - e^{b^2 t} \operatorname{erfc}(\sqrt{b^2 t}) \right], \quad (A-13)$$

where t is the time in seconds and where

$$\operatorname{erfc} x = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-x^2} dx.$$

For an ordinary metal $b^2 t$ is a very large number, so that the following asymptotic expansion for $\operatorname{erfc} x$ is useful:

$$\operatorname{erfc} x = \frac{e^{-x^2}}{x\sqrt{\pi}} \left[1 - \frac{1}{2x^2} + \frac{1 \cdot 3}{2^2 x^4} - \dots \right].$$

When two terms of this expansion are used, (A-13) becomes

$$H_1 = \frac{2H_0}{\sqrt{\pi} b t^{3/2}} \quad (A-14)$$

The wave reflected from the air-to-metal boundary is an impulse occurring at $t = 0$ followed by a tail which decreases inversely as $t^{3/2}$.

The second term is

$$H_0 R_{12}(1 - R_{01}^2)e^{-2\tau_1 d} = H_0 \left(\frac{\sqrt{\frac{\mu_1 \sigma_2}{\sigma_1 \mu_2}} - 1}{\sqrt{\frac{\mu_1 \sigma_2}{\sigma_1 \mu_2}} + 1} \right) \frac{4b \sqrt{s_1}}{(b + \sqrt{s})^2} e^{-a \sqrt{s}} \quad (A-15)$$

where

$$a = 2d \sqrt{\mu_1 \sigma_1}.$$

From the transform table, (A-15) gives

$$H_2 = 4H_0 b^2 \left(\frac{\sqrt{\frac{\mu_1 \sigma_2}{\sigma_1 \mu_2}} - 1}{\sqrt{\frac{\mu_1 \sigma_2}{\sigma_1 \mu_2}} + 1} \right) \left[\frac{e^{-(v^2/\mu)}}{\sqrt{\pi \mu}} (2\mu + 1) - 2(1 + v + \mu) e^{(2v + \mu)} \operatorname{erfc} \left(\frac{v + u}{\sqrt{u}} \right) \right] \quad (A-16)$$

where

$u = b^2 t$ and $v = d \sigma_1 \sqrt{\frac{\mu}{\epsilon}}$. Since both u and v are large, the asymptotic expansion for $\operatorname{erfc} x$ may again be used to give for (A-16):

$$H_2 = 4H_0 b^2 \left(\frac{\sqrt{\frac{\mu_1 \sigma_2}{\sigma_1 \mu_2}} - 1}{\sqrt{\frac{\mu_1 \sigma_2}{\sigma_1 \mu_2}} + 1} \right) \left[\frac{\sqrt{u} e^{-(v^2/u)} (v^2 - u/2)}{(v + u)^3 \sqrt{\pi}} \right] \quad (A-17)$$

$$= \frac{4 H_0 b^2}{(\frac{\mu}{\epsilon})^{3/2} d^3 \sigma_1^3 \sqrt{\pi}} \left(\frac{\sqrt{\frac{\mu_1 \sigma_2}{\sigma_1 \mu_2}} - 1}{\sqrt{\frac{\mu_1 \sigma_2}{\sigma_1 \mu_2}} + 1} \right) e^{-\frac{d^2 \mu_1 \sigma_1}{t}} \left[\left(\frac{d^2 \mu_1 \sigma_1}{t} \right)^{5/2} - \frac{1}{2} \left(\frac{d^2 \mu_1 \sigma_1}{t} \right)^{3/2} \right]$$

Since the cathode-ray oscillograph connected across the probe coil will have indications proportional to the voltage across the coil, and is hence proportional to the derivative of the magnetic field, the observed voltages will be proportional to

$$\frac{dH_1}{dt} = \frac{3H_0}{\sqrt{\pi} b t^{5/2}} \quad (A-18)$$

and

$$\frac{dH_2}{dt} = \frac{4H_0b^2}{\left(\frac{\mu}{\epsilon}\right)^{3/2} d^5 \sigma_1^4 \mu_1 \sqrt{\pi}} \left(\frac{\sqrt{\frac{\mu_1 \sigma_2}{\sigma_1 \mu_2}} - 1}{\sqrt{\frac{\mu_1 \sigma_2}{\sigma_1 \mu_2}} + 1} \right) e^{-\frac{d^2 \mu_1 \sigma_1}{t}} \left[\left(\frac{d^2 \mu_1 \sigma_1}{t} \right)^{9/2} - 3 \left(\frac{d^2 \mu_1 \sigma_1}{t} \right)^{7/2} + \frac{3}{4} \left(\frac{d^2 \mu_1 \sigma_1}{t} \right)^{5/2} \right] \quad (A-19)$$

Again the voltage indication (dH_1/dt) of the wave reflected from the air-to-metal boundary is an impulse at $t = 0$ with a tail which decreases inversely proportional to $t^{5/2}$. The voltage indication (dH_2/dt) of the wave reflected from the metal-to-metal boundary is a pulse with the shape shown in Fig. 3.

Notice that this pulse has a maximum at $\left(\frac{t}{d^2 \mu_1 \sigma_1}\right) = 0.181$, a zero at $\left(\frac{t}{d^2 \mu_1 \sigma_1}\right) = 0.367$, and a minimum at $\left(\frac{t}{d^2 \mu_1 \sigma_1}\right) = 0.560$. There is another zero at $\left(\frac{t}{d^2 \mu_1 \sigma_1}\right) = 3.633$ and another maximum at $\left(\frac{t}{d^2 \mu_1 \sigma_1}\right) = 5.259$, but when t is this large, the amplitude is so small as to be negligible compared to the first maximum. Notice in particular that the first crossing point (or zero) has $t = 0.367 d^2 \mu_1 \sigma_1$. For a finite pulse length there will still be a crossing point with the time t proportional to the square of the cladding thickness, d . Notice also that (dH_2/dt) is proportional to the reflection factor

$$\left(\frac{\sqrt{\frac{\mu_1 \sigma_2}{\sigma_1 \mu_2}} - 1}{\sqrt{\frac{\mu_1 \sigma_2}{\sigma_1 \mu_2}} + 1} \right) \text{ which is zero only if the base metal and coating metal have the same conductivity and permeability constants.}$$

For zirconium as the coating metal and uranium as the base metal, $\sigma_1 = 2.22 \times 10^6 \text{ mhos/meter}$; $\sigma_2 = 3.47 \times 10^6 \text{ mhos/meter}$.

Thus

$$\left(\frac{\sqrt{\frac{\mu_1 \sigma_2}{\sigma_1 \mu_2}} - 1}{\sqrt{\frac{\mu_1 \sigma_2}{\sigma_1 \mu_2}} + 1} \right) = 0.111 \quad (A-20)$$

In order to use the (dH_2/dt) wave to determine d , it would be necessary to cancel out the (dH_1/dt) wave which is large enough to mask (dH_2/dt). An indication of how difficult this cancellation might be may be obtained from the ratio

$$\frac{dH_2}{dt} = - \left(\frac{\sqrt{\frac{\mu_1 \sigma_2}{\mu_2 \sigma_1}} - 1}{\sqrt{\frac{\mu_1 \sigma_2}{\mu_2 \sigma_1}} + 1} \right) e^{-\left(\frac{d^2 \mu_1 \sigma_1}{t}\right)} \left[\frac{4}{3} \left(\frac{d^2 \mu_1 \sigma_1}{t} \right)^2 - 4 \left(\frac{d^2 \mu_1 \sigma_1}{t} \right) + 1 \right] \quad (A-21)$$

A plot of (A-21) is shown in Fig. 4 and indicates that the wave reflected from the metal-to-metal boundary is small compared to the wave reflected from the air-to-metal boundary, especially when $(\frac{t}{d^2 \mu_1 \sigma_1})$ is small. Since

one important point would be the crossing point at $\frac{t}{d^2 \mu_1 \sigma_1} = 0.367$, the curve of Fig. 4 indicates that the H_1 wave should be at least cancelled by a factor

of $1000 \left(\frac{\sqrt{\frac{\mu_1 \sigma_2}{\sigma_1 \mu_2}} + 1}{\sqrt{\frac{\mu_1 \sigma_2}{\sigma_1 \mu_2}} - 1} \right)$. For zirconium coated on uranium, this factor is of

the order of 10,000. Hence the H_1 wave should be cancelled out by some form of bridge circuit by a factor of 10,000 to one. If this were true it is believed that the crossing point of the H_2 wave would be sufficiently good so that it could be used as a measure of the depth d . The factor of 10,000 to one is also an indication of how sensitive the detecting device must be, because the input impulse and the pulse reflected from the air-to-metal boundary must be of the same order of magnitude. Hence if 100 volts are placed upon the probe the detector should be able to detect at least 0.01 volt.

As an indication of the time axis necessary, let it be assumed that the coating of zirconium is 15 mils thick. The time for the crossing point is

$$\begin{aligned} t &= 0.367 d^2 \mu_1 \sigma_1 \\ &= 0.367 (3.81 \times 10^{-4})^2 (4\pi \times 10^{-7}) (2.22 \times 10^6) \\ &= 0.1487 \times 10^{-6} \text{ second.} \end{aligned} \tag{A-22}$$

APPENDIX B

All of the circuits given below use an 884 thyatron tube as the driver pulse tube. The trigger pulse for the thyatron was taken from the Rate Generator Output B of the Tektronix Type 517 oscilloscope. The complete plate and grid circuits of the 884 tube are shown in the first diagram only, but are similar for the other four diagrams. The repetition rate of the pulses was varied between five hundred and ten thousand per second without any change being noticed in the output waveforms. The Type 517 oscilloscope was found to be the most satisfactory oscilloscope available for viewing the short pulses which appear as the output of these circuits.

First Circuit:

Fig. B-1

A and B are connected directly to the vertical plates of an oscilloscope. The sensitivity is low, but it could be improved by designing an amplifier to amplify the difference voltage between A and B before applying it to the oscilloscope. If such an amplifier were made, this would be the simplest and best circuit of all. Two 884 tubes could be used to drive the probes, but this necessitates matching the tubes, which is troublesome.

Second Circuit:

Fig. B-2

The above circuit uses probes each of which has both a primary and secondary. The difficulties are to reduce the interwinding capacitance of the probes and to balance the primary and secondaries of the probes.

Third Circuit:

Fig. B-3

This circuit worked well and is easy to construct. Difficulties arise because of the unbalanced capacitances of the tube and of the rest of the circuit.

Fourth Circuit:

Fig. B-4

The difficulty here was to eliminate the effect of the interwinding capacitance of the output transformer.

Fifth Circuit:

Fig. B-5

Again the difficulties are the presence of the interwinding capacitance of the transformer and the problem of transferring enough power to the probes.

APPENDIX C

ANALYSIS OF THE REFLECTION FROM THE METALPart I Air-to-Metal Boundary

If, in equation (A-12) of Appendix A, a step input magnetic field intensity is employed, the first term of the series representing the reflected field of the clad metal is

$$L [H_1(t)] = \frac{B}{s} R_{01} = \frac{B}{s} \left(\frac{b - \sqrt{s}}{b + \sqrt{s}} \right), \quad (C-1)$$

where B is the amplitude of the input step magnetic field intensity $H_0(t)$. Then the corresponding time function is

$$H_1(t) = L^{-1} \left(\frac{B R_{01}}{s} \right) = B \left\{ 1 - 2\epsilon(t/T_1) \operatorname{erfc} \sqrt{\frac{t}{T_1}} \right\} u(t), \quad (C-2)$$

where $u(t)$ is the unit step function $= \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases}$,

and $T_1 = \left(\frac{1}{b^2} \right) = \frac{\mu_1 \epsilon}{\mu \sigma_1}$. The quantities μ_1 and σ_1 are the permeability and the conductivity of the clad layer of the metal, while μ and ϵ are the permeability and permittivity of the air.

The quantity $(H_0 + H_1)$ then represents the original applied field plus that reflected from the air-to-metal boundary under the conditions of the assumed plane wave type of solution. By the use of (C-2) it is possible to show that $(H_0 + H_1)$ is zero at time $t = 0$ and then increases to B at $t = 0.6 T_1$ approximately and 1.5 B at $t = 5T_1$. As the time becomes very large, $(H_0 + H_1)$ approaches 2 H_0 . Since for metals T_1 is of the order of 10^{-18} sec, this initial rise of the field is extremely rapid. The last part of the rise is much slower and may be calculated much more easily from the asymptotic expression

$$H_0 + H_1 = 2B \left\{ 1 - \sqrt{\frac{T_1}{t\pi}} \left[1 - \frac{1}{2} \left(\frac{T_1}{t} \right) + \frac{1.3}{2^2} \left(\frac{T_1}{t} \right)^2 - \frac{1.3 \cdot 5}{2^3} \left(\frac{T_1}{t} \right)^3 \right] \right\} u(t). \quad (C-3)$$

Thus at time $t = 1000 T_1$, $(H_0 + H_1)$ is 1.96 B. For values of time larger than $1000 T_1$, only two terms of (C-3) need be used for most purposes.

Now let it be assumed that the input magnetic field intensity $H_0(t)$ has the half sinusoidal loop shown in Fig. 8, with the equation

$$\begin{aligned} H_0(t) &= H_m \sin \left(\pi \frac{t}{T} \right) \text{ when } 0 < t < T \\ &= 0 \text{ when } T < t \text{ and } t < 0. \end{aligned} \quad (C-4)$$

Also assume that the voltage $v(t)$ appearing across the pick-up coil is the derivative of the sum of the input wave and the reflected wave from the air-to-metal boundary. Then

$$\begin{aligned} v(t) &= \frac{d}{dt} \left\{ u(0) A(t) + \int_0^t u'(t - \tau) A(\tau) d\tau \right\} \\ &= u(0) A(t) + u'(0) A(t) + \int_0^t u'(t - \tau) A(\tau) d\tau, \end{aligned} \quad (C-5)$$

where $u(t)$ is the input voltage and $A(t)$ is the response of the system to a unit step voltage. Hence $u(t) = H_0(t)$ as given in (C-4) and

$$A(t) = 2 \left[1 - \epsilon^{(t/T_1)} \operatorname{erfc} \sqrt{\frac{t}{T_1}} \right] u(t), \quad (C-6)$$

as determined from (C-2) and (C-3).

For the air-to-metal boundary the voltage developed from the reflected wave is

$$\begin{aligned} v_a(t) &= \left(\frac{\pi}{T} \right) H_m A(t) - \left(\frac{\pi}{T} \right)^2 H_m \left[\left(\sin \frac{\pi t}{T} \right) \int_0^t \left(\cos \frac{\pi t}{T} \right) A(t) dt \right. \\ &\quad \left. - \left(\cos \frac{\pi t}{T} \right) \int_0^t \left(\sin \frac{\pi t}{T} \right) A(t) dt \right]. \end{aligned} \quad (C-7)$$

Now using the first two terms of the asymptotic series of (C-3) for $A(t)$

$$\begin{aligned} \int_0^t \left(\cos \frac{\pi t}{T} \right) A(t) dt &\approx 2 \int_0^t \left(\cos \frac{\pi t}{T} \right) \left(1 - \sqrt{\frac{T_1}{t\pi}} \right) dt \\ &\approx \frac{2T}{\pi} \left(\sin \frac{\pi t}{T} \right) - \sqrt{\frac{8T_1 T}{\pi}} \left(C \frac{\pi t}{T} \right). \end{aligned} \quad (C-8)$$

Similarly

$$\int_0^t \left(\sin \frac{\pi t}{T} \right) A(t) dt \approx \frac{2T}{\pi} \left(1 - \cos \frac{\pi t}{T} \right) - \sqrt{\frac{8T_1 T}{\pi}} S \left(\frac{\pi t}{T} \right), \quad (C-9)$$

where

$$C(z) = \sqrt{\frac{2}{\pi}} \int_0^{\sqrt{z}} \cos V^2 dV$$

$$S(z) = \sqrt{\frac{2}{\pi}} \int_0^{\sqrt{z}} \sin V^2 dV.$$

Tables of these functions are available.

Then from (C-7), (C-8) and (C-9) and putting

$$x_1 = (t/T), \quad a_1 = (T_1/T) \quad ,$$

there is obtained

$$v_a(t) = \frac{2\pi H_m}{T} \left\{ \cos \pi x_1 + \sqrt{a_1} \left[\frac{1}{\sqrt{\pi x_1}} + \sqrt{2\pi} (\sin \pi x_1) C(\pi x_1) \sqrt{2} (\cos \pi x_1) S(\pi x_1) \right] \right\}. \quad (C-10)$$

Since $a_1 \ll 1$, the crossing point of $v_a(t)$ is approximately that of $\cos \pi x_1$, i.e., $x_1 = 0.5$. It is also possible to show that the shift in the point where $v_a(t)$ crosses the t (or x_1) axis is very nearly proportional to the change in $\sqrt{a_1}$. Since $a_1 = \left(\frac{1}{T}\right)\left(\frac{\epsilon}{\mu}\right)\left(\frac{\mu_1}{\sigma_1}\right)$, this shift in the crossing point is inversely proportional to the square root of the cladding conductivity and hence can be used as a measure of the conductivity of the cladding. The results of this first part are true whether or not there is a base metal below the cladding.

Part II. Metal-to-Metal Boundary

Assume now that there is a base metal with the clad-to-base boundary located a depth d below the surface of the cladding. Then, as in (C-1), the second term of the series representing the reflected field of the clad metal is

$$L \left[H_2(t) \right] = \frac{B}{s} e^{-t_1(2d)} R_{12} (1 - R_{01}^2) \quad (C-11)$$

where

$$R_{12} = \frac{\sqrt{\frac{\mu_1}{\sigma_1}} - \sqrt{\frac{\mu_2}{\sigma_2}}}{\sqrt{\frac{\mu_1}{\sigma_1}} + \sqrt{\frac{\mu_2}{\sigma_2}}}$$

$$1 - R_{01}^2 = \frac{4 \sqrt{\frac{s}{T_1}}}{\left(s \sqrt{\frac{1}{T_1}} \right)^2}$$

$$T_1 = \sqrt{s \mu_1 \sigma_1}$$

and μ_2 and σ_2 are, respectively, the permeability and conductivity of the base metal. Then

$$L [H_2(t)] = \frac{4B R_{12}}{T_1} \frac{e^{-2\sqrt{s} T_2}}{\sqrt{s} \left(\sqrt{s} \frac{1}{\sqrt{T_1}} \right)^2}, \quad (C-12)$$

where $T_2 = d^2 \mu_1 \sigma_1$. Now, using the inverse transform,

$$H_2(t) = 8 B R_{12} \left[\sqrt{\frac{t}{\pi T_1}} e^{-\frac{T_2}{t}} - \left(\sqrt{\frac{T_2}{T_1}} \frac{t}{T_1} \right) e^{-\left(2 \sqrt{\frac{T_2}{T_1}} \frac{t}{T_1} \right)} \operatorname{erfc} \left(\sqrt{\frac{T_2}{T_1}} \sqrt{\frac{t}{T_1}} \right) \right] \quad (C-13)$$

An asymptotic expansion of $H_2(t)$ is

$$H_2(t) \approx 8 B R_{12} \sqrt{\frac{t}{\pi T_1}} e^{-\frac{T_2}{t}} \left[\frac{1}{2 \left(\sqrt{\frac{T_2}{t}} \sqrt{\frac{t}{T_1}} \right)^2} - \frac{1.3}{2^2 \left(\sqrt{\frac{T_2}{t}} \sqrt{\frac{t}{T_1}} \right)^4} \right] \quad (C-14)$$

Now the quantity $\left(\sqrt{\frac{T_2}{t}} \sqrt{\frac{t}{T_1}} \right)$ has its minimum value at $t = \sqrt{T_1 T_2}$ and this minimum is $2(T_2/T_1)^{\frac{1}{4}}$. Since $2(T_2/T_1)^{\frac{1}{4}}$ is of the order of 10^3 for the cases to be considered, only the first term of (C-14) need be used. Also for t in the neighborhood of 10^{-6} sec, $\sqrt{\frac{t}{T_1}}$ is much larger than $\sqrt{\frac{T_2}{t}}$. Hence,

$$H_2(t) \approx \left(4 B R_{12} \sqrt{\frac{T_1}{\pi T_2}} \right) \sqrt{\frac{T_2}{t}} e^{-\frac{T_2}{t}} \quad (C-15)$$

This value of $H_2(t)$ increases quickly to a maximum of $0.857 \left(4 B R_{12} \sqrt{\frac{T_1}{\pi T_2}} \right)$

at a time $t = 2T_2$ and then has a long tail which decreases to $0.470 \left(4 B R_{12} \sqrt{\frac{T_1}{\pi T_2}} \right)$

when $t = 16 T_2$. This wave represents then the reflection from the metal-to-metal surface when a step wave is suddenly applied to the surface of the clad metal.

Following the method of equations (C-5), (C-6) and (C-7),

$$A(t) \approx 4 R_{12} \sqrt{\frac{T_1}{\pi T_2}} \sqrt{\frac{T_2}{t}} e^{-\frac{T_2}{t}}, \quad (C-16)$$

and the voltage developed from the metal-to-metal reflection is

$$v_m(t) = \frac{4\pi}{T} H_m R_{12} \left\{ \sqrt{\frac{T_1}{\pi T_2}} \sqrt{\frac{T_2}{t}} e^{-\frac{T_2}{t}} - \frac{2}{T} \sqrt{T_1 T_2} \pi \left[\left(\sin \frac{\pi t}{T} \right) \int_{\sqrt{\frac{T_2}{t}}}^{\infty} \frac{e^{-z^2}}{z^2} \left(\cos \frac{\pi T_2}{T z^2} \right) dz - \left(\cos \frac{\pi t}{T} \right) \int_{\sqrt{\frac{T_2}{t}}}^{\infty} \frac{e^{-z^2}}{z^2} \left(\sin \frac{\pi T_2}{T z^2} \right) dz \right] \right\} . \quad (C-17)$$

Now let $x_1 = (t/T)$, $a_1 = T_1/T$, $a_2 = (T_2/T)$; then

$$v_m(t) = \frac{4\pi}{T} H_m R_{12} \sqrt{a_1} \left\{ \frac{e^{-(a_2/x_1)}}{\sqrt{\pi x_1}} - 2 \sqrt{\pi d_2} \left[\left(\sin \pi x_1 \right) \int_{\sqrt{\frac{a_2}{x_1}}}^{\infty} \frac{e^{-z^2}}{z^2} \left(\cos \frac{\pi a_2}{z^2} \right) dz - \left(\cos \pi x_1 \right) \int_{\sqrt{\frac{a_2}{x_1}}}^{\infty} \frac{e^{-z^2}}{z^2} \left(\sin \frac{\pi a_2}{z^2} \right) dz \right] \right\} . \quad (C-18)$$

Notice that the part of the expression (C-18) with the braces depends only on the values of a_2 and x_1 . For a fixed value of a_2 , $v_m(t)$ increases from zero as t increases, then decreases, crosses the time axis and becomes negative. It was proposed to use this crossing point as a measure of the thickness of the cladding. Hence the function

$$\begin{aligned}
f(a_2, x_1) &= \frac{1}{\sqrt{\pi x_1}} e^{-(a_2/x_1)} - 2\sqrt{\pi a_2} \int_{\sqrt{\frac{a_2}{x_1}}}^{\infty} \frac{e^{-z^2}}{z^2} \sin \pi \left(x_1 - \frac{a_2}{z^2} \right) dz \\
&= \frac{1}{\sqrt{\pi x_1}} e^{-(a_2/x_1)} - \sqrt{\pi a_2} \int_0^{\frac{x_1}{a_2}} \frac{e^{-(1/u)}}{\sqrt{u}} \sin \pi(x_1 - a_2 u) du
\end{aligned}
\tag{C-19}$$

was studied to determine its crossing point as a function of a_2 . The services of an electronic analog computer (REAC) were utilized to obtain a curve of x_1 for the crossing point as a function of a_2 . This curve is shown in Fig. 9 and indicates that the crossing point changes faster (higher sensitivity) for the smaller values of a_2 .

As an example for a 15-mil clad thickness of zirconium,

$$T_2 = d^2 \mu_1 \sigma_1 = (3.81 \times 10^{-4})^2 (4\pi \times 10^{-7}) (2.22 \times 10^6) = 0.407 \times 10^{-6} \text{ second.} \tag{C-20}$$

If high sensitivity is desired, take $a_2 = 0.15 = (T_2/T)$. Then $T = 2.71 \times 10^6$ second. Hence the length of the input pulse should be approximately three microseconds, which is of the order of ten times longer than was indicated by the previous analysis.

APPENDIX D

ANALYSIS OF PULSE CIRCUIT

The approximate equivalent circuit is shown in Figure 12. The capacitor C is assumed to have an initial voltage $e(0)$. Then if $E = L(e)$,

$$CsE - Ce(0) \frac{E}{R} - \frac{E}{Ls} = 0 \quad (D-1)$$

or

$$E = e(0) \frac{s}{(s + a)^2 + B^2} \quad (D-2)$$

where

$$a = \frac{1}{2RC}$$

and

$$B = \sqrt{\frac{1}{LC} - a^2}$$

Since $e(t) = L^{-1}(E)$,

$$e(t) = e(0) e^{-at} \left(\cos Bt - \frac{a}{B} \sin Bt \right) \quad (D-3)$$

Equation (D-3) represents the oscillatory case with the resistor R large in value.

If R is made small enough, the critically damped condition is approached. For this condition $R = \frac{1}{2} \sqrt{\frac{L}{C}}$. It was found experimentally that R should be about four times as large as this critical value or $R \approx 2 \sqrt{\frac{L}{C}}$.

This value of R is large enough to allow the circuit to be oscillatory and still will be small enough to allow the voltage to die away with time. The time T of the pulse length will be approximately $T = (T/B)$. The maximum current in the capacitor will be approximately $[e(0)/BL]$, and will be limited principally by the rated values of maximum current and voltage of the thyatron tube. The average tube current and the time T will determine the repetition rate of the pulses.

As an example, assume that an 884 tube is to be used, for which 300 milliamperes is the peak current and 75 milliamperes is the maximum average current. Also, take $T = 2$ microseconds and $e(0) = 225$ volts.

Then

$$B \approx \frac{\pi}{T} \approx 1.6 \times 10^6$$

$$\text{peak current} = 0.3 \approx \frac{225}{1.6 \times 10^6 L}$$

or

$$L \approx 470 \times 10^{-6} \text{ henry}$$

$$B \approx \frac{1}{\sqrt{LC}}$$

$$C \approx \frac{1}{(1.6 \times 10^6)^2 470 \times 10^{-6}} = 830 \times 10^{-12} \text{ farad}$$

$$R \approx 2 \sqrt{\frac{L}{C}} = 2 \sqrt{\frac{470 \times 10^{-6}}{830 \times 10^{-12}}} = 1500 \text{ ohm}$$

Assume a repetition rate of 1000 times a second, which should give ample time for the transient voltages to die away before the next pulse appears. Then

$$\begin{aligned} \text{the average tube current} &\approx \frac{2}{\pi} (\text{maximum tube current}) T (\text{repetition rate}) \\ &\approx \frac{2}{\pi} (0.3) (2 \times 10^{-6}) (1000) \\ &\approx 0.4 \times 10^{-3} \text{ ampere,} \end{aligned}$$

which is well below the maximum value of 75 milliamperes.

It should be understood that these values could be used as a guide and the actual circuit values might best be determined experimentally. This was done for the values given in Figure 11.

APPENDIX E

ANALYSIS OF THE BRIDGE CIRCUIT

By use of Thevenin's theorem any bridge circuit, such as that of Figure 13(a), may be changed into that of Figure 13(b). In this application Z_1 is the same kind of impedance as Z_2 and similarly for the pair Z_3 and Z_4 . Thus

$$Z_2 = K_1 Z_1 \quad ; \quad Z_4 = K_3 Z_3 \quad ,$$

where the K 's are real positive numbers and

$$E_i' = E_i \left[\frac{Z_2 Z_3 - Z_1 Z_4}{(Z_1 + Z_2)(Z_3 + Z_4)} \right] = E_i \left[\frac{K_1 - K_3}{(1 + K_1)(1 + K_3)} \right] .$$

Hence the applied voltage in Figure 13(b) has the same shape as the applied voltage of Figure 13(a).

The actual bridge circuit has two resistors which are combined to make R_1 , two probes which are combined to give R , C , and L in parallel, and the output impedance which is the input impedance of an oscilloscope. For most practical purposes this last impedance can be taken as that of a capacitance C_1 . The resulting equivalent circuit is shown in Figure 13(c). The output voltage E_0 would be

$$E_0 = \frac{\frac{1}{R_1 C_1} \left(S^2 + \frac{S}{RC} + \frac{1}{LC} \right)}{S^3 + a_2 S^2 + a_1 S + a_0} \quad , \quad (E-1)$$

where

$$a_2 = \frac{1}{RC} + \frac{1}{R_1 C_1} + \frac{1}{R_1 C}$$

$$a_1 = \frac{1}{LC} + \frac{1}{R_1 C_1 RC}$$

$$a_0 = \frac{1}{LCR_1 C_1} .$$

If it were possible to cancel the quadratic term in the numerator with a corresponding term in the denominator, then

$$\frac{E_0}{E_i'} = \frac{1}{R_1 C_1 (S+b)} \quad , \quad (E-2)$$

where b is a real number. This would mean that E_0 would have a decay of the form e^{-bt} as compared to E_1' . If b is sufficiently small, E_0 and E_1' have practically the same shape. Equation (E-2) would be true providing that

$$b + \frac{1}{RC} = a_2 = \frac{1}{RC} + \frac{1}{R_1 C_1} + \frac{1}{R_1 C}$$

$$\frac{b}{RC} + \frac{1}{LC} = a_1 = \frac{1}{LC} + \frac{1}{R_1 C_1 RC}$$

$$\frac{b}{LC} = a_0 = \frac{1}{LCR_1 C_1} \quad .$$

It is impossible to satisfy these three conditions exactly, but a good approximation will result if $C \gg C_1$. Hence the capacitance C shunting the probe should be at least ten times as large as the input capacitance of the oscilloscope. Then $b = \frac{1}{R_1 C_1}$ and this, when multiplied by the length of the pulse T , should be somewhere in the neighborhood of unity. Hence $\frac{T}{R_1 C_1} \approx 1$ or $R_1 C_1 \approx T$. Actually it is difficult in practice to make R_1 sufficiently large without increasing the shunt capacitance across R_1 excessively. It has been suggested that using capacitors in place of the resistors represented by R_1 might help.

Since it is not possible to make the quadratic factors cancel out, the next best thing is to make their resonant frequencies as high as possible and the damping sufficiently large so that the oscillations die out quickly.

Then, using $\left(s^2 + \frac{S}{RC} + \frac{1}{LC}\right)$ as the quadratic factor, the damping factor is $a = \frac{1}{2RC}$ and the natural frequency of oscillation is $f_M = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - a^2}$.

The product of the damping factor and the pulse length T should be equal to or somewhat greater than one or $T \geq 2RC$. Also the natural frequency should be somewhat higher than the fundamental frequency of the pulse or $f_n > \frac{1}{T}$.

As an example, assume that T is two microseconds and $C = 50$ micro-microfarads; then $R \leq (T/2C)$ or $R \leq 20,000$ ohms. Take $R = 20,000$ ohms. Then $a = 5 \times 10^6$, and

$$f_n = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - (5 \times 10^6)^2} > \frac{1}{2 \times 10^{-6}}$$

or

$$\frac{1}{LC} > 27.5 \times 10^{12}$$

Take

$$\frac{1}{LC} = 100 \times 10^{12};$$

then

$$L = 200 \text{ microhenries.}$$

Since $C_1 \ll C$, take $C_1 = 5$ micromicrofarads. Then $R_1 \approx \frac{T}{C_1} = 4 \times 10^5$ ohms.

Actually this value is much too high for the shunt capacitance and hence a value of $R_1 = 50,000$ ohms might be used. If this is done a reduction in R should also be made to about 500 ohms. Each probe itself then would appear to be a parallel combination of 1000 ohms resistance, 400 microhenries inductance, and 100 micromicrofarads capacitance.

A useful method of checking the frequency response of the bridge network is to put $s = j\omega = j2\pi f$ and plot the magnitude of the numerator of (E-1) and also the magnitude of the denominator. The frequency above which there is a significant departure from the d-c (zero frequency) value indicates approximately the band-width of the circuit. This band-width should be several times greater than the fundamental pulse frequency $f = \frac{1}{T}$.

APPENDIX F

POSITION OF INTERSECTION POINTS

The probe is assumed to have the approximate equivalent circuit of Figure 22, where e is the voltage across the probe and i is the current through the circuit. The circuit constants are: C for the shunting capacitance, R for the series resistance and L for the inductance. Assume also that the input current pulse i is an impulse of area Q coulombs. Then the Laplacian transform of the current is

$$L(i) = Q \quad (F-1)$$

The voltage transform is

$$\begin{aligned} L(e) &= L(i) \frac{(R + sL)(1/sC)}{R + sL + (1/sC)} \\ &= \frac{Q}{C} \frac{s + 2a}{(s + a)^2 + B^2} \quad (F-2) \end{aligned}$$

where

$$a = (R/2L), \quad B = \sqrt{(1/LC) - a^2}$$

Then

$$e = \frac{Q}{C} \left(e^{-at} \right) \left(\frac{a}{b} \sin Bt + \cos Bt \right) \quad (F-3)$$

Now the effect of decreasing the spacing between the probe and the metal is to decrease the inductance L of the probe and increase the resistance R of the probe. Let it be assumed then that the decrease in L for a given small change in spacing is λ and that the corresponding increase in R is $K\lambda$, where K is a constant. The problem then is to show that there are points whose amplitude remains essentially constant as λ changes. This means that the value of time at which $\frac{de}{d\lambda} = 0$ has to be found, or

$$\frac{de}{d\lambda} = \frac{\partial e}{\partial a} \frac{\partial a}{\partial \lambda} + \frac{\partial e}{\partial B} \frac{\partial B}{\partial \lambda} = 0 \quad (F-4)$$

From (F-3):

$$\begin{aligned} \frac{\partial e}{\partial a} &= \frac{Qe^{-at}}{CB} [\sin Bt - Bt \left(\frac{a}{b} \sin Bt + \cos Bt \right)] \\ \frac{\partial e}{\partial B} &= \frac{Qe^{-at}}{CB} [-Bt \sin Bt + \frac{a}{B} (Bt \cos Bt - \sin Bt)] \quad (F-5) \end{aligned}$$

Now

$$a = \frac{R}{2L} = \frac{R_0 + K\lambda}{2(L_0 - \lambda)}$$

$$B = \sqrt{(1/LC) - a^2} = \sqrt{\frac{1}{C(L_0 - \lambda)} - a^2}, \quad (F-6)$$

where R_0 and L_0 are the probe resistance and inductance before the probe-to-metal spacing is changed slightly. From (E-6)

$$\frac{\partial a}{\partial \lambda} = \frac{R_0 + KL_0}{2(L_0 - \lambda)^2}$$

$$\frac{\partial B}{\partial \lambda} = \frac{1}{2B(L_0 - \lambda)^2} \left[\frac{1}{C} - a(R_0 + KL_0) \right] \quad (F-7)$$

Using (F-4), (F-5) and (F-7) it is found that

$$(\sin Bt - Bt \cos Bt) \left[\frac{R_0 + KL_0}{B(L_0 - \lambda)} - \frac{a}{b} \right] - Bt \sin Bt = 0 \quad (F-8)$$

Let $x = Bt$ and $b = \left[\frac{R_0 + KL_0}{B(L_0 - \lambda)} - \frac{a}{B} \right]$, then (F-8) becomes

$$b = \frac{1}{\frac{1}{\chi} - \cot \chi} \quad (F-9)$$

A sketch of b as a function of $\chi = Bt$ is shown in Figure 23. For a given value of b , say b_1 , the intersection of the b_1 ordinate (dotted line) with the solid curve would be the solution of (F-9). Such solutions are shown as points A, B, C, D, E of Figure 23. The solutions corresponding to A and B may be observed in Figure 22. According to the solution, A should be near $Bt = (\pi/2)$, while B should be near $Bt = (3\pi/2)$. This appears to be true in Figure 22. In almost all cases b will be a positive number. Evidently, there is too much damping to make points C, D, E, ... appear in Figure 22, although by shunting a capacitor across the primary of the transformer driving the probes, intersections C and D were observed experimentally.

From (F-3), the time derivative of the voltage e is

$$\frac{de}{dt} \frac{Q}{BC^2(L_0 - \lambda)} e^{-at} \sin Bt \quad (F-10)$$

Hence, the slope of the curves passing through the crossing points will vary with the spacing between the probe and the metal. The change of this slope as indicated by (F-10) is small, but after passing through the balancing

part of the circuit the change in slope should be increased. The variation in slope with probe-to-metal spacing is easily observable in Figure 20.

If now the constants of the probe are changed in some other manner than that dictated by the variation in probe-to-metal spacing, the amplitude of the crossing points will change and this change will be observable. This type of change might be caused by a variation in the constants of the metal or by a change in the thickness layer of one metal clad on another metal. Hence, a vertical change in the crossing point position could be used as a measure of cladding thickness.