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AEC RESEARCH AND
DEVELOPMENT REPORT

MASTER

**FO010 -- A TWO-DIMENSIONAL IBM-704
CODE FOR THE CALCULATION OF
THERMAL STRESSES IN A LONG,
HOLLOW CYLINDER WITH INTERNAL
HEAT GENERATION**

May 1958

CONTRACT AT-11-1-GEN-14

**BETTIS PLANT
PITTSBURGH, PENNSYLVANIA**

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CALCULATION OF THERMAL STRESSES IN A LONG, HOLLOW
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by

K. W. Brand, H. Kraus, T. J. Lawton

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I. INTRODUCTION

A. Background of the Problem

In nuclear reactor practice it has been found that heat is generated in the structural portions of the nuclear reactor by absorption of gamma rays which emanate from the core of the reactor. The structural members in question are generally the core barrel, the thermal shields, and the pressure vessel, which surround the nuclear core. Such gamma-ray heating must be considered in the design of these members because it produces thermal stresses in them. It is a particularly serious problem with respect to the pressure vessel because the thermal stresses produced by the gamma-ray heating add to the stresses already present due to the internal pressure which acts in the vessel. The combination of these effects may result in an excessively high stress level and means must therefore be available to determine the contribution of the gamma heating to the total stress acting in the vessel.

Previous investigations of this problem have assumed that the gamma heating is uniform throughout the member or that it varied only with the radius of the hollow cylinder which approximates the members in question. The radial variation selected is the decaying exponential because it is agreed that this variation most closely describes the absorption of the gamma rays in a material. In all cases axial symmetry has been assumed. However, this is not the actual case because it is known that the gamma rays are not emitted uniformly around the longitudinal axis of the nuclear core. Thus, it is felt that a study should be made which will take into account the angular variation of the internal heat generation, as well as the radial variation. Of main interest along this line is the determination of the shear stresses set up in the structural members by the gamma-ray heating, as well as the effect of the angular variation on the other stresses. To date the assumption that the heat generation is axially symmetric has prevented the determination of the shear stresses. It will be the aim in this investigation to determine the effect of the angular variation of the internal heat generation.

B. Method of Solution

The problem is solved by determining the temperature distribution in the hollow cylinder with internal heat generation and subsequently determining a thermo-elastic displacement potential from the temperature distribution (Ref 1). Appropriate derivatives of the displacement potential are then taken to obtain the shear, axial, tangential and radial stresses. Since the shear and radial stresses obtained from the above do not vanish at the inner and outer surfaces of the cylinder as required, an Airy stress function from the ordinary theory of elasticity is superimposed on the displacement potential to cause these stresses to go to zero.

Three sets of boundary conditions are applied to the solution as follows:

Case I - zero temperature on the inner and outer surfaces of the cylinder.

Case II - zero temperature on the inside surface and inside surface insulated.

Case III - zero temperature on the inside surface and outside surface insulated.

II. DERIVATION OF EQUATIONS

A. The Temperature Problem

The temperature distribution is found from the Fourier conduction equation

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial \theta^2} = - \frac{H_i}{k} [f(\theta) + 1] e^{-\beta(r-a)} \quad (1)$$

with

$$f(\theta) = \sum_{n=1}^{\infty} G_n \sin n\theta$$

If one lets $T(r, \theta) = T_1(r) + T_2(r, \theta)$ the above equation is replaced by

$$\frac{\partial^2 T_1}{\partial r^2} + \frac{1}{r} \frac{\partial T_1}{\partial r} = - \frac{H_i}{k} e^{-\beta(r-a)} \quad (2)$$

$$\frac{\partial^2 T_2}{\partial r^2} + \frac{1}{r} \frac{\partial T_2}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T_2}{\partial \theta^2} = - \frac{H_i}{k} f(\theta) e^{-\beta(r-a)} \quad (3)$$

The stresses due to the temperature distribution arising from Eq (2) are the subject of Ref 6. The present code is concerned with the stresses arising from the temperature distribution described by Eq (3).

Equation (3) is non-dimensionalized by the introduction of the following variables:

$$\Theta = \frac{T_K \beta^2}{H_1} \quad \beta_1 = \beta_a \quad t = \beta_1 y \quad y = r/a \quad (4)$$

By use of the above, Eq (3) becomes

$$\frac{\partial^2 \Theta}{\partial t^2} + \frac{1}{t} \frac{\partial \Theta}{\partial t} + \frac{1}{t^2} \frac{\partial^2 \Theta}{\partial \theta^2} = - f(\theta) e^{\beta_1} e^{-t} \quad (5)$$

The above equation is solved by reducing it to an equation in one variable by means of the finite Fourier sine transformation with respect to the variable θ . The transformed equation is an inhomogeneous Euler equation whose complementary solution is well known (Ref 2) and whose particular solution is obtained by variation of parameters (Ref 2). Upon determination of the transformed temperature, the inversion formula for the finite Fourier sine transform is applied to yield the following solution for the dimensionless temperature.

$$\Theta_1 = \frac{2}{\pi} \left\{ A_1 t + B_1 t^{-1} - h_1 e^{-t} (1 + t^{-1}) \right\} \sin \theta \quad (6)$$

$$\Theta_n = \frac{2}{\pi} \left\{ A_n t^n + B_n t^{-n} + \frac{h_n}{2n} \left[t^n \int_t^\infty \frac{e^{-x} dx}{x^{n-1}} + t^{-n} \int_\infty^t e^{-x} x^{n+1} dx \right] \right\} \sin n\theta \quad (7)$$

$$\Theta = \Theta_1 + \sum_{n=2}^{\infty} \Theta_n \quad (8)$$

In the above expression h_n is the finite Fourier sine transform of the quantity $f(\theta)$ which appears in Eq (5) and is defined by

$$h_n = G_n \frac{\pi}{2} e^{\beta_1} \quad (9)$$

To determine the arbitrary constants appearing in the temperature relations, one applies the three sets of boundary conditions as described previously.

Common to all of these is the condition that temperature is zero on the inner surface of the cylinder. This condition gives the following relation between A_n and B_n and is applicable to all the cases of the boundary conditions for $n \geq 1$.

$$B_n = -A_n \beta_1^{2n} - \frac{h_n}{2n} \left[\beta_1^{2n} \int_{\beta_1}^{\infty} \frac{e^{-x} dx}{x^{n-1}} + \int_{\infty}^{\beta_1} e^{-x} x^{n+1} dx \right] \quad (10)$$

A_n is now obtained for the three individual cases.

Case I - Equal temperatures on the inside and outside surfaces of the cylinder.

A_n^I

$$= -h_n \left\{ \frac{(\beta_1 y_o)^{2n} \int_{\beta_1 y_o}^{\infty} \frac{e^{-x} dx}{x^{n-1}} + \int_{\infty}^{\beta_1 y_o} e^{-x} x^{n+1} dx - \beta_1^{2n} \int_{\beta_1}^{\infty} \frac{e^{-x} dx}{x^{n-1}} - \int_{\infty}^{\beta_1} e^{-x} x^{n+1} dx}{2n (\beta_1 y_o)^{2n} (1 - y_o^{-2n})} \right\} \quad (11)$$

Case II - Inside surface insulated.

$$A_n^{II} = \frac{-h_n}{2n} \int_{\beta_1}^{\infty} \frac{e^{-x} dx}{x^{n-1}} \quad (12)$$

Case III - Outside surface insulated.

A_n^{III}

$$= -h_n \left\{ \frac{(\beta_1 y_o)^{2n} \int_{\beta_1 y_o}^{\infty} \frac{e^{-x} dx}{x^{n-1}} - \int_{\infty}^{\beta_1 y_o} e^{-x} x^{n+1} dx + \beta_1^{2n} \int_{\beta_1}^{\infty} \frac{e^{-x} dx}{x^{n-1}} + \int_{\infty}^{\beta_1} e^{-x} x^{n+1} dx}{2n (\beta_1 y_o)^{2n} (1 + y_o^{-2n})} \right\} \quad (13)$$

B. The Stress Problem

The previously described thermo-elastic displacement potential Φ is a solution of Ref (3)

$$\nabla^2 \Phi = \frac{1+\nu}{1-\nu} \alpha \frac{H_1}{k\beta^4} \Theta \quad (14)$$

Equation (14) is reduced to an equation in one variable by means of the finite Fourier sine transformation with respect to the variable Θ . The transformed equation is an inhomogeneous Euler equation whose complementary solution is well known and whose particular solution is obtained by variation of parameters. Upon solution of the transformed potential the inversion formula for the finite Fourier sine transformation is applied to obtain the thermo-elastic displacement potential.

It has also been stated that an Airy stress function F must be superimposed on the solution to cause the boundary shear and radial stress to vanish. The Airy stress function satisfies

$$\nabla^2 (\nabla^2 F) = 0 \quad (15)$$

whose solution is available in the literature (Ref 4).

Having thus obtained the thermo-elastic displacement potential and the Airy stress function one obtains the stresses from the following relations (Ref 5):

$$\sigma_{rr} = \left(\frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} \right) (F - 2G\Phi) \quad (16)$$

$$\sigma_{r\theta} = - \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial}{\partial \theta} \right) (F - 2G\Phi) \quad (17)$$

$$\sigma_{\theta\theta} = \frac{\partial^2}{\partial r^2} (F - 2G\Phi) \quad (18)$$

$$\sigma_{zz} = \nabla^2 (\nu F - 2G\Phi) \quad (19)$$

To simplify the calculations non-dimensional stresses are defined

$$\sigma_{rr} = \xi \sigma_R \quad \sigma_{r\theta} = \xi \sigma_S \quad \sigma_{\theta\theta} = \xi \sigma_T \quad \sigma_{zz} = \xi \sigma_A \quad (20)$$

$$\xi = \frac{4G}{\pi} \frac{(1+\nu)}{(1-\nu)} \frac{\alpha H_1}{k\beta^2} \quad (21)$$

The code yields solutions for σ_R , σ_S , σ_T and σ_A . These are found by use of Eqs (16), (17), (18), and (19) after division of each by ξ , and are given by (for $n = 1$)

$$\sigma_{T_1} = \left\{ \frac{2D_1}{t^3} + 6 \left(E_1 - \frac{A_1}{8} \right) t - \frac{B_1}{2t} + \frac{h_1 e^{-t}}{t^3} (t^3 + 2t^2 + 3t + 3) \right\} \sin \theta \quad (22)$$

$$\sigma_{R_1} = \left\{ -\frac{2D_1}{t^3} + 2 \left(E_1 - \frac{A_1}{8} \right) t - \frac{B_1}{2t} - \frac{h_1 e^{-t}}{t^3} (t^2 + 3t + 3) \right\} \sin \theta \quad (23)$$

$$\sigma_{S_1} = \left\{ \frac{2D_1}{t^3} - 2 \left(E_1 - \frac{A_1}{8} \right) t + \frac{B_1}{2t} + \frac{h_1 e^{-t}}{t^3} (t^2 + 3t + 3) \right\} \cos \theta \quad (24)$$

$$\sigma_{A_1} = 8vE_1 t \sin \theta - \frac{\pi}{2} \quad (25)$$

For $n \geq 2$ the stress equations are

$$\begin{aligned} \sigma_{T_n} = & \left\{ (2+n)(1+n) \delta_n t^n + (1-n)(2-n) \epsilon_n t^{-n} + n(n-1) \rho_n t^{n-2} \right. \\ & + n(n+1) \eta_n t^{-n-2} - \frac{h_n}{8n^2} \left[n(2+n)t^n \int_t^\infty \frac{e^{-x} dx}{x^{n-1}} + n(2-n)t^{-n} \int_\infty^t e^{-x} x^{n+1} dx \right. \\ & \left. \left. + n^2 t^{-n-2} \int_\infty^t e^{-x} x^{n+3} dx + n^2 t^{n-2} \int_\infty^t \frac{e^{-x} dx}{x^{n-3}} \right] \right\} \sin n \theta \quad (26) \end{aligned}$$

$$\begin{aligned} \sigma_{R_n} = & \left\{ (2-n)(1+n) \delta_n t^n + (1-n)(2+n) \epsilon_n t^{-n} + n(1-n) \rho_n t^{n-2} \right. \\ & - n(n+1) \eta_n t^{-n-2} - \frac{h_n}{8n^2} \left[n(2-n)t^n \int_t^\infty \frac{e^{-x} dx}{x^{n-1}} + n(2+n)t^{-n} \int_\infty^t e^{-x} x^{n+1} dx \right. \\ & \left. \left. - n^2 t^{-n-2} \int_\infty^t e^{-x} x^{n+3} dx - n^2 t^{n-2} \int_\infty^t \frac{e^{-x} dx}{x^{n-3}} \right] \right\} \sin n \theta \quad (27) \end{aligned}$$

$$\begin{aligned}
\sigma_{S_n} = & \left\{ -n(n+1) \delta_n t^n - n(1-n) \epsilon_n t^{-n} - n(n-1) \rho_n t^{n-2} \right. \\
& + n(n+1) \eta_n t^{-n-2} + \frac{h_n}{8} \left[t^n \int_t^\infty \frac{e^{-x} dx}{x^{n-1}} + t^{-n} \int_\infty^t e^{-x} x^{n+1} dx \right. \\
& \left. \left. + t^{n-2} \int_\infty^t \frac{e^{-x} dx}{x^{n-3}} - t^{-n-2} \int_\infty^t e^{-x} x^{n+3} dx \right] \right\} \cos n \theta \quad (28)
\end{aligned}$$

$$\sigma_{A_n} = 4\nu \left[(n+1) E_n t^n + (1-n) b_n t^{-n} \right] \sin n \theta - \frac{\pi}{2} \Theta_n \quad (29)$$

Finally, the stresses acting are given by

$$\sigma_T = \sigma_{T_1} + \sum_{n=2}^{\infty} \sigma_{T_n} \quad (30)$$

$$\sigma_S = \sigma_{S_1} + \sum_{n=2}^{\infty} \sigma_{S_n} \quad (31)$$

$$\sigma_R = \sigma_{R_1} + \sum_{n=2}^{\infty} \sigma_{R_n} \quad (32)$$

$$\sigma_A = \sigma_{A_1} + \sum_{n=2}^{\infty} \sigma_{A_n} \quad (33)$$

All the arbitrary constants appearing in the stress relations arise from the combination of the displacement potential and the Airy stress function with the exception of A_n and B_n for which expressions have been presented previously. The remaining constants are obtained from the boundary conditions on the stresses, which are

$$\sigma_S = \sigma_R = 0 \quad \text{at the inner and outer surfaces of the cylinder.} \quad (34)$$

Upon application of Eq (34) one obtains the relations for the arbitrary constants given by

$$2D_1 = -\frac{B_1 \beta_1^2 y_0^2}{2(1 + y_0^2)} - h_1 \left[\frac{y_0^4 e^{-\beta_1} (\beta_1^2 + 3\beta_1 + 3) - e^{-\beta_1 y_0} (\beta_1^2 y_0^2 + 3\beta_1 y_0 + 3)}{(y_0^4 - 1)} \right] \quad (35)$$

$$E_1 = \frac{A_1}{8} + \frac{B_1}{4(1 + y_0^2)\beta_1^2} + h_1 \left[\frac{e^{-\beta_1} (\beta_1^2 + 3\beta_1 + 3) - e^{-\beta_1 y_0} (\beta_1^2 y_0^2 + 3\beta_1 y_0 + 3)}{2\beta_1^4 (1 - y_0^4)} \right] \quad (36)$$

In the above B_1 is found from Eq (10) and A_1 is found from Eqs (11) (12) or (13).

For $n \geq 2$ the conditions Eq (34) lead to four simultaneous equations in four unknowns which are solved for each n by the code. This will yield values of δ_n , ϵ_n , ρ_n and η_n . These in turn yield

$$\delta_n + \frac{A_n}{4(n+1)} = E_n \quad (37)$$

$$\epsilon_n + \frac{B_n}{4(-n+1)} = b_n \quad (38)$$

The improper integrals which occur in the foregoing expressions are computed by recursion relationships. For the incomplete gamma function integration by parts yields

$$\int_0^t \frac{e^{-x} x^n}{x} dx = -t^n e^{-t} + n \int_0^t \frac{e^{-x} x^{n-1}}{x} dx$$

For the E function (Ref 8)

$$\int_t^\infty \frac{e^{-x}}{x} dx = \frac{e^{-t}}{t} \left\{ \frac{a_0 + a_1 t + a_2 t^2 + a_3 t^3 + t^4}{b_0 + b_1 t + b_2 t^2 + b_3 t^3 + t^4} \right\}$$

$$a_0 = 0.2677737343$$

$$b_0 = 3.9584969228$$

$$a_1 = 8.6347608925$$

$$b_1 = 21.0996530827$$

$$a_2 = 18.0590169730$$

$$b_2 = 25.6329561486$$

$$a_3 = 8.5733287401$$

$$b_3 = 9.5733223454$$

Then (Ref 9)

$$t^{n-1} \int_t^{\infty} \frac{e^{-x}}{x^n} dx = \frac{1}{n-1} \left(e^{-t} - t \left[t^{n-2} \int_t^{\infty} \frac{e^{-x}}{x^{n-1}} dx \right] \right)$$

It should be pointed out here that usually the angular variation $f(\theta)$ of the internal heat generation can be adequately described by N terms of the sine series, and thus the infinite sums appearing in all of the preceding expressions for temperature and stress go over into finite sums of N terms. Furthermore, if the variation of the internal heat generation can be adequately described by a single sine term of a given n , the infinite sums go over into single terms involving the given n . For example, in a square nuclear core it is agreed that four peaks occur in the gamma heating around the periphery of the core (one peak at each corner of the square), and a rough approximation of this variation can be achieved by using $n = 2$ only.

Also it is to be noted that the code only provides the stress values for the angularly varying portion of the heat generation. To the stresses obtained herein one must add the stresses arising from the uniform component due to the temperature distribution of Eq (2). These are found from calculations described in (Ref 6) and (Ref 7).

III. USE OF CODE

A. Limitations

Due to inherent limitations of the IBM-704, certain restrictions must be placed on the problems run on this code.

(1) in any of the expressions involving exponentials e^x $|x| \leq 88$,

(2) in any of the expressions involving power of t $t^{|N|} \leq 10^{38}$.

It should also be noted that these are absolute upper limits. Approaches to these magnitudes may cause inaccuracies due to the fact that sums and differences of extremely large and extremely small numbers are taken.

Tests are made during the calculation to halt the problem when values go out of range, but it is impossible to catch every case. It may be observed however, that the boundary conditions imposed on this problem provide an excellent checking device. If at any time the boundary conditions are not fulfilled, then one of the above criteria has been violated.

B. Input and Output

1. Preparation of input.

Title card - Each problem is preceded by a title card containing any desired alpha-numeric information in columns 2-72. Note that column 1 is blank. The problem number should always be part of this information.

Data cards - The first three columns of all subsequent cards are reserved for sequential numbering. The program interrogates these card numbers to check the order of data and any discrepancy will result in a program stop. The cards are numbered starting at 1.

Remember that when a field of so many columns is assigned to some quantity, it should be pushed to the right of the field so that the last digit punched is the last column of the field.

All quantities designated non-integer are punched with the decimal point properly placed. Failure to punch the decimal point will result in erroneous output.

Card number	Columns	Quantity
1	1-3	001 - card number.
	4-6	n - number of terms in Fourier series. $n \leq 25$.
	7-9	m - number of values of β , $m \leq 20$.
	10-11	00 or 01 - No Boundary Condition I or Boundary Condition I.
	12-13	00 or 01 - No Boundary Condition II or Boundary Condition II.
	14-15	00 or 01 - No Boundary Condition III or Boundary Condition III.
	16-17	00 or 01 - Don't print Components or print Components.

Card number	Columns	Quantity
1 (Cont)		
	18-39	ν - Poisson's ratio (non-integer)
	1-3	002 - card number
	4-15	β - non-integer
	16-27	β - non-integer
	28-39	β - non-integer
	40-51	β - non-integer
	52-63	β - non-integer
	64-72	Blank

etc. giving up to five values of β per card until m (when m is specified on card #1) values have been entered. Assume that the m^{th} value of β is given on card #J.

J + 1	1-3	J + 1 - card number
	4-15	a - non-integer, inner radius of cylinder
	16-27	b - non-integer, outer radius of cylinder
	28-30	K - number of positions in wall at which to calculate (including surfaces)
	31-72	Blank
J + 2	1-3	J + 2 - card number
	4-15	θ_o - non-integer, initial angle
	16-27	θ_f - non-integer, final angle
	28-30	L - number of angular positions at which to calculate (including initial and final).
	31-72	Blank
J + 3	1-3	J + 3 - card number
	4-15	G_1 - non-integer, 1 st coefficient of series
	16-27	G_2 - non-integer, 2 nd coefficient of series
	28-39	G_3 - non-integer, 3 rd coefficient of series

Card number	Columns	Quantity
J + 3 (Cont)		
	40-51	G_4 - non-integer, 4 th coefficient of series
	52-63	G_5 - non-integer, 5 th coefficient of series
	64-72	Blank

etc. giving up to five values of G per card until n values have been specified.

Follow the complete deck with a blank card.

An example of an input deck is given in the Appendix II.

2. Output.

Values of the stresses and temperature, both pure and after combination with the coefficients of the Fourier series, are supplied at every point. As an option the requestor may get the individual components of stress and temperature by adjusting his input.

A sample printout of both options is supplied in the Appendix II.

C. Operating Instructions

1. Place program deck labeled F0011000-F0011... in the card reader followed by as many problem decks as desired. Each problem should be followed by one blank and the last by two blanks.
2. Mount blank tapes on logical #9 and #10. There may or may not be output to #9 depending on the input. There will always be output to #10.
3. Clear and load cards.
4. Normal stop is a select on the card reader. In the event that the last two blank cards were omitted, there will be a program stop at 7227 after the problem has been completed.

ERROR STOPS

Location

Why

6123 }
6127 }
6131 }
6133 }

Problem out of range; may be input error.

<u>Location</u>	<u>Why</u>
6135	Input cards out of sequence.
7227	Card end of file; this stop may be caused by incorrectly sequenced cards also.
7315 } 7350 } 7370 } 7404 }	Inappropriate character encountered on card - may be punching error.
7147	Key punch malfunction, non-hollerith character.
11235	Printer echo-check error - press start to continue.

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APPENDIX I

NOMENCLATURE

<u>Symbol</u>	<u>Quantity</u>	<u>Units</u>
a	Inner radius of cylinder	inches
a_n, A_n	Arbitrary constants	none
b_n, B_n	Arbitrary constants	none
c_n, C_n	Arbitrary constants	none
d_n, D_n	Arbitrary constants	none
E	Modulus of elasticity	lb/in ²
F	Airy stress function	none
f(θ)	Arbitrary variation of heat generation with angle	none
G_n	Amplitude of n th term of f(θ)	none
G	Shear modulus	lb/in ²
h_n	Sine transform of f(θ)e ^{β_1}	none
H_i	Heat generation at inner surface	Btu/sec/in ³
k	Thermal conductivity	Btu/sec/in/°F
n	Index of sine series	none
r	Radius to a point in the wall	inches
t	Dimensionless variable	none
T	Temperature	°F
x	Dummy variable appearing in integrals	none
y	Radius to a point in the wall divided by the inner radius of the cylinder	none
y_o	Outer radius divided by inner radius	none

Greek Symbols

α	Coefficient of thermal expansion	in/in/°F
β	Logarithmic decrement of heating	in-l
β_1	Logarithmic decrement times inner radius	none
$\delta_n, \epsilon_n, \eta_n, \rho_n$	Arbitrary constants	none
θ	Angular co-ordinate	degrees
Θ	Dimensionless temperature difference	none
ν	Poisson's ratio	none
ξ	Conversion factor for stresses	lb/in ²

<u>Symbol</u>	<u>Quantity</u>	<u>Units</u>
σ_{rr}	Radial stress	lb/in ²
$\sigma_{r\theta}$	Shear stress	lb/in ²
$\sigma_{\theta\theta}$	Tangential stress	lb/in ²
σ_{zz}	Axial stress	lb/in ²
σ_{Rn}	Dimensionless radial stress	none
σ_{Tn}	Dimensionless tangential stress	none
σ_{Sn}	Dimensionless shear stress	none
σ_{An}	Dimensionless axial stress	none
Φ	Thermo-elastic displacement potential	none

Superscripts

I	Case I of the boundary conditions
II	Case II of the boundary conditions
III	Case III of the boundary conditions

APPENDIX II

SAMPLE PROBLEM

TEST1 F0010 THICK CYLINDER Y0=1.21539 BETA=45

00100200101000001 .3

002 45.

003 1. 1.21539 3

004 15. 30.00000 2

005 1. 1.

TEST1 F0010 THICK CYLINDER $Y_0=1.21539$ BETA=45

THERMAL STRESSES IN A LONG HOLLOW CYLINDER WITH INTERNAL HEAT GENERATION REPRESENTED BY A 2 TERM FOURIER SINE SERIES.
SOLUTION WILL BE MADE FOR THE FOLLOWING BOUNDARY CONDITIONS-

1-EQUAL TEMPERATURES ON INNER AND OUTER SURFACE.

THE COEFFICIENTS OF THE SINE SERIES ARE-
1.000000 1.000000

THE CYLINDER WALL WILL BE TRAVERSED FROM $A= 1.000000$ TO $B= 1.215390$ IN 3 INCREMENTS OF LENGTH 0.107695

THE CIRCUMFERENCE WILL BE TRAVERSED FROM 15.000000 DEGREES IN 2 INCREMENTS OF LENGTH 15.000000 TO 30.000000 DEGREES

THE SUMS.

THETA
G*THETA

SIGMA T
G*SIGMA T

SIGMA R
G*SIGMA R

SIGMA A
G*SIGMA A

SIGMA S
G*SIGMA S

BOUNDARY CONDITION 1

-0.000000	0.653917	-0.000000	0.196175	Y= 1.000000	ANGLE= 15.00
-0.000000	0.653917	-0.000000	0.196175	0.000000	
				0.000000	
0.357915	-0.107691	-0.009412	-0.428680	Y= 1.107695	ANGLE= 15.00
0.357915	-0.107691	-0.009412	-0.428680	0.032155	
				0.032155	
-0.000000	0.268013	-0.000000	0.080404	Y= 1.215390	ANGLE= 15.00
-0.000000	0.268013	-0.000000	0.080404	0.000000	
				0.000000	
-0.000000	1.159616	-0.000000	0.347885	Y= 1.000000	ANGLE= 30.00
-0.000000	1.159616	-0.000000	0.347885	0.000000	
				0.000000	
0.644536	-0.193905	-0.017672	-0.772177	Y= 1.107695	ANGLE= 30.00
0.644536	-0.193905	-0.017672	-0.772177	0.026730	
				0.026730	
-0.000000	0.497025	-0.000000	0.149108	Y= 1.215390	ANGLE= 30.00
-0.000000	0.497025	-0.000000	0.149108	0.000000	
				0.000000	

N
1
2A
-0.074773
-0.000572E
0.001450B BOUNDARY CONDITION 1
223.671860
6010.444397
D
-70658.823242

N	THETA G*THETA	SIGMA T G*SIGMA T	SIGMA R G*SIGMA R	SIGMA A G*SIGMA A	SIGMA S G*SIGMA S		
					Y= 1.000000	ANGLE=	15.00
1	-0.000000	0.135126	-0.000000	0.040538	0.000000		
	-0.000000	0.135126	-0.000000	0.040538	0.000000		
2	-0.000000	0.518791	-0.	0.155637	-0.000000		
	-0.000000	0.518791	-0.	0.155637	-0.000000		
					Y= 1.107695	ANGLE=	15.00
1	0.123163	-0.036933	-0.006853	-0.148561	0.025578		
	0.123163	-0.036933	-0.006853	-0.148561	0.025578		
2	0.234752	-0.070758	-0.002559	-0.280118	0.006577		
	0.234752	-0.070758	-0.002559	-0.280118	0.006577		
					Y= 1.215390	ANGLE=	15.00
1	-0.000000	0.164231	-0.000000	0.049269	0.000000		
	-0.000000	0.164231	-0.000000	0.049269	0.000000		
2	0.000000	0.103782	0.000000	0.031135	0.000000		
	0.000000	0.103782	0.000000	0.031135	0.000000		

N	THETA G*THETA	SIGMA T G*SIGMA T	SIGMA R G*SIGMA R	SIGMA A G*SIGMA A	SIGMA S G*SIGMA S		
					Y= 1.000000	ANGLE=	30.00
1	-0.000000	0.261043	-0.000000	0.078313	0.000000		
	-0.000000	0.261043	-0.000000	0.078313	0.000000		
2	-0.000000	0.898573	-0.	0.269572	-0.000000		
	-0.000000	0.898573	-0.	0.269572	-0.000000		
					Y= 1.107695	ANGLE=	30.00
1	0.237933	-0.071349	-0.013240	-0.286998	0.022932		
	0.237933	-0.071349	-0.013240	-0.286998	0.022932		
2	0.406603	-0.122556	-0.004432	-0.485179	0.003797		
	0.406603	-0.122556	-0.004432	-0.485179	0.003797		
					Y= 1.215390	ANGLE=	30.00
1	-0.000000	0.317269	-0.000000	0.095181	0.000000		
	-0.000000	0.317269	-0.000000	0.095181	0.000000		
2	0.000000	0.179756	0.000000	0.053927	0.000000		
	0.000000	0.179756	0.000000	0.053927	0.000000		