UNITED STATES ATOMIC ENERGY COMMISSION

**GAS COOLED PEBBLE BED REACTOR FOR
A LARGE CENTRAL STATION**

Reactor Design and Feasibility Study

By

**A. Schock
D. F. Bruley
H. N. Culver
P. W. Ianni
W. F. Kaufman
R. A. Schmidt
R. E. Supp**

August 1957

**Oak Ridge School of Reactor Technology
Oak Ridge, Tennessee**

Technical Information Service Extension, Oak Ridge, Tenn.

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

LEGAL NOTICE

This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:

A. Makes any warranty or representation, express or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or

B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.

As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission to the extent that such employee or contractor prepares, handles or distributes, or provides access to, any information pursuant to his employment or contract with the Commission.

This report has been reproduced directly from the best available copy.

Since nontechnical and nonessential prefatory material has been deleted, the first page of the report is page 5.

Printed in USA. Price \$5.00. Available from the Office of Technical Services, Department of Commerce, Washington 25, D. C.

CF-57-8-12

"GAS COOLED PEBBLE BED REACTOR FOR A LARGE CENTRAL STATION"

Prepared by:

A. Schock, Group Leader
D. F. Bruley
H. N. Culver
P. W. Ianni
W. F. Kaufman
R. A. Schmidt
R. E. Supp

OAK RIDGE SCHOOL OF REACTOR TECHNOLOGY

Reactor Design and Feasibility Study

August 1957

PREFACE

In September, 1956, a group of men experienced in various scientific and engineering fields embarked on the twelve months of study which culminated in this report. For nine of those months, formal classroom and student laboratory work occupied their time. At the end of that period, these seven students were presented with a problem in reactor design. They studied it for ten weeks, the final period of the school term.

This is a summary report of their effort. It must be realized that, in so short a time, a study of this scope can not be guaranteed complete or free of error. This "thesis" is not offered as a polished engineering report, but rather as a record of the work done by the group under the leadership of the group leader. It is issued for use by those persons competent to assess the uncertainties inherent in the results obtained in terms of the preciseness of the technical data and analytical methods employed in the study. In the opinion of the students and faculty of ORSORT, the problem has served the pedagogical purpose for which it was intended.

The faculty joins the authors in an expression of appreciation for the generous assistance which various members of the Oak Ridge National Laboratory gave. In particular, the guidance of the group consultants, A. M. Weinberg and R. A. Charpie, is gratefully acknowledged.

Lewis Nelson

for

The Faculty of ORSORT

ACKNOWLEDGEMENT

The authors wish to express their appreciation to Dr. A. M. Weinberg and Dr. R. A. Charpie, ORNL, for their guidance during the preparation of this report. We also acknowledge the assistance given by S. T. Robinson and R. F. Beneati of Sanderson and Porter for their suggestions, and to R. H. Dunham of the Tennessee Valley Authority for his assistance with the turbinecycle calculations. In addition we recognize that much of the work included in this report would have been quite impossible without the efforts of J. G. Sullivan and W. E. Kinney in establishing a code for the ORACLE.

In addition the following gave valuable assistance on various matters:

L. R. Dresner
E. E. Gross
A. M. Perry
A. J. Taylor
R. C. Robertson
R. V. Meghreblian
R. P. Milford
E. L. Nicholson
R. E. Blanco
L. M. Ferris
C. L. Pigott
G. I. Cathers
W. E. Clark
J. W. Ullman
H. G. MacPherson
J. A. Lane
J. C. Cox
W. K. Eister
R. L. Macklin
J. McGuire
A. L. Boch

ABSTRACT

This study is to determine an optimum economic design for a high temperature, helium cooled, central station reactor power plant of about 400 MW of electric power. The core consists of a randomly packed bed of unclad graphite spheres, approximately one inch in diameter, impregnated with U-233 and thorium such that a conversion ratio of near unity is achieved. The high temperature helium permits steam conditions, at the turbine throttle, of 1000°F and 1450 psia.

TABLE OF CONTENTS

Page

1. Introduction	13
2. General Discussion of System	15
2.1 Overall Plant Description	15
2.1.1 Reactor Plant	15
2.1.2 Turbogenerator Plant	21
2.1.3 High Pressure Helium System	21
2.1.4 Vent System	21
2.1.5 Containment Vessel	22
2.1.6 Plant Layout	23
2.1.7 Fuel Loading and Discharge System	25
2.1.8 Fuel Annealing and Decontamination	26
2.1.9 Accessibility for Maintenance	27
2.2 Design Data	31
3. Reactor Components	34
3.1 Pressure Vessel and Core Assembly Design	34
3.1.1 Reflector	34
3.1.2 Thermal Shielding	36
3.1.3 Loading and Discharging Mechanism	36
3.1.4 Structural	37
3.2 Heat Exchanger Design	39
3.2.1 Primary Heat Exchangers	40
3.2.2 Boilers	40
3.2.3 Economizers	41
3.2.4 Superheaters	41
3.3 Blower Design	56
3.4 Valves	56
3.5 Piping	56
4. Physics	58
4.1 Object of Design	58
4.2 List of Symbols	58
4.3 Method of Calculation	59
4.4 Fast Leakage	62
4.5 Thorium and Uranium Resonance Capture	63
4.6 Thermal Poison	64
4.7 Preliminary Calculations	64
4.8 Assumptions for Preliminary Calculations	64
4.9 Results of Preliminary Calculations	65
4.10 Effect of Reflector Thickness and Density	65
4.11 Neutron Temperature	72
4.12 Possibility of Boiling out Fission Products	73
4.13 Change of Average Number of Fissions Neutrons per Uranium Absorption	73
4.14 Fission Product Poison in Fast and Resonance Group	74
4.15 Effect of Gas Hole in Middle of Reactor	76
4.16 Calculation for Final Design	77
4.17 Final Core Design	81
4.18 Temperature Coefficient of Reactivity	85

5. Design Justification	86
5.1 Basis for Selection of Final Design	86
5.1.1 Selection of Coolant	86
5.1.2 Selection of Moderator	87
5.1.3 Selection of Fuel	88
5.1.4 Selection of Fissionable Material	88
5.1.5 Selection of Plant Capacity	88
5.1.6 Selection of Fuel Geometry	88
5.1.7 Selection of Core Size	89
5.1.8 Selection of Core Geometry	90
5.1.9 Reactor Pressure	90
5.1.10 Selection of Power Plant	91
5.2 Design Optimization	92
5.2.1 Revised Set of Equations	106
5.2.2 Final Results	110
5.3 Coolant Decontamination	110
5.4 Emergency Cooling	114
5.5 Power Changes	119
6. Shielding	120
7. Controls and Instrumentation	122
7.1 Power Plant Control	122
7.2 Reactor Control	122
7.2.1 General Discussion	122
7.2.2 Reactor Kinetics	124
7.2.3 Control Rod Requirements	127
7.2.4 Mechanical Design	129
8. Economics of the System	130
8.1 Basis for Cost Estimation	130
8.1.1 Basis for Estimating Nuclear Plant Costs	130
8.1.1.1 Heat Exchangers	130
8.1.1.2 Piping, Fittings and Valves	132
8.1.1.3 Blowers	132
8.1.1.4 Containment Vessel	133
8.1.1.5 Pressure Vessel and Thermal Shields	133
8.1.1.6 Shielding	134
8.1.1.7 Nuclear Control and Instrumentation	134
8.1.1.8 Fuel Charging and Discharging Equipment	134
8.1.1.9 Land	134
8.1.1.10 Miscellaneous Equipment	134
8.1.2 Physical Plant Cost for Nuclear Plant	135
8.1.3 Direct Plant Cost	136
8.1.4 Fixed Capital for Nuclear Plant	136
8.1.5 Power Plant Costs	136
8.1.6 Total Fixed Capital	140
8.1.7 Total Depreciating Investment	140
8.1.8 Working Capital	140
8.1.9 Capital Charges	140
8.1.10 Operating Costs	140
8.1.11 Fuel Rental	141
8.1.12 Graphite Inventory Cost	141
8.1.13 Thorium and Helium Costs	141

8.1.14	Fuel Reprocessing	141
8.1.15	Graphite Replacement	142
8.2	Cost Analysis of Final Reactor Design	142
9.	Conclusions	147
10.	Appendix A	149
10.1	Physics	149
10.1.1	Calculation of Age	149
10.1.2	Thermal Poisons	151
10.1.3	Thorium and Uranium Resonance Absorptions	155
10.1.4	Temperature Coefficient of Reactivity	161
10.1.5	Reactivity Effect Due to Neutron Streaming in the Reactor Gas Duct	164
10.2	Engineering	171
10.2.1	Heat Generation and Thermal Stresses Within the Pressure Vessel Walls	171
A.	Sources of Radiation Heating in the Pressure Vessel Walls	171
(1)	Core Prompt Fission Gammas	171
(2)	Core Capture Gammas	173
(3)	Core Decay Gammas	173
(4)	Thermal Neutron Capture Gammas Within the Pressure Vessel Walls and Thermal Shield	174
(5)	Fast Neutron Scattering Within the Pressure Vessel Walls	175
(6)	Fast Neutron Absorption Gammas in the Pressure Vessel Walls and Thermal Shield	176
(7)	Fast Neutron Inelastic Scattering	177
B.	Thermal Stresses in the Pressure Vessel Walls	178
C.	Heat Generation Symbols	183
D.	Stress Symbols	184
10.2.2	Shielding	185
10.2.3	Containment Vessel Design	191
10.2.4	Calculation of Heat Exchanger Tube Lengths	195
10.2.4.1	Primary Heat Exchanger	195
10.2.4.2	Economizer	195
10.2.4.3	Boiler	197
10.2.4.4	Superheater	201
10.2.5	Calculation of Heat Exchanger Required Areas of Heat Transfer	203
10.2.5.1	Primary Heat Exchanger	203
10.2.5.2	Economizer	203
10.2.5.3	Boiler	203
10.2.5.4	Superheater	204
10.2.6	Cost of Heat Exchangers	205
10.2.6.1	Primary Heat Exchanger	205
10.2.6.2	Economizer	206
10.2.6.3	Boiler	206

10.2.6.4	Superheater	206
10.2.7	Calculation of Pressure Drops	207
10.2.7.1	Reactor	207
10.2.7.2	Primary Heat Exchanger	209
10.2.7.3	Economizer	210
10.2.7.4	Boiler	210
10.2.7.5	Superheater	211
10.2.7.6	Piping	212
10.2.8	Calculation of Power Lost in Pumping	212
11.0	Appendix B	214
11.1	Nomenclature	214
11.2	Heat Balances	216
11.3	Calculation of Flow Rates and Blower Power	218
11.4	Heat Exchanger Equations	221
11.4.1	Nomenclature	221
11.4.2	Boiler Equations	223
11.4.3	Superheater Equations	231
11.4.4	Primary Heat Exchanger Equations	239
11.4.5	Economizer Equations	247
11.5	Rigorous Treatment of Boiler Heat Transfer	254
11.6	Pressure Drop Equations in Stainless Steel Piping	259
11.7	Pressure Drop Equations in Carbon Steel Piping	261
11.8	Pressure Drop Through Reactor	262
11.8.1	Pressure Drop in Central Hole	262
11.8.2	Diffuser Losses	265
11.8.3	Exit Losses	265
11.8.4	Entrance Losses	266
11.8.5	Radial Pressure Drop Across Core	267
11.9	Temperature Coefficient of Reactivity	274
11.9.1	Effect of γ on Temperature Coefficient of Reactivity	274
11.9.2	Effect of f on Temperature Coefficient of Reactivity	275
11.9.3	Effect of B on Temperature Coefficient of Reactivity	277
11.9.4	Effect of M on Temperature Coefficient of Reactivity	278
11.9.5	Effect of Resonance Absorption on Temperature Coefficient of Reactivity	280
11.9.5a	Method of Determining $\partial I / \partial T$	281
11.10	Derivation of U-234, U-235 and U-236 Build Up Equations	289
11.11	Thermal Stresses and Minimum Allowable Thermal Conductivity in Impregnated Graphite Spherical Fuel Elements	292
11.12	Core Temperature Distribution	297
12.0	Appendix C	306
12.1	Thermal Cross Section Data	306
12.2	Eyewash Cross Section Data	307
12.3	Cross Sections for OCUSOL-A Program	308
12.4	Heat Balance Data	309
13.0	Appendix D	310
13.1	Original Set of Equations	310
13.2	Revised Set of Equations	315

CHAPTER 1

INTRODUCTION

Nuclear reactors which have been proposed for central station power plants generally fall into one of two categories; the solid fuel element heterogeneous type, or the circulating fuel type such as the LMFR or aqueous homogeneous reactor. The advantages and disadvantages of these reactor types have been fully discussed in the literature and therefore no attempt will be made to discuss the merits of these systems in detail. In general, the solid fuel type reactors tend to have intricate core designs, clad fuel elements, high fuel costs and severe limitations on temperature and burnup, all of which make the possibility of economical power quite improbable. On the other hand, the circulating fuel type of reactor presents the possibility of a cheap fuel, relatively simple core design and eliminates the necessity of cladding the fuel. However, the circulating fuel type of reactors have the very severe limitation caused by corrosion and until this problem is solved there seems little chance of economical power to compete with the modern day fossil fueled steam plant.

The reactor system proposed in this report attempts to retain the advantages of gas cooled solid fuel type reactors such as chemical compatibility and lack of corrosion while minimizing their usual disadvantages of high fuel fabricating costs, low burnup and the usual temperature limitations imposed by solid fuel rods with some form of cladding. The reactor proposed in this report also incorporates the advantages of the circulating fuel reactors by allowing the fuel to be reprocessed and redistributed in a manner similar to a liquid fuel. Such a scheme allows for a high burnup and makes

for a reduction in the cost of the fuel while attaining a high conversion ratio or a breeding gain.

Since there were no restrictions placed upon the reactor study during the initial phase of the work, the following objectives were established and deemed necessary to attain economical power.

1. High steam temperature and pressure.
(High efficiency and conventional power plant.)
2. No structural materials in core.
(Good neutron economy, high conversion ratio or breeding gain)
3. Simple materials, minimum corrosion, chemical compatability.
4. Low fuel element fabrication cost.
5. Ease of charging and discharging fuel.
(High burnup)
6. No close tolerances or fine machining required for fuel elements
or for the parts making up the reactor core or pressure vessel.
7. Good negative temperature coefficient.

With the above objectives in mind, the reactor system proposed was represented by a series of 51 equations (See Section 13.1), which predicts the most economical system and thus set the design point for the assumptions made in this study.

In all instances there was a firm desire to remain as close as possible to present technology, and where extrapolation of present day capability was made it was based upon data as found in the literature which has been referred to.

CHAPTER 2

GENERAL DISCUSSION OF SYSTEM

2.1 Overall Plant Description

The overall plant layout is shown in Figures 2-1 and 2-2; the reactor plant is shown in Figures 2-3 and 2-4; and the flow sheet for the system is shown in Figure 2-5. Reference will be made to these figures in describing the system.

2.1.1 Reactor Plant

The reactor plant in this system compares to what would normally be the boiler plant in a conventional system. In this portion of the plant the reactor pressure vessel, primary heat exchangers, boilers, superheaters, economizers, blowers and auxiliary reactor equipment are located. The coolant gas enters at the top of the reactor pressure vessel and then flows down the sides of the vessel and radially through the perforated graphite shell toward the central axis of the reactor. A portion of the gas flows down the sides of the vessel and up through the grates at the bottom of the reactor core. The hot gases flow up through the central hole in the reactor and then split into four primary loops (see Figure 2-3), each containing two primary heat exchangers in series. The coolant helium passes through the primary heat exchangers inside the tubes and then is returned again to the reactor pressure vessel by a 27,000 cfm blower. Valving is provided in the primary loop to permit isolation of the equipment if required. (See Figure 2-5).

In traversing the secondary loop, nonradioactive helium, hereafter referred to as "cold helium" to distinguish it from the radioactive helium

OVERALL PLANT LAYOUT (Plan View)
Figure 2.-1

ORNL LR DWG. 23444

OVERALL PLANT LAYOUT (Elevation)
 Figure 2-2

ORNL LR DWG. 23445

Fig. 2.4. Reactor Plant Layout —Elevation.

FLOW DIAGRAM FOR 400MW HELIUM GAS COOLED REACTOR

Figure 2-5

or "hot helium" in the primary loop, passes successively through the primary heat exchangers, superheater, boiler, economizer, and blower. The steam generated in the superheater then goes to the normal Turbogenerator Plant.

(See Figures 2-1 through 2-5).

2.1.2 Turbogenerator Plant

The turbogenerator plant in this system is a conventional design based on the Johnsonville Steam Plant in the TVA system. The steam conditions in this reactor were chosen so that conditions at the turbine were the same as at the Johnsonville Steam Plant. This allowed for utilization of high steam plant efficiencies as well as purchase of equipment at economical costs.

2.1.3 High Pressure Helium System

A high pressure helium system is provided to pressurize between the seals on valves and to prevent leakage at the blowers. To compensate for the "cold helium" leaking into the system a let down stream is taken off of the "hot helium" system to maintain pressure. The "hot helium" stream is sent through a decontamination system and then to the storage tanks. Valving is provided to pass this to a stack if desired.

2.1.4 Vent System

All of the equipment containing radioactive helium are contained in sealed compartments as discussed in Sections 2.1.6 and 2.1.9. A vent system to these hot compartments is provided to circulate air to these sealed compartments. This system serves the purpose of reducing the temperature in the compartments, detecting activity and reducing any activity to a level acceptable to operation of the plant.

Since the primary loops will be contained in a relatively gas tight region, separately ventilated, activity within this container during normal operation will be a minimum, allowing continuous operation of the plant in the event of minor leaks.

The activity in the coolant stream due to fission products which are volatile at 1600°F is about 10^8 curies. If a ventilating stack activity of $2 \times 10^{-4} \mu\text{c/cc}$ is assumed to be the allowable discharge of activity at the stack, a leakage of 1 cubic foot of gas at STP can be tolerated from the primary circuit for every 200,000 cubic feet of air circulated through the sealed compartments containing the primary loops and associated hot equipment.

If the tolerance level within the upper dome of the containment vessel is maintained at a level of $2 \times 10^{-8} \mu\text{c/cc}$, and assuming that the sealed compartment containing the primary loops is at the acceptable stack activity level of $2 \times 10^{-4} \mu\text{c/cc}$, then the allowable leakage of the air-gas mixture, in the sealed compartment, into the upper containment space is about 10 cubic feet at STP for every 200,000 cubic feet of air at STP being circulated through the upper dome in the containment vessel.

2.1.5 Containment Vessel

The reactor system proposed in this report has for the fuel system spherical graphite balls impregnated with both fissionable and fertile material, U^{233} and Th^{232} . These balls are unclad and will therefore allow the escape of millions of curies of fission product activity into the primary coolant loops. Under normal operating conditions this activity is completely contained in the system by having an all welded system and by providing a jacket of non-radioactive helium at a slightly higher pressure wherever leakage

of radioactive helium may occur.

In the event of an unforeseen accident due either to a rapid power rise resulting in equipment failure, or an accident caused by external force, provision must be made for containing the radioactive helium and preventing its escape into the atmosphere. The containment provided for this reactor is a steel tank 80' in diameter into which the reactor and primary loops are installed. Above the ground level this provides a hemisphere with a volume into which the gases will expand should an accident cause the release of the radioactive gases. The hemisphere has been designed to contain the gases mixed with air keeping stresses below allowable values. The temperature and pressure in the containment vessel after an incident are shown in Figure 2-6. (See Section 10.2.3).

2.1.6 Plant Layout

A gas cooled reactor circulating radioactive He presents a great hazard in case of an accident. The layout of equipment in this design has been made to prevent any leakage of hot gases in case of an unforeseen incident.

The reactor pressure vessel, primary heat exchangers, secondary heat exchangers, blowers and hot He storage systems are all located in the containment vessel.

The pressure vessel is centrally located and shielded by seven feet of high density concrete. This concrete, incidentally, would also serve as a blast shield in the event of a major incident. Located in an annular array between this shielding and the containment vessel are individual cells, shielded from each other, which contain the primary loops. The primary

loops have removable shielding overhead to allow for replacement of equipment. The secondary heat exchangers and blowers are also located in this annulus to provide additional safety and to produce the most compact arrangement of equipment. The hot He storage tanks and fuel handling equipment are located in shielded compartments just below the secondary loops. The secondary loops are contained inside the containment vessel but do not have shielding overhead since they do not operate with radioactivity present. Maintenance of this equipment can be done in a normal manner.

Steam lines penetrate the containment vessel walls and terminate in a conventional turbine plant layout. The design and layout of the turbine plant is conventional and is shown only as it might conveniently be located to the reactor discussed in this report.

2.1.7 Fuel Loading and Discharging System

Fuel can be loaded into or withdrawn from the reactor system by passing it through something equivalent to an air lock. This is indicated to some extent by Figures 2-3 and 2-4.

The fuel balls flow into the tank directly above the reactor. A rotating mechanism distributes them into 8 pipes leading to the top of the reactor core. To unload balls, radial grates at the bottom of the annular reactor core are rotated. This allows a pre-determined number of fuel balls to drop out of the reactor. After removal from the core, fuel balls will have to be cooled by forced convection until the radioactive fission products contained therein have decayed to the point where cooling by natural convection is adequate. The fuel balls may then be withdrawn from the reactor again through a gas lock. In order to perform chemical reprocessing, the graphite may be burnt, leaving only the unburnt fuel and non-volatile fission

products.

2.1.8 Fuel Annealing and Decontamination

Another desirable factor of the proposed reactor system is that it affords the opportunity of annealing radiation damage in the fuel and simultaneously removing a substantial fraction of the fission products by, what appear to be, relatively simple and inexpensive means. Although the following is not a necessary part of the proposed system, the advantages it offers suggest that its inclusion be given careful consideration.

It is suggested that a small fraction of the fuel balls be continuously withdrawn from the reactor and temporarily stored in a graphite holdup chamber. The dimensions of this holdup chamber should be such that the time dwell of the fuel balls therein be of the order of several hours. Part of the "hot" helium coolant stream should be passed through this holdup chamber to remove the heat generated by radioactive decay in the fuel balls. The velocity of this coolant stream should be adjusted so as to produce maximum fuel temperatures of 3500 to 4000°F. In other words, the fuel would act as its own heat source with its temperature determined by the rate of coolant flow. A study of References 26 and 45 indicates that at these proposed temperatures a substantial part of the graphite radiation damage will be annealed and simultaneously a surprisingly large fraction of the fission products will be released. The fission products released from the fuel under the proposed scheme would be carried off by the helium coolant by-pass stream from which they would subsequently have to be removed. Since a fraction of the "hot" helium must in any event be passed through a decontamination section to compensate for the amount of "cold" helium leaking into the system, it may be possible to combine this latter purpose

with the proposed scheme of removing fission products.

It has been calculated that if it were desired to pass the entire core through the above described process, at a rate of 1 cycle per week, and if the core were composed of fuel balls 1 inch in diameter, this would be equivalent to a fuel flow rate of 153 balls per minute. Undertaking the problem of designing equipment for handling balls at such a rate through the annealing holdup chamber and subsequently conveying them to the distributing tank of the top of the reactor appears to be warranted by the benefits attainable.

Because of the uncertainty surrounding the operations suggested above, no credit was taken for the possible reduction in radiation damage nor for the decrease in fission product poisoning. In other words, the time cycle for chemically reprocessing the entire core was based on the assumption that all fission products and radiation damage were retained by the fuel.

2.1.9 Accessibility for Maintenance

The ultimate use of nuclear energy for central station power will depend upon the safety, reliability and economics of the system. In general, all reactors must be safe to be utilized for any purpose. However, central station power reactors have the additional requirements, not necessarily required of test reactors or military reactors, that the system must be economical or that it must compete with the present day fossil fueled plants. A power reactor for central station plants must have a low capital cost and must be able to produce power at the low rates presently offered by conventional plants. It is therefore quite important for any proposed nuclear power reactor to be a reliable system, one that is able to be operated for times comparable to conventional plants. In the study of the economics

of this system it was assumed that the plant would operate with a load factor of 80%. This is the normal figure used for a conventional steam plant in a modern system. Any deviation from the load factor of 80% will, of course, affect the power production costs by a large factor and in the case of nuclear reactors, where the capital costs tend to be higher than for conventional plants, this deviation is of even greater significance. It is therefore more important in a power reactor to have a system that is either quite reliable or one which may be maintained with the minimum of time lost to outage.

In the design of this reactor two factors were incorporated into the design to reduce the down time of the reactor. The first provision was to design the system for maximum reliability. The second provision was to locate the equipment such that maintenance could be performed on the equipment without requiring the system to be shut down.

The following factors were incorporated into the design to give the system maximum reliability:

a) Choice of coolant - Possibly the most important factor tending to give reliability to the system is the use of a coolant that is compatible with all materials. The use of helium for the coolant does away with all the problems of corrosion encountered in many of the reactor systems being studied.

b) Simple core and pressure vessel design - The core and pressure vessel have a minimum of moving parts so that mechanical difficulty is greatly reduced.

c) Primary loop designed to operate with activity - The system is designed to operate with activity in the primary loop by controlling the leakage and assuring that all leakage is into the system. The primary loop is shielded to operate with radioactivity in the primary loop. There is not

the possibility of having to shut down because of a ruptured fuel element as is the case with many reactors using fuel elements with cladding.

d) Use of secondary heat transfer loop - A secondary "cold helium" loop was incorporated in this design to reduce the need for maintenance of radioactive equipment to an absolute minimum and to improve the safety of the system. The extra cost of the primary heat exchangers, amounting to about $\frac{1}{4}$ mill per kwh, was considered justified by the two big advantages obtained.

- 1) By incorporation of a secondary loop in the system the primary heat exchangers contain helium on both sides of the tubing walls. This eliminates the danger of corrosion by water which would exist if the "hot helium" were used directly to generate steam.
- 2) This system makes it possible for operations to continue even if some small leaks should develop, since the only result would be the leakage of a small amount of "cold helium" into the primary coolant stream. If the "hot helium" were used directly to generate steam, this would not be true since the steam would be at such a high pressure that the amount leaking in even through a pinhole would be quite substantial. The presence of steam in the reactor core would raise problems of chemical compatability as well as danger of sudden increases in nuclear reactivity.

e) Stringent specifications on materials and fabrication - All materials and fabrication techniques such as welding are subject to the stringent requirements familiar in reactor work. Although this increases the cost of

the components and piping many fold, it is a requirement well justified when it gives the reliability required in a reactor system.

Regardless of how much care goes into the design of a system, and regardless of how stringent specifications are on materials and fabrication techniques, eventually a system will require a certain amount of maintenance. In the case of central station power plants where the lifetime of a plant is many times the present accumulated experience on operating reactors, it is only natural that maintenance is to be expected. The proposed design attempts to reduce the time of maintenance and to decrease outage times to acceptable values by the locating of equipment as shown in Figures 2-3 and 2-4. As may be seen in these figures, the power output from the reactor is divided into four primary loops, each of which is contained in a separate cell, shielded from the adjacent one. It is therefore possible to operate three of these primary loops by closing the valves in the main lines of the loop that requires maintenance. The shielding is adequate enough to allow a maintenance crew to enter a pit and remove equipment while the reactor is still operating.

The equipment in the secondary loops such as the boilers, economizers or superheaters may be removed quite readily since this equipment operates with no activity and therefore there is no shielding above the cells in which they are located. One of the main reasons for installing a secondary helium loop into the system instead of passing the helium containing radio-activity directly to the steam generating equipment was to keep the steam generating equipment free from radiation. This was felt necessary since there is a greater probability of having to do maintenance on the equipment in contact with water and helium as compared to the equipment with helium

on both sides of the tube wall. An overhead crane is provided in the dome to facilitate the removal of the shielding plugs and to remove the equipment from the cells.

2.2 Design Data

The following is a summary of the design data for the proposed reactor system. A complete description of the individual components listed here may be found in another section of this report, along with the assumptions and criteria for picking the final design values listed below.

2.2.1 Overall Plant Performance

Thermal Power Output	MW	1164.5
Net Electric Power Generated	MW	400.0
Plant Efficiency	%	35.5

2.2.2 Reactor Data

Core

Diameter	ft.	16.0
Height	ft.	24.0
Diameter Central Gas Hole	ft.	2.5
Initial U-233 Content	kg	1040
Thorium Content	kg	19,000
Graphite Content	tons	223
Void Volume	%	37.5
Refueling Cycle	years	.86
Conversion Ratio	-	1.02
Thermal Flux	n/cm ² /sec	3.4×10^{13}
Fissions per Atom of U-233	-	0.44

Thermal Fissions	%	60
Resonance and Fast Fissions	%	40
Thermal Eta	-	2.31
Resonance Eta	-	2.10
Temperature Coefficient (Equil. Xe Poisoning)	-	-5.34×10^{-5}
Temperature Coefficient (Max. Xe Buildup)	-	-5.10×10^{-5}
Fuel Element Configuration	-	1" Sphere
Number of Fuel Elements (Spheres)	-	1,541,000

Reflector

Material		Graphite
Thickness	ft.	1.0

2.2.3 Thermal Data on Reactor at Full Power

Operating Pressure	psi	1000
Coolant		Helium
Coolant Inlet Temperature	°F	341
Coolant Outlet Temperature	°F	1400
Coolant Flow (Total)	#/sec	833
Number of Flow Loops	-	4
Maximum Fuel Element Surface Temperature	°F	1680
Maximum Fuel Element Internal Temperature	°F	2480

Pressure Vessel

Material		Main Vessel-Carbon Steel
Wall Thickness	in.	8

Outside Diameter	ft.	20.0
------------------	-----	------

Approximate Overall Length (including head)	ft.	52.0
--	-----	------

Heat Exchangers

(For the design data about heat exchangers
see Section 3.2).

CHAPTER 3

REACTOR COMPONENTS

3.1 Pressure Vessel and Core Assembly Design

In the design of the pressure vessel, the following objectives were desired.

1. Simplified design requiring no fine machining or close tolerances.
2. Conventional material requiring no special fabricating techniques.
3. A minimum of moving parts to minimize or eliminate the requirement of having to do maintenance, inside the pressure vessel.
4. Limit vessel diameter to 20' so that the vessel could be fabricated in shop and shipped by barge.
5. Elimination of any mechanical joints to prevent leakage of helium.

To attain these objectives, several core configurations and pressure vessel designs were studied. The design selected is shown in Figure 3-1. The core and pressure vessel design consists of four main parts; structural, loading and discharging mechanism, thermal shielding, and reflectors.

3.1.1 Reflectors

A twelve inch thick shell is provided between the container shell and the thermal shields. This reflector is either a solid shell perforated with holes and acting as an integral part of the graphite container shell or the reflector may be graphite spheres similar to the graphite spheres inside the reactor core, but of course not impregnated with fissile or fertile material. A reflector is also provided in the cone shaped space between the top of the core and the conical baffle plate. In this area the reflector consists of graphite spheres since this reflector is subjected

REACTOR PRESSURE VESSEL

Figure 3-1

ORNL LR DWG. 23461

to the temperature drop across the reactor and a solid graphite reflector would be subjected to extreme thermal stresses.

3.1.2 Thermal Shielding

In order to reduce the stresses in the pressure vessel walls, thermal shields are required to absorb the heat generation due to neutrons and gamma rays leaking from the reactor core. An analysis of the heat generation within the pressure vessel walls is included in Appendix A, Section 10.2.1, and the results are shown in Figure 3-2. Figure 3-2 indicates that six inches of thermal shields are required. The proposed design consists of three 2-inch thick shells with a one-eighth inch gap between the shells and between the outside shell and the pressure vessel wall. The thermal shields are not shown in Figure 3-1 because of the limitation of space for such a drawing. The thermal shields are to be provided along the sides of the pressure vessel and at both the top and bottom of the pressure vessel. The thermal shields are supported off of the ring indicated in Figure 3-1 and are also connected to the pressure vessel walls for support. The thermal shields are made of plain carbon steel.

3.1.3 Loading and Discharging Mechanism

The fuel loading and discharging system was previously discussed in Section 2.1.7. The loading provision incorporated into the pressure vessel involves only the installation of eight pipes which penetrate the pressure vessel wall and extend through the top graphite reflector and terminate at the top of the reactor core. These pipes are made of stainless steel since they are subjected to a higher temperature than the pressure vessel walls. The discharging mechanism, as well as the support for the core structure, is a radial grate structure at the bottom of the reactor

pressure vessel. The cool gases flow down the walls of the pressure vessel and thermal shields and some go up through the grate structure reducing the temperature in the grates. The grate structure consists of radial rotating grates which are operated from a single drive that penetrates the pressure vessel. The grates are to be designed such that a predetermined number of graphite spheres will be discharged for a given speed on the drive mechanism. Time did not allow a detailed study of the grating structure but the scheme involved seems both adaptable to this reactor and feasible technically.

3.1.4 Structural

The structural portion of the pressure vessel includes not only the main vessel but the graphite shells inside the vessel supporting the graphite spheres and the grating system previously discussed. The graphite shells are provided to maintain the required core configuration and to provide the gas passages for the coolant helium. Since the shell at the center of the reactor is subjected to extreme temperatures, and since the stresses are very low (only the pressure drop stress), graphite was selected for the material of construction.

The gas flow in the reactor was chosen such that the inlet flow would cool the walls of the pressure vessel, allowing the use of carbon steel. Stress calculations in the pressure vessel wall are included in Appendix A, Section 10.2.1, and the results are shown in Figure 3-2. An eight inch wall thickness was chosen based on this data.

The portion of the pressure vessel at the gas outlet is subjected to the highest temperatures in the reactor. Therefore, this portion of the reactor will be stainless steel.

The other structural members in the vessel are the graphite pipes in which the control rods travel. These are supported off the grating structure at the bottom and are given support at the top of the pressure vessel.

3.2 Heat Exchanger Design

In a power reactor, one of the most important factors influencing the cost of the reactor system is the design of the heat exchangers. This is especially the case for the reactor system under study for the following reasons:

1. The heat exchangers represent the only major components whose design is able to be varied to any large degree.
2. The heat exchangers represent a large percentage of the total investment in the nuclear plant.
3. The design of the heat exchangers influences the pumping power required and therefore has a large effect on the net electrical power output.

Since the design of a heat exchanger for this system involved changing many variables (i.e., tube diameter, shell diameter, shell length and the number of tubes) for the many conditions over which the reactor could be designed, equations were derived for each of the heat exchangers and these equations were then used to determine the cost of the exchanger and its influence on the cost of the system.

The heat exchanger equations are derived in Appendix B, Section 11.4 and 11.5.

Heat exchanger tube length, heat transfer area and cost are shown in Appendix A, Sections 10.2.4, 10.2.5, and 10.2.6. The pressure drop

equations for the heat exchangers are shown in Sections 10.2.7.2 - 10.2.7.5.

The assumptions for heat exchanger cost are reported in Section 8.

The final design data for all the heat exchangers is shown in Table 3-1, and Figure 3-3. It should be noted that the final design values used do not correspond to the design values indicated in Figure 3-4 through 3-12. These curves represent data computed on the ORACLE using the original set of equations (see Section 13.1) and are included since the trends shown are significant. After solving the original set of equations on the ORACLE, a new set of equations which were more representative of the system were established. The results in Table 3-1 are from the revised equations (See Section 13.2)

3.2.1 Primary Heat Exchangers

The primary heat exchangers are designed to have radioactive helium circulated through the tubes and non-radioactive helium around the tubes. The non-radioactive helium is at a slightly higher pressure so that any leakage of He will be into the system. Each primary coolant loop is designed to have two primary heat exchangers, a stainless steel exchanger on the hot end and a carbon steel exchanger on the cold end. All exchangers are to be U type heat exchangers to minimize thermal stresses.

The effect of tube diameter and heat exchanger size on inlet temperature and relative cost is shown in Figures 3-4 and 3-5. It was desired to reduce the inlet temperature as much as possible without increasing costs.

3.2.2 Boilers

The boilers are designed to have non-radioactive helium circulated around the tubes and water and steam inside the tubes. Each secondary loop contains one boiler made of stainless steel. The boilers are U type heat exchangers to minimize thermal stresses.

The effect of tube diameter on the boiler shell length to diameter ratio is shown in Figure 3-6. The effect of the boiler tube diameter on relative cost, shell diameter and length to diameter ratio is shown in Figure 3-7 for a constant length.

3.2.3 Economizers

The economizers are designed to have non-radioactive helium circulated around the tubes and water inside the tubes. Each secondary loop contains one economizer made of carbon steel. The economizers are U shaped to minimize thermal stresses.

The effect of tube diameter on the economizer length to shell diameter ratio is shown in Figure 3-8. The effect of the economizer tube diameter on relative cost, shell diameter, and length to diameter is shown in Figure 3-9 for a constant length. The effect of the tube diameter on the relative cost and tube length for a constant shell length to diameter ratio is shown in Figure 3-10.

3.2.4 Superheaters

The superheaters are designed to have non-radioactive helium circulated around the tubes and steam inside the tubes. Each secondary loop contains one superheater made of stainless steel. Like all the other exchangers, the superheater is a U type exchanger to minimize the thermal stresses.

The effect of the superheater tube diameter on the superheater shell length to diameter ratio is shown in Figure 3-11. The effect of the superheater tube diameter on relative cost, shell diameter and shell length to diameter ratio is shown in Figure 3-12 for a constant length.

TABLE 3-1
FINAL DESIGN VALUES FOR HEAT EXCHANGERS

	Symbol	Units	High Temp. Primary H. E.	Low Temp. Primary H. E.	Economizer	Boiler	Superheater
Shell Diameter	D	ft	6.50	6.50	4.20	3.75	3.73
Tube ID	d_i	in	1.009	1.01	0.435	1.334	1.334
Tube OD	d_o	in	1.125	1.25	0.625	1.50	1.50
Tube Thickness	t	in	0.058	0.120	0.095	0.083	0.083
Shell Area	A	ft ²	33.2	33.2	13.85	11.0	10.92
No. of Tubes	N	-	1225	1225	1942	173	163
No. of fins per tube	f_N	-	12	12	12	12	12
Fin width	f_w	in	0.404	0.404	0.218	0.667	0.667
Fin thickness	f_t	in	0.10	0.10	0.043	0.133	0.133
Tube Spacing $\underline{L} - \underline{L}$		in	2.12	2.12	1.09	3.25	3.34
Area Inside tubes	A_i	ft ²	6.81	6.81	2.005	1.68	1.58
Area outside tubes	A_o	ft ²	20.58	18.61	8.20	6.22	6.42
Tube area	A_t	ft ²	5.81	7.79	3.645	3.10	2.92
Surface area inside tubes	S_i	ft ²	324 L	324 L	221.5 L	60.5 L	57 L
Surface Area outside tubes	S_o	ft ²	1350 L	1390 L	1651 L	358 L	338 L
He mass flow per unit	$W_H = W_c$	lb/hr	755,200	755,200	755,200	755,200	755,200
He mass flow per unit	"	lb/sec	205	205	205	205	205
Water mass flow per unit	W_w	lb/hr	-	-	701,500	701,500	701,500

Water Mass Flow per unit	W_w	lb/sec	-	-	195	195	195
Mass velocity Inside tubes	G_i	lb/sec-ft ²	30.10	30.10	97.26	116.0	123.4
Mass velocity Outside tubes	G_o	lb/sec-ft ²	9.96	11.02	25.00	33.0	31.93
Avg temperature inside tubes	T_i	°F	1135	605	398	593	796
Avg temperature Outside Tubes	T_o	°F	1020	490	425	833	1165
Fluid density inside tubes	ρ_i	lb/ft ³	0.233	0.350	53.6	-	1.81
Fluid density outside tubes	ρ_o	lb/ft ³	0.252	0.392	0.421	0.288	0.229
Viscosity of fluid inside tubes	μ_i	lb/ft-hr	0.0932	0.0745	0.35	-	0.064
Viscosity of fluid outside tubes	μ_o	lb/ft-hr	0.0895	0.0695	0.0665	0.0838	0.0944
Prandtl Number inside tubes	Pr_i	-	0.654	0.679	0.85	2.55	1.50
Prandtl number outside tubes	Pr_o	-	0.661	0.682	0.684	0.670	0.653
Thermal conduct- ivity of fluid (inside tubes)	k_i	$\frac{BTU}{hr-ft-°F}$	0.180	0.136	0.38	-	0.035
Thermal conduct- ivity of fluid (outside tubes)	k_o	"	0.169	0.127	0.121	0.155	0.180
Velocity Inside tubes	V_i	ft/sec	129.2	85.9	1.812	-	68.2
Velocity outside tubes	V_o	ft/sec	39.5	28.1	59.4	114.8	139.1
Tube ID	D_i	ft	0.0840	0.0841	0.0362	0.111	0.111
	$4 A_o$	ft ²	82.4	74.5	32.8	24.9	25.7

Tube OD	D_o	ft	0.0610	0.0536	0.0199	0.0695	0.0760
	G_i/μ_i	ft ⁻¹	1,161,000	1,456,000	1,000,000	-	6,940,000
	G_o/μ_o	ft ⁻¹	401,000	572,000	1,352,000	1,420,000	1,218,000
Reynolds No.							
Inside tubes	Re_i	-	97,600	122,300	36,200	-	771,000
Reynolds No.							
Outside tubes	Re_o	-	24,500	30,600	26,900	98,800	92,500
	$P_{ri}^{0.4}$	-	0.844	0.856	0.937	1.454	1.176
	$P_{ro}^{0.4}$	-	0.848	0.858	0.859	0.852	0.844
	$Re_i^{0.8}$	-	9810	11,750	4440	-	51,400
	$Re_o^{0.8}$	-	3250	3890	3490	9910	9390
	$.023 Re_o^{0.8}$	-	226	270	102	-	1181
	$.023 Re_o^{0.8}$	-	74.8	89.5	80.4	228	216
Nusselt No.							
Inside tubes	N_{ui}	-	190.4	231.0	95.5	-	1390
Nusselt No.							
outside tubes	N_{uo}	-	63.4	76.6	69.0	194	182.5
	k_i/D_i	$\frac{BTU}{hr-ft^2-^{\circ}F}$	2.14	1.62	10.50	-	0.315
	k_o/D_o	"	2.77	2.37	6.08	2.23	2.37
Heat transfer							
coeff. inside							
tubes	h_i	$\frac{BTU}{hr-ft^2-^{\circ}F}$	408	374	1002	*	438
Heat transfer							
coeff. outside							
tubes	h_o	"	176	181.5	420	-	432
	s_i/s_o	-	0.240	0.233	0.134	-	0.1685
	$1/h_i$	$\frac{hr-ft^2-^{\circ}F}{BTU}$	.00245	.00267	.000999	-	.00228
	$s_i/s_o h_o$	"	.00136	.00128	.000319	-	.00039

* See Section 11.5 for more detailed information concerning the boiler.

Thermal Con- ductivity of tube metal	k_T	$\frac{\text{BTU}}{\text{hr-ft-}^\circ\text{F}}$	12.8	23.0	25.0	*	12.8
	$1.3 \text{ t}/k_T$	$\frac{\text{BTU}}{\text{hr-ft}^2\text{-}^\circ\text{F}}$	.000490	.000566	.000412	-	.000713
	$1/U_i$	$\frac{\text{hr-ft}^2\text{-}^\circ\text{F}}{\text{BTU}}$	.00430	.004516	.00173	-	.00337
Overall coeff. of heat trans- fer	U_i	$\frac{\text{BTU}}{\text{hr-ft}^2\text{-}^\circ\text{F}}$	232.5	221.5	578	-	296
Temp. Drop	$\Delta \theta$	$^\circ\text{F}$	115	115	65.7	155.3	362
	$U_i \Delta \theta$	$\frac{\text{BTU}}{\text{hr-ft}^2}$	26750	25450	38000	-	107000
Fraction of Power extracted	α_p	-	0.509	0.491	0.375	0.397	0.228
Heat transfer Rate	q	BTU/hr	5.05×10^8	4.87×10^8	3.72×10^8	3.94×10^8	2.26×10^8
Surface area Inside tubes	S_i	ft^2	18900	19150	9790	1930	2110
Tube length	L	ft	58.4	59.1	44.2	31.9	37.0
Friction factor inside tubes	f_i	-	.0055	.0052	.0065	-	.0037
Friction factor outside tubes	f_o	-	.0071	.0069	.0070	.0055	.0055
	G_i^2	$\frac{\text{lb}^2}{\text{sec}^2\text{-ft}^4}$	905	905	9450	13450	15210
	G_o^2	"	99.2	121.9	625	1090	1020
Pressure Drop inside tubes	Δp_i	psi	6.41	4.09	-	-	-
Pressure drop outside tubes	Δp_o	psi	1.148	1.022	0.996	4.12	5.15
Maximum pressure at tubes	p	psi	1010	1010	1450	1450	1450
Heat exchanger cost		\$	550,000	154,000	121,000	147,000	157,000

* See Section 11.5 for more detailed information concerning the boiler.

TYPICAL H.E. TUBE
SHOWING FINS.

Item	Number Req'd.	D (ft)	L (ft)	Total Tube Length	No. of Tubes	Tube diam OD (in)	Tube Thick. (in)	Fin Thickness (in)	Fin Length (in)
PRIMARY H.E. (Stainless Steel)	4	6.5	19.00	58.4	1225	1.125	.058	.10	.404
PRIMARY H.E. (Carbon Steel)	4	6.5	19.35	59.1	1225	1.25	.120	.10	.404
Boiler	4	3.75	10.05	31.9	173	1.50	.083	.133	.667
Economizer	4	4.2	15.50	44.2	1942	.625	.095	.043	.218
Superheater	4	3.73	12.65	37.0	163	1.50	.083	.133	.667

Typical Heat Exchanger
Figure 3-3

Fig. 3.4. Effect of Primary Heat Exchanger Tube Diameter on Power Cost and Inlet Temperature.

Fig. 3.5. Effect of Primary Heat Exchanger Size on Relative Power Cost and Inlet Temperature.

Fig. 3.6. Effect of Tube Diameter On Boiler Shell Length to Diameter Ratio.

Fig. 3.7. Effect of Boiler Tube Diameter on Relative Power Cost, Shell Diameter and Shell Length to Diameter Ratio.

Fig. 3.8. Effect of Tube Diameter on Economizer Shell Length to Diameter Ratio.

Fig. 3.9. Effect of Economizer Tube Diameter on Relative Power Cost, Shell Diameter and Length to Diameter Ratio for a Constant Length.

Fig. 3.10. Effect of Economizer Tube Diameter on Relative Power Cost and Tube Length.

Fig. 3.11. Effect of Tube Diameter on Superheater Shell Length to Diameter Ratio.

Fig. 3.12. Effect of Superheater Tube Diameter on Relative Power Cost, Shell Diameter and Length to Diameter Ratio.

3.3 Blower Design

Each main loop contains a blower rated at 28,000 cfm. Leakage of radioactive helium is prevented by installing two shaft seals, and pressurizing the space between them with "cold helium" at a slightly higher pressure.

3.4 Valves

A valve is contained in each of the lines entering the pressure vessel and in each of the lines leaving the pressure vessel. See Figures 2-1, 2-2 and 2-5. These valves contain two seats with "cold helium" between the seats. Under normal operating conditions these valves are open and will be closed only when maintenance is required in a particular loop.

3.5 Piping

In determining the size of the piping, formulae were established relating the pipe size to cost of the piping system. Relationships were also established regarding pumping power in relation to the pipe diameter. These relationships may be found in Section 10.2.7.6 and the results are shown in Figure 3-13. Pipe sizes for this system were established at 18" for all the main piping in the primary and secondary loops. All piping on the high temperature end of the reactor is to be stainless steel, the piping from the second primary heat exchanger to the reactor, and from the cold end of the boiler to the condenser are to be carbon steel. Piping from the boiler, through the superheater and to the turbine are also stainless steel. All piping is welded to the equipment and, as is the case with the equipment previously discussed, the material fabrication and welding are to stringent specifications to reduce the possibility of leakage or of having to do any unnecessary maintenance work.

Fig. 3.13. Effect of Pipe Diameter of Stainless Steel Pipe on Relative Power Costs Investment and Pumping Power.

CHAPTER 4

PHYSICS

4.1 Object of Design

The object of the nuclear calculations was to design a helium cooled thorium-uranium-233 breeder reactor. The reactor should meet the following general requirements:

- 1) The conversion ratio should be equal or greater than unity. This requirement was the basic criterion of the design.
- 2) The fuel elements were to be graphite spheres impregnated with thorium and uranium oxide.
- 3) The core life should be as long as possible.
- 4) An attempt should be made to accomplish these objectives with a single region reactor and a graphite reflector. The thorium would be located within the reactor fuel elements and not in the reflector.

4.2 Symbols (Symbols apply to Chapter 4 only)

η = number of fission neutrons per absorption in uranium

ϕ_T = thermal flux - neutrons/cm²-sec

Σ_{AR}^P = poison absorption cross section in resonance group, cm⁻¹

Σ_{AT}^P = poison absorption cross section in thermal group, cm⁻¹

Σ_{AR}^{23} = U-233 absorption cross section in resonance group, cm⁻¹

Σ_{AT}^{23} = U-233 absorption cross section in thermal group, cm⁻¹

N_1 = number of fast neutrons

N_T = number of thermal neutrons

B^2 = geometric buckling in cm⁻²

τ = age in cm^2

P_T = resonance escape probability for thorium

P_U = resonance escape probability for uranium-233

Σ_{AT}^T = total absorption cross section in thermal group, cm^{-1}

Σ_{AT}^{O2} = thorium absorption cross section in thermal group, cm^{-1}

Σ_{AT}^{PG} = sum of poison and graphite absorption cross section in thermal group, cm^{-1}

L^2 = diffusion length - cm^2

D = outside diameter of reflector, ft

R = radius to outside of reflector, ft

H = height of reactor, ft

k = reactor reactivity

R_T = atom ratio of graphite to thorium

R_U = atom ratio of graphite to uranium

ρ = graphite density

4.3 Method of Calculation

For the reactor under consideration, the slowing-down behavior of the neutrons is of much greater interest and importance than their spatial behavior. This is true for the following reasons:

1. Graphite is a relatively poor moderator, compared to hydrogenous materials. It therefore requires a relatively large number of collisions to thermalize a neutron.

2. Both thorium and uranium have pronounced resonance peaks.

3. The fuel and moderator are present in a homogeneous mixture. This maximizes the likelihood of resonance absorptions.

4. As a result of the above factors, a majority of the neutron absorptions occur in the resonance region.

5. Eta, the average number of fissions neutrons per uranium absorption, depends on the neutron energy.

6. The absorption cross-section of the fission products is also energy dependent.

7. Because of the reactor's large size, the neutron leakage is small. In view of the reactor's large size and simple geometry, it was possible to perform most calculations by hand rather than by machine. To analyze the neutron slowing down behavior, the fast and resonance energy region were divided into eleven hypothetical barriers. At six of these barriers, uranium absorptions were assumed to take place while at the other five barriers thorium absorptions were assumed. The absorptions at each barrier were computed by means of individual resonance escape factors. The uranium absorption and thorium absorption barriers alternated, and the resonance escape factors were calculated by averaging the respective absorption cross-section curve over the corresponding energy interval. In addition to the neutrons lost by resonance absorption, allowance was made (after each of the above barriers) for neutrons lost by leakage from the reactor. At each energy level, an appropriate value of η and fission product cross-section was used. At the thermal energy, the poisoning effect of Xe, Sm, fission product, and heavy nuclides were taken into account. The method used to determine fuel concentrations and conversion ratio is based on the neutron balance diagram of Figure 4-1. The corresponding formulae for reactivity and conversion ratio are shown in Table 4-1. Calculation based on Figure 4-1 and Table 4-1 are accurate only if the individual factors accurately represent the reactor. The primary problem is to obtain the correct factors for fast leakage, thorium resonance absorption, and uranium resonance absorption.

FIGURE 4-1 NEUTRON BALANCE DIAGRAM

These are discussed in Sections 4.4 - 4.6. In addition to the hand calculations, some machine computations using the 3G-3R ORACLE code were performed in order to evaluate the effect of the graphite reflector (See Reference 16).

TABLE 4-1

Formulae for Criticality and Conversion Ratio, based on Neutron Balance of Figure 4-1.

$$k = \eta e^{-B^2 \tau} P_T \left[(1 - P_u) + \left(\frac{P_u}{1 + B_L^2} \right) \left(\frac{\sum_{aT}^{23}}{\sum_{aT}^T} \right) \right]$$

$$\text{Conversion Ratio} \equiv \frac{\text{Number of Neutrons Absorbed in Th}^{232}}{\text{Number of Neutrons Absorbed in U}^{233}}$$

$$\text{Conversion Ratio} = \frac{(1 - P_T) + \left(\frac{P_T P_u}{1 + B_L^2} \right) \left(\frac{\sum_{aT}^{02}}{\sum_{aT}^T} \right)}{P_T (1 - P_u) + \left(\frac{P_T P_u}{1 + B_L^2} \right) \left(\frac{\sum_{aT}^{23}}{\sum_{aT}^T} \right)}$$

4.4 Fast Leakage

The accuracy of the fast leakage calculation depends mainly on the correct determination of the neutron age. As the basis of this determination, the age in 1.6 density graphite, from fission energy to a thermal energy of 0.025 ev, was taken to be 364 cm^2 (Ref. 3). This value was corrected for several factors. In a graphite ball reactor, the average moderator density is reduced by the voids between the balls; age being inversely proportional to the square of the mean density of the core. Moreover, the age will increase even more than the reduction in average density would lead us to expect. This

is shown by Reference (4) which discusses pebble bed shields. The oxygen contained in thorium and uranium oxide will reduce the age by a small amount. Neutron absorption in the thorium and uranium resonances will also change the age. When all of the factors are included (See Appendix A - Section 10.1.1), the age from fission energy to the thermal energy of this reactor (0.10 ev) was found to be 820 cm².

The buckling was taken to be that of a cylindrical reactor, or $B^2 = \left(\frac{2.405}{R} \right)^2 + \left(\frac{\pi}{H} \right)^2$. Where R is the radius to the outside of the reflector. This assumes that the reflector savings are equal to the reflector thickness. As the age in graphite balls is so large (> 820 cm²) and the diffusion length is also large, the assumption of reflector savings equal to reflector thickness appears justified. A three-group-three region ORACLE code was used to investigate the effects of different reflector dimensions and densities.

4.5 Thorium and Uranium Resonance Capture

Since both uranium-233 and thorium have strong resonance absorptions and since many of their resonance peaks are intermeshed, the respective resonances of the two absorbers are to some extent mutually self-shielding. As a result, the calculation of resonance absorptions for either material depends on the concentration of the other. To accurately include this interdependence, the uranium fast and resonance absorptions were broken into six energy groups, and the thorium resonance absorptions were broken into five groups. The resonance absorption factors to be used in the neutron balance of Figure 4-1 are calculated in Appendix (A) - Section 10.1.3.

4.6 Thermal Poison

The thermal poisoning effects of xenon, samarium, fission products, protactinium and higher isotopes were included. The calculation of thermal poisons is shown in Appendix A - Section 10.1.2.

4.7 Preliminary Calculations

In order to assess qualitatively the magnitude and trend of various effects, preliminary calculations based on some very approximate assumptions were first performed. The final calculations, however, employed more rigorous methods and - as will be shown - all assumptions were modified to make the results correspond to the final design point.

4.8 Assumptions for Preliminary Calculations

For the first calculations the following assumptions were made:

1. The neutron temperature was taken to be 1660°F , which corresponds to a thermal energy of 0.1 ev.

2. The graphite density was assumed to be 1.6 grams/cm^3 , with a void fraction of 37.5%, giving an average graphite density of 1.0 grams/cm^3 .

3. For the calculation of fission product poisoning, the reactor core was assumed to have a life cycle between reprocessing of 1 year at a thermal generation rate of 800 MW, and an 80% load factor.

4. Fission product poisoning in the fast and resonance groups were ignored in the preliminary calculations. This important factor was considered separately in later cases.

5. The average number of fission neutrons per absorption in U-233 was assumed to be constant at a value of 2.31. The decrease in η in the resonance region was considered separately in later cases.

4.9 Results of Preliminary Calculations

Figure 4-2 shows the effect of uranium concentration. For a solid cylindrical reactor, 16 feet in diameter with a 1 ft. side reflector and 18 ft. in height, the conversion ratio, inventory charge, and critical thorium concentration are shown as a function of fuel concentration.

Figure 4-3 shows the effect of core size. The conversion ratio is plotted as a function of diameter and height.

On the basis of these two sets of curves, a reactor of 16 ft. diameter, 1 ft. reflector, and 18 ft. in height was initially selected as the basis of further calculations. The graphite-uranium atom ratio was chosen as 2500. The average conversion ratios for such a reactor is 1.049, and the graphite-thorium atom ratio is 157.

A typical diagram to show the division of 1000 neutrons is shown in Table 4-2.

4.10 Effect of Reflector Thickness and Density

A three-group, three-region ORACLE code was used to evaluate the effect of the reflector. As three group calculations depend upon the accuracy of the group constants, an attempt was made to select group constants that would yield the same results as hand calculations. To obtain an accurate basis by which to compare results, the hand calculations were for a bare reactor. The fast leakage was broken into 9 separate fast leakages. The uranium resonance was again broken into six groups, and the thorium resonance into five groups.

A comparison of neutron balances resulting from hand calculations and ORACLE calculations, for a bare 18 ft. diameter reactor with an average graphite

Fig. 4.2. Effect of Uranium Concentration on Conversion Ratio, Inventory Charge and Critical Thorium Concentration.

Fig. 4.3. Effect of Core Size on Conversion Ratio.

TABLE 4-2

-68-

TYPICAL DIAGRAM TO SHOW DIVISION OF 1000 FAST NEUTRONS

Total Leakage----- 56.6

Fast Thorium----- 407.1

Fast Uranium-----187.7

Thermal Thorium--- 55.8

Thermal Uranium--262.5

Poison & Graphite- 30.3

Total---- 1000 neutrons.

density of 1.0 gm/cm^3 are shown below:

	Results of Hand Calculation	Results of ORACLE Calculations
Total leakage	7.8%	7.05%
Fast absorption	20.8%	18.6%
Resonance absorption	41.3%	36.2%
Thermal absorption	30.1%	38.1%

It is to be noted that the fast and resonance absorptions from the ORACLE calculations are low compared to those obtained by hand calculations. This means that either the fast and resonance group transfer cross-sections were too high, or the fast and resonance group absorption cross-sections were too low. The smaller leakage (7.05% compared to 7.8%) indicate that a high transfer cross-section is at least partially at fault.

The results, however, do agree closely enough so that the group constants adequately represent the reactor to show the effect of a change in reflector thickness and density.

Figure 4-4 shows that for a constant overall diameter (18 ft), the net core leakage goes up as reflector thickness is increased and core diameter is reduced. This is to be expected and only shows that the reflector savings are not quite equal to reflector thickness.

The net core leakage increased from 7.05% for a bare reactor to 8.97% for a reactor with a three foot reflector. In every case the overall diameter of core and reflector was 18 ft.

As the net leakage goes up, the conversion ratio goes down. This disadvantage is counterbalanced by a reduction of the fuel inventory charge as the core volume is reduced.

Fig. 4.4. Effect of Reflector Thickness on Core Leakage and Conversion Ratio.

ORNL-LR-DWG 24602

Fig. 4.5. Effect of Reflector Density on Core Leakage and Conversion Ratio.

The effect of increased reflector density is shown in Figure 4-5. Since the reflector must provide coolant passages, the maximum possible reflector density is probably around 1.5.

4.11 Neutron Temperature

In the early stages of the project it was estimated that the neutron temperature would be 1660°F, which corresponds to a thermal energy of 0.1 ev. Examination of the final engineering design indicates that this estimate is satisfactory.

It should be pointed out that the neutron temperature is considerably above the moderator temperature. An approximate formula for the neutron temperature from Reference 11 is:

$$T_N = T_M (1 + 1.11 AK)$$

where T_N = neutron temperature in °K

T_M = moderator temperature in °K

A = mass of moderator atom/mass of neutron

= 12 for graphite moderator

$$K = \sqrt{\frac{2}{3}} \sqrt{\frac{293}{T_M}} = \frac{14.0}{\sqrt{T_M}}$$

$$T_N = T_M + 15.35 \sqrt{T_M}$$

A neutron temperature of 1660°F corresponds to an average moderator temperature of 1070°F.

4.12 Possibility of Boiling Out Fission Products

References 8, 10, and 53 show that a large percentage of the fission products may be removed by heating the fuel elements to temperature in the range of 2000^oC. Estimates made on the basis of these references and yields and cross-sections of fission products from Reference 14 indicates that as much as 60-90% of the fission product poisoning may be removed by this process.

Some fission products will be released in the reactor. However, to be conservative, calculations were carried out with the assumption that all the fission products remain in the fuel elements until removed by chemical processing.

4.13 Change of Average Number of Fission Neutrons per Uranium Absorption

The initial calculations were performed on the basis of information that eta of U-233 remains constant, at least up to 25 ev (References 6 and 7). Later information indicates, however, that eta of U-233 does change. Based on the fission and absorption cross-section of U-233 in OCUSOL-A program (Ref. 5) an averaged eta was obtained for each of the six uranium groups by weighing the eta of each of the OCUSOL energy groups by the uranium resonance integral of that group.

These averaged values for eta were then used with the uranium absorptions in each group to determine the effect of a reduced eta. This is shown in Table 4-3. As a result, reactivity was found to be decreased by 3.42%, and conversion ratio by 7.9%.

TABLE 4-3

EFFECT OF A REDUCED ETA ON CONVERSION RATIO @ REACTIVITY

U-Group*	η	$U_{\text{absorption}}$	$\eta \cdot U_{\text{abs.}}$
1	2.30	9.1	20.9
2	2.22	17.5	38.9
3	2.22	9.7	21.5
4	2.22	12.2	27.1
5	2.105	145.1	305.4
6	2.20	7.0	15.4
Thermal	2.31	<u>232.3</u>	<u>536.6</u>
		432.9	965.8

* See page 156 for energy limits of each group

$$432.9 \times 2.31 = 1000.0$$

$$\Delta K = -3.42\%$$

$$\Delta CR = -3.42 \times 2.31 = -7.9\%$$

(Based on 16 ft. diam - 18 ft. solid cylindrical reactor with

$$\rho_c = 1.0; R_u = 2500; R_T = 157)$$

4.14 Fission Product Poison in Fast and Resonance Group

The fission product cross-section, in terms of barns per fission, were obtained from OCUSOL-A compilation of 30 lethargy groups (Ref. 5). (See Section 12.3). These fission products cross-sections were then lethargy averaged over each of the six uranium groups used in this report. The fission cross-section in Group 1 was also corrected for the fission spectrum.

For the initially assumed power level of 800 MW, 0.8 load factor, 1 year reprocessing cycle, and 800 Kg of fuel, the number of fissions per atom of original fuel is 0.4. Although the final design is for a thermal power of 1165 instead of 800 MW, the assumed reprocessing cycle is down to 0.86 years, and the fuel charge is up to 1050 kg. As a result, the number of fissions

TABLE 4-4

ABSORPTIONS OF FAST AND RESONANT GROUP NEUTRONS BY FISSION PRODUCTS

U-group*	ave. Barns/ fission	$\int \sigma_a^P du$	$\int \sigma_a^{23} du$	$P = \frac{\sigma_a^P du}{\sigma_a^{23} du}$	No. of ** neutrons absorbed by U-233	No. of neutrons absorbed by fission Pro- ducts
1	0.21	.216	17.8	.012	9.1	0.1
2	1.88	1.13	36.0	.031	17.5	0.5
3	7.9	2.20	25.2	.087	9.7	0.8
4	28.0	7.52	38.4	.196	12.2	2.4
5	34.4	22.7	673.0	.034	145.1	4.9
6	33.5	7.4	80.0	.092	7.0	0.6
Thermal	---	---	---	---	232.3	---
					432.9	9.3

* See page 156 for energy limits of each group

** Based on an original neutron population of 1000.

$$\% \text{ Change in } K = \frac{9.3}{432.9} = 2.15\% \quad ***$$

$$\% \text{ Change in C.R} = (2.31)(2.15) = 5.0\% \quad ***$$

*** This is the effect of fission product poisoning in the fast and resonance groups only. The effect of fission products in the thermal region has been previously taken into account.

Based on 16 ft diam - 18 ft high-solid cylindrical reactor with a power level of 800 MW for 1 year and 80% load factor.

Uranium absorptions based on:

Graphite density (average over core) 1.0 gm/cm³

$$\frac{N_M}{N_{23}} = 2500$$

$$\frac{N_M}{N_{02}} = 157$$

per original fuel atom is still around 0.4, and the effect of fission products calculated from the initial assumptions therefore remains valid. The resonance integrals for fission products in each of the six groups were obtained for an average fission ratio, i.e., 0.2 fission per atom of fuel.

These fission product resonance integrals were compared with the uranium resonance integrals and weighed for the number of uranium absorptions in each group.

The results show that fission product poisoning in the fast and resonance region reduces reactivity by 2.15% and the conversion ratio by 5.0%.

Tabular results are shown in Table 4-4.

4.15 Effect of Central Gas Duct on Reactivity of Core

The effect of the central gas duct on core reactivity was divided into two parts. First, the reactivity for a cylinder of infinite length is decreased due to the shift in the neutron flux curve toward the radial boundary. In addition, the reactivity also decreases (in a finite cylinder) due to neutrons streaming out of the ends of the duct. These two effects should be treated simultaneously for better accuracy, but due to lack of time, the simpler two step method was used.

The general equation for the radial flux profile in a cylindrical reactor takes the form of a Bessel function.

$$\phi(r) = A J_0(\alpha r) + B Y_0(\alpha r)$$

For an infinite length cylinder, two boundary conditions are applicable.

One is that the flux goes to zero at the extrapolated boundary. The extra-

polated boundary was assumed to be at the reflector outside edge. The second boundary condition is that the derivative of the flux is zero at the core-duct interface.

The use of the two boundary conditions gives a solution that is satisfied when the following relationship holds:

$$\frac{J_0(\alpha R)}{Y_0(\alpha R)} = \frac{J_1(\alpha r)}{Y_1(\alpha r)}$$

where R is radius to outside of reflector and a is the gas duct radius. For a fixed R and a, a value of " α " can be found to satisfy the equation and the buckling is directly related to α squared.

The total buckling for a 24 ft long cylinder of 9 ft radius is shown in Figure 4-6 as a function of duct radius. This buckling can be used to calculate leakage resulting from the shifted flux profile.

The computation of the decrease in reactivity due to neutron streaming out of the central duct is contained in Appendix A, Section 10.1.5. The change in effective multiplication constant due to neutron streaming as a function of duct radius is shown in Figure 4-6. The preceding data is based on the assumption that the bottom end of the gas duct is blocked by an "infinite reflector".

4.16 Calculation for Final Design

The decrease in conversion ratio due to the inclusion of fission product poisoning in the fast and resonance groups, the decrease in η in the fast and resonance groups, and the increase in leakage due to the central gas hole necessitated a complete revision of the preliminary design in order to achieve a conversion ratio of unity.

ORNL-LR-DWG 24603

Fig. 4.6. Effect of Gas Duct Radius on Reactor Buckling and Δk .

The original graphite density of 1.6 with 37.5% voids had to be increased to 1.92. This increase in graphite density not only reduces the total leakage but also decreases the number of uranium and fission product absorptions in the resonance region. Since a smaller fraction of the neutrons are absorbed in the resonance region, the average value of eta is increased.

4.16.1 Final Leakage Calculation

As approximately 92% of the total leakage occurs in slowing down to thermal energy, the final leakage calculation was based on fast leakage correction factors which were applied to the previously computed values. The total leakage was calculated in this final calculation as follows:

1. Total leakage for bare 18 ft. diameter, 18 ft. high solid cylindrical reactor with $B^2 = 1.10 \times 10^{-4}$ and graphite density of 1.6 is 7.8%.

As previously explained, the 7.80% total leakage was computed from 9 fast leakage factors between 6 uranium and 5 thorium groups.

2. Increase in leakage resulting from replacement of outer 1 ft of core by reflector, from Figure 4-4 $= 7.388/7.05 = 1.048$.

3. B^2 for 24 ft high reactor with 2.5 ft diameter hole $= 1.01 \times 10^{-4} \text{ cm}^{-2}$.
Therefore a factor of $\frac{\text{New } B^2}{\text{Old } B^2} = \frac{1.01 \times 10^{-4}}{1.10 \times 10^{-4}}$ must be applied.

4. Decrease in leakage due to higher graphite density $= \left(\frac{1.60}{1.92} \right)^2$

5. Total leakage $= (7.80)(1.048) \left(\frac{1.01 \times 10^{-4}}{1.10 \times 10^{-4}} \right) \left(\frac{1.60}{1.92} \right)^2 = 5.2\%$.

6. Additional leakage due to streaming out of 2.5 ft hole, (with bottom end blocked by reflector material) $= 0.74\%$.

7. Total leakage $= 5.20 + 0.74 = 5.94\%$.

4.16.2 Neutron Balance

Considering the resonance integrals from Appendix A, Section 10.1.3 for thorium and uranium, the resonance fission product poisoning from Section 4.14, the decrease in η from Section 4.13 and using 6 uranium and 5 thorium groups, the following fast and resonance group absorptions are calculated.

For a graphite density of 1.92, $R_U = 3000$, $R_T = 164$

Original fast neutrons (assumed)	1,000.0
Total leakage	<u>59.4</u>
Neutrons absorbed in reactor	940.6
Total fast and resonance absorption	<u>598.4</u>
Neutrons reaching thermal	342.2

Division of fast and resonance absorptions -

U	187.5
Th	404.6
Poison	<u>6.3</u>
	598.4

Division of thermal neutrons

U	262.5
Th	55.5
Poison	20.9
Graphite	<u>3.3</u>
	342.2

To compute the number of absorptions in the resonance region, the number of absorptions in each uranium group was calculated. Using an average η for each group, it was determined that 187.5 resonance uranium

absorption produced 394.3 fast neutrons. The 262.5 thermal uranium absorption produced (with $\eta = 2.31$) the remaining 605.7 fast neutrons so that the reactor was critical and k was equal to unity.

The total number of thorium absorptions were 460.1 or a conversion ratio of $460.1/(187.5 + 262.5) = 1.022$.

The thermal poisons were computed as in Appendix A, Section 16.1.2 with this one exception. The cross section of xenon was taken to be 1.5×10^6 barns, which is the correct value for a Maxwell-Boltzman distribution around 0.1 ev instead of the previously used value of 3.5×10^6 barns.

4.17 Final Core Design

Based on the calculation of Section 4.16, the final reactor design has these characteristics:

Configuration	- - -	cylindrical
Height	- - -	24 ft
Core diameter	- - -	16 ft.
Reflector thickness	- - -	1 ft.
Overall diameter	- - -	18 ft.

Fuel elements - Uranium oxide, thorium oxide impregnated graphite balls

Ball diameter	- - -	1 in.
Graphite density	- - -	1.92 gm/cm^3
Void volume	- - -	37.5%
Average graphite density	- - -	1.2 gm/cm^3
Reflector graphite	- - -	60 tons
Core graphite	- - -	163 tons
Total graphite	- - -	223 tons

Graphite-uranium 233 atom ratio	-	3000
Graphite-thorium atom ratio	-	164
Uranium weight	- - -	1040 kg
Thorium weight	- - -	19,000 kg
MWD/ton	- - -	13,200
Reactor power	- - -	1140 MW
Core life	- - -	0.858 yr.
Average conversion ratio	- - -	1.02
Thermal flux	- - -	3.4×10^{13} neutrons/cm ² -sec

Fission/atom of original fuel/ core life	- - -	0.44
Inventory cost of U-233 in core - (based on 4% inventory charge, \$20,000/Kg, 80% load factor)	-	0.104 mils/KWH - Heat
% thermal fissions	- - -	60.0
Average η	- - -	2.22
Thermal η	- - -	2.31
Average Fast and Res. η	- - -	2.10

Neutron Balance

Total leakage	- - -	5.94%
---------------	-------	-------

Fast and Res. Absorptions

U-233 - - - 18.75

Th - - - 40.46

Poison - - .63

59.84%

Thermal absorptions

U-233	- - -	26.25	
Th	- - -	5.55	
Poison	- - -	2.09	
Graphite	- - -	<u>.33</u>	
			<u>34.22%</u>
			100.00%
$\sum_{AR}^P / \sum_{AR}^{23}$	- - -		.034
$\sum_{AT}^P / \sum_{AT}^{23}$	- - -		.016
Total poisons	- - -		.0272

In several parts of the text, the statement has been inferred that a change in reactivity of 1.0 per cent means a 2.0 % change in conversion ratio. The explanation for this statement is most easily seen by an example.

Assume a reactor design gives the following neutron balance:

- a) Uranium absorptions = 450
- b) Thorium absorptions = 450
- c) Leakage, poison,
Graphite = $\frac{100}{1000}$ neutrons

If the average eta is 2.2, then $k = \frac{2.2 \times 450}{1000} = 0.99$, and conversion

ratio = $450/450 = 1.0$.

To increase the reactivity up to 1.00, the number of uranium absorption must^{increase} by the factor of $1/.99$; or the number of uranium absorption is $450/.99 = 454.5$. The only way to increase the number of uranium absorption (for a given size and fuel exposure) is to reduce the thorium absorption. The new reactor design is:

- a) Uranium absorption = 454.5
- b) Thorium absorption = 445.5
- c) Leakage, poison
graphite $= \frac{100.0}{1000 \text{ neutrons}}$

The change in relative number of absorptions in uranium and thorium can be accomplished by changing the uranium and/or thorium concentrations. If this is done, the leakage and poison absorptions will be approximately the same as before the change in concentrations.

The reactivity of the new design is 2.2×454.5 or 1.00 and the conversion ratio is $445.5/454.5$ or 0.98. Thus the reactivity has increased 1.0 per cent and the conversion ratio has decreased 2.0 per cent.

4.18 Temperature Coefficient of Reactivity

As the temperature of a reactor changes, the reactivity of the reactor will also change causing a change in the flux level of the reactor and this will then in turn result in a new power level for the system.

The temperature coefficient of reactivity is defined as the amount of reactivity change per $^{\circ}\text{C}$ temperature change. The determination of the temperature coefficient of reactivity is an important consideration in a power reactor since the coefficient must be negative in order to have a stable system, i.e., one which will not allow a divergent power increase with an increase in temperature.

An expression showing the effect of η , f , B^2 , M^2 , and resonance absorption on the temperature coefficient of reactivity is derived in Section 11.9 of this report. In Section 10.1.4, the temperature coefficient of reactivity has been calculated. For steady state poison level the temperature coefficient of reactivity is -5.34×10^{-5} per $^{\circ}\text{C}$. With maximum poison buildup after shut down, the temperature coefficient of reactivity is decreased to -5.10×10^{-5} per $^{\circ}\text{C}$. An examination of the terms in the expression for the temperature coefficient of reactivity shows that by far the greatest contribution is due to the effect of resonance absorption resulting from the Doppler effect.

CHAPTER 5

DESIGN JUSTIFICATION

5.1 Basis for Selection of Final Design

5.1.1 Selection of Coolant

In the selection of a coolant for a gas cooled reactor, the following characteristics of the possible gas coolants should be considered:

1. Pumping efficiency index*.
2. Cooling efficiency index**.
3. Induced activity in the coolant.
4. Chemical activity or corrosion caused by the coolant.
5. Loss of reactivity due to the coolant flow.
6. Availability and cost of the coolant.

Table 5-1 lists the characteristics of the most probable choice of coolants. It may be seen from this table that the best coolants appear to be hydrogen and helium from the standpoint of pumping and cooling efficiency. As the table indicates, helium has the further advantages of no loss of reactivity and the complete elimination of corrosion problems. Since the system under study is to operate with radioactivity in the coolant stream, induced activity of the gas coolant was not a factor in the selection of the gas to be used. In another section of this report concerning the economics of the system, it may be seen that the cost of the coolant is not a determining factor in the selection of the coolant since it is a negligible cost. Helium was finally chosen instead of hydrogen since it eliminated all problems of corrosion, giving a more reliable system, and since the safety of the system was greatly improved by the use of helium instead of hydrogen.

* Pump work per unit heat transferred - See Table 5-1.

** See Table 5-1 for definition of index relationship.

TABLE 5-1**

SOME CHARACTERISTICS OF POSSIBLE GASEOUS COOLANTS

	Pumping Eff. Index*	Cooling Eff. Index ⁺	Approx. Loss in Reactivity	Induced Activity	Chemical Activity (Corrosion)	Radiation Stability	Cost \$/1000 ft ³
H ₂	652	0.18	0.0007	Negligible	?	-	8.60
He	3315	0.030	0	Acceptable (from im- purities eg A ⁴¹)	inert	-	20.00
CO ₂	6560	0.017	0	high, acceptable C ¹⁴ , O ¹⁷	oxidizing when dissociation	some dis- sociation	0.30
Air	8150	0.013	0.003	Acceptable A ⁴¹ , C ¹⁴ , O ¹⁷	Oxidizing	-	?
H ₂ O	5040	0.0040	0.0007	High, acceptable	Oxidizing	some dis- sociation	?

* Pump work per unit heat transferred $\frac{\mu}{C_p^2 \cdot 2 \cdot K(C_p / K)^{0.4}}$ (lowest number is best)

+ $\rho^2 C_p^3$ @ 100°C and 1 atmos. (highest number is best)

μ = viscosity C_p = specific heat ρ = density K = thermal con-
ductivity

** Table from C. O. Smith, ORSORT

5.1.2 Selection of Moderator

Since it was desired to achieve a conversion ratio of unity, it was necessary to choose a moderator with a very low absorption cross-section. As it was also desired to operate the reactor at very high temperatures in order to achieve good power plant efficiency, graphite was chosen as the moderator.

5.1.3 Selection of Fuel

In line with the above objectives, it was decided that the graphite moderator be homogeneously impregnated with fuel, rather than placing the fuel and moderator in a heterogeneous arrangement. Another attraction of this scheme is that a great deal of previous development work has been done with respect to the impregnation of graphite with various fuel solutions. Moreover, graphite is one of the best structural materials at very high temperatures with regard to retention of strength and resistance to thermal shock. Finally, the use of the above type of fuel elements offers a possibility of annealing radiation damage and boiling out a large fraction of the fission products.

5.1.4 Selection of Fissionable Material

The desire for a thermal reactor with a conversion ratio of unity led automatically to a choice of the thorium U-233 system, rather than U-235 or plutonium.

5.1.5 Selection of Plant Capacity

It was realized that the projected power cost would be very definitely dependent upon the capacity of the plant. In order to study the attractiveness of the proposed reactor under the most favorable circumstances, it was decided to choose the highest capacity which was considered to be of potential interest to power companies, i.e., 400 megawatts.

5.1.6 Selection of Fuel Geometry

In order to eliminate the need for any structural materials within the core, it was decided to introduce the fuel in a random array

of small fuel elements. Various fuel element geometric shapes were considered, such as Raschig rings, Berl saddles, and spheres. Although the former two have the advantage of a much larger ratio of surface area to volume, it was found that this ratio was of no great importance since the gas film temperature drop proved to be relatively small compared with the temperature drop within the fuel element. Spheres, on the other hand, offer the most regular geometric shape and therefore minimize the possibility of any sudden shifting and densification of the fuel bed. Moreover, they offer the advantage that small spheres flowing through large pipes behave somewhat like a liquid and therefore facilitate fuel addition and removal. A further advantage of spheres over the other shapes considered is that they have a lower void fraction (37.5%) and therefore result in smaller Fermi age and lower neutron leakage.

5.1.7 Selection of Core Size

In order to achieve a conversion ratio of unity, it was found necessary to make the core as large as possible. The size of the reactor is limited by the availability of furnaces for heat treatment of the pressure vessel. For this reason, a pressure vessel with an outside diameter of 20 feet was chosen. Such a vessel must, of course, be shipped by barge and the proposed reactor could, therefore, be built only along some waterfront. The possibility of constructing a larger vessel at the site was considered but rejected because of the lack of experience with stress relieving thick-walled pressure vessels by localized heat treatment.

It should be emphasized that the large size of the proposed vessel is dictated by the need for neutron economy and not by the fluid flow or heat transfer requirements.

5.1.8 Selection of Core Geometry

It had at first been planned to place the fuel spheres in a cylindrical basket and have the coolant gas flow through the fuel bed from bottom to top. However, it was found that the gas velocities required to carry off the desired power would result in fluidizing the fuel bed. It would, of course, have been possible to reverse the coolant flow and make the gas pass from top to bottom, but this would have involved passing the very hot gas over the metallic fuel grates at the bottom of the cylinder. To avoid the above difficulties, it was decided to have the fuel in an annular arrangement and have the gas flowing from the outside towards the central axis. (See Figure 3-1). This arrangement not only eliminated the problems discussed above, but also greatly reduced the pressure drop suffered by the gas in passing through the fuel bed. Finally, it offers the advantage that the shell of the pressure vessel only comes in contact with the cold incoming helium stream.

5.1.9 Selection of Reactor Pressure

The stress in the pressure vessel walls was calculated by adding the normal pressure stress to the thermal stress resulting from heat generation in the wall. It was found that for a vessel of the proposed size with 8 inch thick walls and three 2 inch thermal shields, the maximum allowable pressure was 1000 psi. Decreasing the pressure decreases the cost of the reactor pressure vessel but increases the size and cost of the heat transfer equipment and of the pumping power losses. The effect of pressure on power cost is illustrated by Figure 5-4. A pressure of 1000 psi with an 8 inch wall was chosen.

5.1.10 Choice of Power Plant

There are, in general, two major ways in which the thermal energy from a high temperature gas-cooled reactor can be converted into electricity. The first is by direct cycle, using the coolant gas as the working fluid of the power cycle. The second is the indirect method of using the hot gas to boil water, using the resultant steam to drive the turbo-generators.

With the direct cycle, the gas leaving the reactor would pass successively through a turbine, a pre-cooler, and a compressor (which may have several stages with intervening inter-coolers). At first glance, this direct cycle appears to be the better and more logical choice for a gas-cooled reactor since it eliminates the need for a second fluid and for a good deal of additional heat transfer equipment; and for this reason the use of this type of power plant was given serious consideration. However, the following factors favored the indirect steam cycle:

1. Whereas a vast amount of development work has been performed on steam power plants, very little work has been done with closed cycle high pressure gas power systems. In essence, the choice of the direct cycle would have meant piling another development problem on top of the reactor development required. There are very few closed-cycle power plants in the world and their performance is still surrounded by uncertainty.

2. In order to match the thermodynamic efficiency of a steam plant at a given temperature, it is necessary to operate a gas turbine plant at a much higher temperature.

3. Whereas ample performance information about steam plants was available to permit carrying out a design optimization, this would not have

been true of the gas turbine plant. Therefore, the choice of the latter would have forced us to guess at the design parameters whereas with the steam plant, as will be shown, it was possible to carry out a design optimization.

It should be emphasized that our rejection of the gas cycle was not based on the belief that the steam cycle was necessarily superior but, rather, was brought about by the availability of more information about the steam cycle.

In order to achieve a high thermodynamic efficiency, it was decided to employ steam conditions in line with latest modern power plant practices. For this reason the steam temperature and pressure were frozen at 1,000 degrees F and 1450 psia, respectively. The only item permitted to vary in the steam cycle was the number of feed water heaters which, in turn, determines the temperature and enthalpy of the condensate entering the economizer. We were fortunate in receiving some very useful performance data from Mr. Roy Dunham of TVA which enabled us to assess the effect of the number of feed water heaters on over-all plant economics.

5.2 Design Optimization

Having decided on the general system to be employed in the proposed plant, it was desired to arrive at an optimum design. It was felt that the only basis for optimizing the design was to determine the effect of every variable on the cost of power produced. To do this, a set of 51 equations was developed which, when applied to a given set of input variables, led to a final cost in mills per net electric kilowatt hour. These equations were based on fluid flow relations, heat transfer relations, pressure drop,

and power loss formulas, and equipment cost estimate formulas. The following parameters were considered to be independent variables:

N_{FW}	No. of feed water heaters
x	No. of primary coolant loops (in parallel)
y	No. of primary heat exchangers per primary coolant loop (in series)
b	No. of secondary heat exchangers (in parallel), i.e., boilers and super-heaters
θ_3	Temperature of helium leaving reactor
θ_8	Temperature of helium leaving boiler
d_F	Fuel diameter
D_O	Diameter of central gas hole in reactor
p	Pressure of primary and secondary helium
D_X	Shell diameter of primary heat exchanger
L_X	Tube length of primary heat exchanger
d_X	Tube diameter of primary heat exchanger
L_e	Tube length of economizer
L_b	Tube length of boiler
L_s	Tube length of super-heater
P_e/P	Power consumed in pumping gas through economizer
P_B/P	Power consumed in pumping gas through boiler.
P_s/P	Power consumed in pumping gas through super-heater
P	The total thermal power produced by the reactor

The dependent variables which emerged from the use of the above equations were:

M	The power cost in mills per net electric kilowatt hour
θ_1	The temperature of the helium entering the reactor

P_{ne}	The net electric power produced by this system
I	The total investment required for the system
d_e	Tube diameter of economizer
d_b	Tube diameter of boiler
d_s	Tube diameter of super-heater
w_h	Mass flow rate of helium
w_w	Mass flow rate of water

After completing the derivation of the 51 equations, our first impulse was to determine the point of minimum cost by taking partial derivatives of power costs with respect to each of the variables. Setting each of the derivatives equal to 0 would thus result in a set of simultaneous equations equal in number to the independent variables. Such a scheme, however, would not be practical because the minimization of power cost is not the only criterion for choice of the design. In addition to cost minimization, the following other criteria had to be satisfied:

1. The proportions of each piece of equipment had to be reasonable; for example, the ratio of tube length to shell diameter for each of the heat exchangers could not be allowed to fall below a certain reasonable value.
2. The tube diameter of each heat exchanger had to be large enough to facilitate welding. For this reason, a lower limit of 0.5 inches was established for the tube OD.
3. It was desired to keep the temperature of the helium at the blower as low as possible to minimize the need for additional blower development.

Since it was therefore impossible to optimize the design analytically, it was desired to determine the effect of each variable on each of the above criteria. Because of the large number of variables involved and the endless number of combinations possible, it was decided to employ the ORACLE, an Electronic Digital Computer, for this design optimization. Accordingly, a code was prepared incorporating the 51 equations previously referred to. For each set of input variables, the computer was instructed to automatically try various values of P, the thermal power, until it found a value which resulted in a net electric power output of 400 megawatts (plus or minus 0.5%). With this code, the computer was able to complete a calculation corresponding to a given set of input variables, including print-out, in approximately 7 seconds. It is estimated that several thousand combinations were studied in order to arrive at the final design.

To summarize, the design optimization effort had the following objectives in order of decreasing importance:

1. Arrive at sound engineering design.
2. Develop a method of optimizing design with the aid of electric computers. This method would be valid even if the assumptions employed were not.
3. Optimize the design on the basis of the assumptions employed.
4. Estimate power cost for the optimized design.

The results of this study indicate the following effects for each of the variables:

1. Number of Feed Water Heaters - The number of feed water heaters used in a power plant determines the thermal efficiency of the plant. For a conventional fossil fuel plant, where the cost of fuel is appreciable,

it has been determined that the optimum number of feed water heaters would be 6. As explained in the section discussing cost estimates (See Chapter 8), this is not true for a nuclear plant. As shown by Figure 5-1, the power cost would be a minimum for either no heaters or one heater, with very little difference between them. However, since the use of no feed water heaters results in a lower blower temperature, it was decided to design a system with no heaters.

2. Reactor Outlet Temperature, θ_3 - As could have been predicted, increasing the reactor outlet temperature lowers power cost and also decreases the blower temperature. This effect is shown by Figure 5-2. The outlet temperature is limited by the mechanical properties of available materials at high temperatures. Consequently, an outlet temperature of 1400 degrees F was chosen.

3. Plant Capacity, P_{NE} - The net electric power output of a plant of this type has a very pronounced effect on the power cost. This is illustrated by Figure 5-3. It can be seen that while a plant of 400 megawatt capacity could produce power at a competitive cost, this would not appear to be true for a plant of half that size.

4. Number of Primary Coolant Loops, λ - Calculations were performed using 4, 6 and 8 primary coolant loops. The results indicated that 4 loops gave the lowest power cost. It is possible that going to 3 loops would reduce costs even further. However, the design has been predicated on the principle that even if one of the loops should have to be shut down for maintenance work, the reactor could continue to operate at partial capacity by using the other loops. For this reason, it was deemed undesirable to reduce the number of coolant loops below 4.

Fig. 5.1. Effect of the Number of Feedwater Heaters on the Relative Power Cost and Inlet Temperature.

ORNL-LR-DWG 23517

Fig. 5.2. Effect of Reactor Outlet Temperature on Relative Power Cost and Inlet Temperature.

Fig. 5.3. Effect of Power Output of Proposed Plant on Relative Power Cost and Investment.

5. Number of Primary Heat Exchangers in Series per Loop, y - Comparison of systems with 2 and 3 heat exchangers in series indicated that the former resulted in lower costs. The use of just 1 primary heat exchanger per loop was considered undesirable because it would make the equipment excessively bulky, and because the use of 2 units has the advantage of permitting the employment of a less expensive carbon steel exchanger for the lower temperature unit.

6. Number of Secondary Heat Exchangers, b - Systems containing 2, 4, 6 and 8 economizers, boilers, and superheaters were studied. The results of the calculations indicate that the use of 2 units of each type would lead to lower costs than 4. However, the design simplification possible by having one set of secondary heat exchangers per primary loop was felt to justify going to a design with 4 economizers, boilers, and superheaters.

7. System Pressure, p - The effect of the primary and secondary helium pressure is shown by Figure 5-4. As would be expected, increasing the system pressure decreases the size of the required equipment as well as the power lost in pumping the coolant gas through the system. However, increasing the pressure also necessitates the use of a thicker walled pressure vessel. The use of the thicker wall in turn increases thermal stress in the vessel wall and eventually a point of minimum power cost is reached, as shown by Figure 5-4. Consequently, a value of 1000 psi was chosen for the system pressure.

8. Temperature Difference at Cold End of Boiler, $\theta_8 - \theta_{11}$ - The effect of the difference in temperature between the helium leaving the boiler and the water entering it would have been very difficult to predict. This effect is shown by Figure 5-5 where it is seen that the power cost is a

Fig. 5.4. Effect of System Pressure on Power Cost

Fig. 5.5. Effect of Temperature Difference at Cold End of Boiler on Relative Power Cost, Inlet Temperature and Tube Length.

fairly flat minimum as long as the above temperature difference lies between 10°F and 40°F . Increasing the above temperature difference results in an increase of the blower temperature. A value of 30 degrees F was chosen as the design point.

9. Fuel Diameter, d_F - The effect of the fuel diameter is illustrated by Figure 5-6. Increasing the fuel diameter decreases the power loss in passing the coolant through the reactor core. However, it entails the disadvantage of increasing the thermal stress of the fuel sphere and its maximum temperature. Consequently, the maximum size of the fuel balls is dictated by the allowable stress of the graphite. The thermal stress of the fuel sphere is very strongly dependent on the thermal conductivity of the materials. It is known that graphite impregnated with fuel suffers a very substantial decrease in thermal conductivity as a result of radiation damage. There is some uncertainty as to the magnitude of this decrease as well as to the extent to which the damage could be mitigated by high temperature annealing. If it is assumed that as a result of radiation damage the thermal conductivity of the material decreases to $1/8$ its unirradiated value, then for the proposed reactor the maximum allowable fuel diameter would be 1 inch. If it were felt that the decrease in thermal conductivity may be even more drastic, then it would be necessary to reduce the fuel diameter to a smaller value.

10. Diameter of Central Gas Hole Core, D_O - The effect of this variable is shown by Figure 5-6. It can be seen that increasing the hole diameter decreases the pressure drop through the core. It also decreases the velocity of the helium entering the hole from the annular core region and therefore decreases the turning losses. However, as shown in the section on nuclear

Fig. 5.6. Effect of Fuel Diameter and Central Core Hole Diameter on Power Required to Pump Gas Through Reactor.

calculations, increasing the size of the central hole has a substantial effect on reactivity and conversion ratio. Since it is imperative to maintain a high conversion ratio, a compromise choice of $2\frac{1}{2}$ foot diameter was made for the central hole.

11. Pipe Diameter, D_{PS} , D_{PC} - The effect of pipe diameter is shown by Figure 3-13. Increasing the pipe diameter decreases the pressure drop and power loss, while decreasing the diameter decreases the cost of piping and valves. For conventional chemical plants nomographs have been prepared showing the optimum pipe diameter for a given condition. If these nomographs were applied to our reactor, they would indicate an optimum pipe size several feet in diameter. It is interesting to note from Figure 3-13 that because of the higher cost of nuclear piping and valves, the optimum diameter for our system actually lies between 1.4 and 1.7 feet. Consequently, a diameter of $1\frac{1}{2}$ feet was chosen both for the stainless steel and carbon steel piping.

12. Primary Heat Exchanger - Preliminary calculations indicated that power costs are reduced by minimizing the primary heat exchanger tube length and increasing its shell length. However, to satisfy the U-shaped heat exchanger configuration which had been chosen, there was a limit to how much the ratio of tube length to shell diameter could be reduced. This limit was arbitrarily chosen to be 8. Figure 3-5 shows the effect of varying the primary heat exchanger dimensions (keeping a constant ratio of $\frac{L_x}{D_x} = 8$) on the power cost. It is seen that the power cost goes through a minimum. This same minimum occurs at approximately the same point regardless of the tube diameter chosen. For the final design a tube length of 44 feet and shell diameter of $5\frac{1}{2}$ feet was chosen for the primary heat exchangers.

Figure 3-4 shows the effect of varying the tube diameter while keeping the over-all dimensions of the primary heat exchanger constant. It is seen that costs approach a minimum at a tube diameter of 1.2 inches. However, decreasing the tube diameter has the desired effect of decreasing the blower temperature. Consequently, a compromise value of 1 inch ID was chosen for the tube diameter of the primary heat exchanger.

13. Secondary Heat Exchangers - The design optimization for the economizer, boiler, and superheater was complicated by the manner in which the respective equations had been cast. It had been felt that there were good reasons for letting the tube diameters appear as dependent instead of independent variables. However, in retrospect, these reasons do not appear to be valid. By the time this was realized, it had been too late to change the computer code and therefore it was necessary to find the optimum point for each exchanger in two steps. First, the result of the ORACLE computations were used to prepare a cross-plot of $(\frac{L}{D})$ versus tube diameter for each exchanger, as shown by Figures 3-6, 3-8, and 3-11. Subsequently, values were read off from the above figures and replotted in the manner shown by Figures 3-7, 3-9, 3-10 and 3-12. From these figures, the final design parameters for the economizer, boilers and superheaters were chosen.

5.2.1 Revised Set of Equations

As previously explained, the 51 equations listed in Section 13.1 were employed to locate what was judged to be the optimum design for the proposed system. In the course of this work, it soon became apparent that a number of these relationships could have been expressed in a manner more desirable for efficiently determining the optimum design. However, lack of time precluded changing the ORACLE code to take account of these possible

improvements. Consequently, the work leading to the choice of the proposed design was carried out in a somewhat cumbersome and round about manner, using the original set of 51 equations.

After this phase of the work had been completed and a design chosen, a detailed and more rigorous check calculation was performed on a system of the chosen design. At that time, it became apparent that some of the approximations inherent in the original set of equations caused greater errors than had initially been anticipated. It therefore appeared desirable to modify the original equations by making their derivations more rigorous.

Consequently, the original equations were reworked, in accordance with the above paragraphs. All revisions and changes made can be divided into the two categories outlined above:

1. Revisions to Facilitate More Efficient Choice of Optimum Design

a. From the original work, it was readily apparent that the optimum heat exchanger design from the cost standpoint was one having a minimum tube length L and a maximum shell diameter D . Combining these conclusions, costs could be decreased by reducing the ratio (L/D) . However, there was obviously a lower limit the values of (L/D) could assume and still have a heat exchanger having the geometry which had been chosen to minimize thermal stress problems (U-tubes and U-shell). It was therefore decided, somewhat arbitrarily, that the minimum practical value which the ratio (L/D) could assume was 8. Accordingly, the equations for the primary heat exchangers, economizers, and superheaters were modified so as to produce a ratio of $\frac{L_x}{D_x} = \frac{L_e}{D_e} = \frac{L_s}{D_s} = 8$ automatically. The reason this same change was not carried out for the boiler was that the much greater complexity of the boiler heat transfer equations made it impossible to obtain an explicit expression for the ratio $\left(\frac{L_B}{D_B}\right)$.

As a result of this change, it is now no longer necessary to make complicated families of cross-plots of the nature of Figures 3-8 and 3-11, in order to choose an optimum design. Each heat exchanger (except for the boiler) now has but one independent variable, namely its tube diameter. The optimum design for each can now be determined simply by plotting power cost versus tube diameter, the required tube length and shell diameter being automatically defined.

b. Under the system as set up in the original 51 equations, the reactor inlet temperature had been a dependent variable. In other words, after the various independent variables had been chosen, the reactor inlet temperature was automatically determined. This proved to be a rather poor system since changing some of the independent variables would affect not only the relative power costs, but also the inlet temperature. Although the primary objective of the design optimization was to minimize costs, the reduction of gas temperature at the blowers was a desirable secondary objective. Frequently these two requirements were found to conflict, since changing a given variable would lower costs at the expense of a higher inlet temperature, or vice versa - See Figure 3-5. In these cases it was necessary to choose the design point on the basis of qualitative judgement.

It would, of course, be much more desirable to obtain a direct measure of the effect of a given variable on cost, at fixed reactor inlet and outlet temperatures. Accordingly, the revised formulas have been reworked so as to make the inlet temperature an independent variable. This has been done by changing the dimensions of the primary heat exchangers to dependent variables.

c. In the revised equations, the fuel ball diameter is treated as an independent variable and the minimum allowable thermal conductivity is a dependent variable. This reverses their previous roles.

2. Revisions to Make the Equations More Rigorous and Exact

a. In deriving the first set of equations, it had been assumed that the resistance of the tube wall to heat flow was negligible in comparison to the gas film resistance. A more rigorous check of the final design revealed that this was not true and that the tube wall resistance should, in fact, be taken into account. The appropriate changes were, therefore, incorporated in the revised equations.

b. In the previous set of equations it had been assumed that the surface of the fins was essentially at the same temperature as that of the tubing; or, in other words, that the fins had an effectiveness of 100%. The new equations were modified by assuming a fin effectiveness of only 80%.

c. In the original set of equations for heat transfer in the boiler it had been assumed that the heat transfer coefficient of the boiling water surface was 19 times as great as the heat transfer coefficient on the helium side. An exact check on the final design indicated that this assumption was not on the safe side. There are, of course, equations available for calculating the heat transfer coefficient of boiling water, but they are given in terms of the temperature difference between the tube wall and the water boiling point. This makes it extremely difficult to arrive at an explicit expression for the required boiler heating surface area, taking account of the actual boiling heat transfer coefficient. Subsequent to the completion of the machine calculations, an exact derivation for doing this was devised and the resultant formulas have been included in the list of revised equations. (See Equations 10 - 14).

c. As explained in Chapter 8, the cost of heat exchangers had been based on a quotation for a similar unit recently obtained by the REED Division of ORNL. To obtain the cost of a differently sized unit, it had

been assumed that the price varied as the 0.562 power of the heating area. Subsequently, this assumption was checked against a quotation for a much larger unit which the REED Division had obtained, and its cost was found to be considerably higher than the value predicted by the previous equations. Consequently, to be conservative, the revised equations are based on both of the quotations obtained by REED, resulting in an exponent of .676 instead of .562.

d. In the original equations, the capital charges during construction and start-up had been computed by deducting 3% (for depreciation) from the nominal 15% capital charges. In the revised equations another 3% was deducted, since it was considered unlikely that appreciable taxes would be paid before the plant was in operation.

5.2.2 Final Results

The solution of the equations in Section 13.1 gave results as shown in the figures of Section 5.2. Although the curves shown in Sections 5.2 and 3.2 do not represent the final design in some cases, the trends shown on the curves indicate results which could be obtained from the solution of the equations in Section 13.2.

The equations as listed in Section 13.2 were used to arrive at the final design point for the system. Final design data are listed in Section 2.2 and the costs resulting from the selection of this design point are indicated in Chapter 8.

5.3 Coolant Decontamination

To prevent leaks from the highly contaminated system, it is necessary to incorporate a non-radioactive helium jacket at a slightly higher pressure than the hot helium around those areas where leaks are most likely to occur. Therefore, any leak which might develop would be into the hot helium stream,

thus eliminating the possibility of releasing radioactivity into the atmosphere. This, however, poses the problem of helium build-up in the primary hot loop, and therefore requires that a hot stream bypass system be included in the reactor design. In this way the primary loop can be maintained at given operating conditions by allowing a small portion of the hot stream to be removed, decontaminated, and then sent to the cool helium storage vessel for reuse in the cool helium side.

The hot gas clean up system will contain the following seven stages. (See Figure 5-7).

(a) The removal of large particles, such as small pieces of graphite, etc., can be accomplished by the use of AEC high efficiency fiber glass filters. These filters are capable of removing particles down to 0.1 micron size with a 99.9% efficiency. Since the fiber glass is sensitive to high temperatures, it is necessary to decrease the hot gas temperature to about 400⁰F. This, of course, requires a small heat exchanger prior to the fiber glass filters.

(b) Radioactive iodine can be eliminated from the off-gas stream by passing the gas through a packed tower where the packing is coated with silver nitrate. The iodine in the gas reacts with the silver nitrate to form silver iodide. Since iodine retention is not complete after approximately 40% of the silver nitrate has been converted to the iodide it is necessary to renew the bed by flushing it with a silver nitrate solution. This removes the silver iodide and at the same time recoats the packing with silver nitrate. Having two columns in the system might be desirable, thus enabling one column to be in operating while the other is being flushed.

(c) The initial activated charcoal bed can be used for the removal of radioactive Xe. This is accomplished by absorption of the Xe in the highly porous graphite and designing the bed so that the retention time allows the

Xe to decay out. Based on HRT off gas system figures it would require approximately ten tons of activated charcoal to remove the Xe for a 1000 cubic foot per day gas passage (assuming that 1000 ft³/day would probably be the maximum allowable leakage). Problems involving bed geometry are very difficult and will not be discussed in this report due to lack of time.

(d) Since the long lived Kr⁸⁵ can not be removed in the initial charcoal bed, it is necessary to incorporate a secondary charcoal bed system for this purpose. By allowing the gas stream to pass through one of the secondary beds the Kr⁸⁵ will be absorbed. Immediately after the bed is saturated with Kr⁸⁵, the off-gas stream is diverted to an auxiliary charcoal bed where the Kr⁸⁵ continues to be absorbed out. During this time O₂ can be used to strip the Kr⁸⁵ in the saturated bed, thus allowing the Kr⁸⁵ to be sent to a pressurizer where it is compressed in cylinders for storage. After restoring the bed it can again be used for Kr⁸⁵ removal.

(e) To prevent fine particles of charcoal and scale from getting into the reactor system AEC high efficiency fiber glass filters can be used. The cool He gas can then be sent directly to the cool He storage area.

(f) The pressurizer is necessary for compressing the Kr⁸⁵ in cylinders for storage purposes.

(g) The pressure tanks are used for O₂ storage for charcoal bed restoring.

FIGURE 5-7

BY PASS STREAM CLEAN UP SYSTEM

CALCULATION

$$\text{HRT Gas Flow} = \frac{500 \left(\frac{\text{cc}}{\text{min}} \right) \times 1440 \left(\frac{\text{min}}{\text{day}} \right)}{28,100 \frac{\text{cc}}{\text{ft}^3}} = 25.7 \text{ ft}^3/\text{day}$$

$$\text{Size of bed required} = 1000 \left(\frac{\text{ft}^3}{\text{day}} \right) \times \frac{500 \#}{25.7 \text{ ft}^3/\text{day}} = \sim 10 \text{ tons.}$$

5.4 Emergency Cooling

Of the various emergency conditions that might arise, three of the most likely are loss of all electrical power within the plant, loss of pressure, or a combination of the two.

The transient conditions were not analyzed but a few pertinent points should be noted. The core can withstand high temperature transients for shorts periods. Against this must be considered the low heat capacity of the coolant, short coast down time of the helium flow and negligible assistance from natural circulation.

A general simplified equation will be derived to determine the magnitude of the standby power required to remove the 6% shutdown heat of the core, which will be present over a period of several days.

Let

Δp = system pressure drop

V = velocity of coolant

W = mass flow of coolant

ρ = density

l = conditions before incident

2 = conditions after incident

h = feet of head

K = constants.

The pressure drop around the loop is

$$\Delta p = K_1 f v^2 \rho$$

where

$$f \approx \frac{1}{(\rho v)^{.2}}$$

so that

$$\Delta p = K_2 v^{1.8} \rho^{0.8}$$

Since

$$W = \rho AV$$

we may write

$$\Delta p = K_3 v^{1.8} \rho^{.8} \frac{K_4 W^{1.8}}{\rho}$$

and

$$\frac{\Delta p_2}{\Delta p_1} = \left(\frac{v_1}{v_2} \right)^{1.8} \left(\frac{\rho_1}{\rho_2} \right)^{0.8} = \left(\frac{W_2}{W_1} \right)^{1.8} \left(\frac{\rho_2}{\rho_1} \right)$$

The power to drive the blowers is

$$HP = K_5 h W$$

where

$$h = \frac{(144) \Delta p}{\rho}$$

so that

$$HP = \frac{K_6 W \Delta p}{\rho}$$

Hence

$$\frac{H P_2}{H P_1} = \left(\frac{W_2}{W_1} \right) \left(\frac{\Delta P_2}{\Delta P_1} \right) \left(\frac{\rho_1}{\rho_2} \right)$$

or

$$\frac{H P_2}{H P_1} = \left(\frac{W_2}{W_1} \right)^{2.8} \left(\frac{\rho_1}{\rho_2} \right)^2$$

Assuming the temperatures are approximately the same,

$$\rho \propto p$$

and

$$\frac{H P_2}{H P_1} = \left(\frac{W_2}{W_1} \right)^{2.8} \left(\frac{P_2}{P_1} \right)^2$$

The total mass flow required for any given power level will vary directly as the power to maintain the same ΔT across the reactor.

For the case where electrical power is lost to the blowers, the H.P. required to remove 6% of the heat is

$$\frac{H P_2}{H P_1} = (.06)^{2.8} (1) = .0004$$

Hence a stand by power plant having only about $\frac{1}{20}$ of 1% of the rated blower power is required provided rated pressure level is maintained.

In the event that the pressure is lost, the calculations for the containment vessel indicates the pressure level will be about 30 psi. Hence,

$$\frac{H P_2}{H P_1} = (.06)^{2.8} \left(\frac{1000}{30} \right)^2 = .445$$

That is, approximately half of the rated blower capacity will have to be provided to cool the core if the electrical power and pressure are lost.

From the above cursory examination it appears that a small independent power package can be provided for the case of loss of power to the blowers. However, to accommodate the loss of pressure incident, it may be best to provide a standby turbine generator set piped to the main steam generator. The turbine valve would be actuated by a combination of loss of pressure and electrical input.

If only pressure is lost, sufficient output from the main turbine generator exists to provide the blower power or some can be piped in from the mains.

The most likely incident is a loss of pressure due to a large leak. Since the blowers would continue to operate and remove the core decay heat, the pressure and temperature inside the container as a function of time can be calculated numerically. The heat loss through the vessel surface is,

$$\Delta Q = U A \Delta T \Delta t$$

where U = overall heat transfer coefficient due to radiation and conduction.

A = surface area ft²

ΔT = temperature of the vessel gas - temperature of outside air

Δt = time increment in hrs.

The drop in internal gas temperature will be

$$\Delta t = \frac{\Delta Q}{W C_v}$$

$$\Delta t = \frac{U A \Delta T \Delta t}{W C_v}$$

For the 80' sphere case, the overall U can be expressed as:

$$U = 2 - .004 \Delta t \quad , \quad W = 24,600$$

$$C_v = .311 \quad , \quad A = 10,000 \text{ ft}^2$$

So that

$$\Delta t = 1.33 (2 - .004 \Delta t) \Delta T \Delta \theta$$

The heat capacity of the system as a whole, i.e., concrete, walls, etc. is considerable and will be very helpful if the leakage rate approached a step function. The calculations was made neglecting this heat capacity so the results may be considered upper limits.

If a major catastrophe were to occur causing the emergency cooling system to fail, all the shutdown heat output of the core would eventually find its way to the gas inside the container. The heat removal from the sphere surface is strictly limited. For an 80' sphere and a $450^\circ \Delta T$, it is about 11,000,000 BTU/hr. For a 120' sphere, about 25,000,000 BTU/hr. The core decay heat, approximately 5% of the total power, or 50 mw, amounts to 170,000,000 BTU/hr. In order to remove all this heat via convection and radiation from the surface, the required temperatures would be intolerably high, i.e., several thousand degrees. Although the probability of such a catastrophe is very slight, provisions must nevertheless be made to assure the preservation of the structural integrity of the containment vessel, in the event such an incident should occur. It is recommended that this be done by providing a grid of water pipes and spray nozzles surrounding the containment vessel.

The activity released inside the container would be no worse than for other reactors for which core melt down is assumed during similar accidents.

5.5 Power Changes

The power variations in a plant of this size will probably not be very great, since such a plant would presumably be used to handle the base load. However, to operate at part load two schemes present themselves.

First, the pressure could be varied to accommodate the load, with the blowers operating at approximately constant speed. The temperatures across the reactor would remain approximately constant. This control system would be similar to that proposed for closed cycle gas turbines. The disadvantages, particularly for the reactor considered here, is that a great deal of radioactive gas would have to be shuttled, during load changes, from the core to the low or high pressure reservoirs. Such a scheme might increase the leakage of the system. Also, since the helium is radioactive, it is best that it be handled as little as possible. The other disadvantage is that stress cycling of the equipment will occur and thereby bring about leaks much sooner in the life of the plant.

The better control scheme would be to employ variable speed blowers over most of the power range, and to change pressure only over the range where it is uneconomical or impossible to do so with blower speed change. Although this will place a greater engineering demand on the blower and associated drive design, the system is more suitable for the reactor considered here. Changing blower speed is best facilitated by employing steam turbine drives.

CHAPTER 6

SHIELDING

The plant layout for the reactor proposed in this report has been described in Section 2.1.6, and is shown in Figures 2-3 and 2-4. Under normal operating conditions, the reactor pressure vessel and core, primary heat exchangers and the blowers in the primary loop will contain radioactivity. Radioactivity will also be present in the fuel discharging equipment and in the helium storage facilities. Therefore all of this equipment along with the piping containing radioactivity must be shielded.

The two main cases of shielding required for this system are, (1) neutron and gamma shielding and (2) gamma shielding. The reactor pressure vessel and core will contain activity due to both neutrons and gamma rays. All other equipment will contain activity due only to gamma rays, (assuming delayed neutrons contribute only a small percentage of the activity and beta activity is not a factor in this case). Therefore, the amount of shielding required was studied for the maximum case, i.e., shielding adjacent to the reactor pressure vessel when the reactor was operating at full power.

Section 10.2.2 of Appendix A shows that seven feet of high density concrete is sufficient shielding for the system. No attempt was made to optimize the shield design in the limited time available. A further study of the requirements for shields to shield against gamma rays only was not made but a shield of approximately five feet would probably be sufficient. Therefore, the shielding above and adjacent to the reactor pressure vessel would be seven feet of high density concrete. The shielding between the individual compartments, above the primary loops, and adjacent to the fuel discharging

equipment and storage equipment would be five feet of concrete.

All penetrations through the shields are to be such that neutron streaming is either prevented or minimized. The shielding over the primary loops and at places where access to equipment is required, will be removable type shields in the form of plugs. These plugs are to be designed to prevent streaming of activity and the size of these plugs is to be such that they may be easily removed with the maintenance facilities provided.

As was pointed out in Section 10.2.2, there will be a temperature rise in the shield of approximately 50-60°F making it necessary to provide cooling to the inner portion of the shield. This may be provided by embedding piping in the concrete or by laminating the shield and cooling the inner portions with the ventilating air which is used to vent the sealed compartment (see Section 2.1.4).

CHAPTER 7

CONTROLS AND INSTRUMENTATION

7.1 Power Plant Control

An integrated control system which combines the steam system controls with the reactor system controls to give optimum plant performance, both statically and dynamically, is desired. An analysis of the over-all power plants control requirements was not made since the major group effort was spent on reactor system engineering. In addition, linearized approximations of the thermodynamic equations would have to be employed for obtaining a solution. The assumption that the fluid is incompressible is highly questionable for transient analysis of this reactor design.

A qualitative discussion of reactor controls is presented to give some insight into the control requirements for this proposed central power station system.

7.2 Reactor Control

7.2.1 General Discussion

The basic function of the reactor control system is to provide means for adjusting and maintaining reactor thermal power output at a desired level. In addition, absolute safe operation must be provided for in the basic reactor control system.

In a fossil-fueled power plant, the thermal power output is made to deliver heat in accordance with steam demands by controlling the rate of fuel feed to the burners. Analogously, a control system to "match" the fuel-feed control function in a fossil-fuel power plant is required for control of reactor heat generation.

The heat generation in a reactor is determined by the rate at which fissions occur. The rate of fissioning is proportional to the neutron density and to the amount of fissionable material within the reactor. Hence, variation in neutron density results directly in a change in the heat generation of the reactor.

Reactivity (ρ) is the independent variable through which neutron density, and consequently reactor heat generation, is controlled. The reactivity is defined as

$$\rho = \frac{k - 1}{k}$$

where k is the multiplication factor and is the ratio of the number of neutrons of any one generation to the number of corresponding neutrons of the immediately preceding generation.

With zero reactivity, the chain reaction will continue at a constant rate and therefore the power produced will remain constant. If the reactivity is positive or negative, the power will increase or decrease, respectively, with time.

The following methods may be employed for control of reactivity:

1. Variation in the probability of escape of neutrons.
 - a. Variation in reflector effectiveness.
 - b. Variation in moderator effectiveness.
 - c. Variation in the mass and or spatial distribution of the nuclear fuel.
2. Variation in the probability of non-productive absorption of neutrons.

Since the reactor under consideration in this report is a large reactor, variation in reflector effectiveness has only a negligible effect on reactivity. In addition, variation in moderator effectiveness is not applicable for the

homogeneous reactor system under discussion. The provisions for adding additional or reprocessed fuel when the reactor is in operation may be used to offset fuel depletion. The addition of fuel may be employed in an analogously manner as the shim controls are employed in a fixed fuel charge reactor system. It does not appear practical to employ the fuel discharging apparatus as basic control of reactor reactivity. Therefore, non-productive absorbing material or "poisons" for the basic control of reactivity is employed.

The reactivity of a reactor is a function not only of the externally inserted poisons used for control but is also dependent on the poison buildup within the reactor from the fission process, fuel depletion, and reactor temperature. Compensation for fuel depletion has been discussed previously. Compensation for poison buildup within the reactor may be offset by removal of an equivalent amount of poison which is externally controlled. Variation of reactivity with temperature is defined as the reactor temperature coefficient. The reactor system under study has a negative temperature coefficient. A negative temperature coefficient provides self regulation and is therefore mandatory for central station power plant operation. Additional comments on the stabilizing influence of a negative temperature coefficient will be made in the discussion of reactor kinetics. Analytical determination of the temperature coefficient is presented in the Reactor Physic Section of this report.

7.2.2 Reactor Kinetics

The transient behavior of reactor power may be determined if the transient behavior of the neutrons is known. A realistic determination of the transient behavior of neutrons is a formidable problem. However, it is believed that the general observations of the effect of delayed neutrons

drawn from simplified analyses are applicable for a qualitative understanding of the proposed reactor system.

For reactivities less than the total fraction of delayed neutrons the reactor period (time required for neutron density or reactor power to change by a factor e) is dependent upon the weighed mean delay time of the delayed neutrons. This may be approximated as follows:

$$T \text{ (reactor period)} \approx \frac{1}{\rho} \sum_{i=1}^m \beta_i t_i$$

where ρ = reactivity

β_i = fractional yield of ith delay group

t_i = delay time of ith delay group

Since the total fraction of delayed neutrons ($\beta = \sum_{i=1}^m \beta_i$) is approximately 0.25% for U^{233} , the reactivity must be limited to less than 0.25% in the proposed reactor system for the period to be dependent upon delayed neutrons. An estimate of the mean delay time of the delay neutrons for U^{233} is approximately 0.1 second and therefore does not impose any severe time response requirements for reactor control.

For reactivities greater than the total fraction of delayed neutrons, the reactor period is dependent upon the mean neutron lifetime. This may be approximated as follows:

$$T(\text{reactor period}) \approx \frac{l}{\rho}$$

where l = mean neutron lifetime

ρ = reactivity

The mean neutron lifetime is dependent upon the specific reactor design. For the proposed reactor design, a relative long neutron lifetime of approximately 10^{-3} second is obtained. Dangerously fast power excursions should not occur in the proposed reactor system.

The preceding comments have been based on solutions of linear equations. There are a considerable number of factors which introduce non-linearities into the basic reactor kinetic equations. Solutions of non-linear equations do not lend themselves to defining general characteristics of a system. However, for small perturbations, the non-linear equations may be linearized.

A complete discussion of factors which result in the non-linear kinetic behavior of the reactor system is not presented. The following brief discussion is limited to the reactor temperature coefficient.

A change in temperature results in two effects on reactivity. First, the energy distribution of neutrons in equilibrium with their surroundings is altered. Since material cross-sections are energy dependent, the neutron distribution and consequently reactivity is altered. Secondly, the reactor material densities are changed. Since macroscopic cross-sections are proportional to the densities of materials, the neutron distribution and consequently reactivity is altered.

Although the negative temperature coefficient provides inherent regulation, the reactor is rendered susceptible to dynamic oscillations. A negative temperature coefficient of reactivity is required for support of oscillations. Analytical studies of "linearized systems" indicate that delayed neutrons have the effect of damping the oscillations. Postulations have been made that the dynamic stability of a reactor is dependent upon the numerical magnitude of the negative temperature coefficient. It is

not possible to postulate that the proposed reactor system is dynamically stable or unstable without recourse to a thorough dynamic analysis of the over-all power plant system.

7.2.3 Control Rod Requirements

Control rods are generally classified as to their primary functions of regulating, shim, or safety control. In the proposed reactor, breeding or a high conversion ratio is desired. Therefore, the use of non-productive poisons for control must be kept at a minimum.

It is estimated that there would be an excess multiplication factor of 0.22 from reducing the reactor from its nominal operating design condition to a base temperature of 300°K. Assuming a value of 0.9 for the multiplication factor at shut-down conditions, the safety rods must compensate for an excess multiplication factor of 0.32.

To obtain a preliminary estimate of the diameter and number of cylindrical safety control rods required, the formula given in Reference (13) for Δk (the maximum multiplication that can be controlled by a rod) according to the two-group calculation, was used. The product of the correction factors for non-axial position, shadowing, and reactor reflector was assumed to be unity. Assuming a concentric array of 12 safety rods, the maximum worth required per rod is approximately 0.03. The following values were used in determining safety rod diameter:

$$\tau(\text{Fermi Age}) = 816$$

$$L^2(\text{Diffusion Length})^2 = 215$$

$$M(\text{Migration Area}) = 1031$$

A safety rod diameter of 8 inches is required. Although the preceding computation is limited, it is apparent that considerable number of pressure vessel penetrations would have to be provided for installation of safety control rods.

For the proposed reactor system which includes provisions for continuous addition and removal of fuel, the necessity of safety control rods with a large change in multiplication factor is questionable. The large change in the multiplication factor is due to an assumed large temperature change from the normal reactor operating condition to the shut-down condition. For the proposed operation of this reactor system, there is no fundamental requirement of reduction of reactor operating temperature. In addition, provision for emergency cooling of the reactor is required to remove the considerable amount of heat from fission product decay. Establishment of safety rod requirements, therefore, can only be accomplished by a thorough study of the integrated reactor system. Changes in multiplication factor due to reactor loading and startup, cyclic addition and removal of reactor fuel, possible reactor incidents, etc. have to be determined.

The possibility of achieving safe reactor operation with only regulating control rods would be highly desirable. Emergency or backup safety measures would have to be provided for in any event, however, there is considerable difference in mechanical design complexity of provisions for continuous operation or very infrequent operation of the emergency safety controls.

Sufficient time was not allocated to this phase of the proposed reactor system to fully define the reactor control requirements. Consequently, control rod requirements were not determined.

7.2.4 Mechanical Design

The basic mechanical design philosophy in the proposed reactor system was to minimize complex fabrication procedures and to minimize the use of neutron absorbing structural materials that would compromise feasibility of breeding. The installation of reactor control rods and their associated drives is one of the most limiting factors in achieving design simplicity. A possible control rod installation is shown in Figure 3-1. Information required for mechanical design such as heat generation, thermal and mechanical stresses, control rod drive power requirements, etc. was not obtained.

CHAPTER 8

ECONOMICS OF THE SYSTEM

8.1 Basis for Cost Estimation

For purposes of cost estimation, the entire plant was divided into two distinct parts which were called the "nuclear plant" and the "power plant". The former encompasses all equipment needed to produce the steam, including economizers, boilers and superheaters. The power plant includes all equipment necessary for converting the steam to electrical energy, including the feed water heaters.

8.1.1 Basis for Estimating Nuclear Plant Costs

8.1.1.1 Heat Exchangers - Examination of Ref. (54) indicated that the cost of any particular type heat exchanger is proportional to approximately the 0.562 power of its heating area.

An examination of Ref. (55), indicated that a pressure correction factor should also be applied and that a good value for this factor would be $(0.85 + \frac{P}{1000})$. Since the cost of nuclear equipment, because of tighter specifications and inspection requirements, tends to be much higher than that for conventional equipment, it was decided not to use the absolute values recommended by Ref. (54). Instead a coefficient was chosen to make our cost formula fit a quotation the REED Division of ORNL had recently obtained for a specific stainless steel heat exchanger. To this was added a factor of + 10% for finned tubing, - 10% for multiple quantity order, and + 5% for installation. This resulted in the following formula for the cost of a finned tube all stainless steel heat exchanger.

$$\$ = 770 \left(0.85 + \frac{p}{1000} \right) S^{0.562}$$

where p = pressure in psi

S = heating area, ft²

On the basis of information in Ref. (55), it was decided that the cost estimate of an equivalent carbon steel heat exchanger could be obtained by dividing the above formula by a factor of 3.6.

It should be pointed out that the above formulas result in a cost 2.3 times what would have been obtained by applying all appropriate correction factors to the correlation in Ref. (54). Consequently, the use of our formulas implies that the more rigid specifications and inspection required for nuclear plant heat exchangers has the effect of raising costs by a factor of 2.3.

After the completion of the machine computations, it was decided that the previous assumption about the proportionality of cost to the 0.562 power of the heating area was not valid for the special specification equipment under consideration. Instead, a new formula was chosen, based on the previously used quotation plus one new quotation obtained by REED.

1st Quotation: 141 \$/ft² for a 550 ft² unit

2nd Quotation: 38.4 \$/ft² for a 3000 ft² unit

Based on the above prices, the following new and much more conservative cost formulas were arrived at

$$\text{Stainless steel: } \$ = 380 \left(0.85 + \frac{p}{1000} \right) S^{0.676}$$

$$\text{Carbon Steel: } \$ = 105 \left(0.85 + \frac{p}{1000} \right) S^{0.676}$$

8.1.1.2 Piping, Fittings and Valves - From the preliminary layout of the reactor system, the number of valves, length of piping and number of fittings required per loop were established. Based on the cost data made available from the REED Division of ORNL, (Reference 56), the costs of the required piping, fittings and valves were estimated for both stainless steel and carbon steel for various pipe diameters. From these costs the following relationships were established.

Cost of Stainless Steel Piping, Fittings and Valves

$$\$_{PS} = 1.59 \times 10^5 \propto D_{PS}^{1.72}$$

Cost of Carbon Steel Piping, Fittings and Valves

$$\$_{PC} = 4.48 \times 10^4 \propto D_{PC}^{1.72}$$

The costs shown above are for installed cost and include a factor of + 75% for special specifications and inspection.

8.1.1.3 Blowers - The cost of blowers was assumed to follow a formula of the form

$$\text{cost} = C_1 (\Delta p)^{C_2} Q^{C_3}$$

where Q = flow rate in cubic feet per second

Δp = pressure rise in PSI

The exponents C_2 and C_3 were obtained from Ref. (54). The coefficient C_1 was obtained by taking the value given in the same reference and multiplying it by a factor of 2, to take into account the higher cost resulting from more rigid specifications and inspection requirements for nuclear equipment. In addition, the coefficient was multiplied by a cost index of 1.15 to bring it up to date and by a factor of 1.40 to cover installation expense. The final cost formula, after applying the above factors, was:

$$\text{Blower Cost} = 629 (\Delta p)^{0.746} Q^{0.614}$$

8.1.1.4 Containment Vessel - The cost of the containment vessel was arrived at from some estimates made for PWR. The vessel proper was \$564,700; Foundation, \$599,000; Coolant Piping, \$388,000; Ventilation Equipment, \$350,000; Valves and Penetrations, \$150,000; and Miscellaneous Costs, \$350,000. This results in approximately \$2,500,000 as the total containment vessel cost.

8.1.1.5 Pressure Vessel and Thermal Shields - The cost of these items was determined with the aid of unofficial and approximate estimates prepared by Combustion Engineering, Inc. The vessel specified was of all-welded construction, with no flanges, bolts or removable heads. Although the use of various stainless steels and special alloy steels had been considered, it was later decided that the use of less expensive carbon steel was permissible since only cold helium (341°F) came into contact with the vessel walls. Since the system contains no corrosive fluids, there is, of course, no necessity for cladding the vessel. Finally, the vessel costs are reduced because the excessively close tolerances usually required of nuclear reactor vessels should not be needed for the proposed reactor.

On the basis of Combustion Engineering estimates, a value of \$3,200,000 was chosen for the 8" thick C.S. vessel, with three 2" thick thermal shields. Finally, in order to assess the effect of helium pressure on power cost, it was desirable to express the vessel cost as a function of pressure. Unfortunately, lack of time prevented our determining this dependence with any degree of accuracy. A very approximate analysis indicated that the above estimate should be multiplied by a pressure correction factor of $(0.5 + \frac{P}{2000})$.

8.1.1.6 Shielding - The total amount of concrete shielding required for the proposed reactor was estimated at 1700 cubic yards. Multiplying this figure by an installed high density concrete price of \$350 per cubic yard gave an estimated shielding cost of \$600,000.

8.1.1.7 Nuclear Control and Instrumentation - In the absence of any detailed design for control and instrumentation, the cost of the required equipment can only be roughly estimated. Examination of similar cost estimates for other reactors indicated that an estimate of \$2,000,000 would be conservative for our reactor, in view of the fact that we have no distinct fuel channels requiring individual monitoring.

8.1.1.8 Fuel Charging and Discharging Equipment - This equipment includes not only devices for adding and subtracting fuel spheres from the core basket, but also an appropriate gas lock to permit doing this without loss of substantial part of the coolant gas. Since the time available for this study was obviously inadequate to permit mechanical design of such equipment, the cost of this item can only be guessed at. A figure of $1\frac{1}{2}$ million dollars was assumed which, it is hoped, will prove to be conservative.

8.1.1.9 Land - For this item it should be remembered that the cost of land normally associated with a conventional power plant, including roads and yard improvements, is already included in our estimated power plant costs. To pay for the additional exclusion area required for a nuclear power plant, an allowance of \$300,000 was made.

8.1.1.10 Miscellaneous Equipment - This encompasses a variety of equipment whose individual cost is very difficult to estimate without first preparing a mechanical design. It includes such items as high pressure

helium storage equipment, emergency cooling apparatus, equipment for decontaminating radioactive helium at a rate equal to that which leaks into the system, equipment for periodically annealing fuel balls removed from the reactor and simultaneously boiling out and absorbing some of the volatile fission products; also a stack to permit release of some quantity of radioactive gases; finally, cranes, miscellaneous maintenance equipment, and any other items that may have escaped our attention. The total allowance for these items was set at \$5,000,000. In evaluating the validity of of this estimate, it should be remembered that the estimated nuclear plant cost contains an allowance of 20% for contingencies, which can also be used to pay for unexpected items of equipment.

8.1.2 Physical Plant Cost for Nuclear Plant

The physical plant cost normally is the sum of the following items:

- Purchased equipment
- Installation
- Piping
- Instrumentation
- Insulation
- Electrical auxiliaries and wiring
- Building
- Land and yard improvements

The sum of the individual equipment cost estimates previously arrived at already includes purchased equipment cost, installation, piping, instrumentation, land and yard improvements. Since the containment vessel will in effect serve as a building for the nuclear plant, and since the power

plant estimate already includes an allowance for administrative building, no additional provision for a nuclear plant building needs to be made. The estimated cost of insulation for the nuclear plant was examined and found to be negligible. Therefore, the physical plant cost for the nuclear plant can be arrived at simply by adding an allowance for electrical auxiliaries. This allowance was estimated at 5% of the sum of the previously calculated items.

8.1.3 Direct Plant Cost

The cost of plant engineering and construction supervision was estimated at 20% of the physical plant cost. The direct plant cost, being the sum of the physical plant cost plus engineering and construction, was therefore taken to be $1.20 \times$ the physical plant cost.

8.1.4 Fixed Capital for Nuclear Plant

The fixed capital investment in the nuclear plant was estimated by adding 5% for the contractor's fee and 20% for contingencies to the direct plant cost.

8.1.5 Power Plant Costs

The power plant design was based on the assumption that the condition of the steam leaving the superheater is fixed at a temperature of 1000°F and a pressure of 1450 psig; and that the only variable is the number of feedwater heaters (N_{FW}), which in turn determines the temperature and enthalpy of the water entering the economizer (H_{10}).

The basis for estimating the power plant cost was provided by Reference (57), which comprises a complete cost breakdown for the Johnsonville Steamplant. This reference has taken the total cost of the Johnsonville plant and very conveniently split it into two parts, i.e., the "steam plant"

and the "electric plant". The cost of the latter, which corresponds to what we have called the "power plant", is given as \$82.46 per KW capability. This figure includes not only equipment cost but also installation, erection, engineering, contractor's fees, etc. Since the Johnsonville plant had been built in 1951, a factor of 1.30 was applied to bring the cost estimate up to date (1957). This resulted in an estimate of \$107.20 per KW for a plant of the same type, i.e., a plant with six feedwater heaters.

The effect of the number of feedwater heaters on plant performance and costs is examined in Reference (2), a private communication from TVA. According to this reference, for a given steam rate (1,000,000 lbs/hr) changing the number of feedwater heaters has practically no effect on the major equipment cost. This is illustrated by the following table:

No. of feedwater heaters	6	5	4	3
Major equipment cost	4,966,000	4,913,000	4,861,000	4,808,000
Less \$53,000 per heater	318,000	265,000	212,000	159,000
Adjusted equipment cost	4,648,000	4,648,000	4,649,000	4,649,000
Generator output (KW)	123,512	128,699	134,006	139,658
Thermodynamic efficiency	40.46%	40.23%	39.85%	39.15%

The trend to note in the above table is that as the number of heaters is decreased, the enthalpy change per lb of steam increases, and therefore - since the steam rate is constant - the thermal power going into the plant increases proportionally. This increase in thermal power input is more than adequate to counteract the decrease in thermodynamic efficiency, with the result that the electric power output also increases as the number of feedwater heaters is decreased. Consequently, the "power plant" cost per

unit of electric capacity decreases materially as the number of feedwater heaters is lowered. This cost decrease is over and above the direct savings resulting from the elimination of feedwater heaters.

Another way of summarizing the information in the above table would be to say that the cost of the major "power plant" equipment is a function of the steam rate rather than the electric capability. This question, whether to base "power plant" costs on steam rate or on electric capacity was found to have a very pronounced effect on the resultant cost estimates.

On the basis of the information in Reference (2), it had at first been decided to base our cost estimates on the steam rate (lbs/hr). Subsequently, however, this assumption was re-examined and its validity was found to be very questionable. Although there are many items in a power plant whose cost would be proportional to the steam rate, many others would have costs proportional to the electric capability. Examples of the latter type are such items as the generator and transmission plant.

After examination of the power plant cost breakdown contained in Reference (57), it was decided to base our cost estimate 50% on the steam rate and 50% on the electric capability. The 50% part based on the electric capability is obtained simply by dividing the previously obtained figure of \$107.20/KW in half, obtaining \$53.60/KW. In order to obtain an expression for the other 50%, it is necessary to apply a factor equal to the ratio

$$\frac{\eta_{pp} \text{ (Johnsonville)}}{\eta_{pp} \text{ (Proposed plant)}} \times \frac{[H_{13} - H_{10} \text{ (Johnsonville)}]}{[H_{13} - H_{10} \text{ (Proposed Plant)}]}$$

Recalling that

$$\eta = 0.4046 \text{ (Johnsonville)}$$

$$H_{13} = 1492.7 \text{ BTU/lb}$$

$$H_{10} \text{ (Johnsonville)} = 449.2 \text{ BTU/lb}$$

The above factor reduces to:

$$\frac{0.4046}{\eta_{pp}} \times \frac{1492.7 - 449.2}{1492.7 - H_{10}} = \frac{422.2}{\eta_{pp} (1492.7 - H_{10})}$$

Applying the above factor, the total "power plant" cost per KW capability can be expressed as

$$53.60 \left[1 + \frac{422.2}{\eta_{pp} (1492.7 - H_{10})} \right] \text{ \$/KW}$$

Since the electric capability is given by the expression

$$\text{KW} = \frac{\eta_{pp} P}{1000}$$

The total "power plant" cost is equal to

$$\left[0.0536 \eta_{pp} + \frac{22.63}{1492.7 - H_{10}} \right] P$$

One small correction remains to be made. It can be shown that the direct savings resulting from the elimination of feedwater heaters can be accounted for by replacing the number 22.63 in the above expression by $(21.24 + 0.230 N_{FW})$. Making this substitution, we obtain

$$\$_{PP} = \left[0.0536 \eta_{pp} + \frac{21.24 + 0.230 N_{FW}}{1492.7 - H_{10}} \right] P$$

The above procedure implies that for a given type of plant, the cost per unit capacity will be essentially a constant over the range of capacity variation under consideration. Since our attention will be confined to very

large plants, the above assumption is valid.

8.1.6 Total Fixed Capital

Since the estimated power plant cost already includes such items as engineering, construction, and contractor's fee, it really represents an estimate of the fixed capital for the power plant. The total fixed capital required for our proposed plant is, therefore, obtained by adding the corresponding sub-totals for the nuclear plant and power plant.

8.1.7 Total Depreciating Investment

The total depreciating investment includes not only the fixed capital, but also the start-up cost and the capital charges during construction and start-up. The start-up costs were assumed to be equal to six months' operating costs at rated power. The capital charges during construction and start-up were estimated by assuming that the full fixed capital was invested at a linear rate over a period of two years, allowing 12% per year for interest, insurance and taxes during that period.

8.1.8 Working Capital

The working capital was estimated to be equal to 90 days operating costs at rated power.

8.1.9 Capital Charges

Capital charges were assumed to be 15% per year for the total depreciating investment, and 12% per year for the working capital. This covers return to investors, taxes, insurance and depreciation.

8.1.10 Operating Costs

Because of insufficient experience, it is very difficult to estimate operating costs for a nuclear power plant. We, therefore, endeavored to make our estimate consistent with those used in previous reactor evaluations.

Since it does not seem reasonable that the operating cost for such a plant should be directly proportional to its power output, we arbitrarily assumed that the operating cost was proportional to the 0.8 power of the gross electric power production. To make our estimate consistent with previous evaluations, we chose our coefficient such that the operating costs for a 100 gross electric megawatt plant would come out to be 1 mill per kilowatt hour.

8.1.11 Fuel Rental

The cost of renting the uranium fuel, both that present in the reactor core as well as that fraction undergoing chemical reprocessing, was estimated by assuming a fuel cost of \$20,000 per kilogram and a rental charge of 4% per year.

8.1.12 Graphite Inventory Cost

The basis for estimating the cost of the fabricated and impregnated graphite balls is explained in the section on graphite replacement costs. The graphite inventory cost was calculated by assuming an interest charge of 12% per year.

8.1.13 Thorium and Helium Costs

The cost of interest charges for the thorium and helium were examined and found to be negligible in comparison with the cost of fuel rental.

8.1.14 Fuel Reprocessing

The fuel reprocessing cost estimates are based on the assumption of a one year reprocessing cycle for 1,000 megawatt thermal operation, with a proportionately shorter cycle for operation at higher power levels. Our physics calculations indicate that we could maintain criticality and a conversion

ratio of 1 with burn-ups several times as high. However, the above assumption was made because the information on resultant radiation damage in the graphite balls is not adequate for accurate core life predictions.

The cost of actually processing the fuel is based on a centrally located reprocessing plant described in Ref. (58). It was assumed that there were 2 other reactors identical to ours and that the three reactors would jointly ship a fraction of their fuel for each reprocessing campaign. It was assumed that the graphite in the spent fuel elements would be burnt prior to reprocessing, leaving a volume of ashes so small as to make its shipping cost to the reprocessing plant practically negligible. It was assumed that the chemical plant would process 30 kilograms of U-233 per day, that 8 days would be required to clean up the plant after each reprocessing campaign, and that the cost of using the plant would be \$15,000 per day (See Ref. 58).

8.1.15 Graphite Replacement

The cost of graphite replacement was based on actual operating experience in fabricating 600,000 graphite balls for the MTR reflector. We were advised by McPherson to assume a cost of 7.7 cents per one inch ball for material and fabrication, and an additional cost of 0.5 cents per ball for impregnation with fuel. To be consistent with our fuel reprocessing cost estimate, we assumed a consumption of one core per year when operating at a power level of 1,000 megawatts thermal.

8.2 Cost Analysis of Final Reactor Design

The equations of Section 5.2 representing the reactor system were solved using the ORACLE. On the basis of the results of the ORACLE the design was

set as indicated in Section 2.2. The breakdown of the nuclear plant costs, total investment and annual costs is shown in Tables 8-1, 8-2 and 8-3. The final design resulted in giving a power cost of 6.6 mils/kwh.

TABLE 8-1

BREAKDOWN OF NUCLEAR PLANT COSTS

Nuclear Plant - Installed Equipment Costs

Pressure Vessel and Thermal Shields	3,200,000
Primary Heat Exchanger	2,816,000
Economizer	484,000
Boilers	588,000
Superheaters	628,000
Blowers	2,616,000
Piping, Fittings, and Valves	1,637,000
Nuclear Control and Instrumentation	2,000,000
Shielding	500,000
Containment Vessel	2,500,000
Fuel Changing and Discharging Equipment	1,500,000
Extra Land (Exclusion Area)	300,000
Miscellaneous Equipment	5,000,000
Helium Storage and Purification	
Emergency Cooling	
Compressor	
Cranes and Maintenance Equipment	
Electrical Wiring and Auxiliaries	1,188,000
<hr/>	
Nuclear Plant - Physical Plant Cost	24,957,000

TABLE 8-2

BREAKDOWN OF INVESTMENT

Nuclear Plant

Physical Plant Cost	24,957,000
Engineering and Construction	4,999,000
Contractor's Fee	1,498,000
Contingencies	5,991,000
Fixed Capital	37,445,000

Power Plant

Fixed Capital	39,625,000
---------------	------------

TOTAL FIXED CAPITAL	77,070,000
---------------------	------------

Capital Charges during Construction and Startup	9,248,000
--	-----------

Start-up Costs	1,095,000
----------------	-----------

TOTAL DEPRECIATING INVESTMENT	87,413,000
-------------------------------	------------

Working Capital	548,000
-----------------	---------

TABLE 8-3

BREAKDOWN OF ANNUAL COSTS

Capital Charges for Depreciating Investment	13,112,000
Depreciation	
Insurance	
Taxes	
Return to Investors	
Capital Charges for Working Capital	49,000
Operating and Maintenance Costs	2,190,000
Fuel Rental	1,067,000
Graphite Inventory Cost	116,000
Fuel Reprocessing Cost	820,000
Graphite Replacement Cost	881,000
	<hr/>
Total Annual Cost	18,236,000

$$\text{Power Cost} = \frac{18,236,000 \text{ \$/yr} \times 1000 \text{ mills/\$}}{7000 \text{ hrs/yr} \times 400,000 \text{ KW}} = 6.51 \text{ mills/KWH}$$

CHAPTER 9

CONCLUSIONS

The reactor system as proposed in this report has the following advantages:

1. High steam temperature and pressure, making possible the utilization of a high efficiency conventional power plant.
2. Since there are no structural materials in the core having high cross sections for neutron capture, the neutron economy is good. The reactor has a conversion ratio of 1.02 and is therefore self sustaining.
3. The use of helium as the coolant eliminates the normal problems of corrosion. Therefore normal materials of construction are used through-out the system.
4. The fuel element design makes it possible to charge and discharge fuel with relative ease. The fuel may be recycled to give uniform and high burnup. The fuel fabricating cost is low compared to most systems.
5. The core and pressure vessel design eliminates the necessity of requiring close tolerances, fine machining and mechanical closures.
6. There is a good negative temperature coefficient of reactivity. The fuel can stand high temperature overshoots.

The reactor system has the following disadvantages:

1. Radioactive coolant.
2. Large pressure vessel with thick walls.
3. Large fuel charge.

It is recommended that the following study and development work be undertaken in support of the proposed reactor system:

1. Effect of radiation on fuel elements.
2. Reprocessing of fuel.
3. Kinetics of the system.
4. Stresses in the high temperature heat exchangers.
5. Containment of radioactive helium with non-radioactive helium at a slightly higher pressure.
6. Blower seals.
7. Control Rod requirements.

Included in Appendix A are the calculations pertaining to the work described in previous chapters.

10.1 Physics

10.1.1 Calculation of Age

The correct age from fission energies to the thermal energy of 0.025 ev was taken to be 364 cm² in 1.6 density graphite. The 364 cm² age is the value given in the Reactor Physics Handbook (Ref. 3).

In order to obtain the age at any energy, age was computed for eye-wash graphite cross-section by the method of Reference (12). The computed age as a function of lethargy is shown in Figure (10-1). By this second method, the age from fission energy to 0.025 ev is 392.0 cm². Consequently, it was assumed that the age at any energy is equal to the value from Figure 10-1 times the ratio of 364 to 392.

The age in a graphite ball reactor must be corrected for several reasons. First the age must be corrected for the lower average carbon density. For an average carbon density of 1.0 gram per cm³ (corresponding to 37.5% voids), the age will increase by a factor of (1.6)².

In addition, the age will increase even more than what would be expected from the reduced average density. Based on an approximate method, similar to that used in calculation of the attenuation of pebble bed shields, the age for 1 inch spheres would increase by an additional 5%. (See Reference 4).

The change in age due to the scattering from thorium and uranium was neglected. For a graphite to thorium atom ratio of 150, the oxygen present in the form of thorium oxide will decrease the age by 2%.

The amount of fast leakage is reduced by neutron absorption in thorium and uranium. When the total fast leakage was computed in a one group calculation, the age was reduced by 10% to compensate for this effect. After

ORNL—LR—DWG 24604

Fig. 10.1. Calculated Age in Graphite, Including First Flight Correction vs Lethargy.

deciding on the final thorium and uranium concentrations to be used in this reactor, it was realized that the thorium and uranium absorption would reduce fast leakage by 20%, but the 10% figure was used in most hand calculations.

The age from fission energy down to thermal energy (0.10 ev in this reactor) is given as 370 cm^2 by Figure (10-1).

Based on the previously mentioned age corrections, the 370 cm^2 value becomes 818 cm^2 as shown by the following calculation:

$$(370) \left(\frac{364}{392} \right) (1.6)^2 (1.05)(.98)(.90) = 818 \text{ cm}^2$$

In a step-by-step calculation of the overall neutron balance, the reduction of fast leakage due to uranium and thorium absorptions is already included. In that case, to calculate the difference in age between two energy levels, the same method is used without applying the arbitrary 90% factor.

10.1.2 Calculation of Thermal Poisons

The effect of xenon, samarium, fission products, and the higher isotopes were all considered in determining thermal poisoning.

As xenon-135 attains a steady-state concentration in a time measured in hours, xenon poisoning was always assumed to be that determined by its steady state concentration. The equation for steady-state xenon poisoning as developed by reference (13) is

$$P_x = \frac{\sum_{fT}^{23} (.059)(\phi_T \sigma_{AT}^{Xe})}{\sum_{AT}^{23} (2.1 \times 10^{-5} + \phi_T \sigma_{AT}^{Xe})}$$

For a xenon cross section of 3.5×10^6 barns, P_x is .032, for a thermal flux of 10^{13} , and $P_x = .050$ for a thermal flux of 10^{14} .

Samarium-149 reaches an equilibrium concentration in a period of days; therefore samarium poisoning was also assumed to be at its equilibrium value. The equation for samarium poison is from Reference (13).

$$P_s = .014 \frac{\sum_{fT}^{23}}{\sum_{AT}^{23}} = .013$$

The thermal poisoning due to fission products was computed from Reference (14). The thermal fission product poisoning curve (Figure 10-2) is based on maximum cross sections for all fission product except zirconium-93; the cross-section of zirconium-93 was taken at its minimum value.

The heavy nuclide poisoning for a uranium-233 reactor is small. As shown in Reference (14), the heavy nuclide chain produces nearly as many neutrons from fission as it consumes in absorptions. For a reactor with an assumed effectiveness of a fast neutron to a thermal neutron of 0.8, the heavy nuclide chain poisoning is only .029 for a fuel exposure time of 10 fissions per atom of original fuel. As a breeder reactor must have small leakage and small absorption of neutrons by poisons, the relative effectiveness of a fast neutron compared to a thermal neutron will be equal or greater than 0.9, rather than 0.8. For an effectiveness of 0.9 and long exposure, the heavy nuclide poisoning even becomes negative; i.e., the heavy nuclide chain acts as fuel instead of a poison. For this reason, the heavy nuclide poisoning was taken to be that of Protactinium-233, which, at steady state, was shown to be

$$P_{Pa-233} = 2.26 \times 10^{-16} \phi_T$$

Fig. 10.2. Effect of Burn-up on Fission Product Poisoning.

Fig. 10.3. Fission Product and Heavy Nuclide Poisoning vs Flux.

The xenon, samarium, and heavy nuclide poisoning are plotted in Figure 10-3 as a function of thermal flux. The fission product poisoning depends on the total number of fissions per fuel atom and not just on number of thermal fissions. The fission product poisoning has been plotted as a separate curve for one year exposure time. The thermal flux must be multiplied by the ratio of total fissions to thermal fissions to determine the correct value of fission product poisoning.

10.1.3 Calculation of Effective Thorium and Uranium Resonance Escape Factors

The thorium resonance escape factor depends not only on the ratio of graphite to thorium, but also on the graphite to uranium ratio. Fast and resonance absorptions in uranium will reduce the number of resonance neutron absorptions in thorium. In this report, the "effective" thorium resonance escape factor P_T is defined to be "one minus the probability that a neutron is absorbed by thorium before reaching the thermal cutoff energy" for a specific homogeneous mixture of graphite, thorium, and uranium. As with the usual resonance escape probability, the "effective" resonance escape factor is computed for an infinite region. The effective resonance escape factor is larger than the normal thorium resonance escape probability.

The effective uranium resonance escape factor (P_U) is a fictitious index used to specify the number of uranium resonance absorptions. The effective escape factor " P_U " is defined by

$$P_T (1 - P_U) \equiv \text{probability that a neutron will be absorbed by uranium before it reaches the thermal cutoff energy.}$$

It will be noted that the "effective" uranium resonance escape factor would equal the "normal" uranium resonance escape probability if all absorptions

by thorium took place at higher energies than the absorptions by uranium. Since this is of course not the case, the "effective" uranium resonance escape factor is smaller than the "normal" escape probability.

The uranium resonance integral was broken into six groups. The uranium resonance integral was computed from Eyewash cross-section data (Appendix C) for an infinite dilution case. The number of scattering barns per uranium atom in this report range between 8,000 and 22,000 barns per uranium atom, so that the infinite dilution assumption is good. The six energy groups and the computed resonance integrals were as follows:

Group	Energy range (ev)	U^{233} Res. Integral (barns)
U-1	10^7 - 9,118	17.8
U-2	9,118 - 454	36.0
U-3	454 - 112	25.2
U-4	112 - 33	38.4
U-5	33 - 1.2	673.
U-6	1.2 - .4	80.

The thorium resonance integral was computed for a temperature of 1660°F by the method of reference (15). The thorium resonance integral was broken into five energy groups. For a graphite thorium atom ratio of 300 ($R_T = 300$), the division is as follows:

$R_T = 300$			
Group	Energy Range (ev)		^{235}Th Res. Integral
T-1	$10^7 - 2000$	- - - - -	5.8
T-2	$2000 - 230$	- - - - -	22.9
T-3	$230 - 40 \text{ ev}$	- - - - -	23.9
T-4	$40 - 1.2$	- - - - -	26.6
T-5	$1.2 - .4$	- - - - -	3.5

The resonance integral was computed also for $R_T = 100$ and $R_T = 200$ and the curves of Figure 10-4 was made by connecting these three points.

The method of calculating effective resonance escape factor is illustrated by the following numerical example. It is assumed that fast neutrons are first absorbed in the U-1 group, then T-1 group, then U-2 group, and alternating down to the U-6 group.

Assume:

1) $R_U = 1800$ and $R_T = 100$

2) $P_T = \frac{\int \sigma_a^{02} du}{\sum \sigma_s R_T}$ $P_U = \frac{\int \sigma_a^{23} du}{\sum \sigma_s R_U}$

3) $\sum \sigma_s$ for graphite taken as constant value of 0.739

4) For $\int \sigma_a^{02} du$ from Figure 10-4 ;

$$1 - P_{T1} = .0756$$

$$1 - P_{T2} = .255$$

5) For $\int \sigma_a^{23} du$ from page 75

$$1 - P_{U1} = .0134$$

$$1 - P_{U2} = .0270$$

Fig. 10.4. Resonance Integrals for Five Thorium Groups.

Fig. 10.5. Effective Resonance Escape Probability for U^{233} .

Fig. 10.6. Effective Resonance Escape Probability for Th^{233} .

- 6) Assume 1000 fission neutrons
- 7) Absorption in U-1 = $(1000)(.0134) = 13.4$
- 8) Neutrons reaching T-1 = $1000 - 13.4 = 986.6$
- 9) Absorption in T-1 = $(986.6)(.0756) = 74.6$
- 10) Neutrons reaching U-2 = $986.6 - 74.6 = 912.0$
- 11) Absorptions in U-2 = $(912.0)(.0270) = 24.6$
- 12) Neutrons reaching T-2 = $912.0 - 24.6 = 887.4$
- 13) Calculation proceed thru group U-6 with 222.5 neutrons reaching thermal and 544.1 neutrons absorbed by thorium and 233.4 neutrons absorbed by uranium in fast and resonance region.
- 14) Effective thorium resonance escape is then = $1.00 - .5441 = .4559$
- 15) Effective uranium resonance escape is = $\frac{.2334}{.1559} = .486$

When the effective escape factors are plotted for other concentrations, the effective resonance escape factors all fall within a narrow band for the concentrations of interest (Figs. 10-5 and 10-6). This narrow band curve is then used as the correct curve to determine neutron resonance absorption in thorium and uranium.

10.1.4 Calculation of the Temperature Coefficient of Reactivity

The following formulae, derived in Appendix B, Section 11.8, was employed to calculate the temperature coefficient of reactivity:

$$\left(\frac{\partial \rho}{\partial T}\right) = - \left[\frac{1}{1 + \frac{P_s}{P_x} + \frac{1}{P_x} \frac{\sum_a^0}{\sum_a^{23}}} \right] \left[\frac{1}{\sigma_a^x} \left(\frac{\partial \sigma_a^x}{\partial T} \right) - \frac{1}{2T} \right] -$$

$$- \left[\frac{1}{1 + \frac{P_x}{P_s} + \frac{1}{P_s} \frac{\sum a^0}{\sum a^{23}}} \right] \left[\frac{1}{\sigma_a^s} \left(\frac{\partial \sigma_a^s}{\partial T} \right) - \frac{1}{2T} \right] -$$

$$- 4\alpha B^2 M^2 - \frac{B^2 L^2}{2T} + \frac{D_M B^2}{\sum_s M_T} - \frac{1}{\xi R_M \sigma_s^M} \left(\frac{\partial I}{\partial T} \right)$$

The following values were used in evaluating $\left(\frac{\partial \rho}{\partial T} \right)$ for steady state operation

$$P_x = .038$$

$$P_s = .012$$

$$\sum_a^0 = .00703 \text{ cm}^{-1}$$

$$\sum_a^{23} = .0055 \text{ cm}^{-1}$$

$$\sigma_a^x = 1.44 \times 10^6 \text{ b}$$

$$\frac{\partial \sigma_a^x}{\partial T} = -1260 \text{ 1/}^\circ\text{C}$$

$$T = 1100^\circ \text{ K}$$

$$\frac{1}{\sigma_a^s} \frac{\partial \sigma_a^s}{\partial T} = -2.26 \times 10^{-4}$$

$$\alpha = 7.86 \times 10^6 \text{ 1/}^\circ\text{C}$$

$$B^2 = 1.015 \times 10^{-4} \text{ cm}^2$$

$$M^2 = 1031 \text{ cm}^2$$

$$L^2 = 215 \text{ cm}^2$$

$$R_M = 160$$

$$\xi = .158$$

$$\sigma_s^m = 4.8 \text{ b}$$

$$\frac{\partial I}{\partial T} = 6.5 \times 10^{-3} \text{ 1/}^\circ\text{C}$$

$$D_M = 1.43 \text{ cm}$$

$$\sum_s^M = .243 \text{ cm}^{-1}$$

The quantity $\frac{1}{\sigma_a^x} \left(\frac{\partial \sigma_a^x}{\partial T} \right)$ was obtained from Reference (42) which contained a graph showing the effect of neutron temperature upon the microscopic absorption cross-section of xenon; with the cross-section obtained by integrating over the Maxwell-Boltzmann energy distribution corresponding to the neutron temperature. The quantity $\frac{1}{\sigma_a^s} \frac{\partial \sigma_a^s}{\partial T}$ was also integrated over the M.B. distribution at $T = 1100^\circ\text{K}$. The quantity $\left(\frac{\partial I}{\partial T} \right)$ was obtained from Figure 11-2 based on calculations in Section (11.9.5a).

Substituting the above values,

$$\begin{aligned} \left(\frac{\partial \rho}{\partial T} \right) &= +12.1 \times 10^{-6} - 2.10 \times 10^{-6} - 3.29 \times 10^{-6} \\ &\quad - 9.95 \times 10^{-6} + 3.43 \times 10^{-6} - 5.36 \times 10^{-5} \\ &= -5.34 \times 10^{-5} \text{ 1/}^\circ\text{C} \end{aligned}$$

for steady state poison level.

Because of the relatively low flux level of the proposed reactor, the xenon and samarium poison build-up after shutdown is quite small. The maximum poison ratios attained are:

$$\begin{aligned} P_x &\equiv \frac{\sum_a^x}{\sum_a^{23}} = 0.047 \\ P_s &\equiv \frac{\sum_a^s}{\sum_a^{23}} = 0.014 \end{aligned}$$

(See Reference 13)

Substitution of these maximum poison values into the above equations decreases the magnitude of the temperature coefficient of reactivity to:

$$\frac{\partial \rho}{\partial T} = -5.105 \times 10^{-5} \text{ 1/}^\circ\text{C}$$

10.1.5 Calculation of the Reactivity Effect Due to Neutron Streaming in the Reactor Core Gas Duct

The decrease in reactivity due to a duct in a homogeneous reactor is given in Reference (37). The formulation derived for a change in the multiplication constant as function of duct size is given in the following equation.

$$\Delta K = \left[0.99 + u\left(\frac{a}{h}\right) \right] \frac{v T \rho^2(a,0)}{\iiint_{\text{Reactor}} \rho^2 dv} \frac{\pi^3 a^3}{4h} \quad 10.1.5-1$$

where $u\left(\frac{a}{h}\right) = 0.636 + 1.6 \frac{a}{h} - 5.4 \frac{a}{h} \ln \frac{h-6a}{6a} - 116 \frac{a^2}{h^2}$ 10.1.5-2

ΔK = change in multiplication constant

a = radius of duct

h = extrapolation height of the reactor note h is approximated by geometrical height

v = avg speed of neutrons

T = mean neutron lifetime in the reactor

$\rho(r,z)$ = neutron density at radius r and height z

dv = element of reactor volume

Evaluate

$$\iiint_{\text{Reactor}} \rho^2 dv$$

-165-

The neutron density spatial variation is given by

$$\rho(r, z) = \rho(0, 0) \cos\left(\frac{\pi}{h} z\right) J_0\left(\frac{2.405}{R} r\right) \quad 10.1.5-3$$

where R is extrapolated radius of reactor

Note R is approximated by geometrical radius

$$\begin{aligned} \iiint_{\text{Reactor}} \rho^2 dv &= \int_{-\frac{h}{2}}^{+\frac{h}{2}} \int_0^R \rho^2(0, 0) \left[\cos\left(\frac{\pi}{h} z\right)\right]^2 \left[J_0\left(\frac{2.405}{R} r\right)\right]^2 2\pi r dr dz \\ &= 2\pi \rho^2(0, 0) \int_0^R \left[J_0\left(\frac{2.405}{R} r\right)\right]^2 r dr \int_{-\frac{h}{2}}^{+\frac{h}{2}} \left[\cos\left(\frac{\pi}{h} z\right)\right]^2 dz \\ &= 2\pi \rho^2(0, 0) \left\{ \frac{R^2}{2} \left[J_1\left(\frac{2.405}{R} r\right)\right]^2 \right|_{r=0}^{r=R} \left[\frac{z}{2} + \frac{1}{4\left(\frac{\pi}{h}\right)} \sin 2\left(\frac{\pi}{h} z\right) \right] \right|_{z=-\frac{h}{2}}^{z=+\frac{h}{2}} \} \\ &= 2\pi \rho^2(0, 0) \left\{ \frac{R^2}{2} [0.5191]^2 - 0 \right\} \left[\left(\frac{h}{4} + 0\right) - \left(-\frac{h}{4} + 0\right) \right] \} \\ &= 2\pi \rho^2(0, 0) \left[\frac{R^2}{2} (0.270) \left(\frac{h}{2}\right) \right] = 0.135 \pi R^2 h \rho^2(0, 0) \quad (10.1.5-4) \end{aligned}$$

Substitute the value of

$$\iiint_{\text{Reactor}} \rho^2 dv$$

from Equation (10.1.5-4) into Equation (10.1.5-1)

$$\Delta K = \frac{[0.99 + u\left(\frac{a}{n}\right)] v_T \rho^2(a, 0)}{0.135 \pi R^2 h \rho^2(0, 0)} \frac{\pi^3 a^3}{4h}$$

Assume

$$\rho(a,0) \approx \rho(0,0)$$

$$\Delta K = \frac{\pi^2}{4(0.135)} \left[0.99 + \mu \left(\frac{a}{h} \right) \right] \frac{v T a^3}{R^2 h^2}$$

or

$$\Delta K = 18.3 \left[0.99 + \mu \left(\frac{a}{h} \right) \right] \frac{v T a^3}{R^2 h^2} \quad 10.1.5-5$$

For dimensional consistency

$$v = \text{ft/sec}$$

$$T = \text{sec}$$

$$a = \text{ft}$$

$$R = \text{ft}$$

$$h = \text{ft}$$

from reactor design data

$$R = 9' \text{ and } h = 24'$$

$$\Delta K = \frac{18.3}{(9)^2 (24)^2} \left[0.99 + \mu \left(\frac{a}{h} \right) \right] v T a^3$$

or

$$\Delta K = 3.92 \times 10^{-4} \left[0.99 + \mu \left(\frac{a}{h} \right) \right] v T a^3 \quad 10.1.5-6$$

Compute average speed (v) of neutrons at duct surface.

$$v = \frac{\phi_F + \phi_R + \phi_{Th}}{\frac{\phi_F}{v_F} + \frac{\phi_R}{v_R} + \frac{\phi_{Th}}{v_{Th}}} \quad 10.1.5-7$$

where ϕ - relative neutron flux

v - average neutron speed

and subscripts

F - fast region (310 ev - 2 Mev)

R - resonance region (0.4 ev - 310 ev)

Th - thermal region (0.1 ev)

The relative neutron flux values were obtained from the section on nuclear calculations:

$$\phi_F = 1.00$$

$$\phi_R = 0.240$$

$$\phi_{Th} = 0.177$$

The estimated average neutron velocities for the three energy regions are as follows:

$$v_F = 148 v_T$$

$$v_R = 4.12 v_T$$

$$v_T = \sqrt{\frac{4}{\pi} \frac{(0.1)}{0.025}} \times 2200 \frac{m}{sec} = 4960 \frac{m}{sec}$$

Substitute the preceding values into Equation (10.1.5-7)

$$v = \frac{1.00 + 0.240 + 0.177}{\frac{100}{148 v_{Th}} + \frac{0.240}{4.12 v_{Th}} + \frac{0.177}{v_{Th}}} = \frac{1.417}{\frac{0.242}{v_{Th}}} = 5.83 v_{Th}$$

$$v = 5.83 (4960) = 28,900 m/sec$$

$$v = 95,000 ft/sec$$

Compute average neutron lifetime using data from the section on nuclear calculations.

Fast region

$$\Delta U (\text{Lethargy change}) = \ln \frac{2 \times 10^6}{310} = 8.76$$

$$N = (\text{number of collisions}) = \frac{\Delta U}{\xi} = \frac{8.76}{0.158} = 55.5$$

$$\lambda_s = (\text{Ave dist. between collisions}) = \frac{1}{\Sigma_s} = \frac{1}{0.244} = 4.1 \text{ cm}$$

$$L_{SD} = (\text{Ave. slowing down path length}) = 55.5 \times 4.1 = 227 \text{ cm}$$

$$T_F = (\text{Ave. fast neutron life time}) = \frac{L_{SD}}{v_F} = \frac{227}{(148)(4960)(100)} = 3.1 \times 10^{-6} \text{ sec}$$

Resonance region

$$\Delta U = \ln \frac{310}{0.9} = 6.65$$

$$N = \frac{6.65}{0.158} = 42.1$$

$$L_{SD} = 42.1 (4.1) = 173 \text{ cm}$$

$$T_R = \frac{173}{(4.12)(4960)(100)} = 8.46 \times 10^{-5} \text{ sec.}$$

$$T_h = \frac{1}{v_{Th} \Sigma_{aTh}} = \frac{1}{(4960)(100)(0.00703)} = 2.87 \times 10^{-4} \text{ sec}$$

The percentage absorptions in the three energy regions are as follows:

Fast absorption 20%
 Resonance absorption - 40%
 Thermal absorption - 40%

$$T(\text{ave neutron lifetime}) = 0.2 T_F + 0.4 (T_F + T_R) + 0.4 (T_F + T_R + T_{Th})$$

10.1.5-8

$$T = 0.2 (3.1 \times 10^{-6}) + 0.4 (3.1 \times 10^{-6} + 84.6 \times 10^{-6}) + 0.4 (3.1 \times 10^{-6} + 84.6 \times 10^{-6} + 287 \times 10^{-6})$$

$$= 0.19 \times 10^{-3} \text{ sec}$$

Substitute the values obtained for v in Equation (10.1.5-7) and T in Equation (10.1.5-8) into Equation (10.1.5-6).

$$\Delta K = 3.92 \times 10^{-4} \left[0.99 + \mu \left(\frac{a}{h} \right) \right] [9.5 \times 10^4] [0.19 \times 10^{-3}] a^3$$

or

$$\Delta K = 7.07 \times 10^{-3} \left[0.99 + \mu \left(\frac{a}{h} \right) \right] a^3$$

10.1.5-9

For one end of the gas duct blocked with infinite reflector

$$\Delta K = 3.53 \times 10^{-3} \left[0.99 + \mu \left(\frac{a}{h} \right) \right] a^3$$

10.1.5-10

Evaluate ΔK for various radii:

a	$\frac{a}{h}$	$\mu \left(\frac{a}{h} \right)$	$\left[0.99 + \mu \left(\frac{a}{h} \right) \right]$	a^3	ΔK
1	0.0417	0.26	1.25	1.0	0.44 %
1.25	0.0520	0.18	1.17	1.95	0.80 %
1.50	0.0625	0.12	1.11	3.38	1.52 %
2.0	0.0833	0	0.99	8.0	2.80 %

10.2 Engineering

10.2.1 Heat Generation and Thermal Stresses Within Pressure Vessel Wall

A. The Sources of Radiation Heating in the Pressure Vessel Walls

- (1) Core prompt fission gammas
- (2) Core capture gammas
- (3) Core decay gammas
- (4) Thermal neutron capture gammas within the pressure vessel wall and thermal shields.
- (5) Fast neutron scattering within the pressure vessel wall.
- (6) Fast neutron absorption gammas in the pressure vessel walls and thermal shields
- (7) Fast neutron inelastic scattering.
- (1) Core Prompt Fission Gammas - Approximately 7.5 mev per fission

(Reference 19) appears as gamma energy. The intensity of radiation for one energy at the core surface is given by

$$S_{\gamma} = \frac{\phi_n \sum N_{\gamma} E_{\gamma} \cos \theta}{4 \pi \mu_c} \left[1 - e^{-\frac{\mu_c b}{\cos \theta}} \right] \frac{\text{Mev}}{\text{cm}^2}$$

where a uniform average flux and isotropic angular distribution of the surface radiation was assumed. Since the core is large, the exponential term is nearly negligible. If the actual flux distribution is used, the surface source is found to differ by only a few per cent. By means of the straight head scattering integral spectrum approximation (Reference 20) the heating rate at any point outside the core is found to be:

$$Q'' = \int_0^{2\pi} \int_0^{\infty} \frac{S_{\gamma} \mu_e e^{-\mu_e \rho} r dr d\psi}{\rho^2} \left(\frac{\text{Mev}}{\text{cc-sec}} \right)$$

Noting that $\cos \theta = \frac{\alpha}{\rho}$

and integrating gives:

$$\ddot{Q} = \frac{\phi_n \sum_f N_\gamma E_\gamma \mu_e}{\mu_c} E_2(\mu_e x) \frac{\text{Mev}}{\text{cc-sec}}$$

But $\phi_n \sum_f$ are the total fissions per cc

or $\frac{(P)(3.1)(10^{10})}{V_R} \frac{\text{fissions}}{\text{cc-sec}}$

With the $1.545 \times 10^{-8} \frac{\text{BTU}}{\text{ft}^3\text{-hr}}$ per $\frac{\text{mev}}{\text{cc-sec}}$ conversion factor, the

heating rate caused by photons of energy E is

$$\ddot{Q} = \frac{(3.1)(1.545)(10^2) P N_\gamma E_\gamma \mu_e B}{V_R \mu_c} E_2(\mu_e x) \frac{\text{BTU}}{\text{ft}^3\text{-hr}}$$

B is the build up factor based on the total number of relaxation lengths from the face of the core. The total heating rate is found by summing over the various energy groups. In this case only one group at 3.5 mev was used because of the nature of radiation. For a 1165 Mw reactor with a one foot graphite reflector,

$$\ddot{Q} = 225,000 B F_2(.67 + .17 x) \text{ BTU/ft}^3\text{-hr}$$

$$N_\gamma = 2$$

$$E_\gamma = 3.5 \text{ mev}$$

$$\mu_e = .17 \text{ 1/cm}$$

$$\mu_c = .021 \text{ 1/cm}$$

$$V_R = (1.24)(10^8) \text{ cc.}$$

(2) Core Capture Gammas - The equation developed for the prompt gamma heating can also be used to estimate the gammas resulting from captures in the core. The absence of any structural material in the core leaves graphite as the only source of capture gammas. One 4.6 mev gamma is given off per capture (Reference 18).

Since

$$\ddot{Q} = \frac{\bar{\Phi}_n \sum_a N_\gamma E_\gamma \mu_e B E_2 (\mu_e x)}{2 \mu_e}$$

and inserting

$$\bar{\Phi}_n = 3.3 \times 10^{14} \text{ n/cm}^2\text{-sec}$$

$$\sum_a = .0071 \text{ cm}^{-1}$$

$$\mu_c = .021 \text{ cm}^{-1}$$

$$N_\gamma E_\gamma = 4.6 \text{ mev}$$

$$\mu_e = .17 \text{ cm}^{-1}$$

$$\ddot{Q} = (44,000) B E_2 (.67 + .17 x) \text{ BTU/ft}^3\text{-hr}$$

(3) Core Decay Gammas - Since this is a core source also, and proportional to the total number of fissions,

$$\ddot{Q} = \frac{(3.1)(1.545)(10^2) P N_\gamma E_\gamma \mu_e B}{2 \mu_c} \left[E_2 (.65 + \mu_e x) \right]$$

Approximately seven mev per fission are emitted as decay gammas if the reactor has operated for a long time (Reference 17). Two groups, one at .3 mev with 15 photons and one at 2 mev with 1.4 photons resulted in an average of

$$\ddot{Q} = 190,000 B E_2 (.65 + .165 x) \text{ BTU/ft}^3\text{-hr}$$

(4) Thermal Neutron Capture Gammas - Thermal neutrons leaking

through the reflector are captured in the thermal shields and vessel walls giving rise to gamma rays. A method for calculating the resulting heat generation has been outlined in Reference (20). For an infinite slab emitting neutrons, the heating rate at a point away from the face is given by:

$$\ddot{Q} = \frac{\sqrt{3 \Sigma_a \Sigma_{tr}} J_T N_\gamma E_\gamma \mu_e B e^{-\sqrt{3 \Sigma_a \Sigma_{tr}} x}}{2} \times \left\{ F_1\left(\mu_e x, \frac{\sqrt{3 \Sigma_a \Sigma_{tr}}}{\mu_e}\right) + F_1\left(\mu_e(b-x), -\frac{\sqrt{3 \Sigma_a \Sigma_{tr}}}{\mu_e}\right) \right\} \frac{\text{Mev}}{\text{cc-sec}}$$

J_T can be approximated by

$$J_T = \frac{(3.1 \times 10^{10})(P)}{A_R} \gamma (1 - P_{FL})(P_{RE})(P_{TL}) \alpha \quad \text{neut/cm}^2\text{-sec}$$

From the physics calculations,

$$(1 - P_{FL})(P_{RE})(P_{TL})(\alpha) \approx .005$$

$$\begin{aligned} \text{Since } A_R &= 1.24 \times 10^6 \\ P &= 1.160 \times 10^9 \end{aligned}$$

$$\sqrt{3 \Sigma_a \Sigma_{tr}} = .755$$

$$\gamma = 2.31$$

$$b = 40 \text{ cm}$$

$$\ddot{Q} = 3650 N_\gamma E_\gamma \mu_e \left\{ F_1(\mu_e x, 4.1) + F_1\left[(\mu_e)(40 - x), -4.1\right] \right\}$$

for each group considered. The iron gamma spectrum was broken into four groups of 2, 4, 6 and 7 mev having .10, .24, .22, and .50 photons respectively per group. Since this contribution was small compared to the core gammas

an average value was used, giving,

$$Q'' = 4000 B e^{-.755x} \left\{ F_1 (.185 x, 4.1) + F_1 \left[(.185(40-x), -4.1) \right] \right\}$$

(5) Fast Neutron Scattering Within the Pressure Vessel Wall

Collisions involving fast neutrons result in captures or elastic collisions. In either case, kinetic energy is imparted to the medium. Inelastic collisions occur also but because of the small inelastic cross section as compared to the elastic and capture cross sections this is ignored.

Reference 17 gives for the heating due to the latter two as

$$Q''' = (CF)(\delta_{ne}) \int_E \phi E_n \Sigma_s dE + \delta_{nc} \int_E \phi E_n \Sigma_a dE \quad \frac{\text{BTU}}{\text{ft}^3\text{-hr}}$$

The flux can be approximated by the expression:

$$\phi = \frac{\sqrt{3 \Sigma_a \Sigma_{tr}} J_F e^{-\sqrt{3 \Sigma_a \Sigma_{tr}} x}}{\Sigma_a} \quad \frac{\text{neuts}}{\text{cm}^2\text{-sec}}$$

δ_n is the fraction of energy transferred as kinetic energy per collision

$$\delta_{ne} = \frac{1-\beta}{2} \quad \beta = \frac{M-1}{M+1}^2 \quad \delta_{nc} = \frac{1}{M+1}$$

Assuming an average energy of 2 mev for the fast neutrons

$$Q''' = \delta_{ne} (CF) \sqrt{3 \Sigma_a \Sigma_{tr}} \frac{\Sigma_s}{\Sigma_a} J_F e^{-\sqrt{3 \Sigma_a \Sigma_{tr}}} \bar{E}_n + \frac{\delta_{nc} (CF) \sqrt{3 \Sigma_a \Sigma_{tr}} \Sigma_a J_F e^{-\sqrt{3 \Sigma_a \Sigma_{tr}}} \bar{E}_n}{\Sigma_a}$$

$$J_F = \frac{(3.1)(10^{10})(P \gamma P_{FL}) f_a}{(1.24)(10^6)}$$

where $P_{FL} \approx .05$

$$\gamma = 2.31$$

$$P = 1.16 \times 10^9$$

$$f_a = \frac{\pi}{2} = \text{axial peak to avg.}$$

giving

$$J_F = 5.5 \times 10^{12} \text{ neut/cm}^2\text{-sec}$$

$$\Sigma_s = .0588$$

$$\Sigma_a = .193$$

$$\delta_{ne} = .0357$$

$$\delta_{nc} = .017$$

$$\bar{E}_n = 2 \text{ mev}$$

$$\sqrt{3 \Sigma_a \Sigma_{tr}} = .585$$

Thus,

$$\ddot{Q} = 1080 e^{-\sqrt{3 \Sigma_a \Sigma_{tr}} x} + 1600 e^{-\sqrt{3 \Sigma_a \Sigma_{tr}} x}$$

$$\ddot{Q} = 2680 e^{-\sqrt{3 \Sigma_a \Sigma_{tr}} x} \text{ BTU/hr-ft}^3$$

due to fast neutron heating effects.

(6) Fast Neutron Capture Gammas - Upon capturing a fast neutron, the iron nucleus emits an average of 4.6 mev in gamma rays due to the binding energy of the neutron. In addition, the initial neutron energy minus the fraction given up in kinetic energy must come off as gamma rays.

This gives $N_\gamma E_\gamma$ of 6.6 mev max. The problem is treated in the same manner as the thermal neutron captures except that J_T is changed to J_F , i.e.,

$$\ddot{Q} = \frac{\sqrt{3 \Sigma_a \Sigma_{tr}} J_F N_\gamma E_\gamma \mu_e B e^{-\sqrt{3 \Sigma_a \Sigma_{tr}} x}}{2} \left\{ F_1 \left[.17 x, c \right] + F_1 \left[.17(40-x), -c \right] \right\} \frac{\text{Mev}}{\text{cm}^3 \cdot \text{Sec}}$$

$$\text{where } c = \frac{\sqrt{3 \Sigma_a \Sigma_{tr}}}{\mu_e}$$

Since

$$\sqrt{3 \Sigma_a \Sigma_{tr}} = .585$$

$$J_F = 5.5 \times 10^{12}$$

$$\ddot{Q} = 12,400 N_\gamma E_\gamma \mu_e B e^{-.585 x} \left\{ F_1 \left[\mu_e x, 3.43 \right] + F_1 \left[\mu_e (40-x), -3.43 \right] \right\}$$

for each energy group. On the average, $N_\gamma E_\gamma \mu_e$ was nearly unity. Without too much error, the heating due to fast neutron gammas may be written as

$$\ddot{Q} = 12,400 B e^{-.585 x} \left\{ F_1 \left[\mu_e x, 3.43 \right] + F_1 \left[\mu_e (40-x), -3.43 \right] \right\}$$

(7) Fast Neutron Inelastic Scattering - Because of the lack of data on inelastic scattering gamma rays, it has been assumed that these are equal to about 1/3 of the neutron capture gammas. Since the neutrons have an average energy of about 2 mev and the capture gammas are 6.6 mev, this assumption is conservative.

$$\ddot{Q} = 3000 e^{-.585 x} \left\{ F_1 \left[.17 x, 3.43 \right] + F_1 \left[.17(40-x), -3.43 \right] \right\}$$

B. Thermal Stresses in Pressure Vessel Walls

Since it is desired to find the stresses as a function of the heating rates, the first step was to fit a curve to the total heat generation plots of Figure (10-7). This was fitted by

$$Q = 150,000 e^{-.163 x(\text{inches})} \text{ BTU/ft}^3\text{-hr}$$

Next, the temperature distribution must be determined. This requires making an assumption about the outer vessel walls. The most conservative and simple case is that for which the vessel is completely insulated. The diameter of the vessel is sufficiently large to be treated as a slab. From the general heat equation,

$$\frac{d^2\theta}{dx^2} = - \frac{Q}{k}$$

and using the boundary conditions that $\frac{d\theta}{dx} = 0$ at $x = c$

$$T = T_1 \text{ at } x = 0$$

the equation for the temperature inside the wall becomes

$$\begin{aligned} \theta_x - \theta_s &= \frac{Q_0}{1728 k m^2} (1 - e^{-mx} - m e^{-mt} x) \\ &= 205 (1 - e^{-.414 x} - .414 e^{-.414 t} x) \end{aligned}$$

The maximum for a given thickness is:

$$\theta_m - \theta_s = 205 (1 - e^{-.414 t} - .414 e^{-.414 t} t)$$

The maximum stress in the vessel wall and thermal shields is the tangential stress at the inner surfaces. This stress was calculated for such an exponential

HEAT GENERATION IN THERMAL SHIELD & VESSEL WALL

Figure 10-7

Heat Generation - Btu/hr-ft² - $\dot{\Phi}''$

heat source in APAE Memo 85 (Reference 21).

$$\sigma_{\theta T} = \left(\frac{\alpha E}{1 - \mu} \right) \left(\frac{Q_o e^{-.16\Delta}}{k m^2} \right) \left(\frac{2}{(a+t)^2 - a^2} \right) \times$$

$$\times \left[a \left(t + \frac{e^{-mt}}{m} - \frac{t^2 m e^{-mt}}{2} - \frac{1}{m} \right) + \frac{t^2}{2} \frac{t e^{-mt}}{m} \right.$$

$$\left. + \frac{e^{-mt}}{m^2} - \frac{t^3 m e^{-mt}}{3} - \frac{1}{m^2} \right] \text{ psi}$$

For carbon steel

$$\left(\frac{\alpha E}{1 - \mu} \right) = 260, \quad k = 25 \text{ BTU/}^\circ\text{F-ft}$$

Also

$$m = (.414)$$

$$t = 8''$$

$$a = 112$$

$$Q_o = 150,000$$

$$\Delta = 6''$$

So

$$\sigma_{\theta T} = 2800 \text{ psi}$$

The pressure stress at the inner surface was calculated from

$$\sigma_{\theta P} = \frac{a P F}{t}$$

where F is a 110% factor to allow for over pressure.

For $a = 112$, $P = 1000$ $t = 8$

$$\sigma_{\theta_P} = 15,500.$$

The combined stress is

$$\sigma_{\theta} = \sigma_{\theta_T} + \sigma_{\theta_P} = 18,300 \text{ psi}$$

The allowable stress at up to 700°F is 18,750 psi.

The heat generation in the wall per unit surface area is

$$\begin{aligned} q &= \int_0^8 150,000 (e^{-.16\Delta}) (e^{-.414 t}) dt \\ &= 14,400 \int_0^8 e^{-.414 t} dt \\ &= 14,400 \int_0^{.75} e^{-4.97 t} dt \\ &= \frac{14,400 (.963)}{4.97} = 2780 \text{ BTU/hr.} \end{aligned}$$

Hence,

$$q = 2780 \text{ BTU/hr-ft}^2$$

The coefficient along the wall will depend on the amount of gas by passed for cooling.

Or,

$$\theta_m - \theta_s = 205 e^{-.16\Delta} (1 - e^{-.414 t} - .414 e^{-.414 t} t)$$

with thermal Δ cm of thermal shields present. With the 6" shields,

$$(\theta_m - \theta_s) = 16.5^{\circ}$$

The maximum temperature
of the wall is

$$\theta_m = (\theta_s - \theta_l) + \theta_l + 16.5^\circ$$

The film temperature drop,

$$(\theta_s - \theta_l) = \frac{q}{h}$$

where q is the heat flux density at the wall surface and h the heat transfer coefficient. The minimum amount required between the thermal shield and wall will be that flow which will result in less than about a 250° rise in the coolant gas temperature. That is,

$$W_c = \frac{(q A)_{\text{wall}} + (q A)_{\text{thermal shields}}}{(C_p)(250)}$$

$$A = (\pi)(D_v)(H_c)$$

The heat flux from the shield surface is $1600 \text{ BTU}/^\circ\text{F-hr-ft}^2$.

Hence,

$$\begin{aligned} W_c &= \frac{(1600 + 2780)(\pi)(D_v)(H_c)}{(1.25)(250)} = \\ &= \frac{6,100,000}{(1.25)(250)} = 19,500 \text{ \#/hr} \end{aligned}$$

The available pressure drop will be approximately that across the core or 3.0 psi. If a .125" gap is allowed between the shield and wall, the velocity will be

$$\begin{aligned} V &= \left[\frac{\Delta P D_e (24)(144)}{\rho (4) f H_c (12)} \right]^{\frac{1}{2}} \\ &= 35 \text{ ft/sec} \end{aligned}$$

and

$$W_c = \frac{(\rho)(\pi)(D_v)(S)(V)(3600)}{(12)}$$

$$= 30,000 \text{ \#/hr}$$

which is sufficient and will give a gas temperature rise of about 170°F. The coefficient at the wall is

$$h = \frac{.023 k}{D_e} (\rho V D_e)^{0.8} \left(\frac{c_p \mu}{k} \right)^{1/3}$$

$$= 39$$

This gives,

$$(\theta_s - \theta_1) = \frac{2780}{37} = 71^\circ$$

Hence,

$$\theta_m = (\theta_s - \theta_1) + \theta_1 + 16.5^\circ$$

$$= 71 + (340 + 170) + 16.5$$

$$\theta_m = 598^\circ$$

is the maximum temperature to be expected in the vessel wall. The allowable stress is 18,750 psi up to 700°F.

C. Heat Generation Symbols

S = source strength, photons (cm²/sec)

ϕ = flux, neutrons/cm²-sec

Σ = capture cross section, cm⁻¹

N_γ = number of γ photons emitted.

E_γ = energy of γ photons, mev.

μ_c = attenuation coefficient for core, cm⁻¹

w = thickness of source, cm

$\dot{Q}$ = heat generation, BTU/ft³-hr

μ_e = energy absorption coefficient, cm⁻¹

CF = conversion factor, 1.545×10^{-8} BTU/ft³-hr per $\frac{\text{mev}}{\text{cc-sec}}$

V_R = core volume, cc

P = core power, watts

M = mass of scattering medium

J = net neutron current, neut/cm²-sec

D. Stress Symbols

σ_θ = tangential stress, total, psi

$\sigma_{\theta T}$ = thermal tangential stress, psi

$\sigma_{\theta P}$ = pressure tangential stress, psi

θ_s = temp. at inner wall surface, °F

θ_m = max wall temperature, °F

α = coefficient of expansion

E = Young's modulus, psi

k = thermal conductivity, BTU/°F-hr-ft.

m = exponent of heat generation curve

t = wall thickness, inches

a = inner radius of vessel, in.

P = pressure, psi

θ_1 = gas temperature at wall surface, °F

W_c = mass flow rate of coolant gas, #/hr

D_v = vessel I.D, ft

H_c = vessel height along core, ft.

ΔP = pressure drop, psi

ρ = gas density, $\#/ft^3$

V = gas velocity, fps

f = friction coefficient $\approx .005$

S = gap thickness, inches

D_e = equivalent dia of gap $\approx 2s, ft.$

10.2.2 Shielding

No attempt was made to optimize the shielding for this system. However, calculations were made to determine the adequacy of 7 ft of concrete around the reactor container for keeping the heat exchanger compartments dose to a level which will permit access for maintenance without shutdown. The geometry used for calculating the dosage due the core sources is shown below.

The radiation sources are the same as discussed in the section on radiation heating. The prompt, capture, and decay core gammas were treated in four groups, as indicated below.

<u>E</u> (mev)	<u>E_γ N_γ[*] (Prompt)</u>	<u>**</u> <u>E_γ N_γ (core)</u>	<u>***</u> <u>E_γ N_γ (decay)</u>	<u>Σ E_γ N_γ</u>
.3	1.9		4.5	6.4
2	4.2		2.8	7.0
4	.9	4.6		5.5
6	.5			.3

* Ref. (17)

** Due to graphite capture only

*** TID-7004, page 39

From Reference (18), the flux at a point outside the shield is given by

$$\phi = \frac{2 p a^2 B}{4(z + t_c)} \left[F(\theta, \mu_s t_s + \mu_c t_c + \mu_{Fe} t_{Fe}) \right]$$

Since $\mu_c t_c \ll \mu_{Fe} t_{Fe} + \mu_s t_s$ Let $\mu t = \mu_s t_s + \mu_{Fe} t_{Fe}$

Since the dose in Mr/hr is

$2 E \phi N \times 10^{-3}$ mr/hr for gammas,

$$D = \frac{p a^2 B}{(z + t_c)} \left[F(\theta, \mu t) \right] E N \times 10^{-3} \quad \text{mr/hr}$$

The number of fissions per cc is,

$$p = \frac{3.1 \times 10^{10} P}{V} \quad \frac{3.1 \times 10^{10} P}{\pi a^2 h}$$

Therefore,

$$\begin{aligned} D &= \frac{3.1 N E P B F(\theta, \mu t) \times 10^7}{\pi h (z + t_c)} \\ &= \frac{3.1 \times 10^7 N E P B [F(\theta, \mu t)]}{\pi (18)(30.5)(8+2.5)(30.5)} \end{aligned}$$

$$D = 59 N E P B [F(\theta, \mu t)]$$

The buildup factor was calculated by

$$B = A e^{-\alpha_1 \mu t} + (1 - A) e^{-\alpha_2 \mu t}$$

where A , α_1 , and α_2 are appropriate constants given in Ref. (17) and (18).

The results of these calculations are summarized below.

E	μ_{Fe}	$\mu_{Fe} t_{Fe}$	μ_s	$\mu_s t_s$	μt	F	B	D
.3	.848	25.5	.375	80.5	106.0		1000	nil
2	.323	18.5	.157	33.6	52.1		96.3	nil
4	.231	7.00	.115	24.6	31.6	3.5×10^{-15}	25.3	.1
6	.192	5.88	.094	20.2	26.0	4×10^{-12}	14.5	1.5

With 7' of concrete the dose from core sources due to 1000 MW operation only, is approximately 1.6 mr/hr. With 6' of concrete the dose from a 1000 mw increases to about 8 mr/hr. The capture gammas in the vessel wall and thermal shields were calculated by assuming that

$$\phi \Sigma_a N E = \sqrt{3 \Sigma_a \Sigma_{tr}} J_0 e^{-\sqrt{3 \Sigma_a \Sigma_{tr}} x} NE$$

Hence the average uniform source inside the steel is

$$S = \frac{N \int_0^w \phi \Sigma_a dx}{\int_0^w dx} = \frac{N J_0 (1 - e^{-\sqrt{3 \Sigma_a \Sigma_{tr}} w})}{\sqrt{3 \Sigma_a \Sigma_{tr}} w}$$

$$= .044 J_0 N$$

The geometry was taken as a slab.

The photon flux is given in Ref. (17), page 353 as

$$\phi = \frac{B S}{2 \mu_{Fe}} \left[E_2(\mu_s t_s) - E_2(\mu_s t_s + \mu_{Fe} t_{Fe}) \right]$$

Since the dose is $2 \times 10^{-2} \phi E$,

$$D = \frac{.001 B S E}{\mu_{Fe}} \left[E_2(\mu_s t_s) - E_2(\mu_s t_s + \mu_{Fe} t_{Fe}) \right] \frac{mr}{hr}$$

But $S = .044 \text{ Jo N for steel.}$

$$\begin{aligned} &= \frac{(.044)(3.1)(10^{10}) P (1 - P_{FL}) P_{RE} P_{TL} \propto N}{\pi a h} \\ D &= \frac{(.0422) B (10^7)(1 - P_{FL}) P_{RE} P_{TL} \propto P N E}{z t_{Fe} \mu_{Fe}} \left[E_2(\mu_s t_s) - E_2(\mu_s t_s + \mu t_{Fe}) \right] \\ D &= \frac{(21.4)(10^6) B N E}{\mu_{Fe}} \left[E_2(\mu_s t_s) - E_2(\mu_{Fe} t_{Fe} + \mu_s t_s) \right] \end{aligned}$$

The spectrum of the capture gammas in the iron was treated in 4 groups with energies as given in Reference (17).

The calculations are summarized below

E	N_γ	μ_{Fe}	μ_s	$\mu_s t_s$	B	$\mu_{Fe} t_{Fe}$	$E_2(\mu_s t_s)$	$E_2(\mu_{Fe} t_{Fe})$	Dose (7')	Dose (6')
2	.10	.323	.157	33.6		18.5	4.2×10^{-14}	nil	nil	nil
4	.24	.231	.115	24.6	25.3	7.0	8×10^{-13}	nil	nil	nil
6	.22	.192	.094	20.3	14.5	5.9	9×10^{-11}	nil	.23	5.8
7	.50	.177	.087	18.6	8.9	5.4	4.2×10^{-10}	nil	2.1	33

For a 1165 mw core, the total dose from capture gammas in the thermal shields and wall is approximately 2.3 mr/hr with 7' of high density concrete. With 6' of concrete the dose is about 39 mr/hr.

The neutron dose was estimated roughly by assuming an equivalent isotropic source S, and a removal cross-section.

From Ref. (18) for fission neutrons

$$D = \frac{S e^{-\sum r^t_s} (.125)}{4\pi R^2} \quad \text{mr/hr}$$

where R was assumed as the distance from the core face to the dose point.

$\sum r$ for heavy concrete is approximately .20/cm.

Hence

$$D = \frac{(3.1)(10^{19})(.125) e^{-.44}}{4\pi (210)^2} = \text{nil.}$$

Based on the preceding estimates it is seen that for limited access 6' may be sufficient. However, since voids due to coolant piping, and capture gammas within the concrete were not accounted for it is best that 7' be used.

Heating of the concrete was estimated roughly and a temperature rise of between 50 and 60° can be expected for a 1160 mw core. This is excessive both from the thermal stress and dehydration stand points. Hence, means of cooling the inner portions of the concrete must be provided. This could be accomplished by piping embedded in the concrete or by laminating the shield near the core face and cooling with ventilating air.

Explanation of Symbols

(This applies to Section 10.2.2 only)

ϕ = radiation flux, neutrons or photons/cm²-sec

p = number of fissions per cc.

a = core radius - cm

B = build-up factor

z = distance from dose point to core surface - cm

t_c = effective core attenuation distance cm (from Fig. 5.13 (Ref. 18))

F = exponential functions of the attenuation as plotted in Ref. (17 and (18)).

E = photon or neutron energy, mev

mr = mil roentgen equivalent man

D = dose at a point - $\frac{mr}{hr}$

N = Number of photons per fission

V = core volume, cc

h = core height, cm

P = core power, watts

Σ_a = capture cross section cm⁻¹

Σ_r = neutron removal cross section, cm⁻¹

S = point source, photon/sec

E₁, E₂ = exponential functions

J₀ = net current, neutrons or photons/cm²-sec

P_{FL} = fast leakage probabilities

P_{RE} = resonance escape probabilities

P_{TL} = thermal leakage probabilities

μ_c = attenuation coefficient for source cm⁻¹

t_{Fe} = thickness of Fe, cm

μ_{Fe} = attenuation coefficient for iron, cm^{-1}

μ_s = attenuation coefficient for concrete shield, cm^{-1}

t_s = thickness of concrete shield

10.2.3 Containment Vessel Design

The containment vessel was designed to accomodate an accident resulting in the release of all the helium from both the primary and secondary systems. An estimate of the pressure and temperature immediately following such an incident was estimated as follows:

Let

P_H = initial helium pressure, psia

P_A = initial air pressure, psia

T_A = initial average helium temperature, $^{\circ}\text{F}$

T_H = final pressure of mixture

P' = final pressure of mixture

T' = final temperature of mixture

V_{cv} = total volume inside vessel, ft^3

W_H = total weight of helium, #

W_A = total weight of air, #

E = internal energy, BTU/#

C^v = specific heat at constant volume $^{\circ}\text{F}/\text{#}$

Since no outside energy sources will be present, the total internal energy of the system will remain constant

$$\frac{dE}{dT} = C^V$$

hence

$$W_A C_A^V T_A + W_H C_H^V T_H = W_A C_A^V T' + W_H C_H^V T'$$

from which

$$T' = \frac{W_A C_A^V T_A + W_H C_H^V T_H}{W_A C_A^V + W_H C_H^V}$$

$$T' = \frac{\frac{W_A C_A^V}{W_H C_H^V} T_A + T_H}{\frac{W_A C_A^V}{W_H C_H^V} + 1}$$

$$T' = T_A \left| \frac{\frac{W_A C_A^V}{W_H C_H^V} + \frac{T_H}{T_A}}{1 + \frac{W_A C_A^V}{W_H C_H^V}} \right|$$

from Daltons Law

$$p' = p_A' + p_H'$$

$$p' = \frac{W_A R_A T'}{V_{cv}} + \frac{W_H R_H T'}{V_{cv}}$$

from which

$$p' = T' \left[1 + \frac{W_A R_A}{W_H R_H} \right] \frac{W_H R_H}{V_{cv}}$$

$$p' = T_A \left[\frac{\frac{W_A C_A}{W_H C_H} + \frac{T_H}{T_A}}{1 + \frac{W_A C_A}{W_H C_H}} \right] \left[1 + \frac{W_A R_A}{W_H R_H} \right] \frac{W_H R_H}{V_{CV}}$$

But

$$W_A = \frac{P_A V_A}{R_A T_A}$$

$$W_H = \frac{P_H V_H}{R_H T_H}$$

so that

$$T' = \left[\frac{\left(\frac{R_H}{R_A} \right) \left(\frac{P_A}{P_H} \right) \left(\frac{V_A}{V_H} \right) \left(\frac{C_A}{C_H} \right) + 1}{1 + \left(\frac{R_H}{R_A} \right) \left(\frac{P_A}{P_H} \right) \left(\frac{V_A}{V_H} \right) \left(\frac{C_A}{C_H} \right) \left(\frac{T_H}{T_A} \right)} \right]$$

and

$$p' = \left(\frac{T'}{T_H} \right) \left[1 + \left(\frac{P_A}{P_H} \right) \left(\frac{V_A}{V_H} \right) \left(\frac{T_H}{T_A} \right) \right] \frac{P_H}{\left(1 + \frac{V_A}{V_H} \right)}$$

For a containment vessel of 80' diameter

$$V_A = 265,000 \text{ ft}^3 \quad V_H = 20,000 \text{ ft}^3 \quad P_A = 14.7 \text{ psia}$$

$$P_H = 1000 \text{ psi} \quad T_H = 1460^\circ \text{R} \quad T_A = 530^\circ \text{R}$$

$$T' = 1460 \left[\frac{\left(\frac{386}{57} \right) \left(\frac{14.7}{1000} \right) \left(\frac{265,000}{20,000} \right) \left(\frac{.18}{.75} \right) + 1}{1 + \left(\frac{386}{57} \right) \left(\frac{14.7}{1000} \right) \left(\frac{265,000}{20,000} \right) \left(\frac{.18}{.75} \right) \left(\frac{1460}{530} \right)} \right] =$$

$$T' = 550^\circ \text{F}$$

$$p' = \left[\frac{1010}{1460} \right] \left[1 + \frac{14.7}{1000} \frac{265,000}{20,000} \frac{1460}{530} \right] \left[\frac{1000}{1 + \frac{265,000}{20,000}} \right] =$$

$$p' = 75 \text{ psi}$$

This gives a loop stress of

$$\sigma = \frac{PD}{2t} = \frac{(75)(80)(12)}{(2)(1)} = 36,000 \text{ psi}$$

for 1" plate, and 24000 psi for 1.5" plate.

10.2.4 Calculation of Heat Exchanger Tube Lengths

10.2.4.1 Primary Heat Exchanger

As previously shown

$$\theta_3 - \theta_6 = 1.948 \frac{P^{0.2} d_x^{1.2} (\theta_3 - \theta_1)^{0.8}}{\chi^{0.2} D_x^{0.4} L_x}$$

Let

$$\frac{L_x}{D_x} = 84$$

$$D_x^{0.4} = \left(\frac{1}{84} L_x \right)^{0.4} = 0.435 \frac{L_x^{0.4}}{y^{0.4}}$$

$$\theta_3 - \theta_6 = 1.948 \times 1.15 \frac{P^{0.2} d_x^{1.2} (\theta_3 - \theta_1)^{0.8} y^{0.4}}{\chi^{0.2} L_x^{1.4}}$$

The factor of 1.15 is introduced to allow for an 85% fin efficiency, as well as for the resistance of the tube wall.

$$L_x = 3.22 \left[\frac{P^{0.2} d_x^{1.2} (\theta_3 - \theta_1)^{0.8} y^{0.4}}{\chi^{0.2} (\theta_3 - \theta_6)} \right]$$

10.2.4.2 Economizer

As previously shown

$$D_e = 0.219 \left[\frac{P^{0.5} d_e^3 \psi_4^{2.5}}{\chi^{0.5} L_e^{2.5} (\Delta \theta_e)^{2.5}} \right]$$

Let $\frac{L_e}{D_e} = 8$, $D_e = \frac{L_e}{8}$

$$L_e^{3.5} = 1.75 \left[\frac{P d_e^6 \psi_4^5}{\chi (\Delta \theta_e)^5} \right]^{0.143}$$

where

$$\psi_4 = 13 \left(\frac{607.1 - H_{10}}{1492.7 - H_{10}} \right) \left[\underbrace{0.164 (1492.7 - H_{10})^{0.8}}_{\text{Resistance of water}} + \underbrace{0.136 (\theta_3 - \theta_1)^{0.8}}_{\text{Resistance of He}} + \underbrace{\left(1.3 \frac{t_T}{h_T} \right) \left(\frac{P^{0.8}}{N_e^{0.8} d_e^{1.8} \chi^{0.8}} \right)}_{\text{Resistance of tube wall}} \right]$$

$$N_e = 20.9 \frac{D_e^2}{d_e^2} = 0.326 \frac{L_e^2}{d_e^2} \cong \frac{0.326 \times 1225}{d_e^2} = \frac{399}{d_e^2}$$

$$N_e^{0.8} \cong \frac{120}{d_e^{1.6}}$$

$$1.3 \frac{t_T}{h_T} = 1.3 \frac{\left(\frac{d_e}{120} \right)}{20} = 5.4 \times 10^{-4} d_e$$

$$\begin{aligned} \left(1.3 \frac{h_T}{t_T} \right) \left(\frac{P^{0.8}}{N_e^{0.8} d_e^{1.8} \chi^{0.8}} \right) &= (5.4 \times 10^{-4} d_e) \left(\frac{P^{0.8}}{\frac{120}{d_e^{1.6}} d_e^{1.8} \chi^{0.8}} \right) = \\ &= 4.5 \times 10^{-6} \left(\frac{P d_e}{\chi} \right)^{0.8} \end{aligned}$$

$$\psi_4 = \left(\frac{607.1 - H_{10}}{1492.7 - H_{10}} \right) \left[2.13 (1492.7 - H_{10})^{0.8} + 2.21 (\theta_3 - \theta_1)^{0.8} + 4.5 \times 10^{-6} \left(\frac{P d_e}{\chi} \right)^{0.8} \right]$$

where the term corresponding to the helium film resistance has been increased by 25% implying an assumed fin efficiency of 80%.

10.2.4.3 Boiler

As previously shown

$$L_B = 791 \frac{\omega_H C_p}{\left(\frac{S_{Bi}}{L_B}\right) b} \left[\psi_o^{-2.035} - \psi_L^{-2.035} \right] + 10920 \frac{\omega_H C_p}{h'_{Bo} \left(\frac{S_{Bo}}{L_B}\right) b} \left[\ln \frac{\psi_L}{\psi_o} \right]$$

where ψ_o and ψ_L are defined by

$$\left(\frac{2.24}{h'_{Bo}} \right) \left(\frac{S_{Bi}}{S_{Bo}} \right) \psi_o^{3.035} + \psi_o - (\theta_B - 1053) = 0$$

$$\left(\frac{2.24}{h'_{Bo}} \right) \left(\frac{S_{Bi}}{S_{Bo}} \right) \psi_L^{3.035} + \psi_L - (\theta_i - 1053) = 0$$

$$\frac{1}{h'_{Bo}} = \frac{1}{h_{Bo}} + 1.30 \left(\frac{S_{Bo}}{S_{Bi}} \right) \left(\frac{t_r}{b_T} \right)$$

where

$$\frac{1}{h_{Bo}} = \text{helium film resistance}$$

$$1.30 \left(\frac{S_{Bo}}{S_{Bi}} \right) \left(\frac{t_r}{b_T} \right) = \text{resistance of tube and fins}$$

$$h_{Bo} = 1.72 \frac{p^{0.8}}{N_B^{0.8} d_B^{1.8} b^{0.8} (\theta_3 - \theta_i)^{0.8}}$$

where

$$D_B^2 = 4.78 \times 10^{-2} N_B d_B^2$$

-198-

$$N_B = 20.92 \frac{D_B^2}{d_B^2}$$

$$N_B^{0.8} = 11.4 \frac{D_B^{1.6}}{d_B^{1.6}}$$

$$h_{BO} = 0.120 \frac{P^{0.8}}{D_B^{1.6} d_B^{0.2} b^{0.8} (\theta_3 - \theta_1)^{0.8}}$$

this term has been reduced by 20% since a fin efficiency of 80% was assumed.

$$W_H = 7.58 \times 10^{-4} \frac{P}{\theta_3 - \theta_1} \text{ lbs/sec}$$

$$c_p = 1.25 \text{ BTU/lb } ^\circ\text{F}$$

$$\left(\frac{S_{Bi}}{L_B} \right) = 0.262 N_B d_B = 5.48 \frac{D_B^2}{d_B}$$

$$\left(\frac{S_{Bo}}{L_B} \right) = 1.31 N_B d_B = 27.4 \frac{D_B^2}{d_B}$$

$$\left(\frac{S_{Bo}}{S_{Bi}} \right) = \frac{1.31}{0.262} = 5.00$$

$$\frac{t_r}{h_T} = \frac{0.1 d_B (\text{in})}{12.8 \frac{\text{BTU}}{\text{hr ft } ^\circ\text{F}} 12 \left(\frac{\text{in}}{\text{ft}} \right)} = 6.51 \times 10^{-4} d_B \frac{\text{hr ft } ^\circ\text{F}}{\text{BTU}}$$

$$1.30 \left(\frac{S_{Bo}}{S_{Bi}} \right) \left(\frac{t_r}{h_T} \right) = 4.23 \times 10^{-3} d_B$$

$$h'_{BO} = \frac{1}{8.93 \left(\frac{D_B^{1.6} d_B^{0.2} L^{0.8} (\theta_2 - \theta_1)^{0.8}}{P^{0.8}} \right) + 4.23 \times 10^{-3} d_B}$$

$$\frac{791 \frac{\omega_H C_P}{\left(\frac{S_{Bi}}{L_B} \right) b}}{=} = \frac{791 \times 7.58 \times 10^{-4} \left(\frac{P}{\theta_2 - \theta_1} \right) \times 1.25}{5.48 \left(\frac{D_B^2}{d_B} \right) b} =$$

$$= 0.1368 \frac{P d_B}{b D_B^2 (\theta_3 - \theta_1)}$$

$$\frac{10920 \frac{\omega_H C_P}{h'_{BO} \left(\frac{S_{Bo}}{L_B} \right) b}}{=} = \frac{10920 \times 7.58 \times 10^{-4} \left(\frac{P}{\theta_3 - \theta_1} \right) \times 1.25}{h'_{BO} \times 27.4 \frac{D_B^2}{d_B} \times b}$$

$$= 0.378 \frac{P d_B}{b D_B^2 (\theta_3 - \theta_1) h'_{BO}}$$

$$L_B = \left(\frac{P d_B}{b D_B^2 (\theta_3 - \theta_1)} \right) \left(0.1368 (\psi_0^{-2.035} - \psi_L^{-2.035}) + \frac{0.378}{h'_{BO}} \ln \frac{\psi_L}{\psi_0} \right)$$

Solving for the value of ψ at the two ends of the boiler

$$\psi_0 \cong 20 (\pm 50\%) \quad , \quad \psi(0)^{0.035} \cong 1.11 (\pm 2\%)$$

$$\psi_L \cong 400 (\pm 50\%) \quad , \quad \psi(L_B)^{0.035} \cong 1.23 (\pm 2\%)$$

$$\left(\frac{2.24}{h'_{BO}} \right) \left(\frac{S_{Bi}}{S_{Bo}} \right)^{3.035} \psi_0^3 = (2.24) \frac{1}{h'_{BO}} (0.200) (1.11) \psi_0^3 = \frac{0.497}{h'_{BO}} \psi_0^3$$

$$\left(\frac{2.24}{h'_{BO}} \right) \left(\frac{S_{Bi}}{S_{Bo}} \right)^{3.035} \psi_L^3 = \frac{0.497}{h'_{BO}} \frac{1.23}{1.11} \psi_L^3 = \frac{0.551}{h'_{BO}} \psi_L^3$$

$$\frac{0.497}{h_{B0}'} \psi_0^3 + \psi_0 - (\theta_B - 1053) = 0$$

$$\frac{0.551}{h_{B0}'} \psi_L^3 + \psi_L - (\theta_7 - 1053) = 0$$

The above equations are now solved using the standard formula for cubic equations.

Let $A_0 = \frac{h_{B0}'}{3 \times 0.497} = 0.671 h_{B0}'$

$$A_L = \frac{h_{B0}'}{3 \times 0.551} = 0.605 h_{B0}'$$

$$C_0 = 1.006 h_{B0}' (\theta_B - 1053)$$

$$C_L = 1.006 h_{B0}' (\theta_7 - 1053)$$

$$\psi_0^3 + 3A_0 \psi_0 + 2C_0 = 0$$

$$\psi_L^3 + 3A_L \psi_L + 2C_L = 0$$

$$\psi_0 = \left[-C_0 + \sqrt{C_0^2 + A_0^3} \right]^{1/3} + \left[-C_0 - \sqrt{C_0^2 + A_0^3} \right]^{1/3}$$

$$\psi_L = \left[-C_L + \sqrt{C_L^2 + A_L^3} \right]^{1/3} + \left[-C_L - \sqrt{C_L^2 + A_L^3} \right]^{1/3}$$

Summarizing

$$h_{B0}' = \frac{1}{8.33 \left(\frac{D_B^{1.6} d_B^{0.2} b^{0.8} (\theta_3 - \theta_1)^{0.8}}{\rho^{0.8}} \right) + 4.23 \times 10^{-3} d_B}$$

$$A_0 = 0.671 h_{B0}'$$

$$A_L = 0.605 h'_{BO}$$

$$C_0 = 1.006 h'_{BO} (\theta_B - 1053)$$

$$C_L = 1.006 h'_{BO} (\theta_1 - 1053)$$

$$\psi_0 = \left[-C_0 + \sqrt{C_0^2 + A_0^3} \right]^{1/3} + \left[-C_0 - \sqrt{C_0^2 + A_0^3} \right]^{1/3}$$

$$\psi_L = \left[-C_L + \sqrt{C_L^2 + A_L^3} \right]^{1/3} + \left[-C_L - \sqrt{C_L^2 + A_L^3} \right]^{1/3}$$

$$L_B = \left[\frac{P_{dB}}{h D_B^2 (\theta_3 - \theta_1)} \right] \left[0.1368 (\psi_0^{-2.035} - \psi_L^{-2.035}) + \left(\frac{0.378}{h'_{BO}} \right) \ln \left(\frac{\psi_L}{\psi_0} \right) \right]$$

10.2.4.4 Superheater

As previously shown

$$D_s = 133 \left[\frac{P_{ds}^6 \psi_s^5}{h L_s^5 (\Delta \theta_s)^5} \right]^{0.5}$$

Let

$$\frac{L_s}{D_s} = 8$$

$$L_s^{3.5} = 1064 \left[\frac{P_{ds}^6 \psi_s^5}{h (\Delta \theta_s)^5} \right]^{0.5}$$

$$L_s = 7.31 \left[\frac{P_{ds}^6 \psi_s^5}{h (\Delta \theta_s)^5} \right]^{0.143}$$

where

$$\psi_5 = \left(\frac{322.8}{1492.7 - H_{10}} \right) \left[\underbrace{0.1246 (\theta_3 - \theta_1)^{0.8}}_{\text{He resistance}} + \underbrace{0.421 (1492.7 - H_{10})^{0.8}}_{\text{steam resistance}} + \underbrace{\left(1.3 \frac{t_T}{h_T} \right) \left(\frac{P^{0.8}}{d_s^{1.8} N_s^{0.8} b^{0.8}} \right)}_{\text{tube wall resistance}} \right]$$

He resistance

steam resistance

tube wall resistance

$$N_s = 0.353 \frac{L_s^2}{d_s^2}$$

$$N_s^{0.8} \approx \left[\frac{0.353 \times 900}{d_s^2} \right]^{0.8} = \frac{100}{d_s^{1.6}}$$

$$t_T \approx 0.1 d_s (\text{in}) \frac{1(\text{ft})}{12(\text{in})} = \frac{d_s}{120}$$

$$\begin{aligned} \left(1.3 \frac{t_T}{h_T} \right) \left(\frac{P^{0.8}}{d_s^{1.8} N_s^{0.8} b^{0.8}} \right) &= \left(\frac{1.3 d_s}{120 \times 12.8} \right) \left(\frac{P^{0.8}}{d_s^{1.8} \left(\frac{100}{d_s^{1.6}} \right) b^{0.8}} \right) = \\ &= 8.45 \times 10^{-6} \left(\frac{P d_s}{b} \right)^{0.8} \end{aligned}$$

$$\psi_5 = \left(\frac{322.8}{1492.7 - H_{10}} \right) \left[0.156 (\theta_3 - \theta_1)^{0.8} + 0.421 (1492.7 - H_{10})^{0.8} + 8.45 \times 10^{-6} \left(\frac{P d_s}{b} \right)^{0.8} \right]$$

where the term corresponding to the resistance of the helium film has been increased by 25%, implying an assumed fin efficiency of 80%.

10.2.5 Calculation of Heat Exchanger Required Areas for Heat Transfer

10.2.5.1 Primary Heat Exchanger

As previously shown

$$S_{xi} = 7.75 \frac{D_x^2 L_x}{dx}$$

$$D_x^2 = \frac{L_x^2}{64 y^2}$$

$$S_{xi} = 0.121 \frac{L_x^3}{y^2 dx}$$

The above gives the area for y primary heat exchangers in one loop.

The area per unit is given by:

$$S_{xi} = 0.121 \left(\frac{L_x^3}{y^2 dx} \right)$$

10.2.5.2 Economizers

As previously shown

$$S_{ei} = 0.262 N_e d_e L_e$$

$$N_e = 20.9 \frac{D_e^2}{d_e^2} = 0.326 \frac{L_e^2}{d_e^2}$$

$$S_{ei} = 0.0854 \frac{L_e^3}{d_e}$$

10.2.5.3 Boiler

$$S_{bi} = 0.262 N_B d_B L_B$$

$$N_B = 20.9 \frac{D_B^2}{d_B^2}$$

$$S_{bi} = 5.48 \frac{D_a^2 L_a}{d_b}$$

10.2.5.4 Superheater

$$S_{si} = 0.262 N_s d_s L_s$$

$$N_s = 0.353 \frac{L_s^2}{d_s^2}$$

$$S_{si} = 0.0924 \frac{L_s^3}{d_s}$$

10.2.6 Cost of Heat Exchangers

For these revised equations, the new and more conservative cost formulas shown on page 131 were employed.

Stainless steel exchanger:
$$\$ = 380 \left(0.85 + \frac{p}{1000} \right) S_i^{0.676}$$

Carbon steel exchanger:
$$\$ = 105 \left(0.85 + \frac{p}{1000} \right) S_i^{0.676}$$

Using the above equations, the following formulas for installed heat exchanger costs are obtained (assuming that in each loop the coldest primary heat exchanger and the economizer are carbon steel and everything else stainless.

10.2.6.1 Primary Heat Exchangers, Stainless Steel

$$\$_{XS} = (X \text{ loops}) (y-1) \frac{\text{units}}{\text{loop}} \times \left[380 \left(0.85 + \frac{p}{1000} \right) S_{xi}^{0.676} \frac{\$}{\text{unit}} \right]$$

$$\$_{XS} = (323 + 0.380 p) X (y-1) S_{xi}^{0.676}$$

10.2.6.2 Primary Heat Exchangers, Carbon Steel

$$\$_{XC} = (X \text{ loops}) \left(1 \frac{\text{unit}}{\text{loop}} \right) \times \left[105 \left(0.85 + \frac{p}{1000} \right) S_{xi}^{0.676} \frac{\$}{\text{unit}} \right]$$

$$\$_{XC} = (89.3 + 0.105 p) X S_{xi}^{0.676}$$

10.2.6.3 Economizers

$$\$e = (x \text{ units}) \left[105 \left(0.85 + \frac{1450}{1000} \right) S_{ei}^{0.676} \right] \frac{\$}{\text{unit}}$$

$$\$e = 241 x S_{ei}^{0.676}$$

10.2.6.4 Boilers

$$\$B = (b \text{ units}) \left[380 \left(0.85 + \frac{1450}{1000} \right) S_{bi}^{0.676} \right] \frac{\$}{\text{unit}}$$

$$\$B = 874 b S_{bi}^{0.676}$$

10.2.6.5 Superheaters

$$\$s = (s \text{ units}) \left[380 \left(0.85 + \frac{1450}{1000} \right) S_{si}^{0.676} \right] \frac{\$}{\text{unit}}$$

$$\$s = 874 s S_{si}^{0.676}$$

10.2.7 Calculation of Pressure Drops

10.2.7.1 Reactor

total = central hole + entrance + diffuser + exit + reactor core

$$\Delta p_T = \Delta p_o + \Delta p_{en} + \Delta p_D + \Delta p_{ex} + \Delta p_R$$

as previously shown

$$\Delta p_{en} = 7.80 \times 10^{-2} D_o^5 \left(\frac{\theta_1}{\theta_2} \right)^2 \Delta p_o$$

$$D_o^5 = (2.5)^5 = 97.7$$

$$\left(\frac{\theta_1}{\theta_2} \right)^2 = \left(\frac{800}{2000} \right)^2 = 0.4^2 = 0.16$$

$$\Delta p_{en} = 1.22 \Delta p_o$$

$$\Delta p_D = 0.25 D_o \Delta p_o = 0.625 \Delta p_o$$

$$\Delta p_{ex} = 2.5 \left(\frac{D_o^5}{D_{ex}^4} \right) \left(\frac{\theta_2}{\theta_3} \right)^2 \Delta p_o$$

$$D_o^5 = 2.5^5 = 97.7$$

$$D_{ex}^4 = 3.5^4 = 150.1$$

$$\frac{\theta_2}{\theta_3} = 1.10^2 = 1.21$$

$$\Delta p_{ex} = \frac{2.5 \times 97.7 \times 1.21}{150.1} \Delta p_o = 1.97 \Delta p_o$$

Therefore,

$$\Delta p_T = (1 + 1.22 + 0.63 + 1.97) \Delta p_0 + \Delta p_R$$

$$\Delta p_T = 4.82 \Delta p_0 + \Delta p_R$$

$$\Delta p_0 = 1.56 \times 10^{-8} \frac{P^2 \theta_2}{P D_o^5 (\theta_3 - \theta_1)^2} \left(\frac{163}{ft^2} \right) \frac{1}{144} \left(\frac{ft^2}{in^2} \right)$$

$$D_o^5 = (2.5)^5 = 97.7$$

$$\Delta p_0 = 1.11 \times 10^{-12} \frac{P^2 \theta_2}{P (\theta_3 - \theta_1)^2} \text{ psi}$$

$$\Delta p_T = 5.34 \times 10^{-12} \frac{P^2 \theta_2}{P (\theta_3 - \theta_1)^2} + \Delta p_R$$

as previously shown

$$\Delta p_R = 1.56 \times 10^{-7} \frac{P^2}{P d_F (\theta_2 - \theta_1)^2} \left[\frac{24.5}{D_o} - \frac{24.5}{D_R} + \ln \frac{D_R}{D_o} \right] \frac{163}{ft^2} \frac{1}{144 in^2}$$

$$\frac{24.5}{D_o} = \frac{24.5}{2.5} = 9.80$$

$$\frac{24.5}{D_R} = \frac{24.5}{16.0} = 1.53$$

$$\ln \frac{D_R}{D_o} = \ln \frac{16.0}{2.5} = \ln 6.4 = 1.86$$

$$9.80 - 1.53 + 1.86 = 10.13$$

$$\Delta p_R = 1.10 \times 10^{-8} \frac{\rho^2}{\rho d_F (\theta_2 - \theta_1)^2} \text{ psi}$$

$$\Delta p_T = 5.34 \times 10^{-12} \frac{\rho^2 \theta_2}{\rho (\theta_3 - \theta_1)^2} + 1.10 \times 10^{-8} \frac{\rho^2}{\rho d_F (\theta_2 - \theta_1)^2}$$

10.2.7.2 Primary Heat Exchangers

As previously shown

$$\Delta p_{xi} = 5.70 \times 10^{-9} \frac{\rho^{1.75} L_x (\theta_1 + \theta_3)}{D_x^{3.5} d_x^{1.25} \rho (\theta_3 - \theta_1)^{1.75} \chi^{1.75}}$$

$$\Delta p_{xo} = 1.463 \times 10^{-9} \frac{\rho^{1.75} L_x (\theta_5 + \theta_6)}{D_x^{3.5} d_x^{1.25} \rho (\theta_3 - \theta_1)^{1.75} \chi^{1.75}}$$

Substituting that

$$\frac{L_x}{D_x} = 8y$$

$$D_x^{3.5} = \left(\frac{L_x}{8y} \right)^{3.5} = \frac{L_x^{3.5}}{1446 y^{3.5}}$$

$$\Delta p_{xi} = 8.24 \times 10^{-6} \frac{\rho^{1.75} y^{3.5} (\theta_1 + \theta_3)}{L_x^{2.5} d_x^{1.25} \rho (\theta_3 - \theta_1)^{1.75} \chi^{1.75}}$$

$$\Delta p_{xo} = 2.115 \times 10^{-6} \frac{\rho^{1.75} y^{3.5} (\theta_5 + \theta_6)}{L_x^{2.5} d_x^{1.25} \rho (\theta_3 - \theta_1)^{1.75} \chi^{1.75}}$$

10.2.7.3 Economizers

As previously shown

$$\Delta p_{eo} = 1.552 \times 10^{-7} \frac{p^{1.75} L_e (\theta_8 + \theta_5)}{N_e^{1.75} d_e^{4.75} \chi^{1.75} p (\theta_3 - \theta_1)^{1.75}}$$

$$A_e = 3.76 \times 10^{-2} N_e d_e^2 = 0.7854 D_e^2$$

$$N_e = 20.9 \frac{D_e^2}{d_e^2}$$

$$N_e^{1.75} = 205 \frac{D_e^{3.5}}{d_e^{3.5}}$$

$$D_e^{3.5} = \left(\frac{L_e}{8} \right)^{3.5} = \frac{L_e^{3.5}}{1446}$$

$$N_e^{1.75} = 0.142 \frac{L_e^{3.5}}{d_e^{3.5}}$$

$$\Delta p_{eo} = 1.093 \times 10^{-6} \frac{p^{1.75} (\theta_5 + \theta_8)}{\chi^{1.75} p d_e^{1.25} L_e^{2.5} (\theta_3 - \theta_1)^{1.75}}$$

10.2.7.4 Boilers

As previously shown

$$\Delta p_{Bo} = 1.61 \times 10^{-7} \frac{p^{1.75} L_B (\theta_2 + \theta_8)}{d_B^{4.75} p N_B^{1.75} b^{1.75} (\theta_3 - \theta_1)^{1.75}}$$

$$N_B = 20.9 \frac{D_B^2}{d_B^2}$$

$$N_B^{1.75} = 205 \frac{D_B^{3.5}}{d_B^{3.5}}$$

$$\Delta p_{B0} = 7.85 \times 10^{-10} \frac{p^{1.75} (\theta_7 + \theta_8) L_B}{h^{1.75} p d_B^{1.25} D_B^{3.5} (\theta_3 - \theta_1)^{1.75}}$$

10.2.7.5 Superheaters

As previously shown

$$\Delta p_{S0} = 1.63 \times 10^{-7} \frac{p^{1.75} L_S (\theta_6 + \theta_7)}{N_S^{1.75} d_S^{4.75} h^{1.75} (\theta_3 - \theta_1)^{1.75} p}$$

$$D_S = 0.218 d_S \sqrt{N_S}$$

$$N_S = 22.59 \frac{D_S^2}{d_S^2}$$

$$D_S^2 = \left(\frac{L_S}{8} \right)^2 = \frac{L_S^2}{64}$$

$$N_S = 0.353 \frac{L_S^2}{d_S^2}$$

$$N_S^{1.75} = 0.162 \frac{L_S^{3.5}}{d_S^{3.5}}$$

$$\Delta p_{S0} = 1.006 \times 10^{-6} \frac{p^{1.75} (\theta_6 + \theta_7)}{h^{1.75} p d_S^{1.25} L_S^{2.5} (\theta_3 - \theta_1)^{1.75}}$$

10.2.7.6 Piping

As previously shown

$$\Delta p_{ps} = 1.75 \times 10^{-11} \frac{p^{1.75} (211 D_{ps} + 40) (\theta_3 + \theta_6)}{\chi^{1.75} p D_{ps}^{4.75} (\theta_3 - \theta_1)^{1.75}}$$

$$\Delta p_{pc} = 1.58 \times 10^{-11} \frac{p^{1.75} (81 D_{pc} + 56) (\theta_1 + \theta_5)}{\chi^{1.75} p D_{pc}^{4.75} (\theta_3 - \theta_1)^{1.75}}$$

10.2.8 Calculation of Power Lost in Pumping

Summing up all the pressure drops for the "hot" helium loop

$$\Delta p_H = \Delta p_r + \Delta p_{xi} + 0.5 \Delta p_{ps} + 0.5 \Delta p_{pc}$$

For the secondary loop

$$\Delta p_c = \Delta p_{xo} + \Delta p_{eo} + \Delta p_{Bo} + \Delta p_{so} + 0.5 \Delta p_{ps} + 0.5 \Delta p_{pc}$$

As previously shown, the ratio of pumping power loss to thermal reactor power is given by

$$\left(\frac{P_H}{P} \right) = 0.498 \frac{\theta_1 \Delta p_H}{p (\theta_3 - \theta_1)}$$

$$\left(\frac{P_c}{P} \right) = 0.498 \frac{\theta_5 \Delta p_c}{p (\theta_3 - \theta_1)}$$

$$\left(\frac{P_w}{P}\right) = 0.01 \eta_{PP}$$

Therefore, the total power loss is given by

$$\left(\frac{P_L}{P}\right) = \left(\frac{0.498}{\phi(\theta_3 - \theta_1)}\right) (\theta_1 \Delta p_H + \theta_5 \Delta p_c) + 0.01 \eta_{PP}$$

(Appendix B)

Included in Appendix B are the derivations of equations used in previous chapters.

11.1 Nomenclature (applies to Sections 11.2-11.8, except as noted)*

<u>Symbol</u>	<u>Units</u>	<u>Definition</u>
θ_x	$^{\circ}\text{R}$	Temperature at point x (See Figure 2-5 for location of x)
H_x	BTU/#	Enthalpy at point x (See Figure 2-5 for location of x)
$W_{H,w}$	#/sec	Mass flow rate of helium (or water)
Q_H	ft^3/sec	Volume flow rate of helium
ρ	#/ft ³	Density
P	#/in ²	System pressure
M	- -	Molecular weight
P	watts	Thermal power output of system
P_H	watts	"Hot" loop pumping power
P_C	watts	"Cold" loop pumping power
ΔP_H	psi	Total pressure drop in "hot" loop
ΔP_R	psi	Pressure drop across reactor
ΔP_Z	psi	Pressure drop across equipment (Z defined Section 11.4.1)
ΔP_C	psi	Total pressure drop in "cold" loop
P_R	watts	Power to pump through reactor
P_L	watts	Total pumping power
$P_{Zi,o}$	watts	Pumping power required for equipment Z; i-inside o-outside
P_{PS}	watts	Pumping power required for stainless steel piping
P_{PC}	watts	Pumping power required for carbon steel piping
η_{PP}	- -	Power plant efficiency
q	BTU/hr ft ²	Heat transfer rate
U_Z	BTU/hr-ft ² - $^{\circ}\text{F}$	Overall Coefficient of heat transfer for exchanger "z"
h_Z	BTU/hr-ft ² - $^{\circ}\text{F}$	Heat transfer coefficient for exchanger "Z"

<u>Symbol</u>	<u>Units</u>	<u>Definition</u>
C_p	BTU/#-°F	Specific heat
μ	#/ft-hr	Viscosity of fluid
k	BTU/hr-ft-°F	Thermal conductivity
$D_{Zi,o}$	ft	Hydraulic diameter for exchanger "Z" i - inside o - outside
G	lb/sec-ft ²	Mass velocity
Re	- -	Reynolds number
f	- -	Friction factor
g	ft/sec ²	Acceleration of gravity
D_Z	ft	Diameter of exchanger "Z"
Pr	- -	Prandtl number
V	ft/sec	Velocity
Nu	- -	Nusselt number

* The subscript z, i, o on the symbols denotes the piece of equipment as defined in Section 11.4.1 and where i or o are also shown outside of the tubes.

11.2 Heat Balance

$$\frac{\theta_6 - \theta_7}{\theta_6 - \theta_8} = \frac{H_{13} - H_{12}}{H_{13} - H_{11}} = \frac{1492.7 - 1169.9}{1492.7 - 607.1} = 0.3645$$

$$\theta_6 - \theta_7 = 0.3645 (\theta_6 - \theta_8)$$

$$\theta_8 = \theta_6 - \left(\frac{\theta_6 - \theta_7}{0.3645} \right) = \theta_6 - 2.743 \theta_6 + 2.743 \theta_7$$

$$\theta_8 = 2.743 \theta_7 - 1.743 \theta_6$$

$$\theta_7 = \theta_6 - 0.3645 (\theta_6 - \theta_8)$$

$$\boxed{\theta_7 = 0.6355 \theta_6 + 0.3645 \theta_8}$$

$$\frac{\theta_6 - \theta_5}{\theta_6 - \theta_8} = \psi_1 = \frac{H_{13} - H_{10}}{H_{13} - H_{11}}$$

$$-\theta_6 + \theta_5 = \psi_1 \theta_6 + \psi_1 \theta_8$$

$$\theta_5 = \theta_9 = \psi_1 \theta_8 - (\psi_1 - 1) \theta_6$$

$$\theta_1 = \theta_3 + \theta_5 - \theta_6$$

$$\frac{\theta_6 - \theta_8}{\theta_3 - \theta_1} = \frac{H_{13} - H_{11}}{H_{13} - H_{10}} = \frac{1492.7 - 607.1}{1492.7 - H_{10}} = \frac{885.6}{1492.7 - H_{10}} = \frac{1}{\psi_1}$$

$$\theta_6 = \theta_8 + \frac{\theta_3 - \theta_1}{\psi_1}$$

$$\theta_7 = 0.6355 \theta_6 + 0.3645 \theta_8$$

$$\theta_3 - \theta_1 = \theta_6 - \theta_5$$

$$\theta_5 = \theta_6 + \theta_1 - \theta_3$$

$$\Delta \theta_x = \theta_3 - \theta_6$$

$$\Delta \theta_e = \frac{\theta_8 + \theta_{10} - \theta_5 - 1053}{\ln \left(\frac{\theta_8 - 1053}{\theta_5 - \theta_{10}} \right)}$$

$$\Delta \theta_8 = \frac{\theta_7 - \theta_8}{\ln \left(\frac{\theta_7 - 1053}{\theta_8 - 1053} \right)}$$

$$\Delta \theta_5 = \frac{\theta_6 - \theta_1 - 407}{\ln \left(\frac{\theta_6 - 1460}{\theta_1 - 1053} \right)}$$

11.3 Calculation of Flow Rates and Blower Power

The mass flow rate and volume flow rate are

$$W_H = 7.58 \times 10^{-4} \frac{P}{\theta_3 - \theta_1} \frac{\text{lb}}{\text{sec}}$$

$$Q_H = \frac{W_H}{P_1} \frac{\text{lb/sec}}{\text{lb/ft}^3}$$

where $P_1 = 0.0931 \frac{\text{PM}}{\theta_1} = 0.372 \frac{P}{\theta_1}$

Substituting

$$Q_H = \frac{7.58 \times 10^{-4} \frac{P}{\theta_3 - \theta_1}}{0.372 \frac{P}{\theta_1}} = 2.04 \times 10^{-3} \frac{P \theta_1}{P(\theta_3 - \theta_1)} \frac{\text{ft}^3}{\text{sec}}$$

The power in watts can now be expressed as

$$P_H = \frac{Q_H \frac{\text{ft}^3}{\text{sec}} \times \Delta p_H \frac{\text{lb}}{\text{in}^2} \times 144 \frac{\text{in}^2}{\text{ft}^2} \times 1.356 \frac{\text{Watt sec}}{\text{ft lb}}}{\eta_B}$$

$$P_H = 195.1 \frac{Q_H \Delta p_H}{\eta_B}$$

Substituting for Q_H (Hot loop pumping power)

$$P_H = 0.398 \frac{P \theta_1 \Delta p_H}{P(\theta_3 - \theta_1) \eta_B} \text{ watts}$$

(Cold loop pumping power)

$$P_C = 0.398 \frac{P \theta_5 \Delta p_C}{P(\theta_3 - \theta_1) \eta_B} \text{ watts}$$

the total pressure drop in the hot loop is

$$\Delta p_H = \Delta p_3 + \Delta p_1 + \Delta p_{xi} + \Delta p_R$$

The pressure drop in the cold side is:

$$\Delta p_c = \Delta p_6 + \Delta p_5 + \Delta p_{x0} + \Delta p_{eo} + \Delta p_{Bo} + \Delta p_{so}$$

The ratio of power to pump through the reactor to the reactor thermal power, assuming a blower efficiency $\eta_p = 0.80$, is given by

$$\frac{P_R}{P} = \frac{0.398}{0.80} \frac{\theta_1 \Delta p_R}{\rho(\theta_3 - \theta_1)}$$

Total pumping power to reactor thermal power is:

$$\begin{aligned} \frac{P_L}{P} = & \left(\frac{P_R}{P} \right) + \left(\frac{P_{xi}}{P} \right) + \left(\frac{P_{xo}}{P} \right) + \left(\frac{P_{eo}}{P} \right) + \left(\frac{P_{Bo}}{P} \right) + \left(\frac{P_{so}}{P} \right) + \left(\frac{P_{ps}}{P} \right) \\ & + \left(\frac{P_{pc}}{P} \right) + 0.02 \eta_{pp} \end{aligned}$$

where $0.02 \eta_{pp}$ is the feed water pump power consumption.

The net electrical power in watts is:

$$P_{NE} = P \left[\eta_{pp} - \left(\frac{P_L}{P} \right) (1 - \eta_{pp}) \right]$$

Calculating Pressure Drop from Power Loss

$$\frac{P_H}{P} = 0.498 \frac{\theta_1 \Delta p_H}{\rho(\theta_3 - \theta_1)} = \left(\frac{P_e}{P} \right) + \left(\frac{P_{xi}}{P} \right) + \left(\frac{P_{pc}}{P} \right)$$

$$\frac{P_c}{P} = 0.498 \frac{\theta_5 \Delta p_c}{\rho(\theta_3 - \theta_1)} = \left(\frac{P_{xo}}{P} \right) + \left(\frac{P_{eo}}{P} \right) + \left(\frac{P_{Bo}}{P} \right) + \left(\frac{P_{so}}{P} \right) + \left(\frac{P_{ps}}{P} \right)$$

The pressure drop through the hot loop

$$\Delta p_H = 2.01 \frac{\rho(\theta_3 - \theta_1)}{\theta_1} \left[\left(\frac{P_R}{P} \right) + \left(\frac{P_{xi}}{P} \right) + \left(\frac{P_{Rc}}{P} \right) \right]$$

Through the cold loop

$$\Delta p_c = 2.01 \frac{\rho(\theta_3 - \theta_1)}{\theta_5} \left[\left(\frac{P_{xo}}{P} \right) + \left(\frac{P_{eo}}{P} \right) + \left(\frac{P_{ao}}{P} \right) + \left(\frac{P_{so}}{P} \right) + \left(\frac{P_{ps}}{P} \right) \right]$$

11.4 Heat Exchanger Equations

General heat exchanger equations were developed so that an optimum heat transfer system could be designed.

11.4.1 Nomenclature

The subscript "Z" symbolizes the four following heat exchangers:

- 1) "Z" = "X" (primary heat exchanger)
- 2) "Z" = "e" (Economizer)
- 3) "Z" = "B" (Boiler)
- 4) "Z" = "S" (Superheater)

The additional subscript i or o indicate inside or outside of tubes

L_Z = Tube length, ft.

d_Z = tube diameter, in.

N_Z = number of tubes per exchanger

P_Z = thermal power transferred by exchanger, watts

P = Reactor thermal power, watts

Assume:

d_Z = inside tube diameter, in.

0.1 d_Z = tube wall thickness, in.

0.1 d_Z = fin thickness, in.

0.4 d_X = fin width, in.

0.5 d_e , 0.5 d_B , 0.5 d_S = fin width, in.

Center to center spacing of tubes = 2.1 d_X , 2.5 d_e , 2.5 d_B , 2.5 d_S

S_{Zi} = total surface area inside tubes, per exchanger, ft^2

S_{Zo} = total surface area outside tubes, per exchanger, ft^2

A_Z = cross sectional area, total, per exchanger, ft^2

A_{Zi} = cross sectional area, inside tube, per exchanger, ft^2

A_{ZT} = cross sectional area, tubes and fins, per exchanger, ft^2

A_{ZO} = cross sectional area, outside tubes, per exchanger, ft^2

$\Delta\theta_z$ = log mean temperature difference of exchanger "Z"

Assume that all heat exchangers are countercurrent with the following fluid arrangement:

<u>Heat Exchanger</u>	<u>Subscript</u>	<u>Fluid Inside Tube</u>	<u>Fluid Outside Tube</u>
Primary heat exchanger	x	Hot helium	Cold helium
Economizer	e	Water	Cold helium
Boiler	B	Water	Cold helium
Superheater	s	Steam	Cold helium

Let x = the number of primary coolant loops per reactor. Assume that the tubes are arranged at the apexes of equilateral triangles, with a center to center distance indicated previously. Each tube is to have 12 external longitudinal fins.

Let y = the number of primary heat exchangers in series.

Note: Each of the x primary loops has y primary heat exchangers in series.

For the purpose of calculating the required surface area, these y exchangers were treated as though they were one. Therefore, the tube length per unit is $\frac{L_x}{y}$, since L_x is the total tube length of all the primary exchangers in one loop.

Let b = the number of secondary coolant loops. Therefore for b secondary loops there will be b boilers and b superheaters. Since the size of the economizers is much greater than the size of the boilers or superheaters, assume that there are $\propto$ economizers in the system.

11.4.2 Boiler Equations

The basic heat transfer equation in terms of the log mean temperature difference $\Delta\theta$ can be represented as:

$$q = U_{Bi} S_{Bi} \Delta\theta_D$$

where the overall heat transfer coefficient is given by

$$U_{Bi} = \frac{1}{\frac{1}{h_{Bi}} + \frac{S_{Bi}}{S_{Bo}} \frac{1}{h_{Bo}}}$$

$$S_{Bi} = \frac{N_B \pi d_B (in) L_B (ft)}{12 (in)/(ft)} = 0.262 N_B d_B L_B (ft^2)$$

$$S_{Bo} = N_B [\pi 12 d_B + 12 \times 2 \times 0.5 d_B] (in) / 12 (in)/(ft) \times L_B (ft) = 1.31 N_B d_B L_B (ft^2)$$

$$A_{Bi} = N_B \pi d_B^2 (in^2) \times \frac{1 (ft^2)}{144 (in^2)} = 5.45 \times 10^{-3} N_B d_B^2 (ft^2)$$

$$A_{BT} = N_B [\pi d_B + 12 \times 0.5 d_B] [0.1 d_B] (in^2) \times \frac{1 (ft^2)}{144 (in^2)} = 6.35 \times 10^{-3} N_B d_B^2 (ft^2)$$

$$A_B = N_B [2.5 d_B]^2 [0.866] (in^2) \times \frac{1 (ft^2)}{144 (in^2)} = 37.6 \times 10^{-3} d_B^2 N_B (ft^2)$$

$$A_{Bo} = A_B - A_{Bi} - A_{BT} = 25.8 \times 10^{-3} N_B d_B^2 (ft^2)$$

$$\frac{S_{Bi}}{S_{Bo}} = \frac{0.262 N_B d_B L_B}{1.31 N_B d_B L_B} = 0.20$$

Therefore,

$$U_{Bi} = \frac{1}{\frac{1}{h_{Bi}} + \frac{0.2}{h_{Bo}}}$$

The thermal power transferred in each of the boilers, in watts, is given by

$$P_B = \frac{\Delta \theta_{Bm} (^{\circ}R) \times U_{Bi} \left(\frac{BTU}{hr ft^2 ^{\circ}F} \right) \times S_{Bi} (ft^2)}{3.413 \left(\frac{BTU}{watt hr} \right)}$$

To determine the outside heat transfer equation (He outside) the Dittus-

Boelter equation is used; avg. temp. = 850°F

$$h_{Bo} = 0.023 \left(\frac{C_p \mu}{k} \right)_{Bo}^{0.4} \left(\frac{D_G}{\mu} \right)_{Bo}^{0.8} \left(\frac{k}{D} \right)_{Bo}$$

$$h_{Bo} = 0.023 (0.675)^{0.4} \left(\frac{D_{Bo} G_{Bo}}{\mu_{Bo}} \right)^{0.8} \left(\frac{k_{Bo}}{D_{Bo}} \right)$$

$$h_{Bo} = 0.0196 \left(\frac{D_{Bo} G_{Bo}}{\mu_{Bo}} \right)^{0.8} \left(\frac{k_{Bo}}{D_{Bo}} \right)$$

$$\mu_{Bo} = 0.084 \text{ #/ft hr}$$

$$k_{Bo} = 0.157 \text{ BTU/ft hr } ^{\circ}F$$

The hydraulic diameter must be used:

$$D_{Bo} = \frac{4 \times \text{area}}{WP} = \frac{4 \times A_{Bo}}{\left(\frac{S_{Bo}}{L_B} \right)} = \frac{4 (25.8 \times 10^{-3} N_B d_B^2)}{1.31 N_B d_B}$$

$$D_{Bo} = 0.079 d_B$$

The mass velocity in #/hr-ft² is given by:

$$G_{B0} = \frac{W_H (\#/sec) \times 3600 (sec/hr)}{A_{B0} (ft^2) \ell}$$

$$G_{B0} = \frac{3600}{25.8 \times 10^{-3}} \frac{W}{N_B d_B^2 \ell}$$

$$G_{B0} = 1.4 \times 10^5 \frac{W}{N_B d_B^2 \ell}$$

$$W = 7.58 \times 10^{-4} \frac{P}{\theta_3 - \theta_1}$$

$$G_{B0} = 106 \frac{P}{N_B d_B^2 \ell (\theta_3 - \theta_1)}$$

The Reynolds number can be expressed as:

$$Re_{B0} = \frac{DG}{\mu} = \frac{(0.079 d_B)(106 P)}{(0.0840)(N_B d_B^2 \ell [\theta_6 - \theta_5])}$$

$$Re_{B0} = 99.75 \frac{P}{N_B d_B \ell (\theta_6 - \theta_5)}$$

$$(Re_{B0})^{0.8} = 39.7 \frac{P^{0.8}}{[N_B d_B \ell (\theta_6 - \theta_5)]^{0.8}}$$

Therefore

$$h_{B0} = 0.778 \left(\frac{h}{D} \right)_{B0} \left(\frac{P^{0.8}}{N_B^{0.8} d_B^{0.8} \ell^{0.8} [\theta_6 - \theta_5]^{0.8}} \right)$$

$$h_{BO} = 0.778 \left(\frac{0.175}{0.079 d_B} \right) \left(\frac{p^{0.8}}{N_B^{0.8} d_B^{0.8} L^{0.8} [\theta_6 - \theta_5]^{0.8}} \right)$$

$$h_{BO} = 1.72 \frac{p^{0.8}}{N_B^{0.8} d_B^{1.8} L^{0.8} [\theta_3 - \theta_1]^{0.8}}$$

Determine the inside heat transfer coefficient.

Assume the steam side $\frac{1}{h}$ is 5% of the He side $\frac{1}{h}$

$$\frac{1}{h_{Bi}} = 0.05 \frac{1}{h_{BO}}$$

therefore

$$U_{Bi} = \frac{1}{\frac{S_{Bi}}{S_{BO}} \frac{1}{h_{BO}} + 0.05 \frac{1}{h_{BO}}} = 4 h_{BO}$$

$$U_{Bi} = \frac{6.88 p^{0.8}}{N_B^{0.8} d_B^{1.8} L^{0.8} [\theta_6 - \theta_5]^{0.8}}$$

Recall that

$$q = U_{Bi} S_{Bi} \Delta \theta_B$$

$$q = \left(\frac{6.88 p^{0.8}}{N_B^{0.8} d_B^{1.8} L^{0.8} [\theta_6 - \theta_5]^{0.8}} \right) \left(0.262 N_B d_B L_B \right) \Delta \theta_B$$

$$q = \frac{1.8 P^{0.8} N_B^{0.2} L_B}{d_B^{0.8} t^{0.8} [\theta_6 - \theta_5]^{0.8}} \Delta \theta_B = \text{BTU/hr}$$

$$\Delta \theta_B = \frac{[\theta_7 - \theta_{12}] - [\theta_8 - \theta_{11}]}{\ln \frac{[\theta_7 - \theta_{12}]}{[\theta_8 - \theta_{11}]}}$$

$$P_B = \frac{0.527 P^{0.8} N_B^{0.2} L_B}{d_B^{0.8} t^{0.8} [\theta_6 - \theta_5]^{0.8}} \frac{[\theta_7 - \theta_{12}] - [\theta_8 - \theta_{11}]}{\ln \frac{[\theta_7 - \theta_{12}]}{[\theta_8 - \theta_{11}]}} = \text{Watts}$$

Rearranging

$$N_B = \left[\frac{P_B d_B^{0.8} t^{0.8} [\theta_6 - \theta_5]^{0.8} \ln \frac{[\theta_7 - \theta_{12}]}{[\theta_8 - \theta_{11}]}}{0.527 P^{0.8} L_B ([\theta_7 - \theta_{12}] - [\theta_8 - \theta_{11}])} \right]^5$$

$$N_B = \frac{P_B^5}{P^4} \frac{d_B^4 t^4}{L_B^5 (0.527)^5} \left[\frac{[\theta_6 - \theta_5]^{0.8} \ln \frac{[\theta_7 - \theta_{12}]}{[\theta_8 - \theta_{11}]}}{[\theta_7 - \theta_{12}] - [\theta_8 - \theta_{11}]} \right]^5$$

$$P_B = \frac{H_{12} - H_{11}}{H_{13} - H_{10}} \frac{P}{t}$$

Thus

$$N_B = 24.6 \frac{P}{t} \frac{d_B^4}{L_B^5} \left[\frac{\frac{[H_{12} - H_{11}]}{[H_{13} - H_{10}]} [\theta_6 - \theta_5]^{0.8} \ln \frac{[\theta_7 - \theta_{12}]}{[\theta_8 - \theta_{11}]}}{[\theta_7 - \theta_{12}] - [\theta_8 - \theta_{11}]} \right]^5$$

The total cross sectional area in the exchanger is

$$A_B = 3.76 \times 10^{-2} N_B d_B^2 = 0.7854 P_B^2$$

the boiler diameter is

$$D_B = \left[4.78 \times 10^{-2} N_B d_B^2 \right]^{\frac{1}{2}} = 0.218 d_B \sqrt{N_B}$$

$$D_B = 1.075 \frac{P^{.5}}{L^{.5}} \frac{d_B^3}{L_B^{2.5}} \left[\frac{\frac{[H_{12} - H_{11}]}{[H_{13} - H_{10}]} [\theta_6 - \theta_5]^{0.8} \ln \frac{[\theta_1 - \theta_{12}]}{[\theta_8 - \theta_{11}]}}{[\theta_7 - \theta_{12}] - [\theta_8 - \theta_{11}]} \right]^{2.5}$$

where

$$\psi_3 = \frac{H_{12} - H_{11}}{H_{13} - H_{10}}$$

$$D_B = 1.075 \frac{P^{.5}}{L^{.5}} \frac{d_B^3}{L_B^{2.5}} \left[\frac{\psi_3 [\theta_6 - \theta_5]^{0.8}}{\Delta \theta_B} \right]^{2.5}$$

$$S_{Bi} = 0.262 N_B d_B L_B = 6.44 \frac{P}{L} \frac{d_B^5}{L_B^4} \left[\frac{\psi_3 [\theta_6 - \theta_5]^{0.8}}{\Delta \theta_B} \right]^5$$

The pressure drop equations for the boiler on the He side were developed as follows.

The pressure drop can be expressed as

$$\Delta p = \frac{2 f G^2 L}{g \rho D}$$

The friction factor f is given approximately by

$$f_{Bo} = 0.079 \left(\frac{\mu_{Bo}}{D_{Bo} G_{Bo}} \right)^{0.25}$$

The He density can be represented by

$$\rho_{Bo_{avg}} = 0.745 \frac{\rho}{[\theta_7 + \theta_8]}$$

therefore

$$\Delta p_{Bo} = \frac{3.53 \times 10^{-12} \mu_{Bo}^{.25} G_{Bo}^{1.75} L_B [\theta_7 + \theta_8]}{\rho D_{Bo}^{1.25}}$$

$$\mu_{Bo} = 0.084$$

$$\mu_{Bo}^{.25} = (0.084)^{.25} = 0.539$$

$$\Delta p_{Bo} = 1.90 \times 10^{-12} \frac{G_{Bo}^{1.75} L_B [\theta_7 + \theta_8]}{\rho D_{Bo}^{1.25}}$$

The mass velocity is

$$G_{Bo} = 106 \frac{\rho}{N_B d_B^2 L [\theta_6 - \theta_5]}$$

Hence

$$\Delta p_{Bo} = \frac{1.90 \times 10^{-12} (106)^{1.75} \rho^{1.75} L_B [\theta_7 + \theta_8]}{\rho D_{Bo}^{1.25} N_B^{1.75} d_B^{3.5} L^{1.75} [\theta_6 - \theta_5]^{1.75}}$$

$$D_{Bo} = 0.079 d_B$$

$$D_{Bo}^{1.25} = 0.042 d_B^{1.25}$$

$$\Delta p_{Bo} = \frac{1.61 \times 10^{-7} \rho^{1.75} L_B [\theta_7 + \theta_8]}{\rho d_B^{4.15} N_B^{1.75} L^{1.75} [\theta_6 - \theta_5]^{1.75}}$$

Let

$$\beta_B = \left[\frac{\frac{[H_{12} - H_{11}]}{[H_{13} - H_{10}]} [\theta_6 - \theta_5]^{.8} \ln \frac{[\theta_7 - \theta_{12}]}{[\theta_8 - \theta_{11}]}}{[\theta_7 - \theta_{12}] - [\theta_8 - \theta_{11}]} \right]^5$$

Then

$$N_B = 24.4 \frac{P}{t} \frac{d_B^4}{L_B^5} \beta_B$$

$$\Delta p_{Bo} = \frac{1.61 \times 10^{-7} L_B^{9.75} [\theta_7 + \theta_8]}{P d_B^{11.75} [\theta_6 - \theta_5]^{1.75} \beta_B^{1.75} (24.4)^{1.75}}$$

$$\Delta \theta_{Bo} = \frac{6.04 \times 10^{-10} L_B^{9.75} [\theta_7 + \theta_8]}{P d_B^{11.75} [\theta_6 - \theta_5]^{1.75} \beta_B^{1.75}}$$

The ratio of the power required to pump the gas through the boiler to the reactor thermal power is given by

$$\frac{P_{Bo}}{P} = 0.497 \frac{\theta_5 \Delta p_{Bo}}{P [\theta_6 - \theta_5]}$$

$$\frac{P_{Bo}}{P} = \frac{0.497 \times 6.04 \times 10^{-10} \theta_5 L_B^{9.75} [\theta_7 + \theta_8]}{P [\theta_6 - \theta_5] P d_B^{11.75} [\theta_6 - \theta_5]^{1.75}} \left[\frac{\Delta \theta_B}{\frac{[H_{12} - H_{11}]}{[H_{13} - H_{10}]} [\theta_6 - \theta_5]^8} \right]^{8.75}$$

$$\frac{P_{Bo}}{P} = \frac{3.00 \times 10^{-10} L_B^{9.75} \theta_5 [\theta_7 + \theta_8] [\Delta \theta_B]^{8.75}}{P^2 [\theta_6 - \theta_5]^{9.75} d_B^{11.75} \frac{[H_{12} - H_{11}]^{8.75}}{[H_{13} - H_{10}]^{8.75}}}$$

Rearranging

$$d_b = \left[\frac{3.00 \times 10^{-10} L^8 \theta_5^{9.75} [\theta_7 + \theta_8] [\Delta \theta_8]^{0.0851}}{\rho^2 [\theta_6 - \theta_5]^{9.75} \left(\frac{P_{80}}{P} \right) \psi_3^{8.75}} \right]$$

where

$$\psi_3 = \frac{H_{12} - H_{11}}{H_{13} - H_{10}} = \frac{562.8}{1492.7 - H_{10}}$$

11.4.3 Super Heater Equations

The tube areas are the same as the general boiler equations.

The Dittus-Boelter equations was used to determine the outside and inside heat transfer coefficients.

A Prandtl number of 0.646 was used for the He side (outside) calculations.

$$h_{so} = 0.023 \left(0.646 \right)^{.4} \left(\frac{D G}{\mu} \right)^{.8} \left(\frac{k}{D} \right)$$

$$h_{so} = (0.023)(0.840) \left(\frac{0.182}{D^{.2}} \right) \left(\frac{G}{\mu} \right)^{.8}$$

$$h_{so} = 0.023 \frac{G^{.8}}{D^{.2}}$$

as before

$$D_{so} = 0.079 d_s$$

Therefore

$$h_{so} = \frac{0.023}{(0.079)^{.2}} G_{so}^{.8} \left(\frac{1}{d_s} \right)^{.2}$$

$$h_{so} = 0.0382 \frac{G_{so}^{.8}}{d_s^{.2}}$$

Noting that

$$G_{so} = \frac{W \times 3600}{A_{so} \times b} = \frac{3600}{25.8 \times 10^{-3}} \frac{W_H}{N_s d_s^2 b}$$

$$G_{so} = 1.4 \times 10^5 \frac{W_H}{N_s d_s^2 b}$$

and that

$$W_H = 7.58 \times 10^{-4} \frac{P}{[\theta_6 - \theta_5]}$$

we find that

$$G_{so} = 106 \frac{P}{N_s d_s^2 b [\theta_6 - \theta_5]}$$

The Reynolds number is therefore given by

$$Re_{so} = 88.2 \frac{P}{N_s d_s b [\theta_6 - \theta_5]}$$

Hence

$$Re^{0.8} = 36.0 \frac{\rho^{.8}}{(N_s d_s k [\theta_6 - \theta_5])^{.8}} \quad -233-$$

The outside heat transfer coefficients is

$$h_{so} = 0.0382(106)^{.8} \frac{\rho^{.8}}{d_s^{1.8} N_s^{.8} k^{.8} [\theta_6 - \theta_5]^{.8}}$$

$$h_{so} = 1.605 \frac{\rho^{.8}}{d_s^{1.8} N_s^{.8} k^{.8} [\theta_6 - \theta_5]}$$

Calculations for the inside coefficient are as follows

$$h_{si} = 0.023 (1.38)^{.4} \left(\frac{DG}{\mu} \right)_{si}^{.8} \left(\frac{k_{si}}{D_{si}} \right)$$

$$h_{si} = 0.023 (1.137) \frac{(.035)}{(.110)} \frac{G_{si}^{.8}}{D_{si}^{.2}}$$

The hydraulic diameter is equal to the I.D.

$$D_{si} = \frac{d_s \text{ (in)}}{12 \frac{\text{in}}{\text{ft}}} = 0.0833 d_s \text{ (ft.)}$$

Substituting into the above expression for h_{si} :

$$h_{si} = 0.0137 \frac{G_{si}^{.8}}{d_s^{.2}}$$

To determine the mass velocity inside the tubes (G_{si}), we must first find the mass flow rate (w_w)

$$w_w = \frac{P_{watts} \times 3.413 \frac{BTU}{watt \cdot hr}}{(H_{13} - H_{10}) \frac{BTU}{\#} \times 3600 \frac{sec}{hr}} = \# / sec$$

$$w_w = 0.918 \times 10^{-3} \frac{P}{(H_{13} - H_{10})}$$

Hence

$$G_{si} = \frac{0.948 \times 10^{-3} \times 3600}{(H_{13} - H_{10}) \ell N_s d_s^2 5.45 \times 10^{-3}}$$

$$G_{si} = 626 \frac{P}{N_s d_s^2 \ell (H_{13} - H_{10})}$$

The inside heat transfer coefficient is thus

$$h_{si} = \frac{0.0137 (626)^{.8} P^{.8}}{N_s^{.8} d_s^{1.8} \ell^{.8} (H_{13} - H_{10})^{.8}}$$

$$h_{si} = 2.37 \frac{P^{.8}}{N_s^{.8} d_s^{1.8} \ell^{.8} (H_{13} - H_{10})^{.8}}$$

The overall heat transfer coefficient based on the inside area is given by:

$$U_{si} = \frac{1}{\frac{S_{si}}{S_{so}} \frac{1}{h_{so}} + \frac{1}{h_{si}}}$$

Noting that

$$\frac{S_{si}}{S_{so}} = 0.2$$

and substituting, we obtain

$$U_{si} = \frac{1}{\frac{.2}{1.605} \frac{d_s^{1.8} N_s^8 L^8 (\theta_6 - \theta_5)^8}{P^8} + \frac{1}{2.37} \frac{d_s^{1.8} N_s^8 L^8 (H_{13} - H_{10})^8}{P^8}}$$

$$U_{si} = \frac{1}{\frac{d_s^{1.8} N_s^8 L^8}{P^8} \left(.1246 (\theta_6 - \theta_5)^8 + .421 (H_{13} - H_{10})^8 \right)}$$

$$U_{si} = \frac{P^8}{d_s^{1.8} N_s^8 L^8 \left(.1246 (\theta_6 - \theta_5)^8 + .421 (H_{13} - H_{10})^8 \right)}$$

The general heat transfer equation in, BTU/hr. is:

$$q = U_{si} S_{si} \Delta \theta_{sen}$$

$$q = \frac{0.262 P^8 N_s d_s L_s \Delta \theta_{sen}}{d_s^{1.8} N_s^8 L^8 \left(.1246 (\theta_6 - \theta_5)^8 + .421 (H_{13} - H_{10})^8 \right)}$$

where

$$\Delta \theta_{sen} = \frac{(\theta_6 - \theta_{13}) - (\theta_7 - \theta_{12})}{\ln \frac{(\theta_6 - \theta_{12})}{(\theta_7 - \theta_{12})}}$$

Therefore the heat transferred in the superheater (in watts) is given by

$$P_s = 0.0768 \frac{P^8 N_s d_s L_s \Delta \theta_{sen}}{d_s^{1.8} N_s^8 L^8 \left(.1246 (\theta_6 - \theta_5)^8 + .421 (H_{13} - H_{10})^8 \right)}$$

$$P_s = 0.0768 \frac{P^8 L_s N_s^2 \Delta \theta_{sen}}{d_s^8 L^8 \left(.1246 (\theta_6 - \theta_5)^8 + .421 (H_{13} - H_{10})^8 \right)}$$

Rearranging

$$N_s^2 = \frac{P_s d_s^8 L^8 (.1246 (\theta_6 - \theta_5)^8 + .421 (H_{13} - H_{10})^8)}{L_s P^8 (0.0768) \Delta \theta_{sen}}$$

$$N_s = \left(\frac{1}{.0768} \right)^5 \frac{P_s^5 d_s^4 L^4 (.1246 (\theta_6 - \theta_5)^8 + .421 (H_{13} - H_{10})^8)^5}{P^4 L_s^5 (\Delta \theta_{sen})^5}$$

$$N_s = 3.727 \times 10^5 \frac{P_s^5 d_s^4 L^4}{P^4 L_s^5} \left(\frac{.1246 (\theta_6 - \theta_5)^8 + .421 (H_{13} - H_{10})^8}{\Delta \theta_{sen}} \right)^5$$

The superheater power can be expressed as (watts)

$$P_s = \frac{P}{L} \left(\frac{H_{13} - H_{12}}{H_{13} - H_{10}} \right)$$

Therefore

$$N_s = 3.727 \times 10^5 \frac{P d_s^4}{L L_s^5} \left(\frac{\left(\frac{H_{13} - H_{12}}{H_{13} - H_{10}} \right) (.1246 (\theta_6 - \theta_5)^8 + .421 (H_{13} - H_{10})^8)}{\Delta \theta_{sen}} \right)^5$$

The cross sectional area per superheater in ft² is given by

$$A_s = 3.76 \times 10^{-2} N_s d_s^2 = 0.7854 D_s^2$$

so that

$$D_s = .218 d_s \sqrt{N_s}$$

Substituting for N_s

$$D_s = \frac{1.33 \times 10^2 P^5 d_s^3}{L^{.5} L_s^{2.5}} \left(\frac{\left(\frac{H_{13} - H_{12}}{H_{13} - H_{10}} \right) (.1246 (\theta_6 - \theta_5)^8 + .421 (H_{13} - H_{10})^8)}{\Delta \theta_{sen}} \right)^{2.5}$$

The inside tube area per superheater (ft²) is

$$S_{si} = .262 N_s d_s L_s$$

$$S_{si} = \frac{9.75 \times 10^4 P d_s^5}{L_s^4} \left(\frac{(H_{13} - H_{12})}{(H_{13} - H_{10})} \left(.1246 (\theta_6 - \theta_5)^8 + .421 (H_{13} - H_{10})^8 \right) \right) \Delta \theta_{sen}$$

The pressure drop equations for the superheater on the He side were developed as follows:

The pressure drop can be expressed as

$$\Delta p_{so} = \frac{2 f_{so} G_{so}^2 L_s}{g \rho_{so} D_{so}}$$

The friction factor is approximated by

$$f_{so} = 0.079 \left(\frac{\mu_{so}}{D_{so} G_{so}} \right)^{.25}$$

$$\mu = 0.095 \frac{\text{lb}}{\text{ft-hr}}, \mu^{.25} = 0.555$$

$$f_{so} = \frac{0.0439}{D_{so}^{.25} G_{so}^{.25}}$$

The He density can be expressed as follows:

$$\rho_{so} = \frac{P \times 144}{\frac{1544}{4} \frac{\theta_6 + \theta_7}{2}} = 0.745 \frac{P}{\theta_6 + \theta_7}$$

Substituting

$$\Delta p_{so} = \frac{9.53 \times 10^{-12} \times .555 \times G_{so}^{1.75} L_s (\theta_6 + \theta_7)}{P D_{so}^{1.25}}$$

As previously shown

$$G_{so} = 106 \frac{P}{N_s d_s^2 L (\theta_6 - \theta_5)}$$

$$\Delta p_{so} = \frac{1.96 \times 10^{-12} \times (106)^{1.75} P^{1.75} L_s (\theta_6 + \theta_7)}{N_s^{1.75} d_s^{3.5} \mu^{1.75} (\theta_6 - \theta_5)^{1.75} \rho D_{so}^{1.75}}$$

since

$$D_{so} = 0.079 d_s$$

we obtain

$$\Delta p_{so} = \frac{1.63 \times 10^{-7} P^{1.75} L_s (\theta_6 + \theta_7)}{N_s^{1.75} d_s^{4.75} \mu^{1.75} (\theta_6 - \theta_5)^{1.75} \rho}$$

Substituting for $N_s^{1.75}$

$$\Delta p_{so} = \frac{1.63 \times 10^{-7} P^{1.75} L_s (\theta_6 + \theta_7) \mu^{1.75} L_s^{8.75} \Delta \theta_s^{8.75}}{(3.727 \times 10^5)^{1.75} d_s^{4.75} \mu^{1.75} (\theta_6 - \theta_5)^{1.75} \rho P^{1.75} d_s^7 \left(\left[\frac{H_{13} - H_{12}}{H_{13} - H_{10}} \right] \left[.1246 (\theta_6 - \theta_5)^8 + .421 (H_{13} - H_{10})^8 \right] \right)^{8.75}}$$

$$\Delta p_{so} = 2.91 \times 10^{-17} \frac{L_s^{9.75} (\theta_6 + \theta_7) \Delta \theta_s^{8.75}}{d_s^{11.75} \rho (\theta_6 - \theta_5)^{1.75} \left(\left[\frac{H_{13} - H_{12}}{H_{13} - H_{10}} \right] \left[.1246 (\theta_6 - \theta_5)^8 + .421 (H_{13} - H_{10})^8 \right] \right)^{8.75}}$$

Let

$$\psi = \left(\left[\frac{H_{13} - H_{12}}{H_{13} - H_{10}} \right] \left[.1246 (\theta_6 - \theta_5)^8 + .421 (H_{13} - H_{10})^8 \right] \right)$$

Thus

$$\Delta p_{so} = 2.91 \times 10^{-17} \frac{L_s^{9.75} (\theta_6 + \theta_7) \Delta \theta_s^{8.75}}{d_s^{11.75} \rho (\theta_6 - \theta_5)^{1.75} \psi^{8.75}}$$

The ratio of the power required to pump the coolant through the superheater to the reactor thermal power is given by:

$$\frac{P_{so}}{P} = 0.497 \frac{\theta_5 \Delta p_{so}}{\rho (\theta_6 - \theta_5)}$$

Substituting for Δp_{so} , we obtain

-239-

$$\frac{P_{so}}{P} = 1.45 \times 10^{-17} \frac{L_s^{9.75} \theta_5 (\theta_6 + \theta_7) \Delta \theta_s^{8.75}}{\rho^2 d_s^{11.75} (\theta_6 - \theta_5)^{2.75} \psi_5^{8.75}}$$

Rearranging and solving for d_s

$$d_s = \left[1.45 \times 10^{-17} \frac{L_s^{9.75} \theta_5 (\theta_6 + \theta_7) \Delta \theta_s^{8.75}}{\rho^2 \left(\frac{P_{so}}{P} \right) (\theta_6 - \theta_5)^{2.75} \psi_5^{8.75}} \right]^{.0851}$$

11.4.4 Primary Heat Exchangers

The power transferred per loop is given by

$$\frac{P}{\gamma} = \frac{(\Delta \theta_{xm})^{\circ R} \times U_{xi} \left(\frac{\text{BTU}}{\text{hr ft}^2 \circ F} \right) \times S_{xi} \text{ ft}^2}{3.413 \frac{\text{BTU}}{\text{Watt hr}}}$$

Rearranging

$$\Delta \theta_{xm} = \frac{3.413 P}{\gamma U_{xi} S_{xi}}$$

As shown on page

$$S_{xi} = 0.262 N_x dx L_x$$

$$N_x = 29.6 \frac{D_x^2}{dx^2}$$

$$S_{xi} = 7.75 \frac{D_x^2 L_x}{dx}$$

The overall heat transfer coefficient based on the inside area is

$$U_{xi} = \frac{1}{\frac{1}{h_{xi}} + \left(\frac{S_{xi}}{S_{xo}} \right) \left(\frac{1}{h_{xo}} \right)}$$

As shown on page

$$\left(\frac{S_{xi}}{S_{xo}} \right) = \frac{0.262}{1.061} = 0.246$$

Thus

$$U_{xi} = \frac{1}{\frac{1}{h_{xi}} + \frac{0.246}{h_{xo}}}$$

Using the Dittus-Boelter equation to determine the heat transfer coefficients

$$h_x = 0.023 \left(\frac{C_p \mu}{k} \right)^{0.4} \left(\frac{D G}{\mu} \right)^{0.8} \left(\frac{k}{D} \right)$$

To determine μ and k , assume $\theta_{x \text{ ave}} = 800^\circ \text{ F}$, then

$$\left(\frac{C_p \mu}{k} \right)^{0.4} = (0.671)^{0.4} = 0.852$$

$$h_x = 0.0196 \left(\frac{D G}{\mu} \right)^{0.8} \left(\frac{k}{D} \right)$$

$$\mu \cong 0.0821 \text{ lbs/ft hr}$$

Inside tubes:

$$D_{xi} = \frac{d_x (\text{in})}{12 \text{ in/ft}} = 0.0833 d_x \text{ ft}$$

$$G_{xi} = \frac{w \text{ \#/sec} \times 3600 \text{ sec/hr}}{A_{xi} \text{ ft}^2 \times b}$$

$$A_{xi} = 5.45 \times 10^{-3} N_x d_x^2$$

Therefore,

$$G_{xi} = 6.46 \times 10^5 \frac{w}{N_x d_x^2} \text{ \#/hr ft}^2$$

As shown on page , the mass flow rate of helium (hot or cold) is given by

$$\dot{W}_H = 7.58 \times 10^{-4} \frac{P}{\theta_6 - \theta_5}$$

The number of tubes per exchanger are (See page)

$$N_x = 29.6 \frac{D_x^2}{d_x^2}$$

therefore

$$G_{xi} = \frac{(6.46 \times 10^5) \left(7.58 \times 10^{-4} \frac{P}{\theta_6 - \theta_5} \right)}{\left(29.6 \frac{D_x^2}{d_x^2} \right) (d_x^2 \mu)}$$

$$G_{xi} = 16.5 \frac{P}{D_x^2 \mu (\theta_6 - \theta_5)}$$

The Reynolds Number is then:

$$Re_{xi} = \frac{DG}{\mu} = \frac{(0.0833 d_x) \left(16.5 \frac{P}{D_x^2 \mu (\theta_6 - \theta_5)} \right)}{0.0821}$$

$$Re_{xi} = 16.75 \frac{P d_x}{D_x^2 \mu (\theta_6 - \theta_5)}$$

$$(Re_{xi})^{0.8} = 9.58 \frac{P^{0.8} d_x^{0.8}}{D_x^{1.6} \mu^{0.8} (\theta_6 - \theta_5)^{0.8}}$$

$$\frac{h}{D} = \frac{0.153 \text{ BTU/hr ft } ^\circ\text{F}}{0.0833 d_x \text{ ft}} = \frac{1.84}{d_x} \text{ BTU/hr ft}^2 \text{ } ^\circ\text{F}$$

thus,

$$h_{xi} = (0.0196) \left(9.58 \frac{P^{0.8} d_x^{0.8}}{D_x^{1.6} \mu^{0.8} (\theta_6 - \theta_5)^{0.8}} \right) \left(\frac{1.84}{d_x} \right)$$

Hence

-242-

$$h_{xi} = 0.345 \frac{p^{0.8}}{D_x^{1.6} \kappa^{0.8} d_x^{0.2} (\theta_6 - \theta_5)^{0.8}}$$

Determine the outside heat transfer coefficient.

The hydraulic diameter is

$$D_{xo} = \frac{4 \times \text{area}}{\text{Wetted Perimeter}} = \frac{4 \times A_{xo}}{\left(\frac{S_{xo}}{L_x}\right)}$$

$$D_{xo} = \frac{4(1.56 \times 10^{-2} N_x d_x^2)}{1.061 N_x d_x} = 0.0588 d_x$$

The mass velocity is

$$G_{xo} = \frac{w_H \text{ #/sec} \times 3600 \text{ sec/hr}}{A_{xo} \text{ ft}^2 \kappa}$$

where

$$A_{xo} = 1.56 \times 10^{-2} N_x d_x^2 \text{ ft}^2 \quad (\text{See page })$$

Substituting above

$$G_{xo} = 2.31 \times 10^5 \frac{w_H}{N_x d_x^2 \kappa} \text{ lbs/hr ft}^2$$

$$w_H = 7.58 \times 10^{-4} \frac{P}{\theta_6 - \theta_5}$$

$$N_x = 29.6 \frac{D_x^2}{d_x^2}$$

$$G_{xo} = \frac{(2.31 \times 10^5) \left(7.58 \times 10^{-4} \frac{P}{\theta_6 - \theta_5}\right)}{\left(29.6 \frac{D_x^2}{d_x^2}\right) d_x^2 \kappa}$$

The Reynolds number is given by

$$Re_{xo} = \frac{DG}{\mu} = \frac{(0.0588 dx) \left(5.91 \frac{P}{D_x \kappa (\theta_6 - \theta_5)} \right)}{0.0821}$$

$$\left(\frac{DG}{\mu} \right)^{0.8} = 3.17 \frac{P^{0.8} dx^{0.8}}{D_x^{1.6} \kappa^{0.8} (\theta_6 - \theta_5)^{0.8}}$$

$$\frac{h}{D} = \frac{0.153 \text{ BTU/hr ft}^2 \text{ } ^\circ\text{F}}{0.0588 dx \text{ ft}} = \frac{2.60 \text{ BTU/hr ft}^2 \text{ } ^\circ\text{F}}{dx}$$

Therefore, the outside heat transfer coefficient is:

$$h_{xo} = 0.1614 \frac{P^{0.8}}{D_x^{1.6} \kappa^{0.8} dx^{0.2} (\theta_6 - \theta_5)^{0.8}}$$

The overall heat transfer coefficient is now

$$U_{xi} = \frac{1}{\frac{1}{h_{xi}} + \frac{0.246}{h_{xo}}}$$

$$\frac{1}{h_{xi}} = 2.90 \frac{D_x^{1.6} \kappa^{0.8} dx^{0.2} (\theta_6 - \theta_5)^{0.8}}{P^{0.8}}$$

$$\frac{0.246}{h_{xo}} = 1.52 \frac{D_x^{1.6} \kappa^{0.8} dx^{0.2} (\theta_6 - \theta_5)^{0.8}}{P^{0.8}}$$

$$\frac{1}{U_{xi}} = 4.42 \frac{D_x^{1.6} \kappa^{0.8} dx^{0.2} (\theta_6 - \theta_5)^{0.8}}{P^{0.8}}$$

Recalling the basic heat transfer equation

$$\Delta\theta_x = \frac{3.413 P}{\kappa S_{xi}} \left(\frac{1}{U_{xi}} \right)$$

$$\Delta T_x = \frac{3.413 P}{\chi \left(7.75 - \frac{D_x^2 L_x}{d_x} \right)} \left(4.42 \frac{D_x^{1.6} \kappa^{.8} d_x^{.2} (\theta_6 - \theta_5)^{.8}}{P^{.8}} \right)^{-244}$$

$$\Delta T_x = 1.948 \frac{P^{.2} d_x^{1.2} (\theta_6 - \theta_5)^{.8}}{\chi^{.2} D_x^{.4} L_x} = \theta_3 - \theta_6$$

Let

$$\psi_1 = \frac{\theta_6 - \theta_5}{\theta_6 - \theta_8} = \frac{1492.7 - H_{10}}{1492.7 - 607.1} = \frac{1492.7 - H_{10}}{885.6}$$

Then

$$\theta_3 - \theta_6 = 1.948 \frac{P^{0.2} d_x^{1.2} \psi_1^{.8} (\theta_6 - \theta_8)^{.8}}{\chi^{.2} D_x^{.4} L_x}$$

Knowing θ_3 and θ_8 , the computer solved for θ_6 by a method of successive approximations.

Pressure Drop

The pressure drop equations inside the tubes are as follows:

$$\Delta p_{xi} = \frac{2(f_{xi}) G_{xi}^2 \left(\frac{lb_m^2}{hr^2 ft^4} \right) L_x (ft)}{32.2 \left(\frac{ft}{sec^2} \right) \rho_{xi} \left(\frac{lb_m}{ft^3} \right) D_{xi} (ft) 1.296 \times 10^7 \left(\frac{sec^2}{hr^2} \right) 144 \left(\frac{in^2}{ft^2} \right)}$$

$$\Delta p_{xi} = 3.33 \times 10^{-11} \frac{f_{xi} G_{xi}^2 L_x}{\rho_{xi} D_{xi}} \quad \text{psi}$$

$$f_{xi} = 0.079 \left(\frac{\mu_x}{D_{xi} G_{xi}} \right)^{0.25}$$

$$\rho_{xi} = 0.0931 \frac{P M}{\theta_{xi}}$$

where

$$M = 4$$

$$\theta_{xi} = \frac{\theta_1 + \theta_3}{2}$$

$$P_{xi} = 0.745 \frac{P}{\theta_1 + \theta_3}$$

Substituting

$$\Delta p_{xi} = \frac{(3.33 \times 10^{-11}) (0.079 \frac{\mu_x^{.25}}{D_{xi}^{1.25} G_{xi}^{.25}}) G_{xi}^2 L_x}{0.745 \left(\frac{P}{\theta_1 + \theta_3} \right) D_{xi}}$$

$$\Delta p_{xi} = 3.53 \times 10^{-12} \frac{\mu_x^{.25} G_{xi}^{1.75} L_{xi} (\theta_1 + \theta_3)}{P D_{xi}^{1.25}}$$

$$\mu_x^{.25} = (0.0821)^{0.25} = 0.536$$

Thus

$$\Delta p_{xi} = 1.89 \times 10^{-12} \frac{G_{xi}^{1.75} L_x (\theta_1 + \theta_3)}{P D_{xi}^{1.25}} \text{ psi}$$

Similarly

$$\Delta p_{xo} = 1.89 \times 10^{-12} \frac{G_{xo}^{1.75} L_x (\theta_5 + \theta_6)}{P D_{xo}^{1.25}} \text{ psi}$$

The mass velocity and exchanger tube diameter are:

$$G_{xi} = 16.5 \frac{P}{D_x^2 \chi (\theta_6 - \theta_5)}$$

$$D_{xi} = 0.0833 d_x$$

substituting into the above equations for Δp_{xi} , we obtain

$$\Delta p_{xi} = \frac{(1.89 \times 10^{-12}) \left(135.2 \frac{P^{1.75}}{D_x^{3.5} \chi^{1.75} (\theta_6 - \theta_5)^{1.75}} \right) L_x (\theta_1 + \theta_3)}{P (0.0449 d_x^{1.25})}$$

$$\Delta p_{xi} = 5.70 \times 10^{-9} \frac{P^{1.75} L_x (\theta_1 + \theta_3)}{D_x^{3.5} d_x^{1.25} P (\theta_6 - \theta_5)^{1.75} \chi^{1.75}}$$

Similarly

$$G_{x0} = 5.91 \frac{P^{-2.46}}{D_{x0}^2 \times (\theta_6 - \theta_5)}$$

$$D_{x0} = 0.0588 d_x$$

Substituting

$$\Delta p_{x0} = \frac{(1.89 \times 10^{-12}) \left(22.4 \frac{P^{1.75}}{D_x^{3.5} d_x^{1.25} (\theta_6 - \theta_5)^{1.75}} \right) L_x (\theta_5 + \theta_6)}{P (0.0289 d_x^{1.25})}$$

$$\Delta p_{x0} = 1.463 \times 10^{-9} \frac{P^{1.75} L_x (\theta_5 + \theta_6)}{D_x^{3.5} d_x^{1.25} P (\theta_6 - \theta_5)^{1.75} \eta^{1.75}}$$

The pumping power to reactor thermal power ratio was found as follows:

$$W_H = 7.58 \times 10^{-4} \frac{P}{\theta_3 - \theta_1} \text{ lbs/sec}$$

$$Q_H = \frac{W_H}{P_1} \frac{\text{lbs/sec}}{\text{lbs/ft}^3}$$

$$P_1 = 0.093 \frac{P M}{\theta_1} = 0.372 \frac{P}{\theta_1}$$

$$Q_H = \frac{7.58 \times 10^{-4} \frac{P}{\theta_3 - \theta_1}}{0.372 \frac{P}{\theta_1}} = 2.04 \times 10^{-3} \frac{P \theta_1}{P (\theta_3 - \theta_1)} \frac{\text{ft}^3}{\text{sec}}$$

$$P_H = \frac{Q_H \text{ ft}^3/\text{sec} \times \Delta p_H \text{ lbs/in}^2 \times 144 \frac{\text{in}^2}{\text{ft}^2} \times 1.356 \frac{\text{watt sec}}{\text{ft lb}}}{\eta_B}$$

$$P_H = 195.1 \frac{Q_H \Delta p_H}{\eta_B} = 0.398 \frac{P \theta_5 \Delta p}{P (\theta_3 - \theta_1) \eta_B}$$

Substituting the pressure drops:

$$\left(\frac{P_{xi}}{P} \right) = 2.84 \times 10^{-9} \frac{P^{1.75} L_x \theta_1 (\theta_1 + \theta_3)}{\rho^2 D_x^{3.5} d_x^{1.25} \eta^{1.75} (\theta_3 - \theta_1)^{2.75}}$$

$$\left(\frac{P_{xo}}{P} \right) = 7.29 \times 10^{-10} \frac{P^{1.75} L_x \theta_5 (\theta_5 + \theta_6)}{\rho^2 D_x^{3.5} d_x^{1.25} \eta^{1.75} (\theta_3 - \theta_1)^{2.75}}$$

11.4.5 Economizer Equations

The heat transfer coefficient for liquid water can be expressed as follows; the equation was obtained from, Glasstone, page 678.

$$h_{ei} \text{ (Btu/hr ft}^2 \text{ } ^\circ\text{F)} = 170 (1 + 10^{-2} t_w - 10^{-5} t_w^2) \frac{v_e^{.8}}{d_e^{.2}}$$

where

t = temperature, $^\circ\text{F}$

v = ft/sec

d = in

$$h_{ei} = 170 (1 + 4.40 - .1 (4.4)^2) \frac{v_e^{.8}}{d_e^{.2}}$$

$$h_{ei} = 170 (1 + 4.40 - 1.935) \frac{v_e^{.8}}{d_e^{.2}}$$

$$v_{ei} (\text{ft/sec}) = w_w (\text{\#}/\text{sec}) \frac{1}{\rho} \left(\frac{\text{ft}^3}{\text{\#}} \right) \frac{1}{A_{ei}} \left(\frac{1}{\text{ft}^2} \right) \frac{1}{\kappa}$$

$$= \frac{w_w}{\kappa} \frac{1}{52.4} \frac{1}{5.45 \times 10^{-3} N_e d_e^2}$$

$$v_{ei} = \frac{3.51 w_w}{\kappa N_e d_e^2}$$

$$w_w = \frac{P_{\text{watts}}}{H_{13} - H_{10}} \frac{3.413 \text{ Btu/watt hr}}{3600 \text{ sec/hr}} = \frac{9.48 \times 10^{-4} P}{H_{13} - H_{10}}$$

$$h_{ei} = \frac{(589)(3.51)^{.8} w_w^{.8}}{\kappa^{.8} N_e^{.8} d_e^{1.8}}$$

Substituting for w_w

$$h_{ei} = \frac{(6.1) P^{.8}}{N_e^{.8} d_e^{1.8} (H_{13} - H_{10})^{.8} \kappa^{.8}}$$

Calculation of Outside Heat Transfer Coefficient

$$D_{eo} = \frac{4 \times \text{area}}{\text{Wetted Perimeter}} = \frac{4 \times A_{eo}}{\frac{S_{eo}}{L_e}}$$

$$D_{eo} = \frac{(4)(25.81) \times 10^{-3} N_e d_e^2 L_e}{1.31 N_e d_e L_e} = 7.8 \times 10^{-2} d_e$$

$$G_{eo} = \frac{w_H (\text{\#}/\text{sec}) 3600 (\text{sec/hr})}{A_{eo} (\text{ft}^2) \kappa} = \frac{(3600)(7.58 \times 10^{-4}) P}{(\theta_3 - \theta_1)(25.81) \times 10^{-3} N_e d_e^2 \kappa}$$

$$G_{eo} = \frac{105.7 P}{N_e d_e^2 \kappa (\theta_3 - \theta_1)}$$

The Reynolds number is given by

$$Re_{eo} = \frac{D G}{\mu} = \frac{(7.8 \times 10^{-2} d_e) 105.7 P}{N_e d_e^2 \kappa (\theta_6 - \theta_5)(.0690)}$$

$$\left(\frac{DG}{\mu}\right)^{.8} = \frac{46 P^{.8}}{N_e^{.8} d_e^{1.8} \gamma^{.8} (\theta_6 - \theta_5)^{.8}}$$

$$\frac{h}{D} = \frac{.126 \text{ BTU/hr ft } ^\circ R}{7.8 \times 10^{-2} d_e} = 1.62/d_e$$

The outside heat transfer coefficient can be represented by the Dittus-Boelter equation:

$$h_{eo} = .023 \left(\frac{c_p \mu}{h}\right)^{.4} \left(\frac{DG}{\mu}\right)^{.8} \left(\frac{h}{D}\right)$$

Substituting

$$h_{eo} = \frac{(.023)(.682)^{.4} (46)(1.62) P^{.8}}{N_e^{.8} d_e^{1.8} \gamma^{.8} (\theta_6 - \theta_5)^{.8}}$$

$$h_{eo} = \frac{1.47 P^{.8}}{N_e^{.8} d_e^{1.8} \gamma^{.8} (\theta_6 - \theta_5)^{.8}}$$

As shown in the case of the boiler and superheater, the overall coefficient is given by

$$U_{ei} = \frac{1}{\frac{1}{h_{ei}} + \frac{.2}{h_{eo}}}$$

$$U_{ei} = \frac{1}{\frac{\gamma^{.8} N_e^{.8} d_e^{1.8} (H_{13} - H_{10})^{.8}}{1.605 \times 10^3 P^{.8} 3.8 \times 10^{-3}} + \frac{N_e^{.8} d_e^{1.8} \gamma^{.8} (\theta_6 - \theta_5)^{.8} .2}{1.47 P^{.8}}}$$

$$U_{ei} = \frac{P^{.8}}{N_e^{.8} d_e^{1.8} \gamma^{.8}} \left(\frac{1}{\frac{(H_{13} - H_{10})^{.8}}{6.1} + \frac{(\theta_6 - \theta_5)^{.8}}{7.35}} \right)$$

The power per economizer is:

$$\frac{P}{K} = \frac{\Delta \theta_e U_{ei} S_{ei}}{3.413} \frac{(H_{13} - H_{10})}{(H_{11} - H_{10})}$$

Rearranging

$$3.413 (H_{11} - H_{10}) P = (H_{13} - H_{10}) K \Delta \theta_e U_{ei} S_{ei}$$

Substitute for U_{ei} and S_{ei}

$$\frac{(3.413)(H_{11} - H_{10})}{(1.262)(H_{13} - H_{10})} = \frac{N_e^{.2} K^{.2} L_e \Delta \theta_m}{P^{.2} d_e^{.8}} \left(\frac{1}{\frac{(H_{13} - H_{10})^{.8}}{6.1} + \frac{(\theta_6 - \theta_5)^{.8}}{7.35}} \right)$$

The number of tubes per economizer

$$N_e = \frac{P d_e^4 \psi_4^5}{K L_e^5 (\Delta \theta_e)^5}$$

where

$$\psi_4 = \left(\frac{(13)(H_{11} - H_{10})}{(H_{13} - H_{10})} \right) \left(\frac{(H_{13} - H_{10})^{.8}}{6.1} + \frac{(\theta_6 - \theta_5)^{.8}}{7.35} \right)$$

$$S_{ei} = 0.262 N_e d_e L_e$$

$$S_{ei} = \frac{.262 \psi_4^5 P d_e^5}{K L_e^4 \Delta \theta_e^5}$$

$$A_e = 37.6 \times 10^{-3} N_e d_e^2$$

$$A_e = \frac{37.6 \times 10^{-3} \psi_4^5 d_e^6 P}{K L_e^5 \Delta \theta_e^5}$$

$$A_e = \frac{\pi D_e^2}{4}$$

$$D_e = \frac{.219 \psi_1^{2.5} d_e^3 P^{.5}}{K^{.5} L_e^{2.5} \Delta \theta_e^{2.5}}$$

Pressure Drop

The outside (He side) pressure drop equation for the economizer was developed as follows:

$$\Delta p_{eo} = \frac{2 f_{eo} G_{eo}^2 \left(\frac{\text{lb}^2}{\text{hr}^2 \text{ft}^4} \right) L_e (\text{ft})}{32.2 \left(\frac{\text{ft}}{\text{sec}^2} \right) \rho_{eo} \left(\frac{\text{lb}}{\text{ft}^3} \right) D_{eo} (\text{ft}) 1.296 \times 10^7 \left(\frac{\text{sec}^2}{\text{hr}^2} \right) 144 \left(\frac{\text{in}^2}{\text{ft}^2} \right)}$$

$$\Delta p_{eo} = \frac{3.33 \times 10^{-11} f_{eo} G_{eo}^2 L_e}{\rho_{eo} D_{eo}} \text{ psi}$$

The friction factor is given approximately by

$$f_{eo} = 0.079 \left(\frac{\mu_e}{D_{eo} G_{eo}} \right)^{0.25}$$

As previously shown

$$\rho_{eo} = \frac{0.0931 p M}{\theta_{eo}}$$

$$M = 4, \quad \theta_{eo} = \frac{\theta_8 + \theta_9}{2}$$

$$\rho_{eo} = \frac{0.745 p}{\theta_8 + \theta_9}$$

Substituting

$$\Delta p_{eo} = \frac{(3.33 \times 10^{-11})(.079) \mu_e^{.35} G_{eo}^2 L_e (\theta_8 + \theta_9)}{D_{eo}^{.25} G_{eo}^{.25} .745 p D_{eo}}$$

11.5 Rigorous Treatment of Boiler Heat Transfer

Let l = distance from cold end of boiler

θ_G = gas temperature at l , $^{\circ}\text{R}$

θ_T = tube inside wall temperature at l , $^{\circ}\text{R}$

θ_w = boiling temperature of water, $^{\circ}\text{R}$

L_B = tube length of boiler, ft

S_{Bi} = surface area inside boiler tubes, ft^2

S_{Bo} = surface area outside boiler tubes, ft^2

dq = rate of heat transferred in segment dl of boiler, BTU/hr

$$dq = h_{Bi} \left(\frac{S_{Bi}}{L_B} \right) dl (\theta_T - \theta_w)$$

also,

$$dq = h'_{Bo} \left(\frac{S_{Bo}}{L_B} \right) dl (\theta_G - \theta_T)$$

where

$$\frac{1}{h'_{Bo}} = \frac{1}{h_{Bo}} + 1.3 \left(\frac{S_{Bo}}{S_{Bi}} \right) \left(\frac{t_T}{k_T} \right)$$

$$\frac{1}{h_{Bo}} = \text{Resistance of gas film}$$

$$1.3 \left(\frac{S_{Bo}}{S_{Bi}} \right) \left(\frac{t_T}{k_T} \right) = \text{Resistance of tube wall}$$

$$\therefore \frac{\theta_T - \theta_w}{\theta_G - \theta_T} = \left(\frac{h'_{Bo}}{h_{Bi}} \right) \left(\frac{S_{Bo}}{S_{Bi}} \right)$$

As shown on page

-253-

$$Pe_o = 0.398 \frac{\rho \theta_5 \Delta p_{eo}}{\rho (\theta_3 - \theta_1) \eta_B}$$

$$\Delta p_{eo} = 1.552 \times 10^{-7} \frac{\Delta \theta_e^{8.75} (\theta_8 + \theta_9) L_e^{9.75}}{\psi_4^{8.75} (\theta_3 - \theta_1)^{1.75} d_e^{11.75} \rho}$$

$$\frac{Pe_o}{\rho} = 6.18 \times 10^{-8} \frac{\theta_5 (\theta_8 + \theta_9) \Delta \theta_e^{8.75} L_e^{9.75}}{(\theta_3 - \theta_1)^{2.75} \rho^2 \eta_B \psi_4^{8.75} d_e^{11.75}}$$

$$d_e = \left(7.72 \times 10^{-8} \frac{\theta_5 (\theta_8 + \theta_9) \Delta \theta_e^{8.75} L_e^{9.75}}{(\theta_3 - \theta_1)^{2.75} \rho^2 \psi_4^{8.75} \left(\frac{Pe_o}{\rho} \right)} \right)^{0.0851}$$

11.5 Rigorous Treatment of Boiler Heat Transfer

Let l = distance from cold end of boiler

θ_G = gas temperature at l , $^{\circ}\text{R}$

θ_T = tube inside wall temperature at l , $^{\circ}\text{R}$

θ_w = boiling temperature of water, $^{\circ}\text{R}$

L_B = tube length of boiler, ft

S_{Bi} = surface area inside boiler tubes, ft^2

S_{Bo} = surface area outside boiler tubes, ft^2

dq = rate of heat transferred in segment dl of boiler, BTU/hr

$$dq = h_{Bi} \left(\frac{S_{Bi}}{L_B} \right) dl (\theta_T - \theta_w)$$

also,

$$dq = h'_{Bo} \left(\frac{S_{Bo}}{L_B} \right) dl (\theta_G - \theta_T)$$

where

$$\frac{1}{h'_{Bo}} = \frac{1}{h_{Bo}} + 1.3 \left(\frac{S_{Bo}}{S_{Bi}} \right) \left(\frac{t_T}{k_T} \right)$$

$$\frac{1}{h_{Bo}} = \text{Resistance of gas film}$$

$$1.3 \left(\frac{S_{Bo}}{S_{Bi}} \right) \left(\frac{t_T}{k_T} \right) = \text{Resistance of tube wall}$$

$$\frac{\theta_T - \theta_w}{\theta_G - \theta_T} = \left(\frac{h'_{Bo}}{h_{Bi}} \right) \left(\frac{S_{Bo}}{S_{Bi}} \right)$$

According to Figure 11.22 of Reference (30), the boiling water heat transfer coefficient h_{Bi} (for vertical tubes) is given by the relationship

$$h_{Bi} = 2.24 (\theta_r - \theta_w)^{2.035} \text{ BTU/hr ft}^2 \text{ } ^\circ\text{F}$$

Substituting

$$\frac{(\theta_r - \theta_w)^{3.035}}{(\theta_6 - \theta_r)} = \frac{h_{Bo}}{2.24} \left(\frac{S_{Bo}}{S_{Bi}} \right)$$

and solving θ_6

$$\theta_6 = \theta_r + \left(\frac{2.24}{h_{Bo}} \right) \left(\frac{S_{Bi}}{S_{Bo}} \right) (\theta_r - \theta_w)^{3.035}$$

Let $\psi = \theta_r - \theta_w$

$$\theta_6 = \theta_w + \psi + \left(\frac{2.24}{h_{Bo}} \right) \left(\frac{S_{Bi}}{S_{Bo}} \right) \psi^{3.035}$$

Differentiating with respect to l

$$\frac{d\theta_6}{dl} = \left(\frac{d\psi}{dl} \right) \left(1 + \frac{6.79}{h_{Bo}} \frac{S_{Bi}}{S_{Bo}} \psi^{2.035} \right)$$

But

$$\omega_H \left(\frac{\text{lb}_m}{\text{sec}} \right) c_p \left(\frac{\text{BTU}}{\text{lb } ^\circ\text{R}} \right) \frac{d\theta_6}{dl} \left(\frac{^{\circ}\text{R}}{\text{ft}} \right) 3600 \left(\frac{\text{sec}}{\text{hr}} \right) = h_{Bi} \left(\frac{\text{BTU}}{\text{hr ft}^2 ^\circ\text{F}} \right) \frac{S_{Bi}}{L_B} \left(\frac{\text{ft}^2}{\text{ft}} \right) [\theta_r - \theta_w] l$$

$$\frac{d\theta_6}{dl} = \left(\frac{h_{Bi}}{3600 \omega_H c_p} \right) \left(\frac{S_{Bi}}{L_B} \right) (\theta_r - \theta_w) l$$

Substituting for h_{Bi} and $(\theta_r - \theta_w)$

$$\frac{d\theta_6}{dl} = \left(\frac{6.22 \times 10^{-4} l}{\omega_H c_p} \right) \left(\frac{S_{Bi}}{L_B} \right) \psi^{3.035}$$

Equating our two equations for $\left(\frac{d\theta_c}{d\ell}\right)$, we obtain

$$\left[1 + \left(\frac{6.79}{h'_{BO}}\right)\left(\frac{S_{Bi}}{S_{Bo}}\right)\psi^{2.035}\right]\left[\frac{d\psi}{d\ell}\right] = \left[\frac{6.22 \times 10^{-4}}{\omega_H C_p}\right]\left[\frac{S_{Bi}}{L_B}\right]\psi^{3.035}$$

$$\left[\frac{1610 \omega_H C_p}{b (S_{Bi}/L_B)}\right]\left[\psi^{-3.035} + \left(\frac{6.79}{h'_{BO}}\right)\left(\frac{S_{Bi}}{S_{Bo}}\right)\psi^{-1}\right]d\psi = d\ell$$

Integrating between $\ell=0$ and $\ell=L_B$

$$L_B = \frac{1610 \omega_H C_p}{b (S_{Bi}/L_B)} \int_{\psi(0)}^{\psi(L_B)} \left[\psi^{-3.035} + \left(\frac{6.79}{h'_{BO}}\right)\left(\frac{S_{Bi}}{S_{Bo}}\right)\psi^{-1}\right]d\psi$$

$$L_B = \frac{1610 \omega_H C_p}{(S_{Bi}/L_B)b} \left[\frac{\psi^{-2.035}(0) - \psi^{-2.035}(L_B)}{2.035} + \left(\frac{6.79}{h'_{BO}}\right)\left(\frac{S_{Bi}}{S_{Bo}}\right) \ln \frac{\psi(L_B)}{\psi(0)} \right]$$

$$L_B = 791 \frac{\omega_H C_p}{(S_{Bi}/L_B)b} [\psi(0)^{-2.035} - \psi(L_B)^{-2.035}] + 10920 \frac{\omega_H C_p}{h'_{BO} (S_{Bo}/L_B)b} \times$$

$$\times \left[\ln \frac{\psi(L_B)}{\psi(0)} \right]$$

To make use of the above equation, the values of ψ at the two ends of the exchanger must be determined by a trial and error solution of the equation.

$$\left(\frac{2.24}{h'_{BO}}\right)\left(\frac{S_{Bi}}{S_{Bo}}\right)\psi^{3.035} + \psi - (\theta_6 - \theta_w) = 0$$

Application to a particular example,

$$W_H = 820 \text{ lbs/sec}$$

$$b = 4$$

$$C_p = 1.25 \text{ BTU/lb } ^\circ\text{F}$$

$$h'_{Bo} = 433 \text{ BTU/hr ft}^2 \text{ } ^\circ\text{F}$$

$$S_{Bi}/L_B = 60.5 \text{ ft}^2/\text{ft}$$

$$S_{Bo}/L_B = 358 \text{ ft}^2/\text{ft}$$

$$S_{Bi}/S_{Bo} = 0.1685$$

$$\frac{791 W_H C_p}{\left(\frac{S_{Bi}}{L_B}\right) b} = \frac{791 \times 820 \times 1.25}{60.5 \times 4} = 3350$$

$$\frac{10920 W_H C_p}{h'_{Bo} \left(\frac{S_{Bo}}{L_B}\right) b} = \frac{10920 \times 820 \times 1.25}{433 \times 358 \times 4} = 18.1$$

$$\left(\frac{2.24}{h'_{Bo}}\right) \left(\frac{S_{Bi}}{S_{Bo}}\right) = \frac{2.24 \times 0.1685}{433} = 8.72 \times 10^{-4}$$

$$L_B = 3350 \left[\psi^{-2.035}(0) - \psi^{-2.035}(L_B) \right] + 18.1 \left[\ln \frac{\psi(L_B)}{\psi(0)} \right]$$

$$8.72 \times 10^{-4} \psi^{3.035} + \psi - (\theta_G - \theta_w) = 0$$

$$b = 0$$

$$\theta_G = \theta_B = 1083$$

$$\theta_w = \theta_{II} = 1053$$

$$\theta_G - \theta_w = 30$$

$$8.72 \times 10^{-4} \psi^{3.035}(0) + \psi(0) = 30$$

$$\psi^{3.035}(0) + 1147 \psi(0) = 34400$$

By trial and error it is found that $\psi(0) = 21.0$.

$$l = L_0$$

$$\theta_0 = \theta_1 = 1504$$

$$\theta_w = \theta_{12} = 1053$$

$$\theta_0 - \theta_w = 451$$

$$8.72 \times 10^{-4} \psi^{3.035}(L_0) + \psi(L_0) = 451$$

$$\psi^{3.035}(L_0) + 1147 \psi(L_0) = 5.18 \times 10^5$$

$$\psi(L_0) = 72.0$$

By trial and error,

$$\psi^{-2.035}(0) = (21.0)^{-2.035} = 0.00202$$

$$\psi^{-2.035}(L_0) = (72.0)^{-2.035} = \underline{0.00017}$$

$$0.00185$$

$$\ln \frac{\psi(L_0)}{\psi(0)} = \ln \frac{72.0}{21.0} = \ln 3.43 = 1.23$$

$$L_0 = (3350)(0.00185) + (18.1)(1.23) =$$

$$L_0 = 6.2 + 22.2 = 28.4 \text{ ft}$$

11.6 Pressure Drop Equations in Stainless Steel Piping

The pressure drop equation is:

$$\Delta p_{PS} = \frac{2 f_{PS} G_{PS}^2 L_{PS}}{g \rho_{PS} D_{PS}}$$

The mass velocity and mass flow rate are

$$G_{PS} \frac{lb_s}{hr ft^2} = \frac{W_H \frac{lb_s}{sec} \times 3600 \frac{sec}{hr}}{0.785 D_{PS}^2 ft^2 \pi} = 1580 \frac{W_H}{D_{PS}^2 lb}$$

$$W_H = 7.58 \times 10^{-1} \frac{P}{\theta_3 - \theta_1}$$

$$G_{PS} = 3.48 \frac{P}{D_{PS}^2 (\theta_3 - \theta_1) \pi} \frac{lb_s}{hr ft^2}$$

The friction factor can be expressed as

$$f_{PS} = 0.079 \left(\frac{\mu_{PS}}{D_{PS} G_{PS}} \right)^{0.25}$$

Substituting into the original pressure drop equation

$$\Delta p_{PS} = \frac{2 \times 0.079 \left(\frac{\mu_{PS}}{D_{PS} G_{PS}} \right)^{0.25} G_{PS}^2 \left(\frac{lb^2}{hr^2 ft^4} \right) L_{PS} (ft)}{32.2 \left(\frac{ft}{sec} \right) \times \rho_{PS} \left(\frac{lb_s}{ft^3} \right) \times D_{PS} (ft) \times 144 \left(\frac{in^2}{ft^2} \right) \times 1.296 \times 10^7 \left(\frac{sec^2}{hr^2} \right)}$$

$$\Delta p_{PS} = 2.63 \times 10^{-12} \frac{\mu_{PS}^{0.25} G_{PS}^{1.75} L_{PS}}{\rho_{PS} D_{PS}^{1.25}} \text{ psi}$$

$$\rho_{PS} = 0.0931 \frac{PM}{\theta_{PS}}$$

where $M = 4$

$$\theta_{PS} \cong \frac{\theta_3 + \theta_6}{2}$$

Substituting in the density equation

$$\rho_{PS} = 0.745 \left(\frac{\rho}{\theta_3 + \theta_6} \right)$$

Viscosity raised to the 0.25 power is

$$\mu_{PS}^{0.25} \approx (0.096)^{0.25} = 0.557$$

The mass velocity raised to the 1.75 power is:

$$G_{PS}^{1.75} = 8.87 \frac{P^{1.75}}{D_{PS}^{3.5} (\theta_3 - \theta_1)^{1.75} \mu^{1.75}}$$

The pressure drop equation therefore becomes

$$\Delta P_{PS} = 1.75 \times 10^{-11} \frac{P^{1.75} L_{PS} (\theta_3 + \theta_6)}{D_{PS}^{4.75} \rho \mu^{1.75} (\theta_3 - \theta_1)^{1.75}} \text{ psi}$$

The ratio of the power for pumping through the stainless piping to the thermal power of the reactor can be expressed as:

$$\frac{P_{PS}}{P} \approx 0.498 \frac{\theta_5 \Delta P_{PS}}{\rho (\theta_3 - \theta_1)}$$

$$\frac{P_{PS}}{P} = 8.70 \times 10^{-12} \frac{P^{1.75} L_{PS} (\theta_3 + \theta_6) \theta_5}{D_{PS}^{4.75} \rho^2 \mu^{1.75} (\theta_3 - \theta_1)^{2.75}}$$

where $L_{PS} = 211 D_{PS} + 40$

$$\frac{P_{PS}}{P} = 8.70 \times 10^{-12} \frac{P^{1.75} (\theta_3 + \theta_6) \theta_5 (211 D_{PS} + 40)}{\rho^2 \mu^{1.75} (\theta_3 - \theta_1)^{2.75} D_{PS}^{4.75}}$$

$$\boxed{\frac{P_{PS}}{P} = 1.83 \times 10^{-9} \frac{P^{1.75} (\theta_3 + \theta_6) \theta_5 (D_{PS} + 0.190)}{\rho^2 \mu^{1.75} (\theta_3 - \theta_1)^{2.75} D_{PS}^{4.75}}}$$

An approximate survey of the proposed layout shows that the equivalent pipe length per loop (including fittings) is given by

$$L_{PS} = 211 D_{PS} + 40$$

11.7 Pressure Drop Equations in the Carbon Steel Piping

The pressure drop equation is

$$\Delta P_{PC} = 2.63 \times 10^{-12} \frac{\mu_{PC}^{0.25} G_{PC}^{1.75} L_{PC}}{P_{PC} D_{PC}^{1.25}} \text{ psi}$$

where $P_{PC} = 0.745 \left(\frac{P}{\theta_1 + \theta_5} \right)$

$$\mu_{PC}^{0.25} \approx (0.065)^{0.25} = 0.505$$

$$G_{PC}^{1.75} = 8.87 \frac{P^{1.75}}{D_{PC}^{3.5} (\theta_3 - \theta_1)^{1.75} \chi^{1.75}}$$

Substituting in the pressure drop equation

$$\Delta P_{PC} = 1.58 \times 10^{-11} \frac{P^{1.75} L_{PC} (\theta_1 + \theta_5)}{D_{PC}^{4.75} P \chi^{1.75} (\theta_3 - \theta_1)^{1.75}}$$

The ratio of pumping power through the carbon piping to the reactor thermal power is

$$\frac{P_{PC}}{P} \approx 0.498 \frac{\theta_1 \Delta P_{PC}}{P (\theta_3 - \theta_1)}$$

Substituting for ΔP_{PC}

$$\frac{P_{PC}}{P} = 7.86 \times 10^{-12} \frac{P^{1.75} L_{PC} (\theta_1 + \theta_5) \theta_1}{D_{PC}^{4.75} P^2 \chi^{1.75} (\theta_3 - \theta_1)^{2.75}}$$

as in the previous case, we find that

$$L_{PC} = 81 D_{PC} + 56$$

Substituting this into the above equation

$$\boxed{\frac{P_{PC}}{P} = 6.34 \times 10^{-10} \frac{P^{1.75} (\theta_1 + \theta_5) \theta_1 (D_{PC} + 0.692)}{P^2 \chi^{1.75} (\theta_3 - \theta_1)^{2.75} D_{PC}^{4.75}}}$$

11.8 Pressure Drop Through Reactor

11.8.1 Pressure Drop in Central Hole

It is assumed that the flow through the core is orificed so as to match the axial flux profile. This will result in the coolant entering the central hole at a constant temperature, θ_2 .

Let $w_Z \equiv$ the mass flow rate of coolant through the central hole (lb/sec) at Z , where

$Z \equiv$ distance from bottom plane of reactor core.

w_Z will be assumed to equal 0 at $Z = 0$, and to equal w_H at $Z = H_R$. In between, its rate of increase will be proportional to the axial heat generation profile. If we assume that the axial flux profile follows a sine distribution

$$\theta(Z) = \left[\phi \left(\frac{H_R}{Z} \right) \right] \sin \left(\frac{\pi Z}{H_R} \right) \quad (11.8.1-1)$$

then the derivative of w_Z will follow the same distribution.

$$\frac{dw_Z}{dZ} = C \sin \left(\frac{\pi Z}{H_R} \right)$$

where $C =$ a constant.

Integrating

$$w_Z = C_1 \cos \left(\frac{\pi Z}{H_R} \right) + C_2$$

where C_1 and C_2 are constants.

Substituting our boundary conditions:

$$z = 0, \quad w_z = C_1 + C_2 = 0$$

$$z = H_R, \quad w_z = -C_1 + C_2 = w_H$$

therefore

$$C_1 = - \frac{w_H}{2}$$

$$C_2 = + \frac{w_H}{2}$$

$$w_z = \left(\frac{w_H}{2} \right) \left[1 - \cos \frac{\pi z}{H_R} \right] \quad (11.8.1-2)$$

The pressure drop across a differential distance is given by,

$$dp = - \frac{f \rho_2 v_o^2 dz}{2 g D_o} \quad (11.8.1-3)$$

but since

$$v_o = \frac{w_z}{\rho_2 A_o}$$

$$dp = \frac{f w_z^2 dz}{2 g D_o \rho_2 A_o^2}$$

Substituting for w_z

$$dp = \frac{f w_H^2}{8 g D_o \rho_2 A_o^2} \left[1 - \cos \left(\frac{\pi z}{H_R} \right) \right]^2 dz \quad (11.8.1-4)$$

Integrating from 0 to H_R , we obtain ΔP_o , the pressure drop through the central hole

-264-

$$\Delta p_o = \frac{f \omega_H^2}{8 g D_o \rho_2 A_o^2} \int_0^{H_R} \left[1 - 2 \cos \left(\frac{\pi z}{H_R} \right) + \cos^2 \left(\frac{\pi z}{H_R} \right) \right] dz$$

$$= \frac{f \omega_H^2}{8 g D_o \rho_2 A_o^2} \left[z - \frac{2 H_R}{\pi} \sin \left(\frac{\pi z}{H_R} \right) + \frac{z}{2} + \frac{H_R}{\pi} \sin \left(\frac{\pi z}{H_R} \right) \cos \frac{\pi z}{H_R} \right]_0^{H_R}$$

$$= \frac{f \omega_H^2}{8 g D_o \rho_2 A_o^2} \left(\frac{3 H_R}{2} \right)$$

$$\Delta p_o = \frac{3 f \omega_H^2 H_R}{16 g D_o \rho_2 A_o^2} \quad \frac{\text{lbs}}{\text{ft}^2} \quad (11.8.1-5)$$

However, as has been shown

$$\omega_H = \frac{(.76)(10^{-3})(P)}{(\theta_3 - \theta_1)}$$

$$A_o = \frac{\pi}{4} D_o^2$$

so that

$$\Delta p_o = \left(\frac{3}{16} \right) \frac{(.76)^2 (10^{-6} f P^2 H_R)}{(.785)^2 g D_o^5 \rho_2 (\theta_3 - \theta_1)^2} \quad (11.8.1-6)$$

Letting, $f = 0.045$, $H_R = 24 \text{ ft}$

$$\Delta p_o = \frac{0.58 \times 10^{-8} P^2}{(\theta_3 - \theta_1)^2 D_o^5 \rho_2}$$

as shown previously

$$\rho_2 = \frac{0.37 P}{\theta_2} \quad (11.8.1-7)$$

where p = pressure in psi

Hence,

$$\Delta p_0 = \frac{(1.56)(10^{-8}) \theta_2 p^2}{(\theta_3 - \theta_1)^2 D_o^5 p} \quad \frac{\text{lbs}}{\text{ft}^2} \quad (11.8.1-8)$$

11.8.2 Diffuser Losses

The diffuser loss is expressed in terms of the exit velocity head in the vertical pipe.

With a 10° diffuser one tenth of a velocity head is lost; one velocity head is

$$\frac{v_o^2}{2g} \rho_2 = \frac{W_H^2}{2g A_o \rho_2} \quad (11.8.2-1)$$

Referring to Equation (11.8.1-5), we see that this is equal to

$$\left(\frac{8 D_o}{3 f H_R} \right) \Delta p_0 = 2.5 D_o \Delta p_0 \quad (11.8.2-2)$$

Hence the diffuser loss is

$$\Delta p_0 = 0.1 \text{ velocity head} = 0.25 D_o \Delta p_0 \quad \frac{\text{lbs}}{\text{ft}^2} \quad (11.8.2-3)$$

11.8.3 Exit Losses

From the diffuser, the gas flows radially outward to the system piping. One velocity head was allowed for turning exit losses, hence,

$$\Delta p_{ex} = \frac{v_{ex}^2 \rho_3}{2g}$$

where the subscript "ex" refers to the vertical pipe after the diffuser.

Referring to Equation (11.8.2.1) we see that

$$\Delta p_{ex} = (2.5 D_o) \left(\frac{v_{ex}}{v_o} \right)^2 \Delta p_o \quad (11.8.3-1)$$

v_{ex} is related to v_o by

$$v_{ex} = v_o \left(\frac{D_o}{D_{ex}} \right)^2 \left(\frac{\rho_2}{\rho_1} \right) = v_o \left(\frac{D_o}{D_{ex}} \right)^2 \left(\frac{\theta_2}{\theta_3} \right) \quad (11.8.3-2)$$

where D_{ex} = diameter of vertical pipe after diffuser

Combining Equations (11.8.3-1) and (11.8.3-2)

$$\Delta p_{ex} = (2.5) \left(\frac{D_o}{D_{ex}} \right)^4 \left(\frac{\theta_2}{\theta_3} \right)^2 D_o \Delta p_o \quad (11.8.3-3)$$

11.8.4 Entrance Losses

Gas enters through 4 two foot diameter entrance nozzles.

A total of two velocity heads was allowed up to the entrance of the core proper.

From the continuity equation, it follows that

$$v_{en} = v_o \left(\frac{A_o}{4A_{en}} \right) \left(\frac{\rho_2}{\rho_1} \right) = \frac{1}{4} \left(\frac{D_o}{D_{en}} \right)^2 \left(\frac{\rho_2}{\rho_1} \right) v_o$$

(11.8.4-1)

$$\Delta p_{en} = 2 \left(\frac{\rho_1 v_{en}^2}{2g} \right)$$

Referring to Equation (11.8.2-1 and 11.8.2-2) we see that

$$\Delta p_{en} = (2)(2.5) D_o \left(\frac{v_{en}}{v_o} \right)^2 \Delta p_o \quad (11.8.4-2)$$

Substituting for $\left(\frac{v_{en}}{v_o} \right)$

$$\Delta p_{en} = \frac{5 D_o}{4} \left(\frac{D_o^4}{D_{en}^4} \frac{\rho_2^2}{\rho_1^2} \right) \Delta p_o \quad (11.8.4-3)$$

Substituting for $\frac{\rho_2}{\rho_1}$ in terms of Equation (11.8.1-7)

$$\Delta p_{en} = 1.248 \frac{D_o^5}{D_{en}^4} \left(\frac{\theta_1}{\theta_2} \right)^2 \Delta p_o$$

Since $D_{en} = 2$

$$\Delta p_{en} = 0.078 D_o^5 \left(\frac{\theta_1}{\theta_2} \right)^2 \Delta p_o$$

11.8.5 Radial Pressure Drop Across Core

The pressure drop across the core was calculated by the correlation in report HPC-35. The axial flow distribution was matched to the axial power variation.

The drop across a small element dr is;

$$\frac{dp}{dr} = \frac{2 f_r G^2}{g D_F \rho_r}$$

$$\frac{dp}{dr} = \frac{24 f_r \omega^2}{g d_F \rho_r A_r^2} \quad (11.8.5-1)$$

where

D_F = fuel diameter in., ft.

d_F = fuel diameter in., in.

Since the gas temperature θ_G varies almost linearly (See Figure 11-4) the density at r (ρ_r) can be approximated by,

$$\rho_r = \frac{.37 p}{\theta_G}$$

$$\rho_r = \frac{.37 p}{800 + 187(\theta - r)} \quad (11.8.5-2)$$

Assuming an inlet temperature $\theta_1 = 800^\circ \text{R}$ and an outlet temperature

$\theta_2 = 2060^\circ \text{R}$.

Also,

$$A_r = 2\pi r H_r$$

Substituting (11.8.5-1) and (11.8.5-2)

$$\frac{dp}{dr} = - \frac{24 f_r \omega^2 [800 + 187(\theta - r)]}{g d_F .37 p (2\pi r H_r)^2}$$

Substituting

$$H_r = 24$$

$$g = 32.2$$

$$f_r = 6.5$$

and multiplying by the square of the ratio of maximum to average axial flux $\left(\frac{\pi}{2}\right)$ we obtain an expression for the pressure gradient at the point of maximum flow, i.e., the midplane of the reactor core.

$$\frac{dp}{dr} = - \frac{\left(\frac{\pi}{2}\right)^2 (.108) w^2 (12.25 - r)}{\rho d_F r^2} = - \frac{\left(\frac{\pi}{2}\right)^2 .108 w^2}{\rho d_F} \left(\frac{12.25}{r^2} - \frac{1}{r} \right) \quad (11.8.5-3)$$

Integrating between $r = \frac{D_R}{2}$ and $r = \frac{D_0}{2}$

$$\Delta P_R = \frac{\left(\frac{\pi}{2}\right)^2 (.108) w^2}{\rho d_F} \left(\frac{24.5}{D_0} - \frac{24.5}{D_R} - \ln \frac{D_0}{D_R} \right)$$

as shown on page

$$w = \frac{.76 \times 10^{-3} P}{(\theta_2 - \theta_1)} \quad (11.8.5-4)$$

so that

$$\Delta P_R = \frac{.156 \times 10^{-6} P^2}{\rho d_F (\theta_2 - \theta_1)^2} \left(\frac{24.5}{D_0} - \frac{24.5}{D_R} - \ln \frac{D_0}{D_R} \right) \quad (11.8.5-5)$$

11.8.6 Pumping Power

The pumping power required is

$$P_R = \frac{\Delta P W}{\eta_B \rho 778} \left(\frac{3600}{3.413} \right)$$

Upon substituting Equation (11.8.5-4) and

$$\rho = \frac{.37 P}{\theta_1}$$

$$P_R = \frac{(2.78)(10^{-3}) P \theta_1 \Delta p_T}{\eta_B P (\theta_3 - \theta_1)}$$

assuming a blower efficiency of 80%

$$P_R = \frac{(3.48)(10^{-3}) P \theta_1 \Delta p_T}{P (\theta_3 - \theta_1)}$$

where Δp_T = total pressure drop through reactor vessel in, lbs/ft^2 .

Summing up the various pressure drops

$$\Delta p_T = \Delta p_{en} + \Delta p_R + \Delta p_o + \Delta p_D + \Delta p_{ef}$$

$$\Delta p_{en} = 7.80 \times 10^{-2} D_o^5 \left(\frac{\theta_1}{\theta_2} \right)^2 \Delta p_o =$$

$$= 1.22 \times 10^{-9} \left(\frac{P^2 \theta_1^2}{P \theta_2 (\theta_3 - \theta_1)^2} \right) \text{ psf}$$

$$\Delta p_R = 1.56 \times 10^{-7} \frac{P^2}{P d_F (\theta_2 - \theta_1)^2} \left[\frac{24.5}{D_o} - \frac{24.5}{D_R} + \ln \frac{D_R}{D_o} \right] \text{ psf}$$

$$\Delta p_o = 1.56 \times 10^{-8} \frac{P^2 \theta_2}{P D_o^5 (\theta_3 - \theta_1)^2} \text{ psf}$$

$$\Delta p_D = 0.25 D_o \Delta p_o = 3.90 \times 10^{-9} \frac{P^2 \theta_2}{P D_o^4 (\theta_3 - \theta_1)^2}$$

$$\Delta p_{ex} = 2.5 \left(\frac{D_o^5}{D_{ex}^4} \right) \left(\frac{\theta_2^2}{\theta_3^2} \right) \Delta p_o =$$

$$= 3.90 \times 10^{-8} \frac{P^2 \theta_2^3}{P D_{ex}^4 \theta_3^2 (\theta_3 - \theta_1)} \text{ psf}$$

therefore

$$\Delta p_r = \left[\frac{P^2}{P(\theta_3 - \theta_1)^2} \right] \left[1.22 \times 10^{-9} \frac{\theta_1^2}{\theta_2} + 1.56 \times 10^{-8} \frac{\theta_2}{D_o^5} \right.$$

$$+ \left(\frac{\theta_3 - \theta_1}{\theta_2 - \theta_1} \right)^2 \left(\frac{24.5}{D_o} - \frac{24.5}{D_R} \ln \frac{D_R}{D_o} \right) 1.56 \times 10^{-7} +$$

$$\left. + 3.90 \times 10^{-9} \frac{\theta_2}{D_o^4} + 3.90 \times 10^{-8} \frac{\theta_2^3}{D_{ex}^4 \theta_3^2} \right]$$

Illustrative calculations for Δp_T are as follows.

Substituting the following values:

$$P = 1.165 \times 10^9 \text{ watts (thermal power)}$$

$$P = 1000 \text{ psi}$$

$$\theta_1 = 801 \text{ } ^\circ R$$

$$\theta_3 = 1860 \text{ } ^\circ R$$

$$\theta_2 = 2060 \text{ } ^\circ R$$

$$D_o = 2.5 \text{ ft}$$

$$D_R = 16 \text{ ft}$$

$$D_{ex} = 3 \text{ ft}$$

$$\frac{P^2}{\rho(\theta_3 - \theta_1)^2} = \frac{(1.165 \times 10^9)^2}{(1000)(1860 - 801)^2} = \underline{1.21 \times 10^9}$$

Substituting into each term in the large bracket

$$1.22 \times 10^{-9} \frac{\theta_1^2}{\theta_2} = 1.22 \times 10^{-9} \frac{(801)^2}{2060} = \underline{38 \times 10^{-8}}$$

$$\begin{aligned} & (1.56 \times 10^{-7}) \left(\frac{\theta_3 - \theta_1}{\theta_2 - \theta_1} \right)^2 \left(\frac{24.5}{D_0} - \frac{24.5}{D_R} + \ln \frac{D_R}{D_0} \right) = \\ & = (1.56 \times 10^{-7}) \left(\frac{1860 - 801}{2060 - 801} \right) \left(\frac{24.5}{2.5} - \frac{24.5}{16} + \ln \frac{16}{3} \right) = \\ & = \underline{10.9 \times 10^{-7}} \end{aligned}$$

$$(1.56 \times 10^{-8}) \frac{\theta_2}{D_0^5} = (1.56 \times 10^{-8}) \frac{(2060)}{(2.5)^5} = \underline{33 \times 10^{-8}}$$

$$(3.90 \times 10^{-9}) \frac{\theta_2}{D_0^4} = 3.90 \times 10^{-9} \frac{2060}{(2.5)^4} = \underline{206 \times 10^{-9}}$$

$$(3.90 \times 10^{-8}) \frac{\theta_2^3}{D_{ex}^4 \theta_3^2} = (3.90 \times 10^{-8}) \frac{(2060)^3}{(3)^4 (1860)^2} = \underline{1.12 \times 10^6}$$

Multiplying these numbers by the first term (1.21×10^9) and dividing by 144 gives the pressure drop in psi.

Entrance Losses	=	3.10 psi
Core Losses	=	9.20 psi
Vertical Pipe Losses	=	2.76 psi
Diffuser Losses	=	1.80 psi
Exit Losses	=	<u>9.00 psi</u>
		25.86 psi

$$\Delta p_T = 25.86 \text{ psi}$$

11.9 Temperature Coefficient of Reactivity

The temperature coefficient of reactivity is defined as

$$\frac{\partial \rho}{\partial T}$$

where $\rho = \frac{k-1}{k}$

$$\frac{\partial \rho}{\partial T} = \frac{1}{k} \frac{\partial k}{\partial T} - \frac{(k-1)}{k^2} \frac{\partial k}{\partial T}$$

$$\frac{1}{k} \frac{\partial \rho}{\partial T} = \frac{1}{k^2} \frac{\partial k}{\partial T}$$

If $k \approx 1$

$$\therefore \frac{\partial \rho}{\partial T} \approx \frac{1}{k} \frac{\partial k}{\partial T}$$

The multiplication constant is:

$$k = \frac{\eta f \rho_{re} e^{-\beta^2 \tau}}{1 + L^2 \beta^2} \approx \eta f \rho_{re} e^{-\beta^2 (\tau + L^2)} \approx \eta f \rho_{re} e^{-\beta^2 M^2}$$

$$\frac{1}{k} \left(\frac{\partial k}{\partial T} \right) = \frac{1}{\eta} \frac{\partial \eta}{\partial T} + \frac{1}{f} \frac{\partial f}{\partial T} + \frac{1}{\rho} \frac{\partial \rho}{\partial T} - M^2 \frac{\partial \beta^2}{\partial T} - \beta^2 \frac{\partial M^2}{\partial T} + \frac{1}{\rho_{re}} \frac{\partial \rho_{re}}{\partial T}$$

11.9.1 The Effect of γ_l on the Temperature Coefficient

$$\eta = \nu \left(\frac{\sigma_f^{23}}{\sigma_a^{23}} \right) = \nu R_F \quad R_F = \frac{\sigma_f^{23}}{\sigma_a^{23}}$$

$$\frac{1}{\eta} \frac{\partial \eta}{\partial T} = \frac{1}{\nu R_F} \frac{\partial (\nu R_F)}{\partial T} = \frac{1}{R_F} \frac{\partial R_F}{\partial T} + \frac{1}{\nu} \frac{\partial \nu}{\partial T} =$$

$$= \frac{\partial \ln R_F}{\partial T} + \frac{\partial \ln \nu}{\partial T} = \left[\frac{\partial \ln R_F}{\partial E} + \frac{\partial \ln \nu}{\partial E} \right] \frac{\partial E}{\partial T} =$$

$$\frac{1}{\eta} \frac{\partial \eta}{\partial T} = \frac{b}{E} \left[\frac{\partial \ln R_F}{\partial \ln E} + \frac{\partial \ln v}{\partial \ln E} \right]$$

For the purpose of this calculation η was assumed to be constant.

This is a conservative assumption, since-if anything- η would decrease with increasing temperature.

11.9.2 Effect of f on Temperature Coefficient

$$f = \frac{\Sigma_a^{23}}{\Sigma_a^{23} + \Sigma_a^M + \Sigma_a^{02} + \Sigma_a^P + \Sigma_a^X + \Sigma_a^S} \equiv \frac{\Sigma_a^{23}}{\Sigma_a^X + \Sigma_a^S + \Sigma_a^O}$$

where

Σ_a^X = xenon

Σ_a^S = samarium

Σ_a^P = other poisons

$\Sigma_a^O = \Sigma_a^{23} + \Sigma_a^M + \Sigma_a^{02} + \Sigma_a^P$, all of which are

assumed to follow a $1/v$ relationship.

$$\frac{1}{f} \frac{\partial f}{\partial T} = \left(\frac{\Sigma_a^X + \Sigma_a^S + \Sigma_a^O}{\Sigma_a^{23}} \right) \left(\frac{\Sigma_a^{23}}{[\Sigma_a^X + \Sigma_a^S + \Sigma_a^O]^2} \right) \left(\frac{\partial \Sigma_a^X}{\partial T} + \frac{\partial \Sigma_a^S}{\partial T} + \frac{\partial \Sigma_a^O}{\partial T} \right) + \left(-\frac{1}{\Sigma_a^{23}} \frac{\partial \Sigma_a^{23}}{\partial T} \right)$$

$$\frac{1}{f} \frac{\partial f}{\partial T} = - \left(\frac{1}{\Sigma_a^X + \Sigma_a^S + \Sigma_a^O} \right) \left(\frac{\partial \Sigma_a^X}{\partial T} + \frac{\partial \Sigma_a^S}{\partial T} + \frac{\partial \Sigma_a^O}{\partial T} \right) + \left(-\frac{1}{\Sigma_a^{23}} \frac{\partial \Sigma_a^{23}}{\partial T} \right)$$

$$\frac{\partial \Sigma_a^X}{\partial T} = N_X \frac{\partial \sigma_a^X}{\partial T} + \sigma_a^X \frac{\partial N_X}{\partial T} = \Sigma_a^X \left(\frac{1}{\sigma_a^X} \frac{\partial \sigma_a^X}{\partial T} + \frac{1}{N_X} \frac{\partial N_X}{\partial T} \right)$$

Let $\frac{1}{N_x} \frac{\partial N_x}{\partial T} = -3\alpha$ then $\frac{\partial \Sigma_a^x}{\partial T} = \Sigma_a^x \left(\frac{1}{\sigma_a^x} \frac{\partial \sigma_a^x}{\partial T} - 3\alpha \right)$

Similarly

$$\frac{\partial \Sigma_a^s}{\partial T} = \Sigma_a^s \left(\frac{1}{\sigma_a^s} \frac{\partial \sigma_a^s}{\partial T} - 3\alpha \right)$$

$$\frac{\partial \Sigma_a^o}{\partial T} = \Sigma_a^o \left(\frac{1}{\sigma_a^o} \frac{\partial \sigma_a^o}{\partial T} - 3\alpha \right)$$

$$\frac{\partial \Sigma_a^{23}}{\partial T} = \Sigma_a^{23} \left(\frac{1}{\sigma_a^{23}} \frac{\partial \sigma_a^{23}}{\partial T} - 3\alpha \right)$$

Assume σ_a^o and σ_a^{23} proportional to $1/v$

$$\therefore \sigma_a^o = C_1 v^{-1} = C_2 E^{-1/2} = C_3 T^{-1/2}$$

$$\frac{\partial \sigma_a^o}{\partial T} = -\frac{1}{2} C_3 T^{-3/2}$$

$$\frac{1}{\sigma_a^o} \frac{\partial \sigma_a^o}{\partial T} = \frac{-\frac{1}{2} C_3 T^{-3/2}}{C_3 T^{-1/2}} = -\frac{1}{2T}$$

similarly

$$\frac{1}{\sigma_a^{23}} \frac{\partial \sigma_a^{23}}{\partial T} = -\frac{1}{2T}$$

$$\frac{\partial \Sigma_a^x}{\partial T} = \Sigma_a^x \left(\frac{1}{\sigma_a^x} \frac{\partial \sigma_a^x}{\partial T} - 3\alpha \right)$$

$$\frac{\partial \Sigma_a^s}{\partial T} = \Sigma_a^s \left(\frac{1}{\sigma_a^s} \frac{\partial \sigma_a^s}{\partial T} - 3\alpha \right)$$

$$\frac{\partial \Sigma_a^o}{\partial T} = \Sigma_a^o \left(-\frac{1}{2T} - 3\alpha \right)$$

$$\frac{\partial \Sigma_a^{23}}{\partial T} = \Sigma_a^{23} \left(-\frac{1}{2T} - 3\alpha \right)$$

$$\begin{aligned} \frac{1}{f} \frac{\partial f}{\partial T} &= \left(\frac{\Sigma_a^x}{\Sigma_a^x + \Sigma_a^s + \Sigma_a^o} \right) \left(- \frac{1}{\sigma_a^x} \frac{\partial \sigma_a^x}{\partial T} \right) + \left(\frac{\Sigma_a^s}{\Sigma_a^x + \Sigma_a^s + \Sigma_a^o} \right) \left(- \frac{1}{\sigma_a^s} \frac{\partial \sigma_a^s}{\partial T} \right) + \\ &\quad + \left(\frac{\Sigma_a^o}{\Sigma_a^x + \Sigma_a^s + \Sigma_a^o} \right) \left(- \frac{1}{2T} \right) + 3\alpha - \frac{1}{2T} - 3\alpha \\ \frac{1}{f} \frac{\partial f}{\partial T} &= - \left(\frac{1}{1 + \frac{P_s}{P_x} + \frac{1}{P_x} \frac{\Sigma_a^o}{\Sigma_a^{23}}} \right) \left(\frac{1}{\sigma_a^x} \frac{\partial \sigma_a^x}{\partial T} + \frac{1}{2T} \right) - \left(\frac{1}{\frac{P_x}{P_s} + 1 + \frac{1}{P_s} \frac{\Sigma_a^o}{\Sigma_a^{23}}} \right) \\ &\quad \left(\frac{1}{\sigma_a^s} \frac{\partial \sigma_a^s}{\partial T} + \frac{1}{2T} \right) \end{aligned}$$

11.9.3 The Effect of B^2 on the Temperature Coefficient

For an infinitely long cylinder

$$B^2 = \frac{4.81^2}{D_R^2} = \frac{23.2}{D_R^2} + \frac{\pi^2}{H_R^2}$$

Calculations indicate that the presence of a $2\frac{1}{2}$ ft. diameter central hole in the core increases the radial leakage by approximately 5%. Therefore the value of 23.2 was multiplied by 1.05, resulting in

$$B^2 \cong \frac{24.4}{D_R^2} + \frac{\pi^2}{H_R^2} = \frac{24.4}{D_R^2} + \frac{9.86}{H_R^2}$$

Differentiating with respect to time:

$$\frac{\partial B^2}{\partial T} = - \frac{48.8}{D_R^3} \frac{\partial D_R}{\partial T} - \frac{19.72}{H_R^3} \frac{\partial H_R}{\partial T}$$

$$\frac{\partial B^2}{\partial T} = - \alpha \left(\frac{48.8}{D_R^2} + \frac{19.72}{H_R^2} \right)$$

$$- M^2 \frac{\partial B^2}{\partial T} = \alpha M^2 \left(\frac{48.8}{D_R^2} + \frac{19.72}{H_R^2} \right) = 2\alpha B^2 M^2$$

11.9.4 Effect of M^2 on Temperature Coefficient

$$M^2 = L^2 + \tau$$

$$\frac{\partial M^2}{\partial T} = \frac{\partial L^2}{\partial T} + \frac{\partial \tau}{\partial T}$$

where $L^2 = \frac{D}{\Sigma_a}$

$$\frac{\partial L^2}{\partial T} = \frac{1}{\Sigma_a} \frac{\partial D}{\partial T} - \frac{D}{\Sigma_a^2} \frac{\partial \Sigma_a}{\partial T}$$

$$\frac{1}{\Sigma_a} \frac{\partial D}{\partial T} = \frac{L^2}{D} \frac{\partial D}{\partial T}$$

$$D \approx \frac{1}{3 \Sigma_s} \approx \frac{1}{3 N_m \sigma_s^m}$$

$$\frac{\partial D}{\partial T} = -\frac{1}{3} \left(\frac{1}{\sigma_s N_m^2} \left[\frac{\partial N_m}{\partial T} \right] + \frac{1}{N_m \sigma_s^2} \left[\frac{\partial \sigma_s^m}{\partial T} \right] \right)$$

where $\left[\frac{\partial \sigma_s^m}{\partial T} \right] \approx 0$

$$\frac{\partial D}{\partial T} = -\frac{1}{3 N_m^2 \sigma_s^m} \left(\frac{\partial N_m}{\partial T} \right)$$

$$\frac{1}{D} \frac{\partial D}{\partial T} = -\frac{3 N_m \sigma_s^m}{3 N_m^2 \sigma_s^m} \left(\frac{\partial N_m}{\partial T} \right) = -\frac{1}{N_m} \frac{\partial N_m}{\partial T} = 3\alpha$$

$$\frac{1}{\Sigma_a} \frac{\partial D}{\partial T} = 3\alpha L^2$$

$$-\frac{D}{\Sigma_a^2} \frac{\partial \Sigma_a}{\partial T} = -\frac{L^2}{\Sigma_a} \frac{\partial \Sigma_a}{\partial T}$$

$$\Sigma_a = \Sigma_a^x + \Sigma_a^s + \Sigma_a^o$$

$$\frac{\partial \Sigma_a}{\partial T} = \frac{\partial \Sigma_a^x}{\partial T} + \frac{\partial \Sigma_a^s}{\partial T} + \frac{\partial \Sigma_a^o}{\partial T}$$

Since thermal leakage is a minor factor, and since $\frac{\Sigma_a^x + \Sigma_a^s}{\Sigma_a} < 0.06$ it will be permissible at this point to consider all Σ_a 's to be proportional to $1/v$.

$$\therefore \frac{1}{\Sigma_a} \frac{\partial \Sigma_a}{\partial T} \approx \frac{1}{\Sigma_a^{23}} \frac{\partial \Sigma_a^{23}}{\partial T} = -\frac{1}{2T} - 3\alpha$$

$$-\frac{D}{\Sigma_a^2} \frac{\partial \Sigma_a}{\partial T} = \frac{L^2}{2T} + 3\alpha L^2$$

$$\therefore \frac{\partial L^2}{\partial T} = \frac{1}{\Sigma_a} \frac{\partial D}{\partial T} - \frac{D}{\Sigma_a^2} \frac{\partial \Sigma_a}{\partial T} =$$

$$= 3\alpha L^2 + \frac{L^2}{2T} + 3\alpha L^2$$

$$\frac{\partial L^2}{\partial T} = 6\alpha L^2 + \frac{L^2}{2T}$$

$$\tau = \int_E^{E_0} \frac{D}{3 \Sigma_s} \frac{dE}{E} \approx \left(\frac{1}{3 N_m^2 \xi} \right) \int_E^{E_0} \frac{1}{\sigma_s^2} \frac{dE}{E}$$

$$\frac{\partial \tau}{\partial T} = \left(\frac{\partial \tau}{\partial E} \right) \left(\frac{\partial E}{\partial T} \right) = h_0 \frac{\partial \tau}{\partial E}$$

$$\frac{\partial \tau}{\partial E} = \left(\frac{1}{3 N_m^2 \xi} \right) \left(-\frac{1}{\sigma_s^2 E} \right) + \left(\frac{1}{3 \xi} \int_E^{E_0} \frac{1}{\sigma_s^2} \frac{dE}{E} \right) \left(-\frac{2}{N_m^3} \frac{\partial N_m}{\partial T} \right)$$

$$\left(\frac{1}{3 \xi} \int_E^{E_0} \frac{1}{\sigma_s^2} \frac{dE}{E} \right) = N_m^2 \tau$$

$$\frac{\partial \tau}{\partial E} = \left(\frac{1}{3 N_m^2 \xi} \right) \left(- \frac{1}{\sigma_s^2 E} \right) - \frac{2 \tau}{N_m} \left(\frac{\partial N_m}{\partial T} \right) \left(\frac{\partial \tau}{\partial E} \right)$$

$$\frac{1}{N_m} \frac{\partial N_m}{\partial T} = - 3 \alpha$$

$$\frac{\partial \tau}{\partial E} = \left(\frac{1}{3 N_m^2 \xi} \right) \left(- \frac{1}{\sigma_s^2 E} \right) + \frac{6 \alpha \tau}{h_B}$$

$$= - \frac{D_m}{3 \xi \Sigma_s^M E} + 6 \alpha \tau$$

$$\frac{\partial \tau}{\partial E} = - \frac{D_m}{3 \xi \Sigma_s^M h_B T} + 6 \alpha \tau$$

$$\frac{\partial \tau}{\partial T} = h_B \frac{\partial \tau}{\partial E} = - \frac{D_m}{3 \xi \Sigma_s^M T} + 6 \alpha \tau$$

$$\frac{\partial M^2}{\partial T} = \frac{\partial L^2}{\partial T} + \frac{\partial \tau}{\partial T} = 6 \alpha L^2 + \frac{L^2}{2 T} - \frac{D_m}{3 \xi \Sigma_s^M T} + 6 \alpha \tau$$

$$\frac{\partial M^2}{\partial T} = 6 \alpha M^2 + \frac{L^2}{2 T} - \frac{D_m}{3 \xi \Sigma_s^M T}$$

$$- B^2 \frac{\partial M^2}{\partial T} = - 6 \alpha B^2 M^2 - \frac{B^2 L^2}{2 T} + \frac{D_m B^2}{3 \xi \Sigma_s^M T}$$

11.9.5 Effect of Resonance Absorption on Temperature Coefficient

$$\rho_{re} = \exp \left(- \frac{N_{20} I}{3 \xi \Sigma_s} \right) = \exp \left(- \frac{I}{3 R_m \sigma_s^M} \right)$$

$$\frac{\partial P_{Re}}{\partial T} = - \cancel{eff} \left(\frac{I}{\sum R_m \sigma_s^m} \right) \left(\frac{1}{\sum R_m \sigma_s^m} \right) \frac{\partial I}{\partial T} = \left(- \frac{P_{Re}}{\sum R_m \sigma_s^m} \right) \frac{\partial I}{\partial T}$$

$$\frac{1}{P_{Re}} \frac{\partial P_{Re}}{\partial T} = - \frac{1}{\sum R_m \sigma_s^m} \frac{\partial I}{\partial T}$$

11.9.5a Method of Determining $\left(\frac{\partial I}{\partial T} \right)$

To evaluate $\left(\frac{\partial I}{\partial T} \right)$ we employed a method outlined by L. Dresner in Reference (15) and in supplemental verbal instructions. The resonance integral I was divided into the sum of the individual resolved resonance integrals I_i , plus a term I' to account for the effect of the unresolved resonances at higher energies.

$$I = \sum_i I_i + I'$$

To calculate each I_i , we used the formula

$$I_i = \frac{\sigma_p \Gamma}{E_i} J(\xi_i, \Delta_i)$$

where

$$\sigma_p = \frac{\sum \sigma_s^{o2} + \sum \sigma_s^m}{N_{o2}} = \frac{N_{o2} \sigma_s^{o2} + N_m \sigma_s^m}{N_{o2}} = \left(\sigma_s^{o2} + R_m \sigma_s^m \right) \text{ barns}$$

Γ_o was assumed to equal 0.030 ev for each resonance

E_i = energy of resonance peak, in ev

$$I_i = \left(\sigma_s^{o2} + R_m \sigma_s^m \right) \left(\frac{0.030}{E_i} \right) J(\xi_i, \Delta_i)$$

$$\xi_i = \Gamma_i / \Delta_i$$

$$\Gamma_i = \Gamma_o + \Gamma_{ni} = 0.030 + \Gamma_{ni}$$

values of Γ_{ni} were obtained from BNL-325

$$\Delta_i = \frac{4 \mu E_i k_B T}{M}$$

$$\left. \begin{array}{l} \mu = 1 \\ M = 232 \end{array} \right\} \text{ for thorium}$$

$$k_B = 8.62 \times 10^{-5} \text{ ev/}^\circ\text{K} \quad (\text{Boltzman's constant})$$

$$T = \text{Temperature in } ^\circ\text{K}$$

$$\Delta_i = 1.22 \times 10^{-3} \sqrt{T E_i}$$

$$\xi_i = \frac{\Gamma_i}{\Delta_i} = \left(\frac{820}{\sqrt{T}} \right) \left(\frac{0.030 + \Gamma_{ni}}{\sqrt{E_i}} \right)$$

$$2^k = \beta \times 10^5$$

$$k_i = 16.54 + 3.31 \log \beta_i$$

$$\text{where } \beta_i = \frac{\sigma_P}{\sigma_{oi}}$$

$$\sigma_P = \sigma_s^{o2} + R_M \sigma_s^M$$

$$\sigma_{oi} = \frac{2.67 \text{ barns}}{(E_i \times 10^{-6}) \text{ Mev}} \times \frac{\Gamma_{ni}}{\Gamma_i}$$

$$\sigma_{oi} = \frac{2.67 \times 10^6}{E_i} \frac{\Gamma_{ni}}{\Gamma_{ni} + 0.030}$$

$$\beta_i = \frac{\sigma_P}{\sigma_{oi}} = \frac{E_i (\sigma_s^{o2} + R_M \sigma_s^M)}{2.67 \times 10^6} \left(1 + \frac{0.030}{\Gamma_{ni}} \right)$$

$$k_i = 16.54 + 1.24 \times 10^{-6} E_i (\sigma_s^{o2} + R_M \sigma_s^M) \left(1 + \frac{0.030}{\Gamma_{ni}} \right)$$

Using the values of ξ_i and k_i calculated from the above equations,

corresponding values of J were obtained from graphs prepared by Mr. Dresner.

These graphs were the result of "Oracle" calculations he had previously performed for the purpose of determining the thorium resonance integral at lower temperatures. (See Figure 11-1).

The contribution of the unresolved resonances was evaluated as follows:

$$I' = \frac{\sigma_p \Gamma_\gamma}{\Delta E} \int_{\beta_n}^{\infty} J(\xi', \beta) \frac{d\beta}{\beta}$$

$$\sigma_p = \sigma_s^{o2} + R_m \sigma_s^m$$

$$\Gamma_\gamma = 0.030$$

$$\Delta E = \text{avg spacing between resonances} = \frac{E_n - E_1}{n-1}$$

$$E_1 = \text{energy of first resonance peak}$$

$$E_n = \text{energy of last resonance peak}$$

$$n = \text{number of resolved resonances}$$

$$\Delta E = \frac{310 - 22}{17} = 11.06 \text{ ev}$$

$$I' = 2.7 \times 10^{-3} (\sigma_s^{o2} + R_m \sigma_s^m) \int_{\beta_n}^{\infty} J(\xi', \beta) \frac{d\beta}{\beta}$$

$$\int J(\xi', \beta) \frac{d\beta}{\beta} = \int J(\xi', \beta) d(\ln \beta)$$

$$\ln \beta = (\ln 2) k - \ln(10^{-5})$$

$$d(\ln \beta) = (\ln 2) dk = 0.693 dk$$

$$I' = 3.9 \times 10^{-3} (\sigma_s^{o2} + R_m \sigma_s^m) \int_{k_n}^{\infty} J(\xi', k) dk$$

where k_n = the value of k corresponding to the last resolved resonance.

For the unresolved resonances, it was assumed that $\xi = 0.1$ at 300°K, and that it varied inversely as the square root of the absolute temperature.

$$\xi' = \frac{C}{\sqrt{T}}$$

$$C = \xi' \sqrt{T} = 0.1 \sqrt{300} = 1.732$$

$$\text{where } \xi' = \frac{1.732}{\sqrt{T}}$$

Summarizing

$$I = (\sigma_s^{o2} + R_m \sigma_s^m) \left(0.030 \sum_i \frac{J(\xi_i, b_i)}{E} + 3.9 \times 10^{-3} \int_{b_n}^{\infty} J(\xi', b) db \right)$$

$$\text{where } \xi_i = \left(\frac{820}{\sqrt{T}} \right) \left(\frac{0.030 + \Gamma_{ni}}{\sqrt{E_i}} \right)$$

$$b_i = 16.54 + 1.24 \times 10^{-6} E_i (\sigma_s^{o2} + R_m \sigma_s^m) \left(1 + \frac{0.030}{\Gamma_{ni}} \right)$$

$$\xi_i' = \frac{1.732}{\sqrt{T}}$$

Since ξ_i and ξ' are functions of temperature, the resonance integral I is also a function of temperature. Using the above equations, we plotted curves of the resonance integral versus temperature, for several different values of R_M (ratio of moderator to thorium) (See Figures 11-2 and Table 11-1).

By measuring their slope at the desired operating temperature, we determined the value of $\left(\frac{\partial I}{\partial T}\right)$, which is required to calculate the contribution of the resonance absorption to the temperature coefficient.

Summing up all contribution to the temperature coefficient, we obtain

$$\frac{\partial \rho}{\partial T} = \frac{1}{f} \frac{\partial f}{\partial T} - M^2 \frac{\partial B^2}{\partial T} - B^2 \frac{\partial M^2}{\partial T} + \frac{1}{\rho_{RC}} \frac{\partial \rho_{RC}}{\partial T} =$$

$$\begin{aligned} \frac{\partial \rho}{\partial T} = & - \left(\frac{1}{1 + \frac{P_s}{P_x} + \frac{1}{P_x} \frac{\Sigma_a^0}{\Sigma_a^{23}}} \right) \left(\frac{1}{\sigma_a^x} \frac{\partial \sigma_a^x}{\partial T} + \frac{1}{2T} \right) - \left(\frac{1}{\frac{P_x}{P_s} + 1 + \frac{1}{P_s} \frac{\Sigma_a^0}{\Sigma_a^{23}}} \right) \left(\frac{1}{\sigma_a^s} \frac{\partial \sigma_a^s}{\partial T} + \frac{1}{2T} \right) - \\ & - \left(4\alpha B^2 M^2 \right) - \frac{B^2 L^2}{2T} + \frac{D_m B^2}{\Sigma \Sigma_s^m T} - \frac{1}{\Sigma R_m \sigma_s^m} \left(\frac{\partial I}{\partial T} \right) \end{aligned}$$

TABLE 11-1

CONTRIBUTION OF RESONANCE PEAKS TO RESONANCE INTEGRAL (barns)

Resonance Energy	$\frac{N_M}{N_T} = 100$					$\frac{N_M}{N_T} = 200$					$\frac{N_M}{N_T} = 300$					
	E_1	300°K	600°K	900°K	1100°K	1300°K	300°K	600°K	900°K	1100°K	1300°K	300°K	600°K	900°K	1100°K	1300°K
	22.0	5.89	6.80	7.55	7.93	8.02	8.21	9.19	10.10	10.64	10.50	9.50	10.70	11.34	11.85	11.99
	23.7	6.75	7.75	8.50	9.03	9.36	9.74	11.20	12.10	12.31	13.38	11.70	13.37	14.35	14.71	15.38
	59.6	2.27	2.65	2.80	2.89	2.86	2.96	3.32	3.50	3.57	3.60	3.40	3.74	3.88	3.90	3.94
	70.1	2.70	3.06	3.32	3.40	3.67	3.98	4.60	6.48	5.10	5.44	5.02	5.70	6.20	6.51	6.75
	114.0	1.18	1.37	1.30	1.44	1.47	1.51	1.66	1.70	1.77	1.76	1.69	1.79	1.84	1.91	1.92
	123.0	1.36	1.61	1.74	1.79	1.83	1.91	2.19	2.42	2.38	2.38	2.20	2.55	2.66	2.72	2.79
	131.0	1.13	1.23	1.30	1.32	1.32	1.41	1.46	1.58	1.59	1.58	1.56	1.64	1.73	1.73	1.75
	152.0	1.02	1.11	1.16	1.19	1.21	1.27	1.35	1.43	1.46	1.47	1.43	1.51	1.57	1.60	1.63
	174.0	0.833	0.962	1.02	1.15	1.145	1.23	1.38	1.46	1.42	1.60	1.48	1.62	1.71	1.81	1.90
	195.0	0.755	0.865	0.945	0.945	1.000	1.00	1.11	1.17	1.17	1.23	1.14	1.25	1.32	1.32	1.36
	202.0	0.715	0.772	0.795	0.812	0.840	0.88	0.92	0.971	0.97	1.01	0.96	0.98	0.90	1.05	1.06
	215.0	0.148	0.188	0.193	0.193	0.195	0.19	0.19	0.202	2.02	0.21	0.20	0.20	0.20	0.20	0.20
	226.0	0.631	0.745	0.790	0.161	0.816	0.84	0.95	1.01	1.02	1.00	0.98	1.08	1.12	1.12	1.16
	235.0	0.093	0.096	0.098	0.097	0.098	0.11	0.11	0.112	0.11	0.11	0.12	0.12	0.15	0.11	0.14
	256.0	0.526	0.610	0.686	0.680	0.698	0.72	0.80	0.866	0.87	0.88	0.83	0.90	0.93	0.95	0.97
	269.0	0.462	0.501	0.530	0.565	0.575	0.57	0.61	0.619	0.64	0.65	0.58	0.65	0.69	0.69	0.68
	290.0	0.448	0.530	0.566	0.570	0.585	0.60	0.67	0.707	0.72	0.74	0.68	0.75	0.79	0.80	0.81
	310.0	0.385	0.439	0.460	0.495	0.511	0.53	0.59	0.618	0.66	0.68	0.62	0.67	0.71	0.74	0.74
Unresolved Resonances	22.4	22.8	24.2	25.0	25.3	25.2	25.4	25.6	25.6	26.0	24.0	25.6	25.6	25.6	25.6	25.6
Total Resonance Integral	49.71	54.09	57.97	59.66	61.50	63.26	67.7	72.65	74.02	74.22	68.09	74.82	77.69	79.34	80.77

Fig. 11.1. Values of $J(\xi, k)$ vs k .

Fig. 11.2. Effect of Temperature on Thorium 232 Resonance Integral (for Different Atomic Ratios of Graphite to Thorium).

11.10 Derivation of U^{234} , U^{235} , and U^{236} Buildup Equations

Assume

1) The 23 concentration is being held constant by conversion of Th^{232}

2) $\theta = \eta \nu = 3.4 \times 10^{13}$ neutrons/cm² sec

To find the N^{24}/N^{23} ratio the following procedure was used. The differential equation is:

$$\frac{dN^{24}}{dt} = N^{23} \sigma_c^{23} \phi - N^{24} \sigma_c^{24} \phi$$

$$dN^{24} = N^{23} \sigma_c^{23} \phi dt - N^{24} \sigma_c^{24} \phi dt$$

$$\frac{dN^{24}}{dt} + N^{24} \sigma_c^{24} \phi = N^{23} \sigma_c^{23} \phi$$

$$N^{24} e^{\int \sigma_c^{24} \phi dt} = \int e^{\int \sigma_c^{24} \phi dt} N^{23} \sigma_c^{23} \phi dt + C$$

$$N^{24} e^{\sigma_c^{24} \phi t} = \int e^{\sigma_c^{24} \phi t} N^{23} \sigma_c^{23} \phi dt + C$$

$$N^{24} e^{\sigma_c^{24} \phi t} = N^{23} \sigma_c^{23} \phi \frac{e^{\sigma_c^{24} \phi t}}{\sigma_c^{24} \phi} + C$$

Solving for the constant of integration c

$$N^{24} = \frac{N^{23} \sigma_c^{23}}{\sigma_c^{24}} + C e^{-\sigma_c^{24} \phi t}$$

$$C = N^{24} e^{\sigma_c^{24} \phi t} - \frac{N^{23} \sigma_c^{23}}{\sigma_c^{24}} e^{\sigma_c^{24} \phi t}$$

$$\textcircled{a} \quad t=0, \quad N^{24} = 0$$

Therefore

$$C = - \frac{N^{23} \sigma_c^{23}}{\sigma_c^{24}}$$

so

$$N^{24} = \frac{N^{23} \sigma_c^{23}}{\sigma_c^{24}} \frac{N^{23} \sigma_c^{23}}{\sigma_c^{24}} e^{-\sigma_c^{24} \phi t}$$

$$\frac{N^{24}}{N^{23}} = \frac{\sigma_c^{23}}{\sigma_c^{24}} \left(1 - e^{-\sigma_c^{24} \phi t} \right)$$

To find the N^{25} and N^{23} ratio the same procedure was followed. The initial equation is

$$\frac{dN^{25}}{dt} = N^{24} \sigma_a^{24} \phi - N^{25} \sigma_c^{25} \phi - N^{25} \sigma_f^{25} \phi$$

The solution to the differential equation is

$$\frac{N^{25}}{N^{23}} = \frac{\sigma_c^{23}}{\sigma_a^{25}} \left(1 - e^{-\sigma_a^{25} \phi t} \right) + \frac{\sigma_c^{23}}{(\sigma_a^{25} - \sigma_c^{24})} \left(e^{-\sigma_a^{25} \phi t} - e^{-\sigma_c^{24} \phi t} \right)$$

The same procedure was followed to obtain the N^{26} to N^{23} ratio. The initial equation is:

$$\frac{dN^{26}}{dt} = N^{25} \sigma_c^{25} \phi - N^{26} \sigma_a^{26} \phi$$

The solution to the differential equation is

$$\begin{aligned} \frac{N^{26}}{N^{23}} = & \frac{\sigma_c^{25} \sigma_c^{23}}{\sigma_a^{25} \sigma_a^{26}} \left(1 - e^{-\sigma_a^{26} \phi t} \right) + \frac{\sigma_c^{23} \sigma_c^{25}}{\sigma_a^{25} (\sigma_a^{26} - \sigma_a^{25})} \left(e^{-\sigma_a^{26} \phi t} - e^{-\sigma_a^{25} \phi t} \right) + \frac{\sigma_c^{23} \sigma_c^{25}}{(\sigma_a^{25} - \sigma_c^{24}) (\sigma_a^{26} - \sigma_a^{25})} \left(e^{-\sigma_a^{25} \phi t} - e^{-\sigma_c^{24} \phi t} \right) \\ & + \frac{\sigma_c^{23} \sigma_c^{25}}{(\sigma_a^{25} - \sigma_c^{24}) (\sigma_a^{26} - \sigma_a^{25})} \left(e^{-\sigma_a^{26} \phi t} - e^{-\sigma_c^{24} \phi t} \right) \end{aligned}$$

 Fig. 11.3. Build Up of U^{234} , U^{235} and U^{236} .

11.11 Thermal Stress and Minimum Allowable Thermal Conductivity in Impregnated Graphite Spherical Fuel Elements

Graphite is weakest in tension. The limiting thermal stress is the tangential stress at the surface of the sphere. From Reference (22), the tangential stress at the surface of a sphere assuming the temperature is a function of the radius only is given in Equation (11.11-1).

$$\sigma(R_f) = \frac{\beta E}{1 - \nu} \left[-\frac{3}{R_f^3} \int_0^{R_f} \Theta(R) R^2 dR - \Theta(R_f) \right] \quad (11.11-1)$$

where

$\sigma(R_f)$ = Tensile stress at spheres surface - lb/in²

β = Coefficient of thermal expansion - in/in-°F

E = Young's modulus - lb/in²

ν = Poisson's ratio

$\Theta(R)$ = Temperature at radius R - °F

R_f = Radius of sphere -ft

R = Variable radius - ft

The temperature distribution in a sphere with uniform heat generation and constant thermal conductivity is given in Equation (11.11-2).

$$\Theta(R) = \frac{P_D}{6K} \left(R_f^2 - R^2 \right) + \Theta(R_f) \quad (11.11-2)$$

where K - Thermal conductivity - BTU/hr ft °F

P_D - Power density - BTU/hr ft³

Substituting the temperature distribution given in Equation (11.11-2) into Equation (11.11-1) and integrating to obtain the thermal stress at the sphere's surface.

$$(11.11-3) \quad \sigma(R_f) = \frac{\beta E R_f^2 P_D}{15 K (1 - \nu)} \left[\frac{\left(\frac{1 \text{ in}}{1 \text{ in } ^\circ \text{F}} \right) \left(\frac{1 \text{ lb}}{1 \text{ in } ^2} \right) \left(\frac{\text{ft}^2}{\text{hr ft}^3} \right) \left(\frac{\text{BTU}}{\text{hr ft } ^\circ \text{F}} \right)}{\left(\frac{\text{BTU}}{\text{hr ft } ^\circ \text{F}} \right)} \right]$$

or as a function of sphere diameter (D_f -ft)

$$\sigma(D_f) = \frac{\beta E D_f^2 P_D}{60 K (1 - \nu)} \quad (11.11-4)$$

Solving for the minimum allowable value of thermal conductivity from Equation (11.11-4), in terms of maximum power density ($P_{D_{\max}}$) and maximum allowable tensile stress ($\sigma_{\max \text{ allow}}$), gives

$$K_{\min \text{ allow}} = \frac{\beta E D_f^2 P_{D_{\max}}}{60 (1 - \nu) \sigma_{\max \text{ allow}}} \quad (11.11-5)$$

Express $P_{D_{\max}}$ in terms of P (reactor thermal power)

$$P_{D_{\max}} = \frac{3.413 \cdot P \cdot \xi}{V_{R_c} \propto} \quad (11.11-6)$$

where P - Reactor thermal power - watts

3.413 - electrical/thermal conversion/factor - $\frac{\text{BTU}}{\text{Watts-hr}}$

V_{Rc} - Reactor core volume - ft^3

α - Solid fraction of reactor core

ξ - Ratio of maximum thermal power to average thermal power in the reactor core

Substituting the expression for $P_{D_{max}}$ given in Equation (11.11-6) into Equation (11.11-5)

$$K_{min_{allow}} = \frac{\beta E D_f^2}{60(1-\nu) T_{max_{allow}}} \left(\frac{3.413 P \xi}{V_{Rc} \alpha} \right) \quad (11.11-7)$$

Dimensional check

$$\frac{BTU}{hr \ ft \ ^\circ F} = \frac{\left(\frac{in}{in^{\circ F}}\right) \left(\frac{lb}{in^2}\right) (ft^2)}{\left(\frac{lb}{in^2}\right)} \frac{\left(\frac{BTU}{Watt \ hr}\right) (watts)}{(ft)}$$

$$(11.11-8) \quad K_{min_{allow}} = \left(\frac{0.0569}{(1-\nu)} \right) \left(\frac{\beta E}{T_{max_{allow}}} \right) \left(\frac{\xi}{\alpha} \right) \left(\frac{P}{V_{Rc}} \right) D_f^2$$

Let R_K be defined as the ratio of the thermal conductivity of an irradiated graphite fuel element to the thermal conductivity of unirradiated graphite.

We may now obtain an expression for the maximum allowable diameter of the graphite spherical fuel element. This expression implies that the thermal conductivity of the irradiated graphite has been reduced to R_K times its unirradiated value.

$$D_F^2 = \frac{R_k (K_{min\ allow})_{unrad} (1-\nu)}{0.0569 \left(\frac{\beta E}{\sigma_{max\ allow}} \right) \left(\frac{\xi}{\alpha} \right) \left(\frac{P}{V_{Rc}} \right)} \quad (11.11-9)$$

or

$$d_F = 50.3 \left[\frac{R_k (K_{min\ allow})_{unrad} (1-\nu)}{\left(\frac{\beta E}{\sigma_{max\ allow}} \right) \left(\frac{\xi}{\alpha} \right) \left(\frac{P}{V_{Rc}} \right)} \right]^{0.5} \quad (11.11-10)$$

where

$$D_F (ft) = \frac{d_F (in)}{12 \left(\frac{in}{ft} \right)}$$

Reducing Equation (11.11-9) by applying reactor design data

$$V_{Rc} = 4440 \text{ ft}^3$$

$$\alpha = 0.63$$

$$\xi = 2.72$$

we find that

$$d_F = 1.61 \times 10^3 \left[\frac{(K_{min\ allow})_{unrad} (1-\nu)}{\left(\frac{\beta E}{\sigma_{max\ allow}} \right)} \right]^{0.5} \left[\frac{R_k}{P} \right]^{0.5} \quad (11.11-11)$$

The expression in the first bracket in Equation (11.11-11) is dependent upon the physical properties of graphite. The physical properties of graphite were obtained from Reference (33). The values given for perpendicular to extrusion axis were used since they result in a more conservative design than the values given for parallel to extrusion axis.

$$V = 0.25$$

$$K = 20 \frac{\text{Btu}}{\text{ft}^2 \text{ } ^\circ\text{F hr}}$$

$$\beta = 5.0 \times 10^{-6} \frac{\text{in}}{\text{in } ^\circ\text{F}}$$

$$E = 0.6 \times 10^6 \frac{\text{lb}}{\text{in}^2}$$

$$\sigma_{\text{mat allow}} = 800 \frac{\text{lb}}{\text{in}^2}$$

Substituting the preceding values into Equation (11.11-11), we obtain the graphite spherical fuel element diameter (d_F) as a function of reactor thermal power (P) and the parameter R_K .

$$(11.11-12) \quad d_F = 1.61 \times 10^3 \left[\frac{20 (1 - 0.25)}{\left(\frac{5.0 \times 10^{-6} \times 0.6 \times 10^6}{800} \right)} \right]^{0.5} \left[\frac{R_K}{P} \right]^{0.5}$$

or

$$(11.11-13) \quad d_F = 1.01 \times 10^5 \left[\frac{R_K}{P} \right]^{0.5}$$

11.12 Core Temperature Distribution

Based on the flux distribution plots, the power generation in the radial direction was approximated as shown in the figure. That is

$$P(x) = P_{max} \sin\left(\frac{\pi x}{2a}\right)$$

or in terms of r , the distance from the center line

$$P(r) = P_{max} \sin \frac{\pi (a-r)}{2a}$$

where a = the distance of the flux peak from the outside of the reflector.

The radial temperature gradient of the gas across a small shell of the core, assuming the flow is metered to match the axial power distribution so as to eliminate any axial temperature gradient, is defined by:

$$\frac{d\theta_a}{dr} = C 2 \pi r \sin \frac{\pi (a-r)}{2a}$$

where $c =$ a constant

Integrating between the known temperature limits:

$$(\theta_2 - \theta_1) = c_1 \int_{r=8}^{r=1.25} r \sin\left(\frac{\pi(9-r)}{2a}\right) dr$$

since

$$r = (9 - x)$$

$$dr = -dx$$

$$(\theta_2 - \theta_1) = c_1 \int_1^{7.75} (9-x) \sin\left(\frac{\pi x}{2a}\right) dx$$

Let

$$y = \frac{\pi x}{2a}$$

$$x = \frac{2ay}{\pi}$$

$$dx = \frac{2a dy}{\pi}$$

$$\frac{\theta_2 - \theta_1}{c_1} = \int_{y=\frac{.5\pi}{a}}^{y=3.88\frac{\pi}{a}} \sin(y) \left(9 - \frac{2ay}{\pi}\right) \left(\frac{2a}{\pi}\right) dy$$

$$= \left(\frac{18a}{\pi} \int \sin y dy - \frac{4a^2}{\pi^2} \int y \sin y dy \right) \Big|_{\frac{.5\pi}{a}}^{3.88\frac{\pi}{a}}$$

$$\frac{\theta_2 - \theta_1}{c_1} = -\frac{18a}{\pi} \cos y - \frac{4a^2}{\pi^2} \sin y + \frac{4a^2}{\pi^2} y \cos y \Big|_{\frac{.5\pi}{a}}^{3.88\frac{\pi}{a}}$$

On the basis of nuclear calculations, the flux peak was estimated at a distance of 6.25 ft from the outside of the reflector.

But $\bar{a} = 6.25$

$$y = .251x$$

Substituting into the above expression, we obtain

$$\frac{\theta_2 - \theta_1}{C_1} = 21.50$$

and $C_1 = .0465(\theta_2 - \theta_1)$

so that

$$\frac{\theta_6 - \theta_1}{\theta_2 - \theta_1} = .0465 \left\{ 35.9 \cos(.251x) - 15.80(\sin(.251x) + .251x \cos(.251x)) + 34.91 \right\}$$

or

$$\frac{\theta_6 - \theta_1}{\theta_2 - \theta_1} = .735 \left\{ -2.27 \cos(.251x) - \sin(.251x) + .251x \cos(.251x) + 2.20 \right\}$$

In terms of r,

$$\left(\frac{\theta_6 - \theta_1}{\theta_2 - \theta_1} \right) = .735 \left\{ [.251(9-r) - 2.27] \cos(.251)(9-r) - \sin(.251)(9-r) + 2.20 \right\}$$

Since

$$\begin{aligned} \theta_1 &= 341^\circ \text{F} \\ \theta_2 &= 1600^\circ \text{F} \end{aligned}$$

$$\theta_G = 341 + 925 \left\{ 2.20 - .251 P \cos(.251)(9-r) - \sin(.251)(9-r) \right\}$$

11.12.1 Film Temperature Drop

The heat transfer coefficient through the core is found from the correlation given in HDC-35, i.e.,

$$h = \frac{.58 C_p G}{\left(\frac{G d_F}{\mu} \right)^{.3}}$$

$$= \frac{.58 C_p \mu^{.3} G^{.7}}{d_F^{.3}}$$

But $G = \frac{W}{A}$

so $h = \frac{(.58 C_p \mu^{.3})}{d_F^{.3}} \left(\frac{W}{A} \right)^{.7}$

The area in the radial direction per unit height is:

$$A = 2\pi r dz$$

and

$$W = \frac{f_a Q (2\pi) (r_2^2 - r_1^2) dz}{C_p (\theta_2 - \theta_1)}$$

at the midpoint of the core

But

$$Q = \frac{(3.413)(P) dz}{\pi (r_2^2 - r_1^2) H_c}$$

so

$$\frac{W}{A} = \frac{(3.413)(f_a P)}{2\pi H_c C_p r (\theta_2 - \theta_1)}$$

Hence

$$h = \frac{(.58)(C_p)(\mu^{.3})}{d_F^{'.3}} \left(\frac{3.413 f_a P}{2\pi H_c C_p r (\theta_2 - \theta_1)} \right)^{.7}$$

$$= \frac{.0205}{d_F^{'.3}} \left(\frac{f_a P}{\theta_2 - \theta_1} \right)^{.7}$$

or, since $f_a = 1.5$

$$h = \frac{.0575 P^{.7}}{d_F^{'.3} (\theta_2 - \theta_1)^{.7} r^{.7}}$$

where d_F is now in inches. The film temperature drop at any point r , is

$$(\Delta T)_G = \frac{(P_r)(3.413)}{A_F h}$$

where

$$A_F = \frac{(35.7)(D_c^2 - D_m^2)(H_c)}{d_F}$$

also

$$P_r = P_m \sin .251(9-r)$$

or, since

$$P_m = f_a f_r P = 2.81 P$$

$$P_r = (2.81)(P) \sin (.251)(9-r)$$

Therefore,

$$\Delta T_G = \frac{(3.413)(2.81)(P) d_F^{.3} r^{.7} (\theta_2 - \theta_1)^{.7}}{(35.7)(.0575)(H_c)(D_c^2 - D_m^2)} \sin(.251)(9-r)$$

since $H_c = 24'$, $(\theta_2 - \theta_1) = 1258$, $D_R = 16$, $D_o = 2.5$

$$(\Delta T)_G = .114 P^{.3} d_F^{1.3} r^{.7} \sin .251 (9-r)$$

For

$$d_F = 1'' \quad , \quad P = 1.123 \times 10^9$$

$$(\Delta T)_G = 59 r^{.7} \sin .251 (9-r)$$

The maximum fuel temperature will be in the center. The difference in temperature between the surface and the maximum is

$$\Delta \theta_F = \frac{P_a}{6 K_F} \left(\frac{d_F}{12} \right)^2$$

The power density in the ball at any point in the core midjoint is

$$P_D = \frac{(2.81)(P)(3.413)}{(.785)(D_R^2 - D_o^2)(H_c)(.63)} \sin .251 (9-r)$$

Substituting

$$= .0081 P \sin (.251)(9-r)$$

So that

$$\Delta \theta_F = \frac{(.0081)(P)(d_F^2)}{(6)(144)(12)(K_F)} \sin (.251)(9-r)$$

and

$$\Delta \theta_F = \frac{4400 d_F^2}{K_F} \sin (.251)(9-r)$$

where the thermal conductivity, K_F , is so inconsistent that it was left as a variable. Assuming a value of 5 BTU/°F-hr-ft and for $d_F = 1''$.

Assuming a value for k_F of 5 BTU/ $^{\circ}$ F-hr-ft

$$\Delta\theta_F = 880 \sin(.251)(9-r)$$

Thus the surface temperature of the fuel is

$$\theta_s = \theta_G = \Delta T_G$$

The maximum internal fuel temperature is therefore

$$\theta_{\max} = \theta_G + \Delta T_G + \Delta \theta_F$$

The ball maximum temperature, surface temperature and gas temperature are shown in Figure 11-4 as a function of distance from the face of the core. Figure 11-5 shows the film ΔT and ball ΔT as a function of distance from the face of the core.

Fig. 11.4. Ball Maximum Temperature, Surface Temperature and Gas Temperature vs Distance from Face of Core.

Fig. 11.5. Film ΔT and Ball ΔT vs Distance from Face of Core.

CHAPTER 12

APPENDIX C

Appendix C includes tabulation of data used in previous chapters of this report.

12.1 Thermal Cross-Section Data*

$$\sigma_{AT}^{23} = 585 \text{ b}$$

$$\sigma_{AT}^{23} \text{ over M-B around } 0.1 \text{ ev} = 274.6 \text{ b.}$$

$$\sigma_{FT}^{23} = 533 \text{ b}$$

$$\lambda = 2.54$$

$$\eta = 2.31$$

$$\sigma_{AT}^G = 3.2 \text{ mb}$$

$$\sigma_{TT}^G = 4.8 \text{ b}$$

$$\sigma_{aT}^{02} = 7.05 \text{ b}$$

$$\sigma_{AT}^{02} = \text{over M-B around } 0.1 \text{ ev} = 3.26 \text{ b}$$

$$\sigma_{TT}^{02} = 19.5 \text{ b}$$

* See Reference 35

12.2 Eyewash Cross-Section Data

Group	Δu	Energy	Carbon			Fission Spectrum	σ_a^{23}
			$\Sigma \sigma_T$	$\bar{\sigma}_a$	$3 \sigma_{TR}$		
0	--	10^7	.159	--	--	--	--
1	.5	6.06×10^6	.167	.0000	2.046	.0210	2.50
2		3.68×10^6	.397		3.27	.1023	1.80
3		2.23×10^6	.250		3.56	.2070	1.82
4		1.35×10^6	.327		3.98	.2316	1.89
5		$.821 \times 10^6$	.420		5.53	.1835	2.00
6	1.5	$.183 \times 10^6$	.653		9.34	.2195	2.13
7	3.0	9120	.739		12.91	.0351	3.41
8	3.0	454		.0000	13.17		13.2
9	1.4	112		2			36.3
10	1.2	33.7		4			74.8
11	.8	15.2		5			103.4
12	.4	10.2		6			115.5
13	.8	4.56		7			99.0
14	1.2	1.38		8			208
15	.4	.921		09			190
16		.618		10			139
17		.414		11	13.17		143
18		.278		13	13.16		161
19	.2	.227		16			188
20		.186		17			208
21		.152		19			229
22		.125		20			249
23		.102	.739	.0023	13.16		277

12.3 Cross-Sections for OCUSOL-A Program*

Group No.	Energy (ev)	Δu	Barns	Barns	Barns
			σ_a^{23}	σ_f^{23}	σ_a^{fp}
0	10^7	--	--	--	--
1	6.06×10^6	.5	2.5	2.5	.043
2	3.68×10^6	.5	1.8	1.8	.050
3	2.23×10^6	.5	1.85	1.85	.060
4	1.35×10^6	.5	1.9	1.9	.072
5	8.21×10^5	.5	1.95	1.95	.09
6	3.88×10^5	.75	2.05	2.05	.10
7	1.83×10^5	.75	2.28	2.2	.14
8	6730	1.0	2.8	2.5	.19
9	2480	1.0	3.2	2.8	.29
10	9120	1.0	4.9	4.3	.38
11	3360	1.0	7.9	6.9	.74
12	1230	1.0	12	10.5	1.6
13	454	1.0	21.2	18.6	3.3
14	320	.35	28.5	25	5.4
15	226	.35	33.1	29	6.9
16	112	.7	44.5	39	9.5
17	61.5	.6	66.1	58	28
18	33.7	.6	84.4	74	28
19	15.2	.8	111	94	19
20	10.2	.4	124	105	20
21	4.56	.8	109	90	52
22	1.38	1.2	229	189	38
23	.921	.4	209	173	30
24	.672	.4	139	126	35
25	.414	.4	137	125	35
26	.278	.4	157	142	23

* See Reference 5

12.4 Heat Balance Data*

N_{FW}	0	1	2	3	4	5	6
θ_{10}	563.6	627.0	682.8	761.9	829.7	881.3	926.5
H_{10}	76.1	138.9	194.7	273.8	343.6	399.5	449.2
η_{pp}	0.3551	0.3693	0.3801	0.3915	0.3985	0.4023	0.4046

* See Reference 2

CHAPTER 13

APPENDIX D

13.1 Original Set of Equations

The following equations represent the reactor system as proposed in this report. All equations are developed in the Appendices and the basis for the costs are outlined in Chapter 8.

Original Set of Equations

(Basis for ORACLE Code)

Assume

$$N_{FW} \longrightarrow (H_{10}, \theta_{10}, \gamma_{pp})$$

$$x, b, y, \phi_v, p, W_F, f_F, R_K, D_{PS}, D_{PC}$$

$$\theta_2, \theta_3, \theta_8, d_x, L_x, D_x, L_e, L_B, L_S \left(\frac{P_{eo}}{P} \right) \left(\frac{P_{Bo}}{P} \right) \left(\frac{P_{so}}{P} \right)$$

(1) Instruct computer to try different values for P, until $(3.98 \times 10^8) < P_{NE} <$

$$(4.02 \times 10^8)$$

(2) Calculate ψ_1

$$\psi_1 = \frac{1492.7 - H_{10}}{885.6}$$

(3) Calculate ψ_2

$$\psi_2 = 1.948 \frac{P^{0.2} d_x^{1.2} 1^{0.8}}{\alpha^{0.2} D_x^{0.4} L_x}$$

(4) Calculate θ_6 by successive approximations, until $|\theta_6^n - \theta_6^{n-1}| < 1$

$$\theta_6' = \theta_3 - \psi_2 (\theta_3 - 75 - \theta_8)^{0.8}$$

$$\theta_6'' = \theta_3 - \psi_2 (\theta_6' - \theta_8)^{0.8}$$

$$\theta_6 = \theta_3 - \psi_2 (\theta_6'' - \theta_8)^{0.8}$$

(5) Calculate θ_7 :

$$\theta_7 = 0.6355 \theta_6 + 0.3645 \theta_8$$

(6) Calculate θ_5 :

$$\theta_5 = \psi_1 \theta_8 - (\psi_1 - 1) \theta_6$$

(7) Calculate θ_1 : $w_H = w_C$

$$\theta_1 = \theta_3 + \theta_5 - \theta_6$$

(8) Calculate $\Delta\theta_e$:

$$\Delta\theta_e = \frac{\theta_8 + \theta_{10} - \theta_5 - 1053}{\ln\left(\frac{\theta_8 - 1053}{\theta_5 - \theta_{10}}\right)}$$

$$(9) \Delta\theta_B = \frac{\theta_7 - \theta_8}{\ln\left(\frac{\theta_7 - 1053}{\theta_8 - 1053}\right)}$$

$$(10) \Delta\theta_s = \frac{\theta_6 - \theta_7 - 407}{\ln\left(\frac{\theta_6 - 1460}{\theta_7 - 1053}\right)}$$

$$(11) \psi_4 = 13 \left(\frac{607.1 - H_{10}}{1492.7 - H_{10}} \right) \left[0.164 (1492.7 - H_{10})^{0.8} + 0.136 (\theta_3 - \theta_1)^{0.8} \right]$$

$$(12) d_e = \left[7.72 \times 10^{-8} \frac{\theta_5 (\theta_5 + \theta_8) (\Delta\theta_e)^{8.75} L_e^{9.75}}{(\theta_3 - \theta_1)^{2.75} p^2 \psi_4^{8.75} \left(\frac{p_{eo}}{p} \right)} \right]^{0.0851}$$

$$(13) \quad \psi_3 = \frac{562.8}{1492.7 - H_{10}}$$

$$(14) \quad d_B = \left[3.00 \times 10^{-10} \frac{\theta_5 (\theta_7 + \theta_8) (\Delta \theta_B)^{8.75} L_B^{9.75}}{(\theta_3 - \theta_1)^{9.75} p^2 \psi_3^{8.75} \left(\frac{P_{Bo}}{P} \right)} \right]^{0.0851}$$

$$(15) \quad \psi_5 = \left(\frac{322.8}{1492.7 - H_{10}} \right) \left[0.1246 (\theta_3 - \theta_1)^{0.8} + 0.421 (1492 - H_{10})^{0.8} \right]$$

$$(16) \quad d_s = \left[1.45 \times 10^{-17} \frac{\theta_5 (\theta_6 + \theta_7) (\Delta \theta_s)^{8.75} L_s^{9.75}}{(\theta_3 - \theta_1)^{2.75} p^2 \psi_5^{8.75} \left(\frac{P_{so}}{P} \right)} \right]^{0.0851}$$

$$(17) \quad d_F = 1.01 \times 10^5 \left(\frac{R_K}{P} \right)^{0.5}$$

$$(18) \quad \left(\frac{P_R}{P} \right) = 1.20 \times 10^{-13} \frac{p^2 \theta_1 (\theta_1 + \theta_2)}{p^2 d_F (\theta_2 - \theta_1)^2 (\theta_3 - \theta_1)}$$

$$(19) \quad \left(\frac{P_{xi}}{P} \right) = 2.84 \times 10^{-9} \frac{p^{1.75} L_x \theta_1 (\theta_1 + \theta_3)}{p^2 D_x^{3.5} d_x^{1.25} \chi^{1.75} (\theta_3 - \theta_1)^{2.75}}$$

$$(20) \quad \left(\frac{P_{xo}}{P} \right) = 7.29 \times 10^{-10} \frac{p^{1.75} L_x \theta_5 (\theta_5 + \theta_6)}{p^2 D_x^{3.5} d_x^{1.25} \chi^{1.75} (\theta_3 - \theta_1)^{2.75}}$$

$$(21) \quad \left(\frac{P_{PS}}{P} \right) = 1.83 \times 10^{-9} \frac{p^{1.75} (D_{PS} + 0.190) \theta_5 (\theta_3 + \theta_6)}{p^2 D_{PS}^{4.75} \chi^{1.75} (\theta_3 - \theta_1)^{2.75}}$$

$$(22) \quad \left(\frac{P_{PC}}{P} \right) = 6.34 \times 10^{-10} \frac{p^{1.75} (D_{PC} + 0.692) \theta_1 (\theta_1 + \theta_5)}{p^2 D_{PC}^{4.75} \chi^{1.75} (\theta_3 - \theta_1)^{2.75}}$$

$$(23) \left(\frac{P_L}{P} \right) = \left[\left(\frac{P_R}{P} \right) + \left(\frac{P_{xi}}{P} \right) + \left(\frac{P_{xo}}{P} \right) + \left(\frac{P_{ps}}{P} \right) + \left(\frac{P_{pc}}{P} \right) \right] + \left(\frac{P_{eo}}{P} \right) + \left(\frac{P_{Bo}}{P} \right) + \left(\frac{P_{so}}{P} \right) + .01 \eta_{pp}$$

$$(24) P_{NE} = P \left[\eta_{pp} - \left(\frac{P_L}{P} \right) (1 - \eta_{pp}) \right]$$

$$(25) S_{xi} = 7.75 \frac{D_x^2 L_x}{d_x}$$

$$(26) S_{ei} = 0.262 \frac{P d_e^5 \psi_4^5}{\gamma L_e^4 (\Delta \theta_e)^5}$$

$$(27) S_{Bi} = 6.44 \frac{P d_B^5 \psi_3^5 (\theta_3 - \theta_1)^4}{b L_B^4 (\Delta \theta_B)^5}$$

$$(28) S_{si} = 9.75 \times 10^4 \frac{P d_s^5 \psi_5^5}{b L_s^4 (\Delta \theta_s)^5}$$

$$(29) Q_H = 2.04 \times 10^{-3} \frac{P \theta_1}{P(\theta_3 - \theta_1)}$$

$$(30) Q_C = 2.04 \times 10^{-3} \frac{P \theta_5}{P(\theta_3 - \theta_1)}$$

$$(31) D_e = 0.219 \left[\frac{P d_e^6 \psi_4^5}{\gamma L_e^5 (\Delta \theta_e)^5} \right]^{0.5}$$

$$(32) D_B = 1.075 \left[\frac{P d_B^6 \psi_3^5 (\theta_3 - \theta_1)^4}{b L_B^5 (\Delta \theta_B)^5} \right]^{0.5}$$

$$(33) D_S = 133 \left[\frac{P d_s^6 \psi_5^5}{b L_s^5 (\Delta \theta_s)^5} \right]^{0.5}$$

$$(34) \quad \Delta p_H = 2.01 \frac{p (\theta_3 - \theta_1)}{\theta_1} \left[\left(\frac{P_R}{P} \right) + \left(\frac{P_{xi}}{P} \right) + \left(\frac{P_{PC}}{P} \right) \right]$$

$$(35) \quad \Delta p_c = 2.01 \frac{p (\theta_3 - \theta_1)}{\theta_5} \left[\left(\frac{P_{xo}}{P} \right) + \left(\frac{P_{eo}}{P} \right) + \left(\frac{P_{Bo}}{P} \right) + \left(\frac{P_{So}}{P} \right) + \left(\frac{P_{PS}}{P} \right) \right]$$

$$(36) \quad \$_{36} = 492 \left(0.85 + \frac{p}{1000} \right) \propto y^{0.438} s_{xi}^{0.562}$$

$$(37) \quad \$_{37} = 214 \propto \left(0.85 + \frac{p}{1000} \right) s_{ei}^{0.562}$$

$$(38) \quad \$_{38} = 770 b \left(0.85 + \frac{p}{1000} \right) s_{Bi}^{0.562}$$

$$(39) \quad \$_{39} = 770 b \left(0.85 + \frac{p}{1000} \right) s_{Si}^{0.562}$$

$$(40) \quad \$_{40} = 1.59 \times 10^5 \propto D_{PS}^{1.72}$$

$$(41) \quad \$_{41} = 4.48 \times 10^4 \propto D_{PC}^{1.72}$$

$$(42) \quad \$_{42} = 629 (\Delta p_H)^{0.746} \propto Q_H^{0.614}$$

$$(43) \quad \$_{43} = 629 (\Delta p_c)^{0.746} \propto Q_c^{0.614}$$

$$(44) \quad F_N = 1.575 \left[\$_{36} + \$_{37} + \$_{38} + \$_{39} + \$_{40} + \$_{41} + \$_{42} + \$_{43} + \$_V + 11.80 \times 10^6 \right]$$

$$(45) \quad F_{PP} = \left[\frac{21.24 + 0.230 N_{FW}}{1492.7 - H_{10}} + 0.0536 \gamma_{pp} \right] P$$

$$(46) \quad I = 1.12 (F_N + F_{PP}) + 0.14 (\gamma_{pp} P)^{0.80}$$

$$(47) \quad C_{co} = 0.168 (F_N + F_{PP}) + 0.307 (\gamma_{pp} P)^{0.80}$$

$$(48) \quad f_F = \left[\frac{4 \times 10^{-5} P}{9.10 \times 10^4 800 W_F} \right]^{0.5}$$

$$(49) \quad C_{MI} = \left[800 W_F + 9.10 \times 10^4 \right] (1.05 + f_F)$$

$$(50) \quad C_F = \left[5 \times 10^{-7} W_F \frac{4 \times 10^{-5}}{f_F} + 7.56 \times 10^{-4} \right] (P)$$

$$(51) \quad M = 142.9 \left(\frac{C_{co} + C_{MI} + C_F}{P_{NE}} \right)$$

13.2 List of Revised Equations

Choose input values for:

$N_{FW}, \theta_1, \theta_2, \theta_3, \theta_8, x, y, b, d_x, d_e, d_B, d_s, D_B, d_F, p$ *

Choice of N_{FW} automatically determines values of θ_{10}, H_{10} , and γ_{pp}

$$(1) \quad \theta_6 = \theta_8 + \left(\frac{885.6}{1492.7 - H_{10}} \right) (\theta_3 - \theta_1)$$

$$(2) \quad \theta_7 = 0.6355 \theta_6 + 0.3645 \theta_8$$

$$(3) \quad \theta_5 = \theta_6 + \theta_1 - \theta_3$$

* Instruct computer to carry out the following procedure with regard to P:

First trial: Let $P' = 4.40 \times 10^8$; let $P_{NE}' \equiv$ corresponding value of P_{NE} obtained

Second trial: Let $P'' = 4.00 \times 10^8 \frac{P'}{P_{NE}'}$; let $P_{NE}'' \equiv$ " " " " "

Third trial: Let $P''' = 4.00 \times 10^8 \frac{P''}{P_{NE}''}$; let $P_{NE}''' \equiv$ " " " " "

Continue until: $(3.99 \times 10^8) < P_{NE}^n < 4.01 \times 10^8$

Print results of final trial only.

$$(4) \Delta \theta_e = \frac{\theta_8 + \theta_{10} - \theta_5 - 1053}{\ln \left(\frac{\theta_8 - 1053}{\theta_5 - \theta_{10}} \right)}$$

$$(5) \Delta \theta_B = \frac{\theta_7 - \theta_8}{\ln \left(\frac{\theta_7 - 1053}{\theta_8 - 1053} \right)}$$

$$(6) \Delta \theta_s = \frac{\theta_6 - \theta_7 - 407}{\ln \left(\frac{\theta_6 - 1460}{\theta_7 - 1053} \right)}$$

$$(7) L_x = 3.22 \left[\frac{p^{0.2} d_x^{1.2} (\theta_3 - \theta_1)^{0.8} y^{0.4}}{\chi^{0.2} (\theta_3 - \theta_6)} \right]^{0.714}$$

$$(8) \psi_4 = \left(\frac{607.1 - H_{10}}{1492.7 - H_{10}} \right) \left[2.13 (1492.7 - H_{10})^{0.8} + 2.21 (\theta_3 - \theta_1)^{0.8} + 4.50 \times 10^{-6} \left(\frac{p d_e}{\chi} \right)^{0.8} \right]$$

$$(9) L_e = 1.173 \left[\frac{p d_e^6 \psi_4^5}{\chi (\Delta \theta_e)^5} \right]^{0.143}$$

$$(10) h_{Bo}' = \left[8.33 \left(\frac{D_B^{1.6} d_B^{0.2} b^{0.8} (\theta_3 - \theta_1)^{0.8}}{p^{0.8}} \right) + 4.23 \times 10^{-3} d_B \right]^{-1}$$

$$(11) A_o = 0.671 h_{Bo}'$$

$$A_L = 0.605 h_{Bo}'$$

$$C_o = 1.006 h_{Bo}' (\theta_8 - 1053)$$

$$C_L = 1.006 h_{Bo}' (\theta_7 - 1053)$$

$$(12) \psi_o = \left[-c_o + \sqrt{c_o^2 + A_o^3} \right]^{1/3} + \left[-c_o - \sqrt{c_o^2 + A_o^3} \right]^{1/3}$$

$$(13) \psi_L = \left[-c_L + \sqrt{c_L^2 + A_L^2} \right]^{1/3} + \left[-c_L - \sqrt{c_L^2 + A_L^2} \right]^{1/3}$$

$$(14) L_B = \left[\frac{P d_B}{b D_B^2 (\theta_3 - \theta_1)} \right] \left[0.1368 (\psi_o^{-2.035} - \psi_L^{-2.035}) + \left(\frac{0.378}{h_{Bo}} \right) \ln \left(\frac{\psi_L}{\psi_o} \right) \right]$$

$$(15) \psi_5 = \left(\frac{322.8}{1492.7 - H_{10}} \right) \left[0.156 (\theta_3 - \theta_1)^{0.8} + 0.421 (1492 - H_{10})^{0.8} + 8.45 \times 10^{-6} \left(\frac{P d_s}{b} \right)^{0.8} \right]$$

$$(16) L_s = 7.31 \left[\frac{P d_s^6 \psi_5^5}{b (\Delta \theta_5)^5} \right]$$

$$(17) \Delta P_T = 5.34 \times 10^{-12} \left[\frac{P^2 \theta_2}{p(\theta_3 - \theta_1)^2} \right] + 1.10 \times 10^{-8} \left[\frac{P^2}{p d_F (\theta_2 - \theta_1)^2} \right]$$

$$(18) \Delta P_{xi} = 8.24 \times 10^{-6} \left[\frac{P^{1.75} y^{3.5} (\theta_1 + \theta_3)}{\chi^{1.75} p d_x^{1.25} L_x^{2.5} (\theta_3 - \theta_1)^{1.75}} \right]$$

$$(19) \Delta P_{xo} = 2.115 \times 10^{-6} \left[\frac{P^{1.75} y^{3.5} (\theta_5 + \theta_6)}{\chi^{1.75} p d_x^{1.25} L_x^{2.5} (\theta_3 - \theta_1)^{1.75}} \right]$$

$$(20) \Delta P_{eo} = 1.093 \times 10^{-6} \left[\frac{P^{1.75} (\theta_5 + \theta_8)}{\chi^{1.75} p d_e^{1.25} L_e^{2.5} (\theta_3 - \theta_1)^{1.75}} \right]$$

$$(21) \Delta P_{Bo} = 7.85 \times 10^{-10} \left[\frac{P^{1.75} (\theta_7 + \theta_8) L_B}{b^{1.75} p d_B^{1.25} D_B^{3.5} (\theta_3 - \theta_1)^{1.75}} \right]$$

$$(22) \Delta p_{so} = 1.006 \times 10^{-6} \left[\frac{p^{1.75} (\theta_6 + \theta_7)}{b^{1.75} p d_s^{1.25} L_s^{2.5} (\theta_3 - \theta_1)^{1.75}} \right]$$

$$(23) \Delta p_{PS} = 1.75 \times 10^{-11} \left[\frac{p^{1.75} (\theta_3 + \theta_6)(211 D_{PS} + 40)}{\chi^{1.75} p D_{PS}^{4.75} (\theta_3 - \theta_1)^{1.75}} \right]$$

$$(24) \Delta p_{PC} = 1.58 \times 10^{-11} \left[\frac{p^{1.75} (\theta_1 + \theta_5)(81 D_{PC} + 56)}{\chi^{1.75} p D_{PC}^{4.75} (\theta_3 - \theta_1)^{1.75}} \right]$$

$$(25) \Delta p_H = \Delta p_T + \Delta p_{xi} + 0.5 \Delta p_{PS} + 0.5 \Delta p_{PC}$$

$$(26) \Delta p_c = \Delta p_{xo} + \Delta p_{eo} + \Delta p_{Bo} + \Delta p_{so} + 0.5 \Delta p_{PS} + 0.5 \Delta p_{PC}$$

$$(27) \left(\frac{P_L}{P} \right) = 0.498 \left[\frac{\theta_1 \Delta p_H + \theta_5 \Delta p_c}{p (\theta_3 - \theta_1)} \right] + 0.01 \gamma_{pp}$$

$$(28) S_{xi} = 0.121 \left(\frac{L_x^3}{y^3 d_x} \right)$$

$$(29) S_{ei} = 0.0854 \left(\frac{L_e^3}{d_e} \right)$$

$$(30) S_{Bi} = 5.48 \left(\frac{L_B D_B^2}{d_B} \right)$$

$$(31) S_{si} = 0.0924 \left(\frac{L_s^3}{d_s} \right)$$

$$(32) P_{NE} = P \left[\gamma_{pp} - \left(\frac{P_L}{P} \right) (1 - \gamma_{pp}) \right]$$

$$(33.1) Q_H = 2.04 \times 10^{-3} \left[\frac{P \theta_1}{p (\theta_3 - \theta_1)} \right]$$

$$(33.2) \quad Q_C = 2.04 \times 10^{-3} \left[\frac{P \theta_5}{P (\theta_3 - \theta_1)} \right]$$

$$(34) \quad R_K = 0.98 \times 10^{-10} P d_F^2$$

$$(35) \quad \$_{xs} = (323 + 0.380 P) \chi (y - 1) S_{xi}^{0.676}$$

$$(36) \quad \$_{xc} = (89.3 + 0.105 P) \chi S_{xi}^{0.676}$$

$$(37) \quad \$_e = 241 \chi S_{ei}^{0.676}$$

$$(38) \quad \$_B = 874 b S_{Bi}^{0.676}$$

$$(39) \quad \$_S = 874 b S_{si}^{0.676}$$

$$(40) \quad \$_v = 1.60 \times 10^6 \left(1 + \frac{P}{1000} \right)$$

$$(41) \quad \$_{PS} = 1.59 \times 10^5 \chi D_{PS}^{1.72}$$

$$(42) \quad \$_{PC} = 4.48 \times 10^4 \chi D_{PC}^{1.72}$$

$$(43) \quad \$_{B\ell, H} = 629 (\Delta P_H)^{0.746} \chi^{0.386} Q_H^{0.614}$$

$$(44) \quad \$_{B\ell, C} = 629 (\Delta P_C)^{0.746} \chi^{0.386} Q_C^{0.614}$$

$$(45) \quad F_N = 1.575 \left[\$_{xs} + \$_{xc} + \$_e + \$_B + \$_S + \$_v + \$_{PS} + \$_{PC} + \$_{B\ell H} + \$_{B\ell C} + 1.18 \times 10^7 \right]$$

$$(46) \quad F_{PP} = \left[\frac{21.24 \quad 0.230 N_{FW}}{1492.7 - H_{10}} + 0.0536 \gamma_{pp} \right] (P)$$

$$(47) \quad I = 1.09 (F_N + F_P) + 0.14 (\gamma_{pp} P)^{0.80}$$

$$(48) \quad C_{co} = 0.1635 (F_N + F_{PP}) + 0.307 (\gamma_{pp} P)^{0.80}$$

$$(49) \quad f_F \quad \frac{P}{2.275 \times 10^9 \quad 2.00 \times 10^7 W_F} \quad 0.5$$

$$(50) \quad C_{MI} \quad 800 W_F \quad 9.10 \times 10^4 \quad (1.05 \quad f_F)$$

$$(51) \quad C_F \quad 5.00 \times 10^{-7} W_F \quad \frac{4 \times 10^{-5}}{f_F} \quad 7.56 \times 10^{-4} \quad (P)$$

$$(52) \quad M \quad 142.9 \quad \frac{C_{co} \quad C_{MI} \quad C_F}{P_{NE}}$$

REFERENCES

1. An ORACLE Code for Calculation of Fermi Ages by Numerical Integration, ORNL-56-4-53.
2. Memo from R. H. Dunham - Tennessee Valley Authority.
3. Reactor Handbook - Physics
4. CNL-21, "Absorption and Scattering of Radiation in Random Aggregates of Pebbles".
5. ORNL-57-6-5, "Cross Sections for OCUSOL-A Program".
6. Geneva Conference Paper P/425, "Neutron Yields from Fissile Nuclei".
7. Discussion of Geneva Conference Paper, P/425, Vol. IV, Session 18A, page 353.
8. NAA-SR-72, "Diffusion of Fission Fragments from Uranium-Impregnated Graphite", by C. A. Smith and C. T. Young.
9. NAA-SR-895, "A Pebble Bed Reactor for Stationary Power Plants".
10. NAA-SR-247, "High Temperature Diffusion of Individual Fission Products from Small Uranium-Impregnated Graphite Samples Under Deuteron Bombardment", C. T. Young.
11. ORNL-1958, "Effect of Moderator Temperature Upon Neutron Flux in Infinite Capturing Medium".
12. ORNL-CF-56-4-53, "ORACLE Code for Fermi Age by Numerical Integration".
13. Glasstone and Edlund, "Elements of Nuclear Reactor Theory".
14. KAPL-1226, "Poisoning in Thermal Reactors Due to Stable Fission Products".
15. Dresner paper.
16. ORNL-CF-55-1-76, "Description and Operating Manual for the Three Group, Three Region Reactor Code for ORACLE".
17. Rockwell, T., "Reactor Shielding Manual", TID-7004.
18. Blizard, E. P., "Nuclear Radiation Shielding".
19. Alexander, L. G., "Heat Release in Nuclear Reactors, ORNL-CF-56-10-24.
20. Alexander, L. G., "The Integral Spectrum Method for Gamma Heating", ORNL-CF-56-11-82.

21. Astra Associates, Effect of Gamma Heating on APPR-1 Pressure Vessel, APAE memo 85.
22. Smith, C. O., "Stress Analysis for ORSORT Students".
23. Fairchild, H. P., "Pressure Drop Through a Pebble Pile".
24. Lyon, R. N., "Heat Transfer Experiments for Helium Plant".
25. BNL-20, "The Escape of Fission Products from an Uranium Rod".
26. ANL-4153, "Volatilization of Uranium from Uranium Impregnated Graphite".
27. NP-4794, "Generation of Steam from Liquid Metal at High Heat Fluxes".
28. ATC-54-12, "Design Study - 60 MW Closed Cycle Gas Turbine Nuclear Power Plant".
29. McAdams, "Heat Transmission", 3rd Edition.
30. Glasstone, "Principles of Nuclear Engineering".
31. Timoshenko and Goodier, "Theory of Elasticity".
32. AECD-3646, Reactor Handbook, Engineering.
33. AECD-3647, Reactor Handbook, Materials.
34. Binder, "Fluid Cross Sections".
35. BNL-325, "Neutron Cross Sections".
36. ORNL-1387, "Cost Estimates for Seven 200 MW Turbine Plants for Operation with Nuclear Reactors at Various Steam Conditions".
37. Critoph, E., and Pearce, R. M., "The Reactivity Effect Due to Neutron Streaming in an Empty Tube", Journal of Nuclear Energy, Vol. 4, No. 4, April, 1957.
38. ORNL-CF-55-9-73, "The Effective Resonance Integrals of U^{238} and Th^{232} ".
39. Nuclear Science and Engineering, "The Effective Resonance Integrals of U^{238} and Th^{232} ", 1, 68-79, (1956).
40. WAPD-PA-3, "PWR Vapor Container Design Criteria".
41. NAA-SR-213, "The Use of Bromine Trifluoride in the Recovery of Uranium from Uranium Impregnated Graphite".
42. ORNL-CF-54-8-235, "Gas Cooled Ceramic Power Breeder Reactor".

43. Mon N-308, "Steam Boiler for Gas-Cooled Pile".
44. Mon N-299, "Heat Transfer in Gas-Cooled Pile".
45. NAA-SR-194, "Diffusion of Xe from U Impregnated Graphite at High Temperatures".
46. TID-2004, F. E. Faris
47. TID-278, C. F. Leyse, M. P. Paleczny, R. S. Cole.
48. NAA-SR-5, F. R. Gilmore.
49. ORNL-171, E. B. Ashcraft.
50. KAPL-1440, P. Greebler, H. Hurwitz, Jr.
51. ANL-4448, F. Daniels.
52. ANL-4242.
53. NAA-SR-232, C. T. Young, C. A. Smith, J. B. McClelland.
54. Aries and Newton, "Chemical Engineering Cost Estimation".
55. Chemical Engineering, July, 1953.
56. Private Communication from C. A. Burchsted, ORNL, to H. N. Culver, July, 1957.
57. Johnsonville Steam Plant Cost.
58. Kratzer, Myron, TID-7534, "Symposium on the Reprocessing of Irradiated Fuel", May 1957.