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LIQUID ROCKET PLANT

DYNAMIC CALIBRATION STUDY

NERVA Contract 0718

Technical Memorandum 161 LRP

23 October 1963

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DYNAMIC CALIBRATION STUDY

NERVA Contract 0718

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ABSTRACT

Since transducers used in rocket engine applications measure time varying quantities, the characteristics which define the behavior of a transducer under dynamic conditions are extremely important in their applications by the design engineer. Proper calibration of transducers in the dynamic or frequency domain required a thorough understanding of the dynamic characteristics of a transducer. Analytical models are generated in this report, and the techniques based on these models for determining frequency response are discussed. The effects of the system elements, such as tubing and ports, on dynamic characteristics in addition to the effect of the measurand environment itself are examined. Analog and digital techniques to compensate system performance for the improvement of frequency response are presented. Additional considerations in system design are discussed, especially the effect of a sampled data acquisition system. The special problems of temperature sensor dynamic characteristics are covered in detail. Finally, experimental methods for determining transducer dynamic characteristics and recommendations for test facilities are given.

FOREWORD

The objective of this study is to determine the approach and equipment necessary to perform dynamic calibrations of the transducers to be used on the NERVA engines.

This report is prepared in accordance with Paragraph 4.2.a(10) of NERVA TAD ST 2.4. It is submitted in fulfillment of LRP PD 101-c, dated 1 March 1963, and in partial fulfillment of NERVA Contract 0718.

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I. INTRODUCTION

The testing, evaluation, and specifications of transducer characteristics can be divided into three main categories: (1) environmental, (2) static, and (3) dynamic. Environmental testing is designed to evaluate the characteristics of a transducer under the various conditions of environment that may be encountered in operation. Vibration, acceleration, acoustic level, temperature range, weather effects, propellant compatibility, altitude range, and nuclear environment are examples of environmental conditions that may affect transducer operation.

Static testing provides data on the response of the transducer to controlled input signals under steady-state conditions; i.e., input signal is applied in discrete steps and the output reading is obtained only after equilibrium of the input is assured. Linearity, hysteresis, repeatability, and zero and full-scale offset are examples of the characteristics that are determined by static testing.

Dynamic testing is designed to determine the nature of the response of a transducer to an input that is fluctuating with respect to time. Frequency response, rise time, damping ratio, and resonant frequency are examples of dynamic characteristics that define the behavior of a transducer when subjected to transient or time-varying inputs.

II. TECHNICAL DISCUSSION

A. ANALYTICAL MODELS

It is desirable to establish a relationship between the static or steady-state conditions and dynamic conditions. Such a relationship is necessary because of the relative accuracies inherent in the two types of testing--static and dynamic. It is possible to establish the steady-state or static characteristics with a high degree of accuracy whereas characteristics under dynamic conditions are far more difficult to obtain with a very good degree of accuracy. Ideally, a one-to-one relationship would simplify the problem greatly. This means that characteristics obtained from static testing would be valid for dynamic conditions.

Static testing establishes an input-versus-output relationship as shown in Figure 1.

Figure 1. Static Test Input-Versus-Output Curve for a Typical Pressure Transducer

II, A, Analytical Models (cont.)

For the sake of simplicity, a perfectly linear input-output relationship has been assumed. In the case of pressure transducers, this is a justifiable approximation since this discussion will not involve high orders of accuracy, and the effects of linearity, hysteresis and repeatability will not be considered here in detail.

A relationship between static response and dynamic response is desired. Since under dynamic conditions, the input is fluctuating with time, it would be suspected that a relationship involving changes in input with respect to time, or simply frequency would be in order. Clearly, the ratio of output to input (amplitude ratio) at steady-state or zero frequency can be established by static testing. In particular, Point A in Figure 2 could be established by static testing. It only remains to establish how the amplitude ratio varies under different frequencies, and a curve such as the one shown in Figure 2 could be

Figure 2. Typical Frequency Response Curve for a Pressure Transducer

II, A, Analytical Models (cont.)

generated. Hopefully, the frequency response curve (amplitude ratio versus frequency) would reveal no change in amplitude ratio over a wide frequency range; thus the static calibration would be valid over a wide dynamic frequency range. In the case of electrical circuits, it is a relatively simple matter to establish a frequency response curve.

A sinusoidal generator applies a constant amplitude signal to the input terminals of the device under test (Figure 3), and output readings are taken as the frequency of the input signal is varied through known values. In the case of pressure transducers, the issue becomes cloudy because of the lack of a device for generating a sinusoidal pressure input with a constant known amplitude. Therefore a different approach must be attempted.

Figure 3. Test Setup for a Frequency Response Test

II, A, Analytical Models (cont.)

Classic analysis of an engineering problem may be divided into three phases:

1. Generation of a suitable mathematical model to describe the real device under investigation;
2. Manipulation of the mathematical model to yield or predict solutions;
3. Interpretation of the mathematical results in terms of the behavior of the real device.

Using the classic analysis we can proceed as follows:

Consider the device shown in Figure 4. A piston restrained by a spring is shown sliding inside of a cylinder. The piston is connected to a strain

Figure 4. Typical First-Order System Model for a Pressure Transducer

II, A, Analytical Models (cont.)

gage; variations in the position of the piston or the piston itself produce changes in the output of the strain gage. It will be assumed that the output of the strain gage is directly proportional to the displacement of the piston. It will be further assumed that the displacement of the spring is directly proportional to the magnitude of the force $F(t)$ applied.

A mathematical model of the device shown in Figure 4 may now be generated.

$$\text{output} = K x$$

$$x = k F(t)$$

where,

K = arbitrary constant

x = displacement of piston

lumping K and k together,

$$\text{output} = k F(t)$$

or

$$\text{output} = k \text{ input.}$$

The relationship between output and input may be drawn as in Figure 5. As shown in Figure 5, the effect of the piston mass was neglected. This mathematical model simulates the static characteristics of an actual pressure transducer. If F psia of pressure are exerted, E output units are indicated; if $2F$ input is applied, $2E$ output is produced.

II, A, Analytical Models (cont.)

Figure 5. Static Input-Versus-Output Relationship
for a First-Order System Model

If the dynamic mathematical model were generated (Appendix A) for the transducer shown in Figure 5 and a step function of pressure applied (Figure 6a), the response curve would be as shown in Figure 6b.

II, A, Analytical Models (cont.)

Figure 6. Driving Function and Response of a First-Order System Model

II, A, Analytical Models (cont.)

Up to this point the first two rules of analysis have been applied; i.e., (1) generation of a mathematical model, and (2) manipulation of the mathematical model to predict a solution. It remains to interpret the results in terms of the known behavior of an actual pressure transducer.

The curve shown in Figure 7 approximates the response of an actual pressure transducer to a unit step pressure application. The mathematical model of Figure 5 does approximate the static characteristics of an actual transducer, but it does not allow for overshoot or the oscillatory nature of actual transducers; therefore, the model fails to be adequate for dynamic purposes.

Figure 7. Typical Response of a Second-Order System to Step Input

II, A, Analytical Models (cont.)

By considering the mass of the piston (Figure 8) a new mathematical model can be generated (Appendix B). The static and dynamic responses of the new model are shown in Figures 9a and 9b. It can be seen that this new mathematical model does approximate the actual transducer under both static and dynamic conditions.

Figure 8. Model of a Second-Order System

II, A, Analytical Models (cont.)

Figure 9. Static Calibration and Dynamic Response of a Second-Order System to Step Input

II, A, Analytical Models (cont.)

It has been implied that a known step of pressure can be applied to actual pressure transducers without great difficulty. This is the case here; it may be done with a shock tube or other devices. The remaining problem is to deduce the frequency response from the results of a shock tube test.

The mathematical model for the device shown in Figure 8 yields an equation for the output response to a step function of the form.

$$x(t) = 1 - F(\delta)e^{F(\delta, \omega_n)} \cos(F^1(\delta, \omega_n)) \quad (\text{Eq 1})$$

(Appendix B, Eq. 10a)

where,

$x(t)$ = amplitude (relative)

$F(\delta)$ = function of damping ratio

$F(\delta, \omega_n)$ = function of damping ratio and natural frequency

δ = damping ratio

ω_n = natural frequency

e = natural logarithm base

The damping ratio is a function of three of the system parameters.

$$\delta = F(c, k, m)$$

(Appendix B, Eq. 5b)

where,

c = coefficient of friction

k = spring constant

m = mass of piston

and the natural frequency (ω_n) is a function of the two system parameters

$$\omega_n = F(k, m)$$

(Appendix B, Eq. 5a)

II, A, Analytical Models (cont.)

Manipulation of Equation 1 shows that the damping ratio (δ) is a function of the maximum overshoot only (Appendix B).

$$\delta = F(A_1) \quad (\text{Eq 1a})$$

(Appendix B, Eq. 16)

where, A_1 = maximum overshoot of first peak of response (Fig. 9b).

Further manipulation of Equation 1 reveals that the natural frequency is a function of the time to reach the maximum overshoot and the damping ratio:

$$\omega_n = F(\delta, t_{\max}) \quad (\text{Eq 1b})$$

(Appendix B, Eq 13).

In order to determine the frequency response curve, it is necessary to transform Equation 1, which is written in the time domain, into an equation in the frequency domain. Such an equation is written:

$$X(j\omega) = \frac{\omega_n^2}{(j\omega)^2 + 2\delta\omega_n(j\omega) + \omega_n^2} \quad (\text{Eq 2})$$

where,

$X(j\omega)$ = amplitude ratio

$(j\omega)$ = frequency

(Appendix B, Eq 7b)

Substituting different values of frequency ($j\omega$) into Equation 2 will generate a curve as shown in Figure 10.

II, A, Analytical Models (cont.)

Generally it is safe to assume that the curve shown in Figure 10 is flat (amplitude ratio constant and equal to static ratio) to a frequency ω_1 where $\omega_1 = \omega_n/5$. The maximum error introduced in amplitude ratio at this frequency is $\pm 5\%$.

Figure 10. Typical Frequency Response Curve for a Second-Order System

In summary, the following points are reiterated:

1. A typical pressure transducer may be approximated mathematically by a second-order system as illustrated in Figure 8 and defined in Equations 1 and 2.
2. By analyzing the second-order system mathematically, two equations (one in the time domain, Equation 1; and one in the frequency domain, Equation 2) are obtained which relate the output to damping ratio and natural frequency.

II, A, Analytical Models (cont.)

3. The damping ratio and natural frequency of a transducer may be determined from an experimentally obtained curve (Figure 7).

4. The damping ratio and natural frequency obtained from experimental data may be substituted into a theoretical equation (Equation 2) to predict the frequency response of the transducer.

5. The frequency response may be defined by specification of the damping ratio and the natural frequency. Since the natural frequency is a function of damping ratio and the time required to reach maximum overshoot (t_{max}),

$$\text{where, } \omega_n = F(\delta, t_{max}), \quad (\text{Eq 3})$$

the transducer could be defined by specifying δ and t_{max} . The rise time, which is defined as the time required for the output to go from 10% to 90% of the final value in response to a step input may be expressed as

$$t_{rise} = F(\delta, \omega_n) \quad (\text{Eq 4})$$

so that the transducer could be defined as to frequency response by specifying t_{rise} and δ .

This may be summarized by saying that the specification of any two of the three parameters $\omega_n, \delta, t_{rise}$ define the frequency response of a transducer. It should be pointed out, however, that of the three combinations

- a. ω_n, δ
- b. δ, t_{rise}
- c. t_{rise}, ω_n

the last, c., can lead to physically impractical values for the damping ratio. In practice, the damping ratio (δ) is usually fairly well fixed by the nature of

II, A, Analytical Models (cont.)

the transducer so that a specification of ω_n (natural frequency) is sufficient to define the frequency response of the transducer. A handy rule of thumb then applies,

$$\begin{aligned} \text{maximum usable frequency} &= \omega_n / 5 \\ (\text{maximum inaccuracy} &= \pm 5\%). \end{aligned}$$

In practice, the so-called ring frequency or resonant frequency is approximately equal to the natural frequency for strain gage pressure transducers.

A specification of the frequency response of a transducer clearly obviates the necessity of calling out any additional dynamic parameters. The fact is that there are few facilities available to evaluate the frequency response of a transducer directly, whereas the ring frequency may be determined by relatively simple means.

It must be specified that the foregoing is based on approximations and assumptions, which are reasonable in a majority of cases, and can only serve as a guide at best.

Actual shock tube data is analyzed by a more exact mathematical method (Guillemin Technique) than the one presented here: however, the basic approach-- generation of a mathematical model, mathematical manipulation of the model to obtain equations, and substitution of experimental data into theoretical equations to predict behavior--is the same as the method shown on the foregoing pages.

II, Technical Discussion (cont.)

B. DISCUSSION OF THE METHODS OF EVALUATING AND ANALYZING TRANSIENT RESPONSE DATA OF PRESSURE TRANSDUCERS FOR THE DETERMINATION OF DYNAMIC CHARACTERISTICS

In the preceeding discussion of dynamic characteristics, a method for evaluating the frequency response of a pressure transducer using the analog time domain response (transient response) of the transducer was presented. This method is commonly referred to as a graphical analysis and is performed by the following steps:

1. The pressure transducer is excited by a step input of pressure, and the resulting output is recorded by oscilloscope and camera, or some other equivalent method.
2. The analog output is presented as a curve in the time domain, i.e., a plot of amplitude versus time (Figure 11a).
3. The curve of the analog output is measured for pertinent values--in particular, the frequency of the curve and the overshoot (Figure 11b).
4. The pertinent measurements are then manipulated by use of formulas to obtain the frequency response characteristics.

The application of this method is illustrated in a report in Appendix C. The mathematical derivation of the method appears in Appendix B under the discussion of dynamic characteristics.

In essence, the graphical analysis method provides a technique of obtaining the transfer function of a transducer by calculating the natural frequency and damping ratio from transient response data.

II, B, Discussion of the Methods of Evaluating and Analyzing Transient Response Data of Pressure Transducers for the Determination of Dynamic Characteristics (cont.)

Knowledge of the transfer function defines the frequency response. The graphical analysis method presupposes that the behavior of the transducer under consideration may be adequately defined by a mathematical model based on a linear second-order system, such as a system using a spring, mass, and friction as shown in Figure 11. In other words, the validity of the graphical analysis depends upon the system meeting two requirements:

1. That system is linear, and
2. That the system is capable of being described by a linear second-order differential equation.

Since the representation of a pressure transducer by a linear second-order system is a fair approximation, in actual practice the graphical method provides a good quick-look method of approximating the frequency response of a pressure transducer.

A more exact method of evaluating the frequency response of a pressure transducer from transient data is the Guillemin Impulse Approximation. This method assumes only that the system is linear (no restriction on the order, such as second order or third order). The rigorous derivation of the Guillemin technique is presented in numerous texts and articles. A brief example is included in Appendix D. A quasi-intuitive presentation will be given here to provide a better insight into the approximations and assumptions inherent in the method.

II, B, Discussion of the Methods of Evaluating and Analyzing Transient Response Data of Pressure Transducers for the Determination of Dynamic Characteristics (cont.)

(2)

(b)

Figure 11. Second-Order System Model with a Dynamic Response Curve

II, B, Discussion of the Methods of Evaluating and Analyzing Transient Response Data of Pressure Transducers for the Determination of Dynamic Characteristics (cont.)

Consider the "black box" in Figure 12. In order to determine the frequency response of the box, a calibrated sinusoidal generator is connected to the input, and the amplitude of the input voltage is monitored by voltmeter B. The amplitude of the output is monitored by voltmeter C. By applying sinusoidal inputs of different frequencies and constant amplitudes a tabulation as shown in Figure 12 results. A graphical plot of the tabulated data, giving the frequency response characteristics, is also presented in Figure 12.

As a variation of this method, suppose instead of applying each sinusoidal individually, a number of sinusoidal inputs are applied simultaneously and by means of a hypothetical instrument; the resulting output is recorded as a number of simultaneous sinusoidal waves (Figure 13a). By measurement of the input and output waves, a tabulation of the results could be made (Figure 13b) and the frequency response curve drawn (Figure 13c).

To carry the hypothetical case a step further, suppose the sinusoidal waves, E0, E1, E2, E3 of Figure 13a are combined to form a wave having the shape shown in Figure 13d. If this wave is applied to the box G, an output as shown in Figure 13e would result.

From the previous example, it is known that the output wave shape shown in Figure 13e is actually the combination of the sinusoidal output voltages shown in Figure 13a.

In a practical case it would be desirable to apply a wave made up of a great number of frequencies to the input. With practical instruments we could easily measure the resulting output. A tool is needed whereby the input waves and the output waves can be separated into their sinusoidal components. Such a tool is furnished by the Fourier series.

II, B, Discussion of the Methods of Evaluating and Analyzing Transient Response Data of Pressure Transducers for the Determination of Dynamic Characteristics (cont.)

(a)

FREQ. ω	E	C
0	1.00	1.00
1	1.00	.99
2	1.00	.90
3	1.00	.80
4	1.00	.70

(b)

(c)

Figure 12. Frequency Response Test Setup, and Typical Test Results

II, B, Discussion of the Methods of Evaluating and Analyzing Transient Response Data of Pressure Transducers for the Determination of Dynamic Characteristics (cont.)

Figure 13. Typical Test Setup for Applying Simultaneous Sinusoidal Inputs to Determine Frequency Response

II, B, Discussion of the Methods of Evaluating and Analyzing Transient Response Data of Pressure Transducers for the Determination of Dynamic Characteristics (cont.)

Consider the series,

(Eq 5)

$$F(t) = a_0 + a_1 \cos \omega t + a_2 \cos 2 \omega t + \dots + b_1 \sin \omega t + b_2 \sin 2 \omega t + \dots$$

$$\text{or, } F(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos \omega t n + b_n \sin \omega t n)$$

where, $F(t)$ = function of time

and a_n, b_n = specific coefficients

ω = angular frequency

t = time

Graphically, this shown in Figure 14.

By mathematical manipulation, the coefficients, $a_0, a_1, a_2, a_3, \dots, a_n$ may be found as

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} F(t) \cos n \omega t dt$$

(Eq 6)

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} F(t) \sin n \omega t dt$$

It can be seen that Equation 5 relates the function of time $F(t)$ to the sinusoidal frequency components that made up the function; i.e., if the harmonics which make up the time function were given, the time function could be generated.

II, B, Discussion of the Methods of Evaluating and Analyzing Transient Response Data of Pressure Transducers for the Determination of Dynamic Characteristics (cont.)

Figure 14. Nonsinusoidal Wave with Sinusoidal Components

II, B, Discussion of the Methods of Evaluating and Analyzing Transient Response Data of Pressure Transducers for the Determination of Dynamic Characteristics (cont.)

In other words, Equation 5 is said to define the function in the time domain. Likewise, if the function were given in the time domain (such as an oscilloscope picture displaying amplitude versus time), the sinusoidal harmonics making up the time function could be defined from Equation 6. Equation 6 is said to define the function in the frequency domain.

A simple example will illustrate the complete procedure:

1. Apply a known step input to the system G, and record the transient (time domain) output (Figure 15a).
2. By use of Equation 6, reduce the input step and the output transient to their component sinusoidals (transform to the frequency domain) (Figures 15d and 15e). A tabulation as shown in Figure 15b will result and a frequency response curve such as Figure 15c may be made.

The actual Guillemin technique involves a number of sophisticated mathematical shortcuts, but is described in essence by the foregoing example. As might be suspected at this point, the transformation of the transient output into the frequency domain must be performed graphically in actual practice. This consequently introduces some approximation.

The actual transient is approximated by straight lines. By taking a large number of small segments, the error due to this approximation can be held to under 5%. It should be pointed out that in spite of the fact that the unit step function is rich in harmonics, the amplitudes of the higher frequencies become increasingly small. In addition, in actual practice a step is not achieved, but rather a relatively long pulse. This truncated step limits the useful low-frequency range of most physical systems.

II, B, Discussion of the Methods of Evaluating and Analyzing Transient Response Data of Pressure Transducers for the Determination of Dynamic Characteristics (cont.)

Figure 15. Typical Test Setup for Applying Set Input to Determine Frequency Response

II, B, Discussion of the Methods of Evaluating and Analyzing Transient Response Data of Pressure Transducers for the Determination of Dynamic Characteristics (cont.)

To sum up, neglecting the accuracy of the measuring equipment, the Guillemin impulse approximation as generally applied to shock tube data reduction is subject to the following limitations:

1. The transient output is graphically approximated by straight line segments. By choosing a large number of straight lines this error can be held to under 5%.
2. Analysis of the high frequency response of the system under test is limited by the relatively low-amplitude components of higher harmonics present in the physically realizable step functions used.
3. Low frequency (under 100 cps) information is limited by the truncation of the step waves in the shock tube.
4. The system under consideration is assumed to be linear.
5. In practice, a digital computer is required to handle the large number of computations necessary for high accuracy.

II, Technical Discussion (cont.)

C. SYSTEM FACTORS AFFECTING TRANSDUCER DYNAMIC CHARACTERISTICS

In the foregoing discussion (Section I, B), various methods of determining frequency response from transient response data were presented. The methods given may be applied to any "black-box" system as a whole. An actual system, however, may be divided into a number of distinct parts as shown in Figure 16. Each of the component parts should be examined individually to see what effect, if any, they have on the dynamic characteristics of the system as a whole. In this way, parts which seriously affect response may be dealt with independently.

Figure 16. Schematic of the Pressure Measuring System

II, C, System Factors Affecting Transducer Dynamic Characteristics (cont.)

The discussion following is based on the condition that the transmission medium is a gas.

The transducer diaphragm and connected elements (beam, strain gages, damping medium, etc.) form the basic sensing unit. This sensing unit will have a frequency response determined by the mechanical design of the diaphragm and connected components. The mechanical ring frequency of this unit is usually specified by the manufacturer as the ring frequency of the transducer. Obviously the frequency response of this individual unit would determine the theoretical maximum response of the unit.

The transducer connecting port and chamber form the inlet unit, which has a resonant frequency based on acoustical properties. For relatively short lengths of connecting port (or tube), the inlet configuration may be considered as a Helmholtz resonator whose resonant frequency is approximately*

$$F_r \approx \frac{c}{2\pi} \sqrt{\frac{\pi r^2}{V(L + 1.7r)}}$$

where,

F_r = resonant frequency

c = velocity of sound in the medium

r = radius of tube

L = length of tube

V = volume of chamber

If capillary action may be neglected, this unit can be considered as a second-order system with approximately zero damping; it will have a frequency response which is flat to approximately one-fifth of the resonant frequency.

* "On the Problem of Dynamic Pressure Analysis," G. J. Schick, A. W. Langill, M. A. Henry, S. J. Spataro; Aerojet-General Memorandum from G. J. Schick to L. E. Finden, dated 15 April 1963

II, C, System Factors Affecting Transducer Dynamic Characteristics (cont.)

The connecting tubing is usually more accurately described by an organ pipe approximation

$$Fr = \frac{c}{4L}$$

where, c = velocity of sound in the medium
 L = length of tubing
 Fr = resonant frequency

It can be seen from the above, that for a 3 in. piece of tubing and where $c = 1000$ ft/sec ,

$$Fr = \frac{1000}{4 \times .25} = 1000 \text{ cps.}$$

Again the maximum flat frequency response is considered as 20% of the resonant frequency, or in this example, the maximum flat frequency response with only 3 in. of tubing is 200 cps. The connecting port to the measurand is usually considered as an extension of the connecting tubing.

The above discussion concerns the use of gas as the transmission medium. In the case of liquid filled lines the behavior of the system with gas must be modified by the following considerations:

1. mass of the fluid in the system
2. viscosity of fluid
3. transducer "impedance"

The last of these considerations, transducer "impedance", refers to the relative amount of energy required from the source to drive the transducer. In the case of a very springy diaphragm that requires a relatively long travel, the imped-

II, C, System Factors Affecting Transducer Dynamic Characteristics (cont.)

ance would be low, whereas in the case of a crystal type transducer with extremely low displacement and requiring relatively little power from the source, the impedance would be high. While it is possible to improve system response with fluid or grease filled connecting lines in some applications, the reverse is often true, and care must be exercised in the application of fluid filled lines.

Experimental verification of system performance is most desirable also.

In summing up, components used to connect the transducer diaphragm to the measurand (inlet ports, connecting tubing, etc.) may be thought of as low-pass filters connected to the input of the measuring instrument. In general, system response is degraded by the addition of any additional tubing or parts. Frequency response of the complete system can be no greater than the response of the element with the lowest resonant frequency.

II, Technical Discussion (cont.)

D. EFFECT OF MEASURAND ENVIRONMENT ON DYNAMIC CHARACTERISTICS

Up to now, the effect of the condition of the measurand itself on the dynamic characteristics of the pressure transducer has not been considered. The temperature of the measurand will have a definite effect on the mechanical parts of the transducer, possibly introducing a change in spring constants, damping ratios, etc., and thus changing the mechanical natural resonance of the element. Proper design may eliminate this possible source of error. If the diaphragm of the transducer is mounted flush to the measurand, this error will be the main effect to consider.

If an inlet port, or other conduit is used to connect the transducer to the measurand, the following factors must be considered:

1. Effect of temperature on the velocity of sound in the medium
2. Velocity of sound in the measurand medium

In Section II, C it was stated that for a system using ports or tubing, the resonant frequency is determined by the acoustical properties of the connecting parts. The acoustical resonance is dependent on the velocity of sound in the particular medium present in the connecting conduit. Therefore, the characteristics of the measurand which effect the velocity of sound also affect the frequency response of the pressure measuring system. As an example, consider a pressure measuring system calibrated in air at 25°C and showing a resonant frequency of 1000 cps. The pressure measuring system consists of a transducer with connecting tubing. The inlet tubing may be considered as an organ pipe.

$$Fr_1 = \frac{c_1}{4L} = \frac{12 \times 1000}{4 \times 3} = 1000 \text{ cps}$$

II, D, Effect of Measurand Environment on Dynamic Characteristics (cont.)

where, c_1 = velocity of sound in air ≈ 1000 fps
 L = length of tube = 3 in.
 Fr_1 = resonant frequency in air at 25°C

$$\text{maximum usable range} = 1000 \text{ cps} \times 1/5 = 200 \text{ cps}$$

If this same pressure measuring system is used to measure pressure in hydrogen at -100°C, the new resonant frequency may be found as follows.

$$c_2 = \sqrt{\frac{T_2}{T_1}} (c_4)$$

$$c_2 = \sqrt{\frac{173}{298}} (4000) = 3050$$

$$Fr_2 = \frac{c_2}{4L} = \frac{3050 \times 12}{4 \times 3} = 3050 \text{ cps}$$

where, c_2 = velocity of sound in hydrogen at -100°C (173°K)
 T_2 = -100°C = 173°K
 T_1 = 25°C = 298°K
 c_4 = velocity of sound in hydrogen at 25°C ≈ 4000 fps

$$\text{Maximum usable range} = 3050 \times 1/5 = 610 \text{ cps}$$

In the above example the maximum range has been increased. However, the mechanical ring frequency must be above the resonant frequency of the tubing.

The absolute pressure of the measurand may have some effect on the velocity of sound and must sometimes be considered; usually it is negligible.

II, D, Effect of Measurand Environment on Dynamic Characteristics (cont.)

In summary, the properties of the measurand and its environmental effect on the transducer must be considered when predicting dynamic response of a pressure measuring system.

II, Technical Discussion (cont.)

E. TECHNIQUES TO COMPENSATE TRANSDUCER SYSTEM DYNAMIC CHARACTERISTICS FOR IMPROVED FREQUENCY RESPONSE (DATA RECONSTRUCTION)

As pointed out in Section II, C, "System Factors Affecting Transducer Dynamic Characteristics", it is quite often necessary to introduce a coupling between the transducer diaphragm and the measurand. This seriously degrades the frequency response of the transducer. Connecting conduit and fittings should be held to a minimum in good designs for pressure measuring systems; but, sometimes complete elimination or even modification of connecting conduit is not possible. The next alternative to improve frequency response is to apply some form of dynamic compensation.

Compensation for the undesirable characteristics introduced by a transducer that affect the faithful reproduction of the measurand signal is not a new idea in itself. Temperature compensation of electrical circuits in transducers has reached a high level of reliability and accuracy. This type of compensation is usually done by adding elements to the original circuit which have mirror image temperature coefficients in comparison to the elements to be compensated. That is, if a particular resistor had a resistance-versus-temperature curve as shown in Figure 17a, then another resistor with a curve as in Figure 17b would be added and the compensated characteristic would result as in Figure 17c.

Dynamic compensation is effected in basically the same way. Consider a transducer G with the output-versus-frequency characteristic in Figure 18a. If a transducer H with the frequency characteristics of Figure 18b were obtained and added in cascade to transducer G as shown in Figure 18c, the output of the combination would result in the flat frequency response curve of Figure 18c.

II, E, Techniques to Compensate Transducer System Dynamic Characteristics for Improved Frequency Response (Data Reconstruction) (cont.)

(a)

(b)

(c)

Figure 17. Resistance Temperature Compensation

II, E, Techniques to Compensate Transducer System Dynamic Characteristics for Improved Frequency Response (Data Reconstruction) (cont.)

Figure 18. Transducer Dynamic Compensation

II, E, Techniques to Compensate Transducer System Dynamic Characteristics for Improved Frequency Response (Data Reconstruction) (cont.)

To be more specific, consider the system G (Figure 19) with a frequency response characteristic defined by the transfer function

$$G(S) = \frac{1}{S+a}$$

where, $G(S)$ = transfer function of G in S domain

S = Laplace operator $j\omega$

$$j = \sqrt{-1}$$

ω = frequency

a = constant

Figure 19. Typical Transducer System

II, E, Techniques to Compensate Transducer System Dynamic Characteristics for Improved Frequency Response (Data Reconstruction) (cont.)

If the output of G is fed into a second system H with a transfer function

$$H(S) = S+a$$

where, $H(S)$ = Transfer function of
H in S domain

then the resultant output C will be

$$C(S) = E(S) \frac{1}{S+a} \quad S+a = E(S)$$

where, $E(S)$ = input in S domain
 $C(S)$ = output in S domain

or in other words, the final output C will look exactly like the original input E. The main question is how the inverse transfer function of H is generated.

The transfer function

$$H(S) = S+a$$

represents the relationship between the input C' and the output C as follows:

$$\frac{C(S)}{C'(S)} = S+a$$

$$C(S) = C'(S) (S+a)$$

This can be transformed into the time domain as

$$C(t) = \dot{C}'(t) + C'(t) a \quad (\text{Eq. 7})$$

where, $C(t)$ = output in time domain
 $\dot{C}'(t)$ = first derivative of $C'(t)$
 $C'(t)$ = input in time domain

It is possible that the transfer function $S+a$ could be programmed on a digital computer as a recursion formula for the solution of the differential Equation 7.

II, E, Techniques to Compensate Transducer System Dynamic Characteristics for Improved Frequency Response (Data Reconstruction) (cont.)

Another possible way of obtaining the inverse transform, $H(s) = s+a$ would be analog simulation. The circuit in Figure 20 would provide this.

Figure 20. Analog Simulator of Inverse Transfer Function

II, B, Techniques to Compensate Transducer System Dynamic Characteristics for Improved Frequency Response (Data Reconstruction) (cont.)

An alternate solution could be the feedback loop shown in Figure 21.

$$1. E_o(s) = K E(s)$$

$$2. E(s) = \frac{1}{K} E_o(s) - G(s) E_o(s) + E_{in}(s)$$

$$3. E(s) = \frac{E_o(s)}{K}$$

$$4. \frac{E_o(s)}{K} = E_{in}(s) + E_o(s) \left(\frac{1}{K} - G(s) \right)$$

$$5. E_o(s) \left(\frac{1}{K} - \frac{1}{K} + G(s) \right) = E_{in}(s)$$

$$6. \frac{E_o(s)}{E_{in}(s)} = H(s) = \frac{1}{G(s)} = s \tau_a$$

WHERE: $E_{in}(s)$ = LAPLACE TRANSFORM OF INPUT SIGNAL
 $E_o(s)$ = LAPLACE TRANSFORM OF OUTPUT SIGNAL
 K = GAIN OF AMPLIFIER
 $G(s)$ = GIVEN TRANSFER FUNCTION
 $H(s)$ = SYSTEM TRANSFER FUNCTION

Figure 21. Feedback Simulation of Inverse Transfer Function

II, E, Techniques to Compensate Transducer System Dynamic Characteristics for Improved Frequency Response (Data Reconstruction) (cont.)

Other solutions are undoubtedly possible, since the solutions outlined above only allude to the actual solutions.

The analog simulation circuit is obviously a noise amplifier and would require modification for application. The feedback network shown in Figure 21 would tend to be unstable; further refinement or modification would be required for a working model. The digital computer program method has been applied with some success at Aerojet-General, Sacramento. Working compensating units have been built using analog methods.

It suffices to say that dynamic compensation techniques are becoming available and could be applied to a system for the improvement of frequency response. Another apparent way for compensating a system to extend the usable frequency range would be the use of dynamic calibration curves. The calibration curves could be obtained by shock tube or sinusoidal pressure generator methods.

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II, Technical Discussion (cont.)

F. ADDITIONAL CONSIDERATIONS IN THE SELECTION OF DYNAMIC CHARACTERISTICS TO FIT OVERALL SYSTEM DESIGN

The necessity of knowing the characteristics that define transducer behavior in response to time-varying inputs is self evident in view of the fact that most actual applications require the measurement of time-varying quantities. With this criteria in mind, it could be assumed that a transducer with the highest frequency response possible would be the most ideal for any given situation. Paradoxically, however this is not always the case. The fault in this line of reasoning is the data acquisition system used. Specifically, a sampled-data system is sensitive to the frequency of the input signal. To elaborate further, the following hypothetical example is used:

The circuit shown in Figure 22a consists of a sinusoidal generator, a sampler, and a readout device E. The sampler is defined to be an ideal switch closing at regular intervals instantaneously. If the continuous wave form $C(t)$ is sampled at a cyclic rate $T/8$, the series of impulses $C^*(t)$ would be recorded on the readout device E. An observer at E with no prior knowledge of the continuous wave $C(t)$ could connect his impulse values by straight lines as shown in Figure 22a and have a fair approximation of the actual wave, $C(t)$.

Suppose now, the cyclic rate of the sampler is slowed until the sampling rate is equal to exactly $T/2$. In other words, the sampler would sample twice each cycle of $C(t)$. The output $C^*(t)$ could be as shown in Figure 22b. The straight line approximation is still vaguely similiar to the continuous wave $C(t)$. Now carry the example a step further and set the sampling rate to be exactly equal to the frequency of $C(t)$ or T . The readout device E could record a series of impulses as shown in Figure 22c, and the observer by connecting the impulses with straight lines would conclude that the original input was a pure dc voltage of magnitude $C(\max)$. If the observer had been told that he was to receive sine wave information, he would conclude that a sine wave of zero frequency and $C(\max.)$ dc offset had been transmitted. A more involved case is pictured in Figure 23a where a wave

II, F, Additional Considerations in the Selection of Dynamic Characteristics to Fit Overall System Design (cont.)

Figure 22. Sample Data Reconstruction

II, F, Additional Considerations in the Selection of Dynamic Characteristics to Fit Overall System Design (cont.)

Figure 23. Sampled Data--Signals of Different Frequency

II, F, Additional Considerations in the Selection of Dynamic Characteristics to Fit Overall System Design (cont.)

$C_1(t)$ with period T and a wave $C_2(t)$ with period $T/3$ are each being sampled at a rate $T/2$. Figure 23b shows $C_1^*(t)$ and $C_2^*(t)$ reconstructed by connecting straight lines to the impulses. The reconstructed data clearly indicated a sine wave of period T in both cases.

The above examples serve to introduce the subject of periodicity in sampled data systems. If the above example was extended to a complete frequency response test where inputs comprising a wide band of frequencies were monitored through a sampler operating at a fixed sampling rate w_s , an amplitude spectrum as shown in Figure 24 would result. Figure 24 illustrates the idea of folding, i.e., higher frequencies are folded back into the primary band. For instance, if $w_s/2 = 50$ cps, then 100 cps would resemble 0 cps; 85 cps would resemble 15 cps, 60 cps would resemble 40 cps, etc. The frequencies above $w_s/2$ are folded back into the band between 0 and $w_s/2$. This implies that the sampler cannot distinguish between a signal of $w_s/2$ and a signal of $Kw_s + w_s/2$.

Figure 24. Sampled Data Frequency Spectrum

II, F, Additional Considerations in the Selection of Dynamic Characteristics to Fit Overall System Design (cont.)

To lend mathematical credence to what has been shown intuitively, consider the sampler shown in Figure 25.

Figure 25. Sampled Data Example

$$C^*(t) = C(t) \mu_0(t)$$

where, $c(t)$ = continuous function

$C^*(t)$ = sampled function

$\mu_0(t)$ = train of unit impulses occurring at a frequency equal to the sampling rate

To examine the frequency characteristics it is necessary to transform Equation 8, which is written in the time domain, into the frequency domain

II, F, Additional Considerations in the Selection of Dynamic Characteristics to Fit Overall System Design (cont.)

$$C^*(s) = C(s) \star U_o(s) \quad (\text{Eq 9})$$

where $C^*(s)$ = Laplace transform of $C^*(t)$

$C(s)$ = Laplace transform of $C(t)$

$U_o(s)$ = Laplace transform of $Mo(t)$

$\star$ = indicates complex convolution in the s domain

Since convolution in the s domain involves evaluation of a complicated integral expression, the reader is directed to the references for a formal derivation of the following relationships which are presented here without further proof.

$$C^*(s) = \sum_{n=1}^{\infty} C(nT)e^{-nTs} \quad (\text{Eq 10})$$

$$C^*(s) = \frac{1}{T} \sum_{n=1}^{\infty} C(S + jn\omega_s) \quad (\text{Eq 11})$$

where n = integer

T = sampling period

ω_s = sampling frequency

s = Laplace operator j

e = natural logarithm base

Equation 10 demonstrates the pulse modulated character of the sampler output.

Equation 11 demonstrates the important periodicity characteristic of the sampler output.

Two important criteria emerge from the foregoing discussion:

II, F, Additional Considerations in the Selection of Dynamic Characteristics
to Fit Overall System Design (cont.)

1. The sampling rate should be at least twice the frequency of the signal to be measured.

2. High frequency noise must be filtered or otherwise eliminated to avoid folding back into the primary measurement band and adding to or degrading measure and signals.

In selecting pressure transducer systems, it is sometimes necessary for the design engineer to purposely introduce elements which downgrade frequency response so as to be compatible with the data acquisition system.

II, Technical Discussion (cont.)

G. TEMPERATURE SENSORS--DYNAMIC CHARACTERISTICS

From the viewpoint of dynamic response, temperature sensors fall into two general cases:

1. Sensors, such as thermocouples, in which the measuring element is directly exposed to the measurand.
2. Sensors, such as resistance temperature devices, which have the sensing element enclosed in a well.

In the first case, a bare thermocouple (Figure 26), the response can be considered to be first order in nature and of the form

$$E_o = 1 - \varepsilon^{-t/\tau}$$

where, E_o = output voltage

ε = logarithm base

t = time

τ = time constant

Figure 26. Thermocouple Response

II, G, Temperature Sensors--Dynamic Characteristics (cont.)

In the second case, where the sensing element is encased in a well as shown in Figure 27, the response will be more exactly defined by a second-order expression of the form

$$E_o = 1 - \epsilon^{-t/\tau_1} - \epsilon^{-t/\tau_2}$$

where, τ_1 = time constant

τ_2 = time constant

Figure 27. Response of Sensor in Well

II, G, Temperature Sensors--Dynamic Characteristics (cont.)

The first case may be represented by a transfer function of the form

$$G(s) = \frac{1}{\tau_1 s + 1}$$

where, $G(s)$ = transfer function in
Laplace domain

$S = \sigma + j\omega$ - the complex
Laplace operator

The second case may be represented by a transfer function of the form

$$G(s) = \frac{1}{(\tau_1 s + 1)(\tau_2 s + 1)}$$

In current practice, temperature transducers are usually approximated by a first-order system and dynamic characteristics are specified in terms of one time constant. Generally, the time constant τ is a function of the following

$$\tau = \frac{\rho_w C_w V}{h A}$$

where, τ = time constant

ρ_w = density of thermocouple

C_w = specific heat of thermocouple

V = volume of the thermocouple

h = heat transfer coefficient between
the measurand and the junction

A = surface area of the junction

The heat transfer coefficient h is generally difficult to determine. This coefficient is a function of the fluid properties and velocity of the measurand and the conduit configuration.

II, G, Temperature Sensors--Dynamic Characteristics (cont.)

Since the time constant is a function of the heat transfer coefficient, it is apparent that care must be taken to account for flow conditions of the measurand. This dependence of the time constant on the fluid properties of the measurand makes correlation between different flow rates in different mediums difficult. Prediction of response of a specific thermocouple under general conditions based on tests at specific conditions can be very difficult, especially where wide ranges of temperature and flow conditions are encountered.

Another factor to be considered at cryogenic temperatures is the dependence of the time constant on the specific heat of the probe material. This specific heat C_w changes at very low temperatures and consequently a probe will have at least two time constants that are not related linearly when used in cryogenic measurements.

In summary then, since flow conditions of the measurand affect the time constant of a temperature probe, response cannot be predicted from the physical characteristics of the probe itself; hence, extensive testing under near operational environmental conditions is highly desirable in dynamic studies.

It can be pointed out that when dynamic characteristics are determined under operating conditions, compensation techniques such as discussed in Section II, E can be applied to improve response time. The application of methods presented in the previous sections to determine frequency response may be applied when dynamic characteristics can be defined at a particular operating condition.

II, Technical Discussion (cont.)

H. EXPERIMENTAL APPROACHES TO DEFINE THE DYNAMIC CHARACTERISTICS OF TRANSDUCERS

An ideal calibration would yield information for completely defining the input-versus-output relationship of a transducer under all possible operating conditions. To experimentally verify this calibration would require laboratory testing that would impose all possible operating conditions on the test transducer. It is rarely possible to achieve the ideal calibration so a close approximation must be made.

The usual laboratory calibration furnishes an input-versus-output relationship over a specified operating range. Since dynamic characteristics are best specified in the frequency domain, that is, amplitude versus frequency, a method of applying continuously variable inputs of known amplitude and frequency would be an ideal calibration tool. Another approach would be to subject the transducer to certain limited known inputs and from the data obtained predict behavior under all operating conditions.

The approaches to calibration are therefore divided into two cases:

1. Application of continuously variable inputs of known amplitude and frequency, and
2. Application of limited known inputs and mathematical prediction of behavior in the specified operating range.

It should be kept in mind that a general definition of the input-versus-output relationship of the transducer in the frequency domain, i.e., the transfer function, would allow more complete analysis of the basic measurement problem. If the transfer function of the transducer under calibration could be defined exactly, compensation techniques could be applied with a great deal of accuracy. Definition of the transfer function of the test transducer is therefore a desirable product of the actual calibration procedure.

II, H, Experimental Approaches to Define the Dynamic Characteristics of Transducers (cont.)

1. Pressure Transducers--Sinusoidal Pressure Generators

A number of different approaches to the problem of developing a sinusoidal pressure generator have been attempted to date. The methods investigated for this study are itemized as follows:

1. Siren
2. Rotating disk generator
3. Piston driven generator
4. Piezo-Electric driven generator

The first method simply involves the generation of high power sound waves that serve as a calibrated input to the transducer. This method has not proven successful because of the relatively low amplitudes available.

In the second method a rotating disk with holes placed in the perimeter has been developed at Princeton University and is evidently being used with success at present. Figure 28 shows the basic configuration used. Frequency range of the device is in excess of 50 kc; peak-to-peak amplitudes of nine psi with biases of over 100 psi are possible.

The third method involves the use of a shaker table driven by a power amplifier and controlled by a feedback loop. The system configuration is shown in Figure 29. This system has not been built to date. Mechanical vibration problems might be encountered, but otherwise the approach seems feasible.

II, H, Experimental Approaches to Define the Dynamic Characteristics of Transducers (cont.)

Figure 28. Rotating Disk Sinusoidal Pressure Generator

Figure 29. Piston Driven Pressure Generator

II, H, Experimental Approaches to Define the Dynamic Characteristics of Transducers (cont.)

In the fourth method a Piezo-Electric element furnishes the driving power. The system is pictured in Figure 30. This device has been developed by the Lockheed Corporation and has been evaluated by Aerojet-General engineers.

Figure 30. Piezo-Electric Driven Generator

II, H, Experimental Approaches to Define the Dynamic Characteristics of Transducers (cont.)

Preliminary investigation has shown that the device has the following characteristics:

1. A frequency range up to five kc
2. Peak-to-peak amplitude output of approximately two psi maximum
3. Adjustable bias pressures up to approximately 1000 psi

Further improvement of the device should result in higher output amplitudes.

2. Shock Tubes

The generation of a unit step of pressure is usually accomplished with a shock tube. A shock tube consists of a rigid tube (usually cylindrically shaped) that is divided into two parts by a suitable burst diaphragm. One part is called the compression chamber; the other part is called the expansion chamber (Fig. 31). When the diaphragm bursts, the pressure tends to equalize by forming a shock wave that travels into the expansion chamber. This happens in the following way:

The part of the wave at high pressure travels faster than the low pressure part and thus the wave tends to steepen as it progresses down the expansion chamber. Figures 31b through 31d show the progress down the tube. If the high pressure continued to travel faster than the low pressure, a situation such as shown in Figure 31e would result. This figure of course shows one point at three different pressures, which is physically impossible. The effects of viscosity and heat conduction prevent this occurrence, and in actual conditions a very steep shock front is formed. Detailed analysis shows that the shock front is a stable system that does not change its shape with time. In actual practice, it is possible to generate shock waves with rise times less than one microsec and with a duration of up to 10 millisec. It is possible to obtain these shock waves with accurately known amplitudes up to 5000 psi.

II, H, Experimental Approaches to Define the Dynamic Characteristics of Transducers (cont.)

Figure 31. Shock Tube Pressure Distribution

II, H, Experimental Approaches to Define the Dynamic Characteristics of Transducers (cont.)

3. Temperature Dynamic Calibration

At present, there is no method available for generating sinusoidal temperature variations of known amplitude. Since temperature sensors are generally characterized by behavior that may be defined by first-order differential equations, this is not a serious deficiency. A measurement of the rise time is sufficient to define the frequency response.

The main difficulty involved in the dynamic testing of temperature sensors hinges on the fact that the rise time of the transducer is dependent on the flow conditions of the measurand. A unit step of temperature may be applied by quickly moving the sensor from ambient conditions into a container of boiling water or liquid nitrogen or other medium. Such a procedure does not give data that may be easily used to predict rise times under various flow conditions. A rotating bowl such as pictured in Figure 32 may be used to simulate flow conditions for

Figure 32. Rotating Bowl for Simulating Flow Conditions

II, H, Experimental Approaches to Define the Dynamic Characteristics of Transducers (cont.)

various media from liquid nitrogen to boiling water. The bowl is rotated, forcing the liquid to form around the edges by centrifugal action. The test sensor is then quickly immersed in the moving fluid giving a step input of temperature change under different flow conditions.

For gas environments, wind tunnel setups are used to generate operational flow conditions. Other schemes using quick-opening insulated valves installed in pipes may sometimes be used.

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III. RECOMMENDATIONS

A. GENERAL

1. Recommendations based on this study are divided into three parts:

- a. A proposed system for the dynamic calibration pressure transducers
- b. A proposed system for the dynamic calibration of temperature sensors
- c. Recommendations for future study

The systems proposed would serve to perform dynamic calibration of transducers for evaluation, acceptance, and reinspection testing. All phases of dynamic characteristics would be examined with the proposed systems.

2. Problem Areas

The anticipated areas of design concentration would involve the following:

- a. The addition of environmental conditions to test transducers on the shock tube
- b. Development of a sinusoidal pressure generator
- c. Development of laboratory methods for applying environmental conditions and variable flow rates to temperature sensor testing.

At present, most dynamic testing of pressure transducers has been done at ambient conditions. Application of an environmental chamber to a shock tube will require additional design concentration.

III, A, General (cont.)

Successful approaches to the fabrication of sinusoidal pressure generators have been realized and additional development should yield a workable instrument.

Present methods of simulating flow rates in temperature sensor testing are not completely adequate. Since temperature sensors rise times are sensitive to flow conditions, additional development of this type test equipment is necessary.

B. PROPOSED SYSTEM FOR DYNAMIC CALIBRATION OF PRESSURE TRANSDUCERS

The system for dynamic calibration of pressure transducers is divided into two parts:

A shock tube facility, and

A sinusoidal pressure generator

1. Shock Tube Facility

The shock tube facility presented here was designed by Houston Engineering Research Corporation specifically to meet the needs of the Nerva Program based on requirements of this study. A block diagram of the complete shock tube facility is shown in Figure 33.

III, B, Proposed System for Dynamic Calibration of Pressure Transducers (cont.)

Figure 33. Shock Tube Facility

III, B, Proposed System for Dynamic Calibration of Pressure Transducers (cont.)

The main components are:

- a. Shock-tube pressure-step generator with the associated gas-handling and shock-velocity measuring equipment
- b. Calibrating wave form generator for generating the calibration information, an oscilloscope for displaying the transducer system transient response, and a camera for recording the response
- c. Flying spot scanner and digitizer for reading the photographic recording and converting the recorded wave form to digital form
- d. High-speed paper tape punch for recording the digitized output data
- e. Environmental chamber

The system would have the following features:

- a. A pressure range of 0 to 5000 psia
- b. An environmental temperature range of (-)300°F to + 600°F
- c. Complete digitizing of output data on paper tape in approximately one minute
- d. Gas combinations--air to air, nitrogen to air, and air to helium
- e. Design shock strength of Mach 1.3

III, B, Proposed System for Dynamic Calibration of Pressure Transducers (cont.)

- f. A step rise time of less than 10^{-8} sec
- g. A step duration of about 5 millisec
- h. Accuracy of step amplitude of about $\pm 1\%$
- i. Digitizing repeatability of about $\pm 0.1\%$

The shock tube would be constructed of 1/2 in. steel plate with the interior surface ground to reduce shock-wave attenuation. The heavy steel construction serves to reduce the vibration in the tube itself. Static pressure and temperature measuring devices would provide initial conditions. The passage of the shock would be sensed at two points by flush-mounted pressure transducers, and the time interval would be measured with an electric counter.

The data recording system would have a calibrating waveform generator, an oscilloscope with a sweep time-delay generator, a lens system, and a Polaroid camera to record the transient output. A flying spot scanner senses the transient response record and a digitizer converts the analog signal to digital form. Digital data would be fed to a computer for high-speed computation of the complex transfer function and associated frequency response. It is proposed to heat, or cool, the transducer mounting and the transducer by external means and to insulate the shock tube from the transducer and mounting plate.

III, B, Proposed System for Dynamic Calibration of Pressure Transducers (cont.)

2. Sinusoidal Pressure Generator

The sinusoidal pressure generator chosen is the LMSC sinusoidal pressure generator developed by Lockheed Aircraft Corporation. A block diagram of the system is shown in Figure 34. Basically, the system consists of an oscillator and a power amplifier which feed to the Piezo-Electric driver. The input is monitored for amplitude and phase. Provision is made to apply bias pressure to the transducer. The transducer is connected to the driver, and the output is measured for amplitude and phase and recorded on an oscilloscope. The environmental chamber would be insulated from the driver chamber. Principal features would be:

- a. A frequency range of 0 - 5 kc
- b. An amplitude of 5 psi peak-to-peak maximum
- c. A bias pressure of up to 2500 psi

Figure 34. Sinusoidal Pressure Generator

III, B, Proposed System for Dynamic Calibration of Pressure Transducers (cont.)

The transducer would be driven through a gas medium or liquid medium as required.

The shock tube would provide for determination of rise time, damping ratios, etc. associated with definition of the complex transfer function. Complete analysis of high frequency performance of the transducers would be possible with the transient data available from shock tube testing. The sinusoidal pressure generator would provide for low cost evaluation of actual system configurations (tubing, ports, etc.). This relatively low-frequency analysis could be carried out with great efficiency because of the ease of reducing data from the sinusoidal pressure generator tests.

C. PROPOSED SYSTEM FOR DYNAMIC CALIBRATION OF TEMPERATURE SENSORS

Two methods are presented for the application of step temperature functions to determine the rise times for temperature sensors. The first method is for fluids and is shown in Figure 35. The rotating bowl could contain the appropriate test fluid; by means of a variable drive the effective flow rates could then be obtained. The sensor is immersed from ambient conditions into the rotating fluid to achieve the step.

Figure 36 shows the wind tunnel configuration used to generate step temperature changes in gaseous mediums. The transducer is first exposed to air or gas at ambient conditions. When the quick opening valve is energized, the heated, or cooled, gas flows into the tunnel and subjects the sensor to a step temperature change. Flow conditions would be monitored by means of measuring equipment attached to the tunnel.

III, C, Proposed System for Dynamic Calibration of Temperature Sensors (cont.)

Figure 35. Temperature Sensor Fluid Dynamic Test Setup

Figure 36. Temperature Sensor Gas Dynamic Test Setup

III, Recommendations (cont.)

D. RECOMMENDATIONS FOR FUTURE STUDY

During this study, certain areas were contacted which seem to merit additional investigation. Itemized specifically they are:

1. Investigation of dynamic characteristics of transducers under operating environmental conditions
2. Additional development of techniques to compensate transducer dynamic characteristics (improve frequency response or rise times)
3. Development of techniques to accurately define the complex transfer function of a transducer
4. Development of a suitable sinusoidal pressure generator.

The first three areas apply to transducers in general, while the fourth item is directed specifically to pressure transducers.

1. Most investigation of the dynamic characteristics of pressure transducers to date has been performed at ambient conditions. Certainly changes in important mechanical and acoustical parameters will occur at the extreme environmental conditions expected in nuclear rocket engine applications. Temperature environments should be relatively easy to simulate and the shock tube facility recommended in this report contains this feature. Nuclear environments would need more sophisticated techniques for successful application.

Extensive testing of temperature sensors at different environmental conditions and flow rates is particularly needed since dynamic characteristics of these devices are inherently sensitive to measured conditions.

III, D, Recommendations for Future Study (cont.)

2. The advantages to be realized from the dynamic compensation of transducers is self evident in view of the fact that direct mounting of pressure transducers on nuclear engines is sometimes impossible. It is feasible to increase the frequency response of pressure measuring systems by a factor of from 2 to 3 by application of either analog or digital techniques. Improvement of the rise times of temperature sensors should be simpler to achieve with compensation techniques. It should also be possible by analog methods to improve the rise time of a thermocouple by a factor of 10 to 100.

3. Development of better experimental and analytical methods for determining the complex transfer function of a particular transducer would allow more extensive analysis of the basic problems of dynamic measurement and open new areas for system improvement. In addition, more exact knowledge of the transfer function would greatly facilitate the application of compensation techniques.

4. At present, the shock tube method of obtaining dynamic characteristics of pressure transducers is well established. The problem of long setup times for tests and the extensive data reduction associated with this type testing could be largely eliminated if a usable sinusoidal pressure generator were available. Particularly, the investigation of system tubing configurations could be done quickly and cheaply with a sinusoidal generator. Savings in initial investment and operating expenses would be also realized.

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APPENDIX A

DERIVATION OF EQUATIONS FOR RESPONSE OF A
FIRST-ORDER SYSTEM TO A STEP INPUT

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FIGURE LISTFigure

Typical First-Order System Model

1

A typical first-order system may be used to represent various physical phenomena. The equations which govern the behavior of this mathematical model can be used to predict actual system response. These equations are derived as follows:

Consider the device in Figure 1

where, k = spring constant
 c = coefficient of friction
 $F(t)$ = applied force (as a function of time)
 x = displacement of piston

Figure 1. Typical First-Order System Model

The mass of the piston is neglected:

$$F(t) = c \frac{dx}{dt} + kx \quad (\text{Eq 1})$$

Transforming to the Laplace domain ($s = \sigma + j\omega$),

$$F(s) = csX(s) + kX(s) \quad (\text{Eq 2})$$

$$\frac{X(s)}{F(s)} = \frac{1}{cs + k} \quad (\text{Eq 3})$$

for

$$F(t) = f_0 \mu(t)$$

$$F(s) = \frac{f_0}{s}$$

where, f_0 = amplitude of applied force

$$\mu(t) = 0 \quad t < 0$$

$$\mu(t) = 1 \quad t > 0$$

$$X(s) = \frac{f_0}{c} \left[\frac{1}{s \left(s + \frac{k}{c} \right)} \right] = \frac{f_0}{c} \left[\frac{A}{s} + \frac{B}{\left(s + \frac{k}{c} \right)} \right] \quad (\text{Eq 4})$$

By partial fractions

$$A = \frac{c}{k} \quad B = -\frac{c}{k}$$

Taking the inverse Laplace transform

$$x(t) = \frac{f_o}{k} \left[1 - e^{-\frac{k}{c} t} \right] \quad (\text{Eq 5})$$

note: $\frac{f_o}{k}$ = static displacement of spring under constant force f_o

So if $\frac{x(t)}{f_o/k}$ = relative amplitude = $x_a(t)$

then,

$$x_a(t) = 1 - e^{-\frac{k}{c} t} \quad (\text{Eq 6})$$

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APPENDIX B

DERIVATION OF EQUATIONS FOR RESPONSE
OF A SECOND-ORDER SYSTEM TO A STEP INPUT

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FIGURE LISTFigure

Typical Second-Order System Model

1

Typical Second-Order System Response to Step Input

2

Since pressure transducers can be described by a model such as shown in Figure 1, the equations which govern the behavior of this model are derived as follows:

Consider the device in Figure 1,

where, k = spring
 c = coefficient of friction (viscous)
 $F(t)$ = applied force as a function of time
 x = displacement of piston
 m = mass of piston

$$m \frac{d^2x}{dt^2} = F(t) - c \frac{dx}{dt} - kx \quad (\text{Eq 1})$$

Figure 1. Typical Second-Order System Model

Transforming to the Laplace domain ($s = \sigma + j\omega$),

$$ms^2 X(s) = F(s) - csX(s) - kX(s) \quad (\text{Eq 2})$$

$$X(s) = \frac{F(s)}{ms^2 + cs + k} \quad (\text{Eq 3})$$

$$\frac{X(s)}{F(s)} = G(s) = \frac{1}{m(s^2 + \frac{c}{m}s + \frac{k}{m})} \quad (\text{Eq 4})$$

$$G(s) = \frac{1}{k} \left[\frac{\frac{k}{m}}{s^2 + \frac{c}{m}s + k/m} \right] \quad (\text{Eq 5})$$

$$\text{Let } \omega_n^2 = \frac{k}{m} \quad (\text{Eq 5a})$$

$$2\delta\omega_n = \frac{c}{m}$$

$$\text{so that } \delta = \frac{C}{2\sqrt{km}} \quad (\text{Eq 5b})$$

Rewriting Equation 5,

$$G(s) = \frac{1}{k} \left[\frac{\omega_n^2}{s^2 + 2\delta\omega_n s + \omega_n^2} \right] \quad (\text{Eq 6})$$

For $F(t) = f_0 \mu(t)$ where, f_0 = amplitude of applied force

$$\mu(t) = 0 \quad t < 0$$

$$\mu(t) = 1 \quad t \geq 0$$

$$X(s) = \frac{f_0}{k} \left[\frac{\omega_n^2}{s(s^2 + 2\delta\omega_n s + \omega_n^2)} \right] \quad (\text{Eq 7})$$

for

$$X(s) = \frac{f_0}{k} \left[\frac{A}{s} + \frac{B}{s + \alpha - j\beta} + \frac{\bar{B}}{s + \alpha + j\beta} \right] \quad (\text{Eq 8})$$

$$\alpha = \delta\omega_n \quad \beta = \omega_n \sqrt{1 - \delta^2} \quad \alpha^2 + \beta^2 = \omega_n^2$$

By partial fractions,

$$A = 1 \quad B = \frac{\omega_n^2}{-\alpha + j\beta} \frac{1}{2j\beta} = \frac{1}{2} \frac{\omega_n}{\beta} \epsilon^{j\phi}$$

$$\phi = \tan^{-1} -\frac{\beta}{\alpha} \equiv \tan^{-1} \frac{\delta}{\sqrt{1-\delta^2}}$$

$$x(t) = \frac{f_o}{k} \left[1 - \epsilon^{-\alpha t} \cos \beta t - \frac{\delta}{\sqrt{1-\delta^2}} \epsilon^{-\alpha t} \sin \beta t \right] \quad (\text{Eq 9})$$

Note that $\frac{f_o}{k}$ = static displacement of spring under constant force, f_o .

So, if $\frac{x(t)}{f_o/k}$ = relative amplitude = $x_a(t)$

$$x_a(t) = 1 - \epsilon^{-\alpha t} \cos \beta t - \frac{\delta}{\sqrt{1-\delta^2}} \epsilon^{-\alpha t} \sin \beta t \quad (\text{Eq 10})$$

$$x_a(t) = 1 - (1 - \delta^2)^{-1/2} \epsilon^{-\alpha t} \cos(\beta t + \phi) \quad (\text{Eq 10a})$$

Equation 10 is plotted in Figure 2.

Figure 2. Typical Second-Order System Response to Step Input

To solve for maximum overshoot, $x_a(t)_{\max}$, note that:

$$\frac{dx(t)}{dt} = \int^{-1} sX(s) = \int^{-1} \frac{f_0}{k} \frac{n^2}{s^2 + 2\delta\omega_n s + \omega_n^2} =$$

$$\int^{-1} \frac{f_0}{k} \frac{\omega_n^2}{\beta} \frac{\beta}{(s+\alpha)^2 + \beta^2} \quad (\text{Eq 11})$$

$$\frac{dx(t)}{dt} = \text{velocity} = v(t) = \frac{f_0}{k} \frac{\omega_n^2}{\beta} \epsilon^{-\alpha t} \sin \beta t$$

$$= \frac{f_0}{k} \omega_n (1 - \delta^2)^{-1/2} \epsilon^{-\alpha t} \sin \beta t \quad (\text{Eq 12})$$

The first maximum of displacement comes at the first zero of velocity after the initial one, or:

$$\beta t = \pi = \omega_n \sqrt{1 - \delta^2} t_{\max}$$

$$t_{\max} = \frac{\pi}{\omega_n \sqrt{1 - \delta^2}} = \frac{\pi}{\beta} \quad (\text{Eq 13})$$

Substituting Equation 13 in Equation 10,

$$x_a(t)_{\max} = 1 + \epsilon^{-\pi \frac{\alpha}{\beta}} = 1 + \epsilon^{-\frac{\delta\pi}{\sqrt{1 - \delta^2}}} \quad (\text{Eq 14})$$

Referring to Figure 2,

$$A_1 = \text{overshoot} = 1 + \epsilon^{-\pi \frac{\alpha}{\beta}} - 1 = \epsilon^{\frac{-\delta\pi}{\sqrt{1 - \delta^2}}}$$

$$\ln A_1 = \frac{-\delta\pi}{\sqrt{1 - \delta^2}} \quad (\text{Eq 15})$$

$$2 \ln A_1 = \frac{\delta^2 \pi^2}{1 - \delta^2}$$

$$\delta = \frac{\ln 1/A_1}{\sqrt{2 \ln 1/A_1 + \pi^2}} \quad (\text{Eq 15})$$

To obtain frequency response, rewrite Equation 7 as follows:

$$X_a(s) = \frac{\omega_n^2}{\omega_n^2 \left[\left(\frac{s}{\omega_n} \right)^2 + 2\delta \left(\frac{1}{\omega_n} \right) s + 1 \right]}$$

$$= \frac{1}{(\tau s)^2 + 2\delta\tau s + 1} \quad (\text{Eq 7a})$$

where, $\tau = \frac{1}{\omega_n}$

To evaluate frequency response, substitute $s = (j\omega)$ in Equation 7a.

$$x_a(j\omega) = \frac{1}{(\tau j\omega)^2 + 2\delta\tau(j\omega) + 1} \quad (\text{Eq 7b})$$

To calculate amplitude ratio, x_a at $\frac{\omega_n}{5} = \omega$ and $\delta = 0$:

$$\left| x_a \left(j \frac{\omega_n}{5} \right) \right| = \left| \frac{1}{\frac{j^2}{25} + 1} \right| = 1.04$$

at $= \frac{\omega_n}{5}$ and $\delta = 1$

$$\left| x_a \left(j \frac{\omega_n}{5} \right) \right| = \left| \frac{1}{.96 + .4j} \right| = .96$$

REFERENCES

Pfeiffer, Paul E., Linear Systems Analysis, McGraw-Hill, New York; 1961 (see especially Chap 6)

Langill, A.W., Jr., "Control System Engineering Course Syllabus", Aerojet-General Engineering Technical Course (see especially Chap 8)

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APPENDIX C

RESULTS OF A TEST TO DETERMINE THE DYNAMIC CHARACTERISTICS
OF A STATHAM PRESSURE TRANSDUCER

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ABSTRACT

A Statham Model PA 732-TC-1M-350 (0 to 1000 psia) Pressure Transducer was subjected to a 40 to 0 psig step input of pressure and the output recorded on an oscilloscope. The resulting analog data is analyzed graphically to predict the maximum usable frequency response of the transducer. This result is compared to the predicted frequency response of the transducer, assuming the inlet port may be considered a Helmholtz resonator whose resonant frequency represents the maximum resonant frequency of the transducer.

FIGURE LISTFigure

Actual Test Setup for Transducer Dynamic Response	1
Test Results for Dynamic Response of the Pressure Transducer	2
Inlet Port Configuration	3

I. SUMMARY

A. TEST OBJECTIVES

The purpose of the test was to determine the ring frequency and damping ratio of a Statham Pressure Transducer. These parameters were then used in a formula based on a linear second-order system to predict frequency response.

B. CONCLUSIONS

The maximum usable frequency response (5% degradation of the signal) was determined to be 375 cps. For 2% data, the maximum frequency response is predicted to be 300 cps. The ring frequency was determined to be 1500 cps, and the damping ratio was 0.448. Calculations are presented below.

The test setup used (Figure 1) was far from ideal; however, the results do indicate a distinct ring frequency and decay envelope. It is believed that the results when analyzed graphically are indicative of the frequency response that may be expected from this particular transducer. This viewpoint is reinforced by comparison of the measured ring frequency and the calculated ring frequency based on a Helmholtz approximation. The natural mechanical ring frequency was observed to be approximately 20 kc or greater.

II. PROCEDURE

A. PHYSICAL TEST SETUP

The transducer under test was connected directly to a Wiancko Dynamic Test Calibrator, Model Q1003. The output of the transducer was displayed on a Tecktronic Model 545 Oscilloscope and recorded on a Polaroid camera. Excitation to the transducer was regulated 10v dc current.

II, A, Physical Test Setup (cont.)

Figure 1. Actual Test Setup for Transducer Dynamic Response

II, A, Physical Test Setup (cont.)

The transducer was pressurized to 40 psig; then the test diaphragm of the Wiancko was burst to give a step decrease of from 40 psig to 0 psig. To observe natural mechanical ring frequency, the transducer was pressurized at atmospheric pressure and then gently tapped. The resulting output was observed on the oscilloscope.

B. CALCULATIONS

1. Graphical analysis of analog data to determine frequency response was made.

It was assumed that the transducer could be approximated by a linear second-order system.

Figure 2 shows a sketch representing the actual results. For convenience, the pertinent dimensions from the test data have been placed on Figure 2.

Figure 2. Test Results for Dynamic Response of the Pressure Transducer

II, B, Calculations (cont.)

The ring frequency, ω_n , was calculated as follows:

$$\omega_n = \frac{1}{t_1} = \frac{1}{.66 \text{ millisec}} = 1500 \text{ cps}$$

The damping ratio, δ , was calculated in the following way:

$$\delta = \frac{\ln_e 1/A_1}{\sqrt{21 \ln_e 1/A_1 + \pi^2}} = \frac{\ln 5}{\sqrt{21 \ln 5 + \pi^2}} = .448$$

The amplitude ratio of a linear second-order system response to a step input may be written (in the frequency domain) as:

$$\left| X_a(j\omega) \right| = \left| \frac{1}{\frac{j\omega}{\omega_n}^2 + 2 \frac{j\omega}{\omega_n} + 1} \right| \quad (\text{Eq 1})$$

where, $X_a(j\omega)$ = relative amplitude

and ω = frequency

$$\text{For } \omega = \omega_n/5 \quad \left| X_a(j\omega) \right| = \left| X_a(j \omega_n/5) \right| = \left| \frac{1}{.96 + .179j} \right| = 1.02$$

$$\text{For } \omega = \omega_n/4 \quad \left| X_a(j \omega_n/4) \right| = \left| \frac{1}{.938 + .224j} \right| = 1.04$$

Response, therefore, was up 2% at 300 cps and up 4% at 375 cps.

II, B, Calculations (cont.)

2. Frequency response of the transducer inlet port was calculated as follows:

$$Fr \approx \frac{c}{2\pi} \sqrt{\frac{\pi r^2}{V(L + 1.7r)}}$$

Where, c = speed of sound in in./sec

fr = resonant frequency in cps

r = radius of tubing in in.

V = volume of chamber

$$Fr \approx \frac{1090 \times 12}{6.28} \sqrt{\frac{3.14(.0875)^2}{.02(2.0 + 1.7(.0875))}} \approx 1550 \text{ cps}$$

Figure 3. Inlet Port Configuration

Note: Figure 3 gives the dimensions of the inlet port as taken from actual hook-up and manufacturer's specifications. Chamber volume is approximated from manufacturer's specifications.

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APPENDIX D

EXAMPLE OF THE APPLICATION OF THE
GUILLEMIN IMPULSE APPROXIMATION

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FIGURE LISTFigure

Graphic Example of the Guillemin Impulse Approximation
Typical Frequency Response Curve

1
2

Transient response data obtained from the shock tube testing of pressure transducers is frequently analyzed by the Guillemin Impulse Approximation to determine the frequency response. An example of this technique follows.

(The following relationships, which are pertinent to the discussion that follows, are presented without proof. The reader may check the enclosed references for formal derivations.)

$$C(s) = \int_0^{\infty} C(t) e^{-st} dt = \mathcal{L}\{C(t)\} \quad (\text{Eq 1})$$

by fundamental definition of the Laplace transform

where,

$C(s)$ = Laplace transform of $C(t)$

$C(t)$ = function of time

t = time

s = system parameter

If $s = j \omega$

where, ω = angular frequency

$$j = \sqrt{-1}$$

then:

$$C(j \omega) = \int_0^{\infty} C(t) e^{-j \omega t} dt \quad (\text{Eq 2})$$

Differentiation of $C(t)$ with respect to time corresponds to multiplication of $C(s)$ by s or $C(j \omega)$ by $(j \omega)$.

Therefore:

$$C'(j\omega) = \frac{1}{j\omega} \int_0^{\infty} C'(t) e^{-j\omega t} dt \quad (\text{Eq 3})$$

$$C''(j\omega) = \frac{1}{(j\omega)^2} \int_0^{\infty} C''(t) e^{-j\omega t} dt \quad (\text{Eq 4})$$

where, $C'(j\omega)$ = first derivative of $C(j\omega)$ with respect to time

$C''(j\omega)$ = second derivative of $C(j\omega)$

$C'(t)$ = first derivative of $C(t)$

$C''(t)$ = second derivative of $C(t)$

Figure 1b represents a hypothetical transient response curve obtained by applying a unit step to a system as shown in Figure 1a. In Figure 1c, $C(t)$ has been approximated by a series of straight lines, and the new function denoted $C^*(t)$. $C^*(t)$ is differentiated graphically (Figures 1d and 1e) to obtain a series of impulses (note the ease of differentiation of a straight line function). Since differentiation in the time domain corresponds to multiplication by $(j\omega)$ in the frequency domain (Equation 3), we may write:

$$C^*(j\omega) = \frac{1}{(j\omega)^2} \int_0^{\infty} (C^*)''(t) e^{-j\omega t} dt \quad (\text{Eq 5})$$

$(C^*)''(t)$ as shown in Figure 1e is a series of impulses and may be expressed as

$$(C^*)''(t) = \sum_{k=1}^n a_k \mu_0(t - t_k) \quad (\text{Eq 6})$$

Figure 1. Graphic Example of the Guillemin Impulse Approximation

where, a_k = amplitude of impulse, and
 $\mu_o(t - t_k)$ = unit impulse function

since:

$$C^*(j\omega) = \frac{1}{(j\omega)^2} \int \left\{ (C^*)''(t) \right\} \quad (\text{Eq 7})$$

by Equations 5 and 1, and

$$\begin{aligned} \int \left\{ (C^*)''(t) \right\} &= \int \left\{ \sum_{k=1}^n a_k \mu_o(t-t_k) \right\} \\ &= \sum_{k=1}^n a_k e^{-j\omega t_k} \end{aligned} \quad (\text{Eq 8})$$

then,

$$C^*(j\omega) = \frac{1}{(j\omega)^2} \sum_{k=1}^n a_k e^{-j\omega t_k} \quad (\text{Eq 9})$$

$C^*(j\omega)$ may now be written by inspection from Figure 1e as

$$C^*(j\omega) = \frac{1}{(j\omega)^2} (1 - 1.6e^{-j\omega^{1.5}} + .866e^{-j\omega^{3.0}} - .266e^{-j\omega^{4.5}}) \quad (\text{Eq 10})$$

To determine frequency response, the transfer function, $G(s) = \frac{C(s)}{E(s)}$ must be defined for $S=j\omega$ at all values of ω . Since E is a unit step, then

$$E(s) = \frac{1}{s}, \quad E(j\omega) = \frac{1}{j\omega}$$

Therefore:

$$G^*(j\omega) = \frac{C^*(j\omega)}{E(j\omega)} = \frac{1}{j\omega} \left(1 - 1.6e^{-j\omega^{1.5}} + .866e^{-j\omega^{3.0}} - .266e^{-j\omega^{4.5}} \right) \quad (\text{Eq 11})$$

Since $G^*(j\omega)$ = the amplitude ratio expressed as a function of frequency,

$$\text{then, } |G^*(j\omega)| = \left| \frac{1}{j\omega} (1 - 1.6e^{-j\omega^{1.5}} + .866e^{-j\omega^{3.0}} - .266e^{-j\omega^{4.5}}) \right| \quad (\text{Eq 12})$$

= The absolute value of the amplitude ratio at any frequency, ω .

By substituting different values of ω into the right hand side of Equation 12, the corresponding value of amplitude ratio may be found, and the frequency response graph shown in Figure 2 may be generated. The frequency response in Figure 2 is the

Figure 2. Typical Frequency Response Curve

exact frequency response of the straight line approximation of Figure 1c. The only approximation introduced was the representation of the original respons (Figure 1b) by the straight line curve in Figure 1c.

In practice, Equation 12 is usually expressed in rectangular form and programmed for digital computer solution.

REFERENCES

Tallman, Charles R.	"Transducer Frequency Response Evaluation for Rocket Instability Research," <u>ARS Journal</u> No. 29, 1959
Guillemin, E. A.,	"Computational Techniques Which Simplify the Correlation between Steady State and Transient Response in Filters and other Networks" Proceedings of the National Electronics Conference, 1953; Volume 9, 1954
Truxal, John G.,	<u>Control System Synthesis</u> , McGraw-Hill, New York, 1955

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APPENDIX E

SOME COMMENTS ON A DYNAMIC TEST
PERFORMED ON A DIFFERENTIAL PRESSURE TRANSDUCER

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FIGURE LISTFigure

Low-Frequency Shock Tube Test Setup	1
Response of Differential Pressure Transducer	2
Model of Differential Pressure Transducer	3
Electrical Analog of a Differential Pressure Transducer	4
Response of Electrical Analog Components	5
Response of Differential Pressure Transducer Electrical Analog to Step Input	6

During the time this study was being conducted, a number of transient pressure tests were performed on evaluation transducers for the Nerva Program. One of the tests was performed on a differential pressure transducer. The arrangement of the test equipment and the test procedure are outlined as follows:

1. The test setup is shown schematically in Figure 1.

Figure 1. Low-Frequency Shock Tube Test Setup

2. The high side was subjected to a pressure step with the low side exposed to ambient pressure.
3. The low side was subjected to a pressure step with the high side exposed to ambient pressure.
4. Both the high side and low side were simultaneously exposed to a pressure step.

The transient response of each test is shown in Figure 2.

Figure 2. Response of Differential Pressure Transducer

The complete results of the test are presented in a report from M. A. Henry of the Aerojet Solid Rocket Plant technical staff at Sacramento, California.*

The purpose here is to show how an approximate analytical model may be verified by experimental results.

Consider the model in Figure 3a. The transducer consists of three main parts:

1. A high-side inlet port and cavity
2. A low-side inlet port and cavity
3. A diaphragm and connected mechanical elements (referred to as the mechanical sensing system).

As a first approximation, each of these three parts may be considered as a spring mass system interconnected as shown in Figure 3b. If the mechanical sensing system has a very high natural frequency with respect to the acoustical ring frequencies of the two cavities, then its effect might be neglected and the model shown in Figure 3c would result. In addition, since the low-side cavity is relatively large, a first-order system approximation may be made and the mass, M_1 , neglected.

With the mechanical model of Figure 3c chosen, it remains to define the behavior of the point x_3 , as this would represent the output of the transducer. An electrical analog of the model in Figure 3c would be easier to work with. Such an electrical analog is shown in Figure 4.

* Aerojet-General Corp. Test Engineering Report - "Dynamic Response Test of Statham Model 732-TC-100-350 Pressure Transducer", M. A. Henry, Technical Staff, Measurement Engineering Department, Test Division, Solid Rocket Plant, Sacramento, Calif.

Figure 3. Model of Differential Pressure Transducer

Figure 4. Electrical Analog of a Differential Pressure Transducer

The electrical equivalents of the mechanical quantities are:

- E is proportional to F
- R1 is proportional to c_1
- R2 is proportional to c_2
- C1 is proportional to K_1
- C2 is proportional to K_2
- L2 is proportional to M_2
- V is proportional to displacement of x_3

Because of the relatively small cavity on the high side, an underdamped response would be expected; therefore, the circuit comprised of R_2 , L_2 , and C_2 would be defined to have the following characteristics:

Second-order, underdamped, oscillatory-type response

Resonant frequency equal to first acoustical resonance of high-side cavity

Damping ratio equal to acoustical damping of high-side chamber.

These are graphically illustrated in Figure 5a. Likewise, since the low side cavity is represented by a first-order system, the following characteristics would apply to the circuit comprised of R_1 , C_1 :

First-order exponential rise response

Rise time equal to acoustical response of low side cavity.

These are graphically illustrated in Figure 5b.

With the circuit elements of Figure 4 so defined, the response of voltage V to a step input of voltage would be as shown in Figure 6. It can be seen that this response does approximate the response of the actual transducer (Figure 2).

In summary, if the dynamic characteristics of each element in a system can be approximated, it is possible to approximate the actual output of the elements when interconnected to form a system. Such approximations serve as useful guideposts to setting up the format for actual dynamic calibration procedures.

RESPONSE OF R_2, L_2, C_2 TO STEP INPUT, E .

(a)

RISE TIME = ACOUSTICAL RESPONSE
OF LOW SIDE CAVITY

RESPONSE OF R_1, C_1 TO STEP INPUT E

(b)

Figure 5. Response of Electrical Analog Components

Figure 6. Response of Differential Pressure Transducer Electrical Analog to Step Input