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THE DESIGN OF A FOCUSING AND ANALYZING SYSTEM
FOR THE 27" CYCLOTRON BEAM

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1. INTRODUCTION

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In planning for the utilization of the rebuilt 27" cyclotron for precision scattering experiments, we were faced with the common problem of obtaining an intense collimated beam of approximately monoenergetic ions at a location which could be shielded from the background radiation from the cyclotron. The requirement of high intensity suggested that the beam be focused, and high energy resolution required some sort of energy dispersion.

Both these properties, i.e., two-directional (or "double") focusing and energy analysis, can be obtained simultaneously by the use of a sector-shaped, uniform magnetic field (See Part III). However, a sharp image can be obtained only from a sharply-defined object, and it is well known that the apparent source of ions in the cyclotron does not have a position as sharply defined as the desired image. This suggests, then, that in order to produce a suitable object for the analyzing magnet, we introduce a second magnet whose sole function is to focus as much of the beam as possible on a precision slit. To use the optical analogy, we first use a "condenser" lens to focus as much light as possible on a slit, and we next use this illuminated slit as the object for an "analyzer," which separates the light into component frequencies and brings the rays of a given frequency to a sharp focus.

The space requirements which the magnet system must satisfy can be described with the aid of Figure I, which shows the proposed locations of the cyclotron, shield wall, and analyzer image. The condenser should be as near to the beam exit port as possible without producing fringe field interference, in order that the

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horizontal spread of the beam be as small as possible. Also, it is desirable that the condenser image point be within the shield enclosure, or at least inside the shield wall, so that shielding is provided around the defining slit. (A considerable amount of undesirable radiation will be produced by that part of the beam intercepted by the slit system.) It is clear that space limitations require the analyzer to be outside the enclosure, but it is also desirable that the analyzer image distance be large, in order to reduce the angular spread of the beam at the image point. This argues for placing the analyzer near the shield wall. More general considerations of convenience, which we shall not discuss, show the desirability of placing the analyzer near the south-east wall of the room.

The choice of the type of analyzer to use was greatly simplified by the existence in this laboratory of a C-shaped magnet, having a pole face area of about 1000 cm², capable of producing a uniform field of up to 14 kilogauss across a 4 cm gap. This magnet was, in fact, being used as an analyzer with the "old" cyclotron, and it seemed likely that only minor modifications would be required to produce a suitable analyzer for the new system. Consequently, it was decided early to design a magnet system, if possible, which would enable us to use this C-magnet (modified) as the analyzer.

The type of magnet to use as a condenser was not so easily settled upon. Our first inclination was to use another sector-shaped magnet, thus obtaining moderate energy resolution as well as focusing. Several possible designs were made and considered,

but all had several disadvantages in common. Their size would have had to be comparable with the existing C-magnet (about 3T), which seemed uncomfortably large for the function to be fulfilled. This bulk was partly due to the highly asymmetric pole piece required, which further complicated matters by the unconventional and inconvenient vacuum plumbing which would be necessary. The bulky magnet yoke also would not permit the magnet to be near the cyclotron port, which fact, in turn, then required larger condenser pole pieces in order to accommodate the horizontally spreading beam. Because of these considerations, thought was given to other types of condenser.

Several ideas had extremely short lives, primarily because the effects of the magnets were not subject to practical mathematical analysis. We then considered the possibility of a magnet system based on the so-called Strong Focusing Principle, employing successive reversals of field gradients to focus the beam to a point on the beam axis, which remains undeflected. Approximate calculations indicated that a suitable condenser could be constructed in this way, and that the weight and power requirements would each be less than the corresponding sector-magnet requirements by at least a factor of ten. These promising initial results were immediately followed by detailed calculations and design.

Once the types of analyzer and condenser were decided upon, we were able to plan a general layout of the equipment and then

FIGURE I-1(a)

SCALE FOR BOTH FIGURES $\sim 1"=8'$

FIGURE I-1(b)

to try to fit this layout by the appropriate design of the individual magnets. Such a proposed arrangement is shown in Figure I - 1(b). This arrangement, of course, served merely as a guide and not as the source of a set of rigid specifications. Also, we tried to keep in mind the desirability of a flexible system, i.e., one which could fit in the room satisfactorily in a number of different ways, as insurance against the need for changes in the future.

II. CONDENSER

A. Strong Focus Principle:

The strong focusing principle which we have used is that described by Courant, Livingston and Snyder primarily for use in designing high energy accelerators. The reader is referred to their paper (1) for details of the principle since only a few equations will be quoted herein. Following Courant et.al., we consider a beam of particles of momentum B_p traveling in the x direction in a magnetic field having components $B_z = 0$, $B_y = kz$ and $B_x = ky$ (where k is a constant).

The equations of motion are then

$$\frac{d^2 y}{dx^2} + \left\{ \frac{1}{B_p} \frac{dB_x}{dy} \right\} y = 0 \quad \text{or} \quad \frac{d^2 y}{dx^2} + Ky = 0 \quad \text{II-1}$$

$$\frac{d^2 z}{dx^2} - \left\{ \frac{1}{B_p} \frac{dB_y}{dz} \right\} z = 0 \quad \text{or} \quad \frac{d^2 z}{dx^2} - Kz = 0 \quad \text{II-2}$$

(1) Courant, E. D., Livingston, M. S. and Snyder, H. S.
Phys. Rev. 88, 1190 (1952)

where we define $K = \frac{1}{B_p} \frac{\partial B_z}{\partial y} = \frac{1}{B_p} \frac{\partial B_y}{\partial z}$

II-3

By solving these equations we express the lens properties of the magnetic system in terms of this parameter K . Note that the results will be energy independent provided we change the equal field gradients to compensate for changes in the particle momentum

B_p , thus keeping K constant. The lens properties may be illustrated as in Figure II-1

Here the entering beam is parallel to the axis and distant y_0 units from it. If the length of the inhomogeneous field is l then this beam will cross the axis at F , thus defining the effective focal length f of the system as well as the position of the principle plane P .

If we now define a new parameter $\lambda = K^{\frac{1}{2}} l$ and agree to measure all distances in units of the magnet length l

then we must find solutions to the equations

$$\frac{d^2 y}{dx^2} + \lambda^2 y = 0 \quad \text{II-4}$$

$$\frac{d^2 z}{dx^2} - \lambda^2 z = 0 \quad \text{II-5}$$

The change in sign in these equations means, unfortunately, that a doubly divergent magnetic field with uniform and equal gradients is convergent in one plane but divergent in a perpendicular plane. Figure II-1 represents the convergent plane. To determine the locations of the foci and principal planes of the lens, we solve equations II-4,5 for the equations of the emergent rays and find the point of intersection with the axis of a ray initially parallel to, but displaced from, the axis.

From II-5, for example, we find

$$z = \frac{1}{2} z_0 \{ \lambda (e^{\lambda} - e^{-\lambda}) (x-1) + e^{\lambda} + e^{-\lambda} \}$$

as the equation for the convergent ray which was initially displaced z_0 units from the axis. This ray intersects the $z = z_0$ line at $x = x'_p$ say where x'_p is the abscissa of the principal plane.

$$x'_p - 1 = \frac{2 \cdot e^{\lambda} - e^{-\lambda}}{\lambda (e^{\lambda} - e^{-\lambda})} = \frac{1 - \cosh \lambda}{\lambda \sinh \lambda}$$

It is convenient, however, to measure the position of the principal plane in terms of a distance measured from the back face of the magnet unit; i.e., $x = \ell$ or $x = 1$ in our units. Making

this change, and introducing the superscript or subscript to indicate quantities in the divergent plane and in the convergent plane, we have for this distance

$$x_P^D = \frac{\cosh \lambda - 1}{\lambda \sinh \lambda} = f_D (1 - \cosh \lambda)$$

since $f_D = -\frac{1}{\lambda \sinh \lambda}$ measured from x_P^D as in

Figure II-1. Similarly, considering equation II-4 we get

$$x_P^C = \frac{1 - \cos \lambda}{\lambda \sin \lambda} = f_C (1 - \cos \lambda)$$

whereas above f_C is measured from x_P^C and is

$$f_C = \frac{1}{\lambda \sin \lambda} \quad \text{II-7}$$

In order to achieve focusing in all azimuthal planes, we note from elementary optics that if we have two thin lenses with distance δ between their principal planes, then the focal length of the combination F is given by

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{\delta}{f_1 f_2} \quad \text{II-8}$$

If we have $f_1 = -f_2 = f$ say, then

$$F = f^2 / \delta$$

and is positive so that the combination of two converging and diverging lenses is convergent.

If we note that for small $|f_0| = |f_c| \approx \frac{1}{\lambda^2}$

we are led to the consideration of two identical magnetic lenses rotated through 90° with respect to one another.

It is immediately clear, however, that the resultant lens combination will be, in general, strongly astigmatic since, as noted, the position of the principal plane in each magnetic lens depends on whether the lens is converging or diverging. To restate this as it applies to our problem, if the first lens focuses horizontally it will defocus vertically, but the focusing and defocusing do not occur with the same focal length, and furthermore this focal length is measured from a different position in each case.

The major problem then lies in the design of a pair of magnetic lenses which will satisfy our boundary conditions and yet be nonastigmatic, i.e., the image point in both the horizontal and vertical planes should coincide.

Before continuing to the actual design of the lens it is worthwhile to indicate some of the qualitative advantages which a pair of strong focus lenses will have over the more conventional sector magnet. These will be listed in the following section.

B. Specific Qualitative Advantages:

1. Space will be at a premium within our shielded enclosure. Use of the lens pair will reduce the space required for the condenser by a factor of 10 or more.

2. Preliminary calculations of the pole profile for a suitable sector condenser indicated that the amount of iron,

copper, and excitation power required would be at least 20 times that required for the lens pair.

3. The vacuum plumbing for the lens pair is extremely simple, consisting only of an appropriate straight length of tubing whereas with a sector, in order to minimize power requirements, the gap tank must be made with iron top and bottom--with consequent machining and gasketing intricacies and problems.

4. Once designed for a particular particle B_p a sector magnet cannot be readily adjusted in an attempt to maximize the focused current since for any given B_p , change in B will result in a change in ϕ , the total angle of beam deflection (see Section III) which in turn would require that all equipment following the condenser would have to move to correspond. With the lens pair, since there is no deflection, we can readily change both field B and intermagnet spacing d without affecting the position of the image slit or analyzing magnet and associated equipment.

6. Derivation of Practical Design Equations.

In order to calculate the requirements for a nonastigmatic lens at least to a first approximation, it is reasonable to replace each of the magnetic lenses by an "equivalent" thin lens of equal focal length with center at the principle plane of the magnetic lens and then to use the simple optical thin lens equations.

Before beginning the actual calculation we should point out that although we have no exact knowledge of the effective particle source in the cyclotron, simple arguments based on considerations

of the vertical and radial oscillations of the particles in acceleration orbits indicate that the effective "point" source in the vertical plane does not coincide with that in the horizontal plane and is such that the vertical-plane object distance $\checkmark$ is greater than the corresponding horizontal-plane object distance H (see Figure II-2). This, then, will be one of the boundary conditions which we must satisfy. The second is simply that the common image be real and beyond the magnets themselves. We then consider the horizontal and vertical planes separately but require that the image be the same distance from the back of the second magnet unit in each case. Since it is known that the angular spread is greater in the horizontal than in the vertical plane, we make the first lens convergent in the horizontal plane. In a subsequent section we shall calculate the resultant vertical defocusing and shall show that our system has sufficient vertical aperture to prevent loss of beam from this cause before the second lense reconverges it vertically.

Let us call case (a), the horizontal plane study and case (b) the vertical.

From the figure II-2

$$A = d + x_p^C + 1 - x_p^D \quad \text{II-9}$$

$$B = d + x_p^D + 1 - x_p^C \quad \text{II-10}$$

$$A + B = 2(d + 1) \quad \text{II-11}$$

$$A - B = 2(x_p^C - x_p^D) \quad \text{II-12}$$

For a non-astigmatic combination we also note

$$q_v = q_H + x_p^C - x_p^D = q_H + \frac{A - B}{2} \quad \text{II-13}$$

By applying the thin lens approximation to the two combinations, we get expressions for

Case (a)

$$q_H = f_o \frac{A(H - f_c) - Hf_c}{A(H - f_c) - H(f_c + f_o) + f_c f_o} \quad \text{II-14}$$

Case (b)

$$q_v = f_c \frac{B(V - f_o) - Vf_o}{B(V - f_o) - V(f_c + f_o) + f_c f_o} \quad \text{II-15}$$

Substituting II-14 and II-15 into II-12 we get

$$f_c \frac{B(V - f_o) - Vf_o}{B(V - f_o) - V(f_c + f_o) + f_c f_o} - f_o \frac{A(H - f_c) - Hf_c}{A(H - f_c) - H(f_c + f_o) + f_c f_o} = \frac{A - B}{2} \quad \text{II-16}$$

After considerable algebraic manipulation this becomes

$$V = \frac{CH + D}{EH + F} \quad \text{II-17}$$

where

$$C = f_D \{ 2f_c^2 f_D + f_c (f_c - f_D)(A+B) - f_c [(A+B)(A-B) + 2AB] \\ + B(A+B)f_D + AB(A-B) \}$$

$$D = f_c f_D \{ (A-B)f_c f_D + A(A+B)f_c - B(A+B)f_D - AB(A-B) \}$$

$$E = f_c^2 (A+B) - f_D^2 (A+B) + 2f_c f_D (A-B) - f_c [(A+B)(A-B) + 2AB] \\ - f_D [(A+B)(A-B) - 2AB] + AB(A-B)$$

$$F = f_c \{ 2f_c f_D^2 - f_D (f_c - f_D)(A+B) + f_D [(A+B)(A-B) - 2AB] \\ + A(A+B)f_c - AB(A-B) \}$$

From equation II-17, by assuming values for λ and d we are able to plot curves which give pairs of values of V and H which will give non-astigmatic operation.

Corresponding to each of these pairs we are able, using equations II-13, II-14 and II-15, to calculate q' , the distance from the back of the second magnet to the image plane, from two separate equations, thus checking the overall algebra.

2) In this particular application, there are additional restrictions which may be used. These are the following:

1. V must be greater than H , and for practical reasons concerning the cyclotron layout, both must be greater than about 100 cm.

2. q , the distance from the back of the magnet to the focus must be greater than about 100 cm and must, of course, be positive.

3. In order to achieve a focus at all, both q_v and q_H must be positive. Applying this condition to equations II-14 and II-15 we find that the following limitations hold:

Case (a) (i) for $A > f_c$ or $A < f_c + f_D$

$$\frac{A - f_D}{A - f_c - f_D} < H < \frac{A}{A - f_c} \quad \text{II-18}$$

(ii) for $f_c + f_D < A < f_c$

$$H > \frac{A - f_D}{A - f_c - f_D} \approx f_c$$

Case (b)

$$\text{for } f_c + f_D > B > f_D$$

$$V > \frac{B - f_c}{B - f_c - f_D} \approx f_D$$

II-19

There is only this single limiting expression for V instead of two as was the case for H which is of interest.

Use of these equations enables us to restrict the range of values of H and V considered in the actual calculations. As mentioned earlier, it is possible to compute the spread in the

vertical plane due to defocusing in the first lens of the pair.
The situation in the vertical plane is as shown in Figure II-3.

$$\tan \beta = \frac{Y}{q+B} = \frac{V \tan \alpha}{q} = \frac{(V+f_0) \cdot V \tan \alpha}{V f_0} = \frac{(V+f_0) \tan \alpha}{f_0}$$

$$Y = \left\{ \frac{V f_0}{V+f_0} + B \right\} \frac{V+f_0}{f_0} \tan \alpha = \left\{ B + V + \frac{B V}{f_0} \right\} \tan \alpha$$

--- II-20

This equation gives an upper limit on Y since the beam enters the second lens and focusing begins before it traverses the entire distance B to the second equivalent lens.

D. Calculations

Detailed calculations have been carried out for $\lambda = 0.5$, 0.75 and 1.0. For λ below 0.5 the lens is relatively weak and not of interest in this application. For λ above 1.0, the absolute magnitude, and the range of acceptable values, of μ are small, thus limiting the usefulness of the system. In each case we have calculated, for various lens separation values d , pairs of V and H which result in non-astigmatic operation as well as q' the distance from the back of the magnet unit to the focal plane of the system. These results are shown graphically in Figures II-4,5,6,7,8,9.

As an example, we include the calculation for $\lambda = 0.5$, $d = 0.1$.

For $\lambda = 0.5$

$$f_c = 4.1718$$

$$f_D = -3.838$$

$$x_p^c = 0.5106$$

$$x_p^D = 0.4897$$

$$A = d + 1.0209$$

$$B = d + 0.9791$$

Substituting these values into II-17 gives

$$C = 61.323d^2 - 133.13d + 318.26$$

$$D = -255.83d^2 - 511.88d - 245.34$$

$$E = -15.978d^2 - 26.635d - 11.988$$

$$F = 66.657d^2 + 389.19d + 835.24$$

Substituting $d = 0.1$ we get

$$V = \frac{305.56H - 299.09}{-14.812H + 874.82}$$

$$A = 1.1209$$

$$B = 1.0791$$

II-21

Note that for $V = 0$, $H = 0.9788$

$$V = \infty, H = 59.062$$

For positive focal lengths

$$H > \frac{A - f_D}{A - f_c f_D} f_c = 26.22$$

Thus the useful range of H lies between 26.22 and 59.06. As an example we include calculations for $H = 40$.

$$V = 42.229 \text{ from equation II-21}$$

From equation II-14

$$q_H = 45.046$$

$$q' = q_H - x_p^D \text{ (from Figure II-2)}$$

$$= 44.556$$

As a check on the algebra:

From equation II-15

$$q_v = 45.069$$

$$\text{From equation II-13 } q_H + \frac{A-B}{2} = q_v \text{ Here } q_H + \frac{A-B}{2} = 45.067$$

while q_v is 45.069, thus checking within the accuracy of our calculations.

On each of the families of curves for V as a function of H , we have drawn the $V=H$ line. The intersection of this line with the curves then gives the focusing conditions for the special case of a point source. We have also drawn in a line which divides the V,H plane into regions which give positive and negative values of q' . Similarly, in the q',H plots we have drawn in the curve whose intersections with the q' curves give focusing conditions for a point source as well as the curve whose intersections give focusing conditions for a source which has a finite horizontal, but an infinite vertical object distance. This would roughly correspond to a vertical line source neglecting vertical crossfire.

It is obvious from the curves that for a given particle rigidity and field, i.e., λ constant, the focusing properties are very sensitive functions of the inter lens spacing d .

E. Design Values

The maximum energy protons for which this system is designed is about 8 Mev. As a design value for the magnetic rigidity $B\rho$ we have taken, therefore $B\rho = 4 \times 10^5$ gauss-cm.

On the basis of our calculations, we have decided on a design based on $\lambda = 0.75$ as being best suited to our boundary conditions. In arriving at a final design we used a cyclic process, choosing one variable, computing the remainder, and repeating the process until the complete set of parameters seemed practical from a physical point of view and satisfied our boundary conditions.

The beam dimensions on entering the magnet unit were taken to be less than 2 cm in height and less than 6 cm in width. This information, which we had obtained during our work on the extraction system for the old machine, determines the requisite aperture of the lens system. As noted in Reference (1), a doubly divergent magnetic field with equal and constant gradients is obtained in a quadrupole magnet with hyperbolic pole profile, (Fig. II-10), providing that the magnet poles are magnetic equipotentials. Having decided to use this form of magnet, we had then to calculate the equations of the rectangular hyperbola which would form the pole profile and give adequate aperture.

The parameter a shown on the figure is that used in Reference (1). The effective aperture of a magnet of this sort is a circle of diameter $2a$ centered at the origin, and the field gradient is given by Fig. II-10. Note that

$$c = \frac{a}{2}$$

$$\frac{2B}{a} = \frac{\partial B_z}{\partial y} = \frac{\partial B_y}{\partial z}$$

II-21

If we use 50% safety factors, we want a lens which will accommodate an elliptic tube with major and minor axes 4.5 and

1.5 cm respectively. We wish to find a value C such that the curves

$$xy = c^2 \quad \text{--- magnet poles}$$

and

$$\frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2} = 1 \quad \text{--- beam tube}$$

are tangent at their points of intersection. This requirement gives $c^2 = \frac{\alpha\beta}{2} = 0.844 \text{ cm}^2$. With this as a guide we apply an extra factor of safety and take $\alpha = 3$, $c = 1.5$, hence the magnet poles are given by $xy = \pm 2.25 \text{ cm}^2$ (1) II-22

For reference, the equation of the contained ellipse, the axes of which are in the ratio 3:1 is

$$2x^2 + 18y^2 = 27\text{cm}^2$$

Our final choice of parameters were $\alpha = 0.15$, $H = 6.5$, $d = 0.45$. The choice of $H = 6.5$ fixes the length of the magnet unit. Measurements have shown that the effective length of the magnets is as much as 20% greater than the physical length. (2) The physical length of our magnet units is 18.1 cm and we have

(1) Morley Machine Company, Rochester, N.Y., produced milling cutters conforming to this equation which were used in generating the pole faces. Subsequent work has shown that circular arcs would have been satisfactory.

(2) David Nelson--private communication.

taken 20 cm as the effective length. Our design figures then are

$$d = 0.45 = 9.0 \text{ cm}$$

$$H = 6.5 = 130 \text{ cm}$$

$$V = 8.87 = 177 \text{ cm}$$

$$q' = 6.10 = 122 \text{ cm}$$

Using equation II-20 we find (see Figure II-2)

$$Y = \left\{ B + V + \frac{BV}{f_D} \right\} \tan \theta$$

$$= 0.35 \text{ cm}$$

$$B = 28.06 \text{ cm}$$

$$V = 177.0 \text{ cm}$$

$$y \approx 3 \text{ mm}$$

$$f_D = 32.43 \text{ cm}$$

This insures that we have an adequate aperture in our beam tube.

Recalling that $\lambda = 0.75 = K^{\frac{1}{2}} l$ we have that

$$K = \frac{1}{B_p} \times \frac{\partial B_z}{\partial y} = \frac{1}{B_p} \times \frac{2B}{a} = \left(\frac{\lambda}{l} \right)^2 \quad \text{or}$$

$$B = \left(\frac{\lambda}{l} \right)^2 \times \frac{a}{2} \times (B_p)$$

$$\approx 1 \text{ kilogauss}$$

This low field avoids saturation difficulties in the magnet iron and high excitation power requirements. Furthermore, it enables us to dispense with water cooling in the windings. Note that for a given particle and energy λ is proportional to $B^{\frac{1}{2}}$, thus giving a convenient parameter for adjustment in use.

Knowing the required B, the ampere turns of excitation were computed from

$$NI = \frac{10}{4\pi} \frac{l_i}{\mu_i}$$

where l_i is the length of the section of the magnetic circuit and μ_i its permeability. A field of 1 kilogauss requires $NI = 5000$ ampere turns. As a safety factor, we have designed for 6000 ampere turns.

Since we had on hand a large stock of #22 heavy formax coated magnet wire, we have designed coils using this wire. The cross sectional area available for coils around each pole in our final design is 2.67 cm^2 . For a random wound coil this permits about 3000 turns per coil or 12000 turns per unit, thus requiring a convenient maximum current of 500 ma. If we allow 1000 cm per ampere, we could safely operate the coils at 625 ma. The resistance of each coil at 20°C is about 72.5 ohms.

Each coil will have exactly 3000 turns and the coils in each unit are connected in series since otherwise we would have extreme difficulty in maintaining uniform gradients. The excitation will be supplied by a 500 volt, 2 kw motor generator in series with a bank of 6AS7 pass tubes. A voltage from a series shunt and a comparison voltage from an adjustment pot on the control desk will be applied to a 6SL7 difference amplifier which drives the 6AS7 grids.

For reasons of convenience we have designed a 12" diameter cylindrical yoke to hold the poles. In use these cylinders

will be mounted so that they can be moved relative to one another (changing d) or moved as unit along the beam axis (changing H and V).

From our design dimensions we estimate that the volume of iron will be less than 0.21 cu. ft. per unit and that of copper will be less than 0.14 cu. ft. per unit. This gives an estimated total weight for the pair of about 355 pounds which is to be compared to an estimated 3 tons for a sector condenser.

III. ANALYZER

(A) Focusing Principle.

The manner in which double focusing can be obtained by using a sector-shaped magnet can be seen qualitatively as follows. (For the full treatment of this subject, see the paper by Cross, Rev. Sci. Instr. 22, 717 (1951).) Consider a

beam of charged particles entering the gap between horizontal sector-shaped pole pieces. The particles will follow horizontal circular orbits in the uniform vertical field, and, if the beam is initially divergent horizontally, those particles nearest the vertex of the sector will undergo the least total angular deflection, and the particles farthest from the vertex will undergo the greatest deflection. This is simply because of the relative amounts of time spent in the field. It is clear, then, that the uniform field produces a horizontal focusing effect.

Since a uniform vertical field can exert no vertical force upon an ion, the vertical focusing effect must come from the fringe field (primarily outside the boundaries of the poles.) Suppose the path of an approaching ion is above the median plane of the gap and makes an angle ϵ_1 with the normal to the pole piece boundary. If $\epsilon_1 > 0$ (we measure positive ϵ_1 toward the sector vertex), the reader can easily convince himself that the effect of the curved fringe field lines will be to exert a downward force on the particle. Similarly, ions approaching below the median plane will experience an upward force for $\epsilon_1 < 0$. Thus, there is a convergence of the beam for $\epsilon_1 > 0$, and it can easily be seen that $\epsilon_1 < 0$ produces a divergence of the beam. A similar situation exists on the exit side of the magnet. We call the corresponding angle ϵ_2 , and with the same sign convention the same focusing conditions obtain. Clearly, we require a magnet such that the net effect of ϵ_1 and ϵ_2 is one of convergence.

We now follow Cross: "Figure III-2 (a) shows the path of particles in the median plane of the magnet. The magnetic field is in the 'vertical' direction, perpendicular to the plane of the paper. Let A H K B be the path of a 'mean' particle, of momentum p_0 . Consider the path A' H' K' B' (also in the median plane) of a particle whose initial position and direction differ by small quantities s and Δ from those of the mean particle, and whose momentum is $p_0 (1 + \beta)$ where $\beta \ll 1$. After it has left the region of magnetic field, the displacement x from the mean path of such a particle will be,...."

$$x = \Delta \left\{ \frac{(1-pq) \cos E_1 \cos E_2}{\sin \Omega} \right\} - s \left\{ q \frac{\cos E_2}{\cos E_1} \right\} + \beta \left\{ \frac{2 \cos E_2 \sin \phi/2}{\sin \Omega} [\cos(E_1 - \phi/2) + q \cos(E_2 - \phi/2)] \right\} \quad \text{III-1}$$

+ second-order terms in Δ , β , and s .

Here

$$p = \frac{l_{1H} \sin \Omega}{\cos^2 E_1} = \frac{\cos(\phi - E_2)}{\cos E_1}$$

$$q = \frac{l_{2H} \sin \Omega}{\cos^2 E_2} = \frac{\cos(\phi - E_1)}{\cos E_2} \quad \text{III-2}$$

$$\Omega = \phi - E_1 - E_2$$

and the remaining symbols are identified in Figure III-2 (a). All distances are measured in terms of the radius of curvature of the mean particle in the uniform field, and the sign convention for E_1 and E_2 is that given earlier.

Equation III-1 holds for relativistic velocities as well, except that in this case β must be multiplied by $(1 - \frac{v_0^2}{c^2})$ as compared with the non-relativistic case.

Provided the pole gap is small compared with the length of the particle's path in the uniform field, the only first-order effects of the fringe field are to displace slightly the whole image and to change slightly the magnification and dispersion of the magnet.

"Now consider motion in the vertical plane, as shown in Figure III-2(b). Here ω and y_0 define the initial direction and position of the particle (in the vertical plane) with respect to the mean particle. After a particle has passed out of the field, its distance y from the median plane is given, to first order, by"

$$y = y_0 \left\{ 1 - \phi \tan \epsilon_1 - l_{2v} [\tan \epsilon_1 + (1 - \phi \tan \epsilon_1) \tan \epsilon_2] \right\} \\ + \omega \left\{ \phi + l_{1v} [1 - \phi \tan \epsilon_1] - l_{1v} l_{2v} [\tan \epsilon_1 + (1 - \phi \tan \epsilon_1) \tan \epsilon_2] \right. \\ \left. + l_{2v} [1 - \phi \tan \epsilon_2] \right\}$$

As is well known, for motion in the median plane, a non-parallel beam from a line source cannot be focused to a point, i.e., it is impossible to find an arrangement which will make the coefficients of Δ and s (in Equation III-1) vanish simultaneously. Similarly it can be shown easily from Equation III-3 that it is impossible to make the coefficients of ω and y_0 vanish simultaneously."

(b) Focusing Equations

Horizontal (angular) focusing occurs when the coefficient of $\mathcal{L}$ in Equation III-1 vanishes (for a discussion of types of focusing other than angular, see Cross' paper)

$$\mathcal{L}_{2H} = \cos E_1 \frac{\sin \phi + \frac{\mathcal{L}_{1H} \cos(\phi - E_1)}{\cos E_1}}{\frac{\mathcal{L}_{1H} \sin \phi}{\cos E_1} - \cos(\phi - E_2)} \quad \text{III-4}$$

and vertical (angular) focusing occurs when the coefficient of ω in Equation III-3 vanishes, i.e., when

$$\mathcal{L}_{2V} = \frac{\phi + \mathcal{L}_{1V}(1 - \phi \tan E_1)}{\mathcal{L}_{1V}(\tan E_1 + \phi) + \phi + \tan E_1 - (1 - \phi \tan E_2)} \quad \text{III-5}$$

Equations III-4 and III-5 can be re-written as

$$\frac{1}{\mathcal{L}_{2H}} = \tan(\phi - \psi) - \tan E_2 \quad \text{III-6}$$

and

$$\frac{1}{\mathcal{L}_{2V}} = \tan E_2 - \frac{1}{\phi - \cot \eta} \quad \text{III-7}$$

by defining ψ and η from the equations

$$\tan \psi = \tan E_1 + \frac{1}{\mathcal{L}_1} \quad \text{III-8}$$

and

$$\tan \eta = \tan E_1 - \frac{1}{\mathcal{L}_1} \quad \text{III-9}$$

The condition for double-focusing is that $\mathcal{L}_{2H} = \mathcal{L}_{2V}$; this then requires that

$$\tan E_2 = \frac{1}{2} \left\{ \tan(\phi - \psi) + \frac{1}{\phi - \cot \eta} \right\} \quad \text{III-10}$$

and it then follows from III-7 and III-10 that ℓ_2 is given by

$$\frac{1}{\ell_2} = \frac{1}{2} \left\{ \tan(\phi - \psi) - \frac{1}{\phi - \cot \eta} \right\} \quad \text{III-11}$$

It should be noted that negative values of ℓ_2 are possible, indicating a virtual image. In this case the emergent beam is divergent and, in general, of no practical value.

We remark, also, that it is possible to obtain second-order horizontal focusing in addition to double focusing. The additional condition which must be satisfied, that of making the coefficient of ℓ^2 in Equation III-3 vanish, is given by

$$\frac{\cos^2 \eta}{\ell_1^2} \left\{ 1 + \frac{1}{2} \left(\frac{1}{\ell_1} \right) - \frac{\cos^2(\phi - \psi)}{\ell_1^2} \left\{ 3 \tan(\phi - \psi) - \frac{2}{\ell_2} \right\} \right\} \quad \text{III-12}$$

This is satisfied only by a few isolated values of ℓ_1 and E_1 . We shall discuss this further in a subsequent section.

(c) Resolving Power.

If we neglect the contribution of terms beyond the first order in Equation III-1, an indication of the optimum resolving power of the system is given by R, the absolute value of the ratio of the momentum dispersion (coefficient of β) to the magnification (coefficient of s): this is given by

$$R = \frac{2 \cos E_1 \sin \phi / 2}{\beta \sin \phi} \cos(E_1 - \phi / 2) \tan(E_2 - \phi / 2) \quad \text{III-13}$$

"If we define the reciprocal of the resolving power of a momentum spectrometer as the fractional change in momentum

which, in the plane of the image, displaces a particle by half the geometrical image width (measured in the median plane), then the resolving power is R/S , where S is the source width in units of the radius of curvature of the mean particle."

We take this opportunity to emphasize that the contents of the preceding section (III-B-1) were obtained in their entirety from Cross' paper.

(2) Special case of $\epsilon_1 = \epsilon_2 = \epsilon$.

If it can be arranged that $\epsilon_1 = \epsilon_2 = \epsilon$, some simplification of the design is achieved. As we shall see, this condition implies that the sector be symmetric with respect to the bisectors of both ϕ and η ; a corollary is that these bisectors must be colinear. The practical advantages are that the amount of iron is reduced, and that the machining on the pole pieces and vacuum plumbing is simplified.

If we require $\epsilon_1 = \epsilon_2 = \epsilon$, the Equation III-11 can be reduced, after some tedious algebra, to the following transcendental equation for ϕ

$$\phi = \frac{\tan \phi [Z + \tan \psi + \tan \psi \tan \eta - 1] + 2Z}{\tan \phi [Z \tan \psi \tan \eta - \tan \eta] + Z \tan \eta + \tan \psi \tan \eta} \quad \text{III-14}$$

where $Z = 2 \tan \epsilon$. In practice, one solves this equation numerically for ϕ , choosing values for ℓ_1 and ϵ .

Having determined ϕ , one can then find ℓ_2 from any of Equations III-6, 7 or 11; it is generally well to compute ℓ_2 from both Equation III-6 and Equation III-7 as a check on the

calculation. Both ϕ and l_2 can then be plotted as a function of ϵ , for a given l , and from this graph suitable values for all the variables can be sought.

We should remark here that these values of l_1 and l_2 are in terms of ρ , the radius of curvature of the mean particle in the uniform field, and, therefore, the actual lengths involved are dependent on the determination of the value of ρ , about which we shall say more later.

(3) Geometric Equations

We shall now consider what requirements are made on the geometry of the magnet by the focusing conditions. To our knowledge the equations which we shall derive do not appear in print elsewhere.

Figure III-3 is a drawing of a sector which is completely general except for one feature. The base of the sector, labeled X, has been drawn parallel to the chord of the mean particle orbit in the uniform field. This has the sole purpose of simplifying some of the mathematical analysis and involves no permanent loss of generality; X merely serves as a base line for the geometric measurements, and whether or not the magnet base coincides with X is of no importance.

See page 38 for Figure III-3

FIGURE III-3

Some simple relations which can easily be verified are:

$$\Delta = \phi - \epsilon_1 - \epsilon_2 \quad \text{III-15}$$

$$D = 2r \sin \phi/2 \quad \text{III-16}$$

$$\delta_1 + \delta_2 = \Delta + i \neq 0 \quad \text{III-17}$$

Now by application of the law of sines, we have

$$\frac{x}{\sin \delta_2} = \frac{A}{\sin(180 - \delta_2)} = \frac{A}{\sin \delta_2} \quad \text{III-18}$$

and by the same law

$$\begin{aligned} \frac{D}{\sin \Omega} &= \frac{A + Y_1}{\sin \delta_2} \\ &= \frac{X}{\sin \Omega} + \frac{Y_1}{\sin \delta_2} \end{aligned} \quad \text{III-19}$$

by III-18. It follows clearly then that

$$\frac{D}{\sin \Omega} = \frac{X}{\sin \Omega} + \frac{Y_1}{\sin \delta_2} = \frac{X}{\sin \Omega} + \frac{Y_2}{\sin \delta_1} \quad \text{III-20}$$

from which we have:

$$Y_1 \sin \delta_1 = Y_2 \sin \delta_2 \quad \text{III-21}$$

From the diagram we see that

$$\epsilon_{1,2} = \beta + 90 - \delta_{1,2}$$

and since $\beta = \phi/2$, this gives us the δ 's in terms of ϕ and the ϵ 's:

$$\begin{aligned} \delta_1 &= \phi/2 + 90 - \epsilon_1 \\ \delta_2 &= \phi/2 + 90 - \epsilon_2 \end{aligned} \quad \text{III-22}$$

Note that $\epsilon_1 = \epsilon_2$ requires that $\delta_1 = \delta_2$, and, therefore, by III-21 $Y_1 = Y_2$; the magnet is now symmetric.

These are actually all the relations we need, but we should note one correction to III-20, which we re-write as

$$D = X + \frac{\sin \Omega}{\sin \delta_2} Y_1 \quad \text{III-20(b)}$$

The effect of the fringe field is to shift the effective pole boundary outward, and this is taken into account empirically by adding to the right-hand side of III-20(b) a term

$0.4 G (\csc \gamma_1 + \csc \gamma_2)$; we then have

$$D = X + \frac{\sin \Omega}{\sin \gamma_2} Y_1 + 0.4 G (\csc \gamma_1 + \csc \gamma_2) \quad \text{III-23}$$

This, together with III-21, constitutes the basic relation.

The γ 's are given by III-22.

(C) Old magnet + boundary conditions.

The dimensions of the pole face of the old analyzer are

shown in Figure III-3. Our desire was to modify these dimensions as little as possible, and the simplest way to do this was to require retention of the symmetry with respect to the median vertical plane. (As we shall see, this requires

that $\epsilon_1 = \epsilon_2$.) Other boundary conditions, following from the proposed layout in Figure I-1, were that the object and image distances be about 150 cm and 350 cm, respectively, and that the total angle of deflection of the beam be about 45° .

(D) Sample calculation for $\ell_1 = 3$.

For fields above about 8 kilogauss, saturation effects begin to set in, and the field becomes less uniform near the pole boundaries. It is desirable, therefore, to keep the field below this value, which means that the radius of curvature should be at least 47 cm ($B\rho = 3.8(10)^5$ for 7 mev protons). If we take

$\rho \approx 50$ cm, then we should try $\ell_1 = \frac{150}{50} = 3$ in order to satisfy the proposed layout. For $\ell_1 = 3$, then Equations III-8, 9 become $\tan \psi = \tan \epsilon + \frac{1}{3}$ $\tan \eta = \tan \epsilon - \frac{1}{3}$. We have here applied the restriction $\epsilon_1, \epsilon_2 = \epsilon$. The procedure we follow is to assume a value of ϵ , and solve Equation III-14 numerically for ϕ . Suppose we take, as an example, $\epsilon = 0.225$. We then find the following values:

$$\epsilon = 0.225 = 12^\circ 53.15'$$

$$\tan \epsilon = 0.22888$$

$$Z = 2 \tan \epsilon = 0.45776$$

$$\tan \psi = \tan \epsilon + \frac{1}{3} = 0.56221$$

$$\tan \eta = \tan \epsilon - \frac{1}{3} = -0.10445$$

Substituting these values into Equation III-14, we get

$$\begin{aligned} \phi &= \frac{\tan \phi (Z \tan \psi + \tan \psi \tan \eta - 1) + 2 Z}{\tan \phi (Z \tan \psi \tan \eta - \tan \eta) + Z \tan \eta + \tan \psi \tan \eta} \\ &= \frac{\tan \phi (0.25736 - 0.05872 - 1) + 0.91552}{\tan \phi (-0.026881 + 0.10445) - 0.047813 - 0.058723} \\ &= \frac{-0.80136 \tan \phi + 0.91552}{0.07757 \tan \phi - 0.10654} \end{aligned}$$

The remainder of the solution for ϕ is a trial-and-error process. We can, however, narrow down the range of ϕ in which to search by determining the limits on $\tan \phi$ which make the fraction positive. If both numerator and denominator are to be positive, we must have $1.142 < \tan \phi < 1.373$; it is clear that no value of $\tan \phi$ will make both numerator and denominator negative. The determination of these limits actually results in a consider-

able saving of time. We find, then, that $\phi = 0.8594$ ($-49^{\circ} 14.'4$). Next we compute ℓ_2 from both Equations III-6 and III-7 thus obtaining a check on all the calculations up to this point.

$$\begin{aligned}\frac{1}{\ell_{2H}} &= \tan(\phi \cdot \gamma) - \tan \epsilon \\ &= \tan(49^{\circ}14' - 29^{\circ}21') - \tan 12^{\circ} 53.'5 \\ &= 0.36167 - 0.22888\end{aligned}$$

$$\tan \psi = 0.56221$$

$$\psi = 29^{\circ}21'$$

$$\ell_{2H} = 7.531$$

Next:

$$\begin{aligned}\frac{1}{\ell_{2V}} &= \tan \epsilon - \frac{1}{\phi \cdot \omega \eta} \\ &= 0.22888 - \frac{1}{0.8594 + 9.5740} \\ &= 0.22888 - 0.09585\end{aligned}$$

$$\ell_{2V} = 7.517$$

The two values for ℓ_2 differ by less than 2 parts in 1000, confirming the accuracy of the calculations.

These calculations are repeated, using different values for ϵ , and the results are shown graphically in Figure III-4. (A similar set of calculations was made for $\ell_1 = 2$, corresponding to $\rho = 75$ cm, and the results are shown in Figure III-5.

(E) Determination of Physical Dimensions.

After an examination of these graphs, we chose as the optimum set of parameters the ones for which we made the sample

calculation, viz., $\ell_1 = 3$, $\epsilon = 0.225$. The values of ϕ and ℓ_2 satisfied the proposed layout nicely. Our next step was to determine whether or not the old analyzer could be easily modified to conform with the new design. To do this we apply the geometric equations III-21-23.

$$\begin{aligned} \delta &= \phi / 2 + 90 - \epsilon & \text{and } \Omega &= \phi - 2\epsilon \\ &= 24^\circ 37' + 90^\circ - 12^\circ 53' & &= 49^\circ 14' - 25^\circ 47' \\ &= 101^\circ 44' & &= 23^\circ 27' \end{aligned}$$

We note that, if we can manage to keep X and Y the same in the new magnet as in the old, the only modification required will be the shaving of about 5° off the sides of the pole pieces. To continue:

$$D = 2\rho \sin \phi / 2 = \frac{\sin \Omega}{\sin \gamma} Y + X + 2 (0.4) G \csc \delta \quad \text{III-24}$$

For G we take the present value of 4 cm. Substituting the known values for ϕ , Ω , G , and δ we have

$$0.83310 \rho = \frac{0.39795}{0.97910} Y + X + \frac{3.2}{0.97910}$$

Suppose we see what value of ρ is required in order to be able to retain the present values of $X = 32.7$ cm, $Y = 13.3$ cm.

$$\begin{aligned} 0.83310 \rho &= 5.406 + 32.7 + 3.268 \\ &= 41.374 \\ \rho &= 49.66 \text{ cm} \end{aligned}$$

This is excellent, because the necessary field is then

$$B = \frac{3.8(10)^5}{49.66}$$

$$= 7.6 \text{ Kilogauss}$$

Furthermore, the object and image distances are also satisfactory:

$$\ell_1 = 3(49.66) = 149.0 \text{ cm} \approx 4' 11''$$

$$\ell_2 = 7.517(49.66) = 373.6 \text{ cm} \approx 12' 3''$$

It should be emphasized that this entire design hinged on the value of ρ just calculated. If it had proved unsuitable, it would have been necessary to start afresh, using a new set of values of ϵ, ℓ . This would not be the case if we were designing a completely new magnet, in which event X and Y could be assigned any desired values.

We now assemble the values chosen for the new design.

$\rho = 49.7 \text{ cm}$	$\epsilon = 12^\circ 54'$	
$\ell_1 = 149 \text{ cm}$	$\phi = 49^\circ 14'$	
$\ell_2 = 374 \text{ cm}$	$\gamma = 101^\circ 44'$	III-25
$X = 32.7 \text{ cm}$	$\mu_1 = 23^\circ 27'$	
$Y = 13.3 \text{ cm}$	$B = 7.6 \text{ kilogauss}$	

At this point we mention briefly one further precautionary calculation which was made. This was a check that the beam would not emerge from the rear of the magnet! The margin of safety for the present design is (see Fig. III-3)

$$26 - Y \sin \gamma - \rho (1 - \cos \phi/2) = 26 - 13.3(0.979) - 49.7(1 - 0.9091)$$

$$= 8.5 \text{ cm}$$

This sample calculation was made for the mean particle, but it is easily extended to cover all the particles in the extended beam. The margin of safety for an incident beam 6 cm across was found to be about 5 cm.

In conclusion, we note that serious consideration was given to the possibility of obtaining second-order horizontal angular focusing and dropping the requirement that $\epsilon_1 = \epsilon_2$. Several calculations were carried out, but we did not make use of the results for two main reasons. In the first place, the conditions for second-order focusing depend much more strongly on ϵ than do those for first order. The focusing equations were derived for the ideal case of no fringe field, and in our opinion the methods for correcting for the fringe fields effects on ϵ are not sufficiently precise to warrant basing the magnet design on the second-order calculations. In the second place, the designed magnet could not be obtained by an easy modification of the existing analyzer, and our conclusion was that there would probably be no net gain made by constructing a new magnet according to the second-order, double-focusing design. For the sake of completeness, however, we indicate how the calculations were carried out.

In order to obtain second-order horizontal as well as first-order double focusing, we must satisfy Equations III-11 and III-12 simultaneously. Eliminating ℓ_2 from both, we get

$$\cos^3 \eta \frac{2 + \tan^2 \eta - 1/\ell_1}{\ell_1} = \cos^3(\phi - \mu) \left\{ 2 + \tan^2(\phi - \mu) + \frac{1}{\phi - \cot \eta_1} \right\} \frac{1}{4} \left\{ \tan^2(\phi - \mu) - \frac{1}{\phi - \cot \eta_1} \right\} \quad \text{III-26}$$

The procedure is then as follows. We choose values for ϕ and ℓ_1 . Equations III-26 then involves only ψ and η , and we plot the values which satisfy III-26 as graph with axes ψ and η . Now ψ and η are also related by Equations III-8 and III-9 which when combined give

$$\tan \psi = \tan \eta + 2/\ell_1 \quad \text{III-27}$$

Equation III-27 is plotted on the same graph, and the intersection determines the values of ψ and η which satisfy the necessary conditions. We then find ϵ_1 from either III-8 or 9, ℓ_2 from III-6, 7 or 11, and ϵ_2 from III-10.

As an example, for $\ell_1 = 3$, $\phi = 45^\circ$, Equations III-26 and III-27 intersect at $\eta = 11.97^\circ$, $\psi = 41.31^\circ$. We then find, in the manner explained above, $\epsilon_1 = 28^\circ 37'$, $\epsilon_2 = -5^\circ 25'$, $\ell_2 = 6.27$

(F) Expected Resolution

When we substitute the design values III-25 in the displacement equation III-1, we obtain $\chi = -2.326s + 6.646 \beta$

Then, by III-13, we have $R = \frac{6.646}{2.326} = 2.86$

and we should expect that, if S is the source width in cm, the "half-width" of a line will be $\frac{S}{49.66 R} = 7.0(10)^{-3} S$

for a monoenergetic beam. For example, if $S = 1$ mm, the "half (momentum) width" is 0.07%. Hence, for 7 Mev particles, and $S = 1$ mm, the "half-width" of the energy spread in the image plane should be about $2(0.07\%) 7 \text{ Mev} = 10 \text{ kev}$.

Supplementary Notes (October 10, 1954)

At this writing the system described in this report has been assembled and has undergone initial testing with the extracted beam from the rebuilt cyclotron.

It was found that the initial alignment of the individual lenses was quite critical. Once aligned however the condenser system will focus the emergent beam from the cyclotron into a vertical line of approximately 2 mm. width, a horizontal line of approximately 2 mm. width or a spot of approximately 3 mm. diameter; depending only on the excitation current to the coils. (We have installed a balance control which allows us to regulate the total current to the condenser and change the division of this current between the individual lenses.) Our measurements indicate that we can focus between 60 and 70 percent of the beam which enters the condenser aperture through a vertical slit 1 mm. wide and 4 mm. high.

The operation of the wedge analyzer has been checked using the standard current carrying wire technique.* These measurements indicated that beam entering the wedge over about 80 percent of the horizontal aperture was focussed in the horizontal plane. The image distance as measured from the experimentally determined focus to the wedge was about 15 percent longer than that calculated above. At least part of this discrepancy may be due to a small change in the effective wedge angle from that used in the calculations.

* R.A. Campbell, M.S. Thesis, University of Rochester, 1954 (unpublished).

We have used the wedge to image the output slit of the condenser magnets to the target position.

Our preliminary measurements indicate that we can expect beam currents of order 0.1 μ amp with an energy spread of about 0.2 percent, however we have not determined this resolution experimentally as yet.

This is a preliminary report on our focussing system. Additional operational data will be included in a report describing the complete cyclotron installation which will be published later by H.W. Fulbright, D.A. Bromley and J.A. Bruner.

Recent publications dealing with the use of strong focus condenser magnets are the following:

(a) Shull, McFarland and Bretscher

Rev. Sci. Inst. 25, 364(1954).

(b) Dayton, Shoemaker and Mozley

Rev. Sci. Inst. 25, 485(1954).

A somewhat similar focussing system is that described by B. Cork and E. Zajec, Phys. Rev. 92, 853(A)(1953).