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Multiple Scattering Enhancement of X Rays

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E. W. Bennett

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MULTIPLE SCATTERING ENHANCEMENT OF X RAYS

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ABSTRACT

A Monte Carlo calculation is described for the determination of the contribution of multiple scattering to x rays scattered at 90° in various thicknesses of beryllium and carbon plates. The plates are inclined at an angle of 45° to the incident radiation. The photon energy range considered is 1 keV to 100 keV. Scatterer thicknesses range from 0.0046 g/cm² to 1.204 g/cm². Two sets of results are given: one from a formulation neglecting the polarization, the other including polarization. Results are presented in the form of an enhancement factor defined as the ratio of the number of scattered photons escaping from the scatterer at 90° to the number escaping at 90° after scattering only once. Enhancement factors are given in both tabular and graphical form.

INTRODUCTION

We frequently observe x rays scattered at right angles from foils of beryllium or polystyrene (CH) with the foil surface tilted at 45° to the incident beam. Occasionally these "foils" would be better described as "slabs", to increase the counting rate; in these circumstances we are interested in the contribution made to the observed signal by photons which have been multiply scattered in the foil.

This report describes a Monte-Carlo code (called MORAX for Monte Carlo Right Angle X Rays) written to investigate this special case. Calculations were done for several thicknesses of the elements beryllium and carbon over a range of photon energies from 1 keV to 100 keV. Results are presented in numerical listings and also in graphical forms as an "enhancement factor" plotted against scatterer thickness for constant energy and against energy for constant thickness.

Initially the polarization of the scattered photon was neglected to minimize machine time; subsequently the code was modified to include polarization. Results of both calculations are given here. The energy shift due to incoherent scattering is neglected in all cases.

MORAX

The general Monte-Carlo technique will not be discussed here; however, each weight function used will be developed in detail.

Let $p(x)$ represent a function proportional to the probability that a variable take on the value x . The weight function, $w(x)$, is defined by

$$w(x) = \frac{\int_a^x p(z) dz}{\int_a^A p(z) dz},$$

where the interval $a \leq x \leq A$ is the range of possible values of x . Note that w takes on values restricted to the interval $0 \leq w \leq 1$.

A diagram illustrating the cartesian coordinate system used is shown in Figure 1.

The scattering foil is represented by an infinite plane slab of thickness t tilted at an angle of 45° with respect to the y -axis and the x -axis. In the figure the x -axis is sensed downward and the y -axis to the right; this puts the z -axis in the upper surface of the slab sensed out of the paper. Photons are incident from the negative y direction (traveling from left to right). A detector located at 90° from the photon direction would lie on the x -axis; the $+x$ direction is referred to as the "back" side and the $-x$ direction as the "front" side.

We are interested in comparing the total number of photons reaching a detector located on either the front side or the back side with the number of photons reaching that detector after being scattered only once. Accordingly, rather than starting with photons incident and computing the (probably small) fraction scattered, we can save computing time by making the source once-scattered photons distributed properly within the scatterer. Thus, weight functions must be determined to select the location, $y = a$, at which a source photon is started (scattered), and the angles θ, φ required to specify its direction.

Selection of source point

Let the total mass absorption coefficient be denoted by $\mu(E)/\rho$, $\text{cm}^2 \text{gm}^{-1}$, where E is the photon energy. The probability that a photon penetrates to a depth " a " cm without interaction is given by

$$p(a) = e^{-(\mu/\rho)a}$$

where ρ is the density, gm cm^{-3} . The range of values of a is $0 \leq a \leq \sqrt{2} t$. The weight function, then, is given by

$$w(a) = \frac{\int_0^a e^{-(\mu/\rho)\rho x} dx}{\int_0^{\sqrt{2}t} e^{-(\mu/\rho)\rho x} dx}.$$

A random number A , $0 \leq A \leq 1$, is generated in the computer and a found from the equation

$$A = w(a).$$

Performing the integrations and solving for a we have

$$a = -\frac{1}{(\mu/\rho)\rho} \log [(1 - A) + A e^{-(\mu/\rho)\rho \sqrt{2}t}].$$

Selection of direction

Consider a polar coordinate system r, θ, φ with origin at a and the polar axis coincident with the y -axis. The "zenith" angle θ is measured from the $+y$ -axis and the "azimuth" angle φ in the x - z plane. Let $\varphi = 0$ be along the $+x$ -axis and the direction of increasing φ from $+x$ to $+z$. The "back" then corresponds to $\theta = 90^\circ, \varphi = 0^\circ$ and the "front" to $\theta = 90^\circ, \varphi = 180^\circ$. The angle θ is the scattering angle for a first scattering.

The probability of a photon being scattered into a particular solid angle is given by the differential scattering cross section; this cross section is in two parts: (1) the "incoherent" cross section, $\sigma_{\text{INC}}(\theta)$, due to individual electrons and (2) the "coherent" cross section, $\sigma_{\text{COH}}(\theta)$, due to the whole atom. The net cross section is the sum of these two. Thus, taking the element of solid angle as $2\pi \sin \theta d\theta$, the weight function is

$$w(\theta) = \frac{\int_0^\theta [\sigma_{\text{INC}}(\alpha) + \sigma_{\text{COH}}(\alpha)] \sin \alpha d\alpha}{\int_0^\pi [\sigma_{\text{INC}}(\alpha) + \sigma_{\text{COH}}(\alpha)] \sin \alpha d\alpha}.$$

A table of numbers, $w(\theta)$ vs θ , is computed for the photon energy of interest and θ found by generating a random number B , setting

$$w(\theta) = B$$

and looking up the corresponding value of θ .

If effects of polarization are neglected, all values of φ are equally likely. Hence, φ is found by generating a random number C and computing

$$\varphi = 2\pi C.$$

Selection of distance

The probability that a photon will penetrate a distance r is given by

$$e^{-(\mu/\rho)\rho r}, \quad 0 \leq r \leq \infty.$$

Determination of the weight function is similar to that for selection of the source point; the value of r is found by generating a random number D and computing

$$r = - \frac{1}{(\mu/\rho)_p} \log (1 - D).$$

In this formulation the Compton shift in photon energy has been neglected in the interest of a simpler, faster running code. The energy range of interest is from 0.001 MeV to 0.1 MeV; the worst case, 0.1 MeV and $\theta = 180^\circ$, results in an energy shift of $\sim 30\%$ and the more important 90° case at this energy has only a 15% shift. Changes in cross section are small in Be and C at these energies so the net uncertainty should be of the order of 15% at 0.1 MeV and less than this at lower energies.

Photon location

After the above operations have been completed the photon has undergone one scattering and has arrived at a new location (b_1, a_1, c_1).

The first item to be determined is whether the photon is still within the scatterer or not. The scatterer is between the planes defined by

$$y = -x$$

$$\text{and } y = -x + \sqrt{2}t.$$

Computing

$$x = r \sin \theta \cos \varphi$$

$$y = r \cos \theta,$$

we find the new position from

$$a_1 = a + y$$

$$b_1 = x$$

and test if $-b_1 \leq a_1 \leq -b_1 + \sqrt{2}t$.

If the test is not satisfied then the photon escaped from the foil. A count is added to the appropriate $\Delta\theta, \Delta\varphi$ "box" and the code returns to select a new source point.

If the test is satisfied, then the photon is still within the scatterer. The next step is to determine if the photon is absorbed or scattered.

The contribution to the total mass absorption coefficient from scattering is given by μ/ρ_{COH} for the coherent case and by $\mu/\rho_{\text{INC}, s}^{\text{BD}}$ for the incoherent. The reason for this notation is to conform with that used in the report from which the absorption coefficients were obtained.¹

Thus the fraction, R, of the photons which is absorbed is given by

$$R = \frac{\mu/\rho_{\text{TOT}} - \mu/\rho_{\text{COH}} - \mu/\rho_{\text{INC}, s}^{\text{BD}}}{\mu/\rho_{\text{TOT}}}$$

and the selection made by generating a random number F and comparing:

$F < R$ photon absorbed.

$F \geq R$ photon scattered.

When absorption occurs the event is recorded and the code returns to select a new source point.

When scattering occurs new values θ' , φ' , and r' for direction and distance are found in the same manner as before. However, the new polar axis is directed along r (the previous photon direction) and it is necessary to transform back to the original system.

The new location is given by

$$x' = r' \sin \theta' \cos \varphi'$$

$$y' = r' \cos \theta'$$

$$z' = r' \sin \theta' \sin \varphi'.$$

If we consider r a vector with components r_x, r_y, r_z then we can rotate the y -axis into the y' -axis by (1) rotating first about the z -axis, y toward x , through an angle α given by

$$\sin \alpha = \frac{r_x}{\sqrt{r_x^2 + r_y^2}},$$

and (2) then rotating about the x -axis, y toward z , through an angle β given by

$$\sin \beta = \frac{r_z}{r}.$$

The first rotation has the matrix

$$\begin{pmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

and the second the matrix

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \beta & -\sin \beta \\ 0 & \sin \beta & \cos \beta \end{pmatrix}.$$

Taking the matrix product, the result is

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cos \alpha & -\sin \alpha & 0 \\ \cos \beta \sin \alpha & \cos \beta \cos \alpha & -\sin \beta \\ \sin \beta \sin \alpha & \sin \beta \cos \alpha & \cos \beta \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

which has the solution

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \cos \alpha & \cos \beta \sin \alpha & \sin \beta \sin \alpha \\ -\sin \alpha & \cos \beta \cos \alpha & \sin \beta \cos \alpha \\ 0 & -\sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}.$$

It should be noted that if another scattering occurs the values θ_1, φ_1 , must be used in the transformation matrix, replacing θ, φ .

Using the identities

$$\sin \alpha = \frac{\sin \theta \cos \varphi}{\sqrt{1 - \sin^2 \theta \sin^2 \varphi}}, \quad \cos \alpha = \frac{\cos \theta}{\sqrt{1 - \sin^2 \theta \sin^2 \varphi}}$$

$$\sin \beta = \sin \theta \sin \varphi, \quad \cos \beta = \sqrt{1 - \sin^2 \theta \sin^2 \varphi},$$

where θ and φ are the previous angles, we have finally,

$$x = \frac{\cos \theta}{\sqrt{1 - \sin^2 \theta \sin^2 \varphi}} x' + \sin \theta \cos \varphi y' + \frac{\sin^2 \theta \sin \varphi \cos \varphi}{\sqrt{1 - \sin^2 \theta \sin^2 \varphi}} z',$$

$$y = \frac{-\sin \theta \cos \varphi}{\sqrt{1 - \sin^2 \theta \sin^2 \varphi}} x' + \cos \theta y' + \frac{\sin \theta \sin \varphi \cos \theta}{\sqrt{1 - \sin^2 \theta \sin^2 \varphi}} z',$$

$$z = -\sin \theta \sin \varphi y' + \sqrt{1 - \sin^2 \theta \sin^2 \varphi} z'.$$

The new value for θ, θ_1 , in the original polar coordinate system, is given by

$$\theta_1 = \cos^{-1} \left(\frac{y}{r} \right)$$

and the new value for φ, φ_1 , by

$$\cos \varphi_1 = \frac{x}{r' \sin \theta}.$$

The position of the photon, (b_2, a_2) , is found from

$$\begin{aligned} b_2 &= b_1 + x \\ a_2 &= a_1 + y. \end{aligned}$$

The test is made to determine if the photon got out; if so, a count is added to the proper $\Delta\theta, \Delta\varphi$ box, in this case determined by θ_1, φ_1 , and the code returns to select a new source point; if not, new determinations of absorption or scattering angles θ', φ' and distance r' are made and the process continued until either absorption occurs or the photon escapes. The code repeats this procedure until a specified number, usually 100,000, source points have been selected.

Changes to include polarization

The differential cross section for plane-polarized incident radiation interacting with stationary, unbound electrons with randomly oriented magnetic moments is proportional to 2

$$\frac{v_0}{v'} + \frac{v'}{v_0} - 2 + 4 \cos^2 \gamma$$

where γ is the angle between the electric vectors of the incident and the scattered photon.

For our purposes the first three terms tend to cancel and we have the cross section approximately proportional to $\cos^2 \gamma$.

If we take the incident electric vector to be vertically downward (along the + x-axis in our coordinate system) then

$$\cos \gamma = \cos \delta \sin \xi$$

where ξ is the angle between the incident polarization and the scattering direction (between +x and $\vec{r}$) and δ is the angle between the scattered polarization and the vertical plane containing $\vec{r}$.

It turns out that $\cos \xi = \sin \theta \cos \varphi$, so we have, finally, that the scattering cross section is proportional to

$$\cos^2 \gamma = \cos^2 \delta (1 - \sin^2 \theta \cos^2 \varphi)$$

for Klein-Nishina scattering. We will assume all scattering of polarized photons in MORAX follows this relationship.

We will take the initial incident beam as unpolarized and all scattered photons as polarized. Thus there is no change in procedures for finding the first values of θ , φ , and r . If more scattering occurs, however, the procedure for selecting φ is different and a direction for $\varphi = 0$ (determined by the angle δ from the downward direction) must be determined. Both φ and δ may take on values between 0 and 2π .

When multiple scattering occurs a value for δ must be found to establish the initial direction of the polarization vector. The weight function is given by

$$w(\delta) = \frac{\int_0^\delta \cos^2 \alpha d\alpha}{\int_0^{2\pi} \cos^2 \alpha d\alpha}$$

$$w(\delta) = \frac{\frac{1}{2} \delta + \frac{1}{4} \sin 2\delta}{\pi}$$

A table of numbers, $w(\delta)$ vs δ is computed and δ found by generating a random number G , setting

$$w(\delta) = G$$

and looking up the corresponding value of δ .

Now new values, θ' , φ' , and r' must be found for direction and distance. The angle θ' is found in the same manner as before. To find φ' we proceed as follows.

A weight function is generated by

$$w(\varphi_p) = \frac{\int_0^{\varphi_p} (1 - \sin^2 \theta \cos^2 \alpha) d\alpha}{\int_0^{2\pi} (1 - \sin^2 \theta \cos^2 \alpha) d\alpha}$$

$$= \frac{\varphi_p - \frac{1}{2} \varphi_p \sin^2 \theta - \frac{1}{4} \sin^2 \theta \sin 2\varphi_p}{2\pi - \pi \sin^2 \theta}$$

A table of numbers is found, $w(\varphi_p)$ vs φ_p , a random number H generated, and φ_p found from

$$w(\varphi_p) = H$$

The value of φ' is, then,

$$\varphi' = \varphi_p + \delta$$

If $\varphi' > 2\pi$, then replace by $\varphi' - 2\pi$.

The value of r' is found in the same manner as before followed by the tests for escaping and absorption.

If still another scattering takes place, values for the new θ' , φ' , and r' are found as just described. An angle, δ_p , is found as δ was before and φ' found from

$$\delta' = \delta_p + \delta,$$

where δ is the previous value. If $\delta' > 2\pi$, it is replaced by $\delta' - 2\pi$.

Bookkeeping

In any Monte Carlo calculation the tabulated results usually represent a compromise between fineness of detail and running time. Results are subject to statistics and, if $\Delta\theta$ and $\Delta\varphi$ are made too small the running time required to obtain a large enough number of "counts" in the $\Delta\theta$ and $\Delta\varphi$ will be prohibitively large.

In this case interest is primarily in the front and back cases. Thus, relatively small "square" boxes $\Delta\theta = 10^\circ$, $\Delta\varphi = 10^\circ$, centered on front and back are provided; each box, in the likely event that too few photons are included, is made part of a larger box, $\Delta\theta = 30^\circ$, $\Delta\varphi = 30^\circ$, centered about the same direction.

The rest of the available directions are arbitrarily divided to give some information on front and back totals separated into forward scattering and back scattering. Boxes included are listed below.

$\Delta\theta$	$\Delta\varphi$	Remarks
from 85° to 95°	from 355° to 5°	Back Side
85° 95°	175° 185°	Front Side
75° 105°	345° 15°	Back Side
75° 105°	165° 195°	Front Side
0° 45°	all φ	Forward scattering, back side
135° 180°	all φ	Back scattering, front side
45° 75°	90° 270°	Forward scattering, front side
45° 75°	270° $-(360) - 90^\circ$	Forward scattering, back side
75° 105°	15° 165°	Right angle, but not to detector
75° 105°	195° 345°	Right angle, but not to detector
105° 135°	90° 270°	Back scattering, front side
105° 135°	270° $-(360) - 90^\circ$	Back scattering, back side

The code tabulates the number of photons in a particular box arriving directly from the source point (i.e., were scattered once) and the number getting to the box after further scattering. The ratio of the total number of photons in a box to the number arriving from the source point without further scattering is the "multiple scattering enhancement" factor. This factor is computed by the code for each case.

RESULTS

Calculations were performed for the following thicknesses of Be and C.

THICKNESSES					
Be $\rho = 1.848 \text{ g/cm}^3$			C $\rho = 0.948 \text{ g/cm}^3$		
inches	cm	g/cm^2	inches	cm	g/cm^2
0.001	0.0025	0.0046	0.003	0.0076	0.0072
0.003	0.0076	0.014	0.010	0.0254	0.0241
0.010	0.0254	0.0469	0.020	0.0508	0.0482
0.020	0.0508	0.0939	0.030	0.0762	0.0722
0.030	0.0762	0.1408	0.050	0.1270	0.1204
0.050	0.1270	0.2341	0.100	0.2540	0.2408
0.100	0.2540	0.469	0.150	0.3810	0.361
0.150	0.3810	0.704	0.250	0.6350	0.602
			0.375	0.9525	0.903
			0.500	1.2700	1.204

* Density of carbon was arbitrarily set equal to that of polystyrene

In general the number of photons escaping in the small boxes is too small to give good statistics so the values listed are from the larger boxes. The enhancement factor shown is given by

$$\frac{S + M}{S} \pm \frac{\sqrt{S + M}}{S},$$

where S represents the number escaping directly from the source without further interaction (i.e., scattered once) and M represents those escaping in the box after one or more scatterings after leaving the source.

Of the two ways to plot results, that of enhancement factor against thickness seemed to be the smoother; these plots are shown in Figs. 2 through 4. "Eyeball" smooth curves have been drawn through the points; obviously many other smooth curves

could be drawn to suit the draftsman. Points were then picked off the eyeball curves at constant thickness and plotted against energy; these results are given in Figs. 5 and 6. In some cases, lack of consistency of one of these points with its neighbors led to redrawing the appropriate curve in the first set of graphs.

There appears to be no consistent difference between "front" and "back" factors except possibly for the two thickest carbon calculations. This is illustrated in Fig. 7 where values are taken from the listings. Even if the effect is real the difference appears negligible for most purposes.

It is encouraging to note that values appear to be equal for equal thicknesses, g/cm^2 , of Be and C at photon energies such that incoherent scattering is dominant for both elements.

The contribution of multiple scattering to observations is less than 10% for Be thicknesses $< \sim 30$ mils and polystyrene thicknesses $< \sim 50$ mils.

Acknowledgements

Programming of MORAX for the CDC6600 was done by Richard W. Humphrey. All cross sections were provided on computer ready punched cards by Harvey I. Israel.

REFERENCES

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2. Robley D. Evans, "Compton Effect," Encyclopedia of Physics XXXIV pp. 244F (1958).

TABLE I
MORAX RESULTS FOR BERYLLIUM, FRONT SIDE

E keV	0.0046 g/cm ²	0.014	0.0469	0.0939	0.1408	0.2347	0.469	0.704
1	1.00 ± ?	1.00 ± ?	1.00 ± ?	1.00 ± ?	1.00 ± ?	1.00 ± ?	1.00 ± ?	1.00 ± ?
1.5	1.00 ± ?	1.00 ± ?	1.00 ± ?	1.00 ± ?	1.00 ± ?	1.00 ± ?	1.00 ± ?	1.00 ± ?
2	1.00 ± ?	1.00 ± ?	1.00 ± ?	1.00 ± ?	1.00 ± ?	1.00 ± ?	1.00 ± ?	1.00 ± ?
3	1.007 ± .028	1.008 ± .029	1.016 ± .034	1.016 ± .037	1.012 ± .037	1.013 ± .038	1.015 ± .037	1.023 ± .038
4	1.004 ± .027	1.016 ± .029	1.018 ± .031	1.040 ± .034	1.028 ± .035	1.033 ± .038	1.039 ± .039	1.045 ± .040
5	1.007 ± .027	1.017 ± .028	1.023 ± .030	1.034 ± .032	1.041 ± .034	1.065 ± .035	1.060 ± .039	1.065 ± .039
6	1.004 ± .027	1.017 ± .027	1.026 ± .029	1.041 ± .030	1.038 ± .031	1.080 ± .033	1.094 ± .037	1.101 ± .040
8	1.001 ± .027	1.010 ± .027	1.030 ± .027	1.052 ± .028	1.042 ± .028	1.100 ± .030	1.130 ± .032	1.178 ± .035
10	1.007 ± .026	1.013 ± .026	1.031 ± .026	1.041 ± .027	1.073 ± .028	1.099 ± .029	1.172 ± .030	1.227 ± .033
15	1.004 ± .026	1.015 ± .026	1.030 ± .026	1.052 ± .026	1.076 ± .027	1.125 ± .028	1.206 ± .029	1.285 ± .031
20	1.007 ± .026	1.013 ± .026	1.032 ± .026	1.058 ± .026	1.080 ± .027	1.115 ± .027	1.190 ± .029	1.273 ± .031
30	1.005 ± .026	1.015 ± .025	1.028 ± .026	1.045 ± .026	1.064 ± .026	1.106 ± .027	1.189 ± .028	1.253 ± .030
40	1.004 ± .025	1.010 ± .025	1.035 ± .026	1.038 ± .026	1.057 ± .025	1.108 ± .027	1.168 ± .028	1.240 ± .029
50	1.002 ± .025	1.007 ± .025	1.030 ± .026	1.039 ± .026	1.051 ± .025	1.101 ± .027	1.163 ± .028	1.209 ± .028
60	1.004 ± .025	1.006 ± .025	1.021 ± .025	1.038 ± .026	1.054 ± .025	1.088 ± .026	1.149 ± .027	1.215 ± .029
80	1.006 ± .025	1.007 ± .025	1.024 ± .026	1.033 ± .026	1.051 ± .025	1.075 ± .026	1.140 ± .027	1.216 ± .029
100	1.003 ± .025	1.008 ± .025	1.016 ± .025	1.029 ± .026	1.042 ± .025	1.073 ± .026	1.138 ± .027	1.189 ± .029

TABLE II
MORAX RESULTS FOR BERYLLIUM, BACK SIDE

E, keV	0.0046 g/cm ²	0.014	0.0469	0.0939	0.1408	0.2347	0.469	0.704
1	1.00 ± ?							
1.5	1.00 ± ?	1.00 ± ?						
2	1.00 ± ?	1.008 ± .037	1.00 ± ?					
3	1.003 ± .027	1.008 ± .029	1.010 ± .039	1.044 ± .060				
4	1.005 ± .027	1.011 ± .028	1.015 ± .032	1.033 ± .037	1.048 ± .045			
5	1.002 ± .027	1.010 ± .028	1.023 ± .030	1.037 ± .032	1.042 ± .035	1.104 ± .046	1.126 ± .075	
6	1.001 ± .027	1.011 ± .027	1.025 ± .029	1.044 ± .029	1.056 ± .032	1.109 ± .037	1.126 ± .049	1.157 ± .067
8	1.004 ± .026	1.011 ± .027	1.034 ± .027	1.057 ± .028	1.060 ± .029	1.086 ± .030	1.151 ± .035	1.204 ± .040
10	1.003 ± .026	1.014 ± .026	1.026 ± .026	1.050 ± .027	1.071 ± .028	1.108 ± .029	1.195 ± .032	1.233 ± .034
15	1.004 ± .026	1.006 ± .026	1.031 ± .027	1.053 ± .027	1.069 ± .027	1.100 ± .028	1.204 ± .030	1.288 ± .031
20	1.003 ± .026	1.012 ± .025	1.037 ± .026	1.056 ± .027	1.069 ± .026	1.099 ± .027	1.206 ± .029	1.273 ± .031
30	1.003 ± .026	1.006 ± .025	1.027 ± .025	1.047 ± .025	1.071 ± .026	1.094 ± .027	1.185 ± .029	1.253 ± .030
40	1.004 ± .025	1.006 ± .025	1.027 ± .026	1.046 ± .026	1.068 ± .027	1.095 ± .027	1.165 ± .028	1.242 ± .030
50	1.001 ± .025	1.005 ± .025	1.027 ± .025	1.044 ± .025	1.065 ± .026	1.096 ± .027	1.172 ± .028	1.227 ± .029
60	1.001 ± .025	1.004 ± .025	1.027 ± .025	1.042 ± .025	1.058 ± .026	1.085 ± .026	1.152 ± .028	1.204 ± .028
80	1.003 ± .025	1.007 ± .025	1.018 ± .025	1.031 ± .025	1.054 ± .026	1.080 ± .026	1.154 ± .028	1.197 ± .028
100	1.001 ± .025	1.006 ± .025	1.018 ± .026	1.027 ± .025	1.059 ± .026	1.077 ± .026	1.154 ± .028	1.186 ± .028

TABLE III

MORAX WITH POLARIZATION FOR BERYLLIUM, FRONT SIDE

E keV	0.0046 μ/cm^2	0.014	0.0469	0.0939	0.1408	0.2347	0.469	0.704
1	1.001 $\pm$.034	1.001 $\pm$.036	1.004 $\pm$.035	1.001 $\pm$.035	1.004 $\pm$.035	1.000 $\pm$.035	1.001 $\pm$.034	1.002 $\pm$.035
1.5	1.003 $\pm$.030	1.002 $\pm$.033	1.004 $\pm$.035	1.003 $\pm$.036	1.006 $\pm$.036	1.001 $\pm$.035	1.001 $\pm$.035	1.007 $\pm$.035
2	1.003 $\pm$.028	1.005 $\pm$.033	1.008 $\pm$.036	1.004 $\pm$.036	1.010 $\pm$.036	1.007 $\pm$.036	1.000 $\pm$.036	1.012 $\pm$.035
3	1.006 $\pm$.027	1.012 $\pm$.029	1.018 $\pm$.034	1.029 $\pm$.037	1.022 $\pm$.037	1.025 $\pm$.038	1.017 $\pm$.038	1.028 $\pm$.037
4	1.004 $\pm$.027	1.021 $\pm$.029	1.028 $\pm$.032	1.042 $\pm$.034	1.049 $\pm$.035	1.041 $\pm$.038	1.042 $\pm$.039	1.043 $\pm$.038
5	1.008 $\pm$.027	1.023 $\pm$.028	1.043 $\pm$.030	1.041 $\pm$.032	1.060 $\pm$.034	1.081 $\pm$.036	1.087 $\pm$.039	1.088 $\pm$.040
6	1.007 $\pm$.027	1.021 $\pm$.027	1.037 $\pm$.029	1.060 $\pm$.030	1.086 $\pm$.032	1.076 $\pm$.034	1.110 $\pm$.037	1.136 $\pm$.039
8	1.001 $\pm$.026	1.013 $\pm$.027	1.039 $\pm$.027	1.064 $\pm$.028	1.087 $\pm$.029	1.142 $\pm$.031	1.187 $\pm$.033	1.220 $\pm$.037
10	1.007 $\pm$.026	1.016 $\pm$.026	1.032 $\pm$.026	1.063 $\pm$.027	1.091 $\pm$.028	1.129 $\pm$.029	1.202 $\pm$.030	1.343 $\pm$.036
15	1.003 $\pm$.026	1.017 $\pm$.026	1.045 $\pm$.026	1.055 $\pm$.026	1.095 $\pm$.027	1.169 $\pm$.028	1.277 $\pm$.030	1.388 $\pm$.033
20	1.006 $\pm$.026	1.012 $\pm$.026	1.037 $\pm$.026	1.063 $\pm$.026	1.098 $\pm$.027	1.171 $\pm$.029	1.272 $\pm$.030	1.378 $\pm$.032
30	1.005 $\pm$.026	1.010 $\pm$.026	1.035 $\pm$.026	1.064 $\pm$.027	1.090 $\pm$.026	1.159 $\pm$.028	1.260 $\pm$.030	1.357 $\pm$.031
40	1.003 $\pm$.025	1.013 $\pm$.025	1.032 $\pm$.025	1.062 $\pm$.026	1.079 $\pm$.026	1.138 $\pm$.028	1.254 $\pm$.029	1.329 $\pm$.030
50	1.003 $\pm$.025	1.011 $\pm$.026	1.029 $\pm$.025	1.061 $\pm$.026	1.074 $\pm$.026	1.131 $\pm$.027	1.229 $\pm$.029	1.310 $\pm$.030
60	1.003 $\pm$.026	1.011 $\pm$.025	1.032 $\pm$.025	1.056 $\pm$.026	1.077 $\pm$.026	1.135 $\pm$.027	1.218 $\pm$.028	1.291 $\pm$.029
80	1.004 $\pm$.025	1.008 $\pm$.025	1.032 $\pm$.026	1.048 $\pm$.025	1.076 $\pm$.026	1.108 $\pm$.027	1.180 $\pm$.027	1.275 $\pm$.029
100	1.004 $\pm$.025	1.011 $\pm$.025	1.020 $\pm$.025	1.049 $\pm$.026	1.059 $\pm$.026	1.109 $\pm$.026	1.180 $\pm$.028	1.280 $\pm$.029

TABLE IV

MORAX WITH POLARIZATION FOR BERYLLIUM, BACK SIDE

E, keV	0.0046 μ/cm^2	0.014	0.0469	0.0939	0.1408	0.2347	0.469	0.704
1	1.00 $\pm$?							
1.5	1.002 $\pm$.033	1.008 $\pm$.064						
2	1.002 $\pm$.028	1.008 $\pm$.037						
3	1.006 $\pm$.027	1.017 $\pm$.029	1.023 $\pm$.040	1.031 $\pm$.059				
4	1.005 $\pm$.027	1.014 $\pm$.028	1.026 $\pm$.032	1.036 $\pm$.037	1.064 $\pm$.045	1.109 $\pm$.071		
5	1.004 $\pm$.027	1.009 $\pm$.027	1.025 $\pm$.030	1.041 $\pm$.032	1.066 $\pm$.036	1.137 $\pm$.046	1.139 $\pm$.075	
6	1.003 $\pm$.027	1.018 $\pm$.027	1.035 $\pm$.029	1.048 $\pm$.030	1.081 $\pm$.032	1.117 $\pm$.037	1.180 $\pm$.050	1.212 $\pm$.066
8	1.005 $\pm$.026	1.014 $\pm$.026	1.042 $\pm$.027	1.068 $\pm$.028	1.109 $\pm$.029	1.119 $\pm$.031	1.215 $\pm$.036	1.264 $\pm$.040
10	1.008 $\pm$.027	1.021 $\pm$.026	1.036 $\pm$.026	1.071 $\pm$.027	1.110 $\pm$.029	1.121 $\pm$.029	1.236 $\pm$.033	1.297 $\pm$.035
15	1.003 $\pm$.026	1.016 $\pm$.026	1.042 $\pm$.026	1.070 $\pm$.026	1.101 $\pm$.027	1.198 $\pm$.029	1.257 $\pm$.030	1.367 $\pm$.032
20	1.006 $\pm$.026	1.019 $\pm$.026	1.038 $\pm$.026	1.068 $\pm$.026	1.098 $\pm$.027	1.178 $\pm$.029	1.263 $\pm$.030	1.367 $\pm$.032
30	1.004 $\pm$.025	1.015 $\pm$.026	1.043 $\pm$.026	1.068 $\pm$.026	1.090 $\pm$.027	1.171 $\pm$.028	1.236 $\pm$.029	1.323 $\pm$.030
40	1.006 $\pm$.025	1.012 $\pm$.025	1.036 $\pm$.026	1.063 $\pm$.026	1.084 $\pm$.027	1.147 $\pm$.027	1.245 $\pm$.029	1.306 $\pm$.030
50	1.004 $\pm$.025	1.010 $\pm$.025	1.030 $\pm$.025	1.059 $\pm$.025	1.078 $\pm$.026	1.137 $\pm$.027	1.232 $\pm$.029	1.329 $\pm$.030
60	1.004 $\pm$.025	1.013 $\pm$.026	1.033 $\pm$.026	1.058 $\pm$.025	1.070 $\pm$.026	1.135 $\pm$.027	1.236 $\pm$.029	1.292 $\pm$.030
80	1.003 $\pm$.025	1.010 $\pm$.025	1.036 $\pm$.026	1.057 $\pm$.026	1.073 $\pm$.026	1.119 $\pm$.027	1.220 $\pm$.028	1.267 $\pm$.029
100	1.005 $\pm$.025	1.008 $\pm$.025	1.028 $\pm$.026	1.051 $\pm$.025	1.070 $\pm$.026	1.127 $\pm$.027	1.200 $\pm$.028	1.245 $\pm$.029

TABLE V
MORAX RESULTS FOR CARBON, BACK SIDE

$E, \text{ keV}$	0.0072 g/cm^2	0.0241	0.0482	0.0722	0.1204	0.2408	0.361	0.602	0.903	1.204
1	1.00 ± ?	1.00 ± ?	1.00 ± ?	1.00 ± ?	1.00 ± ?	1.00 ± ?	1.00 ± ?	1.00 ± ?	1.00 ± ?	1.00 ± ?
1.5	1.00 ± ?	1.00 ± ?	1.00 ± ?	1.00 ± ?	1.00 ± ?	1.00 ± ?	1.00 ± ?	1.00 ± ?	1.00 ± ?	1.00 ± ?
2	1.00 ± ?	1.00 ± ?	1.00 ± ?	1.00 ± ?	1.00 ± ?	1.00 ± ?	1.00 ± ?	1.00 ± ?	1.00 ± ?	1.00 ± ?
3	1.005 ± ?	1.006 ± ?	1.006 ± ?	1.003 ± ?	1.004 ± ?	1.004 ± ?	1.005 ± ?	1.007 ± ?	1.003 ± ?	1.004 ± ?
4	1.007 ± ?	1.013 ± ?	1.004 ± ?	1.012 ± ?	1.004 ± ?	1.007 ± ?	1.009 ± ?	1.009 ± ?	1.012 ± ?	1.013 ± ?
5	1.006 ± .029	1.005 ± .033	1.012 ± .036	1.035 ± .039	1.017 ± .038	1.029 ± .041	1.017 ± .042	1.009 ± .040	1.026 ± .040	1.025 ± .040
6	1.011 ± .029	1.020 ± .032	1.024 ± .035	1.029 ± .036	1.028 ± .038	1.042 ± .042	1.033 ± .042	1.036 ± .047	1.041 ± .041	1.032 ± .041
8	1.008 ± .029	1.024 ± .031	1.031 ± .032	1.040 ± .032	1.044 ± .035	1.082 ± .039	1.062 ± .039	1.086 ± .043	1.089 ± .043	1.079 ± .042
10	1.006 ± .028	1.023 ± .029	1.028 ± .030	1.045 ± .030	1.061 ± .031	1.085 ± .034	1.105 ± .038	1.130 ± .040	1.134 ± .041	1.135 ± .041
15	1.005 ± .027	1.023 ± .027	1.031 ± .027	1.035 ± .028	1.065 ± .029	1.100 ± .029	1.154 ± .031	1.213 ± .034	1.249 ± .036	1.312 ± .039
20	1.008 ± .026	1.019 ± .026	1.034 ± .027	1.036 ± .027	1.065 ± .028	1.108 ± .029	1.150 ± .029	1.229 ± .031	1.320 ± .034	1.355 ± .035
30	1.007 ± .026	1.020 ± .026	1.024 ± .026	1.045 ± .026	1.063 ± .027	1.124 ± .028	1.169 ± .028	1.243 ± .031	1.386 ± .033	1.455 ± .034
40	1.006 ± .025	1.019 ± .026	1.024 ± .026	1.042 ± .027	1.063 ± .026	1.099 ± .027	1.155 ± .028	1.227 ± .029	1.370 ± .032	1.425 ± .033
50	1.007 ± .025	1.017 ± .026	1.033 ± .026	1.042 ± .026	1.055 ± .026	1.101 ± .026	1.145 ± .028	1.234 ± .030	1.309 ± .031	1.394 ± .032
60	1.004 ± .025	1.017 ± .026	1.030 ± .026	1.039 ± .026	1.056 ± .026	1.097 ± .026	1.132 ± .027	1.228 ± .030	1.288 ± .030	1.356 ± .031
80	1.006 ± .025	1.009 ± .025	1.025 ± .025	1.037 ± .026	1.046 ± .025	1.086 ± .026	1.132 ± .027	1.203 ± .029	1.267 ± .029	1.348 ± .031
100	1.007 ± .025	1.009 ± .026	1.029 ± .026	1.031 ± .025	1.041 ± .025	1.080 ± .026	1.124 ± .027	1.183 ± .029	1.261 ± .030	1.330 ± .031

TABLE VI
MORAX RESULTS FOR CARBON, BACK SIDE

$E, \text{ keV}$	0.0072 g/cm^2	0.0241	0.0482	0.0722	0.1204	0.2408	0.361	0.602	0.903	1.204
1										
1.5										
2										
3										
4										
5	1.005 ± .029	1.012 ± .033	1.020 ± .041	1.031 ± .051						
6	1.005 ± .029	1.020 ± .031	1.027 ± .035	1.027 ± .039	1.058 ± .051					
8	1.008 ± .029	1.020 ± .030	1.028 ± .032	1.041 ± .032	1.039 ± .037	1.162 ± .049	1.173 ± .062			
10	1.008 ± .028	1.016 ± .029	1.029 ± .030	1.042 ± .030	1.066 ± .032	1.108 ± .037	1.145 ± .042	1.221 ± .054	1.302 ± .078	
15	1.003 ± .027	1.024 ± .027	1.033 ± .027	1.041 ± .027	1.067 ± .028	1.095 ± .030	1.152 ± .032	1.190 ± .035	1.286 ± .039	1.399 ± .046
20	1.007 ± .027	1.019 ± .027	1.032 ± .027	1.045 ± .027	1.071 ± .028	1.107 ± .028	1.163 ± .030	1.248 ± .033	1.360 ± .036	1.390 ± .037
30	1.005 ± .026	1.017 ± .026	1.031 ± .026	1.045 ± .027	1.060 ± .027	1.111 ± .028	1.169 ± .029	1.254 ± .030	1.345 ± .032	1.447 ± .034
40	1.004 ± .025	1.013 ± .025	1.028 ± .026	1.043 ± .026	1.072 ± .027	1.104 ± .027	1.160 ± .029	1.204 ± .029	1.375 ± .033	1.475 ± .035
50	1.005 ± .025	1.011 ± .025	1.027 ± .026	1.039 ± .026	1.060 ± .026	1.100 ± .027	1.155 ± .028	1.231 ± .030	1.331 ± .031	1.412 ± .031
60	1.002 ± .025	1.011 ± .025	1.024 ± .026	1.029 ± .025	1.057 ± .026	1.096 ± .027	1.141 ± .028	1.209 ± .029	1.310 ± .030	1.398 ± .031
80	1.004 ± .025	1.010 ± .025	1.018 ± .025	1.027 ± .025	1.049 ± .026	1.092 ± .026	1.136 ± .027	1.199 ± .028	1.279 ± .029	1.372 ± .031
100	1.004 ± .025	1.011 ± .025	1.016 ± .026	1.030 ± .025	1.056 ± .026	1.088 ± .026	1.143 ± .027	1.173 ± .028	1.272 ± .030	1.375 ± .031

TABLE VII
NORAX WITH POLARIZATION - CARBON - FRONT SIDE

E, keV	0.0072 g/cm^2	0.0241	0.0482	0.0722	0.1204	0.2408	0.361	0.602	0.903	1.204
1	1.000 ± ?									
1.5	1.003 ± ?									
2	1.001 ± ?									
3	1.007 ± .031	1.008 ± .036	1.006 ± .037	1.008 ± .037	1.012 ± .037	1.006 ± .037	1.004 ± .037	1.009 ± .036	1.007 ± .036	1.009 ± .036
4	1.008 ± .030	1.018 ± .035	1.015 ± .038	1.017 ± .038	1.019 ± .038	1.021 ± .039	1.012 ± .039	1.024 ± .039	1.021 ± .038	1.021 ± .039
5	1.006 ± .029	1.015 ± .033	1.028 ± .036	1.051 ± .039	1.021 ± .039	1.038 ± .040	1.031 ± .042	1.032 ± .040	1.048 ± .040	1.026 ± .040
6	1.016 ± .029	1.019 ± .032	1.035 ± .035	1.043 ± .036	1.046 ± .038	1.057 ± .042	1.074 ± .044	1.040 ± .041	1.061 ± .041	1.049 ± .042
8	1.007 ± .029	1.03 ± .021	1.048 ± .032	1.050 ± .033	1.068 ± .036	1.112 ± .040	1.112 ± .040	1.121 ± .043	1.108 ± .042	1.118 ± .044
10	1.010 ± .029	1.029 ± .029	1.048 ± .031	1.058 ± .030	1.088 ± .032	1.122 ± .035	1.150 ± .038	1.178 ± .042	1.180 ± .043	1.174 ± .043
15	1.009 ± .027	1.035 ± .027	1.043 ± .027	1.061 ± .028	1.085 ± .028	1.141 ± .030	1.199 ± .031	1.282 ± .036	1.306 ± .036	1.455 ± .041
20	1.008 ± .026	1.031 ± .027	1.043 ± .027	1.065 ± .027	1.084 ± .028	1.159 ± .029	1.208 ± .030	1.317 ± .034	1.387 ± .035	1.526 ± .038
30	1.009 ± .026	1.029 ± .026	1.039 ± .026	1.060 ± .026	1.093 ± .027	1.176 ± .029	1.231 ± .029	1.328 ± .032	1.433 ± .033	1.537 ± .035
40	1.007 ± .026	1.023 ± .026	1.037 ± .026	1.061 ± .026	1.087 ± .027	1.159 ± .028	1.219 ± .029	1.328 ± .031	1.430 ± .032	1.531 ± .034
50	1.007 ± .025	1.022 ± .026	1.041 ± .026	1.055 ± .026	1.082 ± .026	1.142 ± .028	1.205 ± .028	1.311 ± .030	1.411 ± .032	1.494 ± .033
60	1.006 ± .025	1.018 ± .026	1.034 ± .026	1.047 ± .026	1.076 ± .026	1.158 ± .028	1.184 ± .028	1.299 ± .030	1.391 ± .031	1.492 ± .033
80	1.008 ± .026	1.018 ± .025	1.032 ± .025	1.047 ± .026	1.063 ± .025	1.135 ± .027	1.175 ± .027	1.275 ± .029	1.370 ± .031	1.464 ± .032
100	1.004 ± .025	1.019 ± .026	1.029 ± .025	1.041 ± .026	1.055 ± .026	1.132 ± .027	1.158 ± .027	1.276 ± .029	1.346 ± .031	1.441 ± .032

TABLE VIII
NORAX WITH POLARIZATION - CARBON BACK SIDE

E, keV	0.0072 g/cm^2	0.0241	0.0482	0.0722	0.1204	0.2408	0.361	0.602	0.903	1.204
1	1.000 ± ?									
1.5	1.000 ± ?									
2	1.004 ± .061									
RATIO INDETERMINATE										
3	1.009 ± .033	1.029 ± .065	1.000 ± .213							
4	1.006 ± .029	1.021 ± .039	1.061 ± .063	1.028 ± .096	1.167 ± .235					
5	1.011 ± .029	1.028 ± .033	1.046 ± .042	1.067 ± .052	1.116 ± .077	1.000 ± .289				
6	1.011 ± .029	1.030 ± .031	1.037 ± .035	1.048 ± .040	1.095 ± .051	1.115 ± .092	1.000 ± .183			
8	1.004 ± .029	1.025 ± .030	1.042 ± .032	1.055 ± .032	1.094 ± .037	1.186 ± .050	1.158 ± .062	1.439 ± .132	1.345 ± .215	1.833 ± .553
10	1.008 ± .028	1.017 ± .028	1.049 ± .030	1.058 ± .030	1.085 ± .032	1.170 ± .038	1.159 ± .042	1.256 ± .054	1.329 ± .078	1.412 ± .111
15	1.005 ± .027	1.026 ± .027	1.050 ± .018	1.057 ± .028	1.094 ± .030	1.142 ± .031	1.213 ± .033	1.283 ± .035	1.360 ± .040	1.518 ± .046
20	1.006 ± .026	1.030 ± .027	1.041 ± .027	1.059 ± .027	1.091 ± .028	1.162 ± .030	1.221 ± .030	1.333 ± .033	1.450 ± .037	1.573 ± .040
30	1.009 ± .026	1.019 ± .025	1.043 ± .026	1.057 ± .026	1.090 ± .027	1.185 ± .029	1.226 ± .030	1.317 ± .030	1.483 ± .034	1.610 ± .036
40	1.007 ± .026	1.019 ± .026	1.047 ± .026	1.052 ± .026	1.082 ± .027	1.165 ± .028	1.213 ± .029	1.300 ± .030	1.455 ± .033	1.555 ± .034
50	1.004 ± .025	1.019 ± .025	1.049 ± .026	1.059 ± .026	1.066 ± .026	1.173 ± .028	1.206 ± .029	1.271 ± .029	1.447 ± .033	1.527 ± .034
60	1.008 ± .025	1.022 ± .025	1.034 ± .026	1.057 ± .026	1.064 ± .026	1.163 ± .028	1.197 ± .028	1.267 ± .029	1.421 ± .032	1.487 ± .033
80	1.007 ± .025	1.019 ± .025	1.032 ± .026	1.055 ± .025	1.061 ± .026	1.147 ± .028	1.195 ± .028	1.262 ± .029	1.377 ± .031	1.471 ± .033
100	1.006 ± .025	1.012 ± .025	1.032 ± .025	1.047 ± .026	1.072 ± .026	1.141 ± .027	1.161 ± .028	1.238 ± .029	1.347 ± .031	1.462 ± .033

Fig. 2 Enhancement factor vs thickness for Be at various photon energies.

Fig. 3 Enhancement factor vs thickness for C at various photon energies.

Fig. 4 Enhancement factor vs thickness for C at various photon energies.

Fig. 5 Enhancement factor vs photon energy for several thicknesses of Be.

Fig. 6 Enhancement factor vs photon energy for several thicknesses of C.

Fig. 7 Enhancement factor vs photon energy for a thick carbon scatterer showing a possible difference between observing the front surface and the back surface.