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UPSTREAM DIFFUSION OF RADIOACTIVE GASES

A. H. Fox

Problems of leakage of radioactive gases into moving airstreams are encountered in several places in the ^{project} ~~ARD~~ Xenon and Krypton which are expected to diffuse across the pump seals and accumulate in the oil reservoir, whence they might leak into the auxiliary equipment pit is one example. The pit is swept by an airstream down into the tunnel and up the stack, but the possibility of upstream leakage into the rest of the building must be considered in determining leakage tolerances. Another place in which a similar effect may be considered is leakage from the main radiators into the blower room, with possible contamination of the atmosphere outside the building through the louvers.

The problem can be represented by a simplified model of a one dimensional airstream in the direction of the negative z-axis in a three dimensional representation, with a uniform source at a point, chosen as the origin. This is also equivalent to a system consisting of a point source in a stationary medium moving along the positive x-axis with a corresponding velocity. Diffusion and convection are considered simultaneously.

The case of uniform velocity, V , of the airstream is considered first, since velocity profiles in the actual installations would be of very complicated forms. Under this assumption, we represent the system by the differential equation for the concentration of activity:

$$D \nabla^2 C = - V \frac{\partial C}{\partial z} + \frac{\partial C}{\partial t} \quad (1)$$

with boundary conditions of zero concentration at an infinite distance from the origin. A uniform source q_0 at the origin may be represented by the condition

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$$\lim_{r \rightarrow 0} \left(-4\pi r^{2D} \frac{\partial C}{\partial r} \right) = q_0.$$

We first introduce cylindrical coordinates to simplify the convection problem. Equation (1) becomes:

$$\frac{\partial^2 C}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial C}{\partial \rho} + \frac{\partial^2 C}{\partial z^2} = -\frac{v}{D} \frac{\partial C}{\partial z} + \frac{1}{D} \frac{\partial C}{\partial t} \quad (3)$$

We now substitute the form:

$$C = e^{-\frac{v}{2D} z} U(\rho, z),$$

and (3) becomes, after simplification,

$$\frac{\partial^2 U}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial U}{\partial \rho} + \frac{\partial^2 U}{\partial z^2} - \frac{v^2}{4D^2} U = \frac{1}{D} \frac{\partial U}{\partial t} \quad (4)$$

We change this to spherical coordinates for easier treatment of the source terms, because of the spherical symmetry of the system, and have:

$$\frac{\partial^2 U}{\partial r^2} + \frac{2}{r} \frac{\partial U}{\partial r} - \frac{v^2}{4D^2} U = \frac{1}{D} \frac{\partial U}{\partial t} \quad (5)$$

We assume an initial condition of no activity in the system,

$$U(r, 0) = 0.$$

The boundary conditions on U are

$$\lim_{r \rightarrow \infty} U(r, t) = 0, \quad (6)$$

and

$$\lim_{r \rightarrow 0} -4\pi r^2 D e^{-\frac{v}{2D} r \cos \phi} \frac{\partial U}{\partial r} = q_0. \quad (7)$$

If we take the Laplace transform of this system with respect to t , we find the differential equation for the transform $u(r,s) = \int_0^\infty \{U(r,t)\} e^{-st} dt$

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} - K^2 u = -\frac{s}{D} u, \quad (8)$$

where we write K for $\frac{V}{2D}$, and the boundary conditions become:

$$\lim_{r \rightarrow \infty} u(r,s) = 0, \quad (9)$$

and

$$\lim_{r \rightarrow 0} -4\pi r^2 D e^{-K r \cos \phi} \frac{\partial u}{\partial r} = \frac{q_0}{s}. \quad (10)$$

This equation in r may be solved by elementary methods to give:

$$u(r,s) = \frac{A}{r} e^{-\sqrt{K^2 + \frac{s}{D}} r} + \frac{B e^{\sqrt{K^2 + \frac{s}{D}} r}}{r} \quad (11)$$

The application of condition (9) makes $B = 0$, and then (10) determines

$$A = \frac{q_0}{4\pi D s}.$$

Hence, we find the transform of our function to be

$$u(r,s) = \frac{q_0}{4\pi D r} \frac{e^{-r\sqrt{K^2 + \frac{s}{D}}}}{s}. \quad (12)$$

We may change this by simple substitutions to a form found in the tables of inverse transforms [Erdelyi, p. 246 (10)] as follows:

Set $s' = s + K^2 D$, then

$$u(r,s') = \frac{q_0}{4\pi D r} \frac{e^{-r\sqrt{s'/D}}}{s' - K^2 D} {}^{1/2} \quad (13)$$

From this the inverse transform of $u(r, s)$ takes the form

$$\begin{aligned}
 u(r, t) = & e^{-K^2 Dt} \frac{q_0}{4\pi Dr} \frac{1}{2} e^{+K^2 Dt} \left[e^{KD^{1/2} r/D^{1/2}} \operatorname{Erfc}\left(\frac{r}{2D^{1/2}t^{1/2}} + KD^{1/2}t^{1/2}\right) \right. \\
 & \left. + e^{-KD^{1/2} r/D^{1/2}} \operatorname{Erfc}\left(\frac{r}{2D^{1/2}t^{1/2}} - KD^{1/2}t^{1/2}\right) \right] \\
 = & \frac{q_0}{8\pi Dr} \left[e^{Kr} \operatorname{Erfc}\left(\frac{r}{2D^{1/2}t^{1/2}} + KD^{1/2}t^{1/2}\right) \right. \\
 & \left. + e^{-Kr} \operatorname{Erfc}\left(\frac{r}{2D^{1/2}t^{1/2}} - KD^{1/2}t^{1/2}\right) \right].
 \end{aligned} \tag{14}$$

The concentration $C(r, \phi, t)$ then is given by

$$\begin{aligned}
 C(r, \phi, t) = & \frac{q_0 e^{Kr \cos \phi}}{8\pi Dr} \left[e^{Kr} \operatorname{Erfc}\left(\frac{r}{2(Dt)^{1/2}} + K(Dt)^{1/2}\right) \right. \\
 & \left. + e^{-Kr} \operatorname{Erfc}\left(\frac{r}{2(Dt)^{1/2}} - K(Dt)^{1/2}\right) \right].
 \end{aligned} \tag{15}$$

The rate of build up of concentration

$$\begin{aligned}
 \frac{\partial C}{\partial t} = & \frac{q_0}{8\pi Dr} e^{Kr \cos \phi} \left[e^{Kr} \frac{2}{\sqrt{\pi}} \left(\frac{r}{4D^{1/2}t^{3/2}} - \frac{KD^{1/2}}{2t^{1/2}} \right) e^{-\left(\frac{r}{2(Dt)^{1/2}} + K(Dt)^{1/2}\right)^2} \right. \\
 & \left. + e^{-Kr} \frac{2}{\sqrt{\pi}} \left(\frac{r}{4D^{1/2}t^{3/2}} + \frac{KD^{1/2}}{2t^{1/2}} \right) e^{-\left(\frac{r}{2(Dt)^{1/2}} - K(Dt)^{1/2}\right)^2} \right]
 \end{aligned} \tag{16}$$

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$$\begin{aligned} \frac{\partial c}{\partial t} &= \frac{q_0}{8\pi^{3/2} D r} e^{Kr \cos \phi} \left[\left(\frac{r}{2D^{1/2} t^{1/2}} - KD^{1/2} t^{1/2} \right) \frac{e}{t} - \frac{r^2}{4Dt} - K^2 Dt \right] \\ &\quad + \left(\frac{r}{2D^{1/2} t^{1/2}} + KD^{1/2} t^{1/2} \right) \frac{e}{t} \left[- \frac{r^2}{4Dt} - K^2 Dt \right] \\ &= \frac{q_0}{8\pi^{3/2} D^{3/2} t^{3/2}} e^{Kr \cos \phi - \frac{r^2}{4Dt} - K^2 Dt}. \end{aligned}$$

Since this rate is positive for all r and t , and the initial concentration is zero, the limiting value of the concentration is that for large t , this is the same as the steady-state solution of the same problem.

For steady-state operation, the right hand side of equation (5) vanishes, and the solution of the resulting equation takes the form

$$U(r, t) = \frac{q_0}{4\pi D r} e^{-\frac{V}{2D} r}. \quad (17)$$

From this the concentration is given by:

$$C(r, \phi) = \frac{q_0}{4\pi D r} e^{-\frac{V}{2D} (1 + \cos \phi) r}. \quad (18)$$

This may be represented as a surface above a plane section through the flow axis, giving the concentration at any point of the plane. The upstream side corresponds to $\phi = 0$, and the downstream side to $\phi = \pi$. The maximum concentration can be seen to fall off with the distance from the source, and the rate of decrease upstream is very rapid. For a specific concentration, $C = C_0$, a level line can be obtained, and the greatest distances upstream and downstream at which this concentration

will be observed can be calculated.

The upstream concentration is determined by

$$C(r, 0) = \frac{q_0}{4\pi D r} e^{-\frac{V}{D} r} \quad (19)$$

The maximum distance upstream for a concentration $C = C_0$ is determined by solving the equation

$$C_0 = \frac{q_0}{4\pi D r_u} e^{-\frac{V}{D} r_u} \quad (20)$$

This can be changed to the form

$$\log r_u = \log \frac{q_0}{4\pi D C_0} - \frac{V}{D} r_u \quad (21)$$

For small values of V , r_u approaches the pure diffusion value of

$$r_u = \frac{q_0}{4\pi D C_0}$$

This also turns out to be the downstream value, r_D , since for $\phi = \pi$

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the steady-state concentration is independent of Y .

Numerical examples may be used to illustrate the orders of magnitude of the quantities involved.

$$\text{Let } q_0 = 10^{-6} \text{ cu/sec}$$

$$D = .10 \text{ cm}^2/\text{sec}$$

$$V = 10^4 \text{ cm/sec}$$

$C_0 = 10^{-9} \text{ cu/cc}$, this corresponds approximately
8 mr/hr of 1 Mev gammas.

Then

$$C(r, \phi) = \frac{10^{-5}}{4\pi} \frac{e^{-10^5 r \frac{(1 + \cos \phi)}{2}}}{r}$$

On the downstream side, $C < C_0$ for all $r > r_D = \frac{10^4}{4\pi} = 796 \text{ cm}$.

On the upstream side, $C < C_0$ for $r > r_u$ where r_u is the root of the equation

$$\log r_u = \log 796 - 10^5 r_u$$

$$r_u = .000155 \text{ cm.}$$

To show the effect of the value of the diffusion constant D , change D to 10 and leave other values unchanged.

$$C(r, \phi) = \frac{10^{-7}}{4\pi} e^{-10^3 r \frac{(1 + \cos \phi)}{2}}$$

Then

$$r_D = \frac{100}{4\pi} = 8 \text{ cm}$$

and

$$r_u = .007 \text{ cm.}$$

The effect of velocity of the stream is illustrated by changing V to 10^2 cm/sec

$$C(r, \phi) = \frac{10^{-5}}{4\pi} - \frac{e}{r} \frac{-10^3 r(1 + \cos \phi)}{2}$$

$$r_D = 796 \text{ cm as before}$$

$$r_u = .0113 \text{ cm.}$$

In the extremely slow flow of $V = 10$ cm/sec and with $D = 10$, we have

$$C(r, \phi) = \frac{10^{-7}}{4\pi} - \frac{e}{r} \frac{-r(1 + \cos \phi)}{2}$$

$$r_D = 8 \text{ cm}$$

$$r_u = 1.6 \text{ cm}$$

Since r_D is proportional to q_0 , any increase in the leak will produce a direct increase in r_D . For instance, with $q_0 = 10^{-3}$ cu/sec in the last example

$$C(r, \phi) = \frac{10^{-4}}{4\pi} - \frac{e}{r} \frac{-r(1 + \cos \phi)}{2}$$

$$r_D = \frac{10^5}{4\pi} = 7.96 \text{ Km}$$

$$r_u = 8.35 \text{ cm}$$

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SUMMARY OF RESULTS

Example	q_0	D	V	c_0	r_D	r_u
1	10^{-6}	0.10	10^4	10^{-9}	796.	.00016
2	10^{-6}	10.0	10^4	10^{-9}	8.	.007
3	10^{-6}	0.10	10^2	10^{-9}	796.	.011
4	10^{-6}	10.	10	10^{-9}	8.	1.6
5	10^{-3}	10	10	10^{-9}	7960	8.4

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CONCLUSIONS

The situation of a leak of active material into a flowing air-stream presents the problem of determining possible upstream diffusion of the activity. In a constant velocity one directional flow involving three dimensional diffusion from a point source, the upstream concentration is severely limited compared to that downstream. For all practical values of the flow velocity and diffusion coefficients, this upstream concentration is negligible.

It is to be noted, however, that the velocity profile of a flow in a finite channel involves points of very low velocity, and at these points the diffusion effect might predominate, resulting in some upstream diffusion. Even in this case, unknown convection effects would be more important than diffusion effects.

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