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UCRL - 76317
PREPRINT

CONF-741215--2

LAWRENCE LIVERMORE LABORATORY
University of California/Livermore, California

MASTER

SOME MAGNETIC EFFECTS IN
STELLAR COLLAPSE AND ACCRETION

James R. Wilson

January 6, 1975

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This paper was prepared for submission to
7th Texas Symposium on
Relativistic Astro Physics

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INTRODUCTION

In the preceding talk (T. Damour) some magnetic effects to be expected when magnetized material is accreting onto a black hole were discussed. In order to obtain analytic solutions, it was necessary to ignore material and magnetic pressures. The work to be described here was done numerically with a computer so that finite fields and pressures could be treated. These calculations demonstrate actual energy extraction from the black hole in the case of accretion, and show heuristically how a charged black hole might form in collapse.

COLLAPSE CALCULATIONS

MASTER

For the study of stellar collapse, it is necessary to solve the gravitational field equations as well as the magneto-hydrodynamic equations. The gravitational field equations are used in their stationary form. At any instant, the gravitational field is calculated as if the momentary configuration had existed forever. This is felt to be a good approximation because the high velocities are in the radial direction and in the direction of axial symmetry. For the example given below, $1/m \int (\ddot{Q})^2 dt < 10^{-6}$. The expected radiated gravitational energy is very small.

* This work was performed under the auspices of the United States Atomic Energy Commission.

The equations to be solved are from Bardeen and Wagoner, extended to include perfect conducting material with an imbedded magnetic field. The gravitational field equations are (see Wilson 1972 for notation):

Equation of motion of material

$$\text{Let } S_\mu = (\rho + P + \epsilon) U_\mu U^t \text{ and } v^i = U^i/U^t$$

$$\frac{\partial \dot{S}_\mu}{\partial t} + \frac{1}{\sqrt{-g}} \frac{\partial}{\partial X^i} (\sqrt{-g} v^i S_\mu) = \frac{1}{2} \frac{\partial g^{\alpha\beta}}{\partial X^\mu} \frac{S_\alpha S_\beta}{S^t} + J^\alpha F_{\alpha\mu}$$

$$\text{current } J^\mu = \frac{\partial}{\partial X^\alpha} (\sqrt{-g} F^{\alpha\mu})$$

Describe field by two functions H_ϕ and A_ϕ . Then condition of perfect conductivity $U^\alpha F_{\alpha\beta} = 0$ yields field tensor components

$$F_{tR} = H_\phi V^Z - \Omega A_{\phi,R}$$

$$F_{RZ} = H_\phi$$

$$F_{tZ} = -H_\phi V^R - \Omega A_{\phi,Z}$$

$$F_{R\phi} = -A_{\phi,R}$$

$$F_{t\phi} = V^R A_{\phi,R} + V^Z A_{\phi,Z}$$

$$F_{Z\phi} = -A_{\phi,Z}$$

By Maxwell's equation and perfect conductivity condition the magnetic field equations of motion are:

$$\frac{\partial H_\phi}{\partial t} = - (H_\phi V^Z)_{,Z} - (H_\phi V^R)_{,R} + \Omega_{,Z} A_{\phi,R} - \Omega_{,R} A_{\phi,Z}$$

$$\frac{\partial A_\phi}{\partial t} = - A_{\phi,R} V^R - A_{\phi,Z} V^Z$$

Gravitation equations use metric in Bardeen-Wagoner form

$$ds^2 = B^2 e^{2F} \left[e^{2c} (dR^2 + dZ^2) + R^2 (d\phi - \omega dt)^2 \right] - e^{-2F} dt^2$$

$$\nabla \cdot (B \vec{\nabla} F) = -\frac{1}{2} R^2 B^3 e^{4F} \nabla \omega \cdot \nabla \omega - 4\pi B^3 e^{2F+2c} \left[\frac{(\rho+\epsilon+p)(1+v^2)}{1-v^2} + 2p \right] - 4\pi B e^{-2F} \left\{ H_\phi^2 e^{-2c} + \frac{(1+v^2)}{R} \left[\left(\frac{\partial A_\phi}{\partial R} \right)^2 + \left(\frac{\partial A_\phi}{\partial Z} \right)^2 \right] \right\}$$

$$\nabla \cdot (R^2 B^3 e^{4F} \nabla \omega) = -16RB^4 e^{4F+2c} \frac{(\rho+\epsilon+p)}{(1-v^2)} v - \frac{8\pi e^{-4F-2c}}{R^2 B^2} v \left[\left(\frac{\partial A_\phi}{\partial R} \right)^2 + \left(\frac{\partial A_\phi}{\partial Z} \right)^2 \right]$$

$$\nabla \cdot (R \nabla B) = 16\pi R B^3 e^{2F+2c} + \frac{8\pi B e^{-2F}}{R} (1+v^2) \left[\left(\frac{\partial A_\phi}{\partial R} \right)^2 + \left(\frac{\partial A_\phi}{\partial Z} \right)^2 \right]$$

$$\frac{\partial c}{\partial R} = \left[\left(1 + \frac{R}{B} \frac{\partial B}{\partial R} \right) S_1 + \frac{R}{B} \frac{\partial B}{\partial Z} S_2 \right] / \left[\left(1 + \frac{R}{B} \frac{\partial B}{\partial R} \right)^2 + \left(\frac{R}{B} \frac{\partial B}{\partial Z} \right)^2 \right]$$

$$S_1 = R \left\{ \left(\frac{\partial \log B}{\partial Z} \right)^2 - \left(\frac{\partial \log B}{\partial R} \right)^2 - \frac{1}{2B} \left(\frac{\partial^2 R}{\partial Z^2} - \frac{\partial^2 B}{\partial R^2} \right) + \left(\frac{\partial F}{\partial R} \right)^2 - \left(\frac{\partial F}{\partial Z} \right)^2 - \frac{R^2 B^2}{4} e^{4F} \left[\left(\frac{\partial \omega}{\partial R} \right)^2 - \left(\frac{\partial \omega}{\partial Z} \right)^2 \right] \right. \\ \left. + \frac{B^2 e^{-4F-2C}}{R} (1-v^2) \left[\left(\frac{\partial A_\phi}{\partial R} \right)^2 - \left(\frac{\partial A_\phi}{\partial Z} \right)^2 \right] \right\}$$

$$S_2 = R \left\{ \frac{1}{B} \frac{\partial^2 B}{\partial R \partial Z} - \frac{2}{B^2} \frac{\partial B}{\partial R} \frac{\partial B}{\partial Z} + 2 \frac{\partial F}{\partial R} \frac{\partial F}{\partial Z} - \frac{R^2 B^2}{2} e^{4F} \frac{\partial \omega}{\partial R} \frac{\partial \omega}{\partial Z} \right\} + \frac{B^2 e^{-4F-2C}}{R} \frac{\partial A_\phi}{\partial R} \frac{\partial A_\phi}{\partial Z} (1-v^2)$$

The calculation we wish to recount is the collapse of a supermassive star of such size that the thermal properties of the gas are well represented by a perfect gas with $\gamma = 4/3$. The initial stellar configuration is a spherically symmetric density distribution with an exponential radial fall-off. The initial mean radius is the order of 100. (The mass of the star is taken as one). The star was given a rotational energy equal to ten percent of its internal energy, and a poloidal magnetic field whose energy is equal to one percent of the internal energy. The energies are taken so that the star is a few percent undersupported so that collapse promptly ensues. The boundary conditions are imposed that no material or magnetic field lines leave the calculational grid. This assures that the total electric charge of the system be zero. The gravitational field boundary conditions are those used in Wilson (1972). The hydrodynamic effect of the boundary is checked by comparing the virial $\int_S \rho \underline{S} \cdot \underline{x}$ with $\int \rho dV$. The former is at most a few percent of the latter.

The final configuration is shown in Figure 1, where isodensity lines and magnetic field lines are shown. At this time the star is only mildly distorted so that no non-axial disturbances are expected at this time. In Figure 2 the time histories of several integral quantities are presented. It should be noted that the initial collapse is almost halted at a time of 1500 (see the RZ kinetic energy). However, the toroidal magnetic field has risen sufficiently high that the outside is able to brake the inside rotations enough to allow the collapse to proceed. The rotation and magnetic field act like a homopolar generator and produce a charge separation. A charge separation

is induced with an inner charge of one sign and an outer charge of opposite sign. The total charge is zero. In Figure 2 the charge, e , of the inner region is shown as a function of time. It should be noted that e rises to about 20% and then appears to level out, even though the inner regions continue to change rapidly. The mass associated with the inner charged region is about .9 and the specific angular momentum of the inner charged region is also about .9. It seems likely therefore that the collapse will continue. The calculations were terminated when the central red shift was one.

BLACK HOLE CALCULATIONS

For the calculations of material motion near a black hole we assume the metric is given by the metric of black hole alone. Hence, we must consider the mass of gas in the vicinity of the hole to have a mass much less than the hole. In all these calculations the material is allowed to fall into the calculation grid, which extends out to 10 to 20 m , with a velocity corresponding to free fall from infinity, zero internal energy, specific angular momentum, A , with respect to the hole, and a magnetic flux with a ratio of magnetic field energy to material density given by a parameter, H . The magnetic field is taken parallel to the axis of rotation. The calculations are symmetric with respect to the $z = 0$ plane.

Three calculations were made with a Schwarzschild metric. In the first of these, the accretion parameters are $A = .0001$ and $H = .0001$. The magnetic field and rotation are able to deflect the material only very slightly from

radial. The electric charge, e , induced on the hole is found to grow linearly with time with a magnitude given by $e \simeq 2AA_\phi$, where A_ϕ is the magnetic flux at the equator and hence the magnetic moment of the hole. Figure 3 shows the field structure at late time. The radial electric field has two terms, $U_\phi H_\theta$ and $U_\theta H_\phi$, arising from the condition of perfect conductivity. To produce charge, it is necessary to break the spherical symmetry by either having H_θ or U_θ not equal zero. In the low-field low-rotation example the charge arises primarily from the $U_\phi H_\theta$ term. It may be noted that in the case of steady accretion discussed by Damour H_θ is zero, but in the present example H_θ is not zero, which is possible with spherical infall only because $\frac{\partial A_\phi}{\partial t} \neq 0$. In the other examples cited below the $U_\phi H_\theta$ term is dominant in producing the radial electric field. For all subsequent calculations the field and/or rotation are sufficient to cause large deviations from spherical infall and $e \gg 2AA_\phi$.

The second calculation has $A = .0001$ and $H = .01$. This field is large enough to strongly deflect the material from its free fall motion. Figure 4 shows how the charge and magnetic flux at the equator vary with time. The charge changes sign, because the velocity, U_θ , near the hole flops about with time. In Figure 5 the field configuration is shown. The magnetic field deflects the material towards the waist. Then near the hole the gravitational field becomes dominant and the motion becomes again almost radial. Note reversal of sign of H_ϕ .

In the last Schwartzchild calculation the rotational momentum is given a large value, $A = 2$, but the magnetic field has a small value of $H = .0001$.

In this case the centrifugal forces deflect material to the waist, enhancing the magnetic field. The combination of magnetic and rotational forces is sufficient to form a shock wave at a radius of about 5 m. Calculations with $A = 2$ and $H = 0$ form no shock waves. We thus see the complicated field structure of Figure 6.

Two calculations were performed with an extreme Kerr metric ($a = m$). In the first calculation $A = 0$ and $H = .0001$. The twisting of the field lines impedes the material enough that the angular momentum extracted from the hole (see Damour's paper) does not fall immediately back into the hole. The rise of specific angular momentum with time is shown in Figure 7. This demonstrates the extraction of energy from the hole. Finally, in Figure 8 the late time magnetic field configuration for the case of $a = m$, $A = 2$, and $H = .0001$ is shown. Again, as in the earlier calculation with $A = 2$, a strong shock wave is formed, behind which material slowly falls into the hole. Again, angular momentum is flowing out of the hole.

CONCLUDING REMARKS

The non-linear character of the flows with non-negligible fields and rotation produce partially stagnated clouds of material about black holes with induced electric charge and an outward flow of energy from the hole. For the case of accretion, the electric charge is of the order of the mass of accreting mass and so is gravitationally unimportant. For collapsing magnetized stars the charge may be of the order of the mass. In real situations a collapsed star of the order of one solar mass, the charge would

be limited to a value much smaller than the mass by pair production. Super massive black holes ($M > 10^7 \theta$) would not be so limited and hence might be expected to be highly charged. This work was done with the help of J. LeBlanc, R. Hanni, and R. Leach.

FIGURE CAPTIONS

**Fig.
No.**

- 1 The solid lines are isodensity contours at the end of the collapse calculation. The central density value is .2. The ratio of density between curves is 2. The dashed curves are the magnetic field lines.
- 2 The charge of central region, e , the total internal energy, IE , the kinetic energy of ϕ motion, KE_ϕ , the kinetic energy of the motion in the R, Z direction, $KE_{R,Z}$, the energy of the toroidal magnetic field, HE_ϕ , and the energy of the poloidal magnetic field, $H_{R,Z}$, are plotted as functions of time. The energies are calculated as if the system were Newtonian. They are only to be used qualitatively to see the general behavior.
- 3 The solid lines are the magnetic flux lines and the dashed curves are the isotoroidal field contours. The figure is for the case, $a = 0$, $A = .0001$, $H = .0001$ at a time of 90 min.
- 4 This graph is for the case of $a = 0$, $A = .0001$, and $H = .01$. Q_{hole} is the charge on the black hole, Q_{TOT} is the total charge in the whole calculational grid, A_ϕ is the magnetic flux at the equator of the hole. The integral of $(H^2 + E^2)$ is performed using the coordinate values of the field quantities.
- 5 This figure is the same as Figure 3 except $a = 0$, $A = .0001$, and $H = .01$.
- 6 This figure is the same as Figure 3 except $a = 0$, $A = 2$, $H = .0001$, and time = 388 min.
- 7 $\bar{A} = \int \rho U_\phi dV / \int \rho dV$ is the mean specific angular momentum of the material in the calculational grid exterior to the black hole; t is the time.

Figure Captions - cont.

Fig.

No.

- 8 This figure is the same as Figure 3 except $a = m$, $A = 2$,
 $H = .0001$, and $t = 100$ min.

REFERENCES

Bardeen, J. M., and Wagoner, R. V., 1971, Ap.J., 167, 359.

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FIGURE 1

FIGURE 2

SCHWARZSCHILD

FIGURE 3

FIGURE 4

SCHWARZSCHILD

FIGURE 5

SCHWARZSCHILD

FIGURE 6

FIGURE 7

FIGURE 8