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TWO-BODY NEUTRINO REACTIONS: FORM FACTORS*

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ABSTRACT

A review of experiments to measure hadronic form factors in two-body and quasi-two-body neutrino reactions is presented.

INTRODUCTION

I will review experiments to measure hadronic form factors in two-body and quasi-two-body charged-current neutrino reactions. There is data on seven reactions:

$$\nu n \rightarrow \mu^- p \quad (1)$$

$$\bar{\nu} p \rightarrow \mu^+ n \quad (2)$$

$$\bar{\nu} p \rightarrow \mu^+ \Lambda^0 \quad (3)$$

$$\nu p \rightarrow \mu^- \Delta^{++} \quad (4)$$

$$\nu n \rightarrow \mu^- \Delta^+ \quad (5)$$

$$\bar{\nu} p \rightarrow \mu^+ \Delta^0 \quad (6)$$

$$\bar{\nu} n \rightarrow \mu^+ \Delta^- \quad (7)$$

The data¹ on reactions (2), (5), (6), and (7) are so limited that little physics analysis is possible. The only results are that the cross sections are consistent with what one would expect based on form factors measured in the charge symmetric neutrino reactions; I will not discuss them further.

The data on processes (1), (3), and (4) come from spark chamber experiments at CERN, Brookhaven, and Argonne, and from bubble chamber exposures at CERN and Argonne. Fig. 1 shows the energy distribution of the various ν beams, which all employ one or two magnetic focusing devices for the parent pions and kaons. The uncertainty in the normalization was about $\pm 30\%$ in the old (< 1968) experiments. The second and third generation experiments

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have less than $\pm 15\%$ flux errors, although the errors are often larger at the low and high energy extremes of the spectrums. The neutrino energy in these experiments is essentially below 6 GeV. Within the next few months, we can expect new low energy results from the 7-ft bubble chamber at BNL and very high energy data from the 15-ft chamber at the National Accelerator Laboratory.

THE REACTIONS

The simplest charged-current neutrino reaction is $\nu n \rightarrow \mu^- p$. The hadronic current matrix element for the reaction is²

$$J_\lambda = \frac{G}{2} \bar{U}_p \left\{ \gamma_\lambda F_1^V + \frac{i\sigma_{\lambda\nu} q^\nu F_2^V}{2M} (\mu_p - \mu_n) + \frac{q_\lambda F_3^V}{M} \right. \quad (8)$$

$$+ \gamma_\lambda \gamma_5 F_1^A + \frac{q_\lambda \gamma_5 F_P^A}{M}$$

$$\left. + \frac{\gamma_5 (P_n + P_p)_\lambda}{M} F_3^A \right\} U_n,$$

where q is the four-momentum transfer between the leptons, M is the nucleon mass, μ_p and μ_n are the proton and neutron magnetic moments, and the F 's are the hadronic form factors. The form factors are complex functions of q^2 . Present day event samples consist of only a few hundred events, so one must make some assumptions to simplify the above expression if any progress is to be made.

- (1) The Conserved Vector Current Hypothesis - This hypothesis states that the divergence of the vector current is zero, and therefore the F_3^V form factor must be zero.
- (2) Time Reversal Invariance - This forces all the form factors to be relatively real.
- (3) Charge Symmetry - This symmetry requires all the form factors to be real except F_3^A which is purely imaginary. Thus assumptions (2) and (3) force F_3^A to be zero. At this point, it should be noted that F_3^A is also required to be zero if there are no Second Class axial currents. Remember that First Class currents transform under G parity like $G V G^{-1} = V$ and $G A G^{-1} = -A$; Second Class axial currents transform like $G A G^{-1} = A$.

If Second Class currents do exist, they will be quite difficult to detect in $\nu n - \mu^+ p$; one must measure the final state baryon polarization³ or measure precisely the differential cross section at very large momentum transfers.

- (4) Isotriplet Current Hypothesis - This well-known hypothesis allows one to identify F_1^V and F_2^V with the Dirac and Pauli isovector form factors measured in electron-nucleon scattering experiments. F_1^V and F_2^V are usually written in terms of the magnetic and electric form factors of the proton and neutron. To an accuracy of 7%, these functions are well represented by dipoles

$$\frac{G_m^p}{\mu_p} = \frac{G_m^n}{\mu_n} = G_e^p = \frac{1}{(1 - q^2/m_v^2)^2} \quad (9)$$

with the vector mass m_v equal to 0.84 ± 0.03 GeV. Figure 2 shows some recent data on G_m^p and a comparison of G_m^p with eq. (9).

- (5) Negligible F_p^A - Induced Pseudoscalar Form Factor - Since the axial current is believed to be only partially conserved, $F_p^A \neq 0$. Assuming F_p^A to be dominated by the pion pole, it can be expressed in terms of known coupling constants and a pion propagator. Near $q^2 = 0$, F_p^A is quite large, but since it enters the cross section multiplied by m_μ^2 , it is at most a 1% effect.
- (6) The last assumption is that the q^2 dependence of F_1^A is that of a dipole:

$$F_1^A(q^2) = 1.23 / (1 - q^2/m_A^2)^2 \quad (10)$$

This shape is used only because a dipole fits the vector form factors so well.

With these six assumptions, there is only one number, m_A , to measure. Present experiments have used the two obvious methods to measure m_A -- the shape of the differential cross section and the value of the total cross section. Theoretically, the shape method is not nearly as sensitive as the rate method, but because of the ν flux errors, the two methods are nearly equal in worth in current experiments.

Since there are no bare neutron targets available, one must use some nucleus as a target and then correct for Fermi motion and the Pauli exclusion principle. The ratio R of the nuclear cross section to that off a free neutron has been calculated by many authors.^{4,5} Figure 3 shows results for Fe^{56} and d . The theoretical error in heavy nuclei calculations such as for Fe is about $\pm 7\%$; and in d , the error is $\pm 4\%$. While the ratio R is of course strongly dependent upon q^2 , it is only weakly dependent upon the ν beam energy.

Nearly all attempts to measure the axial form factor suffer from common problems. They must have low contamination of photons and neutrons in the beam to prevent large backgrounds of $\gamma n \rightarrow \pi^+ p$ and $nn \rightarrow \pi^+ p$. The two serious neutrino-induced backgrounds are $\nu n \rightarrow \mu^- p \pi^0$ and $\nu p \rightarrow \mu^- p \pi^+$, since both of these cross sections are comparable with that of $\nu n \rightarrow \mu^- p$; if the photons from the π^0 decay go undetected or the π^+ scatters and is absorbed in a heavy nucleus target, one-pion production can result in a quite large contamination. Also in heavy nucleus experiments, the final state proton may scatter in the nucleus so that its momentum is not well determined.

Table 1 summarizes the five experiments to measure $\nu n \rightarrow \mu^- p$.⁶⁻¹⁰ The first three suffer from large π^0 and π^+ contamination and large ν flux errors; they measured m_A by using the shape of the dN/dq^2 distribution. The last two experiments used dN/dq^2 and the magnitude of the cross section. Let me discuss these two experiments in some detail.

The CERN experiment used a C_3H_8 fill of 0.5 m^3 bubble chamber. The candidates for $\nu n \rightarrow \mu^- p$ had one muon track and one or two protons; the second proton can come from rescattering effects in the nucleus. In order to remove the neutron, photon, and charged hadron background, two cuts were made: the visible momentum along the known neutrino beam direction had to be greater than $0.3 \text{ GeV}/c$ and the visible energy had to be greater than 0.3 GeV . After these cuts 130 events remain. In an attempt to remove $\nu p \rightarrow \mu^- p \pi^+$ background, one considers the quantity $m_c^2 = (P + P_T - P_\mu)^2$ the missing-mass squared recoiling from the muon; Fig. 4a, b shows the distribution of this variable for events with one and two protons, respectively. The one-proton distribution looks reasonable compared with the solid curve -- a Monte Carlo calculation based on the Fermi gas model of the nucleus. However, the two-proton data is shifted to high mass values. These events are believed to be strongly contaminated by inelastic events and were, therefore, removed from the physics sample; corrections were made to account for their removal. In

the one-prong m^2 distribution, the dotted line indicates the estimated π^0 contamination. In order to reduce this background, a cut was made on the m^2 distribution at 1 GeV^2 . Sixty-six events remain. Figures 5 and 6 show the differential cross section and energy variation of the production cross section after corrections were made for all of the cuts. A fit to the shape and rate gives $m_A = (0.7 \pm 0.2) \text{ GeV}$; the curves in Figs. 5 and 6 show the results of the fit.

A second analysis of this data was also performed. In order to remove the effects of the Pauli exclusion principle, events with q^2 less than $0.3 (\text{GeV}/c)^2$ were removed; and to reduce some of the uncertainties in the ν flux, an $E_\nu \geq 1 \text{ GeV}$ cut was made. Only 26 events remain. These events represent a $\nu n \rightarrow \mu^- p$ rate that also corresponds to $m_A = (0.7 \pm 0.2) \text{ GeV}$.

Next consider the continuing Argonne experiment using a deuterium fill of the 12-foot bubble chamber (11.1 m^3 fiducial volume). By using deuterium as a target, the experiment avoids many of the problems of these early studies. The candidates appear in the 12-foot chamber as 1-, 2-, or 3-prong events depending upon the spectator proton momentum and q^2 . The 1-prong events are not used in this analysis. About 30% of the proton spectators are seen (3-prongs) in the 12-foot chamber, as compared to about 40% usually seen in small deuterium bubble chambers.

In order to separate the $\mu^- p$ events and to investigate background contamination, consider the 0-constraint "fit" to $X^0 + d \rightarrow \mu^- pp_s$, where X^0 has unknown mass, energy, and direction. The mass-squared of X^0 is plotted in Fig. 7 versus the angle of X^0 relative to the known neutrino beam direction for the events with a seen proton spectator. A clear enhancement is seen at zero mass and angle. Figure 8 shows the same distribution for the unseen spectator events. Again, a clear signal is observed, although it is somewhat washed out compared to that in Fig. 7. Most of the other events in these plots are charged hadron scatters (eq. $\pi^+ n \rightarrow \pi^0 p$). By carefully examining the distributions, the non-neutrino induced background is determined to be less than 2%. The contamination from the $\nu d \rightarrow \mu^- pp_s \pi^0$ process is determined to be $(1 \pm 1)\%$. This value was measured by selecting $\nu d \rightarrow \mu^- p \pi^+ n_s$ events in the same film, deleting the π^+ track, and treating the events as $\nu d \rightarrow \mu^- p n_s$. (The $\Delta I = 1$ rule was used to estimate the $\mu^- pp_s \pi^0$ cross section relative to that of $\nu d \rightarrow \mu^- p \pi^+ n_s$.)

Now, for all of the 2- and 3-prong events, 3-constraint kinematic fits were attempted to $\nu d \rightarrow \mu^- pp_s$. For the unseen spectator events, the standard bubble chamber method was used of assigning a momentum of $0 \pm 50 \text{ MeV}/c$ to the

spectator proton. Figure 9 shows the spectator proton momentum distribution. (The spectator proton is defined to be the one with lower momentum.) There is a small excess of events at high momentum, which can be attributed to rescattering effects. Figures 10 and 11 display the dN/dq^2 and $\sigma(E)$ distributions, respectively. A fit just to the shape of the differential cross section gives $m_A = 0.79^{+0.14}_{-0.15}$ GeV. A fit just to the rate gives $m_A = 1.04^{+0.20}_{-0.17}$ GeV. Since these results are consistent, we do an overall shape plus rate fit and obtain $m_A = (0.90 \pm 0.10)$ GeV. This result is stable against subdivisions of the data into low/high energy, low/high q^2 , seen/unseen spectator, etc. If one allows vector mass parameter m_V to be variable as well as m_A and makes a two-parameter fit (using dipole form factors for both), we find $m_V = 0.70^{+0.19}_{-0.14}$ GeV and $m_A = 1.14^{+0.18}_{-0.32}$ GeV. So the data is consistent with the conserved vector current hypothesis. One can also try fitting the data with a monopole axial vector form factor, with the vector form factors fixed by the electron-scattering results. The monopole form gives an equally acceptable fit with $m_A = (0.54 \pm 0.08)$ GeV.

The weighted average value of m_A from the five experiments shown in Table I is (0.87 ± 0.08) GeV. Thus, the vector and axial mass parameters are equal to each other within errors.

Theoretically, Riazuddin¹¹ has derived a relation between F_1^A and the magnetic form factor of the proton. He assumes scaling to hold and the Llewellyn-Smith sum rule¹¹, which relates the structure functions of inclusive neutrino-proton and neutron scattering to those of electron nucleon scattering. At $q^2 \rightarrow \infty$, one obtains the simple relation $F_1^A/G_M^p = 1/2 \beta (1 + \mu_n/\mu_p)$, where $\beta = 6$ in the quark model and 2 in the Sakata model. If one makes the strong assumption that both form factors are dipoles at large q^2 values, then one obtains the result that $m_A = 1.01$ GeV and 0.76 in the quark and Sakata models, respectively. So in this simple calculation, the data lies between the two predictions.

The measurement of m_A in neutrino reactions is also of interest because one can use it to test current-algebra and PCAC ideas. This comes about because m_A can be determined in the reaction $cp \rightarrow en\pi^+$ at threshold. The differential cross section for the latter can be written as

$$\frac{d\sigma}{dE'd\Omega} = s \sigma_{\text{Mott}} q_\pi^* + \left(\frac{d^2\sigma}{dE'd\Omega} \right)_{\Delta 1236} + \dots, \quad (11)$$

where the first term is the s-wave contribution and the second term is the p-wave Λ_{1236}^+ contribution (q_π is the π^+ momentum in the π^+n rest frame). Now a measurement of the slope parameter s can be used along with current-algebra and PCAC to determine F_1^A in the soft pion limit. In fact, s is given simply by

$$s = \frac{g^2}{16\pi^2 (M+m_\pi)^2} \left\{ 2 \left(\frac{F_1^A}{F_1^A(0)} - \frac{q^2}{2M^2 - q^2} G_m^n \right)^2 - 8M^2 q^2 \left(\frac{G_e^n}{2M^2 - q^2} \right)^2 \right. \\ \left. + 4 \tan^2 \theta/2 \left(1 - \frac{q^2}{4M^2} \right) \left(\frac{F_1^A}{F_1^A(0)} - \frac{q^2}{2M^2 - q^2} G_m^n \right)^2 \right\} \quad (12)$$

The problem is that one needs to correct this expression for the finite pion mass, and there seems to be no unique way to do this at present. I have chosen to use the recent analysis of Brian Read.¹² Figure 12 shows measurements of F_1^A by four groups¹³⁻¹⁶ studying $ep \rightarrow en\pi^+$. The curve is the average value of m_A obtained from $\nu n \rightarrow \mu^- p$. There appears to be some disagreement and the disagreement persists no matter what currently popular formulation of the finite mass corrections is used.

The next two-body reaction I want to review is $\bar{\nu}p \rightarrow \mu^+ \Lambda^0$. Only one experiment has observed this process -- the current CERN experiment¹⁷ using a freon fill of the Gargamelle bubble chamber. They observe 13 events which corresponds to a cross section off freon of $(1.3^{+0.7}_{-0.5}) 10^{-40}$ cm²/proton. The important question is, what is the cross section off free protons? Detailed Monte Carlo studies show that the probability for Λ absorption is $\sim 15\%$, and the probability that a Σ will convert into a Λ in the freon nucleus is $\sim 20\%$. Now the cross section for Σ production divided by that for Λ^0 production is about 0.75, independent of the value of m_A , assuming exact SU(3) to hold. So the cross section scale factor to transform the freon result to a free proton result is determined to be 1.0 ± 0.5 . The $\bar{\nu}p \rightarrow \mu^+ \Lambda$ cross section is thus $(1.3^{+1.0}_{-0.8}) 10^{-40}$ cm². To do better, one clearly needs data in a hydrogen or propane bubble chamber.

Figure 13 shows the energy distribution of the events as a function of q^2 . Note that below 3 GeV, essentially all of the antineutrinos come from π^- decays; only above ~ 4 GeV do K^- decays contribute strongly to the $\bar{\nu}$ flux. So presumably $\bar{\nu}_{\pi^-}$ can produce strange particles.

While the data in Fig. 13 looks reasonable, there is clearly not enough events to determine the form factors from the dN/dq^2 shape. To determine m_A from the cross section, one can write the matrix element for the reaction as

$$J_\lambda = -\frac{G}{2} \sqrt{\frac{3}{2}} \sin \theta_c \bar{U}_\Lambda \left[\gamma_\lambda F_1^p + \frac{i\sigma_{\lambda\nu} q^\nu}{2m} \mu_p F_2^p \right. \\ \left. + \frac{1+2x}{3} \gamma_\lambda \gamma_5 F_1^A \right] U_p \quad (13)$$

where I have made the usual assumptions to simplify the expression. Furthermore, if one assumes $m_A = 0.84$ GeV, exact SU(3) symmetry, and an F/D ratio of $0.45/0.70$ independent of q^2 , then the cross section averaged over the CERN $\bar{\nu}$ flux distribution is $2.4 \times 10^{-40} \text{ cm}^2$. So the data is consistent with this simple theory and an axial vector form factor equal to that measured in $\Delta S = 0$ reactions.

The final reaction I want to review is $\nu p \rightarrow \mu^- \Delta_{1236}^{++}$. This process was first studied using a "free proton" target in the old CERN propane bubble chamber.¹⁸ In this experiment, there was 85 3-prong events with track ionization and delta rays consistent with the $\mu^- \pi^+ p$ final state. To separate out the $\mu^- \pi^+ p$ events off free protons, the group used longitudinal and transverse momentum unbalance. Figure 14 shows a scatter plot of these two variables for the $\mu^- \pi^+ p$ candidates. An enhancement is certainly present at low momentum unbalance. Cuts were made at $|\Delta P_x| \leq 120 \text{ MeV/c}$ and $P_\perp < 240 \text{ MeV/c}$. Fifty-two events remain. To estimate the contamination from $\nu p \rightarrow \mu^- \pi^+ p \pi^0$, one uses the 12 events where one or both photons from the π^0 decay convert into $e^+ e^-$ pairs; after applying the longitudinal and transverse momentum cuts, the background from π^0 production is 4.3 ± 2.5 events. The contamination from carbon interactions is measured by comparing the propane events with data from a freon fill of the chamber; the carbon interaction background is thus estimated as 4.6 ± 1.5 events. So the $\mu^- \pi^+ p$ data sample contains 15% contamination.

Figure 15 shows a scatter plot of the $\pi^+ p$ invariant mass as a function of q^2 for the 52 events. A strong Δ_{1236}^{++} signal is seen above a low background; the events are mainly produced at low q^2 values.

Before discussing the CERN data further, let's introduce the Argonne data¹⁹ obtained from hydrogen and deuterium fills of the 12-foot chamber. To separate the $\mu^-\pi^+p$ events, consider all 3-prong events whose tracks reconstruct as $\mu^-\pi^+p$. For the 410 such events in H_2 , we perform the 0-constraint "fit" to $X^0 + p \rightarrow \mu^-\pi^+p$ where, as before, X^0 has unknown mass, energy, and direction. Figure 16 shows a scatter plot of $(MM)_{X^0}^2$ versus $(\phi)_{X^0}$; a clear signal at zero mass and angle is seen. The background is mainly from π^0 production and is estimated to be 2%. In D_2 , one has the corresponding reaction $X^0 + d \rightarrow \mu^-\pi^+pn_s$, where the neutron spectator is undetected. Figure 17, which shows a histogram of $(MM)_{X^0}^2$ when (ϕ) is less than 20° . The cross-hatched events give a 3-constraint fit to $\mu^-\pi^+p$; the background is seen to be quite low, and by carefully examining the events in the wings, is determined to be $\leq 5\%$. In order to perform the 3-constraint fit in D_2 , the starting value for the momentum of the neutron spectator was set at 0 ± 50 MeV/c. As expected, there is then a loss of high momentum spectators. This loss is estimated to be $(17 \pm 5)\%$ based on the seen proton spectators in the $\nu d \rightarrow \mu^-\pi^+pn_s$ reaction.

Figure 18 shows the three invariant mass combinations for the combined H_2 and D_2 data. As in the CERN data, the final state is dominated by Δ_{1236}^{++} production. A fit to the mass spectrum gives the resonant fraction as $(95 \pm 5)\%$. There is no evidence for μ^-p or $\mu^-\pi^+$ mass enhancements. Figure 19(a) shows the q^2 distribution for the events; the mean value is 0.3 (GeV/c)². However, q^2 is not a very interesting variable to look at because the kinematic limits of q^2 depend upon the neutrino energy, and this is a variable energy experiment. Figure 19(b) shows instead the $\cos \theta_{\nu\mu}^*$ distribution; the data is forward peaked, but not as strongly as suggested by the q^2 plot.

Figure 20 shows the energy variation of the cross section for the CERN and ANL data. All flux uncertainties are included in the error bars. The Argonne results are somewhat lower than that of CERN, but perhaps consistent within the error bars.

More information about the production process, and therefore the form factors, can be learned by studying the decay angular distribution of the Δ^{++} . Figure 21 shows the coordinate system defined by S. Adler; it has the y-axis along the production plane normal and the z-axis along the three-momentum transfer direction between the two leptons. In this system, the decay distribution can be represented by the sum of four terms, assuming time reversal invariance

to hold in the production process and the absence of background:

$$D(\theta\phi) = \frac{1}{\sqrt{4\pi}} \left\{ Y_0^0 - \frac{2}{\sqrt{5}} (\rho_{33} - \frac{1}{2}) Y_2^0 - \frac{4}{\sqrt{10}} \rho_{3-1} \operatorname{Re} Y_2^2 + \frac{4}{\sqrt{10}} \rho_{31} \operatorname{Re} Y_2^1 \right\} \quad (14)$$

where the ρ_{nm} are the density matrix elements and the Y_L^M are the usual spherical harmonics. Figure 22 shows histograms of $\cos \theta$ and ϕ in the CERN and ANL experiments. The distributions are relatively featureless, although there are some weak indications of "illegal" moments in two of the plots.

Now the question is, what information about form factors can be learned from all of these distributions? To answer that we must first examine the hadronic current matrix element. There are several sets of variables used in the literature to study Δ_{1236} production: multi-pole moments, helicity amplitudes, and Rarita-Schwinger form factors. Since the title of this talk is "Form Factors," let me use the form factor notation, and to be precise, I will use that given by Llewellyn-Smith in his review paper.² The matrix element is thus

$$M = \sqrt{3} G \bar{\psi}_\alpha \left\{ \left[\frac{C_3^V}{m} \gamma_\lambda + \frac{C_4^V}{m^2} (P_\Delta)_\lambda + \frac{C_5^V}{m^2} (P_P)_\lambda \right] \gamma_5 F^{\lambda\alpha} + C_6^V j^\alpha \gamma_5 + \left[\frac{C_3^A}{m} \gamma_\lambda + \frac{C_4^A}{m^2} (P_\Delta)_\lambda \right] F^{\lambda\alpha} + C_5^A j^\alpha + \frac{C_6^A}{m^2} (q)_\alpha q \cdot j \right\} U_p \quad (15)$$

where j_α is the lepton current, $F^{\lambda\alpha} = q^\lambda j^\alpha - q^\alpha j^\lambda$, and the $C_i^{V,A}$ functions are the four vector and four axial form factors. One would, of course, like to simplify the expression. First, assume time reversal invariance; this forces all the form factors to be relatively real. Second, assume the isotriplet current hypothesis so that the vector form factors can be determined from electroproduction data on the Δ_{1236} . One finds²⁰ that $C_5^V = C_6^V = 0$, $C_4^V \approx -\frac{M}{W} C_3^V$ and C_3^V is well represented by the dipole expression:

$$C_3^V = \frac{2.05}{(1 - q^2/0.54)^2} . \quad (16)$$

However, note that the mass parameter is 0.73 GeV instead of 0.84 GeV as in elastic scattering. Figure 23 compares C_3^V with G_m^D .

Consider next the axial form factors. By measuring the differential cross section and three density matrix elements at different values of neutrino energy, the four axial form factors are far over determined.²¹ But present day experiments just do not have enough events to perform the measurements. So one must consider model predictions for the form factors. C_3^A and C_4^A are completely model dependent. C_6^A , the induced pseudoscalar form factor, is always taken to contain a pion pole and is given by:

$$C_6^A = \frac{m g_{\Delta P\pi\pi} F_\pi}{2\sqrt{3} (m_\pi^2 - q^2)} . \quad (17)$$

Note that the cross section is not sensitive to C_6^A since it enters multiplied by m_μ^2 . To determine C_5^A , one may use the PCAC hypothesis. Letting $m_\pi^2 \rightarrow 0$ and $q^2 \rightarrow 0$ and forcing the divergence of the axial current to equal 0, one has

$$C_5^A(0) = 1.2 .$$

Table II summarizes the predictions for the dominant axial form factors at $q^2 = 0$ in several different models.²¹⁻²⁵ The q^2 dependence of the form factors in the various models are somewhat different, but they all contain a common dipole factor of $(1 - q^2/m_A^2)^{-2}$, where m_A is the same mass parameter measured in elastic scattering.

To compare these models with data, we have performed maximum likelihood fits to the experimental production and decay angular distributions and the energy dependence and magnitude of the cross section. m_A was allowed to be a free parameter in these fits. Table III gives our fitted values of m_A and the corresponding density matrix elements, averaged over all production angles (only a subsample of 121 events were available at the time these fits were done). Salin's and Bijtebier's models disagree badly with the data. However, Adler and Zucker do give reasonable density matrix elements and cross sections. For these two models, the production and differential cross sections are compared in Figs. 20 and 24 respectively. As can be seen, the data

is not yet good enough to separate the two models. The revised Zucker model²⁶ (which has $C_5^A = 1.2$ and thus satisfies PCAC) also gives a good fit to the data.

PROGRESS IN THE NEAR FUTURE

Considering the possible progress one can expect on form factor measurements in the next few years, the major limitation will still be neutrino flux. With more data from Argonne and new data from Brookhaven and NAL, one can hope for the following:

- $\nu N \rightarrow \mu N$
 - test if the dipole axial form factor is the correct shape.
 - resolve the possible discrepancy with $e p \rightarrow e n \pi^+$ measurements of m_A .
 - test the isotriplet current hypotheses that $m_V = 0.84 \pm 0.03$ GeV.
- $\bar{\nu} N \rightarrow \mu^+ Y$
 - test the SU_3 and $\Delta I = 1$ predications for $\bar{\nu} n \rightarrow \mu^+ \Sigma^-$, $\bar{\nu} p \rightarrow \mu^+ \Sigma^0$, and $\bar{\nu} p \rightarrow \mu^+ \Lambda^0$.
- $\nu N \rightarrow \mu \Delta_{1236}$
 - actual measurements of the axial form factors, not just tests of model predictions.
 - test of $\Delta I = 1$ rule for the $(\nu n \rightarrow \mu^- \Delta^+)/(\nu p \rightarrow \mu^- \Delta^{++})$

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TABLE I - Measurements of the Axial Vector Mass

Experiment	No. Events	Contamination	Flux Error	Method	M_A (GeV)
CERN BC CF_3Br	88	15%	$\pm 30\%$	$\frac{dN}{dq^2}$	0.75 ± 0.24
CERN Sp C Al	74	45%	$\pm 30\%$	$\frac{dN}{dq^2}$	0.65 ± 0.42
ANL Sp C Fe	~ 200	25%	$\pm 30\%$	$\frac{dN}{dq^2}$	1.05 ± 0.20
CERN BC C_3H_8	26 66	10%	$\pm 15\%$	$\frac{dN}{dq^2}, \sigma$	0.70 ± 0.20
ANL BC D_2	288	3%	$\pm 15\%$	$\frac{dN}{dq^2}, \sigma$	0.90 ± 0.10

**TABLE II - Predications for the Dominant Axial Vector
Form Factor at $q^2 = 0$**

Model	$C_3^A(0)$	$C_4^A(0)$	$C_5^A(0)$
Salin	0.0	2.7	0.0
Static	0.0	- 0.3	1.2
Quark			
Adler			
Bijtebier	0.0	~ 3.0	1.2
Zucker	1.8	- 1.8	1.9

TABLE III - Experimental density-matrix elements and, for each model: fitted values of m_Λ , integrated density-matrix elements and chi-squared (over the number of data points) for density matrix elements.

	$m_\Lambda(\text{GeV})$	$\tilde{p}_{33}$	$\tilde{p}_{3-1}$	$\tilde{p}_{31}$	$\chi^2(p_{mn})$
Data		0.58 ± 0.09	-0.24 ± 0.11	-0.18 ± 0.11	
Salin	0.73 ± 0.14	0.46	0.05	0.22	$\frac{22}{3}$
Adler	0.96 ± 0.16	0.67	-0.01	-0.13	$\frac{6}{3}$
Bijtebier	0.69 ± 0.12	0.66	-0.05	0.13	$\frac{12}{3}$
Zucker	0.74 ± 0.13	0.65	-0.03	-0.19	$\frac{4}{3}$

Fig. 1 Neutrino Flux distributions at Argonne, Brookhaven, and CERN.

Fig. 2 Measurements of the vector form factor G_m^p in electron-nucleon scattering experiments.

Fig. 3 Pauli exclusion ratios for iron and deuterium - see Ref. 4.

Fig. 4 Missing-mass squared distribution for $\nu n \rightarrow \mu^- MM$ in CERN propane experiment.

Fig. 5 Differential cross section for $\nu n \rightarrow \mu^- p$ in CERN propane experiment; solid curve is V-A theory with $m_A = 0.7$ GeV.

Fig. 6 Cross section for $\nu n \rightarrow \mu^- p$ as a function of neutrino energy as measured in the CERN propane experiment; solid curves correspond to V-A theory.

Fig. 7 $M_{X^0}^2$ versus ϕ_{X^0} in $X^0 + d \rightarrow \mu^- pp_s$ for 3-prong events in the Argonne deuterium experiment.

Fig. 8 $M_{X^0}^2$ versus ϕ_{X^0} in $X^0 + d \rightarrow \mu^- p (p_s)$ for 2-prong events in the Argonne deuterium experiment.

Fig. 9 Spectator proton momentum distribution from $\nu d \rightarrow \mu^- pp_s$ in the Argonne deuterium experiment. The curve corresponds to a Hulthén distribution normalized to the number of events.

Fig. 10 Differential cross section for $\nu d \rightarrow \mu^- pp_s$ in the Argonne deuterium experiment. The curve corresponds to the V-A theory with $m_A = 0.90 \text{ GeV}$.

Fig. 11 Cross section for $\nu n \rightarrow \mu^- p$ as a function of neutrino energy as measured in the Argonne experiment. The data is corrected for scanning loss at small q^2 and the Pauli exclusion principle. The curve corresponds to the V-A theory with $m_A = 0.90 \text{ GeV}$.

Fig. 12 Measurements of the axial vector form factor in threshold $ep \rightarrow en\pi^+$ experiments. The curve is the result from $\nu n \rightarrow \mu^- p$ experiments.

Fig. 13 Anti-neutrino energy versus q^2 for $\bar{\nu} p \rightarrow \mu^+ \Lambda^0$ events observed in the Gargamelle experiment.

Fig. 14 Longitudinal versus transverse momentum unbalance for $\nu p \rightarrow \mu^- \pi^+ p$ candidates as measured in the CERN propane experiment.

Fig. 15 Chew-Low plot $\nu p \rightarrow \mu^- \pi^+ p$ in the CERN propane experiment.

Fig. 16 $M^2_{X^0}$ versus ϕ_{X^0} in $X^0 + p \rightarrow \mu^- \pi^+ p$ in the Argonne hydrogen experiment.

Fig. 17 Histogram of $(MM)_{X^0}^2$ when $\phi_{X^0} \leq 20^\circ$ in the process $X^0 + d \rightarrow \mu^- \pi^+ p n_s$ in the Argonne deuterium experiment.

Fig. 18 Two-body invariant mass combinations for 220 $\nu p \rightarrow \mu^- \pi^+ p$ events in the Argonne hydrogen and deuterium experiments.

Fig. 19 Differential cross section for $\nu p \rightarrow \mu^- \Delta^{++}$ in the Argonne experiment plotted in terms of (a) q^2 and (b) $\cos \theta_{\nu\mu}^*$.

Fig. 20 Energy variation of the cross section for $\nu p \rightarrow \mu^- \Delta^{++}$ in the CERN and Argonne experiments. Curves correspond to models of S. Adler and P. Zucker.

Fig. 21 Coordinate system for Δ^{++} decay in the reaction $\nu p \rightarrow \mu^- \Delta^{++}$.

Fig. 22 Δ^{++} decay angular distributions in the CERN and Argonne experiments.

Fig. 23 Comparison of C_3^V with the dipole form factor measured in elastic electron scattering experiments.

Fig. 24 Differential cross section for $\nu p \rightarrow \mu^- \Delta^{++}$ as measured in Argonne experiment.