EXPERIMENTAL STUDY OF THE STRESS-STRAIN PROPERTIES
OF LEAD UNDER SPECIFIED IMPACT CONDITIONS

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ABSTRACT

A study was conducted to determine the dynamics stress-strain properties of lead subjected to gross compressive strains at strain rates comparable to those experienced by lead shielding in shipping containers for radioactive materials under accident conditions.

Lead specimens were placed on a load cell and impacted with a free falling weight. It was found that chemical lead is not significantly strain-rate sensitive in the strain-rate range of 100-800 in/in/sec for strains up to 0.5 in/in. However, when the dynamic stress-strain properties are compared with static and quasi-static properties, the lead exhibits a significant degree of rate sensitivity.

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NOMENCLATURE

\[ a(t) \quad = \quad \text{acceleration (function of time)} \]
\[ b \quad = \quad \text{constant} ; \quad 7800 \text{ psi } (5.38 \times 10^7 \text{ N/m}^2) \]
\[ c \quad = \quad \text{constant} ; \quad 0.004 \]
\[ f(t) \quad = \quad \text{force (function of time)} \]
\[ g \quad = \quad \text{acceleration of gravity, 386 in/sec}^2 \quad (9.80 \text{ m/sec}^2) \]
\[ h \quad = \quad \text{drop height} \]
\[ k \quad = \quad \text{constant} ; \quad 2.02 \]
\[ M \quad = \quad \text{mass of falling weight} \]
\[ m \quad = \quad \text{constant} ; \quad 0.44 \]
\[ m_e \quad = \quad \text{effective mass of specimen} \]
\[ n \quad = \quad \text{constant} ; \quad 0.56 \]
\[ V_c \quad = \quad \text{velocity of falling weight at impact} \]
\[ V_e(t) \quad = \quad \text{velocity of top surface of lead specimen (function of time)} \]
\[ X(t) \quad = \quad \text{displacement of top surface of lead specimen (function of time), in.} \]
\[ \varepsilon \quad = \quad \text{engineering strain, in/in.} \]
\[ \dot{\varepsilon} \quad = \quad \text{strain-rate in/in-sec.} \]
\[ \sigma_a \quad = \quad \text{apparent stress} \]
\[ \sigma_s \quad = \quad \text{true static compressive stress} \]
\[ \sigma_t \quad = \quad \text{true stress} \]
INTRODUCTION

The Federal Government, through the United State Atomic Energy Commission and the Department of Transportation, has established regulations governing shipment of radioactive materials. One of the regulations specifies that the container must be able to withstand a free drop through a distance of 30 feet (9.14 m) onto a flat essentially unyielding horizontal surface, striking the surface in a position in which maximum damage is expected.

Analyses of shielded containers have generally resulted in ultraconservative design in that most of these analyses were based on material properties determined by static tests. However, many materials have impact properties which vary significantly from their static properties. Since lead is frequently used as the shielding material in shipping casks, the determination of its stress-strain properties under impact conditions is of importance. A survey of the literature revealed that a considerable amount of work had been done to determine the impact properties of lead. However, the previous work did not cover the strain-rates fall. Consequently, the study reported here was undertaken.

Previous Work

Several investigators studied the effects of impacting lead bodies with bodies of harder materials. Among these were
Vincent [1], Tabor [2], Crook [3], and Mok and Duffy [4, 7]. They determined a factor called "flow pressure" based on test parameters and the geometry of the impact-affected area of a specimen after impact. The magnitude of this factor varied from about 8800 to 16,850 psi (6.07x10^7 to 1.16x10^8 N/m^2), depending upon test conditions, parameters, and specimen geometry.

Tests were conducted by Mok and Duffy [5] and by Gondusky and Duffy [6], in which a Hopkinson bar apparatus was used to determine the dynamic stress-strain characteristics of lead at strain rates of 150, 1110, 1190, and 1500 in/in per sec. The maximum strains achieved in these tests were about 0.12 in/in. Mok and Duffy [5] derived an empirical equation which described their data as follows.

$$\sigma_t = b\varepsilon^n$$  \hspace{1cm} (1)

where

- $\sigma_t$ = true stress in psi (N/m^2)
- $b = 7800$ for a strain rate of 150 in/in-sec
  ($5.38x10^7$ for stress in N/m^2)
- $\varepsilon$ = engineering strain
- $n = 0.432$ for a strain rate of 150 in/in/sec.

Impact tests in which 1.0 in (0.0254 m) diameter right circular cylinders 1 in (0.0254 m) long were impacted with a free

*Numbers in brackets designate references.*
falling weight were conducted by Slater, Johnson, and Aku [7]. A mass of 22.5 lb. (10.19 kg) was allowed to free fall through a distance which was varied from 2 to 12 ft. (0.608 to 3.65 m) and impact a guided plunger resting on the specimen. The applied force was measured with a load cell and the displacement was measured with a capacitance type of displacement transducer. Those values were recorded with respect to time by using a cathode ray oscilloscope and Polaroid photography equipment. Slater, Johnson, and Aku proposed the approximate equation.

\[ \sigma_+ = \sigma_0 \left( 1 + C \varepsilon^m \right) K \varepsilon^n, \]

where:

- \( \sigma_+ \) = dynamic true stress,
- \( \sigma_0 \) = static true compressive stress,
- \( C \) = constant with mean value of 0.004,
- \( \varepsilon \) = average compressive engineering strain rate,
- \( n \) = index with mean value of 0.66,
- \( K \) = constant with mean value of 2.02,
- \( \varepsilon \) = compressive engineering strain at any instant, and
- \( m \) = index with mean value of 0.44.

**Test Program and Apparatus**

The general test philosophy of impacting specimens with a free falling weight was selected as being most applicable to the problem. It was felt that this approach offered the most promising means of obtaining the desired data within the limit;
of engineering accuracy at a minimum cost. It was also recognized that the successful conclusion of this test program was contingent on the ability to measure and record or determine the force applied to the specimen, the velocity of some point or points in the specimen, and the displacement of the specimen with respect to time. Considerable difficulty and expense were anticipated if either velocity and displacement were to be measured and recorded directly. In contrast, the techniques for measuring and recording force with respect to time are well developed, universally accepted, and obtainable at reasonable costs. The other parameters, velocity and displacement, can be derived from the force-time data by integrating the equation of motion obtained from Newton's second law.

The test facility was designed, fabricated, and installed at the Oak Ridge National Laboratory Drop Tower. Three steel rods 1/2 in (0.013 m) in diameter were suspended the length of the tower and aligned with respect to each other by spacers located at appropriate intervals. This assembly served as a guide for the cylindrical weights used to impact the specimens. These weights consisted of three right circular mild-steel cylinders which weighed 6.94, 8.69, and 10.56 lb. (3.15, 3.94, and 4.79 kg) and they were dropped from a maximum height of 432 in (11.97 m).

Force was measured with a load cell which was a 4 in (0.10 m) long cylinder with an outside diameter of 2.0 in (0.051 m) and an
inside diameter of 1.80 in (0.0457 m) that was fabricated of Type 6061-T6 aluminum alloy. Bonded resistance strain gages were cemented to the cell in a full bridge, as illustrated in Fig. 1. This arrangement effects a gage output amplification of approximately 2.6 as compared with a single compression gage. Temperature compensating foil gages with a gage length of 1/8 in (0.0032 m) were used. The output of the Wheatstone bridge was fed to a cathode ray oscilloscope after amplification. The battery powered trace trigger was used to initiate the single-sweep scope trace, and a Polaroid camera was used to record the force signal displayed on the screen of the scope at a known sweep rate. A schematic diagram of this apparatus is illustrated in Fig. 1.

The load cell, amplifier, and oscilloscope package were calibrated to the design capacity of the load cell (12,000 lb., 5441 kg) by using a conventional compression testing machine. The free fall velocities of weights released from different heights were determined by using electronic equipment to measure the time required for a weight to travel a fixed distance between two conductors just prior to impact. The results of these tests demonstrated that the friction effects of the guide rails on the weights and air resistance were negligible. Consequently, the velocity of the free falling bodies could be calculated by the well-known equation
\[ V_0 = (2gh)^{1/2}, \quad (3) \]

where

\[ g = \text{acceleration of gravity} \]
\[ h = \text{the height or distance through which the weight falls.} \]

A survey of shipping container design practice revealed that cast-in-place chemical load is generally specified as the shielding material. The American Society for Testing and Materials Specification ASTM B29 is also frequently used to further specify the material. Chemical lead conforming to ASTM B29 was therefore selected as the test material.

Right circular cylinder test specimens with diameters of 1.0 (0.0254 m) and 1.25 in (0.0318 m) and length-to-diameter ratios of 1, 1.5, and 2 were cast in high-quality graphite molds. The ends were machined so that they were parallel to each other and perpendicular to the center line of each cylinder within ±0.001 in (2.54x10^-6 m) in 4 in (0.1016 m). The diameters did not require machining.

Similar static tension and compression test specimens were also fabricated from the same lot of material. Tensile and compressive stress-strain tests were conducted with these specimens to establish the actual static properties of the impact test material, and the results of these tests are illustrated in Figs. 2 and 3.

**Performance of Impact Tests**

Sixty-eight specimens were impact tested, and the force-
time data for each impact were measured and photographically recorded with the equipment previously described. Cylindrical lead specimens with five different geometries were impact tested with three different weights dropped from several different elevations; and the combinations of nominal specimen geometry, weight, and drop height for which data were taken are given in Table I. Specimens before and after impact testing are shown in Fig. 4 for visual comparison.

The procedures used to conduct the impact tests for each of the 68 specimens were as follows. A test specimen of the desired geometry was selected, and its diameter and length were measured to the nearest thousandth of an inch and recorded. The machined ends of the specimen were lubricated with Garlock "Lubal" molybdenum disulfide high pressure lubricant to reduce friction between the weight, specimen, and load cell. The specimen was then placed on the load cell and visually centered with the scribed alignment rings on the top surface of the cell.

The desired sweep rate and force scale were set on the oscilloscope. Sweep rates of 0.2 and 0.5 msec/cm and force scales of 2080 and 3125 lb/cm were used. Selection of the sweep rate and force scale was a function of the weight dropped, drop height, and the geometry of the specimen. The Polaroid camera was set for time exposure with the scale illumination off and the shutter open.

Then the weight was hoisted until the release mechanism contacted the trip, releasing the weight. The free falling
weight first contacted the trace trigger, initiating the single-
sweep trace. The falling weight then contacted the specimen
and transmitted force to the load cell. As the specimen deflected
and load was applied to the cell, the previously balanced bridge
became unbalanced and effected a change in the scope voltage
that deflected the beam by an amount proportional to the load.

A typical force-time curve appeared on the film as is shown
in Fig. 5.

Data Transformation and Evaluation

For each of the 68 specimens the force was scaled from the
Polaroid pictures (example: Fig. 5) at increments of 0.0001
seconds. The time of the impact event varied between 0.0013 and
0.0025 seconds, thus, between 13 and 25 discrete points were
used to describe the force-time relationship of the impact event.
Since the mass of the free falling weight was much greater than
the mass of the lead specimen and the impact was essentially
plastic, the force recorded by the load cell could be used with
a free body diagram and D'Alembert's principle to calculate the
acceleration of the top surface of the lead specimen.

\[ a(t) = \frac{f(t)}{M + m_e} \]  

where:

- \( a(t) \) = acceleration of the top surface of the lead specimen
- \( f(t) \) = force recorded by the load cell
- \( M \) = mass of falling weight
- \( m_e \) = effective mass of lead specimen (for a linear strain
rate use 1/3 the total mass of specimen, see ref. 8)
A computer program was written using Simpson's rule to numerically integrate the discrete force-time data to obtain velocity and displacement as functions of time.

\[ V(t) = \int_0^t a(\tau) d\tau \quad (5) \]

\[ x(t) = V_o t - \int_0^t \int_0^\tau a(t_1) dt_1 d\tau \quad (6) \]

where:

- \( V(t) \) and \( x(t) \) = velocity and displacement of top surface of the lead specimens respectively
- \( V_o \) = initial velocity at impact.

It was assumed that the velocity of the lead specimen varied linearly from the top surface to the base which indicated a constant strain-rate throughout the specimen. The two major factors that would cause a deviation from a linear velocity distribution are the inertial and end effects. The nearly uniform cross section of the impacted specimens (Fig. 4) indicated that the lubricated ends had a small effect. The acceleration levels of the impacted specimens were sufficiently low to make the additional stress required to accelerate the specimen be a maximum of 4 percent of the total stress. This means the struck end of the specimen (having a highest acceleration) would have a slightly higher strain rate than the base end. However, the difference in the two extreme strain rates are small and the...
average value is used in the calculations. Since the specimen is only in the linear stress-strain range for a very small portion of the total time, stress waves will have essentially no effect on the results.

It was observed that the volume of the specimen and the variation of the cross-section along the length remained approximately constant after impact. These approximations along with the force, velocity, and displacement data were used in the computer program to calculate apparent stress, true stress, and strain rate for the 1197 discrete points taken from the 68 specimens.

It was observed that the computed velocity (Equation 5) did not quite reach zero at the point where the actual velocity was known to be zero. A measure of the accuracy of the data acquisition and numerical transformation was based on the calculated kinetic energy of the moving weight immediately after impact. The relatively small portion of test data from specimens that did not account for at least 95% of the available energy was discarded. It was estimated that less than 2 percent of the total energy was associated with the rebound of the falling weight, residual wave motion in the cell and falling weight, and other minor effects.

The following equation was derived using a least-squares method and describes the experimentally measured true compressive stress for all measured strain rates with an average absolute
error of less than 267 psi. The strain-rate range for which Equation 7 is valid is between 100 and 800 in/in-sec.

\[ \sigma_+ = 20,900 - 14250 \epsilon^{-0.093} \text{ (psi)} \]

\[ \sigma_+ = 1.20 \times 10^6 - 9.825 \times 10^7 \epsilon^{-0.093} \text{ (N/m}^2\text{)} \]

\[ \epsilon = \text{strain (0.03 ≤ } \epsilon \leq 0.5) \quad \epsilon > 0.05 \quad (7) \]

The standard deviation between the values calculated by Equation 7 and all experimental values is approximately 342 psi (2.36x10^6 N/m^2). A five percent Chauvenet's rejection criterion [9] was applied to all experimental data. This resulted in rejecting less than 0.4% of all data points.

There is reasonable agreement between the plot of Equation 7 developed from the impact test data and the plot of Equation 1 reported by Mok and Duffy [5], as is illustrated in Fig. 6. However, the plot of Equation 2 reported by Slater, Johnson, and Akwu [7] shows a significant deviation from that of Equation 7, as is also illustrated in Fig. 6.

For a constant volume specimen, the following equation gives the relationship between the apparent (engineering) compressive stress and strain.

\[ \sigma_a = \frac{20900 - 14250 \epsilon^{-0.093}}{1 - \epsilon} \text{ psi} \quad \epsilon > 0.05 \quad (8) \]

where:

\[ \sigma_a = \text{force/original area}. \]
Equation 8 is nonlinear and can result in awkward mathematical manipulation. When a lower level of computational accuracy can be tolerated, the following linear equation which was derived using a least-squares method can be used.

\[ \sigma_a = 1.7470 \varepsilon + 1914 \text{ (psi)} \quad 0.1 \leq \varepsilon \leq 0.45 \]  \hspace{1cm} (9)

\[ \sigma_a = 1.20 \times 10^6 \varepsilon + 1.32 \times 10^7 \text{ (N/m}^2\text{)} \quad 100 \leq \varepsilon \leq 800 \text{ in/in-sec} \]

The absolute average error of using Equation 9 for all measured strain rates is less than 309 psi (2.13x10^6 N/m^2) and the standard deviation between the calculated and measured apparent stress is approximately 405 psi (2.79x10^6 N/m^2).

The effect that strain rate had on the stress-strain relationship was investigated by dividing the 1197 data points by strain rate into seven equally spaced groups ranging from 100 to 800 in/in/sec. The data from each separate group was compared with Equation 7 which was derived by using data from all strain-rate groups. It was found that Equation 7 accurately represented each group. This meant there was essentially no difference between the stress-strain relationship of each strain rate group and the lead specimens were not appreciably strain-rate sensitive in the range of 100 to 800 in/in/sec. However, when the data from the quasi-static compression tests (Fig. 2) is compared with the results from the drop tests the lead specimens are significantly strain-rate sensitive. This is shown by Figure 8 where the strain-rate range is plotted over seven decades.
Conclusions

The results of the impact tests show that chemical lead is not significantly strain-rate sensitive in the range of 100 to 800 in/in/sec. for strains up to 0.5 in/in. However, when the dynamic compressive stress-strain properties are compared with static and quasi-static properties (Fig. 2), chemical lead exhibits a significant degree of rate sensitivity as shown by Fig. 8. Equation 7, developed empirically, adequately represents the dynamic compressive stress-strain relationship for chemical lead within the limits of the impact data. Therefore, this equation for true stress, the equation for engineering stress (Eq. 8), and the curves plotted from these equations (shown in Fig. 7) can be used to estimate the dynamic compressive stress-strain properties of lead for strains less than 0.5 and strain-rates between 100 and 800 in/in/sec. When a lower level of accuracy can be tolerated, the approximate linear equation for apparent stress, Eq. 9 can be used.
REFERENCES


**TABLE 1**

*Combinations of Specimen Geometry, Weight, and Drop Height Used in Impact Tests*

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<th>Specimen Geometry</th>
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<td>(m)</td>
<td>(m)</td>
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