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A MONTE CARLO CALCULATION OF THE
FERMI AGE AND OTHER PROPERTIES
OF THE SLOWING DOWN DISTRIBUTION

By
Henry C. Honeck

August 1955

Oak Ridge School of Reactor Technology
Oak Ridge, Tennessee

Technical Information Service Extension, Oak Ridge, Tenn.

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OAK RIDGE SCHOOL OF REACTOR TECHNOLOGY

F. C. VonderLage, Director

" A MONTE CARLO CALCULATION OF THE FERMI AGE AND OTHER
PROPERTIES OF THE SLOWING DOWN DISTRIBUTION."

By: Henry C. Honeck

OAK RIDGE NATIONAL LABORATORY
Operated by
UNION CARBIDE NUCLEAR COMPANY
OAK RIDGE, TENNESSEE

August 1955

PREFACE

In September, 1954, a group of men representing various scientific and engineering fields embarked on the twelve months of study which culminated in this report. For nine of these months, formal classroom and student laboratory work occupied their time. At the end of that period, this student was presented with a problem in reactor design.

This is a summary report of the study, the research, the problems and the solutions which developed during the final ten-weeks period of the school term. It must be realized that, in so short a time, a study of this scope can not be guaranteed complete or free of error. This "thesis" is not offered as a polished engineering report but rather as a record of the work done by this student under the leadership of the advisor. It is reproduced for use by those persons competent to assess the uncertainties inherent in the results obtained in terms of the preciseness of the technical data and analytical methods employed in the study. In the opinion of the student and faculty of ORSORT, the problem has served the pedagogical purpose for which it was intended.

As a matter of historical fact and pride we point out that similar investigations by student groups of previous ORSORT classes have led to sufficiently encouraging results to warrant more exhaustive studies; in at least one instance, a reactor first investigated by a student group is soon to become a physical reality. There is also recorded an instance in which calculations contained in a similar report were uncritically abstracted and applied to a study for which they were never intended. It is to avoid the recurrence of the latter experience that we have taken some pains to acquaint the reader with the character of this report.

The faculty wishes to join the author in an expression of appreciation for the assistance which various members of the Oak Ridge National Laboratory have so generously contributed. In particular, the guidance of the consultant, Robert R. Coveyou, is gratefully acknowledged.

F. C. VonderLage
for
The Faculty of ORSORT

ABSTRACT

A program was developed for the ORACLE which computes the second moment of the slowing down distribution of neutrons in a mixture of elements. The program allows mixtures of up to eight elements, four of which have scattering, absorption, and transport cross sections, and four which have only absorption cross sections. Provision is made to calculate other properties of the slowing down system, such as the average diffusion coefficient, the resonance escape probability, and the squared first and last flight mean free paths.

ACKNOWLEDGEMENTS

The author is indebted to R. R. Coveyou and R. R. Bate for the derivation of the method, and for their many helpful suggestions during the programming.

I. INTRODUCTION

The design of nuclear reactors requires a knowledge of the energy and spatial distribution of the neutrons slowed down by elastic collisions with nuclei. Often it is sufficient to know only the second moment of the slowing down distribution, a quantity proportional to the Fermi Age.

A program was developed for the ORACLE which computes the Age in a mixture of elements by Monte Carlo techniques. The general expression for the squared distance a neutron travels from birth to the n^{th} collision is set up and reduced to a set of recursion formulae, which depends only on the energy history of the neutron. The energy history of each neutron is traced, collision by collision, until its energy falls below the desired lower energy limit. The Age is calculated, and the procedure repeated until enough histories are accumulated to produce the desired statistical accuracy.

The program allows mixtures of up to eight elements, four of which have scattering, absorption, and transport cross sections, and four which have only absorption cross sections. Provision is made to calculate other properties of the slowing down system, such as the average diffusion coefficient, the resonance escape probability, and the squared first and last flight mean free paths. Finally, methods are presented to extend the calculation to include inelastic scattering.

II. THEORY

A. Calculation of The Age

Assume that in the history of a neutron, the values of the speed after each collision and the mass of the scattering nucleus have been determined by Monte Carlo techniques. The values of the scattering angles, and the mean free flight times can now be computed. There remains only to find an expression for the expected value of the squared distance from the source to the n^{th} collision.

The following notation is used:

- $\underline{V}_n$ - The velocity of the neutron after the n^{th} collision.
- S_n - The speed of the neutron after the n^{th} collision.
- $\underline{\lambda}_n$ - A unit vector in direction $\underline{V}_n$.
- μ_n - The cosine of the scattering angle of the n^{th} collision in the laboratory system.
- M_i - The mass of the i^{th} type of scatterer in atomic mass units.
- u_n - The fraction of the mean flight time between the n^{th} and $n + 1^{\text{st}}$ collision. u is a random number from the distribution $e^{-u} du$. $0 \leq u < \infty$.
- λ_n - The mean flight time after the n^{th} collision.
- $\underline{X}_n$ - The position of the neutron at the n^{th} collision.
- $\alpha_i = 1/(M_i - 1)$
- $\beta_i = M_i/(M_i - 1)$
- $E\{\}$ - The expected value of $\{\}$

The position of the p^{th} collision is,

$$\underline{X}_P = \sum_{n=0}^{P-1} V_n \lambda_n u_n = \sum_{n=0}^{P-1} S_n \lambda_n u_n \underline{\rho}_n \quad (1)$$

The square of the distance to the p^{th} collision is,

$$(\underline{X}_P \cdot \underline{X}_P) = \sum_{n=0}^{P-1} \sum_{m=0}^{P-1} S_n \lambda_n S_m \lambda_m u_n u_m (\underline{\rho}_n \cdot \underline{\rho}_m) \quad (2)$$

The expected value of $(\underline{X}_P \cdot \underline{X}_P)$ is,

$$E \left\{ \underline{X}_P \cdot \underline{X}_P \right\} = \sum_{n=0}^{P-1} \sum_{m=0}^{P-1} S_n \lambda_n S_m \lambda_m E \left\{ u_n u_m (\underline{\rho}_n \cdot \underline{\rho}_m) \right\} \quad (3)$$

$$= \sum_{n=0}^{P-1} \sum_{m=0}^{P-1} S_n \lambda_n S_m \lambda_m E \left\{ u_n u_m \right\} E \left\{ \underline{\rho}_n \cdot \underline{\rho}_m \right\} \quad (4)$$

The term $E \left\{ u_n u_m \right\}$ can be evaluated as follows.

$$E \left\{ u_n u_m \right\} = \int_0^\infty u_n e^{-u_n} du_n \int_0^\infty u_m e^{-u_m} du_m = 1 \quad n \neq m \quad (5)$$

$$= \int_0^\infty u_n^2 e^{-u_n} du_n = 2 \quad n = m \quad (6)$$

$$= 1 + \delta_n^m \quad (7)$$

where δ_n^m is the Kronecker delta. Equation (4) becomes,

$$\begin{aligned} E \left\{ \underline{X}_P \cdot \underline{X}_P \right\} &= \sum_{n=0}^{P-1} \sum_{m=0}^{P-1} S_n \lambda_n S_m \lambda_m (1 + \delta_n^m) E \left\{ \underline{\rho}_n \cdot \underline{\rho}_m \right\} \\ &= 2 \sum_{n=0}^{P-1} \sum_{m=0}^n S_n \lambda_n S_m \lambda_m E \left\{ \underline{\rho}_n \cdot \underline{\rho}_m \right\} \quad (8) \end{aligned}$$

The following proof is necessary to evaluate $E \left\{ \frac{1}{v_n} \cdot \frac{1}{v_m} \right\}$. Assume the distribution of neutrons before a collision is expanded in a set of Legendre polynomials.

$$D(w) dw = \sum_{\ell=0}^{\infty} \frac{2\ell+1}{2} D_{\ell} P_{\ell}(w) dw \quad (9)$$

Similarly, the scattering kernel is expanded.

$$K(\mu) d\mu = \sum_{n=0}^{\infty} \frac{2n+1}{2} K_n P_n(\mu) d\mu \quad (10)$$

The resultant distribution after the collision is,

$$D(w_1) dw_1 = \int_{w=-1}^{w=1} \sum_{\ell=0}^{\infty} \sum_{n=0}^{\infty} \frac{2n+1}{2} \frac{2\ell+1}{2} D_{\ell} K_n P_{\ell}(w) P_n(\mu) dw dw_1 \quad (11)$$

From the addition theorem of Legendre polynomials,⁽¹⁾

$$P_n(\mu) = \sum_{m=-n}^n \frac{(n-m)!}{(n+m)!} P_n^m(w) P_n^m(w_1) \cos m(\phi - \phi_1) \quad (12)$$

Substituting into equation (11),

$$\begin{aligned} D(w_1) dw_1 &= \sum_{n=0}^{\infty} \sum_{\ell=0}^{\infty} \frac{2\ell+1}{2} \frac{2n+1}{2} D_{\ell} K_n dw_1 P_n(w_1) \int_{-1}^1 P_{\ell}(w) \\ &\quad \sum_{m=-n}^n \frac{(n-m)!}{(n+m)!} P_n^m(w) P_n^m(w_1) \cos m(\phi - \phi_1) dw_1 \\ &= \sum_{\ell=0}^{\infty} \frac{2\ell+1}{2} D_{\ell} K_{\ell} P_{\ell}(w_1) dw_1 \end{aligned} \quad (13)$$

Now let $w = w_m = 1$, and let $\mu = \mu_{m+1}$, the cosine of the scattering angle in the laboratory system of the $m+1^{\text{st}}$ collision. Then from equations (10) and (11),

$$D(w_m) dw_m = \delta_{w_m}^1 dw_m = \sum_{\ell=0}^{\infty} \frac{2\ell+1}{2} P_{\ell}(1) P_{\ell}(w_m) dw_m \quad (14)$$

$$K(\mu) d\mu = \delta_{\mu_{m+1}}^{\mu} d\mu = \sum_{\ell=0}^{\infty} \frac{2\ell+1}{2} P_{\ell}(\mu_{m+1}) P_{\ell}(\mu) d\mu \quad (15)$$

The resultant distribution is then

$$D(w_{m+1}) dw_{m+1} = \sum_{\ell=0}^{\infty} \frac{2\ell+1}{2} P_{\ell}(1) P_{\ell}(\mu_{m+1}) P_{\ell}(w_{m+1}) dw_{m+1} \quad (16)$$

$$D(w_{m+2}) dw_{m+2} = \sum_{\ell=0}^{\infty} \frac{2\ell+1}{2} P_{\ell}(1) P_{\ell}(\mu_{m+1}) P_{\ell}(w_{m+2}) dw_{m+2} \quad (17)$$

and in general,

$$D(w_n) dw_n = \sum_{\ell=0}^{\infty} \frac{2\ell+1}{2} P_{\ell}(1) \prod_{i=m+1}^n P_{\ell}(\mu_i) P_{\ell}(w_n) dw_n \quad (18)$$

But $w_n = \mu_n \cdot \mu_m$

$$\begin{aligned} \text{and } E\{w_n\} &= E\{\mu_n \cdot \mu_m\} \\ &= \int_{-1}^1 \sum_{\ell=0}^{\infty} \frac{2\ell+1}{2} P_{\ell}(1) \prod_{i=m+1}^n P_{\ell}(\mu_i) P_{\ell}(w_n) dw_n \\ &= \prod_{i=m+1}^n \mu_i \end{aligned} \quad (19)$$

Equation (8) becomes,

$$E\left\{\frac{X}{P} \cdot \frac{Y}{P}\right\} = E_P = 2 \sum_{n=0}^{P-1} \sum_{m=0}^n S_n \lambda_n S_m \lambda_m \prod_{i=m+1}^n \mu_i \quad (20)$$

$$\text{Let } \gamma_n = \frac{S_n}{S_{n-1}} = (\alpha_n^2 + \beta_n^2 + 2\alpha_n \beta_n w_n)^{\frac{1}{2}} \quad (21)$$

$$\text{Then } S_n = S_0 \prod_{i=1}^n \gamma_i \quad (22)$$

The scattering angle in the laboratory system can be written,

$$\mu_n = \frac{a_n + \beta_n w_n}{\gamma_n} \quad (23)$$

where w_n is the scattering angle in the center of mass system. Substituting equations (22) and (23) into equation (20),

$$\begin{aligned} E_P &= 2 \sum_{n=0}^{P-1} \sum_{m=0}^n \lambda_n \lambda_m S_o^2 \prod_{i=1}^n \gamma_i \prod_{j=1}^m \gamma_j \prod_{k=m+1}^n \left(\frac{a_{k+\beta_k} w_k}{\gamma_k} \right) \\ &= 2 \sum_{n=0}^{P-1} \sum_{m=0}^n \lambda_n \lambda_m S_m^2 \prod_{k=m+1}^n \delta_k \end{aligned} \quad (24)$$

where $\delta_k = a_k + \beta_k w_k$ (25)

$$E_P = 2 \sum_{n=0}^{P-2} \sum_{m=0}^n \lambda_n \lambda_m S_m^2 \prod_{k=m+1}^n \delta_{k+2} \lambda_{P-1} \sum_{m=0}^{P-1} \lambda_m S_m^2 \prod_{k=m+1}^{P-1} \delta_k \quad (26)$$

Define $Q_P = \sum_{m=0}^{P-1} \lambda_m S_m^2 \prod_{k=m+1}^{P-1} \delta_k$ (27)

$$Q_P = \delta_{P-1} Q_{P-1} + \lambda_{P-1} S_{P-1}^2 \quad (28)$$

From equation (26),

$$E_P = E_{P-1} + 2 \lambda_{P-1} Q_P \quad (29)$$

Equations (28) and (29) along with the conditions

$$E_0 = Q_{-1} = 0 \quad (30)$$

are used to calculate the expected squared distance E_P , collision by collision, to the P^{th} collision.

B. Calculation of the Diffusion Coefficient

Assume the average diffusion coefficient to be defined by,

$$\bar{D} = \frac{\int D(E) \phi(E) dE}{\int \phi(E) dE} \quad (31)$$

$$= \frac{\int S^2 \lambda_T(E) \lambda_{Tr}(E) q(E) dE}{3 \int S \lambda_T(E) q(E) dE} \quad (32)$$

where $q(E)$ is the collision density. Since $q(E) dE$ is the weighting function supplied by the Monte Carlo calculation,

$$\bar{D}_P = \frac{\sum_{n=0}^{P-1} S_n^2 \lambda_{Tn} \lambda_{Trn}}{\sum_{n=0}^{P-1} S_n \lambda_{Tn}} \quad (33)$$

define $F_P = \sum_{n=0}^{P-1} S_n^2 \lambda_{Tn} \lambda_{Trn} \quad (34)$

and $G_P = \sum_{n=0}^{P-1} S_n \lambda_{Tn} \quad (35)$

Then $F_P = F_{P-1} + S_{P-1}^2 \lambda_{TP-1} \lambda_{TrP-1} \quad (36)$

$$G_P = G_{P-1} + S_{P-1} \lambda_{TP-1} \quad (37)$$

Equations (36) and (37) along with the conditions,

$$F_0 = G_0 = 0 \quad (38)$$

can be used to calculate F_P and G_P .

C. Calculation of the Collision Parameters

Assume that a neutron has suffered a scattering collision, and that the type of scattering nucleus has been determined. The parameters describing the n^{th} collision are S_n^2 and $\mathcal{S}_n^1$. The parameter $\mathcal{S}_n^1$ is defined by,

$$\mathcal{S}_n^1 = \alpha_1 + \beta_1 w_n \quad (25)$$

where w_n is the cosine of the scattering angle in the center of mass system, and has a frequency function,

$$f(w) \, dw = \frac{1}{2} (1 + a_1 w) \, dw \quad -1 \leq w \leq 1 \quad (39)$$

A random number is generated from the distribution given by equation (39) and $\mathcal{S}_n^1$ calculated from equation (25). The energy after the collision is then calculated from,

$$S_n^2 = S_{n-1}^2 (b_1 + c_1 w) \quad (40)$$

where

$$b_1 = \frac{M_1^2 + 1}{(M_1 + 1)^2} \quad (41)$$

$$c_1 = \frac{2 M_1}{(M_1 + 1)^2} \quad (42)$$

D. Monte Carlo Calculations

1) Generation of Random Numbers

The random numbers used for the Monte Carlo calculations are of the congruence type defined by,

$$C^n = (C^1)^n \text{ Mod } 2^{40} \quad (43)$$

where $C^1 = 5^{15}$. C^{n-1} and C^1 , each of which are 40 binary digits, are multiplied together forming an 80 digit number, the last 40 digits are selected as the random number, the first digit being the sign digit. The numbers thus generated are uniformly distributed over the interval from -1 to 1.

2) Selection of the Cosine of the Scattering Angle

The frequency function for the cosine of the scattering angle in the center of mass system, (including p scattering), is given by,

$$f(w) dw = \frac{1}{2} (1 + a w) dw \quad (44)$$

where a is the p scattering coefficient.

Consider two random variables, x and y , with frequency functions,

$$q(x) dx = \frac{1}{2} dx \quad -1 \leq x < 1 \quad (45)$$

$$h(y) dy = \frac{1}{2} dy \quad -1 \leq y < 1 \quad (46)$$

It will be shown that having selected x and y , if $ay \geq x$, then y is the desired value of w and is distributed according to equation (44). The distribution of the selected values of y is,

$$f(y) dy = \frac{h(y) dy \int_{-1}^{ay} g(x) dx}{\int_{-1}^1 h(y) dy \int_{-1}^{ay} g(x) dx} = \frac{1}{2} (1 + ay) dy \quad (47)$$

Then $w = y$ and is distributed according to equation (44).

A schematic flow chart of the procedure is shown in Figure (1).

3) Selection of the Fission Neutron Energies

It has been shown by Watt⁽³⁾ that the velocity spectrum of fission neutrons in the laboratory system can be represented as the sum of two vectors. The first is the vector velocity of the fission fragment (f) from which the neutron is emitted. The second vector is the velocity vector (V) of the neutron emitted isotropically with a Maxwell-Boltzmann distribution of speeds from the fission fragment. The resultant velocity spectrum is then of the form,

$$N(v) dv = \text{Const.} \times e^{-\frac{v^2}{K-1}} \sinh \sqrt{K_2 v^2} \quad (48)$$

A velocity V is first selected from the Maxwell-Boltzmann distribution,

$$P(v) dv = \sqrt{\frac{2}{\pi}} v^2 e^{-\frac{v^2}{2}} dv \quad (49)$$

by the following method.

Let t , u , w , and s , be random variables with distribution functions,

$$\begin{aligned} P(t) dt &= e^{-t} dt & 0 \leq t < \infty \\ P(u) du &= e^{-u} du & 0 \leq u < \infty \\ P(w) dw &= e^{-w} dw & 0 \leq w < \infty \\ P(s) ds &= e^{-s} ds & 0 \leq s < \infty \end{aligned} \quad (50)$$

Then if $y = w + s$

$$P(y) dy = dy \int_0^y P(w) P(y-w) dw = ye^{-y} dy \quad (51)$$

and if $z = u + y = u + w + s$

$$P(z) dz = dz \int_0^z P(y) P(z-y) dy = \frac{1}{2} z^2 e^{-z} dz \quad (52)$$

Finally, if $(z - 1)^2 \leq 2t$

$$Q(z) dz = \frac{\int_0^\infty P(z) dz \int_{\frac{1}{2}(z-1)^2}^\infty P(t) dt}{\int_0^\infty P(z) dz \int_0^\infty P(t) dt} = \sqrt{\frac{2}{\pi}} z^2 e^{-\frac{z^2}{2}} dz \quad (53)$$

Note that if C is distributed uniformly over the interval -1 to 1 , and

$t = -\ln C$, then,

$$P(t) dt = e^{-t} dt \quad 0 \leq t < \infty \quad (54)$$

The method is then to select four random numbers uniformly distributed

from -1 to 1 , and find their logarithms, If

$$(\ln C_1 + \ln C_2 + \ln C_3 - 1)^2 \leq 2 \ln C_4 \quad (55)$$

Then $V = \left| \ln C_1 + \ln C_2 + \ln C_3 \right|$ and is distributed according to equation (49). The actual velocity $S = \alpha V$, where α is a scale factor.

The resultant velocity (v) in the laboratory system is given by,

$$v^2 = f^2 + S^2 - 2 f S \cos \phi \quad (56)$$

where $\cos \phi$ is the cosine of the angle between $\underline{f}$ and $\underline{V}$.

Since the emission of the neutron is assumed isotropic with respect to the fission fragment, a random number C is selected from a uniform distribution from -1 to 1 , and v^2 calculated from

$$v^2 = f^2 + S^2 - 2 f S C \quad (57)$$

A schematic flow chart of the method is shown in Figure (1).

Selection of w from a p-scattering distribution

Selection of v^2 from a fission distribution

Figure 1. Schematic Flow Diagrams of the Monte Carlo Calculations

E. Calculation of the First Flight Mean Free Path

When the calculated Age is to be compared with analytical results, the values of $\ell^2(0)$, the squared first flight mean free path, and $\ell^2(u)$, the squared last flight mean free path, are often useful. The first quantity is defined by,

$$\ell^2(0) = \int_0^{\infty} S^2 \lambda_T^2(E) f(E) dE \quad (58)$$

where $f(E)$ is the normalized fission spectrum.

$$\ell^2(0) = \sum S_n^2 \lambda_T^2(E_n) f(E_n) \Delta E_n \quad (59)$$

where the summation includes the 640 energy points between 0.00931 Mev and 19.1 Mev.

The value of λ_T corresponding to $S_{\min}^2$ is found from the table, and the last flight mean free path calculated from,

$$\ell^2(u) = \lambda_T^2(E_{\min}) S_{\min}^2 \quad (60)$$

F. Statistics

Let X_1 be a number from a set of numbers such as the quantities E_P , F_P , or G_P . The mean value of X_1 is defined by,

$$\bar{x} = \frac{1}{h} \sum_{i=1}^h X_i \quad (61)$$

and the mean squared value of X_1 by

$$\bar{x}^2 = \frac{1}{h} \sum_{i=1}^h X_i^2 \quad (62)$$

Then the relative error is defined by

$$R = \frac{1}{h (\bar{x})^2} \sum_{i=1}^h (X_i - \bar{x})^2 \quad (63)$$

$$= \frac{(\bar{x}^2) - (\bar{x})^2}{h (\bar{x})^2} \quad (64)$$

The relative error as defined by equation (64) is used to test the convergence of the calculation.

III. DESCRIPTION OF THE PROGRAM

A. General Description

The program is divided into three parts.

- 1) Preparation routine (Proto-Chronos)
- 2) Correction routine
- 3) Main program (Chronos)

All of the programs are stored on magnetic tape. When one program is completed, the next program is automatically read in. The sequence is as follows.

- 1) The preparation routine is read in and calculates and stores on magnetic tape the table of flight times, scattering probabilities, and p scattering coefficients.
- 2) The correction routine is read in and modifies the main program for the desired problem.
- 3) The main program is read in and calculates the age and other properties of the slowing down system. The results are punched out.

In the most general case, the following quantities are computed and punched out.

- 1) τ_T - The mean squared distance traveled by a neutron from birth to lethargy u or absorption.
- 2) τ_S - The mean squared distance traveled by a neutron from birth to lethargy u excluding neutrons absorbed.
- 3) τ_a - The mean squared distance traveled by a neutron from birth to absorption at a lethargy less than u .
- 4) τ_F - The mean value of the product of the total mean free path and the transport mean free path.

- 5) τ_G - The mean value of the total mean free path.
- 6) $\tau_T^2, \tau_S^2, \tau_a^2, \tau_F^2, \tau_G^2$ - The mean squared values of the quantities 1 - 5.
- 7) R_T, R_S, R_a, R_F, R_G - The relative errors associated with quantities 1 - 5.
- 8) h_a - The total number of histories ending in absorptions.
- 9) h_T - The total number of histories.
- 10) $\ell^2(0)$ - The average squared first flight mean free path.
- 11) $\ell^2(u)$ - The average squared last flight mean free path.

B. Proto-Chronos

The purpose of Proto-Chronos is to prepare, for a mixture of elements, the table of flight times, scattering probabilities, and p scattering coefficients from a master cross section tape. The program allows for a mixture of eight elements, four of which have scattering, transport, and absorption cross sections, and four which have only capture cross sections. The cross sections on the master tape are arranged to fit the following group structure.

- 1) 25 groups equally spaced in lethargy.

$$\Delta u = \ln 2.$$

- 2) Each group is divided into 64 subgroups of cross sections equally spaced in energy. $0.0080 \leq \Delta u \leq 0.016$

The master tape then represents a 1600 point fit to the cross section curves between the limits of 19.1 Mev ($S^2 = 2^{65}$) and 0.284 ev ($S^2 = 2^{39}$ cm²/sec²).

At each energy point the following equations are applied.

$$\lambda_T = \frac{1}{S \sum_T} \quad (65)$$

$$\lambda_{Tr} = \frac{1}{S \sum_{Tr}} \quad (66)$$

$$P_1 = \frac{N_1 \sigma_{S1}}{\sum_T} \quad (67)$$

$$\sum_T = \sum_{i=1}^4 (\sigma_{S1} + \sigma_{a1}) N_i + \sum_{k=1}^4 \sigma_{ak} N_k \quad (68)$$

$$\sum_{Tr} = \sum_{i=1}^4 \sigma_{Tr1} N_i \quad (69)$$

The flight times and probabilities are calculated for one group and stored

in a block on tape. The procedure is repeated until all groups have been stored on the magnetic tape in order of increasing lethargy.

Note that $\sum_{i=1}^4 P_i = 1 - P_a$

where P_a is the probability of absorption. When a capture occurs, the type of capturing nucleus is not recorded.

C. Chronos

The general method is to trace the parallel histories of J neutrons until all have slowed down below $S_{\min}^2$ or have been absorbed. The various means are advanced and the process repeated until enough histories have been accumulated for the statistical errors to be less than ϵ . The value of J is determined by the problem, but is always greater than 40.

A schematic diagram of Chronos is presented in Figure (2).

The program proceeds as follows:

- 1) J fission neutron energies are generated by Monte Carlo techniques using Watt's formula.
- 2) A block of flight times, probabilities, and a_1^{ls} are read into the fast memory beginning with the highest energy.
- 3) The energy S_j^2 of the j^{th} neutron is tested. If the energy does not lie within the energies represented by the block of flight times in the memory, $2S_j^2$ is put into S_j^2 and step (3) is applied to the next neutron of energy S_{j+1}^2 .
- 4) If the energy S_j^2 lies within the table, the appropriate values of $\lambda_T, \lambda_{Tr}, P_1, P_2, P_3, P_4, a_1, a_2, a_3$, and a_4 are found from the table. The quantities Q_j, E_j, F_j , and G_j are calculated.
- 5) The neutron now suffers a collision of type determined by P_1, P_2, P_3 and P_4 . If a capture occurs the means $\tau_a, \tau_T, \tau_F, \tau_G, \tau_a^2, \tau_T^2, \tau_F^2$ and τ_G^2 are advanced. The history is ended and the $(j+1)^{\text{st}}$ neutron is selected. The calculation returns to step (3).
- 6) If the collision was a scattering, the new energy and δ_j are calculated.

The calculation returns to step (3).

- 7) The history of each of the J neutrons is followed in the manner described in steps (3) through (6) until all energies fall below the lowest energy represented by the table. The calculation returns to step (2).
- 8) When the last group of flight times has been read in, the S_j^2 are tested against $S_{\min}^2$ rather than $\frac{1}{2}$. When all S_j^2 are less than $S_{\min}^2$ all means are advanced. They are: $\tau_T, \tau_S, \tau_a, \tau_F, \tau_G, \tau_T^2, \tau_S^2, \tau_a^2, \tau_F^2, \tau_G^2$.
- 9) The counters h_T, h_S, h_a are advanced. If $h_T < H$, the calculation returns to step (1).
- 10) If $h_T \geq H$, the relative errors R_a, R_S, R_T, R_F , and R_G are computed. The last four are tested against ϵ . If all are less than ϵ , and also were all less than ϵ the last time tested, the calculation is ended and results printed. If any test fails, or failed on the preceeding test, the calculation returns to step (1).

Figure 2. Schematic Flow Chart of Chronos

D. Correction Routine

It is apparent from the flow chart of Chronos that if certain quantities are not needed, such as the diffusion coefficient, considerable savings in speed and space can be realized by by-passing these calculations. It is the purpose of the correction routine to modify Chronos to effect these savings. The following modifications are possible.

- 1) The fission source may be replaced by a monoenergetic source.
- 2) The calculation of the diffusion coefficient may be omitted.
- 3) The calculation of $\ell^2(o)$ and $\ell^2(u)$ may be omitted.
- 4) The age calculation may be omitted.
- 5) The convergence of the calculation may be tested by any one, or combination of, the following relative errors: R_T , R_S , R_F , R_G .
- 6) An estimate of the resonance escape probability can always be deduced from h_a and h_T . If this is the only quantity desired, modifications (1) through (4) are applied, the relative error tests are by-passed, and a new H is used. The calculation ends when $h_T \geq H$.

IV APPENDIX - INELASTIC SCATTERING

It is possible to extend the method to include inelastic scattering if the exit energy of the neutron can be calculated. Two methods are possible.

If the energy levels in the target nucleus are many and closely spaced, the evaporation model may be used. The kinetic energy of the system after the collision, E_s , has the distribution,

$$P(E'_s) dE'_s = K E'_s e^{-\beta E'_s} dE'_s \quad (70)$$

where K and β are functions of the type of nucleus and the incident energy of the neutron.

If there are few energy levels in the target nucleus, the levels can be given as a table or an analytical expression. The exit energy of the neutron is then,

$$E'_s = E_s - E_j \quad (71)$$

where E_j is a function of the energy levels.

With either method the emission of the neutron is assumed isotropic in the center of mass system. If the target nucleus is heavy, the emission is also isotropic in the laboratory system. The equations for E_p and Q_p are then decoupled and the collision characterized by $Q_p = 0$.

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