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HIGH THERMAL-NEUTRON FLUX FROM FISSION-OVERSIMPLIFIED CASES

W. K. Ergen

ABSTRACT

Two very much simplified models are discussed in order to gain insight into the problems of a flux-trap reactor. The first model consists of source of fission neutrons, embedded into D_2O , Be, BeO, or C moderator, or a moderator of zero absorption cross section. The source is assumed not to absorb any neutrons. This model gives five to eight times the thermal flux obtainable at the center of the flux-trap reactor. The other model is a flux-trap reactor, idealized as in two previous memoranda, Refs. 1 and 2, and further idealized by neglecting all absorption in the moderator. This model does not differ markedly from the D_2O -moderator case.

In two previous memoranda^{1,2} we investigated an idealized flux-trap reactor consisting of fissionable spherical shell, embedded in an infinite moderator. Though this model is too simple to give accurate quantitative results for actual reactors, it may be of interest to investigate still simpler cases to point out qualitatively the importance of some of the factors involved.

If we had a point source of fission neutrons, which does not absorb any neutrons of whatever energy, and if this source were embedded in a moderator, then the highest thermal neutron flux would occur at the location of the source. This is intuitively obvious and also follows from the reasoning given below. In the flux-trap reactor this maximum of the thermal flux is suppressed by the absorption in the shell, and it is of interest to see how much lower the maximum is in the flux-trap reactor than in the case of the non-absorbing source. That the flux is highest at the location of the source can be seen as follows. The slowing-down kernel for a point source

$$K_1(r') = (4\pi\tau)^{-3/2} e^{-r'^2/4\tau} \dots \quad (1)$$

is a monotonically decreasing function of the distance r' between the fission-neutron source and the point at which the neutrons reach the thermal age, τ . Also the point-source diffusion kernel

$$K_2(r'') = \frac{1}{4\pi D r''} e^{-\kappa r''} \quad (2)$$

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1. W. K. Ergen, Flux Distribution in a Reactor Consisting of Spherical Shell of Fuel in an Infinite Moderator, ORNL CF-57-12-100 (Dec. 24, 1957).
 2. W. K. Ergen, Fluxes Obtainable in a Flux-Trap Reactor, ORNL CF-58-1-4 (Jan. 15, 1958).
 3. S. Glasstone and M. C. Edlund, The Elements of Nuclear Reactor Theory, D. Van Nostrand Co., Inc., pp. 129 and 180, Toronto, New York, London, 1952.

is a monotonically decreasing function of the distance r'' between the source of thermal neutrons and the field point. The total thermal flux at the field point is obtained by putting the fission-neutron source at the origin, in which case (r', ϑ, ϕ) become the coordinates of the point of thermalization, substituting for r'' the distance between the point (r', ϑ, ϕ) and the field point, and integrating over r', ϑ , and ϕ . If the field point is identical with the location of the fission-neutron source, then $r'' = r'$ and the two functions K_1 and K_2 are "similarly ordered," that is, the largest value of K_1 occurs at the same value of (r', ϑ, ϕ) as the largest value of K_2 , the next to the largest value of K_1 at the same (r', ϑ, ϕ) as the next to the largest value of K_2 , etc. If the field point is not identical with the source point, K_1 and K_2 are not similarly ordered. According to a theorem on inequalities, the integral over the product of two functions is greater when the two functions are similarly ordered than if they are ordered in any other manner.⁴

The result that the thermal flux is highest at the location of the fission neutron source could of course also be obtained from the expression for this flux,⁵ and by the usual way of setting the derivative of the expression equal to zero. However, this turns out somewhat tedious due to the form of the expression.

The absolute value of thermal flux at the location of the source can also be found from the general expression for this flux, but it turns out that this requires a somewhat involved transition to the limit $r \rightarrow 0$, where r is the distance from the source to the field point. The following procedure is simpler. Equation (1) gives the slowing down density at r' , caused by the fission source at the origin. If this expression is multiplied by $4\pi r'^2 dr'$ it gives the number of thermal neutrons originating in the spherical shell of radius r' and thickness dr' . The thermal neutron flux caused by such a thermal neutron source is given, for instance, by Glasstone and Edlund.⁶ Specifically for the center of the shell the thermal neutron

4. G. H. Hardy, J. E. Littlewood and G. Polya, Inequalities, Cambridge University Press, 2nd Edition, 1952, Theorem 368, p. 261.

5. The Reactor Handbook, AECD-3645, Vol. 1, Physics, p. 404 (Mar. 1955).

6. S. Glasstone and M. C. Edlund, loc. cit., p. 129.

flux becomes $e^{-\mathcal{K}r'}/4\pi r' D$ per neutron originating in the shell. We thus have to evaluate

$$\begin{aligned}\phi &= \int_0^\infty (4\pi r')^{-3/2} e^{-r'^2/4\tau} \times 4\pi r'^2 dr' \times e^{-\mathcal{K}r'}/4\pi r' D \\ &= \frac{1}{4\pi^{3/2} \tau^{1/2} D} - \frac{1}{4\pi} \frac{\mathcal{K}}{D} e^{\mathcal{K}^2 \tau} \operatorname{cerf} \mathcal{K} \sqrt{\tau} \dots\end{aligned}\quad (3)$$

For $\mathcal{K} = 0$, this reduces to

$$\phi_0 = \frac{1}{4\pi^{3/2} \tau^{1/2} D} \quad (4)$$

which indicates that the thermal flux per fission neutron emitted is inversely proportional to the slowing-down length ($\sqrt{\tau}$) and the diffusion constant D . If absorption is present, ($\mathcal{K} > 0$) the flux is, of course, smaller. Table 1 gives for the most common moderators the thermal neutron flux ϕ at the fission neutron source for the very much idealized case to which Eq. (3) applies. The table also gives the flux ϕ_0 for the still further idealized case of zero absorption ($\mathcal{K} = 0$, Eq. 4). The numerical values for D , τ , and $\mathcal{K}$ are taken from Ref. 1, Table 1.

The ratio ϕ/ϕ_0 is also tabulated to give an idea of the influence of the absorption. It may be seen that ϕ_0 is a fair approximation to ϕ only in the case of D_2O . For H_2O , the influence of the absorption is very appreciable. For Be, BeO, and C, the absorption reduces the flux roughly by a factor of 2.

Also tabulated is the ratio ϕ_c/ϕ , where ϕ_c is the maximum flux in the flux-trap reactor, as given in Table 2 of Ref. 2. It may be seen that the flux-trap reactor gives with Be, BeO, and C roughly one-fifth of the flux of the oversimplified model used here, and that with D_2O the flux-trap reactor gives only about one-eighth of the flux of the present model. This is apparently due to the low absorption cross section of D_2O for thermal neutrons, the advantage of which is drastically reduced by the absorptions in the shell of the flux-trap reactor. Conversely, the oversimplified case

TABLE 1. FLUXES UNDER IDEALIZED CONDITIONS

Moderator	Density	ϕ_0	ϕ	ϕ/ϕ_0	ϕ_c/ϕ	Power (Mw) for 10^{16} n/cm ² sec
H ₂ O	1	0.05007	0.00490	0.098		27
D ₂ O	1.1	0.00454	0.00384	0.85	0.130	34
Be	1.85	0.00948	0.00446	0.47	0.213	29
BeO	3.0	0.00876	0.00463	0.52	0.195	28
C	1.6	0.00255	0.00144	0.56	0.185	91
C	2.0	0.00399	0.00225	0.56	0.185	58

discussed here would give 10^{16} n/cm²sec with the power listed in the last column. These powers are of the order of tens of megawatt.

It should be mentioned that the oversimplified case could be approximated, at least theoretically, by the use of a spontaneously fissionable nuclide of low thermal absorption cross section. Cf²⁵⁴ would probably be the most "practical" such nuclide. It decays predominantly by spontaneous fission with a 60-day half life. Its thermal absorption cross section is given as less than 2 barns.⁷

Starting from U²³⁸ it takes, even theoretically, 16 neutrons to produce Cf²⁵⁴, and in practice the number of neutrons is still greater because some of the nuclei intermediate between U²³⁸ and Cf²⁵⁴ fission or alpha-decay in which case neutrons expended to build some of the nuclei are lost. Since it takes about one fission to produce one excess neutron, usable in the building of higher nuclei, it would take more than 16 fissions to produce one atom of Cf²⁵⁴, and more than 16 Mw to get 1 Mw in the Cf²⁵⁴ flux producer. It takes only 5 Mw in a flux-trap reactor to give the flux delivered by a 1 Mw Cf²⁵⁴ flux producer.

If Cf²⁵⁴ had a thermal absorption cross section of 1 barn, a flux of 1.3×10^{17} n/cm²sec would cause it to burn out as fast as it decays by spontaneous fission. At this, or higher fluxes, the use of Cf²⁵⁴ would be quite unfavorable. More important, Cf²⁵⁴ undoubtedly has epithermal resonances which will depress the epithermal, and hence to some extent the thermal flux at the fission neutron source, thus decreasing the advantage which the Cf²⁵⁴ flux producer would otherwise have.

Another interesting oversimplification of a flux-trap reactor is obtained by neglecting the thermal absorption in the moderator. This gives an idea on how important this absorption is and to what extent it can be neglected in the calculation of more complicated models.

For $K \rightarrow 0$, the equation for ϕ_c of Ref. 1 assumes the following form:

$$\phi_c = \frac{1}{4\pi D\sqrt{\tau}} \left[\frac{1}{\sqrt{\pi} \rho^2} (1 - e^{-\rho^2}) - \frac{1}{\rho} (\text{erf } \rho - \text{erf } \frac{\rho}{2}) \right] \dots \quad (5)$$

7. W. H. Sullivan, Trilinear Chart of Nuclides, U. S. Government Printing Office, Washington, D.C., 1957.

whereas the multiplication factor ηf is defined by the limit, for $K \rightarrow 0$, of Eq. (14) of this reference

$$1 = \frac{\eta f}{\rho} \left(\rho \operatorname{erf} \rho + \frac{1}{\sqrt{\pi}} e^{-\rho^2} - \frac{1}{\sqrt{\pi}} \right) \dots \quad (6)$$

The values of $4\pi D\sqrt{\tau} \phi_c$, as computed from Eq. (5), have been recorded in the first column of Table 2, for comparison with the corresponding values of $4\pi D\sqrt{\tau} \phi_c$ for D_2O , Be, BeO, and C, as computed from Table 1 of Ref. 1, and from Table 1 of Ref. 2. The values of $4\pi D\sqrt{\tau} \phi_c$, for the idealized case of Eq. (5), and for the case of Be, are also plotted in Fig. 1.

It may be seen that D_2O very closely corresponds to the idealized case. The deviation from the idealized case increases, as is to be expected, as K increases, that is, in the order D_2O , C, BeO, and Be. The deviation also increases as ρ increases, because the neutrons are more subject to absorption for greater ρ , and greater distance of travel of the average thermal neutron.

Table 3 lists the ρ values for which the maximum of ϕ_c is obtained together with the $4\pi D\sqrt{\tau} \phi_c$. As already noted in Ref. 2 these ρ values decrease, but only slightly, with increasing K . The values of $4\pi D\sqrt{\tau} \phi_c$ at the maximum of ϕ_c are likewise listed. They also decrease slightly as K increases. However, the required multiplication factor, for the idealized case of $K = 0$, is 1.369 according to Eq. (6). It increases rapidly as K increases as discussed already in Ref. 2.

TABLE 2. NORMALIZED FLUXES

ρ	K = 0	D_2O	Be	BeO	C
		K = 0.0964	K = 0.474	K = 0.394	K = 0.351
1.2	0.04359	0.04352	0.04201	0.04250	0.04272
1.4	0.05124	0.05115	0.04876	0.04952	0.04987
1.6	0.05695	0.05679	0.05343	0.05417	0.05497
1.8	0.06055	0.06034	0.05589	0.05726	0.05792
2.0	0.06215	0.06189	0.05634	0.05803	0.05884
2.2	0.06204	0.06172	0.05513	0.5712	0.05806
2.5	0.05942	0.05902	0.05105	0.05339	0.05455
3.0	0.05139	0.05089	0.04123	0.04396	0.04533
3.5	0.04225	0.04167	0.03114	0.03399	0.03545
4.0	0.03409	0.03345	0.02269	0.02546	0.02692

TABLE 3. MULTIPLICATION FACTORS

	K = 0	D_2O	C	BeO	Be
		K = 0.0964	K = 0.351	K = 0.394	K = 0.474
ρ_{max}	2.09	2.08	2.00	1.99	1.95
$4\pi D\bar{r}\phi$ at max ^c	0.06229	0.06201	0.05884	0.5805	0.05640