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Fluid Analog in High Energy Collisions

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Abstract:

We have used the generating functions which represent the p p data from 50 to 303 GeV/c as analog grand partition functions and explored notions of analog pressure, specific volume, equation of state and critical point. We also discuss ℓ -plane singularities in this connection.

1. Introduction

There is recent interest in the possibility that the multiplicity data on p p collisions above 100 GeV/c may be interpreted in terms of a language familiar in statistical mechanics¹⁾. This idea is reinforced when we note the similarity between the generating function $Q(z, Y)$ in particle production and the grand partition function in statistical mechanics. Both form a medium from which one can derive physical quantities.

The generating function is defined by the equation

$$Q(z, Y) = \sum_n \sigma_n(Y) z^n / \sigma_{in}(Y) \quad (1.1)$$

where $\sigma_n(Y)$ is the topological cross section for n charged particles, $\sigma_{in}(Y) = \sum_n \sigma_n(Y)$ is the inelastic cross section. A variable Y is defined by $Y = \ln(S/S_0)$ where $S^{1/2}$ is the center of mass energy of the p p system and z is a parameter.

A particular choice of the parameter, $z = 1$, fixes the normalization of Q .

$$Q(1, Y) = \sum_n \sigma_n / \sigma_{in} = 1. \quad (1.2)$$

The first derivative of Q at $z = 1$ yields the average multiplicity $\langle n \rangle$:

$$\left. \frac{dQ(z, Y)}{dz} \right|_{z=1} = \langle n \rangle, \quad (1.3)$$

where

$$\langle n \rangle = \sum_n n \sigma_n / \sigma_{in}.$$

Similarly, higher order derivatives at $z = 1$ give the binomial moments; for example

$$\left. \frac{d^2 Q(z, Y)}{dz^2} \right|_{z=1} = \langle n(n-1) \rangle,$$

where $\langle n(n-1) \rangle = \sum_n n(n-1) \sigma_n / \sigma_{in}$.

Although z is merely a parameter here, it may be interpreted as the square of the coupling constant in some Regge-type models. In statistical mechanics σ_n corresponds to the canonical partition function and z is the fugacity. The variable Y plays the role of volume.

In particular we are interested in studying the viability of such analog variables with the aid of the generating functions which represent the data on high energy $p p$ collisions from 50 to 303 GeV/c. In view of the fact that the analog grand partition $Q(z, Y)$ is obtained in a closed form, it is possible to make definite statements based on the experimental facts. At these energies, there is some theoretical justification for Koba, Nielsen, Olesen (KNO) scaling²⁾. There is recent interest in the fluid analog^{3) 4)}.

The analog pressure P , the single particle density ρ and the specific volume v are defined in the following as a function of z and Y

$$P(z, Y) = \frac{1}{Y} \ln Q(z, Y) \quad (1.4)$$

$$\begin{aligned} \rho(z, Y) &= v^{-1}(z, Y) \\ &= \frac{1}{Y} z \frac{\partial}{\partial z} [\ln Q(z, Y)] \end{aligned} \quad (1.5)$$

In statistical mechanics, the limit $Y \rightarrow \infty$ is taken. Here Y is considered to be arbitrary large and kept fixed. The explicit Y dependence in the physical quantities may sometimes be suppressed. For a fixed Y , we can construct an equation of state, which is the equation expressing $P(z, Y)$ as a function $v(z, Y)$, by eliminating the explicit z dependence. Then we consider the limit $Y \rightarrow \infty$ in order to discuss the possible existence of critical points.

It is interesting to examine whether the hadronic fluid has a critical point which is a singularity or discontinuity in the equation of state. According to thermodynamics P is a decreasing function of v , i.e.,

$$\frac{\partial P}{\partial v} < 0$$

At the critical point we must have

$$\frac{\partial P}{\partial v} = 0 \quad (1.4)$$

Furthermore, the equilibrium of a state with $\frac{\partial P}{\partial v} = 0$ requires the second derivative to vanish simultaneously⁵⁾

$$\frac{\partial^2 P}{\partial v^2} = 0. \quad (1.5)$$

In our hadronic case $\frac{\partial P}{\partial v} < 0$ is satisfied in the models considered here. We shall examine the possible existence of critical points by constructing explicitly the equation of state.

A more direct approach is to investigate the analytic property of $Q(z, Y)$ when z is regarded as a complex variable^{5) 6)}. This

4.

approach becomes particularly powerful, when the form of the generating function is compactly known such as those given in section 2.

Present empirical fits do not seem to support the occurrence of a phase transition in the hadronic fluid.

We also investigate the singularity structure in the complex ℓ -plane, based on a fluid analog. Existence of a cut contribution in the Regge theory can be understood as an explicit (or finite) Y dependence of analog pressure.

In section 2 we take up the exponential model of σ_n and in section 3 the gaussian model of σ_n . In section 4 we briefly discuss the equation of state. In section 5 we discuss singularities in the complex ℓ -plane.

2. Exponential model of σ_n .

According to Slattery ⁷⁾ the following KNO scaling function $\psi(n/\langle n \rangle)$ can fit the high energy p p collision data:

$$\sigma_n = \sigma_{in} \frac{1}{\langle n \rangle} \psi(n/\langle n \rangle),$$

where

$$\psi(x) = \sum_{L=1, 3, 5, 7} N_L x^L e^{-ax} \quad (2.1)$$

Here, σ_{in} is the inelastic cross section $\sigma_{in} = \sum_n \sigma_n$, $x = n/\langle n \rangle$ and $\langle n \rangle$ is the average charged multiplicity. Lack of even values of L come from the experimental data. Numerically, $a = 3.04$.

$N_1 = 3.79$ $N_3 = 33.7$ $N_5 = -6.64$ and $N_7 = 0.332$. We note that experimentally $\langle n \rangle$ is a function of Y and increases as $Y \rightarrow \infty$.

Construction of the generating function and the relation with correlation function for the above empirical fit are given in reference ⁸⁾. In order to simplify our discussion we first present a case of $L = 0$ and will consider a realistic case later.

For $L = 0$, we have

$$\sigma_n(Y) = \sigma_{in} \frac{N_0}{\langle n \rangle} e^{-an/\langle n \rangle} \quad (2.2)$$

Normalization of σ_n determines N_0 as

$$N_0 = \langle n \rangle (1 - e^{-a\langle n \rangle}) \quad (2.3)$$

The generating function $Q(z, Y)$ is obtained from eqs. (1.1), (2.2) and (2.3)

$$Q(z, Y) = (1 - e^{-a\langle n \rangle}) / (1 - z e^{-a\langle n \rangle}) \quad (2.4)$$

Parameter a can be determined from eq. (1.3) as

$$a = \langle n \rangle \ln \left(1 + \frac{1}{\langle n \rangle} \right) . \quad (2.5)$$

Pressure and specific volume are calculated with the aid of (1.2), (1.3) (2.4) and (2.5) to be

$$P(z, Y) = - \frac{1}{Y} \ln \left[1 + \langle n \rangle (1 - z) \right] , \quad (2.6)$$

$$v(z, Y) = \left(\frac{Y}{z \langle n \rangle} \right) \left[1 + (1 - z) \langle n \rangle \right] . \quad (2.7)$$

The elimination of z from eqs. (2.6) and (2.7) yields the equation of state

$$P(z, Y) = \frac{1}{Y} \ln \left[\left(1 + \frac{Y}{v(z, Y) \langle n \rangle} \right) / (1 + \langle n \rangle) \right] . \quad (2.8)$$

One notes from eq. (2.7) that $v(z, Y)$ requires $\frac{1}{\langle n \rangle} + 1 > z \geq 0$ which is also the condition for the convergence of the original series $Q(z, Y)$, i.e. $|z e^{-a/\langle n \rangle}| < 1$. This means that the analog specific volume must be non negative for the grand partition function $Q(z, Y)$ to exist.

It follows from eqs. (2.6) and (2.7) that for the case $z = 1$,

$$P(1, Y) = 0 \quad , \quad (2.9)$$

$$v(1, Y) = Y / \langle n \rangle . \quad (2.10)$$

The value of the pressure depends on the normalization of $Q(z, Y)$ ³⁾.

If one replaces the σ in eq. (2.1) by some function $\sigma_c(Y)$, one

obtains $P'(z) = P(z) + P_c(Y)$ where $P_c(Y) = \ln[\sigma_m/\sigma_c]/Y$.

Note that P_c is independent of z and $v'(z) = v(z)$, so that the effect on the equation of the state is to replace $P(z)$ by $P'(z) - P_c(Y)$.

The different forms of $Q(z, Y)$ leads to a shift in the $P(z)$.

We obtain from eq. (2.8)

$$\frac{\partial P(z, Y)}{\partial v(z, Y)} = - [v^2(z, Y) + v(z, Y)Y]^{-1} < 0,$$

$$\frac{\partial^2 P(z, Y)}{\partial v^2(z, Y)} = \frac{2v(z, Y) + Y}{[v^2(z, Y) + v(z, Y)Y]^2} > 0,$$

which do not satisfy the conditions for a critical state when Y is finite. In order to find the critical points we have to take the limit $Y \rightarrow \infty$. Experimentally, the average charged multiplicity seems to increase linearly with Y , although there are other possibilities. Here we accept $\langle n \rangle \propto Y$ as $Y \rightarrow \infty$. When we take the limit $Y \rightarrow \infty$ maintaining $v(1) = Y/\langle n \rangle$ a constant, we find a rather trivial result

$$P(z, \infty) = 0$$

$$\frac{\partial P(z, \infty)}{\partial v(z, \infty)} = \frac{\partial^2 P(z, \infty)}{\partial v^2(z, \infty)} = 0$$

, meaning no critical point.

We can understand this situation in a slightly different way. Let us regard $Q(z, Y)$, given in eq. (2.4) and reproduced here, as a function of the complex variable z : 5), 6)

$$Q(z, Y) = \frac{1 - e^{-a\langle n \rangle}}{1 - z e^{-a\langle n \rangle}} \quad (2.4)$$

As is discussed earlier, the positivity of the specific volume

$v(z, Y)$ requires

$$|z e^{-a\langle n \rangle}| < 1 \quad (2.11)$$

For the region where eq. (2.11) is satisfied, eq. (2.4) is analytic, and so is $P(z, Y)$ and $V(z, Y)$. Since our argument is independent of the magnitudes of Y we can let $Y \rightarrow \infty$.

For $|z| < 1$, $P(z, \infty)$ and $v(z, \infty)$ are analytic. Thus no singularity or discontinuity can appear in the equation of state, which means non-existence of critical points.

Next we discuss the case with an arbitrary L :

$$\sigma_n = \sigma_{in} \frac{N_L}{\langle n \rangle} \left(\frac{n}{\langle n \rangle} \right)^L e^{-a n \langle n \rangle}$$

The generating function can be similarly constructed ⁸⁾

$$Q^{(L)}(z, Y) = \frac{\partial^L}{\partial \left(-\frac{a}{\langle n \rangle}\right)^L} (1 - z e^{-a\langle n \rangle})^{-1} \bigg/ \frac{\partial^L}{\partial \left(-\frac{a}{\langle n \rangle}\right)^L} (1 - e^{-a\langle n \rangle})^{-1}$$

$$\text{with } Q^{(L)}(1, Y) = 1 \quad (2.12)$$

Here a superscript specifies the contribution from a particular L .

Taking derivatives explicitly, we find

$$Q^{(L)}(z, Y) = z^L [Q^{(0)}(z, Y)]^{L+1}$$

where

$$Q^{(0)}(z, Y) = \frac{(1 - z e^{-a\langle n \rangle})^{-1}}{(1 - e^{-a\langle n \rangle})^{-1}} \quad (2.13)$$

Pressure and specific volume can be similarly calculated

$$P(z, \gamma) = \frac{L}{\gamma} \ln z + (L+1) P^{(0)}(z, \gamma) \quad (2.14)$$

$$\frac{1}{v(z, \gamma)} = \frac{L}{\gamma} + (L+1) \frac{1}{v^{(0)}(z, \gamma)} \quad (2.15)$$

where $P^{(0)}(z, \gamma)$ and $v^{(0)}(z, \gamma)$ are pressure and specific volume for $L = 0$:

$$P^{(0)}(z, \gamma) = \frac{1}{\gamma} \ln Q^{(0)}(z, \gamma) \quad (2.16)$$

$$\frac{1}{v^{(0)}(z, \gamma)} = \frac{z}{\gamma} \frac{e^{-a \langle n \rangle}}{1 - z e^{-a \langle n \rangle}} \quad (2.17)$$

where parameter a is determined from

$$e^{-a \langle n \rangle} = \frac{\langle n \rangle - L}{\langle n \rangle + 1} \quad \text{that it is} \quad (2.18)$$

An essential feature of the arbitrary L case is the same as the $L = 0$ case. This is particularly true if we can neglect the first term of eqs. (2.14) and (2.15).

We now construct a generating function corresponding to the empirical fit of eq. (2.1):

$$\begin{aligned} Q(z, \gamma) &= \sum_{\text{odd } L}^{\gamma} N_L Q^{(L)}(z, \gamma) / \sum_{\text{odd } L}^{\gamma} N_L \quad (Q^{(L)}(1, \gamma) = 1) \\ &= \sum_{\text{odd } L}^{\gamma} N_L z^L [Q^{(0)}(z, \gamma)]^{L+1} / \sum_{\text{odd } L}^{\gamma} N_L \quad (2.19) \end{aligned}$$

Parameter a is now determined by

$$e^{-\frac{a}{\langle n \rangle}} = \frac{\langle n \rangle - \langle L \rangle}{1 + \langle n \rangle} \quad (2.20)$$

where

$$\langle L \rangle = \frac{\sum_{\text{odd } L}^7 N_L L}{\sum_{\text{odd } L}^7 N_L}$$

Pressure and specific volume can be similarly calculated.

$$P(z, Y) = \frac{1}{Y} \left(\ln \left[\sum_{\text{odd } L}^7 N_L z^L (Q^{(0)}(z, Y))^{L+1} \right] - \ln \left[\sum_{\text{odd } L}^7 N_L \right] \right), \quad (2.21)$$

$$v(z, Y)^{-1} = \frac{1}{Y} \frac{\sum_{\text{odd } L}^7 N_L z^L [Q^{(0)}(z, Y)]^{L+1} (L + z e^{-a \langle n \rangle})}{(1 - z e^{-a \langle n \rangle}) \sum_{\text{odd } L}^7 N_L z^L [Q^{(0)}(z, Y)]^{L+1}} \quad (2.22)$$

By eliminating z in $P(z, Y)$ and $v(z, Y)$ we can construct an equation of state for fixed Y . We note that the valid range of z is the same as that for the $L = 0$ case, i.e.

$$0 \leq z \leq 1 + \frac{1}{\langle n \rangle}.$$

This is because the $L \neq 0$ case is constructed from the $L = 0$ case by taking derivatives.

We will discuss the $P-v$ diagram in section 4.

3. Gaussian model of σ_n .

An alternative form for the KNO scaling function has been proposed by Weisberg ⁹⁾ in order to fit the pp collision data:

$$\sigma_n = \sigma_m N_m \frac{n^m}{\langle n \rangle^{m+1}} e^{-\alpha n^2 / \langle n \rangle^2}, \quad (3.1)$$

where

$$N_m = 4 \alpha^{(m+1)/2} / \Gamma[(m+1)/2], \quad (3.2)$$

$$\alpha^{1/2} = \Gamma[(m+2)/2] / \Gamma[(m+1)/2]. \quad (3.3)$$

Here n is an even integer and m is an adjustable positive parameter (experimentally $m = 1.215$).

After the summation in the generating function is approximated by an integral, we have

$$Q(Z, Y) = N_m \langle n \rangle^{-(m+1)} 2^m \int_0^\infty dt t^m Z^{2t} e^{-\frac{4\alpha}{\langle n \rangle^2} t^2}. \quad (3.4)$$

We can write this also as ⁸⁾

$$Q(Z, Y) = \exp \left[\langle n \rangle^2 (\ln Z)^2 / 8\alpha \right] \times \\ U \left(m + \frac{1}{2}, -\langle n \rangle \ln Z / (2\alpha)^{1/2} \right) / U \left(m + \frac{1}{2}, 0 \right) \quad (3.5)$$

where U is a parabolic cylinder function ¹⁰⁾

We obtain from eqs. (1.2), (1.3) and (3.5)

$$P(Z, Y) = \frac{\langle n \rangle^2 (\ln Z)^2}{8\alpha Y} + \frac{1}{Y} \ln \left[U \left(m + \frac{1}{2}, -\langle n \rangle \ln Z / (2\alpha)^{1/2} \right) / U \left(m + \frac{1}{2}, 0 \right) \right], \quad (3.6)$$

$$\frac{1}{U(Z, Y)} = \frac{\langle n \rangle}{Y} \frac{(m+1)}{(2\alpha)^{1/2}} U \left(m + \frac{3}{2}, -\langle n \rangle \ln Z / (2\alpha)^{1/2} \right) / U \left(m + \frac{1}{2}, -\langle n \rangle \ln Z / (2\alpha)^{1/2} \right) \quad (3.7)$$

where the following recurrence relation for $U(a, x)$ was used:

$$\frac{dU(a, x)}{dx} = -\frac{1}{2} x U(a, x) - (a + \frac{1}{2}) U(a+1, x). \quad (3.8)$$

It is appropriate to consider the three cases $z = 1$, $z > 1$ and $z < 1$ separately.

In the case $z = 1$, we have

$$P(1, Y) = 0 \quad (3.9)$$

$$v(1, Y) = Y / \langle n \rangle \quad (3.10)$$

In the case $z > 1$, when $\frac{\langle n \rangle |\ln z|}{(2\alpha)^{1/2}} \gg m + \frac{1}{2}$ is satisfied, we have the following asymptotic forms

$$P(z, Y) \approx \frac{1}{Y} \left\{ \ln \left[2^{\frac{m+2}{2}} \frac{\Gamma(\frac{m+2}{2})}{\Gamma(m+1)} \right] + m \ln \left[\langle n \rangle \frac{\ln z}{(2\alpha)^{1/2}} \right] \right. \quad (3.11)$$

$$\left. + \frac{\langle n \rangle^2 (\ln z)^2}{4\alpha} \right\},$$

$$v(z, Y) \approx 2\alpha Y / (\langle n \rangle^2 \ln z). \quad (3.12)$$

The equation of state is

$$P(z, Y) = \frac{1}{Y} \left\{ \ln \left[2^{\frac{m+2}{2}} \frac{\Gamma(\frac{m+2}{2})}{\Gamma(m+1)} \right] + m \ln \left[\frac{(2\alpha)^{1/2} v(1, Y)}{v(z, Y)} \right] \right. \quad (3.13)$$

$$\left. + \frac{v^2(1, Y) \alpha}{v^2(z, Y)} \right\}.$$

In the case $z < 1$, when $\frac{\langle n \rangle |\ln z|}{(2\alpha)^{1/2}} \gg m + \frac{1}{2}$ is satisfied, we have the following asymptotic forms

$$P(z, Y) \approx -\frac{(m+1)}{Y} \ln \left[-\langle n \rangle \ln z / (2\alpha)^{1/2} \right] - \frac{1}{Y} \ln [U(m+\frac{1}{2}, 0)] \quad (3.14)$$

$$v(z, Y) \approx -Y \ln z / (m+1). \quad (3.15)$$

The equation of state is

$$P(z, Y) \approx -\frac{1}{Y} (m+1) \ln \left[\frac{(m+1) u(z, Y)}{(2\alpha)^{1/2} u(1, Y)} \right] - \frac{1}{Y} \ln [U(m+\frac{1}{2}, 0)] \quad (3.16)$$

The above condition $\frac{\langle n \rangle \ln z}{(2\alpha)^{1/2}} \gg m + \frac{1}{2}$ will be met as $\langle n \rangle \rightarrow \infty$ as far as $\ln z \neq 0$.

In practice, however, $\langle n \rangle$ is finite and it is sometime necessary, specially for small $\ln z$, to use the following expansion

$$P(z, Y) \approx \frac{1}{Y} \left[(m+\frac{1}{2})^{1/2} y + \left(\frac{1}{4} - \frac{1}{16(m+\frac{1}{2})} \right) y^2 + \frac{1}{24} \frac{1}{(m+\frac{1}{2})^{1/2}} y^3 + \dots \right] \quad (3.17)$$

$$u(z, Y) \approx \frac{Y}{\langle n \rangle} \frac{(2\alpha)^{1/2}}{(m+1)} \frac{U(m+\frac{1}{2}, 0)}{U(m+\frac{3}{2}, 0)} \exp \left[\left\{ (m+\frac{1}{2})^{1/2} - (m+\frac{3}{2})^{1/2} \right\} y \right. \\ \left. + \frac{1}{16} \left\{ -\frac{1}{(m+\frac{1}{2})} + \frac{1}{(m+\frac{3}{2})} \right\} y^2 + \frac{1}{24} \left\{ \frac{1}{(m+\frac{1}{2})^{1/2}} - \frac{1}{(m+\frac{3}{2})^{1/2}} \right\} y^3 + \dots \right] \quad (3.18)$$

where $y = \frac{\langle n \rangle \ln z}{(2\alpha)^{1/2}}$.

These expressions are valid for $m + \frac{1}{2} \gg y^2$.

4. Discussions

We now evaluate $P(z, Y)$ at the maximally available energy 303 GeV.

For numerical analysis, we use $Y = \ln s - 1.5$ ³⁾.

The experimental value of $\langle n \rangle$ is ⁽⁷⁾

$$\langle n \rangle = 8.86 \pm 0.15$$

We find that the exponential model and the gaussian model yield almost identical results. (See Figs. 1 and 2.)

The main difference appears in the range of validity of z . In the exponential model z must be restricted to

$$0 \leq z \leq 1.12$$

while the gaussian model is not required to have any restriction on z :

$$0 \leq z \leq \infty$$

The $p - v$ diagram at 303 GeV is plotted in Fig. 3.

We observe in Fig. 3 that $\frac{\partial p}{\partial v} < 0$ and that there is no point at this energy satisfying $\frac{\partial p}{\partial v} = 0$. We would like to add a remark about $z = 1$ in connection with a phase transition. In the limit $Y \rightarrow \infty$, physical quantities such as $v(z, Y)$ could become singular at $z = 1$. However, $z = 1$ can not be a critical point, since in the limit $Y \rightarrow \infty$ the allowed region for z is shrunk to

$$0 \leq z < 1$$

which is the range of convergence of series $\mathcal{Q}(z, Y)$ in both models.

Thus the present empirical fits do not seem to support the occurrence of a phase transition.

When there is a simple Regge pole exchange in multiperipheral models, one has short range correlations whereas eqs. (2.1) and (3.1) lead to long range correlations i.e. the two particle correlation function $f_2 \sim \langle n \rangle^2$.

Equations (5.4) and (5.5) indicate the complex exchange process.

In conclusion, if pressure $P(z, Y)$ depended only on z , such as $P(z, \infty)$, then we should have obtained Regge poles rather than cuts.

Importance of cut contributions in Regge phenomenology may indicate that Y dependence of the pressure can not be ignored.

5. Singularities in complex ℓ plane

In order to understand the long range correlations which are present in the $p p$ data as exemplified by the KNO scaling model, we are going to express our generating function $Q(z, Y)$ in the Regge language. Then the asymptotic energy dependence of $Q(z, Y)$ is controlled by a few singularities in the complex ℓ plane. According to our definition (1.4), we have

$$Q(z, Y) = e^{PY} \quad (5.1)$$

In order to study the ℓ dependence, we construct its Laplace transform defined by

$$Q(z, \ell) \equiv \int_0^\infty dY e^{-\ell Y} Q(z, Y) \quad (5.2)$$

$$= \int_0^\infty dY e^{-(\ell - P)Y} \quad (5.3)$$

Since P depends upon Y also, $Q(z, Y)$ does not produce poles, but a cut.

Substituting eqs. (1.2) and 2.6) for the exponential σ_n with $L = 0$, for instance, we get

$$Q(z, \ell) = \int_0^\infty dy \frac{e^{-y}}{\ell - [(z-1)y / v(1)]} \quad (5.4)$$

Equation (5.4) exhibits a cut in the ℓ -plane for $z > 1$ and $\text{Re } \ell \geq 0$.

In a similar manner, we get from (1.2) and (3.4) for the gaussian σ_n

$$Q(z, \ell) = \frac{1}{2^{\frac{m-1}{2}} \Gamma(\frac{m+1}{2})} \int_0^\infty ds \frac{s^m e^{-\frac{1}{2}s^2}}{\ell - s [\ln z / v(1)(2\alpha)^{1/2}]} \quad (5.5)$$

in which $Q(z, Y)$ again exhibits a cut when $z > 1$, $\ell \geq 0$.

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Figure captions

Fig. 1 Specific volume $\mathcal{V}$ as a function of z at $\sqrt{s} = 303$ GeV

———— : Exponential model

----- : Gaussian model

Fig. 2 Pressure P as a function of z at $\sqrt{s} = 303$ GeV

———— : Exponential model

----- : Gaussian model

Fig. 3 Pressure versus specific volume at $\sqrt{s} = 303$ GeV

———— : Exponential model

----- : Gaussian model

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z)

5

4

3

2

1

0

0.5

1.0

1.5

2.0

z

Fig

$\beta = 303$

$P(z)$

+1

0

-1

z

0

0.5

1.0

1.5

Fig