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ABSTRACT

Many elements exhibit anisotropic scattering at energies of interest in reactor calculations. A method is presented for the calculation of transport cross sections including the observed anisotropy.

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CALCULATION OF TRANSPORT CROSS SECTIONS

C. W. Nestor

There is a considerable amount of information now available¹ concerning the variation of the differential elastic scattering cross section with energy for many elements. It is clear that the scattering is anisotropic in the center-of-mass system at energies of interest in reactor calculations. This report presents a method for calculating the elastic transport cross section as a function of energy, taking account of the observed anisotropy.

The elastic transport cross section is taken to be of the form²

$$\sigma_{tr}(E) = \sigma_t(E) \left[1 - \bar{\mu}(E) \frac{\sigma_e(E)}{\sigma_t(E)} \right] \quad (1)$$

(Symbols are defined in Nomenclature at end of report.)

The transformation between the laboratory and center of mass systems is

$$\mu = \frac{1 + A \eta}{\sqrt{1 + 2A\eta + A^2}} \quad (2)$$

For convenience the expression for μ is expanded into a series of Legendre polynomials in η , making use of the generating function

$$T(t, \eta) = \frac{1}{\sqrt{1 - 2t\eta + t^2}} = \sum_{\ell=0}^{\infty} t^{\ell} P_{\ell}(\eta) \quad (3)$$

¹ "Neutron Cross Sections - Angular Distributions," BNL 400, by D. J. Hughes and R. S. Carter, 1956.

² "Notes on Reactor Analysis - Part II - Theory," CF-54-7-88, by D. K. Holmes and R. V. Meghreblan, 1954.

and the recursion relation³

$$\eta P_{\ell}(\eta) = \frac{\ell+1}{2\ell+1} P_{\ell+1}(\eta) + \frac{\ell}{2\ell+1} P_{\ell-1}(\eta) \quad (4)$$

The result is

$$\mu = \sum_{\ell=0}^{\infty} \frac{(-1)^{\ell} P_{\ell}(\eta)}{A^{\ell}} \left[\frac{1}{A} \frac{\ell+2}{2\ell+3} - \frac{A\ell}{2\ell-1} \right] \quad (5)$$

The frequency function for η may be written⁴

$$f(\eta) = \frac{\sigma(\eta)}{\int_{-1}^1 \sigma(\eta) d\eta} = \sum_{m=0}^{\infty} \frac{2m+1}{2} f_m P_m(\eta) \quad (6)$$

where

$$f_m = \frac{\int_{-1}^1 P_m(\eta) \sigma(\eta) d\eta}{\int_{-1}^1 \sigma(\eta) d\eta}.$$

These coefficients can be obtained by numerical integration using experimental values for the differential cross section $\sigma(\eta)$.

The average value $\bar{\mu}$ of μ is then of the form

$$\bar{\mu} = \int_{-1}^1 \mu(\eta) f(\eta) d\eta = \sum_{\ell=0}^{\infty} \frac{(-1)^{\ell}}{A^{\ell}} \left[\frac{1}{A} \frac{\ell+2}{2\ell+3} - \frac{A\ell}{2\ell-1} \right] f_{\ell} \quad (7)$$

If f_{ℓ} is assumed to be zero for $\ell > 0$, i.e., isotropic scattering in the center of mass system, the conventional value of $\bar{\mu}$ is obtained, namely

$$\bar{\mu} = \frac{2}{3A}.$$

A convenient form for $\bar{\mu}$ is obtained assuming that terms above

$P_2(\eta)$ are negligible:

³ See, for example, "Introduction to Quantum Mechanics," by L. Pauling and E. B. Wilson, p 126, McGraw-Hill (1935).

⁴ Anisotropic Elastic Scattering of Neutrons, CF 57-3-128, by C. M. Copenhaver. It is to be understood that $\sigma(\eta)$ and the f_m are functions of E .

$$\bar{\mu} = \frac{2}{3A} + \left(1 - \frac{3}{5A^2}\right) f_1 - \frac{2}{A} \left(\frac{1}{3} - \frac{2}{7A^2}\right) f_2 \quad (8)$$

The functions $\frac{2}{3A}$, $1 - \frac{3}{5A^2}$ and $-\frac{2}{A} \left(\frac{1}{3} - \frac{2}{7A^2}\right)$ are plotted in Fig. 1 for $1 \leq A \leq 12$.

NOMENCLATURE

$\sigma(E)$	microscopic cross section at energy E
subscripts:	t total
	e elastic
	tr transport
$\sigma(\eta)d\eta$	differential cross section for scattering into $d\eta$ about η
$\bar{\mu}(E)$	average cosine of lab system scattering angle
η	cosine of center-of-mass system scattering angle
μ	cosine of laboratory system scattering angle
A	mass number of scattering nucleus

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Fig. 1

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