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VISCOUS ATTENUATION OF EXPLOSIVE SHOCKS IN TUBES*

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Civil Defense Research Project

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VISCOUS ATTENUATION OF EXPLOSIVE SHOCKS IN TUBES

Lawrence Dresner

1. INTRODUCTION

This is the second of a series of papers dealing with the propagation of explosive shocks in a bare tube. (Explosive shocks are shocks produced by detonation of small amounts of high explosive in the closed end of a tube.) In the first paper,¹ measurements of shock arrival time and peak overpressure were reported for distances up to 55 diameters down the tube. At such short distances down the tube, viscous drag at the wall has very little effect on shock propagation. The present paper reports additional measurements made at distances up to 165 diameters down the tube; at these greater distances, viscous drag at the walls markedly attenuates shock waves.

When a small piece of high explosive is detonated in the closed end of a tube, the following sequence of events unfolds. Following detonation, the high explosive is converted into a bubble of very hot, very highly compressed gas. This gas expands in all directions, striking the wall and closed end of the tube. A complicated series of reverberations ensues that defies description, but after a short while the flow assumes the form of a plane blast wave. (A plane blast wave is the flow that would be produced in an infinite, homogeneous atmosphere by the detonation of a plane sheet of explosive.)

During the approach of the flow field to that of a plane blast wave, the gaseous detonation products lose energy to the wall of the tube. Only after the flow approximates a plane blast wave does its total energy remain fairly constant. (In the absence of viscous drag, its energy would be conserved). The total energy in air blast is thus only a fraction of the total yield of the explosive.

During the next phase of shock propagation, the flow field changes with time according to the laws governing plane blast waves. By studying the flow field during this phase of shock propagation, we can determine the total energy in air blast. Eventually, viscous drag at the walls begins to affect the flow field, and in the last phase of shock propagation, the simple theory of plane blast waves is no longer adequate to describe the flow field in the tube.

2. PLANE BLAST WAVES

The laws of propagation of plane blast waves can be worked out explicitly in the limiting cases of very strong and very weak shocks. (Shocks are strong or weak according to whether their peak overpressure is large or small compared to the pressure of the undisturbed air in front of them.)

The propagation of strong explosive shocks is described by the similarity theory of Taylor,² von Neumann,³ and Sedov.⁴ According to this theory, the time of arrival t_* of the shock front at any distance X from the explosion is related to the energy E per unit face area of the blast wave and the density ρ_0 of the undisturbed air in front of the shock in the following way:

$$t_* = 0.733 \left(\frac{\rho_0 X^3}{E} \right)^{1/2} \quad (1)$$

The peak pressure p_* is given by

$$p_* = 0.689 \frac{E}{X} \quad (2)$$

The numerical constants in Eqs. (1) and (2) depend on the ratio of the specific heats γ of the gas in which the explosion takes place; those shown above are appropriate for $\gamma = 1.4$.

The propagation of weak shocks is described by the asymptotic theory of Bethe,⁵ Chandrasekhar,⁶ and Landau.⁷ According to this asymptotic theory,⁸ the peak overpressure Δp_* of a weak shock wave varies with distance traveled according to the law

$$\frac{\Delta p_*}{p_0} = \left(4\gamma \frac{\gamma-1}{\gamma+1} \frac{E}{p_0 X} \right)^{1/2} \quad (3)$$

where p_0 is the pressure of the undisturbed air in front of the shock. The velocity of weak shocks is nearly the velocity c_0 of sound in the undisturbed air. Hence, asymptotically, the arrival time is given by

$$t_* = \frac{X}{c_0} \quad (4)$$

The uppermost curve in Fig. 1 shows the variation of peak overpressure with distance for plane explosive shocks of any strength. The curve approaches the asymptotes given by Eqs. (2) and (3). The curve was calculated numerically using von Neumann and Richtmyer's method of artificial viscosity.⁹ More will be said later about these numerical calculations. The lowest curve in Fig. 2 gives the time of arrival at any position. The curve approaches the asymptotes given by Eqs. (1) and (4).

If the energy E per unit face area of the blast wave is known, the upper curve in Fig. 1 and the lower curve in Fig. 2 may be used to predict the arrival time and peak overpressure at any station down the tube as long as viscous drag is unimportant. Conversely, if the arrival time and peak overpressure are known at various stations down the tube, the energy E per unit face area of the air blast may be found. This is what was done in reference 1. According to the experiments reported there, the fraction of the total explosive yield Y appearing in air blast in a pipe of diameter D equals $3.1 \times (p_0 D^3 / Y)^{1/3}$ for values of the quantity in parentheses between 4×10^{-5} and 5×10^{-3} .

3. SHOCK PROPAGATION WITH VISCOUS WALL DRAG: THEORY

Recently, Slepicka¹⁰ gave a simple derivation of the empirical formula of Emrich and Wheeler¹¹ for viscous attenuation in gas-driven shock tubes. His method is applied here to explosively driven shock tubes.

Slepicka's derivation is based on the use of the following formula for the drag force on a slug of fluid moving in a round pipe:¹³

$$F = 2\pi RL \cdot \frac{1}{2} \rho v^2 \cdot f \quad (5)$$

where F is the total drag force on the slug of fluid, R is the radius of the pipe, L is the length of the slug of fluid, ρ is the density of the fluid, v is the flow velocity, and f is the Fanning friction factor. f is an empirical factor defined by Eq. (5) which owes its usefulness to the fact that for turbulent pipe flow it depends only on the Reynolds number and weakly at that. Use of Eq. (5) to describe the effect of wall drag on shock propagation presupposes that the flow behind the shock front is turbulent (also see section 4).

Fig. 1. The variation of peak overpressure with distance down an explosively driven shock tube. Δp_* is the peak overpressure, p_0 is the initial pressure, X is the distance down the tube, and E is the energy per unit face area of the blast wave. The highest curve corresponds to no viscous wall drag. The partly dashed and partly solid curves correspond to various amounts of wall drag; the numbers labeling these curves are values of $p_0 D/Ef$, where D is the tube diameter and f is the friction factor. The dotted curve corresponds to a case in which $p_0 D/Ef = 0.1$ but in which the transition to turbulent flow occurs 50 diameters down the tube. The points are experimental.

Fig. 2. The variation of arrival time with distance down an explosively driven shock tube. t is the arrival time, c_0 the speed of sound in the undisturbed air, X is the distance down the tube, p_0 is the initial pressure, and E is the energy per unit face area of the blast wave. The lowest curve corresponds to no viscous wall drag. The partly dashed and partly solid curves are values of $p_0 D / E f$, where D is the tube diameter and f is the friction factor. The dotted curve corresponds to a case in which $p_0 D / E f = 0.1$ but in which the transition to turbulent flow occurs 50 diameters down the tube. The points are experimental.

The Reynolds number in a four-inch shock tube, such as the one used in our experiments, is of the order of 10^6 . For such a high Reynolds number, the friction factor of all but the smoothest pipes is virtually independent of the Reynolds number and depends only on the roughness of the tube wall. In the theory to follow, it will be assumed that the friction factor f is a constant.

The second assumption of the theory given below is that the force F , which in actual fact acts mainly in the boundary layer near the pipe wall, can be replaced by a force per unit mass F_0 equal to

$$F_0 = v^2 f / R \quad (6)$$

With this assumption, the viscous wall drag is smeared uniformly over the pipe cross section, and the theory remains one-dimensional, an invaluable simplification. It is impossible to assess the error introduced by Eq. (6), but when the disturbance is many diameters long, it should not be very great.

3.1 STRONG SHOCKS

The problem of explosive shock propagation in the presence of the drag force (6) can be solved approximately in analytic terms by use of the so-called "shell" model of strong, explosive shocks.¹² The "shell" model of strong, explosive shocks is based on idealization of certain features of the similarity theory of Taylor,² von Neumann,³ and Sedov⁴ mentioned above. If we examine the similarity solution for gases for which $\gamma-1$ is not too large, we see that most of the gas behind the shock front can be found in a region of small extent (the "shell") just behind the shock front. For example, when $\gamma = 1.4$, approximately 75% of the gas behind the shock front can be found in a region just behind the shock front whose thickness is one-fifth of the distance traveled by the shock, i.e., in the region $0.8 \leq \xi \leq 1.0$. About 50% of the shocked gas can be found between $\xi = 0.9$ and $\xi = 1.0$. When $\gamma = 1.1$, the percentages of the shocked gas found in these shells are 99% and 90%, respectively. Behind the shell is a region of highly rarified gas in which the pressure is roughly constant. For example, when $\gamma = 1.4$, the pressure changes by only 40%

between $\xi = 0$ and $\xi = 0.8$ and by only 4% between $\xi = 0$ and $\xi = 0.5$.

Based on this picture, we introduce the following simple idealized model of strong, explosive shocks:

All but a negligible portion of the shocked gas lies in a shell just behind the shock front whose thickness is small compared to the distance X of the shock front from the site of the explosion. The gas in the shell is characterized by a single pressure p_* and a single flow velocity v_* related by the Rankine-Hugoniot equation

$$\frac{1}{\gamma} \rho_0 v_*^2 = \frac{p_*}{\gamma + 1} \quad (7)$$

which is a limiting form valid for strong shocks. Owing to the extreme thinness of the shell, the internal energy of the gas in the shell is a negligible fraction of the total internal energy of all the shocked gas. The region behind the shell contains a negligible fraction of the total mass of shocked gas. The pressure p in this region is uniform. Owing to the extreme rarefaction of the gas in this region, its kinetic energy is negligible compared to its internal energy and its momentum is negligible compared to the momentum of the gas in the shell. Finally, the pressure p_0 of the undisturbed atmosphere is negligible compared to the pressures p_* and p .

The detailed calculations based on the "shell" model are given in Appendix I. As a by-product of these calculations, we obtain formulas for the arrival time and peak overpressure of a constant-energy blast wave as a function of X, E , and p_0 . These formulas have the same form as their exact counterparts, Eqs. (1) and (2), and differ only in the value of the constant. The constant in the approximate version of Eq. (1) is too low by about 8% and that in the approximate version of Eq. (2) is too high by about 16%. This close agreement supports the reliability of the "shell" model.

The results of the shell-model calculations can be expressed as correction factors C_t and C_p to be applied respectively to the arrival time and peak overpressure calculated from Eqs. (1) and (2). Formulas for these correction factors are given below:

$$C_t = \frac{3}{2} y^{-3/2} \int_0^y \frac{y' dy'}{(1 - e^{-y'}) y^2} \quad (8a)$$

$$C_p = \frac{1 - e^{-y}}{y} \quad (8b)$$

where

$$y = \frac{8f}{\gamma+1} \frac{X}{D} \quad (8c)$$

C_t and C_p are shown in Fig. 3 plotted against y .

3.2. NUMERICAL CALCULATIONS

Numerical calculations of explosive shock propagation in the presence of the frictional force (6) have been carried out using von Neumann and Richtmyer's method of artificial viscosity.⁹ These calculations have been carried out in order to check the theory given in Sec. 3.1 for strong shocks and in order to obtain results valid when the shocks are no longer strong. The results are shown in Figs. 1 and 2. Each curve is labeled with a value of the parameter $p_0 D / fE$.

The numerical calculations are beset by a number of difficulties. In the first place, in the method of artificial viscosity, the shock is smeared out over several lattice points of the calculational mesh. Consequently, it is impossible to locate precisely the position of the shock front. In the calculations being described here, the error in the position of the shock front was of the order of 1%. Secondly, the pressure in the neighborhood of the shock front exhibited some minor fluctuations, which in the calculations being reported here were of the order of a few percent. Neither of these two sources of error is at all serious.

A third, more serious source of error arises in the following way. We wish the numerical calculations to apply to the blast wave produced by a plane explosion. This means that the initial blast energy should be introduced in an infinitesimally thin region or, in other words, that the initial pressure distribution should be a delta function. Of course, such a pressure distribution cannot be represented numerically. The closest we can come to it is to put all the initial blast energy in

Fig. 3. The correction factors C_p and C_t to be applied to the peak overpressure and arrival time_p to account for the effects of viscous wall drag according to the "shell" model. y is equal to $8fX/(\gamma + 1)D$, where f is the friction factor, X is the distance down the tube, γ is the ratio of the specific heats of air, and D is the tube diameter.

in the interval between the first two integral mesh points, i.e., to make the pressure very high in that interval. (The space coordinate is defined at integral mesh points, the pressure and density at half-integral mesh points; see reference 9).

A few time steps after the start of the calculation, the shock front will have advanced a few mesh intervals. Thus only a few mesh widths are available to represent the flow field behind the shock. Since at first, the flow behind the shock consists of three regions of uniform flow separated by a rarefaction wave and a contact discontinuity, a few mesh widths are not enough to represent it accurately. Thus at the outset, the numerical calculations may be badly in error. Only when the region behind the shock front is many mesh widths wide, do the numerical calculations approach the correct results closely. Experience has shown that the region behind the shock front must be some fifty or more mesh widths wide for the error in the pressure and arrival time to be only a few percent or less.

The calculations are actually done in the following manner. A length L is chosen and divided into 100 mesh widths. Next a value of $p_0 L/E$ is chosen, this being the greatest value of $p_0 X/E$ that can be reached in this calculation. From these two values we may calculate the value of the ratio p_1/p_0 of the source pressure p_1 at the first half-integral mesh point to the pressure p_0 of the undisturbed air. With the source so determined, the calculation is begun. The overpressure resulting from such a calculation if plotted on Fig. 1 would be an arc of a curve extending out to an abscissa $p_0 L/E$. The right-hand half of this arc represents reasonably well the overpressure due to a plane explosion at the particular value of $p_0 X/E$. The overpressure curve for a plane explosion is then constructed as the envelope of the right-hand halves of the arcs produced in several suitable calculations of the type just described.

Shown in Fig. 1 are curves corresponding to values of $p_0 D/fE$ of 0.05, 0.1, 0.2, 0.4, and 0.8. Certain portions of these curves are shown solid and certain portions are shown dashed. The solid portions are made up of the right-hand halves of numerically calculated arcs; the dashed portions are estimates. Shown in Fig. 2 are corresponding curves for the arrival time.

It can be verified without difficulty that the strong-shock theory described in Sec. 3.1 reproduces the curves shown in Figs. 1 and 2 within about ten percent for values of $p_0 X/E$ less than about 0.1. Thus for strong shocks, the simple analytic "shell" theory is acceptable for all practical purposes. (See, however, the remark following Eq. (A18) in the appendix.)

4. EXPERIMENTS AND THEIR COMPARISON WITH THEORY

Shown in Figs. 1 and 2 are the results of measurements carried out in the four-inch shock tube described in reference 1. The total explosive yield in these experiments was 4.65 g TNT, exactly as in reference 1. The arrival time and overpressure were measured at distances of 418, 525, 540, 641, and 660 inches from the explosion, i.e., at distances of approximately 105, 130, 135, 160, and 165 diameters down the tube. Three different initial air pressures were used in these experiments, namely, 760, 150, and 30 mm Hg.

Comparison of the experimental overpressures with those calculated from the theory outlined in Sec. 3 shows that the value of the parameter $p_0 D/fE$ is about 0.1 when $p_0 = 30$ mm Hg, about 0.4 when $p_0 = 150$ mm Hg, and about 0.8 when $p_0 = 760$ mm Hg. The respective values of f that follow from these values of $p_0 D/fE$ are 0.0088, 0.0064, and 0.0097. The experimental overpressures at points more than 100 diameters down the tube thus seem consistent with a constant value of the friction factor f of about 0.008. For high Reynolds numbers, this value of f corresponds to a relative roughness (height of the protuberances divided by the pipe diameter) of about 0.006.

If the curve corresponding to $p_0 D/fE = 0.1$ really fits the data taken with $p_0 = 30$ mm Hg for all values of $p_0 X/E$, then when the data reported here are combined with the data reported in reference 1, all the data points should lie on a curve that is nearly straight on a log-log plot. However, that turns out not to be the case. The points given in reference 1 lie too high; in fact, they fall very close to the upper curve in Fig. 1, which corresponds to no viscosity at all.

An explanation of this anomaly is that the flow does not become turbulent until the shock front has progressed some distance down the tube. According to Bradley,¹³ the transition to turbulent flow occurs from 20 to 140 diameters down the tube. Shown in Figs. 1 and 2 as a dotted curve are the results of a calculation in which there was no viscous drag until the shock

front traveled 50 diameters down the tube. From there on, the friction factor was chosen to make $p_0 D / f E = 0.1$. ($p_0 D / E$ was chosen to be 0.001, and f was chosen to be 0.01. Under these circumstances, the overpressure and arrival time curves are the same as those with no viscous wall drag until $p_0 X / E = 0.05$.) The dotted curve fits the data taken with $p_0 = 30$ mm Hg, indicating that the transition to turbulent flow occurs of the order of fifty diameters down the tube.

Analysis of the arrival time data is more difficult than analysis of the overpressure data because viscous wall drag has a smaller effect on arrival time than on overpressure. This means that the various arrival-time curves are much closer to each other than the corresponding curves of overpressure. On the whole the values of $p_0 D / f E$ deduced from the arrival times are consistent with those deduced from the overpressures.

It seems unlikely that additional curve-fitting would improve the values of f and the distance of the transition to turbulent flow deduced above. We conclude that flow in explosively driven shock tubes is well described by the model presented in this paper. Furthermore we conclude that in our four-inch shock tube the friction factor f is about 0.008 and the transition to turbulent flow occurs of the order of 50 diameters down the tube (this latter figure is of course very rough).

APPENDIX

The equations of the "shell" model are the following:

$$\frac{d}{dt} (\rho_0 X v_*) = p - F_0 (\rho_0 X) \quad (\text{Ala})$$

$$\frac{d}{dt} \left(\frac{pX}{\gamma-1} \right) = -F_0 v_* (\rho_0 X) \quad (\text{Alb})$$

where

ρ_0 is the initial density of the gas,
 X is the position of the shock front at any time t ,
 v_* is the flow velocity of the gas in the shell,
 p is the uniform pressure of the gas behind the shell,
 $F_0 = f v_*^3 / R$ is the frictional force per unit mass
 f = the Fanning friction factor, assumed constant,
 R = the pipe radius, and
 γ = the ratio of the specific heats of the gas.

Eqs (Ala) and (Alb) express the change with time of the total momentum and energy, respectively. In addition to the simplifications described in the text, the kinetic energy of the gas in the shell has been ignored compared to the internal energy of the gas behind the shell (see reference 12), and the frictional force has been assumed to act only on the shell.

If we eliminate p between Eqs. (Ala) and Alb) and use the strong-shock Hugoniot relation

$$v_* = \frac{2}{\gamma+1} \dot{X} \quad \left(\dot{X} = \frac{dX}{dt} \right) \quad (\text{A2})$$

we obtain the following differential equation for X :

$$\frac{d}{dt} \left(X \frac{d}{dt} (\dot{X}) \right) = -\frac{2f}{\gamma+1} \left[\frac{d}{dt} \left(\frac{\dot{X}^2 X^2}{R} \right) + \frac{2(\gamma-1)}{\gamma+1} \frac{\dot{X}^3 X}{R} \right] \quad (\text{A3})$$

Let us now introduce the dimensionless variables

$$\eta = \frac{X}{R} \quad (\text{A4a})$$

$$\tau = \frac{3}{2} \left[\frac{(\gamma+1)(\gamma-1)E_0}{\rho_0 R^3} \right]^{1/2} t \quad (A4b)$$

where E_0 is the initial energy per unit face area of the blast wave, i.e.,

$$E_0 = \lim_{t \rightarrow 0} \left(\frac{pX}{\gamma-1} \right)$$

Eq. (A4b) has been normalized in a convenient way as we shall see shortly.

In terms of η and τ , Eq. (A3) can be written

$$\frac{d}{d\tau} \left(\eta \frac{d}{d\tau} (\eta \dot{\eta}) \right) = -\frac{2f}{\gamma+1} \left[\frac{d}{d\tau} (\eta^2 \dot{\eta}^2) + \frac{2(\gamma-1)}{\gamma+1} \eta \dot{\eta}^3 \right] \quad (A5)$$

When $f = 0$, Eq. (A4b) may be replaced by Eq. (A6) below:

$$\frac{pX}{\gamma-1} = E_0 \quad (A6)$$

If p is eliminated between Eqs. (A4b) and (A6), we then obtain after some manipulation

$$\frac{d}{dt} (X \dot{X})^2 = \frac{(\gamma+1)(\gamma-1)E}{\rho_0} \dot{X} \quad (A7a)$$

or

$$\frac{d}{d\tau} (\eta \dot{\eta})^2 = -\frac{4}{3} \eta \dot{\eta} \quad (A7b)$$

Eqs. (A7a) and (A7b) follow from integrating Eqs. (A3) and (A5), respectively, with respect to time when $f = 0$. They have the advantage that the constant of integration is given explicitly in terms of the physical parameters of the problem.

Eq. (A7b) may be integrated again to give

$$(\eta \dot{\eta})^2 = \frac{4}{3} \eta + C \quad (A8)$$

where C is an arbitrary constant. According to the similarity theory,^{2,3,4}

the shock velocity varies as $X^{-1/2}$, i.e., $\dot{\eta} \sim \eta^{-1/2}$. Therefore if the shell model is to agree even approximately with the similarity theory, C must equal zero. If we now integrate Eq. (A8) once more, we find

$$\tau = \eta^{3/2} \quad (A9)$$

since $\eta = 0$ when $\tau = 0$.

As long as η is not too large compared to $1/f$, the relationship (A9) between τ and η should be approximately correct. We can use it to compare the two terms on the right-hand side of Eq. (A5) with each other. If $\tau = \eta^{3/2}$, then $\frac{d}{d\tau} (\eta^2 \dot{\eta}^2) = \eta \dot{\eta}^3$. Hence if $\gamma-1 \ll 1$, we can neglect the last term on the right-hand side of Eq. (A5). If we do this, we can immediately integrate Eq. (A5) to get

$$\eta \frac{d}{d\tau} (\eta \dot{\eta}) + \frac{2f}{\gamma+1} \eta^2 \dot{\eta}^2 = \frac{2}{9} \quad (A10)$$

Here the constant of integration has been obtained by requiring the solution to approach $\tau = \eta^{3/2}$ for small τ .

Since $d\tau = d\eta / \dot{\eta}$, we can write this differential equation as

$$\frac{d}{d\eta} (\eta \dot{\eta})^2 + \frac{4f}{\gamma+1} (\eta \dot{\eta})^2 = \frac{4}{9} \quad (A11)$$

This is a linear, first-order differential equation with constant coefficients for $(\eta \dot{\eta})^2$; and under the boundary condition $(\eta \dot{\eta})_{\eta=0}^2 = 0$ (required if $\tau = \eta^{3/2}$ for small τ) it has the solution

$$(\eta \dot{\eta})^2 = \frac{\gamma+1}{9f} \left[1 - \exp \left(-\frac{4f}{\gamma+1} \eta \right) \right] \quad (A12)$$

From Eq. (A12) it follows that

$$\tau = \left(\frac{9f}{\gamma+1} \right)^{1/2} \int_0^\eta \frac{\eta' d\eta'}{[1 - \exp(-\frac{4f}{\gamma+1} \eta')]^{1/2}} \quad (A13)$$

Thus

$$C_t = \frac{\tau}{\tau_{f=0}} = \left(\frac{9f}{\gamma+1} \right)^{1/2} \eta^{-3/2} \int_0^\eta \frac{\eta' d\eta'}{[1 - \exp(-\frac{4f}{\gamma+1} \eta')]^{1/2}} = \frac{3}{2} y^{-3/2} \int_0^y \frac{y' dy'}{(1 - e^{-y'})^{1/2}} \quad (A14)$$

where $y = \frac{4f}{\gamma+1} \eta = \frac{8f}{\gamma+1} \frac{X}{D}$. Furthermore, from Eqs. (7), (A2), and (A12) it follows that

$$C_p = \frac{\dot{\eta}^2}{\dot{\eta}_{f=0}^2} = \frac{\gamma+1}{4f} \frac{1 - e^{-\frac{4f}{\gamma+1} \eta}}{\eta} = \frac{1 - e^{-y}}{y} \quad (A15)$$

If the transition to turbulent flow occurs at a distance X_0 , Eq. (A5) applies for $X \geq X_0$, while the solution (A9) applies for $X \leq X_0$. If we again neglect the last term on the right-hand side of Eq. (A5), we can integrate it to give

$$\eta \frac{d}{d\tau} (\eta \dot{\eta}) + \frac{2f}{\gamma+1} \eta^2 \dot{\eta}^2 = \frac{2}{9} \left(1 + \frac{4f}{\gamma+1} \eta_0 \right) \quad (A16)$$

The value of the constant on the right-hand side has been determined by the condition that the solution be given by Eq. (A9) at $\eta = \eta_0 = \frac{4f}{\gamma+1} \frac{X_0}{R}$. Eq. (A16) may be solved in the same manner as Eq. (A10) with the boundary condition just mentioned. The solution corresponding to Eq. (A12) is

$$(\eta \dot{\eta})^2 = \frac{\gamma+1}{9f} \left[1 + \frac{4f}{\gamma+1} \eta_0 - \exp\left(-\frac{4f}{\gamma+1} \{\eta - \eta_0\}\right) \right] \quad (A17)$$

Thus

$$C_p = \frac{\dot{\eta}^2}{\dot{\eta}_{f=0}^2} = \frac{1 + \frac{4f}{\gamma+1} \eta_0 - \exp\left(-\frac{4f}{\gamma+1} \{\eta - \eta_0\}\right)}{\frac{4f}{\gamma+1} \eta} \quad (A18a)$$

$$= \frac{1 + y_0 - e^{-(y-y_0)}}{y} \quad (A18b)$$

When we attempt to reproduce the dotted curve in Fig. 1 using Eq. (A18), we obtain only qualitative agreement, the results based on Eq. (A18) being about 50% too high.

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