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A PARAMETER STUDY OF B_z LINER IMPLOSIONS FOR POSSIBLE CTR APPLICATIONS

William C. Condit

January, 1974

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A PARAMETER STUDY OF B_z LINER IMPLOSIONS FOR POSSIBLE CTR APPLICATIONS

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ABSTRACT

Results of a preliminary parameter study of near-breakeven B_z liner implosions are presented. The calculations were done using magnetic fields of up to 400 kg and liner radii in the range 45-90 cm, with plasma preheats in the 100-200 eV range. Breakeven is found to occur near 100 MJ/meter input to the system for copper liners with the 400 kilogauss drive field. For lighter metals, the energy requirement is roughly proportional to $1/\rho$. For lower drive pressure the yield is only slightly decreased.

The results, though reasonably complete in their coverage of the different parameters, are only preliminary. Suggestions for further studies are made.

I. INTRODUCTION

Following the resurgence of interest in liner implosion schemes for plasma compression (see References 1-3), we have undertaken a survey of the field to identify and evaluate new ideas for controlled thermonuclear fusion. We have started by doing calculations on the cylindrical implosion geometry proposed for the original "LINUS" concept. A sketch of the geometry is given in Fig. 1: a thin metal cylinder is imploded by magnetic field pressure, compressing a preheated D-T plasma to burn conditions. Preheat to $T_0 =$

50-100 eV minimum is required to allow temperatures in the vicinity of 10 keV to be achieved with reasonable compression ratios.

Magnetic insulation is necessary in such systems to reduce plasma heat losses — this can take the form of a gap filled with field, as in Fig. 1, or a field throughout the plasma. The latter is less desirable, since fields strong enough to inhibit heat conduction tend to reduce the plasma density at peak compressions. Figures 2 and 3 show this effect: in the example chosen, a 2.5 kg field throughout the plasma is only moderately successful in reducing heat losses, yet contributes significantly to the total pressure at peak compression. The performance of the "no-gap" problem is obviously inferior to the "with-gap" problem. We have therefore decided to do all our calculations with a standard gap of 1/8 the bulk plasma radius, having a density of 1/10 to 1/100 of the bulk plasma density.

In the limit of final plasma pressure $P_f \leq 1$ megabar, the liner will behave almost incompressibly, serving merely as a means of energy transfer between the initial magnetic field and the compressed plasma⁴. Looked at this way, a given final liner-plasma state can be reached from a large number of initial states, differing in P_0 , T_0 driving pressure P_D , liner radius, liner thickness, and drive-gap volume. To the extent that radial heat loss is negligible and implosion efficiency is similar for these different initial states, we should be able to estimate the energy required for a given thermonuclear yield or yield ratio from the plasma energy in the final state alone. The energy per unit length $dE/d\ell$, can be written:

$$(1) \quad dE/d\ell = 3 n_f kT_f (\pi R_1^2)$$

where n_f , T_f , R_1 , the plasma parameters at maximum compression. The yield ratio $Y = E_{out}/E_{in}$ can be written

$$(2) \quad Y = \frac{1}{6} n_f^2 \overline{\sigma v} Q \tau / P_f$$

where we take $Q = 17.6$ MeV/reaction, $n_f = 2 n_{Deut} = 2 n_{Trit}$, and $P_f =$ sum of ion and electron pressures.

The effective burn time, τ , in Eq. (2) can be estimated by noting that $\overline{\sigma v}$ is proportional to T^3 , in the 5 keV - 15 keV range, and then using the adiabatic 2-D expansion laws to note that the burn rate is proportional to R^8 . Since the burn depends so strongly on radius, we can approximate the pressure as constant in the equations of motions during burn -- i.e., $R = R_1 + \frac{1}{2} (P_f / \rho_w \delta) t^2$ for a liner with density ρ_w and thickness $\delta / R_1 \ll 1$. Upon integrating the burn rate over $t = (-\infty, \infty)$ and setting $E_{out} = \dot{E}_{max} \tau$, we get

$$(3) \quad \tau = .93 \sqrt{\frac{R_1 \rho_w \delta}{P_f}}$$

(See Appendix A for the details of calculating the coefficient.) One can check that during the time $\pm \tau/2$ around maximum burn the radius changes by only 11%. For a thick incompressible liner the thickness δ is replaced by $R_1 \ln (R_2/R_1)$ in Eq. (3). R_2 is the outer radius at maximum compression. Letting the logarithm equal 2 (about the maximum allowed by the need to let neutrons escape through the heavy-metal liner), we get

$$(3a) \quad \tau = 1.31 R_1 / \sqrt{P_f / \rho_w}$$

Combining this with Eq. (2) and eliminating R_1 by means of Eq. (1), we find:

$$(4) \quad Y = 1.31 \sqrt{\rho_w (dE/d\ell)/864\pi} \quad Q \overline{\sigma v}/k^2 T_f^2$$

We are implying that $dE/d\ell$ is the total energy per unit length put into the system, but this expression remains valid for less than perfect efficiency if Eq. (2) is taken as the definition of Y , and $dE/d\ell$ represents energy actually getting into the plasma.

This formula is in a very convenient form to estimate energy requirements for any required yield ratio. Note that the equation is separated into a temperature factor and one dependent only on the energy input and liner density. The elimination of the plasma properties from Eq. (4) is an accident of the 2-D geometry — strictly speaking, a very weak dependence on the final plasma radius should be retained by reinstating the logarithm in Eq. (3a). In Appendix A, the case of a 3-D compression is examined, and it is found that Y is proportional to the same temperature factor times $(\rho_w E_{in}/R_1)^{1/2}$, which gives one a way to vary the yield ratio independently of the energy input through the (admittedly weak) radius dependence.

One immediate consequence of Eq. (4) is that liner experiments require large energies. The temperature factor in the equation is plotted in Fig. 4 — it has a broad maximum around 10 keV. Setting $Y = 1$, $\rho_w = 8.929$ gms/cc (copper) converting to MKS units ($Q \overline{\sigma v}/k^2 T^2 = 1.324 \times 10^{-4}$) we get

$$(5) \quad dE/d\ell = 10 \cdot 1 \text{ MJ/meter (Breakeven).}$$

Noting that $dE/d\ell \propto Y^2$, we see that reactors with $Y = 5-10$ will be very sizeable devices. Using lead or mercury instead of copper is probably

advantageous both from an inertial and neutronic point of view: it does not, however, reduce the energy required in a significant way.

To close out this introduction, we should estimate the extent to which non-ideal processes affect the estimates above. Three such processes are heat conduction across the "insulating" gap, plasma diffusion into the gap, and flux diffusion through the liner. As a criterion of the importance of these processes, we will evaluate them for a 40 cm radius plasma, starting out at $n_{\text{plasma}} = 8 \times 10^{15}/\text{cc}$, $n_{\text{gap}} = 4 \times 10^{14}/\text{cc}$, $T_e = T_i = 100 \text{ eV}$, with a copper liner of thickness 1 cm, insulating gap thickness 5 cm, and energy input 10 MJ/meter. The implosion velocity of this system is about $3 \times 10^4 \text{ cm/sec}$, so the dissipation effects will be negligible if their characteristic times are slow compared to the implosion time of about 1.3 millisecond. This represents an extremely slow implosion, and is a worst case.

The heat-loss time for a mass M_p of isothermal plasma conducting through a gap of width w , length ℓ , circumferential area $A = 2\pi R_p \ell$ is:

$$(6) \quad \tau_{\text{HL}} = M_p C_p w / K_{\perp} \quad A = w \rho_p R_p C_p / 2K_{\perp}$$

Now, in the gap there is about 10 kilogauss to balance the bulk plasma pressure, as shown in Fig. 1. Thus the heat conductivity is given by

$$K_{\perp} = \rho_{\text{gap}} C_{p(\text{gap})} (R_{\text{gyro}}^2 / \tau_{ii})$$

in the gap, with $C_{p(\text{gap})} = C_p = 3 \text{ k/m}_1$. Equation 6 thus reduces to

$$(7) \quad \tau_{\text{HL}} = (R_p w / 2R_{\text{gyro}}^2) (\rho_{\text{plasma}} / \rho_{\text{gap}}) \tau_{ii}$$

For the gap conditions listed above, $\tau_{11} = 5$ microseconds, and $R_{\text{gyro}} = .287$ cm. This gives $\tau_{\text{HL}} \approx .12$ second, which is completely negligible.

The diffusion of plasma into a strong magnetic field has been discussed by Spitzer⁶ and by Dawson⁷. Spitzer gives $\tau = 2 \times 10^{-13} T_K^{3/2} L^2$, which for a gap $L = 5$ cm, gives $\tau = 6.2$ milliseconds. This may have to be reduced by a factor of two to account for the effect of the magnetic field on the electrical resistivity. Dawson, et al., give

$$(8) \quad \ell^2 = (2.5 \times 10^{-4} n_e \bar{Z} / B^2 \sqrt{T_e}) \tau \text{ (eV-gauss-cgs)}$$

for the resistive diffusion of a plasma into a "vacuum". T_e is in eV and B in gauss.* This should be approximately applicable to our case, with n_e being the bulk plasma density, not the gap density. This gives, with $\ell = 5$, $\tau = 12.5$ millisec. Taking the pessimistic Spitzer case (3 milliseconds), we can see that very slow implosions of the geometry of Fig. 1 may suffer significant diffusion into the gap if they approach 1 millisecond in duration or if thinner gaps are used. It should be noted that the above processes are non-linear -- the colder the gap gets, the higher its diffusivity. Thus the use of linear diffusion theory is only approximate. Also, thermo-electric effects, such as the Nernst effect, have been omitted from this discussion. Tests with the computer runs have, to date, indicated that these effects have only a small influence on calculations for the geometry of Fig. 1, having mainly to do with the thickness of the current sheet between the bulk plasma and the "gap" plasma. The effect on bulk plasma temperature, yield, etc., appears to be small, but will be the subject of a later report.

* If $B^2/8\pi = 2nkT_e$, this is essentially the same as the Spitzer formula.

Finally, flux diffusion through the liner should be considered. Let the coil radius be a and the liner radius b . Then the drive field will diffuse through the liner in a time, in MKS units,

$$(9) \quad \tau = \frac{1}{2} \mu_0 a \delta \sigma \left(1 - \frac{a^2}{b^2} \right)$$

For $a = 45$ cm, $b = 50$ cm, $\delta = 1$ cm, $\sigma = 10^8/2$ (MKS) for copper, $\tau = 26.6$ milliseconds. Thus, except for very thin liners, loss of drive flux will not be a problem. One might also worry about losses of the "insulating" flux into the liner -- this occurs only when the gap field has been compressed above the value of the drive field (prior to that, the diffusion is the other way, and may help). The time for this process is

$$(10) \quad \tau = \mu_0 \sigma w \delta$$

which, for $w = \delta = 1$ cm, is about 6 milliseconds. This, again, will be a problem only for very thin liners and very slow implosions -- in general when the gap has gotten as small as 1 cm, the liner will have thickened to greater than 1 cm, and in any case, the implosion will be nearly completed, making a 5-10 millisecond diffusion process unimportant.

In summary, the dissipative process most likely to affect the implosion we envision is the diffusion of the bulk plasma into the insulating gap.

II. PRELIMINARY CALCULATIONAL RESULTS

Computer runs with the LASNEX code⁵ have been made to check the above theory. The code was modified to handle simple analytic equations of state. Due to the large number of runs, coarse but adequate zoning was used.

Equation (4) and the assumptions which led to it imply several very specific ways in which the yield ratio should vary with initial conditions. For example, initial drive pressure should not matter, so long as one communicates the required $dE/d\ell$ to the liner. For any configuration in which the drive gap is very small compared to the liner radius, this efficiency will be very close to one -- i.e., comparing a system with a 4 cm drive gap at 400 kilogauss with one which had a 1 cm gap at 800 kilogauss, little difference should be observed. A comparison of two such runs is shown in Fig. 5. The results are very similar except for the slower start with the weaker drive. The weaker drive yielded only 18% less energy, for a 300% reduction in drive pressure.

Equation (3) suggests that for thin liners, $Y \propto (\rho_w \delta)^{1/2}$. Equation (3a) suggests that for thick liners, other things being equal, there is an upper limit to liner thickness, beyond which only liner material (i.e., density) matters. These two predictions are examined in Fig. 6. For very thin liners, a fairly weak dependence of Y on δ is observed -- however it is not exactly a square-root dependence. Also, the curve for copper crosses that for aluminum. Examination of the details of these calculations shows that there is a significant amount of magnetic diffusion during the implosion for the very thin cases. Using Eq. (9), we find that L/R is within a factor of two of the implosion time for the smallest δ on Fig. 6 -- note that aluminum has a longer L/R than copper for a given $\rho_w \delta$ product, which is consistent with the fact that it has a higher Y at low $\rho_w \delta$.

For large thicknesses, a plateau is indeed reached, as would be expected from Eq. (3a). The peak value scales as $\sqrt{\rho_w}$ for Cu-Al but varies more rapidly for Al-Li. At very large values of $\rho_w \delta$, the yield ratio starts to decrease again, but rather slowly. This appears to be due in part to the decrease in efficiency of conversion of magnetic energy to kinetic energy for thick liners. There are two reasons for this. First, Eq. (8) assumes that the flux in the liner itself is negligible -- it assumes that the flux diffuses from the annular gap of thickness $b - a$ into the cylinder of radius a , neglecting the volume in between occupied by the liner. For thick liners, the initial diffusion into the liner (which is much more rapid) represents a small but noticeable loss of drive flux. Secondly, for thick liners the outer radius, R_2 , still encloses a significant fraction of the coil area, at the point of maximum compression. Since we have approximately (Drive Energy) $\propto 1/(R_{\text{coil}}^2 - R_2^2)$ in the drive gap, the effect of not letting $R_2 \rightarrow 0$ is to hinder the efficiency of conversion of drive energy to kinetic energy. Both of these effects are worse for lithium and aluminum than for copper, consistent with the fact that the lighter metals reach their peak value of Y at lower values of $\rho_w \delta$. A third possible reason for degradation of yield for very thick liners is plasma diffusion into the gap. At $\rho_w \delta = 10 \text{ gms/cm}^2$, the implosions shown on Fig. 6 take about 600 microseconds. Plasma diffusion in such a case would be expected to be essentially negligible, and such seems to be the case. It would be the same for all liner materials at given $\rho_w \delta$, which is not the case.

Plots of yield ratio versus preheat temperature and initial fill density are shown in Figs. 7 and 8. There are optimum values of Y on both

curves. We would expect that, given $dE/d\ell$, the implosion would proceed until $T_p = (dE/d\ell)/\pi R_0^2 (3n_0 k)$, at which time all the energy would be in the plasma, independent of the preheat temperature. We would thus, on the basis of Eq. (4), predict that there would be an optimum density for given implosion geometry and $dE/d\ell$, namely, that density which would result in a final temperature (12 keV) of maximum $\overline{Q}/k^2 T^2$. More generally, given $dE/d\ell$, there is an optimum mass/unit length leading to this temperature. This is seen to be approximately the case in Fig. 7. The optimum in preheat temperature can be explained by the behavior of the flux in the gap: $\phi = \pi B_0 (2R_{p0} w_0)$ is roughly conserved, as indicated in Eq. (10), and since magnetic energy rises more rapidly than plasma energy during the implosion, the gap tends to soak up an increasing amount of the implosion energy as the compression ratio increases. Thus for very low preheats the implosion energy is essentially all converted to magnetic energy before the adiabatic increase of plasma temperature allows burn to start. Additionally, a sizeable amount of energy is lost to liner internal energy for large compression ratios (low preheats). A third problem with very low preheat is that the heat-loss times may become important (see Introduction). These effects reduce the real $dE/d\ell$ for Eq. (4).

Excessive preheat causes the implosion to lead to temperatures above the optimum if the DT mass/unit length is lower than the optimum. This leads to the drop-off for high T_0 shown in Figs. 8 and 9. However, the peak Y is achieved at $T_I = 20$ keV in Fig. 9, instead of 12 keV. The details of these runs show that the higher preheats lead to better coupling into the plasma, partially offsetting the $\overline{Q}/k^2 T^2$ effect by increasing $dE/d\ell$.

For mass values greater than the optimum, increasing the preheat should always make the calculated yield ratio increase monotonically. This is shown by the high-density curve in Fig. 8.

As a final test of the validity of the lossless incompressible predictions, we kept $dE/d\ell$, liner mass/unit length, DT mass/unit length, drive pressure, and preheat constant, while varying the initial radius. This was done for both aluminum and copper liners. The results are shown in Fig. 11. Again, there is an optimum way to distribute the mass, contrary to the predictions of Eq. (4). The drop-off at low radii is caused by inefficiency of the conversion of drive energy to kinetic: to get the given $dE/d\ell$ at 45 cm takes roughly twice the drive gap as at 90 cm, making the gap a larger fraction of the total volume ($\eta = (A_f - A_i)/A_f$, where A_i , A_f are the initial and final values of the drive gap cross-sectional area). For the 45 cm liner radius in Fig. 10, the initial gap area was 31% of the area inside the coil, giving an upper limit of 69% conversion efficiency to kinetic energy. The actual efficiency was about 65%. For the 90 cm radius, the efficiency of this part of the process was over 90%. The decrease in yield for large initial radii comes from overheating -- the mass used in the calculations for Fig. 10 is below the optimum for the $dE/d\ell$ used, and the larger the radius, the higher the temperature attained (due both to the increased efficiency of coupling to the plasma, and to the decreased plasma heat losses for larger dimensions). Presumably this drop-off would disappear for larger values of the mass/unit length, as in Fig. 8.

III. CONCLUSIONS AND SUGGESTIONS FOR FURTHER WORK

The central theoretical result of this paper is Eq. (4), which gives an expression for the yield ratio in terms of the wall density, the energy per unit length in the plasma, and a temperature factor with an optimum at 12 keV. This formula represents what should be a very close (20%-30%) calculation of the behavior of cylindrically compressed plasma system with negligible heat and flux loss. The calculational results in Part II show fairly good agreement with the $\sqrt{\rho_w dE/d\ell}$ behavior predicted in Eq. (4), with $dE/d\ell$ being taken now as the total input to the system. The optimum temperature in various parameter variations is sometimes 12 keV (as in Fig. 7) and sometimes higher (Fig. 9). The deviations in behavior from Eq. (4) can, in all cases to date, be traced to the fact that the efficiency of coupling of input energy to plasma energy is a function of the parameters being changed. Thus, in Fig. 9, the input $dE/d\ell$ is constant, but the actual $dE/d\ell$ getting into the plasma increases with preheat temperature, offsetting the $\sigma v/k^2 T^2$ dropoff up to about 20 keV. The result is that Eq. (4), coupled with the adiabatic gas laws, offers a good framework to discuss liner implosions with, but does not provide the final word.

The predicted breakeven level for copper liners is 10 MJ/meter. The actual level found was 90 MJ/meter (Fig. 6). This discrepancy is not as puzzling as one might think. The efficiency for Fig. 6 runs around 40%. Thus of the input 90 MJ/meter, only about 36 MJ/meter gets into the plasma. Using Eq. (4) with this value of $dE/d\ell$, we predict an output of 68 MJ/meter, which is within 25% of the observed output.

The coupling efficiencies ranged from 20%-60%, with most being in the 40% range. The chief sources of inefficiency were the magnetic energy left in the drive gap and that coupled into the insulating gap (see Fig. 1). For copper liners at pressures up to 2 megabars, the liner internal energy never exceeded 20% of the total system input. Thus liner compression is not at present a serious problem.

Suggested work for the future includes

- A. Attempt to increase the efficiency by making the insulating gap smaller. Attention must be paid to the loss processes discussed in Part I.
- B. Consider the implications of Eq. (4) for a CTR reactor system. For $Y = 10$, $dE/dl = 9000$ MJ/meter would be required by scaling up the calculations of Part II. Even assuming improvements in efficiency 2000-4000 MJ/meter would seem necessary for a reactor.
- C. Give considerable attention to the coil strength problem. Since, apart from efficiency, high drive pressures have no advantage over moderate drive pressures, there is some freedom here (see Fig. 5). However, moderate drive pressures lead to very large system radii and large convergence ratios, which require greater care in liner preparation than economics may allow. The transient stresses in the coil are similar to those in a pressure vessel containing an explosion — scaling laws indicate that containing 2×10^3 MJ/meter will require about 10^6 lbs of structural steel/meter⁴, which could be a significant part of the cost of a reactor.

D. Consideration must be given to end losses. Most of the runs for which computations were made involved pressures in the vicinity of 1 megabar -- the Lawson criterion is approximately given by $P\tau = 2$ megabar-microseconds for a breakeven device of 10-20 microseconds for a reactor are implied. Such a device will be very long without some type of end-stoppering.

Finally, two operational suggestions need to be made. First, some additional attention needs to be paid to zoning -- comparison runs on 2 of the approximately 40 runs made indicated less than 5% variation in all significant quantities except the electron temperatures in the insulating gap, and the liner internal energy. These quantities were unimportant in the work to date, but may be more significant in future studies. Secondly, as a matter mostly of academic interest, it would be worth repeating some of the calculations with arrangements made to get constant $dE/d\ell$ into the plasma, rather than into the system as a whole, to check on the validity of the analytical results of Part I. As stated in the text, Eq. (4) should be very close to the computer calculations, being somewhat (10%-15%) pessimistic in the calculation of the burn time, due to the assumption $P = P_{\max}$ during liner turnaround.

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APPENDIX A

Yield Ratios and Lawson Criterion:

Calculational Details and Applications to 3-D Compressions

For 3-D compressions with thin liners such as the "Flying Cusp", one finds a similar effective containment time to that given in Eq. (3), namely, $\tau = K \sqrt{R_1 \delta \rho_w / P}$. The constant K is evaluated by assuming $R = R_1 + \frac{1}{2} (P/\rho_w \delta) t^2$ around turnaround, with P being a constant ($P = P_{\max}$). In 3-D implosions, the burn rate varies as R^{-12} ($n^2 \propto R^{-6}$, $\overline{\sigma v} \propto R^{-6}$ in the 5-15 keV range. Since the pressure varies as R^{-5} , the assumption of constant pressure should be a good one, although a little pessimistic.

We set $E_{\text{out}} \equiv \dot{E}_m \tau = \int_{-\infty}^{\infty} \dot{E} dt$, where the thermonuclear power rate $\dot{E} = n^2 \overline{\sigma v} / 4$, n being the electron or total ion density (D + T). Then, as above, $\dot{E} = K/R^{12}$, and we get

$$(A-1) \quad \dot{E}_m \tau = \frac{K \tau}{R_1^{12}} = \int_{-\infty}^{\infty} \frac{K}{(R_1)^{12} \left(1 + \frac{1}{2} (P/\rho_w \delta R_1) t^2\right)^{12}} dt$$

Doing the integral (reduced to $2\sqrt{2} \int_0^{\pi/2} \cos^{22} \theta d\theta$) we get

$$(A-2) \quad \tau = .747 \sqrt{R_1 \rho_w \delta / P}$$

For a thick liner, the dependence on outer radius will be even weaker. For incompressible liners, which will have the strongest dependence in any case, $r^2 v_r = w(t)$ is a constant through the liner. The equation of motion near turnaround then can be linearized to

$$(A-3) \quad \left[\frac{1}{R_1} - \frac{1}{R_2} \right] \frac{dw}{dt} = \Delta P / \rho_w$$

so that if $R_2 \gg R_1$, $v_r = (\Delta P / \rho_w R_1) t$, or $R = R_1 + \frac{1}{2} (P / \rho_w R_1) t^2$,

Thus δ should be replaced by R_1 in (A-2) for thick liners.

For the cylindrical case, the same steps can be carried out.

LINUS INITIAL CONDITIONS

FIGURE 1

Note: The no-gap run had 20% less density, plus steep ∇T_i , $\therefore$ About 8 times less energy produced.

Note Strong Heat Loss For $B_z = 2500$ Gauss. Weaker B_z made this worse. Stronger B_z stopped implosion unless concentrated in a gap.

Figure 5: R and TI History for Two Runs
at $dE/dL = 180 \text{ MJ/M}$ (Drive = 400, 800 Kg)

Figure 6: Yield Ratio

vs.

Shell Thickness

Figure 7: Yield Ratio and
Max. Ion Temp. Vs.
Fill Density

Fig. 8: Yield Vs. T_0

Yield
Ratio

En. Input:
170-180
MJ/M

Figure 9: Ion Temp. and
Yield Ratio Vs.
Preheat Temp.

TI
At
 $R_{min.}$
(Kev)

Fig. 10: Thermonuc. Yield Vs. Liner Radius

(Constant Mass
and Input E)

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