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LIMIT OF NONADIABATIC ENERGY IN 2XIIB

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November 7, 1974

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Prepared for U. S. Atomic Energy Commission under contract no. W-7405-Eng-48

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LIMIT OF NONADIABATIC ENERGY IN 2XIIB

ABSTRACT

Limits for adiabatic behavior in the 2XIIB magnet have been studied. Numerical calculations are made to test an analytic expression for the adiabatic limit. The presence of finite- β plasma is found to reduce the limit to where nonadiabatic losses may be significant in 2XIIB.

INTRODUCTION

The limits for adiabatic behavior of ions in the 2XIIB experiment have been examined. In the vacuum field these limits lie well above the planned injection energies. The steep, radial magnetic-field gradients present in a high- β plasma, combined with the enhanced mirror ratio, lower the adiabatic limits to the point where nonadiabatic losses may be important in 2XIIB for plasma conditions near the β limit of hydro-magnetic instability.

NUMERICAL CALCULATIONS

Although an analytical expression for the adiabatic limit¹ was available, it was felt that the applicability of this expression to 2XIIB should be tested by numerical calculations. We have followed Foote's approach in tracing particle orbits. Experience has shown that nonadiabatic behavior occurs first for particles whose velocity vectors lie near the loss cone. Using the Tibro code,²⁻⁴ the orbits of particles of various energies, whose velocity vectors were initially just inside the loss cone, were followed. This process was repeated for several different values of the initial guiding-center radius in the central plane.

The field values were those of the Mafco⁵ representation of 2XIIB at design level, i. e., 3-kG dc field plus approximately 6-kG pulsed field giving a central-field minimum value of 9.12-kG. The mirror ratio as a function of central-plane radius is shown in Fig. 1. Recent measurements have shown that the actual field differs somewhat in shape from the Mafco-derived fields: the actual field is about 3% higher in the mirrors and the central minimum 2% lower than the computer field. No attempt was made to obtain a better fit of the Mafco code to the actual field, since it was felt that the effects of finite β would mask these differences.

Tibro is used to examine the behavior of the normalized magnetic moment,

$$\mu = \frac{W_{\perp}}{WB} = \frac{\sin^2 \theta}{B},$$

Fig. 1. Mirror ratio vs central-plane radius. $R = \frac{B_{\max}}{B_{\min}}$, $R^* = \frac{B_{\max}}{B_{z=0}}$.

where θ is the instantaneous angle the particle makes with the magnetic field and B is field strength at the particle. Typical behavior of μ with time is shown in Fig. 2. In this field, μ varies at twice the cyclotron frequency with an amplitude that is a maximum in the low field near the center and a minimum in the mirrors. To observe long-term changes in μ , the code outputs $\bar{\mu}$ for every 500 orbit points. In all but the most nonadiabatic cases, $\bar{\mu}$ exhibits a random walk behavior. If we observe the maximum fractional deviation in $\bar{\mu}$ as a function of time, we find

$$\frac{\Delta\bar{\mu}}{\bar{\mu}_0} \approx Kt^{1/2},$$

where $\Delta\bar{\mu} = \bar{\mu}_{\text{Max}} - \bar{\mu}_{\text{Min}}$ is the difference between the extreme values of $\bar{\mu}$ from the start of orbit tracing to time t , and $\bar{\mu}_0$ is the initial value of $\bar{\mu}$.

From plots of $\Delta\bar{\mu}/\bar{\mu}$ vs time for the individual computer runs, we obtain the best $t^{1/2}$ fits, and from them the value of $\Delta\bar{\mu}/\bar{\mu}$ at $30 \mu\text{s}$ (to compare with Foote's results). These values of $\Delta\bar{\mu}/\bar{\mu}$ are plotted vs velocity in Fig. 3. The range on the individual points indicates how well the data, from which each point was obtained, was fitted by a $t^{1/2}$ relationship. The behavior is quite similar to that reported by Foote, where, as the velocity is reduced, there is a rapid drop in $\Delta\bar{\mu}/\bar{\mu}$ to about the 1% level where it then remains constant. This is apparently the noise level in the numerical calculations. If, following Foote, we take the adiabatic boundary as being when the steep-sloped portion of the $\Delta\bar{\mu}/\bar{\mu}$ vs velocity curve intercepts the $1.5 \pm 0.5\%$ level (for the $30\text{-}\mu\text{s}$ time), we obtain Fig. 4, a plot of adiabatic limit for deuterons as a function of guiding-center radius in the central plane.

ANALYTICAL CALCULATIONS

Foote has given an analytical expression for the maximum energy (in keV) of adiabatic behavior in a mirror machine:

$$W_{\text{max}} = (8.5 \times 10^{-4}) \frac{z^2}{A} \frac{(B_M L)^2}{R(R-1)^3}, \quad (1)$$

where

$$\frac{1}{L^2} = \frac{1}{1_z^2} + \frac{1}{1_r^2 (R-1)},$$

and where B_M is the maximum magnetic field (in kilogauss) acting on the confined particle ($B_M = 1/\bar{\mu}$). The parameters 1_z and 1_r are the distances, in centimeters, from the center to the B_M constant-magnetic-field contour, along and perpendicular

Fig. 2. Behavior of μ with time.

Fig. 3. Adiabatic limit for deuterons with orbits centered (a) on or near $r = 0$, (b) at $r = 5$ cm, (c) at $r = 10$ cm, and (d) at $r = 15$ cm.

Fig. 4. Adiabatic limit vs guiding-center-central-plane radius for deuterium in 2XIIB.

to the magnetic axis. Values of l_z and l_r are obtained from a parabolic fit of the field in the central region. The effective mirror ratio R is B_M divided by the magnetic field at the center of the well; z^2/A , the charge-squared-to-mass ratio, is 1 for protons and 1/2 for deuterons.

The results of the numerical calculation of adiabatic limit as a function of central-plane, guiding-center radius were compared with the values obtained from Eq. (1). Table 1 lists the values used in Eq. (1) and compares the results of the two types of calculations. We see that for the smaller values of r , the numerical values of adiabatic limit are higher than the analytical; whereas for larger values of r , the reverse is true.

Table I. Vacuum-field adiabatic limit for deuterium.

r	B_m (kG)	R	l_z	l_r	L	Eq. (1) $W_{\max}$ (keV)	Numerical $W_{\max}$ (keV)	$W_{\max}$ numerical
								$W_{\max}$ analytical
0	17.0	1.86	76	53	41.3	177	208-239	1.18-1.35
5	16.6	1.82	74	52	39.7	184	228-244	1.24-1.33
10	15.3	1.68	68	47	33.7	214	175-189	0.82-0.88
15	15.6	1.71	69	48	37.5	238	173-189	0.73-0.79

There appears to be an abrupt change between $r = 5$ and $r = 10$. This may somehow be related to the fact that for the smaller values of guiding-center radius, the orbits link the axis, whereas for the larger value they do not. Another possible explanation is that the change is due to the greater curvature of the lines of force at larger r .

FINITE- β ADIABATIC LIMIT

In some cases early in the injection cycle, 2XII operated near the boundary for magnetostatic equilibria.^{6,7} Under such high- β ($=4\pi p_{\perp}/B_{\text{vac}}^2$) conditions, there is a lowering of the central field by the plasma diamagnetism and a consequent increase in the effective mirror ratio and the radial field gradient. It is possible that 2XIIB may be maintained by beam injection at these same high- β conditions, and it is necessary to make some estimate of the adiabatic limits under such conditions.

Since we have shown that for the vacuum-field case, the numerical and analytical results agree fairly well, we assume that Eq. (1) can be used to estimate the adiabatic limit in the finite- β case when we use the proper value for mirror ratio and scale length. In doing this we are following the same method that has been used to estimate the adiabatic limit for FERF.⁸

For a needle-shaped plasma in 2XIIB, $R_w \approx 3$ at the mirror stability boundary. The central field is thus depressed to about 6 kG from its vacuum value of 9 kG, yet at the plasma surface, the field must have approximately its vacuum value. Thus in the distance of a plasma radius the field may vary by a large amount, and, if our assumptions are correct, it is likely that the adiabatic behavior is governed by this radial gradient. There is some question whether radial gradients, especially when created by imbedded currents ($\nabla \times H \neq 0$), are as important as we now assume; but until these questions are resolved, we are forced to live with this approach, pessimistic as it may be.

For a long skinny plasma, the fractional change in the magnetic field in the central region of the plasma, because of plasma diamagnetism, is given by

$$B/B_{\text{vac}} \approx \sqrt{1 - \beta} \quad (2)$$

We use this relationship to obtain values of R , L , and β , by means of Eq. (1), W_m as a function of β . To obtain L we assume that the plasma-depressed field has a parabolic shape to the peak of the mirrors in the z direction, and to the plasma surface in the radial direction where it merges with the vacuum field. Using the prescription described earlier for obtaining 1_r and 1_z , we calculate L for two possible values of plasma radius. A plot of W_m vs β for the two values of r is shown in Fig. 5. The end points of the curves correspond to the mirror stability boundary. We see that the combination of higher mirror ratio and smaller scale length greatly reduce the adiabatic limit from the $\beta = 0$ vacuum-field value. For a 10-cm-radius plasma, β would have to be below 0.27 for 20 keV deuterons to behave adiabatically.

If the average perpendicular energy of the plasma is 10 keV, this then puts an upper limit on density of 5×10^{13} . If the density rises above this level, 20-keV particles will begin to behave nonadiabatically. This does not necessarily mean that all 20-keV particles will be lost, but only that they will have a different angular distribution from the adiabatic particles held by the maximum mirrors. The higher-energy particles will be contained by mirrors lower than the maximum value. As the angular distribution tries to spread towards the loss cone, the lower-angle particles will be lost nonadiabatically.

If it should turn out to be true that the radial gradients of the field are not important, then it has been shown⁹ that

$$W_m \sim \frac{z^2}{A} \frac{(B_M 1_z)^2}{R(R-1)^2};$$

since $B_M 1_z$ remains constant, the reduction in the adiabatic limit by a finite- β plasma will be

$$f = \frac{R_v(R_v - 1)^2}{R(R - 1)^2} = 0.167$$

Fig. 5. Adiabatic limit vs β in 2XIIB.

for the case of 2XIIB at the magnetostatic equilibria limit. Applying this value to the numerical values of $W_{M \text{ vac}}$ gives $W_{\text{Max}} \approx 37 \text{ keV}$ for deuterium.

CONCLUSIONS

We have examined the limits for adiabatic behavior in the 2XIIB magnet. Numerical calculations agree with the results of Foote's equation within about 30%, in general giving higher values for particles with guiding centers near the axis. The vacuum-field adiabatic limit for deuterium is in the neighborhood of 200 keV.

The adiabatic limit is reduced by the presence of finite- β plasma. This occurs both because of the enhanced mirror ratio and the decrease in magnetic scale length. If adiabaticity depends upon radial, magnetic-field gradients as indicated in Eq. (1), then in a plasma of 10-cm radius, 20 keV deuterium ions will be nonadiabatic for plasma β 's above 0.27.

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