MASTER

UNCOLLIDED FLUX FROM FINITE CIRCULAR AND RECTANGULAR CYLINDERS

December 1958

CONTRACT AT-11-1-GEN-14

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(UC-34, Physics and Mathematics
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AND RECTANGULAR CYLINDERS

D. C. Anderson and B. D. O'Reilly

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Technical Services, U. S. Department of Commerce
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ABSTRACT

Expressions are derived for the uncollided flux from finite cylindrical sources with circular and rectangular cross sections. The source density is assumed to be completely arbitrary in the rectangular case and arbitrary to within independence of the azimuth in the circular case. The circular cylinder is assumed surrounded by an arbitrary number of concentric cylindrical shields and by plane circular shields above (or below) its upper (or lower) face. The rectangular source is treated with plane laminar shields parallel to one of its faces and with coaxial cylindrical shields. For both kinds of source geometry the point of observation is taken exterior to the source but without further restriction. The final expressions for the fluxes are in the form of volume integrals; they have not been reduced.

INTRODUCTION

Most of the source geometries commonly applied to practical shielding calculations are restricted to one dimension or else field points must be taken only along the axes of symmetry. If shield contouring is desired or, in the case of radiation heating, axial and azimuthal variations of the energy deposition must be obtained, one-dimensional and symmetry-restricted descriptions of the flux emanating from a volume source distribution become inadequate. It is hoped that the formal solutions presented herein for fluxes from the often encountered cylindrical geometries will prove interesting to those working in shield design and core structural component design. They can serve as the basis for the numerical evaluation of radiation fluxes by mechanical quadrature.

THE CIRCULAR CYLINDER

We wish to find a formal solution for the uncollided flux from a right circular cylinder within which there is a source distribution $S(r,z)$ particles/unit volume-unit time. Note that the source distribution is independent of the azimuthal angle.

Field Point in the Region $r > R_0$, $|z| < h/2$

We begin by seeking the solution for the flux at a field point P_f due to a point isotropic source at P_s (see Fig. 1). The n^{th} concentric shield has inner radius R_n and attenuation coefficient μ_n . The Cartesian coordinates of the source point P_s and the observation point P_f are (x',y',z') and (x,y,z) , respectively. P_n ($n < N$) denotes the point (x_n, y_n, z_n) on the inner surface of the n^{th} region at which that surface is pierced by the line $P_s P_f$.

The immediately succeeding derivation is restricted by the conditions $r > R_0$, $|z| < h/2$.

Figure 1

Note that the direction cosines of $P_s P_f$ are given by

$$\left. \begin{aligned} \alpha &= \frac{x-x'}{\rho} \\ \beta &= \frac{y-y'}{\rho} \\ \gamma &= \frac{z-z'}{\rho} \end{aligned} \right\} \quad \rho \equiv \left[(x-x')^2 + (y-y')^2 + (z-z')^2 \right]^{1/2} \quad (1)$$

The coordinates of the points P_n are determined by the following conditions:

$$\left. \begin{aligned} x_n^2 + y_n^2 &= R_n^2 \\ \frac{x_n - x'}{\alpha} &= \frac{y_n - y'}{\beta} = \frac{z_n - z'}{\gamma} \end{aligned} \right\} \quad (2)$$

In setting down Eqs. (1) and (2), we have taken advantage of the uniformity of our solution in the azimuthal angle and have chosen to make an observation at a point P_f in the x-z plane.

$$\left. \begin{aligned} x_n &= \frac{\alpha^2}{\beta^2 + \alpha^2} \left\{ \frac{\beta}{\alpha} (\frac{\beta}{\alpha} x' - y') + \sqrt{R_n^2 + \left[\left(\frac{\beta}{\alpha} \right)^2 - 1 \right] \left(\frac{\beta}{\alpha} x' - y' \right)^2} \right\} \\ y_n - y' &= \frac{\beta}{\alpha} (x_n - x') \\ z_n - z' &= \frac{\gamma}{\alpha} (x_n - x') \end{aligned} \right\} \quad (3)$$

Let us introduce the transformation from Cartesian to cylindrical polar coordinates:

$$\left. \begin{aligned} x &= r & x' &= r' \cos \phi' \\ y &= 0 & y' &= r' \sin \phi' \\ z &= z & z' &= z' \end{aligned} \right\} \quad (4)$$

Under this transformation of coordinates, Eq. (1) becomes

$$\left. \begin{aligned} \alpha(r', \phi', z', r, z) &= \frac{r - r' \cos \phi'}{\rho} \\ \beta(r', \phi', z', r, z) &= \frac{-r' \sin \phi'}{\rho} \\ \gamma(r', \phi', z', r, z) &= \frac{z - z'}{\rho} \\ \rho &\equiv \rho(r', \phi', z', r, z) = \left[(r - r' \cos \phi')^2 + r'^2 \sin^2 \phi' + (z - z')^2 \right]^{1/2} \end{aligned} \right\} \quad (5)$$

With the help of Eq. (3) we find that the length of the line $P_s P_n$ is given by

$$\rho_n = \frac{x_n - x'}{\alpha} \quad (6)$$

The number of mean-free-paths from source to field point is given by

$$\left. \begin{aligned} \lambda_{sf} &= \mu_0 \rho_0 + \sum_{n=1}^N \mu_n (\rho_n - \rho_{n-1}) = \sum_{n=0}^N (\mu_n - \mu_{n+1}) \rho_n \\ \mu_{N+1} &\equiv 0 \\ \rho_N &\equiv \rho \end{aligned} \right\} \quad (7)$$

From the preceding derivations we see that the flux at $P_N \equiv (r, z)$ such that $r > R_0$, $|z| < h/2$ is

$$\Phi(r, z) = \int_0^{2\pi} d\phi' \int_0^{R_0} r' dr' \int_{-h/2}^{+h/2} dz' \frac{e^{-\sum_{n=0}^N (\mu_n - \mu_{n+1}) \rho_n(r', \phi', z', r, z)}}{4\pi \rho^2(r', \phi', z', r, z)} \beta(r', \phi'). \quad (8)$$

B. Field Point in the Region $r < R_0, |z| > h/2$

We next seek a solution for $\Phi(r, z)$ which is valid under the restrictions $r < R_0, |z| > h/2$. Above (or below) the cylinder plane slab laminae are situated as illustrated in Fig. 2. The cylinder has its upper surface at $z = h/2$ and has attenuation coefficient $m_0 (= \mu_0)$. The lower face of the ℓ^{th} slab has elevation $z = h_\ell$ and attenuation coefficient m_ℓ . P_s denotes the point (x', y', z') , and $P_f \equiv P_N$ denotes (x, y, z) . P_ℓ denotes (x_ℓ, y_ℓ, z_ℓ) on the lower surface of the ℓ^{th} region at which that surface is pierced by the line $P_s P_f$.

The direction cosines of $P_s P_f$ are given by Eq. (1) with the understanding that $P_s P_f$ is as shown in Fig. 2.

The coordinates of the points P_ℓ are determined by

$$\left. \begin{aligned} z_\ell &= h_\ell \\ \frac{x_\ell - x'}{\alpha} &= \frac{y_\ell - y'}{\beta} = \frac{z_\ell - z'}{\gamma} \end{aligned} \right\}, \quad (9)$$

the solutions to which are

$$\left. \begin{aligned} x_\ell - x' &= \frac{\alpha}{\gamma} (h_\ell - z') \\ y_\ell - y' &= \frac{\beta}{\gamma} (h_\ell - z') \\ z_\ell - z' &= (h_\ell - z') \end{aligned} \right\}. \quad (10)$$

The length of $P_s P_\ell$ is seen to be

$$\rho_\ell = \frac{h_\ell - z'}{\gamma}. \quad (11)$$

Figure 2

In this case the number of mean-free-paths from source to field point is

$$\lambda_{sf} = m_0 \rho_0 + \sum_{\ell=1}^L m_{\ell} (\rho_{\ell} - \rho_{\ell-1}) = \sum_{\ell=0}^L (m_{\ell} - m_{\ell+1}) \rho_{\ell}$$

$$\mu_{L+1} \equiv 0 \quad (12)$$

$$\rho_L \equiv \rho = \left[(r - r' \cos \phi')^2 + (r' \sin \phi')^2 + (z - z')^2 \right]^{1/2}$$

It is seen that the flux at $P_L = (r, z)$ such that $r < R_0$, $|z| > h/2$ is

$$\Phi(r, z) = \int_0^{2\pi} d\phi' \int_0^{R_0} r' dr' \int_{-h/2}^{+h/2} dz' \frac{e^{-\sum_{\ell=0}^L (m_{\ell} - m_{\ell+1}) \rho_{\ell}(r', \phi', z', r, z)}}{4\pi \rho^2(r', \phi', z', r, z)} S(r', z'). \quad (13)$$

Field Point In the Region $r > R_0$, $|z| > h/2$

We now consider the derivation of a formula for $\Phi(r, z)$ which applies in the region $r > R_0$, $|z| > h/2$. To do this we must first determine the flux resulting from a point source located at any point within the cylinder.

We must distinguish, however, between points P_s for which line $P_s P_f$ pierces the end of the cylinder and those for which line $P_s P_f$ pierces the side of the cylinder. Consider the conical surface swept out by a line fixed to point P_f and moved along the periphery of the upper (or lower) surface of the cylinder. Source points, common to the inside of the cone and the cylinder, connect with point P_f through the upper surface of the cylinder. Source points outside of the cone connect with point P_f through the side of the cylinder (see Fig. 3).

Note that the cone intersects the plane $z=z'$ in a circle with center at c . To locate c , solve for the intersection of the line $\vec{\mathcal{L}}_c$ with the plane $z=z'$. The direction cosines of $\vec{\mathcal{L}}_c$, which connect P_f with the center of the upper face of the cylinder, are given by

Figure 3

$$\left. \begin{aligned} a &= \frac{x}{\mathcal{L}_c} \\ b &= 0 \\ c &= \frac{z}{\mathcal{L}_c} \\ \mathcal{L}_c &= \left[x^2 + (z-h/2)^2 \right]^{1/2} \end{aligned} \right\} \quad (14)$$

The coordinates of c are determined by

$$\left. \begin{aligned} z_c &= z' , \quad y_c = 0 \\ \frac{x-x_c}{a} &= \frac{z-z_c}{c} \end{aligned} \right\} \quad (15)$$

or

$$\left. \begin{aligned} z_c &= z' \\ y_c &= 0 \\ x_c &= x - a/c(z-z') \end{aligned} \right\} \quad (16)$$

The radius, R_c , of the circle centered about c can be obtained by solving for the coordinates of a point c' on the periphery of the circle of intersection. To do this, consider the vector $\vec{\mathcal{L}}_c$, connecting point P_f with $(R_o, 0, h/2)$. Its direction cosines are given by

$$\left. \begin{aligned} a' &= \frac{x-R_o}{\mathcal{L}_{c'}} \\ b' &= 0 \\ c' &= \frac{z-h/2}{\mathcal{L}_{c'}} \end{aligned} \right\} \quad \mathcal{L}_{c'} = \sqrt{(x-R_o)^2 + (z-h/2)^2} \quad (17)$$

The coordinates of c' are determined by

$$\left. \begin{aligned} z_{c'} &= z' , y_{c'} = 0 \\ \frac{x - x_{c'}}{a'} &= \frac{z - z_{c'}}{c'} \end{aligned} \right\} \quad (18)$$

or

$$\left. \begin{aligned} x_{c'} &= x - a'/c' (z - z') \\ y_{c'} &= 0 , z_{c'} = z' \end{aligned} \right\} \quad (19)$$

From Eqs. (16) and (19) it follows that

$$R_c = |x_c - x_{c'}| = (z - z') |a/c - a'/c'| \quad (20)$$

From Fig. 3 it can be seen that v is the distance between c and $(0,0,z')$.

$$v = |x - a/c (z - z')| = x_{c'} \quad (21)$$

To find the values of r' which bound the points in the plane $z=z'$ that are common to the cylinder and the cone, we apply the law of cosines to the triangle formed by lines R_c , v , and r' . It is found that

$$r'(\phi', z') = \begin{cases} -v \cos \phi' + \sqrt{R_c^2 - v^2 \sin^2 \phi'} & , x_{c'} > 0 \\ -v \cos \phi' - \sqrt{R_c^2 - v^2 \sin^2 \phi'} & , x_{c'} < 0 \end{cases} \quad (22)$$

$\phi_1(z')$ is determined by the intersection of the circle centered on c and the cross section of the cylinder. With the aid of Fig. 3, we see that the intersection is given by

$$\left. \begin{aligned} r' &= R_0 \\ R_c^2 &= r'^2 + x_c^2 + 2r'x_c \cos \phi_1 \end{aligned} \right\} \quad (23)$$

Solving Eq. (23) for ϕ_1 , we obtain

$$\phi_1(z') = \cos^{-1} \left[\frac{R_c^2 - R_o^2 - x_c^2}{2R_o x_c} \right], \quad 0 < \phi_1(z') < \pi \quad (24)$$

Now note that the elevation of the intersection of the cone with the side of the cylinder away from point P_f is determined by

$$x'_c = -R_o = x - a'/c' (z - \hat{z}), \quad (25)$$

so that

$$\hat{z} = z - c'/a' (x + R_o). \quad (26)$$

Having defined the bounds of integration, we must now seek the form of the point source solution for points within the cone which are common with the cylinder and for points within the cylinder which lie outside the cone.

Let the coordinates of points in the region $r < R_1$, $|z| < h/2$ be denoted by (ξ, η, ζ) . The coordinates in $r > R_1$ are denoted by (x, y, z) .

We now consider field points such that $r > R_1$, $|z| > h/2$. As in the case of $r < R_o$, the slabs above the cylinder are pierced at points $(\xi_\ell, \eta_\ell, \zeta_\ell)$ such that

$$\left. \begin{aligned} \xi_\ell - x' &= \alpha/\gamma (h_\ell - z') \\ \eta_\ell &= \beta/\gamma (h_\ell - z') \\ \zeta_\ell &= z' = h_\ell - z' \end{aligned} \right\} \quad (27)$$

In this case, however, line $P_s P_f$ intersects the cylinder surface $r = R_1$ such that the distance from point P_s to this surface is ρ_1 (see Eq. (6)). The z -coordinate of this point is such that $h_k < z_1 < h_{k+1}$ for some k between 0 and L (see Fig. 3).

Now let

$$S_{\ell} = \sqrt{(\xi_{\ell} - x')^2 + (\eta_{\ell} - y')^2 + (\zeta_{\ell} - z')^2} \quad (28)$$

$$= \sqrt{\left(\frac{1-\gamma^2}{\beta^2}\right) (r' \sin \phi')^2 + (h_{\ell} - z')^2}$$

The number of mean-free-paths from source to field point for radiation penetrating the upper face of the cylinder is

$$\left. \begin{aligned} \lambda_{\text{of}}^{\text{top}} &= m_0 S_0 + \sum_{\ell=1}^k m_{\ell} (S_{\ell} - S_{\ell-1}) + m_{k+1} (\rho_1 - S_k) + \sum_{n=2}^N \mu_n (\rho_n - \rho_{n-1}) \\ &= \sum_{\ell=0}^k (m_{\ell} - m_{\ell+1}) S_{\ell} + (m_{k+1} - \mu_2) \rho_1 + \sum_{n=2}^N (\mu_n - \mu_{n+1}) \rho_n \\ \mu_{N+1} &\equiv 0, \quad m_0 = \mu_0, \quad m_1 = \mu_1 \\ \rho_N &\equiv \rho(r', \phi', z', r, \phi) \end{aligned} \right\} \quad (29)$$

The number of mean-free-paths from source to field point for radiation penetrating the side of the cylinder is, as in Eq. (7),

$$\left. \begin{aligned} \lambda_{\text{sf}}^{\text{side}} &= \sum_{n=0}^N (\mu_n - \mu_{n+1}) \rho_n \\ \mu_{N+1} &\equiv 0 \\ \rho_N &\equiv \rho(r', \phi', z', r, z) \end{aligned} \right\} \quad (30)$$

From the preceding derivations the flux at a field point such that $r > R_0$, $|z| > h/2$ is given by

$$\begin{aligned} \Phi(r, z) = & \int_{-h/2}^{+h/2} dz' \left(\int_{-\phi_1(z')}^{+\phi_1(z')} d\phi' \int_0^{r'(\phi', z')} r' dr' + \int_{\phi_1(z')}^{2\pi - \phi_1(z')} d\phi' \int_0^{R_0} r' dr' \right) \\ & \frac{S(r', z')}{4\pi\rho^2(r', \phi', z', r, z)} e^{-\left[\sum_{k=0}^k (m_k - m_{k+1}) S_k + (m_{k+1} - \mu_2) \rho_1 + \sum_{n=2}^N (\mu_n - \mu_{n+1}) \rho_n \right]} \\ & + \int_{-h/2}^{+h/2} dz' \int_{-\phi_1(z')}^{+\phi_1(z')} d\phi' \left(\int_0^{R_0} - \int_0^{r'(\phi')} \right) r' dr' \frac{S(r', z')}{4\pi\rho^2(r', \phi', z', r, z)} e^{-\sum_{n=0}^N (\mu_n - \mu_{n+1}) \rho_n} \end{aligned} \quad (31)$$

THE RECTANGULAR PRISM

Next we seek an expression for the uncollided flux from a rectangular prism within which the source distribution is denoted by $S(x, y, z)$. In this case the source distribution is a completely arbitrary function of the spatial coordinates within the prism.

Description of the Source Geometry

We now consider simultaneously the cases of plane shield laminae parallel to one face of the prism and of cylindrical shields concentric about an axis of the prism (see Fig. 4). In the case of the plane parallel shields, note that those faces of the prism with normals not parallel to the normals of the shields radiate into an infinite homogeneous medium. Similarly, for the case of cylindrical shields, those faces of the prism with normals parallel to the common axis of the shields radiate into an infinite medium.

We first examine the case in which the field point is exterior to the source and attempt to keep the analysis general enough to include the

Figure 4

following three field point positions: (1) P_f is exterior to the source prism but interior to the projection of the prism taken in the direction normal to the shield laminae; (2) P_f is exterior to the projections of all faces of the prism and opposite an edge; and (3) P_f is exterior to the projections of all faces of the prism and opposite a corner.

Let the coordinates of the field point P_f be labeled by (x, y, z) and those of the source point P_s by (x', y', z') . In the integration over the source it is found convenient first to find the contribution of the $x'-z'$ planes as functions of y' after which the integration over y' is carried out.

Since P_f is a general field point, the uncollided flux arriving at P_f consists of three parts:

- (a) that which leaves the source prism through the plane $y' = y_2$;
- (b) that which leaves the source prism through the plane $x' = x_0$; and
- (c) that which leaves the source prism through the plane $z' = z_0$.

Using the above integration scheme, we will determine the portions of any plane cross section $Y = y'$ which emit direct flux through the prism surfaces defined in (a), (b), and (c). Consider the plane cross section $Y = y'$ shown in Fig. 5. The portions (a), (b), and (c) of Fig. 5 correspond to the respective flux divisions discussed above. Observe that the point P_i is the intersection of the line between P_f and (x_0, y_2, z_0) and the $y = y'$ plane.

The dashed line connecting P_i and (x_i, y', z_1) is a segment of the line of intersection between the plane $Y = y'$ and the plane determined by P_f , E, and F. Similarly the dashed line connecting P_i with (x_3, y', z_i) is a segment of the line of intersection between the plane $Y = y'$ and the plane determined by P_f , E, and H. The dashed line connecting P_i and (x_0, y', z_0) is the line of intersection between the plane $Y = y'$ and the plane determined by P_f , E, and G. Note that, if $x_i > x_3$, there is no contribution to the flux at P_f (resulting from radiation passing through the plane $y' = y_2$). It is still necessary to determine the point (x_i, y', z_i) , however, since the subdivision between portions (b) and (c) must be established. If $x_i > x_3$ and simultaneously $z_i > z_1$, then the problem reverts to the simple case (1) above in which P_f lies in the projection of the prism in the direction of the shield normal. Hence for this case all flux must leave the prism through its $X = x_0$ face.

Figure 5

At this point we solve for the coordinates of P_i . Let the direction cosines of the line EP_f be denoted by a , b , and c :

$$\left. \begin{aligned} a &= \frac{x_o - x}{\rho(EP_f)} \\ b &= \frac{y_2 - y}{\rho(EP_f)} \\ c &= \frac{z_o - z}{\rho(EP_f)} \end{aligned} \right\} \rho(EP_f) \equiv \sqrt{(x_o - x)^2 + (y_2 - y)^2 + (z_o - z)^2} \quad (32)$$

The equation of the line EP_f is

$$\frac{X - x}{a} = \frac{Y - y}{b} = \frac{Z - z}{c} \quad (33)$$

The intersection of EP_f with the plane $y = y'$ occurs at (x_i, y_i, z_i) where

$$\left. \begin{aligned} x_i &= \frac{a}{b} (y' - y) + x \\ y_i &= y' \\ z_i &= \frac{c}{b} (y' - y) + z \end{aligned} \right\} \quad (34)$$

From Eqs. (32) and (34) the equations of the dashed lines within the cross section of the source can be determined in terms of the coordinates of P_f , P_s , E , F , G , and H .

We now set about finding the contribution to the uncollided flux at P_f from a point source at P_s , where P_s is arbitrarily situated within the source prism. Thus we must find the path lengths from P_s situated in each of the regions defined by conditions (a), (b), and (c) described previously.

If P_s is in region (a), then the direct flux to P_f crosses the plane $Y = y_2$ at the point $P_{(a)} \equiv (x_{(a)}, y_{(a)}, z_{(a)})$ where $x_{(a)}$, $y_{(a)}$, and $z_{(a)}$ are given by

$$\left. \begin{aligned} x_{(a)} &= \frac{\alpha}{\beta} (y_2 - y) + x \\ y_{(a)} &= y_2 \\ z_{(a)} &= \frac{\gamma}{\beta} (y_2 - y) + z \end{aligned} \right\} \quad (35)$$

where α , β , and γ are direction cosines of the line $P_s P_f$.

$$\left. \begin{aligned} \alpha &= \frac{x' - x}{\rho} \\ \beta &= \frac{y' - y}{\rho} \\ \gamma &= \frac{z' - z}{\rho} \end{aligned} \right\} \quad \rho \equiv \sqrt{(x' - x)^2 + (y' - y)^2 + (z' - z)^2} \quad (36)$$

If we let $P_{(a)}$ represent the path length in the source for radiation leaving region (a) along $P_s P_f$, then $P_{(a)}$ is given by

$$P_{(a)} = \left| \frac{y_2 - y'}{\beta} \right| \quad (37)$$

If P_s is in region (b) we must determine the path length between P_s and $P_{(b)} \equiv (x_{(b)}, y_{(b)}, z_{(b)})$, the point at which $P_s P_f$ penetrates the plane $X = x_0$. The coordinates of $P_{(b)}$ are

$$\left. \begin{aligned} x_{(b)} &= x_0 \\ y_{(b)} &= \frac{\beta}{\alpha} (x_0 - x) + y \\ z_{(b)} &= \frac{\gamma}{\alpha} (x_0 - x) + z \end{aligned} \right\} \quad (38)$$

In this case let $P_{(b)}$ denote the path length through the shield.

$$P_{(b)} = \left| \frac{x_0 - x'}{\alpha} \right| \quad (39)$$

Similarly, $P_{(c)}$, the path length in the source region, is found by finding the point $P_{(c)} \equiv (x_{(c)}, y_{(c)}, z_{(c)})$ at which $P_s P_f$ pierces the plane $Z = z_0$.

$$\left. \begin{aligned} x_{(c)} &= \frac{\alpha}{\gamma} (z_0 - z) + x \\ y_{(c)} &= \frac{\beta}{\gamma} (z_0 - z) + y \\ z &= z_0 \end{aligned} \right\} \quad (40)$$

and

$$P_{(c)} = \left| \frac{z_0 - z'}{\gamma} \right| \quad (41)$$

Thus far we have confined our attention to locating the length of the segment of $P_s P_f$ which resides within the source prism. We must now find the length of each segment of $P_s P_f$ which passes through a given shield lamina. At this point it is convenient to discuss the problems of plane and concentric cylindrical laminae separately.

Plane Shield Laminae

Let us denote by ρ_n and x_n , respectively, the distance from P_s to the inner surface of the n^{th} shield (counting the innermost shield as the zeroth and moving sequentially away from the prism) along the line $P_s P_f$ and its projection along the X-axis. Then we see that the path length through the n^{th} shield is

$$\ell_n \equiv \rho_n - \rho_{n-1} = \frac{x_n - x_{n-1}}{\alpha} \quad (42)$$

where

$$n = 1, 2, \dots, N.$$

We have gathered sufficient information to this point to describe the flux emanating from a point arbitrarily portioned on the $Y = y'$ plane

within the source prism. Such a differential flux is given by

$$d\Phi \equiv \frac{S(x', y', z') e^{-\left[\mu_s \rho_{\text{source}}(x', y', z') + \frac{1}{\alpha} \sum_{n=1}^N \mu_n (x_n - x_{n-1})\right]}}{4\pi \rho^2(x, y, z; x', y', z')}, P_s \text{ in region (a)} \quad (43)$$

where $\rho_N \equiv \rho$. The total flux at P_f from all regions (a) over y' is

$$\Phi_{(a)}(x, y, z) = \int_{y_0}^{y_2} dy' \int_{x_1(y')}^{x_3} dx' \int_{z_1(y')}^{z_1} dz' \cdot \frac{S(x', y', z') e^{-\left[\mu_s \rho_{(a)}(x', y', z') + \frac{1}{\alpha} \sum_{n=1}^N \mu_n (x_n - x_{n-1})\right]}}{4\pi \rho^2(x, y, z; x', y', z')} \quad (44)$$

The total flux at P_f from all regions (b) over y' is

$$\Phi_{(b)}(x, y, z) = \int_{y_0}^{y_2} dy' \int_{x_0}^{x_1(y')} dx' \left\{ \int_{z_1(y')}^{z_1} dz' + \int_{\tilde{z}(x', y')}^{z_1(y')} dz' \right\} \cdot \frac{S(x', y', z') e^{-\left[\mu_s \rho_{(b)}(x', y', z') + \frac{1}{\alpha} \sum_{n=1}^N \mu_n (x_n - x_{n-1})\right]}}{4\pi \rho^2(x, y, z; x', y', z')} \quad (45)$$

where $\tilde{z}(x', y')$ represents the Z-coordinate of the dividing line between regions (b) and (c). Its dependence on x' and y' is given by

$$\tilde{z}(x', y') = \frac{x' - x_1(y')}{x_1(y') - x_0} [z_1(y') - z_0] + z_1(y') \quad (46)$$

The total flux at P_f from all regions (c) over y' is

$$\Phi_{(c)}(x, y, z) = \int_{y_0}^{y_2} dy' \left\{ \int_{x_i(y')}^{x_3} dx' \int_{z_0}^{z_i(y')} dz' + \int_{x_0}^{x_i(y')} dx' \int_{z_0}^{\tilde{z}(x', y')} dz' \right\} \cdot$$

$$\frac{- \left[\mu_s \rho_{(c)}(x', y', z') + \frac{1}{\alpha} \sum_{n=1}^N \mu_n (x_n - x_{n-1}) \right] S(x', y', z') e}{4\pi \rho^2(x, y, z; x', y', z')} \quad (47)$$

Concentric Cylindrical Shields

In the cylindrical shields problem we consider the same three regions (a), (b), and (c) as before and the various path lengths $\rho_{(v)}$ ($v = a, b, c$) are, of course, given by the same expressions as above. The only quantity which differs from its definition in the preceding discussion is ℓ_n , the path length on $P_s P_f$ through the n^{th} shield lamina. Let the common axis of the shields be the line

$$X = x_c, \quad Z = z_c \quad (48)$$

Let R_n denote the inner radius of the n^{th} shield. Taking $(x_0, y_0, z_0) = (0, 0, 0)$, we obtain for the equation of the n^{th} shield's inner surface

$$(X - x_c)^2 + (Z - z_c)^2 = R_n^2 \quad (49)$$

The path length of the ray $P_s P_f$ through the n^{th} shield is determined by the direction cosines α , β , and γ and equations for the inner and outer surfaces of the n^{th} slab. Formally it is still true that

$$\ell_n = \frac{x_n - x_{n-1}}{\alpha} \quad (50)$$

However, in the present case x_n and x_{n-1} are then respectively, the larger and smaller positive values of X satisfying

$$\left. \begin{aligned} (X - x_c)^2 + (Z - z_c)^2 &= R_{n-1}^2 \\ \frac{X - x}{\alpha} &= \frac{Z - z}{\gamma} \end{aligned} \right\} \quad (51)$$

and

$$\left. \begin{aligned} (X - x_c)^2 + (Z - z_c)^2 &= R_n^2 \\ \frac{X - x}{\alpha} &= \frac{Z - z}{\gamma} \end{aligned} \right\} \quad (52)$$

Explicitly, we get

$$x_n = \frac{\alpha^2}{\alpha^2 + \gamma^2} \left\{ x_c + \frac{\gamma}{\alpha} \left(\frac{\gamma}{\alpha} x + z_c - z \right) + \right. \quad (53)$$

$$\left. \sqrt{\left[x_c + \frac{\gamma}{\alpha} \left(\frac{\gamma}{\alpha} x + z_c - z \right) \right]^2 + \left(\frac{\alpha^2 + \gamma^2}{\alpha^2} \right) R_n^2 - x_c^2 - \left(\frac{\gamma}{\alpha} x + z_c - z \right)^2} \right\};$$

x_{n-1} is obtained from Eq. (53) by replacing n with $n-1$.

Taking the above differences in the definitions of symbols into account, the formal expressions for $\Phi_{(a)}$, $\Phi_{(b)}$, and $\Phi_{(c)}$ given previously represent the desired solutions for the case of cylindrical shield also.

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Errata made in master - 9/25/59 - JB

Figure 3 on page 8 was erroneously inserted in
this report. Please replace it with the attached, correct
Figure 3.