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CONF-661019-
ARGONNE NATIONAL LABORATORY

MASTER

PROCEEDINGS OF THE
INTERNATIONAL CONFERENCE ON
FAST CRITICAL EXPERIMENTS
AND THEIR ANALYSIS

October 10-13, 1966

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Printed in the United States of America
Available from

Clearinghouse for Federal Scientific and Technical Information
National Bureau of Standards, U. S. Department of Commerce
Springfield, Virginia 22151

Price: Printed Copy \$3.00; Microfiche \$0.65

ANL-7320

Reactor Technology (TID-4500)
AEC Research and
Development Report

Argonne National Laboratory

9700 South Cass Avenue
Argonne, Illinois 60439

CFSTI 8210.5

H.C. 3.00; MN 65

PROCEEDINGS OF THE
INTERNATIONAL CONFERENCE ON
FAST CRITICAL EXPERIMENTS
AND THEIR ANALYSIS

October 10-13, 1966

Chairman

W. C. Redman

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Preface

This document contains the papers presented at the third of a series of topical conferences sponsored by Argonne National Laboratory and the U. S. Atomic Energy Commission on specific topics related to the development of fast power breeder reactors.

The proceedings of the first of these conferences, held in October 1963 and entitled "Breeding, Economics and Safety in Large Fast Power Reactors," were issued as ANL-6792. The proceedings of the second, held in October 1965 on "Safety, Fuels and Core Design in Large Fast Power Reactors," were issued as ANL-7120. A related conference on "The Physics of Breeding" was held at Argonne in October 1959, with the proceedings issued as ANL-6122.

We at Argonne realize that the success of these topical conferences is the direct result of active participation by many of the scientists and engineers involved in the development of fast reactors. We express our thanks to the U. S. Atomic Energy Commission and the participating organizations from abroad for making possible such meetings in which the recent developments on specific topics are reviewed and the basis for a cooperative effort on the remaining problems established.

These proceedings have been edited by A. Glassner, Joyce Kopta, and W. C. Redman. In the interest of prompt publication we have relied on the scientific secretaries for the specific sessions to present in brief form the discussions stimulated by the papers presented. The editors trust that the individual speakers will find that the essential aspects of these discussions have been maintained.

In order to provide a complete record of this conference, the talk presented at the banquet by Chauncey Starr on "The Fast Reactor Development Program," and a timely report presented by Eldon L. Alexanderson, "Preliminary Report on Fuel Damage in the Fermi Reactor," have been included in this document.

I wish to acknowledge the able guidance and willing assistance provided by William G. Davey and George J. Fischer in formulating the program. Finally, on behalf of all participants I express our appreciation of the efficient manner in which Dorothy Carlson, Florence O'Neill, Cyrilla Hytry, and Elizabeth Wallis handled the myriad of administrative details and thereby contributed to an effective meeting.

W. C. Redman
Conference Chairman

Cover photo: Assembly No. 5 in ZPR-6

ZPR-6 is one of Argonne's four split bed type critical facilities used in support of the fast power breeder reactor development program. Assembly 5 is a depleted uranium reflected core having a composition representative of a large dilute uranium carbide fueled fast breeder reactor. The 2600-liter core contains 1600 kg of U^{235} , the largest quantity of fissionable material ever used in a single critical assembly.

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Table of Contents

SESSION I

October 10, 1966

DIFFERENTIAL AND GROUP CROSS SECTIONS

Chairman: I. F. Zartman, U. S. Atomic Energy Commission

Status of Basic Nuclear Data Required for Fast Breeder Reactor Development S. A. Cox, <i>Argonne National Laboratory</i>	3 ✓
Fission Cross-section Measurements of ^{233}U , ^{235}U , ^{239}Pu , and ^{241}Pu in the Energy Range from 1 to 25 keV G. D. JAMES, <i>UKAEA-Harwell</i>	16 ✓
Ratio of Capture to Fission in ^{239}Pu at keV Neutron Energies A. LOTTIN, L. W. WESTON, G. DE SAUSSURE, and J. H. TODD, <i>Oak Ridge National Laboratory</i>	22 ✓
The Automated Preparation of Multigroup Cross Sections for Fast Reactor Analysis Using the MC ² Code D. M. O'SHEA, <i>Combustion Engineering</i> , B. J. TOPPEL and A. L. RAGO, <i>Argonne National Laboratory</i>	27 ✓
The Atomic International Evaluated Nuclear Data Files and Associated Computer Programs for the Automated Preparation of Multigroup Constants HARRY ALTER, <i>Atomic International</i>	33 ✓
Presentation of the Multigroup Cross-section Set Prepared at Cadarache J. RAVIER and J. M. CHAUMONT, <i>CEN-Cadarache</i>	47 ✓

SESSION II

October 10, 1966

TESTS OF CROSS-SECTION SETS

Chairman: W. B. Loewenstein, Argonne National Laboratory

Intercomparison of Calculations for a Dilute Plutonium-fueled Fast Critical Assembly (ZPR-3 Assembly 48) W. G. DAVEY, <i>Argonne National Laboratory</i>	57 ✓
Implications of Recent Fast Critical Experiments on Basic Fast Reactor Design Data and Computational Methods P. GREEBLER, G. L. GYOREY, B. A. HUTCHINS, and B. M. SEGAL, <i>General Electric Co.</i>	66 ✓
The Evaluation of Basic Cross-section Data by Analysis of Fast Critical Assemblies F. L. FILLMORE, E. R. SPECHT, A. R. VERNON, and E. H. OTTEWITTE, <i>Atomic International</i>	80 ✓
The Optimisation of Neutron Cross-section Data Adjustments to Give Agreement with Experimental Critical Sizes PAMELA C. E. HEMMENT and E. D. PENDLEBURY, <i>UKAEA-Aldermaston</i>	88 ✓
Comparison between Experimental and Theoretical Integral Data on Fast Critical Facilities CALI, a Program for Generating "Effective" Nuclear Group Constants by a Correlation Method G. CECCHINI, A. GANDINI, <i>CNEN-Casaccia</i> , I. DAL BONO, and B. FALESCHINI, <i>CNEN-Bologna</i>	107 ✓
Comparative Studies of the Criticality of Fast Critical Assemblies ARTHUR R. BAKER, <i>UKAEA-Winfrith</i>	116 ✓
Comparison of Multigroup Cross section Sets Used in Reactor Calculations M. E. BATTAT and R. J. LABAUVE, <i>Los Alamos Scientific Laboratory</i>	130 ✓

SESSION III-A

October 11, 1966

CLEAN CRITICAL EXPERIMENTS AND THEIR ANALYSIS

Chairman: G. T. Petersen, General Electric

Evaluation of Reactor Physics Experiments on the Coupled Fast-Thermal Argonaut Reactor STARK	137	✓
L. BARLEON, A. BAYER, CHR. BRUCKNER, K. BURKART, G. FIEG, D. KUHN, G. KUSSMAUL, H. MEISTER, H. SEUFERT, D. STEGEMANN, and H. WERLE, <i>Kernforschungszentrum Karlsruhe</i>		
Experimental and Theoretical Work at the Zero-energy Fast Reactor FRO.	159	✓
T. L. ANDERSSON, K. BJORÉUS, E. HELLSTRAND, H. HAGGBLOM, S-O. LONDEN, L. I. TIRÉN, <i>AB Atomenergi</i> , and J. KOCKUM, <i>Research Institute of National Defence</i>		
Measurements and Analysis of Al-, Al ₂ O ₃ -, and BeO-reflected Fast Critical Experiments	186	✓
D. K. BUTLER, R. C. DOERNER, and W. G. KNAPP, <i>Argonne National Laboratory</i>		
Recent Fast Critical Experiments in the MSCA and 710-CE	195	✓
G. T. PETERSEN, J. F. KUNZE, I. B. WALL, J. W. HALLAM, W. B. HENDERSON, and F. G. WARZEK, <i>General Electric Co</i>		
ZPR-3 Assembly 48 Studies of a Dilute Plutonium-fueled Assembly	205	✓
A. M. BROOMFIELD, P. I. AMUNDSON, W. G. DAVEY, J. M. GASIDLO, A. L. HESS, W. P. KEENEY, and J. K. LONG, <i>Argonne National Laboratory</i>		
Zebra 6· A Dilute Plutonium-fuelled Assembly	216	✓
J. ADAMSON, R. M. ABSALOM, A. R. BAKER, G. INGRAM, S. K. I. PATTENDEN, and J. M. STEVENSON, <i>UKAEA-Wmfrth</i>		
Physics Parameters of Large Dilute Uranium Carbide Cores	231	✓
G. K. RUSCH, R. A. KARAM, W. Y. KATO, and G. W. MAIN, <i>Argonne National Laboratory</i>		
Prompt-neutron Time Behavior in Delayed-critical Coupled Uranium-metal Cylinders.	237	✓
J. T. MIHALCZO, <i>Oak Ridge National Laboratory</i>		

SESSION III-B

October 11, 1966

EVALUATION AND COMPUTATION TECHNIQUES

Chairman: E. D. Pendlebury, UKAEA-Aldermaston

Theory of the Neutron Strength Function	245	✓
N. AZZIZ, <i>Westinghouse Electric Corp.</i>		
Doppler Broadening of the U ²³⁵ Fission Cross Section—Comparison of Numerical Treatment with Conventional Single Level Analysis	247	✓
S. C. COHEN, <i>General Atomic</i>		
Preliminary Fast Reactor Cross section Sensitivity Analysis	263	✓
F. M. HECK, <i>Westinghouse Electric Corp.</i>		
Use of Integral Measurements as Supplementary Data in Neutron Cross-section Evaluation	270	✓
A. PAZY, G. RAKAVY, Y. REISS, and Y. YEIVIN, <i>The Hebrew University of Jerusalem</i>		
Monte Carlo Analysis of Small Fast Critical Assemblies	278	✓
V. BENZI, E. CUPINI, and A. DE MATTEIS, <i>CNEN-Bologna</i>		
Monte Carlo Calculations of Fast Multicomponent Critical Systems	299	✓
J. T. THOMAS, <i>Oak Ridge National Laboratory</i>		
Analysis of Fast Reactors by the CIAP and GLOBPERT Codes Using Improved Perturbation Methods	304	✓
A. GANDINI, M. SALVATOIRES, G. SENA, <i>CNEN-Casaccia</i> , and I. DAL BONO, <i>CNEN-Bologna</i>		

SESSION IV

October 11, 1966

DOPPLER AND SODIUM REACTIVITY EFFECTS

Chairman: F. Storrer, CEN-Cadarache

ZPR-6 Doppler Measurements and Comparisons with Theory	319	✓
C. E. TILL, R. A. LEWIS, and R. N. HWANG, <i>Argonne National Laboratory</i>		

Dependence of the Doppler Coefficient of Reactivity for Heavy Elements on Chemical Form, Surface-to-mass Ratio, and Neutron Spectrum S. G. CARPENTER, L. A. MOUNTFORD, T. H. SPRINGER, and R. J. TUTTLE, <i>Atomics International</i>	334 ✓
Doppler Coefficient Measurements in the Cryogenic Temperature Region L. A. MOUNTFORD, <i>Atomics International</i>	341 ✓
Results of Recent Doppler Experiments in ZPR-3 J. M. GASIDLO, <i>Argonne National Laboratory</i>	345 ✓
Interpretation of Doppler Coefficient Measurements in Fast Critical Assemblies E. A. FISCHER, <i>Kernforschungszentrum Karlsruhe</i>	350 ✓
Some Theoretical Studies Concerning the Interpretation of the Doppler Loop Measurements in ZEBRA J. CODD, P. J. COLLINS, and T. D. BEYNON, <i>UKAEA-Winfrith</i>	358 ✓
Calculation of the Sodium-void Effect in Large Carbide Cores F. HELM and A. TRAVELLI, <i>Argonne National Laboratory</i>	369 ✓

SESSION V-A

October 12, 1966

SPECIAL EXPERIMENTS AND THEIR ANALYSIS

Chairman: R. D. Smith, UKAEA-Winfrith

Calculational Studies of Sodium-void Reactivity Variations due to Thin Slab Heterogeneities in Fast Critical Assemblies D. MENEGHETTI, <i>Argonne National Laboratory</i>	377 ✓
A Preliminary Investigation of the Effect of Heterogeneity on Activation Doppler Experiments R. B. NICHOLSON, <i>Argonne National Laboratory</i>	386 ✓
Calculation of the Sodium-void and Doppler Reactivity Coefficients in Fast Reactors and Critical Assemblies, with Heterogeneity Taken into Account A. KHAIRALLAH and F. STORRER, <i>CEN-Cadarache</i>	394 ✓
Experimental Verification of Reactor Physics Parameters of Zoned Cores R. A. KARAM, A. ANCARANI, F. HELM, W. Y. KATO, G. W. MAIN, and G. K. RUSCH, <i>Argonne National Laboratory</i>	403 ✓
Studies of Analytical Problems in Interpretation of Zone-loading Experiments, and Basic Considerations in Zone-loaded Core Design G. J. FISCHER and A. TRAVELLI, <i>Argonne National Laboratory</i>	421 ✓
Progressive Substitution Method in Fast Multizone Experiments M. BARBERGER, P. CAUMETTE, P. GOVAERTS, and F. STORRER, <i>CEN-Cadarache</i>	429 ✓
The Application of Space-Energy Synthesis to the Interpretation of Fast Multizone Critical Experiments F. STORRER and J. M. CHAUMONT, <i>CEN-Cadarache</i>	439 ✓
A P_N Approximation Using Integral Transforms and Its Application to Control-rod Calculations for the Zero-energy Fast Reactor FRO H. HÄGGBLOM, <i>AB Atomenergi</i>	448 ✓
The Balanced Oscillator Tests in SEFOR M. AUDOUX, L. CALDAROLA, P. GIORDANO, H. ROHRBACHER, <i>Kernforschungszentrum Karlsruhe</i> , and C. RUSSELL, <i>General Electric Co.</i>	456 ✓

SESSION V-B

October 12, 1966

SPECTRUM MEASUREMENTS

Chairman: H. Meister, Kernforschungszentrum Karlsruhe

Spectrum Measurements in a Large Dilute Plutonium-fueled Fast Reactor E. F. BENNETT, R. GOLD, and R. J. HUBER, <i>Argonne National Laboratory</i>	477 ✓
A Comment on the Comparison of Theoretical with Experimental Neutron Spectra in Fast Critical Assemblies A. TRAVELLI, <i>Argonne National Laboratory</i>	481 ✓
Comment on Spectrum Measurements in a Large, Dilute Plutonium-fueled Fast Reactor P. S. BROWN, <i>General Electric Co.</i>	486 ✓

Wall-effect Corrections in Proportional-counter Spectrometers	490 ✓
N. L. SNIDOW and H. D. WARREN, <i>The Babcock & Wilcox Company</i>	
A Comparison of Measured and Calculated Energy Spectra in Two VERA Cores	501 ✓
J. W. WEALE, P. W. BENJAMIN, and W. J. PATERSON, <i>UKAEA-Alder-maston</i>	
Measurements of Neutron Spectra and Decay Constants with the Fast Subcritical Facility SUAK	506 ✓
M. KUCHLE, F. MITZEL, E. WATTECAMPS, and H. WERLE, <i>Kernforschungszentrum Karlsruhe</i>	
Time-of-flight Measurements of Neutron Spectra in U ²³⁵ and Tungsten	524 ✓
A. E. PROFIO, J. C. YOUNG, K. L. CROSBIE, R. HACKNEY, H. M. ANTUNEZ, and J. L. RUSSELL, JR., <i>General Atomic</i>	
Measurement of Epithermal Spectra in Fast Assemblies Using Resonance Sandwich Detectors	535 ✓
A. WEITZBERG, <i>General Electric Co.</i>	

SESSION VI

October 12, 1966

REACTOR DESIGN AND PERFORMANCE MEASUREMENTS

Chairman: H. A. Morewitz, *Atomics International*

The Utilization of Fast Critical Assemblies for Fast Reactor Design	543 ✓
W. B. LOEWENSTEIN, <i>Argonne National Laboratory</i>	
The Critical Experiments and Preliminary Analysis of Multifueled, Nonuniform Core Loadings for the FARET Program	550 ✓
P. J. PERSIANI, A. L. HESS, and D. KUCERA, <i>Argonne National Laboratory</i>	
Critical Assembly Mockup of the SEFOR Reactor in ZPR 3	560 ✓
R. L. McVEAN, R. W. GOIN, A. WEITZBERG, A. LERIDON, and J. KREMSER, <i>Argonne National Laboratory</i>	
Application of the SEFOR Critical Experiments at ZPR-3 to SEFOR	569 ✓
A. B. REYNOLDS and S. L. STEWART, <i>General Electric Co.</i>	
Critical Experiments with a Mockup of the Repetitively Pulsed Reactor SORA	586 ✓
G. KISTNER and J. T. MIHALCZO, <i>Oak Ridge National Laboratory</i>	
Theoretical and Experimental Analysis of Reaction Rate Distribution in the Dounreay Fast Reactor	590 ✓
J. C. SMITH, H. ATKINSON, <i>UKAEA-Dounreay</i> , and M. H. McTAGGART, <i>UKAEA-Alder-maston</i>	
Static Physics Measurements in the Fermi Reactor	605 ✓
R. E. MUELLER, J. B. NIMS, and R. C. CALLEN, <i>Atomic Power Development Associates</i>	
Reactor Dynamics Measurements on the Enrico Fermi Reactor	629 ✓
A. E. KLINKMAN, M. L. BATCH, N. C. PAIK, S. U. ZAMAN, <i>Atomic Power Development Associates</i> , H. A. WILBER, <i>Power Reactor Development Co.</i> , and B. D. VANDUSEN, <i>Chrysler Corp.</i>	

SESSION VII

October 13, 1966

EXPERIMENTAL TECHNIQUES AND EQUIPMENT

Chairman: J. D. Orndoff, *Los Alamos Scientific Laboratory*

Some Recent Experimental Techniques Developed in Idaho Division of Argonne National Laboratory	657 ✓
W. G. DAVEY, P. I. AMUNDSON, W. P. KEENEY, and J. M. GASIDLO, <i>Argonne National Laboratory</i>	
Special Experimental Techniques Developed Recently for Application in Fast Zero-power Assemblies	663 ✓
A. BAYER, H. SEUFERT, and D. STEGEMANN, <i>Kernforschungszentrum Karlsruhe</i>	
An International Comparison of Fission Detector Standards	679 ✓
P. I. AMUNDSON, A. M. BROOMFIELD, W. G. DAVEY, <i>Argonne National Laboratory</i> , and J. M. STEVENSON, <i>UKAEA-Winchuth</i>	

Errors and Optimization of Neutron Importance Measurements R. J. TUTTLE, <i>Atomics International</i>	688 ✓
Pulsed-source, Rossi- α and Variance-to-mean Measurements Performed at the FRO Reactor A. BERGSTROM, B. BRUNFELTER, J. KOCKUM, and S. SÖDERBERG, <i>Research Institute of National Defence</i>	694 ✓
Precision Pulsed Measurements with Reflected Fast Critical Assemblies G. S. BRUNSON and R. J. HUBER, <i>Argonne National Laboratory</i>	706 ✓
The Theory of Fast Reactor Integral Measurements by Propagation Methods R. B. PEREZ, R. S. DENNING, and M. J. OHANIAN, <i>University of Florida</i>	714 ✓
Construction and Experimental Equipment of the Karlsruhe Fast Critical Facility, SNEAK P. ENGELMANN, W. BICKEL, U. DÄUNERT, F. W. A. HABERMANN, P. L. VAN VELZE, H. WALZE, and G. WITTEK, <i>Kernforschungszentrum Karlsruhe</i>	725 ✓

SESSION VIII

October 13, 1966

FUTURE PROGRAMS

Chairman: W. C. Redman, Argonne National Laboratory

The Future United States Fast Reactor Physics Program I. F. ZARTMAN, <i>U. S. Atomic Energy Commission</i>	737 ✓
Recent Developments in the SEFOR Experimental Program L. NOBLE, G. R. PFLASTERER, C. D. WILKINSON, <i>General Electric Co.</i> , and L. CALDAROLA, <i>Kernforschungszentrum Karlsruhe</i>	746 ✓
Initial Experiments in the Karlsruhe Fast Critical Facility, SNEAK P. ENGELMANN, R. BÖHME, G. BRUHN, R. DURAND-SMET, W. K. FOELL, F. W. A. HABERMANN, R. SCHRODER, and D. WINTZER, <i>Kernforschungszentrum Karlsruhe</i>	759 ✓
The JAERI Fast Critical Facility J. HIROTA, S. NOMOTO, T. NAKAMURA, T. IJIMA, T. HIRAKAWA, and N. HIRAKAWA, <i>Japan Atomic Energy Research Institute</i>	776 ✓
Fast Test Reactor: Critical Experiments Program R. A. BENNETT, P. L. HOFMANN, W. W. LITTLE, and L. L. MAAS, <i>Battelle-Northwest</i>	785 ✓
Comments on Other Planned Critical Assembly Programs: FRO, MASURCA, VERA, ZEBRA, ZPR-3 and ZPPR, ZPR-6 and -9	794

Appendix A, Banquet Speech:

The Fast Reactor Development Program...	800
---	-----

CHAUNCEY STARR, *North American Aviation*

Appendix B, Special Report:

Preliminary Report on Fuel Damage in the Fermi Reactor.....	805
---	-----

Appendix C:

Non-ANL Attendees	808
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SESSION I

October 10, 1966

DIFFERENTIAL AND GROUP CROSS SECTIONS

Chairman: I. F. Zartman
U. S. Atomic Energy Commission

Secretaries: E. M. Pennington
J. W. Meadows
Argonne National Laboratory

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✓ Status of Basic Nuclear Data Required for Fast Breeder Reactor Development*²

S. A. Cox

Argonne National Laboratory
Argonne, Illinois

S. A. Cox

ABSTRACT

The present precision and availability of nuclear data are discussed. Fission cross sections are examined in the light of recent U^{235} measurement at A.W.R.E., Aldermaston. Since the U^{235} fission cross section is used as a normalization standard for many cross-section measurements, its status is examined in detail. The normalization of neutron capture cross sections depends not only on the U^{235} fission cross section, but also on the $B^{10}(n,\alpha)$ cross section and measurements of spherical shell transmission. Each normalization procedure is discussed in detail. It is concluded that much of the existing disparity in results for fission and capture cross sections would be greatly reduced by a critical renormalization of all of the data to currently acceptable values of the U^{235} fission cross section and the $B^{10}(n,\alpha)$ cross section. Recent re-evaluations of measurements of spherical shell transmission also tend to reduce the disparity. Neutron-scattering cross sections from 0.3–1.5 MeV are in good shape. Because of the recent observation of intermediate structure in elastic neutron-scattering cross sections, measurements must be made in great detail especially below ~ 3 –5 MeV. More data are needed from 1.5–5 MeV. Above 5 MeV the optical model can be used to interpolate between measurements rather widely spaced in energy. Recent $\bar{\nu}$ measurements confirm the nonlinearity of $\bar{\nu}$ vs. E_n and also suggest the presence of an anomaly at ~ 0.4 MeV. Requests for nuclear data are examined according to the feasibility of the measurement and the man-year requirement necessary to achieve the requested precision.

1. Introduction

The title of this talk refers to "Basic Nuclear Data." This is a very large subject and obviously there is not sufficient time to discuss the entire area, nor do I have the competence to do so. I will therefore take my cue

from the title of this session and concentrate in the area of cross-section data. Since the next talk concerns fission cross sections from 1 keV to 25 keV, my discussion of fission cross sections will be restricted to measurements from 25 keV and up. Further, since the subject is specifically the status of nuclear data, only pertinent details of the experimental techniques will be given.

2. Fission Cross Section

Fission cross sections have a position of great importance not only in reactor physics but also in the related field of cross-section measurement. The U^{235} cross section has been utilized extensively for the measurement of neutron flux in the determination of many neutron cross sections, e.g., capture cross sections and other fission cross sections. In view of its importance as a standard, recent changes in the U^{235} fission cross section deserve special consideration.

The most comprehensive recent measurements are those made at Aldermaston by White,⁽¹⁾ who used a hydrogen recoil counter to monitor the neutron flux. White's recoil counter was capable of yielding good results for neutron energies as low as 40 keV and was used for all of his measurements of neutron flux up to 2.25 MeV. Perkin *et al.*⁽²⁾ have made very accurate measurements of the fission cross section of a number of isotopes of uranium, neptunium, and plutonium at 24 keV with a calibrated Sb-Be source. The Sb-Be source was independently calibrated at three different laboratories. The weighted mean of the three determinations yielded a neutron source strength with an uncertainty less than 1%. Both White and Perkin *et al.* quote $\sim 2.5\%$ accuracy for their measurements. Perkin and White's results for U^{235} together with the results of other workers are given in Fig. 1. The open circles are due to White and the "winged" open circle at 24 keV is due to Perkin *et al.* It is clear from the figure that the White results which extend down to 40 keV are consistent with the Perkin measurement at 24 keV and that the cross sections determined by both

* This work performed under the auspices of the U. S. Atomic Energy Commission.

FIG. 1. Measurements of the U^{235} Fission Cross Section. Data labeled present measurements are those of White.⁽¹⁾

FIG. 2. Ratios of P. H. White,⁽¹⁾ Allen and Henkel,⁽³⁾ and Perkin *et al.*,⁽²⁾ BNL-325 Fission Cross Sections Showing Magnitude of Differences.

experiments are lower than previously accepted values, especially at the low-energy end.

In the past, the most extensively used values of the U^{235} cross section were undoubtedly the "best fit" curve given in BNL-325 and the "best fit" curve given in Allen and Henkel.⁽³⁾ The "best fit" curves do not agree with each other or with the Aldermaston results. Probably the most direct method of representing the difference between the BNL-325, Allen and Henkel, and Aldermaston results is to plot the ratio of the cross sections. This is done in Fig. 2, where the solid curve labeled R_1 is a least-squares fit to the ratio of White's

cross sections and the Allen and Henkel cross sections, and the solid curve labeled R_2 is the ratio of the BNL-325 cross section to the Allen and Henkel cross sections. The single point is the ratio of the Perkin 24-keV cross section and the Allen and Henkel cross section. It is clear from the figure that the Aldermaston hydrogen recoil and Sb-Be determinations are in very good agreement. It is also clear that past cross-section measurements in the vicinity of 20–30 keV were probably in error by 6–7% if the BNL-325 curve was used as a standard and by ~18% if the Allen and Henkel curve was used as a standard.

I have spent some time on the U^{235} fission cross section because it is basic to the determination of many other cross sections used in reactor physics calculations. Almost all measurements of fission cross section are made as ratio measurements using the U^{235} fission cross section as a standard. Virtually all neutron capture cross sections above 200 keV use the U^{235} fission cross section for absolute determinations of neutron flux. Considering the quoted precision of the recent cross-section results it may be possible for the first time, to make reliable cross-section measurements in the region between 24 and 200 keV, a region which has presented many difficulties in the past.

Figure 3 gives results for the Pu^{239} fission cross section taken from a report by White *et al.*⁽⁴⁾ The upper portion of the figure gives some measurements of the ratio of Pu^{239} to U^{235} , and the lower portion gives the White fission cross-section data together with other measurements. The White data given by the closed circles, as well as the Perkin 24-keV measurement given by the double concentric circles, are considerably lower than

FIG. 3. Pu^{239} Fission Cross Sections.⁽⁴⁾

previously accepted values. The BNL-325 curve is indicated by the solid line. Plots of the cross-section data for U^{233} and Pu^{241} are similar. White explains the difference in the fission cross sections as due entirely to the different values of the U^{235} fission cross section used for normalization. It is undoubtedly true that differences in normalization are a large contributing factor in fission cross-section discrepancies; however, a study of the recent barn book supplement yields an interesting result. The fission cross sections of the fertile elements are in much better shape than the fission cross sections of the fissile elements. This may be an indication that an inaccurate determination of thermal and epithermal background is another factor contributing to discrepancies in cross sections.

3. Capture Cross Sections

During the past several years much effort has been devoted to the measurement of neutron radiative capture cross sections. The experimental techniques have involved activation measurements, measurements with large liquid scintillators, and measurements of spherical shell transmission. The cross sections are normalized in various ways. Many Sb-Be activation measurements use a manganese bath for neutron source calibration. Other activation measurements are based on values of the U^{235} fission cross section, the n-p cross

section, and the $\text{B}^{10}(n,\alpha)$ cross section. The liquid scintillator results are based on either the U^{235} fission cross section or measurements of spherical shell transmission, which in principle are independent of an absolute determination of neutron flux. Capture cross sections for the fissile elements are generally contained in a measurement of α (the capture to fission ratio). Some results for U^{235} are given in Fig. 4.^(5, 6) The agreement between the two sets of measurements is rather good, possibly because a measurement of α does not require a measurement of neutron flux. However, if the capture cross section is to be extracted, the fission cross section must be known and this does depend on a neutron-flux measurement. Although at first sight the data in the barn book looks worse than the data in Fig. 4, a little study reveals that the scattering in the points is due mainly to data which are derived from measurements of η and therefore do not constitute a direct measurement of α .

Recent measurements of the capture cross section of fertile materials seem to have made matters worse rather than better. Figure 5 shows the situation for U^{238} as of early 1964.⁽⁷⁾ The most recently measured points on the graph, given by the open circles, agree well with what had been until then the accepted values, given by the solid curve. There are three points near 1 MeV which are anomalously high and three points

in the several-MeV region which are anomalously high, but the preponderance of the data is in reasonably good agreement. For comparison, Fig. 6 shows the latest

FIG. 4. α Measurements for U^{235} . (5, 6)

compilation of data taken from the recent barn book. The profusion of data is a little overwhelming. Again the discrepancies are probably largely due to differences in neutron-flux measurement and normalization. The situation for Th^{232} is somewhat worse. It is clear that the task of renormalizing all of the data to currently accepted values of the standard cross sections on which the data are based would be a large one. Even so the effort might well be worthwhile and would probably go a long way toward resolving the discrepancies.

It still remains to discuss the status of neutron capture cross sections for lighter elements, including structural materials. This is a troublesome area and is probably best divided into two parts: measurements above 200 keV and measurements below 200 keV. Above 200 keV most measurements are made relative to either the U^{235} fission cross section or the n-p cross section. The agreement between various workers is generally quite good, and there seems to be little difficulty associated with absolute normalization in this case. Differences which still remain can largely be ascribed to different choices in the fission cross section used as a standard.

In the region below 200 keV the situation is quite

FIG. 5. Neutron Capture Cross Sections of U^{235} . (7)

FIG. 6. Neutron Capture Cross Sections of U^{238} from BNL-325, Second Edition, Supplement 2.

different. Possibly because of the ease of availability of a pure isotope and because of its simple decay scheme, gold was the first element to fall victim to the profusion of measurements. Since most people like to pick on gold, I will do the same; however, gold is not a unique case. Figure 7 gives the status of gold as of early 1964. The figure contains only some of the data. Most of the cross-section discrepancies which are apparent above 200 keV are removed if the data are all normalized to the same fission cross section. Below 200 keV the situation is very complex. A number of different approaches were used in determining the absolute cross section. For the activation measurements some groups used calibrated Sb-Be sources,^(8, 9) some used the U^{235} fission cross section,⁽¹⁰⁻¹³⁾ some the $B^{10}(n,\alpha)$ cross section,⁽¹²⁾ and some spherical shell transmission measurements.⁽¹⁴⁾

The measurements⁽¹⁵⁾ with large liquid scintillators were normalized to the shell measurements at 24 keV and extrapolated to 170 keV using the $B^{10}(n,\alpha\gamma)$ cross sections. I have previously described the recent changes in the U^{235} fission cross section. The $B^{10}(n,\alpha)$ cross section has also undergone some recent changes. Figure 8 gives recent results for the $B^{10}(n,\text{absorption})$ cross section measured by two quite independent techniques, the closed circles by a spherical shell⁽¹⁶⁾ transmission

method and the open squares by a simultaneous measurement of the total cross section and total scattering.⁽¹⁷⁾ The dashed line represents a $1/v$ extrapolation from the thermal absorption cross section. Both measurements are in good agreement with a $1/v$ extrapolation up to ~ 250 keV. This is in contrast to previous measurements which indicated that the cross section departed from a $1/v$ behavior at ~ 120 keV. At 200 keV, the present cross section is $\sim 25\%$ higher than the previously accepted value. When the activation measurements based on the $B^{10}(n,\alpha)$ and U^{235} fission cross sections are renormalized to present values, the cross sections near 30 keV are lowered appreciably, in one case by $\sim 30\%$. There are some indications that the 24-keV absorption cross section derived from the spherical shell measurement for gold is too low. A recent Monte Carlo analysis⁽¹⁸⁾ of sphere transmission data for gold yields a cross section $\sim 30\%$ higher than the one originally quoted. If this analysis is correct, the cross-section discrepancies in the vicinity of 24 keV are reduced to $\sim 15\%-20\%$.

The situation for a number of elements⁽¹²⁾ is given in Fig. 9. The closed circles are ANL activation measurements based on the Allen and Henkel U^{235} fission cross section; the triangles are Los Alamos liquid

Fig. 7. Neutron Capture Cross Section of Au^{197} as of Early 1964.

Fig. 8. B^{10} ($n, \text{absorption}$) Measurements.⁽¹⁶⁾

scintillator measurements based on the Allen and Henkel U^{235} fission cross section and the Los Alamos measured values of α . The open circles are the ORNL liquid scintillator measurements which are based on the spherical shell transmission measurements at 24 keV and the shape of the $\text{B}^{10}(n, \alpha\gamma)$ cross section. If the ANL and LASL data are renormalized to the recent fission cross section, the values in the vicinity of 150

keV are reduced by 6–8%. If, further, the ORNL data are renormalized to sphere transmission results as indicated by the recent Monte Carlo analysis, the ORNL data would be raised by perhaps as much as 15%. If future work bears out these considerations, the serious discrepancies in neutron capture cross section measurements may be greatly alleviated.

4. Scattering

Now we came to a somewhat happier field of nuclear data, that of neutron scattering. During the last 3 or 4 years, an enormous quantity of data has been acquired. The increase in data production is due partly to advances in experimental techniques, e.g., improvements in T.O.F. systems and increases in neutron source strengths on accelerators, and partly to advances in automation of data acquisition and the increased use of computers in collecting, processing, and analyzing the data.

In the energy region from 300 to 1500 keV there is a profusion of data for elastic and inelastic scattering. The state of the art for inelastic scattering may be demonstrated with U^{238} and W^{186} as examples. Figure 10 gives results from various laboratories for U^{238} .⁽¹⁹⁾

FIG. 9. Comparison of Activation and Large Liquid Scintillator Capture Cross Section Measurements.⁽¹²⁾

The level structure as reported by the Harwell group is given at the left and the inelastic scattering excitation measurements corresponding to some of the levels are indicated by the appropriate arrows. The results of the several laboratories are in reasonable agreement and the attainable resolution is quite good, as is indicated by the excellent quality of the data for inelastic scattering to the 45-keV first excited state. The solid curves, which are optical model calculations by Wilmore,⁽²⁹⁾ agree rather well with the data.

Figure 11 gives some ANL results for W^{186} .⁽¹⁹⁾ The inelastic excitation of a number of levels is illustrated. The solid curves are the results of deformed-potential calculations by Dunford.⁽³⁰⁾ The agreement between the data and calculations is reasonably good. By contrast, the dashed curve is included to demonstrate the kind of fit obtained from a spherical-well calculation. Clearly the spherical-well calculation does not agree either with the data or with the deformed-potential calculation.

Figure 12⁽¹⁹⁾ shows a composite of results for an incident energy of 1.0 MeV which extends over most of the periodic table. All data are plotted against the

mass number A . The upper set of data points gives the elastic scattering cross section, the lower set gives the total inelastic scattering cross section, and the middle two give the first- and second-order Legendre polynomial coefficients obtained from a fit to the angular distribution. The solid and dashed curves are obtained from an optical model plus Hauser-Feshbach calculation using a real potential of 46 MeV and the three imaginary potentials indicated. The prominent minima in the inelastic scattering cross sections are a striking illustration of the effect of shell closure. Clearly an imaginary potential of 14 MeV represents the best overall fit. The reasonably good agreement between the data and theory indicates that the scattering properties can be estimated by interpolation fairly reliably for unknown nuclei.

Recently, intermediate structure has made its appearance in elastic-scattering cross sections. This could lead to some difficulty in using the optical model for the aforementioned interpolation since the optical model does not take intermediate structure into account; therefore, some caution should be used, especially for energies below ~ 2 MeV and for nuclei near closed

FIG. 10. Neutron Inelastic Scattering from U^{238} .⁽¹⁹⁾

FIG. 11. Neutron Inelastic Scattering from W^{186} .⁽¹⁹⁾

FIG. 12. Résumé of Neutron Scattering Data. The fit to the data with three imaginary potentials is shown.⁽¹⁹⁾

shells. Intermediate structure appears even in nuclei with sufficiently high level density that it is unlikely that the observed structure could arise from random fluctuations. A good example of this kind of structure is shown in Fig. 13 which gives elastic-scattering data for Mn.⁽²⁰⁾ The total elastic-scattering cross section is given at the bottom. Above the scattering cross section are the coefficients of a 4th-order Legendre polynomial fit to the angular distribution data. Whether the structure is related to the "doorway state" hypothesis of Feshbach⁽³¹⁾ or whether it arises from some other source is still an open question.

There has been some excellent work at higher neutron energies. Figure 14⁽²¹⁾ gives an example of scattering of 5-MeV neutrons from iron measured by time of flight. Many inelastic neutron groups are resolved. It is evident that, with recent improvements in ion-source technology, and the consequent increase in the neutron

source strength, this kind of measurement will yield some very valuable information.

A recent addition to the field of cross-section measurement is the Li-Ge detector. The detection efficiency is not so good as that for NaI, but with its superior resolution the Ge-Li detector is an invaluable tool in measurements of γ rays following neutron capture, and promises to be a very useful tool in the measurement of γ rays following inelastic neutron scattering.

Future needs for neutron-scattering data probably lie mostly in the region between 1.5 MeV and ~ 4 or 5 MeV. The region between 0.3 and 1.5 MeV has been extensively studied, and measurements in the region are continuing, so that the supply of elastic- and inelastic-scattering data in this region should be ample to meet the demand. In the region above 4 or 5 MeV a few good measurements for one nuclide should suffice. for then the intermediate data can be obtained by

interpolation using the optical model. The region between 1.5 MeV and 4 or 5 MeV is an awkward one. This is the region that in many cases bridges the gap between resonance structure and the continuum. The optical model does not work as well here as at higher energies and because of the presence of structure in the cross section of many elements the data must be taken in much more detail.

5. $\bar{\nu}$ Measurements

Most of the recent effort in $\bar{\nu}$ measurements has gone into a study of the energy dependence and into precision measurements of $\bar{\nu}$ for spontaneous fission of Cf^{252} . The nonlinear relationship between $\bar{\nu}$ and the neutron energy is confirmed by recent measurements. Figure 15, taken from Hopkins and Diven,⁽²²⁾ shows this effort very clearly for U^{235} . It is not possible to make a linear fit to all of the data. The low-energy region has been fitted with a slope of 0.085 neutron/MeV, and the data at higher energies have been fitted with a slope of 0.160 neutron/MeV. Certainly it is not necessary to use two straight lines to fit the data. Mather *et al.*⁽²³⁾ use a quadratic expression to fit their $\bar{\nu}$ data for U^{235} . This is given in Fig. 16 where the solid line is calculated from the quadratic expression in the figure.

There is some evidence for more complicated structure below 1 MeV. Figure 17 is from Meadows and Whalen.⁽²⁴⁾ Here there is an apparent anomaly in the energy dependence at ~ 0.4 MeV. The scatter of the points is well outside the standard deviations. An anomaly at ~ 0.4 MeV was also observed by Blyumkina *et al.*⁽²⁵⁾ Evidence of such structure suggests that the two-slope fit of Hopkins and Diven and the quadratic fit of Mather *et al.* are both only approximations to the actual energy dependence.

Some measurements of $\bar{\nu}$ for Pu^{240} were reported recently by Vroey *et al.*⁽²⁶⁾ Measurements of Pu^{240} are complicated by the high spontaneous fission rate, so the accuracy obtained ($\sim 5\%$) is not so good as for other $\bar{\nu}$ measurements. The data of Vroey *et al.* together with other data is given in Fig. 18. Their data are in reasonable agreement with previous measurements, although more data are needed before anything definitive can be said about the energy dependence.

Precision measurements of $\bar{\nu}$ from the spontaneous fission of Cf^{252} have been made principally at four laboratories. A table of the final values is given in Table 1 together with the associated laboratories. An absolute value of $\bar{\nu}$ for Cf^{252} is important because it is the basis for the absolute scale for other $\bar{\nu}$ measurements. The data groups into two pairs, the Stockholm and Los Alamos values being in the higher pair, and the Harwell and Aldermaston values in the lower pair. When the quoted uncertainties in the measurements are considered, the

FIG. 13. Elastic Neutron Scattering Cross Section and Legendre Polynomial Coefficients for Mn.⁽²⁰⁾

FIG. 14. Scattering of 5-MeV Neutrons from Fe.⁽²¹⁾

agreement cannot be said to be entirely satisfactory. It should be noted that the Aldermaston result is based on a recent revision of the Harwell Pu^{240} standard which reduced the Aldermaston result from a previous value of 3.77.

6. Supply and Demand

I would like to spend a few moments on the supply-and-demand aspects of nuclear data. With the ever-increasing industrial effort in the reactor business, it is inevitable that the engineering side of the reactor field should increase at a more rapid rate than the physics side. In such a situation the demands for more cross-

FIG. 15. $\bar{\nu}$ for U^{235} , showing Nonlinearity of $\bar{\nu}$ vs. E_n .⁽²²⁾

FIG. 16. $\bar{\nu}$ for U^{235} , showing Quadratic Fit of Mather *et al.*⁽²³⁾

section information of greater precision are likely to increase. The cross-section measurers are not very numerous and in some cases are only half-heartedly in the business of cross-section measurement. Most cross-

section measurers are physicists and as such have broader interests than just the measurement of neutron cross sections. These men often spend their time doing things which are of little interest to the cross-section

FIG. 17. $\bar{\nu}$ for U^{235} showing Anomalous Behavior in the Vicinity of 0.4 MeV. ⁽²⁴⁾

user. Despite the work habits of the physicist, which may at times seem somewhat incompatible with the demands of the cross-section user, I believe that physicists will continue to supply more and better cross-section data in the future. Perhaps the way to reconcile the increased demand from the cross-section user with the limited number of cross-section measurers is a more efficient utilization of automation in data acquisition and a more widespread use of both on-line and off-line computers in data processing and analysis.

In all fairness to the cross-section measurer the cross-section requests should not demand a precision which is not attainable by present techniques. The

compilation of requests for nuclear data contains a rather large number of such items. I recall coming across any number of requests for neutron capture cross sections in the keV region with a precision from 2% to 5%. One in particular is worthy of separate mention. It was for the capture cross section of U^{238} from 10 keV to 1 MeV with a required precision of 2%. In the light of present knowledge concerning capture cross sections in this region, the request is a little optimistic.

Associated with the increasing demands for improved precision is the manpower consideration. How many man-years is a given measurement worth? Even though the precision of the capture cross sections leaves something to be desired, there has been an enormous amount of manpower expended in their measurement. In recent years alone, there have been well over 100 man-years devoted to the measurement of capture cross sections. The measurement of $\bar{\nu}$ has also required the expenditure of more than 100 man-years; however, the result is more satisfactory. In short, any serious request for a precision cross-section measurement should be made in the knowl-

TABLE 1. RECENT VALUES OF $\bar{\nu}$ FOR SPONTANEOUS FISSION OF Cf^{252}

Stockholm	3.799 ± 0.034
Los Alamos	3.771 ± 0.030
Harwell	3.705 ± 0.015
Aldermaston	3.685 ± 0.040

FIG. 18. $\bar{\nu}$ for Pu^{240} . ⁽²⁶⁾

edge that a request for 2–5% precision might take 4–5 times as many man-years as a request for 10% precision, or it might not be feasible at all. All aspects of neutron cross-section measurements will be greatly aided by the increasing use of local computers for data acquisition and processing, and the rapidly increasing capability of large, high-speed computers for the often very complicated corrections necessary to achieve high precision.

7. Summary

To summarize: There are a large number of measurements of fission cross sections and capture cross sections. The principal need is not for more, but for better data. I believe that a careful evaluation of existing data concerned mainly with a critical examination of the standards used for neutron-flux measurements would appreciably improve existing precisions. In the case of neutron-scattering data the same problems of normalization do not arise. Most scattering cross sections are determined relative to scattering from either hydrogen or carbon. The hydrogen cross section is well known, and the carbon cross section can be obtained from a combination of a total-cross-section measurement and a relative measurement of the elastic-scattering angular distribution. There is no absolute measurement of neutron flux necessary. Present techniques permit very good precision in $\bar{\nu}$ measurements. In the region of lower energy, say <1 MeV, the relative measurements can be made with uncertainties as small as 0.75%. The absolute scale, however, depends on $\bar{\nu}$ for Cf^{252} spontaneous fission, which at present is not known to better than 1.5–2.0%.

Finally, I would like to apologize for omitting much of the nuclear data pertinent to fast reactor design. In particular, I would like to mention the very excellent work in the resonance region by pulsed accelerators. These machines are providing vast amounts of data on total, fission, and capture cross sections.

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Discussion

Mr. Smith (ANL): I would like to make a remark addressed, not to precision measurements such as of $\bar{\nu}$ or fission cross sections, but to general things such as capture or scattering. There are two ways of looking at this problem. One is the way of looking at a precise requirement for a given reactor concept, or one or two reactor concepts, and to promote a program of quite limited scope, but of high precision. The other, which I think is a more general and better approach, is to look at a long-range effort in which you seek the data for a whole series of future reactors. This is really a basic understanding of the process. If you follow the latter approach, then you will not only meet many of the

requirements for the immediate designs, but also will provide for the future that you cannot foresee now. I think that is a very important guideline to measurers of cross sections. Too often you are discouraged by these ultra-precision measurements, and too often you design an experiment which is too narrow and too restricted; once finished, you do not get the maximum value out of either your effort or your equipment.

Mr. Smith (U. K.-Winfrith): I would also like to comment on the need for these precision measurements. I don't altogether agree with the suggestion that the reactor physicist should only ask for measurements that the measurers can at this instant measure, rather than measurements that he actually wants. I do think

that these precision measurements which have been requested come from a genuine need. I think that the reactor calculators understand that these needs are not going to be met in the near future. On the other hand, there is no point in measuring, to greater accuracy, cross sections which are easy to measure, but which are of relatively little use to the reactor designer. So I think the accurate cross sections should be kept as a target at which to aim and which will, as Smith has just said, ultimately form a wide basis for accurate calculation of a reactor system.

Mr. Cox: I was referring only to cross sections required on a short-term basis. Cross-section needs a number of years from now may be quite different.

G. D. James

Fission Cross-section Measurements of ^{233}U , ^{235}U , ^{239}Pu , and ^{241}Pu in the Energy Range from 1 to 25 keV

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ABSTRACT

Fission cross-section measurements have been carried out at Harwell, by time-of-flight experiment, over the energy range from 1 eV to 25 keV for the nuclides ^{233}U , ^{235}U , ^{239}Pu , and ^{241}Pu . The data obtained from 1 keV to 25 keV are presented and compared with existing data. It is shown that in this energy range the available data for ^{235}U are in good agreement and that the data for ^{233}U and ^{239}Pu agree fairly well.

1. Introduction

The fission cross sections of ^{233}U , ^{235}U , ^{239}Pu , and ^{241}Pu in the neutron energy range near 1 keV become increasingly important in the design of fast breeder reactors as these systems are made larger and become characterised by softer spectra. Few measurements^(1, 2) of fission cross sections had been made in this energy range until recently, when the results of several experiments⁽³⁻⁵⁾ became available, including the measurements of Perkin *et al.*⁽⁶⁾ at 24 keV and a number of experiments using a nuclear explosion as a source of neutrons.⁽⁷⁻⁹⁾ The fission cross sections of all four nuclides, over the energy range from 1 eV to 25 keV, have been measured by time-of-flight experiment using the electron linear accelerator and neutron booster source of the Harwell Neutron Project.⁽¹⁰⁾ This paper contains a brief description of the experimental methods used in these measurements (see Sect. 2) and a comparison of the results obtained in the keV energy region with the data from other experiments (see Sect. 3). An estimate of the error in the mean fission cross section has been obtained for each nuclide from the spread of the measurements. These estimates should help to decide which cross sections are sufficiently well-known and which are still required to greater accuracy.

2. Experimental Method

2.1 The Fission Detector and Time-of-flight Equipment

The fission cross sections of ^{233}U , ^{235}U , ^{239}Pu , and ^{241}Pu were measured using the continuous-flow gas

scintillation fission-fragment detector depicted in Fig. 1. A mixture of helium with 0.3% nitrogen, which acts as a wavelength shifter, was used as the scintillating medium. Pulses from the 56AVP photomultiplier attached to the gas scintillator were passed through a fast tunnel diode discriminator set at 150 mV to remove unwanted alpha pulses. Output pulses from the discriminator were then taken to a 16,384-channel magnetic tape time-of-flight recorder.⁽¹¹⁾ A PDP-4 computer was used to convert the times of flight recorded during the experiment to counts per timing channel. The neutron spectrum was measured using a BF_3 counter (20th Century Electronics type 5 EB40/13) which was operated below 1500 V to prevent the amplifier being overloaded by the gamma flash from the neutron source. Background measurements and the calculations of the cross section and neutron energy were carried out as described previously.^(12, 13) An IBM-7030 computer was used to calculate the fission cross section and to record the final data on magnetic tape. Table 1 gives details of the experiments on all four nuclides. For ^{235}U , ^{239}Pu , and ^{241}Pu two independent measurements of the fission cross section were made, one using a cadmium filter (0.22 gm cm^{-2}) and one using a boron carbide filter (0.27 gm cm^{-2}) to prevent the overlap of slow neutrons from one cycle to the next. In most experiments a layer of cobalt (0.27 gm cm^{-2}) was kept in the neutron beam to give a permanent indication of the background at 132 eV. Only one measurement, with a cadmium filter, has been completed for ^{233}U .

2.2 Normalisation and Accuracy of the Data

The present relative measurements of the fission cross sections of ^{233}U , ^{239}Pu , and ^{241}Pu were normalised by equating the integral of the cross section over the energy ranges shown in Table 2 to the values obtained from the data of Moore *et al.*⁽¹⁵⁾ for ^{233}U , Bollinger *et al.*⁽²⁾ for ^{239}Pu , and James⁽¹²⁾ for ^{241}Pu , after re-normalising these data to the thermal-fission cross sections of Westcott *et al.*⁽¹⁴⁾ This process of normalisation is not complete for ^{235}U , and the data over the

FIG. 1. Continuous-flow Gas Scintillation Fission-fragment Detector.

energy range above 3 keV have been normalised to the mean of the data of Michaudon *et al.*⁽¹⁾ and Wang *et al.*⁽³⁾

Table 2 also gives the total error in the average fission cross section over 1-keV intervals, in the energy range from 1 to 10 keV. This error is composed of the statistical accuracy of the data, the error in the process of normalisation, and the error in the data to which the results are normalised. A comparison of the separate measurements made with different filters enabled the calculated statistical accuracy of the data to be checked. For ^{239}Pu the two measurements agreed within the statistical error, but for ^{235}U and ^{241}Pu the 'statistical' error was increased to allow for the discrepancy between the measurements. For each of the energy ranges listed in Table 2, the ratio of the present data to the data chosen for normalisation was calculated, and the statistical spread in this ratio was used as a measure of the error in the process of normalisation. It is seen from Table 2 that most of the error in the ^{233}U data comes from the error in the data chosen for normalisation, whereas in the ^{235}U measurement, most of the error is 'statistical' and comes from a discrepancy between the two measurements made. All contributions to the total error in the ^{239}Pu and ^{241}Pu data are roughly equal.

3. Comparison of Data with other Measurements

Tables 3, 4, 5, and 6 show the results of the present experiments as average values of the fission cross section over 1-keV intervals from 1 to 10 keV and as average values over the intervals from 10 to 20 keV and from 20 to 25 keV. For ^{235}U , ^{239}Pu , and ^{241}Pu the data below 3 keV are not given because of scattering resonances at 1.98 keV and 2.75 keV in the copper and zinc forming the brass back of the fission chambers used in these experiments. Also shown in the tables are the results of other measurements in these energy ranges. Most of the data were obtained from the ENEA Neutron Data Compilation Centre, but the data of Bergen *et al.* and of Shunk *et al.* were provided by the authors.⁽¹⁹⁾

Table 3, which gives the fission cross section for ^{233}U , includes the data of Albert⁽⁷⁾ and Bergen *et al.*⁽⁸⁾ Albert measured the ratio of the ^{233}U to the ^{235}U fission cross sections from 30 eV to 8 MeV in an experiment with an atmospheric nuclear explosion as a source of neutrons and a flight path length of 1280 km. The ratio measured was normalised to that found by Allen and Ferguson⁽¹⁶⁾ at 550 keV and converted to a ^{233}U fission cross section by multiplying by the ^{235}U fission cross section given by Stehn *et al.*⁽¹⁷⁾ above 10 keV and by the ^{235}U data of Michaudon *et al.*⁽¹⁾

below 10 keV. Bergen *et al.* used the Petrel underground nuclear explosion as a source of neutrons and obtained the fission cross section from the calculated efficiency of their solid-state fission-fragment detector,

TABLE 1. EXPERIMENTAL DETAILS

	Material			
	²³³ U	²³⁵ U	²³⁹ Pu	²⁴¹ Pu
Weight of material, mg	40	40	40	40
Thickness, mg/cm ²	0.49	0.49	0.49	0.49
Isotopic Composition, %				
²³³ U	99.2	—	—	—
²³⁴ U	0.8	1.14	—	—
²³⁵ U	—	92.9	—	—
²³⁶ U	—	0.02	—	—
²³⁸ U	—	5.98	—	—
²³⁹ Pu	—	—	99.92	0.53
²⁴⁰ Pu	—	—	0.072	1.18
²⁴¹ Pu	—	—	0.003	96.95
²⁴² Pu	—	—	<0.003	1.33
Neutron filter				
(1)	Cd and Co	Cd and Co	Cd	Cd
(2)	—	B ₄ C and Co	B ₄ C and Co	B ₄ C
Electron pulse width, μ s	0.2	0.2	0.1	0.1
Channel width, μ s	1/16	1/16	1/16	1/16
Pulse repetition frequency, pps	192	192	192	192
Total running time (non-black samples only), hr				
Filter (1)	75	100	124	91
Filter (2)	—	130	88	172

situated about 185 m from the neutron source, and from an absolute determination of the neutron flux using the ⁶Li(*n*, α) reaction below 100 keV and the ²³⁵U(*n*,*f*) reaction above 10 keV. From the spread of the results, the average error in the mean of the data given in Table 3 is $\pm 5.2\%$.

Table 4 gives the ²³⁵U data of Michaudon *et al.*,⁽¹⁾ measured using a 50-m flight path and the Saclay electron linear accelerator neutron source, and the data of Wang *et al.*,⁽³⁾ measured using the pulsed fast reactor of the Joint Institute for Nuclear Research (Dubna) and a 1000-m flight path. Michaudon *et al.* used a large gas scintillation fission-fragment detector and normalised their data to the integrated cross section of Shore and Sailor⁽¹⁸⁾ from 8 to 10 eV. Wang *et al.* detected fission neutrons in a large cadmium-loaded liquid scintillator and normalised their data by extending the measurement to thermal-neutron energy. The average error in the mean of the combined data presented in Table 4 is $\pm 2.7\%$.

Together with the present results, Table 5 presents the ²³⁹Pu data of Bollinger *et al.*,⁽²⁾ de Saussure *et al.*,⁽⁴⁾ Dubrovina and Shigin,⁽⁵⁾ and Shunk *et al.*⁽⁹⁾ The measurements of Bollinger *et al.* were made, with a 25-m flight path, with the Argonne fast chopper and normalised directly at thermal-neutron energy. de Saussure *et al.* used a 19-m flight path with the Saclay linear accelerator and normalised their data to Bollinger's at 7.8 eV. In the experiment of Dubrovina and Shigin, neutrons were produced in the reaction *T*(*p*,*n*) by an electrostatic generator and the cross section was found from an absolute determination of the neutron

TABLE 2. ERRORS IN THE FINAL DATA

	Isotope			
	²³³ U	²³⁵ U	²³⁹ Pu	²⁴¹ Pu
Data used for normalisation	Moore <i>et al.</i> ⁽¹⁵⁾	Michaudon <i>et al.</i> ⁽¹⁾ and Wang <i>et al.</i> ⁽³⁾	Bollinger <i>et al.</i> ⁽²⁾	James ⁽¹²⁾
Energy ranges used to calculate error in process of normalisation, eV	1.082-2.82 2.82-5.35 5.35-8.0 8.0-14.64 14.64-16.0	1-keV intervals from 3 keV to 10 keV	6.66-9.0 9.0-12.5 12.5-14.44	4-8 8-16 16-32
Error in process of normalisation, %	5	1.8	3.1	3.2
Error in data chosen for normalisation, %	10	2.4	2	3.0
Statistical error in average fission cross section over 1-keV interval from 3 keV to 10 keV, %	2	5	1.5	2.3
Total error in average data over 1-keV interval, %	11.5	6	4.0	4.9

flux, using a calibrated long counter. Shunk *et al.* made their measurements on the Petrel underground nuclear explosion and obtained the cross section in the way used by Bergen *et al.* for the measurement of the ^{233}U fission cross section. The average error in the mean of the ^{239}Pu data given in Table 5 is $\pm 7.3\%$.

Finally, Table 6 gives the fission cross section of ^{241}Pu and the data obtained previously by James.⁽¹²⁾ These two sets of data barely overlap, and it is not possible to quote a value for the error in the mean cross section.

In Table 7, data near 24 keV are compared with the results of Perkin *et al.*⁽⁶⁾ The present data agree with those of Perkin *et al.* for ^{233}U , but disagree with the

high value given by Albert. For ^{235}U the present data are higher than the result of Perkin *et al.* by twice the assigned error, whereas the result of Wang *et al.* is higher by eight times the assigned error. All data are in good agreement for ^{239}Pu , but the present data for ^{241}Pu are higher than the result of Perkin *et al.* by more than twice the assigned error.

4. Conclusion

A comparison of the fission cross-section data available for ^{233}U , ^{235}U , ^{239}Pu , and ^{241}Pu in the energy range from 1 to 25 keV may be summarised as follows. The present data for ^{233}U agree well with the data of Albert *et al.* below 4 keV and with the data of Bergen

TABLE 3. ^{233}U FISSION CROSS-SECTION DATA FROM 1 KEV TO 25 KEV^(a)

Energy range (keV)	Present data σ_f	Albert ⁽⁷⁾ σ_f	Bergen <i>et al.</i> ⁽⁸⁾ σ_f	Mean ^(b) σ_f
1-2	9.39 ± 0.9	9.97 ± 0.30	12.44 ± 1	10.60 ± 0.93
2-3	7.61 ± 0.8	7.93 ± 0.24	8.60 ± 1	8.04 ± 0.29
3-4	5.95 ± 0.6	6.16 ± 0.18	6.84 ± 0.5	6.32 ± 0.27
4-5	—	6.35 ± 0.19	5.71	6.03 ± 0.32
5-6	—	5.78 ± 0.17	4.79 ± 0.5	5.28 ± 0.5
6-7	5.19 ± 0.5	5.56 ± 0.16	4.75	5.17 ± 0.23
7-8	4.42 ± 0.4	4.91 ± 0.15	4.21	4.51 ± 0.21
8-9	4.20 ± 0.4	5.11 ± 0.15	4.40	4.57 ± 0.28
9-10	4.31 ± 0.4	4.70 ± 0.14	4.07 ± 0.09	4.36 ± 0.18
10-20	3.49 ± 0.3	4.03 ± 0.12	—	3.76 ± 0.32
20-25	2.98 ± 0.3	3.60 ± 0.11	—	3.29 ± 0.40

^(a) σ_f is in barns.

^(b) The quoted error is the standard deviation in the mean. All data were given equal weight.

TABLE 4. ^{235}U FISSION CROSS-SECTION DATA FROM 1 KEV TO 25 KEV^(a)

Energy range (keV)	Present data σ_f	Michaudon <i>et al.</i> ⁽¹⁾ σ_f	Wang <i>et al.</i> ⁽³⁾ σ_f	Mean ^(b) σ_f
1-2	—	7.70	7.55 ± 0.15	7.62 ± 0.08
2-3	—	5.81	5.56 ± 0.10	5.68 ± 0.12
3-4	5.04 ± 0.30	4.90	4.88 ± 0.10	4.94 ± 0.07
4-5	4.27 ± 0.26	4.51	4.47 ± 0.08	4.41 ± 0.10
5-6	3.83 ± 0.23	4.26	3.97 ± 0.08	4.02 ± 0.18
6-7	3.51 ± 0.21	3.79	3.49 ± 0.07	3.60 ± 0.13
7-8	3.58 ± 0.21	3.55	3.40 ± 0.09	3.51 ± 0.08
8-9	3.50 ± 0.21	3.51	3.23 ± 0.10	3.41 ± 0.13
9-10	3.61 ± 0.21	3.41	3.37 ± 0.11	3.46 ± 0.10
10-20	3.08 ± 0.18	2.77	3.26 ± 0.11	3.04 ± 0.20
20-25	2.65 ± 0.16	—	3.29 ± 0.11	2.97 ± 0.32

^(a) σ_f is in barns.

^(b) The error quoted is the standard deviation in an individual reading to allow for the fact that the present data are normalised to the mean of the data of Michaudon *et al.*⁽¹⁾ and Wang *et al.*⁽³⁾

TABLE 5. ^{239}Pu FISSION CROSS-SECTION DATA FROM 1 KEV TO 25 KEV^(a)

Energy range (keV)	Present data σ_f	Bollinger <i>et al.</i> ⁽²⁾ σ_f	de Saussure <i>et al.</i> ⁽⁴⁾ σ_f	Dubrovina and Shigin ⁽⁵⁾ σ_f	Shunk <i>et al.</i> ⁽⁹⁾ σ_f	Mean ^(b) σ_f
1-2	—	3.69	5.43	—	—	4.56 ± 0.87
2-3	—	3.12	3.88	—	2.63 ± 0.3	3.21 ± 0.36
3-4	2.81 ± 0.08	3.33	3.40	—	2.75 ± 0.3	3.07 ± 0.17
4-5	2.48 ± 0.08	2.67	2.91	—	2.32	2.59 ± 0.13
5-6	2.37 ± 0.07	2.79	3.21	—	2.71	2.77 ± 0.17
6-7	2.09 ± 0.07	2.70	2.70	2.70	2.21	2.48 ± 0.14
7-8	2.21 ± 0.07	1.80	—	—	2.23	2.08 ± 0.14
8-9	2.32 ± 0.08	2.0	—	—	2.46	2.26 ± 0.14
9-10	2.00 ± 0.07	2.2	—	—	2.12 ± 0.3	2.11 ± 0.06
10-20	1.90 ± 0.05	3.22 (2.03)	—	1.88	—	2.33 ± 0.43
20-25	1.79 ± 0.05	1.85	—	1.88	—	1.84 ± 0.03

^(a) σ_f is in barns.

^(b) The error quoted is the standard deviation of the mean. All data were given equal weight.

^(c) Results in parentheses were obtained by omitting two very high points in Bollinger's data.

TABLE 6. ^{241}Pu FISSION CROSS-SECTION DATA FROM 1 KEV TO 25 KEV^(a)

Energy range (keV)	Present data σ_f	James ⁽¹²⁾ σ_f	Mean ^(b) σ_f
1-2	—	9.47 ± 0.4	—
2-3	—	7.67 ± 0.3	—
3-4	7.10 ± 0.35	6.45 ± 0.35	6.77 ± 0.33
4-5	6.80 ± 0.33	—	—
5-6	6.15 ± 0.30	—	—
6-7	5.70 ± 0.47	—	—
7-8	5.25 ± 0.26	—	—
8-9	5.50 ± 0.27	—	—
9-10	4.81 ± 0.24	—	—
10-20	4.03 ± 0.26	—	—
20-25	3.49 ± 0.17	—	—

^(a) σ_f is in barns.^(b) The error quoted is the standard deviation of the mean. Both data were given equal weight.

TABLE 7. COMPARISON OF DATA NEAR 24 KEV

Isotope	Experimenter	Energy (keV)	σ_f (b)
^{233}U	Perkin <i>et al.</i> ⁽⁶⁾	24	2.73 ± 0.11
	Present data	23.9	2.73 ± 0.26
	Albert ⁽⁷⁾	23.5	3.53 ± 0.10
^{235}U	Perkin <i>et al.</i> ⁽⁶⁾	24	2.36 ± 0.06
	Present data	23-25	2.67 ± 0.16
	Wang <i>et al.</i> ⁽³⁾	20-25	3.29 ± 0.11
^{239}Pu	Perkin <i>et al.</i> ⁽⁶⁾	24	1.66 ± 0.07
	Present data	23-25	1.70 ± 0.09
	Bollinger <i>et al.</i> ⁽²⁾	23.6	1.70
	Dubrovina and Shigin ⁽⁵⁾	25.7	1.68 ± 0.17
^{241}Pu	Perkin <i>et al.</i> ⁽⁶⁾	24	2.83 ± 0.21
	Present data	23-25	3.23 ± 0.15

et al. above 6 keV, and are in good agreement with the results of Perkin *et al.* at 24 keV. The data of Albert are consistently higher than other results above 4 keV.

All the data for ^{235}U are in good agreement below 10 keV. At 24 keV, however, the results of Wang *et al.* are substantially higher than those of the present measurement and those of Perkin *et al.*

The present measurements give data for ^{239}Pu in good agreement with those of Shunk *et al.* and agree fairly well with the data of Bollinger *et al.* All measurements at 24 keV are in good agreement. From 1 to 10 keV the average error in the average cross section over 1-keV intervals is 7.3%. Much of this error comes from the discrepancy between results below 2

keV. Considering only data above 3 keV, the error in the mean is reduced to $\pm 5.4\%$.

A previous measurement for ^{241}Pu between 3 and 4 keV agrees with the present measurement within their combined errors and similarly, at 24 keV, the present data agree with the result of Perkin *et al.*

The data compared derive from experiments which show a wide diversity of time-of-flight equipment and methods of data reduction and normalisation. By finding the average fission cross section for each isotope studied it has been shown that the data for ^{235}U , between 1 and 25 keV, are in good agreement, and that the data for ^{233}U and ^{239}Pu agree fairly well.

It is unlikely that better mean values would be obtained by calculating weighted mean cross sections because it is clear from the tables presented that the errors quoted on some data are too small in view of the observed spread of the results.

Acknowledgments

The author is grateful to Dr. M. J. Poole and Dr. E. R. Rae for their advice and encouragement, to Mr. D. Boreham for preparing the ^{241}Pu foil, to Mr. W. Baker for preparing the ^{233}U and ^{235}U foils, to Mr. D. A. J. Endacott for his help with the experiments, to ENEA Neutron Data Compilation Centre for listing of fission cross-section data, and to Drs. D. W. Bergen and E. R. Shunk for results from the Petrel experiment.

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Discussion

Mr. Butler (ANL): I'm not completely familiar with the technique you used with the linear accelerator, but I believe you mentioned using a monitor of boron. Now if that is so, it implies you used some boron cross section, and I would like to know what cross section and whether this is a thoroughly reliable standard at present.

Mr. James: We used a BF_3 counter and, as I explained, the measurements are relative and have to be normalized at thermal-neutron energy to some other value. The only assumption made is that the boron (n, α) cross section below 25 keV behaves as $1/v$.

Mr. Butler: Is that currently a valid assumption?

Mr. James: At that low energy I think it is.

Mr. Hummel (ANL): In the Pu^{239} fission cross section there are certain energy ranges in which the discrepancies seem to be very large indeed. I think your values are rather low compared to some of the others. Do you have any explanation for this? The differences seem much larger than the 6% average discrepancy we are talking about. Around several keV the discrepancies were really enormous.

Mr. James: The Petrel results of Shunk *et al.*, which are very, very low, were taken from LADC-7620, but the results they sent me, which are average values over one-keV ranges, are not as low as those which I presented. The errors that they claim are all very large. I agree that the situation for Pu^{239} at the moment is in poor shape. We had hoped that the results would be much better than the 6% which I set as the final average error over a 1-keV interval.

Mr. Broido (GA): Were all of the 24-keV Pu^{239} measurements similarly normalized? They are in very

good agreement. But were they from different experimenters?

Mr. James: My results were normalized to the results of Bollinger *et al.* between 6 eV and about 16 eV, so those are similarly normalized. The results of Wang Shih-Ti *et al.* were an absolute measurement using a long counter to determine the flux from their electrostatic generator. The results of Perkin *et al.*, of course, are completely independent, as they measured the flux from an antimony-beryllium source by a manganese bath using several methods. So, the answer is no, except for my own results and Bollinger's.

Mr. Smith (ANL): How do you define the error, particularly on the mean values? You are quoting errors of six percent. What do you mean?

Mr. James: This is done from three measurements. Over three measurements it may not mean very much, but it was merely a method for getting an idea of the spread.

Mr. Smith: Was this a root-mean-square value quoted from the experimental measurements?

Mr. James: This was the spread of the three measurements, without weighting or taking anybody's errors into account. I was going to comment on my findings. Since the spread is so much larger than claimed errors, I didn't think it very reasonable to include them in any way.

Mr. Greebler (GE): Did you attempt your multilevel fit to the high-resolution data at higher energy, say between 100 and 200 eV? If so, was it better than a single-level fit of those data?

Mr. James: No. Our resolution does not allow us to do any fitting much above about 60 eV, and so far I've only gone up to 30 eV. The multilevel parameters there never vary by more than 10% from the single-level parameters, but they do yield a better fit.

Mr. Paik (APDA): I have a question on the fission cross section of U^{235} . I noticed about 12% discrepancy between your measurement and Perkin's at 24 keV, whereas there is practically no discrepancy between your measurement and that of Perkins *et al.* for U^{233} and plutonium. Do you believe that the fission cross section of U^{235} measured by Perkins is on the low side, as other people generally believe, or do you have any explanation for this large discrepancy?

Mr. James: No, I have no explanation for this.

Presented by A. Lottin

Ratio of Capture to Fission in ^{239}Pu at keV Neutron Energies*^e

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ABSTRACT

The neutron capture-to-fission ratio, α , was measured for ^{239}Pu for incident neutron energies from 20 to 600 keV. A pulsed beam of neutrons was collimated on a sample of plutonium placed in the center of a large homogeneous gamma-ray scintillator poisoned with gadolinium. A capture event in the sample was characterized by a single pulse of the scintillator due to the cascade of capture gamma rays, while a fission event was characterized by a pulse due to the prompt fission gamma rays followed, a few microseconds later, by additional pulses due to the gamma rays produced when the thermalized fission neutrons were captured in the gadolinium of the scintillator. Below 100 keV the neutron energies were measured by the time-of-flight technique with a resolution of 7 nsec/m. Above 100 keV approximately monoenergetic neutrons were used. Similar measurements were also performed with ^{235}U and the results are included for completeness. The uncertainties in the values of α obtained are approximately 10%. The data are consistent with measurements performed by other laboratories.

1. Introduction

The neutron capture-to-fission ratio, α , is of particular importance in the design of breeder reactors,⁽¹⁾ and reactor designers have requested this parameter for ^{239}Pu in the energy range from 10 keV to 1 MeV to an accuracy of 5%.⁽²⁾ Hopkins and Diven⁽³⁾ have measured α for ^{239}Pu with approximately monoenergetic neutrons at nine neutron energies between 30 keV and 1 MeV, including two measurements below 150 keV, namely, at 30 ± 10 keV and at 65 ± 15 keV. The values of α obtained at these two energies differ by more than a factor of two. The technique used in the present measurements was very similar to that used by Hopkins and Diven, the main difference being that

below 100 keV the incident neutron energies were obtained by the time-of-flight technique, making it possible to determine α for many different energies, and with a relatively good energy resolution, in the range from 20 to 100 keV.

2. Experimental Technique

Since the experimental technique is described in detail elsewhere,⁽⁴⁾ only the principle of the measurement will be given here. A pulsed beam of neutrons was collimated into a sample of the isotope under investigation (^{239}Pu or ^{235}U) placed in the center of a 1.25-m-diam, homogeneous gamma-ray scintillator poisoned with gadolinium. The tank containing the liquid scintillator is described in Ref. 5. This tank was filled with approximately 250 gal of NE-224, a commercial liquid scintillator,[‡] in which gadolinium 2-ethylhexoate had been dissolved to give a concentration of 16 g/liter. This scintillator was viewed by eight 58 AVP photomultipliers. A capture event in the sample was characterized by a single pulse of the scintillator due to the cascade of capture gamma rays, while a fission event was characterized by a pulse due to the prompt fission gamma rays followed, a few microseconds later, by additional pulses due to the gamma rays produced when the thermalized fission neutrons were captured in the gadolinium of the scintillator.

Two microseconds after a "prompt" pulse was detected, a delayed-coincidence circuit was activated to inspect, during a time interval of 32 μs , for additional pulses due to the capture of fission neutrons in the gadolinium. The probability, ϵ , of detecting a pulse in a 32- μs interval just following a fission was approximately 88%, whereas the probability, P , of detecting a pulse during a "random" 32- μs interval was approximately 7%. This technique for identifying fission and capture events was also used in the measurements reported in Refs. 3, 4, and 6, where more detailed discussions may be found.

* Research sponsored by the U. S. Atomic Energy Commission under contract with Union Carbide Corporation.

[†] On assignment from Centre d'Etudes Nucléaire, Saclay.

[‡] NE-224 produced by Nuclear Enterprises, Ltd., Winnipeg, Canada.

The electronics used was similar to that used in Ref. 4 except that a "fast-slow" coincidence arrangement was used to gather pulse-height and time information. The pulse-height discriminator of the slow system (cross-over pick-off) determined if an event was to be analyzed. The fast pulses from the eight photomultipliers were summed in a passive network and triggered a tunnel diode discriminator with a bias of approximately $\frac{1}{4}$ that of the slow system. The output of the tunnel diode discriminator was used for time information if it was in coincidence with the slow discriminator output.

The time distributions and pulse-height distributions of the pulses from the scintillator were stored simultaneously in two modified 400-channel analyzers. The time analyzer was divided into two sections to store separately the distributions of pulses followed by delayed coincidences and not followed by delayed coincidences. Figure 1 shows an example of time-of-flight spectra. The pulse-height analyzer was divided in four sections to store separately the events "time-gated" over two time-of-flight intervals, followed and not followed by delayed coincidences. The position of these two time intervals is shown in Fig. 1. The prompt interval corresponds to the time during which neutrons are arriving at the sample, whereas the background interval is of equal duration but at a later time, when the neutron burst has passed the sample. Figure 2 shows an example of pulse-height distributions after background subtraction of the coincidence and anticoincidence events in ^{239}Pu .

The ORNL 3-MV pulsed Van de Graaff was used to accelerate protons onto a lithium target; neutrons were produced via the $^7\text{Li}(p,n)$ reaction. The proton bursts

FIG. 2. Pulse-height Spectra for ^{239}Pu .

were 2-ns wide FWHM and repeated every 32 μs . Neutrons of the required energies were obtained by changing the high voltage of the Van de Graaff, and hence the energy of the proton impinging upon the lithium target. Near the threshold for the $^7\text{Li}(p,n)$ reaction ($E_p = 1.88$ MeV) the 30-keV neutrons produced were kinematically collimated in the forward direction by the motion of the center of mass of the reaction.⁽⁷⁾ The experimental arrangement for measurements at 30-keV neutron energies is sketched in Fig. 3. At higher proton energies the neutrons were emitted more isotropically and a neutron collimator was necessary. The arrangement used in this case is sketched in Fig. 4.

The time-of-flight distributions of the various events were obtained with a time-to-pulse-height converter. Figure 1 shows the time-of-flight spectra for 30-keV incident neutrons on ^{239}Pu . The right-hand side of the figure corresponds to events followed by delayed coincidences (fission events and accidental delayed coincidences) whereas the left-hand side corresponds to events not followed by delayed coincidences (capture events, background, and fissions not followed by delayed coincidences). The narrow peak to the right of the time spectra is caused by the gamma rays produced when the proton burst arrives on the lithium target. A lead shield was placed around the target, as shown in Fig. 3, to reduce the intensity of this peak. The time of flight was measured relative to the target

FIG. 1. Time-of-flight Spectrum for ~ 30 -keV Neutrons of ^{239}Pu .

FIG. 3. Experimental Arrangement for Measurement of ~ 30 -keV Neutron Energy

FIG. 4. Experimental Arrangement for Time-of-flight Measurements of α .

gamma-ray peak. The width of this peak is a measure of the time resolution of the system. The height of the target gamma-ray peak in the delayed-coincidence spectrum is a measure of the accidental delayed-coincidence probability. The width of the "sample events" peak in Fig. 1 is determined by the energy spectrum

of the neutron beam, hence by the lithium target thickness, angle of collimation, etc. The time resolution of the system during operation was 7 ns. The measurements at 1-m flight path yielded time-of-flight and pulse-height spectra similar to those in Figs. 1 and 2 but with somewhat poorer signal to background.

The samples used were 100 g of ^{235}U and 20 and 40 g of ^{239}Pu . The ^{235}U sample consisted of ten 2-in.-dia discs that were each sandwiched between 0.005-in.-thick aluminum discs. The aluminum discs were spaced in a 5-in.-OD aluminum can so that the spacing between the ^{235}U plates was 0.10 in. The aluminum can had 0.020-in.-thick end windows. The 20-g ^{239}Pu sample was one disc 3 in. in diameter and similarly canned. The 40-g ^{239}Pu sample consisted of four 2-in. discs, also similarly canned. The 40-g ^{239}Pu sample was used for measurements at 1 m and the 20-g sample was used for the short-flight-path measurement.

The isotopic composition of the ^{235}U sample was 97.5% ^{235}U , 1.6% ^{234}U , 0.06% ^{236}U , and 0.73% ^{238}U . The ^{239}Pu samples contained 0.07% ^{240}Pu and 0.01% ^{241}Pu . The ^{239}Pu was reduced to metal, rolled, and canned by Los Alamos Scientific Laboratory.

3. Analysis of the Data

It was shown in Ref. 4 that α can be obtained from the ratio, γ , of the sample events not followed by delayed coincidence to the sample events followed by delayed coincidence by the relation:

$$\alpha = \frac{\epsilon_f \gamma [1 - C_0 (1 - P)] - C_0 (1 - P)}{\epsilon_c (1 - P - \gamma P)}, \quad (1)$$

where $C_0 = 1 - \epsilon$ is the probability that no delayed coincidence would be detected following a fission, P is the random delayed-coincidence probability defined previously, and ϵ_f/ϵ_c is the ratio of the efficiency of the scintillator for detecting a fission event to the efficiency for detecting a capture event. In Ref. 4 it was also shown how the values of C_0 and ϵ_f/ϵ_c can be obtained from the pulse-height spectra of the events followed and the events not followed by delayed coincidences. The sample events from which γ is obtained must be corrected for two types of backgrounds: a neutron-beam-independent background associated with the specific gamma activity of the sample and with the general background of the detector under operating conditions (cosmic rays, detector activation), and a neutron-dependent background due to the capture gamma rays emitted when neutrons were absorbed in the 25 mls of aluminum in which the samples were canned and when neutrons scattered by the sample were eventually captured in the solution or in some structural material.

The neutron-independent background was measured

by placing a quartz window in front of the Van de Graaff lithium target, thereby preventing the proton beam from reaching that target and producing neutrons. The neutron-associated background was obtained by replacing the plutonium sample with a lead sample (also canned in aluminum) whose thickness was computed to "mock up" the scattering of the plutonium. Pulse-height spectra and time-of-flight spectra of the two kinds of backgrounds were meas-

ured. For the measurements with the two uranium isotopes, an additional small background correction was necessary to correct for ²³⁸U and ²³⁴U isotopic impurities in the sample. These corrections were computed from reported cross sections of those isotopes.⁽⁸⁾

For neutron energies above 100 keV and at 30 keV [near threshold of the ⁷Li(p,n) reaction] the values of γ were obtained as the ratios of the areas corresponding to sample events (see Fig. 2). The neutron energies

TABLE 1. α OF ²³⁹Pu AND ²³⁵U

E (keV)	α		E (keV)	α	
	²³⁹ Pu	²³⁵ U		²³⁹ Pu	²³⁵ U
17.7	0.395 ± 0.108	0.325 ± 0.049	38.5	0.253 ± 0.017	0.340 ± 0.016
18.3	0.490 ± 0.109	0.274 ± 0.044	40.5	0.226 ± 0.016	0.360 ± 0.016
18.8	0.443 ± 0.097	0.337 ± 0.043	42.3	0.246 ± 0.016	0.365 ± 0.016
19.4	0.442 ± 0.089	0.342 ± 0.041	44.5	0.244 ± 0.017	0.365 ± 0.015
20.2 ± 0.6	0.350 ± 0.075	0.445 ± 0.043	46.7	0.286 ± 0.017	0.361 ± 0.015
21.0	0.353 ± 0.071	0.406 ± 0.038	48.5	0.199 ± 0.027	0.335 ± 0.015
21.7	0.406 ± 0.071	0.340 ± 0.031	51.0 ± 2.5	0.198 ± 0.028	0.339 ± 0.016
22.4	0.409 ± 0.048	0.360 ± 0.031	54.5	0.195 ± 0.030	0.322 ± 0.013
23.1	0.371 ± 0.040	0.391 ± 0.031	57.5	0.178 ± 0.032	0.329 ± 0.012
23.9	0.353 ± 0.036	0.373 ± 0.027	60.7	0.176 ± 0.025	0.300 ± 0.011
24.8	0.350 ± 0.034	0.382 ± 0.025	64.0	0.174 ± 0.022	0.355 ± 0.012
25.7	0.355 ± 0.030	0.368 ± 0.024	68.0	0.169 ± 0.021	0.315 ± 0.012
26.8	0.327 ± 0.027	0.372 ± 0.022	72.0	0.165 ± 0.020	0.329 ± 0.013
27.9	0.289 ± 0.025	0.349 ± 0.020	77.0	0.160 ± 0.021	0.350 ± 0.016
29.0	0.281 ± 0.023	0.372 ± 0.020	82.0 ± 5.0	0.172 ± 0.034	0.349 ± 0.022
30.1 ± 1.2	0.329 ± 0.033*	0.384 ± 0.033*			
31.0	0.297 ± 0.020	0.370 ± 0.018	200.0 ± 7.0	0.127 ± 0.008	0.254 ± 0.010
32.3	0.303 ± 0.019	0.367 ± 0.018	300.0 ± 6.0	0.116 ± 0.011	0.215 ± 0.010
33.8	0.288 ± 0.019	0.376 ± 0.018	400.0 ± 6.0	0.078 ± 0.011	0.164 ± 0.008
35.3	0.299 ± 0.019	0.374 ± 0.017	500.0 ± 6.0	0.065 ± 0.005	0.154 ± 0.005
37.0	0.228 ± 0.017	0.374 ± 0.017	600.0 ± 5.0	0.035 ± 0.005	0.129 ± 0.004

* The total error of α is given at 30.1 keV; values at other energies were normalized to this value, and only the statistical error is given for the other energies.

FIG. 5. Comparison of Various Values of α of ²³⁹Pu in the Range from 10 to 1000 keV.

FIG. 6. Comparison of Various Values of α of ²³⁵U in the Range from 1 to 1000 keV.

were computed from the measured proton energies,⁽⁷⁾ and the energy resolution was determined by the thickness of the lithium target.

In the energy range from 20 to 100 keV, the ratio γ was computed channel by channel from the spectra similar to that shown in Fig. 1. The neutron energy corresponding to each channel was obtained by the time of flight over 1 m with a time resolution of 7 ns/m.

The major uncertainties in the values of α were a 6 to 10% uncertainty associated with the uncertainty in the relative efficiencies of the scintillator to detect fission and capture events (ϵ_f/ϵ_c in Eq. 1) and an uncertainty in the value of α of 0.007 associated with the uncertainty in the probability ϵ to detect a pulse in a 32- μ s time interval after a fission (C_0 in Eq. 1). The exact values of these uncertainties were functions of the neutron energy and of the isotope considered.

4. Results and Comparison with Other Measurements

Table 1 lists the values of α obtained for ^{239}Pu and ^{235}U for 40 neutron energies. The values for ^{239}Pu are compared in Fig. 5 with values obtained by Hopkins and Diven⁽³⁾ by a very similar technique, and with values derived from measurements of η by Spivak *et al.*⁽⁹⁾ The figure also shows the result of a measurement by Kanne *et al.*⁽¹⁰⁾ made by a foil irradiation in a reactor spectrum. The agreement between the various data is considered satisfactory, especially considering the different resolutions with which the measurements were performed.

Figure 6 is a similar comparison among values of α for ^{235}U . Again the agreement between various sets of data is fair. Unfortunately, the most accurate data are all taken by very similar techniques so that a possibility remains that some systematic error may be associated with all these measurements. In addition, the uncertainties are generally larger than the 5% requested by reactor designers.

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The Automated Preparation of Multigroup Cross Sections for Fast Reactor Analysis Using the MC² Code

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ABSTRACT

The fully automated multigroup-cross-section preparation program MC² makes use of an evaluated nuclear data file (ENDF) as its basic input. The degree to which the ENDF data is initially subdivided preparatory to averaging is variable in MC². Cross sections in the resolved resonance region are calculated through use of Doppler-broadened line shapes and include effects due to interference scattering and the influence of overlapping resonances in various isotopes in the mixture. Unresolved resonance cross sections are obtained by means of averages over suitable Porter-Thomas distributions of the neutron and fission widths. Quantities which are smoothly varying with respect to energy are represented in the library by coordinates of end points of linear segments taken from $\ln E$ vs $\ln \sigma$, $\ln E$ vs σ , or E vs σ graphs. Inelastic scattering and $n,2n$ matrices are computed from excitation functions for individual levels and by use of a nuclear evaporation model above the resolved region. Elastic-scattering cross sections are computed from Legendre coefficients for the expansions of the angular distribution data for scattering. A fundamental-mode-weighting spectrum for the problem composition may be obtained for either the P1 or the consistent B1 or P1 approximations. Iteration on buckling to criticality is optional. The generated spectrum is used to contract the fine-group cross sections to a specified set of broad-group multigroup cross sections.

The growing availability of accurate basic nuclear data coupled with a desire to achieve an objective standardization in deriving multigroup cross sections has motivated several groups to develop automated computer programs to accomplish this task. Several important examples are the GALAXY code,⁽¹⁾ the

A. I. Cross Section Program,⁽²⁾ and the RBU code.⁽³⁾ These efforts have been guided by recent developments in the theory of cross sections exemplified by the studies of resonance effects.⁽⁴⁻⁶⁾ However, the automated cross-section programs now in existence have a number of limitations. In particular, all lack a total system capability, by which we mean the ability to generate a complete cross-section set, using an Evaluated Nuclear Data File (ENDF), suitable for direct input to neutronics codes without performing ancillary calculations or manipulations. For example, the GALAXY code does not include a detailed calculation in the resonance region. Another serious limitation is the lack of a calculation of a weighting spectrum appropriate to a particular composition. These codes also are generally inflexible in regard to how many groups may be used to average over a given energy range. All of these deficiencies have been eliminated in the fully automated cross-section-preparation system, MC².

In order to attain the present degree of sophistication, the MC² program has had to cope with the problems which arise in proceeding from data retrieval through preliminary averaging, calculation of a weighting spectrum, and then final cross-section averaging. A general solution to these problems will tax the capability of even the most advanced of present day computers. A pedestrian approach would have led to a situation in which the routine preparation of multigroup constants would have been economically prohibitive.

Some of the general features of MC² will now be described.

The degree to which the basic ENDF data is initially subdivided preparatory to averaging is variable in MC². Ideally, sufficiently many groups should be involved so that when they are used for contracting the cross sections there is no effect on the final multigroup set. In the practical case, one is required to conserve

FIG. 1. Schematic Diagram of MC² Operation.

both memory space and computer time so that MC² initially evaluates cross sections on both a fine and an ultra-fine scale.

MC² has a group structure hierarchy consisting of three levels: ultra-fine groups, fine groups, and broad groups. The code computes fine-group cross sections for reactions which are slowly varying with energy. Ultra-fine-group cross sections are generated for resolved resonant reactions and elastic moderation unless the user chooses to make all calculations at a fine-group level.

Fine- and ultra-fine-group fluxes are calculated by use of either a P1 or consistent P1 or B1 fundamental-mode calculation for the specified problem composition with either an input buckling, or iterating on buckling, until criticality is achieved. If a fine-group-only problem is being run, the user may choose to input arbitrary fine-group fluxes rather than have the code calculate them. The code uses the ultra-fine- and fine-group fluxes to collapse the ultra-fine- and fine-group cross sections to equivalent broad-group cross sections.

As a part of the input, the user specifies the number of broad groups, the broad-group energy boundaries, and the fine-group lethargy width. With the present library, the ultra-fine-group lethargy width is 1/120. A typical problem might have 22 broad groups, 60 fine groups, each with a lethargy width of 0.25, and 1800 ultra-fine groups with 30 ultra-fine groups per fine group.

The structure of the code is related to that of the data on the library tape, which consists of five main cate-

gories. The first contains unresolved and resolved resonance parameters. The second consists of linear segment data for various nonresonant quantities; by linear segments is meant a series of energies and corresponding cross sections, for example, with a linear-interpolation law to be used in obtaining values of the cross section at intermediate energy values. The third file on the library tape contains discrete and statistical-level data for the calculation of inelastic and $n,2n$ cross sections. The fourth file contains data pertinent to the calculation of the fission spectra. The last file contains Legendre coefficient data for elastic scattering.

Figure 1 schematically represents the operation of MC² with the flow proceeding from the basic library data at the left to the finally produced broad-group cross sections at the right. Cross sections in the resolved resonance region are calculated by use of Doppler-broadened line shapes with a rational approximation to account for heterogeneities. The interference between resonance and potential scattering, and the influence of overlapping resonances, in other isotopes is allowed.

The calculation of the effective ultra-fine-group cross sections in the resolved resonant region involves a flux weighting assuming $\phi(E) \sim 1/\Sigma_t(E)$, so that for group i and material m

$$\bar{\sigma}_{c_i}^m = \frac{\sum_j \int_{E_i}^{E_{i+1}} \frac{\sigma_{c_j}^m(E)}{\Sigma_t(E)} dE}{\int_{E_i}^{E_{i+1}} \frac{dE}{\Sigma_t(E)}}. \quad (1)$$

The E_i and E_{i+1} are, respectively, the lower and upper energy boundaries of group i , and the summation is over all resonances j belonging to material m which contribute to the group- i cross section. The $\Sigma_i(E)$ in Eq. (1) includes the total resonant plus smooth background scattering for all materials in the mixture and includes the interference scattering contributions.

Cross sections in the unresolved resonance region are computed by taking averages over suitable Porter-Thomas distributions of the neutron and fission widths. The program does the calculation for both s - and p -wave neutrons, and includes a summation over spin states in each case. The program also permits energy variation of the fission width over the unresolved region. A flux-correction factor is used consistent with the treatment used in the resolved resonance region.

The effective capture cross section at energy E^* in the unresolved region is evaluated using

$$\sigma_c^m(E^*) = \sum_s^m \frac{\sigma_p \frac{\langle \Gamma_\gamma J^s \rangle}{\langle D^s \rangle}}{1 - \frac{\langle \Gamma_\gamma J^s \rangle}{\langle D^s \rangle}}, \quad (2)$$

where the summation is over all resonance sequences belonging to material m . The angular bracket $\langle \rangle$ denotes an average over the chi-squared distribution and the other symbols have their usual meaning.

The Doppler line-shape functions used in the resonant-cross-section manipulations are obtained from 6-point bivariate interpolation in a previously generated table of the complex probability integral.⁽⁷⁾ Outside the range of the table, various analytical approximations are utilized consistent with the value of the argument.

For the unresolved region, cross sections are calcu-

lated at discrete energy points and then converted to fine-group cross sections by linear-segment interpolation as described below.

Quantities which are smoothly varying with respect to energy are represented in the library by the coordinates of end points of linear segments taken from $\ln E$ vs. $\ln \sigma$, $\ln E$ vs. σ , or E vs. σ graphs, where σ may be a cross section or other quantity of interest which varies with energy E . Since the quantities tabulated are then linear functions of the energy, they may be easily integrated analytically, by means of an assumed flux shape, to obtain a suitable average over a fine group of arbitrary width. The user may designate a $1/E$, constant, or E -weighting spectrum for the fine-group averaging calculation.

These discrete point data are converted to effective fine-group quantities by use of

$$\overline{\sigma}_{x_i}^m = \frac{\int_{E_i}^{E_{i+1}} \sigma_x^m(E) \phi(E) dE}{\int_{E_i}^{E_{i+1}} \phi(E) dE}, \quad (3)$$

where the user specifies the $\phi(E)$ as $1/E$, constant, or E , and the $\sigma_x(E)$ are represented as linear segments.

The $\sigma_x(E)$ are any of: σ_s , $\bar{\mu}$, σ_{inel} , σ_f , σ_c , $\sigma_{n,2n}$, $\sigma_{n,p}$, $\sigma_{n,\alpha}$, or the data generated for the unresolved resonant region. The σ_f and σ_c are intended to supply any discrepancy between observed cross sections and the values generated by use of the resonance parameters.

Inelastic scattering and $n,2n$ matrices are computed for the fine groups from excitation functions for individual levels and by using a nuclear evaporation model above the region of resolved levels.

Elastic-scattering matrices are generated by use of the methods of the ELMØE code,⁽⁸⁾ and a fundamental-mode spectrum is calculated for the macroscopic homogenized composition cross sections and for a given buckling. The code will iterate on buckling to criticality if desired.

Figure 2 gives a schematic representation of the MC² energy structure as related to the calculation of the fundamental-mode spectrum.

Normally, EMAX will coincide with the energy at the top of the Legendre library data, which is 10 MeV at present.

The fine-group fission spectrum used in evaluating the fundamental-mode spectrum is evaluated by a direct numerical integration of the generalized fission spectrum, given by

$$\chi(E) = \alpha \frac{E}{\tau^2} \exp(-E/\tau) + (1 - \alpha) \sqrt{\frac{4E}{\pi\beta^3}} \exp(-E/\beta). \quad (4)$$

FIG. 2. MC² Energy Structure.

TABLE 1. DIMENSION LIMITATIONS

Quantity	Limit
No. Broad Groups	49
No. Fine Groups	70
No. Ultra-fine Groups	2100
No. Materials	20

The constants α , β , and τ are stored in the library for various fissile isotopes.

The thermal-group data, if any, are supplied by the user and are not generated by MC² directly.

The flux spectrum is utilized to collapse the ultra-fine and fine-group cross sections to be consistent with the specified broad-group structure. Both homogenized macroscopic broad-group cross sections and the individual material, microscopic, broad-group cross sections are computed.

Table 1 lists some of the more important dimension limitations of the MC² code.

By way of interest, it may be mentioned that MC² operates within the overlay capability of the CDC-3600 and, except for one machine-language subroutine, is written exclusively in 3600 FORTRAN. The code utilizes approximately 50000 locations and requires 9 magnetic tapes in addition to the usual monitor-system requirements.

A detailed description of the MC² code will be presented in a forthcoming ANL topical report which is in preparation.

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Discussion

Mr. Lantz (NASA): When you use the ratio of the capture widths to fission widths, don't you preclude allowing accurate calculation of possible interference effects with other materials?

Mr. Toppel: We are specifically using a single-level formulation, so any multilevel effects are omitted. This is a deficiency of our approach.

Mr. Codd (U. K.-Winfrith): Could you repeat what you said about the 20-min computing time? Was that the total time to produce a complete set of broad-group multigroup constants?

Mr. Toppel: The particular problem that I quoted required about 20 min, and it involved 228 resolved resonances for U²³⁸ and the one sodium resonance. It had 51 fine groups and 1530 ultra-fine groups, since there are 30 ultra-fine groups per fine group. There were four materials: nickel, oxygen, sodium, and U²³⁸. It is a very unrealistic problem, but is used just for running-time considerations.

Mr. Codd: Could you give an idea of the running time to produce a 30-broad-group set of constants for the materials in a typical fast power reactor: plutonium, U²³⁸, steel, sodium, and so forth?

Mr. Toppel: I think that the number of broad groups is irrelevant. It is the number of fine groups that matters. The fine groups could be collapsed to 2, 3, 20, etc., broad groups without much change in running time. The time depends strongly on whether one is iterating on buckling to criticality, and how many resonances are involved. This particular problem included U²³⁸, which involves many resonances. We have run problems with more materials, typical of a fast reactor composition. Iterating on buckling, these problems have run as long as 45 min or so.

Mr. Reynolds (GE): You indicated that the fine groups were condensed on the fundamental-mode spectrum. Do you have any plans to condense on a spatially dependent spectrum?

Mr. Toppel: The spatially dependent problem is an historic one. One is constantly faced with the problem of generating cross sections appropriate for a blanket, for example. In the immediate future we don't plan to do more than this. On the other hand, the longer-range plans involve improving the resonance calculation by using a method typified by the RIFF-RAFF code of Kier, in which a heterogeneous-cell calculation is done.

Eventually one would like to do a space-dependent consistent B_N calculation, where N is arbitrary. I think that after we get this system implemented fully we can turn our attention to that, but we are not involved in the conversion as yet.

Mr. Levine (BNL): When you put in your own weighting spectrum, do you put it in on the broad-group or the fine-group scale?

Mr. Toppel: The fine group, typically quarter lethargy units.

Mr. Olhoeft (WAPD): I agree that spatial calculations would be very ambitious. Would it be possible to put in the buckling as group dependent? I understand from your remarks that it would be constant.

Mr. Toppel: It is constant in the present version.

Mr. Olhoeft: Certainly, for certain regions of the reactor, there is a rather substantial change in the buckling and sometimes a change in sign. If this could be incorporated, it would be very helpful.

Mr. Toppel: Yes.

Mr. Olhoeft: Another question I have is with regard to doing a set of calculations. Would there be provision that you could do a super-fine calculation, followed by several fine calculations in which the temperature is varying when you take account of broadening of the resonances? Do you have to go through the complete calculation each time?

Mr. Toppel: At present, if one is trying to generate temperature-dependent cross sections, one goes through the entire calculation.

Mr. Olhoeft: For certain survey-type calculations, one might be a little hesitant to use 20 min for each generation of cross-section sets, but would like to be able to do the ELMOE-type calculation followed by some calculations for temperature-dependent cross sections, assuming that the ELMOE flux would be fairly satisfactory.

Mr. Toppel: Yes. One can always run two MC² problems at the limits of the two temperatures, and then do interpolations. Typically, for survey calculations one will use a single cross-section set for the range of problems, and then only at the end do a more refined job.

Mr. Olhoeft: Do you have an idea of what the relative time would be between the resonance part of the calculations and the ELMOE calculations?

Mr. Toppel: It depends on whether the problem involves many resonances, and whether or not there is iteration on buckling. Roughly, for a large problem, half the time might be spent in the resolved-resonance region and half in the Legendre elastic-scattering treatment.

Mr. Gyorey (G. E.): I would like to express concern over writing a code as important as MC² in a highly machine-dependent form. You pointed out that you are using something like 50K of memory, and that CDC FORTRAN is used. What effort, if any, is being made to make this code available to those who do not have CDC machines?

Mr. Toppel: The code is going to be made available universally in its present form, with the idea that the installation accepting the code takes on the responsibility of converting it. I do not think CDC 3600 FORTRAN presents a big problem because MC² began as a 704 code, which involved FORTRAN II. We haven't particularly attempted to take advantage of some of the elegance of 3600 FORTRAN. We don't have eight characters in the names, for example. There are buffer statements, but this is the kind of problem that you have to overcome. It would be fairly simple to convert to some other FORTRAN. The size of the program is really machine-dependent. Very many places do not have two banks of memory. I think that this means that the user would have to cut down on dimensions for example. I'm sorry.

Mr. Russell (General Atomic): Are there any provisions for taking into account the effect of self-shielding of scattering resonances when calculating the group transport cross section?

Mr. Toppel: That is done just as in the ELMOE code. The Legendre data are put in at a constant lethargy spacing of 1/120, which is a little bit smaller than in the original ELMOE program. We are not doing anything like Doppler broadening of these numbers, for example.

Mr. Hummel (ANL): In the original ELMOE code we did have this supplementary routine called the Narrow Resonance Approximation where we did this. We are not using it currently. For later versions we are planning self-shielding calculations for all of the resonance isotopes using the Narrow Resonance Approximation with all of the scattering resonances immediately at the beginning of the calculation. That will not be available in the present version of MC² or in the first version of the ARC system which will use the present MC².

Mr. Moldauer (ANL): You seem to suggest that 20 Legendre coefficients for elastic scattering were not available as input information for most of the materials of interest. It seems to me that, particularly in the region of unresolved resonances and above, the most advantageous way to obtain this information is to fit elastic-scattering data with an optical model, and then obtain Legendre coefficients from the optical model. In this way one can obtain all the Legendre coefficients

needed for all the materials and, in fact, very good information on them can be obtained for materials for which measurements are not available. This is probably one of the few instances where reliable theoretical information is available for which there are no measurements.

Mr. Toppel: I only meant to say that we do not have these data in the library at the moment. The reason we extended the code capability to 20 is because we have heard from Alter and others that these data are not only necessary but available. We anticipate that we will shortly have the numbers.

Mr. Pitterle (APDA): Is the lethargy width of 1/120

fixed by the code, or is it fixed by the data that you are now using?

Mr. Toppel: At the moment it is fixed by both. The code expects that is what it is going to be.

Mr. Pitterle: In other words, in conjunction with ENDF/B, it should be set at 1/120?

Mr. Toppel: Yes. The ENDF/B is a national effort to set up an evaluated nuclear-data file which will contain most, if not all, of the isotopes of interest. Mr. Pitterle is currently concerned with generating a conversion program which will accept the ENDF/B format and convert it to conform to the MC² library requirement.

21.19.1

The Atomics International Evaluated Nuclear Data Files and Associated Computer Programs for the Automated Preparation of Multigroup Constants*

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ABSTRACT

Input data from the Atomics International Evaluated Nuclear Data File (AIENDF) tape are processed to give spectrum-weighted group-averaged neutron cross sections and other quantities required for the solution of the neutron transport equation by the methods of multigroup diffusion and transport theory. The calculation of multigroup Doppler-broadened effective resonance integrals and cross sections in the resonance region for a heterogeneous or homogeneous resonance absorber is based on the single-level Breit-Wigner theory with no overlapping of neighboring resonances. Inelastic- and elastic-scattering transfer matrices are calculated by means of differential scattering-cross-section data stored on the AIENDF Angular Distribution Data Tape (ADDT). Multigroup libraries are punched in any of the formats required by the one-dimensional multigroup, diffusion-theory codes, ULCER, FAIM, FAIM-CELL, CAESAR, and CAESAR IV; the one-dimensional multigroup S_n transport theory code, DTF; and the spectrum codes, FORM and AILMOE.

1. Introduction

The preparation of nuclear data for input to neutronic codes is a process consisting of several steps. Initially, existing experimental and theoretical data on neutron cross sections for a given element or isotope are surveyed, compiled, evaluated, and a set of best values determined. Where gaps in data exist, calculations based on a nuclear model are used to provide needed information.⁽¹⁻⁴⁾

The second step involves the preparation of the "best" data so that it may be stored on punched cards, magnetic tape, or disk. The Atomics International Cross Section Project Computer Library consists of a number of computer programs⁽⁵⁻⁸⁾ which manipulate the basic microscopic data preparatory to storage on magnetic tape or disk. These programs, in addition to

constructing the nuclear data file, list selected elements/isotopes from the data file; update, add, or correct any parameter on the data file; and provide graphical display of any or all parameters for any or all nuclides on the data file.

In the final step, the stored nuclear data are converted to multigroup constants for use with diffusion theory, transport theory, and/or Monte Carlo neutronic codes. This conversion is carried out by a number of computer programs⁽⁹⁻¹⁵⁾ which mutually interface with the nuclear data files. These programs provide means of producing region-dependent as well as region-independent multigroup constants, provide various techniques for treating resonance data in the resolved as well as the statistical energy regions, and provide means of generating elastic, inelastic, and total transfer matrices which account for anisotropy in both elastic and inelastic differential scattering.

2. Atomics International Evaluated Nuclear Data Files (AIENDF)

The 11 energy-dependent cross-section parameters stored on the AIENDF tape⁽¹⁶⁾ include the total, elastic-scattering, radiative-capture, fission, inelastic-scattering, (n,α) , (n,p) , and $(n,2n)$ cross sections along with μ_L , the average value of the cosine of the scattering angle in the laboratory system, ξ the average logarithmic energy decrement per elastic collision, and $\bar{\nu}$, the average number of neutrons emitted per fission.

The values of the 11 cross-section parameters are tabulated between 10^{-3} and 10^7 eV at the following energy points:

$$10^{-3}(10^{-4})10^{-2}(10^{-3})10^{-1} \cdots 10^5(10^4)10^6(10^5)10^7 \text{ eV};$$

i.e., the energy interval is one-tenth of the lower energy of the decade. (Energy grid point number 106 is 0.0253 eV rather than 0.025 eV.) Angular-distribution data⁽¹⁷⁾ is analyzed in a Legendre series expansion and the data stored in the form of Legendre coefficients for energies greater than 10 keV. Since elastically scattered neutrons

* Work supported by U. S. Atomic Energy Commission

are scattered symmetrically about the azimuthal angle, the distribution may be represented by an expansion in Legendre polynomials of the form

$$\frac{d\sigma}{d\Omega}(E, \mu) = \frac{\sigma(E)}{4\pi} \sum_{l=0}^{\infty} (2l+1) f_l(E) P_l(\mu), \quad (1)$$

where μ is the cosine of the scattering angle in the laboratory or center-of-mass system and E is the energy of the incident neutron. A 9th-order Legendre expansion was made for all angular distributions; in most cases, the input data were reconstituted to within 0.1 % (integral error). Where a 9th-order fit proved to be inadequate to reconstitute the measured angular distribution and ensure a positive-definite distribution, a 14th-order fit was used. This is the maximum number of coefficients presently employed by neutronic codes in use at Atomics International. For several elements, calculated, rather than measured, distributions are included. (For example: angular data for plutonium-238 and curium-244 were calculated by means of a non-spherical optical code.⁽³⁾)

The AIENDF cross-section libraries contain smooth data in the absorption resonance region for the heavy isotopes. These data are intended to supplement the results of resonance-theory calculations in order to compensate for any deficiencies in the Breit-Wigner single-level formula. Such a smooth cross section contributes an unselfshielded, unbroadened component to group constants in the resonance region of an isotope.

At the present time these smooth-resonance data are zero for the capture cross section and an average of potential plus resonance scattering for the scattering cross section for the fertile isotopes. The single-level

formula breaks down for the fissile isotopes, for which there is strong interference between fission channels. The defect is most clearly illustrated by the underestimation of the fission resonance integral by a single-level calculation. Therefore, smooth cross sections have been added in the resonance region for the fission cross section of fissile isotopes. These smooth cross sections are obtained from the resonance-parameter analysis of H. Hennes⁽¹⁸⁾ and are assumed to be constant over various energy intervals. These constant cross sections were obtained from the difference between an infinitely dilute group-averaged fission cross section determined from measured differential data and a resonance integral calculation for the best set of single-level parameters. Whenever this difference was negative, a value of zero was chosen for the smooth cross section.

Inelastic-scattering data are broken into level-excitation components when such information is available. Otherwise, evaporation-model parameters are assumed.

Nuclides, for which cross section and angular distribution data are currently available on the AIENDF, are given in Table 1. Angular-distribution data for the nuclides in BNL-400 are also available.⁽¹⁷⁾

3. Preparation of Multigroup Constants

GRISM⁽¹⁵⁾ is a computational system originally written in FORTRAN II for the IBM 7094, and since converted to FORTRAN IV for the IBM 360, which produces multigroup cross sections averaged over any given flux spectrum for use in calculations by multigroup diffusion and transport theory. For fast reactor calculations the required flux and current spectra can be conveniently obtained from the fast-spectrum code AILMOE,⁽¹⁴⁾ which produces punched card output in the format required by the GRISM code.

The system links together four codes which have been in operational use for some time at AI: GRAVE,⁽⁹⁾ TRIxie,⁽¹⁰⁾ PRISM,⁽¹¹⁾ and CONTRACT.^(12, 13)

In the GRAVE link, the microscopic cross sections are read from the Atomics International Evaluated Nuclear Data File (AIENDF) tape. Up to 20 group-averaged cross sections can be formed from 11 cross-section parameters read from the tape.

The TRIxie link, a modified version of the TRIx⁽¹⁰⁾ code, computes multigroup Doppler-broadened resonance integrals and cross sections for homogeneous or lumped-resonance absorbers based on the single-level Breit-Wigner theory with no overlapping of neighboring resonances. Zero-degree resonance integrals are calculated according to the approximate analytical formulation of Chernick and Vernon⁽¹⁹⁾ as extended to intermediate resonances by Cohen and Goldstein.⁽²⁰⁾ The Doppler broadening of the symmetrical component of a resonance is calculated from a table of the J function.⁽²¹⁾

TABLE 1. ELEMENTS/ISOTOPES AVAILABLE ON AIENDF CROSS-SECTION DATA TAPES

Isotope	ID Number	Resonance Region on Tape	Isotope	ID Number	Resonance Region on Tape
Hydrogen	011	no	Nickel	289	no
Deuterium	012	no	Zirconium	401	no
Lithium-6	036	no	Molybdenum	426	no
Lithium-7	037	no	Thorium-232	902	yes
Beryllium	049	no	Uranium-233	923	yes
Boron-10	050	no	Uranium-234	924	yes
Boron	051	no	Uranium-235	925	yes
Carbon	062	no	Uranium-236	926	yes
Nitrogen	074	no	Uranium-238	928	yes
Oxygen	086	no	Plutonium-238	948	yes
Sodium	113	no	Plutonium-239	949	yes
Aluminum	137	no	Plutonium-240	940	yes
Potassium	199	no	Plutonium-241	941	yes
Titanium	228	no	Plutonium-242	942	yes
Chromium	242	no	Curium-244	964	yes
Iron	266	no			

The scattering interference component is broadened through use of a tabulation of Rothenstein's formulation for small scattering interference.⁽²²⁾ Temperature-independent contributions from $1/v$ tails of resonances and from a negative-energy resonance where applicable are included. Unresolved resonance integrals are computed in the narrow-resonance approximation using average resonance parameters and statistical distributions of scattering and fission level widths. The equivalence of heterogeneous to homogeneous cases is made by means of a modified form of the Wigner rational approximation to the escape probability in lumps.⁽²³⁾ Multigroup cross sections are computed by weighting the resonances by $E\varphi(E)$, summing over each energy group, and dividing by the respective integrated group flux. The $\varphi(E)$ spectrum used may be either a $1/E$ spectrum or the $\varphi(E)$ spectrum input to the GRAVE link (e.g., an AILMOE spectrum). Depletion of the group flux by the resolved resonance is taken into account by a flux-depression correction to the group lethargy width by which the resonance integrals in the group are divided to form the effective group cross section. Integrals of broad resonances are split among the groups to which they contribute significantly according to the $T = 0^\circ\text{K}$ line shape.

In the PRISM link, the inelastic- and elastic-scattering transfer matrices are calculated from differential scattering-cross-section data read from the Angular Distribution Data Tape (ADDT). Two complementary procedures are used to calculate the matrix elements for inelastic scattering:

- 1) When both the levels and their probabilities of excitation as functions of the energy of the incident neutron are known, the energy levels of the target nucleus system are treated as discrete.
- 2) When such detailed information is lacking, the calculation of the inelastic-scattering matrices is based on the Weisskopf evaporation model.

Elastic-scattering matrices are calculated by use of the discrete-level model with zero threshold energy.

In the CONTRACT link, the multigroup constants from GRAVE, PRISM, and TRIXIE are reformatted and are punched in the formats required by the following multigroup neutronic codes in use at Atomic International: the one-dimensional diffusion theory codes, ULCER,⁽²⁴⁾ FAIM,⁽²⁵⁾ FAIM-CELL,⁽²⁶⁾ CAESAR,⁽²⁷⁾ and CAESAR IV;⁽²⁸⁾ the one-dimensional S_n transport theory code, DTF;⁽²⁹⁾ and the spectrum codes, FORM⁽³⁰⁾ and AILMOE.⁽¹⁴⁾

4. Non-resonance-region Group Averages

Given the group limits $E_1, E_2, \dots, E_{N+1}$ for N groups, the spectrum-weighted group-averaged cross sections are defined as

$$\sigma_j = \frac{\int_{E_j}^{E_{j+1}} \sigma(E) \varphi(E) dE}{\int_{E_j}^{E_{j+1}} \varphi(E) dE}, \quad j = 1, \dots, N, \quad (2)$$

where $\varphi(E)$ is the flux spectrum and $\sigma(E)$ represents any of the given cross-section parameters with the exception of μ_L , ξ , $\bar{\nu}$, and σ_r (elastic removal cross section).

The elastic removal cross section from group j to group $j + 1$ is calculated from the following formula, based on the age approximation:

$$\sigma_r^j = \frac{\xi \sigma_s}{\Delta u_j}. \quad (3)$$

σ_r^j is useful only for broad groups and/or heavy scatterers, i.e., when ξ is small. The methods used for calculating rigorously the inelastic and elastic-scattering transfer matrices are described in Ref. 31.

5. Flux and Current Spectra

The user of GRISM must specify the flux spectrum to be used in forming the group averages of the cross-section parameters. The user must also input a second spectrum if the transport cross section is to be averaged over the current spectrum. Flux and current spectra obtained from the AILMOE code (an adaptation by Atomic International of the ANL code ELMOE⁽¹⁴⁾) or other spectral codes such as FORM⁽³⁰⁾ and TEMPEST,⁽³²⁾ can be entered as tabulated data over designated parts of the spectrum while other parts of the spectra can be designated as $1/E^n$, Maxwellian, or one of several fission spectra which are generated by the GRISM code.

The spectrum is generated at each energy point of the 901-point energy grid covering the range specified, and if the spectrum is given in tabular form at energy points other than the AIENDF energy grid points, linear interpolation is used to obtain spectrum values at the grid points. The normalization of the spectrum in each specified region is such that the spectrum is continuous over the entire energy range.

6. Resonance-region Calculations

The effective resonance integral,

$$I = \int \sigma_{\text{eff}} \frac{dE}{E},$$

where σ_{eff} is the effective resonance-absorber microscopic cross section, is simplified by assuming that in neutron collisions with the fuel, the lethargy gain of the neutron is either very large (NN approximation) or very small (NW approximation) compared to the lethargy width of the resonance. If the energy de-

pendences across the resonance of the neutron wavelength and scattering level width are ignored, the resonance integral is analytically integrable for homogeneous mixtures of resonance absorbers and moderators. Even with the above assumptions, the integral cannot be analytically evaluated for lumped-resonance absorbers. However, if the integrands are formulated through use of escape probabilities in Wigner's rational approximation, a heterogeneous system is equivalent to a homogeneous one if the moderator scattering cross section is taken to be

$$\Sigma_m = \frac{1}{l} + \Sigma_m^i,$$

where Σ_m^i is the scattering cross section of any moderator mixed in the lump, and l is the average chord length of the lump. For convex lumps,

$$l = 4V/S,$$

where V is the volume and S is the surface area of the lump. In the case of Dancoff shadowing, S is the effective surface.

Analytic expressions for resonances for which energy degradation in fuel collisions is intermediate between the NN and NW approximation have been developed by Goldstein and Cohen.⁽²⁰⁾ They determined a location between the extreme approximations by using successive approximations to the flux or by variational principles. Their treatment of resonance absorption intermediate between the narrow and wide resonance approximations was extended to heterogeneous geometry using the equivalence relationship.

7. Unresolved Resonances

Resonance integrals in the energy region where resonances are unresolved are calculated by use of average resonance parameters and statistical distributions of level widths. Integrals for orbital-angular-momentum states both with $l = 0$ and $l = 1$ are calculated. Since the resolved-resonance calculation is made for $l = 0$ only, the unresolved $l = 1$ calculation is made down to a low-energy cutoff below which the $l = 1$ contribution is negligible. The cutoff is presently chosen as 20 eV.

Average resonance parameters for $l = 0$ neutrons are taken from the resonance parameter library. $\langle \Gamma_n^0 \rangle / \langle D \rangle$ for $l = 1$ neutrons is also taken from the library, where D is the spacing between resonances and the reduced level width is $\Gamma_n^0 = \Gamma_n / \sqrt{E}$.

The recommended group-averaged fission widths for fissile isotopes in the unresolved region have been correlated with mean group lethargy.⁽¹⁸⁾ These correlations are used by TRIxie when calculating group fission cross sections in the unresolved region. The resonance

integral at each energy point is found by averaging over the statistical distributions of scattering and fission level widths. The Porter-Thomas distribution (the chi-squared distribution with one degree of freedom) was adopted for scattering widths. The fission-width distribution was taken to be the chi-squared distribution with three degrees of freedom. The scattering-width distribution has good experimental verification, whereas the fission-width distribution was chosen from several which are fairly equally supported by experimental evidence. The resonance integral is then taken to be the average of the integrals calculated by use of a set of samples from the level-width distributions. It is given by

$$I_g = \sum_K \frac{H_K}{\langle D \rangle} \frac{1}{n_1 n_2} \sum_{i=1}^{n_1} \sum_{j=1}^{n_2} I(\Gamma_{ni}, \Gamma_{fj}, E_K), \quad (4)$$

where the set of Γ_{ni} or Γ_{fi} are chosen so that all the widths are equally probable, n_1 is the number of sample points taken in the distribution of scattering widths, and n_2 is the number of sample points taken in the distribution of fission widths. $1/v$ and negative-energy resonance contributions to the resonance integral are calculated independently of temperature and concentration. Groupwise integrals for one negative-energy resonance are calculated for the resolved energy range if the pertinent parameters are included in the binary library.

Group cross sections are formed by weighting the resonance cross sections by $E\varphi(E)$, summing over each energy group, and dividing by the respective integrated group flux:

$$\sigma_{ag} = \frac{\sum_i E_i \varphi(E_i) I_i(T)}{\frac{\Delta_g}{\Delta u_g} \int_{E_L}^{E_u} \varphi(E) dE}, \quad (5)$$

where

Δu_g = the lethargy width of the group;

$$\Delta_g = \Delta u_g - \frac{\pi}{2} \sum_i \frac{\Gamma_i I_i(T)}{E_i \beta_i I_{\infty i}}, \quad (6)$$

and $\varphi(E)$ is the input neutron spectrum. $\varphi(E)$ can be either the spectrum input to the GRAVE link (e.g., an AILMOE spectrum) or an internally generated $1/E$ spectrum. The quantity Δ_g accounts for the depression in the spectrum caused by the resonance absorber itself. It is taken equal to Δu_g for the unresolved resonances, since the flux depletion due to the resonances is small there.

Resolved resonances which are sufficiently broad to have significant resonance integrals in more than one group have their integrals split into those groups. The splitting is also accounted for in the summation term of

the integrated flux. A split is made whenever the contribution to a group, other than that in which the resonance lies, is greater than 0.1 barn. The split is based on the $T = 0$ line shape excluding scattering interference, $\psi = 1/(1 + x^2)$, so that the fractional contribution to a group is

$$I_g = \frac{I(T)}{\pi} \left\{ \tan^{-1} \left[\frac{I(0)}{I_\infty} x \right] \right\}_{E_{Lg}}^{E_{Ug}}. \quad (7)$$

Absorption and fission cross sections are calculated by TRIxie for a resonance energy region ($E_{\min}$, $E_{\max}$) which is specified for each isotope in the resonance-parameter library.

Outside of the resonance region these cross sections are taken from the AIENDF tape. For many isotopes there exists a single low-lying resonance with E_r below the lower limit of the band of energies ($E_{\min}$, $E_{\max}$) which defines the resonance region for the given isotope. In general, its high-energy tail persists into the resonance region with noticeable magnitude. The contribution to the total resonance integral from this tail is included.

8. Scattering Transfer Matrices

The averaging of the differential transfer cross section for scattering via the l -th reaction channel over incident energies in transmitter group j and the integration over all contributions to final energies in receptor group i , is expressed by

$$\begin{aligned} \bar{\sigma}_\lambda^l(i \leftarrow j) &= \frac{\int_{E_{j-1,1-1}}^{E_{j,1}} dE' \int_{\epsilon_{i-1}(E')}^{\epsilon_i(E')} dE \sigma_\lambda^l(E \leftarrow E') \psi_\lambda(E')}{\psi_{j\lambda}}, \quad (8) \\ & \quad i \leq j, \end{aligned}$$

where

$$\sigma_\lambda^l(E \leftarrow E') = 2\pi \left\{ \frac{d\sigma^l[E', \eta(E, E')]}{d\Omega} \right\}^{\text{LAB}} \frac{\partial \eta(E, E')}{\partial E} P_\lambda[\eta(E, E')], \quad (9)$$

(ϵ_{i-1} , ϵ_i) are the effective lower and upper limits of the i -th receptor group consistent with the requirement that it is physically possible for neutrons in group j to reach some part of group i , and ($E_{j-1,1-1}^*$, $E_{j,1}^*$) is that portion of group j from which neutrons are able to make transitions to group i .

Because the data for the differential scattering cross section on the AI Angular Distribution Data Tape are in the form of tabulations of Legendre coefficients versus energy, it is convenient to re-express the integration with respect to final energies in terms of the cosine of

the scattering angle η . This change of integration variables gives

$$\begin{aligned} \bar{\sigma}_\lambda^l(i \leftarrow j) &= \int_{E_{j-1,1-1}^*}^{E_{j,1}^*} dE' \sigma_{s0}(E') \sum_{\lambda'=0}^{\infty} \\ & \quad \frac{2\lambda' + 1}{2} f_{\lambda'}(E') \int_{\eta[\epsilon_{i-1}(E'), E']}^{\eta[\epsilon_i(E'), E']} d\eta P_{\lambda'}(\eta) P_\lambda(\eta), \quad (10) \end{aligned}$$

where the differential scattering cross section has been given by the Legendre series:

$$\frac{d\sigma^l(E', \eta)}{d\Omega} = \sigma_{s0}(E') \sum_{\lambda'=0}^{\infty} \frac{2\lambda' + 1}{4\pi} f_{\lambda'}(E') P_{\lambda'}(\eta). \quad (11)$$

The number of neutrons which suffer scattering collisions in group j must equal the sum of all neutrons making transitions from group j to all physically possible receptor groups. This balance condition for the P_0 scattering matrix is expressed by

$$\sum_{i \leq j} \bar{\sigma}_0(i \leftarrow j) = \bar{\sigma}_0(j). \quad (12)$$

The percent deviation from balance is given by

$$\delta_j = \left[\frac{\sum_{i \leq j} \bar{\sigma}_0(i \leftarrow j)}{\bar{\sigma}_0(j)} - 1 \right] 100. \quad (13)$$

The balance condition has been utilized to detect errors introduced by numerical procedures and approximations. A particular example of such a usage appears on p. 44 of Ref. 31.

In both the GRAVE and PRISM links, all integrations with respect to energy are approximated by the trapezoidal rule. It should be noted that, whenever light resonance scatterers comprise a significant fraction of the core medium, both the microscopic scattering cross sections and the neutron spectrum exhibit strong and rapid variations with energy. In many cases the energy profiles of these quantities have slope discontinuities at their peaks and troughs. Serious inaccuracies can result if a quadrature rule based on fitting a polynomial arc through several points is used. For example, were a point of slope discontinuity to fall between the two end points of a 3-point sample through which a quadratic arc has been fit, serious over- or under-estimation can result.

9. Multigroup Library Punched Output

The CONTRACT link of GRISM outputs CRT displays of the flux and current spectra, and punches decimal cards for use with various AI neutronic codes. These cards contain multigroup cross sections for input to the library generation routines for DTF, FAIM, FAIM-CELL, CAESAR, CAESAR IV, FORM, ULCER, and AILMOE. In addition, a complete binary

library and/or tape can be produced for use by CAESAR IV.

Any combination of punched multigroup libraries for the above codes can be obtained based on a single GRISM calculation. For example, decimal cards for DTF, AILMOE, and CAESAR and a binary library for CAESAR IV might be obtained in one GRISM calculation.

10. GRISM Sample Output

The GRISM output for a 16-group plutonium-239 calculation is given in Appendix 1. The output data

includes the inelastic- and elastic-scattering matrices, the resonance-region self-shielded absorption and fission group cross sections, the group-averaged cross-section parameters, and a CRT display of the flux and current spectra used to weight the group-averaged cross sections (see Figures 1 and 2). Sample listings of punched-card output for the diffusion-theory code FAIM and the transport-theory code DTF are presented.

The plutonium-239 sample calculation took 1.5 min on the IBM 7094. This is typical of the GRISM running time for those isotopes for which resonance (TRIXIE) calculations are required. The running time on the

FIG. 1. Flux and Current Spectra between 10 and 10^7 eV.

FIG. 2. Flux and Current Spectra between 0.001 and 10 eV.

IBM 7094 for isotopes which do not require resonance (TRIXIE) calculations is approximately 40 sec.

II. Summary

The preparation of nuclear data for input to neutronic codes has been automated to a high degree at Atomic International. Experimental and theoretical data on neutron cross sections for a large number of isotopes have been surveyed, compiled, and evaluated, and a set of best values has been determined. Where gaps in data exist, nuclear-model calculations are used to

provide needed information. Eleven cross-section parameters have been specified at 90 uniform points per decade, over a 10-decade span (0.001 eV to 10.0 MeV) for each isotope. Angular-distribution data in the form of Legendre coefficients have been specified in a similar fashion, with the lowest energy point at 10 keV.

A number of computer programs which manipulate the basic microscopic data preparatory to storage on magnetic tape or disk have been added to the Atomic International Cross Section Project Computer Library. These programs in addition to constructing the nuclear data file, list selected elements/isotopes from the data

file; update, add, or correct any parameter on the data file; and provide graphical display of any or all parameters for any or all nuclides on the data file.

Stored basic data have been converted to multigroup constants for use with diffusion-theory, transport-theory and/or Monte Carlo neutronic codes. This conversion was carried out by a number of computer programs which mutually interface with the nuclear data files. These programs provide means of producing region-dependent as well as region-independent multigroup constants; provide various techniques for treating

resonance data in the resolved as well as the statistical energy regions; provide means of generating elastic, inelastic, and total transfer matrices which account for anisotropy in both elastic and inelastic differential scattering.

By these techniques multigroup libraries in varied group structures have been prepared for the analysis of several uranium-235-fueled ZPR-III critical assemblies, the plutonium-239-fueled SEFOR mockup, and the plutonium-239-fueled ZPR-III critical assembly 48.

APPENDIX I

CROSS SECTION OPTIONS ARE -

I CROSS SECTION OPTION

1	TOTAL	1
2	ELASTIC	1
3	N, GAMMA	1
4	FISSION	1
5	NU	1
6	MU BAR	1
7	XI	1
8	INELASTIC	1
9	N, ALPHA	1
10	N, P	1
11	N, 2N	1
12	ABSORPTION	1
13	TRANSPORT	5
14	NON-ELASTIC	1
15	ALPHA	1
16	XI*ELASTIC	1
17	NU *SIGMAF	1
18	MU BAR* EL	1
19	REMOVAL	1
20		1
21		1
22		0

GRISM - GROUP CROSS SECTION AVERAGING CODE

GRISM SAMPLE OUTPUT

SPECTRUM 1 HAS 3 REGION(S)

	ILOW	IHIGH
1/E**ENN SPECTRUM	1	452
INPUT SPECTRA	452	837
FISSION SPECTRUM	837	901

NUMBER OF ELEMENTS TO HAVE CROSS SECTIONS GROUP AVERAGED IS 1

NUMBER OF GROUPS IS 16

SPECTRUM 1

GROUP INDEX	E LOW	INTEGRAL OF SPECTRUM
1	3.000E 06	0.26870E 09
2	1.400E 06	0.90997E 09
3	9.000E 05	0.71847E 09
4	4.000E 05	0.21196E 10
5	1.000E 05	0.37249E 10
6	1.700E 04	0.29368E 10
7	3.000E 03	0.64376E 09
8	5.550E 02	0.64849E 08
9	1.000E 02	0.10444E 07
10	3.000E 01	0.45500E 04
11	1.000E 01	0.40000E 03
12	3.000E 00	0.45500E 02
13	1.000E 00	0.40000E 01
14	4.000E-01	0.42000E-00
15	1.000E-01	0.75000E-01
16	10.000E-04	0.49995E-02

SPECTRUM 2

GROUP INDEX	E LOW	INTEGRAL OF SPECTRUM
1	3.000E 06	0.10014E 10
2	1.400E 06	0.33644E 10
3	9.000E 05	0.20193E 10
4	4.000E 05	0.54482E 10
5	1.000E 05	0.69243E 10
6	1.700E 04	0.49043E 10
7	3.000E 03	0.80933E 09
8	5.550E 02	0.82995E 08
9	1.000E 02	0.11100E 07
10	3.000E 01	0.45500E 04
11	1.000E 01	0.40000E 03
12	3.000E 00	0.45500E 02
13	1.000E 00	0.40000E 01
14	4.000E-01	0.42000E-00
15	1.000E-01	0.75000E-01
16	10.000E-04	0.49995E-02

SPECTRUM 2 HAS 3 REGION(S)

	ILOW	IHIGH
1/E**ENN SPECTRUM	1	452
INPUT SPECTRA	452	837
FISSION SPECTRUM	837	901

TRIXIE(LINK 2) RESONANCE CALC FOR ISOTOPE ID 949.0

EMAX RES = 0.10000E 05 TOP RES GROUP = 7

NOOG = 7 EMIN RES = 0.10000E-00 ELR = 0.45000E 01

D1 = 1 D2 = -1 D3 = 1 D4 = 1 D5 = 3 D6 = 0 D7 = 0 D8 = 0 D9 = 0 D10 = 0
D11 = 0 D12 = 1 D13 = 0 D14 = 1 D15 = 1 D16 = 0 D17 = 0 D18 = 0 D19 = 0 D20 = 0

EFF. MOD. SCAT. X-SEC. PER ATOM OF RES. ABS. = 0.510000E 02

GEOMETRICAL TERM OF EFF. MOD. SCAT. X-SEC. = 0.

POT. SCAT. X-SEC. PER ATOM OF RES. ABS. = 0.108000E 02

SIGMA A 2200

MAXIMUM ENERGY = 0.100000E 05

HALF NO. OF DIVISIONS IN ESCAPE PROB. CORR. = 0.

ALPHA SLOWLY VARYING X-SEC. = 0.358000E-00

STRENGTH FUNCTION FOR P-WAVE NEUTRONS = 0.250000E-03

NEG. ENERGY RES. - E RES. = 0.1200E 01 GAMMA NO = 0.7710E-03 GAMMA G = 0.3900E-01 GAMMA F = 0.2010E-00

UNRES. RES. PAR. - AVE. GAMMA NO = 0.42000E-03 AVE. GAMMA G = 0.45000E-01 AVE. D = 0.22000E 01

GROUPWISE ABSORPTION INTEGRALS AT 26.90 DEG. C

GROUP	E MIN	RESOL. I	UNR I L=0	UNR I L=1	1/V	TOTAL I	INT. FLUX	ABS X-SEC
7	.30000E 04	.	.17618E 01	.12180E-00	.	.18836E 01	.12040E 01	.15645E 01
8	.55500E 03	.	.90760E 01	.21501E-00	.	.92910E 01	.16874E 01	.55061E 01
9	.10000E 03	.	.15370E 02	.98934E-01	.	.15469E 02	.17138E 01	.90259E 01
10	.30000E 02	.94888E 01	.73165E 01	.29422E-01	.20611E-01	.16853E 02	.10068E 01	.16739E 02
11	.10000E 02	.20635E 02	.	.14285E-01	.22341E-00	.20852E 02	.56695E 00	.36780E 02
12	.45000E 01	.92101E 01	.	.55916E-02	.91401E 00	.99763E 01	.66429E 00	.15018E 02
TOTAL			.39334E 02	.33524E 02	.48505E-00	.11580E 01	.74325E 02	

GROUPWISE FISSION INTEGRALS AT 26.90 DEG. C

GROUP	E MIN	RESOL. I	UNR I L=0	UNR I L=1	1/V	TOTAL I	INT. FLUX	FIS X-SEC.
7	.30000E 04	.	.12082E 01	.80931E-01	.	.12891E 01	.12040E 01	.10707E 01
8	.55500E 03	.	.59780E 01	.13789E-00	.	.61159E 01	.16874E 01	.36244E 01
9	.10000E 03	.	.97924E 01	.61389E-01	.	.98538E 01	.17138E 01	.57497E 01
10	.30000E 02	.49981E 01	.45830E 01	.17710E-01	.13523E-01	.96109E 01	.10068E 01	.95457E 01
11	.10000E 02	.12312E 02	.	.85986E-02	.16628E-00	.12472E 02	.56695E 00	.21999E 02
12	.45000E 01	.53278E 01	.	.33657E-02	.70466E 00	.59176E 01	.66429E 00	.89082E 01
TOTAL			.22638E 02	.21562E 02	.30989E-00	.88446E 00	.45259E 02	

GRISM SAMPLE OUTPUT

ELEMENT ID IS 949

PLUTONIUM 239

ELEMENT IS FISSIONABLE

INELASTIC SCATTERING THRESHOLD IS 631

N, 2N THRESHOLD IS 858

GROUP	I	ENERGY (EV)	TOTAL	ELASTIC	N,GAMMA	FISSION	NU
	901	0.100E 08					
1	831	0.300E 07	0.7851E 01	0.4392E 01	0.5966E-02	0.1958E 01	0.3420E 01
2	815	0.140E 07	0.7559E 01	0.4441E 01	0.1746E-01	0.1976E 01	0.3138E 01
3	801	0.900E 06	0.7269E 01	0.4652E 01	0.4094E-01	0.1706E 01	0.3019E 01
4	751	0.400E 06	0.8307E 01	0.5840E 01	0.1061E-00	0.1583E 01	0.2951E 01
5	721	0.100E 06	0.1097E 02	0.8820E 01	0.1814E-00	0.1515E 01	0.2900E 01
6	638	0.170E 05	0.1333E 02	0.1138E 02	0.3469E-00	0.1355E 01	0.2878E 01
7	561	0.300E 04	0.1541E 02	0.1239E 02	0.9324E 00	0.2004E 01	0.2872E 01
8	497	0.555E 03	0.1961E 02	0.1411E 02	0.1882E 01	0.3624E 01	0.2871E 01
9	451	0.100E 03	0.2407E 02	0.1466E 02	0.3276E 01	0.6130E 01	0.2871E 01
10	381	0.300E 02	0.4704E 02	0.1822E 02	0.7193E 01	0.2162E 02	0.2871E 01
11	361	0.100E 02	0.5061E 02	0.1288E 02	0.1478E 02	0.2295E 02	0.2871E 01
12	291	0.300E 01	0.2740E 02	0.1043E 02	0.6285E 01	0.1069E 02	0.2871E 01
13	271	0.100E 01	0.3045E 02	0.1134E 02	0.2559E 01	0.1655E 02	0.2871E 01
14	211	0.400E-00	0.1570E 03	0.1354E 02	0.4759E 02	0.9585E 02	0.2871E 01
15	181	0.100E-00	0.2523E 04	0.1829E 02	0.9669E 03	0.1538E 04	0.2871E 01
16	1	1.000E-03	0.7695E 03	0.1091E 02	0.2166E 03	0.5420E 03	0.2871E 01

GROUP	I	ENERGY (EV)	MU BAR	ABSORPTION	TRANSPORT	INELASTIC	ALPHA
	901	0.100E 08					
1	831	0.300E 07	0.7417E 00	0.1986E 01	0.4606E 01	0.1473E 01	0.3049E-02
2	815	0.140E 07	0.5553E 00	0.1994E 01	0.5074E 01	0.1124E 01	0.8937E-02
3	801	0.900E 06	0.4369E-00	0.1747E 01	0.5238E 01	0.8695E 00	0.2419E-01
4	751	0.400E 06	0.3691E-00	0.1689E 01	0.6115E 01	0.7776E 00	0.6723E-01
5	721	0.100E 06	0.2061E-00	0.1697E 01	0.9069E 01	0.4571E-00	0.1198E-00
6	638	0.170E 05	0.3824E-01	0.1702E 01	0.1295E 02	0.2490E-00	0.2449E-00
7	561	0.300E 04	0.2789E-02	0.2936E 01	0.1551E 02	0.8850E-01	0.4612E-00
8	497	0.555E 03	0.2789E-02	0.5506E 01	0.1956E 02	0.	0.5192E 00
9	451	0.100E 03	0.2789E-02	0.9406E 01	0.2402E 02	0.	0.5698E 00
10	381	0.300E 02	0.2789E-02	0.2881E 02	0.4699E 02	0.	0.7536E 00
11	361	0.100E 02	0.2789E-02	0.3773E 02	0.5057E 02	0.	0.6719E 00
12	291	0.300E 01	0.2789E-02	0.1697E 02	0.2737E 02	0.	0.6859E 00
13	271	0.100E 01	0.2789E-02	0.1911E 02	0.3042E 02	0.	0.1432E-00
14	211	0.400E-00	0.2789E-02	0.1434E 03	0.1569E 03	0.	0.4315E-00
15	181	0.100E-00	0.2789E-02	0.2505E 04	0.2523E 04	0.	0.6142E 00
16	1	1.000E-03	0.2789E-02	0.7586E 03	0.7695E 03	0.	0.4054E 00

PLUTONIUM 239

ELEMENT MASS NUMBER = 239.127

INELASTIC SCATTERING CALCULATION
SCATTERING ISOTROPIC IN C M SYSTEM.

NO. OF DISCRETE LEVELS = 7

ENERGY CUTOFF FOR DISCRETE LEVELS = 0.550 MEV.

LEVEL	Q VALUE (MEV)
1	-0.010
2	-0.057
3	-0.076
4	-0.164
5	-0.283
6	-0.331
7	-0.392

ELASTIC SCATTERING CALCULATION
SCATTERING ANISOTROPIC IN C M SYSTEM.

NO. OF LEGENDRE COEFFICIENTS = 9

INELASTIC SCATTERING MATRIX

I	J	SIGMA(I TO J)	I	J	SIGMA(I TO J)	I	J	SIGMA(I TO J)	I	J	SIGMA(I TO J)
(1, 1)	0.95045E-02	(1, 2)	0.21872E-00	(1, 3)	0.31113E-00	(1, 4)	0.56491E 00				
(1, 5)	0.33256E-00	(1, 6)	0.35444E-01	(2, 2)	0.53539E-01	(2, 3)	0.16650E-00				
(2, 4)	0.46769E-00	(2, 5)	0.38563E-00	(2, 6)	0.49349E-01	(2, 7)	0.17214E-02				
(3, 3)	0.46000E-01	(3, 4)	0.34031E-00	(3, 5)	0.41539E-00	(3, 6)	0.65297E-01				
(3, 7)	0.24188E-02	(3, 8)	0.78385E-04	(4, 4)	0.29919E-00	(4, 5)	0.40951E-00				
(4, 6)	0.66313E-01	(4, 7)	0.25713E-02	(4, 8)	0.80328E-04	(4, 9)	0.27807E-05				
(5, 5)	0.40433E-00	(5, 6)	0.52016E-01	(5, 7)	0.52860E-03	(5, 8)	0.29155E-04				
(5, 9)	0.39484E-05	(5,10)	0.50627E-07	(6, 6)	0.19251E-00	(6, 7)	0.56205E-01				
(6, 8)	0.11497E-03	(6, 9)	0.12624E-04	(6,10)	0.16485E-05	(6,11)	0.15146E-06				
(7, 7)	0.57805E-01	(7, 8)	0.27028E-01	(7, 9)	0.27347E-02	(7,10)	0.35753E-03				
(7,11)	0.99057E-04	(7,12)	0.23502E-04	(8, 8)	0.	(8, 9)	0.				

BALANCE FOR INELASTIC SCATTERING MATRIX

I	E LOW	SUM SIGMA (I TO J)	CATCH-BUCKET	SUM SIGMA + CATCH-BUCKET	SIGMA AVE.
1	0.300E 07	0.1472269E 01	0.1211202E-02	0.1473480E 01	0.1473481E 01
2	0.140E 07	0.1124428E 01	0.5717247E-04	0.1124485E 01	0.1123601E 01
3	0.900E 06	0.8694990E 00	0.2794428E-05	0.8695018E 00	0.8595570E 00
4	0.400E 06	0.7776664E 00	0.9102622E-07	0.7776665E 00	0.7775954E 00
5	0.100E 06	0.4569050E-00	0.	0.4569050E-00	0.4568991E-00
6	0.170E 05	0.2488481E-00	0.	0.2488481E-00	0.2488995E-00
7	0.300E 04	0.8804804E-01	0.	0.8804804E-01	0.8723327E-01
8	0.555E 03	0.	0.	0.	0.
9	0.100E 03	0.	0.	0.	0.
10	0.300E 02	0.	0.	0.	0.
11	0.100E 02	0.	0.	0.	0.
12	0.300E 01	0.	0.	0.	0.
13	0.100E 01	0.	0.	0.	0.
14	0.400E-00	0.	0.	0.	0.
15	0.100E-00	0.	0.	0.	0.
16	1.000E-03	0.	0.	0.	0.

ELASTIC SCATTERING MATRIX

I	J	SIGMA(I TO J)	I	J	SIGMA(I TO J)	I	J	SIGMA(I TO J)	I	J	SIGMA(I TO J)
(1, 1)		0.43609E 01	(1, 2)		0.40041E-01	(1, 3)		0.	(1, 4)		0.
(1, 5)		0.	(1, 6)		0.	(2, 2)		0.44089E 01	(2, 3)		0.30967E-01
(2, 4)		0.	(2, 5)		0.	(2, 6)		0.	(2, 7)		0.
(3, 3)		0.45980E 01	(3, 4)		0.55631E-01	(3, 5)		0.	(3, 6)		0.
(3, 7)		0.	(3, 8)		0.	(4, 4)		0.58074E 01	(4, 5)		0.38279E-01
(4, 6)		0.	(4, 7)		0.	(4, 8)		0.	(4, 9)		0.
(5, 5)		0.87760E 01	(5, 6)		0.45034E-01	(5, 7)		0.	(5, 8)		0.
(5, 9)		0.	(5,10)		0.	(6, 6)		0.11356E 02	(6, 7)		0.30196E-01
(6, 8)		0.	(6, 9)		0.	(6,10)		0.	(6,11)		0.
(7, 7)		0.12367E 02	(7, 8)		0.21513E-01	(7, 9)		0.	(7,10)		0.
(7,11)		0.	(7,12)		0.	(8, 8)		0.14100E 02	(8, 9)		0.57197E-02
(8,10)		0.	(8,11)		0.	(8,12)		0.	(8,13)		0.
(9, 9)		0.14660E 02	(9,10)		0.12866E-02	(9,11)		0.	(9,12)		0.
(9,13)		0.	(9,14)		0.	(10,10)		0.18167E 02	(10,11)		0.18498E-01
(10,12)		0.	(10,13)		0.	(10,14)		0.	(10,15)		0.
(11,11)		0.12874E 02	(11,12)		0.16602E-01	(11,13)		0.	(11,14)		0.
(11,15)		0.	(11,16)		0.	(12,12)		0.10382E 02	(12,13)		0.18390E-01
(12,14)		0.	(12,15)		0.	(12,16)		0.	(13,13)		0.11314E 02
(13,14)		0.26319E-01	(13,15)		0.	(13,16)		0.	(14,14)		0.13479E 02
(14,15)		0.60564E-01	(14,16)		0.	(15,15)		0.18282E 02	(15,16)		0.12113E-01
(16,16)		0.10915E 02									

BALANCE FOR

ELASTIC SCATTERING MATRIX

I	E LOW	SUM SIGMA (I TO J)	CATCH-BUCKET	SUM SIGMA + CATCH-BUCKET	SIGMA AVE.
1	0.300E 07	0.4400919E 01	0.	0.4400919E 01	0.4392165E 01
2	0.140E 07	0.4439887E 01	0.	0.4439887E 01	0.4440910E 01
3	0.900E 06	0.4653636E 01	0.	0.4653636E 01	0.4652017E 01
4	0.400E 06	0.5845655E 01	0.	0.5845655E 01	0.5840044E 01
5	0.100E 06	0.8821055E 01	0.	0.8821055E 01	0.8820492E 01
6	0.170E 05	0.1138606E 02	0.	0.1138606E 02	0.1137960E 02
7	0.300E 04	0.1238844E 02	0.	0.1238844E 02	0.1238735E 02
8	0.555E 03	0.1410539E 02	0.	0.1410539E 02	0.1410526E 02
9	0.100E 03	0.1466105E 02	0.	0.1466105E 02	0.1466057E 02
10	0.300E 02	0.1818506E 02	0.	0.1818506E 02	0.1822471E 02
11	0.100E 02	0.1289052E 02	0.	0.1289052E 02	0.1288332E 02
12	0.300E 01	0.1040059E 02	0.	0.1040059E 02	0.1042927E 02
13	0.100E 01	0.1134060E 02	0.	0.1134060E 02	0.1134062E 02
14	0.400E-00	0.1353968E 02	0.	0.1353968E 02	0.1353930E 02
15	0.100E-00	0.1829371E 02	0.	0.1829371E 02	0.1829412E 02
16	1.000E-03	0.1091505E 02	0.1932075E-04	0.1091507E 02	0.1090905E 02

PLUTONIUM 239

GRAVE-PRISM CROSS SECTIONS TO BE PUNCHED FOR - FAIM

GROUP	SIGMA	SIGTR	SIGF	NUSIGF	SIG I-I	SIG I-I+1	SIG I-I+2	SIG I-I+3	SIG I-I+4	SIG I-I+5
1	1.986E 00	4.606E 00	1.958E 00	6.702E 00	4.370E 00	2.588E-01	3.111E-01	5.649E-01	3.326E-01	3.666E-02
2	1.994E 00	5.074E 00	1.976E 00	6.204E 00	4.462E 00	1.975E-01	4.677E-01	3.856E-01	4.935E-02	1.779E-03
3	1.747E 00	5.238E 00	1.706E 00	5.152E 00	4.644E 00	3.959E-01	4.154E-01	6.530E-02	2.419E-03	8.118E-05
4	1.689E 00	6.115E 00	1.583E 00	4.673E 00	6.107E 00	4.478E-01	6.631E-02	2.571E-03	8.033E-05	2.872E-06
5	1.697E 00	9.069E 00	1.515E 00	4.395E 00	9.180E 00	9.705E-02	5.286E-04	2.915E-05	3.948E-06	5.063E-08
6	1.702E 00	1.295E 01	1.355E 00	3.899E 00	1.155E 01	8.640E-02	1.150E-04	1.262E-05	1.649E-06	1.515E-07
7	2.936E 00	1.551E 01	2.004E 00	5.754E 00	1.242E 01	4.854E-02	2.735E-03	3.575E-04	9.906E-05	2.350E-05
8	5.506E 00	1.956E 01	3.624E 00	1.041E 01	1.410E 01	5.720E-03	0.	0.	0.	0.
9	9.406E 00	2.402E 01	6.130E 00	1.760E 01	1.466E 01	1.287E-03	0.	0.	0.	0.
10	2.881E 01	4.699E 01	2.162E 01	6.208E 01	1.817E 01	1.850E-02	0.	0.	0.	0.
11	3.773E 01	5.057E 01	2.295E 01	6.588E 01	1.287E 01	1.660E-02	0.	0.	0.	0.
12	1.697E 01	2.737E 01	1.069E 01	3.068E 01	1.038E 01	1.839E-02	0.	0.	0.	0.
13	1.911E 01	3.042E 01	1.655E 01	4.752E 01	1.131E 01	2.632E-02	0.	0.	0.	0.
14	1.434E 02	1.569E 02	9.585E 01	2.752E 02	1.348E 01	6.056E-02	0.	0.	0.	0.
15	2.505E 03	2.523E 03	1.538E 03	4.416E 03	1.828E 01	1.211E-02	0.	0.	0.	0.
16	7.586E 02	7.695E 02	5.420E 02	1.556E 03	1.092E 01	0.	0.	0.	0.	0.

PLUTONIUM 239

GRAVE-PRISM CROSS SECTIONS TO BE PUNCHED FOR - DTF

GROUP	SIGMA	SIGTR	NUSIGF	SIG G-G	SIG I-I	SIG I-I+1	SIG I-I+2	SIG I-I+3	SIG I-I+4	SIG I-I+5
1	1.986E 00	4.606E 00	6.702E 00	1.116E 00	4.370E 00	2.588E-01	3.111E-01	5.649E-01	3.326E-01	3.666E-02
2	1.994E 00	5.074E 00	6.204E 00	1.978E 00	4.462E 00	1.975E-01	4.677E-01	3.856E-01	4.935E-02	1.779E-03
3	1.747E 00	5.238E 00	5.152E 00	2.612E 00	4.644E 00	3.959E-01	4.154E-01	6.530E-02	2.419E-03	8.118E-05
4	1.689E 00	6.115E 00	4.673E 00	3.909E 00	6.107E 00	4.478E-01	6.631E-02	2.571E-03	8.033E-05	2.872E-06
5	1.697E 00	9.069E 00	4.395E 00	7.274E 00	9.180E 00	9.705E-02	5.286E-04	2.915E-05	3.948E-06	5.063E-08
6	1.702E 00	1.295E 01	3.899E 00	1.116E 01	1.155E 01	8.640E-02	1.150E-04	1.262E-05	1.649E-06	1.515E-07
7	2.936E 00	1.551E 01	5.754E 00	1.253E 01	1.242E 01	4.854E-02	2.735E-03	3.575E-04	9.906E-05	2.350E-05
8	5.506E 00	1.956E 01	1.041E 01	1.405E 01	1.410E 01	5.720E-03	0.	0.	0.	0.
9	9.406E 00	2.402E 01	1.760E 01	1.461E 01	1.466E 01	1.287E-03	0.	0.	0.	0.
10	2.881E 01	4.699E 01	6.208E 01	1.816E 01	1.817E 01	1.850E-02	0.	0.	0.	0.
11	3.773E 01	5.057E 01	6.588E 01	1.283E 01	1.287E 01	1.660E-02	0.	0.	0.	0.
12	1.697E 01	2.737E 01	3.068E 01	1.038E 01	1.038E 01	1.839E-02	0.	0.	0.	0.
13	1.911E 01	3.042E 01	4.752E 01	1.128E 01	1.131E 01	2.632E-02	0.	0.	0.	0.
14	1.434E 02	1.569E 02	2.752E 02	1.344E 01	1.348E 01	6.056E-02	0.	0.	0.	0.
15	2.505E 03	2.523E 03	4.416E 03	1.823E 01	1.828E 01	1.211E-02	0.	0.	0.	0.
16	7.586E 02	7.695E 02	1.556E 03	1.088E 01	1.092E 01	0.	0.	0.	0.	0.

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Discussion

Mr. Razminus (Savannah River): Can you punch multigroup cross sections in the format required by GAM?

Mr. Alter: The CONTRACT link of GRISM will not punch cross sections for GAM, since it punches cross sections only for the AI codes. A subroutine for GAM could easily be added since CONTRACT is a series of subroutines.

Mr. Hutchins (G. E.): Why do you not treat negative cross sections?

Mr. Alter: Our CRT plotting routines involve log-log scales. Thus zero or negative values of cross sections could not be used with these routines.

Mr. Moldauer (ANL): In your treatment of the unresolved resonance region do you have provision for both s- and p-wave strength functions?

Mr. Alter: Yes. Both s- and p-wave strength functions are included in the TRIxie link of GRISM in the treatment of the statistical region.

Presentation of the Multigroup Cross-section Set Prepared at Cadarache*

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France*

Presented by J. Ravier

ABSTRACT

A set of multigroup cross sections has been developed at Cadarache.

After a short review of the sources of basic nuclear data, the report explains how the multigroup set was developed and how it is currently used; then a brief comparison between experimental and calculated results is given.

1. Introduction

The multigroup cross-section set presently prepared at Cadarache for calculations involving large fast reactors is a 25-group set covering the energy range from 14 MeV to 0.4 eV, plus a thermal group.

The present paper reports on the basic nuclear data used in this set, the methods of weighting and the comparisons between experimental results on critical assemblies, and calculated results using this and other sets.

2. Basic Nuclear Data

The basic nuclear data used here come mainly from the Winfrith Nuclear Data File.⁽¹⁾ Some data have been modified, taking into account new experimental results indicated in the second edition of BNL-325⁽²⁾ for the most part. These modifications are chiefly:

the Pu^{239} and U^{235} fission cross sections from 40 keV to

1 MeV measured at Aldermaston;⁽³⁾

the value of α U^{235} measured at Oak Ridge;⁽⁴⁾

the Pu^{239} fission cross section from 10 eV to 30 keV measured at Harwell⁽⁵⁾ and Saclay;⁽⁶⁾

the U^{238} capture cross section below 100 keV;⁽²⁾

the sodium capture cross section from 10 to 100 keV measured at Cadarache;⁽⁷⁾

the nickel and chromium cross sections according to a new evaluation made at Cadarache;⁽⁸⁾

the integral and angular differential elastic cross sections of light or intermediate nuclides below 1

MeV given by the Argonne File for the code ELMOE;⁽¹⁰⁾
the U^{238} total cross section below 4 keV measured at Columbia.⁽⁹⁾

3. Flux Weighting

The group energy intervals agree with the first 14 groups from the YOM set.⁽¹¹⁾ Ten groups have been added below with a lethargy increment multiple of 0.5. The 25th group is a thermal group (see Table 1).

The use of a single set is inadequate to deal with the widely different energy spectra; however, the fine structure of the flux due to absorption or scattering resonances may be treated as a perturbation from an average flux.

A standard spectrum without any fine structure is chosen corresponding to a large fast reactor. The English Code GALAXY⁽²⁰⁾ calculates average cross sections for each group. Except for the elastic transfer cross sections, these average values do not fluctuate much with reasonable variation of the standard flux. At any rate, the deviations are always smaller than the experimental uncertainty of basic data.⁽²²⁾

Superimposed on this slowly varying flux, the fine structure of the flux has been dealt with by means of correction factors.

3a. For *fission* or *capture* cross sections for heavy and light nuclei, the resonance self-shielding has been calculated without taking into account the overlapping effect of resonances of different isotopes. With this assumption the total effective cross section for all the other isotopes is supposed to be a continuum without fine structure when calculating microscopic effective cross sections of an isotope. Other multigroup sets use a similar method, for example, the Hansen and Roach 16-group set⁽¹²⁾ or the Russian 26-group set.⁽¹³⁾

By means of two numerical codes the self-shielding correction factors are computed as functions of the temperature and dilution, the flux having a structure in $1/\Sigma_i(E)$ (narrow resonance approximation) multiplied

* Work performed under the EURATOM-C.E.A. Association for fast reactors.

TABLE 1. ENERGY BOUNDARIES OF THE 25-GROUP CADARACHE SET

Group	Energy Boundaries	Δu	Fission Spectrum	v , cm/sec
1	14.5-3.68 MeV	1.37	0.13120	0.3049 10^{10}
2	3.68-2.23 "	0.5	0.21070	0.2320 "
3	2.23-1.35 "	0.5	0.23200	0.1799 "
4	1.35-0.821 "	0.5	0.17910	0.1401 "
5	821-498 keV	0.5	0.11480	0.1092 "
6	498-302 "	0.5	0.06480	0.8503 10^9
7	302-183 "	0.5	0.03410	0.6729 "
8	183-111 "	0.5	0.01710	0.5208 "
9	111-67.4 "	0.5	0.00840	0.4057 "
10	67.4-40.9 "	0.5	0.00410	0.3161 "
11	40.9-24.8 "	0.5	0.00200	0.2494 "
12	24.8-15.0 "	0.5	0.00090	0.1936 "
13	15.0-9.12 "	0.5	0.00041	0.1503 "
14	9.12-5.53 "	0.5	0.00021	0.1173 "
15	5.53-3.36 "	0.5	0.00010	0.9243 10^8
16	3.36-2.04 "	0.5	0.00005	0.6746 "
17	2.04-1.23 "	0.5	0.00002	0.5520 "
18	1.23-0.748 "	0.5	0.00001	0.4317 "
19	748-454 eV	0.5	0	0.3365 "
20	454-275 "	0.5	0	0.2620 "
21	275-101 "	1	0	0.1842 "
22	101-22.6 "	1.5	0	0.1024 "
23	22.6-3.06 "	2.0	0	0.5095 10^7
24	3.06-0.414 "	2.0	0	0.1875 "
25	Thermal	—	0	0.218 10^6

by a slowly varying function of E . Here $\Sigma_t(E)$ is the total macroscopic cross section of the medium.

The first code, called RDT, deals with nonoverlapping resonances. The second code, called REDITE, is more general and treats overlapping resonances as well (for one isotope). Microscopic cross sections are generated from resonance parameters by the Breit-Wigner multi-level theory with Doppler broadening. Effective cross sections are calculated by numerical integration.

The effective cross sections for U-238 and Pu-239 have been tabulated at several temperatures: 300, 600, 900, 1500, and 3000°K, for seven dilutions varying from ∞ to 0.

Between 0 and 4 keV, the basic data for U-238 are given by an evaluation made at Cadarache on the resonance parameters, taking into account the last measurement of Columbia.⁽⁹⁾ From 4 to 50 keV, a sample of 80 resonances ($l = 0$ and $l = 1$) located near the mean energy of each group is used for the calculation. The s-wave parameters are derived from statistical laws given by the experimental data at lower energy. For $l = 1$ neutrons, the neutron widths Γ_n are fitted to the experimental values of capture cross sections.⁽¹⁴⁾

For Pu-239 in the range of resolved resonances, 0 to 200 eV, the sample used includes 80 resonances of which nearly all of the parameters are the result of measure-

ment [chiefly those of J. Blons⁽⁶⁾ and Uttley⁽¹⁵⁾]. These parameters are shifted upwards to the mean energy of each group from 200 eV to 5 keV. This shift is merely an energy translation, since the average level spacings vary very little in this energy range and since it can be assumed that the reduced neutron, fission and capture widths to be constant. Only the contribution of s waves is taken into account.

A similar method is in progress for U-235. The self-shielding factors used now have been evaluated from those published in the Russian 26-group set. For light nuclei, the narrow resonance approximation is not very well justified (see, however, 3b below).

3b. *The calculation of elastic transfer cross sections* is more complicated because there is an overlapping effect of wide scattering resonances of different isotopes. For the E_g - E_{g+1} group, the elastic transfer cross section is defined by

$$\sigma_{el\ g \rightarrow g+1} = \frac{\int_{E_g}^{E_g + \Delta E} \phi(E) dE \int_{E - \Delta E}^{E_g} \sigma(E \rightarrow E') dE'}{\int_{E_g}^{E_{g+1}} \phi(E) dE}.$$

Except for very light or heavy nuclides, the integration range ΔE is about the same order of magnitude as the scattering width and smaller than the group width (see Table 2).

For the elastic transfer cross sections, two different corrections are applied successively in an independent way:

The first one deals with the fine structure of the flux in the range of integration, ΔE . The flux is supposed to vary as $1/\Sigma_t(E)$, where $\Sigma_t(E)$ is the total macroscopic cross section of the medium. This simple treatment was chosen rather than a more rigorous treatment, such as that used in the ELMOE Code, after a numerical comparison showed that the simple treatment did not lead to appreciable errors on the quantities of practical interest. Of course, an ELMOE-type treatment should be preferred if the objective is to calculate the fine spectrum.

The second correction tries to take into account the overall variation of the flux within each group and

TABLE 2. COMPARISON BETWEEN Γ_t AND AVERAGE ENERGY LOSS BY ELASTIC SCATTERING FOR LIGHT ELEMENTS

Energy	O		Na		Fe		Group Width, keV
	$\overline{\Delta E}$, keV	Γ_t , keV	$\overline{\Delta E}$, keV	Γ_t , keV	$\overline{\Delta E}$, keV	Γ_t , keV	
1 MeV	115	350	60	55	25	22	900
100 keV	12		8.5		3.4		70
1 keV	0.12		0.085	(0.8)	0.035		0.8

evaluates the ratio of the flux at the lower group boundary over the average flux, from the broad variation of the standard flux (a similar treatment is applied with the Russian 26-group set).

3c. *The transport macroscopic cross sections* are computed by the same weighting function $1/\Sigma_t(E)$.

There are around 600 elementary energy intervals between 2 MeV and 100 eV; the basic data come from the ELMOE library.

Following is a diagram which summarizes the scheme used for preparing the data needed for a reactor calculation:

Actually the set includes the following 25 nuclei: H, Be, B-10, B_{nat}, C, O, Na, Al, Cr, Fe, Ni, Cu, Zr, Nb,

Mo, Ta, Th-232, U-233, U-235, U-238, Pu-239, Pu-240, Pu-241, Pu-242, and fission products.

4. Calculation of Critical Assemblies

This new multigroup set has been tested by calculating eight ZPR-III critical assemblies with core volumes ranging from 70 to 500 liters, with critical masses from 100 to 600 kg of U-235, and U-238/U-235 ratios from 10.3 to 0.07. In Tables 3-9 are compared the experimental and the calculated data. Calculations have been carried out in spherical geometry by a diffusion code (MUDE), with a core assumed homogeneous. Criticality was obtained by varying the core radius.

Table 3 gives the corrected critical masses (corrected to account for heterogeneity and cylindrical core geometries as indicated in Ref. 16). The calculated critical masses are smaller than the experimental ones for U-238/U-235 ratios smaller than 7. Figure 1 shows this tendency.

The calculated and experimental⁽¹⁶⁾ central fission

TABLE 3. CALCULATED CRITICAL MASS

Assembly No. of ZPR-III	U-238/U-235	$E^{(a)}$	$[E - C^{(b)}]/E, \%$			
			Cad.	ANL ⁽¹⁶⁾	H.R. ⁽¹⁸⁾	Russ. ⁽¹⁹⁾
25	10.3	581.6	-4.1	+0.3	-10.3	+15.0
36	5.3	242.7	+2.9	+2.4	—	—
41	4.9	489.9	+11.4	+5.7	—	—
20	3.1	431.0	+6.3	—	-5.3	+7.0
34	2.2	503.0	+16.5	+9.7	-2.0	+23.0
29	2.0	421.0	+16.8	+11.6	-0.9	+16.1
14	0.07	136.1	+13.9	+7.3	—	—
33	0.07	238.0	+15.9	+10.6	—	—

^(a) E = Experimental critical mass in kg U-235.

^(b) C = Calculated mass corrected for shape and heterogeneity.

TABLE 4. CENTRAL FISSION AND CAPTURE RATIO

Assembly No. of ZPR-III	σ_f U-238/ σ_f U-235				σ_f Pu-239/ σ_f U-235				σ_c U-238/ σ_f U-235			
	$E^{(a)}$	$E/C^{(b)}$			$E^{(a)}$	$E/C^{(b)}$			$E^{(a)}$	$E/C^{(b)}$		
		Set 1 ^(c)	Set 2 ^(c)	Set 3 ^(c)		Set 1	Set 2	Set 3		Set 1	Set 2	Set 3
33	0.052	0.96	0.96	—	1.21	1.01	0.99	0.99	—	—	—	—
25	0.0315	1.09	1	0.96	1.17	1.03	1	1	0.113	0.95	0.99	—
29	0.0384	1.05	1.07	0.88	1.06	1	0.98	0.98	0.141	1.07	1.14	—
30	0.0461	1.03	—	0.89	1.20	1	—	0.97	—	—	—	—
36	0.0443	1.04	0.96	—	1.19	1.02	0.98	0.99	0.119	1.03	1.10	—
20	0.0411	1.03	—	0.86	1.15	1.01	—	0.97	—	—	—	—
41	0.0395	1.02	0.99	—	1.164	1.02	0.99	—	—	—	—	—
14	0.060	1.09	0.97	—	1.05	1	0.98	1	0.134	1.01	1.15	—

^(a) E = Experimental Data.

^(b) E/C = Experimental Data/Calculated Data.

^(c) Set 1: 25-group Cadarache set; Set 2: 26-group ANL set;⁽¹⁶⁾ Set 3: 16-group Hansen and Roach set.⁽¹⁸⁾

TABLE 5. CALCULATED PROMPT-NEUTRON LIFETIMES
COMPARED WITH EXPERIMENTAL VALUES

Assembly No.	l_p (Exp.) $\times 10^8$ sec	$E/C^{(a)}$			Hansen and Roach
		Ca- darache	ANL- 7133	Davey	
14	18.7	1.16	1.22	1.30	—
20	16.2	1.07	—	1.24	1.12
25	8.1	1	1.08	1.30	1.06
29	23.5	1.03	1.11	1.34	1.09
30	17.8	1	—	1.27	1.06
33	12.5	0.98	1.11	1.26	—
36	7.8	1	1.08	1.38	—
41	13.1	1.02	1.10	1.38	—

^(a) E/C : Experimental value/Calculated result.TABLE 6. RELATIVE CENTRAL REACTIVITY DATA
FOR BORON-10

Assembly No.	Experimental Reactivity (mb $\pm$ 13)	$E/C^{(a)}$		Hansen and Roach
		Cadarache	Davey	
36	—966	0.94	1.32	—
33	—1226	0.86	1.26	—
41	—1125	0.96	1.38	—
25	—866	0.89	1.26	0.90
20	—1244	0.99	1.33	1.05
29	—1744	0.86	1.33	0.94

^(a) E/C = Experimental value/Calculated result.

TABLE 7. RELATIVE CENTRAL REACTIVITY DATA FOR U-235 AND U-238

Assembly No.	U-235				U-238			
	Experimental E (mb) ^(a)	Set			Experimental E (mb) ^(a)	Set		
		Cadarache E/C	Davey E/C	Hansen- Roach E/C		Cadarache E/C	Davey E/C	Hansen- Roach E/C
36	1847	0.95	0.99	—	—97	1.05	1.23	—
33	1783	0.97	1	—	—7	—0.58	∞	—
41	1925	0.98	1.03	—	—85	0.85	1.04	—
25	1925	0.96	1	0.95	—98	0.77	0.90	0.81
20	1918	0.99	1.04	0.96	—83	0.97	1.15	1.08
29	2012	1	1.06	0.95	—104	0.84	1.12	0.78

^(a) E : experimental value normalized at $(\nu-1-\alpha)\sigma_f$ for Pu-239, in millibarn; E/C : experimental value/calculated result.

TABLE 8. CENTRAL REACTIVITY COEFFICIENT FOR STAINLESS STEEL-Nb-Mo

Assembly No. of ZPR-III	Stainless steel				Niobium				Molybdenum			
	$E(\pm 2)^{(a)}$	$E/C^{(b)}$			$E^{(a)}$	$E/C^{(b)}$			$E^{(a)}$	$E/C^{(b)}$		
		Set 1 ^(c)	Set 2 ^(c)	Set 3 ^(c)		Set 1	Set 2	Set 3		Set 1	Set 2	Set 3
36	—25	—4	—2		—99	0.94	1.09	—	—70	0.94	0.89	—
33	+2	—8	—1		—43	0.46	0.77	—	—23	0.55	0.82	—
41	—18	0	+2		—95	0.83	1.07	—	—64	0.76	0.86	—
25	—23	+1	+2		—83	0.79	0.90	0.81	—69	0.80	0.83	0.79
20	—14	—2	0		—	—	—	—	—63	0.85	1.0	0.71
29	—7	—5	+1		+117	0.64	1.04	0.56	—74	0.79	1.09	0.61

^(a) E = Experimental data.^(b) E/C = Experimental data/Calculated data.^(c) Set 1 = 25-group Cadarache Set; Set 2 = 26-group YOM Set; ⁽¹⁷⁾ Set 3 = 16-group Hansen-Roach Set. ⁽¹⁸⁾

and capture ratios are compared in Table 4. The calculated U-238/U-235 fission ratios are systematically too small and deviate from experiment by about 4%. The Pu-239/U-235 fission and U-238 capture/U-235 fission ratios are not in a bad agreement.

Table 5 compares prompt-neutron lifetimes; the Cadarache set gives a better agreement than other multigroup sets. This may be due to the fact that the calculated flux and adjoint flux at low energy are higher than for other sets. Figures 2 and 3 compare the flux and adjoint flux for ZPR-III assembly 34 with those obtained by ANL 26-group set.⁽¹⁶⁾ The same conclusion may be drawn from comparison of calculations for Assembly 48.

TABLE 9. RELATIVE CENTRAL REACTIVITY DATA FOR Al, O, C, AND Na

Assembly No. of ZPR-III	Ratio U^{238}/U^{235} for Core	Al		O		C		Na	
		$E - C$		$E - C$		$E - C$		$E - C$	
		Set 1 ^(a)	Set 2 ^(b)	Set 1	Set 2	Set 1	Set 2	Set 1	Set 2
25	10.3	-2	-3	-6	-5	-7	-7	—	—
36	5.3	-5	-5	-6	-2	-9	-5	-1	-2
41	4.9	+1	-1	-1	-3	-0	-2	+5	+3
20	3.1	-6	-5	—	—	-7	-3	—	—
29	2.0	-6	0	+8	20	0	17	9	21
33	0.07	+12	+9	+19	16	24	24	12	11

(a) Set 1: 25-group Cadarache set.

(b) Set 2: 16-group Davey YOM set.⁽¹⁷⁾

Calculated central reactivity coefficients are compared in Tables 7 to 9. The experimental coefficients are normalized to the central average $(\nu-1-\alpha)\sigma_f$ for Pu-239. All the results are presented in millibarn per atom. The data of Table 7 show that the calculated central reactivity coefficient of U-238 seems to be too large in absolute value.

Our set was checked for ZEBRA Assembly 3,⁽²¹⁾ containing Pu-239. Table 10 compares experimental with calculated data.

The Cadarache and the 26-group Russian set give better agreement than the Winfrith set.

5. Conclusions

Although this set is now being used, work on this subject is far from being completed, and we intend to pursue it in the following directions:

new evaluation of the sensitivity of the main reactor parameters to uncertainty of fundamental data;

extensive evaluation of microscopic data for all the nuclei of interest for fast reactors, in the light of always new and more accurate measurements. This part of the work is being performed in close cooperation with other laboratories (mainly Winfrith and Karlsruhe);

refinement in the use of multigroup sets which, on the one hand, may be more completely automated, and which, on the other hand, could be produced in a more correct way (for example, one should think of the possibility of treating elastic moderation with a continuous or analytical model).

FIG. 1. Excess Reactivity versus Ratio of U-238 to U-235 in ZPR-III Assemblies.

Fig. 2. Central Neutron Spectrum of ZPR-III Assembly 34.

Fig. 3. Central Importance Spectrum of ZPR-III Assembly 34.

TABLE 10. COMPARISON FOR ZEBRA ASSEMBLY 3

	Experimental	Experimental/ Calculated			
		Set 1 ^(a)	Set 2 ^(b)	Set 3 ^(c)	Set 4 ^(d)
Critical Mass, kg Pu 239	79.6	0.94	1.30	1.12	1.01
Central fission ratio					
σ_f Pu-239/ σ_f U-235	1.190 ± 0.015	1.02	0.95	0.95	1.01
σ_f Pu-240/ σ_f U-235	0.373 ± 0.005	1.11	0.92	0.92	1
Central reactivity coefficient					
U-235	1	1	1	1	1
Pu-239	1.61 ± 0.04	1.01	0.92	0.94	0.99
B-10	-0.532 ± 0.024	1.13	1.44	1.53	1.40
U-238	-0.050 ± 0.0019	0.87	1.11	1.11	0.98
C	-0.20 ± 0.0015	1.54	2.50	2.44	1.63
Na	-0.013 ± 0.0015	1.18	1.86	1.86	1.20
Al	-0.014 ± 0.001	1.32	1.75	1.75	1.04

(a) Set 1: 25-group Cadarache set.

(b) Set 2: 33-group Winfrith set.⁽²¹⁾(c) Set 3: 16-group YOM set.⁽¹⁷⁾(d) Set 4: 26-group Russian set.⁽¹³⁾

It is hoped that the discrepancy between experimental and calculated integral results will thus be reduced, or at least understood.

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Discussion

Mr. Baker (U. K.-Winfrith): You obtain more low-energy neutrons than do the experimenters in your calculations for ZPR-3, whereas other people usually obtain the opposite result. Have you an explanation?

Mr. Ravier: This may be a result of our U-238 cross sections, with the effects of inelastic scattering being important.

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SESSION II

October 10, 1966

TESTS OF CROSS-SECTION SETS

Chairman: W. B. Loewenstein
Argonne National Laboratory

Secretaries: J. K. Long
R. L. McVean
Argonne National Laboratory

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Intercomparison of Calculations for a Dilute Plutonium-fueled Fast Critical Assembly (ZPR-3 Assembly 48)

W. G. DAVEY

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W. G. Davey

ABSTRACT

In connection with the present Conference, an international group of establishments which are active in the field of fast reactors was invited to participate in a comparison of the calculated and measured parameters of a plutonium-fueled, soft-spectrum, simple-geometry critical assembly to be constructed in Argonne National Laboratory's Zero Power Reactor-3 (ZPR-3) in the summer of 1966. The study of this assembly, No. 48, in the ZPR-3 series, is still in progress. A summary of the calculated data and the measurements obtained, up to the present date, is presented here.

1. Introduction

ZPR-3 Assembly 48 is the first of a series of comparatively simple, plutonium-fueled critical assemblies which, although not of large critical mass, will have spectra broadly similar to fast power reactors of several thousand-liter core volumes. This is achieved by the use of graphite to moderate the neutron spectrum.

One of the major criteria adopted in the design of this series was the maximum simplicity of core design consistent with obtaining maximum utilization of the plutonium inventory, suitable neutron spectra, and compositions of reactor interest. Assembly 48, therefore, contains the same composition in every drawer in the core, a feature which is believed to be of great importance in aiding the analysis. The core contains both plutonium and plutonium-uranium alloy as fuel, and depleted uranium is added to bring the uranium-to-plutonium ratio up to a value of approximately 4.

2. Specification of Assembly 48

The specification of Assembly 48, which was issued for the purposes of this intercomparison, is given in the Appendix. Part A is the original specification of the core, reflector dimensions, and composition, and a listing of the experimental data to be measured. Part B contains supplemental information issued in response to specific requests for clarification of the original specification.

3. Participants in the Intercomparison

The participating organizations are listed below in alphabetical order.

- (a) Atomic Energy Establishment, Winfrith, England: designated AEEW. Two sets of data are given; those given with the FD2 cross-section set (AEEW FD2) and those adjusted in the light of experiments on the ZEBRA reactor (AEEW Adjusted).
- (b) Atomics International: designated AI.
- (c) Argonne National Laboratory: designated ANL. It should be noted that these were carried out by the Reactor Physics Division in Illinois, completely independently of Idaho Division which executed the experiments. All ANL calculations were completed prior to the critical assembly.
- (d) Atomic Power Development Associates: designated APDA.
- (e) Battelle Northwest Laboratories: designated BNWL. A modified version of the Russian data set published by Abagyan *et al.* was used.
- (f) The Babcock and Wilcox Company: designated B & W.
- (g) Centre d'Etudes Nucleaires de Cadarache: designated Cadar. Three data sets were used: the Los Alamos Hansen-Roach data [Cadar. (H-R)], the Russian data of Abagyan *et al.* [Cadar. (USSR)], and a new 25-group set generated at Cadarache [Cadar. (25-group)].
- (h) The General Electric Company: designated GE.
- (i) The Kernforschungszentrum Karlsruhe, Germany: designated Karlsruhe.
- (j) Los Alamos Scientific Laboratory: designated LASL. The Hansen-Roach data [LASL(H-R)] and the Russian data [LASL(USSR)] were used.
- (k) Westinghouse Electric Corporation: designated W.

The results of the calculations were sent to Idaho in

TABLE 1. CRITICAL MASS OF ASSEMBLY 48 (kg Pu²³⁹)

Organization	Calculated Data				Best Estimate of Heterogeneous Cylindrical Mass	Percentage Deviation from Experiment	
	Sphere		Cylinder			In Mass	In $k^{(g)}$
	Homog.	Hetero.	Homog.	Hetero.			
AEEW FD2	314.1 ^(a)		271	250	250 ^(b)	-8.1	-1.2
AEEW Adjusted			292	267	267 ^(b)	-1.8	-0.3
AI			339.0 ^(a)		314 ^(c)	+15.4	+2.4
ANL				263.3	263.3 ^(b)	-3.2	-0.5
APDA	264 ^(a)		283 ^(a)		258 ^(c)	-5.1	-0.8
BNWL			271		246 ^(c)	-9.6	-1.5
B & W			250		225 ^(c)	-17.3	-2.7
Cadarache (H-R)	286.8		309 ^(e)		284 ^(c)	+4.4	+0.7
Cadarache (USSR)	237.2		256 ^(e)		231 ^(c)	-15.1	-2.3
Cadarache (25GRP)	225.6		243 ^(e)		218 ^(c)	-19.9	-3.1
GE			293	291 ^(f)	268 ^(c)	-1.5	-0.2
Karlsruhe	257 ^(a)	228	277 ^(a)	246 ^(d)	252 ^(c)	-7.4	-1.1
LASL (H-R)	262		283 ^(e)		258 ^(c)	-5.1	-0.8
LASL (USSR)	217		234 ^(e)		209 ^(c)	-23.2	-3.6
W			242		217 ^(c)	-10.2	-3.1
Experiment (Heterogeneous Cylinder)					272		

(a) Both AI and Karlsruhe calculate a shape factor of 0.927. APDA calculates 0.930.

(b) Directly calculated.

(c) Subtracting 25 kg from the homogeneous cylindrical mass. This is equivalent to 1.4% in k .(d) Karlsruhe calculates a rather high Δk for heterogeneity of 2.5%.

(e) Using a shape factor of 0.927.

(f) The heterogeneous calculation only allowed for heterogeneity at low energies.

(g) Assuming $(\Delta M/M)/(\Delta k/k) = 6.5$.

TABLE 2. CENTRAL FISSION RATIOS IN ASSEMBLY 48

Organization	Fission Ratios					
	U ²³⁸ / U ²³⁵	Pu ²³⁹ / U ²³⁵	Pu ²⁴⁰ / U ²³⁵	U ²³⁴ / U ²³⁵	U ²³⁶ / U ²³⁵	U ²³⁸ / U ²³⁵
AEEW FD2	1.58	0.932	0.214	0.195	0.0630	0.0288
AEEW Adjusted	1.42	0.945	0.240	0.200	0.065	0.0320
AI	1.47	0.872	0.216	0.188	.0607	0.0281
ANL	1.44	1.0				0.032
APDA		0.919				0.0295
BNWL	1.45	0.962	0.242	0.228	0.0763	0.0343
B & W	1.45	0.958	0.206	0.184	0.0680	0.0309
Cadarache (H-R)	1.42	0.958	0.217	0.232	0.0762	0.0339
Cadarache (USSR)	1.42	0.958	0.232	0.219	0.0726	0.0328
Cadarache (25GRP)	1.44	0.959	0.213	0.207	0.0645	0.0301
GE		0.92	0.21			0.032
Karlsruhe		0.938	0.223		0.070	0.031
LASL (H-R)		0.962	0.224			0.036
LASL (USSR)		0.954	0.232			0.033
W	1.38	0.99	0.23	0.22	0.077	0.033
Experiment ^(a)	1.48	0.976	0.243	0.204	0.0670	0.0307

(a) Errors are $\pm 1\%$ with the exception of that for Pu²⁴⁰/U²³⁵, for which a provisional error of $\pm 5\%$ has been assigned.

TABLE 3. CENTRAL PERTURBATION CROSS SECTIONS
IN ASSEMBLY 48

Organization	Isotopic Cross Sections						
	Pu ²³⁹ , b	Pu ²⁴⁰ , b	U ²³⁵ , b	U ²³⁸ , b	Ta, b	B ¹⁰ , b	Na, mb
AEEW FD2	3.75	0.31	2.75	-0.199	-1.01	-2.76	-10.0
AEEW Ad-justed	3.85	0.56	2.75	-0.208	-1.15	-3.59	-8.0
AI	3.37	0.263	2.64	-0.221		-3.00	-2.6
ANL	3.42	0.465	2.42	-0.155	-0.64	-2.68	-3.0
APDA	3.46	0.415	2.46	-0.153		-2.41	
BNWL	3.70	0.306	2.56	-0.182	-1.16	-2.64	-5.2
B & W	3.65	0.378	2.46	-0.227		-2.16	-8.2
Cadarache (H-R)	3.83	0.714	2.88	-0.204	-1.38	-2.59	-7.5
Cadarache (USSR)	3.76	0.344	2.67	-0.182	-0.99	-2.41	-5.7
Cadarache (25GRP)	3.46	0.344	2.37	-0.197	-1.16	-2.82	+5.5
GE	3.28	0.21	2.32	-0.17		-2.90	-1.4
Karlsruhe	3.69	0.392	2.71	-0.175		-2.48	-10.5
LASL (H-R)	3.87	0.754	2.88	-0.190	-1.38		-8.9
LASL (USSR)	3.78	0.289	2.67	-0.181	-1.10	-2.54	-5.5
W	3.77	0.42	2.72	-0.15	-0.88	-2.2	-9.0
Experiment ^(a)	3.50	0.640	2.62	-0.194	-0.976	-2.94	-4.8

^(a) Normalized to 2.62 b for U²³⁵. This is the average value of all the calculated values for U²³⁵.

TABLE 4. ADDITIONAL CENTRAL PERTURBATION
CROSS SECTIONS IN ASSEMBLY 48

Organization	Isotopic Cross Section, mb						
	Fe	Ni	Cr	O	C	Mn	Mo
AEEW FD2	-18.0	-37.0	-23.0	-7.0	-6.0	-78.0	
AEEW Ad-justed	-26.0	-42.0	-28.0	-7.0	-1.0	-78.0	
AI	-18.4	-25.4	-16.8	-0.8	+4.2		-95.0
ANL	-16.3	-29.7	-19.2		+2.9		-133.0
APDA	-22.3	-27.9	-24.3				
BNWL	-26.1	-38.4	-23.7	-4.3	-1.6		-168.0
B & W	-25.8	-33.5	-16.5	-32.4	+5.4		-220.0
Cadarache (H-R)	-20.9	-70.8	-62.7		-1.1		-198.0
Cadarache (USSR)	-19.5	-39.1	-22.4		-2.4		-152.0
Cadarache (25 GRP)	-13.3	-14.6	-13.5	+6.0	+12.7		-175.0
GE	-22.0	-29.0	-19.0		+4.3		-143.0
Karlsruhe	-24.2	-42.9	-24.1	-7.5	-7.4		-142.0
LASL (H-R)	-22.2	-73.5	-64.2				-199.0
LASL (USSR)	-21.7	-39.2	-22.8				-164.0
W	-20.0	-27.0	-26.0	-7.0	-9.0		-120.0
Experiment ^(a)	-22.0	-35.1	-16.1	-3.6	-1.8	-38.7	-137

^(a) Normalized to 2.62 b for U²³⁵. This is the average value of all the calculated values for U²³⁵.

TABLE 5. CENTRAL PERTURBATION CROSS SECTIONS
IN ASSEMBLY 48^(a)

Organization	Isotopic Cross Sections						
	Pu ²³⁹	Pu ²⁴⁰	U ²³⁵	U ²³⁸	Ta	B ¹⁰	Na(× 10 ³)
AEEW FD2	1.36	0.11	1.00	-0.072	-0.37	-1.00	-3.6
AEEW Ad-justed	1.40	0.20	1.00	-0.076	-0.42	-1.31	-2.9
AI	1.28	0.100	1.00	-0.084		-1.14	-0.98
ANL	1.41	0.192	1.00	-0.064	-0.26	-1.11	-1.2
APDA	1.41	0.169	1.00	-0.062			-0.98
BNWL	1.45	0.119	1.00	-0.071	-0.45	-1.03	-2.0
B & W	1.48	0.154	1.00	-0.092		-0.88	-3.3
Cadarache (H-R)	1.33	0.248	1.00	-0.071	-0.48	-0.90	-2.6
Cadarache (USSR)	1.41	0.129	1.00	-0.068	-0.37	-0.90	-2.1
Cadarache (25GRP)	1.46	0.145	1.00	-0.083	-0.49	-1.19	+2.3
GE	1.41	0.09	1.00	-0.07		-1.25	-0.6
Karlsruhe	1.36	0.145	1.00	-0.065		-0.92	-3.9
LASL (H-R)	1.34	0.262	1.00	-0.066	-0.48		-3.1
LASL (USSR)	1.42	0.108	1.00	-0.068	-0.41	-0.95	-2.1
W	1.39	0.15	1.00	-0.06	-0.32	-0.8	-3.3
Experiment	1.34	0.244	1.00	-0.074	-0.372	-1.12	-1.8

^(a) Normalized to 1.00 for U²³⁵.

TABLE 6. ADDITIONAL CENTRAL PERTURBATION CROSS
SECTIONS IN ASSEMBLY 48^(a)

Organization	Isotopic Cross Sections (× 10 ³)						
	Fe	Ni	Cr	O	C	Mn	Mo
AEEW FD2	-6.5	-13.5	-8.4	-2.5	-2.2	-28.4	
AEEW Ad-justed	-9.5	-15.3	-10.2	-2.5	-0.4	-28.4	
AI	-7.0	-9.6	-6.4	-0.3	+1.6		-36.0
ANL	-6.7	-12.3	-7.9		+1.2		-55.0
APDA	-9.1	-11.3	-9.9				
BNWL	-10.2	-15.0	-9.3	-1.7	-0.6		-65.6
B & W	-10.1	-13.6	-6.7	-13.2	+2.2		-89.4
Cadarache (H-R)	-7.3	-24.6	-21.8		-0.4		-68.7
Cadarache (USSR)	-7.3	-14.6	-8.4		-0.9		-56.9
Cadarache (25GRP)	-5.6	-6.2	-5.7	+2.5	+5.4		-73.8
GE	-9.5	-12.5	-8.2		+1.9		-61.6
Karlsruhe	-8.9	-15.8	-8.9	-2.8	-2.7		-52.4
LASL (H-R)	-7.7	-25.5	-22.3				-69.1
LASL (USSR)	-8.1	-14.7	-8.5				-61.4
W	-7.4	-9.9	-9.6	-2.6	-3.3		-44.1
Experiment	-8.4	-13.4	-6.1	-1.4	-1.1	-14.8	-52

^(a) Normalized to 1.0 for U²³⁵.

TABLE 7. CENTRAL LOW-ENERGY INDICES IN ASSEMBLY 48

Organization	Index Relative to U^{235} Fission				
	B^{10}/U^{235}	He^3/U^{235}	$\frac{U^{238}(n\gamma)}{U^{235}} \times 10^{-1}$	$\frac{Mn(n\gamma)}{U^{235}} \times 10^{-2}$	$\frac{Na(n\gamma)}{U^{235}} \times 10^{-3}$
AEW FD2	1.32		1.30	3.5	1.0
AEW adjusted	1.61		1.30	3.5	1.0
AI	1.39		1.36		0.78
ANL	1.32		1.27		
APDA	1.35		1.26		1.15
BNWL	1.26		1.32		0.94
B & W			1.29	3.1	1.15
Cadarache (H-R)	1.21	1.39	1.33	2.6	0.89
Cadarache (USSR)	1.14	1.37	1.30	3.9	0.86
Cadarache (25GRP)	1.28	1.55	1.32	3.6	1.12
GE	1.39		1.24		0.97
Karlsruhe	1.23		1.31		0.84
LASL (H-R)					1.42
LASL (USSR)					1.10
W	1.12	1.37	1.3		1.0
Experiment			1.38		

TABLE 8. PROMPT-NEUTRON LIFETIME IN ASSEMBLY 48

Organization	Rossi α (sec^{-1}) $\times 10^4$	$B_{\text{eff}} \times 10^{-3}$	Prompt lifetime (sec) $\times 10^{-7}$
AEW FD2	1.19	3.32	2.79
AEW adjusted			
AI	1.07	3.50	3.28
ANL			
APDA			
BNWL	1.1	3.45	3.13
B & W			
Cadarache (H-R)			2.62
Cadarache (USSR)			2.42
Cadarache (25GRP)	1.14		2.96
GE	1.20	3.55	3.0
Karlsruhe	1.37	3.48	2.53
LASL (H-R)			
LASL (USSR)			
W			
Experiment			

September, 1966, and collected into the form in which they are presented here.

It should be emphasized that in all cases the quantity of data and of explanatory text exceeds that presented here. Pressure of time and a certain degree of nonuniformity in presentation of the results precluded the presentation of more than the summary given here.

4. Experimental Data

The experimental data given here were obtained by members of both Idaho Division and of Reactor Physics Division, Illinois. Only the numerical data are summarized here, and fuller details are given elsewhere in this Conference in papers by Broomfield *et al.*, Bennett and Huber, and Gasidlo and Till.

Results of the Intercomparison

The calculated and experimental data are presented in Tables 1 through 14. All of the experiments have not yet been completed.

TABLE 9. DOPPLER EFFECTS OF U^{238} AND Pu^{239} IN ASSEMBLY 48

Organization	k for U^{238} Reactivity Doppler (300°K to 1100°K), $\times 10^6$	k for Pu^{239} Reactivity Doppler (300°K to 1100°K), $\times 10^6$	Change in U^{238} - activation Doppler (300°K to 750°K), %
AEW FD2	-3.15		
AEW Adjusted			
AI	-4.8	+1.04	
ANL	-5.16	+2.90	
APDA	-4.9	+3.7	
BNWL			+4.0
B & W			
Cadarache (H-R)			
Cadarache (USSR)			+3.3
Cadarache (25GRP)	-6.1	+7.7	+3.4
GE	-4.1	+0.58	+3.4
Karlsruhe	-3.22	+4.69	
LASL (H-R)			
LASL (USSR)			
W			
Experiment ^(a)	-4.9 $\pm$ 0.2	-1.3 $\pm$ 0.1	

^(a) Not corrected for expansion effects

TABLE 10. CALCULATED CENTRAL SPECTRA IN ASSEMBLY 48

Organization	Flux below 9.1 keV (%)	Flux below 1.23 keV (%)
AEW FD2	8.38	2.03
AI	8.87	2.21
ANL	8.27	1.93
APDA	8.76	1.98
BNWL	7.73	2.61
B & W	8.75	2.24
Cadarache (USSR)	7.0	1.6
Cadarache (25GRP)	7.8	1.8
GE	7.83	2.04
Karlsruhe	7.75	1.89
LASL (H-R)	8.56	2.01
LASL (USSR)	7.53	2.62
W	6.28	1.33

TABLE 11. CALCULATED AND MEASURED CENTRAL SPECTRA IN ASSEMBLY 48, PART 1

Group	AEFW FD2			ANL			GE		
	E_L	ϕ_c	ϕ_E	E_L	ϕ_c	ϕ_E	E_L	ϕ_c	ϕ_E
1	3.68 MeV	1.82		3.68 MeV	1.85		3.70 MeV	1.929	
2	2.23	3.87		2.23	3.53		2.20	3.815	
3	1.35	5.98		1.35	6.04		1.35	6.203	
4	0.821	7.31		0.821	8.55		0.825	8.557	
5	0.498	10.74	10.53	0.498	10.57	10.43	0.50	11.204	10.32
6	0.302	12.11	11.02	0.302	11.76	10.92	0.30	12.141	11.05
7	0.183	11.06	10.53	0.183	10.78	10.43	0.18	10.322	10.52
8	0.111	9.33	10.00	0.111	9.76	9.91	0.11	10.250	9.63
9	67.4 keV	8.57	8.70	67.4 keV	8.28	8.62	67.0 keV	7.985	8.46
10	40.9	7.06	6.92	40.9	6.96	6.85	41.0	6.673	6.66
11	24.8	5.59	5.66	24.8	5.44	5.60	25.0	5.032	5.48
12	15.0	4.64	4.62	15.0	4.65	4.58	15.0	4.689	4.63
13	9.12	3.54	3.74	9.12	3.56	3.71	9.1	3.366	3.69
14	5.53	2.28	2.72	4.31	3.21	3.72	5.5	2.210	2.69
15	3.36	1.68	1.77	2.61	0.83	1.24	3.9	1.159	1.32
16	2.04	0.835	1.23	2.04	0.91	0.70	2.5	0.551	1.01
17	1.23	1.554		1.23	1.39		2.0	0.510	0.64
18	748 eV	0.970		961 eV	0.61		1.3	1.305	
19	454	0.547		583	0.64		800 eV	0.891	
20	275	0.327		275	0.46		502	0.574	
21	167	0.125		101	0.18		310	0.330	
22	101	0.046		29	0.04		150	0.214	
23	61.4	0.012					55.6	0.075	
24	37.3	0.004					Thermal	0.015	

TABLE 12. CALCULATED AND MEASURED CENTRAL SPECTRA IN ASSEMBLY 48, PART 2

Group ^(a)	APDA			B & W			W		
	E_L	ϕ_c	ϕ_E	E_L	ϕ_c	ϕ_E	E_L	ϕ_c	ϕ_E
1	3.68 MeV	2.225		3.68 MeV	2.174		3.67 MeV	2.10	
2	2.23	3.587		2.23	3.960		2.22	3.85	
3	1.35	5.684		1.35	6.344		1.35	6.75	
4	0.821	7.621		0.821	8.108		0.818	9.15	
5	0.498	10.048	10.52	0.498	9.122	10.27	0.496	11.90	10.26
6	0.302	11.453	10.01	0.302	10.250	10.74	0.301	12.77	10.69
7	0.183	10.858	10.52	0.183	9.920	10.26	0.183	11.20	10.15
8	0.111	9.793	9.99	0.111	9.878	9.75	0.111	9.65	9.71
9	67.4 keV	8.398	8.69	67.4 keV	8.811	8.48	67.2 keV	7.70	8.50
10	40.9	7.270	6.91	40.9	7.718	6.74	40.7	6.75	6.73
11	24.8	5.705	5.66	24.8	5.570	5.51	24.7	4.80	5.48
12	15.0	4.864	4.62	15.0	5.405	4.51	15.0	4.10	4.44
13	9.12	3.736	3.74	9.12	3.992	3.65	9.09	3.00	3.66
14	5.53	2.374	2.72	5.53	2.580	2.65	4.00	2.71	4.96
15	3.36	1.686	1.76	3.36	1.643	1.72	1.00	2.64	
16	2.04	1.199	1.22	2.04	.600	1.19	300 eV	0.84	
17	1.23	1.522		1.23	1.680		100	0.09	
18	748 eV	0.909		748 eV	1.107		30	0.00	
19	454	0.543		454	0.602				
20	275	0.303		275	0.323				
21	167	0.133		167	0.132				
22	101	0.057		101	0.046				
23	61.4	0.024		61.4	0.021				
24	37.3	0.006		37.3	0.009				
25	22.6	0.002		22.6	0.004				
26	13.7	0.000		13.7	0.001				

^(a) Note that (1) with APDA data the top four groups have been combined to give Group 1, and thereafter each group here represents two APDA groups; (2) with B & W the top two groups have been combined here.

Certain of the calculated values have been modified to facilitate comparison with each other and with experiment. In particular, where necessary, the calculated critical masses have been modified to give the corresponding critical masses for heterogeneous, cylindrical cores. A mass difference of 25 kg of Pu²³⁹ between the homogeneous and heterogeneous critical masses corresponds to a difference in k of 1.4%. This is based on calculated values of 1.4% and 1.1% by AEEW and ANL, respectively.

This correction for heterogeneity is *not* well established.

Additional Information

Final experimental data will be published in an Argonne National Laboratory report.

Further information on the computations should be obtained by direct contact with the participants. These are listed below.

(a) AEEW	Mr. J. Codd
(b) AI	Dr. E. R. Specht
(c) ANL	Dr. A. Travelli
(d) APDA	Dr. T. A. Pitterle
(e) BNWL	Dr. W. W. Little, Jr.
(f) B & W	Dr. G. E. Wolfe
(g) Cadarache	Mr. J. M. Chaumont, Mr. J. Ravier
(h) GE	Dr. P. Greebler
(i) Karlsruhe	Dr. M. Metzenroth
(j) LASL	Dr. M. E. Battat
(k) W	Dr. J. E. Oelhoeft, Dr. L. E. Strawbridge

APPENDIX—THE CIRCULATED SPECIFICATIONS OF ASSEMBLY 48

Part A: Characteristics of ZPR-3 Assembly 48

The compositions of the core and reflector of the ZPR-3 assembly are given in Table I. The core length will be fixed and the radius increased until criticality is reached. Thus, the true core will be a pseudo-cylinder and the length-to-diameter ratio, although known to be approximately unity, will not be known exactly until criticality is reached. However, since most of the measurements will be made at the center of the core where the spectrum is not sensitive to the exact geometry, calculations in spherical geometry will be of similar value to those carried out in the more correct cylindrical geometry.

We propose, where possible, to reduce the experimental data to those appropriate to a homogeneous reactor so that a homogeneous model may be assumed for computation. For those wishing to consider heterogeneity effects, the approximate dimensions and masses of the unit core cell are given in Table II.

We anticipate you will select your preferred data and computational techniques for this study. Any ex-

TABLE I. HOMOGENEOUS COMPOSITIONS IN ASSEMBLY 48 ($\times 10^{24}$ atoms/cm³)

Isotope	Core	Reflector
Pu ²³⁹	0.001644	0.000
Pu ²⁴⁰	0.000106	0.000
Pu ²⁴¹	0.000010	0.000
Pu ²⁴²	0.000000	0.000
U ²³⁵	0.000016	0.000084
U ²³⁸	0.007406	0.039976
C	0.020765	0.000
Na	0.006230	0.000
Fe	0.009899	0.004453
Cr	0.002658	0.001196
Ni	0.001308	0.000588
Mo	0.000206	0.000
Al	0.000110	0.000

The core length will be 30.06 in. (76.35 cm).

The reflector thickness will be a nominal 30.0 cm, both axially and radially.

Note: This homogeneous composition is calculated on the basis of the *actual* plate configuration that will be used and is much more precise than figures which can be derived from the nominal dimensions given in Table II.

TABLE II. CELL ARRANGEMENT IN ASSEMBLY 48

The plate arrangement will be repeated in each core drawer in Assembly 48. Each plate is of 2-in. nominal height. The plates are loaded in perforated steel drawers which contain 10.36 g of stainless steel per inch of length. Each drawer slides in a steel matrix tube with 0.040-in.-thick walls and outer dimensions 2.1835 in. by 2.1755 (i.e., 44.64-g/in. length). The plate sequence and nominal thicknesses (including cladding) are as follows:

	C	Na	U	Pu	U	C	Na	C	C
(in.)	1/8	1/4	1/8	1/8	1/8	1/8	1/4	1/8	1/8
	Alloy				Na		C		
(in.)	1/4				1/4		1/8		

Each sodium plate is clad in 0.015-in.-thick stainless steel.

The plutonium plate contains 64.5% of the total plutonium and it is clad in 0.012-in.-thick stainless steel.

The alloy contains 35.5% of the total plutonium and 34.9% of the depleted uranium, and it is clad in 0.012-in.-thick stainless steel.

The depleted uranium contains 0.206 w/o U²³⁵.

The stainless steel is Type 304 with nominal weight percentages of Fe, Cr, and Ni of 72, 18, and 10, respectively.

It should be noted that the use of accepted densities of materials and the above dimensions will not lead to the *exact* homogenized compositions given in Table I because of the effects of cladding on the ends of plates, etc.

The general characteristics of ZPR-3 are described by Cerutti, Nucl. Sci. Eng., **1**, 126 (1956).

planatory notes you wish to add to your contribution will be welcome.

To facilitate comparison of calculated data, we sug-

gest that the results of perturbation calculations of reactivity worths should be quoted as microscopic effective cross sections in barns.

The following data will be measured:

- (1) Heterogeneous, cylindrical, critical mass, corrected for edge roughness.
- (2) Critical-mass correction for heterogeneity of the core will be estimated, probably by computation.

$$\sigma_p = \frac{\sum_i \nu_i \sigma_{fi} \Phi_i \sum_j \chi_j \Phi_j^+ - \sum_i \sigma_{ai} \Phi_i \Phi_i^+ + \sum_i \Phi_i \left[\sum_j \sigma_{i \rightarrow j} (\Phi_j^+ - \Phi_i^+) \right]}{\sum_i \Phi_i \Phi_i^+}.$$

- (3) Central reactivity worths of Pu^{239} , Pu^{240} , U^{235} , U^{238} , Na, Fe, Ni, Cr, O, C, Mn, Mo, Ta, and B^{10} .
- (4) Central fission ratios, $\text{U}^{238}/\text{U}^{235}$, $\text{U}^{236}/\text{U}^{235}$, $\text{U}^{234}/\text{U}^{235}$, $\text{Pu}^{240}/\text{U}^{235}$, $\text{U}^{233}/\text{U}^{235}$, and $\text{Pu}^{239}/\text{U}^{235}$.
- (5) Central count rate ratios of $\text{B}^{10}(n\alpha)/\text{U}^{235}(\text{nf})$, $\text{He}^3(\text{np})/\text{U}^{235}(\text{nf})$, and $\text{U}^{238}(n\gamma)/\text{U}^{235}(\text{nf})$.
- (6) The ratio of the sodium capture cross section and the manganese capture cross sections to the fission cross section of U^{235} at the core center.
- (7) Reactivity Doppler effects of U^{238} and Pu^{239} in the range from 300 to 1100°K.
- (8) The activation Doppler effect in a thin U^{238} foil from approximately 300°K to 750°K.
- (9) The central neutron spectrum using a (np) recoil spectrometer and a Li^6 spectrometer.
- (10) The "Rossi- α " (by the pulse source method).
- (11) The reactivity effects of removal of sodium at or near the core center.

Part B: ZPR-3 Assembly 48—Supplemental Information

1. Reactivity Measurements

The standard size of sample for measurement of central reactivity coefficients is a cube of 2-in. side. Central measurements with this sample size will be made with sodium, carbon, iron, chromium, nickel, Type 304 stainless steel, and natural uranium. The sodium and chromium samples will be canned in thin stainless steel, and the reactivity of the can will be measured separately. The other materials listed in Item 3 of the original note will be measured in a range of smaller sizes to investigate sample-size effects and it is hoped to deduce the reactivity that would be measured with a very small sample size.

2. Reactivity Calculations

Central reactivity calculations will probably be made by a number of computational techniques, e.g.,

direct comparisons of k with and without the sample in a central void, and by perturbation theory. In view of this probable diversity, it is essential to have some common feature to make intercomparison of data practicable. It is urged that in addition to any other form of quoting calculated data, the results should be given as a simple atomic perturbation cross section in barns.

This we define as σ_p , where

$$\sigma_p = \frac{\sum_i \nu_i \sigma_{fi} \Phi_i \sum_j \chi_j \Phi_j^+ - \sum_i \sigma_{ai} \Phi_i \Phi_i^+ + \sum_i \Phi_i \left[\sum_j \sigma_{i \rightarrow j} (\Phi_j^+ - \Phi_i^+) \right]}{\sum_i \Phi_i \Phi_i^+}.$$

In this definition $\sum_j \chi_j$ is normalized to unity. The real and adjoint fluxes need not be normalized. The fluxes can, of course, be point fluxes or could be averaged over a central sample volume if so desired.

If the principal method of computation is of Δk on insertion of the sample (either by two k calculations or some other technique), then calculation of the simple value of σ_p for one sample will permit normalizing all the calculated values to this figure. This does imply that the sample sizes used in all calculations are "small."

3. Measurements of Sodium Reactivity

These will be at or near the core center. Apart from the 2-in. cube described in Section 1, there will be measurements of the effect of removal of 1/4-in.-thick sodium plates at and near the core center. For the purposes of the intercomparison, however, calculations should be only of central values since the exact specification of all the experiments cannot be made at this time.

4. Reactivity Doppler Measurements

The samples will be PuO_2 and natural UO_2 . Each Doppler element is two cylinders of 1.270-cm diameter and 15.28 cm long. The total PuO_2 weight is 131.91 g and that of natural UO_2 is 137.2 g. The cans, which are principally Inconel, each weigh 97.5 g. At a temperature of 500°K, the atomic densities (atoms/cm³ $\times 10^{-24}$) are as follows:

	Pu^{239}	Pu^{240}	U^{238}	U^{235}	O
PuO_2	0.01471	0.00038	0	0	0.03016
UO_2	0	0	0.01560	0.00011	0.03142

In the reactor, the entire element is enclosed in a stainless steel can 2.54 cm square with walls 0.103 cm thick.

5. U^{238} Activation Doppler Measurement

The measurement will be made with a depleted uranium foil 0.500 in. in diameter and 0.005 in. thick. The U^{235} content is 0.2 a/o.

Discussion

Mr. Engelder (B & W): I presume that the experimental values of the perturbation cross sections for Pu^{240} assume some value of the adjoint spectrum. Is there any way of knowing which of the calculated values may have used an adjoint spectrum that was the same as was used in the interpretation of the Argonne experiments.

Mr. Davey: There is no assumption of either adjoint or real spectrum in determining the experimental Pu^{240} cross section. This is merely the reactivity effect per atom of Pu^{240} relative to the reactivity effect per atom of U^{235} normalized to 2.62 barns for U^{235} . (Ed. note: The 2.62-barn figure, however, is the result of averaging all the calculated U^{235} worths, and as such is an average from all the real and adjoint calculations.) I should emphasize that in all these experiments there are considerable sample size effects. They have been to some extent taken care of in these data.

Mr. Weale (UK-Aldermaston): What normalization was used in the tables of measured and calculated central spectra?

Mr. Davey: The normalization involved summing the calculated spectrum over the region of overlap for each experiment. I normalized the total measured flux in that energy range with the total calculated flux in the same energy range. You can, of course, adjust these any way you want, since only the shapes matter. I thought that a direct group by group comparison of the experimental and calculated spectra would be of value.

Mr. Specht (AI): Do you have a value for the experimental percentage of flux below 9.1 keV?

Mr. Davey: No, I don't. The measurements extended only down to about 1.5 keV. You must make your own extrapolation below 1.5 keV.

Mr. Weale: I would like to again consider the normalization of the spectra. It is true that only the shape matters if you measure the complete spectrum and compare these. But it would be misleading to conclude from this that one group is predicting too low a spectrum in any one particular energy region, because you don't know what is happening in the region above about 1 MeV.

Mr. Davey: I agree it could be misleading when the entire energy range is considered and clearly the spec-

trum above 1 MeV should be measured, as it is perhaps 15 or 20% of the flux. This is the best presentation I could think of for the data which we have. We would like to be able to duplicate your work on VERA in order to extend the data to lower as well as higher energies.

Mr. Loewenstein (ANL): Has everyone used the low Pu fission cross sections in this series of calculations?

Mr. Davey: To the best of my knowledge they have not. For example, the Russian data and Hansen-Roach data are included in this intercomparison.

Mr. Loewenstein: But these are nonmodified.

Mr. Davey: I presume that they are nonmodified, but you would have to ask each individual analyst.

Mr. Karam (ANL): Do you intend to produce cross-section sets by the MC² code or ELMOE code for this particular core and make these same comparisons?

Mr. Davey: Yes, we certainly wish to use the MC² cross-section sets. I believe that the APDA calculations were strongly based on the MC² cross-section set. I believe that MC² itself is not the complete answer. The problem of determining the effective scattering cross section per absorber atom for a heterogeneous system is not straightforward.

Mr. Karam: I am concerned about the influence of different regions. For instance, what happens to the spectrum in a reflector and how does that compare with a typical cross-section set. What is the comparison between the MC²-produced cross-section set and the cross-section set which you presented?

Mr. Davey: Let me clarify my position. I haven't done any of this analysis or these calculations myself. I am summarizing the calculations and experiments of others.

Mr. Campise (AI): In one of your tables there is a reference to the Hansen-Roach cross-section set from the French as well as from Los Alamos and yet the results are different. Is there a difference in these cross sections even though the same reference is used?

Mr. Davey: This question should be directed to the individuals who performed the calculations summarized here. It is quite true that there are differences in the separate calculations based on both the Hansen-Roach and Russian sets. I would be very surprised indeed if the data used were identical.

Presented by P. Greebler

Implications of Recent Fast Critical Experiments on Basic Fast Reactor Design Data and Computational Methods*

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ABSTRACT

Recent experimental information from dilute, plutonium-fueled critical assemblies was used to test and to provide guidance for improving fast reactor design data and calculational methods. Most of the experimental data used in this study are taken from ZPR-III Assembly 47 (the SEFOR core mockup). The important nuclear reactor parameters were calculated with a number of variations in nuclear data and calculational techniques. An analysis of the experimental and calculated results shows that by careful adjustment of the important cross sections, well within experimental uncertainties, and by improved accuracy in the calculational methods, a much closer agreement can be achieved between calculations and experiments than that heretofore reported.

A complete evaluation was made of the cross-section and resonance parameter data for Pu-239, the most important isotope in this case. The new data yield good agreement with the critical mass of Assembly 47. They result in a calculated Pu-239 Doppler effect that is essentially zero, in agreement with the measured values. A two-dimensional calculation of the neutron lifetime, using group constants that adequately account for the spatial variation of the neutron spectrum, yields a significant improvement over that based on a one-dimensional model.

Those adjustments in nuclear data which had the greatest effect on Assembly 47 results were then used to calculate the important parameters of ZPR-III Assembly 48. Their effects on Assembly 48 results can then be analyzed as the experimental results for this assembly become available.

1. Introduction

Recent experimental information obtained from dilute plutonium-fueled critical assemblies^(1, 2) provides

* Work performed under Contract AT(04-3)-189 Project Agreement 10 between the U. S. Atomic Energy Commission and the General Electric Company.

a basis for testing the accuracy of cross-section data and calculational methods, and for indicating improvements in the data and methods where large uncertainties now exist. In this paper, experimental results from ZPR-III Assembly 47⁽¹⁾ are analyzed and used to guide such improvements of data and methods which, in turn, are applied in the precalculation of the results for ZPR-III Assembly 48.⁽²⁾

Large discrepancies have persisted between certain measured and calculated physics quantities in large, dilute critical assemblies. These discrepancies have been most pronounced for parameters that are strongly dependent on the low-energy portion of the neutron flux and adjoint spectra, such as neutron lifetime, boron-10 worth, and Doppler effects. The Pu-239 Doppler coefficient, in particular, has been measured as small and negative in several assemblies,^(1, 3, 4) at variance with a calculated large, positive value. Recently, theoretical models of the Pu-239 resonance structure have been proposed^(5, 6) that appear to account substantially for the measured Doppler results. The analysis of Assembly 47 results here is focused largely on those data mentioned above that are highly sensitive to the uncertainties in the Pu-239 and U-238 cross sections, and where large discrepancies have persisted between measurement and calculation. Other quantities measured in Assembly 47, such as critical mass, fission ratios, sodium-void reactivity, and spatial fission-rate profiles, are also used in the analysis to support conclusions reached on which cross-section data and calculational methods give better general agreement with the experimental results.†

Analysis of cross-section information obtained from the Assembly-47 integral measurements was done for Pu-239 and, to a lesser extent, for U-238. The concentrations of the higher plutonium isotopes in Assembly

† Many other quantities were measured in Assembly 47 in support of the SEFOR reactor core design and operational planning. The analysis of all Assembly 47 data with emphasis on its application to the design of the SEFOR reactor is covered in a separate paper at this conference.⁽⁷⁾

47 were not large enough, and the influence of other individual material cross-section uncertainties on the integral measurements were too small,^(8, 9) to allow similar analyses for these materials in Assembly 47. Variations considered in the Pu-239 data, in light of recent measurements and theory, included s-wave resonance parameters, fission cross section and alpha values at intermediate and low energies, and fission spectrum parameters χ and $\bar{\nu}$. Only $\sigma(n, \gamma)$ variations were considered for U-238. Several common approximations in multigroup computational methods were evaluated for their effects on the calculated results for Assembly 47, including one versus two dimensions, and spatial variation of neutron spectra used in the condensation of fine-group cross sections to group constants for coarser energy intervals.

2. Computational Model and Methods

2.1. Description of the Critical Assemblies

Assembly 47 has both a core composition and neutron spectrum which are representative of large, dilute, Pu-U-ceramic-fueled, fast power reactors. The description of Assembly 47 in Ref. 1 is generally consistent with the model used for our calculations. Figure 1 shows a schematic diagram of the assembly.

The main difference between core regions 1 and 2 is the degree of heterogeneity of Pu-239. Region 1 has $\frac{1}{4}$ -in.-thick mixed U-Pu alloy plates, whereas in region 2 the uranium and plutonium are separated, with the metallic plutonium in $\frac{1}{8}$ -in.-thick plates. This results in considerably different σ_p values for Pu-239 in the two regions, even though the homogeneous concentrations are almost identical.

Assembly 48 has been described.⁽²⁾ It is simpler in geometry than Assembly 47 in that the core is composed of a single region, and has axial and radial blankets which are of identical composition, which are placed directly adjacent to the core. Carbon is the main light-element elastic-scattering material in Assembly 48; beryllium and oxygen are in Assembly 47. These differences in core composition, and in the configuration and composition of the reflector (blanket), give Assembly 48 an appreciably harder core-neutron spectrum than that of Assembly 47. The fuel plates in Assembly 48 are the same as those used in Assembly 47, with the alloy and pure plutonium plates mixed as well as possible. Table 1 compares several calculated physics parameters for these two critical assemblies.

2.2. Basic Cross-section Data

Several different sets of cross-section data were used in the calculation of physics parameters for Assembly 47. The data sources for these sets are described here

FIG. 1. Dimensions and Composition of ZPR-III Assembly 47

TABLE 1. COMPARISONS BETWEEN PHYSICS PARAMETERS* IN ASSEMBLIES 47 AND 48

	Assembly 47	Assembly 48
Critical Mass (Pu-239), kg	314†	276
Core Radius (Equivalent cylinder), cm	44.1	42.0
Mean Fission Energy, keV	150	250
Fissions below 10 keV, %	22	14
Neutron Lifetime, sec $\times 10^6$	0.67†	0.3
U-238 Doppler $[-T(dk/dT)]$	~ 0.01	~ 0.008

* Assembly 47 has no blankets, and different compositions of axial and radial reflectors. Assembly 48 has thick (30-cm) axial and radial blankets, primarily composed of U-238.

† Measured value.

and identified with the nomenclature used later in the left-hand columns of Tables 5 and 6.

The cross-section set used as a basis for the data identified in Table 5 as "Old Pu-239 Data" was essentially the same as the G.E. 60-group set used in our reactor parameter uncertainty study reported at the 1965 Argonne Fast Reactor Conference and published in GEAP-4471.⁽¹⁰⁾ This 60-group set is very similar to the ANL-224 set,⁽¹¹⁾ with identical group structure and ELMOE-corrected values for an oxide core, down to 180 keV. Below 180 keV the 60-group structure is very fine (0.1 to 0.3 lethargy unit) down to several

TABLE 2. SOURCES FOR σ_f AND $\sigma(n, \gamma)$ IN "NEW PU-239 DATA" CROSS SECTIONS

	"New Pu-239 Data"	Main Differences in "Old Pu-239 Data"
σ_f		
Up to 300 eV	Resolved resonances taken from Saclay measurements ⁽¹⁸⁾ and compiled by Schmidt. ⁽⁵⁾ Increased to agree with Petrel bomb data ⁽¹⁹⁾ between 200 and 300 eV.	Resolved resonances to 55 eV from BNL-325, Supplement No. 1. ⁽²⁵⁾
0.3 to 3 keV	Used combination of Petrel bomb data (up to 1 keV), ⁽¹⁹⁾ James data, ⁽¹⁵⁾ and strength functions based on Uttley's σ_i measurements. ⁽²⁰⁾	James data. ⁽¹⁵⁾
Above 3 keV	Data recommended by Davey ⁽²¹⁾ up to 1 MeV. BNL-325 supplement 2 ⁽²²⁾ curve above 1 MeV.	James data to 15 keV and White <i>et al.</i> ⁽¹⁴⁾ to 0.1 MeV. White data gives 5 to 15% lower σ_f between 25 and 100 MeV.
$\sigma(n, \gamma)$		
Up to 300 eV	Resolved resonances. ^(5, 18) Small upward adjustment 200 to 300 eV.	Resolved resonances to 55 eV from BNL-325, Supplement 2. ⁽²²⁾
0.3 to 40 keV	Alpha calculated from s-wave resonance parameters (see Table 3). At low energies alpha curve is higher than that recommended by Schmidt ⁽⁵⁾ and slightly below that recommended by Sampson and Molino. ⁽²³⁾ *	Hopkins-Diven ⁽²⁴⁾ alpha values used down to 30 keV. Alpha close to new data below 30 keV.
Above 40 keV	Hopkins-Diven ⁽²⁴⁾ alpha data extrapolated to zero at 2 MeV.	

* The alpha curve calculated from our s-wave parameters is in good agreement with the recent ORNL alpha measurements made down to 20 keV.⁽³⁸⁾

TABLE 3. s-WAVE RESONANCE PARAMETERS BETWEEN 0.3 AND 100 keV AND FISSION SPECTRUM IN "NEW PU-239 DATA" CROSS SECTIONS

"New Pu-239 Data"	Main Differences in "Old Pu-239 Data"
$\langle \Gamma_n^{(0)} \rangle_{J=0} = 0.94 \times 10^{-3} (\text{eV})^{1/2}$ above 2 keV.* $\langle \Gamma_n^{(0)} \rangle_{J=1} = 0.334 \times 10^{-3} (\text{eV})^{1/2}$ above 2 keV.* $\langle D \rangle_{J=0} = 8.78 \text{ eV}$ $\langle D \rangle_{J=1} = 3.12 \text{ eV}$ $\langle \Gamma_\gamma \rangle_{J=0} = \langle \Gamma_\gamma \rangle_{J=1} = 0.0387 \text{ eV}$ $\langle \Gamma_f \rangle_{J=0} = 2.8 \text{ eV}$ $\langle \Gamma_f \rangle_{J=1} = \frac{\langle D \rangle_{J=1}}{2\pi}$. $\left[1 + \exp \left(2\pi \frac{50 - E_{\text{keV}}}{150} \right) \right]^{-1}$ $(\nu_n)_{J=0} = (\nu_n)_{J=1} = (\nu_f)_{J=1} = 1$ $(\nu_f)_{J=0} = 2$ $(\nu_\gamma)_{J=0} = (\nu_\gamma)_{J=1} = \infty$ Maxwellian $\chi(E)$, $T = 1.41 \text{ MeV}$ $\bar{\nu} = 2.87 + 0.135 E_{\text{MeV}}$	$\langle \Gamma_f \rangle$ values—see Table I of Ref. 26—were not spin dependent. $\langle \Gamma_f \rangle$ was computed to give the best estimate of α , assuming $\nu_f = 3$. ⁽²⁶⁾ This did not give highly self-shielded resonances with high alphas, as in the $J = 1$ sequence of the "New Pu-239 Data." Maxwellian Spectrum, $T = 1.31 \text{ MeV}$

* From 0.3 to 3 keV, the reduced neutron widths were adjusted for cross-section fluctuations using a combination of Uttley's σ_i data,⁽²⁰⁾ Petrel data,⁽¹⁹⁾ and James data.⁽¹⁵⁾

hundred eV, providing direct calculation of small variations in the neutron-energy spectrum. Improvements applied to this 60-group set in going to the "Old Pu-239 Data" set included some minor corrections in the elastic removal cross sections for sodium, oxygen, and beryllium, and the following larger changes:

(1) Reduction of Pu-239 $\bar{\nu}$ values to give a com-

promise between the old values and the lower measurements of Colvin and Sowerby⁽¹²⁾ in substantial agreement with the recommended values of Wescott *et al.*⁽¹³⁾ in the thermal range.

(2) Lowering of the Pu-239 fission cross sections below 0.1 MeV, corresponding to the measurements of White *et al.*⁽¹⁴⁾ and James.⁽¹⁵⁾

(3) Changes in the U-238 capture cross section above 0.1 MeV to conform to the data of Barry, Bunce, and White.⁽¹⁶⁾

A recent evaluation was made of the Pu-239 cross sections in connection with the compilation of the ENDF/B file.⁽¹⁷⁾ Cognizance was taken of new Pu-239 data obtained in the last two years and of their more recent evaluation. The cross-section data identified as "New Pu-239 Data" in Tables 5 and 6 differ from the "Old Pu-239 Data" in that newer Pu-239 information is utilized. Unless specified otherwise (as in Tables 5 and 6), the calculated results reported here are based on the "New Pu-239 Data" cross sections. Table 2 lists the data sources for Pu-239 σ_f and $\sigma(n, \gamma)$ in the "New Pu-239 Data" set, and indicates the main differences between it and the "Old Pu-239 Data" set.

Table 3 lists the s-wave average resonance parameters in the "New Pu-239 Data" set. These are derived from the analysis by Schmidt⁽⁵⁾ of resolved Pu-239 resonances, and from other recent measurements with Pu-239, with adjustments as noted in Table 3. The use of the new Pu-239 resonance parameters with widely different $\langle \Gamma_f \rangle$ values for the two compound-nucleus spin states is probably the most important change between the "Old Pu-239 Data" and the "New Pu-239

Data" sets, especially in its effect on the Pu-239 Doppler coefficient as discussed in Section 3.1. Table 3 also lists the new parameters used for the fission spectrum.

2.3. Variations in the Data

Variations from the "New Pu-239 Data" set considered in Tables 5 and 6 are described below.

Pu-239 (600–900 eV); Higher σ_f

The Petrel and James σ_f data and the Uttley σ_i data upon which the "New Pu-239 Data" values are based show a sharp dip in the region from 600 to 900 eV. This variation is introduced to determine the effect on the calculated physics parameters of making the dip only half as great as indicated by the above data. The term "dip" is used here to denote the departure on the low side from the calculated σ_f , assuming a constant s-wave strength function of $1.07 \times 10^{-4} (\text{eV})^{-1/2}$. Alpha values were held fixed for this variation, i.e., $\sigma(n, \gamma)$ was adjusted in proportion to σ_f and the Pu-239 Doppler $\delta\sigma$ values were changed accordingly.

Pu-239 (<10 keV); Lower α

This variation lowers the alpha values below 10 keV to the curve recommended by Schmidt⁽⁵⁾ (about 9% lower than the "New Pu-239 Data" values). Values of $\delta\sigma(n, \gamma)$ for the Pu-239 Doppler-effect calculation were reduced proportionally to α .

Pu-239 (20–100 keV); Lower σ_f

This variation on the "New Pu-239 Data" set used the σ_f data of White *et al.*,⁽¹⁴⁾ which is 5 to 15% lower in the indicated range. The alpha value was again held constant.

U-238 (40–300 keV); Lower σ_c

This variation lowers the U-238 $\sigma(n, \gamma)$ values in the "New Pu-239 Data" set in the indicated energy range to the curve in BNL-325 Supplement 2.⁽²²⁾ Our U-238 $\sigma(n, \gamma)$ values between 40 and 300 keV are 5 to 20 percent higher than given by the curve in BNL-325, joining the Barry, Bunce, and White⁽¹⁶⁾ measurements above 0.1 MeV and the BNL-325 curve (representing roughly an average of the available data at 40 keV).

U-238 (4–40 keV); Higher σ_c

This variation raises the U-238 $\sigma(n, \gamma)$ values in the "New Pu-239 Data" set in the indicated energy range to the data of Macklin and Gibbons⁽²⁸⁾ measured at Oak Ridge. Our values correspond to the earlier Moxon and Chaffey⁽²⁷⁾ measurements at Harwell using the Moxon-Rae detector and are 10 to 40 percent below

the Oak Ridge data [see curve in BNL-325 Supplement No. 2⁽²²⁾].

$$\langle \Gamma_f \rangle_{J=0} 2.8 \rightarrow 1.5 \text{ eV}$$

The value of $\langle \Gamma_f \rangle$ is changed from 2.8 eV to 1.5 eV. The 2.8-eV value follows from channel theory as discussed by Schmidt.⁽⁵⁾ Pitterle⁽⁶⁾ obtains a value around 1.5 eV, and Schmidt 1.3 eV from analyses of the resolved resonance parameters, but Schmidt justifies using a higher value than indicated by resolved resonances. We find that the 2.8-eV value gives slightly better agreement with integral measurements and consider the 1.5-eV value as an alternative. Since this large value of $\langle \Gamma_f \rangle_{J=0}$ tends to dominate Γ_n , the physics results are rather insensitive to this parameter, and only small changes in the important physics quantities result from a change by a factor of 2.

$$\langle \Gamma_f \rangle_{J=1} 0.055 \rightarrow 0.06 \text{ eV}$$

This variation is introduced to determine the sensitivity of calculated parameters to $\langle \Gamma_f \rangle_{J=1}$. The values 0.055 and 0.06 eV are those which apply in the low-energy region ($\ll 50$ keV) where $\langle \Gamma_f \rangle_{J=1}$ is given by the expression in Table 3. Since the change from 0.55 to 0.6 eV involves the coefficient $\langle D \rangle_{J=1}/2\pi$, the same fractional change applies at all energies, and $\langle \Gamma_f \rangle_{J=1}$ was adjusted by the ratio 6/5.5 over the entire energy range from 300 eV to 100 keV.

$$\bar{\nu}_{\text{therm}}^{239} 2.87 \rightarrow 2.82$$

This variation lowers $\bar{\nu}$ to the value indicated by the measurements of Colvin and Sowerby.⁽¹²⁾

$$\chi^{239} (\text{Lower } T)$$

This variation lowers the neutron "temperature" for the Maxwellian fission spectrum distribution from 1.41 to 1.31. The latter value is close to the U-235 temperature,⁽²⁹⁾ i.e., for the U-236 compound nucleus.

2.4. Methods of Calculation

All calculations were made in multigroup diffusion theory. The aim throughout was to obtain, effectively, 60-group, two-dimensional results, by minimizing and correcting for the effects of group-collapse and one-dimensional representations wherever applicable. The detailed procedures used for Assemblies 47 and 48 differed somewhat, as Assembly 48 is geometrically far simpler than Assembly 47.

Space-dependent group-collapses were made using 60-group, one-dimensional calculations for each assembly with seven spatial regions for Assembly 47 and two regions for the simpler Assembly 48. Region-dependent cross sections were obtained in 16 groups for Assembly 47 and in 24 and 8 groups for Assembly 48.

FIG. 2. Measured and Calculated Doppler Reactivity Changes in Assembly 47.

A basic two-dimensional CRAM⁽³⁹⁾ calculation was run for each assembly, using 16 groups for Assembly 47 with its known critical configuration, and 8 groups for Assembly 48 in conjunction with a critical-radius search. Multigroup perturbation calculations were performed in both cases for the calculation of material worths, prompt-neutron lifetime, effective delayed-neutron fraction, and Doppler coefficients of reactivity. A supporting set of one-dimensional calculations was made for each assembly to study the effects of cross-section variations, group-collapse errors, and mesh spacing.

All one-dimensional calculations for Assembly 47 were in slab or cylindrical geometry, using bucklings obtained from the two-dimensional results. Those for Assembly 48 were in spherical geometry with a core volume equal to that of the critical core as computed in two dimensions, and a 30-cm-thick blanket. Corrections for two-dimensional effects were made to the one-dimensional results where appropriate, by reference to the base two-dimensional models. Areas where this correction are important include the perturbation denominator and any other cases where two-dimensional integration is required, such as for the Doppler effect in the extended Doppler sample. Corrections for effects of group-collapse and mesh spacing were found to be very small for both assemblies treated.

Self-shielding corrections to the cross sections of the basic 60-group file were applied as follows:

(1) In addition to the ANL 224⁽¹¹⁾ ELMOE corrections for an oxide core applied above 180 keV, self-shielding for the broader light-element resonances below 180 keV was accounted for by the fine-group structure of the 60-group set. Additional light-element self-shielding factors were computed from Bondarenko *et al.*⁽³⁰⁾

(2) Self-shielded cross sections and Doppler changes for the fuel isotopes U-238 and Pu-239 were computed by the RAPTURE code.⁽³¹⁾ The σ_p values were obtained by use of Bell's two-region heterogeneous approximation.⁽³²⁾ For Assembly 47, two-region cells typical of the inner and outer fuel regions were calculated, and cross sections were evaluated for both regions separately. For Assembly 48, σ_p 's and cross sections were calculated for the two basic cell types appearing in all core drawers, representing the "alloy" and the "U/Pu plate" fuel arrangements, each immersed in the remaining drawer material. These cross sections were then mixed in "alloy"/"plate" proportions of 0.355/0.645 for plutonium and 0.349/0.651 for uranium.

(3) Small corrections for high-energy heterogeneities (first-flight fission effect) was applied to k_{eff} .

(4) Remaining heavy isotopes were used at effectively infinite dilution.

(5) Cross sections for fissile and fertile samples used to measure capture and fission ratios, material worths, and Doppler coefficients were self-shielded with σ_p values for their homogeneous core concentrations. In the case of Doppler calculations, small-sample effects were also investigated.⁽³³⁾

Two computational approximations were studied for Assembly 47. The calculated physics quantities in Table 5 were computed by (1) iterating energy-dependent bucklings in axial and radial, one-dimensional problems, and (2) condensing to 16 groups in only the core spectrum. These results can be compared with the more exact two-dimensional CRAM results.

3. Analysis of Results with Assembly 47

3.1. Pu-239 and U-238 Doppler Effects

Figure 2 compares the measured sample Pu-239 Doppler effects between 300 and 800°K with those calculated for the $J = 1$ sequence of s-wave resonances. Also shown in Fig. 2 are measured and calculated U-238 Doppler effects. These Pu-239 and U-238 sample Doppler-effect comparisons between experiment and calculation are shown for the case for which the Doppler sample is surrounded by a thin steel can and also for the case where the sample and can are surrounded by a boron sleeve.

The substantially negative measured Doppler effect for U-238 is reduced by about a factor of six when the boron sleeve is put around the sample. The calculated results, both with and without the boron sleeve, are in excellent agreement with the experiments. For Pu-239 the measured Doppler effect is negative for temperatures up to about 750°K. (Above this temperature a non-Doppler effect appears to dominate the experimental results.) Placement of the boron sleeve around the Pu-239 Doppler sample makes the coefficient appreciably positive. These Pu-239 results are qualitatively in good agreement with the calculated Doppler effects of the $J = 1$ sequence of resonances, even though the calculated coefficient is not negative at low temperatures. The $J = 0$ resonance sequence can contribute either a small positive or negative coefficient of the same order as the $J = 1$ effect, depending upon the statistical fluctuations of resonance parameters and the degree of overlap with other resonance sequences (mainly with the cold U-238 resonances). For the present, we will assume a zero Doppler effect for $J = 0$

and proceed with a theoretical interpretation of the results shown in Fig. 2.

The U-238 Doppler-effect results follow directly from the perturbation-theory expression shown in Fig. 2. Insertion of the boron sleeve depresses both the low-energy flux and adjoint flux in the Doppler region between 100 eV and a few keV. This leads directly to the substantial reduction of the U-238 Doppler effect obtained in both the measurement and the calculation.

Interpretation of the Pu-239 Doppler effects is somewhat more complicated. Figure 3 shows plots of the adjoint flux versus energy for both the normal and boron-sleeve cases. The perturbation-theory expression for the Pu-239 Doppler reactivity effect is also shown in Fig. 3. The net Doppler-reactivity effect for the $J = 1$ resonance sequence is seen to be the difference between two large terms. The positive term is proportional to $\bar{\nu} \phi_{\text{high}}^{\dagger}$, the average number of neutrons per fission times the fission-spectrum-averaged adjoint flux. The negative term is proportional to $(1 + \alpha_{\text{low}}) \phi_{\text{low}}^{\dagger}$, the absorption-to-fission ratio in the low-energy Doppler

FIG. 3. Assembly 47 Central Importance.

region times the low-energy adjoint flux. (ϕ_{low}^+ is energy-averaged over the perturbed Doppler reaction rate, $\delta\sigma_a\phi$.) Values of $\bar{\nu}$, $(1 + \alpha_{\text{low}})$ for $J = 1$, ϕ_{high}^+ , and ϕ_{low}^+ are indicated in Fig. 3. The high value of alpha for the $J = 1$ sequence at low energies, α_{low} , results from the resonance model selected, as suggested by Schmidt⁽⁵⁾ and discussed in Sect. 2.2, that the $J = 1$, positive-parity compound nucleus is below its lowest fission threshold at energies less than 50 keV. The boron-sleeve adjoint curve in Fig. 3 was normalized to give the same ϕ_{high}^+ as the normal adjoint curve.

The magnitudes of both the positive and the negative $J = 1$ Pu-239 Doppler terms, and the net $\delta k/k$, are listed in Fig. 3. In the normal case the calculated net Doppler effect is the difference between the two very large and nearly equal terms. Hence, the normal $\delta k/k$ is sensitive to variations in cross-section data (or computational methods) that alter the ratio of ϕ_{low}^+ to ϕ_{high}^+ . A negative Pu-239 Doppler effect, in better agreement with experiment, can be achieved by raising ϕ_{low}^+ , as would result from effects such as increasing the Pu-239 fission cross section below 1 keV or decreasing the U-238 (and/or other material) radiative capture cross sections in the low-energy region.

For the boron-sleeve case, the severe depression of ϕ_{low}^+ makes the positive term by far the dominant one, and the sign of the coefficient is not in such delicate balance as for the normal case. The calculated increase in the Pu-239 Doppler effect upon insertion of the boron sleeve is smaller than that measured. A possible explanation for the difference between calculation and experiment for the normal case and the change upon insertion of the boron sleeve involves the $J = 0$ resonance sequence.

A calculation of the average, isolated-resonance Doppler effect contributed by the very broad and shallow $J = 0$ resonances gave a value which is about twice the calculated $J = 1$ contribution for the normal case, but which drops by a factor of 2.5 upon insertion of the boron sleeve. A crude analysis indicated that these calculated values for $J = 0$ were extremely sensitive to the numerical sampling of the random parameter distributions, since the major portion of the Doppler effect is contributed by resonances which have large Γ_n and small Γ_f values. Moreover, the overlapping of the $J = 0$ resonances with U-238 resonances (and also $J = 1$ resonances) can inject either a positive or negative effect depending upon the relative positions of the resonances, and the small number of important $J = 0$ resonances makes the variance of this effect very large. Studies by Kelber and Kier⁽³⁴⁾ showed that a large uncertainty occurs ($\sim 40\%$ of the average value) even with the much more numerous $J = 1$ resonances due to the randomness of the parameters and the

overlapping with U-238 resonances. Also, calculations by Nevier and Ravier⁽³⁵⁾ at Cadarache gave a negative Pu-239 Doppler coefficient for resolved $J = 0$ resonances (< 300 eV) due to U-238 overlap. Thus the average (expectation value), isolated-resonance $J = 0$ Doppler effect which was computed can be expected to have a large uncertainty, and a combination of overlap and parameter randomness effects could account for the negative Doppler effect which was measured. This possibility is strengthened by the fact that, as seen in Fig. 2, the calculation of only the $J = 1$ contribution with the boron sleeve agrees well with the experimental value. This would be expected if the $J = 0$ effect were negative since the $J = 1$ contribution increases by a factor of 3.5 with addition of the boron sleeve and the expectation value of the $J = 0$ contribution decreases by a factor of 2.5. A quantitative confirmation of the variance in the $J = 0$ Doppler effect produced by parameter randomness and overlap should be obtained using an approach similar to that used by Kelber and Kier for $J = 1$ resonances.

It is not obvious that the $J = 1$ Doppler reactivity effect will be essentially zero (i.e., cancellation of the two perturbation terms shown in Fig. 3) for all power reactor conditions. A large power reactor, with low leakage and more low-energy absorption due to Pu-240 and fission products, will have an enhanced neutron importance at high energies and a degraded importance at low energies. This opens the potential for a much more positive Pu-239 Doppler effect than measured to date in the small, high-leakage critical assemblies. Table 4 lists the ratio $\phi_{\text{low}}^+/\phi_{\text{high}}^+$ and calculated Doppler coefficients for Assembly 47 and for several fast power reactors covering a broad range of geometries and compositions. Reactor No. 2 has clean fuel with no Pu-240, similar to Assembly 47, while the last three have equilibrium plutonium and fission products for fuel at 50,000 MWD/T burnup. Reactors 4 and 5 have considerably lower neutron leakage and, hence, lower fissile "enrichment" than reactors 2 and 3. Reactor 5 has a σ_p (potential scattering cross section per Pu-239 atom) the same as the high-leakage reactors (as, for example, would be the case for a low-leakage gas-cooled reactor having a high fuel-to-structure ratio).

The $J = 1$ Pu-239 Doppler coefficient increases by a factor of about 3.5 in going from the "clean," high-leakage reactor 2 to the low-leakage reactor 5 containing appreciable Pu-240 and fission products. The Doppler effect per Pu-239 atom has increased by about a factor of 8 as between these reactors. (The expectation value of the $J = 0$ Doppler coefficient increases only by 75% in going from reactor 2 to 5.) It is seen in Table 4, however, that the calculated positive Pu-239 Doppler coefficient is extremely small compared to the

calculated negative U-238 coefficient for all of the reactors. It appears, therefore, that the Pu-239 Doppler effect in large, relatively low enriched fast power reactors can be virtually disregarded.

3.2. Effect of Variations in Cross-section Data and Computational Methods on the Comparison between Measurements and Calculations

Tables 5 and 6 compare measured and calculated results in Assembly 47 with several variations considered in the Pu-239 and U-238 cross-section data as described in Sections 2.2 and 2.3. Comparison of the measured values with those calculated using the "New Pu-239 Data" set shows good agreement for all of the measured quantities listed, better than expected in light of the considerable uncertainties remaining in the cross-section

data and calculation methods. The underprediction of neutron lifetime (l/β_{eff} was the actual measured quantity) may be indicative of slightly too hard a calculated flux and/or adjoint spectrum; but the underprediction is only about 5% (actually within the experimental uncertainty range) compared with 20 to 40% reported in earlier comparisons between measured and calculated neutron lifetimes for dilute, fast critical assemblies.⁽³⁶⁾

Comparison of the "New Pu-239 Data" results with those obtained using the "Old Pu-239 Data" in Table 5 shows that the major difference is in the Pu-239 Doppler effect, with the old data (using the same $\langle \Gamma_f \rangle$ for both s-wave compound-nucleus spin states) predicting the familiar large positive Doppler reactivity. Some of the other data variations in Tables 5 and 6 changed the magnitude of the calculated Pu-239 Dop-

TABLE 4. CALCULATED $J = 1$ COMPONENT Pu-239 DOPPLER COEFFICIENTS FOR TYPICAL FAST OXIDE POWER REACTORS^(a)

Reactor Type	$N^{239} \times 10^{-21}$	N^{238}/N^{239}	σ_p^{239} (b)	Fission Fraction %10 keV	$\phi^{\text{plus}}/\phi^{\text{high}}$	$[T(dk/dT)] \times 10^3 (\Delta k)$	
						Pu-239 ^(c)	U-238
1. ZPR III, Assembly 47	1.5	5.2	120	0.22	1.10	+0.2 (est)	-9.5 (est)
2. High-leakage, clean Pu	1.8	3.6	240	0.11	1.14	+0.22	-4.0
3. High-leakage Equilibrium Pu ^(b)	1.6	3.5	240	0.09	0.92	+0.32	-2.4
4. Low-leakage Equilibrium Pu ^(b)	0.78	8.6	480	0.17	0.83	+0.50	-9.5
5. Low-leakage Equilibrium Pu ^(b)	0.78	8.6	240	0.17	0.80	+0.71	-9.5

^(a) Cores contain 40 v/o UO₂-PuO₂, 40 v/o Na, and 20 v/o SS.

^(b) Pu-239/Pu-240/Pu-241/Pu-242 = 1.00/0.41/0.07/0.03. Fission products at 50,000 MWD/T.

^(c) $J = 0$ contribution to Doppler ranges from $A_{\text{Dop}} = +0.0002$ for high-leakage cores to $A_{\text{Dop}} = +0.00035$ for core 5. (Calculated disregarding overlap with U-238 and $J = 1$ Pu-239 resonances.)

TABLE 5. EFFECTS OF DATA VARIATIONS ON COMPARISON BETWEEN MEASURED AND CALCULATED RESULTS WITH ASSEMBLY 47

Description ^(a)	Reactivity ($k - 1$)	$10l^{(b)}/\beta_{\text{eff}}$	Doppler, lh/kg		$\sigma_{f28}^{(c)}/\sigma_{f40}$	$\sigma_{c28}^{(c)}/\sigma_{f49}$	Sodium ^(d) Void $\Delta k(\%)$
			Pu-239	U-238			
Measured Data	0 ± 0.000	2.05 ± 0.1	(-0.5 ± 0.1)	(-1.7 ± 0.17)	0.0288	0.150	0 ± 0.3
New Pu-239 Data	+0.001	1.96	+0.17 ^(e)	-1.66	0.0279	0.138	-8 ^(d)
Old Pu-239 Data	-0.013	1.98	+3.13	-1.67	0.0278	0.139	-7
Pu ²³⁹ (600-900 eV) Higher $\sigma_f^{(f)}$	+0.002	1.95	+0.15 ^(e)	-1.65	0.0278	0.137	-13
Pu ²³⁹ (<10 keV) Lower α	+0.003	1.98	+0.31 ^(e)	-1.70	0.0278	0.138	-12
Pu ²³⁹ (20-100 keV) Lower $\sigma_f^{(f)}$ (White Data)	-0.008	2.01	+0.18 ^(e)	-1.70	0.0282	0.141	-9
U ²³⁸ (40-300 keV) Lower σ_c	+0.019	1.98	+0.17 ^(e)	-1.67	0.0276	0.132	-8
U ²³⁸ (4-40 keV) Higher σ_c (Macklin-Gibbons Data)	-0.025	1.85	+0.17 ^(e)	-1.66	0.0288	0.156	-3

^(a) See Sections 2.2 and 2.3

^(b) $\beta_{\text{eff}} = 0.0033$. Two different methods were used to measure l/β_{eff} .^(37, 40) These both gave $(2.05) \times 10^{-4}$ with an error of the order of 5%. The average measured value is listed, and the experimental error may be assumed to extend over the error limits of the two measurements.

^(c) The precision of the fission ratio is much greater than that of the capture-to-fission ratio and should be given a higher weighting in the evaluations.

^(d) Central 13% of core volume 100% voided of sodium. All sodium-void calculations done with a one-dimensional spherical model; therefore, calculated absolute values are less significant than the changes due to the data variations.

^(e) For $J = 1$ spin state only.

^(f) $\alpha = \sigma_c/\sigma_f$ held constant.

TABLE 6. EFFECT OF VARIATIONS OF DATA AND CALCULATIONAL METHODS ON COMPARISON BETWEEN MEASURED AND CALCULATED RESULTS FOR ASSEMBLY 47^(a)

Description	Reactivity ($k - 1$)	$10^4 l / \beta_{eff}$	Doppler $l h / kg^{(g)}$		$\sigma_{f238} / \sigma_{f49}$	$\sigma_{c238} / \sigma_{c49f}$	Sodium Void $\Delta k^{(c)}$
			Pu-239	U-238			
Measured Data	0 ± 0.000	2.05 ± 0.1	-0.5 ± 0.1	-1.7 ± 0.17	0.0288	0.150	0 ± 0.3
New Pu-239	+0.001	1.96	+0.17 ^(b)	-1.66	0.0279	0.138	-8
$\langle \Gamma_f \rangle_{J=0}$, 2.8 $\rightarrow$ 1.5 ^(e)	-0.001	1.95	+0.10 ^(b)	-1.64	0.0279	0.138	-6
$\langle \Gamma_f \rangle_{J=1}$, 0.055 $\rightarrow$.06 ^(e)	+0.002	1.97	+0.24 ^(b)	-1.68	0.0279	0.138	-10
$\bar{\nu}_{Therm}^{239}$, 2.87 $\rightarrow$ 2.82	-0.019	1.96	+0.07 ^(b)	-1.66	0.0279	0.138	—
$\chi(E)$ of U-235	-0.003	2.02 ^(d)	+0.18 ^(b)	—	0.0274	0.138	—
One-dimensional Calculation ^(e)	-0.006	1.86	—	—	—	—	—
Single Spectrum ^(f)	+0.006	1.97	—	—	—	—	—

^(a) See Sections 2.3 and 2.4.

^(b) For $J = 1$ spin state only.

^(c) In effect, this is only a change in α (by about 0.03 below 10 keV).

^(d) The softer U-235 fission neutron spectrum reduces the fraction of fissions in U-238 and decreases β_{eff} by 3%.

^(e) Iterative, energy-dependent buckling approach in successive radial and axial problems.

^(f) Equivalent one-dimensional calculation in 16-groups using cross sections condensed only in inner core region spectrum.

^(g) For a temperature rise from 300 to 750°K for the Doppler sample.

pler effect; but none of the changes are significant in comparison with the difference between the new and old Pu-239 data.

The “New Pu-239 Data” set predicts a 1.4 % higher reactivity than the “Old Pu-239 Data,” 0.4 % of which is due to the use of a higher nuclear temperature in the fission spectrum (see Table 6). The other data variations produce individual reactivity effects that range from -2 % to +2 %.

Increasing the Pu-239 fission cross section between 600 and 900 eV (reducing by one-half the “dip” in σ_f deduced from the σ_f and σ_t measurements in this energy region) produces little effect on the calculated quantities. Lowering the Pu-239 alpha values below 10 keV by 8 % to agree with Schmidt’s⁽⁵⁾ recommended alpha curve also has little effect. Lowering the U-238 $\sigma(n, \gamma)$ between 40 and 300 keV increases reactivity by about 2 %, but otherwise has a very minor effect on the calculated values. Using the White *et al.*⁽¹⁴⁾ σ_f data between 20 and 100 keV appears to give better overall agreement between the calculated and measured quantities except for reactivity, which is decreased by about 1 %. Increasing the U-238 capture cross section between 4 and 40 keV significantly lowers reactivity and reduces the calculated l / β_{eff} in disagreement with the experimental values, but gives better agreement between calculation and measurement on the U-238 fission and capture rates relative to Pu-239 fission.

The calculated U-238 Doppler effect is not significantly altered by the data variations considered. Calculated ratios of U-238 fission and U-238 capture to Pu-239 fission are all on the low side (except for the higher U-238 σ_c below 40 keV), but are believed to be within the limits of experimental uncertainty. The

variations in cross-section data produced only small changes in the central sodium-void reactivity. This probably results from the fairly flat adjoint-flux spectrum due to the high neutron leakage from the core and lack of substantial Pu-240 and fission products, conditions that would not necessarily prevail in a large fast power reactor.

Table 6 shows the effect on the calculated quantities of variations in $\langle \Gamma_f \rangle$ for each spin state, in the fission spectrum parameters $\bar{\nu}$ and χ , and in calculational methods that are less exact than those used for the reference calculation with the “New Pu-239 Data.” The reduction of $\langle \Gamma_f \rangle_{J=0}$ from 2.8 to 1.5 eV slightly increases the low-energy alpha values and, hence, has an effect opposite to that of the “Pu-239 (<10 keV) Lower α ” variation of Table 5. The calculated positive $J = 0$ component of the Pu-239 Doppler effect increases by about 75 %; but this is not significant because of the randomness of the resonance parameters and the overlap effect discussed in Sect. 3.1. The increase of the low-energy $\langle \Gamma_f \rangle_{J=1}$ from 0.055 to 0.06 eV increases the $J = 1$ Pu-239 Doppler effect; but otherwise has an effect that is opposite in sign, but about equal to, the decrease of $\langle \Gamma_f \rangle_{J=0}$ from 2.8 to 1.5 eV. A reduction of $\bar{\nu}$ as indicated by the Colvin and Sowerby measurements⁽¹²⁾ reduces reactivity in direct proportion to $\bar{\nu}$. Lowering the temperature for the fission-spectrum distribution from 1.41 MeV (valid for Pu-239) to 1.31 MeV (valid for U-235) reduces the calculated multiplication factor by 0.4 %, slightly decreases the U-238 fast fission rate, and hence β_{eff} , and increases the calculated l / β_{eff} .

The “One-dimensional Calculation” variation used one-dimensional multigroup with several radial and

Fig. 4. Radial Fission Traverses for Pu-239 in Assembly 47.

axial iterations to improve perpendicular bucklings. This gave a 5% lower calculated neutron lifetime and a 0.7% lower reactivity than the "New Pu-239 Data" result, which was based on the two-dimensional CRAM multigroup computation as described in Sect. 2.4. It should be noted here that Assembly 48 with a much simpler core and reflector (blanket) arrangement was amenable to a one-dimensional spherical model and did not have a significant difference in calculated neutron lifetime and reactivity between the one- and two-dimensional multigroup calculations.

The case in Table 6 identified as "Single Spectrum Condensation" did not take into account the spatial variation of the spectrum in the condensation of the group constants from the 60-group to the 16-group set. The central core spectrum was used, for this one case, in the condensation for all the core and reflector regions. This resulted in a 0.5% higher calculated reactivity.

Figure 4 compares calculated and measured radial fission rate traverses for Pu-239. It is seen that good agreement is obtained in the Pu-239 spatial fission rate distribution when the foil self-shielding correction is appropriately calculated for each region.

3.3. General Conclusions from the Assembly-47 Analysis

The new theoretical model of Pu-239 resonance parameters⁽⁵⁾ adequately accounts for the near-zero Pu-239 Doppler coefficient measured in dilute critical

assemblies. Calculations of the Doppler effect in a broad range of large, dilute fast power reactor compositions and geometries show that the Pu-239 Doppler coefficient can be virtually disregarded for relatively low enriched systems, since for all the reactors considered it was less than 15 per cent of the negative U-238 Doppler coefficient.

The agreement among calculated and measured reactivity, neutron lifetime (l/β_{eff}), and U-238 Doppler effect is very good in terms of the new Pu-239 cross-section data. Uncertainties in $\bar{\nu}$ and σ_f of Pu-239 and in σ_c of U-238 over several energy ranges each contribute 1 to 2% potential error to the calculated reactivity. These errors are uncorrelated, which accounts for the good agreement between calculation and experiment.

The comparison of calculated and measured neutron lifetime indicates that the calculated flux and/or adjoint spectra may be too hard. However, the discrepancy in neutron lifetime ($\sim 5\%$) is appreciably less than has been reported previously, and the calculated value actually lies within the range of experimental error.

For reactors that do not have a simple geometry, an accurate diffusion-theory calculation requires a two-dimensional model (two-dimensional multigroup code) and coarse-group constants that are generated from fine-energy data using a good approximation to the actual spectrum of each spatial region.

Refinement of cross-section data and calculational methods by integral measurements was not the primary purpose of Assembly 47. As a result the assembly had a more complicated geometry, both local and overall, than is desired for integral cross-section experiments, and several important measurements such as neutron spectrum and boron-10 self-shielding were not performed. It is important, therefore, that the conclusions on improvements of data and computational methods derived from the Assembly-47 analysis be substantiated by results from other plutonium-fueled clean critical assemblies. The data from ZPR-III Assembly 48 were not available at the writing of this paper, but the predictions of the results are given below, based on calculations using the "New Pu-239 Data" set and for several of the data variations that were used in the Assembly-47 analysis. The comparison between the measurements and the calculations on the two assem-

blies will be correlated when the Assembly-48 measurements become available to put the selection of optimum Pu-239 and U-238 cross-section data on a firmer basis than is now possible.

4. Prediction of Assembly-48 Results

The calculated predictions for ZPR-III Assembly No. 48 are given in Table 7. Calculated values are also shown for three of the cross-section data variations that gave the most significant effects in Table 5. These results will be analyzed in detail when the experimental data are available.

Results of our earlier calculations for Assembly 48 were sent to W. Davey of ANL for inclusion in the intercomparison of experiment and calculation for ZPR-III Assembly 48, in connection with the October, 1966 International Conference on Fast Critical Experiments and Their Analysis.⁽²⁾ These earlier calculations

TABLE 7. ZPR-III ASSEMBLY 48 CALCULATED RESULTS WITH DATA VARIATIONS^(a)

Item of Data	"New Pu Data"	Lower Pu-239 σ_f (20-100 keV) "White"	Lower U-238 σ_c (40-300 keV)	Higher U-238 σ_c (4-40 keV) "Macklin-Gibbons"
% $\delta k/k$ vs. Col. 1	0.0	-1.0	+1.2	-2.3
<i>Cylindrical Pu-239 Mass</i>				
Heterogeneous (kg)	276	293	255	316
Homogeneous (kg)	283	300	262	323
<i>Heterogeneous, Cylindrical Critical Radius (cm)</i>	42.0	43.2	40.6	44.7
<i>Neutron Lifetime $\div \beta_{eff}$ (sec $\times 10^4$)^(b)</i>	0.83	0.83	0.83	0.75
<i>Central Fission Ratios vs. U-235^(c)</i>				
Pu-239	0.987	0.965	0.983	1.000
Pu-240	0.224	0.222	0.221	0.234
U-238	0.0342	0.0339	0.0337	0.0356
<i>Central Count Rates vs. U-235 Fission</i>				
B ¹⁰	1.50	1.51	1.51	1.45
U-238	0.134	0.136	0.127	0.149
<i>Reactivity Doppler Effect 300 $\rightarrow$ 750°K</i>				
U-238 (lh/kg)	-1.23	-1.26	-1.23	-1.11
Pu-239 (lh/kg)	+0.18	+0.19	+0.18	+0.17
<i>Central Worths (lh/kg)</i>				
Pu-239	459	459	452	473
Pu-240	29	29	25	40
U-235	325	333	319	339
U-238	-23	-24	-22	-25
Na	-2.0	-2.5	-1.7	-4.7
Fe	-13	-13	-13	-13
Steel	-13	-13	-13	-14
B ¹⁰	-8550	-8750	-8600	-8000

^(a) See Sections 2.2 and 2.3; also Table 5.

^(b) $\beta_{eff} = 0.0036$.

^(c) U-235 σ_f above 1 keV based on Davey's⁽²¹⁾ recommended values.

used the "New Pu-239 Data" except for the fission spectrum, for which a nuclear temperature of 1.33 MeV (instead of 1.41 MeV) and a $\bar{\nu}_{\text{thermal}}$ of 2.86 (instead of 2.87) were used. The earlier calculations also did not include a reactivity advantage for the high-energy heterogeneity, which was computed to be 0.003 on the basis of a "first flight" correction. These three effects gave a combined reactivity addition of 0.009 to the "New Pu-239 Data," or a reduction of 15 kg of Pu-239 from the earlier calculated critical mass (291 kg) of Assembly 48.

Additional corrections made in the "New Pu-239 Data" (first data column of Table 7) since the earlier calculations include the following:

- a) 10% higher U-238 Doppler effect due to overlap correction.
- b) Lower U-235 σ_f (using Davey's⁽²¹⁾ recommended σ_f curve above 1 keV). This lowered the U-235 fission rate by 7.5% and produced a corresponding increase in the other material fission and activation rates relative to U-235 fission.
- c) Less negative B-10 reactivity worth (by 4%), higher U-238 fast fission rate (by 3%), and less negative worth (by 7%) due to change in nuclear temperature (for fission spectrum) from 1.33 to 1.41 MeV. This change also increased the calculated magnitude of the negative reactivity worths of several light materials: sodium (3%), iron (4%), nickel (5%), and chromium (4%). This fission-spectrum correction for the reactivity worths of Pu-240 and U-235 has not yet been calculated; the values listed in Table 7 for these materials are those computed earlier for the intercomparison study. The U-235 reactivity worth also has not yet been corrected for the lower σ_f values based on Davey's⁽²¹⁾ recommended curve.

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Discussion

Mr. Okrent (ANL): First, let me thank Dr. Greebler for his support of clean criticals, and I hope that all those connected with projects, including Dr. Greebler, remember these words when they are thinking about asking for engineering-type experiments on critical facilities.

Mr. Greebler: My endorsement was intended to indicate that we need more critical-assembly facilities.

Mr. Okrent: Did I understand correctly that the new plutonium cross sections add reactivity?

Mr. Greebler: Yes, about 1.5 %.

Mr. Okrent: I recall that the cross sections used for the first calculations for Assembly 47 were about 1.5 % too reactive, and to this is now added another 1.5 %. What changes have offset this? I ask this because you indicated a k_{eff} very close to unity. Consequently I assume that there is now good agreement obtained with the new plutonium data.

Mr. Greebler: There have been a number of changes over the past year. One is the use of the Barry-Bunce-White data for U²³⁸. We are now using higher U²³⁸ capture cross sections over a portion of the spectrum. But there have been so many different changes, which we discuss in our paper, that it would be difficult to

say that any one of the cross-section variations compensated another.

Mr. Gandini (CNEN): You have shown the Doppler effect and the spectral dependence of this quantity. The sodium-void coefficient is also a spectrally dependent quantity. Do you believe that you can predict the sodium-void coefficient with confidence using such a simple approximation?

Mr. Greebler: The sodium-void effect is one of the quantities that we have calculated for Assembly 47. But in our paper we have indicated that we did not predict this as well as we would like to be able to do.

Mr. Campise (AI): You varied a number of parameters in your study. Did you look at the difference between diffusion and transport theory, both in one and two dimensions? This is of particular interest because this is a high-leakage core.

Mr. Greebler: We did run one transport calculation with a code called ANACIN, which is something like DTF4. We obtained 0.1 % lower reactivity with transport theory than with diffusion theory. We are very suspicious of this because this is in the opposite direction from what we expected.

Mr. Kato (ANL): In your Doppler-effect analysis, you mentioned that in the case of Pu²³⁹ you obtained very good agreement between the calculated and experimental values with the boron sleeve around the sample. I wonder if this is fortuitous or real, in view of the omission of a correction for expansion. Furthermore, were there any perturbation measurements made? This would be a much easier way to check your method of calculation with and without the boron sleeve present.

Mr. Greebler: Any exact agreement between measured and calculated Pu²³⁹ Doppler effect would be fortuitous. Just the fact that you both calculate and measure something that is very small represents good agreement. Perhaps the change from the normal case to the boron-sleeve case and how that measured change compared with the calculated change is more significant than the degree of agreement for either one of these. I would certainly encourage central perturbation measurements, with and without the boron sleeve, if they have not already been made, as a very good way to check this method of calculation.

Mr. Pitterle (APDA): You neglected the $J = 0$ Doppler effect. This makes a difference in the net Doppler effect by at least a factor of three, and results in positive and negative terms of very similar magnitude. I find it hard to justify your neglect of this in view of Kelber and Kier's analysis. If the statistical problem is worse for the $J = 0$ state or bad for fissionable isotopes, I would consider it to be worse for the

$J = 1$ state for which the Doppler effect is dominant, the resonances are narrow, and the Doppler effect is sensitive to the distribution. I can't advance any good argument for neglecting $J = 0$, although it does give good agreement with experiment.

Mr. Greebler: When we completed all of these calculations, we believed that we could probably neglect both $J = 1$ and $J = 0$. In regard to $J = 0$, you have about $\frac{1}{3}$ as many resonances, and they are very broad. The degree of interaction with the U^{238} will be much larger. We weren't quite so sure about neglecting $J = 1$.

Mr. Pitterle: The resonances are so broad that I wouldn't expect the U^{238} overlap effect to make much of a difference. But if there are significant overlap effects, then narrow resonances of $J = 1$ state would still be dominant. Other than the fact that the Doppler measurements results in a very small value, I just don't see a good argument for neglecting $J = 0$. I do agree that, if you drop it, you can conceivably get a negative value.

Mr. Hummel (ANL): How well can you calculate the Doppler effect in a large fast reactor?

Mr. Greebler: Considering what we found for Assemblies 48 and 47, it appears that our accuracy is now $\pm 30\%$. Admittedly, neither of these critical assemblies had large amounts of Pu^{240} or fission products present, so we would have to increase any uncertainty that we now have, on the basis of our agreement with the critical

assemblies, because of the much larger uncertainties in the Pu^{240} and fission product data.

Mr. Fluharty (INC): The reason why you can neglect the Doppler effect for the $J = 0$ levels is because the levels are very wide and the Doppler broadening is small compared with the level width, so that the effect is not large.

Mr. Pitterle: All calculations agree that the Doppler effect of the $J = 0$ levels is small, but it is not negligible. When weighted by 2.9 or so in the fission term, this makes the difference between the results reported by Greebler and those reported by the others. It is easily different by a factor of 3, and this is just enough to switch it from a very close balance between positive and negative contributions. While intuitively it sounds correct, calculations show that it is not negligible.

Mr. Greebler: We expected there would be a large error in neglecting $J = 0$ states, but even with the large error, it will result in a very small effect. For $J = 1$, we weren't sure that would be the case, and it was only after we did some calculations on power reactors that we also concluded that the contribution of $J = 1$ levels, whether or not large errors, are involved, would also be small. There are probably some reactors for which one would not want to jump to this conclusion. I could visualize some cermet reactors, highly enriched in plutonium, for which one might still be concerned about the Pu^{239} Doppler effect.

The Evaluation of Basic Cross-section Data by Analysis of Fast Critical Assemblies*

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ABSTRACT

Calculations of the properties of a number of fast critical assemblies are being carried out as part of a continuing program for testing and evaluating multigroup cross sections for fast reactor calculations. The experiments included in the present analysis have been chosen to cover a wide range of fuel materials and reactor spectra. This increases the ability to identify errors according to the material and energy range in which they occur. The assemblies studied include a number of U^{235} -fueled ZPR-III assemblies; ZPR-III-45 and 45A, driven plutonium assemblies which were mainly used for Doppler measurements; the SEFOR mockup, ZPR-III-47; the plutonium-fueled assembly, ZPR-III-48; ZEBRA Core 5; the small fast assemblies POPSY and JEZEBEL (fueled with metallic plutonium), and GODIVA (fueled with metallic uranium). The properties examined include critical mass, Doppler effect, fission ratios, and the reactivity worths of various material replacements.

The properties of the small fast assemblies depend only on the high-energy data; they are practically independent of cross sections below 100 keV. However, the larger assemblies require accurate data down to about 100 eV. All of the multigroup cross sections have been constructed using basic microscopic cross-section data taken from the Atomics International Evaluated Nuclear Data File and have not been adjusted to force agreement with the critical-assembly data. For the large, soft-spectrum assemblies, the cross sections were weighted by a fine-group spectrum calculated for each assembly.

Earlier results disclosed certain deficiencies in the cross sections contained in the data file. Subsequently, many of these cross sections were revised and updated, resulting in significant improvements in the agreement between the calculations and the experimental results.

*Work performed under AEC Contract AT(11-1)-GEN-8.

1. Introduction

An automated system has been developed at Atomics International (AI) to generate a multigroup library for any homogeneous reactor, starting from a basic microscopic cross-section data file (Atomics International Evaluated Nuclear Data File, AIENDF, updated to August 1, 1966). The desired objective of this system is to obtain sufficient accuracy in the basic microscopic cross-section data and averaging techniques so that the multigroup cross sections being used in nuclear design calculations for fast reactors will result in adequate predictions of the nuclear parameters of interest. The main nuclear parameters of interest for fast breeder reactors are critical mass, breeding ratio, Doppler worth, and sodium-void worth.

In order to have a data file and averaging system which is valid over a fairly broad range of reactor designs, and to determine the areas which need further investigation, it is necessary to test the system with experiments encompassing many assembly sizes and compositions. For this reason correlation was performed with experiments on assemblies fueled with U^{235} and Pu^{239} which cover a wide range of median fission energies (0.1 to about $1\frac{1}{2}$ MeV). Thermal and near-thermal systems are not included in this investigation since the objective is to develop a system with good accuracy for fast reactor designs. Also, calculations for thermal reactors necessitate the use of methods which are somewhat different than those needed for fast reactors.

The nuclear parameters calculated included eigenvalues, fission ratios, central reactivity worths, Doppler worths, prompt-neutron lifetimes, and fission traverses.

2. Assemblies

The assemblies used in this study are listed in Table 1. The type of fuel is indicated along with the median fission and flux energies in the core of each assembly.

As described by Hansen,⁽¹⁾ the GODIVA assembly can be approximated as a perfect bare sphere of uranium metal with density 18.75 g cm^{-3} and U^{235} enrichment of 93.8%. The critical mass is 52.04 kg U^{235} . Thus, the calculational model used is a bare sphere of radius 8.718 cm.

Similarly, JEZEBEL, as described by Jarvis *et al.*,⁽²⁾ is a bare sphere of plutonium metal with density 15.66 g cm^{-3} and containing 4.5% Pu^{240} . The critical mass is 16.28 kg plutonium, leading to a radius of 6.284 cm for the idealized critical assembly.

POPSY⁽²⁾ is also a plutonium-metal sphere, reflected by 9.5 in. of natural uranium. The critical mass is 5.73 kg plutonium at a density of 15.63 g cm^{-3} and with 1.5% Pu^{240} . The critical radius is thus 4.44 cm.

The compositions and geometries of ZPR-III-29, -31, -34, and -36 are given in Ref. (3). Core volumes corrected for heterogeneity and shape factors were taken from this reference. These assemblies were fueled with U^{235} -enriched uranium and contained various amounts of carbon, oxygen, sodium, and aluminum to simulate reactor designs with different types of fuel, structure, and coolant.

Assemblies 45 and 45A⁽⁴⁾ were zoned critical experiments. Each had a central test region containing plutonium fuel, stainless steel, sodium, and carbon; a buffer region; and a driver region which was fueled with uranium. Assembly 45 contained more carbon in the test region than assembly 45A (to soften the central region spectrum). The composition of ZPR-III-48 was obtained from Redman.⁽⁵⁾ A shape factor of 0.927 and a heterogeneity correction of $-27 \text{ kg}^{(6)}$ were used with this assembly. The assembly was fueled with plutonium and contained carbon, stainless steel, and sodium (to simulate a carbide-fueled, sodium-cooled, fast breeder reactor).

The composition of the plutonium-fueled SEFOR mockup is given in References (7) and (8). A shape factor of 0.911 was used with this assembly and no correction was made for heterogeneity. Calculations in spherical geometry were made with the radial reflectors surrounding the core and then repeated with the axial reflectors. The k_{eff} values from these two calculations were weighted by the surface areas of the side and ends of the core, respectively, to adjust for the difference in radial and axial reflectors.

ZEBRA core 5⁽⁹⁾ was a multiregion assembly which had a central test region, a buffer which contained plutonium, and a driver which contained U^{235} . The central test region was fairly large (21.5 cm in diameter) and was insulated from the rest of the reactor so that its temperature could be varied. Various compositions were placed in the central test region, such as fuel pins

TABLE 1. ASSEMBLIES STUDIED

Assembly	Fuel	Core Median Fission Energy (Mev)	Core Median Flux Energy (Mev)
Godiva	U^{235}	—	—
Popsy	Pu^{239}	—	—
Jezebel	Pu^{239}	1.40	—
ZPR-III-36	U^{235}	0.379	0.316
-31	U^{235}	0.272	0.311
-34	U^{235}	0.175	0.244
-29	U^{235}	0.156	0.226
-45A and 45	Pu^{239} (center)	—	—
-48	Pu^{239}	0.223	0.175
-47 (SEFOR Mockup)	Pu^{239}	0.118	0.134
ZEBRA Core 5	Pu^{239} (buffer)	—	—

containing only plutonium and fuel pins containing plutonium and U^{238} .

3. Cross Sections and Methods

The present revision of the AIENDF data includes a new evaluation of the fission spectrum and the number of neutrons per fission. A Maxwellian fission spectrum is assumed with $\bar{E} = 1.95 \text{ MeV}$ for U^{235} and $\bar{E} = 2.085 \text{ MeV}$ for Pu^{239} . The values of ν are normalized to the value 3.77 for the spontaneous fission of Cf^{252} given by Westcott.⁽¹⁰⁾

The Pu^{239} fission cross sections between 10 and 700 keV are based on evaluation of the data from Perkin and White,⁽¹¹⁾ White *et al.*,⁽¹²⁾ and the PETREL test.⁽¹³⁾ Above 700 keV, data from the 1965 edition of BNL-325⁽¹⁴⁾ are used. Capture cross sections above 250 keV are obtained from the curve of $\sigma_c(49)/\sigma_f(49)$ in BNL-325 (1965) and from data of Diven and Hopkins below this energy. The U^{235} fission and capture cross section above 10 keV is from BNL-325 (1965). Below 10 keV the results of an evaluation by Hennies⁽¹⁵⁾ are used in combination with a resonance calculation for both Pu^{239} and U^{235} .

The U^{238} capture and fission cross sections above 40 keV are taken from BNL-325 (1965). Below 40 keV a resonance calculation is used. A smooth background component (which is not self-shielded) is added to the capture cross section between the energies of 4 and 40 keV to normalize the dilute resonance cross section to the BNL-325 (1965) result.

A number of revisions were made to the cross-section data of other nuclides. A more detailed description of these revisions is being prepared by H. Alter.

Multigroup libraries for each of the assemblies were generated from the AIENDF data. The first step in

TABLE 2. REACTIVITIES AND NEUTRON LIFETIMES

Assembly	Fuel	k_{eff}	Lifetime (μsec)	
			Calc	Exp
Godiva	U ²³⁵	1.037	—	0.006
Popsy	Pu ²³⁹	1.0081	—	—
Jezebel	Pu ²³⁹	1.0005	0.003	0.003
ZPR-III-36	U ²³⁵	0.979	0.078	0.079
-31	U ²³⁵	1.006	0.168	0.164
-34	U ²³⁵	1.002	0.240	0.250
-29	U ²³⁵	1.010	0.230	0.232
-48	Pu ²³⁹	0.977	0.315	—
-47	Pu ²³⁹	1.002	—	—

generating a library is to calculate the fundamental-mode neutron spectrum for the critical assembly with the AILMOE code⁽¹⁶⁾ in the P1 approximation. The GRISM code⁽¹⁷⁾ is used to weight the AIENDF cross sections with this spectrum to form group averages as described in a paper by H. Alter being presented at this conference.

The calculation of resonance absorption and fission cross sections is based on the analytical methods introduced by Chernick and Vernon, and others. A statistical sample is used to calculate the contribution of the unresolved resonances and a smooth background is added where required, in order to normalize the infinitely dilute cross sections to experimental data. Resonance cross sections are also weighted by the AILMOE spectrum.

The elastic and inelastic transfer cross sections are calculated by integrating the weighted cross sections over broad energy groups. Legendre polynomial expansions of up to the 9th order are used in this calculation. The multigroup library can be generated for either diffusion-theory or transport-theory codes, both of which have been employed in these studies.

Calculations by one-dimensional diffusion theory were made for the larger assemblies using the CAESAR⁽¹⁸⁾ code. This code includes a first-order perturbation calculation which was used to obtain reactivity coefficients and prompt-neutron lifetimes. The cross-section libraries had groups whose widths were either 0.50 or 0.693 lethargy unit.

Transport theory is required for analysis of the small fast assemblies. In addition, transport calculations are also needed as a check on the accuracy of diffusion theory for the larger assemblies. The DTF-II Program⁽¹⁹⁾ was used to carry out these calculations. For the small metal assemblies, the double-P₃ (DP3) transport approximation was selected as the optimum compromise between accuracy and simplicity, while for checking the results of diffusion theory, the S₄ ap-

proximation was used. The present SEFOR results were checked by this method.

Material replacement values for JEZEBEL have been calculated for small spheres equal in mass to the 1/2 x 1/2-in. cylinders used in those experiments.⁽²⁰⁾ The results were obtained by eigenvalue differencing and compared to experimental worths for this sample size. For GODIVA, results were obtained in the same way, but very small samples were used in the calculations and in the experimental results. This technique required a modification of DTF which was not available when the JEZEBEL calculations were made. The required modification consists only of printing more significant figures of the computed eigenvalues, allowing differences to be computed for smaller perturbations.

4. Results

4a. Reactivity and Neutron Lifetime

The results for k_{eff} and the prompt-neutron lifetime are presented in Table 2. The calculated values of k_{eff} show good agreement with experiment except for the GODIVA, ZPR-III-36, and ZPR-III-48 assemblies. The calculated prompt-neutron lifetimes agree well with the experimental values. Although the computed eigenvalue for GODIVA is surprisingly high, it is in good agreement with a calculated value of 1.03 for GODIVA recently reported by Meneley and White.⁽²¹⁾

The calculated eigenvalues will be changed a little by new revisions being made in the molybdenum capture cross section and somewhat more by the high-energy capture cross sections of U²³⁸ (above 10 keV). The U²³⁸ capture cross section revisions (based on optical model theory⁽²⁰⁾) increase the eigenvalue of ZPR-III-48 by 1.2%, ZPR-III-47 by 0.9%, ZPR-III-29 by 1.1%, and ZPR-III-36 by 1.3%. Realistic changes in the present AIENDF values for the capture cross section of molybdenum reduce the eigenvalue of ZPR-III-48 by 0.3% and of ZPR-III-47 by 0.6%. The revised eigenvalue for ZPR-III-48 and 47 thus become 0.986 and 1.005, respectively.

4b. Central Fission Ratios

Calculated and experimental values for central fission ratios are presented in Table 3. There is considerable scatter in the E/C ratios for the threshold detectors. Since these ratios are sensitive to the neutron spectrum they may be more of an indication of shortcomings in the calculated spectra than in the cross-sections of the detectors. For the fissile nuclides, the E/C ratios are more consistent, although two of the ratios deviate from the average by over 10%. From

TABLE 3. CENTRAL FISSION RATIOS

	F_{23}/F_{25}	F_{24}/F_{25}	F_{26}/F_{25}	F_{28}/F_{25}	F_{40}/F_{25}	F_{40}/F_{25}	C_{28}/F_{25}
ZPR-3-29 Exp	1.475	0.269	0.0870	0.0385	1.060	0.300	—
Calc	1.522	0.261	0.0792	0.0359	0.968	0.298	—
E/C	0.969	1.030	1.099	1.072	1.095	1.006	—
-31 Exp	1.520	0.347	0.112	0.0475	1.181	0.326	—
Calc	1.564	0.343	0.105	0.0460	1.069	0.388	—
E/C	0.972	1.011	1.07	1.033	1.105	0.840	—
-34 Exp	1.455	0.257	0.0802	0.0366	1.070	0.283	—
Calc	1.528	0.265	0.0809	0.0369	0.978	0.304	—
E/C	0.952	0.970	0.991	0.992	1.095	0.931	—
-36 Exp	1.415	0.325	0.100	0.0443	1.191	0.350	—
Calc	1.579	0.334	0.096	0.0423	1.103	0.371	—
E/C	0.896	0.973	1.04	1.047	1.080	0.943	—
-47 Exp	1.451	—	—	0.0264	0.916	0.2006	—
Calc	1.443	—	—	0.0222	0.826	0.1703	—
E/C	1.005	—	—	1.191	1.109	1.178	—
-48 Exp	1.480	0.204	0.0670	0.0307	0.976	0.243	—
Calc	1.480	0.193	0.0624	0.0289	0.877	0.221	—
E/C	1.00	1.08	1.074	1.063	1.113	1.100	—
-45A Exp	1.44	0.189	0.0662	0.0282	1.047	0.227	0.152
Calc	1.48	0.191	0.0574	0.0257	0.892	0.221	0.135
E/C	0.97	0.99	1.153	1.097	1.175	1.027	1.127
Godiva Exp	—	—	—	0.154	1.43	—	0.0704
Calc	—	—	—	0.138	1.30	—	0.0897
E/C	—	—	—	1.11	1.10	—	0.785
Zebra Exp	—	—	—	0.0207	0.904	0.178	—
Calc	—	—	—	0.0248	0.893	0.184	—
E/C	—	—	—	0.835	1.012	0.967	—
Popsy Exp	—	—	—	0.175	1.41	—	0.79
Calc	—	—	—	0.149	1.29	—	0.90
E/C	—	—	—	1.17	1.09	—	0.88
Jezebel Exp	—	0.90 ^a	—	0.195	1.5	—	—
Calc	—	0.87 ^a	—	0.172	1.33	—	—
E/C	—	1.03 ^a	—	1.13	1.13	—	—

(a) Results for Np^{237}

these results it might be inferred that $\sigma_f(25)$ and $\sigma_f(23)$ are on the average about 9% and 12% high, respectively, relative to $\sigma_f(49)$. Also, there is a general trend for all of the ratios (except for U^{233}) to have the values of E/C greater than unity.

The heterogeneous nature of the ZPR-III assemblies (drawers containing stacked 1/8- and 1/4-in.-thick plates of fuel, depleted uranium, and several diluents) has been reduced to a homogeneous mixture for these calculations. Any variations which may be caused in the reaction ratios by the relative position of the detec-

tors with respect to the adjacent materials have been neglected in these calculations.

4c. Central Reactivities

Relative values for central reactivities are presented in Table 4. The results are given in barns per atom as in Ref. (3) and normalized to the Pu^{239} experimental value. Core self-shielded cross sections were used for Pu^{239} , U^{235} , and U^{238} , whereas dilute cross sections were used for the other nuclides. This results in a signifi-

TABLE 4. RELATIVE CENTRAL REACTIVITIES FOR ZPR-3-29, 31, 34, 36, 47, and 48

Nuclide	29		31		34		36		47		48	
	Calc	Exp	Calc	Exp	Calc	Exp	Calc	Exp	Calc	Exp	Calc	Exp
Pu ²³⁹	3.388	3.388	3.386	3.386	3.391	3.391	3.356	3.356	3.296	3.296	3.370	3.370
Th ²³²	-0.291	-0.228	-0.197	-0.163	-0.283	-0.232	-0.220	-0.175	—	—	—	—
U ²³⁸	4.172	3.741	3.820	3.385	4.160	3.744	3.797	3.485	—	—	—	—
U ²³⁵	2.343	2.012	2.180	1.837	2.337	1.961	2.183	1.847	2.770	2.41	2.64	—
U ²³⁸	-0.157	-0.104	-0.070	-0.050	-0.156	-0.108	-0.096	-0.081	-0.241	-0.143	-0.221	—
Pu ²⁴⁰	—	—	—	—	—	—	—	—	—	—	0.263	—
C	0.020	0.033	0.019	0.030	0.020	0.034	-0.002	0.000	-0.0051	-0.0033	0.0042	—
O	0.018	0.029	0.019	0.023	0.019	0.020	0.003	-0.001	—	—	-0.008	—
Na	0.014	0.027	0.014	0.020	0.014	0.013	-0.006	-0.003	-0.0037	-0.0055	-0.0026	—
Al	0.005	0.002	0.007	0.003	0.005	0.002	-0.011	-0.012	-0.013	-0.012	—	—
SS	-0.004	-0.007	-0.006	-0.006	-0.005	-0.006	-0.026	-0.025	-0.019	-0.018	—	—
Zr	-0.011	-0.010	-0.013	—	-0.012	-0.009	-0.038	-0.024	—	—	—	—
Mo	-0.031	-0.074	-0.014	—	-0.033	-0.070	-0.051	-0.070	-0.102	-0.107	-0.095	—
B ¹⁰	-2.63	-1.74	-1.78	—	-2.56	-1.59	-1.38	-0.97	-16.0	-10.28	-3.00	—
Cr	—	—	—	—	—	—	—	—	0.017	0.016	-0.017	—
Ni	—	—	—	—	—	—	—	—	-0.024	-0.024	-0.025	—
Fe	—	—	—	—	—	—	—	—	-0.019	-0.015	-0.018	—

cant error in the calculated value for nuclides where self-shielding for the sample is appreciable.

By employing self-shielded U²³⁸, capture cross sections calculated for a 2 x 2-in. block (the size used in many of the ZPR-III assemblies), instead of the core U²³⁸ cross sections, significantly lower reactivity worths are obtained for U²³⁸. For example, the calculated worth in assembly 47 becomes -0.200 (versus -0.241).

The calculated reactivity for U²³⁵ is too high relative to that for Pu²³⁹. This result is consistent with the behavior of the central fission ratios for these nuclides. The calculated reactivities for U²³⁸ and Th²³² are too negative, which may indicate a requirement for more sample self-shielding than was used. The calculated reactivities for light elements are in general of the correct magnitude and sign except that the values for molybdenum are quite a bit lower than the measured values. The present capture cross section for molybdenum in the AIENDF is low by more than a factor of two. No outstanding discrepancies are in evidence for the other light nuclides.

Part of the large overprediction of the worth of B¹⁰ may also be due to the assumption of an infinitely thin sample in the calculations. An experiment was conducted in ZPR-III-29⁽²³⁾ in which the worth of a 3-gm B¹⁰ sample was 30% more per gram than for a 24-gm sample. Also, the large sample size used for most of the measurements on the light elements may cause significant perturbations of the flux as it travels through the samples. This is particularly true of materials which have large scattering cross sections (such as sodium at 2.85 keV).

Central reactivities for GODIVA and JEZEBEL

are presented in Table 5. As explained in the previous section, the values computed for GODIVA are for small samples whereas the JEZEBEL results are for finite samples equal in mass to the experimental samples. Thus, the JEZEBEL results have been compared to the experimental results for 1/2 x 1/2-in. cylinders, but the GODIVA results are compared to experimental results corrected to infinitesimal sample sizes.⁽²⁰⁾

4d. Additional SEFOR Results

A set of calculations was made to evaluate the effect of taking the Pu²³⁹ fission and absorption cross sections from various data sources. The material composition and the cross sections of other nuclides were held constant in all of these calculations. The identification of the Pu²³⁹ cross sections and the corresponding values of k_{eff} are presented in Table 6. The rather large spread in these results illustrates the dependence of calculated values of k_{eff} on the source of the Pu²³⁹ cross-section data.

To evaluate the error due to use of diffusion theory instead of transport theory, radial calculations were made with the same cross sections and compositions and with both diffusion theory and the S₄ transport theory. The value of k_{eff} from the S₄ calculation was 0.011 greater than that obtained from the diffusion theory. Although this is a significant difference, it is evident that the error due to use of diffusion theory is no greater than that due to some of the cross-section uncertainties.

In order to examine the dependence of calculated central fission ratios on the Pu²³⁹ cross sections, the central fission ratios were recalculated using the flux

obtained with the Karlsruhe cross sections. All cross sections except those for Pu^{239} capture and fission are taken from the AIENDF. Table 7 shows the central fission ratios obtained from this calculation and from the reference calculation using AIENDF Pu^{239} cross sections.

The U^{233} ratio is affected very little by the change in Pu^{239} cross sections, but the remaining ratios are changed considerably. Since the changes in the U^{238} and Pu^{240} ratios are due only to the change in the flux in the two calculations, this shows the sensitivity of the fission ratios for threshold detectors to the neutron spectrum. It seems probable that the large variation in the threshold detector ratios shown in Table 3 is an indication of incorrect calculated spectra.

Radial traverses for Pu^{239} fission, U^{238} fission, and boron(n,α) were calculated for the reflected cylinder ZPR-3-47. The traverses shown in Fig. 1 are plotted across a diameter since the experimental values show a lack of symmetry with respect to the axis. Agreement between measured and calculated values is reasonably good. The lack of symmetry in the measured values may indicate a significant deviation of the actual assembly from the homogeneous cylinder model used for the calculations.

4e. Doppler Effects

Plutonium Doppler measurements are still quite sketchy, but calculations have been performed for most of the measurements that have been made. The data reported here are for several of the ZPR-III assemblies and for ZEBRA Assembly 5.

Tables 8 and 9 list the ratio of plutonium and iron Doppler worths to Doppler worths of uranium samples. Only Doppler ratios were quoted because some of the data were not available in absolute units, and the difficulty of correctly calculating U^{238} Doppler worths

TABLE 5. CENTRAL REACTIVITIES FOR GODIVA AND JEZEBEL
(Cents per mole)

Nuclide	GODIVA		JEZEBEL	
	Calc	Exp	Calc	Exp
Pu^{239}	<373	285	1579	1439
U^{235}	<208	149	—	—
U^{238}	22.6	24.3	60	94
Be	<9.4	7.3	19	13
B	<-14	-6.9	—	—
C	>1.9	2.4	-7	-6
Al	—	—	-13	-12
Fe	—	—	-18	-18
Ni	<-6.0	-4.4	-44	-40
Mo	—	—	-45	-37

TABLE 6. SEFOR MOCKUP REACTIVITY FOR SEVERAL SETS OF Pu^{239} CROSS SECTIONS

Pu^{239} Cross Section	k_{eff}
Karlsruhe ⁽²⁴⁾	1.065
Aldermaston ⁽²⁵⁾	1.044
New BNL-325 (1965) ⁽¹⁴⁾	1.030
Perkin-White (Upper Estimate) ^(11, 12)	1.005
Perkin-White (Lower Estimate) ^(11, 12)	0.987

TABLE 7. CENTRAL FISSION RATIOS FOR THE SEFOR MOCKUP RELATIVE TO U^{235}

Nuclide	Exp	Calc	E/C
With AIENDF Pu^{239} Cross Sections			
U^{233}	1.451	1.443	1.005
U^{238}	0.0264	0.0222	1.191
Pu^{239}	0.916	0.826	1.109
Pu^{240}	0.2006	0.1703	1.178
With Karlsruhe Pu^{239} Cross Sections			
U^{233}	1.451	1.451	0.9997
U^{238}	0.02640	0.02478	1.065
Pu^{239}	0.9163	0.9689	0.9457
Pu^{240}	0.2006	0.1876	1.069

FIG. 1. Radial Traverses for SEFOR Mockup.

TABLE 8. CALCULATED AND MEASURED DOPPLER RATIOS

ZPR-III Assembly No.	Pu ²³⁹ /U ²³⁸ Doppler Worth	
	Calc.	Exp.
45	-0.074	-0.17
45A	-0.36	-0.07
47	-0.21	—
48	-0.242	—

TABLE 9. ZEBRA 5 RELATIVE DOPPLER WORTHS

Type of Central Region	Doppler Worth of Test Region Relative to Worth of Uranium-Plutonium Region ^(a)	
	Calc.	Exp.
Pu ²³⁹	0.03	0.14
Iron	0.33	0.18

^(a) $N_{cc} \text{ U}^{238} / N_{cc} \text{ Pu}^{239} = 5.5$

is not nearly as great as the difficulty of calculating plutonium Doppler worths. Also, calculated values are shown for ZPR-III Assemblies 47 and 48 since the experimental results will probably be available in the near future. The limited results available indicate that the present method of calculating plutonium Doppler worth with channel theory is much better than the previous calculations which yielded positive Doppler worths with absolute magnitudes about twice as large as the U²³⁸ Doppler worths.

5. Conclusions

Although most of the calculated values of k_{eff} are satisfactory, the value of $k_{\text{eff}} = 1.0375$ for GODIVA stands out as an anomaly. It can only be noted here that other investigators calculated a similarly high k_{eff} for this assembly.

There is evidence from the optical-model theory of nuclear cross sections⁽²²⁾ that the AIENDF $\sigma_c(28)$ values are somewhat high. In addition the fission ratios and central reactivity coefficients for U²³⁵ and Pu²³⁹ indicate that a further revision of the fission cross sections for these nuclides may be required. An evaluation of new data recently obtained from the PETREL test is being carried out in connection with the determination of further revision of these cross sections. A small reduction in $\sigma_c(28)$ and $\sigma_f(25)$ would tend to produce better results for the calculated k_{eff} values. The k_{eff} for ZPR-III-48 is increased to 0.986 by use of the smaller $\sigma_c(28)$ based on optical-model theory.⁽²⁰⁾

Although it is expected that the suggested cross-section revisions will improve some of the k_{eff} results, it appears that several of the central fission ratios will

still show a considerable error. However, this may be indicative of incorrectly calculated high-energy neutron spectra rather than of cross-section errors. The fission ratios for the threshold detectors show considerable scatter, and it does not appear possible to attribute much of the scatter to cross-section errors.

The significant sample-size effects which exist in many of the ZPR-III experiments make the correlation of calculated central reactivity worths quite difficult. These corrections are of particular importance for strong absorbers; in some cases the corrections may be as high as 50%. Although the use of more sophisticated calculational techniques, such as Monte Carlo, may result in improved correlations, the funding and time required for these studies is seldom available. Obviously, there is a great need for experimental results which can be correlated with simpler methods. Current measurements on ZPR-III with smaller samples should do much to alleviate difficulties experienced in the past.

On the other hand, with the exception of molybdenum, the reactivities for the light elements show no striking disagreement with the measurements. The observed differences of a few millibarns are not considered to be very significant.

Much progress has been made in providing adequate cross sections for reactor calculations. Many of the calculated results show satisfactory agreement with the measured values, and the analysis of critical assemblies provides needed guidance in the evaluation of the data. Nevertheless, it is still not always possible with present data and methods to make accurate predictions for new assemblies. It is especially difficult to make accurate predictions for quantities which are sensitive to the neutron spectrum. It is to be hoped that new basic cross-section data together with the analysis of measurements presently contemplated for Pu²³⁹ critical assemblies will provide further improvement in the ability to calculate accurately the characteristics of fast reactors.

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The Optimisation of Neutron Cross-section Data Adjustments to Give Agreement with Experimental Critical Sizes

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ABSTRACT

A method is described which enables adjustments of group cross sections to be calculated in an optimum way to fit experimental critical sizes when resonance self-shielding is not important. The method uses a least-squares fitting procedure and takes into account the experimental uncertainties on the cross-section data and the critical sizes. It is fully mechanized for use with the IBM-7030 and has been shown to work satisfactorily. The machine programs involved are briefly described and an account given of some results obtained. The extension of the method to take into account other integral data, such as spectra and reaction-rate measurements, in critical systems is discussed along with a way of dealing with resonance self-shielding.

1. Introduction

It is well-known that a set of neutron-cross-section data for fissile nuclei, based on the evaluation of microscopic neutron-cross-section measurements, do not in general give calculated critical sizes, spectra, reaction rates, and perturbation measurements in agreement with measured values. One may therefore wish to adjust the cross-section data as far as possible within limits implied by their experimental errors, so that they are consistent with these various integral quantities. In carrying out such adjustments one has, of course, to ensure that errors due to the method of calculation are acceptably small. When a sufficiently fine group structure is used and resonance self-shielding is not important, it is possible to make the adjustments directly to the group cross sections. When resonance self-shielding is important and shielded constants have to be used, the adjustments must also involve the formulae used in deriving the shielded constants.

A method is described which enables adjustments of group cross sections to be calculated in an optimum way to fit experimental critical sizes. With a suitable extension, spectra, reaction rates, and perturbation

measurements in critical systems in the cases where resonance self-shielding is not important could also be taken into account. If a suitable approximation was used to take into account resonance self-shielding where necessary, the method could be further extended to include this in the adjustment process. As far as fitting critical-size data is concerned, the method is fully mechanized for use with the IBM-7030 (Stretch) and has been shown to work satisfactorily. Machine programs involved are DUNDEE⁽¹⁾ and PENICUIK, and it is the program PENICUIK which actually carries out the optimisation procedure based on a least-squares technique. The DUNDEE-PENICUIK system is linked to the AWRE one-dimensional S_n program STRAINT⁽²⁾ and to the group-cross-section program GALAXY,⁽³⁻⁵⁾ which calculates group cross sections from the basic neutron-cross-section data in the UKAEA Data Library.⁽⁶⁾ The program DUNDEE uses group cross sections calculated by GALAXY with the perturbation functions (on tape)^(1, 2, 7) calculated by STRAINT for all the systems of interest, and calculates by first-order perturbation theory⁽⁷⁾ the sensitivity of the inverse reactivity B and time constant λ to changes in the group-cross-section data. The output from DUNDEE is on tape which is used as part of the input to the program PENICUIK, along with specified standard deviations of the cross-section data and of the integral quantities which are being used in the fitting procedure. The difference between the calculated and measured integral quantities are, of course, also included in the input to PENICUIK. Factors can also be included in PENICUIK to allow some correlation, if required, between the adjustments made in adjacent energy groups.

It should be noted that though critical systems may be assembled experimentally, the models used in the calculation do not have a reactivity of exactly unity. Uncertainties exist, for example, in material densities and material compositions, and some geometrical extrapolation is usually required from the experimental

arrangement to the model. Thus the reactivity of a given model must be specified as $1 \pm \eta$, where η represents the uncertainty from these various factors. These are the standard deviations of the integral quantities referred to in the above paragraph. An attempted exact fit to the experimental critical sizes is likely to lead to unrealistic adjustments to the cross-section data. This is demonstrated by a numerical example in Sect. 3.6.

The program PENICUIK and an earlier version of DUNDEE have been in use since about April, 1964, though to begin with the link between the programs was not mechanized. It is interesting to note that a similar method, also based on least-squares fitting, of using integral data has been suggested independently by other authors.⁽⁸⁻¹⁰⁾ The method as used at Aldermaston has been found to work satisfactorily in practical cases, and an account of some results obtained is given in Sect. 3.

In view of the success in using the method in its present form, it is currently being extended to enable spectra and reaction-rate measurements at the centre of critical systems to be taken into account. A brief account of these extensions is given in Sect. 5.1. (In principle, if a sufficiently large number of critical sizes covering a wide enough range of spectra are used in the present fitting procedure, this should go some way towards obtaining data which give correct spectra, reaction rate, and perturbation effects.) When the new extensions are working, it will then be possible simultaneously to fit spectra-reaction rate ratios *and* critical sizes. The development to take into account spectral measurements at the centre of critical systems is tied in with the work on spectrum measurements being carried out on the VERA reactor at Aldermaston.⁽¹¹⁾

The DUNDEE-PENICUIK system gives the adjustments which have to be made to system-independent group cross sections and from a specification of these adjustments on cards together with a tape containing the unadjusted data, the program GALAXY can be used to produce a tape containing *adjusted* data for use by the AWRE one-dimensional S_n program STRAINT and the two-dimensional S_n program TURTLE.⁽¹²⁾ This procedure is satisfactory as long as one only wants to continue using the same group structure. As soon as one wishes to use a different group structure, one is faced with the problem of how to convert the adjustments to the new group structure, bearing in mind that the *unadjusted* group cross sections in the new structure can readily be obtained by GALAXY from the basic data in the UKAEA Data Library. The most convenient and universal way of dealing with this problem would therefore be to derive adjusted basic data from the adjustments calculated for the group

cross sections. A method for doing this is described in Sect. 4.

2. The Method of Adjusting Group Cross Sections to Fit Experimental Critical Sizes

2.1. General Description of the Method

Consider a set of L critical systems. Let B_l ($l = 1, 2, \dots, L$) be the value of the inverse reactivity B calculated for the experimentally determined critical system l . If the cross-section data used in the calculations were accurate and there were negligible errors due to the method of calculation and to the model used for the experimental system, then B_l would be unity. In practice it is likely that B_l will differ from unity by as much as 0.05. If a very accurate method of calculation is used, the error in B_l from this source is probably less than 0.003. The error in B_l from the uncertainties in the model of the experimental system (due to density, nuclear composition, geometry, etc.) is usually less than about 0.006 for small systems, which corresponds to about 2% of the critical mass. The remaining discrepancy is then due to inaccuracies in the cross-section data used in the calculations.

Without considering at this stage how the values are to be obtained, let x_{mjg} be the *percentage* increase which has to be made to the group cross section in group g for reaction j in nuclide m in order to give agreement between the calculated and experimental critical sizes. It is convenient to exclude the elastic cross section from this and to use the elastic cross section as a dependent variable which is chosen to preserve the balance between the total and partial cross sections. (The total cross section could just as well be treated as the dependent variable if desired.) Let α_{lmjg} be the change in B for system l arising from a 1% increase in the group cross section characterized by the subscripts mjg , with a compensating change made to the elastic cross section. The α_{lmjg} are the quantities calculated by the perturbation program DUNDEE.⁽¹⁾ Write $B_l - 1 = \Delta B_l$, so that to get exact agreement between the calculated and experimental critical sizes B_l must be increased by an amount $-\Delta B_l$. This is given in terms of x_{mjg} by

$$\sum_{mjg} \alpha_{lmjg} x_{mjg} = -\Delta B_l. \quad (1)$$

In view of the uncertainties which exist in the models of the experimental critical systems which have to be used in the calculations, it is illogical to aim at *exact* agreement with the experimental critical sizes. Thus if b_l represents the residual discrepancy from unity for the B value for system l after the adjustments x_{mjg} have been made, eq. (1) can be replaced by

$$\sum_{m,j,g} \alpha_{lm,j,g} x_{m,j,g} = -\Delta B_l + b_l. \quad (2)$$

The total number of unknowns $x_{m,j,g}$ is, of course, very much greater than L , the number of equations (2), and in addition there are L values of b_l . Hence, in the absence of any further constraints, there are an infinite number of different solutions to the L equations (2) for the $x_{m,j,g}$ and b_l . Not all these solutions will be acceptable and some will be more acceptable than others. It is therefore necessary to introduce some criterion which gives a "best" solution. A convenient indication of the goodness of the solution involves the sum of the squares of $x_{m,j,g}$ and b_l with appropriate weights given to the various terms according to the experimental uncertainties. Hence it is reasonable to aim to minimise this quantity. Thus if $\xi_{m,j,g}$ is the percentage standard deviation of the cross section with which $x_{m,j,g}$ is associated and η_l is the standard deviation of the inverse reactivity of system l , then on the basis of the above discussion one should aim to minimise

$$S = \sum_{m,j,g} \frac{x_{m,j,g}^2}{\xi_{m,j,g}^2} + \sum_l \frac{b_l^2}{\eta_l^2}. \quad (3)$$

Minimisation of S , subject to the constraints imposed by (2), leads to the following N equations corresponding to the N values of $x_{m,j,g}$:

$$\frac{x_{m,j,g}}{\xi_{m,j,g}^2} + \sum_l \frac{b_l \alpha_{lm,j,g}}{\eta_l^2} = 0, \quad (4)$$

which together with the L equations (2) can be solved to give a unique solution for the $x_{m,j,g}$ and b_l . This then gives a method of obtaining a "best set" of group-cross-section adjustments to fit experimental critical sizes. It should be noted that it is stable in the sense that if the systems studied are not sensitive to a particular item of cross-section data, then the adjustment calculated for that item of data is zero. This can be seen from equation (4).

The method described above can be generalised so that other integral quantities such as spectra, reaction rates, and perturbation measurements in critical systems (and also in subcritical systems driven by a known neutron source) can be taken into account in the fitting procedure. In this case one includes the appropriate integral quantities in eq. (2) in place of ΔB_l and b_l , and the corresponding α values give the sensitivity of the calculated integral quantity to changes in the cross-section data (see Sect. 5).

It is interesting to note that if it is assumed that the $x_{m,j,g}$ are independent and normally distributed about zero with standard deviation $\xi_{m,j,g}$ and that the b_l are also independent and normally distributed about zero with standard deviation η_l , then the combined multi-

variate probability distribution function $P(x, b)$ for the $x_{m,j,g}$ and the b_l is

$$C \exp \left[-\frac{1}{2} \sum_{m,j,g} \left(\frac{x_{m,j,g}}{\xi_{m,j,g}} \right)^2 - \frac{1}{2} \sum_l \left(\frac{b_l}{\eta_l} \right)^2 \right], \quad (5)$$

C being the normalisation factor. The most probable value of this distribution is given when

$$S = \sum_{m,j,g} \frac{x_{m,j,g}^2}{\xi_{m,j,g}^2} + \sum_l \frac{b_l^2}{\eta_l^2}$$

is a minimum, which is the same condition as suggested above on plausibility arguments. This is just the well-known result that the most probable values are those obtained by least-squares fitting when all the quantities involved are independent and normally distributed.

It is inevitable that correlations will exist between some of the $x_{m,j,g}$ and a particular case of this is between the $x_{m,j,g}$ for different energy groups g . For instance, if $x_{m,j,g}$ is positive for $g = g_0$, say, then in general $x_{m,j,g}$ is more likely to be positive for $g = g_0 \pm 1$. This kind of correlation can easily be taken into account by the introduction of further terms involving the product $x_{m,j,g} (x_{m,j,g+1})$ in the expression to be minimised. Thus, the S of eq. (3) is modified to become

$$S = \sum_{m,j,g} \left\{ \frac{x_{m,j,g}^2}{\xi_{m,j,g}^2} - 2 \frac{x_{m,j,g} (x_{m,j,g+1})}{\xi_{m,j,g} \xi_{m,j,g+1}} p_{m,j,g,g+1} \right\} + \sum_l \frac{b_l^2}{\eta_l^2} \quad (6)$$

with $p_{m,j,g,g+1} > 0$ for positive correlation, the actual form of the expression being chosen for mathematical convenience. It is essential that the expression (6) should be positive definite; otherwise minimisation would not be possible. It can be shown that a *sufficient* condition for (6) to be positive definite is that $|p_{m,j,g,g+1}| < 0.5$. The *necessary* condition is relatively complicated, and the present application does not justify using anything other than the simple sufficient condition referred to. Thus, for positive correlation the restriction

$$0 < p_{m,j,g,g+1} < 0.5$$

needs to be applied. With this restriction, the minimisation of (6) subject to the constraints imposed by eq. (2) leads to

$$\begin{aligned} & \frac{-x_{m,j,g-1}}{\xi_{m,j,g} \xi_{m,j,g-1}} p_{m,j,g-1,g} + \frac{x_{m,j,g}}{\xi_{m,j,g}^2} \\ & - \frac{x_{m,j,g+1}}{\xi_{m,j,g} \xi_{m,j,g+1}} p_{m,j,g,g+1} + \sum_l \frac{b_l \alpha_{lm,j,g}}{\eta_l^2} = 0, \end{aligned} \quad (7)$$

which together with the L equations (2) can be solved as before to give a unique solution for the $x_{m,j,g}$ and b_l .

The method of optimisation is not unique and weak-

nesses in the above method became apparent in the light of experience. Variations can easily be obtained by weighting the various terms appearing in the expression to be minimised in different ways and give rise to minimisation procedures superior to the one described above. This problem is discussed in more detail in Sect. 2.2 below.

2.2. Variations of the Minimisation Procedure

The use of the minimisation procedure described in Sect. 2.1 can sometimes lead to what appear to be anomalous features in the results. For instance, if one introduces more items of cross-section data into the fitting procedure for a given amount of integral data, one would expect to get a better fit to the integral data, but this is not always the case. To illustrate this, consider the case involving one critical system and the cross section for one reaction for a given nuclide. In cases where the systems are fairly sensitive to a given cross section, one would usually allow changes to be made as a function of energy group, but where the systems are not very sensitive to the cross section, only a uniform change in all energy groups may be allowed in order to save machine storage, and it is desirable that these two approaches should not lead to inconsistent results. Returning to the example of one critical system and the cross section for one reaction for a given nuclide, if the cross section is allowed to change as a function of energy group, correlations between groups being neglected, the equations to be solved for the x_g (suffices m and j have been omitted for convenience) and b_1 are

$$\frac{x_g}{\xi_g^2} + \frac{b_1 \alpha_{1g}}{\eta_1^2} = 0; \quad (8)$$

$$\sum_{g=1}^G \alpha_{1g} x_g - b_1 = -\Delta B_1, \quad (9)$$

where G is the number of energy groups involved. If the ξ_g are all the same and equal to ξ , and the α_{1g} are all equal to α_0 say, then the solution of these equations gives

$$x_g = -\frac{\xi^2 \alpha_0 \Delta B_1}{\eta_1^2 \left[1 + \frac{G \xi^2 \alpha_0^2}{\eta_1^2} \right]} \text{ (independent of } g); \quad (10)$$

$$b_1 = \frac{\Delta B_1}{\left[1 + \frac{G \xi^2 \alpha_0^2}{\eta_1^2} \right]}. \quad (11)$$

If the alternative approach had been taken of specifying that the adjustments should be the same in all groups, then if $\bar{x}$ is the percentage change made in the cross section at all energies and b'_1 the resulting value of b_1 , the equations to be solved for $\bar{x}$ and b'_1 would have been

$$\frac{\bar{x}}{\xi^2} + \frac{b'_1 G \alpha_0}{\eta_1^2} = 0; \quad (12)$$

$$G \alpha_0 \bar{x} - b'_1 = -\Delta B_1, \quad (13)$$

which gives

$$\bar{x} = -\frac{G \xi^2 \alpha_0 \Delta B_1}{\eta_1^2 \left[1 + \frac{G \xi^2 \alpha_0^2}{\eta_1^2} \right]}, \quad (14)$$

and

$$b'_1 = \frac{\Delta B_1}{\left[1 + \frac{G \xi^2 \alpha_0^2}{\eta_1^2} \right]}. \quad (15)$$

It will be noted that these are different from the solutions (10) and (11) given by the previous approach. Since $G > 1$,

$$x_g < \bar{x} \text{ and } b_1 > b'_1.$$

Thus when the cross-section changes are specified as a function of energy group, the resulting fit to the integral experiment is worse than when the cross-section change is specified to be the same in all energy groups, and the corresponding adjustment to the cross section is less. It would appear that when one allows more items of cross-section data to change, the fit to the integral experiment(s) becomes worse, which is opposite to what one would expect. The difference between the two results in the above example arises because in the first approach the cross-section changes in the different groups are assumed to be independent (no correlation) and in the second approach they are implicitly assumed to have unit correlation.

In view of the difficulty of assessing correlations, it would be preferable if one did not need to worry about them, and this requires that the method should be modified so that one obtains much the same answer whether the correlation is included or not. A simple way of achieving this for the extreme cases of zero and unit correlation (as considered in the above examples) is to minimise the function

$$\frac{1}{G} \sum_{g=1}^G \frac{x_g^2}{\xi_g^2} + \frac{b_1^2}{\eta_1^2}. \quad (16)$$

An extension of this to more items of cross-section data and more critical systems leads to the minimisation of

$$\sum_{m=1}^M \frac{1}{G} \sum_{g=1}^G \frac{x_{mzg}^2}{\xi_{mzg}^2} + \sum_{l=1}^L \frac{b_l^2}{\eta_l^2}. \quad (17)$$

This thus represents an alternative function for minimisation to that given by eq. (3). In view of the foregoing discussion, (17) has certain advantages over (3); further, one does not have to worry about the correlations between the x_{mzg} for different g .

It will be noted that

$$\frac{1}{G} \sum_{g=1}^G \frac{x_{mjg}^2}{\xi_{mjg}^2}$$

represents the average value of x_{mjg}^2/ξ_{mjg}^2 over the different values of g for fixed mj . One might then ask if it would be better to average the mj terms in a similar way and also the terms involving b_l . In this way one arrives at a further alternative function to minimise, namely,

$$\frac{1}{N_{mj}} \sum_{mj} \frac{1}{G} \sum_{g=1}^G \frac{x_{mjg}^2}{\xi_{mjg}^2} + \frac{1}{L} \sum_{l=1}^L \frac{b_l^2}{\eta_l^2}, \quad (18)$$

where N_{mj} is the number of mj values. An alternative function based on the concept of average values of x_{mjg}^2/ξ_{mjg}^2 and b_l^2/η_l^2 is

$$\frac{1}{N_{mjg}} \sum_{mjg} \frac{x_{mjg}^2}{\xi_{mjg}^2} + \frac{1}{L} \sum_{l=1}^L \frac{b_l^2}{\eta_l^2}, \quad (19)$$

where N_{mjg} is the total number of mjg values.

In considering these various alternative forms of the function to be minimised one should bear in mind that there is no unique correct function to be minimised as might be implied, for example, by a variational principle. One must rely largely on plausibility arguments to decide upon possible functions, and quite clearly there are a large number of possible functions. For instance, one might wish to give the integral systems a different overall weight relative to the cross-section data and so introduce another factor $q < 1$ into (18) to give

$$\frac{1-q}{N_{mj}} \sum_{mj} \frac{1}{G} \sum_{g=1}^G \frac{x_{mjg}^2}{\xi_{mjg}^2} + \frac{q}{L} \sum_{l=1}^L \frac{b_l^2}{\eta_l^2} \quad (20)$$

as the function to be minimised. The ultimate test is, of course, that whatever the function chosen to be minimised, all the values of the x_{mjg} and the b_l should be acceptable in relation to their standard deviations and one can ask no more.

2.3. The PENICUIK Programs

The first PENICUIK program was written to solve the equations resulting from the minimisation of (6). It was originally written to take the α_{lmjg} as decimal input but was later modified to take a tape input produced as an output option from the program DUNDEE. Versions of PENICUIK were later written to minimise (19) and (20), and recently a version has been written to minimise (17). All these versions are really very similar since all the different functions reduce in form to that given by (6) with $p_{mjg,g+1} = 0$ and modified ξ_{mjg} and η_l . The programs are called PENICUIK 0, 1, 2, and 3 and for convenience, the functions they minimise are listed below.

Program	Function Minimized
PENICUIK 0	$\sum_{mjg} \left\{ \frac{x_{mjg}^2}{\xi_{mjg}^2} - \frac{2x_{mjg}x_{mjg+1}}{\xi_{mjg}\xi_{mjg+1}} p_{mjg,g+1} \right\} + \sum_l \frac{b_l^2}{\eta_l^2}$
PENICUIK 1	$\frac{1}{N_{mjg}} \sum_{mjg} \frac{x_{mjg}^2}{\xi_{mjg}^2} + \frac{1}{L} \sum_l \frac{b_l^2}{\eta_l^2}$
PENICUIK 2	$\frac{(1-q)}{N_{mj}} \sum_{mj} \frac{1}{G} \sum_g \frac{x_{mjg}^2}{\xi_{mjg}^2} + \frac{q}{L} \sum_l \frac{b_l^2}{\eta_l^2}$
PENICUIK 3	$\frac{1}{G} \sum_g \frac{x_{mjg}^2}{\xi_{mjg}^2} + \sum_l \frac{b_l^2}{\eta_l^2}$

Historical Note:

Experience with PENICUIK 0 had shown that in practical problems the inclusion of correlation between the x_{mjg} for different g did not appreciably affect the results and in view of the discussion in Sect. 2.2 the correlation terms were not included in PENICUIK 1, 2, and 3.

In trial calculations with PENICUIK 0 it was found that the fit to the integral systems sometimes became worse as more items of cross-section data, obtained by allowing cross sections to change as a function of energy group (see discussion in Sect. 2.2), were introduced into the fitting procedure, though for all cases studied the results were acceptable. PENICUIK 1 was written to carry out a few test calculations to see if the consideration of the sum of the average values of x_{mjg}^2/ξ_{mjg}^2 and b_l^2/η_l^2 gave any improvement. An improvement was, in fact, found, and to make the minimisation procedure a little more logical (see Sect. 2.2) PENICUIK 2 was afterwards written; its results were an improvement on those from PENICUIK 1. In general it was found that the best results were obtained with $q = 1/2$, giving equal weight to the cross-section data and the integral quantities. In all subsequent production calculations PENICUIK 2 with $q = 1/2$ has been used.

PENICUIK 3 has recently been written to assist with a re-examination of minimisation functions with a view to choosing a suitable function for the case where spectra, reaction rates, and critical sizes are simultaneously taken into account.

3. Data-adjustment Calculations Using U-235/U-238 Critical Systems

3.1. Systems Studied

The systems considered in the present calculation are all spherical and consist of 4 bare and 21 reflected cores. A detailed specification of them is given in Tables

TABLE 1. CRITICAL SYSTEMS INVOLVING U-235 AND U-238

System Reference Number used in this Report	Core Radius (cm)	Core Density (g/cc)	Core Composition (Proportion by nuclei)		Reflector Thickness (cm)	Reflector Density (g/cc)	Reflector Composition (Proportion by nuclei)			
			U-235	U-238			U-235	U-238	C	Be
1.1 (Godiva)	8.71	18.75	0.94	0.06	0	—	—	—	—	—
1.2	7.725	18.75	0.94	0.06	1.765	19.0	0.0072	0.9928	—	—
1.3	6.962	18.75	0.94	0.06	4.47	19.0	0.0072	0.9928	—	—
1.4	6.391	18.75	0.94	0.06	8.95	19.0	0.0072	0.9928	—	—
1.5	6.312	18.75	0.94	0.06	9.97	19.0	0.0072	0.9928	—	—
1.6 (Topsy)	6.045	18.75	0.94	0.06	22.9	19.0	0.0072	0.9928	—	—
2.1	16.85	18.80	0.293	0.707	0	—	—	—	—	—
2.2	14.61	18.75	0.380	0.620	0	—	—	—	—	—
2.3	12.14	18.70	0.539	0.461	0	—	—	—	—	—
2.4	8.99	18.80	0.476	0.524	19.70	19.0	0.0072	0.9928	—	—
2.5	7.314	18.75	0.679	0.321	21.60	19.0	0.0072	0.9928	—	—
2.6	6.62	18.70	0.805	0.195	22.2	19.0	0.0072	0.9928	—	—
2.7	20.32	18.75	0.1634	0.8366	7.62	19.0	0.0072	0.9928	—	—
3.1	7.381	18.7	0.94	0.06	5.08	1.69	—	—	1.00	—
3.2	6.915	18.7	0.94	0.06	10.2	1.69	—	—	1.00	—
3.3	6.667	18.45	0.94	0.06	15.2	1.69	—	—	1.00	—
3.4	6.422	18.75	0.94	0.06	20.3	1.69	—	—	1.00	—
3.5	6.158	18.5	0.94	0.06	43	1.69	—	—	1.00	—
3.6	6.699	18.5	0.94	0.06	4.7	1.84	—	—	—	1.00
3.7	5.616	18.6	0.94	0.06	11.8	1.84	—	—	—	1.00

TABLE 2. VERA CRITICAL SYSTEMS INVOLVING U-235 AND U-238

System Reference Number	Core Radius (cm)	Core Density (g/cc)	Core Composition (Proportions by nuclei)							
			U-235	U-238	U-234	U-236	C	Fe	Cr	Ni
1B	19.14	5.025	0.7349	0.0455	0.0092	0.0014	5.7540	0.6282	0.1635	0.0688
2A	22.02	4.384	0.5669	0.0351	0.0071	0.0011	6.0850	0.6282	0.1635	0.0688
3A	21.64	9.191	0.6186	1.3255	0.0076	0.0011	3.5766	0.6282	0.1635	0.0688
4A	15.53	5.211	0.8066	0.0499	0.0101	0.0015	5.0964	0.6282	0.1635	0.0688
5A	17.39	4.667	0.6605	0.0409	0.0083	0.0012	5.4610	0.6282	0.1635	0.0688

System Reference Number	Reflector Thickness (cm)	Reflector Density (g/cc)	Reflector Composition (Proportions by nuclei)				
			U-235	U-238	Fe	Cr	Ni
1B	35	14.508	0.025	3.440	0.6464	0.1682	0.0708
2A	35	14.508	0.025	3.440	0.6464	0.1682	0.0708
3A	35	14.508	0.025	3.440	0.6464	0.1682	0.0708
4A	35	14.508	0.025	3.440	0.6464	0.1682	0.0708
5A	35	14.508	0.025	3.440	0.6464	0.1682	0.0708

1 and 2. The systems in Table 1 are from U.S. laboratories; those in Table 2 have been studied at Alderaston. It will be noted that the reflectors in the case of the VERA systems are natural uranium plus a small amount of stainless steel. The cores of the VERA sys-

tems are made up of 93 % U-235, or 37 % U-235 in the case of core 3A, with carbon and stainless steel and the addition of polythene in cores 4A and 5A. A small amount of hydrogen in all the cores comes from the lacquer with which the U-235 components are coated.

TABLE 3. ENERGY GROUP BOUNDARIES

32-group Set 5		12-group Set 5		8-group Set 5	
Group No.	Boundaries	Group No.	Boundaries	Group No.	Boundaries
1	14.0 MeV				
2	13.5				
3	12.5				
4	11.0		11.0 MeV		11.0 MeV
5	9.5				
6	8.0				
7	6.5	1		1	
8	5.0				
9	3.4				
10	2.4		2.4		
11	1.6	2			1.6
12	1.1		1.1	2	
13	0.78	3			0.78
14	0.55		0.55		
15	0.37	4		3	
16	0.26		0.26		
17	0.18	5			0.18
18	0.13		0.13		
19	80 keV	6			
20	43		43 keV	4	
21	22	7			
22	10		10		10 keV
23	4	8			
24	1.6		1.6	5	
25	0.65	9			
26	0.26		0.26		0.26
27	0.11	10			
28	42 eV		42 eV	6	
29	15	11			15 eV
30	5.5		5.5		
31	2.0	12		7	
	0.4		0.4		0.4
		13		8	
	Thermal		Thermal		Thermal

3.2. General Notes on the STRAINT and DUNDEE Calculations with Particular Reference to the Group Cross Sections Used

The calculations to find the inverse reactivity B of the critical systems using the unadjusted data [from which the ΔB required in eq. (2) can be obtained] were nominally carried out using the S_n program STRAINT in the S_8 approximation with the anisotropic-scattering option using the AWRE 32-group Set 5 data (these are the same as 32-group Set 4 described in reference (13) except for the boundaries of group 1). The 32-group data set is an AWRE set covering the energy range from 14 MeV down to thermal. Where it was known from previous calculations that neutrons below one keV or so had negligible effect on a given type of system, then only the top 23 energy groups were retained in the calculation; the lower energy limit of group 23 is 1.6 keV, and neutrons which would other-

wise have been scattered below 1.6 keV were retained in this group. In the case of the VERA systems all 32 energy groups were retained in the calculations, but it was known from previous calculations that, because the systems were reasonably large, the S_4 approximation was adequate. Also, in some of the systems it was not necessary to use the anisotropic scattering option, the anisotropy of scatter being taken into account by the transport approximation. These various approximations lead to a saving of machine time without introducing any significant loss of accuracy.

In calculating the α_{lmjg} factors giving the sensitivity of the B values to changes in the cross-section data, it is not necessary to achieve as high an accuracy as with the calculation of the B values. This can be illustrated as follows. The calculated value of B for a given system should be equal to $1 \pm \epsilon$ after the cross-section data have been changed by an amount given by $\delta(\ln \sigma)$; thus, symbolically,

$$1 \pm \epsilon = B + \frac{\partial B}{\partial (\ln \sigma)} \delta(\ln \sigma).$$

Now B will be of order 0.95–1.05. If then ϵ is 0.01, B must be determined to 1% accuracy but the

$$[\partial B / \partial (\ln \sigma)] \delta(\ln \sigma)$$

term only needs to be determined to 20% accuracy. The $\partial B / \partial (\ln \sigma)$ is analogous to the α_{lmjg} factors, and so it is that they do not need to be determined to the same accuracy as the B values.

As a result of the above argument, and for historical reasons, the AWRE 13-group Set 5 data⁽¹³⁾ have been used with anisotropic scattering taken into account by the transport approximation to derive the α_{lmjg} factors. The 13-group Set 5 data are derived from the 32-group Set 5 data by condensation and cover the energy range from 11 MeV to thermal. Early perturbation calculations using the 13-group data gave results which were within 5% (in some cases within 1%) of corresponding results obtained using 8-group Set 5 data, which are also derived from the 32-group Set 5 data by condensation and cover the same energy range as the 13-group data. The energy boundaries of the 13- and 8-group data sets in relation to those of the 32-group data are given in Table 3. The fact that such close agreement was obtained between the perturbation results with the 8- and 13-group data leads one to conclude that the difference between 13-group and 32-group results would also be small and therefore gives some confidence in the adequacy of the 13-group treatment of the perturbation calculations to determine the α_{lmjg} .

The consequence of using the 13-group representation for the α_{lmjg} is that the adjustment factors x_g which are subsequently calculated by PENICUIK

also apply to the 13-group representation, and the problem then remains of deriving the corresponding 32-group x_g values from them. To illustrate this problem, consider the case where 2 groups of the 32-group data are condensed to give just 1 group of the 13-group data. Suppose groups i and $i + 1$ of the 32-group data condense to group j of the 13-group data; then

$$a_i\sigma_i + a_{i+1}\sigma_{i+1} = \sigma_j,$$

where σ_i and σ_j are the group cross sections, and a_i and a_{i+1} represent the condensing spectrum with $a_i + a_{i+1} = 1$. If x_i , x_{i+1} , and x_j are the adjustment factors corresponding to σ_i , σ_{i+1} , and σ_j , then

$$a_i\sigma_i x_i + a_{i+1}\sigma_{i+1}x_{i+1} = \sigma_j x_j.$$

There are clearly an infinite number of ways of determining the x_i and x_{i+1} , but the simplest is to take $x_i = x_{i+1} = x_j$. This is in fact what has so far been done to derive the 32-group adjustment factors from the 13-group values. The simple recipe generalises to the case where more than 2 groups of the 32-group set are condensed into 1 group of the 13-group set.

A reasonable alternative way of determining the x_i and x_{i+1} might be to minimise $(x_i^2 + x_{i+1}^2)$, which in the above example would lead to

$$x_i = \frac{a_i\sigma_i\sigma_j x_j}{[(a_i\sigma_i)^2 + (a_{i+1}\sigma_{i+1})^2]};$$

$$x_{i+1} = \frac{a_{i+1}\sigma_{i+1}\sigma_j x_j}{[(a_i\sigma_i)^2 + (a_{i+1}\sigma_{i+1})^2]}.$$

3.3. Summary of the Basic Data used in the Calculations

The basic cross-section data from which the group cross sections used in the calculations have been obtained are from the UKAEA Nuclear Data library.⁽¹⁴⁾ The Data File Numbers (DFN's) identifying the data are given in Table 4.

3.4. Results of "32-group" STRAINT Calculations Using Unadjusted Data

As stated in Sect. 3.2, all the so-called 32-group calculations did not necessarily use all 32 energy groups, nor did they all use the anisotropic S_n method in the S_8 approximation. The calculated B values are given in Table 5 together with a statement of the actual number of energy groups used, of whether the anisotropic or isotropic versions of the S_n program were used, and whether S_4 or S_8 approximations were used. The basic data used are the unadjusted data listed in Table 4.

From the B values given in Table 5 the values of $\Delta B = B - 1$ were calculated for use in the PENICUIK adjustment calculation. If the (unadjusted) cross-section data had been adequate, the values of ΔB would have been more or less within the range ± 0.005 ; a

TABLE 4. BASIC DATA USED IN THE CALCULATIONS⁽¹⁴⁾

Nuclide	DFN
H	107
Be	8
C	117
Cr	132
Fe	162
Ni	134
U-234	174
U-235	155
U-236	173
U-238	156

graphical illustration of the inadequacy is given in Fig. 1, where ΔB is plotted as a function of the core radius. It should be noted that a value of $|\Delta B| = 0.04$ corresponds to a 12% error in the critical mass. The values of B for the VERA systems 2A, 3A, 4A, and 5A, and even possibly VERA 1B, are acceptably close to unity, but in view of the large discrepancies for the other systems the agreement must be considered fortuitous.

3.5. The DUNDEE Calculations

The perturbation functions for the DUNDEE calculations were obtained by calculating the critical dimensions of the cores of the various systems with the compositions, densities and reflector thicknesses as given in Tables 1 and 2. The results of the DUNDEE calculations giving the α_{lmjg} factors are too numerous to include in full in this paper. They are available on machine print-outs and on magnetic tape for use with PENICUIK. The DUNDEE results for systems 1.1–1.6, reproduced from machine print-outs, are given in Ref. (1). The results from DUNDEE giving the change in B due to 1% changes in the cross section of U^{235} in all energy groups are given in Table 6 for systems 1.1–2.7, and similar results for U^{238} are given in Table 7. The reaction type numbers (RTN's) used to identify the various reactions are described in references (6) and (14), but the key to them is included in Tables 6 and 7 for convenience. The results in Tables 6 and 7 indicate which are the important cross sections for these systems.

3.6. The Results of Some PENICUIK Calculations

The results of the PENICUIK calculations which will now be given and discussed are intended to show how the method works under normal conditions and also the kind of results which are obtained under abnormal conditions, such as if errors occur in some of the numbers used or if one is unrealistic in attempting to get an *exact* fit to the experimental critical sizes, thereby ignoring the experimental uncertainties.

TABLE 5. B VALUES OF SYSTEMS LISTED IN TABLES 1 AND 2 OBTAINED WITH THE UNADJUSTED DATA

System	Brief Specification of Systems		No. of Groups Used	Program: A = Anisotropic I = Isotropic	S ₄ or S ₈ Approximation	B Value
	Core	Reflector				
1.1	94% U235	None	23	A	S ₈	0.9707
1.2	94% U235	Nat. U, 1.8 cm thick	23	A	S ₈	0.9627
1.3	94% U235	Nat. U, 4.5 cm thick	23	A	S ₈	0.9550
1.4	94% U235	Nat. U, 9.0 cm thick	23	A	S ₈	0.9555
1.5	94% U235	Nat. U, 10.0 cm thick	23	A	S ₈	0.9572
1.6	94% U235	Nat. U, 23 cm thick	23	A	S ₈	0.9620
2.1	29% U235	None	23	A	S ₈	0.98025
2.2	38% U235	None	23	A	S ₈	0.9729
2.3	54% U235	None	23	A	S ₈	0.9623
2.4	48% U235	Nat. U, 19 cm thick	23	A	S ₈	0.9746
2.5	68% U235	Nat. U, 22 cm thick	23	A	S ₈	0.9659
2.6	81% U235	Nat. U, 22 cm thick	23	A	S ₈	0.9652
2.7	16% U235	Nat. U, 7.62 cm thick	23	A	S ₈	0.9900
3.1	94% U235	C, 5.1 cm thick	32	A	S ₈	0.9656
3.2	94% U235	C, 10.2 cm thick	32	A	S ₈	0.9584
3.3	94% U235	C, 15 cm thick	32	A	S ₈	0.9633
3.4	94% U235	C, 20 cm thick	32	A	S ₈	0.9605
3.5	94% U235	C, 43 cm thick	32	A	S ₈	0.9616
3.6	94% U235	Be, 4.7 cm thick	32	A	S ₈	0.9421
3.7	94% U235	Be, 12 cm thick	32	A	S ₈	0.9352
Vera 1B	93% U235 + C + steel	Nat. U + steel, 35 cm thick	32	I	S ₄	0.9917
2A	93% U235 + C + steel	Nat. U + steel, 35 cm thick	32	I	S ₄	0.9943
3A	37% U235 + C + steel	Nat. U + steel, 35 cm thick	32	I	S ₄	0.9955
4A	93% U235 + C + CH ₂ + steel	Nat. U + steel, 35 cm thick	32	A	S ₄	0.9956
5A	93% U235 + C + CH ₂ + steel	Nat. U + steel, 35 cm thick	32	A	S ₄	0.99765

In PENICUIK calculations the cross sections to which the B values are most sensitive are allowed to be adjusted as a function of energy group, but the cross sections to which the systems are not very sensitive are usually only allowed to be adjusted uniformly over all energy groups to save computer storage. In all the examples given in this paper the PENICUIK 2 program (see Sect. 2.3) has been used with $q = \frac{1}{2}$. The standard deviations which have been taken for the cross-section data have been deduced as far as possible from the quoted accuracies on experimental measurements and the *differences between different experimental measurements*.

In Tables 8 and 9 the results of two PENICUIK calculations are given in which (Run 1) all the systems listed in Table 5 are taken into account and (Run 2) only the first 13 systems (1.1–2.7) listed in Table 5 are taken into account; as some of the systems in Run 1 contain nuclides not present in Run 2, more items of cross section data were allowed to be changed in Run 1. The calculated cross-section adjustments for all the items of cross-section data included in the adjusting

process along with their assumed standard deviations are given in Table 8, and the resulting b_i values along with their assumed standard deviations are given in Table 9. In these calculations only the cross sections in groups 1–8 of the 13-group data were allowed to be adjusted, that is the data above 1.6 keV.

Some of the b_i values given in Table 9 are greater than the assumed standard deviations, but they are all acceptable from a practical point of view. The adequacy of the b_i values is illustrated graphically in Fig. 2, where the b_i values from Run 1 are plotted as a function of core radius. Remembering that the b_i values correspond to the values of $(B_i - 1)$ with the adjusted data, Fig. 2 can be compared with Fig. 1, which gives the corresponding values with unadjusted data. Calculations have been carried out using adjusted 32-group data calculated from the 13-group adjustments in accordance with the simple recipe given in Sect. 3.2. The B_i values obtained tend to be slightly less than the values predicted by PENICUIK, but the difference does not affect the overall good agreement between calculation and experiment indicated by Fig. 2.

FIG. 1. ΔB as a Function of Core Radius with Unadjusted Data.

It will be noted that the b_l values in Table 9 for systems 1.1–2.7 as calculated in Runs 1 and 2 are in very close agreement. The adjustments of cross-section data common to Runs 1 and 2, namely, those for U^{235} and U^{238} , are in general in fairly close agreement in the higher-energy groups, but often differ significantly in the lower-energy groups, showing the influence of the softer-energy systems taken into account in Run 1. Most of the cross-section adjustments are within the assumed standard deviations, but there are some exceptions; for example, the total cross section of beryllium is changed by more than 2 assumed standard deviations in 3 of the energy groups, and the total cross section of U^{235} is changed by nearly 2 standard deviations in group 4. Even so, the resulting data adjustments are not unacceptable.

It is interesting to consider the effect of specifying a ΔB value wrongly in a PENICUIK calculation. Thus, in RUN 3 the ΔB value of System 1.6 was taken as -0.0038 in place of -0.038 ; otherwise the calculation was identical to Run 2. A selection of the resulting cross-section adjustments are given in Table 10 along with the corresponding values from Run 2. It will be

noted that, on the whole, the cross-section adjustments calculated in Run 3 are greater than those in Run 2.

The b_l values are given in Table 11 along with those from Run 2 for comparison. It will be seen that the b_l value for System 1.6 is unacceptably large in the case of Run 3, but apart from that the Run 3 values are in general not quite so good as the Run 2 values. These are desirable features since they help to identify errors in the calculations.

A question of some interest is what effect do the assumed standard deviations of the cross sections have on the results of the PENICUIK calculation, since it is sometimes difficult to decide upon accurate standard deviations. To give some idea of this, Run 4 was carried out identical to Run 2 except for different standard deviations for the fission cross section of U^{235} . The resulting cross-section adjustments are given in Table 10 and the b_l values in Table 11. The b_l values are almost identical with those in Run 2, and so are many of the calculated data changes. As would be expected, there are some slight changes in the adjustments calculated for the fission cross section of U^{235} , but in relation to the standard deviations the changes are quite small and

TABLE 6. EFFECT OF INCREASES IN U^{235} CROSS SECTIONS AND OF $\bar{\mu}$ AND $\bar{\nu}$ BY 1% IN ALL ENERGY GROUPS
(Elastic cross sections changed to compensate where necessary)

System→		$\Delta B \times 10^2$												
		1.1	1.2	1.3	1.4	1.5	1.6	2.1	2.2	2.3	2.4	2.5	2.6	2.7
Cross Section Changed	RTN ^(e)													
1001	Total CS	-0.1727	-0.1204	-0.0860	-0.0661	-0.0640	-0.0560	-0.0677	-0.0872	-0.117	-0.0379	-0.0466	-0.0513	-0.0194
1005	Excitation of single lev- els	-0.00013	-0.00010	-0.00009	-0.00008	-0.00008	-0.00007	-0.00007	-0.00009	-0.00011	-0.00006	-0.00007	-0.00007	-0.00001
1006		-0.0029	-0.0024	-0.0021	-0.0018	-0.0018	-0.0017	-0.0015	-0.0019	-0.0024	-0.0013	-0.0015	-0.0016	-0.0027
1007		-0.0014	-0.0012	-0.0010	-0.00093	-0.00092	-0.00084	-0.00075	-0.00093	-0.0011	-0.00066	-0.00077	-0.00081	-0.0015
1008		-0.00054	-0.00047	-0.00041	-0.00037	-0.00036	-0.00033	-0.00029	-0.00036	-0.00044	-0.00025	-0.00030	-0.00032	-0.00006
1009		-0.00061	-0.00054	-0.00049	-0.00044	-0.00043	-0.00039	-0.00033	-0.00042	-0.00051	-0.00030	-0.00036	-0.00038	-0.00007
1010		-0.0017	-0.0015	-0.0014	-0.0012	-0.0012	-0.0011	-0.00099	-0.0012	-0.0014	-0.00087	-0.0010	-0.0011	-0.00031
1015	(n, n') Cont.	-0.0288	-0.0225	-0.0166	-0.0118	-0.0111	-0.0085	+0.0024	-0.0013	-0.0088	+0.0021	-0.0024	-0.0053	+0.0077
1016	($n, 2n$)	-0.0023	-0.0021	-0.0018	-0.0016	-0.0016	-0.0015	-0.00062	-0.00086	-0.0013	-0.00079	-0.0011	-0.0013	-0.00025
1018	(n, f)	-0.630	-0.611	-0.592	-0.577	-0.575	-0.568	-0.523	-0.539	-0.568	-0.527	-0.546	-0.557	-0.498
1102	(n, γ)	+0.0408	+0.0428	+0.0450	+0.0467	+0.0469	+0.0476	+0.0490	+0.0480	+0.0459	+0.0502	+0.0492	+0.0484	+0.0496
4002	($\bar{\mu}$ for n, n)	+0.0937	+0.0685	+0.0518	+0.0421	+0.0410	+0.0371	+0.0288	+0.0391	+0.0564	+0.0214	+0.0286	+0.0328	+0.0083
4018	($\bar{\nu}$)	-0.989	-0.973	-0.952	-0.932	-0.929	-0.917	-0.827	-0.857	-0.902	-0.846	-0.881	-0.899	-0.765

TABLE 7. EFFECT OF INCREASES IN U^{238} CROSS SECTIONS AND OF $\bar{\mu}$ AND $\bar{\nu}$ BY 1% IN ALL ENERGY GROUPS
(Elastic cross sections changed to compensate where necessary)

System $\rightarrow$		$\Delta B \times 10^2$												
		1.1	1.2	1.3	1.4	1.5	1.6	2.1	2.2	2.3	2.4	2.5	2.6	2.7
Cross Section Changed RTN ⁽⁶⁾														
1001	Total CS	-0.0126	-0.100	-0.167	-0.198	-0.200	-0.191	-0.185	-0.161	-0.114	-0.219	-0.208	-0.200	-0.202
1005	Excitation of single levels	-0.00067	-0.0041	-0.0071	-0.0079	-0.0076	-0.0033	-0.0123	-0.0105	-0.0070	-0.0054	-0.0043	-0.0038	-0.0070
1006		-0.00024	-0.0016	-0.0026	-0.0029	-0.0028	-0.0022	-0.0025	-0.0024	-0.0019	-0.0024	-0.0024	-0.0023	-0.0012
1007		-0.00005	-0.00027	-0.00045	-0.00052	-0.00050	-0.00029	-0.00065	-0.00059	-0.00042	-0.00038	-0.00035	-0.00032	-0.00028
1008		-0.00022	-0.0010	-0.0016	-0.0017	-0.0017	-0.00075	-0.0020	-0.0021	-0.0017	-0.00091	-0.00093	-0.00086	-0.00068
1009		-0.00023	-0.00095	-0.0012	-0.00075	-0.00060	+0.00045	+0.00068	-0.00036	-0.0010	+0.0013	+0.00063	+0.00049	+0.0055
1010		-0.00015	-0.00070	-0.00081	-0.00038	-0.00027	+0.00036	+0.0016	+0.00056	-0.00032	+0.0014	+0.00067	+0.00046	+0.0052
1015	(n,n') Cont.	-0.0017	-0.0097	-0.0121	-0.0077	-0.0065	+0.00074	+0.0214	+0.0088	-0.0022	+0.0148	+0.0053	+0.0024	+0.0631
1016	$(n,2n)$	-0.00019	-0.00054	-0.00086	-0.0010	-0.0011	-0.0011	-0.0021	-0.0019	-0.0015	-0.0019	-0.0015	-0.0013	-0.0021
1018	(n,f)	-0.0073	-0.0204	-0.0351	-0.0476	-0.00493	-0.0562	-0.111	-0.0930	-0.0645	-0.101	-0.0793	-0.0677	-0.146
1102	(n,γ)	+0.0020	+0.0060	+0.0165	+0.0354	+0.0392	+0.0613	+0.0774	+0.0527	+0.0275	+0.0983	+0.0777	+0.0689	+0.178
4002	$(\bar{\mu}$ for n,n)	+0.0060	+0.0446	+0.0712	+0.0827	+0.0833	+0.0816	+0.0704	+0.0643	+0.0485	+0.0906	+0.0880	+0.0852	+0.0728
4018	$(\bar{\nu})$	-0.0107	-0.0275	-0.0484	-0.0681	-0.0708	-0.0828	-0.173	-0.143	-0.0976	-0.154	-0.119	-0.101	-0.235

Optimization of Neutron Cross Sections

TABLE 8. CALCULATED PERCENTAGE ADJUSTMENTS
IN CROSS-SECTION DATA IN PENICUIK RUNS
1 AND 2
(See Sect. 3.6)

Nuclide DFN	RTN	Cross Section	Group	Per- cent- age s.d.	Percentage Data Changes Calcu- lated by PENICUIK	
					Run 1 (All Sys- tems)	Run 2 (Sys- tems 1.1-2.7)
107 (H)	1001	Total C.S.	1	2	+0.01	—
				2	+0.05	—
				3	+0.07	—
				4	+0.07	—
				5	+0.06	—
				6	+0.08	—
				7	+0.08	—
				8	+0.05	—
8 (Be)	1001	Total C.S.	1	5	-4.5	—
				2	-13	—
				3	-12	—
				4	-12	—
				5	-7.3	—
				6	-5.5	—
				7	-3.7	—
				8	-2.4	—
	1024	$(n,2n)\alpha$ C.S.	*	15	-7.8	—
				50	+3.9	—
	1102	(n,γ) C.S.	*	20	+3.3	—
				20	+2.3	—
	1107	(n,α) C.S.	1	5	+3.4	—
				2	+10	—
				3	+6.6	—
				4	+6.2	—
				5	+3.8	—
				6	+0.5	—
117 (C)	1001	Total C.S.	1	5	-3.7	—
				2	-6.2	—
				3	-0.39	—
				4	-1.0	—
				5	-0.76	—
				6	-2.5	—
				7	-2.4	—
				8	-2.6	—
	1015	(n,n') Cont	*	20	-0.2	—
				10	+1.4	—
	4002	$\bar{\mu}$ for (n,n)	1	20	+6.1	—
				20	+6.9	—
				20	+9.4	—
				20	+8.0	—
				10	+1.7	—
				10	+1.7	—
132 (Cr)	1001	Total C.S.	*	10	+0.96	—
	1015	(n,n') Cont.	*	30	+0.48	—
	1102	(n,γ) C.S.	*	40	-0.55	—
	4002	$\bar{\mu}$ for (n,n)	*	5	-0.07	—
				5	-0.07	—

TABLE 8. Continued

Nuclide DFN	RTN	Cross Section	Group	Per- cent- age s.d.	Percentage Data Changes Calcu- lated by PENICUIK	
					Run 1 (All Sys- tems)	Run 2 (Sys- tems 1.1-2.7)
162 (Fe)	1001	Total C.S.	1	7	+1.5	—
			2	20	+28	—
			3	20	+21	—
			4	20	+17	—
			5	20	+8.6	—
			6	10	+1.6	—
			7	10	+1.5	—
			8	5	+0.24	—
	1015	(n,n') Cont.	*	40	+6.2	—
	1102	(n,γ) C.S.	*	50	-1.0	—
	4002	$\bar{\mu}$ for (n,n)	*	5	-0.23	—
				5	-0.23	—
134 (Ni)	1001	Total C.S.	*	10	+0.59	—
	1015	(n,n') Cont.	*	40	+0.56	—
	1102	(n,γ) C.S.	*	40	-0.26	—
	4002	$\bar{\mu}$ for (n,n)	*	5	-0.03	—
				5	-0.03	—
155 (U^{235})	1001	Total C.S.	1	3	+1.1	+0.74
			2	3	+2.9	+1.8
			3	2	+1.5	+0.84
			4	4	+7.1	+3.9
			5	4	+4.8	+2.5
			6	3.5	+3.25	+1.3
			7	3.5	+2.0	+0.15
			8	3.5	+1.6	-0.01
			*	25	+0.12	+0.07
			*	10	+0.32	+0.16
	1005 1006 1007 1008 1009 1010 1015	Cross Sections for excita- tion of single levels	*	25	+0.75	+0.32
			*	60	+1.7	+0.72
			*	60	+1.6	+0.56
			*	65	+4.4	+0.95
			1	20	+15	+9.1
			2	20	+2.2	+2.7
			3	20	+2.9	-1.9
			4	20	+1.2	+0.19
			*	15	-0.13	-0.12
			1	6	-7.8	-5.3
	1016 1018	$(n,2n)$ C.S. (n,f) C.S.	2	6	-7.6	-7.8
			3	7.5	-5.3	-7.4
			4	10	-3.4	+1.5
			5	5	-0.22	+0.15
			6	6	-4.0	-3.8
			7	10	+1.1	-8.5
			8	10	+5.1	-1.4
	1102	(n,γ) C.S.	2	10	+0.58	+0.57
			3	10	+2.3	+1.9
			4	10	+2.2	+1.8
			5	10	-0.83	+0.19
			6	15	-13.7	-7.1
			7	15	-18.4	-3.6
			8	15	-11.9	-0.26

TABLE 8. *Continued*

Nuclide DFN	RTN	Cross Section	Group	Per- cent- age s.d.	Percentage Data Changes Calcu- lated by PENICUK				
					Run 1 (All Sys- tems)	Run 2 (Sys- tems 1.1-2.7)			
155 (U ²³⁵)	4002	$(\bar{\mu}$ for n,n)	1	3	-1.4	-0.93			
			2	6	-4.4	-2.8			
			3	10	-9.4	-5.2			
			4	10	-7.3	-4.0			
			5	10	-2.7	-1.4			
	4018	$\bar{\nu}$	1	2	-1.7	-1.1			
			2	1.5	-0.92	-0.92			
			3	1.5	-0.73	-0.72			
			4	1.5	-0.43	-0.26			
			5	1.5	+0.07	-0.01			
			6	2	+0.44	+0.04			
			7	2	+1.0	-0.15			
			8	2	+0.75	-0.04			
			156 (U ²³⁸)	1001	Total C.S.	1	4	-3.0	-3.1
						2	2	-1.9	-2.2
						3	2	-1.5	-1.7
4	5	-7.4				-8.4			
5	5	-3.9				-5.8			
6	3	-1.8				-2.9			
7	5	+0.19				-2.8			
8	5	+1.0				-0.28			
*	15	-6.0				-5.3			
*	20	-2.3				-2.3			
*	50	-3.0				-2.8			
*	40	-5.2				-4.8			
*	50	-9.0				-9.3			
*	40	-4.6				-5.0			
1005 1006 1007 1008 1009 1010 1015 1016 1018 1102 4002 4018	Cross Sections for excita- tion of single levels	(n,n') Cont.				1	20	-23	-27
						2	20	-7.5	-8.1
			*	4	-0.01	-0.00			
			1	6	-0.65	+0.21			
			2	4	-0.07	+0.20			
			3	5	-0.01	+0.02			
			1	10	-0.17	-0.30			
			2	10	-0.84	-1.2			
			3	8	-2.1	-2.6			
			4	12	-5.7	-7.1			
			5	25	-7.9	-12			
			6	25	-6.3	-5.2			
			7	25	-2.1	+12			
			8	25	-3.7	+2.0			
			4002 4018	$\bar{\mu}$ for (n,n)	1	3	+2.5	+2.6	
					2	6	+6.6	+7.5	
3	10	+8.1			+9.3				
4	10	+5.2			+6.0				
5	10	+1.7			+2.7				
6	10	+0.71			+1.3				
1	2	+0.01			+0.17				
2	2	+0.08			+0.14				
3	2	+0.00			+0.01				

* Implies uniform change made in all groups.

TABLE 9. CALCULATED b_i VALUES FROM PENICUIK
RUNS 1 AND 2

System	s.d.	Calculated b_i Values	
		Run 1	Run 2
1.1	0.003	+0.0023	+0.0025
1.2	0.003	+0.0022	+0.0021
1.3	0.003	-0.0020	-0.0020
1.4	0.003	-0.0023	-0.0023
1.5	0.003	-0.0011	-0.0011
1.6	0.003	-0.0003	-0.0003
2.1	0.003	+0.0041	+0.0038
2.2	0.003	+0.0011	+0.0009
2.3	0.003	-0.0062	-0.0061
2.4	0.003	+0.0034	+0.0036
2.5	0.003	-0.0002	+0.0000
2.6	0.003	+0.0012	+0.0013
2.7	0.003	-0.0027	-0.0026
3.1	0.003	+0.0033	—
3.2	0.003	-0.0031	—
3.3	0.003	+0.0017	—
3.4	0.003	-0.0012	—
3.5	0.003	-0.0003	—
3.6	0.003	-0.0001	—
3.7	0.003	-0.0004	—
Vera 1B	0.003	+0.0013	—
2A	0.003	-0.0005	—
3A	0.003	-0.0003	—
4A	0.003	-0.0003	—
5A	0.003	+0.0004	—

consistent with the differences between the standard deviations in Runs 2 and 4.

Finally, it is useful to demonstrate what happens if one attempts to make an exact fit to the critical systems. This is achieved by specifying a very small standard deviation for the B_i values. In Run 5 the standard deviation of all the B_i values was set equal to 0.00001. The resulting b_i values are given in Table 11, and it will be seen that they are all very close to zero, thereby giving an almost exact fit; but the corresponding data changes given in Table 10 are meaningless and demonstrate that one cannot ignore the experimental uncertainties associated with the critical systems used in the fitting procedure.

4. The Determination of Adjusted Basic Cross-section Data from Adjustments Calculated for Group Cross Sections

If by the DUNDEE-PENICUIK system one has calculated the percentage adjustments x_{mg} to be made to the system-independent group cross sections in

FIG. 2. ΔB as a Function of Core Radius after Data Adjustment.

group g for reaction j and nuclide m , and one later wants to use group cross sections based on a different energy-group structure, then one must either repeat the adjustment calculations using the new group cross sections or one must devise a method of getting from one adjusted set of group cross sections to another. In the UKAEA, group cross sections are often calculated by the group-cross-section program GALAXY⁽³⁻⁵⁾ from the basic cross-section data in tabular form on magnetic tape. If, therefore, adjusted basic cross-section data could be obtained using the adjustments calculated for a set of group cross sections, then the adjusted basic data could be used to calculate group cross sections in any group structure. There is no unique way of obtaining the adjusted basic data from the group-cross-section adjustments, but it would seem desirable to do this in the simplest way consistent with

- (a) obtaining smooth curves for the cross section as a function of energy with no discontinuities corresponding to energy group boundaries,
- (b) preservation of detailed structure in the cross-section curves.

A convenient way of obtaining adjusted cross-section

curves satisfying these conditions is to introduce an adjustment factor $X_{mj}(E)$, which is a continuous function of energy. The simplest form of $X_{mj}(E)$ which can be made continuous and have continuous derivatives at energy-group boundaries (leading to a smooth curve for the adjusted cross section

$$\sigma_{mj}(E) = \sigma_{mj}(E)[1 + X_{mj}(E)],$$

where $\sigma_{mj}(E)$ represents the unadjusted cross section) is a quadratic in E within each of the energy ranges covered by the groups. Thus, dropping subscripts mj one has

$$X(E) = a_g + b_g E + c_g E^2; \quad E_g^L < E < E_g^U, \quad (21)$$

where a_g , b_g , and c_g are constants, and E_g^L and E_g^U are the lower and upper energy-group boundaries of group g . If a total of G energy groups are involved, then there will be G values of each a_g , b_g , and c_g , and one therefore requires $3G$ equations to determine them. To begin with, one has the G equations

$$x_g \sigma_g = \int_{E_g^L}^{E_g^U} X(E) \sigma(E) f(E) dE, \quad (22)$$

where $f(E)$ is the (system-independent) spectrum normalised to unity over the range (E_g^L, E_g^U) used in the calculation of the group cross sections. Substituting (21) in (22) leads to

$$x_g \sigma_g = a_g \int_{E_g^L}^{E_g^U} \sigma(E) f(E) dE + b_g \int_{E_g^L}^{E_g^U} E \sigma(E) f(E) dE + c_g \int_{E_g^L}^{E_g^U} E^2 \sigma(E) f(E) dE. \quad (23)$$

Making $X(E)$ and $dX(E)/dE$ continuous at energy-group boundaries ensures that the adjusted cross-section curve is smooth and leads to the $2G - 2$ equations

$$\begin{aligned} a_g + b_g E_g^L + c_g (E_g^L)^2 &= a_{g+1} + b_{g+1} E_g^L \\ &\quad + c_{g+1} (E_g^L)^2; \\ b_g + 2c_g E_g^L &= b_{g+1} \\ &\quad + 2c_{g+1} E_g^L; \end{aligned} \quad (24)$$

$$g = 1, 2 \dots G - 1$$

(note that $E_g^L = E_{g+1}^U$). Two more equations are still required, and these are obtained by setting

$$X(E_1^U) = 0;$$

$$X(E_G^L) = 0,$$

or setting them equal to some other preferred value. If, however, they are not zero, then one might argue that the energy range considered in the fitting procedure is not great enough.

The method outlined above can be applied to all cross sections and $\bar{\nu}$ values. The cross section in tabular form for the adjusted compensating reaction can be computed from the corresponding tabular values in the other reactions. A program is currently being written at Aldermaston which will take a tape containing unadjusted data in the format of the UKAEA Data Library⁽⁶⁾ and from specified x_g values will produce a corresponding tape containing adjusted data also in the format of the UKAEA Data Library.

The problem of obtaining basic angular-distribution data corresponding to adjustments calculated for the $\cos \theta$ values in the various energy groups is more complicated than that of obtaining adjusted basic cross-section and $\bar{\nu}$ data from the adjustments calculated for the group cross sections. Account has to be taken of whether the angular distributions are represented pointwise or rangewise,⁽⁶⁾ of how many different pointwise or rangewise representations are included in a given energy group, and of whether the angular distributions are represented in the centre-of-mass or labora-

TABLE 10. A SELECTION OF THE CALCULATED PERCENTAGE DATA ADJUSTMENTS FROM PENICUIK RUNS 2, 3, 4, AND 5

Energy Group	Run 2 (Normal Calculation)		Run 3 (Using in-correct ΔB for System 1.6)		Run 4 (s.d. for U-235 fission; C.S. changed)		Run 5 (Exact fit to critical sizes attempted)	
	s.d. (%)	x(%)	s.d. (%)	x(%)	s.d. (%)	x(%)	s.d. (%)	x(%)
DFN 155(U-235); RTN 1001 (Total C.S.)								
1	3	+0.74	3	-0.56	3	+0.68	3	+38
2	3	+1.8	3	-0.74	3	+1.7	3	+36
3	2	+0.84	2	+0.26	2	+0.80	2	-53
4	4	+3.9	4	+3.5	4	+3.8	4	+173
5	4	+2.5	4	+2.2	4	+2.4	4	-8.3
6	3.5	+1.3	3.5	-1.2	3.5	+1.3	3.5	-3.2
7	3.5	+0.15	3.5	-1.1	3.5	+0.15	3.5	-25
8	3.5	-0.01	3.5	-0.14	3.5	-0.01	3.5	-7.4
DFN 155(U-235); RTN 1015 (n, n') Continuum								
1	20	+9.1	20	-50	20	+7.9	20	+720
2	20	+2.7	20	-37	20	+1.4	20	-64
3	20	-1.9	20	-8.2	20	+2.3	20	+455
4	20	+0.19	20	+1.8	20	+0.12	20	+62
DFN 155(U-235); RTN 1018 ((n, f) C.S.)								
1	6	-5.3	6	+2.6	5	-5.1	6	+305
2	6	-7.8	6	-1.2	5	-7.0	6	+71
3	7.5	-7.4	7.5	-9.0	6	-6.0	7.5	+310
4	10	+1.5	10	-4.3	8	-0.18	10	+280
5	5	+0.15	5	-3.0	4	-0.03	5	+110
6	6	-3.8	6	-19	5	-2.8	6	+570
7	10	-8.5	10	-27	8	-5.6	10	+450
8	10	-1.4	10	-3.7	8	-0.92	10	-11
DFN 156(U-238); RTN 1001 (Total C.S.)								
1	4	-3.1	4	-2.9	4	-3.1	4	-46
2	2	-2.2	2	-1.9	2	-2.2	2	+110
3	2	-1.7	2	-1.3	2	-1.7	2	-4.6
4	5	-8.4	5	-8.2	5	-8.4	5	+315
5	5	-5.8	5	-9.2	5	-5.8	5	+260
6	3	-2.9	3	-5.5	3	-2.9	3	+140
7	5	-2.9	5	-5.5	5	-2.9	5	+170
8	5	-0.29	5	-0.55	5	-0.29	5	-10
DFN 156(U-238); RTN 1102 ((n, γ) C.S.)								
1	10	-0.30	10	-0.08	10	-0.32	10	+0.93
2	10	-1.2	10	-0.86	10	-1.3	10	+30
3	8	-2.6	8	-5.3	8	-2.7	8	+310
4	12	-7.1	12	-15	12	-7.1	12	+990
5	25	-12	25	-16	25	-11	25	+340
6	25	-5.2	25	-38	25	-4.2	25	+310
7	25	-12	25	+12	25	+12	25	+560
8	25	+2.0	25	+2.2	25	+2.0	25	+435

TABLE 11. CALCULATED b_l VALUES FROM PENICUIK RUNS 2, 3, 4 AND 5

System	Run 2 (Normal calculation)		Run 3 (Using incorrect ΔB for System 1.6)		Run 4 (s.d. for U-235 fission; C.S. changed)		Run 5 (Exact fit to critical sizes attempted)	
	s.d.	b_l	s.d.	b_l	s.d.	b_l	s.d.	b_l
1.1	0.003	+0.0025	0.003	+0.0010	0.003	+0.0025	0.00001	+0.00002
1.2	0.003	+0.0021	0.003	+0.0000	0.003	+0.0021	0.00001	+0.00005
1.3	0.003	-0.0020	0.003	-0.0045	0.003	-0.0020	0.00001	-0.00003
1.4	0.003	-0.0023	0.003	-0.0063	0.003	-0.0023	0.00001	-0.00002
1.5	0.003	-0.0011	0.003	-0.0055	0.003	-0.0011	0.00001	+0.00003
1.6	0.003	-0.0003	0.003	+0.0263	0.003	-0.0003	0.00001	-0.00053
2.1	0.003	+0.0038	0.003	+0.0070	0.003	+0.0038	0.00001	-0.00016
2.2	0.003	+0.0009	0.003	+0.0036	0.003	+0.0009	0.00001	+0.00029
2.3	0.003	-0.0061	0.003	-0.0050	0.003	-0.0061	0.00001	-0.00017
2.4	0.003	+0.0036	0.003	-0.0007	0.003	+0.0037	0.00001	-0.00013
2.5	0.003	+0.0000	0.003	-0.0060	0.003	+0.0000	0.00001	-0.00086
2.6	0.003	+0.0013	0.003	-0.0055	0.003	+0.0013	0.00001	+0.00123
2.7	0.003	-0.0026	0.003	-0.0043	0.003	+0.0026	0.00001	+0.00001

tory reference systems. The program, referred to in the previous paragraph, which is being written to calculate adjusted basic cross-section and $\bar{\nu}$ data may later by extended to calculate adjusted angular-distribution data for the case of heavy nuclides where the distinction between centre-of-mass and laboratory reference systems is not important.

5. Possible Extension of the Method Described in Section 2 to Take into Account Other Integral Data and Resonance Self-shielding

In principle the method described in Sect. 2 can readily be extended to deal with additional integral data, such as spectra and reaction-rate measurements in critical systems. It can also be extended to include the effects of resonance self-shielding. The possible extension of the method to deal with these additional features will now be discussed.

5.1. Extension to Take into Account Additional Integral Data

In extending the method of Sect. 2 to take into account spectra and reaction-rate measurements in critical systems, it is first of all necessary to be able to calculate the sensitivity of these additional quantities to changes in the group cross sections. The basic perturbation theory giving the change in reaction rates due to changes in the cross-section data has been developed by Usachev,⁽¹⁵⁾ and this can readily be extended to give the change in spectra due to changes in the cross-section data. The calculation of the change in spectra is actually more fundamental than the change in reaction rate because the latter can be calculated from the former. This is easily seen from

$$\delta \sum_g \sigma_g \phi_g = \sum_g \sigma_g \delta \phi_g + \sum_g \phi_g \delta \sigma_g, \quad (25)$$

in which σ_g is a group cross section and ϕ_g is a scalar flux. The second term on the right-hand side is trivial; the first term is calculated from the change in spectrum as given by the $\delta \phi_g$. In a critical system one can, of course, calculate only meaningful ratios of reaction rates, since ϕ_g and $\phi_g + \delta \phi_g$ are subject to an arbitrary constant of multiplication. Thus

$$\delta \left(\frac{\sum_g \sigma_g(1) \phi_g}{\sum_g \sigma_g(2) \phi_g} \right) = \frac{\left(\sum_g \sigma_g(1) \delta \phi_g + \sum_g \phi_g \delta \sigma_g(1) \right)}{\sum_g \sigma_g(2) \phi_g} - \frac{\left(\sum_g \sigma_g(1) \phi_g \right) \left[\sum_g \sigma_g(2) \delta \phi_g + \sum_g \phi_g \delta \sigma_g(2) \right]}{\left(\sum_g \sigma_g(2) \phi_g \right)^2}, \quad (26)$$

where $\sigma_g(1)$ and $\sigma_g(2)$ represent the group cross sections for different reactions. In order to calculate reaction-rate ratios specific values for ϕ_g and $\phi_g + \delta \phi_g$ have to be used, and this means that they must be normalised in some way, but there is no need for any *particular* normalisation. When comparing calculated and measured spectra, however, it is essential that the calculated and measured spectra should be normalised in the same way. The most convenient ways of doing this are either to normalise to the same value at a specific energy or over a specified energy range, or to normalise to the same integrated value. The disadvantage of the latter approach is that the experimental spectra may not have been measured at all relevant energies, and so the spectrum would have to be estimated in some way in the unmeasured regions in order to obtain the normalisation factor. For the present discussion it will therefore be

assumed that the calculated and experimental spectra will be given in terms of the same energy groups and will be normalised to the same value in one of them. There will be no loss of generality as far as the mathematics is concerned if this is assumed to be group 1, though in practice it would usually be taken to be some other group, as group 1 would often not contain many neutrons. On this assumption $\delta\phi_1 = 0$, and one requires to know the sensitivity of ϕ_g/ϕ_1 for the systems of interest to changes in the cross-section data. Given this information, one then needs to specify some suitable function analogous to, say, (17) or (20), which can be minimised to determine cross-section adjustments to give improved agreement between the calculated and measured spectra, but one would really like to be able to take into account critical size data and spectra simultaneously.

Let ζ_{lg} be the standard deviation of the measured value of the spectrum in group g relative to the value in group 1, and let f_{lg} be the difference between the calculated and measured component of the spectrum after the data have been adjusted. There will generally be correlations between the f_{lg} for different g ; so it seems reasonable to introduce a $1/G$ factor in the terms involving f_{lg} . Thus a suitable function for minimisation analogous to (17) is

$$\sum_{mj} \frac{1}{G} \sum_{g=1}^G \frac{x_{mjg}^2}{\xi_{mjg}^2} + \sum_{l=1}^L \frac{b_l^2}{\eta_l^2} + \sum_{l=1}^L \frac{1}{G} \sum_{g=1}^G \frac{f_{lg}^2}{\zeta_{lg}^2}. \quad (27)$$

Terms taking into account reaction-rate ratios can also be added to the above expression. For instance, if d_{lk} represents the difference after adjustment between the calculated and measured reaction-rate ratio, denoted by k for system l , and if χ_{lk} is the standard deviation of the measured value, then the term

$$\sum_{l=1}^L \sum_{k=1}^K \frac{d_{lk}^2}{\chi_{lk}^2}$$

can be added to (27) to give

$$\sum_{mj} \frac{1}{G} \sum_{g=1}^G \frac{x_{mjg}^2}{\xi_{mjg}^2} + \sum_{l=1}^L \frac{b_l^2}{\eta_l^2} + \sum_{l=1}^L \frac{1}{G} \sum_{g=1}^G \frac{f_{lg}^2}{\zeta_{lg}^2} + \sum_{l=1}^L \sum_{k=1}^K \frac{d_{lk}^2}{\chi_{lk}^2}. \quad (28)$$

As will be appreciated from the discussion in Sect. 2.2, many variations of this function are possible, and in particular factors q_1 , q_2 , q_3 , and q_4 say, could be introduced as coefficients of the 4 terms to allow different overall weights to be given to the different terms. Let ΔF be the difference before data adjustment between the calculated and experimental spectrum in group g relative to group 1 for system l and let $\beta_{lmjg'g}$ be the change in the calculated spectrum component in group g for system l due to a 1% change in the cross section for reaction j in nuclide m in group g' . Then

$$\sum_{mjg'} \beta_{lmjg'g} x_{mjg'} - f_{lg} = -\Delta F_{lg}. \quad (29)$$

Similarly, if γ_{lmjgk} is the change in the calculated reaction rate ratio k at a point in system l due to a 1% increase in the cross section for reaction j in nuclide m in group g , then

$$\sum_{mjg} \gamma_{lmjgk} x_{mjg} - d_{lk} = -\Delta D_{lk}, \quad (30)$$

where ΔD_{lk} is the difference before data adjustment between the calculated and measured value of the reaction rate ratio k at the particular point in system l . The γ_{lmjgk} can in fact be derived from the $\beta_{lmjg'g}$ through eq. (24). In addition, one has the formula from Sect. 2.1 connecting the x_{mjg} and b_l :

$$\sum_{mjg} \alpha_{lmjg} x_{mjg} - b_l = -\Delta B_l. \quad (31)$$

Minimisation of (28) subject to (29), (30), and (31) gives the cross-section adjustment factors x_{mjg} and determines b_l , f_{lg} , and d_{lk} . Machine programs are being written to allow this to be carried out in a fully mechanized way.

5.2. Extension to Take into Account Resonance Self-shielding

The inclusion of resonance self-shielding in reactivity and spectrum calculations usually involves the use of some empirical or other approximate formula. A particular example is the empirical formula influenced by the narrow-resonance approximation⁽¹⁶⁾ developed by Rowlands *et al.*⁽¹⁷⁾ A shielded group cross section σ_g is given by

$$\sigma_g = \frac{\sigma_{\infty,g}}{\left(1 + \frac{\alpha_g^s}{\sigma_{sg}} + \frac{\beta_g^s}{\sigma_{sg}^2}\right)^{1/2}}, \quad (32)$$

where $\sigma_{\infty,g}$ is the unshielded group cross section and σ_{sg} is the total scattering cross section in group g per nucleus for which the shielded constant is determined. The α_g^s and β_g^s factors are obtained by an empirical fit to results obtained in simple geometry with detailed energy dependence. The superscript s is included here to distinguish the quantities from the α and β factors referred to in Sect. 5.1. If one uses an approximation of this kind, the adjustment procedure must be able to allow changes in all the factors which appear in the formulae; in the above case this means in $\sigma_{\infty,g}$, σ_{sg} , α_g^s , and β_g^s . Approximate terms can easily be included in the function to be minimised to allow this, and the problem only remains to calculate the sensitivity of B and spectra to changes in these quantities. Perturbation theory will give the sensitivity of B and spectrum for a given system to changes in the value of σ_g for a particu-

lar reaction. These are α_{lmjg} and $\beta_{lmjg'g}$ factors introduced in Sections 2.1 and 5.1. From α_{lmjg} and $\beta_{lmjg'g}$ one can obtain through equation (32) the sensitivity to changes in the $\sigma_{\infty,g}$. This follows from relations of the kind

$$\begin{aligned}\frac{\partial B}{\partial \sigma_{\infty,g}} &= \frac{\partial B}{\partial \sigma_g} \times \frac{\partial \sigma_g}{\partial \sigma_{\infty,g}}; \\ \frac{\partial B}{\partial \alpha_g^s} &= \frac{\partial B}{\partial \sigma_g} \times \frac{\partial \sigma_g}{\partial \alpha_g^s},\end{aligned}\quad (33)$$

the $\partial B/\partial \sigma_g$ being the α_{lmjg} determined by perturbation theory. The $\partial \sigma_g/\partial \alpha_g^s$ are determined from eq. (32). Thus equations analogous to equations (29), (30), and (31) can be obtained, and minimisation of the chosen function subject to the constraints which these equations impose will determine the adjustments to be made to the $\sigma_{\infty,g}$, α_g^s , etc., and will also determine the corresponding fit to the integral quantities.

Acknowledgments

Thanks are due to Mr. J. B. Parker and Dr. K. Parker for many helpful discussions, and to Mr. P. J. Jordan and Mr. W. H. G. Wilson for organising most of the calculations. Mr. M. H. McTaggart supplied the details of the VERA critical systems. The graphs were drawn by Mrs. S. M. Offord.

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Discussion

Mr. Loewenstein (ANL): When you make these adjustments do you check whether certain perturbation measurements are confirmed?

Mr. Pendlebury: We haven't done much checking yet. In carrying out the calculations we have now reached the stage where we can go back and do check calculations. We have done some. For instance, having gone through all this, we have the B_i 's from the perturbation calculations and the fitting. We find that these are consistent with the values that we get when we make those adjustments and do further S_N calculations. But the investigation of whether perturbation cross sections are improved and whether spectrum measurements are improved is something which we need to do and will soon be doing. In the one VERA system that we investigated, it does alter the spectrum slightly in the right direction. For this reason we really need to extend the method to deal with spectra because the spectra are dependent on quantities to which critical sizes are not very sensitive. For example, you may get critical sizes using only two energy groups, but you do not get very good spectra by using only two energy groups.

Mr. Vernon (AI): Would you explain further how you account for the correlations between neighboring groups?

Mr. Pendlebury: One introduces terms in subscripts m, j, g , and $m, j, (g + 1)$. I introduce another positive quantity, p , which may be a function of all the parameters. Thus,

$$\sum \left(\frac{\chi_{mjg}^2}{\xi_{mjg}^2} + \frac{\chi_{mjg}\chi_{mjg+1}}{\xi_{mjg}\xi_{mjg+1}} p \right).$$

We minimize the expression resulting from the introduction of the cross terms. There results a quadratic expression which is positive definite, but which must be less than a certain value, thereby producing a limitation on p . These routines were built into earlier PENICUIK versions. We tried various correlations and found that they did not have a significant effect. Our main reason for dropping it was that we really didn't know what correlation (p) to use. It was somewhat gratifying, however, to find that its effect was not too significant. Taking the average deviation here eliminates some of the need for having those correlations.

Mr. Tirén (AB Atomenergi): Have you been able to make any particular assessments of the cross sections such as White's U^{235} fission cross sections?

Mr. Pendlebury: The data that we have used here did not have White's fission cross-section measurements, but the PENICUIK adjustments that we get do in fact give us a reduction, more or less consistent with White's reduced values.

Comparison between Experimental and Theoretical Integral Data on Fast Critical Facilities. CALI, a Program for Generating "Effective" Nuclear Group Constants by a Correlation Method*

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ABSTRACT

In order to treat simultaneously a large number (about 20) of experimental data relative to a given material, a program (CALI) has been written which determines a new set of "effective" multigroup constants consistent with the experiments considered and as close as possible to a reference set obtained by best-value criteria. Proper allowance is given to the uncertainties of the reference cross sections and to the errors associated with the integral data themselves. This allowance removes most of the difficulties encountered when correlating data obtained in assemblies with spectral characteristics not sufficiently different from each other with respect to the precision of the experimental techniques.

A systematic comparison between experimental and theoretical data relative to the critical facilities ZPR-III, ZPR-VI, and ZEBRA is then performed. The quantities so far considered are the reactivity and fission ratios of Pu-239 and U-233 with respect to U-235. The theoretical predictions are obtained using the APDA 24-energy-group cross-section set and the Russian 26-group one. The new White data of plutonium are also considered. The experimental results are corrected for sample size and shape effect. The corrected APDA plutonium values and the Russian ones are more consistent with the experimental data than are the uncorrected values of the APDA set, when proper corrections for sample size are introduced.

1. Introduction

This work has been originated by the need for correlating in a more consistent way the most significant measurements so far obtained on fast critical facilities

* Work performed under the auspices of the CNEN-Euratom Association.

with the different nuclear data presently available. This correlation is intended to produce a new set of "effective" group constants as close as possible to a given cross-section library defined with best-value criteria. The term "effective" here stands for the ability of specified constants to predict, with reasonable accuracy, integral quantities of interest for a broad class of fast reactors. The present situation seems to be suitable for developments in this direction. On the one hand there exists the great mass of data obtained from ZPR-III, ZPR-VI, ZEBRA, and BFS facilities, as well as experimental data obtainable in the near future; on the other hand, large uncertainties still exist in fast reactor characteristics and behavior predictions.

The approach presented in this paper consists in the adoption of a model based on one-dimensional, conventional perturbation theory, not accounting so far for more sophisticated geometries and other perturbative terms. A serious difficulty was encountered in the beginning by the impossibility of using two or more experimental values of the same kind obtained in real and adjoint-flux environments not very different from each other with respect to the experimental precision. This situation may call for corrections that may not seem to be physically sensible if proper allowance is not given for the experimental errors of the measurements themselves. This difficulty has been removed by proper choice of the experimental data within their uncertainty limits.

An extensive analysis has then been made of the fission ratios of U-233 and Pu-239 to U-235, and of reactivity worths of the same materials measured in several critical experiments. As a result, only few corrections were needed to the cross sections of Pu-239 selected for the calculations in order to obtain an agreement with the experimental values.

Improvement of the present method is obtained by use of a two-dimensional and an improved perturbation analysis. These improvements are now at an advanced stage of development and will be used to interpret, in one batch and in a relatively short time, a big quantity of data. A comprehensive correlation analysis of experiments strongly dependent on geometry and spectrum characteristics (as the Doppler and sodium-void reactivity coefficients) will then be feasible.

2. The Correlation Method Adopted

The basic philosophy of the method adopted is described in Ref. (1) and will be here recalled briefly. Some significant improvements are also indicated.

Assuming a set of M experimental integral data T_m , i.e., reactivity worths and/or reaction rates, given for one specified material inserted in different points of the systems considered and given a set of N group nuclear constants σ_n selected for the same material with best-value criteria, by which values T'_m can be calculated, the discrepancies δT_m are interpreted in terms of conventional perturbation formulae from which corrections $\delta\sigma$ can be obtained. This assumption implies that the theoretical model of interpretation is correct within specified limits of precision and that the $\delta\sigma_n$ values are not of such value as to invalidate the approximation adopted. In this case an iteration procedure may be assumed in some cases. In matrix notation,

$$A\delta\sigma = \delta T + \gamma, \quad (1)$$

where γ is an undetermined function with limits $|\gamma| \leq \epsilon$. The elements of matrix A , of order $M \times N$, are perturbation coefficients calculated from the real and adjoint fluxes of the systems. The vector ϵ represents the uncertainties associated with the experimental data and with the procedure followed for their reduction to the theoretical model adopted (i.e., accounting for shape, size, heterogeneity effects, etc.). Because system (1) is in general undetermined, further conditions have to be chosen in order to obtain a unique solution. The criterion adopted here is that the functional

$$F = \sum \left(\frac{\delta\sigma_n}{\alpha_n} \right)^2 \quad (2)$$

shall be a minimum. The values α_n are input parameters which influence the corrections $\delta\sigma_n$. In general, they are assumed proportional to the absolute errors associated with the cross sections or are zero if a specified value σ_n has to be kept fixed. This procedure may be used iteratively if corrections within the specified errors of the cross sections are wanted. The philosophy on which the choice of the least-norm condition on F is based stems from the following points:

- 1). the relative simplicity of the calculations involved;
- 2). the requirement of obtaining "effective" group constants σ'_n as close as possible to the reference ones, σ_n , proper allowance being given to the error limits assumed;
- 3). the convergence of the "effective" group constants σ'_n to the "physical" solution when the quantity and quality of the integral and differential information are improved (within, of course, practical limits of precision).

It can be said, then, that the method adopted, although not strictly based on physical grounds, operates within the area of uncertainty of what can be assumed to be the present state of the "physical" knowledge both of differential and integral parameters and gives "effective" group parameters consistent with it.

As regards point 3), the convergence of the values σ'_n to the "physical" solution may be accelerated if an improvement of the interpretation model is performed, obtainable, for instance, by introducing improved perturbation techniques.⁽²⁾ The program for this new method is under development, and results will be available in the near future. The use of this new perturbation formulae is mandatory if full exploitation of present integral data is to be done consistently, including those relative to materials characterized by errors σ_n out of those acceptable for the conventional perturbation formulae, and/or those strongly related to spectral characteristics for which the interdependence with other parameter corrections influencing these characteristics has to be considered. In particular, this technique will be of great help in analysing zoned sodium-void coefficients, especially the spectrum-dependent part of it. Another advantage of this new method lies in the possibility of correlating other integral quantities, in particular the prompt-neutron lifetime (measured, for instance, by Rossi- α techniques) and spectral indices (for instance, boron worth and threshold detector measurements). The discrepancies between the experimental and theoretical values of these quantities will be then interpreted in terms of correction of the scattering parameters and/or fission neutron distribution, etc.

In order to include second-order perturbation techniques in the correlation program, only a redefinition of matrix A has to be performed; the remainder is unchanged.

Further considerations are worth mentioning with respect to two points. The first is related to the need of accounting for "physical" conditions other than those specified in system (1), and the second to the need for accounting for the fine structure of the neutron spectrum. In the first case, as far as the known physical

relationships between nuclear constants are linear, i.e., of the type

$$\sum_j a_{ij} \sigma_j = c_i + \gamma_i; \quad i = 1, \dots, I; j = 1, \dots, J, \quad (3)$$

where σ_j need not belong to the same material, a_{ij} are given coefficients, c_i are constants known with uncertainty ϵ_i , and γ_i is an undetermined function with limits $|\gamma_i| \leq \epsilon_i$, there is no difficulty to extend the matrix A [related to the second-order perturbation technique if more than one material is involved in (3)] in order also to comprehend conditions derived from (3). For what concerns the second point, if one considers a library as is given by Abagjan *et al.*,⁽¹⁶⁾ where shielding factors f are tabulated, and introduces new values $\alpha_0 = \alpha/f$, corrections of the group constants σ_0 relative to infinite dilution can be obtained.

Finally, it is worth noticing that the correlation method proposed here may also have applications other than that specified above. For instance, it may be used for cross-checking different libraries of nuclear constants or for bracketing possible inconsistencies existing among integral data assumed with their relative errors.

3. The CALI Code

3.1. General description

As has been mentioned before, the problem consists of solving a linear system of the type

$$Ad = b_0,$$

of order $M \times N$, where M is the number of experimental values and N is the number of the group nuclear parameters to be corrected. In general $N > M$; among the infinite number of solutions of the system the one with least norm is chosen. Once the values are obtained, new theoretical integral values are calculated for a final cross-check with the experimental ones.

In order to perform all these calculations the following programs are used:

TAIM—which produces the real and adjoint fluxes, and other system parameters;

CALI—which performs the proper corrections of the group parameters.

The present version of the CALI code has the following specification limits:

- one-dimensional geometry (slab, cylinder, sphere);
- maximum number of mesh points: 200;
- maximum number of energy groups: 26;
- maximum number of groups of downscattering: 10;
- maximum number of experimental data: 20.

The TAIM program⁽³⁾ is derived from the AIM-6 Code,⁽⁴⁾ but has a higher number of energy groups,

obtained by dropping out some options not considered essential to our purpose. The TAIM Code stores on magnetic tape the following output data:

NO	—problem identification number
M	—number of regions
N	—number of mesh point
N1	—geometry indicator
	—N1 1 = sphere
	—N1 2 = cylinder
	—N1 3 = slab
LB	—mesh number of boundaries
$\phi(r, E)$	—neutron flux
$\phi^*(r, E)$	—adjoint flux
$D(r, E)$	—diffusion coefficient
$\nu \Sigma_f(r, E)$	—fission cross section
$\chi(E)$	—fission spectrum
Δr	—mesh spacing in each region.

These data are required by the CALI code. In addition, the following data must be provided in the input:

NOG	—number of energy groups
NDZ	—number of groups of downscattering
BETA	—elastic downscattering coefficients
	[(NOG-1) values]
ALF	— α values (for each group)
SIGMA	—cross-section values given by group with the following order: capture, fission, fission times ν , elastic and inelastic cross sections.

The following input data are needed for each experimental value:

NO	—problem identification number (cf. TAIM)
NR	—number of perturbed regions
BI	—inner radius of region to be perturbed
B2	—outer radius of region to be perturbed
VS	—experimental values
DVS	—absolute error of experimental value
TYPE	—type of experiment
	TYPE 1 = capture
	TYPE 2 = fission
	TYPE 3 = reactivity

Finally a criterion of independency of measurements, EPS, is input. Two values are needed if at least one value $DVS \neq 0$; otherwise one value is required. As the terms b_0 in (1) are affected by specified errors, the values d are to be found with b_0 varying within these limits. Such problems have been solved with the generalized inversion numerical technique⁽⁵⁾ and with an empirical method for minimum point search.⁽⁶⁾ The system-independency criterion EPS can be estimated empirically. If there is good reason to believe that all the rows of the matrix A are linearly independent, $EPS = 0$ is selected. The CALI program is articulated by chains with the following functions:

A —For each experimental value a search is made of

the problem data on the TAIM output tape, and then the elements of matrix A and of vector b_0 are evaluated by a first-order perturbation technique. It is followed by B.

B.1—If the experimental values are given without errors, the generalized inverse matrix A^+ is evaluated from A and the solution of minimum norm $d = A^+b_0$ is found. This is followed by D.

B.2—If the experimental values are given with errors, the matrix

$$Q = (AA')^+ = A'^+A^+,$$

where A' is the transpose of A , is calculated.

The elements of Q are the coefficients of the quadratic form

$$f(b) = \sum d_i^2 = d'd = b'Qb.$$

This is followed by C.

C —New values of b_0 are determined which minimize the quadratic form $f(b)$, with b varying in the rectangular domain defined by $(b_0 \pm DVS)$. It should be noticed that at the origin of the coordinate system, $f(b) = 0$. This is followed by B.1.

D —With the new corrected group cross sections, the theoretical values are recalculated, using the same perturbative method used in A, in order to check the consistency of the results.

3.2. Input description

The input includes:

1—On tape unit 11, data from the TAIM code.

2—On punched cards, the groups of values:

NOG, NDZ	(Format 2I2)
BETA	(" 8E10.0)
ALF	(" 8E10.0)
SIGMA	(" 8E10.0)

For each experimental value a card:

NO, NR, BI, B2, VS, DVS, TYPE (Format I5, I2, 4E12.0, I1)

A blank card follows, in order to allow for the automatic counting of experimental data.

The last two input cards contain the EPS 1 and EPS 2 values.

3.3. Output description

The output includes:

1—The input data

2—The coefficients of matrix A and vector b , ordered by TYPE 1, 2, and 3.

3—Values of comparison with EPS and coefficients of matrix Q .

4—Minimum values of the quadratic form $f(b)$ and new values of the vector b .

5—Like in 2, with the new values of b .

6—Values of comparison with EPS and coefficients of matrix AA^+ (the sum of the elements of the main diagonal is the rank of A).

7—Solutions d of system (1) and relative corrections of the cross sections.

8—Integral values:

VS —Experimental, as given in input

VSC—Experimental, corrected within given errors

VT —Theoretical, calculated with cross sections given in input

VTC—Theoretical, verified with modified group constants.

An example of output is given in Appendix 2. It refers to 6 experimental values and six energy groups.

3.4. Operational procedure

CALI has been programed to run under control of the IBM 7094 FORTRAN 2 Monitor System. A JOB consists of the program deck with the associated monitor control cards followed by the input data.

The logical tape numbers used in the code are the following:

—Tape 6, output

—Tape 5, input

—Tape 8, scratch

—Tape 10, scratch

—Tape 11, input from TAIM Code.

Another tape is used by the monitor for storage of the program chain links.

—Calculation time: a typical problem of 15 experimental values, 26 energy groups, and 10 groups of downscattering requires a time of 3–4 min.

4. Analysis of Some Experimental Data on ZPR-III, ZPR-VI, and ZEBRA Facilities

4.1. Systems considered

A series of integral experiments with U-235, Pu-239, and U-233 performed with ZPR-III,⁽⁷⁻¹¹⁾ ZPR-VI,⁽¹²⁾ and ZEBRA⁽¹³⁾ have been analyzed. The systems considered are illustrated in Table 1. These systems were chosen so as to satisfy a criterion of complementarity insofar as it was possible with respect to neutron-spectrum characteristics. The quantities considered are the central reactivity worths and, when available, the fission rates.

4.2. Theoretical calculations and reduction of systems to homogeneous composition and spherical geometry

In order to perform the calculations in spherical geometry and up to 26 energy groups, the TAIM diffusion code⁽³⁾ has been used. The central fission rates and reactivity worths have been calculated with the PERT

TABLE 1. COMPOSITIONS OF ASSEMBLIES

Assembly	L/D	Core composition (volume fraction)							Blanket composition (volume fraction)			Experimental Critical Mass (kg)
		U-235	U-238	SS	Al	Na	C	O	U-235	U-238	SS	
Zebra 1	0.842	0.097	0.7152	0.0668	—	—	—	—	0.0063	0.8540	0.0566	228
“ 2	0.989	0.0523	0.3223	0.0604	—	—	0.4512	—	0.0062	0.8540	0.0555	416
ZPR-III 29	0.792	0.0468	0.1031	0.2693	0.2410	—	—	0.1429	0.0019	0.8330	0.0731	441
“ 32	1.17	0.0926	0.0066	0.8100	—	—	—	—	0.0019	0.8330	0.0731	234
“ 33	1.14	0.0937	0.4951	0.1268	—	0.1823	—	—	0.0019	0.8330	0.0731	244
“ 34	0.940	0.0467	0.1035	0.2464	0.2546	—	0.1064	—	0.0019	0.8330	0.0731	521
“ 36	1.44	0.0941	0.5050	0.1218	—	0.1840	—	—	0.0019	0.8330	0.0731	250
“ 38	Sphere	0.0757	0.7290	0.0930	—	—	—	—	0.0019	0.8330	0.0731	424
“ 39	Sphere	0.0581	0.0913	0.2447	0.2333	—	—	—	0.0019	0.8330	0.0731	426
ZPR-VI 2	0.990	0.0476	0.2052	0.1751	—	0.3595	0.1540	—	0.0017	0.8318	0.0712 ^(a)	553
											0.1146	

^(a) Two volume fractions of stainless steel are given; the inner portion of the blanket is contained in stainless steel drawers, while the outer portion is inserted directly in the system matrix.

Code,⁽¹⁴⁾ which has been modified in order to accept a maximum of 30 energy groups. The cross-section sets considered have been the YOM-modified APDA 24-energy-group library⁽¹⁵⁾ and the 26-group Russian library.⁽¹⁶⁾ With the former library the Elmoe treatment⁽¹⁷⁾ has been performed to account for the resonant structure of the light-material scattering cross sections, while for the latter the proper self-shielding factors have been adopted for each composition by interpolation of the values given in Ref. 16. To perform these iterations, the RUSSKI Code⁽¹⁸⁾ has been utilized. This simpler procedure seems to be well justified in some cases when significant quantities of more than two resonant light elements are present in the system⁽¹⁹⁾ (besides, of course, the trivial case of one resonant element).

For reducing the real systems to homogeneous ones in spherical geometry, proper corrections have been evaluated, in order to account for shape factors,^(20, 21) heterogeneity effects, boundary irregularities, gaps, etc.⁽²¹⁾ The calculated values of k_{eff} using the two libraries do not differ much from unity, as may be seen in Table 2, if proper consideration is given to the uncertainties of the corrections introduced. As regards the shape factor, discrepancies of the order of 7% between experimental and theoretical values have been found in some cases.⁽¹¹⁾ Because of these uncertainties, the differences between the values relative to the APDA and the Russian sets seem difficult to assess in terms of merit.

It is worth noticing the tendency of the Russian set to be more reactive.

4.3. Analysis and comparison of experimental and theoretical values of U-233, U-235, and Pu-239

In order to correlate the experimental data with the theoretical values, proper data reductions have been

TABLE 2. CALCULATED VALUES OF k_{eff}

Assembly	Corrections of Experimental Critical (mass %)			Shape Factor	Calculated k_{eff}	
	Heterogeneous	Boundary irregularity	Central Gap		APDA set	Russian set
ZEBRA 1	+6.9	-0.08	-1.3	0.94	0.993	0.995
“ 2	+4.1	-0.07	-1.0	0.92	0.981	1.005
ZPR-III 29	+5.7	-0.05	-0.7	0.91	1.039 ^(a)	1.032
“ 32	+4.3	-0.09	-1.3	0.91	1.011	1.028
“ 33	+4.1	-0.08	-1.3	0.92	1.012	1.026
“ 34	+4.9	-0.08	-1.2	0.91	1.035 ^(a)	1.026
“ 36	+4.5	-0.08	-1.2	0.87	0.996	1.002
“ 38	+7.7	-0.09	-1.3	—	0.995	0.993
“ 39	+5.6	-0.04	-0.7	—	1.034 ^(a)	1.023
ZPR-VI 2	+2.3	-0.05	-0.5	0.91	1.009	1.023

^(a) Values obtained by substituting sodium for aluminium in the calculations, as the latter element is not represented in the APDA library.

performed. For the fission rates, given only for the ZPR systems, corrections have been introduced to the experimental values to account for the presence of different materials in the fission chamber.^(22, 23) The corrected fission ratios of U-233 and Pu-239 with respect to U-235 appear to be in good agreement with the theoretical predictions; as shown in Table 3. Besides the APDA and the Russian sets, a corrected version of the APDA set accounting for the new White data of plutonium has been used. For the reactivity worths, corrections have been introduced to account for the aluminum coating of the samples.^(3, 7) Only rough estimates have been made for heterogeneity and shape effects, either because of lacking information on environmental conditions of experiments or because of difficulty of theoretical interpretation. With respect to

TABLE 3. EXPERIMENTAL AND CALCULATED FISSION RATIOS

Assembly	Experimental values		Theoretical values				
	$\sigma_f^{239}/\sigma_f^{235}$	$\sigma_f^{233}/\sigma_f^{235}$	$\sigma_f^{239}/\sigma_f^{235}$			$\sigma_f^{233}/\sigma_f^{235}$	
			APDA	Mod. APDA	RUSS.	APDA	RUSS.
ZEBRA 1	—	—	—	—	—	—	—
“ 2	—	—	—	—	—	—	—
ZPR-III 29	1.07 ($\pm 4\%$)	1.48 ($\pm 4\%$)	1.083	1.047	1.058	1.511	1.470
“ 32	1.21 “	1.52 “	1.236	1.174	1.180	1.576	1.518
“ 33	1.22 “	1.52 “	1.238	1.175	1.187	1.572	1.517
“ 34	1.08 “	1.47 “	1.095	1.057	1.066	1.518	1.474
“ 36	1.20 “	1.48 “	1.244	1.180	1.180	1.582	1.517
“ 38	1.18 “	1.45 “	1.224	1.166	1.154	1.586	1.512
“ 39	—	—	—	—	—	—	—
ZPR-VI 2	1.07 “	1.49 “	1.101	1.052	1.056	1.534	1.464

TABLE 4. CALCULATED REACTIVITY WORTHS

Assembly	Experimental reactivity worth of U-235 (arbitrary units)		Difference (%)	Experimental reactivity worth of Pu-239 (arbitrary units)		Difference (%)
	Exp.	Theor.		Exp.	Theor.	
ZEBRA 1	3.081 $\pm$ 0.123	3.363	9	4.953 $\pm$ 0.198	5.909	19
“ 2	1.832 $\pm$ 0.073	2.122	16	2.562 $\pm$ 0.102	3.254	27
ZPR-III 29	1.375 $\pm$ 0.055	1.937	41	2.221 $\pm$ 0.089	3.213	45
“ 32	1.017 $\pm$ 0.041	1.356	33	1.869 $\pm$ 0.075	2.514	35
“ 33	0.963 $\pm$ 0.039	1.251	30	1.777 $\pm$ 0.071	2.330	31
“ 34	1.074 $\pm$ 0.043	1.465	36	1.772 $\pm$ 0.071	2.445	38
“ 36	1.117 $\pm$ 0.045	1.380	24	1.995 $\pm$ 0.080	2.448	23
“ 38	0.666 $\pm$ 0.027	0.945	42	1.175 $\pm$ 0.047	1.616	38
“ 39	0.484 $\pm$ 0.016	0.607	25	0.883 $\pm$ 0.035	1.089	23

sample-size effect, there appears to exist some difference of opinion^(13, 21) on how to account for the self-absorption and self-multiplication of the sample. The published ZPR experimental values have not been changed as regards this aspect, while the ZEBRA values have been corrected, using semiempirical formulae, by relevant factors (up to 10%). Now, if one looks at the relatively good agreement of k_{eff} and of the fission ratios and then looks at the results shown in Table 4, where the theoretical values are calculated using the APDA set,^(a) it seems reasonable to impute these large discrepancies between computed and experimental data to the uncertainty in the correction factors mentioned above and probably to some systematic errors introduced in the reactivity measurements, rather than to the cross-section errors. A further check of this assumption is obtainable by considering the reactivity ratios of Pu-239 and U-233 with respect to U-235 instead of absolute quantities. The values obtained are shown in Table 5, where the experimental results with and without size correction

are given. A much better consistency is now obtained between experimental and theoretical values, and this agrees with the aforementioned assumption.

Considering the two sets of corrected and uncorrected experimental values and the three sets of theoretical ones, as illustrated in Table 5, the following considerations can be made:

1. The YOM-modified, uncorrected APDA set checks well with all the uncorrected experimental values of Pu-239 reactivity ratios, while the uncorrected U-233 values are consistently underpredicted. The agreement with the corrected U-233 values is good.
2. The YOM-modified APDA set corrected with the White values of Pu-239 checks well with the experimental values of plutonium reactivity ratios corrected for size effect.
3. The Russian set checks well with the plutonium and with the U-233 values of reactivity ratios corrected for size effect. It is worth noticing that the U-235 cross sections used in the calculations account for the self-shielding factors due to system composition.

(a) With the Russian library the disagreement is even higher.

TABLE 5. REACTIVITY RATIOS

Assembly	Experimental values				Theoretical values				
	$\frac{\Delta k/k(\text{Pu-239})}{\Delta k/k(\text{U-235})}$		$\frac{\Delta k/k(\text{U-233})}{\Delta k/k(\text{U-235})}$		$\frac{\Delta k/k(\text{Pu-239})}{\Delta k/k(\text{U-235})}$			$\frac{\Delta k/k(\text{U-233})}{\Delta k/k(\text{U-235})}$	
	(a)	(b)	(a)	(b)	APDA	Mod APDA	RUSS	APDA	RUSS
ZEBRA 1	1.721 ($\pm 3\%$)	1.608 ($\pm 3\%$)	—	—	1.727	1.616	1.602	—	—
“ 2	1.497 “	1.399 “	—	—	1.508	1.341	1.437	—	—
ZPR-III 29	1.675 “	1.565 “	1.876 ($\pm 3\%$)	1.721 ($\pm 3\%$)	1.658	1.525	1.563	1.760	1.743
“ 32	1.838 “	1.717 “	1.901 “	1.744 “	1.822	1.703	1.715	1.785	1.726
“ 33	1.845 “	1.725 “	1.863 “	1.709 “	1.831	1.702	1.722	1.679	1.719
“ 34	1.701 “	1.590 “	1.925 “	1.766 “	1.669	1.545	1.572	1.757	1.742
“ 36	1.787 “	1.670 “	1.902 “	1.745 “	1.745	1.615	1.626	1.735	1.701
“ 38	1.762 “	1.647 “	1.805 “	1.656 “	1.681	1.593	1.596	1.617	1.690
“ 39	1.778 “	1.661 “	—	—	1.764	1.632	1.662	—	—
ZPR-VI 2	1.581 “	1.478 “	—	—	1.600	1.426	1.515	—	—

(a)—Not corrected for sample-size effect.

(b)—Corrected for sample-size effect.

TABLE 6. RUSSIAN GROUP CONSTANTS, IN BARNS

Group i	σ_c	σ_f	$\nu\sigma_f$	σ_{el}	$\sigma_{i \rightarrow i}$	$\sigma_{i \rightarrow i+1}$	$\sigma_{i \rightarrow i+2}$	$\sigma_{i \rightarrow i+3}$	$\sigma_{i \rightarrow i+4}$	$\sigma_{i \rightarrow i+5}$	$\sigma_{i \rightarrow i+6}$	$\sigma_{i \rightarrow i+7}$	$\sigma_{i \rightarrow i+8}$	$\sigma_{i \rightarrow i+9}$
1	0.01	2.20	8.492	0.616	0.0	0.0	0.04	0.15	0.26	0.32	0.18	0.07	0.03	0.01
2	0.02	1.85	6.4935	0.91	0.01	0.05	0.20	0.36	0.36	0.18	0.08	0.03	0.03	0.0
3	0.03	1.97	6.4419	1.3485	0.04	0.15	0.31	0.40	0.22	0.09	0.03	0.01	0.0	0.0
4	0.04	1.95	6.084	1.8675	0.10	0.20	0.42	0.27	0.12	0.04	0.01	0.0	0.0	0.0
5	0.06	1.80	5.418	2.365	0.30	0.43	0.23	0.13	0.04	0.01	0.0	0.0	0.0	0.0
6	0.11	1.68	4.956	3.4775	0.59	0.44	0.09	0.03	0.01	0.0	0.0	0.0	0.0	0.0
7	0.17	1.66	4.8306	5.4824	0.76	0.16	0.02	0.01	0.0	0.0	0.0	0.0	0.0	0.0
8	0.24	1.68	4.9552	7.5081	0.60	0.15	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
9	0.40	1.85	5.328	9.021	0.43	0.12	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
10	0.71	2.15	6.1705	10.4544	0.11	0.14	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
11	1.08	2.40	6.888	11.1916	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
12	1.25	2.50	7.175	12.6225	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
13	1.90	3.40	9.758	12.7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
14	2.60	4.20	12.054	12.7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
15	5.00	7.60	21.812	13.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
16	9.10	13.00	37.31	14.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
17	14.00	19.00	54.53	15.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
18	46.00	61.00	175.07	20.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	—
19	31.00	22.00	63.14	15.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	—	—
20	72.00	110.00	315.7	12.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	—	—	—
21	35.00	45.0	129.15	10.5	0.0	0.0	0.0	0.0	0.0	0.0	—	—	—	—
22	1.00	12.0	34.44	10.0	0.0	0.0	0.0	0.0	0.0	—	—	—	—	—
23	3.00	24.0	68.88	11.0	0.0	0.0	0.0	0.0	—	—	—	—	—	—
24	73.00	103.0	295.61	13.3	0.0	0.0	0.0	—	—	—	—	—	—	—
25	1210.0	1670.0	4792.9	15.7	0.0	0.0	—	—	—	—	—	—	—	—
26	286.0	742.0	2129.54	9.5	0.0	—	—	—	—	—	—	—	—	—

4.4. Correction of the libraries

The analysis of the preceding chapter has indicated that the Russian set may be assumed *a posteriori* as one of the best sets to be used with the correlation method illustrated in this chapter. We have also assumed that the U-235 cross sections are sufficiently correct to allow this analysis. Looking at the results of Tables 3 and 5 obtained with the CALI Code, it appears that the

theoretical values for U-233 fall within the uncertainty limits of the experimental ones and that no corrections are required for this agreement. On the contrary, some of the theoretical values for Pu-239 fall outside of the assumed uncertainty limits. In Tables 6 and 7 are shown the Russian library adopted for plutonium and the corrections needed for full agreement with the 17 integral experiments considered (10 reactivity and 7

TABLE 7. CORRECTIONS FOR RUSSIAN GROUP SETS, IN BARNS

Group i	$\delta\sigma_c$	$\delta\sigma_f$	$\delta(\nu\sigma)_f$	$\delta\sigma_{el}$	$\delta\sigma_{i \rightarrow i}$	$\delta\sigma_{i \rightarrow i+1}$	$\delta\sigma_{i \rightarrow i+2}$	$\delta\sigma_{i \rightarrow i+3}$	$\delta\sigma_{i \rightarrow i+4}$	$\delta\sigma_{i \rightarrow i+5}$	$\delta\sigma_{i \rightarrow i+6}$	$\delta\sigma_{i \rightarrow i+7}$	$\delta\sigma_{i \rightarrow i+8}$	$\delta\sigma_{i \rightarrow i+9}$
1	.0	.04078	.06731	-.00002	.0	.0	-.00001	-.00033	-.00186	-.00298	-.00097	-.00014	-.00004	.0
2	-.00002	.00728	.20233	-.00001	.0	-.00001	-.00107	-.01295	-.01594	-.00378	-.00085	-.00017	-.00002	.0
3	-.00004	.01795	-.08459	-.00019	.0	-.00110	-.02048	-.04247	-.01236	-.00210	-.00044	-.00004	.0	.0
4	-.00001	.03783	-.08741	-.00173	.0	-.01186	-.07693	-.03276	-.00770	-.00086	-.00008	.0	.0	.0
5	.00033	.03000	-.04530	-.00238	-.00001	-.04634	-.00954	-.00334	-.00049	-.00004	.0	.0	.0	.0
6	-.00058	.06039	.12547	.00564	.00001	.06400	.00449	.00046	.00004	.0	.0	.0	.0	.0
7	-.00206	.11425	.11079	.01243	.00001	.00851	.00005	.00002	.0	.0	.0	.0	.0	.0
8	-.01709	.03363	.03077	-.00847	.0	-.00254	.0	.0	.0	.0	.0	.0	.0	.0
9	-.04960	-.01920	.07965	.00278	.0	.00038	.0	.0	.0	.0	.0	.0	.0	.0
10	.12097	-.02942	-.03778	-.00488	.0	-.00066	.0	.0	.0	.0	.0	.0	.0	.0
11	.07615	-.06471	-.08145	.00116	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
12	.09228	-.11281	-.07076	.00223	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
13	-.16036	-.16249	.28518	.00217	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
14	-.21246	-.03868	-.19799	-.00015	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
15	-.00785	-.05939	-.07057	.00035	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
16	-.38278	-.07346	.37802	-.00005	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
17	-.42418	-.04929	.34080	.00007	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
18	-1.48434	-.03458	1.09453	-.00010	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
19	-.12753	-.00306	.02922	-.00002	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
20	-.09190	-.00975	.13076	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
21	-.00249	-.00019	.00306	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
22	-.0	-.00001	.00008	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
23	-.0	-.00001	.00009	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
24	-.00050	-.00004	.00027	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
25	-.01041	-.00080	.00184	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
26	-.00001	.0	.00002	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0

fission ratios). As may be noticed, the corrections are not needed for the slight disagreement between some measured and calculated quantities. These corrections, although broadly distributed, have appreciable values for some peculiar reactions and energy bands than for others, depending on the integral data information which has been fed into the program. In particular, because the fission ratios have resulted in better agreement than the reactivity values, there appears a tendency for correction of the capture and ν values rather than those for fission.

5. Conclusion

As a consequence of the discrepancies between experimental and theoretical integral values resulting from the present analysis and in view of a more comprehensive knowledge of fast reactor physics, a few recommendations may be formulated with respect to experiments on critical facilities:

1. cleaner experiments should be performed with respect to both geometry and composition, in connection with a specified fast reactor program;
2. improvements of experimental techniques for reaction rate and reactivity measurements should be pursued.

With respect to theoretical methods for the inter-

pretation and utilization of experimental data, the following items should be emphasized:

3. theoretical methods for accounting for heterogeneity, size, and shape effects;
4. comprehensive methods of correlation with differential data to exploit fully the experimental information available.

In terms of point 1, an example is represented by a series of zoned experiments at different central dilutions, with compositions covering the range of interest, in order to study the physical properties of fast systems at conditions of high burnup. Another example is represented by a series of clean experiments for studying the nuclear properties of sodium in the neutron environments of interest.

With regard to 2, it seems that, far from being satisfactory, the present accuracies of many group constants, if proper calculational techniques are used, are often sufficient with respect to the precision by which the reduced experimental values have been so far obtained. This situation calls for an improvement of either the experimental techniques themselves or of the methods for their reduction to calculable quantities, as is stressed in point 3.

It is a fact that new integral experiments (i.e., reaction rates, reactivity worths, Rossi- α measurements, etc.) will add more and more data to those already

existing. It seems that we are getting to a point where it will be extremely difficult to analyse and intercorrelate "by hand" all these experimental data with theoretical predictions whose accuracy depends upon the ability of evaluating the neutronic environment of the systems and on the knowledge of the nuclear parameters under direct investigation. In some cases, as with the sodium void and Doppler effects, the discrepancies between experiments and theory may be not as much dependent on the relative accuracy of sodium and fuel cross sections as they are on that of cross sections relative to other constituent materials. If full exploitation of the critical facilities is to be obtained, more sophisticated methods of analysis and utilization of data are needed, as stressed in point 4. The use of intercorrelation methods in terms of realistic geometries and improved perturbation techniques might represent an approach for which the CALI program is an initial step.

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Discussion

Mr. Kohler (Texas A&M): You said that the number of integral data that you want to fit is usually much smaller than the number of cross sections that you have. What method do you use to determine which cross sections you want to change?

Mr. Gandini: We introduce cross values. We minimize something which is $\Delta\sigma_{0,n}$ and then make that proportional to the absolute values. If the absolute value is zero, this cross section won't change at all, because otherwise, this is not the minimum we seek. If this is a big value, you cause the values to change very much but this would have a very small influence on the value of σ .

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Comparative Studies of the Criticality of Fast Critical Assemblies

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ABSTRACT

Multigroup calculations of the reproduction factor k for 38 fast critical assemblies built with ZPR-III, Zebra, and Vera are compared. The data sets used are Hansen and Roach; Yiftah, Okrent and Moldauer; Abagyan, Bazazyants, Bondarenko, and Nigolaev; ANL Set 800; FD1 and FD2. A least-squares fit has been made to the computed values of k , assumed to be a linear function of the atomic composition of the core, for each set and the standard deviation of the results significantly reduced. The coefficients of this regression formula give clues to the sources of error in each set and show, among other things, that the success achieved by nearly all sets is partly due to off-setting overreactive U^{235} and Pu^{239} data by under-reactive U^{238} data. The regression formula may be used as an interpolation formula to correct the prediction of critical mass for an unknown system. As an example of this use, the critical mass of ZPR-III Assembly 48 is predicted. It is also shown that the standard error of experimental estimates of k for homogeneous spherical cores is 0.003. The paper includes previously unpublished data on the correction necessary to diffusion-theory estimates of k to get transport-theory values in S_4 and S -infinity approximation.

1. Introduction

For some years, the writer has been a protagonist of the building of fast critical assemblies with cores containing only a small number of constituents, because such assemblies give the best evidence for the accuracy of the data for individual constituents used in multigroup calculations. For example, an examination of the reproduction factor k calculated for the ZPR-III⁽¹⁾ critical assemblies 25, 24, and 11 gives evidence on U^{235} and U^{238} data, and an examination of k for the sequence of ZPR-III assemblies 11, 16, 12, 17, and 14 gives evidence on data for carbon. In this paper, this approach is put on a systematic basis which enables use to be made of critical-size data from all critical assemblies. A linear regression formula is

fitted to the calculated values of k for a given data set with one term for each core constituent. Only critical-mass data are included in this approach, but critical mass is one of the few quantities that can be determined really accurately. Other integral quantities may provide supporting evidence for any conclusions reached.

Section 2 gives the statistical theory of the least-squares fit to a regression formula. Section 3 gives the models of ZPR-III, Zebra, and Vera⁽¹⁾ accepted as the best available. The diverse origin of the multigroup calculations using 9 different data sets or variants are described. Some useful information on the difference in k between diffusion- and transport-theory calculations is published here for the first time. Section 4 gives the complete results of the least-squares fits, including special investigations to study sources of systematic error. An estimate is obtained of the accuracy of experimental criticality results. In Section 5, the results are assessed to see what clues there are on the errors in different data sets and different materials. The data sets considered are Hansen and Roach;⁽²⁾ Yiftah, Okrent, and Moldauer;⁽³⁾ FD1;⁽⁴⁾ FD2;⁽⁵⁾ ANL Set 800;⁽⁶⁾ and the Russian set of Abagyan *et al.*⁽⁷⁾ The use of the regression formulas to predict criticality for unknown systems is described in Section 6 and a prediction made for ZPR-III-48⁽⁸⁾ (the subject of check calculations at this conference). In the final section, the conclusions are summarised.

2. Theory of the Fitting Process

The starting information for the fitting process is a table of values of k , calculated using a certain data set, for just-critical models of the fast critical assemblies. A least-squares fit is made to these values by a linear regression formula, as follows:

$$k_f = 1 + a_0 C_0 + a_1 C_1 + a_2 C_2 + \dots$$

The C_i are atomic compositions in the core for Pu^{239} , U^{235} , U^{238} , etc., normalised to $C_1 = 1$ for U^{235} -fuelled systems or $C_0 = 0.6$ for Pu^{239} -fuelled systems. The a_i are

the regression coefficients which are calculated to make the sum of the squares of the residuals, $k - k_f$, a minimum. This is a standard procedure as described by Kempthorne.⁽⁹⁾

A computer programme for this purpose, KFIT, was written in FORTRAN for the KDF9 by D. W. B. Bale. The programme yields the initial standard deviation about the mean value:

$$\sigma_i = \sqrt{\frac{S(k - \bar{k})^2}{n - 1}},$$

where n is the number of values; the standard deviation after fitting is

$$\sigma = \sqrt{\frac{S(k - k_f)^2}{n - m}},$$

where m is the number of coefficients a_i . The programme also yields the standard error of each coefficient a_i , namely $\sigma\sqrt{C_{ii}}$, where σ is defined above and the C_{ii} are the diagonal elements of the variance-covariance matrix as defined by Kempthorne.⁽⁹⁾

It was apparent, by inspection of the data, that the error in k depends on the presence or absence of certain core constituents. A linear regression formula is chosen because it is the simplest choice. Core compositions normalised to a fixed value for the fuel are used in the regression formula to eliminate any dependence on average core density. The composition of Pu²³⁹ is normalised to 0.6 compared with unity for U²³⁵ because it is expected that a Pu²³⁹-fuelled core with a diluent/Pu²³⁹ ratio of $D/0.6$ will have similar properties to that of a U²³⁵-fuelled core with a diluent/U²³⁵ ratio of D . By this assumption, it becomes possible to fit cores of either fuel by the same formula. (A more complicated normalisation would be necessary for cores with mixed fuels.) In Sect. 4.3 it is shown that this approximation makes no significant difference by comparing coefficients obtained with C_0 normalised to 0.6 and 0.7. In Tables 5, 6, and 7, the regression coefficients a_i are given except in the case of Pu²³⁹ for which a_0C_0 is given, so that an easy visual comparison may be made between the errors associated with fuel data for Pu²³⁹ and U²³⁵.

All core constituents are included in the regression formula except those present in very small quantities (Pu²⁴⁰, Pu²⁴¹, Ga, Cu). All Fe, Cr, and Ni are lumped together and called stainless steel, SS. Since molybdenum and zirconium occur in only one assembly, ZPR-III-20, it is impossible to distinguish the errors due to these two. Only the atomic composition of molybdenum is included in the formula, but it must be remembered that any error could be associated with molybdenum or zirconium.

For each set of data, two runs of KFIT were per-

formed, with all materials in the core included in the formula and with the series terminated after U²³⁸. This was done to check the hypothesis that all the errors were due to overreactive data for Pu²³⁹ or U²³⁵ offset by underreactive U²³⁸ data. In general, the three coefficients of a three-term regression formula will not be equal to the corresponding coefficients when the series is extended. In one case (see Sect. 5.6), the coefficients are found to agree. This is evidence of negligible additional error due to data other than for Pu²³⁹, U²³⁵, and U²³⁸.

Tables 5, 6, and 7 also include the average deviations of k from unity separately for Pu²³⁹- and U²³⁵-fuelled systems. Formally, this corresponds to a one-term fit to the values of k for Pu²³⁹- and U²³⁵-fuelled systems separately. The standard deviations of the coefficients for Pu²³⁹ and U²³⁵ are evaluated in this case from the initial standard deviation σ_i , defined above corresponding to the residuals with respect to a common mean for assemblies with both fuels. The standard deviation of the coefficient is $\sigma_i/\sqrt{p}$, where p is the number of assemblies fuelled with the particular fuel.

3. Description of the Criticality Calculations

3.1. Models of Assemblies Used for Calculations

The values of k used in deriving the regression formula are obtained from multigroup calculations for spherical models of the reactors, with homogeneous core and blanket zones. These models are estimated to be just-critical from measurements on the as-built assemblies. There are three important calculations or corrections in establishing such models:

- (i) The true average compositions have to be evaluated. The fuel concentration in the core is, by far, the most important number.
- (ii) Certain corrections must be applied to arrive at the dimensions of a just-critical, homogeneous, cylindrical core. The most important is the correction to allow for the heterogeneous structure of the experimental assembly.
- (iii) A correction, usually called a shape factor, must be applied to the critical mass for a cylindrical core to obtain the critical mass of the equivalent spherical core.

For ZPR-III assemblies, the spherical models evaluated by Davey⁽¹⁰⁾ have been assumed, and all calculations have been corrected to refer to these models as given in Table 1. Some of the more recent calculations may have better estimates of corrections for heterogeneity and shape factor, but the differences are not large enough to make an appreciable change to this paper.

The spherical models for Zebra given in Table 2 are

taken from internal Winfrith reports. The spherical models for Vera are also given in Table 2 as supplied by McTaggart⁽¹¹⁾ for U²³⁵-fuelled assemblies and by Paterson⁽¹²⁾ for Pu²³⁹-fuelled assemblies. The shape factors used in deriving spherical models of Zebra and Vera were obtained from a comparison of multigroup calculations for spherical and cylindrical models of the assemblies.

3.2. Transport Theory and Other Corrections to Multigroup Calculations

Comparison of different cross-section sets and different assemblies are easiest made when all multigroup calculations are made to yield a calculated multiplication factor k for the just-critical model. All the calculations have been evaluated to yield a value of k (in Table 4) that would have been obtained if an S_N

transport-theory calculation had been performed in S_4 approximation. Two types of correction are necessary which will be considered in turn below. These are:

- (i) In a few cases, k is calculated for a system not exactly critical. In others, the calculation method yields the core radius that makes k exactly unity.
- (ii) An estimate must be made of the difference, called h , in k obtained by S_4 and diffusion-theory calculations.

The first correction can be made by means of the formula relating reactivity and the mass of core material added at the edge:

$$\frac{\Delta k}{k} = \frac{1}{f} \frac{\Delta M}{M}.$$

The values of the parameter f for ZPR-III given in

TABLE 1. COMPOSITIONS AND DIMENSIONS OF SPHERICAL MODELS OF ZPR-III ASSEMBLIES

Assembly number	Composition, nuclei/cm ³ × 10 ^{-22(b)}											Reflector type ^(a)	Critical mass of sphere (kg U ²³⁵)
	U ²³⁵	U ²³⁸	Mo	Zr	Ni	Fe	Cr	Al	Na	O	C		
2A	0.6706	0.049			0.254	1.669	0.423	1.896				A	144
5	0.6720	0.763			0.112	0.740	0.187	1.899				D	156
6F	0.6720	0.763			0.113	0.746	0.189	1.893				A	133
9A	0.5616	1.824			0.131	0.863	0.219	1.296				A	151
10	0.5688	2.779			0.178	1.171	0.296					A	153
11	0.4565	3.443			0.084	0.555	0.140					A	239
12	0.4502	1.699			0.084	0.555	0.140				2.678	A	172
14	0.4502	0.034			0.084	0.555	0.140				5.356	A	132
16	0.4502	2.405			0.084	0.555	0.140				1.517	A	202
17	0.4502	0.989			0.084	0.555	0.140				3.828	A	151
20	0.2923	0.911	0.322	0.184	0.131	0.869	0.220	1.516				B	408
22	0.4522	3.365			0.086	0.565	0.143					A	242
23	0.4450	0.034			0.083	0.551	0.139	2.582				A	249
24	0.3634	3.499			0.086	0.566	0.143					A	456
25	0.3442	3.560			0.084	0.555	0.140					A	564
29	0.2386	0.479			0.227	1.499	0.380	1.471		1.392		A	402
30	0.2837	0.434			0.226	1.487	0.376	1.408		0.697		A	377
31	0.2789	0.439			0.226	1.489	0.376	1.416				A	437
32	0.4445	0.032			0.747	4.917	1.245					A	213
33	0.4450	0.033			0.586	3.861	0.977		0.400			A	225
34	0.2242	0.494			0.226	1.493	0.387	1.538			0.761	A	475
35	0.1949	0.014			0.456	3.009	0.761		0.782	0.398		C	485
36	0.4498	2.376			0.117	0.770	0.195		0.401			A	217
41	0.2861	1.397			0.130	0.856	0.217	1.079				A	462
<i>Reflector</i>													
A	0.0091	3.998			0.067	0.440	0.112					—	—
B	0.0048	2.179	0.157		0.179	1.179	0.298	0.608				—	—
C	0.0043	1.901	0.140		0.173	1.145	0.290		0.715			—	—
D ₁	0.0038	1.906			0.068	0.444	0.112	1.483				—	—
D ₂	0.0091	3.998			0.068	0.444	0.112	0.137				—	—

^(a) For reflector D, the inner zone D₁ and outer zone D₂, are each 18.1 cm thick. For all other reflectors, the thickness is 30 cm.

^(b) Compositions for assemblies calculated from volume fractions given in Appendix II of Davey,⁽¹⁶⁾ except for assemblies 22 and 41 from Thalgott *et al.*⁽¹⁾ Critical-mass data from Davey⁽¹⁰⁾ except for assembly 22, which is derived from the unadjusted critical mass given in Ref. 13 with the corrections as given by Davey⁽¹⁰⁾ for assembly 11, which is virtually identical.

TABLE 2. COMPOSITIONS AND DIMENSIONS OF SPHERICAL MODELS OF ZEBRA AND VERA ASSEMBLIES

Assembly number	Composition, nuclei/cm ³ × 10 ⁻²²														Reflector type ^(e,d)	Critical mass of sphere (kg U ²³⁵ or Pu ²³⁹)
	Pu ²³⁹	Pu ^{240(a)}	Pu ²⁴⁰	U ²³⁵	U ^{238(b)}	Ga	Cu ^(e)	Ni	Fe	Cr	Al	Na	C	H		
<i>Zebra</i>																
1				0.4655	3.433			0.032	0.447	0.082					E	214.9
2				0.2523	1.555			0.034	0.402	0.087			3.795		E	390.9
3	0.3465	0.0183	0.0016	0.0229	3.156	0.022	0.457	0.032	0.456	0.082					E	76.5
6A	0.1879	0.0144	0.0016	0.0046	0.635	0.012	0.083	0.044	0.453	0.127	0.250	0.447	2.959		F	172.2
<i>Vera</i>																
1B				0.7349	0.056			0.069	0.628	0.164			5.754	0.0058	G, H	84.2
2A				0.5669	0.043			0.069	0.628	0.164			6.085	0.0045	G, H	98.9
3A				0.6186	1.334			0.069	0.628	0.164			3.577	0.0148	G, H	102.5
4A				0.8065	0.062			0.069	0.628	0.164			5.096	0.7989	G, H	49.4
5A				0.6605	0.050			0.069	0.628	0.164			5.461	0.6551	G, H	56.8
6A				0.6691	1.442			0.069	0.628	0.164			2.926	0.6646	G, H	63.2
7A				0.5676	1.224			0.069	0.628	0.164			3.580	0.5753	G, H	72.0
9A	1.0687	0.0544	0.0042			0.067	1.125	0.066	0.609	0.158	0.001		3.422			22.7
10A	0.8632	0.0441	0.0034			0.054	0.909	0.066	0.609	0.158	0.001		4.105			27.2
11A	0.7213	0.0370	0.0028			0.045	0.759	0.066	0.609	0.158	0.001		4.620			32.8
<i>Reflector</i>																
E				0.0301	4.099			0.036	0.350	0.091					—	—
F				0.0194	2.681			0.032	0.325	0.091			2.341		—	—
G				0.025	3.440			0.071	0.646	0.168					—	—
H				0.026	3.610			0.072	0.658	0.171					—	—

(a) Pu²⁴² included with Pu²⁴⁰.(b) U²³⁴ and U²³⁶ included with U²³⁸.

(c) For Zebra, axial and radial blanket is 30 cm thick.

(d) For Vera, axial blankets of composition G are 33–39 cm thick, depending on assembly. Radial blankets are 34–42 cm thick, depending on assembly, of inner composition G and outer H. The boundary between G and H is a square of side 56.2 cm. In the equivalent cylindrical model, the minimum radial thickness of G is 7.3 cm (for assembly 3A). For most calculations, a complete blanket of composition G may be assumed.

(e) A small mass of solder is included with Cu. For Vera, Sn and Pb are 1.1 and 1.5% by mass of total Cu. For Zebra, the percentages are approximately double.

Table 3 are the averages taken from S_N calculations reported by Davey⁽¹⁰⁾ and by Avery.⁽¹³⁾ The values of f for Vera are obtained from diffusion-theory calculations for spherical models by Brickstock.⁽¹⁴⁾ The values of f for Zebra are also from diffusion-theory calculations for spherical models.

All the known data on the difference, h , in k obtained by S_4 and diffusion-theory calculations is assembled in Table 3. When calculating the difference h , great care is necessary to ensure that, in the pair of transport- and diffusion-theory calculations, both use exactly the same cross-section and composition data, and both are adequately converged. Wardleworth's FD2 calculations are particularly reliable in this first respect since a common input tape containing compositions and cross sections was used for his diffusion-theory (SCRAMBLE) and transport-theory (WDSN) calculations. He used 15 equally spaced mesh spaces in the core and 15 equally spaced in the blanket in each case for ZPR-III

and Zebra; for Vera he used 10 and 20, respectively. The spread in the values of h for a single assembly is possibly due to inadequate convergence, inadequate numbers of mesh points, and differences in data libraries intended to be identical rather than to a real dependence of h on the data set used for calculations. The mean value of h from all available calculations was used for a particular assembly when adjusting diffusion-theory calculations except that Wardleworth's FD2 values of h ⁽¹⁵⁾ were used to correct his own diffusion-theory calculations using FD1.

For certain of the calculations in Table 7, values of k are required in S -infinity approximation (infinite angular mesh). From unpublished calculations by Wardleworth⁽¹⁵⁾ for Vera 9A, 10A, and 11A, in S_2 , S_4 , S_8 , and S_{16} approximation, it was concluded that k for S -infinity was smaller than k for S_4 by $h/6$. This relationship was assumed to hold for all assemblies. In ad-

TABLE 3. COMPARISON OF THE DIFFERENCE, $h = k(S_4) - k(\text{DT})$, OBTAINED BY VARIOUS WORKERS AND VALUES OF $f = (\Delta M/M)/(\Delta k/k)$

Assembly	R (cm) ^(a)	<i>h</i>						<i>f</i> ^(10, 13)
		Set						
		LO ⁽¹⁷⁾	HRC ⁽¹⁸⁾	YIFB	FD1 ⁽¹⁵⁾	FD2 ⁽¹⁵⁾	Mean (Best value)	
<i>ZPR-III</i>								
2A	24	0.025				0.022	0.023	3.3
5	24	0.033				0.032	0.032	4.1
6F	23	0.024				0.023	0.023	4.2
9A	25	0.018	0.017			0.015	0.017	4.7
10	28	0.015				0.011	0.013	5.3
11/22	32	0.009	0.008			0.007	0.008	7.3
12	29	0.015		0.007		0.011	0.011	5.6
14	26	0.020	0.018	0.016		0.017	0.018	3.9
16	30			0.011		0.009	0.010	4.8
17	28			0.013		0.014	0.014	4.5
20	45		0.010				0.010	5.0
23	33			0.010		0.016	0.013	4.5
24	43					0.003	0.003	9.4
25	47					0.002	0.002	9.5
29	48		0.007			0.006	0.006	5.4
30	44		0.009			0.009	0.009	5.7
31	47		0.009			0.009	0.009	5.6
32	32			0.012		0.013	0.013	4.2
33	32					0.014	0.014	4.2
34	52		0.007	0.005		0.006	0.006	5.3
35	54					0.011	0.011	4.8
36	32					0.010	0.010	5.4
41	47		0.007			0.008	0.008	5.8
<i>Zebra</i>								
1	31			0.007		0.007	0.007	7.0
2	45			0.004		0.002	0.003	7.4
3	24			0.015		0.014	0.014	5.2
6A	38			0.010		0.011	0.010	4.8
<i>Vera</i>								
1B	19				0.027	0.026	0.026	3.9
2A	20				0.021	0.021	0.021	4.1
3A	21				0.020	0.019	0.020	4.6
4A	16					0.031	0.031	4.0 ^(b)
5A	17					0.026	0.026	4.2
6A	18					0.023	0.023	4.0 ^(b)
7A	20					0.020	0.020	4.9
9A	11				0.078	0.076	0.077	3.2
10A	12				0.064	0.062	0.063	3.2
11A	14				0.054	0.052	0.053	3.2

^(a) R is radius of critical core calculated by transport theory (S_4).

^(b) Interpolated values.

dition, calculations with a finer mesh suggested that k should be further reduced by 0.001 for Vera only. These two corrections were used in deriving the regression coefficients for S -infinity given in Table 7.

3.3. Origin of the HR Results

The Hansen and Roach set has 16 groups. These are especially broad at low energies. A graphical system of making allowance for self-shielding is provided. Three variants of this set are extant, called in this note HRA, HRB, and HRC.⁽²⁾ Extensive comparisons with experiment are only available for HRB and HRC.

Diffusion-theory calculations of ZPR-III using HRB have been reported by Martin⁽²⁰⁾, and diffusion- and transport-theory calculations using HRC by Kazi.⁽¹⁸⁾ Both used Davey's models. The values of k for HRB for Zebra are from diffusion-theory calculations by A. M. Broomfield.

3.4. Origin of the YOMD Results

The Yiftah, Okrent, and Moldauer set⁽³⁾ has 16 groups with a finer energy structure than HR at high energies, but it is inadequate below 10 keV in systems where low-energy neutrons are important, especially as it has no provision for self-shielding of resonant reactions of heavy elements. Davey's version of the YOM set is called ANL set 635, or, in this paper, YOMD. This set incorporates the modifications to the printed YOM set circulated by the authors. In addition, it incorporates revised estimates of transport and elastic removal cross sections for aluminium and steel, revised capture cross sections for U^{233} , U^{235} , and Pu^{239} , and slightly modified values of ν for U^{235} .

The values of k obtained by Davey for ZPR-III using DSN⁽¹⁰⁾ are used without modification. Davey's calculations for Vera⁽¹⁹⁾ have been revised to agree with the improved models of Table 2.

3.5. Origin of the YIFB Results

YIFB is the name given to the YOM set when incorporating the modifications suggested by the authors. It is thus very similar to YOMD, but it does not include the important changes made by Davey (see Sect. 3.4).

The calculations of k using DSN and YIFB given in Table 4 were performed by D. Gooch and J. M. Stevenson. The calculations for Zebra and Vera were performed by J. Adamson and A. M. Broomfield using the transport theory code, STRAINT. These S_N calculations were compared with diffusion-theory calculations using the codes CRAM or SCRAMBLE, in order to produce the values of h for YIFB in Table 3.

3.6. Origin of the FD1 Results

FD1 is a 33-group version⁽⁴⁾ of the YOM set. The 14 high-energy groups are the same as in YIFB; a further 19 groups were derived and resonance self-shielding in heavy elements treated. The data for structural materials was taken from the GAM library.⁽²¹⁾

TABLE 4. THE k CALCULATIONS IN S_4 APPROXIMATION USED FOR THE FITS

System	Assembly	HRB	HRC	YOMD	YIFB	FD1	ANL-800	ABBN	Modified ABBN	FD2
ZPR-III	2A			1.005		1.021				1.023
	5			1.005		1.025				1.025
	6F			1.003		1.019	1.007			1.021
	9A		0.994	0.998		1.014	1.002			1.011
	10	0.985		0.999		1.008				1.005
	11	0.983	0.986	0.994		1.006		1.011	0.993	0.991
	12	0.999		0.998	1.005	1.011	1.001	1.016	1.002	1.003
	14	1.016	1.020	0.995	1.007	1.006	1.013			1.014
	16	0.995		0.999	1.011	1.012				0.999
	17	1.007		0.998	1.010	1.011				1.009
	20	0.989	0.989	0.992						
	22						0.998			
	23	0.999		1.004	1.016	1.026	1.009	1.026	1.011	1.023
	24	0.981	0.987	0.995		1.007		1.005	0.987	0.981
	25	0.981	0.986	0.991		1.006	0.998	0.999	0.978	0.977
	29	1.001	1.006	1.003		1.024	1.020	1.032	1.017	1.014
	30	1.004	1.005	1.009		1.030		1.039	1.023	1.021
	31	0.995	0.996	1.006		1.026		1.037	1.022	1.018
	32	0.982		1.015	1.030	1.033	1.016	1.034	1.026	1.032
	33	0.988		1.014		1.032	1.019	1.035	1.018	1.033
	34	0.993	0.998	0.997	1.018	1.018	1.014	1.023	1.008	1.007
	35			0.994		1.013				1.027
	36	0.982		0.994		1.006	0.998			0.999
	41		0.995	1.002		1.002	1.003			0.994
Zebra	1	0.979			0.997	1.002				0.987
	2	1.015			0.996	1.006				0.992
	3	0.969			1.023	1.026				0.995
	6A					1.044				1.010
Vera	1B					1.009				1.014
	2A					1.005				1.013
	3A					1.008				1.007
	4A					0.989				0.994
	5A					0.985				0.992
	6A					0.990				0.998
	7A					0.987				0.996
	9A			1.034	1.033	1.032				1.032
	10A			1.026	1.022	1.024				1.022
	11A			1.031	1.030	1.030				1.025

The calculations were performed by Wardleworth,⁽¹⁵⁾ mostly by diffusion theory with the correction to produce a corresponding S_4 result taken from his FD2 calculations (see Table 3). In a few cases, he actually performed an S_4 calculation too. The assemblies concerned can be seen from Table 3 because a value of h is given for FD1.

3.7. Origin of the Results for ANL Set 800

Set 800 is a recent set⁽⁶⁾ from Argonne National Laboratory based on the latest microscopic data. It is said to be equivalent for most systems to ANL set 224. The results of S_4 calculations for ZPR-III are reported by Meneghetti.⁽²²⁾ Minor corrections have been made to make them refer to the models in Table 1.

3.8. Origin of the ABBN Results

The ABBN set is a 26-group set⁽⁷⁾ due to Abagyan *et al.* The modified ABBN set includes reduced fission cross sections for U^{235} in certain groups. Diffusion-theory results have been reported. Minor corrections have been made for model as well as for transport theory.

3.9. Origin of the FD2 Results

FD2 is a data set⁽⁵⁾ with the same group boundaries as FD1, reduced, for most materials, from the UKAEA data library using the code GALAXY. For some reactor constituents of less importance, this set is identical with FD1. A small amount of data is taken from other sources. The shielding parameters are the same as in

FD1. The complete set of FD2 results was obtained by Wardleworth⁽¹⁵⁾ using the Winfrith version of the S_N code WDSN.

4. Evaluation of the Regression Coefficients

4.1. Results in General

The regression coefficients a_i obtained on making a fit to the complete set of values of k (see Table 4) are given in Table 5, the fit to ZPR-III only in Table 6.

Three separate fits are given in each case. The first is merely a fit to an average k separately for Pu^{239} - and U^{235} -fuelled assemblies. The resulting standard deviation (called the 'initial' standard deviation) varies from 0.6 to 1.6% according to data set. The second fit allows for dependence on U^{238} also and amounts to an allowance for the variation with $\text{U}^{238}/\text{Pu}^{239}$ and $\text{U}^{238}/\text{U}^{235}$ atom ratio in Pu^{239} - and U^{235} -fuelled assemblies, respectively. This reduces the standard deviation to 0.5 to 1.3%, according to data set. The final fit allows for

TABLE 5. REGRESSION COEFFICIENTS AND STANDARD DEVIATIONS FOR FIT TO S_4 CALCULATIONS FOR ALL ASSEMBLIES^(a)

	YOMD	YIFB	FD1	FD2
<i>Coefficients ($\times 10^4$)</i>				
Pu^{239} and U^{235} ; only Pu^{239}	303 ± 67	270 ± 61	312 ± 63	168 ± 67
U^{235}	4 ± 14	100 ± 41	109 ± 25	71 ± 27
Pu^{239} , U^{235} , and U^{238} ; $\text{Pu}^{239} \times C_0$	304 ± 31	301 ± 37	325 ± 55	224 ± 42
U^{235}	44 ± 16	173 ± 34	135 ± 30	185 ± 22
U^{238}	-13 ± 4	-24 ± 8	-9 ± 7	-40 ± 5
All materials: $\text{Pu}^{239} \times C_0$	329 ± 18	299 ± 26	337 ± 34	266 ± 24
U^{235}	79 ± 21	144 ± 45	135 ± 37	254 ± 25
U^{238}	-17 ± 3	-18 ± 6	-11 ± 5	-48 ± 3
Mo/Zr	-74 ± 29			
SS	6 ± 2	10 ± 4	9 ± 4	4 ± 3
Al	-10 ± 4	0 ± 7	12 ± 7	-8 ± 5
Na	-65 ± 11		-28 ± 23	-14 ± 16
O	1 ± 6		-4 ± 12	0 ± 9
C	-10 ± 2	-6 ± 3	-3 ± 3	-8 ± 2
H			-237 ± 40	-204 ± 27
<i>Standard deviations ($\times 10^4$)</i>				
Pu^{239} and U^{235} only	116	122	140	150
Pu^{239} , U^{235} and U^{238}	53	72	121	92
All materials	29	46	65	45

^(a) See also Table 6 for certain data sets used for ZPR-III only.

TABLE 6. REGRESSION COEFFICIENTS AND STANDARD DEVIATIONS FOR FIT TO S_4 CALCULATIONS FOR ZPR-III ONLY

	HRB	HRC	YOMD	YIFB	FD1	ANL-800	ABBN	Modified ABBN	FD2
<i>Coefficients ($\times 10^4$)</i>									
U^{235} only: U^{235}	-71 ± 25	-35 ± 31	4 ± 14	139 ± 40	162 ± 20	75 ± 23	235 ± 42	77 ± 48	103 ± 33
U^{235} and U^{238} : U^{235}	-4 ± 31	64 ± 33	44 ± 16	186 ± 38	226 ± 22	138 ± 21	351 ± 25	214 ± 25	249 ± 15
U^{238}	-19 ± 7	-24 ± 6	-13 ± 4	-28 ± 10	-21 ± 5	-20 ± 5	-33 ± 5	-39 ± 5	-48 ± 3
All materials: U^{235}	-14 ± 41	15 ± 52	78 ± 20	149 ± 47	231 ± 37	42 ± 31	310 ± 93	186 ± 84	309 ± 19
U^{238}	-17 ± 5	-15 ± 7	-17 ± 3	-19 ± 7	-22 ± 5	-11 ± 4	-29 ± 10	-37 ± 9	-54 ± 2
Mo/Zr	-34 ± 35	-53 ± 48	-74 ± 27						
SS	-9 ± 3	-3 ± 14	6 ± 2	10 ± 4	8 ± 3	9 ± 2	4 ± 7	7 ± 6	2 ± 2
Al	6 ± 5	-1 ± 20	-10 ± 3	0 ± 7	-4 ± 6	4 ± 5	1 ± 11	-5 ± 10	-15 ± 3
Na	-28 ± 35		-65 ± 11		-67 ± 20	9 ± 31	-7 ± 100	-93 ± 90	-21 ± 10
O	22 ± 7	21 ± 7	1 ± 5		5 ± 10	13 ± 6	9 ± 14	9 ± 12	4 ± 5
C	16 ± 4	15 ± 5	-10 ± 2	-6 ± 3	-12 ± 4	5 ± 3	-11 ± 12	-9 ± 10	-13 ± 2
H									
<i>Standard deviations ($\times 10^4$)</i>									
U^{235} only	104	104	66	106	96	82	139	159	155
U^{235} and U^{238}	86	69	54	76	73	53	59	59	49
All materials	34	32	28	45	51	31	68	61	26

all materials, and the standard deviation is markedly reduced to 0.3 to 0.7%, according to data set.

Before drawing conclusions from these results, possible disturbing features that might lead to systematic errors are considered in the following subsections.

4.2. Accidental Selection of Assemblies

The criticality calculations used for this study were drawn from many sources and none were specially commissioned. When comparing results for different data sets, it must be remembered that different investigators have chosen a different selection of assemblies from the complete set. One check that can be made is to compare the results for all assemblies in Table 5 with those in Table 6 for ZPR-III only. The latter group eliminates all the small Vera assemblies, all assemblies containing hydrogen, and all the plutonium-fuelled assemblies. Every one of these differences might have been thought to make a difference in the coefficients.

It can be seen from Tables 4 and 5 that any standard deviation for all assemblies tends to be slightly larger than that for ZPR-III only, but the difference is not marked enough to be important for three of the four sets compared. For FD1, on the other hand, there is a substantial increase in standard deviation when all assemblies are taken into account. This arises because the formula does not achieve as good a fit to the pluto-

nium-fuelled assemblies as for U^{235} -fuelled ones. The standard deviations after fitting are 0.013 and 0.006, respectively. Similarly, for three of the sets the differences in the regression coefficients are not important when their errors are taken into account. When comparing the fits for different data sets it is best, nevertheless, to take the regression coefficients for ZPR-III only, because all the investigators have tried to make a selection typical of the full set of 24.

4.3. Influence of the Plutonium Assemblies

As there are only 5 Pu^{239} -fuelled assemblies in a total of 38, it was thought unlikely that the inclusion of these 5 would affect significantly the coefficients for all other constituents. This was found to be so in comparisons of the coefficients for all assemblies and for U^{235} -fuelled assemblies only.

In order to put the Pu^{239} -fuelled assemblies on the same basis as the others, the core compositions were normalised to give 0.6 for Pu^{239} (that is $C_0 = 0.6$). If this somewhat arbitrary choice had made a difference, the usefulness of the fits would have been much reduced. Table 7 includes a fit to the FD2 results for which C_0 was set equal to 0.7. On comparing the coefficients and standard deviations with the corresponding results in Table 5, it can be seen that there are no significant differences.

TABLE 7. REGRESSION COEFFICIENTS AND STANDARD DEVIATIONS FOR FIT IN CERTAIN SPECIAL CASES

	S ∞ All assemblies YOMD	S ∞ ZPR-III only YOMD	S ∞ All assemblies FD1	S ∞ ZPR-III only FD1	S ∞ All assemblies FD2	S ∞ ZPR-III only FD2	$C_0 = 0.7$ S_4 All assemblies FD2
<i>Coefficients ($\times 10^4$)</i>							
Pu^{239} and U^{235} only: Pu^{239}	200 $\pm$ 52		238 $\pm$ 61		100 $\pm$ 62		168 $\pm$ 67
U^{235}	-16 $\pm$ 13	-16 $\pm$ 13	85 $\pm$ 25	141 $\pm$ 20	45 $\pm$ 25	83 $\pm$ 31	71 $\pm$ 27
Pu^{239} , U^{235} , and U^{238} : $Pu^{239} \times C_0$	199 $\pm$ 31		244 $\pm$ 59		152 $\pm$ 44		233 $\pm$ 42
U^{235}	16 $\pm$ 16	16 $\pm$ 17	98 $\pm$ 32	197 $\pm$ 23	194 $\pm$ 24	220 $\pm$ 15	184 $\pm$ 22
U^{238}	-9 $\pm$ 7	-9 $\pm$ 4	-5 $\pm$ 8	-18 $\pm$ 5	-35 $\pm$ 7	-45 $\pm$ 3	-40 $\pm$ 5
All materials: $Pu^{239} \times C_0$	224 $\pm$ 21		244 $\pm$ 38		185 $\pm$ 22		279 $\pm$ 24
U^{235}	41 $\pm$ 25	42 $\pm$ 25	72 $\pm$ 41	183 $\pm$ 37	194 $\pm$ 24	261 $\pm$ 18	249 $\pm$ 24
U^{238}	-12 $\pm$ 3	-12 $\pm$ 3	-5 $\pm$ 5	-18 $\pm$ 5	-41 $\pm$ 3	-49 $\pm$ 2	-47 $\pm$ 3
Mo/Zr	-77 $\pm$ 34	-77 $\pm$ 33					
SS	7 $\pm$ 2	7 $\pm$ 4	11 $\pm$ 5	10 $\pm$ 3	6 $\pm$ 3	4 $\pm$ 5	4 $\pm$ 3
Al	-9 $\pm$ 4	-9 $\pm$ 4	16 $\pm$ 8	-3 $\pm$ 6	-4 $\pm$ 5	-13 $\pm$ 3	-7 $\pm$ 5
Na	-60 $\pm$ 13	-60 $\pm$ 13	-24 $\pm$ 25	-70 $\pm$ 20	-12 $\pm$ 15	-24 $\pm$ 10	-11 $\pm$ 15
O	1 $\pm$ 2	1 $\pm$ 2	-5 $\pm$ 14	6 $\pm$ 10	-1 $\pm$ 8	5 $\pm$ 5	0 $\pm$ 8
C	-10 $\pm$ 3	-10 $\pm$ 3	-2 $\pm$ 4	-11 $\pm$ 4	-7 $\pm$ 2	-12 $\pm$ 2	-8 $\pm$ 2
H			-235 $\pm$ 44		-210 $\pm$ 26		-2202 $\pm$ 27
<i>Standard deviations ($\times 10^4$)</i>							
Pu^{239} and U^{235} only	90	61	137	92	139	147	150
Pu^{239} , U^{235} , and U^{238}	54	55	129	75	97	49	91
All materials	34	34	72	51	42	25	44

4.4. Use of S_4 rather than S -infinity Approximation

Most of the fits were completed at a time when only S_4 results were available. Later, the method was devised for making corrections to yield the values of k corresponding to the S_N approximation with an infinite number of angles (S -infinity). Fits to the k -values for S -infinity for YOMD, FD1, and FD2 are given in Table 7.

The standard deviations have been little altered by the change to S -infinity. It is still apparently somewhat easier to fit ZPR-III than the full set of assemblies. The regression coefficient associated with Pu^{239} is most affected, as expected since the correction to S -infinity is greatest for small cores and the three smallest cores are all Pu^{239} -fuelled. The changes in the regression coefficients associated with U^{235} and U^{238} are noteworthy though comparable with their errors. It may be concluded that general observations on the sources of error in a certain set may be based on the fit to S_4 calculations, but the regression formula for S -infinity is necessary if a predicted k is made for a new system, especially if Pu^{239} -fuelled.

4.5. Significance of the Fits

It can be seen from Tables 5 and 6 that the fitting of a regression formula produces a large reduction in the standard deviation. It can be shown that the reduction is highly significant by the standard method of analysis of variance described by Kempthorne.⁽⁹⁾ The fits to the YOMD results for ZPR-III only are taken as an example. The variance is reduced by the smallest ratio, in this case, and therefore it was expected that this would be a severe test.

The analysis of variance for the fit to only two coefficients is given in Table 8. The hypothesis to be tested is that the regression coefficients are both zero. The variance ratio, S_r^2/S^2 , would be expected to be unity. In fact, the value obtained, 5.76, has a 1% prob-

TABLE 8. ANALYSIS OF VARIANCE FOR THE REGRESSION ON U^{235} AND U^{238} ONLY FOR THE S_4 DATA FOR ZPR-III USING YOMD

	Degrees of freedom	Sum of squares ($\times 10^8$)	Mean square ($\times 10^8$)
Regression on U^{235} and U^{238}	2	33712	16856 = S_r^2
Remainder	21	61488	2928 = S^2
	23	95200	

Variance ratio, $S_r^2/S^2 = 5.76$

1% point 5.78

5% point 3.47

10% point 2.57

TABLE 9. ANALYSIS OF VARIANCE FOR THE REGRESSION ON 8 MATERIALS FOR THE S_4 DATA FOR ZPR-III USING YOMD

	Degrees of freedom	Sum of squares ($\times 10^8$)	Mean square ($\times 10^8$)
Regression on 8 materials	8	83860	10482 = S_r^2
Remainder	15	11340	756 = S^2
	23	95200	

Variance ratio, $S_r^2/S^2 = 13.9$

1% point 4.00

5% point 2.64

10% point 2.12

ability of occurring as high as this by chance. Thus there is a significant regression on U^{235} and U^{238} alone.

The analysis of variance for the fit to the eight coefficients corresponding to all materials is given in Table 9. In this case, the value of S_r^2/S^2 is highly significant.

This exact analysis can be checked by comparing the coefficients in Table 6 with their standard errors. For the fit to YOMD with 8 materials, all the coefficients are significantly different from zero except that associated with oxygen.

4.6. Systematic Errors in the Data Used for Fitting

The discrepancies remaining after fitting, $k - k_f$, were examined to see if there were any evidence for systematic errors in the data used. Table 10 shows these discrepancies for ZPR-III for six data sets. There is a hint that certain assemblies always give positive discrepancies and others negative ones.

The analysis of variance shown in Table 11 was performed to check significance. The writer was unable to find a standard procedure for performing the analysis of variance in this case. The difficulty arises because there are 104 items of information in Table 10 (not counting the 4 results where an exact fit was obtained), but only 64 degrees of freedom are left after fitting the regression formula. A standard form of the analysis of variance as described by Kempthorne⁽⁹⁾ is used in Table 11 with the numbers of degrees of freedom multiplied by the factor 64/104. This arbitrary choice apparently has the correct form.

It can be seen that, in Table 11, there is a significantly greater variation between assemblies than within individual assemblies. Any error due to the arbitrary choice mentioned above is very unlikely to alter this conclusion. The variation within individual assemblies could be due (among other things) to:

- (i) the inability of the simple regression formula to allow completely for errors in the data sets, and
- (ii) mistakes in calculations.

The variation between assemblies could be due (among other things) to:

- (i) the inability of the simple regression formula to allow completely for errors in data sets *plus* the use of data of common origin in different sets;
- (ii) errors in calculation methods (such as use of S_4 instead of S_{-inf});
- (iii) errors in assessing shape factors;
- (iv) errors in heterogeneity corrections;
- (v) mistakes in evaluating experimental critical mass, compositions, and dimensions.

The last two in this list seem the most likely causes of the variation.

The residual may be written

$$k - k_f = A_i + e_{ij},$$

where A_i is the systematic error for a particular assembly i due to factors (i) to (v) above, and e_{ij} is the error for assembly i with data set j due to errors in the data set. Both A_i and e_{ij} are assumed normally distributed, and the standard deviation corresponding to A_i is given by

$$\sigma_A^2 = S_3^2 - S_1^2.$$

TABLE 10. REMAINING DISCREPANCIES, $k - k_f$ ($\times 10^4$), AFTER FITTING TO S_4 CALCULATIONS FOR ZPR-III

Assembly	HRB	YOMD	FD1	ANL-800	Modified ABBN	FD2	Average
2A		-21	-34			-41	-32 $\pm$ 22
5		8	44			40	31 $\pm$ 22
6F		-11	-15	16		-1	-3 $\pm$ 19
9A		-35	-26	-15		4	-18 $\pm$ 19
10	-27	-24	-66			-3	-30 $\pm$ 19
11	-12	-23	-18	5 ^(a)	9	2	-6 $\pm$ 16
12	-9	15	23	-33	13	-1	1 $\pm$ 16
14	5	-16	-35	18		-10	-8 $\pm$ 17
16	18	24	36			9	22 $\pm$ 19
17	4	14	20			8	12 $\pm$ 19
20	exact	exact					
23	-14	8	43	10	-55	6	0 $\pm$ 16
24	8	19	35		23	13	20 $\pm$ 17
25	21	-10	40	33	-41	11	9 $\pm$ 16
29	-26	-13	-20	exact	-23	-13	-19 $\pm$ 17
30	63	39	54		54	31	48 $\pm$ 17
31	27	11	28		68	15	30 $\pm$ 17
32	-26	-21	-21	-19	-26	-14	-21 $\pm$ 16
33	29	48	56	32	exact	21	37 $\pm$ 17
34	-29	-30	-2	-4	-22	6	-14 $\pm$ 16
35		-9	-10			0	-6 $\pm$ 22
36	-29	-7	-13	-33		-22	-21 $\pm$ 17
41		33	-120	-10		-62	-40 $\pm$ 19

^(a) The value for assembly 22 has been entered against the almost identical assembly 11.

TABLE 11. ANALYSIS OF VARIANCE FOR THE REMAINING DISCREPANCIES, $k - k_f$, FOR THE S_4 DATA FOR ZPR-III

	Degrees of freedom, n	Sum of squares ($\times 10^8$)	Mean square ($\times 10^8$)	Standard deviation ($\times 10^4$) S^2
1. Variation within individual assemblies	50	38466	769	2.8
2. Variation between assemblies	14	53183	3799	
3. Total	64	91649	1432	3.8

Assumption made: $n_1 = 64 \times (82/104)$

$n_2 = 64 \times (22/104)$

$n_3 =$ total degrees of freedom remaining after fitting regression

Variance ratio 4.9

1% point 2.4

5% point 1.9

10% point 1.6

Hence from Table 11

$$\sigma_A = 0.0026.$$

This standard deviation is measure of the accuracy of experimental values of k for homogeneous spherical cores of ZPR-III. The above procedure is equivalent to the rigorously exact procedure described by Kempthorne.⁽⁹⁾ Here also, the arbitrary choice mentioned at the start of this subsection is unlikely to have affected the estimation of σ_A appreciably. Since the standard deviation obtained in Table 6, after fitting, is little different from σ_A , for YOMD, FD2, and ANL-800, it may be concluded that the k calculations plus regression formula fit the experimental data almost as well as can be expected.

The errors on the average discrepancies in Table 10 were derived from the lumped standard deviation in Table 11 ($\sigma_i = 0.0038$) form the formula

$$\sigma_m = \sigma_i / \sqrt{n},$$

where n is the number of entries for an individual assembly. The average discrepancies for assemblies 30, 33, and 41 are separately significant. The discrepancy for assembly 41 may be due to mistakes in the FD1 or FD2 calculations or both.

5. Comments on Accuracy of Each Data Set

5.1. Hansen and Roach Set

Either HRB or HRC gives a standard deviation before fitting (that is, fit to mean only) as small as many more recent sets; the standard deviation after fitting is among the smallest.

The coefficients for U^{235} and U^{238} in the fit to all materials for HRC are as small as for any set, as expected, since this set is known to have been adjusted

to fit critical sizes of small concentrated critical assemblies. The set would be improved by reducing the effective absorption in U^{238} . This is consistent with the conclusion by Martin⁽²⁰⁾ that the set would be improved by increasing $\nu\sigma_f$ for U^{238} . The present analysis also suggests altering the constants for oxygen and carbon in a way that would reduce reactivity.

The main changes in going from HRB to HRC (according to Table 6) are increases in the reactivity effect of U^{235} and reduction in the absorption in steel.

When the list of assemblies for HRB is extended to include the 3 Zebra assemblies, the Pu^{239} data are apparently shown as insufficiently reactive. This suggestion is based on only one calculation, for Zebra 3. Further calculations are necessary to confirm this point.

5.2. Davey's Modification of the Set by Yiftah, Okrent, and Moldauer

This set clearly sets the standard against which others must be judged, as far as U^{235} -fuelled systems are concerned. It has a smallest standard deviation before fitting for ZPR-III (see Table 6) of any set. The Pu^{239} data are in error, as can be seen from Table 5.

Despite the small initial standard deviation, the regression coefficients suggest significant errors associated with all core constituents except oxygen. The success of this set must be due to the fact that errors associated with different materials are small and vary in sign. For most assemblies there will be compensation of errors. An important effect is the compensation of overreactive U^{235} cross sections offset by underreactive U^{238} cross sections. This effect is, however, less marked than for any other set except HRC and ANL-800. The Pu^{239} data are considerably overreactive.

5.3. YIFB

Although YIFB is very similar to YOMD (see Sect. 3.5), the superiority of YOMD is demonstrated by the smaller standard deviation before fitting. The regression coefficients for YIFB are similar to those for YOMD, except in the case of steel and aluminium. The regression coefficients for U^{235} differ perhaps more than might be expected.

5.4. FD1

The standard deviations and regression coefficients for YIFB and FD1 were expected to be very similar since these sets differ only in the cross sections below 5.5 keV and most of the assemblies studied have hard spectra. The differences in standard deviations between YIFB and FD1 are probably not significant in view of the small number of calculations available for

YIFB. The regression coefficients are in good agreement with those for YIFB, except for that for U^{235} , which suggests that FD1 has slightly more reactive U^{235} data.

5.5. ANL Set 800

This set gives the initial standard deviation next smallest after YOMD. The mean value of k , 1.0075, for ZPR-III differs more from unity than that obtained with YOMD, namely, 1.0004.

The regression coefficients reveal that this set, like all other sets except HRC, achieves its success by offsetting positive errors in U^{235} with negative ones in U^{238} , but this effect is less marked than in other sets. The main source of error is associated with the diluent materials, two of which have coefficients significantly different from zero. It is noteworthy that the coefficients are positive for all 5 diluents. This is consistent with the observation by Meneghetti⁽²²⁾ that the positive error in k increases with increasing quantities of diluent material.

5.6. ABBN and Modified ABBN Sets

This set gives poor estimates of critical size. Even after fitting the regression formula, the standard deviation is the largest in Table 6. The modified ABBN has a better average k but no better standard deviation.

For both forms of the set, the only significant regression coefficients are those for U^{235} and U^{238} . The coefficients for the other 5 diluents are positive in some cases and negative in others. The net result is that the standard deviation is not reduced on extending the regression formula from 2 to 7 terms. The coefficients for U^{235} and U^{238} are unaltered (in the unmodified form) by such an extension. Both those facts demonstrate that, for this set, its limited success is achieved by offsetting positive errors in U^{235} by negative ones in U^{238} .

The modified set only differs in the fission cross-sections for U^{235} . As would be expected, the main change in regression coefficients, in consequence, is the U^{235} coefficient. The change for U^{238} is perhaps larger than would be expected.

5.7. FD2 Set

This new set based on unadjusted microscopic data gives, before adjustment, as poor a standard deviation as any set. It does, however, have the advantage that it calculates Pu^{239} -fuelled systems no worse than U^{235} -fuelled systems. The other sets, except for FD1, have not been adequately tested for Pu^{239} -fuelled systems. FD2 can give estimates of k to accuracy ± 0.003 (s.d.) for both U^{235} - and Pu^{239} -fuelled systems when

used with the regression formula evaluated in this paper.

FD2 achieves a measure of success by offsetting overreactive cross sections for Pu^{239} and U^{235} against underreactive cross sections for U^{238} . There are also significant errors associated with aluminium, sodium, and graphite. As for FD1, cores containing hydrogen are poorly calculated.

5.8. Comments on Data Classified by Elements

All the data sets (except HRB) apparently have overreactive U^{235} data, with the error smallest for YOMD and greatest for the ABBN set.

Pu^{239} data have only been tested adequately for FD2 and the three versions of the YOM set (YOMD, FD1, and FD2). FD2 has overreactive Pu^{239} data, but overreactive to an extent similar to that of U^{235} . For this reason, errors that compensate in U^{235} -fuelled cores also compensate in Pu^{239} -fuelled reactors equally well. All the variants of the YOM set have Pu^{239} data that are considerably more overreactive than the U^{235} data. The calculated k for a critical Pu^{239} -fuelled core is about 2% higher than that of a similar critical U^{235} -fuelled core. There is a hint that HRB may have underreactive Pu^{239} data, but this is based on only one calculation (as discussed in Sect. 5.1). Evidence from other integral experiments suggests that Pu^{239} and U^{235} data may be overreactive because the values of $\alpha = \sigma_c/\sigma_f$ may be too small.⁽²³⁾

The U^{238} data used in all data sets lead to underestimation of reactivity, with ANL-800 the best and FD2 the worst in this respect. Since U^{238} is present in most of the critical assemblies used for check calculations, there is some compensation of overreactive U^{235} by underreactive U^{238} .

Data for molybdenum and zirconium has only been tested in the cases of HRB, HRC, and YOMD. The data for one or other or both of these underestimate reactivity. The data for the constituents of stainless steel are variable in effects for different sets. FD2 apparently has good data. The data for aluminium are also variable. The data for sodium are also variable, with that in FD2 or ANL-800 perhaps the best.

No set apparently deals with both carbon and oxygen well. FD2 and YOMD do well with oxygen, and ANL-800 with carbon. There are substantial errors in calculations for systems containing hydrogen for the sets tested (FD1 and FD2).

Nonzero regression coefficients associated with diluents, including U^{238} , could be due to four possible types of error:

- (i) errors in group absorption cross sections of the diluent (and in fission cross sections for U^{238});
- (ii) errors in group total cross sections, affecting leakage;
- (iii) errors in group inelastic and elastic cross sections that affect reactivity by changing the absorption in other core constituents;
- (iv) errors in the spectral dependence of group absorption cross sections of constituents other than the diluent concerned.

In all the foregoing discussion in this section, a non-zero regression coefficient has been assumed to imply an error in the data for the corresponding diluent as if that coefficient could only be due to errors of the first three types. But it should be realised that the error could be the fourth type. If so, this means that the representation by the simple regression formula is inadequate to represent the change in errors with changing core spectrum.

The substantial error, 2 to 3% in k , associated with hydrogen for FD1 and FD2, might be due to such a spectrum-dependent error. The sign of the coefficient for hydrogen would imply that the low-energy data for the major constituents (U^{235} and U^{238}) are underreactive. On the other hand, the coefficient associated with hydrogen might arise because of an error in the handling of hydrogen in multigroup calculations. The data for hydrogen are presumably accurate.

In a similar manner, nonzero coefficients for diluents other than U^{238} might be due to spectrum-dependent errors in the data for the heavy elements. If this were the explanation, then one might expect all the coefficients to have the same sign as, in fact, is observed with ANL-800. For other sets, the coefficients have varying signs, suggesting that there are really errors in the data for individual diluents.

6. Predictions of Critical Mass for New Assemblies

The regression formula corresponding to any data set can be used to correct the prediction of k for a future fast critical assembly. From Tables 5 and 6, it can be seen that the standard error in k can be as low as 0.003 for certain data sets. This is close to the accuracy with which k can be measured in a reactor like Zebra. Several data sets might be used, each giving a prediction of accuracy of about 0.003. Since these estimates are approximately independent, they may be averaged with consequent reduction in the standard error.

To achieve accuracy as good as this, it is necessary that the data set used for the multigroup calculation of the unknown system should be identical with that used for the check calculations upon which the regression formula is based. In the course of this work, the writer has discovered calculations that differed in k by

as much as 1%, even though the calculations purported to be for identical models with identical data!

The regression formula is really an interpolation formula, and accurate predictions cannot be expected if the core of the unknown reactor differs markedly in composition and neutron spectrum from the majority of those used in compiling the formula. Only 5 of the 38 fast critical assemblies in Table 4 are Pu²³⁹-fuelled, and few have a core neutron spectrum as soft as that in power reactors proposed for the future.

It should also be noted that all but a very few of the assemblies in Table 4 have a similar blanket (solid metallic uranium in steel tubes for Zebra and ZPR-III; the same with slight dilution with stainless steel for Vera). This is ideal when the regression formula is to be used for spotting the sources of error in data sets. On the other hand, when the formula is to be used to predict future systems, no allowance can be made for a new type of blanket. That the blanket is not negligible can be seen from the raw results in Table 4. Assemblies 5 and 6F of ZPR-III have identical core but different blanket compositions. The predicted values of k for the same data can differ by as much as 0.006 (for FD1).

After stating the possibilities of systematic error in the method, the regression formula will now be used to correct the multigroup calculation, using FD2, of the critical mass of ZPR-III assembly 48. This is a severe test because assembly 48 is Pu²³⁹-fuelled and only 5 Pu²³⁹-fuelled assemblies are used in compiling the regression formula. However, since the residuals $k - k_f$ for these 5 assemblies using FD2 do not imply a higher standard deviation than that from the U²³⁵-fuelled assemblies, the estimated standard deviation from all assemblies (S-inf, Table 7) can be taken as appropriate to ZPR-III assembly 48. The calculation is summarised in Table 12. Taking two standard deviations as 95% confidence limits, the resulting predicted critical mass for the homogeneous cylindrical core is

$$278 \pm 18 \text{ kg Pu}^{239} \text{ (95\% confidence limits).}$$

The fit to the FD1 data is not used to make a second prediction because of the poor fit to the plutonium-

TABLE 12. PREDICTION OF CRITICAL MASS FOR ZPR-III ASSEMBLY 48 (HOMOGENEOUS MODEL) USING FD2 CALCULATIONS

Predicted k for S-inf using regression formula	1.0026 ± 0.0042
Predicted k for S ₄	1.0036 ± 0.0042
$f = (\Delta M/M)/(\Delta k/k)$	5.9
Core mass for $k(S_4) = 1$, kg Pu ²³⁹	271
Predicted critical mass, kg Pu ²³⁹ (homogeneous cylindrical core)	277 ± 9 (s.d.)

fuelled assemblies in that case (see Sect. 4.2), especially for Zebra 6A, which is close in composition to ZPR-III-48. In fact, the regression formulas predict that the calculated k for ZPR-III-48 should be 1.9% higher using FD1 than FD2, whereas multigroup calculations give a difference of 4.1%.

7. Conclusion

It has been shown that a linear regression formula, using atom ratios, can fit the complete results of criticality calculations using a particular cross-section set for 38 fast critical assemblies based on Zebra, Vera, and ZPR-III. Nine cross-section sets, from the U.K., U.S.A., and U.S.S.R., have been considered. The fitting process reduces the standard deviation from about 1% in k for unadjusted data to about 0.3%. This reduced standard deviation is close to the standard error expected for the experimental results, after reduction to refer to a homogeneous spherical core.

The regression coefficients give clues to the changes necessary to improve each cross-section set. The most common faults are errors in data for U²³⁵ and Pu²³⁹ that lead to an overestimate of reactivity, offset partially by data for U²³⁸ that lead to an underestimate. Detailed comments are given for each cross-section set in Sect. 5. This approach cannot say how to alter the data to force agreement. The study of other integral quantities will be necessary to distinguish, for example, between increasing the reactivity effect of U²³⁸ by reducing the group capture cross sections or by reducing the inelastic and elastic cross sections to increase the median energy of the neutron spectrum.

The regression formula for any data set may be used as in interpolation formula for correcting the critical-mass prediction for a new critical assembly. Provided the system is not very different (especially in composition and neutron spectrum in the core) from those used in producing the formula, it should be possible to predict k for a given reactor with a standard deviation of 0.003. If several data sets and corresponding regression formulas are used, the predictions are approximately independent and may be averaged with consequent reduction in the standard deviation. Further checks on further systems fuelled with plutonium and study of the likely error in k due to uncertainty in blanket calculations are necessary before the method can be used with confidence.

As a first trial of the method, the corresponding regression formula was used to correct the prediction using FD2 of the critical mass of the plutonium-fuelled ZPR-III assembly 48. (The experimental result was unknown at the time this report was written.) The predicted result for the critical mass, corrected to

refer to a homogeneous cylindrical core, was 278 ± 18 kg Pu²³⁹ (95% confidence limits).

Acknowledgments

The author would like to thank Mr. D. W. B. Bale who wrote the computer programme KFIT without which this work would not have been possible; Mr. D. J. McKenna who ran all the production runs with the programme; Mr. I. C. Pull for advice on statistics and Mr. D. Wardleworth for allowing the use of his extensive results calculated with FD1 and FD2 in advance of publication.

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Discussion

Mr. Gandini (CNEN): Do you believe that fitting the critical masses could also improve perturbation calculations?

Mr. Baker: I hope that it will, and the next step is to see whether there is any correlation between the coefficients that are obtained and the errors that are found in other types of measurements, in particular, in perturbation measurements. As yet we have not done this.

Mr. Gandini: We have found that k_{eff} agrees well but perturbations do not.

Mr. Baker: Obviously in trying to predict critical masses of unknown systems we don't have to know what it does to other integral quantities. If you are looking at this type of evaluation as a clue to errors in cross-section sets, then you must make such a study.

Mr. Campise (AI): Was there some reason that you used Bondarenko's cross sections but not the critical-assembly results that he reported at the 1964 Geneva Conference?

Mr. Baker: I used the Russian cross-section sets and the Russian calculations. I did correct them to get the equivalent S_4 results since they were diffusion-theory calculations. I didn't use the Russian assemblies and their calculations. I was unable to make an independent evaluation of their experiments since enough detail about exactly how those systems were built was not available. Furthermore, there was quite enough work with 9 cross-section sets and ZPR-3, ZEBRA, and VERA results.

Presented by M. E. Battat

Comparison of Multigroup Cross-section Sets Used in Reactor Calculations*

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ABSTRACT

In order to provide a common basis for comparing various neutron multigroup cross-section sets, calculational models of two well-defined critical experiments have been specified. The critical assemblies chosen for this study were the Jezebel (Pu) assembly and the 610-liter ZPR-VI (Assembly No. 2) uranium carbide core. The Hansen-Roach 16-group and Russian 26-group cross-section sets were used for the initial comparisons. Calculations were made to determine the critical core volumes and central reactivity worths by use of the DTF-IV transport code in the S_8 approximation and assuming spherical geometry. In addition, two types of perturbation-theory calculations were made to arrive at central reactivity worths for "small" samples. The first of these is used to compute worths in terms of simple atomic perturbation cross sections. The second is based on transport perturbation theory, using real and adjoint currents and fluxes, with results expressed in terms of $\Delta k/g\text{-atom}$.

Wherever available, experimental data are presented for comparison with calculations. Central reactivity worths have been calculated for the following materials: ^{239}Pu , ^{240}Pu , ^{238}U , ^{235}U , depleted U, Fe, Ni, Cr, Na, Ta, Al, Mo, and B. In the case of ZPR-VI, the reactivity effect of voiding the core sodium was also calculated.

1. Introduction

Before one can confidently use a multigroup cross-section set in reactor calculations, it is necessary to compare results of theory with those of experiment. In general, comparison of calculated critical masses with critical experiment results are difficult to interpret for two reasons. One is the possibility of compensating errors when cross sections for several elements are needed to calculate the system. The second is the uncertainty introduced, and the additional calculations required, to account for heterogeneity ef-

fects. Comparing calculated central reactivity worths with experiment allows one to examine a single element at a time and thus simplify the evaluation. Adopting the latter approach, we have initiated a study to compare two multigroup cross-section sets: The 16-group Hansen-Roach set⁽¹⁾ and the 26-group Russian set.⁽²⁾ The critical experiments chosen for comparison were the Jezebel (all plutonium) assembly⁽³⁾ and the 610-liter ZPR-VI (Assembly No. 2) UC core.⁽⁴⁾ This approach provides a common basis for intercomparison and can, in principle, indicate more specifically the reason for discrepancies between the various multigroup sets.

2. Calculational Methods

The methods used in arriving at central reactivity worths were (1) direct comparison of k_{eff} with and without the sample in a central void, (2) calculation of simple atomic perturbation cross sections, and (3) transport perturbation-theory calculations.

Calculations were made by the DTF-IV code⁽⁵⁾ (multigroup transport) for spherical geometry, S_8 option, and convergence criterion equal to 10^{-7} . In arriving at the $\Delta k/g\text{-atom}$ for finite sample sizes, the following procedure was used. For a given sample size:

- 1) Specify a central void region with radius equal to the sample size, and adjust core radius to achieve criticality ($k_{eff} = 1.000000$). Radius of void region and, in the case of ZPR-VI, blanket thickness are held constant.
- 2) By use of the critical configuration determined above, compute k_{eff} with sample substituted for void.
- 3) $\Delta k = k_{eff}(\text{sample in}) - 1.000000$. By use of Δk , volume of sample, and atom density of sample, compute $\Delta k/g\text{-atom}$.

Simple atomic perturbation cross sections, σ_p , for "small" samples were computed from the formula suggested by W. G. Davey for the ZPR-3 (Assembly 48) intercomparison calculations for this conference.[†] The formula used was

[†] Private communication.

* Work done under the auspices of the U.S. Atomic Energy Commission.

$$\sigma_p = \frac{\sum_i \nu_i \sigma_{fi} \Phi_i \sum_j \chi_j \Phi_j^+ - \sum_i \sigma_{ai} \Phi_i \Phi_i^+ + \sum_i \Phi_i \left[\sum_j \sigma_{i \rightarrow j} (\Phi_j^+ - \Phi_i^+) \right]}{\sum_i \Phi_i \Phi_i^+}$$

The real and adjoint fluxes were obtained from DTF-IV calculations ($k_{eff} = 1.000000$) for a fully fueled core.

"Small" sample worths ($\Delta k/g\text{-atom}$) were also obtained by use of the DAC code written by B. M. Carmichael of this Laboratory.* This code is based on transport perturbation theory, and uses real and adjoint fluxes and currents obtained from DTF-IV calculations ($k_{eff} = 1.000000$) for a fully fueled core.

3. Description of Calculational Models

Atom densities used in the specifications of the Jezebel and ZPR-VI systems are shown in Table 1. Also shown in this table are the calculated critical configurations, assuming homogeneity and spherical geometry, for several void radii. For the Jezebel assembly, unshielded cross sections were used in the DTF-IV calculations; in the Case of ZPR-VI, unshielded cross sections were used for all materials

TABLE 1. ATOM DENSITIES AND CALCULATED CRITICAL CONFIGURATIONS—JEZEBEL AND ZPR-VI ASSEMBLY No. 2.

(A) Atom Densities ($\times 10^{24}$ atoms/cc)			
Element	ZPR-VI Assembly No. 2		Jezebel Core
	Core	Blanket	
^{235}U	0.002285	8.104×10^{-5}	
^{238}U	0.009853	0.03993	
C	0.01290	—	
Na	0.007910	—	
Fe	0.01038	0.006782	
Cr	0.002983	0.001950	
Ni	0.001390	0.000909	
^{239}Pu	—	—	0.037684
^{240}Pu	—	—	0.001776

(B) Calculated Critical Configurations (spherical geometry)

Central Void Radius (cm)	ZPR-VI Assembly No. 2 ^(a) Core Radius (cm)		Jezebel Radius (cm)	
	H-R	RUSSIAN	H-R	RUSSIAN
0	51.2192 ^(b)	48.4455 ^(c)	6.2807	6.1353
1.50	51.2217	48.4483	—	—
2.54	51.2314	48.4592	—	—
0.7268	—	—	6.3041	6.1597

(a) Blanket thickness = 30.0 cm.

(b) Critical core volume = 563 liters.

(c) Critical core volume = 476 liters.

Measured critical core volume (cylinder) = 610 liters.

TABLE 2. SAMPLE SIZES AND ATOM DENSITIES USED IN CALCULATIONS

(A) Sample Sizes for DTF-IV Calculations		
Material	Radius of sample (cm)	
	ZPR-VI Assembly No. 2	Jezebel
^{239}Pu	—	0.7268
^{240}Pu	—	0.7268
^{238}U	—	0.7268
^{235}U	1.50	0.7268
Depleted U	2.54	—
Fe	2.54	0.7268
Ni	2.54	0.7268
Cr	2.54	0.7268
Na	2.54	—
Ta	—	0.7268
^{10}B	1.50	—
B (nat)	1.50	—

(B) Atom Density for DAC and DTF-IV Calculations

Material	Atom Density ($\times 10^{24}$ atoms/cc)
^{239}Pu	0.048
^{240}Pu	—
^{238}U	—
^{235}U	—
Depleted U:	
^{235}U	1.056×10^{-4}
^{238}U	4.8944×10^{-2}
Fe	0.0847
Ni	0.0913
Cr	0.0801
Na	0.0222
Ta	0.055249
B (nat):	
^{10}B	0.0291
^{11}B	0.1073

except core ^{235}U (200 b/atom), core ^{238}U (40 b/atom), and blanket ^{238}U (20 b/atom).

In specifying the sample sizes used in the DTF-IV calculations, an attempt was made, where possible, to match the volumes of the samples used in the experiments. The sample dimensions and densities chosen for the calculations are shown in Table 2.

For the "small" sample calculations (σ_p and DAC) of reactivity worth, fluxes and currents for the sample were appropriate to the first mesh point specified for the DTF-IV problems. These corresponded to radii of 0.1 and 0.1434 cm for ZPR-VI and Jezebel, respectively.

TABLE 3. CENTRAL REACTIVITY WORTHS FOR ZPR-VI ASSEMBLY NO. 2

Material	σ_p (mb)		$\Delta k/g\text{-atom}$ (DAC code) ^(d)		$\Delta k/g\text{-atom}$ (DTF-IV code) ^(d)		$\Delta k/g\text{-atom}$
	H-R ^(a)	RUSSIAN	H-R	RUSSIAN	H-R	RUSSIAN	Measured
²³⁵ U	2279	2245	5.836 -04	6.710 -04	6.51 -04	7.63 -04	5.44 -04
²³⁸ U	-143.6	-134.9	-3.764 -05	-4.132 -04	—	—	—
Depleted U	(-138.3) ^(b)	(-129.6)	-3.579 -05	-3.924 -05	-3.77 -05	-4.19 -05	-3.29 -05
Fe	-16.0	-15.6	-4.167 -06	-4.719 -06	-3.83 -06	-4.04 -06	-3.55 -06
Ni	-55.9	-33.0	-1.453 -05	-1.004 -05	-1.30 -05	-8.84 -06	—
Cr	-55.7 ^(c)	-18.5	-1.448 -05 ^(c)	-5.611 -06	-1.39 -05 ^(c)	-4.71 -06	—
Na	6.1	4.3	1.595 -06	1.095 -06	1.7 -06	1.4 -06	0.82 -06
B (nat)	-357.8	(-358.6)	-9.288 -05	-1.011 -04	-8.43 -05	-9.68 -05	(-8.53 -05)
¹⁰ B	—	-1719	—	—	—	-4.38 -04	-4.09 -04
¹¹ B	—	10.4	—	—	—	—	—
²³⁹ Pu	3424	3479	—	—	—	—	—
²⁴⁰ Pu	826.8	514.7	—	—	—	—	—
Al	-2.02	-4.55	—	—	—	—	—
Mo	-134.5	-106.5	—	—	—	—	—
Ta	-819.1	-580.3	—	—	—	—	—

(a) H-R = Hansen-Roach.

(b) Figures in parentheses inferred from isotopic values.

(c) Aerojet cross sections.

(d) See text.

TABLE 4. CENTRAL REACTIVITY WORTHS FOR JEZEBEL

Material	σ_p (mb)		$\Delta k/g\text{-atom}$ (DAC code) ^(d)		$\Delta k/g\text{-atom}$ (DTF-IV code) ^(d)		$\Delta k/g\text{-atom}$
	H-R ^(a)	RUSSIAN	H-R	RUSSIAN	H-R	RUSSIAN	Measured
²³⁹ Pu	3707	3768	3.047 -02	3.357 -02	3.097 -02	3.367 -02	—
²⁴⁰ Pu	2519	2161	2.078 -02	1.935 -02	1.990 -02	1.803 -02	—
Pu (93.5% Pu-239)	(3651) ^(b)	(3693)	(3.00 -02)	(3.29 -02)	(3.05 -02)	(3.29 -02)	2.79 -02
²³⁸ U	246.5	283.7	2.031 -03	2.551 -03	1.720 -03	2.10 -03	—
²³⁵ U	1882	1964	1.546 -02	1.750 -02	1.484 -02	1.656 -02	—
Normal U	(258.4)	(296.0)	(2.129 -03)	(2.66 -03)	(1.82 -03)	(2.21 -03)	1.83 -03
Fe	-31.2	-37.6	-2.573 -04	-3.365 -04	-2.17 -04	-2.74 -04	-3.51 -04
Ni	-108.3	-94.6	-9.000 -04	-8.521 -04	-7.63 -04	-7.01 -04	-7.80 -04
Cr	-58.0 ^(c)	-39.3	-4.801 -04 ^(c)	-3.527 -04	-4.11 -04 ^(c)	-2.90 -04	—
Na	-11.1	-15.3	-8.945 -05	-1.539 -04	—	—	—
Ta	-235.3	-246.1	-1.957 -03	-2.221 -03	-1.72 -03	-1.88 -03	-1.63 -03
Mo	-74.7	-91.9	—	—	—	—	—
Al	-15.2	-22.6	—	—	—	—	—
¹⁰ B	—	-504.7	—	—	—	—	—
¹¹ B	—	-5.1	—	—	—	—	—
B (nat)	-108.7	(-111.7)	—	—	—	—	—
²⁴¹ Pu	—	3132	—	—	—	—	—

(a) H-R = Hansen-Roach.

(b) Figures in parentheses inferred from isotopic values.

(c) Aerojet cross sections.

(d) See text.

4. Results and Conclusions

Calculated central reactivity worths for ZPR-VI Assembly No. 2 and Jezebel are shown in Tables 3 and 4, respectively. Consider first the "small" sample reactivity worths as given by the σ_p and DAC calculations. Note that the normalization factor used in

DAC is integrated over the entire system, whereas the one used in σ_p is not. Hence, for a given material the different critical radii (see Table 1) obtained using the Hansen-Roach and Russian cross sections must be considered in comparing the $\Delta k/g\text{-atom}$ ratios (Hansen-Roach/Russian) for each system. From our

Fig. 1. Jezebel Assembly. Neutron spectrum for 0.7268-cm central region.

calculations, we infer that the Hansen-Roach $\Delta k/g$ -atom values should be multiplied by 1.17 (for ZPR-VI) and 1.08 (for Jezebel) in order to effect a comparison on the same basis as σ_p . These factors are approximately the ratios of the core volumes (Hansen-Roach/Russian) for each of the systems. Furthermore, the same considerations apply to the calculations (DTF-IV) for finite sample size. Finally, in comparing calculated worths—except for σ_p —with experiment, the same type of adjustment must be made. The σ_p calculations, however, are advantageous in that the computed perturbation cross sections are independent of the dimensions of the system.

The utility of calculations of central reactivity worth for cross-section comparison is illustrated by considering the elements nickel and chromium. In the hard spectrum of Jezebel (see Fig. 1), the nickel σ_p 's for the Hansen-Roach and Russian cross section sets differ by only 15 percent. However, in the softer spectrum of ZPR-VI Assembly No. 2 (see Fig. 2), the corresponding difference is 70 percent. Exami-

nation of the DTF-IV results for nickel in ZPR-VI indicates that the major part of the latter discrepancy can be attributed to the much higher capture cross section assigned by Hansen-Roach in the 2–17-keV energy range as compared to that in the Russian set. In a similar fashion, the large discrepancy in σ_p for chromium is due to major differences in the capture cross sections assigned in the 17–400-keV energy range.

One other calculation of interest is the reactivity effect of voiding the entire ZPR-VI (Assembly No. 2) core of sodium. The calculated percent $\Delta k/k$, for spherical geometry, using the Hansen-Roach and Russian cross section sets, were -2.21 and -2.17 , respectively. These figures are to be compared with -2.14 as determined experimentally for the actual system.

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Discussion

Mr. Gandini (CNEN): You mentioned the importance of an improvement in the transport approximation for perturbation theory with respect to sodium-void experiments. It is certainly an improvement, but it seems that this refinement may be largely overshadowed by the fact that variations of the real and adjoint

fluxes are not considered. These variations may be important in many cases as, for example, in the various sodium-void experiments when large quantities of sodium are removed. This can be shown by complete first-order perturbation techniques of the type developed by Usachev. We have used them and they are described in a subsequent paper at this conference.

Mr. Battat: The reason for showing this was to satisfy ourselves that we could do very well. It showed that certainly you can get into real difficulty if you believe it for the central reactivity worth.

Mr. Gandini: We calculated the sodium-void coefficient as a ratio of two functions of the neutron flux, using improved perturbation techniques. The flux-change rates are very important for the sodium-void coefficient, much higher than indicated by transport theory.

SESSION III-A

October 11, 1966

CLEAN CRITICAL EXPERIMENTS AND THEIR ANALYSIS

Chairman: G. T. Petersen
General Electric Co.

Secretaries: R. W. Goin
P. I. Amundson
Argonne National Laboratory

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21:17:33

Evaluation of Reactor Physics Experiments on the Coupled Fast-Thermal Argonaut Reactor STARK*

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Presented by H. Meister

ABSTRACT

STARK is a coupled zero-power reactor consisting of a subcritical fast core driven by a surrounding light-water-moderated thermal zone. Four different fast core loadings, containing up to 107 kg ^{235}U , have been investigated so far, one of them with a CH_2 -admixture to simulate the spectrum of a steam-cooled reactor.

A number of experiments have been performed to obtain information on the neutron spectrum in the fast zone. Spatial distributions of reaction rates were determined by fission-chamber and foil techniques; differential neutron spectra were studied by proton-recoil proportional counters; reactivity worths of samples were measured in the fast core with a square-wave pile oscillator. Also, integral parameters of the entire system were obtained by various kinetic methods.

The experimental results are compared with one-dimensional diffusion and S_4 calculations using the Russian 26-group cross-section set. The experimental data give no indication of larger deviations from the calculated spectra in the keV- and MeV-regions, whereas discrepancies were found in the region of lower eV. The integral reactor parameters (neutron lifetime, critical mass, and partition of reactor power between the zones) were found to agree reasonably well with the calculation.

1. Introduction

The zero-power facility STARK is a flexible fast-thermal reactor consisting of a subcritical fast core and a surrounding Argonaut-type thermal driver zone. The basic concept of the system⁽¹⁾ dates back to 1962 when it was recognized that a flexible source reactor would be needed in the earlier stages of the Karlsruhe fast reactor program and, therefore, it was decided to convert the Argonaut Reactor at Karlsruhe into a

coupled fast-thermal assembly. Compared to the ZPR-V experiments,⁽²⁾ only moderate fast-core power contributions ($\lesssim 0.40$) were planned such that the safety characteristics of the reactor still correspond to those of an all-thermal system.

Since the operation of STARK began in 1964, a number of reactor physics experiments have been completed. The experimental program was aimed at two main points:

- (a) development of experimental techniques for investigating fast zero-power systems, especially fast reactor mockups on the critical assembly SNEAK,⁽³⁾ and
- (b) investigation of the reactor physics properties of the coupled system STARK and comparison of experimental data with multigroup calculations.

The present paper primarily deals with the second point of this program, whereas the first point is partly treated in Ref. (4).

Compared to large all-fast systems, where the spectrum closely approaches the equilibrium spectrum throughout a major part of the assembly, coupled fast-thermal systems offer greater difficulties to a calculational treatment. Here the neutron spectrum shows a strong space dependence, especially in the low-energy region, and steep flux gradients occur so that certain standard methods (e.g., diffusion approximation) may not be adequate. Therefore, an analysis of experiments on fast-thermal systems is expected to give some general insight into the applicability and limitations of various calculational methods.

Besides this, a discussion of the results may also help to answer the question under which conditions can reactor physics data of all-fast systems (e.g., spectral indices and reactivity worths of samples) be obtained from experiments with fast-thermal assemblies?

2. Description of the Reactor

The basic geometrical structure of STARK is shown in Fig. 1. The fast core (of 37.2-cm average diameter)

* Work performed within the association in the field of fast reactors between the European Atomic Energy Community and Gesellschaft für Kernforschung m.b.H., Karlsruhe.

- a—fast core
 b—natural uranium zone
 c—graphite region
 d—thermal core (fuel region)
 e—thermal core (graphite pieces)
 f—graphite reflector
 v—vertical channel
 19; 25; 26 matrix positions for fission chamber
 and foil measurements
 R₁, R₂, R₃—fine-control plates
 S₁, S₂, S₃—safety plates
 S₄—fuel-poison safety rod

FIG. 1. Schematic Cross Section of STARK.

is formed by an array of 37 vertical stainless steel matrix tubes which are fixed by a bottom grid plate. The tubes can be filled with platelets (5.1 by 5.1 cm) of various core materials (cf. Sect. 3). In order to prevent strong peaks in the ^{235}U -fission-rate at the edge of the fast zone, the core structure is enclosed in a 5-cm-thick natural uranium casing which absorbs slow neutrons incident from the driver zone. Thus, coupling between the zones is maintained mainly by the exchange of fast neutrons.

The thermal core consists of up to 370 Argonaut-type fuel plates and light water as a moderator. The fuel plates, each of them containing 20.83 g ^{235}U (20 percent enriched), are arranged in 24 groups with a 6.3-mm spacing inside an annular aluminium tank; the volume between the fuel region and the outer tank wall is filled with graphite plates and wedges. Normally the thermal zone is operated at 80°C in order to guarantee a high degree of inherent shutdown capability of the reactor.

The control system acts primarily upon the thermal driver zone. Twelve cadmium-plate control units are

evenly distributed around the thermal core and provide a total shutdown reactivity of at least 2% k . Three of them can be used as fine-control units. An additional fuel-poison safety rod is installed at an eccentric position in the fast core. It consists of a vertically driven matrix tube which is filled with the normal core materials and to the upper part of which a boron carbide absorber is attached.

The regular neutron instrumentation is conventional. It comprises several ionisation chambers and BF_3 counters located in the graphite reflector. Additional neutron detectors for experiments can be placed into various channels of the reactor and into specially loaded matrix tubes of the fast core.

A more detailed description of STARK and its safety aspects is given in an earlier paper.⁽¹⁾

3. Core Configurations

Up to now, four reactor configurations have been investigated which differ largely in the material composition of the fast core and the number of fuel plates in the thermal zone.

In the first assembly the matrix tubes of the central zone were filled with natural uranium metal. Since the contribution of fast fissions to the total reactor power was only about 5 percent, this configuration may still be considered as an all-thermal system.

With the following assemblies the amount of fuel in the fast zone was raised stepwise up to 107 kg ^{235}U , while the ^{235}U content of the thermal zone had to be reduced from 7.6 kg to 4.5 kg ^{235}U to keep the system critical. With the most reactive fast core a fast fission contribution of ≈ 40 percent was reached so that the dependence of reactor parameters on the reactivity partition of the fast core could be studied over a relatively large interval.

The core materials for STARK are 3.15-mm-thick platelets of natural uranium, 20 percent enriched uranium, aluminium, and aluminium oxide, which are stacked horizontally inside the matrix tubes. As shown in Fig. 2, relatively simple loading patterns have been chosen, and no attempt has been made, because of the limited number of materials available, to simulate more "realistic" fast reactor core compositions.

In Assembly 2, a small amount of polyethylene (H/U ratio = 0.053) was added to the fast core in order to obtain a soft neutron spectrum similar to that of a steam-cooled fast reactor. The polyethylene was used in the form of discs (2.0 cm in diameter and 1 mm thick) which were placed into recesses machined into the aluminium platelets. Assemblies 3 and 4 had no polyethylene admixture, and were designed to render hard spectra and larger fast core power fractions. Table 1 gives the material composition of these assemblies.

FIG. 2. Loading Pattern of the Fast-core Elements.

TABLE 1. MATERIAL COMPOSITION OF THE STARK ASSEMBLIES (ATOMS PER CM³)

Zone No.	1				2	3	4	5	6	7
Zone	Fast core				Natural Uranium	Graphite + Air Gap	Thermal Core	Graphite	Graphite + Air Gap	Graphite Reflector
Outer radius, cm	18.6				24.2	30.5	38.5 to 45	46.5	50.5	86.5
Assembly	1	2	3	4	1 to 4	1 to 4	4	1 to 4	1 to 4	1 to 4
Material:										
H-1		$1.3081 \cdot 10^{21}$				$3.680 \cdot 10^{20}$	$3.726 \cdot 10^{22}$		$2.527 \cdot 10^{20}$	
C-12		$6.706 \cdot 10^{20}$				$4.922 \cdot 10^{22}$	$1.468 \cdot 10^{22}$	$8.526 \cdot 10^{22}$	$4.127 \cdot 10^{22}$	$8.526 \cdot 10^{22}$
O-16		$1.1637 \cdot 10^{22}$	$1.1543 \cdot 10^{22}$	$1.4428 \cdot 10^{22}$			$2.003 \cdot 10^{22}$			
Al-27		$1.7000 \cdot 10^{22}$	$7.695 \cdot 10^{21}$	$9.619 \cdot 10^{21}$		$9.264 \cdot 10^{21}$	$1.343 \cdot 10^{22}$		$3.484 \cdot 10^{21}$	
Fe-56	$6.042 \cdot 10^{21}$	$6.042 \cdot 10^{21}$	$6.042 \cdot 10^{21}$	$6.042 \cdot 10^{21}$						
U-235	$3.0318 \cdot 10^{20}$	$1.7659 \cdot 10^{21}$	$3.3863 \cdot 10^{21}$	$4.1597 \cdot 10^{21}$	$3.437 \cdot 10^{20}$		$1.0495 \cdot 10^{20}$			
U-238	$4.1805 \cdot 10^{22}$	$2.2822 \cdot 10^{22}$	$2.9134 \cdot 10^{22}$	$2.6330 \cdot 10^{22}$	$4.739 \cdot 10^{22}$		$4.193 \cdot 10^{20}$			

As shown in Fig. 2, 8.4-cm-thick axial reflectors of natural uranium metal were arranged at both ends of the fast core region. The height of the core filling was about 60 cm. It was chosen in such a way so as to render approximately equal axial bucklings in both core regions and to make a comparison with one-dimensional radial calculations meaningful.

4. Experiments

The objectives of the experimental work were to obtain data on the neutron spectra in the fast core, on the spatial distribution of reaction rates through-

out the reactor, and on central reactivity worths of various sample materials. Furthermore, the variations of reactor parameters, e.g., neutron lifetime and reactivity coefficients, were to be studied as a function of the partition of reactor power between both core regions.

In this section the experimental methods and procedures, some of them standard techniques, are outlined briefly. A discussion of the results and a comparison with calculations are given in the following section. Some of the more recent experimental methods used at STARK are described in a separate paper.⁽⁴⁾

4.1. Approach to Critical

For each assembly a critical configuration with given geometry and material composition of the fast zone was to be established, whereas the number of fuel plates in the thermal region was taken as a free parameter to adjust criticality.

The approach to critical usually started from a configuration in which the fast zone was completely filled with natural uranium and the thermal zone with about 80 percent of the critical fuel loading predicted by multigroup calculations. Then the natural uranium elements were replaced with loaded fast core elements. The thermal core was filled with moderator and half of the control plates were withdrawn to ensure complete control of multiplication during the loading process. Finally, the loading of the thermal zone was completed until the reactor went critical.

The critical masses of the thermal zone are listed in Table 2 for the various assemblies. Agreement with calculated values was found within ± 6.5 percent.

4.2. Measurement of Reaction Rates

4.2.1. Fission Rates

Three different methods were employed to determine fission rates in the fast zone of STARK:

- the use of parallel-plate absolute fission chambers;
- the use of small cylindrical fission chambers;
- γ -counting of irradiated uranium foils.

In the thermal zone only measurements with a 3-mm-diameter ^{235}U -chamber could be made because of the small spacing between the fuel plates. However, pairs of gold and copper foils were irradiated there to investigate axial, radial, and azimuthal distributions of the thermal and epithermal neutron fluxes.

As a basis for a calibration of the other counters, absolute measurements with parallel-plate fission chambers were performed in the center of the fast core. The chambers (4.0 cm in diameter, with a 0.5-cm plate spacing) were similar to those described by Kim⁽⁵⁾ and were operated as flow counters with an argon-methane

TABLE 2. EXPERIMENTAL REACTOR PARAMETERS OF STARK COMPARED WITH THE 26-GROUP DIFFUSION CALCULATION

	Relative Exp Error, %	Assembly 1		Assembly 2		Assembly 3		Assembly 4	
		Exp	Calc	Exp	Calc	Exp	Calc	Exp	Calc
<i>Criticality data:</i>									
Fuel mass, fast core, kg ²³⁵ U	±1	7.84		44.59		87.23		106.90	
Fuel enrichment, fast core, a/o		0.72	0.72	7.06	7.06	10.36	10.36	13.57	13.57
Fuel mass, thermal core (20°C), kg ²³⁵ U		7.30	7.759	6.53	6.911	5.56	5.829	4.43	4.429
Deviation of calc <i>k</i> _{eff} from exp, % <i>k</i>			-1.53		-1.28		-0.86		-0.01
<i>Power contributions %:</i>									
Fast core, γ ₁	±3	2.0	1.82	9.3	8.51	20.0	19.42	31.1	33.49
Uranium zone, γ ₂	±10	9.2	8.14	9.6	9.15	10.2	10.07	10.9	11.36
<i>Reactivity contributions %:</i>									
Fast core, α ₁			1.22		7.60		24.95		49.24
Uranium zone, α ₂			7.37		9.39		11.14		11.75
<i>Reactivity worth (central element), %:</i>									
Core material vs. void	±5			+0.028	+0.019	+0.213	+0.203	+0.483	+0.627
Worth of safety rod, S ₄	±3	-0.06		-0.296	-0.30	-0.60	-0.63	-1.24	-1.17
<i>Reactivity worths, thermal core:</i>									
Worth of outer fuel plate, % <i>k</i>	±5	+0.075	+0.0677	+0.071	+0.0701	+0.067	+0.0653	+0.051	+0.0545
Void coefficient, % <i>k</i> / % void	±8	-0.19	-0.235	-0.16	-0.212	-0.114	-0.165	-0.076	-0.103
Worth of temp change, 80 → 20°C, % <i>k</i>	±3	+1.05		+0.89		+0.625		+0.394	
Worth of 1 control plate, % <i>k</i>	±3	-0.48		-0.44		-0.340		-0.252	
Worth of all 12 control plates, % <i>k</i>	±8			-4.7		-3.5		-2.1	
<i>Kinetic parameters:</i>									
Generation time Λ, × 10 ⁻⁴ sec	±2	1.20	1.123	1.13	1.078	0.94	0.902	0.75	0.655
Delayed neutron fractions, × 10 ⁻³									
β _{eff} ²⁵			7.070		6.734		5.999		5.228
β _{eff} ²⁸			0.491		0.762		1.381		2.054
β _{eff} = β _{eff} ²⁵ + β _{eff} ²⁸			7.561		7.496		7.380		7.282

TABLE 3. CALIBRATION OF ABSOLUTE FISSION CHAMBERS

Sample	Material	Gravimetric Value, No. of Nuclei	Low-geometry α -Counting, No. of Nuclei
1	^{232}Th	$3.415 \cdot 10^{18}$	—
2	^{233}U	$7.43 \cdot 10^{17}$	$7.37 \cdot 10^{17}$
3	^{233}U	$3.16 \cdot 10^{18}$	$3.18 \cdot 10^{18}$
4	^{235}U	$8.38 \cdot 10^{17}$	—
5	^{238}U	$9.64 \cdot 10^{18}$	—
6	^{238}U	$3.74 \cdot 10^{18}$	$3.62 \cdot 10^{18}$

mixture. Two of these counters were placed back-to-back in a void volume inside the central matrix tube. The layers of fissionable material were commercially available samples of ^{232}Th , ^{233}U , ^{235}U , and ^{238}U ($\approx 300 \mu\text{g}/\text{cm}^2$ thick) of gravimetrically known mass,* which was found to agree with low-geometry α -counting within ± 1 percent for ^{233}U and within ± 3 percent for ^{238}U (see Table 3). A calibration by absolute β - γ -coincidence counting of a gold foil irradiated close to the sensitive area of the counter in the thermal column of the reactor gave 6 percent lower effective masses, probably due to fission fragment losses in the sample layer. Therefore, the absolute values of the fission rates and ratios should be considered as preliminary until measurements with thinner samples have been made.

To study the spatial distribution of fission rates in the fast zone, small cylindrical fission chambers (20th Century Electronics, FC-4, 6-mm outer diameter) were used which were coated with ^{232}Th , ^{233}U , ^{235}U , ^{238}U , ^{237}Np , and ^{239}Pu . The coating thickness was of the order of $500 \mu\text{g}/\text{cm}^2$. These chambers were calibrated against the parallel-plate absolute fission chambers in the center of the fast core or, in the cases of ^{237}Np and ^{239}Pu , with a 14-MeV neutron generator of known intensity.

Four cylindrical chambers were combined into a unit which was introduced into a 1.8-cm-wide vertical channel placed in one of the matrix tubes or in the natural uranium zone, so that they were exposed to nearly the same neutron field and data of four substances could be taken simultaneously. For the measurement of axial traverses the chambers could be moved vertically by an automatic driving mechanism. Radial traverses were also obtained by using different matrix positions.

Some results of the fission-chamber measurements are given in Figures 3 through 5. The ordinate values are fission rates n^x per atom in material x ($x = ^{235}\text{U}$, ^{238}U), normalized to a reactor power of 1 watt. Figure 3 shows an axial ^{235}U traverse taken in Assembly 4 between the fuel plates of the thermal zone at a dis-

* The foils were fabricated and weighed by the EURATOM Central Bureau for Nuclear Measurements, Geel, Belgium

FIG. 3. Axial Distribution of the ^{235}U -fission Rate in the Thermal Zone of STARK ($r = 34.8 \text{ cm}$, Assembly 4) at 1-watt Reactor Power.

FIG. 4. Axial Distribution of ^{235}U -fission Rate, n^{235} , per Atom at ≈ 1 -watt Reactor Power in the Central Matrix Tube of the Fast Core.

FIG. 5. Radial Distribution of ^{235}U - and ^{238}U -Fission Rates, n^{25} and n^{28} , in the Midplane of Assembly 4 at 1-watt Reactor Power

TABLE 4. EFFECTIVE CORE HEIGHTS H_{eff} (in cm) DERIVED FROM ^{235}U -FISSION RATE TRAVERSES

Assembly No.	Thermal zone	Fast zone
1	83 ± 2.5	85 ± 3
2	82.5 ± 2.5	86 ± 2
3	85 ± 2	85 ± 2
4	85 ± 2	83 ± 2

tance $r = 34.8$ cm from the center. An approximate cosine shape was found in the core region, except for perturbations close to the axial water reflectors and a bump at 2 cm below the reactor midplane, which is caused by an annular water channel in the graphite pieces around the fuel region. A cosine function was fitted to the unperturbed parts of the curve, leading to an effective core height $H_{\text{eff}} = 85.0 \pm 2$ cm.

Axial traverses in the central position of the fast core are shown in Fig. 4. Table 4 gives the effective core heights obtained by fitting cosine functions to the traverses. Deviations of ≤ 3.5 cm from the H_{eff} -value in the thermal zone were found. Also it is noted that in both core regions the maximum of the fission distribution is shifted by ≈ 1 cm towards the bottom with respect to the geometrical reactor midplane, probably due to control-plate shadowing and neutron losses in the slightly conical air gap at the outer wall of the thermal core.

Radial ^{235}U - and ^{238}U -traverses measured in the reactor midplane of Assembly 4 are plotted in Fig. 5.

In addition to the fission chamber data, the ^{235}U -fission rate in the thermal zone was also derived from the thermal flux ϕ_{th} and the epithermal flux ϕ_{ep} per unit lethargy at 4.9 eV found by the Au+Cu-foil activation:

$$n^{25}(r) = \sigma_{\text{th}}^{25} g^{25} \phi_{\text{th}} + \sigma_{\text{ep}}^{25} \phi_{\text{ep}}, \quad (4.1)$$

where

$$\sigma_{\text{th}}^{25} = 582 \text{ b} = \text{thermal fission cross section of } ^{235}\text{U};$$

$$g^{25} = 0.962 = \text{Westcott factor for } ^{235}\text{U at } 80^\circ\text{C};$$

the effective epithermal fission cross section

$$\sigma_{\text{ep}}^{25} = \frac{\sum_{i=1}^{25} \phi_i \sigma_i^{25}}{\phi_{21}/\Delta u} = 429 \text{ b} \quad (4.2)$$

was determined from the calculated 26-group fluxes ϕ_i in the thermal zone ($\phi_{21}/\Delta u$ = flux per unit lethargy at 4.9 eV, i.e., in group 21).

The radial distribution of the ^{235}U -fission rate, n^{25} , shows a steep gradient in the natural uranium and the outermost region of the fast core due to the strong absorption of slow neutrons coming from the thermal zone. In the inner region of the fast core relatively flat shapes of the ^{235}U - and ^{238}U -fission rates were found.

From the fission rates n^{25} and n^{28} , the number of fissions, Z , per unit volume in the actual reactor is derived:

$$Z(r) = N^{25} n^{25}(r) + N^{28} n^{28}(r), \quad (4.3)$$

where N^{25} , N^{28} = number of ^{235}U and ^{238}U nuclei per cm^3 of the core mixture (see Table 1).

Integration of $Z(r)$ leads to the power contributions

$$\gamma_i = \bar{Z}_i / \sum_{\text{all zones}} \bar{Z}_i; \quad \bar{Z}_i = 2\pi \int_{\text{zone } i} Z(r) r dr \quad (4.4)$$

of the various zones, which are compared in Table 2 with calculated values.

4.2.2. ^{238}U Capture-to-fission Ratio

Capture-to-fission ratios have been determined by irradiation of 0.1-mm-thick uranium foils at different positions between the platelets of the fast core and subsequent γ -counting of both the induced ^{239}Np and fission product activities.

To separate the γ activity caused by neutron capture in ^{238}U from the fission background, the γ -X-ray cascade of ^{239}Pu at 105 keV was observed by a coincidence technique in connection with an absolute calibration, using the α -decay of ^{243}Am as a standard.⁽⁴⁾ At the same time, fission product γ activities above 660 keV, which are proportional to the fission rate, were measured and the contributions of ^{235}U and ^{238}U were separated by using foils of different enrichment. A calibration of the foil activities in terms of absolute fission rates was achieved by a comparison with parallel-plate fission chambers.

Foil counting and data processing were carried out with an automatic sample changer⁽⁶⁾ in connection with a computer code, both specially developed for work with a large number of fission foils.

Figure 6 shows some of the experimental results found for Assembly 4 in comparison with the 26-group calculations (cf. Sect. 5.2).

4.2.3. Measurements with Resonance and Threshold Detectors

Additional information on the neutron spectrum in the eV- and keV-region was obtained from an investigation of (n, γ) and (n, n') reaction rates found by the foil-activation technique.

In Assemblies 2 and 4 several indium-foil measurements were performed, employing the reactions $^{115}\text{In}(n, \gamma)^{116m}\text{In}$ (first resonance at 1.46 eV), $^{113}\text{In}(n, \gamma)^{114m}\text{In}$ (main resonances at 4.7, 14.7, 21.7, 25.2, and 32.5 eV), and the reaction $^{115}\text{In}(n, n')^{115m}\text{In}$ (threshold at 335 keV). The foils were irradiated inside 0.6-cm-thick aluminium drawers which were embedded in the normal plate structure of the fast core at different matrix positions. The decay of the foil activity was observed with 2π - β -counters and analyzed, on the basis of the known half-lives, for the contributions of the three reactions to saturation activity.

The radial traverses of the reaction rates in Assembly

FIG. 6. Radial Dependence of the ^{238}U -capture Rate n^{28} and the $^{238}\text{U}/^{235}\text{U}$ Capture-to-fission Ratio, $\sigma_c^{28}/\sigma_f^{25}$, in the Midplane of Assembly 4.

4 (see Fig. 7) show a fairly good agreement with the 26-group diffusion calculation in the case of the (n, n') reaction, but larger discrepancies for the resonance reaction $^{115}\text{In}(n, \gamma)$, which are discussed in Sect. 5.2.

Experiments were also made with indium sandwiches, composed of three identical foils, in order to determine the contribution of the 1.46-eV resonance to the total foil activity. The experimental activity-ratio of outer to inner foil, C_o/C_i , is compared in Table 5 with values calculated from the 26-group spectra (cf. Sect. 5.1) on the basis of the known $^{115}\text{In}(n, \gamma)$ cross section, neglecting the effect of Doppler broadening. The experimental quantity $(C_o/C_i) - 1$, which increases with the ratio of the resonance flux at 1.46 eV to the total $^{115}\text{In}(n, \gamma)$ reaction rate, is found to be 3 to 6 times as high as calculated, showing that the intensity of eV-neutrons in the spectrum is much higher than expected from theory.

Further measurements to check the spectrum in the

keV-region were made in Assembly 2, using the resonance detectors ^{19}F , ^{23}Na , and ^{27}Al . The foil materials, their dimensions, the experimental setup including the rabbit irradiation system, and the procedure of data evaluation have been described.⁽⁴⁾ Irradiations of NaF foils were performed inside the loaded matrix tubes (positions 19, 25, and 30 of Fig. 1) of the fast core and in the empty channel of the natural uranium zone. The foils were covered by 1-mm cadmium to prevent

FIG. 7 Radial Distribution of Indium-foil Activity $C(r)$ in the Fast Zone of Assembly 4.

TABLE 5. ACTIVITY RATIO C_0/C_i (OUTER TO INNER FOIL) FOR INDIUM (^{115}In) SANDWICHES COMPARED WITH RESULTS OF 26-GROUP DIFFUSION CALCULATIONS

Foil Thickness, mg/cm ²	Assembly No.	Radial Position r , cm	Activity Ratio C_0/C_i		Exp Flux at 1.46 eV ÷ Calc Flux at 1.46 eV
			Experimental	Calculated	
$3\sqrt{\times} 36.4$	2	0	1.168 ± 0.020	1.027	9.4
$3\sqrt{\times} 11.3$	4	0	1.038 ± 0.020	1.001	—
3×87.7	4	0	1.012 ± 0.020	1.000	—
$3\sqrt{\times} 89.0$	4	16.2	1.396 ± 0.020	1.147	6.9

FIG. 8. (n, γ) Activation Integral $\int_{E_{\text{Cd}}}^{E_{\text{fiss max}}} \phi(E) \sigma_{n, \gamma}(E) dE$ for ^{23}Na , ^{27}Al , and ^{19}F as a Function of Radial Position r in the Midplane of Assembly 2.

activation by thermal neutrons. The irradiations of "Teflon" (^{19}F) and aluminum foils were performed in the empty matrix positions 19 and 34 by means of the pneumatic rabbit system.

The activation integral from cadmium cutoff to the maximum fission energy has been chosen as a basis for comparison of experimental and theoretical results. Figure 8 shows the activation integral derived from foil activities and the 26-group diffusion calculation as a function of radial position in the reactor midplane. The calculations have shown that in the spectrum of Assembly 2 the activation due to neutrons in the energy region of the main resonance compared to the activation due to all other neutrons is $\approx 81\%$ for ^{23}Na (main resonance at 2.95 keV), 63% for ^{19}F (main resonances at 15.3 and 27.3 keV), and for $^{27}\text{Al} \approx 46\%$ (first resonance at 5.9 keV) and $\approx 16\%$ (second resonance at 35.0 keV). The calculated curves were normalized at $r = 0$ to the experimental points. The experimental value at $r = 19.3$ cm in Fig. 8 is too large because the irradiation was made in the empty channel. The approximate shape of the curve in the full channel is also indicated (dashed-dotted line).

FIG. 9. Neutron Spectrum in the Center of Assembly 4.

Experimental and calculated curves agree in their shapes fairly well for ^{23}Na and ^{19}F , indicating no large discrepancies between calculation and experiment in the keV-region of the spectrum. The relatively large deviation for ^{27}Al is believed to be of systematic nature and has to be investigated further.

4.3. Differential Spectrum Measurements with Spherical Hydrogen Counters

Differential neutron spectra were measured in the fast core of Assembly 4 by the proton-recoil technique using hydrogen-filled spherical proportional counters.*⁽⁷⁾

* Counters fabricated by 20th Century Electronics, Great Britain.

Four counters, each 4 cm in diameter, with different fillings and pressures were used to cover the energy range from 25 keV to 1 MeV (see Fig. 9). The γ - n -discrimination technique,⁽⁸⁾ by which the low-energy limit of the 1-atm hydrogen counter could be lowered, has not been employed in the measurements performed so far.

Spectra were measured in the reactor midplane at various radial positions where the counter was placed in an empty volume inside the matrix tube. By operating the reactor at a low power level (≈ 10 mW), the counting rate was kept below 2000 cps so that deadtime losses could easily be corrected for by the live timer of the analyzer.

The well-known relationship

$$\phi(u) = E\phi(E) = -\frac{E^2}{NT\sigma(E)} \frac{dD(E)}{dE} \quad (4.5)$$

was used to convert the measured distribution of recoil protons per unit energy, $D(E)$, into the neutron flux $\phi(u)$ per unit lethargy. $\sigma(E)$ is the known n,p -scattering cross section, N the number of hydrogen atoms in the sensitive region of the detector, and T the live counting time. The energy calibration was made with monoenergetic neutrons from a Van de Graaff using the ${}^7\text{Li}(p,n){}^7\text{Be}$ reaction.

To analyze the counting data on the basis of eq. (4.5), a computer code was used which determines the logarithmic derivative of $D(E)$ and its error from a section of the experimental proton-recoil distribution. The slope-taking interval was chosen to be 25 percent, i.e., about twice the energy resolution of the detectors.

Wall effects were corrected for, as described by Benjamin,⁽⁷⁾ by first subtracting from the measured recoil spectrum the contribution of neutrons with energies above the upper limit of the counter, as calculated by a Monte Carlo code, and then multiplying the slope of the resulting recoil distribution with wall-effect factors, also derived from Monte Carlo calculations.

In Fig. 9 the experimental and calculated spectra are compared for the central position of Assembly 4.

4.4. Reactivity Measurements

4.4.1. Methods

To determine larger reactivity worths ($|\Delta\rho| \gtrsim 0.05$ \$) in STARK, positive-period, rod-drop, and "continuous run"⁽⁹⁾ techniques have been employed. For measurements of central reactivity worths of small samples a square-wave pile-oscillator was also used. In all cases the time dependence of the neutron population, $n(t)$, was observed and subsequently analyzed on the basis of the space-independent reactor model.

The neutrons were detected by a ${}^{10}\text{B}$ - or ${}^3\text{He}$ -ionization chamber normally located near the thermal core in the graphite reflector. The chamber signal was amplified by an electrometer amplifier (Keithley, Model 417) and then fed to a voltage-to-frequency converter, the output of which consisted of a pulse chain with frequency proportional to the input signal. The pulses were counted in a 256-channel multiscaler time analyzer (TMC, Model CN-110).

4.4.2. Calibration of Control Plates

For a fast and reliable control-plate calibration the rod-drop technique was found to be advantageous. Basically, the reactivity change was determined by

comparing the experimental decay of the neutron signal $n(t)$ with a set of $n(t)$ curves calculated for various reactivities.

For a source-free point reactor, the neutron signal after a step change in reactivity applied at $t = 0$ to the stationary and critical reactor is given by

$$\frac{n(t)}{n_0} = \sum_{j=0}^N A_j e^{s_j t}, \quad (4.6)$$

where

n_0 is the initial neutron signal at critical;

s_j ($j = 0, \dots, N$) are the solutions of the inhour equation

$$s \left(\Lambda + \sum_{i=1}^N \frac{\beta_{\text{eff } i}}{s + \lambda_i} \right) - \rho = 0; \quad (4.7)$$

Λ is the prompt-neutron generation time;

$\beta_{\text{eff } i}$, and λ_i ($i = 1, \dots, N$) are the delayed-neutron parameters;

$$A_j = \frac{\Lambda + \sum_{i=1}^N \frac{\beta_{\text{eff } i}}{s_j + \lambda_i}}{\Lambda + \sum_{i=1}^N \frac{\beta_{\text{eff } i}}{(s_j + \lambda_i)^2}}; \quad (j = 0, \dots, N); \quad (4.8)$$

and N is the number of delayed-neutron groups. In order to avoid systematic errors due to the differences in the λ_i values for ${}^{235}\text{U}$ and ${}^{238}\text{U}$,⁽¹⁰⁾ 12 delayed-neutron groups were used, six for each isotope in eq. (4.6) to (4.8). The effective parameters $\beta_{\text{eff } i}$ were derived from a 26-group perturbation calculation [cf. eq. (5.2) and Table 2].

A detailed analysis of the experimental $n(t)$ curves revealed a strong variation in the amplitude of the prompt transient with detector position, whereas a few seconds after completion of the reactivity change a common, space-independent time behavior was found. To eliminate spatial effects from the experimental data, $n(t)$ was normalized, not to the initial value n_0 , but rather to a value $n(T)$ some seconds after the drop ($T \gtrsim 5$ sec), which is not influenced by higher modes. The reactivities ρ/β_{eff} were obtained by comparison of the experimental ratios $n(t)/n(T)$, ($t > T$), with those calculated by eq. (4.6) and were found to be independent of detector position within ± 2 percent.

Compared to the space-independent model, the prompt transient is larger for detector positions close to the control plate and smaller for positions far away. On the boundary of both regions, positions exist (at an azimuthal angle of $\approx 60^\circ$ from the plate) where $n(t)$ was found to agree well with the space-independent calculation in its entire time dependence. In these positions complete control-plate characteristics were determined by the continuous-run technique in a single

experiment, and no systematic deviations from the other methods were seen.

Table 2 and Fig. 10 shows some results of the reactivity measurements for the safety rod S_4 in the fast zone and for various groups of control plates (see Fig. 1). A decrease of the thermal control-plate worths and an increase for the rod S_4 are noted as the power contribution of the fast core grows. The worth per control plate is seen to increase initially with the number of plates inserted and then to decrease again for larger numbers. This negative coupling effect, which results from a flux shifting towards the fast zone, increases with the importance of the fast zone.

4.4.3. Reactivity Coefficients in the Thermal Zone

For investigating the inherent shutdown mechanisms of the thermal zone, the overall reactivity effects of moderator temperature and void formation were measured.

The temperature coefficient was determined by continuously increasing the moderator temperature while the reactor was kept critical with a calibrated control plate. The calibration was performed in the same experiment by stepwise withdrawal of the plate and determination of the positive period.

In order to measure the reactivity effect of voids in the light-water moderator, a flat aluminium tube (cross section $0.4 \times 1.0 \text{ cm}^2$) was placed between two fuel plates, and the reactor was made critical while the tube (volume $\Delta V = 24 \text{ cm}^3$) was filled with air. By a sudden pressure release water was allowed to enter the tube, and the resulting reactivity $-\Delta\rho$ was derived from a period measurement. The moderator void coefficient $\Delta\rho/\Delta V$ was found to be negative throughout the thermal core.

Results of the average void coefficient $\Delta\rho/\Delta V$ and the temperature coefficient $\Delta\rho/\Delta T$ are given in Table 2.

4.4.4. Measurements of Reactivity Worth with a Pile Oscillator

Measurements of reactivity worths of small material samples (3 to 300 g) in the central channel of the fast zone have been carried out with a pile oscillator for a number of substances. The samples were fixed inside an aluminium tube which was connected to the piston of a driving mechanism actuated by compressed air. The stroke of the oscillator was 40.5 cm, so that the sample could be moved from the center of the core to a position outside the blanket. The transient time between both positions was 0.1 to 0.2 sec, a time such that the reactivity changed in a square-wave manner with a period $T = 25.4$ or 38.4 sec.

In order to study the effect of neutron streaming through the empty channel, measurements were per-

Fig. 10. Reactivity Worth per Control Plate Relative to the Worth of a Single Plate for Various Control-plate Configurations.

formed in Assembly 4, where the oscillating aluminium tube was substituted by a square stainless steel tube filled with 4.6- by 4.6-cm platelets of either core material or aluminium. Two aluminium containers, one for the sample and the other empty, were embedded in the oscillator rod at the two positions corresponding to the core center. With this apparatus the transient time was ≈ 1 sec and the period chosen as 64 sec.

The oscillation of the neutron flux was detected by a ^{10}B -ionization chamber placed near the core-reflector interface. The DC part of the chamber signal was suppressed and only the oscillating part was recorded, as described in Sect. 4.4.1, by a 256-channel time analyzer which was synchronized with the oscillator in such a manner that two consecutive periods were stored in each analyzer sweep. In this way the presence and the influence of a drift component on the signal could be observed. The mean reactor power was 10 watts; small reactivity drifts during the experiment were compensated by a slowly moving fine-control rod.

The experimental data were Fourier-analyzed on the IBM-7074 digital computer. From the Fourier amplitude, c , of the fundamental frequency $\omega = 2\pi/T$, the reactivity amplitude $\Delta = \rho/2$ of the sample was derived (assuming a square-wave oscillation) by the expression

$$\Delta = \frac{\pi}{4} \frac{c\Lambda}{n_0 |G_0(j\omega)|}, \quad (4.9)$$

where

Λ = generation time;
 n_0 = mean power level;

$$G_0(j\omega) = \frac{\Lambda}{j\omega \left(\Lambda + \sum_{i=1}^N \frac{\beta_{eff,i}}{j\omega + \lambda_i} \right)}$$

= calculated reactor transfer function.

In an ideal square-wave oscillation only odd harmonics should be present. In the actual experiment, however, even modes are also excited by reactor noise and by the neutron-scattering effect during the sample motion. When the oscillator tube is filled with core material, a strong additional component of twice the fundamental frequency is observed, which is caused by the periodical removal and reinsertion of irradiated fuel containing delayed-neutron precursors. All these effects, however, are easily eliminated by the Fourier analysis, since they

do not contribute to the fundamental mode of the oscillation.

The theoretical sensitivity of the oscillator experiment was calculated to be $1.2 \cdot 10^{-7} \Delta k/k$ for an integrated power of 10^4 watt-sec if only reactor noise is the limiting factor.⁽¹¹⁾ Measurements without sample oscillation gave 1 to 3 times this value.

Tables 6 and 7 give the experimental reactivity coefficients of several materials compared with calculations based on a first-order perturbation theory. Effects of sample size on the reactivity coefficient were found for CH_2 , B_4C , Ni, C, and (less pronounced) for U (19.2% enriched) and U_{nat} . They were eliminated by extrapolation to zero sample mass.

The comparison of sample worths (relative to ^{235}U) for Assembly 4 in the empty channel (ρ_{empty}) and the channel filled with core material (ρ_{core}) shows remarkable differences (see Table 7), especially for the scattering materials Al, C, and CH_2 . These differences are due to the anisotropy of the neutron flux $\phi(\vartheta)$ and the

TABLE 6. REACTIVITY COEFFICIENTS $\Delta\rho/\Delta N$ (IN Δk PER 10^{30} ATOMS) OF SAMPLE MATERIALS MEASURED IN THE CENTRAL POSITION OF THE FAST ZONE OF STARK; DATA CORRECTED FOR SAMPLE-SIZE EFFECT

Material	Assembly 2	Assembly 3	Assembly 4		
	Channel Empty	Channel Empty	Channel Empty	Channel filled with Al	Channel Filled with Core Material
Al	-4.95 ± 0.15	-11.9 ± 0.15	-18.6 ± 0.2	-13.46 ± 0.2	-13.44 ± 0.2
^{10}B	-281 ± 2	-386 ± 2	-715 ± 3	-741 ± 3	-821 ± 3
C	-4.73 ± 0.15	-12.1 ± 0.15	-14.7 ± 0.15	-9.83 ± 0.15	-9.55 ± 0.15
Fe	-5.62 ± 0.2	—	-22.8 ± 0.3	-17.4 ± 0.3	-19.5 ± 0.3
Ni	-8.0 ± 0.15	—	-32.1 ± 0.3	$+26.6 \pm 0.3$	-28.8 ± 0.3
^{235}U	$+327 \pm 2$	$+639 \pm 3$	$+1115 \pm 5$	$+1169 \pm 5$	$+1298 \pm 5$
^{238}U	-29.2 ± 0.2	-48.8 ± 0.3	-80.8 ± 0.5	-74.8 ± 0.5	-77.4 ± 0.5
B_4C	-244 ± 2	—	—	—	—
CH_2	$+66 \pm 1$	$+46.3 \pm 0.5$	$+169 \pm 2$	$+202 \pm 2$	$+221 \pm 2$

TABLE 7. CENTRAL REACTIVITY COEFFICIENTS $\Delta\rho/\Delta N$ OF SAMPLE MATERIALS IN STARK RELATIVE TO ^{235}U IN COMPARISON WITH 26-GROUP DIFFUSION CALCULATIONS

	Assembly 2			Assembly 3			Assembly 4		
	Experimental		Calculated	Experimental		Calculated	Experimental		Calculated
	Empty Channel	Scattering Corrected		Empty Channel	Scattering Corrected		Empty Channel	Channel Filled with Core Material	
Al	-0.0152	-0.011	-0.00821	-0.0186	-0.013	-0.00917	-0.0167	-0.0104	-0.00724
^{10}B	-0.860	-0.85	-0.765	-0.604	-0.60	-0.454	-0.641	-0.633	-0.489
C	-0.0145	-0.010	-0.00694	-0.0189	-0.014	-0.00645	-0.0132	-0.0073	-0.00186
Fe	-0.0172	-0.0128	-0.01192	—	—	-0.01290	-0.0204	-0.0150	-0.01224
Ni	-0.0245	-0.018	-0.01864	—	—	-0.02036	-0.0288	-0.0222	-0.02084
^{235}U	-0.0894	-0.079	-0.0664	-0.0763	-0.065	-0.0594	-0.0725	-0.0596	-0.0542
B_4C	-0.747	-0.74	-0.6053	—	—	-0.3703	-0.605	—	-0.3790
CH_2	+0.202	+0.218	+0.04455	+0.0728	+0.090	-0.00873	+0.152	+0.170	+0.0857

adjoint $\phi^+(\vartheta)$ in the empty channel, which leads to a negative net reactivity change even for a pure isotropic scatterer at the reactor center. The measured differences were found to be approximately proportional to the importance-weighted transport cross section

$$\rho_{\text{empty}} - \rho_{\text{core}} = \text{const} \sum_{i=1}^{26} \phi_i^+ \sigma_{tr,i} \phi_i \quad (4.10)$$

of the sample material, as obtained from the perturbation code (cf. Sect. 5.2), so that corrections could be made on this basis for the empty-channel measurements in Assembly 2 and 3 (see Table 7).

4.5. Kinetic Experiments

Pulsed-source and noise-analysis techniques were employed to study the variation of the prompt-neutron decay constant and the generation time with reactor configuration.

In the pulsed-source experiments a periodical sequence of d,t -neutron bursts was injected into the system, and the decay of the prompt-neutron population, as observed with a BF_3 counter in the air gap between thermal core and reflector, was analyzed for the decay constant α of the fundamental mode. The experimental setup and the data-evaluation procedure have been described.⁽¹²⁾

The decay constant α , which is related to reactivity ρ/β_{eff} and generation time Λ by

$$\alpha = \frac{\beta_{\text{eff}}}{\Lambda} [1 - (\rho/\beta_{\text{eff}})], \quad (4.11)$$

was measured in Assemblies 1 to 4 at delayed critical and at several subcritical states obtained by inserting various groups of control plates (see Fig. 1) into the critical reactor. The experimental α values are shown in Fig. 11 as a function of ρ/β_{eff} . The reactivity scale was determined by the rod-drop method (see Sect. 4.2), which is known to be insensitive to changes in generation time.

The curvature of the α -vs.-reactivity curves, increasing in magnitude from Assembly 1 to 4, shows that the generation time Λ decreases with subcriticality, due to the shielding of the thermal core and the reflector when cadmium plates are inserted. The inverse effect is observed when the system is made subcritical by insertion of the safety rod S_4 into the fast zone, by which the thermal core gains importance and the generation time grows.

Figure 11 also shows the values $\gamma = \beta_{\text{eff}}/\Lambda$ found by the method of Garelis and Russell⁽¹³⁾ from the prompt- and delayed-neutron intensities, $n_p(t)$ and $n_d(t)$, as solution of the equation

$$\int_0^\infty n_p(t)(e^{\gamma t} - 1) dt = \int_0^\infty n_d(t) dt. \quad (4.12)$$

Except for Assembly 2 the γ values vary only slightly,

FIG. 11. Prompt-neutron Decay Constant α and $\gamma = \beta_{\text{eff}}/\Lambda$ Obtained by Pulsed Neutron Experiments for Different Control-plate Configurations vs. Reactivity ρ/β_{eff} Determined by Rod-drop Measurements.

TABLE 8. PROMPT-NEUTRON DECAY CONSTANT AT DELAYED CRITICAL, $\alpha_c = \beta_{\text{eff}}/\Lambda_c$ (SEC^{-1})

Assembly No.	Pulsed-source method	Noise-analysis methods		
		Cross power spectral density	Rossi- α	Probability distribution
1	62.9 ± 0.8	63.4 ± 2	—	63 ± 2
2	66.4 ± 1.5	67 ± 2	71 ± 5	67 ± 2
3	78.3 ± 2	74.5 ± 2	—	—
4	96.7 ± 2	93 ± 4	93 ± 4	—

due to an appropriate choice of detector position, and converge against the decay constant $\alpha_c = \beta_{\text{eff}}/\Lambda_c$ measured directly at delayed critical.

Besides the pulsed-source measurements, noise-analysis experiments have been performed with the two-detector crosspower spectral density technique,^(12, 14) the Rossi- α method, and the probability distribution technique.^(4, 12) The results for the prompt-neutron decay constant (fundamental mode) at delayed critical found by the four methods are given in Table 8. The data agree within the error limits.

5. Discussion of the Results

5.1. Multigroup Calculations

As a basis for a discussion of the experimental results a number of one-dimensional multigroup calculations have been performed for the various STARK assemblies, using the Russian 26-group ABN-cross-section set.⁽¹⁵⁾

Most calculations were done in cylindrical geometry, with the reactor treated as a system of seven concentric zones, equal in area and material composition to the corresponding actual zones of the reactor:

- zone 1: fast core
- zone 2: natural uranium zone
- zone 3: graphite, aluminum-tank walls, air gap
- zone 4: thermal core
- zone 5: graphite inside the annular tank
- zone 6: tank walls, air gap, graphite
- zone 7: external graphite reflector.

Table 1 gives the geometry of the zones and the atom numbers per cm³ of the homogeneous mixture as derived from the material weights.

A constant axial buckling $B_z^2 = 14.68 \cdot 10^{-4} \text{ cm}^{-2}$ independent of zone and energy group was assumed, which corresponds to the effective core height $H_{\text{eff}} = 82 \text{ cm}$ found by fission-chamber traverses in the thermal core of Assembly 2. Experimental values of H_{eff} of the other assemblies deviate up to 4 cm from this value (see Table 4).

The calculations were carried out with the Karlsruhe nuclear code system NUSYS.⁽¹⁶⁾ As a first step, a diffusion calculation was performed to obtain spectra and reaction rates throughout the reactor. In order to find a critical system with given radius and material composition of the fast core, the thickness of the thermal zone was varied by radius iteration, taking into account the dependence of the average fuel concentration on the zone thickness due to the geometry of the graphite wedges (see Fig. 1). The results were carefully checked for influences of mesh width and number of iteration steps.

Then a perturbation calculation was made to derive the overall neutron-generation time

$$\Lambda = \frac{\int \sum_{i=1}^{26} \phi_i^+ \frac{1}{v_i} \phi_i dV}{\int \sum_{i,j=1}^{26} \phi_i^+ \chi_{i,j} \sum_{f,j} \phi_j dV} \quad (5.1)$$

and the effective delayed-neutron fractions of the fissionable isotopes ²³⁵U and ²³⁸U,

$$\beta_{\text{eff } k}^M = \beta_k^M \frac{\int \sum_{i,j=1}^{26} \phi_i^+ \chi_{i,k} \sum_{f,j}^M \phi_j dV}{\int \sum_{i,j=1}^{26} \phi_i^+ \chi_{i,j} \sum_{f,j} \phi_j dV}, \quad (5.2)$$

where

- $\beta_{\text{eff } k}^M, \beta_k^M$ = effective and actual delayed-neutron fractions of material M , kinetic group k ,
- $\sum_{f,j}^M$ = macroscopic fission cross section of material M , energy group j ,
- $\chi_{i,k}$ = energy distribution of the delayed neutrons, kinetic group k .

Some of the resulting kinetic parameters are given in Table 2.

Furthermore, the relative importance α_m of the core zones (index m) has been evaluated, using the equation

$$\alpha_m = \frac{\int_{\text{zone } m} \sum_{i,j} \phi_i^+ \chi_{i,j} \sum_{f,j} \phi_j dV}{\int_{\text{reactor}} \sum_{i,j} \phi_i^+ \chi_{i,j} \sum_{f,j} \phi_j dV}. \quad (5.3)$$

The α_m are identical with the coefficients of reactivity partition in the theory of coupled systems⁽¹⁷⁾ and describe the effect on the overall reactivity,

$$\Delta \rho = \sum_m \alpha_m \frac{\delta \nu_m}{\nu_m}, \quad (5.4)$$

of a fictitious change $\delta \nu_m$ of the number of fission neutrons in zone m . As shown in Table 2, the importance α_1 of the fast zone varies between 0.0122 (for Assembly 1) and 0.4924 (for Assembly 4).

Also, reactivity worths of different materials were determined by the perturbation code. The sample volume (usually at core center) was assumed to be filled with the normal core mixture to which the sample material was added in low concentration (10^{20} to 10^{21} atoms/cm³). In this way, the ideal worth of an infinitely small and dilute sample should be obtained and effects resulting from a variation of the resonance self-shielding factors should be excluded. Table 7 shows results for the central reactivity worths of various sample materials, relative to ²³⁵U.

Similar calculations were made for critical all-fast systems of cylindrical shape, having the same composition as the fast zone of STARK. These assemblies are large enough to render an equilibrium spectrum in the inner core region such that deviations of the STARK spectrum from the equilibrium spectrum can be studied. In addition, S_4 calculations with isotropic downscattering were carried out for STARK in order to get some information about the magnitude of systematic errors arising from the diffusion approximation.

5.2. Neutron Spectrum and Reaction Rates

The general behaviour of the neutron spectrum in STARK is illustrated in Fig. 12, where the calculated 26-group spectra are given for various radial positions

FIG. 12. Neutron Spectrum $\phi_i(r)$ and Its Deviation from the Equilibrium Spectrum ϕ_i^{as} at Different Radial Positions r in Assembly 2.

r in Assembly 2. The spectrum in the thermal zone follows roughly a $1/E$ -shape. With penetration into the fast zone, a strong decrease of low-energy neutrons ($E \lesssim 100$ eV) is seen, especially for the thermal group and at the main ^{238}U resonance (group 21), whereas the fast and intermediate neutrons show a relatively flat spatial distribution.

In the inner region of the fast core, the spectrum ϕ_i gradually approaches the equilibrium spectrum ϕ_i^{as} , as calculated for the critical one-zone system. The relative deviation $(\phi_i - \phi_i^{as})/\phi_i^{as}$ from the equilibrium spectrum, also given in Fig. 12 as a function of energy, indicates a surplus of slow neutrons ($E < 1$ keV) and of fission neutrons with respect to ϕ_i^{as} near the edge of the fast core. With further penetration into the core, these excess neutrons die out and a gradual softening occurs in

the main part of the spectrum (1 keV to 1 MeV). In the center of Assembly 2, the equilibrium spectrum is not yet completely reached, and deviations from ϕ_i^{as} of the order of 10 percent still remain for energies above 10 eV.

Figure 13 gives the same diagram for Assembly 4, which has a higher ^{235}U content and, therefore, leads to a faster decay of the spatial transients. Larger deviations of the central spectrum $\phi_i(0)$ from the equilibrium spectrum are only seen in the low-energy region; they are less than 2 percent in the main part of the spectrum ($E \gtrsim 10$ keV) where 99 percent of the ^{235}U -fission processes occur.

The results of the S_4 calculation included in Fig. 12 and 13 show that the number of slow neutrons in the fast zone is up to 80 percent higher than given by diffu-

FIG. 13. Neutron Spectrum $\phi_i(r)$ and Its Deviation from the Equilibrium Spectrum ϕ_i^{as} at Different Radial Positions r in Assembly 4.

sion theory, especially in those energy regions where the spectra differ strongly from ϕ_i^{as} , i.e., where strong spatial gradients are present. Above 10 keV, a good agreement between S_4 and diffusion calculations is found for the central region of the fast zone.

A direct comparison between calculated and experimental spectra in the energy range from 20 keV to 1 MeV has been made for Assembly 4 where hydrogen counter data are available. The experimental spectrum (see Fig. 9) shows resonance dips caused by the aluminum contained in the fast core, which are not seen in the 26-group spectrum because of its rough group structure. The experimental spectrum appears to be somewhat softer than the calculated one.

Further information on the neutron spectrum and its spatial variation can be gained from an analysis of reaction rates in a set of different substances.

The radial distribution of fission rates found with the cylindrical fission chambers (cf. Sect. 4.2.1) is compared in Fig. 14 with the 26-group calculation. The calculated fission rate $\bar{n}^x$ per atom of material x has been derived from the group fluxes ϕ_i and the fission cross section σ_i^x of material x according to

$$\bar{n}^x(r) = \sum_{i=1}^{26} f_i^x \sigma_i^x \phi_i(r), \quad (5.5)$$

taking into account the correction factor f_i^x for resonance self-shielding in the core mixture; $f_i^x = 1$ was

assumed for all materials not contained in the core. The calculated curves were normalized in such a way that the ^{238}U -fission rate coincides with the experiment in the center of the fast zone.

The experimental data are subject to two main sources of error. First, calibration uncertainties of less than ± 5 percent are estimated for the parallel-plate chambers, and of the order of ± 10 percent for the cylindrical ^{239}Pu and ^{237}Np chambers. The calibration errors may affect the absolute values of fission rates and ratios, but not the shape of the traverses and the variation of spectral indices from one assembly to the other. Second, the deviation of the actual zone boundaries from the cylindrical shape gives rise to an uncertainty in the effective radial distance to be attributed to the chamber position and leads to a scatter of the experimental points near the edge of the fast core.

The ^{235}U -fission rate in Fig. 14 shows, as a result of the strong absorption of slow neutrons in the natural uranium and at the edge of the fast zone, a steep gradient which goes over into a relatively flat curve in

FIG. 14. Radial Distribution of Fission Rate for ^{232}Th , ^{235}U , ^{238}U , and ^{237}Np Obtained with Cylindrical Fission Chambers.

FIG. 15 $^{239}\text{Pu}/^{235}\text{U}$ Fission Ratio in the Fast Zone of STARK.

the interior of the fast core. The experimental data are seen to agree with the theoretical curve for $r < 10$ cm, but in the outer region they are higher than calculated, indicating that slow neutrons penetrate deeper into the fast zone than predicted by the calculation.

As a sensitive indicator for the low-energy end of the spectrum, the $^{239}\text{Pu}/^{235}\text{U}$ fission ratio, σ^{49}/σ^{25} , is plotted in Fig. 15. The maxima in the calculated curves result from the fact that thermal neutrons are more strongly absorbed in the natural uranium than eV-neutrons, which cause fission at the lower plutonium resonances. Towards the center of the assembly the resonance neutrons die out and the fission ratios go over into the characteristic values of the fast spectrum. For all assemblies the experimental $^{239}\text{Pu}/^{235}\text{U}$ ratio in the natural uranium zone was found to be considerably higher than calculated, which means that the number of eV-neutrons in the spectrum is larger than expected from theory.

The same conclusion must be drawn from the $^{115}\text{In}(n,\gamma)$ reaction rate measured by foil activation (see Fig. 7). In order to separate the contribution of the 1.46-eV resonance from the fast-neutron effect, indium sandwiches were irradiated (cf. Sect. 4.2.3). The activity ratio C_o/C_i (outer to inner foil) shows for Assem-

bly 2 even in the central position a measurable 1.46-eV component which is about 9 times as high as calculated. In Assembly 4 with its higher ^{235}U content the effect of 1.46-eV neutrons, again higher than calculated, dies out within experimental error towards the center of the reactor.

The stronger penetration of low-energy neutrons relative to the diffusion calculation can be qualitatively explained by different effects. The deviations in the natural uranium zone may be partly due to a spectral hardening within the thermal group itself and also to the idealizations made in the effective cross-section concept, by which the neutron transmission at energies between the ^{238}U resonances is underestimated. The deviations in the lower-eV region at the reactor center can partly be attributed to transport effects (cf. the S_4 calculations in Fig. 12 and 13); the remaining discrepancies are probably caused by a radial neutron streaming between the platelets of the fast core and, in the case of Assembly 2, by the heterogeneous arrangement of the hydrogenous material.

	exp. points	26-group diffusion calculation
Assembly 1:	+	-----
Assembly 2:	o	-----
Assembly 3:	Δ	-----
Assembly 4:	□	-----
S_4 calculation:	-----	-----

FIG. 16. $^{238}\text{U}/^{235}\text{U}$ Fission Ratio in STARK as a Function of Radial Position r .

As a check of the neutron spectrum in the keV-region, reaction rates of $^{19}\text{F}(n,\gamma)$ (resonances at 15.3 and 27.2 keV), $^{23}\text{Na}(n,\gamma)$ (resonance at 2.95 keV), and $^{115}\text{In}(n,n')$ (threshold at 335 keV) in Assemblies 2 and 4 are compared in Fig. 7 and 8 with calculated radial traverses, and a fairly good agreement was found. Also the ^{238}U -capture rate, mainly originating from the keV-region because of the self-shielding of the lower resonances, is consistent with the calculation. Hence, it can be concluded that the spectrum in the keV-region does not strongly deviate from the 26-group diffusion calculation.

Information on the spectrum in the upper keV- and MeV-region is obtained from fission rates in threshold substances, such as ^{232}Th , ^{238}U , and ^{237}Np . The fission chamber traverses of these materials in Assemblies 2 to 4 (see Fig. 14) agree well in their shapes with the calculation. The absolute values, however, deviate from the calculation by factors of 1.15 for ^{232}Th and of 1.30 for ^{237}Np . Since these factors are constant within ± 1.5 percent for the Assemblies 2 to 4, which have largely different spectra, it is not believed that these discrepancies arise from true spectrum deviations, but rather from systematic errors in the calibration of the cylindrical chambers and/or effects of finite group widths in the 26-group calculation.

In Figs. 16 and 17 the radial variations of the fission ratios $^{238}\text{U}/^{235}\text{U}$ and $^{237}\text{Np}/^{238}\text{U}$ are plotted. The $^{238}\text{U}/^{235}\text{U}$ data for Assemblies 2 to 4 show, relative to the calculation, a somewhat slower transition into the characteristic values of the fast core, which is a consequence of the stronger penetration of slow neutrons. For Assemblies 3 and 4, strictly constant fission ratios are attained in the interior of the fast zone ($r < 10$ cm), which means that an equilibrium spectrum is reached in the energy region that contributes predominantly to the fission rates. The $^{238}\text{U}/^{235}\text{U}$ ratio in Assembly 2 seems to decrease slightly towards the center, as expected from the softening of the calculated spectra (see Fig. 12). For Assembly 1, the fission ratios vary throughout the central uranium zone, and no asymptotic spectrum is reached due to the low multiplication.

The fission ratio σ^{37}/σ^{28} between the threshold substances ^{237}Np and ^{238}U in Fig. 17 shows for Assemblies 2 and 3, in accordance with the calculation, almost no radial variation. For Assembly 4, a slight decrease of the $^{237}\text{Np}/^{238}\text{U}$ ratio is seen, which results from a deficiency of fast neutrons ($E > 1$ MeV) relative to ϕ_i^{as} at the edge of the fast core (see Fig. 13). The inverse effect (much more pronounced) is noted for the data of Assembly 1.

The fission ratios in the central position of all assemblies are summarized in Table 9. Except for the nearly constant deviations in the cases of ^{232}Th and ^{237}Np , a

fairly good agreement with the 26-group calculation was found for Assemblies 3 and 4. In Assembly 2 the ratios $^{239}\text{Pu}/^{235}\text{U}$ and $^{233}\text{U}/^{235}\text{U}$ are 5 percent higher than expected, due to the influence of eV-neutrons. In Assembly 1 characteristic deviations are also seen for the other fission ratios which indicate, relative to the diffusion calculation, a stronger penetration of fission neutrons from the thermal zone into the reactor center. S_4 calculations give here a better agreement with the experimental data.

The results of the measurements of reaction rate can be summarized as follows: In assemblies of high fast-core multiplication, the spectrum above 100 eV does not seem to deviate substantially from the 26-group diffusion calculation, and a space-independent spectrum close to the equilibrium spectrum ϕ^{as} is reached in the inner region ($r < 10$ cm) of the fast core. In the lower-eV region strong flux gradients occur, and the diffusion calculation underestimates the penetration of such neutrons into the fast zone. S_4 calculations do not fully account for this effect, and it is thought that the heterogeneous structure of the fast core may be a reason for that.

For Assembly 2 with moderate fast-core multiplication the spatial variation of the experimental fission ratios as well as the calculations show that ϕ^{as} is not completely reached in the center of the reactor. Resonance neutrons penetrating into the center contribute slightly to the ^{233}U - and ^{239}Pu -fission rates.

In Assembly 1 with the natural-uranium-filled central

FIG. 17. $^{237}\text{Np}/^{238}\text{U}$ Fission Ratio in the Fast Zone of STARK.

TABLE 9. FISSION RATIOS AND FISSION-TO-CAPTURE RATIOS IN THE CENTRAL POSITION OF STARK COMPARED WITH 26-GROUP CALCULATIONS

Materials	Method ^(a)	Relative Statistical Error	Assembly 1			Assembly 2		Assembly 3		Assembly 4	
			Exp	Calc D ^(b)	Calc S ₄ ^(c)	Exp	Calc C ^(b)	Exp	Calc D ^(b)	Exp	Calc D ^(b)
σ_f/σ_x^{25} :											
$^{232}\text{Th}/^{235}\text{U}$	K	±3%	0.00196	0.00130	0.00142	0.00504	0.00436	0.00785	0.00674	0.00922	0.00793
$^{238}\text{U}/^{235}\text{U}$	K	±1%	0.00823	0.00688	0.00747	0.0228	0.02226	0.0351	0.03427	0.0408	0.04023
$^{237}\text{Np}/^{235}\text{U}$	C	±1%	0.145	0.1267	0.1253	0.218	0.1654	0.336	0.2541	0.375	0.2774
$^{233}\text{U}/^{235}\text{U}$	K	±1%	—	1.660	—	1.57	1.432	—	1.490	1.52	1.492
$^{239}\text{Pu}/^{235}\text{U}$	C	±1%	1.064	1.076	1.081	1.014	0.0847	1.054	1.110	1.071	1.123
σ_f/σ_f^{25} :											
$^{232}\text{Th}/^{238}\text{U}$	K	±3%	0.238	0.1889	0.1904	0.221	0.1958	0.224	0.1968	0.226	0.1971
$^{237}\text{Np}/^{238}\text{U}$	C	±1%	17.6	18.4	16.78	9.57	7.43	9.577	7.43	9.18	6.89
σ_c/σ_f^{25} :											
$^{238}\text{U}/^{235}\text{U}$	F	±2%	—	0.1234	—	—	0.1249	—	0.1225	0.127	0.1208

^(a) K: Calibration with Kirn-type absolute fission chamber.

C: Cylindrical fission chamber.

F: Foil activation.

^(b) D: Diffusion calculation.

^(c) S₄: S₄ calculation (agrees with D within ±0.5% for Assemblies 2 to 4).

region the spectrum varies throughout the core and no equilibrium spectrum is attained at all.

5.3. Central Reactivity Worths of Samples

The investigation of reactivity worths of a set of sample materials may give further information on the neutron spectrum ϕ_i , the shape of the adjoint flux ϕ_i^+ , or the nuclear cross sections of the sample materials.

The reactivity worths found with the pile oscillator in the central position of STARK are given in Table 6, already corrected for sample-size effect and scattering contribution (cf. of Sect. 4.4.4). For a comparison with the 26-group perturbation calculation in Table 7 the data for material *x* are normalized to the reactivity effect of ^{235}U ,

$$\Delta\rho^x/\Delta\rho^{25},$$

in order to eliminate the influence of differences in critical mass between the actual and the calculated reactor.

When experimental and calculated sample worths are compared, much higher discrepancies are found than for the fission ratios, and only qualitative conclusions can be drawn. The only values to agree with the calculation within ± 10 percent are the ratios $\text{Ni}/^{235}\text{U}$ and $^{238}\text{U}/^{235}\text{U}$ (for Assemblies 3 and 4). For these materials the main contribution to $\Delta\rho$ comes from nearly the same energy range (group 5 to 15) of the spectrum.

The reactivity ratio of $^{10}\text{B}/^{235}\text{U}$ is about 30 percent higher than calculated (the ^{10}B data of Assembly 2 are possibly affected by a sample-size effect). This deviation can be explained by the higher flux of eV-neutrons (as seen from the ^{115}In activation) which contribute to the ^{10}B -absorption rate.

The large deviations for the scattering materials Al, C, and H cannot be understood alone by spectrum or sample-cross-section effects. These materials are sensitive indicators for the derivative of the adjoint ϕ_i^+ with respect to energy, i.e., for the height of the ^{238}U threshold and the position of the minimum. To explain the experimental data, a somewhat deeper minimum of ϕ_i^+ , which could possibly arise from transport or heterogeneity effects, must be assumed.

5.4. Dependence of Integral Reactor Parameters on the Reactor Loading

The experimental values of various integral reactor parameters of STARK are summarized in Table 2 together with results of the 26-group diffusion calculation.

The calculated critical fuel mass of the thermal zone is up to +6.5 percent higher than the experimental value, which corresponds to a maximum deviation in k_{eff} of -1.5% ; this deviation reduces to -1.0%

when the S_4 approximation is used. Regarding the simplifications in the calculational treatment (e.g., homogenization of thermal core and air gaps) and the deviations of the actual zone bucklings B_z^2 from the value assumed in the calculation, the agreement in k_{eff} must be considered as satisfactory.

For Assemblies 2 and 3 also, the power contributions of the core zones agree well with the calculation; the effect of an underestimation by theory of the ^{235}U -fission rate in the fast core and the natural uranium zone is nearly compensated by the lower fuel mass in the thermal zone. In Assembly 4 the power contribution of the fast zone is about 10 percent lower than calculated, which could be a consequence of the larger differences in the axial zone bucklings (see Table 4).

For the void coefficient in the thermal zone and the reactivity worth of core material in the reactor center a comparison has been made with estimated values based on a 26-group first-order perturbation calculation. These values differ by as much as ± 40 percent from the experimental ones and show also a different variation with reactor loading.

As expected from the simple two-zone model of a coupled fast-thermal reactor, the neutron-generation time and the reactivity worths in the thermal core (e.g., moderator void coefficient, worth of cadmium plates) are seen to decrease considerably with increasing power contribution γ_1 (or importance α_1) of the fast zone, i.e., when going from Assembly 1 to 4. At the same time the critical ^{235}U content M of the thermal zone decreases. The relation between the fuel mass M and the power fraction γ_1 is given in Fig. 18, and an excellent agreement between experiment and calculation is noted. Further calculations have shown that this relation between γ_1 and M is in good approximation independent of the particular fast-core composition so that interpolations between the 4 assemblies can be made.

Also, for a discussion of the other reactor parameters in Fig. 18, the fuel mass M has been chosen as abscissa. The worth of a single cadmium control plate, Δk_{Cd} , shows a variation with M which agrees in its shape with the perturbation calculation for a thermal absorber. A stronger variation is found for the combined effect of all 12 control plates due to their mutual shadowing. The experimental data for the neutron-generation time Λ were found to be 10 percent higher than the values calculated from eq. (5.1). This deviation is far beyond the experimental error and must be investigated further.

6. Conclusions

The analysis of the experimental data has shown that most of the fundamental properties of the coupled system STARK are described fairly well by diffusion calculations using the 26-group ABN-cross-section

FIG. 18. Variation of Reactor Parameters with Reactor Loading.

set. Satisfactory agreement was found, in particular, for the critical mass and the partition of reactor power between the various zones. Large discrepancies, on the other hand, were found for the reactivity worths of sample materials in the fast zone.

To investigate the neutron spectrum in the fast core, fission rates and differential neutron spectra were measured. For systems with sufficiently high fast-core multiplication, the fission rates are consistent with diffusion calculations in the central region of the fast zone, whereas larger deviations were found for ^{235}U and ^{239}Pu in the natural uranium zone. These deviations indicate a stronger penetration of eV-neutrons into the fast core than predicted by diffusion theory; S_4 calculations give a better, but not yet satisfactory, agreement.

Calculations and fission-ratio measurements show that for sufficiently high fast-core multiplication the neutron spectrum closely approaches in the central region of the fast zone the equilibrium spectrum of an all-fast system of the same core composition. Deviations occur only at the low-energy end of the spectrum, which normally gives negligible contributions to the reaction rates.

The large discrepancies for the central reactivity worths of samples cannot be explained by spectral effects alone. Assuming that the 26-group cross sections of the sample materials are not substantially in error, it must be concluded that the energy dependence of the adjoint flux ϕ_i^+ at the center of STARK deviates from the diffusion calculation; a more pronounced minimum of ϕ_i^+ must be assumed to explain the experimental data. Further theoretical and experimental work (e.g., measurement of the neutron importance) appears to be necessary for a better understanding of these discrepancies.

The dependence of the integral reactor parameters (critical mass, neutron lifetime, power fractions of the zones) on the reactor loading were found to be in reasonably good agreement with the calculations so that reliable predictions can be made for other assemblies.

As shown by the results of this paper, useful information on neutron spectra and reaction rates in specific fast reactor systems can be obtained from experiments with coupled fast-thermal assemblies similar to STARK. However, greater effort on the theoretical and experimental side seems necessary for a quantitative evaluation of measured reactivity effects.

Acknowledgments

The authors wish to acknowledge the assistance given by the STARK operation staff during the experimental work. They also wish to thank Miss R. Kurmis for the numerical evaluation of experimental and calculated data. We also gratefully acknowledge the preparation of the fission foils by the EURATOM Central Bureau for Nuclear Measurements in Geel, Belgium.

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Discussion

Mr. Baker (UK-Winfrith): I should like to ask a question about the interpretation of the assembly which contained hydrogen. We have some systems on VERA which contain hydrogen, and this is the reason for my interest. How do you account for the strong anisotropic scattering in hydrogen when you make your diffusion theory and S_4 calculations?

Mr. Meister: With our codes so far it is possible only to use isotropic downscattering. It's a simple transport cross-section-averaged cosine treatment. In the S_4 calculations there is no consideration of the actual angle of distribution of scattering except for this average cosine.

Mr. Baker: Have you reason to think this is good enough?

Mr. Meister: I do not think it is good enough, but it's the only thing we could do so far.

Mr. Kilian (AEG, Germany): Could you please give me two numbers which characterize this mixed-spectrum system: first, the criticality factor of the fast core by itself, and second, the fraction of power in the fast zone?

Mr. Meister: It depends on the particular assembly. With the most reactive fast core we have a reactivity fraction of 50% in the fast zone, a power contribution of 30% in the fast zone, about 10% in the natural uranium zone, and the rest in the thermal core.

Mr. Karam (ANL): What form of sodium did you use for your activation measurement?

Mr. Meister: It was sodium fluoride, and the fluorine activity was eliminated in the measurement.

Experimental and Theoretical Work at the Zero-energy Fast Reactor FRO

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ABSTRACT

The first fast reactor in Sweden, FRO, has been in operation since February 1964. It is a split-table machine with vertical fuel elements. A quantity of 120 kg U^{235} is available as fuel, fabricated into metallic plates of 20% enrichment. Of the first five cores studied three (numbers 1, 2, and 4) consisted of undiluted fuel. Core no. 3 was diluted with graphite (29 v/o) and no. 5 with graphite (29%) and polythene (7.5%). All assemblies had a thick reflector of copper except for no. 4, which had a uranium reflector.

Among the experiments carried out with FRO the following items are dealt with in this paper: critical mass, reaction ratios, central reactivity coefficients, and heterogeneity effects. Measurements of reactivity worths of multiplying as well as absorbing control rods are also summarized, including studies of interaction effects and streaming in empty channels. Recently, spectrometry work with proton-recoil counters and Doppler measurements by means of foil activation have been initiated. The latter have been carried out at 590°C, and preliminary results for U^{235} and U^{238} have been obtained.

The experimental results have been analyzed with various models. The spectrum programme SPENG, which includes a data library, has been used to calculate core and reflector fine-structure spectra and to derive multigroup cross-section sets for the different assemblies. For the critical-size and flux calculations the DSN and TDC transport theory programmes have generally been employed, and a second-order perturbation code has been used to analyze the central reactivity measurements.

The experiments show that the spectrum in the copper reflector is softer than predicted, but apart from this discrepancy good agreement between theory

and experiment has generally been obtained, especially when critical mass is concerned.

Accounts of a P_N method used in connection with control rod calculations and of pulsed-source, Rossi- α , and variance-to-mean measurements are given in separate papers at this conference.

1. Introduction

The zero-energy reactor FRO is the first fast reactor to be built in Sweden. It is located at the Studsvik Research of AB Atomenergi, about 110 km south of Stockholm. The reactor went into operation in February 1964 and is being used for basic studies in fast reactor physics.

1.1. Reactor Design

FRO is a split-table machine containing vertical fuel and reflector elements (see Fig. 1). The fuel element consists of a steel frame in two sections (see Fig. 2), which are screwed together at the ends. The frame contains the fuel plates of uranium metal (20% U^{235}) and reflector plates. The normal reflector material is copper. The reactor core contains a minimum of 6.5% stainless steel and 5.6% void. The fuel plates are either 0.71 or 0.355 cm thick, and most of them have a square section of 4.30×4.30 cm. They are coated with a thin layer of Teflon. The reactor is furnished with three different types of control elements: five safety rods, three start rods, and a fine-control rod. All rods contain a pattern of fuel and reflector material appropriate to their positions in the reactor. They are raised into the core or reflector to their positions in the reactor. They are raised into the core or reflector from below. The safety rods are spring loaded in their action positions and are released, on a scram signal, with an acceleration of about 10 g. The start rods which are used for

FIG. 1. The FRO Reactor.

approach to criticality when the two reactor halves have been brought together, are inserted with a speed of 0.2 cm/sec, corresponding to a reactivity addition rate of about 1×10^{-4} /sec. There is no limit imposed on the speed of movement of the fine-control rod, but its total reactivity worth is limited to a value of 0.3β .

The reactor is located in a large hall designed to resist an overpressure of 0.75 atm. A leak-rate from the hall of about 1% per hour at this pressure has been measured.

The overall design of FRO is similar to that of the British VERA reactor at Aldermaston.⁽¹⁾

1.2. Scope of Work

The aim of the work at the FRO reactor is two-fold: 1) to gain experience in the operation of a fast reactor and to develop methods for measurement of important reactor physics parameters, and 2) to check calculational methods and cross-section data by experiment. The first aim could probably be attained with any fast facility having a reasonable accessibility to experimental equipment. The second aim requires more than that, since the primary interest is to check the

calculational methods for large power reactors and the data for a wide range of neutron energies. The present fuel inventory of FRO, however, is only 600 kg of 20% enriched uranium metal, i.e., 120 kg U^{235} , and this restricts the experimental programme to the study of fairly small cores (≈ 100 litres). The lack of flexibility in enrichment also restricts the range of compositions. Nevertheless, it has been possible to obtain neutron spectra in quite a wide energy range by diluting the fuel with graphite and polythene, and thus to check the data in the energy intervals of interest for fast power reactors.

Tables 1 and 2 summarize information about the 5 assemblies of FRO studied up to August 1966. All cores were surrounded by a thick copper reflector, except that of Assembly 4, which had a natural uranium reflector. Copper is a suitable reflector material insofar as it has a low absorption cross section, does not become strongly radioactive, and is cheap. But studies of Assemblies 1, 2, and 3 showed that the neutron spectrum in the copper reflector was much softer than the core spectrum and that this spectrum softening is difficult to reproduce in calculations (cf. Sect. 4.3). This

was the major reason for using, as a check, a uranium reflector in Assembly 4.

The cores of Assemblies 1, 2, and 4 were undiluted and had rather hard neutron spectra (median fissioning energy at core centre ≈ 360 keV). All cores were cylinders with height-to-diameter (H/D) ratios near 1, except that of Assembly 2, which was a sphere. The investigations of these undiluted cores were fairly comprehensive, involving different core geometries, and reflector materials and thicknesses. In assemblies 3 and 5 a gradual spectrum softening was achieved by diluting the core with graphite (core 3) and graphite + polythene (core 5). Since elastic scattering is the only reaction of importance in graphite, this material is a very convenient diluent. It has a negative reactivity effect, however, and, as seen in Table 1, dilution with graphite to 29 v/o (core 3) nearly exhausted the fuel inventory. The corresponding spectrum softening was not very great (core centre median fission energy 270 keV).

In order to obtain further spectrum softening, without introducing a complicated two-zone arrangement, polythene was added to the extent of 7.5 v/o in core 5. This had a very strong effect on the spectrum, the median fission energy now dropping to about 50 keV. The reactivity effect of polythene is positive in these small systems, due to a strong reduction in neutron leakage as well as a spectral effect. There are some drawbacks in diluting with polythene, however; transport-theory calculations become more complicated due to anisotropic scattering in hydrogen, and experimental results tend to become ambiguous due to the appearance of strong heterogeneity effects.

Figure 3 shows calculated neutron spectra for the centres of cores 1, 3, and 5. It is noteworthy that the core-5 spectrum is similar in general shape to that of a 1000-MWe steam-cooled fast reactor.

In the present paper some of the theoretical and ex-

FIG. 2. FRO Fuel Element.

perimental work on FRO will be presented. In Sect. 2 recent theoretical work is briefly described. Experimental techniques and results are summarized in Sect. 3. In Sect. 4 a comparison of theoretical and experimental results is made for some items which have been studied systematically: critical mass, reaction ratios, fission and capture distributions, and central reactivity coefficients. Finally, conclusions and viewpoints on future work are given in Sect. 5.

Accounts of a P_N method used in connection with

TABLE 1. SUMMARY OF DATA FOR FRO ASSEMBLIES 1-5

Assembly No.	Core Compositions, v/o				Core Height, cm	Equiv Core Radius, cm	Average Reflector Thickness, cm	Critical Mass, kg U^{235}	Median Fission Energy, ^(b) keV
	Uranium (20% U^{235})	Steel ^(a)	Graphite	Polythene					
1	87.7	6.5	—	—	30.1	15.3	35	72.9	360
2	87.7	6.5	—	—	—	17.2	35	70.1	360
3	58.5	6.5	29.2	—	38.7	20.4	35	111.1	270
4	87.7	6.5	—	—	34.4	16.6	12.9 ^(c) 10.6 ^(d)	98.6	360
5	51.1	6.5	29.2	7.5	38.7	17.9	35	78.7	50

^(a) 74% Fe, 18% Cr, 8% Ni.

^(b) Calculated for core centres.

^(c) Axial reflector.

^(d) Radial reflector.

control-rod calculations and of pulsed-source, Rossi- α , and variance-to-mean measurements will be given separately.^(21, 22)

Parts of this paper have been discussed in detail elsewhere.⁽²⁻⁸⁾

2. Recent Theoretical Work for FRO

2.1. Cross Sections

The main portion of the theoretical work is based on a data library compiled from evaluation work at

TABLE 2. ATOMIC DENSITIES IN FRO ASSEMBLIES 1-5

Region	Atomic densities, 10^{22} at/cm ³					
	U ²³⁵	U ^{238(a)}	C	H	SS	Cu
Assembly 1, 2, and 4; cores	0.851	3.352	—	—	0.55 ^(b)	—
Assembly 3; core	0.568	2.234	2.47	—	0.55	—
Assembly 5; core	0.498	1.963	2.77	0.604	0.55	—
Assembly 1, 2, 3, and 5; reflector	—	—	—	—	0.55	7.48
Assembly 4;						
Axial reflector	0.030	4.19	—	—	0.55	—
Radial reflector	0.030	4.16	—	—	0.13	—

(a) Including 0.16% U²³⁴.

(b) Fe: 0.408; Ni: 0.048; Cr: 0.096×10^{22} at/cm³ (same composition in all cases).

UKAEA, KFK, and AB Atomenergi. The library is called SPENG as it is based on a spectrum programme SPENG, a refined version of the earlier NESPCO programme.⁽²⁾

The principle for the compilation work has been that all data should be based on differential measurements or, when experimental values are not available, upon calculations using appropriate models. The data in SPENG are tabulated point-by-point for the following elements: hydrogen, deuterium, beryllium, boron, carbon, oxygen, fluorine, sodium, aluminum, silicon, calcium, chromium, manganese, iron, nickel, copper, zirconium, lead, U²³⁵, U²³⁸, Pu²³⁹, Pu²⁴⁰, and Pu²⁴¹. The number of energy points are chosen so that the resonances are described in detail whenever data are available and when these resonances do not cause the number of points to exceed about 500. The maximum number of points is 2000, but the lower number is generally used to limit the computation time on an IBM-7044 machine to 5-10 min for the calculation of the spectrum and macroscopic group cross sections. The data are tabulated from 0.01 eV to 10 MeV, sometimes to higher energies.

For U²³⁵, the experimental values of the fission cross section obtained by White⁽⁹⁾ are now included in the library. The energy range concerned is 40 keV to 14.1 MeV. The α -value from 10 eV to 1 MeV is taken from

FIG. 3. Calculated Neutron Spectra at Core Centres.

Ref. (10), and the resonance parameters for U^{235} up to 1 keV from (11) and (12). The data between 1 and 40 keV are obtained from (11) and (13).

References (11) and (14) have been used for Pu^{239} . The resonance parameters of U^{238} are obtained from (15) and (16). Other data for U^{238} and other elements are the same as those given in (2).

For uranium, plutonium, and copper (see below) it has not been possible to describe the resonances point by point except for the very lowest ones. Instead, effective cross sections have been calculated for a large number of energy groups in the resonance region. For these calculations the ERIC-2 code⁽²⁴⁾ has been used, complemented by a routine for calculating mean fluxes. Thus different cross-section sets are obtained for a number of "background" cross sections and for different temperatures. The background cross section for an element i is defined as the sum of the total cross sections of all elements other than i per atom of i .

Special interest is devoted to copper because this element is the main constituent of the reflector in most FRO assemblies. Some of this work is reported in (2). Recently the measurements by J.B. Garg *et al.*⁽¹⁷⁾ of total cross sections for copper from 200 eV to 100 keV (140 resolved resonances) have been introduced into the library point by point and partly as effective cross sections. The latter are given within narrow energy groups with the "background" cross section and temperature as parameters.

Elastic-scattering and capture cross sections have been calculated from the copper data given by Garg *et al.* up to 35 keV, using $\Gamma = 0.5$ eV. Above that energy Russian experimental (n,γ) cross sections have been used.^(18, 19) Reference (18) was also used in the compilation of data for nickel.

2.2. Calculation of Fine-structure Spectrum in a Homogeneous Medium

In order to obtain macroscopic group cross sections, there is need for a programme to calculate the spectrum in a given zone with as many details as possible. As mentioned above, SPENG resolves the resonance effects in any material, subject to the condition that the number of data points must not exceed 2000.

The programme solves the Boltzmann equation in B_1 approximation for a homogeneous medium. For the elastic-scattering cross sections, up to 10 Legendre coefficients can be given. The inelastic cross sections can either be given for isolated levels or calculated from the statistical model. The energy values given in the cross-section tables may differ from element to element and, for a certain element, the energies are not necessarily equidistant. In a calculation the intervals of the cross-section tables are used as integration steps.

However, the user can put restrictions on the widths of the integration steps in arbitrary energy regions. The smallest practical lethargy step for the description of the flux is 0.001. It is then convenient to use this shortest step length only in a region where an element has resonance widths which are less than, say, 0.05 lethargy unit.

Group constants for any desired group structure can be obtained from the SPENG spectrum by a condensation routine in the programme. Critical buckling or critical enrichment is calculated by iterations on the flux and on the fission source neutrons.

It should be mentioned that the SPENG (NESPCO) library is being revised continuously. Therefore, in previous comparisons with experimental results some of the cross-section references were different from those stated here. The cross-section set based on the references given in this section is called Set 2B. Two other sets are referred to in this paper, Sets 2 and 2A. Set 2A differs from 2B only in that the Russian copper data^(18, 19) are not included. This is also true for Set 2. The U^{235} -fission cross section in this set was based on KFK 120 rather than on White's new values. Set 2 also contained a punching error in the copper data which distorted the spectrum in the reflector but did not affect calculated reaction rates, etc., at the core centre.

3. Experimental Techniques and Results

3.1. Determination of Critical Mass

After attaining criticality with a new core, the general procedure was to adjust the loading to make the system slightly supercritical with all control rods inserted. The excess reactivity was determined by a measurement of doubling time, and the equivalent reduction in fuel mass was obtained by measuring the difference in reactivity between core and reflector material at the core-reflector interface. Corrections were also made for the reactivity loss caused by the source access channel and the effect of the step irregularity at the core edge. In order to estimate the latter correction the measurements of core-to-reflector material reactivity difference were extended to a number of radial positions in the vicinity of the core edge. The data obtained were used in a computer calculation with a programme, ALIT-1, which gives the irregular core/smooth core mass ratio.

An investigation was also made of the possible error in the loaded mass itself, due to the small uncertainties in total fuel weight, Teflon coating thickness, and uranium enrichment. Of these factors only the last proved to be significant. Measurements of the enrich-

ment were carried out at two laboratories by mass-spectroscopic technique; the average values obtained were 19.99% and 20.10% U^{235} by weight, respectively. The variation in individual values obtained at each laboratory indicates an uncertainty of about $\pm 0.05\%$ U^{235} in both cases. An enrichment of $20.05 \pm 0.05\%$ is therefore consistent with the measured values. The corresponding uncertainty in critical mass is of the order of 0.5%, which is the limit of accuracy of the experimental determination of critical mass.

In the diluted cores, the heterogeneous arrangement of fuel and diluent plates caused the reactivity to differ from what it would be in a homogeneous system. Since all calculations of critical mass were based on a homogeneous model, an experimental estimate of the heterogeneity effect was required. This was achieved by bunching the plates of each of the constituents (uranium, graphite, and polythene) of a fuel element together and measuring the reactivity as a function of plate thickness. Extrapolation to zero plate thickness approximates the desired correction. The effect for the whole core was obtained by numerical integration of the result for a single fuel element placed at different positions in the core or from bunching experiments on a quadrant of the core. Figure 4 shows the result for

FIG. 4. Effect of Heterogeneity on the Reactivity of Core 5. The abscissa is in units of the plate thickness of all constituents in the original loading.

FIG. 5. Fission Chamber.

core 5 (29% graphite, 7.5% polythene). The effect is particularly large in this assembly, corresponding to a correction in critical mass of $4.3 \pm 0.6\%$. The error in this number is the estimated uncertainty in extrapolating to zero plate thickness.

A comparison of experimental and theoretical critical masses is given in Sect. 4.1.

3.2. Reaction Rates and Ratios. Fine Structure

3.2.1. Fission Rates

Fission rates have been determined with fission chambers containing U^{235} , Pu^{239} , U^{238} , or Np^{237} . The detectors were cylindrical with an outer diameter of 6 mm (compare Fig. 5). To reduce the spectrum perturbation caused by the counters, aluminum was chosen as constructional material, the wall thickness being 0.5 mm.

The amount of the fissionable isotope deposited on the anode could not easily be assessed by α counting. An extensive series of calibration runs were therefore made in a thermal flux for the U^{235} , Pu^{239} , and U^{238} (using the small amount of U^{235} present) detectors. The relation between the measured pulse rates and the absolute fission rates was established—in the thermal-column runs—from an absolute flux determination with irradiated gold foils.

The Np^{237} counter was subsequently calibrated relative to the U^{238} counter in the fast-neutron beam from a Van de Graaff machine, using the (p,n) reaction in lithium. The neutron energy (4 MeV) was

chosen in a region where the neptunium and U^{238} fission cross sections vary slowly with energy. The reduction of the Van de Graaff data involves an assumed value of the ratio of the fission cross sections at 4 MeV for Np^{237} and U^{238} . The published values of this ratio show some inconsistency, equivalent to a discrepancy in the Np^{237} fission cross section. The effect of this is partly reduced by the fact that associated values of the ratio referred to above and the neptunium cross section should be used when measured and calculated reaction rates are compared.

The uncertainty of the detector calibrations is estimated as 1.5–1.7% for the U^{235} , U^{238} , and Pu^{239} chambers, and 6% for the Np^{237} detector.

3.2.2. Foil Activations

A large number of foil irradiations have been performed. The foil materials included U^{238} , gold, tungsten, indium, copper, manganese, and sodium. The high concentration of U^{238} in the core results in a strong depletion of the neutron flux at the U^{238} resonances. A similar effect is encountered in the copper reflector at the copper resonances. Consequently, the effective capture cross sections for a U^{238} and a copper foil are quite

different in the core and the reflector. This must be taken into account when calculated and experimental values are compared. Furthermore, strong spatial variations of the neutron spectrum at discrete energies exist in a transition region at the core-reflector interface, making the comparisons particularly difficult there.

Some efforts were made to study the effect of the spectral flux depletion or recovery experimentally. Thus the flux distribution in the reflector was measured with thin U^{238} and copper foils surrounded in their immediate neighborhood either by the ordinary copper plates or by 7.1-mm-thick natural uranium plates. Some results for Assembly 2 are reproduced in Figs. 6 and 7. The values with and without uranium plates were obtained in the same run from foils placed in the axial reflector, either above or below the core. The difference in the reaction rates is considerable.

3.2.3. Fine Structure

The spatial microscopic flux variation was small in the undiluted cores 1, 2, and 4. It was not negligible in core 3 and was very strong in core 5. These variations have been studied by a number of measurements

FIG. 6. Cu^{63} Capture Rate, Radial Distribution in Assembly 2.

FIG. 7. U^{238} Capture Rate, Radial Distribution in Assembly 2.

of reaction rate. A theoretical treatment of the problem is under way, but no results are available at present. We are consequently restricted to presenting only the experimental results. The data given in Fig. 8 for core 3 and in Fig. 9 for core 5 are valid for a loading pattern in which all plates have *twice* their normal thicknesses.

The low-energy-sensitive reactions $\text{Cu}^{63}(n,\gamma)$, $\text{U}^{238}(n,\gamma)$, $\text{Mn}^{55}(n,\gamma)$, and $\text{U}^{235}(n,f)$ are depressed in the fuel plates, whereas the threshold reactions $\text{In}^{115}(n,n')$ and $\text{U}^{238}(n,f)$ are peaked in the same region. The flux variations are very much stronger in the soft-spectrum core 5 than in core 3. In the former, the largest effect occurs in the U^{238} capture rate with a peak-to-peak variation of 2.4:1, compared with 1.05:1 in core 3 (double plate thicknesses). These very strong flux variations in core 5 introduce large uncertainties in experimental results such as reaction ratios, as will be discussed in Sect. 4.2.

Measurements of the microscopic flux variation were generally made at the core centre, but in core 3 a region near the core-reflector boundary was also investi-

FIG. 8. Microscopic Flux Variations in Core 3 (Plate Thicknesses Are Twice the Normal Ones).

FIG. 9. Microscopic Flux Variations in Core 5 (Plate Thicknesses Are Twice the Normal Ones).

gated. In this region the low-energy-sensitive reactions showed 20–100% stronger variations than at the core centre, due to the influx of slow neutrons from the copper reflector. The results for the threshold reactions, on the other hand, were equal to those at the core centre.

3.2.4. Conversion Ratio

Measurements of the modified relative conversion ratio, RCR, have been performed on all systems. RCR is defined as

$$\text{RCR} = \frac{\left[\frac{\text{U}^{238} \text{ capture}}{\text{U}^{235} \text{ fission}} \right] (\text{FRO spectrum})}{\left[\frac{\text{U}^{238} \text{ capture}}{\text{U}^{238} \text{ fission}} \right] (\text{thermal spectrum})}$$

From this quantity the true conversion ratio in the fast spectrum, i.e., $[U^{238} \text{ capture}/U^{235} \text{ absorption}]$ can be found by use of tabulated thermal cross sections and a calculated effective value of α , the capture-to-fission ratio in U^{235} .

The capture rate in U^{238} was detected by measuring the Np^{239} decay in thin foils using the γ -X-ray coincidence technique. The fission rate in U^{235} was obtained with the aid of a small U^{235} fission chamber. It was early recognized that a U^{235} foil could not be substituted for the fission chamber. The difference in fission product yields for fissions induced by thermal and fast neutrons, respectively, introduces a systematic error that is not negligible. A special study was made of this effect by determining, as a function of time, the ratio of the fission product activity induced in foils irradiated simultaneously in FRO and a thermal column. The result is shown in Fig. 10, reproduced from Ref (20). The ratio in question changes by more than 3% over a time period of 50 hr.

The advantage of the fission chamber in the respects discussed above is counteracted, to a certain degree, by the large size of the detector. The problem may be severe for the heterogeneous core 5, for which the chamber perturbation is difficult to assess.

In Assemblies 3 and 5 the comparison with the RCR values calculated for the homogenized assembly is complicated because of the spatial variation of the flux across a cell. The relative distribution of the U^{238} capture and the U^{235} fission rate has been measured with thin foils, and simple volume averages were used to get an approximate experimental RCR value for the homogeneous case.

A comparison of the various measured reaction rates and ratios with theoretical results is postponed to Sect. 4.2.

3.3. Central Reactivity

Measurements of central reactivity worth have been performed in assemblies 1, 3, and 5, the runs in the first two being the most extensive. Besides measurements on small samples, studies were made of the reactivity per unit mass as a function of sample size for certain elements.

No pile-oscillator equipment has as yet been constructed at FRO, and the reactivity measurements were made by the positive-period technique. A remotely controlled sample changer with room for four samples was used, by means of which measurements could be made without shutting down the reactor between the runs. Under certain conditions the results of period measurements are as accurate as those obtained by pile-oscillator technique [see Ref. (1) for

FIG. 10. FRO-to-thermal Column Fission Product Counting Rate Ratio.

a discussion of this question]. However, the latter technique is definitely to be preferred if the long-term stability of the reactivity is not very good. The FRO reactor has an appreciable temperature coefficient (about 3×10^{-5} per $^{\circ}\text{C}$ for Assembly 1, for instance) which imposes high requirements on the temperature stability. These could not be fully met at FRO, and the combined uncertainty caused by the power drift, statistics in the period measurements, possible non-reproducibility of the control rod position, etc., limited the accuracy of the reactivity determination to about 5×10^{-7} in $\delta k/k$. This value does not include the uncertainty in the effective fraction of delayed neutrons in the system.

For the runs in Assemblies 1 and 3, the upper part of the central fuel element was replaced by a copper rod with a sample container at its lower end. Two rods of different sizes were used (cross-sectional areas $2.9 \times 2.9 \text{ cm}^2$ and $4.6 \times 4.6 \text{ cm}^2$, respectively). It was realized that these arrangements perturbed the flux considerably, however, and in the later studies, i.e., those in Assembly 5, a rod designed as a fuel element with reduced thickness (2.2 cm) was made. This rod was loaded with core or reflector material as appropriate. Only the volume taken up by the rod walls and adjacent channel in the central fuel element then differed from the rest of the core, and the flux perturbation was strongly reduced.

When large samples were studied, the reactor was shut down between the runs and the sample loading was made with the halves separated. The uncertainty of these measurements was 1 to 2×10^{-5} in $\delta k/k$, representing generally less than one per cent of the total reactivity effect of the sample.

FIG. 11. Measured and Calculated Reactivity Worth of a Start Rod.

TABLE 3. REACTIVITY AS A FUNCTION OF AREA OF ROD SECTION. CENTRALLY POSITIONED RODS

B ₄ C rod cross section (mm ²)	ρ (%)
42.0 × 42.0	-2.51 ± 0.08
27.5 × 27.5	-1.38 ± 0.08
20.5 × 20.5	-0.90 ± 0.03

TABLE 4. REACTIVITY AS A FUNCTION OF RADIAL POSITION. ROD CROSS SECTION 20.5 × 20.5 mm²

Rod separation ^(a) (cm)	$\frac{\rho(\text{rod 1}) + \rho(\text{rod 2})}{2}$ (%)	$\frac{\rho(\text{rods 1 + 2})}{2}$ (%)
0	0.880 ± 0.015	0.770 ± 0.015
7.0	0.815 ± 0.015	0.805 ± 0.015
16.1	0.670 ± 0.010	0.660 ± 0.010
25.3	0.430 ± 0.010	0.425 ± 0.010
34.4	0.220 ± 0.005	0.215 ± 0.005

^(a) The distance of either rod from the core centre is half the value.

3.4. Reactivity of Control Rods and Central Air Gap

3.4.1. Multiplying Control Rods

The FRO reactor is equipped with three types of control elements—one fine-control rod (FKS), three start rods (STS), and five safety rods (SCS)—all loaded with core and reflector material in a pattern appropriate to their positions. They have different types

of drive mechanisms and control different amounts of reactivity. For these reasons different calibration techniques were preferred for the different types of rods, namely, the positive-period method, the inverse multiplication technique, and the rod-drop method⁽²³⁾ for the fine-control rod, start rods, and safety rods, respectively. These techniques have also been used in Assembly 1 to measure the reactivity worths of central air gaps of different widths and in Assembly 5 to calibrate boron-loaded rods.

In addition, the possibility of using the pulsed-source technique for reactivity measurements in fast reactors of this type has been investigated. The different aspects of these measurements are presented in a separate paper at this conference.⁽²¹⁾

The experimental results have been compared with calculations, both for central and excentric rod positions. Part of this work will be described at this conference.⁽²²⁾ The reactivity worth versus rod position for a start rod is shown in Fig. 11. The agreement between experiment and calculation is particularly good in this case.

When the inverse multiplication, the rod-drop, and the pulsed-source methods were used with equivalent rods, the results generally agreed within the combined experimental uncertainties of the methods compared.

3.4.2. B₄C Rods

In Assembly 5 a set of measurements with absorbing rods has been performed. The absorbing material was natural B₄C powder, compacted into stainless steel (SS) boxes. An effective B₄C density of about 1.71 g/cm³ was attained, which is approximately 70% of the theoretical density. The SS boxes had inner cross-sectional areas of 20.5 × 20.5 mm², 27.5 × 27.5 mm², and 42.0 × 42.0 mm², with 0.5-mm-thick walls. They extended through the whole height of the core as well as through the top and bottom reflectors.

Two series of reactivity measurements were made with these rods. In the first one the inverse multiplication technique was used to measure the reactivity as a function of cross-sectional area for a rod located at the core centre. In the second series the radial dependence of the reactivity equivalence of one and two symmetrically positioned, 20.5 × 20.5-mm² rods was determined by the positive-period method. In both cases the reactivity values, ρ , quoted below refer to boron carbide rod(s) relative to void channel(s) of equal cross-sectional areas.

The results are given in Tables 3 and 4, and are also shown in Figs. 12 and 13. It is evident from Fig. 12 that there is a self-shielding effect as the boron carbide cross-sectional area increases. From Fig. 13 it is seen

FIG. 12. Reactivity as a Function of Cross-sectional Area of B_4C Rod.

that there is an appreciable interaction between the two rods when they are placed close to each other. When the distance between the two rods exceeds about 8 cm, however, this effect is negligible, and the reactivity of the two rods together is equal to the sum of the values of the individual rods.

The reactivity measurements have been complemented by measurements of reaction-rate distributions inside and around a B_4C rod. The $U^{235}(n,f)$, $Au^{197}(n,\gamma)$, and $In^{115}(n,n')$ reactions were used; the results are not yet available.

These experiments will provide check points for calculations which will be made in the near future.

3.4.3. Central Air Gap

The reactivity of central air gaps was measured in Assembly 1, primarily to ensure that the rate of reactivity increase obtained when bringing the reactor halves together was consistent with safety requirements. The measurements were carried out by use of the inverse multiplication and the pulsed-source techniques; the results are given in Fig. 14.

Two sets of results of inverse multiplication measurements are given, corresponding to a detector position in the fixed and in the movable half, respectively. The discrepancy between the results, which persists down to a gap width of about 0.05 mm ($\rho = 0.01\%$) has not been further explored, but may be due to modal effects. For small gap widths the two counters give an approximately linear relationship between reactivity and gap width with slopes equal to 200×10^{-5} mm

FIG. 13. Reactivity as a Function of Rod Position.

FIG. 14. Reactivity of Air Gap, Comparison of Results.

and 250×10^{-5} mm, respectively. In Fig. 14 are also included results of pulsed-source measurements and TDC calculations, which are both in fair agreement with the inverse multiplication measurements.

FIG. 15. Channel Arrangement in Streaming Experiment.

FIG. 16. Channel Arrangement in Interaction Experiment.

3.5. Reactivity Effects of Empty Channels

Two types of experiments were made, concerned with

- the streaming effect in a channel, and
- the interaction between channels as a function of their mutual separation.

These items were carried out as a complement to the measurements of the overall reactivity effect of empty

channels determined in the control-rod calibration (see Sect. 3.4).

The positive-period method was used for measuring the reactivity changes obtained. A typical experiment involved measuring the reactivity difference between two states A and B. In changing from A to B it was necessary to shut down the reactor and reload or interchange two or more fuel elements. The reactor was subsequently made critical and period measurements were made, in most cases by driving the fine-control rod to its "fully in" position. A reproducibility within 1 to 2×10^{-5} of the measured reactivity difference was generally found when making repeated measurements. This error is attributed partly to uncertainties in the reactor temperature and partly to a lack of reproducibility of the exact configuration of the system after removing fuel elements from the table.

All measurements were carried out with Assembly 1.

3.5.1. Experimental Arrangements

a) The Streaming Effect in a Channel

The two situations compared are illustrated in Fig. 15. The channel or cavities were inserted in a normal fuel element by means of thin-walled rectangular copper tubes (wall thickness of 0.5 mm) with interior section 4.2×0.6 cm. Neutron streaming occurs in the open channel—mainly in the axial direction—but is largely prevented in the staggered arrangement. Measurements of the reactivity difference between the two arrangements were made at the core centre (Fa14 position) and in an element about 10 cm from the centre (Ra16 position).

b) The Interaction between Channels

Vertical channels were inserted in two fuel elements which were placed in the reactor at approximately equal distances from the core centre (see Fig. 16). The channels had a rectangular section (4.2×2.1 cm) and extended throughout the reactor, including top and bottom reflectors. During the experiment the position of channel A was fixed and channel B was placed in the positions indicated in the figure. In order to correct for changes in reactivity not due to an interaction effect between the channels, the experiment was repeated with channel A replaced by an ordinary section of fuel.

3.5.2. Results and Comparison with Theory

a) The Streaming Experiment

The result of the streaming experiment is given in Table 5. The values refer to the reduction in the reactivity of the reactor observed when changing from the staggered arrangement to the open channel. The results

obtained show that the streaming effect in the channels considered is small.

These effects were studied theoretically by perturbation calculations in 6 energy groups (diffusion approximation). The change in the diffusion constant was based on the Benoist formula,⁽²⁵⁾ ignoring correlation terms. The following expression holds for a vertical channel through the reactor (one-group formula, two-region problem):

$$\frac{d\nu}{\nu} = \frac{\frac{1}{3} \sum_k \Delta(rQ_k) \int \text{grad}_k \Phi^+ \text{grad}_k \Phi dV_{\text{channel}}}{\int \Phi^+ \nu \sum_f \Phi dV};$$

$$k = x, y, z.$$

In this equation $d\nu/\nu$ is the reactivity change between the channel and the staggered arrangement. The factor $\Delta(rQ)$ denotes the change in the quantity within the parentheses. Here r is the hydraulic radius of the channel or a single cavity in the staggered arrangement, and $Q = \langle l^2 \rangle \langle l \rangle^{-2}$, where l is the chord length in the corresponding geometries.

These reactivity changes were evaluated numerically by means of the above formula in 6-group form in a

TABLE 5. REACTIVITY DIFFERENCE (IN UNITS OF 10^{-5}) BETWEEN CHANNEL AND STAGGERED ARRANGEMENT

Measurement Location R, cm	Experiment	6-group Calculation ^(a)	
		Approach (1)	Approach (2)
0 (Fa14)	7 ± 1	8.0	5.7
10 cm (Ra16)	4 ± 1	4.8	3.4

^(a) Post-conference comment by authors: "These results are in error. The conclusions based thereon are not now considered valid."

first-order perturbation programme.⁽²⁶⁾ Real and adjoint fluxes were taken from two-dimensional S_4 calculations. Analytical formulae and tables for Q and its x , y , and z projections, which depend on the shape of the hole, were derived by Kristiansen *et al.*⁽²⁷⁾ for parallel epipedic boxes and also for infinite tubes of rectangular section. The thin-walled copper boxes and tubes, which make the analysis a matter of three rather than two regions, were treated either (1) as void (compare the table) or (2) as equivalent to fuel or reflector material in the corresponding regions. It was believed that these approaches would give upper and lower limits of the effect.

The experimental values lie between the results for the two approaches; hence the agreement between experiment and theory is satisfactory. In view of the simplifications inherent in the theoretical method, however, the good agreement may be to some extent fortuitous.

b) The Interaction Experiment

The result of the interaction experiment is shown in Fig. 17. The abscissa is the distance between the channels (see Fig. 16) and the ordinate is the reactivity of the system. For convenience, the point of zero reactivity corresponds to the situation in which the channels form a single channel of double section, i.e., $x = 0$. It should be pointed out that all reactivity values given refer to the same total mass of fuel and volume of void in the reactor. Only the distribution of void has been changed.

Figure 17 shows that there is an increase in reactivity with decreasing distance between the channels. The observed maximum increase is about 80×10^{-5} . This is interpreted as due to a reduction in the statistical weight of a region surrounding a channel, which results in a mutual shielding effect. But when the channels

FIG. 17. Variation in the Reactivity Worth of Two Channels as a Function of Their Separation.

form a single channel (zero distance) the reactivity again decreases; it is possible to explain the latter effect as due to strongly enhanced streaming in the wider channel which offsets part of the shielding. By fitting a straight line to the measured points in Fig. 17 and extrapolating to zero distance one may estimate this excess streaming in the wide channel. A value of $(55 \pm 6) \times 10^{-5}$ is obtained. This estimate may be more uncertain than indicated by the limits of error, however, since the fitting of a straight line rather than some other function is somewhat arbitrary.

The suggested interpretation of the experimental results was confirmed qualitatively by calculations. By means of the perturbation method described in the preceding paragraph, the excess streaming in the wide channel compared with that in the two narrow ones was calculated as 98 and 95×10^{-5} , corresponding to approaches (1) and (2), respectively. Although these numbers are considerably higher than the extrapolated value, they agree as to the order of magnitude.

An attempt was also made to determine the interaction effect by theory. To do this accurately, a 3-dimensional transport code would be required. The best tools available, however, were the two-dimensional diffusion codes 20-GRAND (6 groups)⁽²⁸⁾ and EQUIPOISE-3 A (2 groups).^(29, 30) Since the latter code is more than twice as fast as the former, the EQUIPOISE code was chosen for the calculations. It is obvious that the x - y pattern of the interacting channels must be accurately represented in order to obtain useful results. Therefore the code was used in the x - y version, utilizing its full capacity of 1700 mesh points. On the other hand, the vertical leakage effect could then be represented only in terms of an axial buckling component, B_z^2 . This buckling multiplied by the diffusion constant, DB_z^2 , appears as an equivalent capture cross section in the two-dimensional diffusion equation. The leakage of neutrons due to the channel is therefore pictured as captures in an equivalent absorbing rod.

Some support for the use of this simple model is obtained from the experimental flux distributions with various detectors which show that the neutron spectrum is approximately space-independent in a large portion of the core, thus justifying the use of a few-group model. It has also been found that a cosine and a J_0 function can be fitted to the measured flux distributions; there is no significant deviation from these functions at distances less than 10 to 12 cm from the core centre (core radius = 15.3 cm). Thus the separation of the diffusion equation into axial and radial components characterized by buckling parameters seems permissible.

The primary objection to using diffusion theory persists, however. In particular, the streaming effect is difficult to account for in an exact manner when

using the diffusion approximation. The calculations were therefore designed only to correlate the experimental values for a comparison of those situations in which the streaming effect was unaltered, i.e., the channels were separated, rather than to obtain independent theoretical values.

The following procedure was adopted:

- 1) The reactivity of the reactor in the absence of channels was calculated using the experimental value of B_z^2 . The result agreed with the true reactivity within 1%.
- 2) An empty channel having the position and section of a control rod was considered. The diffusion constant in the channel was adjusted so as to obtain a reactivity effect equal to the experimentally determined worth of the control rod. The value of the diffusion constant determined in this way was 2.9 cm.
- 3) Calculations were now made for different configurations of channels, identical with some of those studied experimentally.

The result is shown in Fig. 17. The increase in reactivity when going from large to small separations of the two channels is well reproduced by the calculations. On bunching the channels into a single one, however, the calculations yield a further increase in reactivity, whereas a decrease was found in the experiment. Now an increase in the calculated value is expected, being due to shielding between the channels. The simultaneous increase in neutron streaming occurring when a single wide channel is formed cannot, as mentioned above, be reproduced by means of the simple model at hand.

3.6. Doppler Measurements by Foil Activation

A series of experiments has been carried out in which thin foils of depleted and enriched uranium metal were heated to about 590°C (1090°F) in core 5 of FRO. These experiments give information about the Doppler broadening of the capture resonances of U^{238} and fission resonances of U^{235} . Similar measurements have previously been reported for U^{238} by Pflasterer *et al.*⁽³¹⁾ and by Davey⁽³²⁾ at 420°C and 500°C, respectively.

In the present experiments the foils were located inside a furnace (diameter of 2.9 cm, length of 8 cm) which was heated by means of a thermocoax resistance wire. The wire is embedded in a small steel element and the dissipated heat is transferred by conduction to a foil holder made of aluminium. The foil holder and heating element are surrounded by a heat shield (of steel 0.1 mm thick). The furnace is evacuated to a pressure of about 10^{-3} mm Hg in order to prevent oxidation of the foils and loss of heat. The hot joints of chromel-alumel thermocouples are located in the

foil holder on each side of the foils. This arrangement makes it possible to determine the foil temperature to about $\pm 5^\circ\text{C}$. During the irradiation of the heated furnace in the reactor, a flow of nitrogen is maintained around the furnace in order to limit the surface temperature of the furnace to about 40°C . A power of about 8 W is needed to heat the foil to 600°C .

The enriched (93% U^{235}) and depleted (0.4% U^{235}) uranium foils are 13.5 mm in diameter and, respectively, 0.06 and 0.05 mm thick. They are placed adjacent to each other in the foil holder with a 0.05-mm steel foil between them to prevent fission product contamination of the depleted foil. A 0.01-mm-thick and 18-mm-wide gold foil is wrapped around the furnace at the uranium foil level and is used as a flux monitor.

In a typical Doppler run, two identical furnaces are loaded with foils, selected to have thicknesses as close to each other as possible. The furnaces are placed in symmetrical positions in the central part of the reactor core. One furnace is heated while the other remains at ambient temperature. The monitoring foil of the heated furnace is displaced vertically by 0.5 mm, corresponding to the thermal expansion of the heater and aluminum holder. The irradiation is made at a reactor power of 10 W and lasts for 2 hr. After shutdown, the "cold furnace" is heated in order to ensure that all foils have an identical history.

The γ activity of the irradiated foils is measured with NaI scintillation detectors. Two 3-in.-dia, 1-in.-thick NaI crystals are placed face to face, and their outputs are added and fed to an amplifier and single-channel analyser. The apparatus includes an automatic sample changer, and the counts accumulated during 300-sec intervals are punched on tape. Detector background and unirradiated-foil-background counting are included in the counting sequence. The measurement on one foil type proceeds for 2 to 3 days until about $2 \cdot 10^7$ counts have been collected on each of the irradiated foils. Finally, the information stored on the tape is analysed on the IBM 360 computer, giving a hot-to-cold foil activation ratio and a statistical analysis of the results.

The 106-keV γ ray of Np^{239} is used in the case of the depleted uranium foils, and the single-channel analyser window corresponds to an energy interval of 30 to 40 keV around this peak. There is a fission product background in this window, mainly due to U^{238} fissions. This background is of the order of 5% and is determined in separate measurements. In the case of the enriched foil, gross fission product activity is counted about a suitable (generally 350 keV) discrimination setting. Integral counting is also used in connection with the gold foils.

Some results for U^{238} and U^{235} are shown in Table 6.

TABLE 6. DOPPLER ACTIVATION RATIOS FOR TEMPERATURE INTERVAL 20–590°C

Foil type	Foil Thickness, mm	Activation Ratio Hot/Cold Foil
U^{238}	0.05	1.106 ± 0.005
	0.10	1.100 ± 0.005
	Theory	1.054
U^{235}	0.063	1.0086 ± 0.0043
	0.127	1.0140 ± 0.0080
	Theory	1.014

The stated limits of error are estimated standard deviations obtained by comparing results from two or three runs only and are therefore uncertain. The spread in results is much larger than the observed errors in foil counting, which were generally less than 0.1%. More work is therefore required to find the causes of the relatively large variations. In one run the coincidence technique was used for counting the Np^{239} activity in the U^{238} foils. From this measurement a fission product correction factor of 1.005 ± 0.003 was obtained, but this correction, being preliminary, has not been applied to the U^{238} results given in the table.

The theoretical values included have been obtained by Häggblom⁽³³⁾ for infinitely thin foils in a homogeneous medium.

The observed Doppler effect in U^{238} is about twice as large as the calculated value. If the stated experimental accuracy is accepted, one may conclude that the calculation is in error. This conclusion, however, is tentative, since there may be a systematic uncertainty in the measurements due to heterogeneity effects. The loading pattern of the different plates around the furnaces was made as close as possible to a homogeneous arrangement, but in order to get a quantitative estimate of the uncertainty caused by the heterogeneity, a truly homogeneous mixture of the core materials should be made up.

In the case of U^{235} the experimental errors are too large to check theory, but the calculated values are not contradicted by the present experiments.

4. Comparison Between Theory and Experiment

4.1. Critical Mass

Table 7 lists the values of critical mass obtained for Assemblies 1 to 5. The errors shown in the experimental figures are mainly due to the uncertainty in U^{235} enrichment and, in assemblies 3 and 5, in the heterogeneity correction factor (see Sect. 3.1). In the case of Assembly

TABLE 7. EXPERIMENTAL AND THEORETICAL CRITICAL MASSES FOR FRO ASSEMBLIES 1-5

Assembly No.	Type of Calculation, Cross-section Set	Number of Energy Groups	Critical Mass, kg U ²³⁵		C/E	Core Centre Median Fission Energy, keV
			Calc C	Exp E		
1	TDC, SPENG	6	70.7	72.9 ± 0.3	0.97	360
2	DSN, SPENG	14	66.7	70.1 ± 0.3	0.95	360
	DSN, Russian set	16	61.2		0.87	
	"Corrected" diffusion theory, FD-2	33	61.3		0.87	
3	TDC, SPENG	6	111.9	111.1 ± 0.6	1.01	270
	DSN, $M_s/M_c = 0.96$, SPENG	14	106		0.95	
	DSN, — " — Russian set	16	97.7		0.88	
	"Corrected" diffusion theory, FD-2	33	100.1		0.90	
4	TDC, SPENG	6	104.3	98.6 ± 2.1	1.06	360
5	DSN, $M_s/M_c = 0.95$, SPENG	16	82.0	78.7 ± 0.7	1.04	50
	"Corrected" diffused theory, Russian set	26	64.2		0.82	

4 a large portion of the error is due to a rather bad reflector geometry.

Most of the calculations of critical mass were made by the Carlson code DSN in S_4 approximation (spherical version) with a maximum number of 16 groups.

For the cylindrical assemblies 1, 3, 4, and 5 a shape factor according to Davey⁽³⁴⁾ was applied. From the measured critical masses of cores 1 (cylinder) and 2 (sphere), a shape factor of 0.961 is obtained, which is in good agreement with the value 0.96 obtained from Davey's curves in this case. In some cases the two-dimensional S_4 code TDC was used for 6-group calculations. Finally, a one-dimensional diffusion code was employed in calculations with more than 16 groups, for example when using the British 33-group set FD-2⁽³⁵⁾ or the Russian 26-group set.⁽³⁶⁾ In the results to be given here, however, the diffusion-theory values are "corrected" to equivalent DSN calculations. The correction factor was obtained by performing 14- or 16-group calculations with both the diffusion code and the DSN code for the assembly in question.

The SPENG calculations listed in Table 7 give critical masses within about 5% of the experimental values in all cases. The TDC code has been found to give systematically larger critical masses than the DSN code, as can be seen, for example, in the case of Assembly 3. The reason for this behaviour has not been established but may be associated with poor convergence in the TDC code.

The calculations with the British and Russian sets give less good agreement with experiment in these assemblies. A comparison between the SPENG and the FD-2 sets shows that the latter uses a higher fission cross section for U²³⁵, whereas the differences in other data are small. The smaller values of critical mass cal-

culated with the FD-2 set should therefore be due mainly to the use of higher U²³⁵ fission cross sections in this set.

The DSN results (including Davey's shape factors) with SPENG cross sections on the copper-reflected assemblies 1, 2, and 3 are slightly too low. On the other hand, calculations for systems having a uranium reflector, for example FRO Assembly 4 and ZPR-III Assembly 16, give too large critical masses. The copper data in the SPENG library (and also in the FD-2 set) therefore appear to yield low values of the critical mass.

4.2. Reaction Ratios at Core Centre

Experimental values of reaction ratios at the centre of the different cores are compared with calculated figures in Table 8. Since assemblies 1 and 2 provide the same information, the results for the former system have been excluded.

Theoretical values were obtained on the basis of the SPENG Set 2B cross sections, including U²³⁵ fission data due to White⁽⁹⁾ in the energy region from 0.04 to 10 MeV. In some cases the ratios were also calculated by use of the SPENG Set 2 in which the U²³⁵ fission cross section is based on the Karlsruhe compilation.⁽¹²⁾ In order to be consistent, one should use the same cross-section set for the spectrum calculation as for the reaction-rate calculations, and this has been done except in the case of Assembly 5, in which the spectrum was calculated only with Set 2B. The Set 2 results for the spectrum-sensitive U²³⁸/U²³⁵ and Np²³⁷/U²³⁵ fission ratios in this core will therefore be slightly inconsistent. The Pu²³⁹/U²³⁵ fission ratio and the U²³⁸(n, γ)/U²³⁵(n, f) ratio, on the other hand, are insensitive to small spectrum changes and will not be affected.

The Np²³⁷ fission cross section is not included in the

TABLE 8. REACTION RATIOS AT CORE CENTRE OF FRO

Reaction Ratio		Assembly 2	Assembly 3	Assembly 4	Assembly 5
$\frac{U^{238}(n,f)}{U^{235}(n,f)}$	Exp	0.0583 ± 0.014	0.0523 ± 0.012	0.0612 ± 0.014	0.0386 ± 0.0009
	$\frac{\text{Calc}}{\text{Exp}} : \begin{matrix} \text{Set 2B} \\ \text{Set 2} \end{matrix}$	$\begin{matrix} 1.031 \\ 1.019 \end{matrix}$	$\begin{matrix} 1.004 \\ - \end{matrix}$	$\begin{matrix} 1.005 \\ - \end{matrix}$	$\begin{matrix} 1.142 \\ 1.122 \end{matrix}$
$\frac{Np^{237}(n,f)}{U^{235}(n,f)}$	Exp	0.442 ± 0.028	0.351 ± 0.022	0.430 ± 0.027	0.234 ± 0.015
	$\frac{\text{Calc}}{\text{Exp}} : \begin{matrix} \text{Set 2B} \\ \text{Set 2} \end{matrix}$	$\begin{matrix} 1.016 \\ 0.950 \end{matrix}$	$\begin{matrix} 1.020 \\ - \end{matrix}$	$\begin{matrix} 1.040 \\ - \end{matrix}$	$\begin{matrix} 1.145 \\ 1.124 \end{matrix}$
$\frac{Pu^{239}(n,f)}{U^{235}(n,f)}$	Exp	1.258 ± 0.029	1.159 ± 0.025	1.289 ± 0.028	1.211 ± 0.026
	$\frac{\text{Calc}}{\text{Exp}} : \begin{matrix} \text{Set 2B} \\ \text{Set 2} \end{matrix}$	$\begin{matrix} 1.041 \\ 0.997 \end{matrix}$	$\begin{matrix} 1.069 \\ - \end{matrix}$	$\begin{matrix} 1.022 \\ - \end{matrix}$	$\begin{matrix} 0.922 \\ 0.906 \end{matrix}$
$\frac{U^{238}(n,f)}{U^{235}(n,\gamma)}$	Exp	0.1044 ± 0.0020	0.1187 ± 0.0041	0.1014 ± 0.0020	0.146 ± 0.013
	$\frac{\text{Calc}}{\text{Exp}} : \begin{matrix} \text{Set 2B} \\ \text{Set 2} \end{matrix}$	$\begin{matrix} 1.109 \\ 1.035 \end{matrix}$	$\begin{matrix} 1.084 \\ 1.008 \end{matrix}$	$\begin{matrix} 1.137 \\ - \end{matrix}$	$\begin{matrix} 0.870 \\ 0.849 \end{matrix}$

SPENG library. In this comparison Np^{237} fission data were obtained from Ref. (11). References (37) and (38) have also been used, but less good agreement with experimental values is obtained.

Table 8 shows the experimental values and the ratio of the calculated and the experimental values. When the experimental errors are taken into account, these ratios have fairly consistent values in assemblies 2, 3, and 4. There appear to be no large errors in the calculated spectra. The Set 2B ratios are systematically slightly larger than unity and fairly constant for each reaction ratio. If the Assembly 5 results are included, however, this pattern is disturbed and considerable fluctuations occur.

Because of the strong variations in flux and spectrum caused by the heterogeneity of the polythene-diluted core 5, the experimental values for this assembly may not be representative for the homogeneous system assumed in the calculations. This source of error needs further study.

The ratios between calculated and measured values in Table 8 can be used for a test of the Set 2 and 2B U^{235} fission cross sections, as this quantity occurs in the denominators of all ratios. The other data and the experimental ratios are then assumed to vary at random around their true values. If all ratios R are collected and Assembly 5 results excluded, one obtains

$$\bar{R} = 1.048 \pm 0.023 \quad (\text{Set 2B data});$$

$$\bar{R} = 1.002 \pm 0.023 \quad (\text{Set 2 data}).$$

It is true that the basis for this type of analysis is doubtful, since there may be a systematic effect of

errors in the other data and in the spectrum, but it does suggest that the U^{235} fission cross section of Set 2 is better than that of Set 2B. In other words, the data indicate that the U^{235} fission cross sections given by White are too low.

4.3. Reaction-rate Distributions

Only a limited selection of the many reaction-rate studies made in the different assemblies will be discussed here. The detectors used may be grouped in three classes according to their spectral sensitivity, the first being the threshold detectors, the second those with smoothly varying cross section, and the third those exhibiting strong resonances in the low-energy region. The results of measurements with such detectors could in principle be combined into a set of linear equations with group fluxes as unknowns, in the way proposed, for instance, in Ref. (39) for threshold detectors. This has not been tried, however. Instead, the spatial variations of the measured reaction rates for the different detectors have been directly compared with the calculated ones, arbitrarily normalizing the curves to unit value at the core centre. The requirements for the calculations are quite different depending on which reaction is involved. As outlined in the earlier sections, most calculations have been made with 14-group DSN or 6-group TDC codes. The small number of groups in the TDC calculations should suffice for the threshold reactions. This is confirmed in Fig. 18, which shows the U^{238} fission rate distribution. The TDC-calculated curve fits the experimental points in the cylindrical Assembly 3 equally well as the 14-group data from the DSN run on the spherical Assembly 2. It is true that the two as-

FIG. 18. U^{238} Fission Rate Distributions in Assemblies 2 and 3.

semblies have somewhat different core compositions, but this is insignificant for the spectral region of importance here.

The situation is more complex when the energy region contributing to a particular reaction rate is broad or when low-lying resonances are important. The 6 groups in the TDC code are then definitely inadequate, especially in the latter case. A large number of groups is needed, and then only one-dimensional codes are available. The flux distributions calculated with the spherical DSN code differ from the corresponding axial and radial distributions in the cylindrical assemblies, however. These differences are particularly pronounced at the core-reflector boundaries, because the spherical-core radius differs from the half-height and radius of the corresponding cylindrical system. In some compari-

sons with experiment the abscissa in the DSN result has therefore been rescaled in such a way as to put the core-reflector boundary at the right position. In areas far from this boundary, the errors due to having different geometries are generally small compared with other uncertainties in the calculated reaction-rate distributions.

The Pu^{239} and U^{235} fission rates may serve as examples of measurements on elements with slowly varying cross sections. The U^{235} fission distribution in Assembly 2 is shown in Fig. 19. The deviations between the experimental points and the calculated curves are substantial in the outer parts of the copper reflector (the Set 2 result should be ignored, due to the punching error mentioned in Sect. 2). The same cross-section sets have been used to calculate the Pu^{239}/U^{235} fission ratio

FIG. 19. U^{235} Fission Rate Distribution in Assembly 2. FIG. 20. Pu^{239}/U^{235} Fission Ratio in Assembly 2.

for Assembly 2, shown in Fig. 20. For this spectrum-insensitive quantity the theoretical results are in fair agreement with experiment in the whole assembly. The discrepancies for the U^{235} fission rate distribution therefore suggest that the flux in the copper reflector is not well calculated, and this is confirmed in the measurements to be discussed next.

Thin foils of gold, manganese, sodium, and U^{238} were used as spectrum-sensitive detectors. Figures 21 and 22 illustrate, for Assemblies 2 and 3, the calculated contributions in percent from different energy groups to the capture rates in the former three detectors.

The 14-group set used in the calculations is not very well suited for reproducing the gold and manganese results in the copper reflector. This is obvious from Figs. 23 and 24. The last group in the calculations,

from 120 to 500 eV, covers the strong 330-eV manganese resonance. Since there is no downscattering from this group to a lower one, there will be a certain "pile-up" of neutrons, and this effect will overestimate the manganese reaction rate in the reflector. In the case of gold the cutoff of the spectrum above the strong 5-eV and the fairly important 61.5-eV resonances means that the calculations will miss the contribution from these resonances completely. The flux is low at these energies, but the high cross section makes the contribution appreciable. The calculations consequently underestimate the gold reaction rate.

These complications do not arise in the case of the $U^{238}(n, \gamma)$ and sodium (n, γ) reactions. In both cases, the calculated curves lie below the measured points in the reflector (compare with Fig. 7 for the U^{238} capture

FIG. 21. Calculated Contributions in Percent from Different Energy Groups to the Mn^{55} and Au^{197} Capture Rates in Assembly 2.

FIG. 22. Calculated Contributions in Percent from Different Energy Groups to the Na^{23} Capture Rate in Assembly 3.

FIG. 23. Au^{197} Capture Rate Distribution in Assembly 2.

FIG. 24. Mn^{55} Capture Rate Distribution in Assembly 2.

rate distribution). Since only small errors can arise from the U^{238} capture cross section, which is well known, the results show that the calculated spectrum in the reflector is too hard. This is confirmed by other information discussed in References 3 and 6.

The strong peaking of the low-energy reaction rate in the detectors used is not present with uranium as reflector. Figures 25 and 26, showing the Mn^{55} and Au^{197} capture distributions, demonstrate this very clearly. The agreement with the calculated curves is good.

The comparisons between theory and experiment for Assembly 5 are not yet available. The strong heterogeneity of this assembly requires special caution in drawing any conclusions.

In an attempt to measure the distribution of the flux at certain energies in the eV-region, some foil activations have been made using the sandwich method proposed by, among others, Ehret.⁽⁴⁰⁾ Each sandwich was composed of three equal foils. The following reactions were utilized: $\text{Au}^{197}(n,\gamma)$, $\text{W}^{186}(n,\gamma)$, and $\text{Mn}^{55}(n,\gamma)$,

FIG. 25. Au¹⁹⁷ Capture Rate Distribution in Assembly 4. FIG. 26. Mn⁵⁵ Capture Rate Distribution in Assembly 4. FIG. 27. Results from W¹⁸⁶ Capture Rate Studies Using the Sandwich Technique, Assembly 5.

the main resonances of which are 4.9 eV, 18.8 eV and 330 eV, respectively. As an illustration of the experimental results (no comparison with theory has yet been made), some preliminary curves from the tungsten measurement are given in Fig. 27.

4.4. Central Reactivity Worths

Calculated values are only available for Assemblies 1 and 3 as yet. The Hansen-Maier⁽⁴¹⁾ second-order perturbation model was used for calculating the perturbed flux, ϕ^p , for small material replacements. Two codes for cylindrical and spherical samples, respectively, have been written. The first of these, PERCYL, is intended for use with 6-group two-dimensional S_N

fluxes and cylindrical samples. The output includes the reactivity and the perturbed angular and regular fluxes, $\phi^p(r, z, E, \Omega)$ and $\phi(r, z, E)$. For the experiments using the small copper bar and sample container (compare Sect. 3.4), the S_N fluxes obtained by TDC calculations on the unperturbed system were used for the PERCYL calculations. With the large bar and sample holder, the TDC calculations were made for the perturbed system, i.e., due regard was taken to the experimental equipment introduced into the core.

The small number of groups in the two-dimensional calculations is a severe limitation. A second-order perturbation code, PERS, for spherical samples and systems was therefore developed. The code was used with 17 group fluxes from a diffusion-theory code, MUCC. The cylindrical samples were in this treatment approximated by spheres, the radii of which were chosen to give the same average chord lengths, $\bar{l} = 4V/A$, as those of the true samples.

The group cross sections used for all flux calculations were obtained as described in Sect. 2. For calculating the reactivity coefficients, the necessary group cross sections were either calculated from the SPENG library or the Russian ABN set⁽³⁶⁾ (the first 17 groups). The SPENG cross sections should be more reliable, as the microscopic data are weighted with an accurate fine-structure flux, whereas the ABN values are lethargy averages below 2.5 MeV. However, the f factors given in the ABN set offer a convenient possibility of taking the resonance shielding of any sample into account.

The calculations thus yield two different sets of reactivity coefficients, the first one, C1, being based on the PERCYL code, the second on the PERS code. The PERCYL values were corrected for resonance self-shielding effects before being presented as set C1. In C2, this is done automatically by the use of the shielded ABN cross sections.

The calculated regular flux spectra in the two assemblies are given in Fig. 3 and the adjoints in Fig. 28. Calculated real group fluxes along the central vertical axis of the core are illustrated in Fig. 29 for Assembly

FIG. 28. Core-centre Adjoint Spectra of Assemblies 1 and 3 as Obtained by 14-group S_4 Calculations.

1. The perturbation of the high-energy flux caused by the experimental equipment is clearly seen.

Some of the experimental and calculated reactivity worths are collected in Table 9. A few comments concerning the results will be given here, but for full details consult reference 5.

U^{238}

The reactivity coefficient increases in absolute number with sample size. This is explained by the decrease in the threshold fission rate in the large samples caused by inelastic scattering in U^{238} . This sample-size dependence is not quite satisfactorily described by the calculations. The C1 and C2 values agree well with each other.

W, Ta, Mo

The reactivity worth as a function of sample size has been measured for tungsten in Assembly 3. The effect per unit weight was constant for the samples studied. Only ABN cross sections were available, and the calculations based on these values overestimate the reactivity values by about 15%. The tantalum and molybdenum values are well predicted by the calculations (C2).

Fe, V, Ti

The negative reactivity effect of iron in the hard-spectrum assemblies 1 and 3 is almost entirely caused by inelastic scattering. The calculations give smaller reactivity effects but reproduce the sample-size dependence well. The C1 set yields slightly smaller values than does C2. The results are illustrated on Fig. 30.

Vanadium and titanium are weak neutron absorbers, as is iron. Calculations and experimental results agree satisfactorily for titanium, although the calculations overestimate the effect for vanadium by more than a factor of two. An examination of the inelastic cross sections shows that vanadium, contrary to titanium, has a large inelastic cross section below 2.5 MeV, where the ABN cross sections are lethargy averages. Considering the form of the spectrum in the energy range of interest, the calculations with the ABN set must yield too high inelastic reaction rates.

Boron

The boron results included in this summary are interesting as they are strongly dependent on the boron absorption cross section at high energies. The large difference between the computed values for Assembly 3 is largely caused by different capture cross sections above 800 keV, the C1 values being considerably higher than those (ABN) used in C2.

The calculated values vary less from Assembly 1 to 3

FIG. 29. Calculated Group Fluxes Along the Central Axis with Large Sample Changer Rod Inserted (Assembly 1).

than do the experimental ones. This would indicate that the calculated spectral shift is underestimated.

The lack of sample-size dependence is striking and, at first sight, somewhat queer. It is due to the fact that scattering down to a certain energy level below the flux maximum compensates for the absorption at this level, as the flux decreases rapidly with decreasing energy.

D and H

The mathematical methods used are not really adequate for deuterium and hydrogen. The flux perturbations in the groups of lower energy reach such a magnitude that the whole perturbation approach is invalidated. No close agreement between experimental and calculated values for the finite sample can be expected for these materials.

The differences between the calculated values of heavy water depend on different weightings and on the varying number of groups, as the cross sections used for deuterium are almost identical. The C2 values again give larger downscattering compared to the C1 values. The experimental value obtained in Assembly 1 lies between the calculated values, but the perturbation model used seems to have lost its validity in Assembly 3, as the experimental value is now significantly above both calculated values. The fact that the effective cross section is lower in Assembly 3 may again be explained by the details of the adjoint spectra.

Diphenyl, $C_6H_5-C_6H_5$, was tested in the small sample holder with three different samples in Assembly 1 and with two in Assembly 3. The results are given in Table 9; the sample size dependence is illustrated in Figs. 31 and 32. An examination of the values obtained by extrapolation to zero sample size in Assembly 1 shows that

TABLE 9. REACTIVITY WORTH PER GRAM (IN UNITS OF 10^{-5}).

Isotope	Weight, g	$(N\bar{l})^{-1}$ b	Assembly 1			Assembly 3		
			Exp. ^(a)	C1	C2	Exp. ^(a)	C1	C2
U ²³⁸ -2 ^(b)	1481	7.4	-0.026	0.028	-0.027			
-3	743	9.9	-0.023	-0.024	-0.025			
-4	248	17.2	-0.016	-0.021	-0.022			
Th ²³²	354	18.2	-0.125		-0.136	-0.101		-0.113
W2	272	24.0				-0.082		-0.093
W3	123	35.6				-0.081		-0.094
W4	31	89.0				-0.081		-0.095
Ta	215	23.2	-0.231		-0.216	-0.200		-0.190
Mo	237	11.2	-0.120		-0.124	-0.093		-0.096
Fe2	604	4.3	-0.063	-0.045	-0.050	-0.035	-0.027	-0.031
Fe3	302	5.7	-0.065	-0.049	-0.053	-0.038	-0.030	-0.033
Fe4	201	7.1	-0.068	-0.051	-0.054			
Fe5	101	11.3	-0.073	-0.054	-0.058	-0.039	-0.032	-0.035
V	107	13.3	-0.019		-0.046			
Ti	77	17.3	-0.066		-0.059			
A12	423	4.9	-0.027	-0.022	-0.030			
A13	212	5.9	-0.021	-0.026	-0.031			
A14	106	7.8	-0.009	-0.028	-0.030			
A15	71	9.7	+0.003	-0.027	-0.030			
A16	35	15.8	+0.033	-0.027	-0.029			
B1 (nat B)	6.5	27.7	-2.51	-2.72	-2.03	-2.27	-2.67	-2.15
B2 "	4.6	29.3	-2.52	-2.70	-2.04	-2.34	-2.65	-2.16
B3 "	2.1	36.1	-2.46	-2.72	-2.03	-2.31	-2.68	-2.15
D ₂ O	5.0	$l = 9.3$	+1.43	+1.22	+1.60	+0.768	+0.638	+0.701
Diphenyl 1	9.5	$l = 17.6$	+3.82	+2.73	+3.58	+3.44	+1.52	+2.35
" 2	7.2	$l = 14.1$	+3.64	+2.72	+3.55			
" 3	2.8	$l = 9.3$	+3.38	+2.73	+3.45	+3.19	+1.53	+2.23

^(a) Experimental uncertainties generally less than $\pm 1\%$.^(b) Natural uranium.

C2 overestimates the down-scattering, while C1 gives a relatively correct value. The cross sections used for hydrogen are almost identical in both sets. All calculated values, however, fail to give an acceptable sample-size dependence.

Even the value obtained with the smaller sample in Assembly 3 is significantly larger than the calculated values, but extrapolation to zero sample size probably gives acceptable agreement.

5. Conclusions and Future Plans

The work at FRO is at present not tied to any project work, a fact that has allowed extensive and systematic studies of different quantities to be made. The experi-

mental results for the hard-spectrum cores (Nos. 1 to 4), for which heterogeneity effects are of minor importance, well fulfill the purpose of serving as check points for calculations. Core 5, on the other hand, was far from homogeneous, so that the interpretation of the experimental results is ambiguous to a certain degree. This will also be true for many future cores.

Despite the relative simplicity of the first set of cores, it is difficult to relate a discrepancy between experiment and theory to errors in a single parameter. Experimental uncertainties and lack of suitable methods to isolate specific phenomena contribute to these difficulties. When this and the complexity of future cores are considered, it is vital that old experimental meth-

ods be refined and new ones developed. Areas to be further explored at FRO include the Doppler activation method at high temperatures and spectrometry using the sandwich-foil technique, as well as differential methods.

Most efforts in other countries are concentrated on sodium-cooled systems. As steam cooling has not yet been ruled out as a possibility for fast power reactors, problems related to such systems will be dealt with at FRO in the near future. The limited resources at this reactor cannot make the studies realistic in the sense that data directly relevant to steam-cooled power systems can be produced. However, it is believed that certain experiments can be made that may be of considerable interest. Measurements of the reactivity effect of polythene (simulating steam), systematic studies of the effect of resonance absorbers, and testing group cross sections for nickel by studies on systems containing large quantities of this material, may serve as examples of what can be done along these lines. Furthermore, it must be desirable to investigate the regions in

FIG. 30. Reactivity Worth per Gram of Iron as a Function of Sample Size in Assembly 3.

FIG. 31. Reactivity Worth per Gram of Diphenyl in Assembly 1 as a Function of Sample Size.

FIG. 32. Reactivity Worth per Gram of Diphenyl in Assembly 3 as a Function of Sample Size.

which experiments with small cores can accurately enough yield measurements that can substitute for those from full-scale systems. This will be studied by comparisons between the work done abroad and suitably designed experiments at FRO.

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Discussion

Mr. Davey (ANL): I think that it is a little dangerous to postulate that the U²³⁵ fission cross section of White is low on this basis. I think a much more realistic point of view is that it is exceptionally important to make sure that the ratios of the cross sections used in

analyzing your fission-ratio measurements are indeed consistent with those measured by the original experimenters. In particular, I think it is important that the White data for U²³⁵ should be correctly combined with the other Aldermaston data to make sure that the ratios of their cross sections are maintained in your cross-section sets. A variation of 8% in the Pu²³⁹ to U²³⁵ ratio measured in Assembly 48 of ZPR-3 is possible if that consistency is not maintained.

Mr. Tirén: Is your comment on Pu²³⁹ pertinent here, since this does not enter these calculations?

Mr. Davey: The point is not specifically Pu²³⁹ or U²³⁵. It is that in all interpretations of fission ratios one must ensure that the ratios of the microscopic cross sections are correctly used. Many experimenters, as we know, measure ratios of cross sections rather than absolute cross sections. This consistency in ratio determination is often not carried through to determine the cross sections that go into analyses, and I mentioned the Pu²³⁹ point to illustrate how important this can be. This 8% correction is merely an illustration.

Mr. Tirén: What I say about the White data may be somewhat exaggerated, but if one cannot make conclusions after this kind of work there is little use in doing it.

Mr. Petersen (GE): In addition to studying the effectiveness of the foil thickness in the activation Doppler measurements, we studied the effect of the furnace wall thickness. Have you looked at this, either analytically or experimentally?

Mr. Tirén: We have looked at this experimentally in one case only, but we are about to do more of this. Our approach is very similar to yours.

Mr. Fischer (ANL): Professor Miller of Northwestern University has developed a Monte Carlo code to study Doppler effects. Among the things being investigated are the effects of thickness of wall, the materials of the vacuum chamber, and the adjacent matrix and drawer materials. I believe that the preliminary results indicate rather large effects. He has also made similar studies of the AI measurements which are reported at this conference.

Mr. Petersen: That is very interesting. I believe that the effect on a measurement using foils is even more pronounced because all of the neutrons interacting in the foil have their last collision outside the furnace.

Mr. Tirén: Yes, what we have done is to look at the Pflasterer results and we find that the wall thicknesses in our furnace are rather small. We don't think that this correction is going to have a significant effect on the results we have reported. The one experiment we have done on this so far supports this belief. I think that the major problem here is heterogeneity.

Mr. Spinrad (ANL): With regard to the work by

Professor Miller, I think it is only fair to say that the tool exists, but that the analyses to date have been only on partial effects and that it would be very premature to say anything about unusually large effects. It shows that there are certain large environmental effects due to half of the problem, but we are not sure that they are not cancelled by the other half of the problem.

Mr. Kato (ANL): Would you comment on what effect heterogeneity has on your fission-ratio measurements. I noticed that your fission counters are fairly small. Are you really measuring an average fission ratio in view of the fact that you had heterogeneous plates? When you are comparing with theoretical calculations, I assume you are doing a diffusion calculation which gives you an average over a certain cell. Have you investigated this?

Mr. Tirén: Except for the last assembly, No. 5, these heterogeneity effects have been very small and only just appreciable in Assembly 3, which was diluted with graphite. In these cases we just took volume averages of the various reaction rates. In Assembly 5 so far only volume averages have been used, but it is obvious that this is not good enough. I understand that this has been investigated by theoretical methods in many places. On the experimental side, as I mentioned, we should try to have smaller plate thicknesses in a small region to deal with this problem.

Mr. Tuttle (AI): Will you describe the method of loading the plates into the vertical tube? Is this done in such a manner that there are planes of uniform composition across the reactor or are the plates staggered?

Mr. Tirén: The plates are staggered.

Presented by D. K. Butler

Measurements and Analysis of Al-, Al₂O₃-, and BeO-reflected Fast Critical Experiments

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1. Introduction

The Argonne nuclear rocket program was developed as a backup effort to the graphite rocket reactor program at Los Alamos. As an alternate to the graphite system, Argonne recommended a refractory-metal-based fast reactor, since the refractory metals have potential for operation at very high temperatures. A fast-neutron spectrum would make it possible to use materials with high thermal-neutron cross sections, and to obtain cores of any desired size down to 20 liters or less. Initial studies led to the choice of tungsten as the refractory metal, possibly alloyed with rhenium where added strength or temperature performance was required, and of uranium oxide as the nuclear fuel. The studies also showed that the best system would probably have a reflector that would help flatten the power distribution and improve neutron economy. With weight at a premium the use of light reflector materials, including aluminum oxide, beryllium, and beryllium oxide, was considered most promising. The series of critical assemblies discussed here was performed in connection with the program for development of a fast tungsten reactor for nuclear propulsion.

At the time, almost nothing was known about the neutronics of such systems. A program of critical experiments was therefore planned to obtain some basic information about fast tungsten-based systems. The approach selected was to begin with systems which had already been studied, such as Assemblies 11 and 22 of ZPR-3 and Assembly 1 of ZPR-6. A series of critical configurations was constructed in which only a single change of composition was made from one assembly to the next.

The experiments were performed with the split-bed assembly ZPR-9. The cores were made as close approximations to cylinders as possible in the rectangular drawers of the assembly. Criticality was normally achieved by varying the core radius.

In each assembly a number of measurements were made. In addition to criticality, determinations of fission-rate distribution, neutron lifetime, and central

and spatial reactivity worths were made. Some time was also spent studying properties related to control, including the effect of inserting a ring of boron into the reflector near the core boundary.

Many of the details of the experiments and some of the calculations have been reported previously.⁽¹⁻⁵⁾ Final reports summarizing all experimental results are in preparation.

Rather than go over all details of measurements, we have tried to select some results for which we have made theoretical comparisons and which, we think, show what has been learned and what is yet to be learned.

Included for discussion will be: (1) criticality, (2) the spatial distribution of fission rates, and (3) the worths of various materials at the core center and as a function of radius.

2. Description of Assemblies

The critical configurations determined in the experiments are shown in Tables 1 and 2. The dimensions have been adjusted to smooth the actual "stair step" outline of the experimental radial outline.

The columns in Table 2 labeled core material and reflector show the progression of materials changes. Assembly 1 is the assembly that had the same core that was studied in ZPR-6 and ZPR-3, but instead of the dense U²³⁸ reflector of the previous experiments, dense aluminum metal was substituted. Assemblies 2, 3, and 4 represent stepwise replacement of U²³⁸ by tungsten. In Assembly 4 the replacement was complete.

Tungsten is a strong resonance absorber of neutrons. Assembly 5 was planned to have a somewhat softer spectrum produced by the addition of some carbon to the core. The actual amount of carbon added was only enough to produce a moderate spectral softening. The sixth assembly was made more like an actual rocket system, by simulating void with perforated aluminum plates. The void would correspond to propellant channels in the actual system.

The first six assemblies had dense aluminum metal reflectors. For the seventh assembly, the reflector was

TABLE 1. PHYSICAL DESCRIPTION OF ZPR-9 ROCKET ASSEMBLIES

	Assembly No.								
	1	2	3	4	5	6	7	8	9
Core									
Radius (cm)	31.39	33.74	35.50	24.62	33.89	36.60	27.56	31.71	26.13
Height (cm)	51.10	61.26	71.42	56.18	56.18	61.26	51.10	51.10	51.10
Inner Radial Reflector									
Radius (cm)	66.63	70.28	70.56	65.37	70.83	75.64	65.52	42.54	77.18
Height (cm)	51.10	61.26	71.42	56.18	56.18	61.26	51.10	51.10	127.30
Outer Radial Reflector									
Radius (cm)	—	—	—	—	—	—	77.18	88.95	88.95
Height (cm)	—	—	—	—	—	—	112.06	51.10	127.30
Axial Reflector									
Thickness (cm)	30.0	30.0	30.0	30.0	30.0	38.10	30.48	37.95	38.10
Core Composition (atoms/cm ³ × 10 ²⁴)									
U-235	0.004516	0.004495	0.004473	0.006717	0.004486	0.004519	0.004498	0.004508	0.004497
U-238	0.03561	0.02418	0.01708	0.000485	0.000323	0.000346	0.000325	0.0003259	0.0003255
C/O	—	—	—	— Carbon	0.02972	—	— Oxygen	0.008863	0.008984
Al	0.006784	0.006803	0.006501	0.0067	0.006803	0.01549	0.01594	0.01656	0.0177
W	—	0.0122	0.02129	0.03953	0.02134	0.02152	0.02169	0.02138	0.02126
Fe	0.000258	0.000221	0.000202	0.000415	0.00221	0.000225	0.000298	0.0002556	0.0003337
Inner Radial Reflector (atoms/cm ³ × 10 ²⁴)									
Be	—	—	—	—	—	—	—	0.05809	—
O	—	—	—	—	—	—	0.05449	0.05809	0.05477
Al	0.05658	0.05658	0.05658	0.05658	0.05658	0.05658	0.04324	0.006917	0.04139
Outer Radial Reflector (atoms/cm ³ × 10 ²⁴)									
Al	—	—	—	—	—	—	0.05658	0.0572	0.0573
Axial Reflector (atoms/ cm ³ × 10 ²⁴)									
O	—	—	—	—	—	—	0.05449	—	0.05477
Al	0.05658	0.05658	0.05658	0.05658	0.05658	0.05658	0.04324	0.0572	0.04139

changed to aluminum oxide. With the added scattering by the oxygen, one can see by the change in critical core volume that this was a much better reflecting material. The original plan was to have only one more assembly (No. 8), namely, one with a beryllium oxide reflector. The delay in obtaining safety approval combined with available time made it possible to insert Assembly 9, which had oxygen added to the core to simulate uranium oxide. This was actually done in the critical experiment by replacing some of the perforated aluminum plates with aluminum oxide. The final assembly of the list had a beryllium oxide radial reflector. In order to simplify obtaining safety approval for operation, aluminum metal was used as the axial reflector to insure a negative reactivity coefficient for axial expansion.

3. Criticality

Table 2 shows results of calculations of k_{eff} for the various assemblies, made by conventional techniques of neutron diffusion and transport theory. For convenience the cross-section set of Hansen and Roach⁽⁷⁾

TABLE 2. VALUES OF k_{eff} CALCULATED BY VARIOUS METHODS

Assembly	Core Materials	Reflector	Core Volume, liters	k_{eff} Diffusion	k_{eff} S_4	k_{eff} 26 Groups
1	U ²³⁵ , U ²³⁸	Al	158	0.966	0.980	0.989
2	U ²³⁵ , U ²³⁸ , W	Al	219	0.975	0.987	0.992
3	U ²³⁵ , U ²³⁸ , W	Al	283	0.976	0.987	0.988
4 ^(b)	U ²³⁵ , W	Al	107	0.982	1.004	0.996
5	U ²³⁵ , W, C	Al	202	0.998	1.020	1.007
6	U ²³⁵ , W ^(a)	Al	258	0.999	1.024	1.018
7	U ²³⁵ , W ^(a)	Al ₂ O ₃	117	0.995	1.003	0.984
9	U ²³⁵ O ₂ , W ^(a)	Al ₂ O ₃	110	1.004	1.012	(0.992)
8	U ²³⁵ O ₂ , W ^(a)	BeO-Al	161	1.007	1.021	0.991

^(a) Core diluted to simulate porosity.

^(b) Fuel concentration 50% greater than for other assemblies shown.

were used early in the program. These did not contain tungsten so that a set of tungsten cross sections were assembled from whatever microscopic information was available.

Several cross-section sets were generated from some basic data. On the basis of the first four critical as-

semblies, one cross-section set was selected as giving the best agreement with experiment. In using this cross-section set to calculate properties of the later assemblies, we have continued to get good results.

Cross sections for rhenium were assembled in a similar way. When used in calculations, these gave results in poorer agreement with the measurements.

Throughout the early part of the rocket critical program, we attempted to calculate properties of tungsten-based systems by one-dimensional diffusion and transport theory. One approach was to calculate in spherical geometry and use a shape factor to convert to cylindrical geometry. For shape factors we were forced to rely on data obtained with uranium reflectors. A second approach was to use the usual cylinder-and-slab synthesis in which the buckling obtained from a slab problem is used in the succeeding cylindrical calculation (or vice versa). It was clear that the light reflectors returned low-energy neutrons distributed spatially in a way that violated the separability of variables assumed in this calculational technique.

Only recently has a many-group two-dimensional diffusion-theory code become available. The column labeled k_{eff} diffusion in Table 2 shows the k_{eff} obtained by use of this code. These values proved to be far more consistent with experiment than those from the one-dimensional iteration procedure.

The calculations for the nine assemblies shown have a rms deviation from 1 of 1.8%. The greatest discrepancy can be seen to be associated with the system containing U^{238} . The U^{238} cross-section set that was used results in underprediction of k . Taking the last six assemblies separately, the rms deviation is only about half that of the U^{238} diluted assemblies.

Even though they were of solid metal, the aluminum reflectors of the earlier assemblies were not good reflectors of neutrons. This means that it is likely that they will not satisfy the diffusion approximation very well. In order to check this, a series of S_N transport calculations were made. These were made in one-dimensional spherical geometry, and the results were converted to equivalent change of k_{eff} as shown in the column of Table 2 labeled k_{eff}, S_4 . As a group, these values show a rms deviation from 1 to 1.6% in Δk . As might be expected, the results for the first six aluminum-reflected assemblies were from 1 to 2% more reactive than would be indicated by results of diffusion theory. The corrections by transport theory for Assemblies 7, 8, and 9 were less because of the greater scattering power of the oxides.

By 1965 there were available considerable new basic data on tungsten and other materials of interest in rocket systems. In order to see whether these new data could be used to obtain improved calculational re-

sults, a 26-group cross-section set was assembled, using the MC² code.⁽⁶⁾ Unfortunately, the library available did not have complete cross sections for very low energies, so that it proved necessary to continue use of the older Hansen-Roach cross sections below 30 eV. Values for k_{eff} calculated by two-dimensional diffusion theory are shown in the last column. As a group, these cross sections show the best accuracy; however, this improvement is primarily in the calculation of the first three assemblies for which the new U^{238} cross-section set gives improved results. Results for tungsten-based systems with the new set are actually a little poorer on the average than are the results based on the Hansen-Roach set.

It is perhaps appropriate to point out that these assemblies are no different from other fast assemblies in that the uncertainty in critical volume is typically something like five times as great as the uncertainty in k . To a person loading a critical assembly and working with fuel of essentially fixed enrichment, this can result in a considerable problem. On the other hand, the uncertainty in enrichment is only about twice that in k . Therefore, an uncertainty in critical mass figured on the basis of uncertainty of enrichment rather than volume as the desirable feature is less sensitive to the geometry of the system and hence allows for a better comparison of the ability to calculate different sized systems.

4. Fission Rates

The radial distribution of fission rates at the core midplane was studied, both theoretically and experimentally. Figure 1 shows the results for the U^{235} fission rate in Assembly 7, the assembly with an aluminum oxide reflector. The values are normalized to unity at the core center. Three sets of data are plotted. The triangular dots were obtained from foil irradiations. The squares were measurements with a solid-state fission counter, and the small crosses show the results of a calculation by one-dimensional cylindrical diffusion theory.

All measurements agree rather well except in the reflector beyond 60 cm, where the measured values are higher than the calculated values. This may be primarily due to the return of low-energy neutrons from the materials surrounding the assembly.

There is another significant disagreement at 30 cm, which amounts to about 30%. Part, if not all, of this discrepancy results from the fact that the counter traverse was made vertically through the core and the foil irradiation horizontally. The pieces of material from which the core is assembled are loaded vertically so there may be streaming up and down that does not exist horizontally.

FIG. 1. Radial Distribution of U^{235} Fission Rate at Axial Midplane of Assembly 7. FIG. 2. Radial Distribution of U^{238} Fission Rate at Axial Midplane of Assembly 7.

Figure 2 shows the U^{238} fission rate normalized to one at the core center for the aluminum oxide-reflected Assembly 7. The squares again represent the results obtained with a solid-state counter; the triangle are

results obtained with foils. Calculated results are shown as small crosses. In the reflector the fission rate depends on high-energy neutrons, and the discrepancy is quite acute. Out to about 45 cm, the major discrep-

TABLE 3. CENTRAL WORTHS (Ih/kg)

Assembly	U ²³⁵	W	H
1	240 (276) ^(a)	-22.6 (-26.0)	
4	270 (286)	-19.6 (-22.3)	3300 (3500)
4B	219	-17.0	-2900
5	197 (221)	16.3 (-21.0)	3900 (600)
6	148 (140)	-10.6 (-11.0)	3700 (2500)
7	176 (163)	-11.0 (-12.8)	3300 (2900)
9	207 (195)	-15.9 (-16.3)	4900 (2100)
8	169 (169)	-11.4 (-14.0)	4800 (1800)

^(a) Calculated values in parentheses.

ancy is between the foil irradiations and the solid-state counter results, with the latter following the calculations quite closely. Beyond 45 cm, both sets of experimental results are much higher than the calculated values. It is not likely that this was caused by room return, since not many high-energy neutrons are returned from the room. As with U²³⁵, some of the discrepancy may be due to streaming in the vertical direction. But it is the foils, which should be less effected by streaming, that have the higher relative count rate. Transport theory might give better agreement with experiment. Clearly, this is the situation leaving something to be learned about both experimental and calculational techniques.

5. Reactivity Traverses

The third series of experiments and calculations are those related to the determination of the reactivity worth of various materials in the assemblies. In all of the assemblies, the worths of a wide variety of materials were measured at the core center. In Table 3 are summarized some of the central worth measurements. The figures in parentheses are the calculated values obtained with perturbation theory using the Hansen-Roach cross-section set. The agreement between experiment and theory for U²³⁵ is reasonably good but some discrepancy remains. The results for tungsten show good agreement between theory and experiment, although again the agreement is not as good as to achieve a thoroughly reliable calculational technique.

The measurements with hydrogen were made to determine the effects of hydrogen propellant on the core. In the fast cores listed in this table the introduction of hydrogen increases reactivity in all but one instance, Assembly 4B. This was a variation of Assembly 4 in which less than 4 v/o rhenium was added to the core. The addition of rhenium increased the critical volume about 30% over that of Assembly 4. This affected hydrogen worth by reversing the sign though maintaining about the same magnitude of reactivity effect. In rocket design this could be a very significant consideration. It shows how reactivity effects can be appreciably sensitive to small changes in core composition.

The calculated worths of hydrogen shown in the third column are in serious disagreement with the measured values. The indications are that the U²³⁵ cross sections tend to overpredict reactivity, with the tungsten producing a compensating underprediction.

In the later assemblies a series of spatial reactivity worth distributions were measured. The experimental technique used was simple in principle. A small tube was installed through the center of the assembly along a diameter at the midplane. Small samples of material were removed through this tube and the resulting changes in reactivity recorded. The key feature in this experiment was the use of a sample drive system that transmitted signals indicating sample position to a computer and the simultaneous transmission to the computer of the instantaneous neutron level. The computer was programmed to calculate the instantaneous reactivity and store it as a function of sample position. The results from several sample traverses could be directly combined. The computer would then directly give output reactivity versus sample position.

The samples used were small and thus the reactivity changes were small. Repeated measurements at neutron levels somewhat higher than for normal operation were required to give results relatively free of reactor noise. One uncertainty in any traverse experiment of this type is the effect of neutron streaming through the traverse hole. For these experiments the hole was about 1 cm in diameter, and some effects of streaming could be expected over distances of perhaps 1 or 2 cm for fast neutrons. With aluminum or beryllium oxide reflectors more serious streaming problems might arise because of the thermal neutrons in the reflector region.

An experimental check of streaming was made by filling the hole both in front of and behind the sample with a lead rod.

Figure 3 shows the results obtained with a sample of U²³⁵. The heavy solid curve was obtained with samples in the empty traverse tube. The lighter line was

FIG. 3. Radial Distribution of Worth of U^{235} at Core Midplane in Assembly 8. Thin line represents results obtained with lead rod.

FIG. 4. Radial Distribution of Worth of Tungsten at Core Midplane in Assembly 8. Solid curve shows experimental data, points results of calculation.

FIG. 5. Radial Worth Distribution of Rhenium at Core Midplane of Assembly 8.

FIG. 6. Radial Worth Distribution of B¹⁰ at Core Midplane in Assembly 8.

obtained with the lead scattering rod preceding and following the sample. The agreement is very close. Another measurement with B^{10} showed similar good agreement. The small irregularities in the traces result from reactor noise.

Figure 4 shows results for a traverse of a sample of tungsten. These measurements were made in Assembly 8 after a small amount of hydrogen was added to the core and reflector in order to study the space-dependent reactivity change with the addition of hydrogen. The hydrogen made a very small change in the other properties of the system. The measured reactivity worth is everywhere negative. A fairly sharp minimum in the beryllium oxide reflector results from the significant number of low-energy and thermal neutrons. Also shown are values calculated using diffusion perturbation theory. Calculations shown as the crosses were made with the Hansen-Roach set, labeled 201, and the squares represent calculations made with the 26-group set, labeled 830. Both cross-section sets give values which are too negative, though the 26-group set falls a little closer to the measured value. At least part of the discrepancy is probably due to the self-shielding effect of the tungsten sample (in the form of a solid cylinder of about 50 g). One problem with these measurements is determining the magnitude of sample self-shielding.

Figure 5 shows results obtained by traverses of rhenium samples in the same assembly. Two experimental curves are shown by the continuous lines. The one with the smaller negative specific worth was obtained with a 54.9-g sample; that with the greater worth was obtained with a 3.4-g sample. Again calculated values are shown. In this instance the rhenium cross section assembled from the Hansen-Roach set leads to an underestimation of the effect while the 26-group set very much overestimates it.

Figure 6 shows results for B^{10} . Four experimental measurements are shown, again representing different sample sizes. The uppermost curve was obtained with a sample of 0.775 g, the second with one of 0.113 g, the third with one of 0.045 g, and the bottom curve for a sample of 0.009 g. Again the results of calculations with both cross-section sets are very much too negative in the beryllium oxide reflector. In this instance the series of samples indicates that the value for an infinitesimal sample, which would satisfy the perturbation approximation, probably would not be nearly as negative as the calculations.

6. Conclusions

As mentioned in Sect. 1, many additional results for these and other qualities were measured and calculated. Although they indicate that the neutronic prop-

erties of the major constituents in the core center are reasonably well predicted, the properties of hydrogen and rhenium are less well predicted. Away from the center of the core the predictions are less good, especially for the Al_2O_3 - and BeO -reflected systems. The disagreements are consistent with the uncertainty of the cross-section information used. Improved cross sections could be expected to give better agreement with experiment. It is possible that this would also require local flux averaging of the cross sections in the various regions where the spectra differs so greatly.

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Discussion

Mr. Richards (ANL): Have you thought of replacing tungsten by molybdenum? Molybdenum is very similar to tungsten, somewhat lighter, and possibly easier to manufacture.

Mr. Butler: In the early parts of the program there was a study of molybdenum, tungsten, and other materials. We have not made measurements with molybdenum. As I remember, molybdenum does not quite have the temperature potential, so we turned directly to tungsten. I did make one calculation, again using the Hansen-Roach molybdenum data, and we decided that if we needed additional reactivity in these cores we might use molybdenum at the cold ends. Molybdenum is neutronicallly a better material.

Mr. Richards: The temperature properties of molybdenum are very similar to tungsten except for extremely high temperatures.

Mr. Butler: For rocket applications the interest is in the highest temperature attainable.

Mr. Meister (Karlsruhe): Did you compare the

ratio of reactivity worths of boron to U^{235} with the Hansen-Roach calculations, or make some lifetime measurements?

Mr. Butler: Yes we have made such calculations, and we also have some lifetime measurements. These will be included in the Proceedings.

Mr. Becker (RPI): Would you describe the experimental procedure used to measure the reactivity?

Mr. Butler: The technique was that developed by C. E. Cohn of Argonne. The sample location and flux as a function of time are read into a DDP-24 computer, programmed to calculate the instantaneous re-

activity on the basis of what the reactivity was a short time before. It continuously calculates reactivity changes and then interprets these as absolute reactivity and stores them as such.

Mr. Becker: What I am curious about is whether or not there was any difference in the core lifetime as you move the perturbation from the core into the reflector. For example, if it depressed the flux in the reflector it might affect the core lifetime. Was that considered in the reactivity calculation?

Mr. Butler: I suggest that you contact Mr. Cohn for such details.

Recent Fast Critical Experiments in the MSCA and 710-CE

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1. Introduction

Experimental fast reactor physics activities at General Electric have embraced both space and central-station power applications. At the Vallecitos Atomic Laboratory, studies utilizing the fast core of the Mixed Spectrum Critical Assembly (MSCA) have concentrated on soft-spectra parameters.* Concurrently, at the Idaho Test Station, small hard-spectrum refractory metal systems have been studied in the 710-CE.†

The fast core of the MSCA contains a loading of 405 kg $U^{235}O_2$, 1540 kg $U^{238}O_2$, and 1060 kg of Inconel in its 400-liter fast-core matrix. Both calculation and experiment demonstrate that the neutron spectrum in this assembly is spatially asymptotic and representative of a dilute fast ceramic reactor (having a calculated median fission energy of 200 keV and about 8% of fissions below 9 keV). Recent measurements to be reported in this paper include fission rates of U^{233} , U^{234} , U^{235} , U^{236} , and Np^{237} , and neutron lifetime as determined by: pulsed neutron and $1/v$ poison substitution, and reactivity worths as determined by pile oscillator, and direct period measurements. Additional experiments are reported elsewhere.⁽¹⁻⁴⁾

Work in the 710-CE at the Idaho Test Station, on the other hand, has been concentrated on small, hard-spectrum, refractory metal critical experiments related to space power reactor applications. In particular, beryllium-reflected cores in the 20- to 50-liter range, containing essentially equal volume mixtures of tungsten and fully enriched (93.2% U^{235}) uranium metal and some tantalum, have been studied. Some of these assemblies provide necessary design data for compact space reactors, while others are primarily devoted to basic physics measurements and furnish very useful integral data for evaluation of higher-energy cross sections and specialized transport problems. Data to be reported in this paper are taken from the first

basic critical experiment and include: central fission ratios, reflector effects on power distribution and neutron lifetime, and relative reactivity worths.

2. MSCA Measurements

2.1. Physical Description

The physical design of the MSCA, a full-scale mockup of a 100-MW(e) power plant reactor core, is given in detail in other reports.^(3, 5) The reactor contains four distinct fuel zones: a fast core, a fast buffer, a thermal buffer, and a thermal core. Figure 1 shows the relative position of each zone. The central fast-core zone approximates a right circular cylinder, 32 in. in diameter and 30 in. high, reflected top and bottom by a 9-in. Inconel reflector. The fast-core composition is 48% UO_2 , 33% Inconel, and 19% void by volume. The UO_2 , in pellet form, is loaded into individual fuel rods of either 26.51 w/o U^{235} or natural enrichment. The measurements reported here were conducted with an average fast-core enrichment of 23.1 w/o U^{235} . The thermal core has a typical BWR composition.

2.2. Activation Ratio Measurements

Activation rate measurements were performed within the fast-core region: (1) to verify cross sections in conjunction with other independent spectrum measurements,^(6, 7) and (2) to demonstrate that the neutron spectrum is constant over an appreciable region of the fast core. They included the fission rates of U^{233} , U^{234} , U^{235} , U^{236} , and Np^{237} , as well as the capture rate of U^{238} .

For many years, gas ionization chambers⁽⁸⁾ or "Kirn counters" have been used successfully to determine relative fissions rates. However, recent developments in solid-state detectors provide several advantages which led to their adoption for the MSCA measurements:

1. improved energy resolution for both alpha particles and fission fragments;
2. smaller size, resulting in less flux perturbation;

* Work performed under AEC Contract No. AT(04-3)-189.

† Work performed under AEC Contract No. AT(40-1)-1847.

FIG. 1. Plan View of MSCA.

FIG. 2. Fission-ratio Assembly.

3. electronically faster;

4. elimination of gas flow to counters.

Very thin (6 to $100 \mu\text{g}/\text{cm}^2$) fissile sources, $\frac{3}{8}$ in. in diameter, of isotopically pure fissile material were deposited on 5-mil-thick, $\frac{1}{16}$ -in.-diameter, stainless steel or aluminum foils. Isotopic analyses of the fission sources showed, in most cases, negligible fissile contamination. The sources were prepared by evaporation from a hot filament in a vacuum and were uniform within a few percent across their active area. The mass of each fissile material "seen" by its detector is determined by the alpha count rate.

The fissile sources were carefully mated to solid-state detectors as shown in Figure 2. Each detector was made from 20-mil-thick, $\frac{3}{4}$ -in.-diameter silicon

wafers having a sensitive area of 100 mm^2 and a dead layer $0.25 \pm 0.05 \mu$ thick. Special care was taken to eliminate hydrogen (epoxy) from the detector area (by a multidiffusion process), which could erroneously soften the neutron spectrum incident upon the fissile materials.

Two fuel rods were removed from a central MSCA fast-core fuel bundle to allow insertion of two detectors with their fissile sources back-to-back. The detectors, in turn, were attached to a drive mechanism enabling axial traversing during reactor operation. The simultaneous measurement of fission rate of two fissile materials eliminated the necessity of calibrations of reactor power level. After suitable amplification and stretching, the detector pulses were analyzed

FIG. 3. Typical Energy Spectrum during Irradiation of U^{234} Source.

on a two-parameter analyzer used in the dual (independent) channel mode. A typical pulse-height spectrum, shown in Fig. 3, indicates the excellent delineation between fission fragments, alpha particles, and background events. The fission rates were determined by integrating all events above an arbitrary but systematically determined cutoff.

In the center of the fast core, ten fission ratios were measured using the isotopes U^{233} , U^{234} , U^{235} , U^{236} , and Np^{237} . The measured fission ratios are compared in Table 1 to values obtained from 24-group axial diffusion calculations. By considering all combinations and evaluating the differences between experimental and calculated values, it is possible to draw the following conclusions:

U^{233} fissions underpredicted by $(4.9 \pm 2.2)\%$
 U^{234} fissions overpredicted by $(10.0 \pm 2.2)\%$

U^{235} fissions underpredicted by $(8.6 \pm 2.2)\%$
 U^{236} fissions overpredicted by $(3.4 \pm 2.2)\%$
 Np^{237} fissions underpredicted by $(0.1 \pm 2.2)\%$

This moderate but systematic disagreement between calculated and experimental values provides direct evidence that the MSCA fast-core spectrum is softer than predicted. Thus, these measurements confirm evidence from direct neutron-spectrum,^(6, 7) as well as other spectrum-sensitive, measurements, that the calculated spectrum is too hard.

To investigate the axial variation of the neutron spectrum near the core-reflector interface, traverses were taken with U^{233} , U^{234} , and U^{235} detectors. As shown in Fig. 4, the U^{234} fission rate, which is indicative of the fast-flux profile, approximates a cosine distribution. The U^{233} and U^{235} fission rates, which are sensi-

TABLE 1. COMPARISON OF EXPERIMENTAL AND CALCULATED FISSION RATIOS (A/B)

B	A				
	U-233	U-234	U-235	U-236	Np-237
U-233	— —	0.173 } — 12.3% 0.153 }	0.638 } + 1.7% 0.649 }	0.0528 } — 4.8% 0.0503 }	0.170 } — 3.0% 0.165 }
U-234	5.76 } + 12.7% 6.54 }	— —	3.68 } — 12.5% 4.17 }	0.305 } + 3.0% 0.314 }	0.983 } + 11.9% 1.10 }
U-235	1.57 } — 1.9% 1.54 }	0.272 } — 12.5% 0.240 }	— —	0.083 } — 8.7% 0.0758 }	0.267 } — 11.5% 0.238 }
U-236	18.9 } + 5.2% 19.9 }	3.28 } — 3.1% 3.18 }	12.1 } + 8.7% 13.2 }	— —	3.23 } + 2.8% 3.32 }
Np-237	5.87 } + 3.4% 6.07 }	1.02 } — 11.9% 0.905 }	3.74 } + 11.6% 4.20 }	0.310 } — 2.9% 0.301 }	— —
Average Difference	+ 4.9%	— 10.0%	+ 8.6%	— 3.4%	+ 0.1%

Notes: 1. Fission ratio is A/B.
 2. Upper values are from a 24-group axial calculation.
 3. Experimental accuracy is $\pm \sim 2\%$ in all cases.
 4. Difference shown is $\left(\frac{\text{Experiment}-\text{Calculation}}{\text{Calculation}} \right) \%$.

FIG. 4. Relative Axial Fission Distributions through Center of MSCA Fast Core.

tive to lower-energy flux, experience substantial peaking in the Inconel reflectors.

A preliminary measurement of the U^{238} capture rate (relative to U^{235} fission rate) was made to provide an indication of the conversion ratio present in the fast core. This measurement provided a modified conversion ratio, U^{238} captures/ U^{235} fissions, of 0.437 ± 0.023 compared to a calculated value of 0.393. A softer spectrum would reduce this discrepancy. This measurement was conducted by irradiating natural and fully enriched uranium metal foils and subse-

quently evaluating the Np^{239} and fission product activity, respectively. A more precise measurement would require the use of highly depleted uranium, which was not delivered before the termination of the program.

2.3. Neutron Lifetime and Reactivity Worths

Since the MSCA contains two regions of vastly different neutron spectrum, and hence neutron lifetime, the coupled-system characteristics may be altered significantly by adjusting the relative importance of the two regions. Initially, the assembly was taken critical without the fast core, that is, with a central void. Under these circumstances, the power distribution and neutron lifetime (via pulsed neutron measurement) were measured to provide a starting base. The thermal-core reference parameters observed were

$$l = 60.0 \mu\text{sec}; \quad k\beta/l = 125 \text{ sec}^{-1}.$$

During the fast-core loading the power distribution and neutron-lifetime measurements were repeated until the desired power split of 0.33/0.67 between the fast and thermal regions was obtained. Figure 5 shows the observed decrease in neutron lifetime during this transition. Even though the fast core has about six times the importance of the thermal core at a power split of 0.33/0.67, it can be seen that the thermal core

FIG. 5. MSCA Neutron Lifetime vs Power Split.

still dominates the neutron lifetime, the fast-core lifetime being on the order of $0.2 \mu\text{sec}$.

With the MSCA at the reference 0.33/0.67 power split, several additional dynamic and static neutron-lifetime and reactivity-worth measurements were conducted. Although calculations were performed using either 1-D or 2-D, 24-group diffusion theory for the coupled reactor, it is qualitatively useful to describe the reactor behavior via Avery's two-node model.⁽⁹⁾ In the model, the criticality factor k is expressed as the sum of the contribution from each of the fueled regions:

$$k = \sum_i k_i.$$

For small changes in k , the reactivity is given by

$$\rho = \frac{dk}{k} = \sum_i k_i \left(\frac{dk_i}{k_i} \right).$$

Since k_i is proportional to ν_i ,

$$\frac{dk_i}{k_i} = \frac{d\nu_i}{\nu_i}$$

and

$$\rho = \sum_i k_i \frac{d\nu_i}{\nu_i}.$$

The value of ν_i cannot be varied experimentally, but by adding poison to the various reactor regions it is possible to produce an equivalent change in k_i . These experiments were performed using small amounts of boron. Additional experimental information, available from these measurements, is the neutron lifetime, since

$$\lim_{c \rightarrow 0} \frac{l}{c} = \rho/c,$$

where (c/ν) is the effective cross section of an absorber

added to produce a reactivity change (ρ) and c is a constant. The resulting k_i 's and neutron lifetime inferred from these measurements are compared to calculations and to the neutron lifetime measured by pulsed-neutron techniques in Table 2. It can be seen that the k for the fast core is overpredicted by both the 1-D and 2-D calculations. This is mainly attributed to an overprediction of the fast-core importance (adjoint-flux weighting). The discrepancy in the neutron-lifetime measurements is not unreasonable considering the complexity of the MSCA. The underprediction is consistent with the underprediction of k for the thermal core, since the thermal core clearly dominates in the lifetime determinations.

A rather extensive list of small-sample central reactivity worths were determined via the pile-oscillator technique. Unfortunately, due to the early program termination, analytic comparisons are available for only five materials. These comparisons are given in Table 3. Good agreement was obtained for all but hafnium. The substantial overprediction for this material may be due to omitting self-shielding from these small-sample calculations since hafnium is known to have substantial resonance structure.

3. 710-CE Measurements

3.1. Physical Description

At the Idaho Test Station, critical experiments have been conducted for a number of small, fully enriched, refractory metal assemblies related to space power reactor applications. Core volumes have spanned the 20- to 50-liter range, with a length-to-diameter ratio near unity. Core materials have included uranium,

TABLE 2. CRITICALITY FACTOR AND NEUTRON LIFETIME

	Measurement	Calculation	
		1-D	2-D
k —fast core	0.63	0.83	0.77
k —thermal core	0.37	0.17	0.23
l (poison), μsec	25	12.7	14.2
l (pulsed neutron), μsec	30	12.7	14.2

TABLE 3. COMPARISON OF MEASURED TO CALCULATED REACTIVITY WORTHS

Material	Measured (ϵ/kg)	Calculated (ϵ/kg)
Natural UO_2	-1.0	-1.1
Inconel	-1.4	-1.0
Nickel	-1.1	-1.1
Boron Carbide	-92	-79
Hafnium	-7	-24

FIG. 6. Plan View of 710-CE.

FIG. 7. Elevation View of 710-CE.

tungsten, tantalum, and aluminum metals. The measurements reported here were conducted with a 22.5-liter core having the following composition:

Uranium metal (93.2% U^{235} -Oralloy)	27%
Tungsten metal	26%
Tantalum metal	10%
Aluminum matrix	7%
Void	30%

This small unmoderated core is surrounded by a beryllium reflector. The core and reflector materials are loaded into a split-table assembly consisting of two honeycomb matrices of hexagonally shaped aluminum tubes, 1¾ in. across flats. A plan view is shown for

the basic core in Fig. 6. Figure 7 shows the vertical cross section.

3.2. Fission-ratio Measurements

As in the program of MSCA measurements, fission rates were measured to evaluate the integral behavior of the neutron spectrum and its spatial dependence. The exposure of miniature absolute fission chambers, at alternate core locations, provided the most precise indication of local fission rates. Isotopes evaluated included: Th^{232} , U^{233} , U^{235} , U^{238} , Np^{237} , Pu^{239} , and Pu^{240} .

Rather than using the solid-state technique employed at Vallecitos, small, gas-flow fission chambers were employed in these measurements. This approach offered sufficient accuracy with a minimum extension of past experience. Through the use of very thin fissionable-material coatings and suitable geometry, detectors having an absolute efficiency of 98% in 2π geometry were developed.

The chambers were constructed of aluminum, 40 mils thick, with a circular sensitive area of 0.5-in. diameter. Figure 8 shows their cross section schematically. The sensitive area consisted of a fissionable coating, from 90 to 120 μg , deposited on a polished stainless steel plate, 2.5 mils thick and 1.27 cm^2 in area. At one-atmosphere gas flow, a cathode-to-anode spacing of 0.156 in. provided good relative fission product sensitivity for the chambers. Attempts to operate the chambers as sealed units were unsuccessful due to outgassing from the walls. In the gas-flow mode of operation, a peak-to-valley ratio of 10 to 1 was observed between the fission product pulses and the alpha and gamma background. This compares to a peak-to-valley ratio of approximately 13 to 1 for the solid-state-detector selectivity observed at Vallecitos.

During the measurements, the chambers were located with their active plane parallel to, and 2.5 cm from, the table-separation plane. The separation was introduced to eliminate any low-energy streaming flux between table halves. Comparison of measured and

FIG. 8. Miniature Absolute Fission Chamber.

TABLE 4. COMPARISON OF EXPERIMENTAL AND CALCULATED FISSION RATIOS

	$r = 2.5 \text{ cm}$		$r = 11 \text{ cm}$
	Exp/Calc #1 %	Exp/Calc #2 %	Exp/Calc #2 %
Th-232/U-235	— —	$\frac{0.023}{0.0190} \left\} - 17.4$	$\frac{0.022}{0.0179} \left\} - 18.6$
U-238/U-235	$\frac{0.083}{0.0736} \left\} - 12.8$	$\frac{0.083}{0.0825} \left\} - 0.6$	$\frac{0.075}{0.0776} \left\} + 3.5$
Pu-240/U-235	— —	$\frac{0.525}{0.519} \left\} - 1.1$	$\frac{0.494}{0.482} \left\} - 2.4$
Np-237/U-235	$\frac{0.56}{0.505} \left\} - 10.9$	$\frac{0.56}{0.608} \left\} + 8.6$	$\frac{0.50}{0.563} \left\} + 12.6$
Pu-239/U-235	$\frac{1.19}{1.22} \left\} + 2.5$	$\frac{1.19}{1.24} \left\} + 4.2$	$\frac{1.20}{1.21} \left\} + 0.8$
U-233/U-235	— —	$\frac{1.56}{1.53} \left\} - 1.9$	$\frac{1.59}{1.50} \left\} - 5.7$

calculated fission rates (relative to U²³⁵) are given in Table 4 for a near-central location ($r = 2.5 \text{ cm}$) and for a position 4 cm from the radial reflector ($r = 11 \text{ cm}$). The calculations employed a 16-group S_8 cylindrical model with a single thermal group. The cross sections employed in this calculation were flux weighted as discussed in Sect. 3.3.

Calculation #1 employed early estimates of the tungsten inelastic cross section which were based on the optical model with the exit spectrum Maxwellian down to threshold energies. In calculation #2, each individual level below 1.5 MeV was treated on a resolved basis using ABACUS-II.⁽¹⁰⁾ In both cases, the pointwise cross sections were weighted over a fission spectrum above 100 keV and over a normal-mode slowing-down spectrum below 100 keV as noted in Sect. 3.3. The sensitivity to the tungsten inelastic cross section is evident in this fully enriched unmoderated system. The systematic correlation is generally good with the ABACUS treatment, although individual errors up to 20% are present. The average error for the threshold ratios is only -2%.

3.3. Reflector Effects on Power Distribution and Neutron Lifetime

Since the central neutron spectrum is quite hard, it is possible to consider unself-shielded cross sections averaged over several groups selected to properly span a degraded fission spectrum. Such calculations have been performed and do provide reasonably good predictions of the measured central fission ratios (see Table 4). However, the beryllium reflector is known

TABLE 5. NEUTRON LIFETIME (μs) IN THE 710-CE

<i>Measurements</i>	
Dynamic	5.6 ± 1.0
Static	4.6 ± 0.4
<i>Predictions</i>	
Infinitely dilute (standard 16-group set)	3.50
Homogeneous core	3.98
Homogeneous core with 50% further reduction of resolved resonance cross sections	4.25

to return a very soft neutron current to the outer core surface. Since the outer core surface is quite "important" in these small cores, improper treatment of the transition region would be expected to lead to significant discrepancies in integral properties such as prompt-neutron lifetime and local properties such as fission distributions. This point was explored experimentally.

The neutron lifetime has been measured by dynamic (Rossi- α and pulsed-neutron techniques) as well as by the classical static $1/v$ poison-substitution technique. The dynamic measurements have been especially interesting since three clearly separable decay modes have been identified. They may be viewed qualitatively as the bare fast-core decay, an intermediate mode of decay, and the slow coupled core-reflector (asymptotic mode) decay. It is, of course, this last decay mode which yields the neutron lifetime.

The lifetime was also determined by $1/v$ poison substitution using boron-10. In particular, the spatial dependence of B¹⁰ worth was mapped at discrete points and the data interpolated to give the integral value for

FIG. 9. Axial Power Distributions in 710-CE.

FIG. 10. Radial Power Distribution in 710-CE.

the entire assembly. Table 5 provides a comparison between measured and predicted neutron lifetimes. All three calculations were carried out in 16-group S_4 radial geometry. The first calculation used the standard unself-shielded cross sections.⁽¹¹⁾ The second, referred to a "homogeneous core," used cross sections energy self-shielded for the bare fast-core spectrum generated by the normal-mode slowing-down program,

C-Fine.⁽¹²⁾ The C-Fine spectrum decreased more rapidly than the assumed $1/E$ spectrum, leading to greater weighting of the high-energy portion of the group and a reduction of both the smooth and resonance cross sections. In the lowest energy groups, the resolved resonance absorption in tungsten, tantalum, and U^{238} was decreased by as much as 90%. Of course, what is really required is a one-dimensional cross-section-weighting program to treat properly the additional self-shielding present in the core-reflector region. To provide insight in this direction, a third calculation was performed with an arbitrary doubling of the C-Fine self-shielding. The resulting improved agreement with measured lifetimes is, of course, fortuitous. However, this adjustment provides an adequate description of the system pending a detailed spatially dependent self-shielding treatment.

Radial and axial power distributions were also studied to evaluate the reflector effects. The power distributions were measured by: aluminum fission product-catcher foils, direct scanning of core-length fuel rods, and traversing miniature U^{235} fission chambers. The bulk of the measurements used the catcher-foil technique. Sixteen-energy-group calculations were employed in 1-D and 2-D, S_x transport theory. Figures 9 and 10 show comparisons between the measured fission-rate distributions and 2-D S_4 , 1-D S_4 axial, and S_8 radial calculations, using the adjusted self-shielded cross-section set. The importance of correctly treating the axial variation in the traverse leakage is apparent near the end reflector. Table 6 indicates the effect of introducing self-shielding in the S_8 radial calculations. Clearly, the trend is in the right direction.

3.4. Differential Reactivity Measurements

Measurement of differential reactivity worths and their correlation with predictions is an important integral test of cross sections and the predicted spec-

TABLE 6. RADIAL POWER PEAKING IN 710-CE

	Peaking Factor		
	$P_{\max}/P_{\min}$	$P_{\text{center}}/P_{\min}$	Distance $P_{\min}$ from Reflector (cm)
Measured	5.0 ± 0.2	1.14 ± 0.02	3.7 ± 0.5
Calculated			
Infinitely dilute (standard 16-group set)	4.0	1.22	2.5
Homogeneous core	4.5	1.19	2.9
Homogeneous core with 50% further reduction of resolved resonance cross sections	4.63	1.17	3.1

TABLE 7. WORTH OF CORE-LENGTH MATERIALS NEAR THE CORE AXIS

Material	Worth, Percent $\Delta k/k$ per gram		Error, %
	Exp	Axial S_{16} Calc	
Oralloy	7.00-4	7.00-4 ^(a)	—
Hydrogen	2.06-2	2.39-2 ^(b)	+16
Beryllium	1.02-3	1.26-3 ^(b)	+24
Aluminum	1.94-4	2.16-4 ^(b)	+11
Iron	9.82-5	1.12-4 ^(b)	+14
Nickel	4.86-5	5.41-5 ^(b)	+11
Manganese	1.01-4	1.18-4 ^(b)	+17
304 SS	8.58-5	1.02-4 ^(b)	+19
Tantalum	-4.67-5	-4.21-5 ^(b)	+10
Titanium	9.89-5	1.04-4 ^(b)	+5
B^{10}	-9.00-3	-6.73-3 ^(c)	+25
		-9.07-3 ^(d)	-1
W	1.37-5	2.84-5 ^(b)	+107
		1.26-5 ^(e)	-9
Mo	6.32-5	4.17-5 ^(b)	-34
		6.62-5 ^(e)	+5
Nb	4.65-5	4.11-5 ^(b)	-12
		6.41-5 ^(e)	+38
Re	-5.69-5	-8.78-5 ^(b)	-54
		-3.95-5 ^(e)	+32

^(a) Normalized-to-measured value. Normalization factor = 1.423 vs measured local-to-average Oralloy worth of 1.35.

^(b) NMPO 16-group cross-section set was used. A small positive bias is expected due to the two-dimensional effect of the radial reflector extension.

^(c) NMPO 16-group B^{10} was used.

^(d) Same as (c) except $\sigma_{n,\alpha}$ values above 80 keV, where correction suggested by D. Bogart was used.⁽¹⁴⁾

^(e) Bondarenko cross-section set was used.

trum. The differential reactivity worth for core-length pieces of 15 different materials has been measured. Table 7 provides a comparison between these measured values and axial one-dimensional S_{16} calculation using the standard 16-group cross-section set. A normalization factor of 1.423 was applied to all the predicted core-averaged worths to adjust them to the core centerline average based on the U^{235} measurement.

The worths of the mildly absorbing materials is a balance between positive scattering and negative absorption effects. The small positive bias for these materials, relative to U^{235} , may be due, in part, to two-dimensional reflector effects. The tungsten worth is in the poorest agreement; however, the Bondarenko⁽¹³⁾ cross sections greatly reduce the discrepancy. The Bondarenko (n,γ) cross-section set is about 20 to 30% higher than the standard set over the important energy

range. However, both sets are well within the spread of the measured cross sections and conditions present quite frequently in this type of evaluation.

Acknowledgments

The authors wish to acknowledge the direct participation of Messrs. P. Brown, R. K. Haling, H. F. Johnston, P. Meyer, F. L. Sims, and W. P. Walsh in the specific measurements and analyses tasks reported. Their efforts are fully appreciated.

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Discussion

Mr. Kato (ANL): As regards your MSCA fission-ratio measurements, did you have any difficulty with radiation damage of your solid-state detectors? If so, how long did they last?

Mr. Peterson: Indefinitely, as far as this experiment is concerned. I think that if you were to use them several years, perhaps you might encounter such effects, but these measurements performed at quite low fluxes.

Mr. Kato: For the U^{238}/U^{235} ratio measurements,

we have found that even at low power, the solid-state detectors tend to become noisy.

Mr. Peterson: Our experience did not confirm your observation.

Mr. Becker (RPI): I'm curious about the measurement and calculation of the reactivities of the fast and thermal portions of the core in the mixed-spectrum assembly. How do you define the experimental reactivity that you measured, and how do you correlate that with the essentially two-mode fit to the data?

Mr. Peterson: We followed the relationship where you define the k , of course, as $\sum_i k_i$ and where you look at reactivity effects as $\sum_i k_i(\Delta\nu_i/\nu_i)$ for that region, and we made an addition of boron equivalent to a k change, or a ν change, of $(\Delta\nu/\nu)$ and through the measured reactivity effect of that boron addition

determined the k_i for that region. These are not the k 's for the bare region, of course. The k_{11} is about 0.94 for this system, although the k_1 is lower, about 0.74, I believe.

Mr. Meister (Karlsruhe): Do you have any idea on the origin of the large overprediction of the reactivity of the fast zone in the mixed-spectrum reactor?

Mr. Peterson: We believe it is mostly a geometrical effect between these two rather complicated regions. The geometry, of course, is based on engineering requirements rather than physics considerations in the layout of the assembly. But, because of our ability to predict the central parameters in the fast region, and of our ability to predict the thermal region independently of the fast region, the error must be caused by geometrical problems.

ZPR-3 Assembly 48: Studies of a Dilute Plutonium-fueled Assembly

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ABSTRACT

A series of relatively simple plutonium-fueled assemblies with well-degraded spectra has been designed for study in ZPR-3. Assembly 48, the first in the series, was chosen as the subject of an international comparison of fast reactor calculation techniques which is reported elsewhere at this conference. Each assembly in the proposed program will consist of a cylindrical core surrounded by a depleted uranium blanket. In Assembly 48 the material constituents of the core are limited to plutonium, depleted uranium, sodium (25%), graphite, and the stainless steel present in the structure and canning of the sodium and plutonium plates. Graphite is included to degrade the neutron spectrum. The ratio of uranium to plutonium in the core is 4.2.

The program of measurements with Assembly 48 is still in progress. The results of the critical-mass evaluation, fission cross-section ratio, and central perturbation measurements are described. Also included are a brief statement of the results of the neutron spectrum and Doppler coefficient measurements which are reported in more detail elsewhere in these proceedings.

1. Introduction

Assembly 48 in the ZPR-3 series is the subject at this conference of a comparison of calculation techniques used by various laboratories throughout the world for the design of fast reactors. The assembly was designed to provide core neutron flux and adjoint spectra similar to those expected in large plutonium-fueled fast power reactors while retaining simplicity in composition and arrangement. The material constituents of the core zone are limited to plutonium, depleted uranium, sodium (25%), graphite, and the stainless steel present in the structure and canning of the sodium and plutonium plates. Graphite was included to degrade the neutron spectrum. The ratio of uranium to plutonium in the core is 4.2, a figure limited

by the available fuel inventory of 350 kg plutonium-239. The measured critical mass of the heterogeneous critical cylinder is 272 kg plutonium-239 and the core volume 417 liters.

In recent years since the study of a 17% enriched uranium core in Assembly 41, ZPR-3 has been occupied with a series of zoned criticals and fast test reactor mockups. Assembly 48 marks a return to cores which are relatively simple in composition and arrangement, and it is the first of these to be fueled with plutonium. The uranium-fueled ZPR-3 assemblies up to and including 41 have provided the fast reactor physicist with a comprehensive range of experimental studies against which to test analytical techniques. This important set of results has been supplemented in the past few years by results from ZPR-6 and ZPR-9 of Argonne National Laboratory, and from Vera and Zebra in the United Kingdom.

The volume of integral data which is available for plutonium-fueled assemblies is small, however, and is mainly limited to assemblies with rather hard neutron spectra. Only three assemblies of those so far reported^(1, 2) had a simple composition and arrangement, and were comprehensively studied. These are Vera 11A, a plutonium-graphite assembly with a critical mass of 34 kg Pu-239; Zebra 3, a plutonium-natural uranium assembly with a critical mass of 81 kg Pu-239; and the recent study of Zebra 6, a plutonium-natural uranium-graphite-sodium assembly with a critical mass of 186 kg Pu-239 + Pu-241. The largest assembly built so far is the SEFOR mockup, ZPR-3 Assembly 47.⁽³⁾ This simulated a mixed-oxide-fueled, sodium-cooled core with beryllium oxide added to soften the spectrum and enhance the Doppler effect. The critical mass was 314 kg Pu-239 + Pu-241. However, like all reactor mockups, it was very complicated both in composition and arrangement, and is thus not an ideal source of integral data for the evaluation of nuclear-data compilations and calculation techniques. A number of assemblies has been built with a central plutonium-fueled test zone surrounded by a uranium-

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CORE DRAWER

CONTROL DRAWER

SAFETY DRAWER

FIG. 1. Top Views of Assembly-48 Drawer Loadings.

fueled driver zone. In each case the prime intention of the assembly was the measurement of Doppler coefficients in flux and adjoint spectra typical of large power reactors. The test zone in ZPR-3 Assembly 45 was typical of a large carbide-fueled reactor.⁽⁴⁾ A series of assemblies was studied in Zebra 5⁽⁵⁾ with the aim of evaluating the components of the Doppler coefficient in a mixed-oxide-fueled reactor.

The need for critical studies of dilute plutonium-fueled assemblies which are as simple as possible in composition and arrangement is clear. It is the aim of Assembly 48 and following assemblies to fill this need within the limitations of the present fuel inventory.

The program of measurements with Assembly 48 is still in progress. This paper presents an interim review of measurements made up to the week immediately preceding This Conference. A final statement of the results will be published later. Attention is confined here to a description of the assembly and the measurements made. The results of the critical-mass evaluation, fission cross-section ratio, and central perturbation measurements are described. Also included are a brief statement of the results of the neutron-spectrum

and Doppler-coefficient measurements reported in more detail in companion papers by Bennett *et al.*⁽⁶⁾ and Gasidlo,⁽⁷⁾ respectively. Comparison of measured results with calculated values from various sources is made by Davey.⁽⁸⁾

2. Assembly Design

The assembly has a single cylindrical core zone surrounded by a reflector of depleted uranium with an average thickness of 12 in. The material constituents of the core are limited to plutonium, depleted uranium, sodium, graphite, and the stainless steel present in the structure and canning of the plutonium and sodium plates. The graphite is present to degrade the neutron spectrum. It was preferred over other diluents such as oxygen and beryllium because it has a simpler cross-section structure.

The core composition is based on a 2-in.-wide, or 1-drawer, cell to simplify the problems of interpreting the effect of the plate heterogeneity on the critical mass and sodium-loss coefficient. A number of factors influenced the choice of plate arrangement:

(1) the desire to use a simple cell and to maximize the use of the available inventory of plutonium. The inventory consists of 200 kg Pu-239 in the form of metal plates and 165 kg Pu-239 alloyed with uranium. The optimum use of the inventory requires the use of one alloy plate and one metal plate in each cell. A maximum core loading of 313 kg Pu-239 is possible for this arrangement;

(2) the desire to produce a well-degraded neutron spectrum typical of large power reactors, by the inclusion of graphite;

(3) the desire to include the maximum amount of sodium;

(4) the desire to have a high ratio of uranium to plutonium.

The plate arrangement chosen for the standard core drawer is given in Fig. 1. There are two fuel plates in each cell, one 0.125-in.-thick plutonium metal plate (4.5% Pu-240) and one 0.250-in.-thick 20% plutonium-uranium alloy plate (8.5% Pu-240). A cell is considered here as the plate arrangement across the width of a drawer. In order to achieve the required shutdown margin in ZPR-3 it is necessary in cores of this size to spike the control rods with additional plutonium, since poison control rods are not available. The plate arrangements in the 8 safety and 2 control-rod drawers are also given in Fig. 1.

The general arrangement of the assembly as loaded for the critical mass determination is given in Figures 2 and 3. The loading contained 324 standard core drawers, and 8 safety and 2 control rods.

The compositions of the core and reflector are given

FIG. 2. Assembly-48 Drawer Loading in Half No. 1.

in Table 1. The core composition is based on an assembly fueled only with standard core drawers and is thus consistent with the corrected critical mass. An axial schematic diagram of this final arrangement is given in Fig. 4.

3. The Critical Mass

To obtain the critical mass of the heterogeneous equivalent critical cylinder, a number of corrections must be applied to the loaded fuel mass of 260.78 kg Pu-239. In addition to the normal correction for the partial insertion of one control rod at the critical position, a correction must be applied for the fuel spikes in the control and safety rods. Measurements were made at a selection of drawer positions to determine the reactivity change produced by the replacement of a standard drawer with safety and control-rod drawers. From these measurements it was possible to estimate

the loss in reactivity produced by changing all control- and safety-rod drawers to standard drawers.

The difference in reactivity between the core as built with an irregular stepped radial boundary and a truly cylindrical core of the same cross-sectional area was estimated. Measurements were made of the reactivity worth of standard drawers at different positions around the boundary. These results were used to calculate the balance of reactivity gains and losses consequent on smoothing the outline to that of a true cylinder.

The sum total of the three corrections to the critical mass mentioned above was obtained in units of reactivity. This total was converted to an increment in mass added at the radial boundary by using the measured relationship between reactivity and the addition of edge fuel. The value of the proportionality constant q in the expression

FIG. 3. Assembly-48 Drawer Loading in Half No. 2.

$$\frac{dk}{k} = \frac{1}{q} \frac{dm}{m}$$

implicit in this addition was 6.35. For the purposes of understanding the significance of these three corrections, it is useful to break down the total mass correction into the sum of components. The results of this critical-mass analysis are given in Table 2. The largest correction by far is for the removal of 5.6 kg Pu-239 from the control and safety rods, which requires the addition of 20.2 kg Pu-239 at the core edge. The critical mass of the heterogeneous critical cylinder can be quoted variously as

292.0 kg Pu total;
274.3 kg Pu (239 + 241);
272.4 kg Pu-239.

The error for the critical mass is about 0.5%.

A further correction to these figures is necessary for the effect of the interface gap between the two halves. This correction will be measured in a future experi-

TABLE 1. ASSEMBLY-48 COMPOSITION ($\times 10^{22}$ atoms/cm³)

Isotope	Core	Reflector	
		Axial	Radial
Pu ²³⁹	0.1645		
Pu ²⁴⁰	0.0106		
Pu ²⁴¹	0.0011		
Pu ²⁴²	0.00004		
U ²³⁵	0.0016	0.0082	0.0083
U ²³⁸	0.7427	3.8377	3.9798
C	2.0767		
Na	0.6231		
Fe	0.9985	0.5515	0.4444
Cr	0.2681	0.1481	0.1193
Ni	0.1330	0.0729	0.0587
Mo	0.0206		
Al	0.0109		

ment, but is estimated to reduce the quoted mass by about 2 kg. Calculations will also be made later to evaluate the correction for plate heterogeneity required

to obtain the equivalent homogeneous critical mass. It is estimated provisionally that the critical mass of the truly homogeneous cylinder will be about 297 kg Pu-239.

4. Fission-ratio Measurements

Measurements of fission ratios have been made at positions close to the core center using two sets of U-233, U-234, U-235, U-236, U-238, Pu-239, and Pu-240 fission chambers. The Kirn chambers⁽⁹⁾ are sealed and have a relatively thick stainless steel wall. They have been used for the majority of measurements in past ZPR-3 assemblies. The new Davey spherical thin-wall aluminum flow chambers⁽¹⁰⁾ each contain two foils in back-to-back geometry. Their mass calibrations have been determined from a knowledge of the plated mass and by using a thermal-flux calibration technique.

The measurements were made in two positions close to the core center with the Kirn chambers placed back to back at the front of the drawer pairs 1 O 14-2 O 14, and 1 Q 18-2 Q 18. The Davey chambers were placed at the front of drawers 1 O 14 and 1 Q 18.

The measured fission ratios obtained from the two sets of chambers are compared in Table 3. Correction factors for the effect of inelastic scattering in the Kirn-chamber walls were determined experimentally in this assembly. The accuracy of the measured ratios is typically about 1.5%. The two sets of fission-ratio results agree within the quoted errors. This agreement gives confidence in the independent mass calibrations of the two sets of chambers and in the wall-effect corrections applied to the Kirn chambers.

The corrections applied to the measured fission ratios for the effect of inelastic scattering in the Kirn-chamber walls have been based previously on a small number of measurements with the same U-235 and U-238 foils used in both thin-wall and thick-wall stainless steel flow chambers. Davey⁽¹¹⁾ used these results to deduce corrections to the other measured ratios. Since the foils in the Kirn chambers are sealed in, it was necessary to construct special fission chambers for the measurements in this assembly. A pair of thick-wall stainless steel flow chambers was made with the same dimensions as the Kirn chambers. A pair of thin-wall aluminum flow chambers with the same external dimensions as the flow chambers but with walls about 0.010 in. thick was also made. A set of U-234, U-236, U-235, and U-238 foils was used. A U-235 foil and one other foil were used for fission-ratio measurements in both sets of chambers.

From a comparison of the measured ratios the correction multipliers to be used to correct the Kirn-chamber results can be deduced, since the thin aluminum walls of the second set of chambers have a

FIG. 4. Axial Schematic Diagram of Assembly 48.

TABLE 2. A SUMMARY OF CRITICAL PARAMETERS FOR ASSEMBLY 48

	Pu-239, kg
Fuel loading of assembly as built (324 fuel drawers, 8 safety- and 2 control-rod drawers)	260.78
<i>Corrections to the loaded mass for</i>	
1. One control rod partly inserted	-0.64
2. Control-rod spiking	
(a) replacement of safety and control drawers by fuel drawers	-5.61
(b) fuel added at boundary to balance removal of spikes	+20.19
3. The irregular stepped outline of the core	-2.27
Mass balance	272.45
Critical mass of the heterogeneous cylinder	272 ± 1.5
<i>Critical dimensions</i>	
Core radius, cm	41.71
Core height, cm	76.35
Core volume, liters	417.2
Radial blanket thickness, cm	30
Axial blanket thickness, cm	30

negligible effect on the measured ratios. The results of these measurements are compared with the values previously used in Table 4. The agreement is excellent, giving confidence in the previous evaluation of results from Kirn chambers.

5. Central Perturbation Measurements

The majority of central perturbation measurements in ZPR-3 assemblies has been made using 2 × 2 × 2-in. samples of different materials. Ideally, one requires from an experiment a measure of the reactivity change

TABLE 3. FISSION-RATIO MEASUREMENTS IN ASSEMBLY 48

Ratio	Kirn Chambers Uncorrected	Kirn with Wall Correction	Davey Chambers	Corrected Kirn/ Davey
U-233/U-235	1.454 ± 0.023	1.454	1.480 ± 0.015	0.98 ± 0.02
Pu-239/U-235	0.995 ± 0.015	0.995	0.976 ± 0.010	1.02 ± 0.02
Pu-240/U-235	0.231 ± 0.003	0.241	0.243 ± 0.002	0.99 ± 0.02
U-234/U-235	0.193 ± 0.003	0.201	0.204 ± 0.002	0.99 ± 0.02
U-236/U-235	0.0643 ± 0.0009	0.0679	0.0670 ± 0.0006	1.01 ± 0.02
U-238/U-235	0.0284 ± 0.0005	0.0307	0.0307 ± 0.0003	1.00 ± 0.02

TABLE 4. MEASUREMENTS OF THE CHANGE IN FISSION RATIOS DUE TO SCATTERING IN THE KIRN-CHAMBER WALLS

Fission Ratio	Thick-wall Chamber Ratio Thin-wall Chamber Ratio	Multiplier to Correct Kirn-chamber Measurements	
		Measured	Estimated
U-238/U-235	0.918 ± 0.004	1.082	1.08
U-236/U-235	0.944 ± 0.004	1.055	1.06
U-234/U-235	0.959 ± 0.003	1.041	1.04
Pu-240/U-235	0.957 ± 0.003	1.042	1.04

per atom of the perturbing material, which is independent of the size of the sample used, and of the heterogeneity of the surrounding medium. This aim is difficult to achieve in practice. The sample size can be reduced until an acceptable lower limit in the accuracy of the reactivity measurement is reached. However, the sample should still be large enough to average the fine structure of the flux and adjoint, produced by the heterogeneous plate structure, when placed in the chosen environment.

The variation of the reactivity worth per atom derived from different sizes of samples is dependent on the combination of a number of effects which include:

1. neutron multiplication in fissile samples;
2. resonance self-shielding in fissile, fertile, and absorbing samples;
3. self-shielding in heavily absorbing samples;
4. multiple scattering in all samples.

A study of the effects of sample size and of the heterogeneity of the core on the measured reactivity worth per kg has begun in Assembly 48.

The $2 \times 2 \times 2$ -in. sample changer consists of a modified core drawer with cutaway sides at the interface end to allow the insertion and removal of samples when the drawer is withdrawn from the reactor. There is a large reactivity change on removal of the drawer for the changeover of samples, which limits the rate of drawer motion. The separation in time of two measurements, one with the sample out and the other with

the sample in, is typically about 30 min. During this time significant changes in the reactivity of the assembly can be produced by changes in temperature. The reference measurement with a central void was continually repeated to correct for this temperature drift, which is the prime factor in limiting the measurement accuracy to about $2 \times 10^{-6} dk/k$.

In an attempt to diminish the importance of temperature drift on the measurements, and to permit the use of samples of a smaller size, an additional sample changer was designed for this assembly. This permitted the insertion of small, cylindrical perturbation samples at the core center by using a long steel rod, of 0.45-in. diameter, which was inserted in the radial traverse tube that passes close to the interfacial plane. The rod contained two carrier sections with sample spaces 2 in. long $\times$ 0.42 in. in diameter, one of which was used for the sample and the other as a reference. The reference carrier position was either empty, or contained a dummy can if a canned sample was in the sample carrier. By limiting the reactivity worth of the samples and moving no core material in the sample-change operation it was possible to compare sample and void with a time separation of about 4 min. Sample and void were interchanged several times to improve the accuracy and further reduce the importance of temperature drift. The accuracy of the measurements depends on the number of cycles made, and whether or not the normal control rods were moved. The limiting accuracy was about $10^{-7} dk/k$.

Measurements were made using both sample changers. The reactivity changes were determined from the movement of a calibrated autorod. For certain samples it was also necessary to adjust the position of the normal control rods. In this assembly $1\% dk/k = 1002$ inhours.

The first set of measurements was made using the $2 \times 2 \times 2$ -in. sample changer in the central drawer position 1 P 16. The reactivity worths of 2-in. cubes of iron, chromium, nickel, stainless steel 304, graphite, aluminum, aluminum oxide, manganese, molybdenum, and depleted uranium used in previous ZPR-3 assemblies were measured. In addition, some measure-

ments were made with small-plate, foil, and cylindrical samples of smaller sizes by placing them in a thin-wall (0.015 in.) stainless steel box with outer dimensions $2 \times 2 \times 2$ in. The errors on the measurements with small samples were too large to show any significant differences, due to size effects, from the results obtained with the cubes.

Two new 2-in.-cube samples of sodium canned in stainless steel and two dummy cans were used to determine the sodium worth. The sample changer was modified so that the samples could be cycled without shutting down the reactor and reloading the changer. By waiting for good conditions of temperature stability within the assembly, and by recycling the sodium samples and their dummy cans several times in succession, an accuracy of about $3 \times 10^{-7} dk/k$ was achieved for the sodium measurement.

The measurements with the new cylindrical sample changer were completed immediately before This Conference, and the results must be regarded as provisional. A new set of samples was prepared which includes small cylinders and annular foil samples of the fissile and fertile materials, and tantalum. The cylinders range in diameter from 0.062 to 0.200 in., and the

TABLE 5. CENTRAL PERTURBATION MEASUREMENTS WITH ENRICHED AND DEPLETED URANIUM SAMPLES

Sample Type	Thickness, Diameter or Cube Size, in.	Measured Worth, lh/kg
<i>Enriched uranium</i> ^(a)		
Foils		
Annular	0.0014	315 $\pm$ 14
Annular	0.0034	314 $\pm$ 4
Annular	0.0052	310 $\pm$ 4
Plate	0.0052	292 $\pm$ 38
Annular	0.0106	312 $\pm$ 3
Plate	0.0106	321 $\pm$ 10
Plate	0.02	316 $\pm$ 9
Plate	0.04	319 $\pm$ 5
Plate	0.062	322 $\pm$ 2
Cylinders	0.061	312 $\pm$ 5
	0.10	310 $\pm$ 2
	0.20	310 $\pm$ 2
<i>Depleted uranium</i> ^(b)		
Foils		
Annular ^(c)	0.022	-23.3 $\pm$ 0.5
Plate	0.125	-21.4 $\pm$ 1.6
Cylinders	0.20	-22.2 $\pm$ 0.5
	0.42	-20.8 $\pm$ 0.2
Cube	$2 \times 2 \times 2$	-19.2 $\pm$ 0.1

^(a) Enrichment range: 93.10 to 93.30% U²³⁵.

^(b) 0.21% U²³⁵.

^(c) 0.40% U²³⁵.

TABLE 6. CENTRAL PERTURBATION MEASUREMENTS WITH PLUTONIUM SAMPLES

Sample	Thickness or Diameter, in.	Measured worth, lh/kg
<i>Plutonium</i> ^(a) (4.5% Pu-240)		
Foils		
Annular	0.005	422 $\pm$ 5
Annular	0.010	427 $\pm$ 4
Annular	0.020	430 $\pm$ 4
Plate ^(b)	0.086	462 $\pm$ 3
Cylinders	0.100	434 $\pm$ 5
	0.200	443 $\pm$ 4
<i>Plutonium</i> ^(c) (22% Pu-240)		
Foils		
Annular	0.005	356 $\pm$ 6
Annular	0.010	366 $\pm$ 4
Annular	0.020	367 $\pm$ 3
Cylinders	0.062	356 $\pm$ 11
	0.100	371 $\pm$ 4
	0.200	381 $\pm$ 3

^(a) Composition of 98.6 w/o Pu, 1.4 w/o Al, plus impurities. Plutonium mixture of 95.05 w/o Pu-239, 4.50 w/o Pu-240, and 0.45 w/o Pu-241.

^(b) Measured in $2 \times 2 \times 2$ -in. sample changer.

^(c) Composition of 98.8 w/o Pu, 1.2 w/o Al, plus impurities. Plutonium mixture of 72.24 w/o Pu-239, 22.28 w/o Pu-240, 4.63 w/o Pu-241, and 0.79 w/o Pu-242.

TABLE 7. CENTRAL PERTURBATION MEASUREMENTS WITH BORON AND TANTALUM SAMPLES

Sample	Thickness or Diameter, in.	Measured Worth, lh/kg
<i>Boron (enriched in B¹⁰)</i> ^(a)		
Annular	0.010	-7632 $\pm$ 40
	0.020	-7523 $\pm$ 40
	0.040	-5838 $\pm$ 30
Cylinders	0.060	-7418 $\pm$ 80
	0.100	-7303 $\pm$ 50
	0.200	-6698 $\pm$ 30
<i>Tantalum</i>		
Foils		
Annular	0.005	-164 $\pm$ 5
Annular	0.010	-156 $\pm$ 3
Annular	0.015	-149 $\pm$ 2
Plate	0.005	-169 $\pm$ 42
Plate	0.010	-182 $\pm$ 21
Plate	0.015	-167 $\pm$ 14
Cylinders	0.061	-142 $\pm$ 5
	0.100	-137 $\pm$ 2
	0.200	-125 $\pm$ 1
	0.200 ^(b)	-131 $\pm$ 14

^(a) Composition of 92.8 w/o B and 7.2 w/o impurities; Boron = 92.1 w/o B¹⁰, 7.9 w/o B¹¹.

^(b) Measured in $2 \times 2 \times 2$ -in. sample changer.

TABLE 8. CENTRAL PERTURBATION MEASUREMENTS FOR A SELECTION OF NONFISSILE SAMPLES

Sample Type	Thickness, Diameter or Cube Size, in.	Measured Worth, lh/kg
Fe		
Cylinder	0.42	-12.3 ± 0.4
Plate	0.25	-11.5 ± 2.0
Cube	2.0	-11.3 ± 0.2
Cr		
Cylinder	0.42	-9.4 ± 0.5
Cube	2.0	-10.8 ± 0.6
Ni		
Cylinder	0.42	-18.2 ± 0.2
Plate	0.25	-18.7 ± 1.6
Cube	2.0	-15.9 ± 0.2
SS 304		
Cylinder	0.42	-14.1 ± 0.2
Cube	2.0	-12.7 ± 0.2
Na		
Cylinder	0.42	-7.0 ± 4.5
Cube	2.0	-6.3 ± 0.3
C		
Cylinder	0.42	-4.5 ± 1.2
Cube	2.0	$+1.3 \pm 1.2$
Al ₂ O ₃		
Cylinder	0.42	-11.5 ± 1.2
Cube	2.0	-9.5 ± 0.1
Mo		
Cylinder	0.42	-43.4 ± 0.4
	0.42	$-45.4 \pm 5.1^{(a)}$
Plate	0.25	-46.6 ± 2.9
Cube	2.0	-39.0 ± 0.3
Al		
Cylinder	0.42	-15.7 ± 0.8
Cube	2.0	-14.5 ± 0.6
Mn		
Cylinder	0.42	-21.4 ± 0.5
Cube	2.0	-19.0 ± 0.4
Polyethylene		
Cylinder	0.42	$+1676 \pm 7.0$

(a) Measured in $2 \times 2 \times 2$ -in. sample changer.

foils in thickness from 0.0012 to 0.040 in. Samples of boron enriched in the boron-10 isotope were also prepared in the same geometry, using boron powder canned in stainless steel. Two sets of plutonium samples were made with Pu-240 contents of 4.5% and 22%, respectively. Both sets were canned in stainless steel. The other materials used were either in the form of solid cylinders 2 in. long by 0.42 in. in diameter or were powders contained in stainless steel cans of the same outer dimensions.

The results of the measurements made in both sample changers are summarized in Tables 5, 6, 7, and 8. Comparison of the measured inhours/kg obtained with the 2-in. cubes and the small cylinders for several materials shows clear evidence that the measured worth per unit mass is higher for the smaller sample.

For the plutonium samples the results show that the measured inhours/kg increase as the sample size increases, an effect which can be qualitatively attributed to internal multiplication within the sample. The large increase in the measured inhours/kg with decrease in size of boron and tantalum samples demonstrates the need for a careful understanding of the precise size of the sample used in any measurement when making a comparison with calculated values.

This is necessarily a brief preliminary appraisal of the results. An attempt will be made after This Con-

TABLE 9. SUMMARY OF CENTRAL PERTURBATION MEASUREMENTS

Material or Isotope	Measured Worth, lh/kg	Sample Size used to Derive Worth Value
U-235	$+339 \pm 4$	Isotopic value derived from 0.0034-in. enriched uranium and 0.022-in. depleted uranium annular samples
U-238	-24.8 ± 0.5	Isotopic value derived from 0.0034-in. enriched uranium and 0.022-in. depleted uranium annular samples
Pu-239	$+445 \pm 4$	Isotopic value derived from 0.005-in. annular samples of 4.5% and 22% plutonium
Pu-240	$+81 \pm 20$	Isotopic value derived from 0.005-in. annular samples of 4.5% and 22% plutonium
B ¹⁰	-8926 ± 80	0.010-in. annular sample
Steel (304)	-14.1 ± 0.2	0.42-in. cylinder
Fe	-12.3 ± 0.4	0.42-in. cylinder
Cr	-9.4 ± 0.4	0.42-in. cylinder
Ni	-18.2 ± 0.2	0.42-in. cylinder
Mn	-21.4 ± 0.4	0.42-in. cylinder
Al	-15.7 ± 0.8	0.42-in. cylinder
Na	-6.3 ± 0.3	2-in. cube
C	-4.5 ± 1.2	0.42-in. cylinder
O	-6.8 ± 0.9	Derived from values for 0.42-in. cylinders of Al and Al ₂ O ₃
Ta	-164 ± 5	0.005-in. annular sample
Mo	-43.4 ± 0.4	0.42-in. cylinder
Polyethylene	$+1676 \pm 7.0$	0.42-in. cylinder

TABLE 10. MASSES AND COMPOSITIONS OF THE ZPR-3 OXIDE DOPPLER SAMPLES AT 500°K

Isotope	Composition, 10^{24} atoms/cm ³	
	Natural Uranium	Pu-239
U-238	0.0156	
U-235	0.0001	
Pu-239 + 241		0.0147
Pu-240		0.0004
O	0.0314	0.0302
Total mass, g	274.4	263.8

ference to correlate the measured differences in worth per unit mass obtained from samples of different sizes.

Selected values from the measured results, generally from samples of the smallest size, have been used for comparison with the calculated values in a study of the assembly at this conference. The experimental results used for this purpose are given in Table 9.

6. Measurements of Doppler Effect

The measurements of Doppler effect made in this assembly are described in more detail by Gasidlo;⁽⁷⁾ only a summary will be given here.

TABLE 11. MEASUREMENTS OF REACTIVITY CHANGES PRODUCED BY HEATING OXIDE DOPPLER SAMPLES

T(°K)	Reactivity change, $10^{-6} dk/k$		
	Natural Uranium ^(a)	Pu-239 ^(b)	Empty Can
300	0 ± 0.0		0 ± 0.046
355		0 ± 0.04	
500	-1.75 ± 0.09	-0.29 ± 0.05	$+0.055 \pm 0.052$
650		-0.49 ± 0.06	
800	-3.76 ± 0.07	-0.87 ± 0.06	-0.023 ± 0.056
900		-1.01 ± 0.06	
1052		-1.25 ± 0.06	
1100	-4.89 ± 0.11	-1.16 ± 0.08	-0.015 ± 0.061

^(a) Average of two runs.

^(b) The lowest experimental temperature was 355°K due to the decay heat of the plutonium sample.

The natural uranium and plutonium samples used consisted of 70% theoretical-density oxide pellets, contained in stainless steel cans. A summary of composition data for the samples is given in Table 10. The pellet stack in each sample was 1.27 cm in diameter and 15.28 cm long. Two identical samples were used in any measurement. They were placed on the same axis and separated by a gap of 12.06 cm to form a single sample assembly. During the measurements the sample assembly was placed parallel to the axis of the core and with the gap between the samples at the core midplane. The samples were fixed so that the axial expansion of one sample was directed towards the core midplane, while that of the other was away from the midplane. This results in a first-order correction for axial expansion effects.

The samples were oscillated at different temperatures and the reactivity changes measured relative to a reference material, by using a calibrated autorod.

The results of measurements made with high-purity plutonium-239 and natural uranium oxide samples are given in Table 11.

The natural uranium sample was measured twice to check the reproducibility of the technique. The average of the results is given in Table 11. All but one of the results was within one standard deviation of the average used. The plutonium results have not been corrected for expansion effects; thus the measured negative effect may not in fact be attributable to the

FIG. 5. Central Spectrum of ZPR-3 Assembly 48.

Doppler effect. The results with natural uranium showed a negative Doppler effect as expected. The expansion corrections for the natural uranium sample are believed to be negligible.

7. Measurements of Neutron Spectrum

Measurements of the neutron spectrum at the center of the core were made using both proton-recoil counters and a Li-6 spectrometer. The methods used are described in more detail by Bennett *et al.*⁽⁶⁾

The proton-recoil measurements were made using both hydrogen- and methane-filled counters at a gas pressure of about 5 atmos. The hydrogen-filled counter was used for the range from 1 to 150 keV, and the methane-filled counter was used above 150 keV. The measured spectrum is shown in Fig. 5. The fine structure in the measured spectrum can be attributed to scattering resonances in the light material present in the core.

The Li-6 spectrometer consisted of a Li⁶F layer on a thin organic film mounted centrally between the silicon surface-barrier detectors. The results from the Li-6 spectrometer were compared with those from the proton-recoil counters by calculating the pulse-height distribution for the Li-6 spectrometer using the measured gas-counter results. The agreement between the two is within counting statistics over the majority of the energy range.

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Discussion

Mr. Campise (AI): You mentioned that in extrapolating your sample-size effect to zero thickness that you expected upwards of an additional 5% correction. Do you plan experiments with smaller samples, and also do you plan to vary the arrangement of the drawer around the sample to evaluate the effect of heterogeneity on the measurement?

Mr. Broomfield: In answer to your first question, I think that as far as the boron is concerned we evaluate this 5% correction by considering the collision probabilities within the samples of different sizes. This is very approximate at this stage. In answer to your question about smaller samples, we have difficulty making smaller samples by the present technique. We are compacting powder in a very fine annulus. Ten mils is about the lower limit with this method. However, we are looking at the possibility of making samples by other methods.

Your second question involved varying the degree of heterogeneity around the sample. I didn't mention that in both the fission-ratio and the perturbation measurements the chambers and the samples were placed at the center of a 2-in. cubic cavity with the plates oriented so they pointed into the cavity. We are considering a limited series of additional measurements with plates placed parallel to the cavity and possibly with plates of different types adjacent to the cavity.

One reason for choosing this 2-in. cell, incidentally, is that we hope that by doing this we are averaging to a reasonable degree the variation of flux and adjoint across the cell. When you make measurements in one drawer and the cell in your reactor consists of three drawers, there is some problem of knowing whether you are really measuring what you're comparing with calculation.

Mr. Specht (AI): What is the conversion from in-hours to reactivity for this assembly?

Mr. Broomfield: 1% $\Delta k/k$ is equivalent to 1002 inhours.

Mr. Weale (UK-Aldermaston): I'd like to add a comment related to Dr. Campise's remarks about the fission-ratio measurements. We found in one of the VERA assemblies that we could change the U²³⁸ to

U^{235} fission cross ratio by 10% according to whether we made the measurements in a small hole cut through the fuel plates or made the measurement in a cavity 1.7 in. square which was effectively surrounded by a thickness of steel that amounted to almost 0.1 in. I wonder whether this point has been covered in these ZPR-3 measurements. Is this 2 by 2-in. cavity surrounded on the four sides by steel or aluminum of the matrix?

Mr. Broomfield: There are perforations between the two front faces of the drawers which are opposing, but on the sides there is solid steel, about $\frac{1}{16}$ in. thick.

Mr. Weale: I think it might have a rather important effect.

Mr. Davey (ANL): We recognize that these are not final measurements. We are fully aware of the difficulty of trying to interpret measurements made in the cavity in a soft-spectrum fast reactor in terms of simple homogeneous calculations. One of the steps we are considering is to return to the thermal-reactor techniques of making such measurements, actually within and between plates, probably by foil methods or very small fission chambers.

Presented by A. R. Baker

Zebra 6: A Dilute Plutonium-fuelled Assembly

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ABSTRACT

Zebra 6 was a plutonium-fuelled fast reactor with natural uranium, graphite, and sodium as diluents. The critical mass was 186 kg of ($\text{Pu}^{239} + \text{Pu}^{241}$). The report contains a selection of the measurements performed, including mainly techniques which will be important in the study of the PFR mockup. Thus the critical mass and the reactivity worth of sodium, probably the topics of most direct importance, are discussed. Measurements of the neutron spectrum and the gamma dose rate through the reactor are described. From perturbation and reaction rate measurements with Pu^{239} and B^{10} , the capture-to-fission ratio in Pu^{239} is obtained. Because the reflector contained moderating material, having a composition nearly like uranium carbide, some attention is given to cross-section measurements in the reflector.

The results are compared with calculations using diffusion theory with the FD2 multigroup cross-section set.

1. Introduction

Between October 1965 and April 1966, four variations of a dilute Pu^{239} -fuelled fast reactor were studied on the Zebra zero-energy system at A. E. E. Winfrith. In several ways these Core-6 assemblies and the experiments performed were designed to test methods for studying the Zebra mockup of the Prototype Fast Reactor, Core 7, which has now been in operation for four months. Thus the Core-6 assemblies used almost the whole of the then-available plutonium inventory of 200 kg to build as large a core as possible. To achieve criticality and simultaneously a neutron spectrum as soft as in the PFR, the fuel was diluted with natural uranium and graphite. It was not possible to simulate the PFR adjoint function well, chiefly because the U^{238} volume fraction was rather less than in the power reactor design. The core also contained a large volume fraction (20%) of sodium to provide, not empirical values of coolant reactivity worth for the PFR, but the means for testing methods of measuring it. A 30-

cm-thick reflector containing two parts by volume of natural uranium and one of graphite surrounded the cylindrical core. This composition has approximately the same moderating ratio as uranium oxide and was intended to provide a reasonable simulation of the PFR breeder sections.

Of the four variations, 6A, B, C and D in chronological order, Core 6A, including the transition to 6D, received the most detailed study. All the features just described were contained in Core 6A, and 6D was different simply in the replacement of the sodium by void. In Core 6B and C advantage was taken of the two different isotopic mixtures of plutonium in the Zebra inventory, containing 5% and 10% of Pu^{240} . These systems were two-zone cores: in 6B, the centre zone was filled with the high- Pu^{240} fuel and the outer mainly with the low- Pu^{240} fuel. In 6C the zone compositions were simply interchanged. The diluent fractions were exactly the same as in 6A, and the main object here was to see how the reactivity worth of sodium depended on the Pu^{240} fraction.

Of the measurements discussed below, those with sodium form a distinct group. In this group a notable experiment was the very accurate measurement of buckling in 6A, leading to a reasonably accurate result for the reactivity worth of sodium. The method has application in determining the coolant reactivity worth in a conceptual reactor from measurements in a mixed critical system with only a small zone of the correct composition. Measurements of the core neutron spectrum are described. A Li^6 semiconductor sandwich was employed to determine the spectrum at high energies, with hydrogen-filled proportional counters extending the measured range down to 60 keV. The spatial distribution of fission rates into a reflector containing some moderator were of special interest in Core 6, since most previous Zebra cores have been surrounded by a natural uranium reflector. A short section is included showing how perturbation and reaction-rate measurements on Pu^{239} and B^{10} have been used to give α_9 , the ratio of the capture and fis-

FIG. 1. Reactor Loading Diagrams for Core 6.

sion cross sections for Pu^{239} . Accurate knowledge of this quantity should remove the chief uncertainty in the determination of breeding gain in the plutonium-fuelled reactor.

Finally, measurements of the gamma dose rate in the reactor are described. These include absolute values of some average over the lattice cell obtained with ionisation chambers, and an indication of fine structure using the fluoride of Li^7 as a thermoluminescent detector. They are of interest because absorption of gamma radiation is an important source of heating in structural parts of a power reactor.

This report contains only a selection of the measurements. Many activation and perturbation experiments under the broad category of measurement of fine structure within the lattice cell are not reported here. The importance of such measurements for systems where a significant proportion of reactions takes place in the resonance region is becoming increasingly clear. So is the difficulty in interpreting them, however.

The experimental results are compared with calculations using the FD2 multigroup set.⁽¹⁾ Most of the calculations are based on diffusion theory. The computer programme used was SCRAMBLE, an improved version of CRAM,⁽²⁾ written for the Winfrith KDF9 computer.

2. Critical Parameters

2.1. Description of Assemblies

The general arrangement of the Zebra system has been described by Smith.⁽³⁾ Essentially, the critical assembly consists of plates, mostly 51 mm square by 3.2 mm thick, loaded into square-section stainless steel tubes to achieve the desired composition. The tubes are loaded vertically into the main lattice. The loading

diagrams for 6A, 6B, and 6C are shown in Fig. 1, with the smoothed compositions in Table 1. Core 6D is like 6A except for the removal of the sodium and the loading of 9 more core elements to make it critical. The removal of sodium was accomplished without changing the remaining composition by replacing the 6.4-mm-thick sodium coupons with aluminium rings. These rings had the same mass as the aluminium cladding of the sodium coupons themselves.

The Zebra plutonium inventory contains four kinds of plutonium coupon, all 3.2 mm thick. Three differ mainly in the cladding, a relatively trivial factor, and contain fuel of the same isotopic mixture: 5% Pu^{240} . The other contains 10% Pu^{240} , and, as already seen, use was made of the different isotopic mixtures in 6B and 6C. Any element contained only one of the four kinds, and in 6A and 6D the elements were arranged on a regular pattern to achieve as uniform a composition as possible. It was shown that the pattern was sufficiently uniform that the difference in reactivity between the actual core and a hypothetical core where each fuel element was identical (with the average core composition of Table 1) would have been less than 10^{-4} in dk/k .

The control-rod mechanisms intrude a small amount of steel at the expense of element contents. The rods were designed so that when fully raised (the normal operating position) the section opposite the core contained the same mass of Pu^{239} , natural uranium, and graphite as in the normal element, the remaining volume being taken up with aluminium. The reactivity invested in the core section of the rods was insufficient to meet the safety requirements. So, with an intervening section of reflector composition, an absorbing section was assembled in each rod containing about 110 g B^{10} . The reactivity worth of all the rods together

TABLE 1. COMPOSITIONS (UNITS OF 10^{22} ATOMS/CM³)

Isotope	Cores 6A and 6D	Core 6B		Core 6C		Common Reflector
		Zone 1	Zone 2	Zone 1	Zone 2	
Pu ²³⁹	0.1879	0.1796	0.1935	0.1918	0.1852	—
Pu ²⁴⁰	0.0144	0.0207	0.0102	0.0099	0.0176	—
Pu ²⁴¹	0.0016	0.0027	0.0008	0.0007	0.0022	—
U ²³⁵	0.0046	0.0046	0.0046	0.0046	0.0046	0.0194
U ²³⁸	0.6353	0.6353	0.6353	0.6353	0.6353	2.681
C	2.959	2.959	2.959	2.959	2.959	2.341
Na	0.4474 (6A) 0.0000 (6D)	0.4474	0.4474	0.4474	0.4474	—
Al	0.2504	0.2504	0.2504	0.2405	0.2405	—
Fe	0.4255	0.4648	0.4422	0.4693	0.4391	0.3253
Cr	0.1270	0.1304	0.1241	0.1317	0.1232	0.0913
Ni	0.0443	0.0455	0.0433	0.0459	0.0430	0.0318
Cu	0.0829	0.0604	0.1010	0.0628	0.0993	—

TABLE 2. CRITICAL PARAMETERS FOR CORE 6A

Parameter	Experimental Value after Correction for Heterogeneity, etc.	Transport- theory Calculation with FD2
Core height, cm	60.16	
Core radius, cm	36.15	
Axial reflector thickness, cm:		
above core	30.42	
below core	30.42	
Radial reflector thickness, cm	34.44	
Volume, liters	247.0 $\pm$ 2.0	
Mass (Pu ²³⁹ + Pu ²⁴¹), kg	185.7 $\pm$ 1.5	174.5

was 5.4%, about one-half of which was invested in the absorbing sections. It was determined experimentally that if all nine rods in their fully raised positions had been replaced by normal elements, the reactivity would have been reduced by only $(1.6 \pm 0.2) \times 10^{-4} dk/k$.

2.2. Criticality of Core 6A

The most detailed study of absolute critical parameters was made with Core 6A. First, the actual critical assembly (see Fig. 1) had to be translated into a model suitable for calculation. Thus the radius of the actual core, with stepped outline, plates of discrete thickness, control rods partly inserted, and holes in reflector elements for monitoring instruments had to be changed to convert it into an equivalent homogeneous critical cylinder, with uniform composition as in Table 1. Correction factors for core smoothing, heterogeneity, etc., were determined experimentally. (The small correction mentioned above, for replacement of control rods by normal elements, was also included.) The heterogeneity correction was established by doubling up each plate in the central region of the reactor and measuring the reactivity change. An assumption that the effect

of heterogeneity is linear with plate thickness gave a measured correction representing the homogenization of unit volume of core material in the central region. The correction for the whole core was obtained by multiplying by the core volume and the computed ratio of average to central worth of fuel. The correction obtained in this way was 1.0% in k . It was also shown that this correction was due almost entirely to the heterogeneity of the fuel plates themselves. The dimensions of the homogeneous cylinder thus determined are shown in Table 2. An indication of the magnitude of the total correction applied is given by the fact that the critical mass (Pu²³⁹ + Pu²⁴¹) of the reactor as constructed was 179.7 ± 0.05 kg. By far the biggest single component of the difference is the heterogeneity correction, the estimated error for which is responsible for most of the error on the critical mass of the homogeneous model in Table 2.

The diffusion-theory programme SCRAMBLE⁽²⁾ was then used to estimate k for this model, using the FD2 cross-section set.⁽¹⁾ This 33-group set allows for resonance self-shielding in fissile and fertile isotopes occurring in homogeneous mixtures with other elements. Because of the insufficient storage capacity of the computer, the calculation had to be carried out in two stages. First, a finite cylindrical and a spherical model of dimensions reasonably close to those of the actual reactor were compared, using a 15-group condensation of the cross-section set. A search for the dimensions, setting k the same for each case, yielded a shape factor of 0.935. The correct dimensions of the equivalent sphere could then be determined, and the final calculation was carried out on this spherical model in the full 33 groups. It would have been impossible to do the two-dimensional calculation directly in the full multigroup scheme because more storage space would have been required for the extra dimension. A value $k = 1.000$ was obtained. This was corrected by the calculated difference, 1.1% in k , between transport- and diffusion-theory predictions on an equivalent spherical model. Thus the theoretical multiplication factor with the FD2 set is 1.011. Using the known reactivity worth of fuel at the core edge, this is the same as saying that the FD2 set underestimates the critical mass by 11 kg, just over 6%.

2.3. Reactivity Changes in Cores 6B and 6C

Only changes in reactivity, rather than absolute values, were investigated in the other assemblies. The transition from 6A to 6D giving the sodium reactivity worth is considered later. Cores 6B and 6C were considered only in relation to each other, since they are not related to the other variants in any particularly significant way.

The physical change involved in the transition from 6B to 6C was just the movement of about 5% of the Pu^{239} from the outer to the inner zone accompanied by a movement in the opposite direction of the same amount of Pu^{240} . The reactivity change, measured in terms of the number of elements removed and control rod balances, was $0.58 \pm 0.02\%$ dk/k . Diffusion theory with the FD2 set was used to calculate the reactivity change on an infinite cylindrical model described in Table 3. Compositions are given in Table 1. Here no attempt was made to correct for heterogeneity, etc., since it was assumed that such factors would be identical in both assemblies. The radii are just those of circles of equal area to the different zones (see Fig. 1). A forced axial buckling, the same for both cores, was applied in compensation for the missing dimension.

The axial buckling represents a reflector saving of 16.3 cm, which is slightly less than the true value of 17.7 cm. Thus the absolute value of k in either core has no physical significance within 0.1% or so. The possible difference in axial buckling between 6B and 6C due to change in the diffusion coefficient in either core zone has been considered and is thought unlikely to affect the change in k by more than 0.02% in dk/k . This has been applied as a calculation uncertainty in Table 3. The good agreement between measurement and calculation is gratifying, but not of special significance concerning Pu^{240} , since at least 80% of the change arises from the movement of Pu^{239} . Later, however, it is shown that the reactivity worth of sodium is primarily affected by the change in the Pu^{240} fraction.

3. Reaction Rates

Particular interest was taken in the distribution of reaction rates in the moderating reflector. This was originally intended to have about the same moderating ratio as UO_2 , the fertile material which will form the breeder of the PFR. However, the aliquot proportions of uranium and graphite plates finally used gave an atomic ratio $\text{U}:\text{C} = 0.867$, which is nearly the formula

for uranium carbide. Bondarenko and others⁽⁴⁾ have previously described a nearly identical reflector, studied on the BR1 reactor, and presented integral data on the equilibrium and asymptotic spectra. The ratio of graphite to uranium was 0.884, and the overall atomic density was the same to within 3%. The main difference is that the BR1 reflector contained depleted uranium, whereas the reflector in Core 6 contained natural uranium. The BR1 reflector was also much thicker, whereas in Core 6 the asymptotic spectrum was not reached. This unfortunately restricts the range of comparison.

3.1. Cross Sections in the Reflector

Absolute and relative measurements of the fission rate in U^{235} , Pu^{239} , and U^{238} , and the (n,α) reaction rate in B^{10} were made with pulse counters on the principal axes of Core 6A. The axial distributions are shown in Figure 2, the radial scans presenting similar characteristics. Calculated results obtained from diffusion theory with FD2 are given for comparison. Calculation underestimates the diffusion length in the reflection (see Fig. 2A), but follows the variation of cross-section ratios (see Fig. 2B) very well. The diffusion length as given by the U^{235} fission measurements is 14.3 cm, but Bondarenko finds 10.7 cm for the asymptotic spectrum. It can be shown approximately that half the difference lies in the failure to reach the asymptotic spectrum in the relatively thin reflector of Core 6. The remainder lies in the greater U^{235} content of the Core-6 reflector, the extra fission in which reduces the effective absorption compared with depleted uranium.

It is also worth comparing the uranium-graphite combination with uranium metal with respect to the spectral index provided by the $\text{U}^{238}/\text{U}^{235}$ fission ratio. A U^{235} -fuelled critical assembly similar in size and spectrum to Core 6, but with a uranium metal reflector, has been described elsewhere (Zebra Core 2, see Ref. 5). Since the asymptotic spectrum was not reached in either reflector in the Zebra systems, the fission ratios of 15 cm from the core-reflector interface are shown in Table 4. These are compared with the measured values of Bondarenko in the asymptotic spectrum.

It is interesting to see from the measurements with both reactors that despite the graphite, the $\text{U}^{238}/\text{U}^{235}$ fission rate hardly differs between the two materials. The spectra are nevertheless different, as Bondarenko shows, but this ratio remains constant because both high- and low-energy extremities of the spectrum are enhanced in the uranium-graphite at the expense of the energy range from 10 to 100 keV. This occurs because, although graphite moderation greatly increases the density of very low-energy neutrons, inelastic scatter-

TABLE 3. CALCULATION MODEL FOR CORES 6B AND 6C

Zone	Outer Radius, cm	
1	23.11	
2	35.83	
Reflector	70.07	
Axial Buckling	0.00117 cm^{-2}	
	6B	6C
k (calc)	1.0000	1.0054
dk/k (calc)		$+0.54 \pm 0.05\%$
(measured)		$+0.58 \pm 0.02\%$

FIG. 2. Axial Distributions through Centre of Core 6A: (a) U^{235} fission rate; (b) cross sections relative to U^{235} fission.

TABLE 4. FISSION CROSS-SECTION RATIOS IN URANIUM-GRAPHITE AND URANIUM METAL REFLECTORS

System	Uranium-Graphite		Uranium Metal	
	σ_{f8}/σ_{f5}	σ_{f9}/σ_{f5}	σ_{f8}/σ_{f5}	σ_{f9}/σ_{f5}
Zebra (15 cm into reflector with natural uranium)	0.0081	0.86	0.0089	--
BR-1 (asymptotic spectrum in reflector with depleted uranium)	0.0026	0.79	0.0027	0.93

ing in U^{238} , which is mainly responsible for the initial slowing down from high energy, is reduced. The Pu^{239}/U^{235} fission ratio is somewhat more sensitive to the reduction in this middle range of the spectrum, as Table 4 also shows. Concerning the absolute values of the U^{238}/U^{235} fission ratio, it would not be expected, in a thicker Zebra reflector, to fall quite to such low values as in BR1, because of the extra U^{235} which should maintain a somewhat harder spectrum.

3.2. Reflector Savings

A computer programme was used to fit a cosine form to the U^{235} measurements in the core, thus giving the

extrapolated height and reflector savings. The latter was found to be 17.7 ± 0.1 cm. A similar fit to the calculated U^{235} fission rate gave a reflector saving of 17.6 cm. The good agreement here would not be expected in view of the calculation underestimating the diffusion length in the reflector, and there must be some compensating error in the ratios of the transport cross sections of core and reflector materials.

In Core 6 the reflector saving is considerably larger than in Core 2, reflected with natural uranium, where it was 12 cm. However, the difference is mainly a function of core, not reflector, properties and arises because of the increased leakage from Core 6, the sodium being fairly transparent to neutrons. Calculation shows that the uranium carbide is in fact a slightly less efficient reflector than uranium metal. Thus, comparing equal thicknesses of 30 cm, the reflector saving is 0.6 cm less for uranium-graphite than for the metal for a core like Core 6. Some of this (about one-third) is regained if the reflectors are made infinitely thick.

4. Neutron Spectrum at the Core Centre

Experimental study of the neutron spectrum was confined to Core 6A. Measurements of cross-section

ratios were made with absolute fission chambers and pairs of thermally intercalibrated BF_3 proportional counters and U^{235} fission chambers to give information

TABLE 5. CENTRAL NEUTRON SPECTRUM CALCULATED WITH THE FD2 SET

Energy Group	Lower Limit (keV)	Proportion of Total Flux		Energy Group	Lower Limit (keV)	Proportion of Total Flux	
		Core 6A	Core 7A			Core 6A	Core 7A
1	3680	0.0217	0.0126	14	5.53	0.0259	0.0205
2	2230	0.0465	0.0307	15	3.36	0.0190	0.0139
3	1350	0.0678	0.0469	16	2.04	0.0105	0.0051
4	821	0.0783	0.0533	17	1.23	0.0156	0.0180
5	498	0.1053	0.0997	18	0.748	0.0102	0.0108
6	302	0.1125	0.0977	19	0.454	0.0060	0.0057
7	183	0.1028	0.1184	20	0.275	0.0038	0.0031
8	111	0.0864	0.1111	21	0.167	0.0016	0.0011
9	67.4	0.0814	0.1040	22	0.101	0.0006	0.0004
10	40.9	0.0662	0.0791	23	0.061	0.0002	0.0001
11	24.8	0.0545	0.0645	24	0.037	0.0001	0
12	15.0	0.0469	0.0664	25-33	0.00041	0	0
13	9.12	0.0362	0.0369				

TABLE 6. COMPARISON OF CALCULATED AND EXPERIMENTAL CROSS SECTIONS RELATIVE TO U^{235} FISSION IN CORE 6A

Isotope	Experimental	Calculated (FD2)	Calc/Exp
$\text{Pu}^{239}(n,f)$	0.961 ± 0.013	0.921	0.96
$\text{U}^{238}(n,f)$	0.0364 ± 0.0007	0.0330	0.91
$\text{Pu}^{240}(n,f)$	0.252 ± 0.004	0.226	0.90
$\text{B}^{10}(n,\alpha)$	1.62 ± 0.08	1.34	0.83
$\text{Np}^{237}(n,f)$	0.230 ± 0.003	0.189	0.82
$\text{U}^{233}(n,f)$	1.423 ± 0.019	1.58	1.11
$\text{U}^{234}(n,f)$	0.209 ± 0.003	0.206	0.99
$\text{U}^{236}(n,f)$	0.072 ± 0.002	0.071	0.99

on the central neutron spectrum. Direct measurements of the spectrum were made using a Li^6 spectrometer and two hydrogen-filled proton-recoil counters. All results are compared with calculated values using the FD2 set. The calculated spectrum in Core 6A is shown in Table 5, with the spectrum in the PFR mockup (Core 7A) for comparison. They are similar at low energy.

4.1. Central Cross-section Ratios

Central fission ratios were measured using absolute fission chambers in which the foil masses had been determined by low-geometry alpha assay.⁽⁶⁾ After correcting for fission fragment losses in the fissile film and below the chosen bias setting, also for the presence of other isotopes and for the effect of inelastic scattering in the chamber walls, the results shown in Table 6 were obtained. Each result is the mean from two chambers containing the same isotope, except for U^{234} for which only one chamber was available. Each pair of results agreed within the expected errors (1.5–2.0%). The central $\text{B}^{10}(n,\alpha)/\text{U}^{235}(n,f)$ cross-section ratio (also shown in Table 6) was measured using chambers which had been intercalibrated in a thermal flux. In the same table all the experimental ratios are compared with calculated values.

4.2. Li^6 Spectrometer

The design of the Li^6 spectrometer (see Fig. 3) is based on an instrument used at AERE, Harwell.⁽⁷⁾ The spectrometer had already been used successfully in previous Zebra cores, and has been described in detail in a paper at a recent conference.⁽⁸⁾ The spectrometer yields the spectrum in the range of the first five groups of the FD2 scheme. In Table 7 the composite

FIG. 3. Lithium-6 Spectrometer Used in Spectrum Measurements.

TABLE 7. COMPARISON OF COMPOSITE MEASURED SPECTRUM (Li^6 AND PROTON-RECOIL) WITH CALCULATION FOR CORE 6A

Energy Group	Lower Limit (keV)	Fraction of Flux Totalled over First 9 Groups	
		Measured	Calculated (FD2)
1	3680	0.050 ± 0.015	0.031
2	2230	0.080 ± 0.010	0.066
3	1350	0.099 ± 0.012	0.096
4	821	0.101 ± 0.012	0.111
5	498	0.138 ± 0.017	0.150
6	302	0.153 ± 0.018	0.160
7	183	0.146 ± 0.015	0.146
8	111	0.124 ± 0.015	0.123
9	67	0.019 ± 0.022	0.116

spectrum given by the Li^6 spectrometer and the proton-recoil counters are compared with calculation.

4.3. Proton-recoil Counters

The counters used for these measurements were designed by Benjamin⁽⁹⁾ and comprise a spherical stainless steel shell, 39 mm in diameter and 0.5 mm thick. The general arrangement is shown in Figure 4.

So far, two counters have been in use in Zebra, having filling pressures of 1 and 4 atm absolute. The gas in both counters was pure hydrogen. The energy ranges covered were 50 to 300 keV for the 1-atm counter, and 150 to 650 keV for the other. Energy calibration was obtained by means of a plutonium alpha source mounted on the collecting wire.

The analysis of the pulse distribution to give the neutron spectrum has been described in detail by Benjamin and Kemshall.⁽¹⁰⁾ Briefly, this comprises a calculation of the effect of finite size of the counter (wall effect) followed by a calculation to unfold the corrected proton spectrum.

A detailed description of the techniques employed for these measurements has been given,⁽⁸⁾ so that only an outline of some of the problems encountered need be given here. The electronic circuit employs a Bennett-type cascode head amplifier⁽¹¹⁾ to reduce the effective lead capacitance. The output from this unit has to be transmitted over about 70 m to the main amplifier in the control room. Several problems were attendant on the use of this link, owing principally to the length (about 10 μs) of the counter pulses. Considerable effort was expended to minimize any distortion from the use of a link, and careful measurements indicate that the change in pulse shape was negligible. Checks on linearity and back-bias setting on the analogue-to-digital converter of the pulse-height analyser have been found to be very important, particularly the back-bias setting. A least-squares fitting routine is used to determine these parameters from measurements made before each series of runs.

As described above, an energy calibration point is at present given by means of a plutonium alpha source mounted on the central wire. The accuracy of the calibration is $\pm 5\%$, but it will be considerably improved in future high-pressure (greater than 3 atm) counters by the introduction of a small quantity of He^3 . This

FIG. 4. Hydrogen-filled Proportional Counter Used in Spectrum Measurements.

will emit protons and tritons in the body of the gas when irradiated with thermal neutrons.

At present the lower limit of energy obtainable is about 50 keV and is determined by the onset of gamma-induced events. Special circuitry is being designed to discriminate against the gamma events by relying on the differences in rise time between these and the proton-recoil pulses.

Distortion may also be introduced by the method of analysis. Initially, a Monte Carlo programme was employed to derive a correction for the wall effect. Inaccuracies in this correction resulted in oscillations in the apparent neutron spectrum at high energy, for which the wall effect was large. A more accurate analytical method has eliminated this defect.

4.4. Conclusions

Fission-ratio measurements are somewhat difficult to interpret as they involve both the cross-section curves and the spectrum. The general indication, from U^{238} , Pu^{240} , and Np^{237} cross sections relative to U^{235} fission, are that the overall spectrum calculated with the FD2 set has not enough flux above the various fission thresholds (i.e., about 0.5 MeV). The U^{234} and U^{236} ratios contradict this conclusion, but this may be because the basic cross-section data for these isotopes are not well-known. The B^{10} result also appears to require an enhanced low-energy tail as well as an enhanced high-energy flux.

The direct measurements, presented in Fig. 5, show agreement with the calculated spectrum within the accuracy of the measurements (approximately $\pm 12\%$ in

most groups). However, these experiments only give relative values in the first 9 groups, i.e., down to 67 keV, and do not yield any information about the fraction of the total flux contained in these groups.

5. Measurements of Pu^{239} Alpha

Because of the influence of the capture-to-fission ratio of Pu^{239} on the breeding properties of fast reactors, some investigations were made in Core 6 to determine how accurately this ratio is predicted by FD2 data.

The method used to obtain an experimental result was based on the usual Pu^{239}/B^{10} perturbation and reaction-rate measurements, which are made in all the Zebra cores, and was similar to that employed by Redman and Bretscher.⁽¹²⁾

Let ρ be the worth/atom of a Pu^{239} sample at the core centre and let ρ' be the worth/atom of an absorbing sample at the core centre; then, using perturbation theory,

$$\frac{\rho}{\rho'} = \frac{\sum_j \phi_j^* \chi_j \sum_i (\nu \sigma_f)_i \phi_i - \sum_i (\sigma_c + \sigma_f)_i \phi_i^* \phi_i}{\sum_i (\sigma'_c)_i \phi_i^* \phi_i},$$

where σ denotes a microscopic cross-section; subscript f denotes fission and c denotes capture processes; χ_j is the fraction of fission neutrons born into group j ; and ϕ_i and ϕ_i^* are the flux and adjoint flux in group i , respectively. For simplicity, the elastic- and inelastic-scattering terms have been omitted from the equation. For Pu^{239} and B^{10} , these are small corrections which will be discussed below.

This equation can be rearranged to yield:

$$\alpha = \frac{\sum_j \phi_j^* \chi_j \sum_i (\nu \sigma_f)_i \phi_i}{\sum_i (\sigma_f)_i \phi_i^* \phi_i} - 1 - \frac{\rho}{\rho'} \frac{1}{F},$$

where

$$F = \frac{\sum_i (\sigma_f)_i \phi_i^* \phi_i}{\sum_i (\sigma'_c)_i \phi_i^* \phi_i}.$$

In Core 6 it was decided to evaluate the first term on the right-hand side by calculation, and to obtain the last term from perturbation and reaction-rate measurements with Pu^{239} and B^{10} .

The errors inherent in calculating the first term are small, probably about 1%, since it is essentially ν , 2.88 in Core 6, weighted with the relative worth of fission neutrons to those producing fission. The value of ν is known accurately from nuclear physics measurements and varies only slightly with energy, so that it will be relatively insensitive to errors in the calcu-

FIG. 5. Neutron Spectrum Measured at Centre of Core 6A.

FIG. 6. Measurements of Sodium Reactivity Worth in Units of $10^{-7} dk/k$ per Gram of Sodium.

lated spectrum. The adjoint-flux weighting is a 5% correction in Core 6, making the first term in the above equation 3.03 ± 0.03 .

The perturbation ratio was obtained from measurements with small annular samples of Pu^{239} and B^{10} , 5 cm long, 1.3 cm in diameter, and with the annulus just under 0.02 cm thick. A 3% correction was applied to the ratio to allow for size effects in the B^{10} sample and a 0.5% calculated correction applied to allow for scattering in the samples. The corrected ratio was 0.96 ± 0.01 .

If σ_f and σ_c in F show the same variation with energy, then $1/F$ is given by reaction-rate measurements. However, in practice it is difficult to find an absorber which meets this requirement. In Core 6 a small calculated correction had to be applied to the measurements with Pu^{239} and B^{10} counters to allow for the difference between the measured average value of F with ϕ weighting and the average F with $\phi\phi^*$ weighting required. This changed $1/F$ from 1.69 ± 0.09 to 1.74 ± 0.09 . Inserting these values, the $\phi\phi^*$ -weighted average value of $\alpha = \sigma_c/\sigma_f$ for Pu^{239} is given by $\alpha = 0.36 \pm 0.09$. Unfortunately, the error is large, and work is in progress to minimise the uncertainty, which mainly stems from the measurement of the B^{10} capture rate with proportional counters. However, the experimental figure is considerably larger than the FD2 data value of 0.23. Furthermore, since this trend is also apparent from the analysis of results in other Zebra cores, there is mounting evidence that the FD2 capture to fission data for Pu^{239} are in error.

6. Reactivity Worth of Sodium

The main purpose of the sodium-removal measurements was to provide experimental results against which the theoretical methods applied to investigate sodium coefficients could be checked. These measurements consisted of removing sodium from zones in the core and eventually from the whole core. Because of the small size of the core, the predominant effect in all these measurements was that due to increased leakage from the core. In fact, in the case of total removal, the leakage effect was over five times larger than all the other effects. Consequently, the measurements offered a particularly good opportunity of checking the accuracy of calculated leakage effects.

Some additional measurements were made to investigate other contributions to the overall effect of sodium removal. These included measurements with a small sodium sample to obtain a detailed picture of the worth of sodium throughout the core, measurements to investigate the effect of heterogeneity, and, finally, because the core contained less Pu^{240} than a typical power reactor, measurements were made to establish the effect of Pu^{240} on the worth of sodium.

6.1. Sodium-removal Experiments

The experimental results for the worth of sodium in various zones of the core are shown in Fig. 6, where they are expressed in terms of $10^{-7} dk/k$ per gram of sodium. Three sets of experimental results are shown. The first was obtained by removing all the sodium plates from normal fuel elements (see Fig. 7). In the

second set all the sodium was removed from elements in which the cell size, i.e., the plate thickness, was doubled. The third set was obtained by integrating the results obtained from perturbation scans with a small sodium sample, 7.6 cm long and 2 cm in diameter. All three sets of results are in quite close agreement, indicating that the effects of changes in heterogeneity, i.e., changes in the fine-structure flux in the cells and changes in resonance shielding in the heavy isotopes in the cells brought about by sodium removal, are small in this core.

To investigate whether sodium removal from a zone could be applied in future cores to predict the worth of sodium in a whole core, two sets of experiments were carried out. In the first, the worth of sodium in a 70° sector was measured and compared with the result for total removal. The result, also shown in Fig. 6, indicates that partial removal in future cores would certainly be adequate to establish the correct leakage contribution.

In all the previous measurements the worth of sodium was measured either in terms of the amount of control-rod movement or in terms of the amount of fuel that had to be added at the core edge to keep the system critical. In the next series of measurements, use was made of the formula relating the change in reactivity for a core of fixed size to the changes in material and geometrical buckling:

$$\frac{dk}{k} = \frac{k_{\infty} - 1}{k_{\infty}} \left(\frac{dB_m^2}{B_m^2} - \frac{dB_g^2}{B_g^2} \right).$$

The first term corresponds to the reactivity change for a bare core of the same composition. The second term gives the effect of the reflector due to the change in reflector savings. The effective infinite multiplication factor, k_{∞} , is obtained from calculations using FD2. The term in brackets is obtained from axial and radial scans with a U²³⁵ fission counter. Great care was taken to ensure that the scans were made under exactly the same conditions in the two cores, since an overall accuracy of 0.2% was required for each measurement in a scan. This method yielded $0.99 \pm 0.08\%$ dk/k for the worth of sodium, which compares well with the directly measured value of $1.05 \pm 0.04\%$ dk/k . It was unfortunate because of the small size of Core 6 that the correction term involving the geometric buckling was relatively large and consequently introduced a large error into the final result. Much better accuracy would be obtained in larger cores. The technique is also of interest as it can be applied in mixed critical assemblies provided that the central zone is large enough to have a measurable material buckling.

FIG. 7. Arrangement of Plates in Core Region.

6.2. Calculations

To compare the experimental results with calculation, a two-dimensional k calculation using diffusion theory and 15-group FD2 data was carried out for two models of Core 6, identical in all respects, except that one contained sodium and the other did not. The resonance shielded cross sections used in the calculations were based on homogeneous models of each core, and it was inherently assumed that the same heterogeneity correction would apply to both cores. The measurements with doubled cells carried out in both cores showed this was a reasonable approximation. To obtain the worth of sodium at the centre and edge of the core, a perturbation-theory compiler was run in conjunction with the two calculations.

The results of the calculations are compared with experiment in Figure 6. The agreement is good for sodium removal over the whole core and at the core edge, but is poor at the core centre. A similar trend is shown by the comparison of the small-scale perturbation experiments with calculation (see Table 8).

Recent work by Wardleworth indicates that the 'homogeneous model' treatment of resonance shielding will tend to overestimate the resonance shielding effect in Core 6, possibly by a factor of 10. Fortunately this effect is relatively small. For example, the homogeneous model predicts a contribution of $-0.10 \times 10^{-7} dk/k$ per gram of sodium over the whole core, which is comparable to the experimental error. The magnitude of the contributions from different zones in the core can be seen from Table 8. Consequently the uncertainty in the calculated resonance shielding effect does not affect the general conclusions that can be drawn from the results. The results are consistent with the conclusions advanced by Rowlands and

TABLE 8. COMPARISON OF PERTURBATION EXPERIMENTS WITH FD2 CALCULATIONS, RESULTS EXPRESSED AS 10^{-7} dk/k PER GRAM OF SODIUM

Zone	Height (cm)	Absorption	Moderation	Resonance	Leakage	Total	Experiment
Core Centre	0	-0.53	-0.16	0.47	0.03	-1.13	-0.3 ± 0.1
	10.2	-0.47	-0.20	-0.40	0.95	-0.12	0.8 ± 0.2
	20.3	-0.32	-0.40	-0.21	3.20	2.27	3.2 ± 0.2
	30.5	-0.14	-1.05	0.05	5.37	4.23	4.8 ± 0.2
Core Edge	0	-0.23	-0.40	-0.11	3.72	2.98	3.0 ± 0.2
	10.2	-0.20	-0.37	-0.09	3.68	3.02	3.1 ± 0.2
	20.3	-0.13	-0.33	-0.03	3.58	3.07	3.0 ± 0.2
	30.5	-0.06	-0.46	-0.04	3.26	2.78	2.6 ± 0.2

TABLE 9. CHANGE IN CENTRAL SODIUM PERTURBATION CROSS SECTIONS PRODUCED BY Pu^{240}

	Change in sodium perturbation cross section (mb)	
	Exp	Calc
Core 6A - Core 6B	-1.22 ± 0.15	-0.80
Core 6C - Core 6B	-1.7 ± 0.1	-1.38

Note: Core 6A contained 7.07% Pu^{240} ;
 Core 6B contained 10.21% Pu^{240} ;
 Core 6C contained 4.89% Pu^{240} .

Wardleworth⁽¹³⁾ from a study of similar measurements in the U^{235} -fueled assembly 2 of ZPR-VI, i.e., the FD2 set provides a good estimate of leakage effects, but tends to overestimate the combined effects of moderation, absorption, and change in resonance shielding.

6.3. Effect of Pu^{240} on Central Sodium Worth

Measurements were made with a sodium sample, 15 cm long and 4 cm in diameter, at the centre of Cores 6A, B, and C (see Fig. 1), in which the diluents remained constant but where the Pu^{239}/Pu^{240} ratio was varied. Because of the length of the sample the measured worth included some leakage effect, but this was assumed to be constant in all three cores. However, a correction was applied to the results to allow for the variation in flux and adjoint flux along the sample. This increased the worth in each core by 8%. The change in the corrected worth of sodium from core to core represents a change in the central perturbation cross-section for sodium. These changes expressed in millibarns are shown in Table 9, where they are compared with calculated values using FD2 data. Additional calculations showed that more than 80% of these changes were due to Pu^{240} , the remainder coming from Pu^{239} . On the average, the calculated values are in error by about 25%, showing that calculation would underestimate the negative effect of Pu^{240} on the worth of sodium, i.e., it would underestimate the positive ef-

fect of Pu^{240} on the sodium coefficient in a power reactor. Some unpublished calculations by Ross and Wardleworth show that Pu^{240} can contribute about 1×10^{-6} dk/k per $^{\circ}C$ to the sodium coefficient of a large power reactor, so that the error in this value would amount to 0.2 to 0.3×10^{-6} dk/k per $^{\circ}C$.

6.4. Conclusions

The following conclusions were drawn from the sodium measurements.

- The sodium-removal measurements showed that calculations using FD2 data give a good prediction of leakage effects. The chief uncertainty in calculating sodium-removal effects arises from the moderation, absorption, and resonance effects, which are in error by about a factor of two at the centre of Core 6.
- The experiments showed that accurate estimates of the total worth of sodium in the core could be obtained both from measurements in a sector of the core and from measurements of the change in buckling. Both techniques will prove useful in future cores.
- Measurements in zones containing different Pu^{239}/Pu^{240} ratios showed that FD2 data underestimated the effect of Pu^{240} on the worth of sodium by about 25%.

7. Measurements of Gamma Dose Rate

Energy absorption from the field of gamma radiation in a fast reactor can be the main source of heating in structural materials. This may be particularly important in a materials testing programme, as has been shown on the Dounreay Fast Reactor.⁽¹⁴⁾ In Zebra, ionisation chambers, type IG8, and Li^7F have been used as detectors to measure it. Although the science of gamma dose measurements is much older than for neutrons, the practical problem in a mixed field of photons and neutrons can be difficult. Each environment, even within a single reactor system, has to be considered on its merits in assessing the possible

neutron sensitivity of the detector. Thus, in a moderating region, the prime cause of neutron sensitivity may be through activation of the detector walls (especially aluminium walls), but the design of the type IG8 ionisation chamber (AERE, Harwell) minimises this particular problem. In Core 6, neutron sensitivity is the result of fast-neutron elastic recoils of the nuclei of wall materials and detector substance, whether gaseous or solid as in lithium fluoride. The neutron sensitivity is enhanced by the complete stopping of the recoil particle in the detector substance, which, however, in many forms of detector absorbs only a small fraction of the energy of the photon-induced secondary electrons. Neary⁽¹⁵⁾ has given a terse review of this problem.

7.1. Ionisation Chambers

In the measurements with ionisation chambers we were able to calculate and then correct for neutron response by using different filling gases and different pressures. The method of obtaining the correction is described in an Appendix. Three chambers were used, all with graphite walls; details of the filling are shown in Table 10. This table also shows the apparent dose

TABLE 10. DOSE RATES MEASURED WITH GRAPHITE-WALLED IONISATION CHAMBERS, COMPARED WITH THEORETICAL VALUE AT THE CENTRE OF CORE 6A
(Central neutron flux of 10^9 n/cm²-sec)

	Dose rate (rad/hr)
Apparent dose rate:	
with argon at 150 cm Hg absolute	519
with argon at 45 cm Hg absolute	600
with CO ₂ at 150 cm Hg absolute	638
Measured gamma dose rate in graphite, after correction for neutron response	474 ± 25
Final measured dose rate after correcting for nonsaturation of fission products	499 ± 25
Calculated gamma dose rate in graphite	502

TABLE 11. FRACTION OF TOTAL GAMMA DOSE RATE INDUCED BY NEUTRON REACTIONS IN CORE 6A

Reaction	Fraction, %	
	Core Centre	Midway through Reflector
Fission (prompt gamma)	40.6	12.1
Saturated fission products	37.4	11.1
Capture and activation	12.6	68.0
Inelastic scattering	9.4	8.8

FIG. 8. Axial Distribution of Gamma Dose Rate in Graphite, Core 6A.

rates measured in each, followed by the true gamma dose rate in graphite after applying the corrections. It is clearly seen how, for the same gas, the neutron response is greater at the lower pressure, and that it is also greater for the gas of lower atomic weight, CO₂, than for argon at the same pressure.

The use of argon as a filling gas in a graphite-walled chamber represents a degree of compromise with ideal chamber design⁽¹⁶⁾ which fortunately is not believed significant in Core 6A because of the absence of very hard or soft components in the photon spectrum.

To get the final measured dose rate, Table 10 shows that a further increment has been applied, to correct for nonsaturation of fission product activity at the time of the measurement. The measurements were made about 3 hr after the reactor reached steady power, when it was estimated that the fission product source strength was still 14% below saturation. This was multiplied by the estimated fraction of the total dose rate due to fission products (see Table 11) to give the correction of 5%.

FIG. 9. Fine Structure of Gamma Dose Rate at the Core Centre (N.B., Cell Contains Twice the Normal Mass of Fuel).

A vertical scan with the argon-filled chamber through the centre of Core 6A is shown in Fig. 8.

7.2. Thermoluminescent Detectors

The lithium fluoride thermoluminescent detector⁽¹⁷⁾ with 99.997% of the lithium as the Li^7 isotope, was used in the form of powder contained in polytetrafluoroethylene (PTFE) capsules to determine the fine structure within a lattice cell at the core centre (see Fig. 9). This material has an inherent insensitivity to recoil energy, as discussed by Wingate.⁽¹⁸⁾ Using Wingate's data together with the calculated recoil dose rate (see Appendix), we estimated the neutron response as only 2.1% of the gamma response. An auxiliary irradiation was done with the powder wetted by alcohol, which enhances the neutron response⁽¹⁸⁾ by a factor of about 3. The ratio of wet to dry powder doses in four detectors irradiated at the core centre ranged between 1.06 and 1.09, in good agreement with theory. It is confidently believed therefore that Li^7 is potentially a valuable detector of gamma dose rate in a fast reactor, particularly on account of its small size. However (see Fig. 9), the absolute gamma dose rate measured has a mean level much above that obtained with the ionisation chambers. Most of this difference arises because, in order to provide small rectangular slots to house the detectors within the cell, U^{235} plates had to be substituted for the canned plu-

tonium coupons. The mass of U^{235} in this cell was more than twice that of Pu^{239} in a normal cell. Thus the source strength, mainly fission photons, was enhanced locally. However, it is also believed that the detector may not be thick enough to achieve secondary electron equilibrium, and the detector thus measures a proportion of the dose rate in the surrounding material which, because of the heavy atoms present, averages about twice that of the lithium fluoride. This is likely to be a problem in dosimetry technique, rather than in the physics of lithium fluoride itself, but its solution would be at the expense of smallness of the detector, thus spoiling one of its advantages.

7.3. Calculations

The gamma dose rate was calculated using a 12-group gamma cross-section set.⁽¹⁹⁾ The source strength was based on FD2-calculated neutron reaction rates except in the reflector, where experimental values of the neutron cross sections were used (e.g., see Fig. 2). A computer programme was used to solve basically a set of fundamental-mode equations with a forced buckling. The implicit use of diffusion theory here is not a serious defect because the mean free path of the photons is much smaller than the relaxation length of the source distribution which provides the forced buckling. So the leakage term is very small relative to source and absorption terms. The method fails near the core-reflector boundary, of course. The source strength comprises photons induced in fission, capture, activation, and inelastic scattering in all materials. While the fission process provides most of the source in the core, in the reflector the main source is capture in U^{238} . Table 11 gives the calculated relative importance of the sources in the core and reflector.

The calculated gamma dose rate in graphite at the centre of Core 6A was 502 rad/hr, for a centre neutron flux of 10^9 n/cm²-sec, in excellent agreement with experiment. In the reflector (see Fig. 8) the agreement is also good. This is interpreted to mean mainly that the total energy emitted in the photons from the principal inducing neutron reactions is known to good accuracy.

Appendix

Measurements of Gamma Dose Rate with Ionisation Chambers; the Correction for Neutron Sensitivity

As previously outlined, the correction for neutron recoils in the ionisation-chamber measurements was evaluated by using chambers with different filling gases and pressures. The chambers had been calibrated in the pure gamma field of a large Co^{60} source. On dividing the current observed in the reactor environment by the gamma sensitivity, an apparent dose rate,

A , was obtained. This is related to the desired gamma dose rate, D , by the following formula:

$$D = A - F(D_r)_w - G(D_r)_g,$$

where D_r is the neutron-recoil dose rate in the wall (w) or the gas (g), and F and G are, respectively, the sensitivities to wall and gas recoils, relative to the gamma sensitivity. The subscripts 1, 2, and 3 will be used to refer to the three chambers in the order given in Table 10.

The corrections evolve from the following physical effects. The neutron dose rate in the gas depends on its atomic weight, being less the heavier the gas, because the energy lost to the recoil nucleus is less. It is assumed, however, that G is independent of the identity of the gas. It is in fact equal to W_e/W_r , the ratio of the energies required of a photon-induced secondary electron and a recoil nucleus to create one ion pair. This assumption is, therefore, tantamount to assuming that W_r is independent of the mass of the recoil nucleus. There is only a little information on G ,⁽¹⁵⁾ but it is known to be less than unity and of the order of some tenths. It is clear that G is independent of the gas pressure p . The factor F , however, is inversely proportional to the gas pressure. This is because the wall recoils are stopped completely in the thin layer of gas near the wall so that the chamber current is independent of pressure. But the photon sensitivity is proportional to the pressure, since the gas is nearly transparent to the secondary electrons, and absorbs from them energy proportional to the mass of gas. Thus F may be written as F_0/p' . It seems reasonable to assume that F is independent of the identity of the gas. Since all our chambers have graphite walls, $(D_r)_u$ is clearly common to all.

Consider the first two chambers containing argon at different pressures, p_1 and p_2 . These have $(D_r)_g$ in common; therefore,

$$A_2 - A_1 = F_0(D_r)_w \left[\frac{1}{p_2} - \frac{1}{p_1} \right],$$

from which we obtained $F_0(D_r)_w$. Next consider the first and third chambers, which contain argon and CO_2 at the same pressure ($p_1 = p_3$). These have D , F , and $(D_r)_u$ in common, and subtraction yields

$$\begin{aligned} A_3 - A_1 &= G[(D_r)_{\text{CO}_2} - (D_r)_A] \\ &= G(D_r)_A \left[\frac{(D_r)_{\text{CO}_2}}{(D_r)_A} - 1 \right], \end{aligned}$$

where the first term in the final bracket is a correction whose calculation will be demonstrated presently. Thus the neutron response may be eliminated, and the final

TABLE 12. CALCULATED NEUTRON RECOIL DOSE RATES AT THE CENTRE OF CORE 6A
(Neutron flux 10^9 n/cm²-sec)

Scattering Material	D_r (rad/hr)
Argon	48
CO_2	530
Li^7F	470
Graphite	600

formula is

$$D = A_1 - \frac{p_2}{p_1 - p_2} (A_2 - A_1) - (A_3 - A_1) \left[\frac{(D_r)_{\text{CO}_2}}{(D_r)_A} - 1 \right].$$

In order to calculate the correction to the third term, the recoil dose rate in the gas is calculated from the equation

$$D_r = \left\{ \frac{2AK}{\rho(A+1)^2} \right\} \int E \phi_n(E) \Sigma_s(E) \{1 - \bar{\mu}(E)\} dE,$$

where

A is the atomic weight of the gas;

ρ is its density;

Σ_s is its macroscopic elastic scattering;

$\bar{\mu}(E)$ is the average cosine of the scattering angle in the centre-of-mass system;

$\phi_n(E)$ is the neutron flux as a function of energy;

K is just a constant, relating to the desired dose rate unit, here taken to be rad/hr.

Values so calculated are shown in Table 12 for argon and CO_2 (together with other materials), and the ratio between them facilitates the analysis of the measurements as described.

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Discussion

Mr. Reynolds (GE): Did I understand that the ratio that you have written* to explain $\bar{\alpha}_{49}$ is the ratio of the worth of plutonium to the worth of boron?

Mr. Baker: Yes, that is correct.

Mr. Reynolds: I do not agree with the equation as it is written. The $\nu\sigma_f$ of Pu^{239} should be weighted with the high-energy adjoint and the others should be weighted with the adjoint of the neutrons where they are absorbed. Was this taken into account in the way that this was calculated?

Mr. Baker: Yes. I couldn't write down the multi-

group formula in the time available, so I wrote the one-group analog which is adequate to explain your point. The variations in the adjoint do have to be taken into account by calculation and ν also suitably corrected to get the value of alpha.

Mr. Davey (ANL): With regard to the same point, I question the relevance of the formula, but not in regard to the required multigroup corrections. The ratio in the denominator† is the ratio of two reaction rates in two chambers and as such is a measure of the rate of ionization or particle formation in the chambers. There is evidence that the boron reactivity is not solely that given by the (n,α) reaction and in fact there is an $(n,t - 2\alpha)$ reaction which is energetically possible and has been measured at high energies. Work at Argonne by Cox has indicated that there may be quite serious differences between the boron absorption cross section as measured by chambers and as measured by, for example, spherical transmission techniques. I believe that this difference, if correct, would tend to put the derived alpha value (0.36) in better agreement with the calculated value of 0.23.

Mr. Baker: The point that you made is correct. It will give the wrong answer if there is a change in the ratio of the boron reactions for thermal neutrons and fast neutrons. This is something which we ought to look into, but we haven't done so.

Mr. Davey: Have you made the same comparison of the relative reactivities of Pu^{239} and U^{235} and their fission ratios, and do you find that the ratio of the experimental alphas you deduce is consistent with the ratio of the calculated alphas? If so, this would indicate that the boron may indeed be the reason for this discrepancy.

Mr. Baker: We haven't done this directly, but we have done them separately against boron. When you compare U^{235} with boron in the same way as this, you get a series of numbers, for the three cores for which we have data, in which the experimental value for alpha for U^{235} is again bigger than the calculated value, but the discrepancy is not so great. We are going to extend this work, but the thing to do is to improve the boron measurement. The BF_3 counter has a very poor plateau, so one must look for other ways. We intend to try Li^6 .

* $\frac{\rho_{49}}{\rho_B} = \frac{(\nu - 1 - \alpha)\sigma_f^{49}}{\sigma_B}$.

† $\frac{\rho_{49}/\rho_B}{R_{49}/R_B} = \nu - 1 - \alpha$.

Physics Parameters of Large Dilute Uranium Carbide Cores

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1. Introduction

The Argonne National Laboratory has investigated the physics characteristics of a large, dilute uranium carbide reactor by studying ZPR-6 Assemblies 4Z and 5, and ZPR-9 Assembly 11. Assemblies 4Z and 11 are zoned core systems with a central region having the same composition as Assembly 5, a 2600-liter carbide reactor. Assemblies 4Z and 11 were designed so that both the neutron-energy spectrum and the importance spectrum at the center of the core would be as identical as possible to those of Assembly 5.

The two main purposes of these assemblies were (1) to obtain physics parameters of large, dilute carbide cores, and (2) to determine the areas of validity of reactor physics parameters obtained from zoned-core measurements for a full-scale system. The measurements conducted on these three assemblies may be divided into seven categories:

1. critical mass;
2. neutron energy and spatial distributions of the real fluxes;
3. spatial distribution of the importance of fission neutrons;
4. sodium-void coefficient and the radial reactivity worth of U^{235} , U^{238} , and graphite;
5. reactivity worths of a large number of materials at the center of the core;
6. Rossi-alpha;
7. Doppler-effect measurements.

This paper deals principally with the description of the cores, critical masses, sodium-coefficient studies, and the radial reactivity worths of core materials. A companion paper⁽¹⁾ discusses the measurements made to test the validity of zoned-core measurements for scaling up to full-sized reactors. A third paper⁽²⁾ discusses some of the measurements of the Doppler effect conducted with these cores.

2. Description

The central core regions of Assemblies 11, 4Z, and 5 had the same composition. Assembly 11 had a cen-

tral region radius of 20.9 cm, 4Z of 41.5 cm, and Assembly 5 of 75.6 cm. The compositions and critical masses of the three assemblies are shown in Tables 1 and 2, and the respective dimensions are given in Figs. 1-3. It should be noted that Assemblies 4Z and 5 have a high-density uranium reflector whereas Assembly 11 had a steel reflector. In all three assemblies, the cell size for the central core region consisted of three drawers as shown in Fig. 4a. This was necessary because of the available thicknesses of core materials and because it was desirable to homogenize the core as much as possible.

Figure 4b shows the drawer loadings used in Assemblies 4Z and 11 (4001 Z, 4003 Z, and 4005 Z), and the drawer loadings for Assembly 5 (5001, 5003, and 5005). The slight change in the arrangement of 5003 compared to 4003 Z was due to the lack of sufficient numbers of $\frac{1}{16}$ -in.-thick depleted uranium pieces for Assembly 5. Thus a $\frac{1}{8}$ -in. piece of depleted uranium replaced two $\frac{1}{16}$ -in. pieces of depleted uranium, with an associated $\frac{1}{16}$ -in. shift in the positions of a few materials in drawer 5003 relative to those in drawer 4003 Z.

3. Measurements of Sodium-Void Coefficient

Measurements of the sodium-void coefficient were made with Assemblies 4Z and 5. Both radial and axial traverses were made. Measurements were made by observing the change in reactivity due to the substitution of empty type-304 stainless steel cans for identical sodium-filled cans. The change in reactivity was determined by period measurements and/or by the change in the critical positions of calibrated control rods.

Radial measurements were performed by removing sodium from the central 40 cm (20 cm in each half) of a "one drawer wide by three drawer high" region at a number of positions on a horizontal radius. Table 3 shows the results obtained with 4Z. Reactivity was found to increase when the sodium was removed from the center of the core, i.e., the sodium-void coefficient was positive. Farther out in the central core the coefficient was less positive due to leakage effects. How-

TABLE 1. ATOM DENSITIES ($\times 10^{-22}$) FOR ASSEMBLIES 4Z, 5, AND 11

Region	Assembly	Outer Radius, cm	U ²³⁵	U ²³⁸	Na	C	Fe	Ni	Cr	B ^{nat}
Central	4Z	41.53	0.153	1.06	0.920	1.29	0.904	0.113	0.239	—
	5	76.3	0.153	1.06	0.920	1.29	0.904	0.113	0.239	—
	11	20.90	0.155	1.06	0.903	1.30	0.909	0.115	0.259	—
Buffer	4Z	47.45	0.0046	2.17	—	3.02	0.697	0.087	0.138	—
	5	—	—	—	—	—	—	—	—	—
	11	28.03	0.0025	1.21	0.903	0.041	0.909	0.115	0.259	0.302
Driver	4Z	54.52	0.656	0.524	0.920	1.29	0.904	0.113	0.239	—
	5	—	—	—	—	—	—	—	—	—
	11	39.82	0.450	0.032	—	4.33	0.701	1.89	0.20	—
Blanket	4Z	84.52	0.008	3.99	—	—	0.432	0.054	0.114	—
	5	103.3	0.008	3.99	—	—	0.432	0.054	0.114	—
	11	80.03	—	—	—	—	7.78	0.089	0.204	—

TABLE 2. CRITICAL MASSES FOR ASSEMBLIES 4Z, 5, AND 11

Region	Assembly	Volume, liters	U ²³⁵ , kg
Central	11	125	75.7
	4Z	660	394
	5	2600	~1550
Buffer	11	100	1.0
	4Z	201	3.6
	5	—	—
Driver	11	230	390.5
	4Z	280	704
	5	—	—
Total	11	455	467.2
	4Z	1141	1101
	5	2600	~1550

ever, the larger concentration of U²³⁸ in the buffer region caused the coefficient to become more positive again, both in the central core and in the driver, next to the buffer region. The sodium-void coefficient rapidly changed from positive to negative in the driver, and was quite negative near the interface between driver and radial reflector.

Only the central value of the sodium-void coefficient can be compared with that obtained for the full-size Assembly 5, but the effect of the U²³⁸ in the buffer can be inferred by a comparison with the data from Assembly 5 shown in Fig. 5. A substantial part of the radial distance, at the axial center of the core, had a positive sodium-void coefficient in Assembly 4Z. By contrast Assembly 5 had a sodium-void coefficient which was negative beyond radial distances of 41.5

FIG. 1. Axial Cross Section through Assembly 4Z.

FIG. 2. ZPR-9 Assembly 11.

cm. Again the apparent effect of the U^{238} in the blanket is seen near the core-blanket interface as the curve tends to turn upward.

Table 4 shows the effects on the axial sodium-void coefficient for Assembly 4Z when in one case the sodium cans are surrounded by depleted uranium, and in the second case the positions of these depleted uranium plates are interchanged with carbon plates (not initially adjacent to sodium cans) from the drawer. It is seen that the sodium-void coefficient was less positive (or more negative) when graphite was located next to the sodium cans.

A comparison of the axial sodium-void coefficients obtained from measurements with Assemblies 4Z and 5 is shown in Fig. 6. (It is to be remembered that the half-height of 4Z was about 61 cm while that of 5 was about 71 cm.) The $\frac{1}{8}$ -in. pieces of depleted uranium normally in the Assembly-5 drawers were replaced by two $\frac{1}{16}$ -in. pieces, in the central region where the voiding experiments were performed, to conform exactly with the 4Z loadings in this region. (It is noted that measurements of the sodium-void coefficient with the two loadings, i.e., the 4Z and 5 drawers, tended to indicate a slight difference. However, all of the comparisons except one were the same within the estimated accuracies of the measurements, and the difference in this case was only 0.3 lh.) All measurements are believed to be accurate to about ± 0.1 lh. The influence of the increased axial flux gradient in Assembly 4Z is readily noted.

4. Radial Worth of Core Materials in Assembly 4Z

The radial worth of core materials was measured by removing material from the core drawers at various

FIG. 3. Cross Section through Assembly 5 of ZPR-6.

FIG. 4a. A Three drawer Cell.

positions on a horizontal radius. The change in reactivity resulting from the removal gave the worth of the material.

To accomplish these measurements the 5-cm-high by 5-cm-long pieces in the material columns of interest were replaced by two 2.5-cm-high by 5-cm-long pieces prior to the measurements. In this way one of the two 5-cm by 2.5-cm pieces could be removed and the other continued to hold the remaining drawer materials in their places. A reference run was then made with all the material in place, and a subsequent run with the desired material removed gave the reactivity worth of the material.

The worth of U^{235} was measured by removing a 5-cm by 2.5-cm by 1.6-mm plate from the axial center of the core at the various radial positions. The results are shown in Table 5. The high fuel concentration and the resulting relatively high neutron flux in the driver maintained a relatively high fuel worth over the entire radial distance. By contrast, in Assembly 5 the

4001Z AND 5001

4003Z

5003

4005Z AND 5005

RELATIVE DRAWER POSITION IN MATRIX

FIG. 4b. Material Arrangement in Three-drawer Cell.

TABLE 3. RADIAL SODIUM-VOID COEFFICIENT FOR ZPR-6 ASSEMBLY 4Z

Radius, cm	Region	Worth/kg, Δh
8.3	Central Core	0.91 ± 0.16
24.2	Central Core	0.57 ± 0.16
40.8	Next to Buffer in Central Zone	$+1.32 \pm 0.24$
48.8	Next to Buffer in Driver Region	$+1.30 \pm 0.24$
54.3	Next to Reflector in Driver Region	-4.24 ± 0.24

fuel worth ratio from center to the radial blanket was of the order of 20 rather than 2 for Assembly 4Z.

The gain in reactivity due to the removal of depleted uranium is shown in Table 6. Half of the height of the entire length of the axial column was removed for these measurements. Only in the reflector did the removal of U^{238} decrease the reactivity.

FIG. 5 Radial Sodium-void Coefficient.

TABLE 4 AXIAL SODIUM-VOID COEFFICIENTS FOR ZPR-6 ASSEMBLY 4Z

Center of Voided Region from Axial Center of Core, cm	With Depleted Uranium Next to Sodium, $\Delta h/\text{kg}$	With Graphite Next to Sodium, $\Delta h/\text{kg}$
0.0	$+1.07 \pm 0.12$	$+0.79 \pm 0.12$
5.1	$+0.96 \pm 0.06$	$+0.62 \pm 0.06$
15.5	$+0.56 \pm 0.05$	$+0.40 \pm 0.05$
25.4	$+0.0 \pm 0.10$	$+0.32 \pm 0.05$
35.6	-0.75 ± 0.10	-0.97 ± 0.10
45.7	-1.69 ± 0.10	-1.63 ± 0.10
55.8	-2.02 ± 0.10	-2.25 ± 0.10

FIG. 6. Axial Sodium-void Coefficients.

The carbon measurements were done in a similar manner as those for U^{238} . It is noted from Table 7 that the removal of graphite from the buffer resulted in increased reactivity.

5. Heterogeneity as Regards Critical Mass in Assembly 4Z

Heterogeneity effects on critical mass were investigated by:

1. measuring the worth of U^{235} as a function of thickness;
2. changing the relative position of U^{235} plates to U^{238} plates;
3. breaking up the normally continuous columns of core material.

To measure the worth of U^{235} as a function of thickness, 5-cm by 5-cm by 0.013-cm-thick foils were stacked. The worths of 1, 2, 3, 6, 12, and 23 stacked

TABLE 5. U^{235} WORTH AS A FUNCTION OF RADIUS IN ZPR-6 ASSEMBLY 4Z

Radius, cm	Region	Worth/kg, Ih
1.74	Central Core	50.1 ± 0.5
22.7	Central Core	44.4 ± 0.5
39.3	Near Buffer in Central Zone	36.5 ± 0.5
48.1	Near Buffer in Driver Region	28.4 ± 0.7
49.7	In Driver	25.5 ± 0.7
55.2	In Reflector	26.7 ± 1.1

TABLE 6. WORTH OF DEPLETED URANIUM AS A FUNCTION OF RADIUS IN ZPR-6 ASSEMBLY 4Z

Radius, cm	Region	Worth/kg, Ih
1.59	Central Core	-2.02 ± 0.08
16.6	Central Core	-2.02 ± 0.08
33.1	Central Core	-1.53 ± 0.08
44.2	In Buffer Region	-1.35 ± 0.06
51.7	In Driver Region	-0.55 ± 0.11
60.7	In Blanket Region	$+0.22 \pm 0.03$
88.3	Outer Edge of Blanket Region	$+0.00 \pm 0.03$

TABLE 7. MEASURED WORTH OF GRAPHITE AS A FUNCTION OF RADIUS IN ZPR-6 ASSEMBLY 4Z

Radius, cm	Region	Worth/kg, Ih
5.52	Central Core	$+0.10 \pm 0.2$
22.1	Central Core	$+0.14 \pm 0.2$
39.3	Central Core Near Buffer	-0.02 ± 0.15
44.2	In Buffer	-1.65 ± 0.15
45.9	In Buffer	-1.40 ± 0.15
48.0	In Driver Next to Buffer	-0.24 ± 0.15
52.4	In Driver	$+6.19 \pm 0.15$

TABLE 8. WORTH OF U^{235} AS FUNCTION OF THICKNESS AT CENTER OF CORE

No. of Samples of Size $2 \times 2 \times 0.005$ in.	Weight, g	Net Worth, Ih	Worth/kg, Ih
1	4.511	$+0.226 \pm 0.020$	50.10 ± 4.40
2	9.358	$+0.401 \pm 0.020$	42.85 ± 2.14
3	13.76	$+0.611 \pm 0.020$	44.41 ± 1.45
6	27.63	$+1.246 \pm 0.030$	45.10 ± 1.10
12	55.37	$+2.493 \pm 0.030$	45.03 ± 0.54
23	105.8	$+4.855 \pm 0.010$	45.90 ± 0.10

TABLE 9. CENTRAL WORTHS OF VARIOUS SAMPLES IN ZPR-6 ASSEMBLY 4Z

Sample Wt, g	Worth of Samples with Sodium in, Ih	Worth of Samples with Sodium out, Ih
Na (51.4)	-0.049 ± 0.005	-0.035 ± 0.008
SS (72.2)	-0.122 ± 0.002	-0.115 ± 0.006
C (103.0)	$+0.194 \pm 0.010$	$+0.042 \pm 0.008$
Be (114.3)	$+1.107 \pm 0.010$	$+0.788 \pm 0.008$
U^{235}	$+1.240 \pm 0.035$	$+1.126 \pm 0.005$
U^{238} (1153.8)	-3.857 ± 0.01	-3.191 ± 0.004
B ¹⁰ (29.2)	-23.3 ± 0.20	-19.25 ± 0.010

foils were measured. As shown in Table 8 the specific worth was found to be constant within experimental uncertainties.

As noted in the drawer loadings (see Fig. 4), the columns containing U^{235} were partly adjacent to carbon and partly adjacent to depleted uranium. A measure of the sensitivity of reactivity to U^{238} placement was obtained by interchanging the carbon which was adjacent to U^{235} with U^{238} which was not adjacent to U^{235} . This measurement was performed in the central nine drawers in each half of the reactor. The number of U^{235} - U^{238} interfaces in each half of the reactor was increased from 9 to 18, with a corresponding increase of $+7.6$ Ih or $1.6 \times 10^{-4} \Delta k/k$. It is estimated that the heterogeneity effect of U^{238} placement as compared to a homogeneous system is equivalent to about 5 kg of U^{235} .

Finally, the effect of continuous columns of core materials was investigated by loading the central 12 in. of each of six drawers with 2-in.-long plates. Alternate 2-in. sections were then rotated 90° and the resulting reactivity change measured. The resulting reactivity change was small, i.e., less than 0.1 Ih.

6. Central Worth Measurements with and without Sodium in Assembly 4Z

Central worths were measured by use of a remotely operated sample changer to avoid separating the reactor halves for sample insertion. The sample changer

was in part comprised of a special fuel drawer which had a $5\text{-} \times 5\text{-} \times 2.5\text{-cm}$ voided section in the front. Samples were removed or added remotely by removing the drawer from the reactor and activating the sample-changer mechanism. The drawer was then driven back into the core to its original position. The change in reactivity due to a sample, referenced to the reactivity with no sample, gave the reactivity worth of the sample.

Measurements were made by observing the changes in asymptotic period due to the perturbing sample. Thus, for samples of low worth only the sample-changer drawer had to be moved. Since the sample-changer drawer contained fuel, its removal from the core resulted in a lowering of the flux level, so that successive periods could be started at identical power levels. The measurements were made on line with a small computer which performed a least-squares fit to the exponential. Residuals were printed to provide a precision index. Measurements were repeated several times to check reproducibility.

Table 9 shows the worths of a number of samples obtained with a normal fuel loading and when the sodium was removed from the entire central zone. The hardening of the spectrum can be inferred from these measurements. Calculations indicate that the adjoint spectrum has a minimum in group 9 (67–112 keV) of Argonne National Laboratory Cross Section Set 224. The decrease in worth of the light scattering samples, such as beryllium and carbon, is quite apparent when sodium is removed from the core. The decreased absorption of B^{10} is also evident. Fewer fissions as well as increased absorption accounts for the more negative worth of U^{238} when sodium is in the core.

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June 9
(0171 - P 113)
12-9-66

Prompt-neutron Time Behavior in Delayed-critical Coupled Uranium-metal Cylinders*

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ABSTRACT

Pairs of identical, coaxial, 27.94-cm-diam cylinders of unmoderated and unreflected uranium metal have been assembled to delayed criticality with large spaces between the flat surfaces. The uranium metal was enriched to 93.15 w/o in the ^{235}U isotope. Prompt-neutron decay constants of the assemblies were measured by the Rossi- α technique, and the results agreed with those predicted by a method in which a pair of equations represented the kinetics of a coupled-core assembly. The equations neglected delayed neutrons and assumed space-independent kinetics for each cylinder with a time lag to represent the flight time of neutrons from one cylinder to the other. Multiplication factors calculated for the assemblies using S_8 transport theory also agreed with the experimental values.

1. Introduction

In order to understand the neutron interaction between the two components of a coupled-core assembly that has a single large gap, it is desirable to study the properties of the assembly that depend directly on the neutrons traveling across the gap from one component to the other. One such property is the delayed-critical configuration. Another is the prompt-neutron decay constant at delayed critical, which is equal to the effective delayed-neutron fraction divided by the prompt-neutron lifetime.

In the investigation reported here, these properties were studied in delayed-critical experiments with pairs of unmoderated and unreflected cylinders of uranium metal (93.15 w/o ^{235}U) assembled so that their axes were coincident and their flat surfaces were separated by large gaps. In such assemblies the neutron exchange is principally between the flat surfaces, and the prompt-neutron decay constant is a strong function of the size of the gap between these surfaces and the fraction of the neutron population crossing the gap. This results from the fact that the prompt-neutron lifetime, on which the decay constant depends, is

strongly affected by the gap. In a single, unmoderated, delayed-critical cylinder of uranium metal ($\rho = 18.75 \text{ g/cm}^3$) the prompt-neutron lifetime is 6.2 nsec.⁽¹⁾ In a two-component system, however, the lifetime is increased by the time it takes a neutron to travel across the air gap, an increase which is considerable since the neutrons leaking from a single delayed-critical cylinder have an average speed of about 0.7 cm/nsec.

In the experiments performed in this study, prompt-neutron decay constants were determined by Rossi- α measurements on a number of two-component delayed-critical assemblies, and some of the results were compared with calculated decay constants. The theory on which the calculations are based has been discussed by Baldwin and others.⁽²⁾ A coupled-core experiment using the pulsed-neutron technique has been performed previously by Helmick, Chezem, and Scale⁽³⁾ at Los Alamos.

Transport calculations using S_8 theory were also performed to obtain multiplication factors for comparison with the experiments.

2. Experimental Results

As shown in Fig. 1, the upper cylinder of each assembly was built on a 0.025-cm-thick stainless steel diaphragm held in position by a 76-cm-ID, 2.54-cm-thick aluminum clamping ring, and the lower cylinder was supported on a low-mass aluminum stand. The assembly of the system to delayed critical with the desired spacing was completed when the lower section was raised remotely by a piston having a maximum stroke of 61 cm. (This apparatus has been described previously by Rohrer *et al.*⁽⁴⁾) The thickness of the cylinders and the spacing between the flat faces were the variables that were adjusted until criticality was achieved, the diameters of all the cylinders remaining constant at 27.94 cm. The uranium-metal density in the assembled cores varied from 18.71 to 18.73 g/cm³.

All reactivities were obtained from stable reactor periods. The reactivity contribution of the support structure, including the diaphragm, was accounted for by measuring the effects of appropriate equivalent

* Research sponsored by the U. S. Atomic Energy Commission under contract with the Union Carbide Corporation.

FIG. 1. Typical Uranium-metal Coupled-core Assembly. Upper cylinder is on a stainless steel diaphragm, and lower core is on aluminum support stand.

materials on the reactivity of each assembly and converting the results to an equivalent change in spacing of the cylinders. The conversion was based on measured values of the change in reactivity per unit change in spacing. A plot of the critical separation as a function of the thickness of one cylinder in each assembly is shown in Fig. 2. The maximum separation distance, 36.96 cm, was determined for an assembly in which each cylinder was 8.25 cm thick.

The prompt-neutron decay constant for each critical configuration was measured by the Rossi- α technique, using instrumentation employing two detectors. One was a plastic scintillator (NE-102) which triggered an analyzer of the time-of-flight type, and the other was a spiral ^{235}U fission counter which provided the count signal. (This instrumentation has been described in Ref. 1.) The effect of the detector locations

on the results obtained was investigated by making successive measurements with the fission counter centered immediately below and above the lower cylinder (Positions A and B) and immediately below and above the upper cylinder (Positions C and D). During these measurements the plastic scintillator was located at a point which was about 1 in. above the top surface of the upper cylinder and 4 in. from its center. For the cases in which the fission counter was in flux-symmetric positions (A and D, or B and C), the data collected were indistinguishable, i.e., the collection time, the number of triggers required to collect a given amount of data, and the background level were identical.

Typical decay curves for the 8.25-cm-diam cylinders are given in Fig. 3, where the error listed for the prompt-neutron decay constant is one standard deviation, as obtained from a least-squares analysis of the

data. The agreement of the prompt-neutron decay constants indicates the insensitivity of the measurement to the detector position.

Figure 4 is a plot of the prompt-neutron decay constants of all the coupled-core assemblies as a function of the thickness of one of the cylinders in each assembly. The decay constant decreases rapidly from $1.07 \mu\text{sec}^{-1}$ to about $0.85 \mu\text{sec}^{-1}$ at a separation of about 8 cm and then increases, returning to a value of $1.07 \mu\text{sec}^{-1}$ when the cylinders are infinitely separated. This behavior can be qualitatively explained by an expression obtained previously⁽⁵⁾ for the prompt-neutron lifetime:

$$l = l_s + t(x) \frac{P(x)}{1 - R(x)},$$

where x is the spacing of the cores, l_s is the prompt-neutron lifetime in the assembly if the neutron flight time across the gap were zero, $P(x)$ is the probability that a neutron born in one cylinder leaks into the other, $t(x)$ is the flight time from one cylinder to the other, and $R(x)$ is the probability that a neutron born in one cylinder will be reflected back by the other. The term

FIG. 2. Critical Separation Distance as a Function of the Thickness of One Cylinder in Each Coupled-core Assembly.

FIG. 3. Prompt-neutron Decay in Coupled-core Assembly for Various Detector Locations. The assembly consisted of two 27.94-cm-diam by 8 25-cm-thick cylinders spaced with 36.96 cm between their flat surfaces.

FIG. 4. Prompt-neutron Decay Constant as a Function of the Thickness of One Cylinder in Each Coupled-core Assembly.

$[1 - R(x)]$ in the denominator accounts for the multiple reflections of the neutrons back and forth across the gap. For an assembly with no gap, $t(x)$ is zero and the lifetime is that of a single critical cylinder. As the space between the cylinders approaches infinity, $P(x)$ approaches zero faster than $t(x)$ approaches infinity, since $P(x)$ decreases as $1/x^2$, and the lifetime again approaches that in a single-component assembly. Thus, since the prompt-neutron decay constant is inversely proportional to the prompt-neutron lifetime, it should decrease and then increase as the spacing increases.

3. Calculations of Prompt-neutron Time Behavior by Coupled-core Kinetics

Calculated values of the prompt-neutron decay constants for the assemblies were obtained by a method in

which a pair of equations represented the kinetics of a coupled-core assembly. If delayed neutrons are neglected, these equations can be written as follows:

$$\frac{dN_1(t)}{dt} = \alpha N_1(t) + \gamma N_2(t - \tau) + \delta(t);$$

$$\frac{dN_2(t)}{dt} = \alpha N_2(t) + \gamma N_1(t - \tau),$$

where

- $N_{1,2}$ = neutron population in core 1,2 at time t ;
- α = prompt-neutron decay constant for one core when it is isolated from the other;
- γ = $\epsilon/(l/\beta)$;
- ϵ = reactivity contribution of one core to the other;
- l = prompt-neutron lifetime in the isolated core;
- β = effective delayed-neutron fraction;
- τ = flight time of the neutrons traveling between cores.

The first term on the right represents the decay of the neutron population in one core; the second term represents the neutron source from the other core delayed a time τ . The third term in the first equation is the source term. This term is a delta function in time and represents the neutron which initiates the fission chain process that is studied by the Rossi- α technique. Since the initiation of a single fission chain can take place only in one core at a time and since the cores are identical, the source term appears in one equation only.

Calculations were performed with the IBM 7090 to solve the above equations for $N_1(t)$ and $N_2(t)$ for some of the coupled-core assemblies. The value of α used for each configuration was that obtained from measurements of the prompt-neutron decay constant for an isolated (subcritical) cylinder having the same

TABLE 2. S_8 TRANSPORT-THEORY CALCULATION OF THE MULTIPLICATION FACTOR

Thickness of Fuel in Each Core (cm)	Calculated Multiplication Factor
4.331	0.995
4.454	0.989
4.768	0.993
5.081	0.998
5.399	0.990
5.721	0.994
6.039	0.998
6.357	1.002
6.672	0.998
7.015	0.998
7.309	0.996

dimensions as those used in the assembly.⁽⁶⁾ The value of ϵ was also obtained from the measurements of the prompt-neutron decay constant for the isolated cylinders. Since the measurements on the two-component assemblies were all made at delayed critical, the value of ϵ is just that reactivity by which the isolated cylinder is subcritical. The prompt-neutron lifetime was obtained using the lifetime inferred from measurements of the prompt-neutron decay constant at delayed critical⁽¹⁾ and from calculated changes in the lifetime for subcritical cylinders. The value used for β , the effective delayed-neutron fraction, was 0.0067.

The values of $N_1(t)$ and $N_2(t)$ so obtained were then used to compute the Rossi- α correlation probabilities, which in turn were used to calculate the prompt-neutron decay constants. The calculated and experimental prompt-neutron decay constants are compared in Table 1. The sensitivity of these values to errors in the constants appearing in the equations has been investigated by varying the values of the constants used. These resulting changes in the prompt-neutron decay constant are also given in Table 1. The large sensitivity of the calculations to changes in α or γ is due to the fact that these assemblies are delayed critical and the decay in one cylinder is almost compensated for by the delayed source from the other (i.e., the absolute magnitude of α is just slightly larger than that of γ , but differs in sign). The sensitivity of the solutions to the values of α and γ is larger, the larger the coupling between cylinders. The solutions are not very sensitive to the value of the flight time, τ . A 1% increase in τ will change the solution about 0.2% in all cases.

The largest disagreement between measured and calculated prompt-neutron decay constants is 8%, and in this case calculation would agree with experiment if either α or γ were changed only by 0.4%. The agreement between the measurements and the calculations is quite good considering the sensitivity of the calcula-

TABLE 1. COMPARISON OF CALCULATED AND MEASURED PROMPT-NEUTRON DECAY CONSTANTS AND SENSITIVITY OF CALCULATIONS TO VARIATIONS IN α AND γ

Cylinder Thickness (cm)	Prompt-neutron Decay Constant (μsec^{-1})		Percent Change in Calculated Prompt-neutron Decay Constant Resulting from 1% Change in:	
	Measured	Calculated	α	γ
6.357	-0.872 ± 0.009	-0.94	25.0	24.7
6.672	-0.885 ± 0.008	-0.92	21.8	21.4
7.015	-0.874 ± 0.007	-0.91	17.8	17.5
7.309	-0.878 ± 0.008	-0.90	14.4	14.0
7.627	-0.900 ± 0.008	-0.88	11.3	10.7
7.946	-0.920 ± 0.008	-0.88	8.0	7.3
8.255	-0.932 ± 0.008	-0.91	4.5	3.9

tion to the value of the decay constant, α , of the isolated cylinder or to the coupling coefficient, γ , and indicates that the model used satisfactorily predicts the prompt-neutron time behavior.

4. Transport Theory Calculations of Multiplication Factors

The multiplication factors of the experimental configurations were calculated by multigroup transport theory using the S_n method⁽⁷⁾ in the S_8 approximation and the six-group cross-section values of Hansen and Roach⁽⁸⁾ for the ^{235}U and ^{238}U isotopes. Fission of ^{234}U and ^{236}U was included, and the absorption and scattering properties of these isotopes were assumed to be the same as those of ^{238}U . The calculated values are all between 0.989 and 1.002, as is shown in Table 2. The good agreement with experiment indicates the ability of the transport method for calculation of assemblies with large gaps.

The dependence of the calculated multiplication constants on the order of the S_n calculations was investigated in one case. For the 6.039-cm-thick cylinders, the calculated multiplication factors for 6th, 8th, and 10th-order S_n calculations were 0.991, 0.998, and 1.002, respectively. These results indicate that the S_8 approximation is adequate.

Acknowledgements

The author wishes to acknowledge the assistance of the staff of the ORNL Critical Experiments Facility, in particular the work of J. J. Lynn, J. R. Taylor, and J. F. Ellis in the performance of the experiment. Appreciation is also expressed to P. Stewart, E. Whitesides, and L. Stewart of the Computer Science Center who performed the calculations and least-squares analysis.

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SESSION III-B

October 11, 1966

EVALUATION AND COMPUTATION TECHNIQUES

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Theory of the Neutron Strength Function

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ABSTRACT

A model that describes the nuclear structure of the compound nucleus Ni^{59} is briefly explained. Some gross nuclear parameters, as neutron strength function and imaginary part of the optical potential, are discussed. Predictions from the model are then used to estimate the values of these parameters.

In the last few years the nuclear optical model has been extensively used in cross-section predictions. Once its parameters are determined by obtaining a best fit to available experimental data, the optical potential is used for predictions in regions where the data are of poor quality or not available. Those who have been involved in this procedure understand well that the set of parameters used in this calculation is not unique. A study of the problem of sensitivity and uniqueness of these parameters was done in WCAP-2913. In essence, the empirical fitting often leads to values or combinations of values for the variable parameters which are inconsistent with more fundamental models of the nucleus. Our objective is to use a realistic theoretical model to derive optical-model parameters. Initially we are concentrating on the derivation of the imaginary part of the optical potential.

The more fundamental model we took is a generalization of that used by Brown and Lande.⁽¹⁾ I will not give much detail of the model since it will soon be published in *Nuclear Physics*. The level structure of a compound nucleus (target plus neutron) is predicted by coupling the collective motion of the target to the single-particle states. As a first example, we are studying the structure of Ni^{59} , assuming Ni^{58} to be a vibrational nucleus. The neutron- Ni^{58} system is described by a Hamiltonian which is the sum of the target vibrations plus single-particle kinetic energy and target-particle interaction represented by a deformed potential well $V(r, \theta, \phi)$.

The Hamiltonian is diagonalized in the space product of the single-particle states and target-phonon states. These states were formed with the possible combina-

tions of the seven single-particle states ($3p_{3/2}$, $3f_{5/2}$, $3p_{1/2}$, $4g_{9/2}$, $4d_{5/2}$, $4s_{1/2}$, and $4d_{3/2}$), and the phonon states which rise from 1, 2, 3, 4, 5, and 6 quadrupole and 1, 2, and 3 octupole phonons.

The results so obtained did not require a parameter search. The single-particle energies as well as the single quadrupole and octupole phonon energies were taken from experiment. The parameters which describe the deformation of the target nucleus (β_2 for quadrupole and β_3 for octupole phonon excitations) were assigned values which are the order of those used in collective-model calculations.

From the calculation the spectroscopic factor for each level is predicted. These spectroscopic factors are related to the reduced width Γ by the equation

$$\Gamma_{\lambda}^0 = S_{sp}^{\lambda} \Gamma_{sp}^0,$$

where the subscript sp stands for single particle and λ for the compound-nucleus level λ , Γ_{sp}^0 is the reduced single-particle width, and Γ_{λ}^0 is that of the compound-nucleus level λ .

For the case of Ni^{58} and the single-particle state $4s_{1/2}$, we estimated

$$\Gamma_{sp}^0 = 2.85 \times 10^3 \text{ eV}.$$

Therefore,

$$\Gamma_{\lambda}^0 = 2.85 \times 10^3 S_{4s_{1/2}}^{\lambda}.$$

According to the ideas of Brown, this model may not give the final compound states of the nucleus but rather the states of an intermediate structure. In any case the statistics seem to be preserved. Figures 1 and 2 show the distribution of the level width and the level spacing. The distribution of the widths follows reasonably well the Porter-Thomas distribution, that of the level spacings the Wigner distribution. Both distributions are expected from experiment. Bilpuch *et al.*⁽²⁾ report similar pictures for Ni, Fe, and Ca.

From the spectroscopic factors which are predicted by the model, we may deduce the strength function. For the case of the $l = 0$, $4s_{1/2}$ single particle,

FIG. 1. Distribution of the Level Width for $4s_{1/2}$ Single-particle State

FIG. 2. Distribution of the Level Spacing for $4s_{1/2}$ Single-particle State.

$$S = \frac{\Sigma \Gamma_n^0}{\Delta E}.$$

Taking $\Delta E = 1$ MeV, we obtained Fig. 3.

The value of the strength function at zero neutron binding energy differs by about a factor of 3 from that obtained from neutron-spectroscopy experiments. A small variation of the parameters used in this first calculation or a variation in the single-particle energy of $4s_{1/2}$, which has an experimental uncertainty of about 1 MeV, may very well increase the value of the strength function, as is seen from Fig. 3. It is remarkable that the general shape of the curve is very much like that found by Fulmer *et al.*⁽³⁾

Currently the imaginary part of the optical potential is being calculated using the "Compound Nucleus

FIG. 3. Strength Functions for $4s_{1/2}$ Single-particle State versus Excitation Energy.

States" predicted by our model. It will, as it should, have a nonlocal form.

An explanation of how the imaginary part is determined from these states would entail some algebraic development which is not appropriate for this conference. However, there is at least one conclusion that we may draw. It can be shown that the spectroscopic factor distribution follows a Lorentzian form, namely,

$$S_{sp}^{\lambda} \propto \frac{W}{(E_{sp} - E_{\lambda})^2 + W^2} \quad \text{when } E_{\lambda} \cong E_{sp}.$$

Here E_{sp} is the single-particle energy and W is the imaginary part of the potential, which in general will depend on the energy. Possibly W will increase as the excitation energy increases; in spite of this the width of the spectroscopic factor distribution must be of the order of $2\bar{W}$, where $\bar{W}$ is an average value. For $4s_{1/2}$ as calculated from the model, we see from Fig. 3 that $2\bar{W} \cong 7$ MeV. The experimental values for $\bar{W}$ obtained from fitting low-energy-scattering data are 3.0-3.5 MeV, in agreement with the above result.

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Doppler Broadening of the U^{235} Fission Cross Section— Comparison of Numerical Treatment with Conventional Single Level Analysis*

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ABSTRACT

The U^{235} cross-section data of Michaudon have been Doppler-broadened numerically, and the results are compared with the conventional single-level treatment using a consistent set of level parameters derived by Drawbaugh and Gibson. The analysis was carried out in the energy interval from 2.38 to 29.03 eV, wherein 38 resonance levels have been resolved. The purpose of the investigation was to examine the feasibility of the numerical treatment and to obtain a quantitative estimate of the deviation between the results of the two methods of analysis for a consistent set of level parameters and experimental data.

The total and fission cross-section data for U^{235} were extracted from the SCISRS library. The data were Doppler-broadened numerically to 1500°K and 2500°K using a version of the TEMPO code revised to operate on the IBM-7044 computer. The code numerically evaluates the Doppler-broadened cross section:

$$\sqrt{E} \bar{\sigma}(E, T) = \int_{-\infty}^{\infty} dE' \sqrt{E'} \bar{\sigma}(E', T') \kappa(E', T'; E, T),$$

where

$$\kappa(E', T'; E, T) = K_a(E', E, T - T_{\text{eff}}),$$

the conventional Maxwellian kernel. The experimental resolution was taken into account by defining an effective temperature:

$$T_{\text{eff}} = T' + T^R,$$

where T^R follows from an assumed Gaussian resolution function.

Effective self-shielded fission cross sections were obtained at the two temperatures for homogeneous mixtures of uranium and oxygen and of uranium and hydrogen. Moderator-to-uranium ratios typical of fast reactors and extending into the near-thermal region

were considered. The results indicate a deviation in $\Delta\bar{\sigma}$, between the single-level and numerical treatment of less than 20 % if each set of cross sections is used to compute the appropriate spectrum, and less than 10 % if only the spectrum obtained from the numerical data is used.

1. Introduction

Single-level theory, derived by Breit and Wigner,⁽¹⁾ forms the basis for nearly all resonance-absorption calculations in reactor physics. The single-level fit is an entirely adequate cross-section representation for the great majority of nuclides. High-resolution measurements of the cross section of the fissile nuclides, however, reveal marked deviations from the symmetric Breit-Wigner shape. By virtue of the close packing of resonances and the few exit channels available for fission, considerable interference occurs between levels. Strong interference is characterized by distortion in the cross sections, and a multilevel treatment becomes necessary to analyze the shape of the resonances.

In principle, the multilevel problem is amenable to calculation within the formalism of R-matrix theory, developed by Wigner and Eisenbud.⁽²⁾ Several workers have applied R-matrix theory for the representation of the fissile cross sections.⁽³⁻⁵⁾ In general, the procedure is extremely time-consuming and involves the determination of four empirical constants per level. To the author's knowledge, the uniqueness of the empirical constants has not been established in any of the approximation methods in use. This limitation deters a statistical application of the multilevel analysis to the unresolved resonance region, although some work in this direction is being carried out.⁽⁶⁾ Furthermore, Lynn⁽⁷⁾ points out that the existing multilevel applications do not necessarily rest upon a firmer theoretical framework than the single-level approximation. Lynn predicts strong resonance interference between levels, such that each observable "quasi-resonance" is in fact constructed from

* Work performed under the auspices of the U. S. Atomic Energy Commission, Contract AT(04-3)-167.

several interfering partial cross sections. Garrison⁽⁸⁾ has studied the two-level interference problem in considerable detail.

Despite the limitations of the single-level approximation, this cross-section representation is almost invariably used for Doppler-broadening calculations of the fissile nuclides. The reasons for this are manifest in the above discussion, since for fast-reactor analysis one is interested primarily in the unresolved resonance region. In addition, only the single-level representation is tractable in conjunction with the usual approximations employed for Doppler-broadening calculations.

The present work is an attempt to estimate the error associated with the single-level approximation in Doppler-broadening calculations of the U²³⁵ fission cross section. The investigation is motivated by the generally poor agreement between analysis and experiment for Doppler-effect measurements of fissile samples in fast reactors. Both the measurement and calculation of the fissile Doppler coefficient, however, are fraught with uncertainties and difficulties, so that the single-level approximation may share little of the responsibility for the overall discrepancy.

The "exact" method employed for this study is a numerical technique in which the cross section is broadened directly from the experimental data. Since this method is relatively uncommon, one of the purposes of this investigation was to gain experience with this technique and to define potential problem areas.

For a particular fissile nuclide there exist several sets of single-level parameters and a number of measurements of differential cross section. One consistent set of data and single-level parameters has been chosen for the present comparison using U²³⁵—the differential-cross-section measurements of Michaudon⁽⁹⁾ and the single-level fit to these data obtained by Drawbaugh and Gibson.⁽¹⁰⁾ The comparison was carried out in the energy interval from 2.38 to 29.03 eV, wherein 38 resonances have been resolved.

The conventional single-level technique is reviewed in Sect. 2 and the numerical method is described in Sect. 3. The procedure and results of the treatment of the Michaudon cross-section data are given in Sect. 4. A discussion of the results and the conclusions drawn from this study are presented in Sect. 5.

2. Conventional Single-level Analysis

The single-level Breit-Wigner formula for the reaction cross section of type x in the neighborhood of a resonance at energy E_0 , for a target at rest, is given by

$$\sigma_x = \pi \lambda^2 g \frac{\Gamma_n \Gamma_x}{(E - E_0)^2 + \Gamma^2/4}, \quad (1)$$

where Γ is the total width, Γ_n is the neutron width,

Γ_x is the width for reaction x (Γ_γ for radiative capture and Γ_f for fission), and λ is the de Broglie wavelength. The statistical factor g is given by

$$g = \frac{2J + 1}{2(2I + 1)}, \quad (2)$$

where I is spin of the target nucleus and J is the compound-nucleus spin.

In the presence of thermal motion of the target nuclei, if the target nucleus is only weakly bound to its neighbors, the Doppler-broadened cross section $\tilde{\sigma}_x$ can be computed from the expression

$$\sqrt{E} \tilde{\sigma}_x(E, T) = \int_0^\infty dE' \sqrt{E'} \sigma_x(E', 0) K(E', E, T), \quad (3)$$

where $\sigma_x(E, 0)$ is the zero-temperature reaction cross section, given by Eq. (1). The Maxwellian kernel $K(E', E, T)$ is given by:

$$K(E', E, T) = \frac{1}{2} \sqrt{\frac{A}{\pi k T E}} \left\{ \exp \left[-\frac{A}{k T} (E'^{1/2} - E^{1/2})^2 \right] - \exp \left[-\frac{A}{k T} (E'^{1/2} + E^{1/2})^2 \right] \right\}, \quad (4)$$

where k is the Boltzmann constant, A is the ratio of target mass to neutron mass, and T is target temperature.*

It is common practice to replace the exact Maxwellian kernel by the simplified expression

$$K_a(E', E, T) = \frac{1}{2} \sqrt{\frac{A}{\pi K T E}} \exp \left[-\frac{A}{4 E k T} (E - E')^2 \right], \quad (5)$$

and to extend the lower limit of the integration in Eq. (3) to $-\infty$, yielding

$$\sqrt{E} \tilde{\sigma}_x(E, T) = \int_{-\infty}^\infty dE' \sqrt{E'} \sigma_x(E', 0) K_a(E', E, T). \quad (6)$$

Carrying out the integration in Eq. (6), one obtains the well-known result for the Doppler-broadened reaction cross section:

$$\tilde{\sigma}_x(E, T) = \frac{\Gamma_x}{\Gamma} \sqrt{\frac{E_0}{E}} \sigma_0 \psi(\xi, x), \quad (7)$$

where σ_0 is the total cross section at the resonance peak:

* In reality, the absorber atoms are not in a free state, but are generally bound in a crystal. In this case, it is a good approximation to take a Maxwellian distribution, but at low temperatures an effective temperature which is slightly higher than the actual crystal temperature should be used.

$$\sigma_0 = 4\pi\lambda^2 g \frac{\Gamma_n^0}{\Gamma}. \quad (8)$$

The symmetric line-shape function, ψ is given by

$$\psi(\xi, x) = \frac{\xi}{2\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{e^{-\xi^2(x-y)^2/4}}{1+y^2} dy, \quad (9)$$

where

$$x = 2(E - E_0)/\Gamma \quad (10)$$

and

$$\xi = \frac{\Gamma}{2} \sqrt{\frac{A}{EkT}}. \quad (11)$$

The following approximations are employed to obtain the result given by Eq. (7):

- The total width Γ is taken to be energy independent and equal to the total width at the peak of the resonance.
- The second exponential in the exact Maxwellian kernel, given by Eq. (4), is neglected in comparison with the first.
- The exponent in the first exponential in Eq. (4) approximated by replacing $(E'^{1/2} - E^{1/2})$ by the first term of a Taylor series expansion in terms of $(E' - E)$.
- The lower limit of the integration in Eq. (3) is extended to $-\infty$.
- $\Gamma_n(E) = \Gamma_n^0 \sqrt{E/E_0}$.

Hinman *et al.*⁽¹¹⁾ discuss these approximations in considerable detail and compare results obtained by Eq. (7) with more accurate calculations employing the exact kernel given by Eq. (4). They conclude that the approximate treatment is entirely adequate for nearly all resonances of practical interest.

A considerable effort has been devoted to the evaluation of the symmetric line-shape function ψ , and several methods, aside from direct integration, have been developed for this purpose.⁽¹²⁻¹⁴⁾ The method of Triplett, Merrill, and Burr,⁽¹²⁾ with a refinement by Stevens and Smith,⁽¹⁵⁾ has been used for the single-level evaluations in the present work. The FASDOP subroutine in the GAROL code⁽¹⁵⁾ was employed to compute the cross sections given by Eq. (7).

3. Numerical Treatment

The formulation for the numerical quadrature is identical to that of the single-level analysis:

$$\sqrt{E} \tilde{\sigma}_x(E, T) = \int_{-\infty}^{\infty} dE' \sqrt{E'} \sigma_x(E', 0) K_a(E', E, T), \quad (12)$$

where, as before, the approximate kernel is utilized:

$$K_a(E', E, T) = \frac{1}{2} \sqrt{\frac{A}{\pi kTE}} \exp \left[-\frac{A}{4EkT} (E - E')^2 \right]. \quad (13)$$

It is assumed, however, that $\sqrt{E}\sigma$ is a linear function of the energy for each cross section, σ_x , between every pair of energy points, so that the broadened cross section at energy point i becomes

$$\begin{aligned} \sqrt{E_i} \tilde{\sigma}_x(E_i, T) &= \frac{1}{\pi^{1/2} \Delta_i} \sum_{j=1}^n \sqrt{E_j} \int_{E_j}^{E_{j+1}} dE' \\ &\exp \left[\frac{-(E_i - E')^2}{\Delta_i^2} \right] \left\{ \sigma_x(E_j) \right. \\ &\left. + \left(\frac{\sqrt{E_{j+1}} \sigma_x(E_{j+1}) - \sigma_x(E_j)}{E_{j+1} - E_j} \right) (E' - E_j) \right\}, \end{aligned} \quad (14)$$

where $\sigma_x(E_j)$ is the reaction cross section at energy j , and Δ_i is defined by

$$\Delta_i = \sqrt{\frac{4EkT}{A}}. \quad (15)$$

Performing the integration in Eq. (14), one obtains, after considerable algebra,

$$\sqrt{E_i} \tilde{\sigma}_x(E_i, T) = \sum_{j=1}^n \sqrt{E_j} M_{ij} \sigma_x(E_j), \quad (16)$$

where M_{ij} is given by

$$\begin{aligned} M_{ij} &= \frac{\Delta_i}{2\sqrt{\pi}} \left\{ \frac{e^{-\beta_{i,j-1}^2} - \sqrt{\pi} \beta_{i,j-1} \operatorname{erfc}(\beta_{i,j-1})}{E_j - E_{j-1}} \right. \\ &- \frac{(E_{j+1} - E_{j-1})[e^{-\beta_{i,j}^2} - \sqrt{\pi} \beta_{i,j} \operatorname{erfc}(\beta_{i,j})]}{(E_{j+1} - E_j)(E_j - E_{j-1})} \\ &\left. + \frac{e^{-\beta_{i,j+1}^2} - \sqrt{\pi} \beta_{i,j+1} \operatorname{erfc}(\beta_{i,j+1})}{E_{j+1} - E_j} \right\} + \delta_{ij}. \end{aligned} \quad (17)$$

In Eq. (17), erfc is the conventional complementary error function, δ_{ij} is the Kronecker delta, and $\beta_{i,j}$ is given by

$$\beta_{i,j} \equiv \frac{1}{\Delta_i} (|E_i - E_j|). \quad (18)$$

The expression for M_{ij} , given by Eq. (17), is valid over all points except the first and last, that is, for

$$\begin{aligned} M_{i1} &= \frac{\Delta_i}{2\sqrt{\pi}} \\ &\cdot \left\{ \frac{(e^{-\beta_{i,2}^2} - e^{-\beta_{i,1}^2}) + \sqrt{\pi} [\beta_{i,1} \operatorname{erfc}(\beta_{i,1}) - \beta_{i,2} \operatorname{erfc}(\beta_{i,2})]}{E_2 - E_1} \right\} + \delta_{i1} \end{aligned} \quad (19)$$

and

$$\begin{aligned} M_{in} &= \frac{\Delta_i}{2\sqrt{\pi}} \\ &\cdot \left\{ \frac{(e^{-\beta_{i,n-1}^2} - e^{-\beta_{i,n}^2}) + \sqrt{\pi} [\beta_{i,n} \operatorname{erfc}(\beta_{i,n}) - \beta_{i,n-1} \operatorname{erfc}(\beta_{i,n-1})]}{E_n - E_{n-1}} \right\} + \delta_{in} \end{aligned} \quad (20)$$

The TEMPO code⁽¹⁶⁾ is used for the numerical quadrature outlined above, and further details regarding the method are given in Ref. 16.

In addition to the linear approximation between adjacent energy points, the code assumes that $E^{1/2}\sigma_x(E)$ is constant for $E \leq E_1$ and $E \geq E_n$. In order to diminish running time, the summation in Eq. (16) is computed only for those terms for which

$$|E_i - E_j| \leq 4\Delta_i. \quad (21)$$

The unbroadened cross sections may be placed on an arbitrary energy mesh, and the broadened cross sections are computed at these same energies.

So far, it has been assumed that the unbroadened cross section in Eq. (12) is at the temperature of absolute zero. Since we desire to broaden measured cross-section data which are already Doppler-broadened to some sample temperature, T' , the integral we wish to evaluate numerically is given by

$$\sqrt{E} \tilde{\sigma}_x(E, T) = \int_{-\infty}^{\infty} dE' \sqrt{E} \tilde{\sigma}_x(E', T') \kappa(E', T'; E, T), \quad (22)$$

where $\tilde{\sigma}_x(E', T')$ is the energy-dependent reaction cross section at the sample temperature, T' . A theorem due to Harris⁽¹⁷⁾ permits the evaluation of the integral in Eq. (22) by the identical treatment outlined above. Harris proves the following simple property of the kernel in Eq. (22):

$$\kappa(E', T; E, T) = K(E', E, T - T'). \quad (23)$$

In other words, the kernel used to further Doppler-broaden cross sections to temperature T from temperature T' is identical to the kernel given by Eq. (13), used to broaden zero temperature cross sections, with T replaced by $T - T'$. This useful result permits the broadening of cross sections directly from the experimental data.

One additional effect must be taken into account before measured cross sections may be used directly. In obtaining the experimental data, resonances are further broadened by the finite resolution of the detection system. Thus the Doppler-broadened reaction cross sections are convoluted with the experimental response function, $R(E' - E)$, and the observed reaction cross section, $\tilde{\sigma}_x^{R.B.}(E, T')$, can be written as

$$\tilde{\sigma}_x^{R.B.}(E, T') = \int \tilde{\sigma}_x(E', T') R(E' - E) dE', \quad (24)$$

where the superscript *R.B.* denotes resolution-broadened. It is frequently assumed that the response function is Gaussian:

$$R(E' - E) = \frac{1}{\sqrt{\pi}\gamma} \exp\left[-\left(\frac{E' - E}{\gamma}\right)^2\right], \quad (25)$$

in which case the observed reaction cross section can be written as a successive convolution with two Gaussians:

$$\sqrt{E} \tilde{\sigma}_x^{R.B.}(E, T') = \frac{1}{\pi\Delta'\gamma} \int dE'' \exp\left[-\left(\frac{E'' - E}{\gamma}\right)^2\right] \int dE' \sqrt{E'} \sigma_x(E', 0) \exp\left[-\left(\frac{E' - E''}{\Delta'}\right)^2\right], \quad (26)$$

where

$$\Delta' \equiv \sqrt{\frac{4EkT}{A}}. \quad (27)$$

If we make the following substitutions:

$$g(E) \equiv \sqrt{E} \sigma(E), \quad (28)$$

$$x \equiv E'' - E, \quad (29)$$

and

$$y \equiv E' - E, \quad (30)$$

Eq. (27) may be written as

$$g(E) = \frac{1}{\pi\Delta'\gamma} \int_{-\infty}^{\infty} dy g(y) \int_{-\infty}^{\infty} dx \exp\left[-\left(\frac{y-x}{\Delta'}\right)^2\right] \exp\left[-\left(\frac{x}{\gamma}\right)^2\right]. \quad (31)$$

Taking the Fourier transform of both sides of Eq. (31), and using the Convolution Theorem, we obtain

$$\mathfrak{J}(g) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dy g(y) \exp\left[-\left(\frac{\Delta'^2 + \gamma^2}{4}\right)\xi^2\right]. \quad (32)$$

The inverse transform of Eq. (32) yields

$$\sqrt{E} \tilde{\sigma}_x^{R.B.}(E, T') = \frac{1}{\sqrt{\pi}\sqrt{\gamma^2 + \Delta'^2}} \int_{-\infty}^{\infty} dE' \sigma_x(E', 0) \exp\left[-\frac{(E' - E)^2}{(\Delta'^2 + \gamma^2)}\right]. \quad (33)$$

Thus a Gaussian response function leads to the simple result that the inclusion of resolution broadening is equivalent to Doppler broadening to the following effective temperature:

$$T_{\text{eff}} = \frac{A}{4kE} (\Delta'^2 + \gamma^2) = T' + T^{R.B.} \quad (34)$$

With the above considerations in mind, the experimentally observed reaction cross section, $\tilde{\sigma}_x^{R.B.}(E', T')$, can be numerically Doppler-broadened to temperature T from the following basic formulation:

TABLE 1 U^{235} SINGLE-LEVEL PARAMETERS DERIVED BY DRAWBAUGH AND GIBSON

No	E_0 (eV)	Γ_n ($\times 10^{-3}$ eV)	Γ_n^0 ($\times 10^{-3}$ eV ^{1/2})	Γ_f ($\times 10^{-3}$ eV)	Γ_γ ($\times 10^{-3}$ eV)
1	0.273	0.0032	0.0061	100	35
2	1.135	0.0154	0.0144	115	42
3	2.04	0.0077	0.0054	10	37
4	2.89	0.005	0.0029	100	40
5	3.15	0.023	0.0130	100	37
6	3.61	0.047	0.0247	45	42
7	4.85	0.056	0.0254	3.8	37
8	5.48	0.022	0.0094	330	40
9	5.84	0.012	0.0050	150	45
10	6.20	0.042	0.0169	130	40
11	6.40	0.260	0.1028	10	35
12	7.095	0.120	0.0451	28	36
13	8.795	1.18	0.3979	82	50
14	9.28	0.20	0.0657	95	65
15	9.74	0.04	0.0128	140	40
16	10.18	0.06	0.0188	58	37
17	11.65	0.63	0.1846	4	36
18	12.39	1.29	0.3665	24	44
19	12.86	0.04	0.0112	60	23
20	13.28	0.04	0.0110	51	40
21	13.71	0.044	0.0119	70	40
22	14.00	0.42	0.1123	400	40
23	14.53	0.115	0.0302	23	38
24	15.415	0.250	0.0637	49	49
25	16.095	0.350	0.0872	17	35
26	16.68	0.265	0.0649	86	46
27	18.05	0.39	0.0918	90	70
28	18.97	0.10	0.0229	25	40
29	19.29	3.1	0.7058	52	50
30	20.13	0.13	0.0290	65	55
31	20.60	0.23	0.0507	33	59
32	21.06	1.58	0.3443	21	47
33	22.93	0.45	0.0940	42	50
34	23.41	0.69	0.1426	8	32
35	23.61	0.80	0.1646	85	80
36	24.27	0.33	0.0670	52	40
37	25.62	0.44	0.0869	150	40
38	26.49	0.43	0.0835	150	40
39	27.05	0.10	0.0192	200	40
40	27.79	0.76	0.1442	60	63
41	28.34	0.16	0.0301	96	44
42	29.63	0.18	0.0331	22	45
43	30.59	0.21	0.0380	77	73
44	30.85	0.49	0.0882	17	42
45	32.06	1.95	0.3444	38	52
46	33.51	1.80	0.3109	20	40
47	34.38	1.80	0.3070	40	50
48	34.81	1.05	0.1780	49	60
49	35.19	4.30	0.7250	120	63

The effective temperature to use in the kernel is given by

$$T_0 = T - T_{\text{eff}} = T - T' - T^{R.B.} \quad (36)$$

4. Doppler Broadening of the U^{235} Michaudon Data—Procedure and Results

The U^{235} pointwise cross-section data were extracted from the SCISRS data tape⁽¹⁸⁾ using the UNPACK code.⁽¹⁹⁾ The Saclay experimental data were used throughout. Single-level parameters which were fit to the experimental data by Drawbaugh and Gibson⁽¹⁰⁾ are given in Table 1. These parameters were used for the single-level comparison.

4.1. Fission Cross Section

Below 7.75 eV, the Michaudon data reported in CEA-R2552⁽⁹⁾ and stored as nuclide type 69 ($\sqrt{E}\sigma_f$) on the SCISRS data tape were used. Above 7.75 eV, the earlier J.P.R. 21-429⁽²⁰⁾ Michaudon data, stored as nuclide type 67 (σ_f) on the SCISRS data tape, were used. The two sets of data consist of approximately 2000 points between 1.31 and 37.2 eV. The pointwise fission cross-section data are plotted in Figures 1-4. The solid line compares the fission cross section obtained from the single-level parameters listed in Table 1. The background cross sections suggested by Drawbaugh and Gibson are invariant under Doppler broadening and thus have been disregarded in this study.

The fission cross-section data were obtained with the uranium sample at ambient temperature. Resolution broadening, however, elevates the effective temperature of the sample, as discussed in Sect. 3. Assuming a Gaussian shape for the experimental response function, Drawbaugh and Gibson⁽¹⁰⁾ have derived the effective sample temperature for the Michaudon fission cross-section measurements. Their results are given in Table 2.

The fission cross sections were numerically broadened to 1500°K and 2500°K in two steps. The pointwise cross sections below 20.0 eV were broadened using a mean effective sample temperature of 309.8°K, and the data above 20.0 eV were broadened using a mean effective sample temperature of 316.8°K. In each case, an overlap region of approximately 4 eV was considered in order to evaluate correctly the broadened data near 20.0 eV. The broadened U^{235} fission cross-section data are displayed in Figures 5 through 8.

The adequacy of the energy mesh for the numerical broadening procedure was investigated in the following way. Pointwise cross sections were obtained over the experimental-data energy mesh from the single-level parameters at ambient temperature. These pointwise cross sections were then numerically broadened to 2500°K, and the results were compared with the broad-

$$\sqrt{E} \tilde{\sigma}_x(E, T) = \int_{-\infty}^{\infty} dE' \sqrt{E'} \tilde{\sigma}_x^{R.B.}(E', T') K_a(E', E, T_0). \quad (35)$$

The numerical quadrature is given in Eqs. (16) through (20), and the Maxwellian kernel is defined in Eq. (13).

FIG. 1. U^{235} Fission Cross Section: 1.0 to 5.0 eV

ened results obtained by the conventional single-level analysis outlined in Sect. 2. The comparison demonstrated that the numerical results are within the error bounds of the single-level results at all energies.

4.2. Total Cross Section

The Michaudon data reported in CEA-R2552 and stored as nuclide type 115 on the SCISRS data tape were used over the entire energy range. The data con-

sist of approximately 2700 points between 1.31 and 50.0 eV.*

* Some modifications were made to the raw total cross-section data from the SCISRS tape. Several duplicate energy points were eliminated. Points exhibiting negative cross sections were omitted. In addition, cross sections in the following energy intervals were clearly in error:

8.741-8.815 eV
12.36-12.43 eV
19.25-19.36 eV

FIG. 2. U^{235} Fission Cross Section: 5.0 to 10.0 eV.

The measurements of total cross section were made with the sample at the temperature of liquid nitrogen. The effective sample temperatures, according to the evaluation by Drawbaugh and Gibson, are given in Table 3.

At the time of this evaluation, it was assumed that the data points were too small by an order of magnitude in these intervals. It was recently determined that the error was 1000 barns. This reevaluation has not been taken into account in the present work; however, the error does not influence the final results concerning the fission cross section.

The total cross sections were numerically broadened to 1500°K and 2500°K in two steps, as before. The pointwise data below 20.0 eV were broadened using a mean effective sample temperature of 91.2°K, and the data above 20.0 eV were broadened from a mean effective sample temperature of 96.9°K. An overlap region of 4.0 eV was considered.

The single-level cross sections were broadened to 1500°K and 2500°K using the single-level parameters given in Table 1 and the method outlined in Sect. 2.

FIG. 3 U²³⁵ Fission Cross Section 10.0 to 20.0 eV

These cross sections were punched onto an energy mesh identical to that of the numerically broadened cross-section data. Once both sets of broadened cross sections were in digital form, the foundation for a meaningful comparison between the two was available.

The basis for the comparison is the change in the effective fission cross section between the two temperatures, i.e.,

$$\delta\langle\bar{\sigma}_f\rangle_i = \langle\bar{\sigma}_f(2500^\circ\text{K})\rangle_i - \langle\bar{\sigma}_f(1500^\circ\text{K})\rangle_i, \quad (37)$$

where the effective reaction cross section is defined by

$$\langle\bar{\sigma}_x(T)\rangle_i = \frac{\int_{E_i}^{E_{i+1}} dE \bar{\sigma}_x(E,T) \varphi(E,T)}{\int_{E_i}^{E_{i+1}} dE \varphi(E,T)}. \quad (38)$$

The subscript i denotes the energy interval or group over which the effective reaction cross section is obtained, and $\varphi(E,T)$ is the energy and temperature-

FIG. 4. U^{235} Fission Cross Section: 20.0 to 40.0 eV.

dependent slowing-down flux in the system. The flux is determined by the following integral equation:

$$\sigma_t(E, T) \varphi(E, T) = \frac{1}{1 - \alpha_0} \int_E^{E/\alpha_0} \frac{dE'}{E'} \cdot \sigma_{s_0}(E', T) \varphi(E', T) + \frac{1}{1 - \alpha_1} \int_E^{E/\alpha_1} \frac{dE'}{E'} \sigma_{s_1} \varphi(E', T) + S(E), \quad (39)$$

where the subscript 0 refers to absorber atoms, and 1 refers to moderator atoms. Also, σ_s is the scattering

TABLE 2. EFFECTIVE SAMPLE TEMPERATURES IN FISSION CROSS-SECTION MEASUREMENT

Energy Interval ΔE (eV)	$T_{\text{eff}} = T' + T^{\text{R.B.}}$ ($^{\circ}\text{K}$)
1.85-8.95	307.8
9.00-10.95	308.9
11.00-14.30	310.1
14.35-18.95	311.3
19.00-19.95	311.3
20.00-24.95	313.6
25.00-28.95	315.9
29.00-33.95	317.1
34.00-37.3	320.6

FIG. 5. Doppler-broadened U^{235} Fission Cross Section: 1.0 to 5.0 eV.

cross section per absorber atom (assumed independent of temperature and energy for the moderator), σ_t is the total cross section per absorber atom, S is the external source (determined by a $1/E$ flux above the energy interval of interest), and $\alpha = [(A - 1)/(A + 1)]^2$.

Equation (39) was solved numerically for the flux using the GAROL code⁽¹⁵⁾ for homogeneous mixtures of U^{235} and hydrogen and of U^{235} and oxygen. Since explicit information regarding the capture or scattering cross sections was unavailable from the experimental data, the U^{235} scattering cross section derived from the single-

level fit was used in obtaining the flux for both the single-level and the numerically broadened cross sections. Moderator-to-uranium ratios typical to fast-reactor design and extending up into the near-thermal region were considered.

In the first set of calculations, the slowing-down spectrum was determined in each case by the particular set of pointwise cross sections under consideration. In other words, the fission cross section from the single-level analysis was averaged over a flux distribution computed using the single-level set of cross sections,

FIG. 6. Doppler-broadened U^{235} Fission Cross Section: 5.0 to 10.0 eV.

and similarly for the numerically broadened cross sections. The results, $\delta\langle\tilde{\sigma}_f\rangle$ averaged over the energy interval from 2.38 to 29.03 eV, are presented in Table 4. The potential scattering cross section of the system per absorber atom, defined by

$$\sigma_p = \sigma_{p0} + \frac{N_1}{N_0} \sigma_1, \quad (40)$$

where σ_{p0} is the potential scattering cross section of U^{235} , is given in Column 1 and the associated atom

ratio of moderator to U^{235} is given in Column 2. Columns 3 and 4 compare the effective cross sections, $\langle\tilde{\sigma}_f(1500^\circ K)\rangle$, for the single-level and numerically broadened pointwise cross sections. The primary results, a comparison of $\delta\langle\tilde{\sigma}_f\rangle$ and the percentage deviation, are given in the last three columns.

A second set of calculations was performed in which the slowing-down spectrum pertinent to the numerically broadened cross sections was used for averaging both fission cross sections. The results of this analysis are

FIG. 7. Doppler-broadened U^{235} Fission Cross Section: 10.0 to 20.0 eV.

given in Table 5. A further breakdown of the results by quarter lethargy groups is given in Table 6 for the U^{235} -hydrogen mixture at $\sigma_p = 50$ b.

5. Discussion and Conclusions

The results given in Table 4 pertain to the case of the slowing-down spectrum determined separately for each set of cross sections. For all ratios of moderator to fuel, the percentage deviation in $\delta\langle\bar{\sigma}_f\rangle$ between the single-level and numerical results is less than that

of $\bar{\sigma}_f$. The maximum deviation is -17% , which occurs for a hydrogen-to- U^{235} ratio of 1.9, or $\sigma_p = 50$ b. The percentage deviations demonstrate a positive trend with increasing moderator-to-fuel ratios; however, these numbers include a significant spectrum-weighting effect which precludes a simple interpretation of the results. The magnitude of the spectral effect can be appreciated by referring to Table 7, in which the group fluxes in 1/4 lethargy groups are compared for the oxygen- U^{235} calculations at 1500°K and $\sigma_o = 100$ b.

FIG. 8. Doppler-broadened U^{235} Fission Cross Section: 20.0 to 40.0 eV.

As to be expected, the flux depletion at lower energies is less severe for the spectrum determined by the single-level cross sections.

In order to separate the cross-section contribution from the spectrum-weighting effect, the single-level and numerically broadened cross sections were averaged over identical spectra for each moderator-to-fuel ratio, and the results are compared in Table 5. The spectrum used is that obtained with the numerically broadened

cross sections. This comparison gives agreement between the single-level and numerical results of better than 10% for all moderator-to-fuel ratios. The hydrogen- U^{235} system with $\sigma_p = 1000$ b yields a nearly flat spectrum in lethargy, which provides essentially uniform weighting of the cross sections in equal lethargy intervals over the entire energy range. The oxygen- U^{235} spectra are strongly depleted at lower energies, except for the case with $\sigma_p = 1000$ b, which approaches

a hydrogen- U^{235} spectrum. A further breakdown of the results is provided in Table 6, which compares $\delta\langle\tilde{\sigma}_f\rangle$ by quarter lethargy groups for the U^{235} -hydrogen mixture with $\sigma_p = 50$ b. This comparison demonstrates that the poorest agreement is at low energies, where the single-level approximation considerably overestimates the degree of Doppler broadening. It is also seen that the largest contribution to the change in the cross sections comes in groups which contain the levels with large neutron widths (at $E_0 = 8.795, 12.39, 14.00, 19.29$, and 21.06 eV).

The present study offers an attractive alternative to the multilevel techniques as a means of treating

TABLE 3. EFFECTIVE SAMPLE TEMPERATURES IN MEASUREMENT OF TOTAL CROSS SECTION

Energy Interval ΔE (eV)	$T_{\text{eff}} = T' + T^{\text{R B}}$ ($^{\circ}\text{K}$)
1.85-8.95	91.5
9.00-10.95	89.4
11.00-14.30	90.6
14.35-18.95	91.8
19.00-19.95	92.9
20.00-24.95	94.1
25.00-28.95	95.2
29.00-33.95	97.6
34.00-37.3	101.0

TABLE 4. COMPARISON OF SINGLE-LEVEL WITH NUMERICAL TREATMENT $\langle\tilde{\sigma}_f\rangle$ AND $\delta\langle\tilde{\sigma}_f\rangle$ FOR U^{235} ; 2.38 TO 29.03 eV
Flux Spectrum Determined Separately for Each Set of Pointwise Cross Sections

U^{235} -Hydrogen						
σ_p , b	H/ U^{235} Ratio	$\langle\tilde{\sigma}_f\rangle$, b		$\delta\langle\tilde{\sigma}_f\rangle$, b		Percentage Deviation
		Single Level	Numerical	Single Level	Numerical	
50	1.9	17.47	24.01	1.0956	0.9203	-16%
100	4.3	19.97	26.00	1.0162	0.8375	-17%
200	9.1	22.43	28.01	0.9132	0.7623	-16%
400	19.1	24.87	30.12	0.7713	0.6885	-10%
1000	47.1	27.42	32.59	0.5419	0.5564	+2%

U^{235} -Oxygen						
σ_p , b	O/ U^{235} Ratio	$\langle\tilde{\sigma}_f\rangle$, b		$\delta\langle\tilde{\sigma}_f\rangle$, b		Percentage Deviation
		Single Level	Numerical	Single Level	Numerical	
50	10.5	19.60	30.96	1.0090	0.8665	-14%
100	23.7	23.03	33.09	1.0136	0.8844	-13%
200	50.0	26.22	34.77	1.0493	0.9436	-10%
400	102.6	28.48	35.74	0.9737	0.9687	-1%
1000	260.5	29.91	36.22	0.6467	0.7506	+16%

TABLE 5. COMPARISON OF SINGLE-LEVEL WITH NUMERICAL TREATMENT $\delta\langle\tilde{\sigma}_f\rangle$ FOR U^{235} ; 2.38 TO 29.03 eV
Spectrum from Numerically Broadened Cross Sections Used for Averaging Both Fission Cross Sections

U^{235} -Hydrogen				
σ_p , b	H/ U^{235} Ratio	$\delta\langle\tilde{\sigma}_f\rangle$, b		Percentage Deviation
		Single Level	Numerical	
50	1.9	0.9724	0.9203	-5%
100	4.3	0.8981	0.8375	-7%
200	9.1	0.8265	0.7623	-8%
400	19.1	0.7517	0.6885	-8%
1000	47.1	0.6136	0.5564	-9%

U^{235} -Oxygen				
σ_p , b	O/ U^{235} Ratio	$\delta\langle\tilde{\sigma}_f\rangle$, b		Percentage Deviation
		Single Level	Numerical	
50	10.5	0.8408	0.8665	+3%
100	23.7	0.8333	0.8844	+6%
200	50.0	0.8759	0.9436	+8%
400	102.6	0.9178	0.9687	+6%
1000	260.5	0.7573	0.7506	-1%

TABLE 6. COMPARISON OF SINGLE-LEVEL WITH NUMERICAL TREATMENT $\delta\langle\tilde{\sigma}_f\rangle$ FOR U^{235} ; U^{235} -HYDROGEN MIXTURE WITH $\sigma_p = 50$ b
1/4 Lethargy Groups between 2.38 and 29.03 eV

Energy, eV	$\delta\langle\tilde{\sigma}_f\rangle$, b		Percentage Deviation
	Single Level	Numerical	
29.03-22.60	0.6952	0.6666	-4%
22.60-17.60	1.8814	1.6006	-15%
17.60-13.71	1.0227	1.0282	+1%
13.71-10.68	1.3811	1.3513	-2%
10.68-8.32	1.4984	1.6143	+8%
8.32-6.48	1.0581	0.8583	-19%
6.48-5.04	0.2745	0.1755	-36%
5.04-3.93	0.1587	0.1035	-35%
3.93-3.06	0.8634	0.8952	+4%
3.06-2.38	0.2779	0.1855	-33%

accurately the fission cross section in Doppler-broadening calculations. The numerical technique, as employed here, presents the possibility of a direct treatment of high-resolution experimental data* from a master data file. An intermediate step, consisting of the least-square fitting of the experimental data, has been avoided, although some care was exercised in the

* Presently limited to a rather narrow energy region from the standpoint of fast-reactor calculations.

TABLE 7. COMPARISON OF GROUP FLUXES BETWEEN SINGLE-LEVEL AND NUMERICAL TREATMENT; U^{235} -OXYGEN MIXTURE WITH $\sigma_p = 100$ b, $T = 1500^\circ\text{K}$
 $1/4$ Lethargy Groups between 2.38 and 29.03 eV

Energy, eV	Group Fluxes	
	Single Level	Numerical
29.03-22.60	0.5814	0.6236
22.60-17.60	0.2425	0.2347
17.60-13.71	0.1064	0.0933
13.71-10.68	0.0392	0.0294
10.68-8.32	0.0158	0.0103
8.32-6.48	0.0075	0.0049
6.48-5.04	0.0030	0.0018
5.04-3.93	0.0021	0.0011
3.93-3.06	0.0012	0.0006
3.06-2.38	0.0009	0.0003
Total 29.03-2.38	1.0000	1.0000

indiscriminate use of data points on the SCISRS data tape, particularly with respect to the total cross section. The computation time for the numerical broadening of the fission cross section to 1500°K and 2500°K was approximately 20 min on the IBM-7044. This is about three times the computation time required for the single-level evaluation to a comparable accuracy. The computation time for the two methods is roughly comparable, however, for a less restrictive calculation involving a considerably larger mesh spacing.*

Over a range of moderator-to-fuel ratios covering the very fast to near-thermal range of spectra, a deviation in $\delta\langle\sigma_f\rangle$ of less than 20% has been computed for the case of the slowing-down spectrum determined separately for each set of cross sections. A deviation of less than 10% was computed when only the spectrum obtained from the numerically broadened cross sections was used. It should be pointed out that although the pointwise values of $\delta\sigma_f$ would remain invariant, the spectrum obtained using the numerically broadened cross section could be approached using a smooth-background cross section in conjunction with the single-level fit, as suggested by Drawbaugh and Gibson.⁽¹⁰⁾

Since the fissile Doppler coefficient involves a complex interaction between fission and capture in a particular spectrum, it is not feasible to assign a specific error to the computed Doppler coefficient attributable to the single-level approximation. It is evident from this study, however, that the single-level approximation does not introduce a gross inaccuracy into the calculations. This conclusion is based upon one of many

sets of experimental data and single-level parameters. The sets which have been chosen, however, are consistent and involve a careful evaluation of the single-level parameters. A least-squares evaluation of the parameters, however, may yield better agreement with the numerical results. Furthermore, it would be instructive to compare other sets of experimental data and single-level parameters.

Acknowledgments

The author is indebted to Thomas Emrick for his help in revising the TEMPO code for operation on the IBM-7044, and to Nancy K. Smith for assisting with the calculations. Appreciation is also extended to Charles Stevens for several helpful suggestions, and to Raymond Shanstrom for his encouragement.

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* The present evaluation used all data points of the Saclay set on the SCISRS tape. Far fewer points are necessary for a comparable numerical-broadening accuracy.

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Discussion

Mr. Levine (BNL): When you did the numerical Doppler broadening, did you use a smooth curve through the experimental values?

Mr. Cohen: No, I used the points themselves directly.

Mr. Ohlhoft (Westinghouse): You stated that you thought you could reduce computer time by reducing the number of points in the numerical analyses. What criteria exist to determine how many points are necessary?

Mr. Cohen: This is like asking how many mesh points are needed in a fine-structure calculation, so I don't have any absolute answer. However, I did take some

points on single-level cross sections and numerically broadened them, so in theory I have an exact answer from the single-level evaluation. I took the same energy spacing as I had taken for the numerical study and reduced it by at least a factor of three, and noticed changes of less than 1 %. Thus I concluded that I needed fewer points than I used in the study by a factor of three, but I have no absolute criterion.

Mr. Kelber (ANL): You mentioned that the scattering cross section in each case is taken from the single-level representation. In some evaluations that I have made, I've noticed that this has, when compared with the experimental sigma total, led to negative cross sections. Did you check that there were no such occurrences as this or did you alter the cross sections?

Mr. Cohen: No, I didn't alter them. However, there were no negative cross sections. The capture cross sections predicted by such a procedure did look very unrealistic.

Mr. Kelber: That's what I would expect.

Preliminary Fast Reactor Cross-section Sensitivity Analysis

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1. Introduction

The reactor designer and the safety analyst need to estimate the uncertainties in the calculated nuclear properties of potential commercial reactor designs. Uncertainties in the usual multigroup calculations can arise from several sources: uncertainties in the gross neutron spectrum used in the averaging process; uncertainties in the averaging process, especially in the resonance regions; uncertainties in the heterogeneity corrections; uncertainties in the calculational process.

A number of the uncertainties have been under evaluation. We believed that it could be useful to compute partial derivatives of important reactor characteristics with respect to individual macroscopic cross sections using our basic survey calculational procedures. Such data permit rapid estimation of the effects of changes or uncertainties of various magnitudes resulting from any source. This is in many respects an extension of earlier five-group calculations by R. P. Moorehead (England) to an eighteen-group set, to sodium temperature coefficients, and to a larger core size.

Having the partial derivatives of reactivity, for example, with respect to the macroscopic group fission cross sections, one can determine the sensitivity of reactivity to the microscopic fission cross section of any single isotope present.

The data presented are preliminary in nature. The calculations were performed to explore the usefulness of this procedure in evaluating static and dynamic design values.

Specifically, the objective of this effort was to perform exploratory calculations to assess the value of computing partial derivatives of the nuclear quantities affecting economics and safety, as an aid in evaluating the group structure and the sensitivity of the economic and safety parameters to experimental and calculational uncertainties.

2. Calculational Procedure

A computer program for one-dimensional diffusion theory was used for all the basic calculations. This code

is a development at W-APD based on the AIM-6 code of AI. The studies were performed with eighteen discrete energy groups with five-group downscattering.

For each of the two reactor configurations chosen, a criticality search was performed by varying the plutonium fraction in discrete steps. The data obtained then permitted computing the slope of reactivity versus plutonium fraction, breeding versus plutonium fraction, and breeding versus reactivity.

The computer program takes microscopic cross sections and material number densities and calculates macroscopic cross sections. The program permits changes in the computed macroscopic cross sections.

A reference case was run for each core size with an arbitrarily selected eigenvalue greater than unity to cover approximately various reactivity deficits due to temperature and power. Each macroscopic cross-section value was then varied separately by (usually) 10%. These variations were stacked as change cases and run sequentially, the time per change case being generally less than 20 sec on an IBM-7094.

Note that the changes as calculated in this fashion correspond to partial differentials, not to total differentials. In this initial work, the changes were made in the cross sections for the core region only.

The cross-section set used was a set developed out of the YOM set for survey calculations on carbide cores. It is self-shielded in the lower energy groups for a "typical" fuel-rod size and spacing.

3. Reactor Models

Since this was an exploratory set of calculations, very simple reactor models were used. The cores were assumed to be spherical, of uniform composition, and completely surrounded by an 18-in. blanket and an 18-in. reflector. Two core volumes were arbitrarily chosen, a small one of 1500 liters and a large one of 6000 liters. These are roughly representative of a small prototype and a large commercial plant. Fixed material compositions were chosen which are approximately representative of core, radial blanket, and reflector compositions for existing designs:

FIG. 1. Macroscopic Group Cross Sections for 1500-liter Core

FIG. 2. Fluxes per Unit Lethargy for 1500-liter Core.

Material	Core Volume Fraction	Blanket Volume Fraction	Reflector Volume Fraction
Fuel	0.30	0.55	0
Steel	0.10	0.24	0.50
Sodium	0.60	0.20	0.50
Void	0	0.01	0

The core region fuel was taken to be 90% dense (U, Pu)C with seven a/o (of U + Pu) as fission products. The blanket material was taken to be fresh, fully depleted UO_2 . The core plutonium composition was taken as 73.2% Pu-239, 14.7% Pu-240 and 12.1% Pu-241.

The enrichment level was varied to achieve approximately an eigenvalue of 1.03 for each core size.

4. Results

Figure 1 is a plot of the macroscopic group cross sections generated for the 1500-liter core. There are no unusual features. The characteristic rapid rise in down-scattering above 300 keV can be seen. This, combined with the nearly constant low absorption cross section, tends to establish the flux peak in energy. The characteristically rapid rise in $\nu\Sigma_f$ is evidenced, which contributes to the positive sodium temperature coefficient.

Figure 2 is a plot of the flux per unit lethargy and the adjoint flux per unit lethargy for the 1500-liter core. The flux per unit lethargy is a smooth function of energy. Small variations between groups 8 and 13 (see Table 1 for energy bounds and lethargy widths of the 18 groups) result from fluctuations in the removal cross

TABLE 1. GROUP STRUCTURE

Group Number	Energy Bounds	Lethargy Width
1	9.9700 MeV	1.0
2	3.6678	0.5
3	2.2246	0.5
4	1.3493	0.5
5	0.81839	0.5
6	0.49638	0.5
7	0.30107	0.5
8	0.18261	0.5
9	0.11076	0.5
10	0.067177	0.5
11	0.040745	0.5
12	0.024713	0.5
13	0.014989	0.5
14	9.0915	0.82104
15	4.0000 keV	1.38629
16	1.0000	1.20397
17	0.30000	1.09861
18	0.10000	1.20397
	0.03000	

sections. Groups 10 and 12 have slightly lower removal cross sections; hence, the flux is slightly high in groups 10 and 12 relative to groups 9, 11, and 13.

Figure 3 is a plot of the sensitivity of reactivity to group macroscopic absorption cross sections for the 1500-liter core. The results are generally as would be predicted by a flux per unit lethargy times adjoint-flux weighting. The sharp drop in sensitivity from group 3 to group 4 results from the drop in Σ^a and $\nu\Sigma^f$ at the U-238 fission threshold. Groups 10 and 12 are up slightly because of the flux perturbations, mentioned above, resulting from the downscattering variation.

Figure 4 is a plot of the sensitivity of reactivity to the macroscopic fission cross section for the 1500-liter core. The general shape is similar to that of Fig. 3 and of opposite sign as it should be, with small differences being attributable to differences in the energy dependence of the cross sections. The magnitude of the sensitivity of reactivity to fission cross section is substantially greater at the high-energy end than the sensitivity of reactivity to absorption. The change of fission cross section was made by changing Σ^f in a fashion which left the total absorption unchanged; hence, it corresponded

to a change in α . If a change were made by increasing or decreasing Σ^f and Σ^a by the same number of barns, then the two changes would be largely self-compensating and the net effect on reactivity very small.

Figure 5 is a plot of the sensitivity of the sodium temperature coefficient to changes in macroscopic absorption for the 1500-liter core. These calculations were performed by reducing the sodium number density by 10% and recomputing changes in reactivity for changes in the new macroscopic absorption cross sections. By comparison with similar data for the 6000-liter core (see Fig. 7) one suspects that the group-16 value may be erroneous, but an error has not yet been found. The general shape of the curve provides an explanation for the fact that the addition of absorbers with a $1/v$ -type cross section makes the sodium coefficient more positive.

Figure 6 is a plot of the sensitivity of reactivity to macroscopic absorption cross sections for the 6000-liter core. The general shape is about the same as that for the 1500-liter core, with somewhat greater sensitivity below 300 keV, as would be expected.

Figure 7 is a plot of the sensitivity of the sodium temperature coefficient to the macroscopic cross sections for the 6000-liter core. With the exception of group 16, the general shape of the curve follows that of the 1500-liter core, but at substantially lower levels. The lower levels result primarily from the larger base value of the sodium temperature coefficient.

5. Breeding Ratio

The curves of sensitivity of breeding ratio (see Figs. 8, 9, and 10) are the data as calculated. There is a problem associated with interpreting the calculated numbers. An uncompensated increase in parasitic absorption, which will lower the eigenvalue of the system, will result in calculating an increase in breeding ratio.

It is often desirable to determine the effect on breeding ratio of a cross-section change assuming that the reactor is restored to its original reactivity state by changing the fuel enrichment. From a cross plot of breeding ratio versus reactivity, this relationship is approximately $\Delta BR = 2.6 \Delta k$ for the 1500-liter core. Thus, for the 1500-liter core, the total change in breeding ratio at a fixed reactivity can be computed by adding $1.76 (\Sigma^a/k) (\partial k / \partial \Sigma^a)$ to the calculated values in Figs. 8 and 9. The correction for reactivity will be of opposite sign to the calculated change, and typically about three times as large.

6. Discussion

One shortcoming of this preliminary survey is the fact that the diffusion coefficient was not allowed to

FIG. 3. Sensitivity of Reactivity to Macroscopic Absorption Cross Sections for 1500-liter Core.

FIG. 4. Sensitivity of Reactivity to Macroscopic Fission Cross Sections for 1500-liter Core.

FIG. 5. Sensitivity of Sodium Temperature Coefficient to Macroscopic Absorption Cross Sections for 1500-liter Core.

FIG. 6. Sensitivity of Reactivity to Macroscopic Absorption Cross Sections for 6000-liter Core.

FIG. 7. Sensitivity of Sodium Temperature Coefficient to Macroscopic Cross Sections for 6000-liter Core.

FIG. 8. Sensitivity of Breeding Ratio to Macroscopic Absorption Cross Sections for 1500-liter Core.

FIG. 9. Sensitivity of Breeding Ratio to Fission Cross Sections for 1500-liter Core.

FIG. 10. Sensitivity of Breeding Ratio to Macroscopic Absorption Cross Sections for 6000-liter Core.

vary when the cross-section changes were made. However, since the diffusion coefficient is largely controlled by scattering, this was believed to be adequate for the purpose of this effort. Further effort should include sensitivity to the scattering cross sections and appropriate adjustment of the diffusion coefficient.

Several checks were performed to check the validity of the "small" changes, which were usually 10% of the base value. The agreement was excellent. A few checks were made to establish that the change effects were additive.

No variations were performed for the blanket properties in this study, and, although these effects should be small, they are not negligible.

It is clear from the data presented that if substantial

systematic errors are present in the cross sections, very large errors would appear in the results unless compensating errors occur. If we consider estimates of potential errors in excess of 10% of accepted cross-section values, it is evident from the moderate to good agreement with experiments that compensating errors must be present. This is not new; it was found long ago for thermal reactors. It points up the dangers inherent in quickly adopting new data. The new data may be substantially superior in accuracy to previous numbers, but if fairly good agreement with experiments had been achieved, adopting the new data may yield poorer results. Thus, new data must be adopted with great care to recheck the resulting total cross-section data set against a broad spectrum of experimental values before accepting the new results.

7. Conclusions

This technique is valuable for developing insight into the relative characteristics which will be produced by changes or variations in cross section values.

This technique, if automated, would be an extremely sensitive tool for checking new multigroup cross-section libraries for more subtle errors than straight reactivity calculations, which typically catch only gross errors.

This technique can be a valuable tool in hazards and accident evaluation to provide both quantitative data on the effects of basic uncertainties, and to show qualitatively that there are no ultra-sensitive quantities involved. This requires evaluation of alternate cross-section libraries with alternate group structures to establish that we do indeed understand the sensitivity of the safety characteristics to uncertainties in the computational data.

Discussion

Mr. Loewenstein (ANL): Would you explain just how these derivatives were obtained? Is this a perturbation formalism or a k calculation?

Mr. Heck: These were calculated strictly by brute force. We made changes in the macroscopic cross sections. The program operates very much like AIM-6. You insert number densities and a basic cross-section library. The program then calculates macroscopic cross sections and permits changes in individual macroscopic cross sections. So we simply put in changes and recalculated reactivity for the other quantities. This was done partly because we weren't quite sure that a change in a high-energy cross section is a small perturbation,

since it affects the entire spectrum. Preliminary checking indicates this is a small perturbation, so we could very well use perturbation theory to get essentially the same results. The changes for the most part were $\pm 10\%$ changes in the cross sections. We made a few other checks some with smaller or larger changes without obtaining significant differences.

Mr. Loewenstein: When you made these changes in, for example, a reaction cross section, did you make a change in the total or transport cross section to correspond to this?

Mr. Heck: The changes were, as nearly as possible, changes in only one quantity. That was the reason for writing partial derivatives. For the change in $\nu\sigma_f$, we didn't change σ_a , but the removal cross section did change, so that we were actually calculating partial derivatives. We were not looking at the kinds of effects you would get by changing an isotope. You can derive those effects by combining the partial effects that we calculated. If you want to calculate the effect of making a change in the fraction of the total Pu-239 cross section, for example, you need the two partial factors, so that you could do it for Pu-239, Pu-241, and U-238, etc. It's not obviously a simple calculation to evaluate the effect of a single isotope if you are changing many things simultaneously.

Mr. Vernon (AI): The adjoint function apparently had a dip at the highest energies.

Mr. Heck: It was the adjoint flux per unit lethargy that was plotted. The highest energy group is one lethargy unit in width; the normal is a half lethargy unit.

Presented by J. J. Wagschal

Use of Integral Measurements as Supplementary Data in Neutron Cross-section Evaluation

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1. Introduction

Measurements of critical masses and flux distribution in critical and subcritical assemblies may be considered as indirect, 'integral' measurements of 'elementary' nuclear parameters, cross sections, number and spectra of fission neutrons, etc. So far the information obtained by these integral measurements was scarcely used in the evaluation of the elementary nuclear parameters. The purpose of this paper is to formulate an exact method to improve the microscopic cross-section evaluation by means of such integral data.

In order to use data from discrete measurements, such as critical-mass determination, along with data from direct measurements of nuclear cross sections, which are continuous functions of the energy, the method of least squares is generalized. This generalization is applied in Sections 2, 3, and 4 in order to improve the microscopic cross-section evaluation by means of integral data. In Sect. 5 the method is illustrated by means of a simple numerical example. In the concluding section some remarks on the practical application of the method are made.

2. Statement of the Problem

Let us consider a set of physical quantities ('cross-sections') $\sigma_i(E)$, the experimental estimates of which are

$$\hat{\sigma}_i(E) \pm \frac{\Delta\hat{\sigma}_i(E)}{\sqrt{\omega_i(E)\Delta E}}, \quad (2.1)$$

where $\omega_i(E)$ is the number of measurements of $\sigma_i(E)$ per unit interval of E ; each measurement has an experimental error $\Delta\hat{\sigma}_i(E)$. Circumflexes over symbols denote experimental results. Further, let us consider another set of quantities γ_j , which are functionals of the functions $\sigma_i(E)$,

$$\gamma_j = \gamma_j[\sigma_1(E), \sigma_2(E), \dots] = \gamma_j[\sigma(E)], \quad (2.2)$$

and assume experimental values of these also to be available, so that

$$\gamma_j \approx \hat{\gamma}_j \pm \Delta\hat{\gamma}_j. \quad (2.3)$$

We assume that the experimental estimates of the functions $\sigma_i(E)$ and the functionals $\gamma_j[\sigma]$ have normal distributions, the former with mean $\sigma_i(E)$ (the true, unknown functions) and standard deviations $\Delta\hat{\sigma}_i(E)$ ($\omega_i(E)\Delta E$)^{-1/2}, and the latter with mean $\gamma_j[\sigma]$ and standard deviations $\Delta\hat{\gamma}_j$.

Using this assumption, the probability of simultaneously obtaining the experimental estimates $\hat{\sigma}_i(E)$ and $\hat{\gamma}_j$ is

$$P = C \exp \left\{ - \sum_i \frac{1}{2} \int \left[\frac{\sigma_i(E) - \hat{\sigma}_i(E)}{\Delta\hat{\sigma}_i(E)} \right]^2 \cdot \omega_i(E) dE - \sum_j \frac{1}{2} \left[\frac{\gamma_j[\sigma] - \hat{\gamma}_j}{\Delta\hat{\gamma}_j} \right]^2 \right\}. \quad (2.4)$$

The improved estimate of the functions considered consists of those functions $\sigma_i(E)$ for which the probability P is maximal. This is the fundamental proposition of the least-squares principle. Thus, the functions $\sigma_i(E)$ for which we are looking should minimize the quadratic form

$$Q = \sum_i \int \left[\frac{\sigma_i(E) - \hat{\sigma}_i(E)}{\Delta\hat{\sigma}_i(E)} \right]^2 \omega_i(E) dE + \sum_j \left[\frac{\gamma_j[\sigma] - \hat{\gamma}_j}{\Delta\hat{\gamma}_j} \right]^2. \quad (2.5)$$

The functions $\omega_i(E)$ were introduced as the densities of the differential measurements of the $\sigma_i(E)$. It is now clear that they may be interpreted more generally as the overall significance or relative weights attributed to the estimates $\hat{\sigma}_i(E)$. These weights are not necessarily just the densities of measurements; other considerations might be invoked in assessing the experimental estimates. If, for example, well-established theoretical interpolation formula exist for some cross section and it is measured only around one energy value, then its estimated values at other energies should be just as reliable. Examples of cases in which such theoretical information may be invoked are optical-model calculations for angular distributions and evaporation-theory results for the variation of $\bar{\nu}$ with energy.

We shall henceforth consider the functions $\omega_i(E)$ to be the relative weights of the experimental estimates $\hat{\sigma}_i(E)$, as determined by all pertinent information.

3. Formal Solution of the General Problem

We shall first treat the problem of improving the direct estimate of one function, using experimental data on N of its functionals, and then demonstrate that the general problem with several functions can always be reduced to this case.

We are thus seeking that function $\sigma(E)$ which minimizes the quadratic form

$$Q = \int \left[\frac{\sigma(E) - \hat{\sigma}(E)}{\Delta \hat{\sigma}(E)} \right]^2 \omega(E) dE + \sum_{i=1}^N \left[\frac{\gamma_i[\sigma] - \hat{\gamma}_i}{\Delta \hat{\gamma}_i} \right]^2. \quad (3.1)$$

It will be convenient to choose the square root of the integrand in (3.1):

$$f(E) = \frac{\sigma(E) - \hat{\sigma}(E)}{\Delta \hat{\sigma}(E)} \sqrt{\omega(E)}, \quad (3.2)$$

as our unknown function. We shall also introduce the derivatives $D_i(E)$ of the functionals $\gamma_i[\sigma]$. These derivatives are defined, as usual, by the relations

$$\delta \gamma_i = \int D_i(E) \delta \sigma(E) dE. \quad (3.3)$$

Then, by (3.2) and (3.3),

$$\gamma_i[\sigma] = \gamma_i[\hat{\sigma}] + \int D_i(E) \frac{\Delta \hat{\sigma}(E)}{\sqrt{\omega(E)}} f(E) dE, \quad (3.4)$$

and the quadratic form (3.1) may now be expressed as

$$Q = \int f^2(E) dE + \sum_{i=1}^N \left[\int G_i(E) f(E) dE - C_i \right]^2, \quad (3.5)$$

where

$$\left. \begin{aligned} G_i(E) &= \frac{D_i(E)}{\Delta \hat{\gamma}_i} \frac{\Delta \hat{\sigma}(E)}{\sqrt{\omega(E)}}; \\ C_i &= \frac{\hat{\gamma}_i - \gamma_i[\hat{\sigma}]}{\Delta \hat{\gamma}_i} \end{aligned} \right\}. \quad (3.6)$$

The variation of Q with respect to $f(E)$ is

$$Q\delta = 2 \int \left\{ f(E) + \sum_{i=1}^N \left[\int G_i(E') f(E') dE' - C_i \right] G_i(E) \right\} \delta f(E) dE. \quad (3.7)$$

For that $f(E)$ which minimizes Q , this variation vanishes for any arbitrary $\delta f(E)$. The function we are

looking for is therefore the solution of the integral equation

$$f(E) + \int \sum_{i=1}^N G_i(E) G_i(E') f(E') dE' = \sum_{i=1}^N C_i G_i(E). \quad (3.8)$$

This is a symmetric Fredholm equation with a degenerated kernel. As long as $+1$ is not an eigenvalue of the homogeneous equation,

$$f(E) + \lambda \int \sum_{i=1}^N G_i(E) G_i(E') f(E') dE' = 0 \quad (3.9)$$

(and in all practical calculations this is indeed the case), Equation (3.8) can be solved by the usual procedure.

In order to solve Equation (3.8) we first define

$$\left. \begin{aligned} x_i &= C_i - \int G_i(E) f(E) dE; \\ A_{ij} &= \int G_i(E) G_j(E) dE \end{aligned} \right\}. \quad (3.10)$$

With these notations Eq. (3.8) can be rewritten as

$$f(E) = \sum_{j=1}^N x_j G_j(E), \quad (3.11)$$

which, on multiplying by $G_i(E)$, and integrating over E , becomes

$$x_i + \sum_{j=1}^N A_{ij} x_j = C_i. \quad (3.12)$$

The solution of the integral equation (3.8) thus reduces to that of a system of N linear equations. Solving Equation (3.12) for the x_i and substituting the result in Eq. (3.11), we obtain

$$f(E) = \sum_{i,j=1}^N G_i(E) (B^{-1})_{ij} C_j, \quad (3.13)$$

where the elements of the matrix B is given by

$$B_{ij} = \delta_{ij} + A_{ij}. \quad (3.14)$$

The improved estimate of $\sigma(E)$, by definition (3.2), is therefore

$$\bar{\sigma}(E) = \hat{\sigma}(E) + \frac{[\Delta \hat{\sigma}(E)]^2}{\omega(E)} \sum_{i,j=1}^N \frac{D_i(E)}{\Delta \hat{\gamma}_i} (B^{-1})_{ij} C_j. \quad (3.15)$$

In actual applications, however, functionals may depend on several functions, rather than one. The integral quantities that we propose to employ, in order to improve direct neutron cross-sections estimates, are practically functionals of the fission, capture, inelastic scattering, and elastic scattering cross sections, and the average number, $\bar{\nu}(E)$, of neutrons emitted per fission caused by a neutron of energy E . There are actually

as many sets of these functions as the number of isotopes of which the system under consideration is composed.

We shall refer to all these functions as cross sections and denote them by $\sigma_m^j(E)$, where $j = 1, 2, \dots, 5$ are the types of the functions, in their respective order above, and $m = 1, 2, \dots, M$ indicate the various isotopes composing the system, M being their total number.

The effective energy range of all cross sections (in the context of reactor physics) is finite: $0 \leq E \leq E_{\max}$ (≈ 10 MeV). We may therefore combine the $5M$ cross sections and construct a generalized single cross section for each system, defined over the energy range $0 \leq E \leq 5ME_{\max}$ by

$$\sigma(E) = \sigma_m^j[E - (5m + j - 6)E_{\max}]; \quad (3.16)$$

$$(5m + j - 6)E_{\max} \leq E \leq (5m + j - 5)E_{\max}.$$

Our problem in the general case of functionals depending on several functions thus reduces to the case of functionals of just one function.

The definition (3.16) may now be used to express our results in terms of the separate components $\sigma_m^j(E)$ of the generalized cross section $\sigma(E)$.

Introducing the obvious modifications in the definitions, i.e., $D_{im}^j(E)$ is defined by the relation

$$(\delta\gamma_i)_m^j = \int D_{im}^j(E) \delta\sigma_m^j(E) dE \quad (3.17)$$

and

$$G_{im}^j = \frac{D_{im}^j(E)}{\Delta\hat{\gamma}_i} \frac{\Delta\hat{\sigma}_m^j(E)}{\sqrt{\omega_m^j(E)}}, \quad (3.18)$$

we obtain

$$\bar{\sigma}_m^j(E) = \hat{\sigma}_m^j(E) + \frac{[\Delta\hat{\sigma}_m^j(E)]^2}{\omega_m^j(E)} \sum_{i,k=1}^N \frac{D_{im}^j(E)}{\Delta\hat{\gamma}_i} (B^{-1})_{ik} C_k, \quad (3.19)$$

where now the element of the matrix A is given by

$$A_{ik} = \int \sum_{j=1}^5 \sum_{m=1}^M G_{im}^j(E) G_{km}^j(E) dE \quad (3.20)$$

and the matrix B is defined by Equation (3.14) with this A .

The various partial cross sections introduced above are not independent. There exist comparatively accurate data on the total cross sections,

$$\sigma(\text{fission}) + \sigma(\text{capture}) + \sigma(\text{inelastic}) + \sigma(\text{elastic}) = \hat{\sigma}_i \pm \Delta\hat{\sigma}_i,$$

and this information should be included in the least-squares evaluation. Adding to Q , as defined by Eq. (2.5), a term

$$\int \left[\frac{\left(\sum_{i=1}^4 \sigma_m^i(E) \right) - \hat{\sigma}_{t,m}(E)}{\Delta\hat{\sigma}_{t,m}(E)} \right]^2 \omega_{t,m}(E) dE$$

for each isotope, our final result (3.19) is modified:

$$\bar{\sigma}_m^j(E) = \hat{\sigma}_m^j(E) + \frac{[\Delta\hat{\sigma}_m^j(E)]^2}{\omega_m^j(E)} \left\{ \sum_{i,k=1}^N \frac{D_{im}^j(E) - \langle D_{im}(E) \rangle}{\Delta\hat{\gamma}_i} (B^{-1})_{ik} K_k + \frac{1}{g_m(E)} \left[\hat{\sigma}_{t,m}(E) - \sum_{i=1}^4 \hat{\sigma}_m^i(E) \right] \right\}, \quad (3.21)$$

where

$$g_m^j(E) = \frac{[\Delta\hat{\sigma}_m^j(E)]^2}{\omega_m^j(E)}; \quad g_{t,m}(E) = \frac{[\Delta\hat{\sigma}_{t,m}(E)]^2}{\omega_{t,m}(E)};$$

$$g_m(E) = g_{t,m}(E) + \sum_{j=1}^4 g_m^j(E);$$

$$\langle D_{im}(E) \rangle = \sum_{j=1}^4 D_{im}^j(E) g_m^j(E) / g_m(E);$$

$$K_k = C_k - \sum_{m=1}^M \int \left[1 + \sum_{j=1}^4 \left[\frac{\Delta\sigma_m^j(E)}{\Delta\sigma_{t,m}(E)} \right]^2 \frac{\omega_{t,m}(E)}{\omega_m^j(E)} \right]^{-1} \cdot \left(\frac{\hat{\sigma}_{t,m}(E) - \sum_{j=1}^4 \hat{\sigma}_m^j(E)}{\Delta\hat{\sigma}_{t,m}} \right) \sum_{l=1}^4 G_{km}^l(E) \cdot \frac{\Delta\hat{\sigma}_m^l(E) \omega_{t,m}(E)}{\Delta\hat{\sigma}_{t,m}(E) \sqrt{\omega_m^l(E)}} dE.$$

4. The Multigroup Approximation

In order to apply our theory we need, besides the experimental data on the directly measured cross section and the integral quantities, means and methods to calculate the integral quantities and their functional derivatives from the cross sections.

The integral quantities and their functional derivatives are usually calculated by a multigroup approximation; thus it seems worthwhile to formulate a corresponding multigroup approximation of the theory which was developed in the previous section.

The multigroup approximation is based on the assumption that true cross sections can be replaced by appropriate fictitious cross sections, which are piecewise constant functions, i.e., constant within each energy group.

With such an approximation it seems meaningless to correct the cross sections in greater detail. In other words, if the corrected cross section is given by

$$\bar{\sigma}_i(E) = (1 + p_i(E)) \hat{\sigma}_i(E), \quad (4.1)$$

then the functions $p_i(E)$ can be estimated significantly only as a piecewise constant functions, constant within each energy group.

TABLE 1. CRITICAL ASSEMBLIES WITH ENRICHED URANIUM CORE AND NATURAL URANIUM REFLECTOR

Name	Critical Mass, kg U-235	Density of Core Alloy, g/cm ³	Enrichment of Core, %	Thickness of Reflector, cm	Density of Reflector, g/cm ³	Ref.
U1	48.8	18.75	93.9	—	—	1
U2	34.0	18.75	93.9	1.77	19.0	2
U3	24.9	18.75	93.9	4.47	19.0	2
U4	19.2	18.75	93.9	8.94	19.0	2
U5	18.55	18.75	93.9	9.96	19.0	2
U6	16.65	18.62	93.2	18.11	19.0	2

For simplicity let us consider in this section only the case of one cross section; the generalization performed at the end of Sect. 3 applies also in the present case.

Let us therefore assume that we use the multigroup approximation with group boundaries defined by the energies

$$0 = E_0 < E_1 < \dots < E_g = E_{\max}. \quad (4.2)$$

The function $p(E)$ is constant within each energy group:

$$p(E) = p_g; \quad E_{g-1} < E < E_g, \quad (4.3)$$

and the quadratic form (3.1) to be minimized is transformed to

$$Q = \sum_{g=1}^G f_g^2 + \sum_{i=1}^N \left[\frac{\gamma_i[\sigma] - \hat{\gamma}_i}{\Delta \hat{\gamma}_i} \right]^2 \quad (4.4)$$

with

$$\left. \begin{aligned} f_g &= p_g / \beta_g \\ \frac{1}{\beta_g^2} &= \int_{E_{g-1}}^{E_g} \frac{\omega(E) dE}{(\Delta \hat{\sigma}(E) / \hat{\sigma}(E'))^2} \end{aligned} \right\} \quad (4.5)$$

Defining

$$G_{ig} = \frac{\beta_g}{\Delta \hat{\gamma}_i} \int D_i(E) \hat{\sigma}(E) dE, \quad (4.6)$$

we obtain the following set of linear equations:

$$f_g + \sum_{i=1}^N \sum_{g'=1}^G G_{ig} G_{ig'} f_{g'} = \sum_{i=1}^N C_i G_{ig}. \quad (4.7)$$

This set of equations is the discrete analog of Eq. (3.8), and its solution is obtained in the same way; thus,

$$\bar{\sigma}(E) = (1 + p_g) \hat{\sigma}(E), \quad E_{g-1} \leq E \leq E_g, \quad (4.8)$$

where p_g is given by

$$p_g = \beta_g \sum_{i,j=1}^N G_{ig} (B^{-1})_{ij} C_j. \quad (4.9)$$

The constants C_j are defined by Eq. (3.6), and $B = I + A$, where

$$A_{ij} = \sum_{g=1}^G G_{ig} G_{jg}. \quad (4.10)$$

It should be noted that the number G of energy groups used in Equations (4.2) to (4.10) can be any number less or equal to the number G' of energy groups used in the calculation of the integral quantities and their derivatives $\partial \gamma_i / \partial \sigma_g$.

The estimate of the error ΔE in the corrected cross sections is

$$\Delta \bar{\sigma}_g = \Delta \hat{\sigma}_g (1 - \sum_{i,j=1}^N G_{ig} (B^{-1})_{ij} G_{jg})^{1/2}. \quad (4.11)$$

The quantities G_{ig} defined by (4.6) are actually also calculated by a multigroup approximation and are thus approximated by

$$G_{ig} \cong \frac{\hat{\sigma}_g \beta_g}{\Delta \hat{\gamma}_i} \frac{\partial \gamma_i(\hat{\sigma}_g)}{\partial \hat{\sigma}_g}, \quad (4.12)$$

where $\gamma_i(\sigma_g)$ is a function (and not a functional) of a finite number of group constants $\hat{\sigma}_g$.

5. Sample Calculations

We present some results of an attempt to improve the U^{235} and U^{238} cross-section data by using critical-mass data from 6 assemblies. Although our calculations are rather crude, a significant improvement of the data is achieved. The errors in the reactivity calculated with the 'improved cross sections' are smaller by almost an order of magnitude than the errors obtained with the original cross sections.

Pertinent data on the critical systems included in our least-square evaluation are given in Table 1. In Table 2 are shown the percentagewise corrections to the various cross sections for four cases. Cases LS1 and LS2 were calculated with the relative experimental errors $\beta \cong \Delta \sigma / \sigma$ given in column 2, and cases LS3 and LS4 were evaluated with the β 's given in column 5. In the least-squares evaluations LS2 and LS4 the system U6 was not taken into account. The set of β 's used in LS1 and LS2 was obtained from the literature insofar as possible. The β 's used in LS3 and LS4 were rather arbitrarily reduced.

In Table 3 the reactivity parameters $\gamma - 1$ as calculated, using the various cross-section sets, are given. It is seen that while the errors with the original cross sections are far greater than the estimated accuracy of the measurements, the errors obtained with the corrected ones are usually smaller than the experimental uncertainties. The experimental uncertainty $\Delta \gamma$ is the estimated error in the critical mass or in the reflector thickness 'translated' into an error in the reactivity parameter γ defined below.

The cross-section set LS1 yields very good results for

TABLE 2. CORRECTIONS TO CROSS SECTIONS

Reaction	1	2	3	4	5	6	7
	Energy Range, MeV	$\beta_g, \%$	LS1	LS2	$\beta_g, \%$	LS3	LS4
			$P_g, \%$			$P_g, \%$	
U ²³⁵							
Fission	0-0.6	5	-0.61	0.27	3.5	1.29	1.07
	0.6-1.6	5	1.40	1.07	3.5	0.46	0.71
	1.6-14.	5	2.86	2.35	3.5	1.42	1.56
Capture	0-0.6	18	1.67	0.39	12.5	-5.10	-1.99
	0.6-1.6	30	-1.04	-1.43	21	2.67	-0.14
	1.6-14.	30	-0.10	-0.44	21	0.87	-0.14
Inelastic Scatter- ing	0-0.6	40	7.18	6.13	17.5	3.96	3.10
	0.6-1.6	40	13.03	10.00	17.5	7.91	5.36
	1.6-14.	40	24.94	21.78	17.5	13.59	11.93
Elastic Scattering	0-0.6	12	2.27	2.01	8.5	4.09	3.42
	0.6-1.6	18	-2.11	-1.13	12.5	1.29	1.63
	1.6-14.	36	-8.85	-7.43	17.0	-2.40	-1.97
NU	0-0.6	4	-0.70	-0.05	3	1.78	0.99
	0.6-1.6	4	1.25	1.06	3	-0.08	0.64
	1.6-14.	4	2.30	1.98	3	0.97	1.46
U ²³⁸							
Fission	0-0.6	5	0	0	3.5	0	0
	0.6-1.6	5	1.14	-0.06	3.5	0.01	0
	1.6-14.	5	-1.22	-0.73	3.5	0.06	-0.30
Capture	0-0.6	18	-35.33	-15.81	6.5	-20.95	-6.28
	0.6-1.6	30	4.84	1.98	21	-10.65	-3.75
	1.6-14.	30	0.14	-0.10	21	-2.05	-0.77
Inelastic Scatter- ing	0-0.6	40	-6.78	-10.60	17.5	-14.78	-9.37
	0.6-1.6	40	-25.28	-18.67	12	-10.23	-6.13
	1.6-14.	40	-17.09	-20.60	9	-13.12	-8.16
Elastic Scattering	0-0.6	12	-2.85	-1.52	8.5	-4.87	-3.01
	0.6-1.6	18	7.33	4.93	12.5	-1.06	-1.68
	1.6-14.	36	10.24	13.02	17	2.93	1.09
NU	0-0.6	4	0	0	3	0	0
	0.6-1.6	4	-0.24	-0.08	3	-0.09	0
	1.6-14.	4	-1.26	-0.77	3	0.04	-0.13

the reactivities, but some of the cross sections vary somewhat beyond their estimated errors. Deleting system U6 from the least-squares evaluation or putting somewhat smaller error bars on the cross sections, or both, we obtain cross-section sets (LS2, LS3, and LS4, respectively) which do not differ from the original set by more than the experimental uncertainty. The errors in the reactivities calculated from these sets, although greater, do not fall far beyond the experi-

mental uncertainty (except for the deleted system U6). Reactivities of 3 additional systems not included in the least-squares evaluation of the cross sections were also reproduced within the experimental accuracy with all four cross-section sets LS1 . . . LS4.

The differential data used in the example are all taken from the cross-section compilation of Schmidt.⁽³⁾

The critical parameter γ which we used in our calculations is defined by the transport equation

TABLE 3 REACTIVITY PARAMETER

System	Exp Error $\Delta\gamma, \%$	$(\gamma - 1) \times 1000$ Calculated with Cross Sections Obtained from				
		Original	LS1	LS2	LS3	LS4
U1	0.5	-10 220	0 044	-0.119	-0.270	-0.439
U2	0.41	-6 805	-0.399	-0.191	-0.301	0.037
U3	0.47	-3.494	0 489	0.722	0.970	1.061
U4	0.65	-3.391	0 378	0.043	0.727	-0.051
U5	0.32	-3.579	0 271	-0.202	0.552	-0.392
U6	0.15	-4 967	-0 086	-1 279	-0.231	-1.935

$$[\bar{\Omega} \cdot \nabla + \sum_{m,j} n_m(\bar{x}) \sigma_m^j(E)] \psi(\bar{x}, E, \bar{r})$$

$$= \frac{1}{\gamma} \iint \sum_{m,j} n_m(\bar{x}) \sigma_m^j(E) T_m^j(E', \bar{r}'; E, \bar{r}) \cdot \psi(\bar{x}, E', \bar{r}') dE' d\bar{r}' \quad (5.1)$$

with the condition that no neutrons enter the system from the outside. The summation over j ranges from 1 to 4 (see the definition in Sect. 3) and that over m from 1 to M , the number of isotopes of which the system is composed. The $n_m(\bar{x})$ are number densities of the various isotopes, and $T_m^j(E', \bar{r}'; E, \bar{r})$ are transfer kernels of neutrons with energy E' and direction of flight $\bar{\Omega}'$ to neutrons of energy E and direction of flight $\bar{\Omega}$.

The critical parameter γ and the functional derivatives of γ with respect to the cross section were calculated by the S_8 method of Carlson,⁽⁴⁾ using a 16-group transport approximation.

Acknowledgment

The authors wish to thank Mr. D. Samoucha for performing the numerical calculations.

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Discussion

Mr. Levine (BNL): In one of the cases shown, the errors in the eigenvalue were appreciably smaller than the errors from the integral experiments. Wouldn't it be a sensible criterion to require that the mean square error in your final result should be just about the same as the mean square error in the integral experiment? You don't want the cross sections derived from the experiments to look as though they are more accurate than the experiments themselves.

Mr. Wagschal: You refer to the case of the Godiva assembly, which is a bare reactor. In all other cases, the results were of about the same order of magnitude as the integral measurement itself. Now the requirement of the method is to have something at least within the bounds of the experimental calculations. It was just a coincidence that the first case was much better.

Mr. Okrent (ANL): I am reluctant to accept the difference in the change of cross section depending on whether you take 5 assemblies or 6 assemblies and whether you used the larger or smaller uncertainties. I believe that the suggested corrections make the cross sections very sensitive to your choice of examples, and therefore I question its validity. This is a question that I'm addressing really to all who propose methods of analyzing integral experiments, and therefore I address the question also to the chairman of the session. I think if this technique would really work, it's very important. But now I am unable to tell whether it will have any real validity.

Mr. Pendlebury (UK-Aldermaston): In the case of system U-6, you have the difficulty that when you eliminated that system you didn't get a particularly good fit. When you included it, it led to some rather large cross-section adjustments. Now in this fitting, the thing that really matters is not necessarily the absolute values of the standard deviations that you use, but the relative values that you use for the cross sections and those you use for the integral systems, because in the mathematics you find it's really the ratio of those two that results. What has happened is that in U-6, if you look at the standard deviations that were given, it was given as an error of 0.15%, which is far smaller than any of the other errors for that system. I would suggest that that was too small because our experience is that if you try to fit the systems too accurately, the cross sections do start being unreasonable. We find that when you're doing this fitting the first run done is not necessarily an acceptable one. The odd cross sections are a bit too large for cross-section adjustment. You have to go back, and the ultimate is when the cross-section adjustments and the fits are all reasonable. We have also tried calculations in which we have fit a certain number of systems, then left some out and gone back to see whether those systems have been fitted to the adjusted data. We have found that they have. So you have to use a bit of judgment when you use the method. You can't use just any standard deviations and get sensible answers. When you do get sensible answers it does give you some confidence that the standard deviations used are reasonable. Does Mr. Gandini agree with this?

Mr. Gandini (CNEN): I don't know about the validity of the standard deviation of the cross-section meas-

urements, as there may be systematic errors which have not been taken into account. I think that this method is just a tool to get more information.

Mr. Wagschal: Because of a very thick uranium reflector for this system, the experimental inaccuracy was not given on the critical mass of the core but rather on the reflector thickness, and therefore the numbers stated are really too small. The experimental errors should be much larger, and then it wouldn't depend on the number of sets. It doesn't depend on the number of sets used for calculations, but rather on the experimental error in each set.

Mr. Gandini: The point that should be stressed is that one should always keep this method within the known experimental accuracy of the integral and the differential data. It is very important that we stay within these uncertainties; otherwise we get into difficulty. One has to find a unique standard computational method, chosen as simply as possible, so that a unique solution is obtained even if the philosophy on which that solution is based is arbitrary. Then you have only one solution which is universally valid.

Mr. Wagschal: Dr. Pendlebury, yesterday in your paper you had a $1/N$ term on the integral measurement and $1/G$ over the cross sections. Why do you reduce the importance of the integral measurements? Our method tries to show that the $1/G$ is natural because it is the integral of all energies; however, I do not understand why you have the $1/N$ on the integral measurements.

Mr. Pendlebury: I agree that the $1/G$ is more natural, and we want to determine whether it does give any better results than the $1/N$. The reason that we introduced $1/N$ was that we found that, by introducing a lot of cross-section data, we obtained many terms in the expression. These were not necessarily large terms, but they appeared to give undue weight to the cross sections in relation to the integral systems. When we inserted them we did, in fact, get better results. We did not at first just try the $1/G$; we tried the combination and found better results. At this stage, we committed ourselves to some production work using this program. We were encouraged because we obtained satisfactory results consistently. It did seem more natural to stick with the $1/G$, so we are going to go back to see if it is better just to stick to the $1/G$.

Mr. Wagschal: I suggest that you stick to the $1/G$ and omit the $1/N$ because this reduces the importance of the integral measurements, which are much more interesting than the differential cross sections.

Mr. Pendlebury: We have $1/N$, but we also have a $1/L$ on the integral experiments. I can only give a slight justification for the $1/L$, the number of integral experiments, on the grounds that there is some vague correlation between integral experiments. For instance,

supposing that, without knowing it, you put in the same integral experiment six times. Then if, in fact, you put a $1/L$ in it doesn't matter. If you don't have it in, you can give as much weight as you like to the integral experiments. There is no unique function to minimize in this sort of thing. It is not as though it's a variational principle which is a definite thing, when you say we must minimize this because it is a law of physics. So good judgment enters in regard to what you really do mean by this.

Mr. Wagschal: In reducing the measurement density, one has to overcome the arbitrariness in giving more importance to one cross section or the other, or to integral or differential data. That was the point in having this measurement density in the combination rather than having just $1/G$ or $1/N$.

Mr. Pendlebury: If one goes on to mechanized evaluation, which is now being investigated at Aldermaston, one could be very ambitious and link this with mechanized evaluation, because you end up with a cross section which could be specified as a function of several parameters. You can then determine everything in terms of the sensitivity to these parameters rather than to group cross sections. These parameters could be determined by least-squares fitting, though spline fitting is the one that is most favored. You can connect the two, but this is very involved and is not something we are contemplating, even though one can see vaguely how to proceed.

Mr. Okrent: There seem to be two possible long-range uses of these methods. One is to help the theoretician and the experimentalist assess the accuracy of existing experiments by determining whether changes in assumed experimental accuracy influence correlations of this sort. On a longer range, one can go to the computer and ask what are the experiments that will reveal the most about the parameters that are the most uncertain. In other words, one could look for experiments that test the specific uncertainties that remain, so one could have computer-selected critical experiments.

Mr. Moldauer (ANL): The significant difference between basic cross sections and effective cross sections derived from integral experiments is that when you measure the basic cross sections you're working essentially with known fluxes. With integral experiments you're dealing with systems in which you don't really know all the flux peculiarities which may occur. When you try to adapt basic cross sections to calculations of an integral system, you somehow guess how you're supposed to use them in such a way that they will work correctly in systems which may have irregularities in the fluxes. So perhaps the integral type of measurements are more appropriate for reactor applications. In particular, the stated uncertainties in microscopic

cross-section data may not be very relevant. I question whether it is appropriate to go backward and say what the basic cross sections should be on the basis of integral data.

Mr. Pendlebury: Here there are two approaches: the nuclear physicist is more interested in the cross sections, and the reactor physicist is interested in calculating the integral results. To the latter the cross sections are a means to an end, and a sort of intermediate end. As far as he is concerned, integral experiments are almost more fundamental to him than the cross sections. If he could get all the breeding ratios and reaction rates correct, the things that matter to him are only critical sizes and enrichments. He is more concerned in getting these than getting absolute cross sections, though he realizes that it will help him if he does get those absolute cross sections. He has to rely more on the integral experiments, the things that he wants to calculate and use in reactor design, than he does on the basic cross sections.

Mr. Lantz (NASA-Lewis): You have brought breeding ratio into the argument, but you haven't really shown that you have improved your accuracy on breeding ratio by this work.

Mr. Parker (UK-Aldermaston): I would like to support Mr. Okrent's first remark on the applicability of this method to give some guidance to experimenters on the accuracy that they should attempt to achieve in experiments. One interesting result is that it has taken

over 2 years to do this exercise. This means that the original data used were essentially based on the evaluation as of the spring of 1962. This means that we didn't have the low fission cross sections that were subsequently measured at Aldermaston.

Mr. Vernon (AI): There seems to be some confusion in this discussion between where we now stand in this subject and where it will ultimately lead. Whereas now the present state just consists of the basic recipes for adjustments in the multigroup sets, to get the critical mass, there is no reason why the breeding ratio and other quantities could not be included.

Mr. Pendlebury: I think most things could be included. The main thing in doing all of this is to determine the sensitivity of these quantities to changes in the cross-section data, and no doubt Mr. Gandini's paper in the parallel session on second-order perturbation theory is somewhat relevant here because in calculating some of the sensitivities you need just such a treatment.

Mr. Vernon: There are a good many pieces we just haven't fit together yet.

Mr. Pendlebury: If one started with the ambitious objective of accounting for everything, you may not get anything to work at all. The thing we need is first of all to demonstrate that the procedure is feasible on a somewhat more limited scale. This I think has been demonstrated. One can now develop from there, confident that there is some hope of success.

Presented by A. Gandini

Monte Carlo Analysis of Small Fast Critical Assemblies*

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SUMMARY

An analysis of several critical fast assemblies containing uranium or plutonium as fuel, and various kinds of materials as reflectors, has been performed by means of an IBM-7094 Monte Carlo code in a multigroup approximation. The main features of the Monte Carlo strategy adopted in this code are briefly summarized. Elastic and inelastic scattering are separately considered, and anisotropy for the elastic scattering has been taken into account. The twenty-group cross-section set adopted for the calculations is given. The multigroup set has been obtained on the basis of microscopic data only, without any *a posteriori* adjustment. In this way it is hoped that the comparison between the results of calculation and experiments may be useful in setting the limits of confidence both for the method and for the constants adopted, at least for the kind of critical assemblies here considered.

1. Introduction

The Monte Carlo method has been successfully applied to the analysis of a large variety of fast critical assemblies by various authors. The interest in this kind of calculation is twofold. In fact, the solution of the transport equation by Monte Carlo provides a method for testing cross-section sets without the interference of some frequently used approximations as, for instance, geometrical and heterogeneity approximations. On the other hand, the test of the method on a large number of critical experimental assemblies gives confidence and limits for further applications.

In this paper the results obtained for 39 small fast critical assemblies are reported, together with the multigroup cross-section set used. Results for some U-235 systems have been previously published.⁽¹⁾ However, they are given here for completeness and because some additional calculations have been performed.

2. The Numerical Method

The computations were made with the IBM-7094 computer, using a Monte Carlo code⁽²⁾ in a multigroup approximation. In this code an initial arbitrary spatial distribution of the neutron source is iterated until equilibrium is reached. The equilibrium is recognized by the code on the basis of the χ^2 -test of homogeneity by comparing two successive spatial distributions of the neutrons born from fission. When this test is satisfied, the main computation begins and the multiplication constant is then estimated as the ratio of the number of neutrons born from fission to the number of source neutrons. Information about the space-energy distributions and other quantities of interest are also collected.

Nuclear reactions considered are elastic scattering, inelastic scattering, fission, and radiative capture. Anisotropy in the elastic scattering is described in each energy group by the average angular distribution. For not too light nuclei ($A > 27$), inelastic scattering is described by a transition matrix assuming that the angular distribution of the scattered neutron is isotropic in the laboratory system. For light nuclei, inelastic scattering is treated taking into account the excitation of the discrete energy levels. The average number of fission neutrons is assumed to depend on E through the linear law $\bar{\nu}(E) = \nu_0 + \alpha E$, with ν_0 and α depending on the kind of fissionable nucleus and on the energy range considered.

Many-zoned spherical or cylindrical (r, z) geometries are treated by the code.

A wide use of random access to precalculated probability tables allows the code to treat about 300 and 250 collisions/sec for spherical and cylindrical geometry, respectively. In one minute, 6000 histories for Godiva assembly can be processed.

Execution time does not practically depend on the number of energy groups.

3. The Cross-section Set

A multigroup cross-section set was constructed for the Monte Carlo calculations. The neutron-energy range

* Work performed under the auspices of Euratom-CNEN Association for Fast Reactors, Contract No. 010-63-1 RAAI.

TABLE 1. COMPUTED k_{eff} : BARE AND REFLECTED CRITICAL ASSEMBLIES OF ENRICHED URANIUM

No.	Geometry	Core				Reflector ^(a)	k_{eff} (± 0.003)
		Radius (cm)	Height (cm)	Enrichment (%)	Density (gm/cm ³)	Thickness (cm)	30000 histories
1 ^(b)	Sphere	8.718	—	93.8	18.75	—	1.018
2	"	6.93	—	93.5	18.75	4.75	1.020
3	"	6.86	—	93.5	18.75	5.08	1.019
4 ^(c)	"	6.03	—	94.1	18.75	20.32	1.005
5	Cylinder	6.90	24.3	93.5	18.8	—	1.002
6	"	7.98	15.2	93.5	18.8	—	1.005
7	"	8.78	12.7	93.5	18.8	—	1.003
8	"	9.43	11.5	93.5	18.8	—	1.002
9	"	19.05	8.26	93.4	17.7	—	0.997
10	"	19.05	3.48	93.4	17.7	7.62	1.000
11	"	4.115	70.61	93.2	18.7	6.99	0.995
12 ^(d)	Annular core	(inner) 7.62 (outer) 15.55	7.65	93	18.8	2.54	0.993

^(a) Reflector composition is natural uranium for all except assembly No. 11; reflector of this assembly is depleted uranium. Assemblies No. 2, 3, and 4 have reflector density 18.8 gm/cm³. Assemblies No. 10, 11, 12 and have reflector density 18.9 gm/cm³.

^(b) Godiva.

^(c) Topsy.

^(d) Inner region filled with reflector material.

TABLE 2. COMPUTED k_{eff} : ENRICHED URANIUM CRITICAL SPHERES IN VARIOUS REFLECTORS^(a)

No.	Core radius (cm)	Reflector			k_{eff} (± 0.004)
		Com-position	Density (gm/cm ³)	Thickness (cm)	20000 histories
1	6.467	WC	14.7	5.08	1.010
2	5.939	WC	14.7	10.16	0.998
3	6.729	H ₂ O	1.0	5.08	0.982
4	7.437	Cu	8.88	2.54	1.005
5	6.858	Cu	8.88	5.08	1.000
6	6.406	Cu	8.88	10.16	1.005
7	7.483	Ni	8.88	2.54	1.015
8	6.885	Ni	8.88	5.08	1.012
9	7.639	Al ₂ O ₃	2.76	2.54	1.002
10	7.703	Fe	7.87	2.54	1.005
11	7.192	Fe	7.87	5.08	0.997
12	6.849	Fe	7.87	10.16	1.011
13	7.506	Th	11.48	5.08	0.989
14	7.932	Al	2.7	2.54	1.000
15	7.959	Ti	4.5	2.54	0.998

^(a) Enrichment = 93.5%; density of enriched uranium = 18.8 gm/cm³.

of interest was assumed to be between 200 eV and 10 MeV, and it was subdivided into twenty groups. The limits of these groups are very similar to those adopted in the well-known Russian library.⁽³⁾ However, some groups have been split to obtain more detail in those energy ranges where the maximum contribution to the

TABLE 3. COMPUTED k_{eff} : GRAPHITE-REFLECTED CRITICAL URANIUM CORES IN EXTREME CYLINDRICAL GEOMETRIES

No.	Core				Reflector		k_{eff} (± 0.004)
	Enrichment (%)	Density (gm/cm ³)	Radius (cm)	Height (cm)	Density (gm/cm ³)	Thickness (cm)	20000 histories
1	93.2	18.7	4.115	55.88	1.6	12.32	0.975
2	93.2	18.7	4.115	40.89	1.6	14.61	0.974
3	93.2	18.7	19.05	2.77	1.6	17.78	0.959
4	46	18.44	14.71	7.62	1.8	15.24	0.982

neutron spectrum is expected for small, fast critical assemblies.

The weighting flux has been assumed flat everywhere. As we shall see, this assumption has no appreciable influence for the kind of assemblies here considered.

The microscopic data for U-235 and U-238 cross sections have been obtained as described in Ref. (1). For Ni, Cu, and Pu-239 the evaluations made by Ravier,⁽⁴⁾ Benzi,⁽⁵⁾ and Schmidt,⁽⁶⁾ respectively, have been used. The multigroup set for all other nuclei are based on the UKAEA Nuclear Data Library.

4. Results and Discussion

In Tables 1-4 the values of k_{eff} obtained by our calculations are given together with the description of the assemblies considered. All the data were taken

TABLE 4. COMPUTED k_{eff} : BARE AND REFLECTED CRITICAL PLUTONIUM CORES

No.	Geometry	Core			Reflector		k_{eff} (± 0.004)
		Radius (cm)	Height (cm)	Composition and Density (gm/cm ³)	Thickness (cm)	Composition and Density (gm/cm ³)	
1 ^(a)	Sphere	6.31	—	Pu-39 (15.44)	—	—	1.010
2 ^(a)	“	6.284	—	Pu-39 (14.95) Pu-40 (0.71)	—	—	1.008
3 ^(b)	“	4.45	—	Pu-39 (15.54)	24.1	Nat U (18.8)	1.005
4 ^(b)	“	4.44	—	Pu-39 (15.4) Pu-40 (0.23)	24.1	Nat U (18.8)	1.008
5	“	5.042	—	Pu-39 (14.91) Pu-40 (0.765) Pu-41 (0.048)	4.128	Nat U (18.92)	1.016
6	“	5.042	—	Pu-39 (14.81) Pu-40 (0.765) Pu-41 (0.048)	1.656	U-235 (17.48) U-238 (1.32)	1.011
7	Cylinder	7.62	3.91	Pu-39 (13.6) Pu-40 (0.7)	7.62	Nat U (18.7)	0.993
8	“	7.62	5.92	Pu-39 (13.6) Pu-40 (0.7)	2.54	Graphite (1.6)	0.968

^(a) Jezebel—In assembly No. 1 the total plutonium mass is assigned to pure Pu-239.

^(b) Popsy—In assembly No. 3 the total plutonium mass is assigned to pure Pu-239.

from ANL-5800⁽⁷⁾ with the exceptions of those referring to Jezebel and Popsy, which were taken from Ref. (8). The computations were performed starting from a flat spatial distribution of the neutron source. The number of histories run when the equilibrium distribution was reached (generally after two or three iterations) is shown in each table. The errors quoted are the “probable errors.” In addition to the value k_{eff} , the following quantities have been computed:

- Godiva and Jezebel leakage spectra (see Figs. 1 and 2);^(9, 10)
- detector ratios at core center for assemblies 1 and 4 of Table 1. The theoretical values of the detector ratio $\rho = \bar{\sigma}_f(235)/\bar{\sigma}_f(238)$ were found to be 6.49 and 8.0, whereas the experimental ones are 6.17–6.54 and 7.3 for assemblies 1 and 4, respectively.

As one can see, all the computed quantities are generally in fairly good agreement with the experimental ones.

To test the influence of the choice of the weighting spectrum, for two assemblies (Godiva and Jezebel) the computed internal-energy spectra obtained were used as weighting function for the construction of a

new multigroup set. It was found that differences in group cross sections between the two sets were very small and differences in k_{eff} were well contained in the limits of the probable errors. More precisely the k_{eff} values changed from 1.018 ± 0.003 and 1.010 ± 0.005 to 1.016 ± 0.003 and 1.007 ± 0.005 for Godiva and Jezebel, respectively. Moreover, for each of the two assemblies the internal-energy spectra were found practically identical for both cross-section sets.

In addition, some calculations were carried out with a thirty-group set to test the influence of the group subdivision of the energy interval. Again no appreciable differences were found.

For these reasons it seems to us that the set adopted in our calculations can be used with some confidence for criticality computations of small, highly enriched assemblies like those here considered. However, it must be noted that the largest discrepancies in k_{eff} (about 3–4%) were found for some assemblies with a rather thick reflector of graphite. This can be due to the fact that in these assemblies the low-energy end of the spectrum contains an appreciable number of neutrons, so that a much more detailed group structure should be adopted.

FIG. 1. Godiva Leakage Spectrum — Neutrons per Unit Energy (Arbitrary Units) vs. Energy.

FIG. 2. Jezebel Leakage Spectrum — Neutrons per Unit Energy (Arbitrary Units) vs. Energy.

Acknowledgments

Thanks are due to Mr. G. Panini for his help in preparation of the multigroup set. The Euratom-CNEN permission for the publication of this work is gratefully acknowledged.

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Appendix

In the following tables the group cross-section set adopted in this work is given.

The lower-energy groups, E_L , the Q_i values for (n, n') inelastic scattering, as well as the energy E in the $\bar{\nu}(E)$ -law, are in MeV. The cross sections are given in barns; the elastic-scattering angular distributions are in arbitrary units and are referred to the c.m. system.

For those groups for which no angular distribution is given, isotropic scattering has been assumed. The same angular distribution has been adopted for uranium and plutonium isotopes.

For those elements for which the Q -values are given, namely, C, O, and Al, the values reported in the inelastic-scattering matrix are the average group values of the single excitation functions. However it must be noted that for aluminum the inelastic scattering in the first two groups is treated according to the evaporation model. For all other elements or isotopes the transition matrix is given.

TWENTY-GROUP CROSS-SECTION SET

						$\sigma_{n,n'}^{(l)}$											
						l	1	2	3	4	5	6	7	8	9	10	11
						Q_t											
1	6.5			1.10													
2	4.0			1.58													
3	2.5			2.23													
4	1.4			3.09													
5	0.8			4.23													
6	0.5			5.61													
7	0.4			6.52													
8	0.3			7.81													
9	0.2			9.11													
10	0.1			11.5													
11	0.075			13.6													
12	0.046			15.2													
13	0.0215			17.1													
14	0.0150			18.4													
15	0.0100			18.9													
16	0.00465		0.001	19.2													
17	0.00215		0.001	19.6													
18	0.00100		0.002	19.7		HYDROGEN											
19	0.000465		0.002	19.9													
20	0.000200		0.003	20.1													

HYDROGEN

ANGULAR DISTRIBUTION

j	E_L	$\cos \theta =$										
		-1.0	-0.8	-0.6	-0.4	-0.2	0.0	+0.2	+0.4	+0.6	+0.8	+1.0
1	6.5											
2	4.0											
3	2.5											
4	1.4											
5	0.8											
6	0.5											
7	0.4											
8	0.3											
9	0.2											
10	0.1											
11	0.075											
12	0.046											

						$\sigma_{n,n'}^{(l)}$											
						l	1	2	3	4	5	6	7	8	9	10	11
E_L	σ_f	σ_c	σ_{el}	$\sigma_{n,n'}$		Q_l	4.43	7.28									
1	6.5			0.89	0.357		0.350	0.007									
2	4.0			1.33	0.111		0.111										
3	2.5			1.97													
4	1.4			1.80													
5	0.8			2.58													
6	0.5			3.25													
7	0.4			3.50													
8	0.3			3.72													
9	0.2			3.97													
10	0.1			4.23													
11	0.075			4.44													
12	0.046			4.53													
13	0.0215			4.61													
14	0.0150			4.66													
15	0.0100			4.67													
16	0.00465			4.69													
17	0.00215			4.70													
18	0.00100			4.71													
19	0.000465			4.71													
20	0.000200			4.71													

CARBON

ANGULAR DISTRIBUTION

$\cos \theta =$												
j	E_L	-1.0	-0.8	-0.6	-0.4	-0.2	0.0	+0.2	+0.4	+0.6	+0.8	+1.0
1	6.5	0.285	0.377	0.460	0.548	0.607	0.629	0.639	0.693	0.938	1.602	3.222
2	4.0	1.134	0.894	0.666	0.590	0.549	0.435	0.370	0.432	0.712	1.379	2.839
3	2.5	2.019	1.387	0.874	0.498	0.266	0.179	0.236	0.440	0.800	1.321	1.980
4	1.4	1.242	1.073	0.932	0.818	0.743	0.717	0.743	0.808	0.892	0.976	1.056
5	0.8	0.752	0.762	0.783	0.811	0.847	0.887	0.932	0.980	1.030	1.081	1.135
6	0.5	0.743	0.769	0.801	0.834	0.869	0.905	0.942	0.979	1.016	1.053	1.089
7	0.4	0.758	0.785	0.815	0.846	0.878	0.909	0.940	0.971	1.002	1.033	1.063
8	0.3	0.788	0.811	0.836	0.861	0.885	0.910	0.934	0.958	0.982	1.006	1.029
9	0.2	0.830	0.847	0.863	0.879	0.895	0.910	0.926	0.941	0.955	0.970	0.984
10	0.1	0.880	0.887	0.893	0.899	0.904	0.910	0.915	0.920	0.926	0.931	0.935
11	0.075											
12	0.046											

<div><div></div><div></div><div></div><div></div><div></div><div></div></div> <div>j</div> <div>E_L</div> <div>σ_f</div> <div>σ_c</div> <div>σ_{el}</div> <div>$\sigma_{n,n'}$</div>						$\sigma_{n,n'}^{(l)}$											
						l	1	2	3	4	5	6	7	8	9	10	11
						Q_l	6.065	6.14	6.91	7.12							
1	6.5		0.231	0.84	0.258		0.024	0.202	0.025	0.007							
2	4.0		0.064	1.37													
3	2.5		0.004	2.21													
4	1.4			1.95													
5	0.8			3.81													
6	0.5			3.32													
7	0.4			10.0													
8	0.3			4.12													
9	0.2			3.7													
10	0.1			3.7													
11	0.075			3.7													
12	0.046			3.7													
13	0.0215			3.7													
14	0.0150			3.7													
15	0.0100			3.7													
16	0.00465			3.7													
17	0.00215			3.7													
18	0.00100			3.7		OXYGEN											
19	0.000465			3.7													
20	0.000200			3.7													

OXYGEN

ANGULAR DISTRIBUTION

j	E_L	$\cos \theta =$										
		-1.0	-0.8	-0.6	-0.4	-0.2	0.0	+0.2	+0.4	+0.6	+0.8	+1.0
1	6.5	0.482	0.025	0.217	0.271	0.184	0.153	0.255	0.424	0.690	1.713	5.586
2	4.0	0.273	0.332	0.557	0.573	0.439	0.332	0.372	0.579	0.965	1.767	3.811
3	2.5	0.763	0.846	0.810	0.713	0.607	0.540	0.554	0.686	0.968	1.427	2.086
4	1.4	0.993	0.881	0.819	0.773	0.737	0.722	0.736	0.788	0.899	1.114	1.538
5	0.8	1.634	1.029	0.740	0.579	0.488	0.469	0.536	0.692	0.927	1.237	1.669
6	0.5	0.283	0.443	0.562	0.671	0.784	0.904	1.025	1.141	1.255	1.381	1.551
7	0.4	0.855	0.726	0.620	0.559	0.554	0.609	0.724	0.899	1.138	1.452	1.864
8	0.3	1.545	1.383	1.278	1.175	1.052	0.913	0.774	0.650	0.544	0.435	0.251
9	0.2	1.141	1.083	1.082	1.075	1.039	0.975	0.898	0.822	0.749	0.656	0.480
10	0.1	0.964	0.953	0.952	0.949	0.940	0.925	0.907	0.888	0.869	0.846	0.807
11	0.075											
12	0.046											

j	E_L	σ_f	σ_c	σ_{e1}	$\sigma_{n,n'}$	$\sigma_{n,n'}^{(l)}$										
						l	1	2	3	4	5	6	7	8	9	10
						Q_t	0.842	1.013	2.21	2.73	2.98	3.0				
1	6.5		0.121	0.84	0.856		(a)									
2	4.0		0.029	1.32	0.843		(a)									
3	2.5		0.005	2.04	0.591		0.074	0.163	0.185	0.077	0.028	0.064				
4	1.4		0	2.63	0.286		0.090	0.193	0.003							
5	0.8		0	3.23	0.139		0.051	0.088								
6	0.5		0	3.81												
7	0.4		0.001	4.33												
8	0.3		0.002	3.66												
9	0.2		0.002	4.28												
10	0.1		0.003	5.67												
11	0.075		0.002	10.3												
12	0.046		0.003	2.11												
13	0.0215		0.009	7.30												
14	0.0150		0.002	0.89												
15	0.0100		0.004	1.17												
16	0.00465		0.006	1.49												
17	0.00215		0.010	1.40												
18	0.00100		0.013	1.39												
19	0.000465		0.014	1.39												
20	0.000200		0.014	1.39												

ALUMINUM

ANGULAR DISTRIBUTION

j	E_L	$\cos \theta =$										
		-1.0	-0.8	-0.6	-0.4	-0.2	0.0	+0.2	+0.4	+0.6	+0.8	+1.0
1	6.5	0.367	0.199	0.132	0.126	0.156	0.195	0.231	0.248	0.347	1.335	6.664
2	4.0	0.326	0.236	0.191	0.205	0.231	0.271	0.333	0.441	0.809	1.935	5.022
3	2.5	0.475	0.391	0.361	0.361	0.388	0.454	0.582	0.805	1.195	1.888	3.100
4	1.4	0.425	0.386	0.374	0.400	0.467	0.578	0.743	0.982	1.326	1.816	2.503
5	0.8	0.459	0.438	0.451	0.498	0.581	0.702	0.863	1.069	1.323	1.628	1.988
6	0.5	0.502	0.525	0.563	0.619	0.695	0.792	0.912	1.058	1.232	1.434	1.668
7	0.4	0.572	0.616	0.667	0.725	0.791	0.866	0.949	1.041	1.143	1.254	1.376
8	0.3	0.685	0.720	0.759	0.801	0.846	0.894	0.945	0.999	1.056	1.115	1.180
9	0.2	0.718	0.752	0.787	0.823	0.862	0.901	0.943	0.985	1.030	1.076	1.123
10	0.1	0.802	0.823	0.843	0.864	0.886	0.907	0.929	0.952	0.975	0.998	1.021
11	0.075	0.856	0.866	0.877	0.888	0.898	0.909	0.920	0.930	0.941	0.952	0.963
12	0.046	0.876	0.883	0.889	0.896	0.902	0.909	0.916	0.922	0.929	0.936	0.942

^(a) Evaporation model with $\theta = 0.294 \sqrt{E}$ (E in MeV).

						$\sigma_{n,n'}j \rightarrow j+k$												
j	E_L	σ_f	σ_c	σ_{el}	$\sigma_{n,n'}$	$k = 0$	1	2	3	4	5	6	7	8	9	10	11	
1	6.5		0.061	1.63	1.24	0.051	0.194	0.303	0.345	0.200	0.083	0.022	0.019	0.014	0.009	0.001	0.001	
2	4.0		0.014	2.11	1.26	0.102	0.339	0.406	0.224	0.104	0.028	0.024	0.018	0.012	0.002	0.001		
3	2.5		0.002	2.62	1.04	0.180	0.571	0.158	0.063	0.020	0.017	0.014	0.010	0.002	0.001			
4	1.4		0.001	3.19	0.589	0.025	0.381	0.137	0.036	0.009	0	0.001						
5	0.8		0.002	2.86	0.127	0	0	0	0.041	0.047	0.030	0.004	0.003	0.002				
6	0.5		0.004	2.95														
7	0.4		0.004	2.36														
8	0.3		0.005	2.41														
9	0.2		0.005	3.51														
10	0.1		0.007	3.17														
11	0.075		0.009	4.65														
12	0.046		0.011	10.6														
13	0.0215		0.016	22.2														
14	0.0150		0.023	83.4														
15	0.0100		0.030	40.6														
16	0.00465		0.114	18.4														
17	0.00215		1.92	12.8														
18	0.00100		0.030	4.72		TITANIUM												
19	0.000465		0.036	4.28														
20	0.000200		0.050	4.23														

ANGULAR DISTRIBUTION

j	E_L	$\cos \theta =$										
		-1.0	-0.8	-0.6	-0.4	-0.2	0.0	+0.2	+0.4	+0.6	+0.8	+1.0
1	6.5	0.121	0.039	0.042	0.077	0.098	0.071	0.037	0.054	0.218	1.397	7.846
2	4.0	0.087	0.098	0.138	0.169	0.160	0.120	0.082	0.134	0.558	2.099	6.355
3	2.5	0.442	0.381	0.343	0.290	0.230	0.189	0.208	0.378	0.900	2.126	4.513
4	1.4	0.819	0.580	0.434	0.357	0.338	0.378	0.496	0.729	1.142	1.825	2.902
5	0.8	0.995	0.671	0.464	0.364	0.363	0.454	0.634	0.900	1.251	1.689	2.215
6	0.5	0.729	0.570	0.481	0.460	0.505	0.611	0.777	1.000	1.277	1.606	1.983
7	0.4	0.647	0.574	0.546	0.560	0.616	0.712	0.848	1.022	1.233	1.480	1.762
8	0.3	0.690	0.647	0.634	0.650	0.696	0.771	0.873	1.002	1.157	1.338	1.543
9	0.2	0.730	0.714	0.715	0.735	0.771	0.825	0.896	0.983	1.086	1.205	1.340
10	0.1	0.782	0.787	0.798	0.817	0.842	0.875	0.914	0.960	1.013	1.073	1.139
11	0.075	0.826	0.836	0.848	0.862	0.879	0.898	0.919	0.942	0.968	0.996	1.026
12	0.046	0.847	0.856	0.867	0.878	0.891	0.904	0.919	0.934	0.950	0.968	0.986

j	I_{L_L}	σ_f	σ_c	σ_{c1}	$\sigma_{n,n'}$	$\sigma_{n,n',j \rightarrow j+k}$											
						$k = 0$	1	2	3	4	5	6	7	8	9	10	11
1	6.5		0.440	1.95	1.04	0.028	0.132	0.239	0.302	0.187	0.081	0.022	0.019	0.014	0.009	0.002	0.001
2	4.0		0.365	2.08	1.16	0.084	0.219	0.357	0.263	0.124	0.036	0.031	0.024	0.016	0.003	0.002	
3	2.5		0.174	2.07	1.08	0.094	0.293	0.373	0.137	0.052	0.050	0.041	0.030	0.005	0.004	0.002	
4	1.4		0.036	2.68	0.50	0	0.156	0.178	0.048	0.041	0.038	0.031	0.005	0.003	0.002		
5	0.8		0.007	3.11													
6	0.5		0.008	3.17													
7	0.4		0.009	4.00													
8	0.3		0.009	4.17													
9	0.2		0.008	5.55													
10	0.1		0.007	6.12													
11	0.075		0.007	6.85													
12	0.046		0.007	9.79													
13	0.0215		0.010	10.5													
14	0.0150		0.023	47.1													
15	0.0100		0.027	36.5													
16	0.00465		0.015	13.5													
17	0.00215		0.024	21.0													
18	0.00100		0.024	14.9		NICKEL											
19	0.000465		0.027	15.4													
20	0.000200		0.042	16.8													

ANGULAR DISTRIBUTION

 $\cos \theta =$

j	I_{L_L}	-1.0	-0.8	-0.6	-0.4	-0.2	0.0	+0.2	+0.4	+0.6	+0.8	+1.0
1	6.5	0.072	0.037	0.029	0.062	0.080	0.068	0.031	0.023	0.176	1.231	8.191
2	4.0	0.076	0.061	0.084	0.129	0.152	0.106	0.047	0.103	0.529	1.829	6.884
3	2.5	0.350	0.271	0.318	0.359	0.341	0.241	0.132	0.184	0.650	1.952	5.202
4	1.4	0.926	0.623	0.522	0.465	0.426	0.361	0.329	0.469	0.865	1.676	3.338
5	0.8	1.304	0.928	0.705	0.575	0.504	0.480	0.515	0.639	0.899	1.358	2.093
6	0.5	1.027	0.806	0.693	0.648	0.645	0.672	0.729	0.831	1.001	1.269	1.679
7	0.4	0.883	0.790	0.766	0.774	0.790	0.807	0.829	0.869	0.956	1.123	1.413
8	0.3	1.009	0.817	0.764	0.780	0.818	0.853	0.876	0.895	0.934	1.028	1.226
9	0.2	0.899	0.820	0.803	0.818	0.845	0.873	0.897	0.923	0.959	1.024	1.139
10	0.1	0.790	0.844	0.863	0.869	0.871	0.883	0.899	0.929	0.992	1.022	1.038
11	0.075	0.848	0.866	0.871	0.877	0.875	0.886	0.888	0.903	0.986	0.997	1.003
12	0.046	0.869	0.875	0.877	0.882	0.878	0.889	0.886	0.896	0.978	0.984	0.986

						$\sigma_{n,n'}j \rightarrow j+k$											
j	E_L	σ_f	σ_c	σ_{el}	$\sigma_{n,n'}$	$k=0$	1	2	3	4	5	6	7	8	9	10	11
1	6.5		0.095	1.86	1.50	0.009	0.088	0.258	0.456	0.354	0.173	0.050	0.044	0.035	0.023	0.004	0.003
2	4.0		0.077	1.95	1.60	0.039	0.182	0.453	0.438	0.242	0.074	0.066	0.054	0.037	0.006	0.005	0.002
3	2.5		0.046	1.74	1.47	0.087	0.336	0.435	0.282	0.093	0.806	0.074	0.052	0.009	0.007	0.004	0.001
4	1.4		0.020	2.25	0.625	0.096	0.181	0.146	0.054	0.052	0.047	0.035	0.006	0.005	0.003		
5	0.8		0.013	3.12	0.264	0	0.043	0.034	0.052	0.060	0.055	0.008	0.007	0.004	0.001		
6	0.5		0.015	4.07	0.017	0	0	0	0	0.004	0.005	0.005	0.003				
7	0.4		0.015	4.65													
8	0.3		0.016	4.95													
9	0.2		0.016	5.19													
10	0.1		0.018	5.01													
11	0.075		0.023	6.03													
12	0.046		0.027	8.32													
13	0.0215		0.043	7.96													
14	0.0150		0.065	9.71													
15	0.0100		0.081	10.1													
16	0.00465		0.156	13.0													
17	0.00215		0.216	14.9													
18	0.00100		0.255	25.0		COPPER											
19	0.000465		2.02	10.2													
20	0.000200		0.233	6.84													

ANGULAR DISTRIBUTION

[illegible]

j	E_L	σ_f	σ_c	σ_{cl}	$\sigma_{n,n'}$	$\sigma_{n,n'} j \rightarrow j+k$											
						$k = 0$	1	2	3	4	5	6	7	8	9	10	11
1	6.5		0	2.70	2.01	0.002	0.038	0.201	0.549	0.565	0.323	0.101	0.091	0.075	0.052	0.008	0.007
2	4.0		0.008	3.12	2.62	0.014	0.134	0.566	0.774	0.519	0.174	0.163	0.139	0.100	0.016	0.014	0.007
3	2.5		0.028	3.62	2.65	0.050	0.378	0.744	0.615	0.227	0.223	0.201	0.151	0.026	0.022	0.011	0.002
4	1.4		0.041	3.98	2.52	0.182	0.579	0.632	0.267	0.280	0.270	0.217	0.039	0.034	0.017	0.003	
5	0.8		0.055	4.86	1.84	0.274	0.442	0.225	0.261	0.278	0.247	0.047	0.024	0.022	0.003	0.002	
6	0.5		0.075	5.80	1.18	0.238	0.150	0.197	0.234	0.234	0.048	0.044	0.024	0.004	0.002		
7	0.4		0.089	6.54	0.704	0.091	0.132	0.175	0.195	0.043	0.040	0.022	0.004	0.002			
8	0.3		0.096	7.03	0.465	0.089	0.128	0.154	0.036	0.035	0.020	0.002	0.001				
9	0.2		0.112	7.49	0.261	0.079	0.108	0.027	0.027	0.016	0.003	0.001					
10	0.1		0.137	8.77	0.087	0.048	0.014	0.015	0.009	0.001							
11	0.075		0.175	10.1													
12	0.046		0.229	10.7													
13	0.0215		0.292	11.3													
14	0.0150		0.390	11.4													
15	0.0100		0.467	11.5													
16	0.00465		3.25	9.5													
17	0.00215		8.28	6.75													
18	0.00100		10.1	5.57													
19	0.000465		12.3	5.0													
20	0.000200		35.3	5.0													

TUNGSTEN

ANGULAR DISTRIBUTION

j	E_L	$\cos \theta =$										
		-1.0	-0.8	-0.6	-0.4	-0.2	0.0	+0.2	+0.4	+0.6	+0.8	+1.0
1	6.5	0.031	0.025	0.044	0.040	0.056	0.118	0.118	0.040	0.290	0.251	8.987
2	4.0	0.066	0.045	0.054	0.084	0.113	0.094	0.100	0.072	0.253	0.800	8.319
3	2.5	0.172	0.172	0.179	0.200	0.209	0.169	0.180	0.214	0.450	1.187	6.868
4	1.4	0.305	0.337	0.348	0.344	0.322	0.281	0.295	0.396	0.732	1.535	5.105
5	0.8	0.514	0.520	0.518	0.489	0.447	0.419	0.479	0.613	1.004	1.765	3.232
6	0.5	0.662	0.635	0.594	0.553	0.528	0.538	0.611	0.783	1.097	1.609	2.390
7	0.4	0.741	0.695	0.648	0.612	0.602	0.631	0.715	0.870	1.110	1.455	1.921
8	0.3	0.745	0.720	0.695	0.678	0.679	0.709	0.778	0.898	1.080	1.337	1.681
9	0.2	0.749	0.745	0.741	0.743	0.755	0.786	0.840	0.926	1.050	1.220	1.445
10	0.1	0.753	0.773	0.794	0.816	0.841	0.872	0.910	0.957	1.016	1.089	1.179
11	0.075	0.784	0.809	0.834	0.859	0.884	0.909	0.934	0.959	0.984	1.009	1.035
12	0.046	0.841	0.855	0.869	0.882	0.896	0.909	0.923	0.936	0.950	0.963	0.976

j	E_L	σ_f	σ_c	σ_{cl}	$\sigma_{n,n'}$	$\sigma_{n,n',j \rightarrow i+k}$											
						$k = 0$	1	2	3	4	5	6	7	8	9	10	11
1	6.5	0.384	0.006	3.0	1.13	0	0.012	0.082	0.278	0.329	0.203	0.066	0.060	0.051	0.036	0.006	0.005
2	4.0	0.172	0.013	4.21	2.72	0.006	0.089	0.492	0.796	0.587	0.206	0.197	0.172	0.127	0.021	0.018	0.008
3	2.5	0.141	0.031	4.17	3.06	0.032	0.330	0.799	0.743	0.291	0.294	0.272	0.211	0.037	0.032	0.016	0.002
4	1.4	0.102	0.079	3.72	2.81	0.134	0.552	0.699	0.316	0.344	0.344	0.288	0.053	0.046	0.024	0.004	0.002
5	0.8	0.008	0.196	4.39	2.17	0.245	0.475	0.265	0.322	0.360	0.336	0.066	0.060	0.032	0.005	0.003	0.002
6	0.5		0.213	5.54	1.76	0.292	0.206	0.287	0.367	0.390	0.084	0.078	0.043	0.007	0.004	0.002	
7	0.4		0.182	6.35	1.68	0.183	0.287	0.412	0.492	0.113	0.109	0.062	0.010	0.006	0.004		
8	0.3		0.186	7.19	1.59	0.265	0.416	0.545	0.133	0.132	0.077	0.013	0.007	0.005	0.001		
9	0.2		0.207	8.56	1.42	0.388	0.587	0.157	0.161	0.098	0.016	0.009	0.006	0.001			
10	0.1		0.272	10.1	1.08	0.548	0.173	0.192	0.125	0.022	0.012	0.008	0.002				
11	0.075		0.365	11.4	0.648	0.188	0.231	0.165	0.031	0.018	0.012	0.003					
12	0.046		0.464	12.2	0.233	0.111	0.086	0.017	0.010	0.007	0.002						
13	0.0215		0.714	12.9													
14	0.0150		1.08	13.2													
15	0.0100		1.20	13.2													
16	0.00465		1.64	12.9													
17	0.00215		2.28	12.3													
18	0.00100		3.32	10.6		THORIUM											
19	0.000465		4.86	10.2													
20	0.000200		6.65	10.5													

ANGULAR DISTRIBUTION

j	E_L	$\cos \theta =$										
		-1.0	-0.8	-0.6	-0.4	-0.2	0.0	+0.2	+0.4	+0.6	+0.8	+1.0
1	6.5	0.043	0.040	0.008	0.054	0.014	0.032	0.062	0.093	0.017	0.153	9.484
2	4.0	0.108	0.019	0.053	0.033	0.006	0.058	0.125	0.084	0.026	0.327	9.161
3	2.5	0.115	0.046	0.037	0.010	0.032	0.128	0.225	0.166	0.023	0.734	7.484
4	1.4	0.345	0.117	0.098	0.141	0.236	0.323	0.324	0.217	0.190	1.228	6.781
5	0.8	0.388	0.255	0.329	0.428	0.488	0.488	0.429	0.385	0.593	0.618	4.599
6	0.5	0.310	0.362	0.436	0.499	0.543	0.578	0.641	0.797	1.140	1.792	2.902
7	0.4	0.441	0.437	0.455	0.475	0.500	0.545	0.652	0.869	1.245	1.814	2.566
8	0.3	0.512	0.490	0.485	0.490	0.508	0.559	0.677	0.902	1.264	1.764	2.349
9	0.2	0.537	0.538	0.555	0.582	0.623	0.689	0.799	0.973	1.226	1.553	1.925
10	0.1	0.605	0.614	0.645	0.690	0.745	0.814	0.902	1.013	1.150	1.315	1.507
11	0.075	0.789	0.737	0.729	0.746	0.778	0.820	0.878	0.956	1.061	1.188	1.318
12	0.046	0.883	0.800	0.771	0.774	0.792	0.821	0.862	0.926	1.016	1.127	1.228

$$0 \leq E \leq 10; \quad \bar{\nu} = 1.79 + 0.105 E$$

j	E_L	σ_f	σ_c	σ_{c1}	$\sigma_{n,n'}$	$\sigma_{n,n' \rightarrow j+k}$											
						$k = 0$	1	2	3	4	5	6	7	8	9	10	11
1	6.5	1.76	0.002	3.36	0.671	0	0.006	0.047	0.163	0.196	0.122	0.040	0.036	0.031	0.022	0.004	0.003
2	4.0	1.23	0.003	4.25	1.730	0.004	0.054	0.306	0.504	0.376	0.133	0.127	0.111	0.082	0.014	0.012	0.006
3	2.5	1.26	0.015	4.70	1.80	0.017	0.187	0.466	0.439	0.173	0.175	0.163	0.127	0.022	0.019	0.010	
4	1.4	1.32	0.079	3.97	1.70	0.077	0.332	0.420	0.192	0.210	0.212	0.177	0.032	0.029	0.015	0.002	
5	0.8	1.26	0.114	3.86	1.42	0.331	0.346	0.134	0.164	0.184	0.173	0.034	0.031	0.017	0.003	0.002	
6	0.5	1.18	0.170	4.74	1.18	0.545	0.262	0.133	0.089	0.096	0.021	0.019	0.011	0.002			
7	0.4	1.19	0.195	5.38	1.03	0.223	0.479	0.222	0.066	0.015	0.015	0.008					
8	0.3	1.24	0.221	6.24	0.89	0.234	0.464	0.165	0.010	0.010	0.006						
9	0.2	1.34	0.280	7.32	0.69	0.226	0.408	0.030	0.024	0.002							
10	0.1	1.58	0.414	8.52	0.49	0.222	0.126	0.112	0.026	0.004							
11	0.075	1.84	0.557	9.50	0.271	0.023	0.063	0.185	0								
12	0.046	2.04	0.677	9.92	0.108	0.025	0.032	0.024	0.015	0.010	0.002						
13	0.0215	2.37	0.897	10.6	0.017	0.011	0.004	0.002									
14	0.0150	2.85	1.15	10.7													
15	0.0100	3.04	1.25	10.6													
16	0.00465	3.60	1.54	12.9													
17	0.00215	5.45	2.47	13.9													
18	0.00100	6.82	3.20	16.5													
19	0.000465	10.9	5.17	11.7													
20	0.000200	16.3	8.02	11.8													

U-235

ANGULAR DISTRIBUTION^(a)

j	E_L	$\cos \theta =$										
		-1.0	-0.8	-0.6	-0.4	-0.2	0.0	+0.2	+0.4	+0.6	+0.8	+1.0
1	6.5											
2	4.0											
3	2.5											
4	1.4											
5	0.8											
6	0.5											
7	0.4											
8	0.3											
9	0.2											
10	0.1											
11	0.075											
12	0.046											

$0 \leq E < 3$; $\bar{\nu} = 2.430 + 0.098E$
 $3 \leq E < 4.94$; $\bar{\nu} = 1.974 + 0.250E$
 $4.94 \leq E \leq 10$; $\bar{\nu} = 2.593 + 0.1247E$

^(a) see U-238.

j	E_L	σ_f	σ_e	σ_{el}	$\sigma_{n,n'}$	$\sigma_{n,n'} \rightarrow j+k$											
						$k = 0$	1	2	3	4	5	6	7	8	9	10	11
1	6.5	0.997	0.006	3.31	0.92	0	0.009	0.065	0.224	0.270	0.168	0.055	0.050	0.042	0.030	0.005	0.004
2	4.0	0.625	0.011	4.39	2.47	0.005	0.077	0.436	0.720	0.538	0.190	0.182	0.159	0.117	0.020	0.017	0.008
3	2.5	0.560	0.024	4.65	2.58	0.025	0.268	0.667	0.630	0.247	0.251	0.234	0.182	0.032	0.027	0.014	0.002
4	1.4	0.477	0.061	4.19	2.59	0.225	0.608	0.713	0.297	0.235	0.223	0.187	0.034	0.030	0.016	0.002	
5	0.8	0.040	0.134	4.52	2.47	1.190	0.479	0.111	0.217	0.239	0.167	0.024	0.022	0.014	0.003	0.002	
6	0.5	0.001	0.134	6.16	1.89	1.443	0.363	0.055	0.014	0		0.008	0.005	0.002			
7	0.4		0.126	7.03	1.69	0.791	0.798	0.071	0.030								
8	0.3		0.127	7.86	1.51	0.763	0.706	0.036	0.004	0.004	0.002						
9	0.2		0.141	8.82	1.31	0.711	0.585	0.009	0.005								
10	0.1		0.201	10.2	0.91	0.558	0.211	0.140	0.004								
11	0.075		0.285	11.5	0.44	0.200	0.240										
12	0.046		0.357	12.3	0.04	0.033	0.009	0.002	0.001								
13	0.0215		0.496	13.2													
14	0.0150		0.609	14.1													
15	0.0100		0.642	14.2													
16	0.00465		0.709	14.3													
17	0.00215		0.872	15.7													
18	0.00100		2.24	12.4		U-238											
19	0.000465		4.27	22.8													
20	0.000200		5.80	19.1													

ANGULAR DISTRIBUTION

j	E_L	$\cos \theta =$										
		-1.0	-0.8	-0.6	-0.4	-0.2	0.0	+0.2	+0.4	+0.6	+0.8	+1.0
1	6.5	0.078	0.033	0.028	0.049	0.031	0.033	0.084	0.075	0.033	0.178	9.378
2	4.0	0.099	0.024	0.064	0.025	0.018	0.076	0.0146	0.106	0.030	0.448	8.964
3	2.5	0.142	0.047	0.032	0.019	0.046	0.161	0.251	0.191	0.056	0.888	8.167
4	1.4	0.128	0.134	0.202	0.307	0.377	0.381	0.345	0.295	0.469	1.551	5.811
5	0.8	0.127	0.224	0.362	0.494	0.549	0.476	0.418	0.498	0.832	1.842	4.173
6	0.5	0.340	0.371	0.408	0.455	0.518	0.570	0.648	0.824	1.155	1.775	2.936
7	0.4	0.438	0.452	0.474	0.511	0.578	0.658	0.766	0.934	1.203	1.626	2.360
8	0.3	0.486	0.511	0.544	0.591	0.658	0.734	0.842	0.958	1.194	1.491	1.964
9	0.2	0.598	0.609	0.638	0.683	0.741	0.805	0.900	1.008	1.152	1.326	1.540
10	0.1	0.761	0.718	0.716	0.739	0.772	0.817	0.886	0.963	1.075	1.208	1.345
11	0.075	0.900	0.810	0.778	0.781	0.793	0.821	0.864	0.917	1.005	1.117	1.214
12	0.046	0.910	0.898	0.894	0.0894	0.085	0.898	0.903	0.909	0.920	0.934	0.945

$$0 \leq E \leq 10; \quad \bar{\nu} = 2.33 + 0.15E$$

j	E_L	σ_f	σ_e	σ_{i1}	$\sigma_{n,n'}$	$\sigma_{n,n'} \rightarrow j+k$											
						$k = 0$	1	2	3	4	5	6	7	8	9	10	11
1	6.5	2.26	0.003	3.67	0.34	0	0.003	0.024	0.083	0.099	0.062	0.020	0.018	0.015	0.011	0.002	0.001
2	4.0	1.83	0.006	4.44	1.47	0.003	0.046	0.259	0.428	0.319	0.113	0.108	0.095	0.070	0.012	0.010	0.005
3	2.5	1.93	0.011	4.52	1.49	0.014	0.155	0.385	0.363	0.143	0.145	0.135	0.105	0.018	0.016	0.008	
4	1.4	1.97	0.020	3.91	1.09	0.050	0.211	0.271	0.124	0.135	0.136	0.114	0.021	0.018	0.010		
5	0.8	1.83	0.046	4.33	0.860	0.094	0.186	0.105	0.129	0.144	0.136	0.027	0.024	0.013	0.002		
6	0.5	1.67	0.099	5.58	0.793	0.146	0.136	0.111	0.159	0.159	0.032	0.030	0.017	0.003			
7	0.4	1.66	0.141	6.45	0.693	0.360	0.209	0.020	0.087	0.010	0.005	0.002					
8	0.3	1.66	0.156	7.00	0.593	0.356	0.182	0.015	0.014	0.016	0.009	0.001					
9	0.2	1.66	0.183	7.76	0.484	0.340	0.140	0.002	0.002								
10	0.1	1.70	0.240	8.83	0.380	0.304	0.050	0.022	0.004								
11	0.075	1.79	0.257	9.79	0.293	0.171	0.089	0.028	0.005								
12	0.046	1.89	0.323	10.4	0.253	0.179	0.070	0.001	0.002	0.001							
13	0.0215	2.18	0.705	11.1	0.231	0.158	0.059	0.014									
14	0.0150	2.45	0.99	11.7	0.203	0	0.107	0.096									
15	0.0100	2.43	1.08	12.1	0.145	0.075	0.066	0.004									
15	0.00465	2.40	1.17	12.4	0.020	0	0.014	0.005	0.001								
17	0.00215	3.24	1.74	13.0													
18	0.00100	3.78	2.23	13.6		Pu-239											
19	0.000465	7.21	4.61	14.2													
20	0.000200	12.0	8.32	14.8													

ANGULAR DISTRIBUTION^(a)

j	E_L	$\cos \theta =$										
		-1.0	-0.8	-0.6	-0.4	-0.2	0.0	+0.2	+0.4	+0.6	+0.8	+1.0
1	6.5											
2	4.0											
3	2.5											
4	1.4											
5	0.8											
6	0.5											
7	0.4											
8	0.3											
9	0.2											
10	0.1											
11	0.075											
12	0.046											

 $0 \leq E \leq 10; \quad \bar{\nu} = 2.89 + 0.123E$ ^(a) sec U-238.

j	E_L	σ_f	σ_c	σ_{el}	$\sigma_{n,n'}$	$\sigma_{n,n'}j \rightarrow j+k$												
						$k = 0$	1	2	3	4	5	6	7	8	9	10	11	
1	6.5	2.1	0.016	3.32	0.442	0	0.004	0.031	0.108	0.130	0.081	0.026	0.024	0.020	0.014	0.002	0.002	
2	4.0	1.55	0.023	4.41	1.72	0.003	0.053	0.302	0.501	0.375	0.133	0.127	0.112	0.082	0.014	0.012	0.006	
3	2.5	1.63	0.033	4.68	1.64	0.015	0.169	0.422	0.400	0.158	0.160	0.149	0.116	0.020	0.017	0.009	0.001	
4	1.4	1.62	0.088	4.31	1.50	0.126	0.312	0.344	0.157	0.172	0.173	0.146	0.027	0.024	0.012	0.002	0.001	
5	0.8	1.51	0.227	4.59	0.971	0.466	0.208	0.054	0.066	0.074	0.070	0.014	0.012	0.007				
6	0.5	0.759	0.256	5.98	1.13	0.764	0.257	0.053	0.029	0.020	0.004	0.004	0.002					
7	0.4	0.286	0.221	7.25	1.34	0.625	0.639	0.051	0.022									
8	0.3	0.169	0.223	8.06	1.28	0.657	0.586	0.027	0.003	0.002	0.001							
9	0.2	0.113	0.260	9.18	1.12	0.624	0.482	0.007	0.003									
10	0.1	0.072	0.349	10.6	0.764	0.501	0.169	0.092	0.002									
11	0.075	0.065	0.497	11.3	0.232	0	0.136	0.096										
12	0.046	0.065	0.633	12.0	0.027	0	0.020	0.005	0.002									
13	0.0215	0.041	0.851	13.3														
14	0.0150	0.009	1.03	13.8														
15	0.0100	0.005	1.11	14.0														
16	0.00465	0.001	1.24	14.4														
17	0.00215		1.42	15.6														
18	0.00100		2.90	15.6		Pu-240												
19	0.000465		3.24	13.7														
20	0.000200		1.69	12.2														

ANGULAR DISTRIBUTION^(a)

j	E_L	$\cos \theta =$										
		-1.0	-0.8	-0.6	-0.4	-0.2	0.0	+0.2	+0.4	+0.6	+0.8	+1.0
1	6.5											
2	4.0											
3	2.5											
4	1.4											
5	0.8											
6	0.5											
7	0.4											
8	0.3											
9	0.2											
10	0.1											
11	0.075											
12	0.046											

$$\begin{aligned}
 0 \leq E < 5; & \quad \bar{\nu} = 2.89 + 0.124E \\
 5 \leq E \leq 10; & \quad \bar{\nu} = 3.01 + 0.1E
 \end{aligned}$$

^(a) see U-238.

j	I_L	σ_f	σ_c	$\sigma_{\epsilon 1}$	$\sigma_{n \ n'}$	$\sigma_{n, n' j \rightarrow j+k}$												
						$k = 0$	1	2	3	4	5	6	7	8	9	10	11	
1	6.5	2.26	0.002	3.32	0.179	0	0.002	0.012	0.043	0.053	0.033	0.011	0.010	0.008	0.006	0.001		
2	4 0	1.63	0.004	4 42	1.54	0.003	0.047	0.269	0.448	0.336	0.119	0.114	0.100	0.074	0.012	0 011	0.005	
3	2 5	1.61	0.008	4.68	1.68	0.016	0.172	0.432	0.410	0.162	0.164	0.153	0.119	0.021	0.018	0.009	0.001	
4	1 4	1.80	0.018	4.28	1.42	0.063	0.271	0.352	0.161	0.177	0.178	0.150	0.027	0 024	0.012	0.002	0.001	
5	0.8	1.69	0.045	4.28	1.30	0.140	0.280	0.158	0 195	0.219	0.206	0.041	0.037	0.020	0.003	0 002	0.001	
6	0 5	1.62	0.100	5.42	1.11	0.178	0.128	0.180	0.233	0.251	0.054	0.051	0.028	0.004	0 002	0.002		
7	0.4	1.67	0.141	6 42	0.937	0.099	0.158	0.229	0 278	0.064	0.062	0.036	0.006	0.003	0.002			
8	0.3	1.78	0.168	7.04	0.817	0.132	0.212	0.282	0.069	0.069	0.040	0.007	0.004	0.002				
9	0 2	1 99	0.205	7.95	0.690	0.185	0.285	0.077	0.079	0.048	0.008	0.004	0.003	0.001				
10	0.1	2.45	0.264	8 56	0.544	0.274	0.087	0.097	0.064	0.011	0.006	0.004	0.001					
11	0.075	3.07	0.344	8.31	0.424	0.122	0.151	0 109	0.020	0.012	0.008	0.002						
12	0.046	3.55	0.415	8.56	0.279	0.132	0.104	0.021	0.012	0.008	0.002							
13	0.0215	4 49	0.559	9.16														
14	0.0150	4.89	0.756	9.21														
15	0.0100	5.12	0.865	9.07														
16	0.00465	5 81	1.26	8.97														
17	0.00215	6.61	1.70	8.79														
18	0.00100	9.05	2.39	7.14		Pu-241												
19	0 000465	13.6	4.69	10.7														
20	0 000200	22.8	12.6	9.77														

Pu-241

ANGULAR DISTRIBUTION^(a)

		$\cos \theta =$											
j	I_L	-1.0	-0.8	-0.6	-0.4	-0.2	0.0	+0.2	+0.4	+0.6	+0.8	+1.0	
1	6.5												
2	4 0												
3	2.5												
4	1 4												
5	0 8												
6	0.5												
7	0.4												
8	0.3												
9	0 2
0.1												
11	0 075												
12	0.046												

 $0 \leq E \leq 10; \quad \bar{\nu} = 2.94 + 0.124E$ ^(a) see U-238.

Discussion

Mr. Alter (AI): You stated that at least two group structures were used in the calculations and that no appreciable difference was apparent for the critical calculation. Was any investigation made of the spectral distribution for the varying group structures?

Mr. Gandini (CNEN): The leakage spectrum is predicted well. So as far as these two quantities are concerned, the group structure is good enough; I would not make it better because there is good agreement. I do not know about the fluxes.

Mr. Loewenstein (ANL): Did you say that for all elements above $A = 28$ you assumed isotropy in the laboratory system?

Mr. Gandini: Yes, for the heavier nuclides. For A bigger than 27, inelastic scattering is described by transition methods assuming that the angular distribu-

tion of the scattered neutron is anisotropic in the laboratory system.

Mr. Lantz (NASA-Lewis): What kind of running times did you have, and how many iterations did you do on your source team?

Mr. Gandini: As far as time is concerned, the code allows the user random access to probability tables. The user is allowed about 300 and 250 collisions per second for spherical and cylindrical geometry, respectively. The result is obtained in less than a minute.

Mr. Nakache (UNC): What was the reason to have a Monte Carlo code for spherical geometry using group cross sections?

Mr. Gandini: It is a reasonable approach if you want to assess the validity of cross-section sets. You can do it without making any approximation in the theory and the calculation of the system.

Monte Carlo Calculations of Fast Multicomponent Critical Systems*

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ABSTRACT

Criticality studies have been made of three-dimensional cuboidal systems of uranium metal cylinders enriched to 93.2 w/o ^{235}U . The systems were composed of nearly identical, individually subcritical components spaced in air. The dependence of the system critical mass upon the mass and shape of the components was examined by utilizing units ranging in mass from 10.5 to 26.2 kg of uranium and with height-to-diameter ratios from 0.47 to 1.17.

The calculation of such assemblies by usual analytic techniques is not practicable, but the systems are ideally suited to Monte Carlo methods. The multiplication factors of a number of the experiments have been computed by several codes and the results compare favorably. The one most extensively employed, however, has been the British GEM Monte Carlo code. A brief description of the geometry input and those features of the code applicable to the experimental systems investigated will be given in the paper as well as the results of typical experiments and comparison with computation. The computed values of k_{eff} agree with experiment to within $1\frac{1}{2}\%$.

1. Introduction

A continuing program in support of nuclear criticality safety in the handling of accumulations of individually subcritical units of fissile materials is being conducted at the Critical Experiments Facility of the Oak Ridge National Laboratory. A significant portion of this program deals with the criticality of uranium metal systems. The uranium metal enriched to 93.2 w/o in ^{235}U was utilized in the form of cylinders with masses ranging from 10.5 to 26.2 kg. Arrays were constructed to be as nearly ideal as was possible commensurate with assembly requirements. As a consequence, the experiments serve as a suitable set to evaluate models and methods used to compute the criticality of such systems.

In the present paper prime interest is in computing

the multiplication constant k and in establishing the reliability of Monte Carlo codes which can then be used in safety evaluations. A summary of a number of critical experiments is followed by a brief description of three codes and their applications to the data. A comparison of the calculative results is of particular interest because of the different cross-section input data.

2. Experiments

The units constituting these arrays consisted of one or more cylinders of uranium metal enriched to 93.2 w/o in ^{235}U , designated as U(93.2). The ^{234}U , ^{236}U , and ^{238}U content was 1.0, 0.2, and 5.6 w/o, respectively.

The units were supported, with their axes vertical, on stainless steel rods passing through 0.508-cm-diam holes in each cylinder parallel to the axis and located 8.547 cm apart on a diameter. The vertical separation of the cylinders was established by spacers of appropriate length cut from Inconel tubing closely fitting the support rods. The rods were mounted in sections of aluminum Unistrut attached to the two parts of a split-table apparatus. The horizontal position of the rods was adjustable. The assemblies were designed to provide a minimum of support structure and control was by means of table separation only. A typical assembly of eight 26.2-kg uranium units, illustrating the support features described, is shown in Fig. 1.

Presented in Table 1 is a summary of a number of critical assemblies⁽¹⁾ of various sized units. The surface separation was equal along the three directions of an array which centered the units at the corners of a rectangular parallelepiped and gave a cuboidal shape for the array as a whole. The description of the units represents the average dimensions of the units in an array. The slight variation of mass among the units and the accuracy in spacing within an array resulted in an estimated error for k of ± 0.003 .

3. Monte Carlo Codes

It is evident that the usual analytic techniques are difficult to apply to the systems described above. Monte Carlo techniques, on the other hand, are par-

* Research sponsored by the U. S. Atomic Energy Commission under contract with the Union Carbide Corporation

FIG. 1. A Critical Assembly of Eight U(93.2) Metal Cylinders Each Having a Mass of 26.2 kg. Criticality is achieved when the tables are together providing a uniform surface spacing of 3.543 cm among the units.

ticularly suited to the problem. The GEM code⁽²⁾ originating in the Health and Safety Branch of the UKAEA was initially written to explore and evaluate just such systems in the interest of nuclear criticality safety. The GEM program is written for the IBM 7090, accepts a number of standard geometries, allows for a fair degree of geometrical freedom, and computes directly a measure of criticality with a speed and accuracy comparable to other similar programs.

The GEM program input is a simple description of

the material, unit, cell, and reflector. The density, the number of nuclides, their names, and their proportions by atoms is given for each material present. The basic element of geometry used to specify a unit or cell is the region. A region is that portion of the system described by associated region boundaries. These boundaries are closed nested surfaces which may touch or coincide but must not intersect. The system is divided in two super-regions by a neutronicly important surface separating a 'core' from a 'reflector' which may not necessarily

TABLE 1. CRITICAL ARRAYS OF UNREFLECTED AND UNMODERATED U(93.2) METAL CYLINDERS IN THREE-DIMENSIONAL CUBOIDAL GEOMETRY

Unit Designation	Mass, ^(a) kg U	Diameter, cm	Height-to-Diameter Ratio	Number of Units along Each Direction			Surface Separation, ^(b) cm
				n_1	n_2	n_3	
A ¹	10.480	11.506	0.47	2	2	4	1.349
A ²	10.484	11.509	0.47	3	3	3	2.007
A ⁵	10.458	11.494	0.47	3	3	5	3.442
A ⁶	10.434	11.481	0.47	4	4	4	3.952
A ⁴	10.489	9.116	0.94	3	3	3	2.436
B ¹	15.692	11.494	0.70	2	2	2	0.902
B ²	15.683	11.490	0.70	3	3	3	4.204
C ²	20.960	11.506	0.94	2	2	2	2.248
C ³	20.877	11.484	0.94	3	3	3	6.363
D ¹	26.218	11.509	1.17	2	2	2	3.543
D ²	26.113	11.486	1.17	3	3	3	8.494

^(a) The uranium metal density is 18.76 g/cm³.

^(b) Errors on all surface separations are ± 0.013 cm.

correspond to the true core-reflector boundary. If the program tracking is done by stages, where a stage begins with the passage of a preselected number of neutrons into the core and terminates when all the descendants of those neutrons reenter the core. The numbers of neutrons leaving, crossing, and returning to this boundary yield an estimate of the core multiplication M and the reflection R of the reflector. The parameter MR is an output of the code, is unity for critical systems, and is the principal measure of criticality.

Each cell in the core has its own origin of coordinates and can be divided into any number of regions by a set of nested surfaces. Cells with different geometries or material content are given different type numbers. The complete core is specified by listing the type numbers to give an arrangement of the cells, and the position of each cell is defined by a set of three integral coordinates. When the outer boundary of the system is cuboidal, as is the case with the experiments described above, it is possible to specify reflection parameters on the six outer faces. The specified percentage of neutrons reaching any given face will be specularly reflected. This option permits one to make use of any symmetry present in the lattice and may also be used to avoid tracking neutrons in a reflector if its albedo is known.

As an example of input to GEM, consider the array shown in Fig. 1. Following a title card, the required input is the following:

```

-1 1 1 1      0 0 0 1 1 1      211
 1 Cylinder  1  5.742      + 6.7295 - 6.7295
 2 Cuboid   0  +7.526 + 7.526 + 8.501 - 7.526 - 7.526 - 8.501
Core 3 Cuboid 0  +15.052 + 15.052 + 17.002 - 15.052 - 15.052 - 17.002
Reflector 4 Cuboid 0  +15.052 + 15.052 + 17.002 - 15.052 - 15.052 - 17.002
 2 7EPLB 7EX5 18.76 93.2 6.8

```

The minus one specifies this as a reflected lattice with only one type of unit. The next three digits are the number of cells along the three coordinates. The next six digits represent the degree of reflection on the six outer boundaries. Instead of the entire array being utilized, use is made of the three planes of symmetry by having 100 percent reflection of neutrons on the planes. The remaining three digits give the number of regions in the cell, the number of regions in the reflector, and the number of materials present, respectively. The next line describes region 1 as a cylinder of material 1 with a radius of 5.742 cm and a height of ± 6.7295 relative to the cell coordinates. The second cell region has no material and its six coordinate boundaries are given. The third region given is that of the core-reflector boundary. The fourth region defines the reflector boundary, in this case of no material and zero thickness. The last line gives the two nuclides, the code names for ²³⁵U and ²³⁸U, followed by the density of the material and atomic proportions. An additional line of input, not given above, would specify the number of neutrons to be used per stage and the number of stages to be run.

The simple descriptions given are adequate for the experiments presented. The code has capabilities not touched on here that make it useful for reactor analyses where the geometries and materials can be much more complex.⁽³⁾

A second code used was the 05R, General Purpose Monte Carlo Neutron Code.⁽⁴⁾ Unlike GEM the geometry input to 05R is burdensome in that all the surfaces in the array must be described with respect to a single reference coordinate frame. Multiple media, however, may be treated, having arbitrary shapes which can be described by quadric surfaces. A second distinctive feature is that an eigenvalue is sought for a volume distribution, whereas in GEM a surface distribution provides the solution. No analysis is performed by 05R. Its output consists of one or more collision tapes. Separate routines must be used to extract the desired information. The multiplication constant is determined from the life history of neutrons run in batches. The fission-source distribution in each batch is obtained from the fission distribution generated by the previous batch; the initial batch being a selected spacial distribution. The multiplication constant is computed for each batch as the ratio of the number of neutrons born in the fissions produced by that batch to the number of source neutrons for the batch. The average taken over

all the batches run is the constant for the system. It is usual to exclude from the averaging the first few batches which are still affected by the initial source distribution.

A third code used originates with the Oak Ridge Computing Science Center.⁽⁵⁾ This Monte Carlo code computes the multiplication constant in a fashion similar to 05R but has the attractive feature of simple geometry input quite similar to that of GEM. The coordinate frame is used in a cell, and the cells are given three integral coordinates. When a neutron leaving a cell reaches a cell boundary, the sign of the coordinate is changed and the cell integral coordinates permuted. The present version of this code uses the 16-group cross-section set of Hansen and Roach,⁽⁶⁾ whereas 05R and GEM utilize available point cross-section data and have options for modified data.

4. Application to Experiment

The three codes have been used to compute the multiplication constants of the critical arrays given in Table 1. These results are presented in Table 2. The multiplication constant reported for the GEM code is a close estimate of k_{eff} obtained directly from the code output.⁽²⁾ The code computes the k_{eff} from the relation

$$k_{\text{eff}} = 1 + \frac{S(M - l_0)[M - (1/R)]}{M_{BF}[(1/R) - l_0]},$$

where

M is the core multiplication;

R is the reflection factor;

S is the number of neutrons entering the core;

l_0 is the number of neutrons leaving the core without having fissioned divided by S ; and

M_{BF} is the sum of fissions for all neutrons leaving the core.

This is a valid measure of reactivity which for practical applications can be used in place of the usual definition of k_{eff} .

It has been shown⁽⁷⁾ that when only the multiplication factors of unreflected homogeneous assemblies such as these are desired, a simpler Monte Carlo treatment of monoenergetic neutrons is sufficient. This one-velocity treatment was used to obtain the results presented in the table for the 05R code.

When one considers the error in spacing of the units coupled with slight variations in sizes of the pieces within an array, the agreement between experiment and calculation may be considered as good. It is also evident that there need not be a strong dependence on cross-section structure used when only the multiplication constant is sought.

TABLE 2. COMPUTED MULTIPLICATION CONSTANTS FOR EXPERIMENTAL ARRAYS OF U(93.2) METAL CYLINDERS

Unit ^(a)	Array			Computed ^(b) Multiplication Constant k		
	n_1	n_2	n_3	GEM	05R ^(c)	CSC ^(d)
A ¹	2	2	4	1.000		
A ²	3	3	3	1.002	1.004	0.987
A ⁵	3	3	5	0.992		1.003
A ⁶	4	4	4	0.994	0.971	1.008
A ⁴	3	3	3	0.992		
B ¹	2	2	2	1.034	0.975	
B ²	3	3	3	0.984		
C ²	2	2	2	1.005	0.997	
C ³	3	3	3	0.996	0.995	
D ¹	2	2	2	0.989		
D ²	3	3	3	0.995		

^(a) See Table 1 for description of average unit in array.

^(b) The standard deviation for 10,000 neutron histories does not exceed ± 0.015 for all three codes.

^(c) Monoenergetic group of neutrons used.

^(d) Sixteen-group Hansen-Roach cross-section set used.

Acknowledgments

Appreciation is expressed to G. E. Whitesides and O. W. Herman of the Computing Science Center at Oak Ridge for their operation of the GEM Code.

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Discussion

Mr. Alter (AI): In the comparison calculation, did you use the same set of cross-section data for the 05R and GEM calculations?

Mr. Thomas: No, GEM uses the Aldermaston data library; 05R is an Oak Ridge product.

Mr. Nakache (UNC): One of the features of the GEM code is to use a boundary to reach a computed multiplication factor. This boundary is quite arbitrary; as I understand you can choose it as you like. Do you have some experience or technique to choose that boundary?

Mr. Thomas: The British have explored this and in general one has to go by experience. What one wants for best statistical results is the core to have a multiplication of around 2. Then one doesn't run into distorted distributions of the k_{eff} from 2 to 3. There is no skewness; the distribution is almost normal.

Mr. Nakache: Did you try to change that boundary?

Mr. Thomas: No, my systems are such that it just happens to be that whatever natural boundary exists for the system is adequate. They give very good results. I might add that the place where one probably has to exercise some judgment is in a computation for a single core. There one can take almost any radius, even though it's a single piece of material like Godiva, and still arbitrarily specify that part of it is core and part of it is reflector. One can see some variation in the resulting k , but it is not too big. When you get down to very small sizes for the core then you distort the situation.

Mr. Pendlebury (UK-Aldermaston): When you

have another multiplication constant, say 1.01, or an array of metal cylinders, what does this really mean in terms of criticality from the point of view of separation? How much would you have to move the pieces in order to produce a multiplication constant of one? Is it very sensitive to this?

Mr. Thomas: It is very sensitive to this. The degree of sensitivity is also highly dependent on the size of the piece, the reactivity of the individual piece, but fractions of a centimeter usually account for quite a bit in reactivity.

Mr. Pendlebury: In doing the experiment, you have the pieces mounted on steel rods, presumably under tension so this helps to reduce vibration. If you have a system which is approximately critical, and it is very sensitive to small displacements, you may get vibrations that could carry you over into a supercritical state. Is this any worry from an experimental point of view?

Mr. Thomas: Initially we were concerned about this. The only control on this system is the separation of the two tables. There was some question whether the force which causes separation when one scrams the system would indeed produce these oscillations. Fortunately, the amplitude of the vibration follows the force, so that by the time of the first unfavorable displacement, the displacement of the table is significantly larger and in the opposite direction. Thus there is no problem.

Presented by A. Gandini

Analysis of Fast Reactors by the CIAP and GLOPERT Codes Using Improved Perturbation Methods*

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SUMMARY

In order to improve the analysis of fast reactors by the use of improved perturbation techniques, two codes have been developed which make use of the theory described by Usachev in 1962 with respect to linear functionals of the neutron flux and extended here to linear functionals of the neutron importance. The CIAP Code (Calculation of Importance for Analysis of Perturbations) calculates in one dimension and in multigroup diffusion theory the importance function to be used in the perturbation formulas relative to quantities such as reaction-rate ratios, reactivity coefficients, prompt-neutron lifetimes, and ratios of neutron source worth. The GLOPERT Code has the features and requires the computing time typical of a conventional perturbation code. It uses the importance function given by the CIAP Code and calculates the fractional variation of reaction-rate ratios (e.g., conversion ratios, and fission and capture rate ratios) as functions of position and the fractional variation of reactivity values (e.g., local and overall sodium void coefficient, and Doppler effect) produced by any perturbation which changes or shifts the flux distribution. A few cross comparisons and applications of the codes are presented relative to two representative fast power systems.

1. Introduction

The object of this paper is to illustrate the improved perturbation methods developed recently^(1,2) and to present a few applications of these new techniques. For this purpose two programs have been developed in multigroup diffusion approximation, the CIAP and the GLOPERT codes. The first calculates the importance

function related to specified linear functionals of the neutron flux (or neutron importance); the latter uses this function in the perturbative formulations. The extension of perturbation techniques to other terms has increased consistently the possibility of analysis of nuclear reactors, especially with respect to fast systems. This fact, reciprocally, should suggest new experimental areas to be exploited. Quantities such as reactivity worths, reaction ratios, prompt-neutron lifetimes, sodium-void coefficients, Doppler coefficients, and neutron-source importance ratios, may be easily correlated to differential quantities, thus allowing more detailed understanding of reactor physics, more improved system optimization analysis, and more consistent cross checks of nuclear multigroup constants with integral data.

The analysis presented in this paper is referred to typical reference fast reactor systems. The quantities taken into consideration are the sodium-void coefficient, the Doppler coefficient, and the total and internal breeding ratios.

2. Improved Perturbation Methods

The basic assumption in order to develop these new perturbation methods consists in considering all neutrons of a critical system as belonging to generations and in following a sufficiently large number of these generations until a specified one is reached for which the linear functional of the neutron flux is considered. This assumption was introduced by Usachev,⁽¹⁾ who derived perturbation formulas for functionals of the neutron flux. These formulae, in matrix notation, are the following:

$$\frac{\delta R}{R} = \int \delta g \varphi \, dr d\Omega + \int \psi^* \delta P \varphi \, dr d\Omega, \quad (1)$$

where R is the ratio of two linear functionals of the

* Work performed under the auspices of the CNEN-Euratom Association.

neutron flux:

$$R = \frac{Q_1}{Q_2} = \frac{\int h_1 \varphi \, dr d\Omega}{\int h_2 \varphi \, dr d\Omega} \quad (2)$$

and

$$g = \frac{h_1}{Q_1} - \frac{h_2}{Q_2}. \quad (3)$$

The functions $h_1(r, \Omega)$ and $h_2(r, \Omega)$ are defined point- and angle-wise. The angle dependence may be dropped if a diffusion approximation is applied. δP represents the perturbation introduced in the system and ψ^* is a new neutron importance related to the functional

$$Q_0 = \int g \varphi \, dr d\Omega \equiv 0. \quad (4)$$

In ref. (2), using a simple operational procedure, formulae for functionals of the neutron adjoint flux are also derived. Let us define R^* as the ratio of two linear functionals of the adjoint flux:

$$R^* = \frac{S_1^*}{S_2^*} = \frac{\int s_1 \varphi^* \, dr d\Omega}{\int s_2 \varphi^* \, dr d\Omega}, \quad (5)$$

where $\varphi^*(r, \Omega)$ is the adjoint flux, and $s_1(r, \Omega)$ and $s_2(r, \Omega)$ are two functions defined space and angle-wise. R^* may represent in particular the worth of a given neutron source (positive or negative) with respect to a reference. Since all the processes considered are linear and we are considering asymptotic quantities relative to critical systems, we may think here, without loss of generality, that all neutrons corresponding to the specified sources (necessarily limited in time) are born at the same moment and then belong to the same generation. This allows us to write the following perturbation formula:

$$\begin{aligned} \frac{\delta R^*}{R^*} = & \int \delta g^* \varphi^* \, dr d\Omega + \int \varphi_1 \delta A^* \varphi^* \, dr d\Omega \\ & + \int \psi \delta P^* \varphi^* \, dr d\Omega, \end{aligned} \quad (6)$$

where

$$g^* = \frac{s_1}{S_1^*} - \frac{s_2}{S_2^*}. \quad (7)$$

The term δP^* represents the perturbation introduced into the system and affecting the adjoint matrix operator, while δA^* represents the same perturbation limited to the absorption leakage and downscattering. The function $\varphi_1(r, \Omega)$ represents the uncollided flux while $\psi(r, \Omega)$ is the importance function related to the func-

tional

$$S_0^* = \int g^* \varphi^* \, dr d\Omega \equiv 0. \quad (8)$$

3. The CIAP Code

The CIAP Code (Calculation of Importance for Analysis of Perturbations) calculates on the IBM-7094 in one dimension (slab, cylinder, or sphere) and in multigroup diffusion theory (up to 26 energy groups are allowed) the importance function relative to linear functionals of the neutron flux and of the neutron adjoint flux. This is obtained following an iterative procedure allowed by the following expressions:

3.1. Functionals of the Real Flux

$$\begin{aligned} A^* \psi_i^* &= F^* \psi_{i-1}^*; \\ A^* \psi_1^* &= g, \end{aligned} \quad (9)$$

where $\psi_i^*(r, \Omega)$ represents the importance of the i -th generation neutrons with respect to the functional Q_0 given by eq. (4).

3.2. Functionals of the Neutron Adjoint Flux

$$\begin{aligned} A \varphi_i &= F \varphi_{i+1}; \\ A \varphi_1 &= g^*, \end{aligned} \quad (10)$$

where φ_i represents the neutron flux (in particular φ_1 is relative to the uncollided neutrons) related to the functional S_0^* given by eq. (8).

For what concerns the input sources g and g^* , their expressions are given by eqs. (3) and (7). In the cases of reactivity changes, corresponding to a perturbation P' , h_1 and h_2 become

$$\begin{aligned} h_1 &= \varphi^* \delta P'; \\ h_2 &= \varphi^* F, \end{aligned} \quad (11)$$

while in the case of effective neutron lifetime,

$$\begin{aligned} h_1 &= \varphi^* r^{-1}; \\ h_2 &= \varphi^* F. \end{aligned} \quad (12)$$

The importance functions ψ^* and ψ are given by the expressions

$$\begin{aligned} \psi^* &= \sum_{i=1}^{I_\epsilon} \psi_i^*; \\ \psi &= \sum_{i=2}^{I_\epsilon} \varphi_i. \end{aligned} \quad (13)$$

These sums may be truncated to the I -th term, corresponding to the precision required, since $Q_0 = S_0^* \equiv 0$, and then a power or an importance change does not affect the values of these latter quantities. A provision

is added to the program in order to calculate automatically h_1 and h_2 of eq. (11) once the perturbation P' is input. The main output of the CIAP code consists of a tape with all the information needed in order to run the GLOPERT code, that is, the functions ψ^* and φ_1 , ψ , respectively, for the functionals of the real and adjoint neutron fluxes.

4. The GLOPERT Code

So far the GLOPERT Code includes only perturbative formulae of functionals of the neutron flux. Provisions for perturbation analysis of functionals of the neutron adjoint flux will be completed in the near future. In this code for what concerns perturbations as represented by eq. (1) there is no need to distinguish between reaction rates, prompt-neutron lifetimes, and reactivity coefficients, except that in the two former cases some sort of reset of criticality may be requested. This is allowed for in a separate subroutine which produces an additional perturbation P'' following a specified criterion (for instance, fuel composition or change of core volume) so that the net contribution to reactivity is negligible. This same reset should in principle be considered also for reactivity coefficients, but in the majority of cases it does not seem necessary. The code prints the first and second term of the second member of eq. (1), corresponding to zero- and first-order change of neutron flux. The zero-order term corresponds to the traditional perturbation formula.

5. Sensitivity Study of Two Reference Fast Power Reactors

In order to develop a systematic sensitivity study for some integral quantities of major interest, two reference fast systems have been considered, reflecting, within the limitations of the theoretical model adopted, present trends in this field. One system represents a sodium-cooled pancake, 6000-liter core, oxide reactor; the other is a sodium-cooled 4000-liter core, carbide system corresponding to a cylinder with a length-to-diameter (L/D) ratio close to one. The calculations have been performed in slab geometry for the pancake case, using a radial buckling $B_r^2 = 0.000164$ to allow for the finite radial size, while for the carbide system the spherical geometry has been adopted. Compositions and dimensions of the two reference systems are illustrated in Table 1. They reflect, in the blanket, burnup conditions at 50,000 MWd/t with respect to Pu-239 concentration. The nuclear constants given by Abagjan *et al.*⁽³⁾ have been adopted by interpolation with the RUSSKI code⁽⁴⁾ of the self-shielding factors. The calculation of the real and adjoint fluxes has been performed with the TAIM diffusion code,⁽⁵⁾ which allows use of up to 26 energy groups. Use of an im-

TABLE 1. COMPOSITIONS (IN NUCLEI/CM³ $\times 10^{-24}$) AND DIMENSIONS OF THE TWO REFERENCE FAST SYSTEMS

	Pancake Oxide System		Carbide System	
	Core	Blanket	Core	Blanket
R (cm)	—	—	102.95	148.67
H (cm)	63.56	45.72	—	—
$N^{\text{Pu-238}}$	0.000851	0.000199	0.00075	0.000206
$N^{\text{U-238}}$	0.004253	0.010008	0.006272	0.012142
N^{Fe}	0.01134	0.01015	0.004718	0.007166
N^{Cr}	0.00326	0.00292	0.001355	0.00206
N^{Ni}	0.00152	0.00136	0.000032	0.00096
N^{Na}	0.01012	0.00704	0.01463	0.00946
N^{Be}	0.00462	—	—	—
N^{O}	0.014828	0.020416	—	—
N^{C}	—	—	0.007022	0.012348

proved version of the PERT code⁽⁶⁾ has been made for the criticality-composition resets when these were needed. The radius resets were obtained by use of pairs of values of k_{eff} and R_{core} (or H_{core}) obtained during the iterative criticality search with the TAIM Code. The quantities considered for the sensitivity study are the internal breeding ratio (IBR), the breeding ratio (BR), the reactivity effect of voiding 10 v/o sodium, and the Doppler effect for a temperature rise from 900 to 2100°K.

The two quantities BR and IBR here are defined by the expressions

$$\left\{ \begin{array}{l} \text{BR} = \frac{\int_{\text{syst}} dr N^{238}(r) \sigma_c^{238} \varphi(r)}{\int_{\text{syst}} dr N^{\text{Pu}}(r) \sigma_{c+f}^{\text{Pu}} \varphi(r)} ; \\ \text{IBR} = \frac{\int_{\text{core}} dr N^{238}(r) \sigma_c^{238} \varphi(r)}{\int_{\text{syst}} dr N^{\text{Pu}}(r) \sigma_{c+f}^{\text{Pu}} \varphi(r)} . \end{array} \right. \quad (14)$$

When a variation $\delta\sigma$ is introduced into the system, the criticality is altered and has to be reset by some procedure. Here we have considered two cases, one where a variation δN^{Pu} is evaluated by conventional perturbation techniques, and one where a volume change is made. If the quantities given by (14) are called generally $R(N^{238}, N^{\text{Pu}}, \sigma^{238}, \sigma^{\text{Pu}})$ and if we recall eq. (1) in the former case, we have the following expression for $\delta R/R$ as function of the variation $\delta\sigma$:

$$\frac{\delta R}{R} = \frac{1}{R} \left(\frac{\partial R}{\partial \sigma} + \frac{\partial R}{\partial N^{\text{Pu}}} \frac{\partial N^{\text{Pu}}}{\partial \sigma} \right) \delta\sigma + \int dr \psi^* \left[\left(\frac{\partial P}{\partial \sigma} + \frac{\partial P}{\partial N^{\text{Pu}}} \frac{\partial N^{\text{Pu}}}{\partial \sigma} \right) \delta\sigma \right] \phi. \quad (15)$$

TABLE 2. FINE- AND COARSE-GROUP STRUCTURE

Coarse Groups	1					2					3				
Fine Groups	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
E_1 (keV)	6,500	4,000	2,500	1,400	800	400	200	100	46.5	21.5	10.0	4.65	2.15	1.00	0.465

Coarse Groups	4										
Fine Groups	16	17	18	19	20	21	22	23	24	25	T
E_1 (keV)	0.215	0.100	0.0465	0.0215	0.010	0.00465	0.00215	0.0010	0.000465	0.000215	0.0000252

An analogous expression holds when a change of core volume is made.

For the other two quantities, namely, the sodium and the Doppler effects, no criticality reset has been considered. Then eq. (15) becomes (using the general symbol R)

$$\frac{\delta R}{R} = \frac{1}{R} \frac{\partial R}{\partial \sigma} \delta \sigma + \int dr \psi^* \left(\frac{\partial P}{\partial \sigma} \delta \sigma \right) \phi. \quad (16)$$

In general we will refer to the first term of the second members of eq. (15) and (16) as that corresponding to the direct effect, while the second will correspond to the indirect effect. Besides, in order to give a more detailed information on the influence of the cross sections on the quantities considered, their various contributions have been separated. In other words, a given $\delta \sigma$ has been considered one time as influencing, in general, the fissions, the captures, and the downscattering terms of the perturbation expressions, and another time as influencing only the diffusion terms. In the first case we will obtain quantities $\delta R_s/R$ and in the other case $\delta R_L/R$. The net effect is the arithmetical sum of the two terms:

$$\frac{\delta R}{R} = \frac{\delta R_s}{R} + \frac{\delta R_L}{R}. \quad (17)$$

Finally, for clarity of exposition, the resulting effects have been collapsed into four coarse groups with energy boundaries shown in Table 2. The assigned percentage variations of the nuclear parameters represent a rough guess of their present uncertainties, so the values given reflect somewhat the different contributions to the uncertainty of the integral parameters R studied.

5.1. Sensitivity of Sodium-void Coefficient

The analysis of the reactivity effect produced by removal of 10 v/o sodium from the systems studied has been performed in terms of assigned percentage variations b of group parameters of the component elements. The results are given in Tables 3, 4, 5, and 6. For the carbide system, the net reactivity effect has

been calculated as $-0.01 \times 10^{-2} \delta k/k$. As may be noticed from Table 3, the major contribution to the uncertainty of $\delta R_s/R$ is the inaccuracy of σ_{in} of U-238, followed by that of σ_{el} of C, σ_c of U-238, and σ_{el} of Na. The total contribution of sodium is reduced by a sort of compensating direct and indirect effects. In fact, an increase of the overall value of σ_{el} for sodium at high energies increases the direct positive spectral contribution of the 10 v/o sodium-void effect, which becomes less negative, but the bigger indirect effect makes it more negative by a larger percentage. Analogous considerations apply to σ_{in} .

It is interesting to see the reversal of sign, at the lower group, of the downscattering terms, which is to be referred directly to the derivative change of the adjoint flux at low energies. For what concerns the capture cross sections, the high energy groups show negative relative variations $\Delta R_s/R$ for a given percentage increase of the group parameters. This might be partly attributed to the absolute higher increase of the lower fine groups within a coarse one which contributes to make the sodium spectral void effect more positive.

More subtle inferences are needed to explain the values relative to σ_f . One can generally visualize, in relation to an increase $\delta \sigma_f$, two effects, a production and an absorption one, which are relative to the following perturbation difference:

$$\int_{\text{system}} dr \int_{E_i}^{E_{i-1}} dE \phi \delta \Sigma_f(r, E) [\nu(E) \psi_x(r) - \psi(r, E)],$$

where ψ_x represents the importance function ψ weighted with the fission spectrum. These effects so redistribute neutrons to energies of different importance (even of different sign) with respect to the functional under study.

Table 4 refers to the pancake oxide system. Considerations similar to those for Table 3 may be deduced relative to the sodium-void effect $\delta R_s/R$, although several differences may be noticed with respect to the previous case. The total sodium-void coefficient R for the pancake system appears more negative, i.e.,

TABLE 3 SENSITIVITY COEFFICIENTS RELATIVE TO SODIUM VOID AND DOPPLER EFFECT OF THE CARBIDE SYSTEM (IN % $\delta R_s/R$)

Quantity R	Group	$\delta\sigma/\sigma$ (%)	Plutonium 239					Uranium 238					Sodium		Iron			C
			ν $b = 2$	σ_f $b = 3$	σ_c $b = 5$	σ_{in} $b = 10$	σ_{el} $b = 10$	ν $b = 2$	σ_f $b = 3$	σ_c $b = 5$	σ_{in} $b = 10$	σ_{el} $b = 10$	σ_{in} $b = 10$	σ_{el} $b = 10$	σ_c $b = 10$	σ_{in} $b = 10$	σ_{el} $b = 10$	σ_{el} $b = 10$
Na 10 v/o Void Coefficient	1	b_1											-0.62	-0.28				
	1	b	0	0	0	0.15	0	0	0.08	-0.03	2.65	0.02	0.14	0.07	0	0.62	0.05	0.36
	2	b_1												-0.63				
	2	b	0	-0.42	-0.13	0.04	0.02			-0.92	0.28	0.17		0.95	-0.03		0.28	1.54
	3	b_1												0.51				
	3	b	0	-0.26	-0.22		-0.01			-0.96		-0.06		-0.01	-0.03		-0.13	-0.44
	4	b_1												0.01				
	4	b	0	0.04	0.05					0.06		-0.01		-0.02	0		-0.01	-0.03
	1	b	-0.01	0	0	0.09	0	0.01	0.04	-0.01	1.41	0.01	0.34	0.19	0	0.28	0.02	0.18
	2	b	-0.01	-0.38	-0.13	0.08	0.03			-1.06	0.59	0.35		3.15	-0.03		0.56	3.22
Doppler Effect ($\Delta T = 2100^\circ\text{K} - 900^\circ\text{K}$)	3	b_1	-0.37	-0.38	0.22					4.48		-0.04						
	3	b	-0.37	-1.06	-0.32		0.01			2.03		0.08		1.06	-0.09		0.13	0.86
	4	b_1	-0.50	-0.45	0.44													
	4	b	-0.50	-0.51	0.37		0			1.11		-0.03		-0.21	0.01		-0.11	-0.32

Note b_1 refers to first order coefficients relative to perturbation b when they are pertinentTABLE 4 SENSITIVITY COEFFICIENTS RELATIVE TO SODIUM VOID AND DOPPLER EFFECT OF THE PANCAKE OXIDE SYSTEM (IN % $\delta R/R$)

Quantity R	Group	$\delta\sigma/\sigma$ (%)	Plutonium 239					Uranium 238					Sodium		Iron			Be	O
			ν $b = 2$	σ_f $b = 3$	σ_c $b = 5$	σ_{in} $b = 10$	σ_{el} $b = 10$	ν $b = 2$	σ_f $b = 3$	σ_c $b = 5$	σ_{in} $b = 10$	σ_{el} $b = 10$	σ_{in} $b = 10$	σ_{el} $b = 10$	σ_c $b = 10$	σ_{in} $b = 10$	σ_{el} $b = 10$	σ_{el} $b = 10$	σ_{el} $b = 10$
Na 10 v/o Void Coefficient	1	b_1											-0.59	-0.32					
	1	b	-0.01	-0.01	0	0.01	0	0.03	0.03	-0.01	0.10	0	-0.53	0	0	0.13	0.01	0.03	0.04
	2	b_1												-0.44					
	2	b	0	0.02	0.01	-0.02	0			-0.07	-0.11	-0.04		-0.77	-0.01		-0.01	-0.37	-0.77
	3	b_1												1.38					
	3	b	0	0.12	0.12		0			0.22		-0.03		1.10	-0.03		-0.02	-0.33	-0.43
	4	b_1												0.03					
	4	b	0	0.13	0.15		0			0.13		-0.01		0.02	-0.01		-0.01	-0.02	-0.02
	1	b	0	0	0	0.06	0	0.01	0.03	0	0.65	0	0.14	0.09	0	0.37	0.03	0.09	0.23
	2	b	0	-0.24	-0.08	0.04	0.02			-0.55	0.26	0.14		1.13	-0.03		0.48	1.33	2.41
Doppler Effect ($\Delta T = 2100^\circ\text{K} - 900^\circ\text{K}$)	3	b_1	-0.43	-0.41	0.26					0.48		-0.02							
	3	b	-0.43	-0.89	-0.18		0			0.37		0.03		0.45	-0.11		0.08	0.31	0.6
	4	b_1	-1.04	-0.92	1.61					3.89		0							
	4	b	-1.04	-0.93	1.59		0			2.20		-0.18		-0.17	0		-0.25	-0.33	-0.48

Note b_1 refers to first order coefficients relative to perturbation b when they are pertinent

TABLE 5. SENSITIVITY COEFFICIENTS RELATIVE TO SODIUM VOID AND DOPPLER EFFECT OF THE CARBIDE SYSTEM (IN $\% \delta R_L/R$)

Quantity R	Group	$\delta\sigma/\sigma$ (%)	Plutonium-239					Uranium-238					Sodium		Iron			C
			ν $b = 2$	σ_f $b = 3$	σ_e $b = 5$	σ_{in} $b = 10$	σ_{el} $b = 10$	ν $b = 2$	σ_f $b = 3$	σ_e $b = 5$	σ_{in} $b = 10$	σ_{el} $b = 10$	σ_{in} $b = 10$	σ_{el} $b = 10$	σ_e $b = 10$	σ_{in} $b = 10$	σ_{el} $b = 10$	σ_{el} $b = 10$
Na 10 v/o Void Co-efficient	1	b_1											1.75	4.74				
	1	b									-0.01	0	1.74	4.73		0	0	0
	2	b_1												1.90				
	2	b										0		1.90		0	0	
	3	b_1												1.54				
	3	b										0		1.54		0	0	
Doppler Effect ($\Delta T = 2100^\circ\text{K} - 900^\circ\text{K}$)	4	b_1												0.14				
	4	b										0		0.14		0	0	
	1	b_1																
	1	b									0.01	0	0.01	0.01		0	0	0
	2	b_1																
	2	b										0		0		0	0	0
	3	b_1																
	3	b										0		0		0	0	0
	4	b_1																
	4	b										0		0		0	0	0

Note: b_1 refers to first-order coefficients relative to perturbation b , when they are pertinent.

TABLE 6. SENSITIVITY COEFFICIENTS RELATIVE TO Na VOID AND DOPPLER EFFECT OF THE PANCAKE OXIDE SYSTEM (IN $\% \delta R_L/R$)

Quantity R		$\delta\sigma/\sigma$ (%)	Plutonium-239					Uranium-238					Sodium		Iron			Be	O
			ν $b = 2$	σ_f $b = 3$	σ_e $b = 5$	σ_{in} $b = 10$	σ_{el} $b = 10$	ν $b = 2$	σ_f $b = 3$	σ_e $b = 5$	σ_{in} $b = 10$	σ_{el} $b = 10$	σ_{in} $b = 10$	σ_{el} $b = 10$	σ_e $b = 10$	σ_{in} $b = 10$	σ_{el} $b = 10$	σ_{el} $b = 10$	σ_{el} $b = 10$
Na 10 v/o Void Coeff.	1	b_1											0.59	1.67					
	1	b									-0.01	-0.01	0.58	1.66		-0.02	-0.02	-0.01	-0.02
	2	b_1												6.20					
	2	b								0	0			6.20		0	0	0	
	3	b_1												1.50					
	3	b										0		1.50		0	0	0	
Doppler Effect ($\Delta T = 2100^\circ\text{K} - 900^\circ\text{K}$)	4	b_1												0.18					
	4	b										0		0.18		0	0	0	
	1	b_1																	
	1	b									0.01	0.01	0.01	0.01			0.01	0.01	0.01
	2	b_1																	
	2	b										0	0	0		0	0	0	
	3	b_1																	
	3	b										0	0	0		0	0	0	
	4	b_1																	
	4	b										0	0	0		0	0	0	

Note: b_1 refers to first order coefficients relative to perturbation b , when they are pertinent.

$-0.095 \times 10^{-2} \delta k/k$, because of the higher leakage due to the flat geometry. This fact increases the negative diffusion component of the sodium void effect and at the same time reduces the importance of high-energy

neutrons. Besides, the increased moderation of this system makes the neutron spectrum much softer, thereby increasing the importance of cross section changes at the low energies

TABLE 7 SENSITIVITY COEFFICIENTS RELATIVE TO BR AND IBR OF THE CARBIDE SYSTEM (IN $\% \delta R_s/R$)

Quantity R	Group	$\delta\sigma/\sigma$ (%)	Critical Reset	Plutonium 239					Uranium 238					Sodium		Iron			C
				ν $b = 2$	σ_f $b = 3$	σ_c $b = 5$	σ_{in} $b = 10$	σ_{e1} $b = 10$	ν $b = 2$	σ_f $b = 3$	σ_c $b = 5$	σ_{in} $b = 10$	σ_{e1} $b = 10$	σ_{in} $b = 10$	σ_{e1} $b = 10$	σ_c $b = 10$	σ_{in} $b = 10$	σ_{e1} $b = 10$	σ_{e1} $b = 10$
BR	1	b_1	C	0 45	0 03	-0 01	-0 14	0	0 69	0 49	0 07	-1 05	0	-0 27	-0 14	0	-0 22	-0 01	-0 16
		b		0 43	-0 01	-0 01	-0 13	0	0 73	0 53	0 06	-0 96	0	-0 25	-0 13	0	-0 20	-0 01	-0 15
	2	b_1	C	1 57	0 05	-0 40	-0 01	0			1 24	-0 11	-0 04		-0 35	-0 05		-0 04	-0 37
		b		1 60	0 07	-0 41	-0 01	0			0 75	-0 02	-0 03		-0 39	-0 06		-0 04	-0 24
	3	b_1	C	0 60	-0 02	-0 54		0			1 05		-0 24		-0 20	-0 04		0 04	0 17
		b		0 60	0 02	-0 49		0			0 84		-0 29		-0 65	-0 04		-0 05	-0 23
	4	b_1	C	0 14	-0 05	-0 19		0			0 09		0		0	0		0	0
		b		0 15	0 03	-0 13		0			0 13		0		-0 01	0		-0 01	-0 02
	1	b_1	R	1 54	1 52	-0 10	-0 28	-0 07	2 46	1 72	-0 55	-3 75	-0 10	-1 09	-0 60	-0 11	-0 88	-0 14	-0 63
		b		1 17	1 15	-0 08	-0 21	-0 06	1 86	1 30	-0 42	-2 84	-0 08	-0 82	-0 46	-0 08	-0 66	-0 11	-0 48
	2	b_1	R	5 70	6 21	-0 69	-0 12	-0 08			-4 65	-0 49	-0 19		-1 33	-0 24		-0 24	-1 47
		b		1 32	4 71	-0 52	-0 09	-0 06			-3 53	-0 38	-0 15		-1 01	-0 18		-0 18	-1 11
	3	b_1	R	2 14	2 30	-0 92		-0 06			-3 52		0 01		0 65	-0 18		0 08	0 57
		b		1 62	1 75	-0 70		-0 05			-2 67		0 01		0 49	-0 14		0 06	0 43
	4	b_1	R	0 49	0 44	-0 43		-0 07			-0 53		-0 07		-0 05	-0 09		-0 06	-0 05
		b		0 37	0 33	-0 33		-0 05			-0 40		-0 05		-0 04	-0 07		-0 05	-0 03
IBR	1	b_1	C	0 45	0 03	-0 01	-0 14	0	0 69	0 49	0 12	-1 05	0	-0 27	-0 14	0	-0 22	-0 01	-0 16
		b		0 46	0 04	-0 01	-0 11	0	0 73	0 49	0 12	-0 61	0	-0 17	-0 08	0	-0 14	0	-0 10
	2	b_1	C	1 57	0 05	-0 40	-0 01	0			1 33	-0 11	-0 04		-0 35	-0 05		-0 04	-0 37
		b		1 58	-0 08	-0 43	0 01	0			1 14	0 03	-0 02		0 04	-0 06		0 01	-0 05
	3	b_1	C	0 60	-0 02	-0 54		0			0 96		-0 24		-0 20	-0 04		0 04	0 17
		b		0 60	-0 03	-0 54		0			1 08		-0 30		-0 67	-0 03		-0 06	-0 27
	4	b_1	C	0 14	-0 05	-0 19		0			0 05		0		0	0		0	0
		b		0 14	0 0	-0 13		0			0 17		0		-0 01	0		-0 01	-0 02
	1	b_1	R	-0 47	-0 47	0 03	0 09	0 02	-0 75	-0 53	0 17	1 15	0 03	0 33	0 18	0 03	0 27	0 04	0 19
		b		-0 33	-0 32	0 02	0 06	0 02	-0 52	-0 37	0 12	0 80	0 02	0 23	0 13	0 02	0 19	0 03	0 13
	2	b_1	R	-1 75	-1 91	0 21	0 04	0 02			1 43	0 15	0 06		0 41	0 07		0 07	0 45
		b		-1 21	-1 32	0 15	0 03	0 02			0 99	0 11	0 04		0 28	0 05		0 05	0 31
	3	b_1	R	-0 66	-0 71	0 28		0 02			1 08		0		-0 20	0 06		-0 03	-0 18
		b		-0 46	-0 49	0 20		0 01			0 75		0		-0 16	0 04		-0 02	-0 12
	4	b_1	R	-0 15	-0 13	0 13		0 02			0 16		0 02		0 02	0 03		0 02	0 01
		b		-0 10	-0 09	0 09		0 02			0 11		0 01		0 01	0 02		0 01	0 01

Notes C means reset of criticality by fissile/fertile ratio change

R " " " " " volume change

b_1 refers to first order coefficients relative to perturbation b , when they are pertinent

As concerns the uncertainty of $\delta R_s/R$ as given by Table 4, the major quantity responsible appears to be the σ_{cl} of sodium (mostly direct effect), followed by the σ_{el} of oxygen and beryllium, and σ_{in} for sodium (mostly direct effect).

Finally, referring to the transport effects $\delta R_L/R$, as given in Tables 5 and 6, the results indicate that the variations $\delta\sigma$ have a negligible influence on the indirect effect. On the contrary, an important role is played by σ_{cl} of sodium on the direct effect.

TABLE 8 SENSITIVITY COEFFICIENTS RELATIVE TO BR AND IBR OF THE PANCAKE OXIDE SYSTEM (IN % $\delta R_s/R$)

Quantity R	Group	$\delta\sigma/\sigma$ (%)	Critic	Reset	Plutonium 239					Uranium 238					Sodium		Iron			Be	Ox
					ν $b = 2$	σ_f $b = 3$	σ_c $b = 5$	σ_{in} $b = 10$	σ_{el} $b = 10$	ν $b = 2$	σ_f $b = 3$	σ_c $b = 5$	σ_{in} $b = 10$	σ_{el} $b = 10$	σ_{in} $b = 10$	σ_{cl} $b = 10$	σ_c $b = 10$	σ_{in} $b = 10$	σ_{el} $b = 10$	σ_{cl} $b = 10$	σ_{el} $b = 10$
BR	1	b_1	C		0 43	-0 02	-0 01	-0 05	0	0 38	0 35	0 04	-0 44	0	-0 10	-0 05	-0 01	-0 22	-0 02	-0 05	-0 14
		b			0 42	-0 03	0 01	-0 05	0	0 42	0 39	0 03	-0 41	0	-0 10	-0 05	-0 01	-0 22	-0 02	-0 05	-0 13
	2	b_1	C		1 47	-0 11	-0 35	0	0			1 04	-0 04	-0 02		-0 12	-0 10		-0 02	-0 07	-0 33
		b			1 48	-0 10	-0 36	0	0			0 64	0 02	-0 01		-0 15	-0 10		-0 08	-0 25	-0 18
	3	b_1	C		0 87	-0 09	-0 68		0			1 14		-0 01		0 17	-0 10		0	0 19	0 25
		b			0 83	-0 05	-0 58		-0 01			0 92		-0 05		-0 15	-0 07		-0 17	-0 15	-0 22
	4	b_1	C		0 41	0 04	-0 16		0			0 15		0		0 0	-0 02		0	0 01	0 01
		b			0 39	0 15	0		0			0 23		-0 01		-0 01	-0 01		-0 01	-0 01	-0 02
	1	b_1	R		0 73	0 45	-0 01	-0 06	0	0 64	0 59	-0 01	-0 75	-0 01	-0 17	-0 10	-0 03	-0 39	-0 04	-0 10	-0 24
		b			0 54	0 26	-0 02	-0 04	0	0 53	0 52	0 01	-0 54	-0 02	-0 13	-0 07	-0 02	-0 29	-0 03	-0 07	-0 17
	2	b_1	R		2 53	1 53	-0 63	-0 02	0			0 25	-0 08	-0 03		-0 19	-0 16		-0 04	-0 13	-0 56
		b			1 90	0 90	-0 55	-0 02	0			0 36	0	-0 02		-0 17	-0 12		-0 09	-0 28	-0 28
	3	b_1	R		1 49	0 91	-1 27		0			0 34		0 03		0 28	-0 16		0 17	0 32	0 41
		b			1 13	0 63	-0 92		0			0 47		-0 02		-0 10	-0 10		-0 05	-0 09	-0 17
	4	b_1	R		0 69	0 29	-0 82		0			-0 05		0		0	-0 05		0 01	0 01	0 02
		b			0 52	0 28	-0 58		0			0 13		0		0	-0 02		0	0	-0 02
IBR	1	b_1	C		0 43	-0 02	-0 01	-0 05	0	0 38	0 35	0 04	-0 44	0	-0 10	-0 05	-0 01	-0 22	-0 02	-0 05	-0 14
		b			0 43	-0 02	-0 01	-0 03	0	0 37	0 35	0 04	-0 22	0	-0 05	-0 02	-0 01	-0 08	-0 01	-0 02	-0 06
	2	b_1	C		1 47	-0 11	-0 35	0 0	0			1 04	-0 04	-0 02		-0 12	-0 10		-0 02	-0 07	-0 33
		b			1 51	-0 14	-0 38	0 01	0			0 97	0	-0 02		0 04	-0 09		0 04	0 16	-0 03
	3	b_1	C		0 87	-0 09	-0 68		0			1 14		-0 01		0 17	-0 10		0	0 19	0 25
		b			0 88	-0 09	-0 69		-0 01			1 01		-0 05		-0 14	-0 10		-0 17	-0 12	-0 25
	4	b_1	C		0 41	0 04	-0 16		0			0 15		0		0	-0 02		0	0 01	0 01
		b			0 39	0 12	-0 04		0			0 02		-0 01		-0 01	-0 03		-0 01	-0 01	-0 02
	1	b_1	R		-0 18	-0 46	-0 01	0 01	0	-0 16	-0 15	0 18	0 19	0	0 04	0 02	0 01	0 10	0 01	0 02	0 06
		b			-0 15	-0 43	-0 01	0 03	0	-0 14	-0 12	0 17	0 37	0	0 09	0 05	0 01	0 22	0 02	0 05	0 06
	2	b_1	R		-0 62	-1 69	-0 22	0 0	0			2 47	0 02	0 01		0 05	0 04		0 01	0 03	0 14
		b			-0 51	-1 64	-0 25	0 01	0			2 34	0 06	0 01		0 20	0 04		0 06	0 25	0 41
	3	b_1	R		-0 37	-1 01	-0 41		0			2 87		-0 01		-0 07	0 04		-0 04	-0 08	-0 10
		b			-0 30	-0 94	-0 44		0			2 66		-0 05		-0 38	0 03		-0 20	-0 40	-0 58
	4	b_1	R		-0 17	-0 45	-0 22		0			0 66		0		0	0 01		0	0	0
		b			-0 14	-0 33	-0 13		0			0 42		-0 01		-0 01	0 0		-0 01	-0 02	-0 03

Notes C means reset of criticality by fissile/fertile ratio change
R " " " " " volume change

5.2. Sensitivity of the Doppler Effect

The Doppler effect associated with core temperature changes from 900°K to 2100°K has been calculated to be $-1.3 \times 10^{-2} \delta k/k$ for the carbide system and -1.5×10^{-2} for the pancake oxide one. The results are shown in Tables 7, 8, 9, and 10. The major contribution here to the uncertainty of $\delta R_s/R$ for the systems considered appears to be the σ_{c1} of carbon (or oxygen and beryllium), the σ_{c1} of sodium, the σ_c of U-238, and the σ_{in} of U-238. To be noticed are the competing direct and indirect effects of σ_c of U-238, which reduce

relevantly their net contribution. In fact, an increase $\delta\sigma_c$ on one side increases the negative term of the Doppler effect, while on the other side, it hardens the spectrum, giving a contribution of opposite sign. Similar considerations apply to the coefficients relative to σ_f for which the two effects add. Most important, anyhow, appear the downscattering nuclear parameters which are directly correlated to the softness of the neutron spectrum. Because of the distributed moderation of the pancake system among several constituent elements, the effect of $\delta\sigma_{c1}$ of a single one

TABLE 9. SENSITIVITY COEFFICIENTS RELATIVE TO BR AND IBR OF THE CARBIDE SYSTEM (IN % $\delta R_L/R$)

Quantity R	Group	$\delta\sigma/\sigma$ (%)	Critic. Reset	Plutonium-239					Uranium-238					Sodium		Iron			C
				ν $b = 2$	σ_f $b = 3$	σ_c $b = 5$	σ_{in} $b = 10$	σ_{e1} $b = 10$	ν $b = 2$	σ_f $b = 3$	σ_c $b = 5$	σ_{in} $b = 10$	σ_{e1} $b = 10$	σ_{in} $b = 10$	σ_{e1} $b = 10$	σ_c $b = 10$	σ_{in} $b = 10$	σ_{e1} $b = 10$	σ_{e1} $b = 10$
BR	1	b	C																
	2	b	C																
	3	b	C																
	4	b	C																
	1	b_1	R									0.47	0.36	0.16	0.56		0.07	0.16	0.47
	1	b										0.35	0.27	0.12	0.43		0.05	0.12	0.36
	2	b_1	R										1.69		1.42			0.49	1.05
	2	b											1.28		1.07			0.37	0.80
	3	b_1	R										0.19		0.12			0.01	0.02
	3	b											0.15		0.09			0.01	0.01
	4	b_1	R										-0.06		-0.06			-0.06	-0.06
	4	b											-0.04		-0.05			-0.04	-0.05
IBR	1	b	C																
	2	b	C																
	3	b	C																
	4	b	C																
	1	b_1	R									-0.14	-0.11	-0.05	-0.17		-0.02	-0.05	-0.15
	1	b										-0.10	-0.08	-0.03	-0.12		-0.01	-0.03	-0.10
	2	b_1	R										-0.52		-0.43			-0.15	-0.32
	2	b											-0.36		-0.30			-0.10	-0.22
	3	b_1	R										-0.06		-0.04			0	-0.01
	3	b											-0.04		-0.02			0	0
	4	b_1	R										0.02		0.02			0.02	0.02
	4	b											0.01		0.01			0.01	0.01

Notes: C means reset of criticality by fissile/fertile ratio change.

R " " " " " volume change.

TABLE 10. SENSITIVITY COEFFICIENTS RELATIVE TO BR AND IBR OF THE PANCAKE OXIDE SYSTEM (IN % $\delta R_L/R$)

Quantity R	Group	$\delta\sigma/\sigma$ (%)	Critic. Reset	Plutonium-239					Uranium-238					Sodium		Iron			Be	Ox
				ν	σ_f	σ_c	σ_{in}	σ_{el}	ν	σ_f	σ_c	σ_{in}	σ_{el}	σ_{in}	σ_{el}	σ_c	σ_{in}	σ_{el}	σ_{el}	σ_{el}
				$b=2$	$b=3$	$b=5$	$b=10$	$b=10$	$b=2$	$b=3$	$b=5$	$b=10$	$b=10$	$b=10$	$b=10$	$b=10$	$b=10$	$b=10$	$b=10$	$b=10$
BR	1	b_1	C																	
	2	b_1	C																	
	3	b_1	C																	
	4	b_1	C																	
	1	b_1	R									0.16	0.13	0.05	0.14		0.10	0.15	0.04	0.33
	1	b	R									0.12	0.10	0.04	0.10		0.07	0.11	0.03	0.25
	2	b_1	R										0.60		0.36			0.39	0.10	0.67
	2	b	R										0.45		0.27			0.29	0.08	0.51
	3	b_1	R										0.14		0.08			0.10	0.02	0.09
	3	b	R										0.10		0.06			0.07	0.01	0.07
	4	b_1	R										0.01		0			0.02	0	0.01
	4	b	R										0.01		0			0.01	0	0.01
IBR	1	b_1	C																	
	2	b_1	C																	
	3	b_1	C																	
	4	b_1	C																	
	1	b_1	R									-0.04	-0.03	-0.01	-0.03		-0.02	-0.04	-0.01	-0.08
	1	b	R									-0.03	-0.03	-0.01	-0.03		-0.02	-0.03	-0.01	-0.07
	2	b_1	R									-0.15			-0.09			-0.10	-0.03	-0.17
	2	b	R									-0.12			-0.07			-0.08	-0.02	-0.13
	3	b_1	R									-0.03			-0.02			-0.02	0	-0.02
	3	b	R									-0.03			-0.02			-0.02	0	-0.02
	4	b_1	R									0			0			0	0	0
	4	b	R									0			0			0	0	0

Notes: C means reset of criticality by fissile/fertile ratio change.

R " " " " " volume change.

appears reduced compared to the corresponding values of the carbide system.

5.3. Sensitivity of the BR and of IBR

In Table 7 the quantity $\delta R_s/R$ relative to the total and internal breeding ratios, BR and IBR, are given for the carbide reactor. Also here the terms on lines b_1 are relative to direct effects, while those on lines b are relative to net direct and indirect effects. For what concerns the uncertainties of both the breeding ratios considered, when the composition criticality reset is

applied, the major quantity responsible appears to be the inaccuracy of ν of plutonium, followed by that of σ_c , σ_{in} , ν , and σ_f of U-238, σ_c of plutonium, and σ_{el} of light materials. The main contributions to the terms relative to these parameters are due to direct effects, i.e., they result from the criticality reset, although in some cases the indirect effects influence their values by a fair amount. The most relevant indirect effects are those related to $\delta\sigma_c$ of U-238 with respect to BR and those related to a few downscattering terms with respect to IBR. As expected, this latter quantity

appears much more dependent on neutron-spectrum distribution than the BR, while the former one shows a direct dependence from the full amount of fertile in the blanket. When the volume criticality reset is applied, the major responsible uncertainties for the inaccuracy of BR and IBR appear to be those of ν and σ_f of the fissile and fertile material, together with that of σ_c of the fertile. The downscattering parameters seem also to play a major role.

6. Conclusion

As is evident by the results shown in this paper, these improved perturbation methods may represent a powerful tool for analysing in detail integral quantities and for performing sensitivity and optimization studies. In relation to the sodium-void and the Doppler coefficients, for what concerns at least the system investigated and very probably for other systems of interests, it seems that emphasis should be put on better knowledge of nuclear parameters such as σ_{el} of light elements, with particular reference to sodium, carbon, and oxygen, and σ_{in} and σ_c of the fertile materials. For what concerns the accuracy of BR and IBR, ν and σ_f of the fissile and fertile materials seem to play a major role together with all the above-mentioned nuclear parameters.

7. Addendum

The present addendum consists of a short presentation of a theoretical result derived recently after this work was completed. It is intended to shed some more light on the method adopted and to indicate ways of improving it.

7.1. Theory

Let us consider ratios of bilinear functionals of the real and adjoint fluxes:

$$\rho_{b11}(\Delta P_1, \Delta P_2) = \frac{F_1}{F_2} = \frac{\int \varphi^* \Delta P_1 \varphi \, dr d\Omega}{\int \varphi^* \Delta P_2 \varphi \, dr d\Omega}, \quad (1)$$

where ΔP_1 and ΔP_2 represent linear matrix operators.

If we now put

$$\begin{cases} \varphi^* \Delta P_1 = h_1; \\ \varphi^* \Delta P_2 = h_2, \end{cases}$$

we obtain the Usachev perturbation formulae relative to linear functionals of the neutron real flux, where h_1 and h_2 are linear operators. If we put

$$\begin{cases} \varphi \Delta P_1^* = s_1; \\ \varphi \Delta P_2^* = s_2, \end{cases}$$

we obtain the perturbation formula derived in Ref. 2 relative to linear adjoint operators. By definition it is

implied that in the Usachev expression the adjoint flux is not allowed to change, allowance being given only to changes of the real one. We might also use the perturbation expression derived in Ref. 2 to obtain a similar formula, where in turn the real flux is not allowed to change and only changes of the adjoint flux are considered.

In order to account properly for changes of both the real and adjoint fluxes produced by a perturbation and affecting the functional ρ_{b11} , a new complete perturbation formula has been derived in Ref. 7, which is the following:

$$\begin{aligned} \frac{\delta \rho_{b11}}{\rho_{b11}} = & \int \delta g \varphi \, dr d\Omega + \int \psi^* \delta P \varphi \, dr d\Omega \\ & + \int \varphi^* \delta P \psi \, dr d\Omega, \end{aligned} \quad (2)$$

where:

$$\delta g = \frac{\varphi^* \delta(\Delta P_1)}{F_1} - \frac{\varphi^* \delta(\Delta P_2)}{F_2},$$

and ψ^* and ψ are functions calculable in terms of unperturbed quantities. Also in this case, if a diffusion approximation is applied, the integration over the angle Ω may be dropped.

7.2. Some Applications

As may be seen from Eq. (1), $\rho_{b11}(\Delta P_1, \Delta P_2)$ may represent quantities such as:

- reactivity values: material worths, coolant-void, and Doppler coefficients, etc.; these effects correspond to specific operational changes $\Delta P_1(\Delta P_2 = F)$.
- prompt-neutron lifetimes: $\Delta P_1 = v^{-1}$; $\Delta P_2 = F$;
- reactivity-worth ratios, corresponding to two operational changes ΔP_1 and ΔP_2 with particular reference to those related to highly different energy and/or space distributions with respect to the perturbation under investigation.

For what concerns the self-shielding effect of a material sample of weight w , inserted in a reactor, on its reactivity work k_s , it is now possible to define the following expression (for simplicity we assume a constant sample volume and varying weight w homogeneously distributed):

$$k_s = \rho_0 \left(1 + \frac{\delta \rho_0}{\rho_0} \right) w, \quad (3)$$

where ρ_0 represents the conventional perturbation value per unit weight and $\delta \rho_0 / \rho_0$ is given by $\delta \rho_{b11} / \rho_{b11}$ as defined by Eq. (2), where δP corresponds to a unit weight change, where ψ^* and ψ are related to the ratio $\delta_{b11}(\Delta P_1, \Delta P_2)$ where ΔP_1 corresponds to an arbitrary weight

change Δw and $\Delta P_2 = F$. In this case δg is set equal to zero.

Another application of this improved formula is represented by the intercorrelation methods between integral and differential nuclear data, made feasible by linearity.

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SESSION IV

October 11, 1966

DOPPLER AND SODIUM REACTIVITY EFFECTS

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ZPR-6 Doppler Measurements and Comparisons with Theory

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1. Introduction

The measurements reported here were designed to study various aspects of the Doppler effect in a spectrum relevant to a fast power reactor. The method used was based on measuring the reactivity changes resulting from the in-pile heating of small samples in a zero-power assembly. This method has the considerable advantage of flexibility in allowing measurements with different materials under different configurations, and has the further advantage that many uncertainties can be evaluated or eliminated by the potential control one has over the experimental conditions. It has the disadvantage that very small reactivity changes are involved, but techniques are well in hand that allow the reactivity measurement to be made to the necessary precision.

After proof-tests of various kinds (studying, for example, the effect of void around the sample, orientation of the reactor materials in the immediate environment of the sample, and the presence or absence of material identical to the sample material, but unheated, in the environment immediately around the sample), the measurements concentrated on four main areas:

1. the U-238 Doppler effect as a function of sample size and chemical form;
2. the U-235 Doppler effect, with particular attention paid to evaluation of the extraneous effects of thermal expansion;
3. the Doppler effect of mixtures of U-235 and U-238;
4. the effect of sodium voiding on the Doppler effect of U-238 and U-235.

Some results are previously unreported, and some have been reported piecemeal in the past.^(1,2) This paper presents a consistent intercomparison of the most pertinent results, and a comparison of the results with a consistent set of calculations using current theory.

2. Equipment and Techniques

Reference 3 gives a more complete description of the equipment and techniques used, but in brief, the technique was to oscillate a Doppler sample repetitively in

and out of the low-power fast critical assembly while a small servo-controlled, calibrated boron rod held the reactor critical. The difference in the average position of the calibrated rod, sample-in minus sample-out, gave the reactivity worth of the sample relative to the reference sample-out configuration. Repeating this procedure with the sample at various temperatures traced out the reactivity change of the sample with increasing temperature.

Figure 1 shows a cutaway view of the oscillator and drawer in the ZPR-6 reactor. The oscillator unit consists of an equipment table and an attached drawer assembly which runs through the reactor matrix. The drawer is fitted with positions for a Doppler sample and a dummy sample. Separation of the sample positions is such that when one sample is at the core center the other is outside the reactor. The table carries the equipment necessary to maintain vacuum, heat the elements, monitor the temperature, air cool the outer sheath of the element to the reactor temperature, and detect leaks in the system. During a measurement a master timer automatically controls the sequence of oscillator movement and servo-rod averaging. Data analysis was done by the method of Bennett and Long,⁽⁴⁾ which allows elimination of first- and second-order reactivity drifts. This system gives a precision of about 0.0025 Δk for 80 min of reactor operation at about 20 W. The reactor power is limited in ZPR-6 by a necessity for holding the radiation level in the fuel to levels which will allow safe fuel handling. Even at these relatively low power levels, a number of temperature points can be done in a day with precisions at each temperature ranging from 1% for U-238 up to a few percent for the pure U-235 samples.

The reported measurements were all done with Assembly 4Z of ZPR-6. This was a zoned assembly (core, buffer, driver, reflector) with a central core composition that of a 2600-liter dilute carbide fast power reactor. The composition was 42 v/o sodium and had a 7:1 fuel isotopic ratio of U-238 to U-235. The central core zone was cylindrical with an 81-cm diameter and a 122-cm

FIG. 1. Doppler Equipment in ZPR-6.

length. A detailed description of the assembly and its composition is given elsewhere.⁽⁵⁾

3. Experimental Results

Table 1 summarizes the properties of the various Doppler samples, the conditions under which they were run, and the observed reactivity changes on heating. The sample temperatures listed are volumetric average temperatures.

The sample environments listed in the table refer to the following: The "normal" core is the regular Assembly-4Z core except that the four drawers nearest to the Doppler sample are loaded more homogeneously than in the rest of the core. The "no sodium" environment consists of the normal environment except that the sodium cans have been removed throughout the core and replaced by steel frames (which preserve the core steel concentration). The " $\frac{1}{2}$ -in. B_4C " environment refers to the normal environment with the first $\frac{1}{2}$ in. outside the sample position replaced with natural B_4C plates.

The "isolated" environment is the normal environment but with the U-238 plates in the eight drawers adjacent to the sample replaced by an amount of molybdenum calculated to retain the same broad-group Doppler-range fluxes and adjoints at the sample posi-

tion. The "reference void" environment was a variation of the standard Assembly-4Z loading arranged so that a 1-in. increase in the void radius around the sample position, to the "1-in. void increase" environment, could be made more easily.

The results shown in Table 1 are not corrected for thermal expansion of the sample, but the expansion corrections, calculated in a manner to be described below, are also listed in the table.

3.1. Expansion Effects

Doppler effects measured by measuring the reactivity changes resulting from heating samples in the reactor, or for that matter all or a portion of the reactor itself, are subject to masking effects due to simple thermal expansion. Depending on the experimental arrangement, these effects can be small or can completely swamp the Doppler effect to be measured. In the type of measurement done here, in which a cylindrical sample of the material of interest is heated at the center of a ZPR, two possible effects may enter. The first is mainly an axial effect due to sample expansion into regions of lower worth, which is related to the overall macroscopic reactor flux and adjoint profiles that the sample finds itself in. The second is mainly a radial effect caused by changes in the sample collision prob-

TABLE 1 SUMMARY OF DOPPLER MEASUREMENTS IN ZPR 6 ASSEMBLY 4Z

Date of Measurement	Sample No	Diameter and Length, in	Material	Enrichment %	Total U Mass kg	Sample Type ^(a)	Sample Environment	Calculated Expansion Correction 1100 - 293°K Ih/kg U ^(b)	σ Ih/kg U	Measured Reactivity Changes vs Temperature (468 Ih = 1% $\Delta k/k$) Temp, °K {Data normalized to 0.000 Ih/kg U at 293°K Ih/kg U and uncorrected for expansion
6/7/65	N1	1 × 12	UO ₂	Nat U	1.116	FE	Normal	0.000	±0.003	546 831 1081 - 124 - 213 - 272
8/16/65	N1	1 × 12	UO ₂	Nat U	1.116	FF	Normal	0.000	±0.003	350 393 446 525 617 788 953 1047 - 031 - 057 - 083 - 115 - 157 - 201 - 252 - 271
3/18/66	N1	1 × 12	UO ₂	Nat U	1.116	FF	Normal	0.000	±0.003	496 1027 - 107 - 263
7/14/65	N1	1 × 12	UO ₂	Nat U	1.116	LE	½ in B ₄ C	0.000	±0.003	541 778 1056 - 031 - 038 - 062
3/29/66	N1	1 × 12	UO ₂	Nat U	1.116	FE	No Na	0.000	±0.003	488 760 1056 - 068 - 129 - 170
6/23/65	N5	½ × 12	UO ₂	Nat U	0.244	FE	Normal	0.000	±0.011	471 658 845 1056 - 089 - 182 - 251 - 310
7/9/65	N6	½ × 12	UC	Dep U 22% U ²³⁵	0.450	LE	Normal	0.000	±0.008	514 759 1068 - 114 - 207 - 296
6/29/65	N8	1 × 12	UO ₂	Nat U	1.078	NE	Normal	0.000	±0.003	390 522 676 852 1031 - 043 - 116 - 174 - 223 - 274
8/20/65	N8	1 × 12	UO ₂	Nat U	1.078	NE	Normal	0.000	±0.004	366 448 547 741 1039 - 048 - 080 - 125 - 191 - 274
7/13/65	N9	¾ × 12	UO ₂	Nat U	0.533	FE	Normal	0.000	±0.006	474 668 685 855 1033 - 105 - 194 - 210 - 261 - 314
3/23/66	N11	1 × 10	U Metal	Dep U 4% U ²³⁵	2.396	Metal	Normal	0.000	±0.003	593 855 902 937 1000 - 135 - 211 - 218 - 230 - 241
6/9/65	LE1	1 × 12	UO ₂	12.5	0.981	FE	Normal	+0.002	±0.004	557 665 764 899 1040 - 116 - 148 - 178 - 214 - 250
6/14/65	LE1	1 × 12	UO ₂	12.5	0.981	FF	Isolated	+0.002	±0.004	447 636 817 1040 - 087 - 159 - 205 - 249
6/17/65	LE1	1 × 12	UO ₂	12.5	0.981	FE	Reference Void	+0.002	±0.003	446 647 826 1034 - 081 - 157 - 210 - 263
6/18/65	LE1	1 × 12	UO ₂	12.5	0.981	FE	1 in Void Increase	+0.002	±0.004	461 644 827 1046 1152 - 086 - 151 - 202 - 249 - 274
3/31/66	LE1	1 × 12	UO ₂	12.5	0.981	FE	No Na	+0.002	±0.003	491 778 1029 - 055 - 112 - 152
9/3/65	LE2A	1 × 12	UO ₂	12.5	0.894	NNE	Normal	<+0.002	±0.003	321 359 397 443 - 017 - 034 - 054 - 070
7/6/65	LE3	1 × 12	UO ₂	12.5	0.993	NE	Normal	<+0.002	±0.004	376 459 577 676 797 803 929 1050 - 039 - 070 - 114 - 140 - 176 - 176 - 212 - 238
7/23/65	HE1	1 × 12	UO ₂	97.7	0.965	FE	Normal	+0.025	±0.004	480 659 851 1051 + 031 + 045 + 053 + 059
7/29/65	HE2	1 × 12	UO ₂	97.7	1.004	NE	Normal	+0.021	±0.004	319 362 410 456 589 783 1011 + 005 + 011 + 021 + 027 + 043 + 056 + 063
4/4/66	HE2	1 × 12	UO ₂	97.7	1.004	NE	No Na	+0.021	±0.005	491 772 1063 + 022 + 028 + 032
8/12/65	HE3	1 × 12	U Metal	93.1	2.369	Metal	Normal	+0.235	±0.002	474 706 899 969 1020 - 001 - 026 - 061 - 103 - 114
8/31/65	HE4	½ × 12	UO ₂	97.6	0.241	FE	Normal	+0.008	±0.007	488 1048 + 016 + 035
3/22/66	HE5	1 × 12	UO ₂	97.7	0.712	NNE	Normal	+0.011	±0.002	329 364 417 447 479 734 936 + 017 + 030 + 037 + 040 + 046 + 065 + 072
8/6/65	LE3	1 × 12	UO ₂	33.3	0.990	FE	Normal	+0.005	±0.004	474 487 747 1051 - 055 - 056 - 114 - 162
8/10/65	LE50	1 × 12	UO ₂	50.0	1.007	FE	Normal	+0.009	±0.004	483 575 668 861 1059 - 033 - 049 - 066 - 086 - 107
8/4/65	LE66	1 × 12	UO ₂	66.7	1.015	FE	Normal	+0.014	±0.004	468 477 760 773 1035 - 012 - 009 - 024 - 030 - 041
9/13/65	Empty Sample Can					FE	Normal		±0.003	1062 + 003

(a) FE - freely expanding element type NE - axially constrained element type NNE = axially and radially constrained element type

(b) Expansion correction is proportionately less at lower temperatures above the room temperature reference (see Sect. 3.1)

abilities as the dimensions and density change. The axial effect can be minimized by positioning the sample in a region where the axial profiles are reasonably flat, and the radial effect by using samples of low density in which self-shielding and self-multiplication effects are reduced. Compromises with other requirements are inevitable in measurements of this kind, however, so

particular effort was directed to experimental separation and evaluation of these effects in the measurements in Assembly 4Z.

Measurements were made with four basically different types of samples, each with different thermal expansion properties. In three sample types reduced density UO₂ was used, and the expansion was variously

FIG. 2. Type 2 Element (Expansion Constrained).

constrained axially and radially. The fourth type contained uranium metal, much more strongly shielded, which, although not directly comparable to the others, would be expected to give results consistent with the others when the known differences are taken into account.

The samples in which the expansion was controlled had the following characteristics in common. They consisted of a cylindrical UO_2 sample, about 1 in. in diameter and 12 in. long, enclosed in a 50-mil-thick nickel-alloy heater tube, which in turn was suspended in a 50-mil SS outer container. (Vacuum was maintained in a 95-mil annulus between the heater tube and the outer container to minimize heat loss from the element.) A typical element is shown in Fig. 2. In Table 1, the expansion properties of the elements are denoted as FE (freely expanding), NE (no axial expansion), or NNE (no axial expansion and limited radial expansion). In the FE elements the heater tube material was Inconel and the sample was made up of twelve 1.043-in.-diameter by 1.0-in.-long UO_2 pellets of approximately 70% theoretical density (TD). The coefficient of expansion of Inconel is greater than that of the pellets so that in this freely expanding element the pellets expand freely both axially and radially on heating; the pellets rest on the tube so that the total length of the stack increases with the expansion characteristic of the Inconel tube.

In the NE elements, the heater tube was also of Inconel, but instead of being sealed with welded end caps at each end, the pellet stack was held by a piston at each end. With increase in temperature, the heater tube expanded over the piston at each end, and the

expansion was taken up by bellows that completed the seal. The dimension between the pistons, in which the pellet stack was constrained, was held constant. When this element is heated, the axial constraint causes the low-density pellets to deform inelastically, allowing a fixed axial length to be held.

The design of the NNE element was identical to the NE element except that the heater tube material was Invar. A vibratory compaction process was used to fill this element so that zero clearance could be maintained between the UO_2 and the Invar containing tube. Invar has an essentially zero expansion coefficient over a range from room temperature to about 200°C . Thus over this range the UO_2 sample was constrained both axially and radially. Above 200°C the Invar expansion coefficient almost parallels that of Inconel, and the element then acts very similarly to NE elements. (Actually, the action of the Invar in expanding parallel to the Inconel allows a very useful check on the consistency of the results given by either element type.)

The elements of sizes other than 1-in. diameter were all of the freely expanding type. The uranium metal elements consisted of ten 1-in.-diameter, 1-in.-long uranium pellets in an Inconel heater tube. One pellet from the nominal number of 12 was removed from each end and replaced by a 1-in.-long quartz dummy pellet. This was done to keep the axial temperature profile along the pellet stack as flat as possible, so that when the element temperature was taken through the alpha-beta phase change in the metal all the material in the sample passed through the phase change at the same time. The discontinuous change in volume through the

alpha-beta phase change was used to calibrate the effect of expansion on this element.

3.2. Calculations of Radial Expansion

The effects under discussion here are broadly analogous to the heterogeneity effects in a thermal reactor caused by lumping the fuel: a self-multiplication effect analogous to ϵ and a self-shielding effect analogous to f , or more properly, a change in these factors with expansion of the lump.

In calculations of the sample worth and of changes in the sample worth due to expansion, a perturbation formulation can be used that takes these factors into account to various degrees of approximation. The method used here was to consider the neutron balance in each energy group and weight the net addition or subtraction of neutrons in each group by the (unperturbed) importance in that group. That is, the sample worth is given by

$$\frac{\Delta k}{k} \propto \sum_i \phi_i^* (1 - P_{c_i}) \left\{ \sum_k (\sigma_{k \rightarrow i} + \chi_i \nu_k \sigma_{f_k}) \phi_k' - \sigma_{t_i} \phi_i \right\},$$

where

- ϕ_i^* = unperturbed adjoint in group i ;
- P_{c_i} = flat-source collision probability for group i ;
- $\sigma_{k \rightarrow i}$ = transfer cross section from group k to i ;
- χ_i = fraction of fission neutrons born in group i ;
- ν_k = neutrons per fission in group k ;
- σ_{f_k} = fission cross section for group k ;
- ϕ_k = perturbed flux in group k ;
- σ_{t_i} = total cross section for group i ;
- ϕ_i = unperturbed flux in group i .

The perturbed flux ϕ_i' is calculated from

$$\phi_i' = \frac{P_{c_i} \sum_{k \neq i} \{ \sigma_{k \rightarrow i} + \chi_i (\nu \sigma_f)_k \} \phi_k' + (1 - P_{c_i}) \sigma_{t_i} \phi_i}{\sigma_{t_i} - P_{c_i} (\sigma_{i \rightarrow i} + \chi_i \nu \sigma_i)},$$

which is simply an expression of the neutron balance, following application of the reciprocity relationship. The solution is iterative but converges very rapidly.

The flat-source collision probabilities for a cylindrical rod are calculated from $P_c = (1 - P_0)$, where

$$P_0 = \frac{2}{3} \Sigma_a \{ 2[\Sigma_a \{ K_1(\Sigma_a) I_1(\Sigma_a) + K_0(\Sigma_a) I_0(\Sigma_a) \} - 1] \\ + \frac{K_1(\Sigma_a) I_1(\Sigma_a)}{\Sigma_a} - K_0(\Sigma_a) I_1(\Sigma_a) \\ + K_1(\Sigma_a) I_0(\Sigma_a) \}$$

as given by Case *et al.*⁽⁶⁾

Now consider the way in which expansion affects Σa in the different types of elements. The following symbols are used:

a = radius;

l = length;

α = linear coefficient of expansion;

ΔT = temperature change;

V = volume of sample;

Σ = total macroscopic cross section;

M = mass of sample;

A = atomic mass.

(a) *Freely expanding elements*

At any temperature T , the volume of the sample is given by

$$V(T) = V_0(1 + \alpha_s \Delta T)^3 = \pi a_0^2 l_0 (1 + \alpha_s \Delta T)^3,$$

where the subscript s refers to the sample material.

$$\Sigma a(T) = \frac{0.603}{A} \sigma \frac{M}{V} a = \frac{0.603}{A} \sigma M \frac{a_0(1 + \alpha_s \Delta T)}{V_0(1 + \alpha_s \Delta T)^3},$$

and for $\alpha_s \Delta T \ll 1$,

$$\frac{\Sigma a(T) - \Sigma a(T_0)}{\Sigma a(T_0)} = -2\alpha_s \Delta T.$$

(b) *Axially constrained elements*

The sample volume at temperature T is given by

$$V(T) = V_0(1 + \alpha_s \Delta T)^3 = V_0(1 + \alpha_{\text{eff}} T)^2$$

as $l(T) = l_0$ and α_{eff} becomes the effective expansion coefficient in the radial direction. Therefore,

$$\Sigma a(T) = \frac{0.603}{A} \sigma \frac{M}{V_0} a_0 \frac{(1 + \alpha_s \Delta T)^{3/2}}{(1 + \alpha_s \Delta T)^3},$$

and for $\alpha_s \Delta T \ll 1$,

$$\frac{\Sigma a(T) - \Sigma a(T_0)}{\Sigma a(T_0)} = -\frac{3}{2} \alpha_s \Delta T.$$

(c) *Axially and radially constrained elements*

The sample volume is now given by

$$V(T) = \pi a_0^2 (1 + \alpha_I \Delta T)^2 l_0,$$

where α_I is the expansion coefficient of the Invar heater tube. Then

$$\Sigma a(T) = \frac{0.603}{A} \sigma \frac{M}{V_0} a_0 \frac{(1 + \alpha_I \Delta T)}{\pi a (1 + \alpha_I T)^2 l_0},$$

and for $\alpha_I \Delta T \ll 1$,

$$\frac{\Sigma a(T) - \Sigma a(T_0)}{\Sigma a(T_0)} = -\alpha_I \Delta T.$$

The expansion characteristics of the Doppler element materials are shown in Figs. 3 and 4. Figure 3 compares the expansion characteristics of Inconel and Invar, illustrating the different radial-expansion characteristics of the NE and NNE elements. Figure 4 gives the expansion characteristics of the UO₂ pellets and the uranium metal, both taken from quartz dilatometer

FIG. 3. Thermal Expansion of Heater Tubes.

FIG. 4. Expansion of Doppler Materials.

FIG. 5. U-235 Experimental Results.

measurements of material with identical metallurgical history to the material used in the elements, and shows the Inconel expansion characteristic for comparison.

3.3. U-235 Results

The experimental results for the four U-235 elements, one of each type, taken from Table 1 and adjusted to $1h/kg-U-235$ with a correction for the small U-238 Doppler-effect contribution, are shown for convenience in Fig. 5. The three oxide samples each had about the same equivalent background scattering per absorber atom so that their results are almost directly comparable.

A direct measurement of the expansion effect in the metal element is given by the discontinuous reactivity change observed as the element temperature passed through the alpha-beta phase change of the metal. From Fig. 5, the observed reactivity change through the phase change per kg of U-235 was $-(0.091 - 0.0605) = -0.0305$ $1h$. Figure 4 shows the dilatometer result for a sample with the same metallurgical history, which gave a discontinuous expansion increase of 1.85 mils, giving a reactivity effect of $-0.0305/1.85 = -0.0165$ $1h/mil$ of expansion per kg of U-235.

Table 2 gives the calculated reactivity effect per mil of linear expansion for each of the four element types. The metal, FE, and NE numbers in Table 2 refer to 10^{-3} in./in. expansion of the sample material itself, and, in

the NNE case, to expansion of the Invar containing tube. The calculated effect for the metal element of -0.0125 Ih/mil can be compared to the measured effect of -0.0165 Ih/mil.

The measured result was used to correct the metal-element results for expansion. For the oxide elements the expansion correction used was given by the ratio of the calculated expansion effects for the oxide element and the metal multiplied into the measured number for the metal. These numbers are listed in the second column of Table 2. Thus the calculated expansion numbers were used simply to give a method of prorating to the oxide elements the measured expansion effect in the metal, rather than using them directly. The calculated numbers are sensitive in some degree to the cross sections used, the accuracy of the flux and adjoint calculations, and the perturbation-expression denominator, as well as the approximations of the method itself. However, their use in this way, essentially calibrated against a measured result, should make the effects of the above uncertainties minimal. In any case, the disagreement of 30% between the observed and calculated number is probably as close as could be expected and would not lead to large changes in the inferred Doppler effect of the oxide elements if the calculated numbers were used directly.

Table 3 and Fig. 6 show the U-235 results corrected for expansion as described above. The measurements show no detectable effect due to axial expansion. Any significant effect due to axial expansion should show up directly when the results for the freely expanding oxide element are compared with either of the other two oxide

elements in which no change in the axial dimension was allowed. As Table 3 and Fig. 6 show, the difference between the axially constrained and freely expanding element results is totally accounted for by the radial-expansion corrections, and any further axial-expansion correction would only force disagreement between the two results.

The apparent total absence of an effect due to axial expansion is rather puzzling. It is true that the rather long core length in this particular assembly tends to minimize these effects. It is also true that the procedure used for prorating the measured expansion effect from the metal element to the oxide elements would tend to include, to some extent, any axial effect present. But the axial and radial effects would not be expected to prorate between the metal and oxide in anything like the same way. So whatever axial effect that might be included in the proration from the metal results should be far underestimated in the oxide results and leave a discrepancy between the freely expanding and axially

FIG. 6. U-235 Doppler Effect Corrected for Expansion.

TABLE 2. REACTIVITY EFFECT OF 1 MIL (10^{-3} IN./IN.) EXPANSION (Ih/KG U^{235} /MIL)

Element Description	Calculated	Measured
Metal Element HE-3	-0.0125	-0.0165
FE Element HE-1	-0.00234	(-0.00309)
NE Element HE-2	-0.00187	(-0.00247)
NNE Element HE-5	-0.00085	(-0.00112)

TABLE 3. DOPPLER EFFECTS CORRECTED FOR EXPANSION

Temperature		Measured Reactivity, Ih/kg U^{235}				Expansion Correction, Ih/kg U^{235}				Doppler Effect, Ih/kg U^{235}			
°C	°K	Metal	FE	NE	NNE	Metal	FE	NE	NNE	Metal	FE	NE	NNE
100	373	+0.005	+0.016	+0.016	+0.027	+0.017	+0.002	+0.002	—	+0.022	+0.018	+0.018	+0.027
200	473	+0.004	+0.032	+0.032	+0.046	+0.038	+0.005	+0.004	—	+0.042	+0.037	+0.037	+0.046
300	573	-0.002	+0.042	+0.043	+0.056	+0.063	+0.008	+0.007	+0.001	+0.061	+0.050	+0.050	+0.057
400	673	-0.014	+0.049	+0.052	+0.063	+0.089	+0.012	+0.010	+0.003	+0.075	+0.061	+0.062	+0.066
500	773	-0.030	+0.055	+0.059	+0.069	+0.118	+0.015	+0.012	+0.005	+0.088	+0.070	+0.071	+0.074
600	873	-0.047	+0.058	+0.064	+0.073	+0.147	+0.019	+0.015	+0.007	+0.100	+0.077	+0.079	+0.080
700	973	-0.098	+0.062	+0.067	+0.076	+0.211	+0.022	+0.018	+0.009	+0.113	+0.084	+0.085	+0.085
800	1073	-0.120	+0.065	+0.070	+0.079	+0.244	+0.025	+0.020	+0.011	+0.124	+0.090	+0.090	+0.090

FIG. 7. 1-in. UO_2 Corrected for Expansion. FIG. 8. 1-in.-diameter Natural UO_2 , Expandable.

constrained oxide results. But no such discrepancy exists.

The conclusion is, therefore, that no significant effects of axial expansion are entering for this particular sample and core arrangement.

Figure 6 indicates that the Doppler effect of the U-235 metal sample is about 40% higher than that of the oxide samples. The experimental uncertainty in the result for the metal is probably considerably greater than for the oxide, because of the very large expansion correction. However, the smoothness of the shape of the corrected curve for the metal does increase confidence in its reliability. If the expansion corrections were

badly off, inflections and discontinuities would be expected in the corrected curve.

The various oxide results show some disagreement at low temperature but, in general, are felt to be quite consistent. Within the experimental statistics, all elements give the same results.

3.4. U-235-U-238 Mixtures

Figure 7 shows a comparison of the Doppler effect for samples containing a mixture of U-235 and U-238 ranging from U-235 alone through four intermediate compositions to U-238 alone. The UO_2 samples were all 1 in. in diameter by 12 in. long at 70% TD with free expansion in each case. The expansion corrections made on the basis described above have been applied to these results, and the results are plotted in Ih/kg of uranium metal.

3.5. Doppler Effects on Sodium Voiding

Measurements were made of the Doppler effect in samples of natural uranium, a mixture of U-238 to U-235 in the ratio 7:1, and in enriched UO_2 (70% TD) with all sodium removed from the central core, so that a comparison could be made with previous measurements with the normal sodium concentration in the core. Steel frames replaced the sodium cans so that the same core-steel concentration was retained. Calculations indicated that the spectrum in the vicinity of the Doppler samples was that of a fully voided, full-scale core of the same composition. Figure 8 shows the result for U-238. This result is typical of each of the three samples.

4. Comparisons with Theory

The aim of these comparisons was to illustrate the degree of agreement with the measurements given by current standard methods and the best currently available resonance parameters. The basic method involved use of the one-dimensional theory program MACH-1⁽⁷⁾ for the reactor calculation, with the Doppler sample considered as a central perturbation using the DEL subroutine which is part of the MACH-1 package. Twenty-five energy groups were used with the energy boundaries as listed in Table 4.

4.1. Calculation of Reactor Parameters

Although Assembly 4Z was a zoned assembly, it was unusual in the sense that the large size of the zone made the central spectrum rather insensitive to the spectra in the regions surrounding the test zone. (Calculations and worth measurements agree that the spectrum was asymptotic for a considerable distance away from the center.) More than anything else the Doppler effect is sensitive to the low-energy spectrum. The size of the

test zone in Assembly 4Z meant that the spectrum in the environment around the Doppler element was determined largely by the composition of the test zone itself. The calculated spectrum was therefore very sensitive only to the cross sections in the test zone, and only to a minor degree to the cross sections in the buffer, driver, and reflector regions. (The perturbation normalization integral is affected by the choice of cross sections in the driver region particularly, but as discussed later, this quantity varies only about 10% over a range of cross-section choices and assumptions.)

In the light of the above, particular attention was given to the cross sections for the test zone. The cross sections for the other regions were taken directly from standard Argonne cross-section sets that had previously been shown to give fair agreement with critical mass, at least.

For the test-zone calculations, the prescription of Hummel⁽⁸⁾ for the Dancoff-corrected effective σ_p of a resonance absorber in a cell consisting of a fuel rod and a diluent region was used to calculate the effective potential scattering cross section of the U-238 and U-235 fuel plates:

$$\sigma_p = \sigma_{p0} + \frac{\Sigma_{e0}^*}{N_0},$$

where

α_{p0} = fuel rod potential scattering cross section per absorber atom;

$$\Sigma_{e0}^* = \Sigma_{e0} (1 - C) \frac{\bar{a}}{1 + (\bar{a} - 1)C};$$

N_0 = absorber atom density in the rod;

$$\Sigma_{e0} = (\bar{l}_0)^{-1};$$

$$1 - C = \gamma_B + \gamma_B^4(1 - \gamma_B);$$

$$\gamma_B = \Sigma_1 / (\Sigma_1 + \Sigma_{e1});$$

$$\Sigma_{e1} = (\bar{l}_1)^{-1};$$

$\bar{l}_0$ = mean chord length in the rod;

$\bar{l}_1$ = mean chord length in the diluent;

Σ_0 = macroscopic total cross section of the rod;

Σ_1 = macroscopic total cross section of the diluent;

$$\bar{a} = 1.35.$$

This prescription leads to values of σ_p for the U-238 plates of 25 and of 78 for the U-235 plates.

Two sets of 25-group cross sections were derived for the core region for use in these comparisons. The preferred set will be designated as set MC²-25. This set was derived using the Argonne MC² code,⁽⁹⁾ with the macroscopic data library current in July 1966. In brief, MC² performs initial resonance integral calculations and an ultrafine group ($\Delta u = 0.00833$) ELMOE⁽¹⁰⁾ calculation to form a cross-section set which is then

TABLE 4. CENTRAL SPECTRA IN ZPR-6 ASSEMBLY 4Z (MC²-25, MACH-1)

Group	E_L (eV)	ϕ_i	ϕ_i^*	FSD ^(a)
1	3.679×10^6	1.496	4.63	
2	2.231×10^6	2.777	4.39	
3	1.353×10^6	4.748	4.01	
4	8.209×10^5	7.251	3.48	
5	4.979×10^5	10.104	3.46	
6	3.020×10^5	12.708	3.50	
7	1.832×10^5	12.170	3.50	
8	1.111×10^5	11.544	3.49	
9	6.738×10^4	9.527	3.50	
10	4.087×10^4	7.605	3.53	
11	2.479×10^4	5.856	3.61	0.240
12	1.503×10^4	4.823	3.73	0.181
13	9.119×10^3	3.304	3.95	0.131
14	4.307×10^3	2.721	4.28	0.089
15	2.613×10^3	5.366×10^{-1}	4.48	0.079
16	2.035×10^3	6.879×10^{-1}	4.51	0.064
17	1.234×10^3	1.095	4.54	0.036
18	961	3.230×10^{-1}	4.50	
19	583	4.021×10^{-1}	4.71	
20	275	2.430×10^{-1}	5.13	
21	101.3	6.791×10^{-2}	4.16	
22	29.02	7.662×10^{-3}	3.73	
23	13.71	1.613×10^{-4}	2.83	
24	3.059	1.397×10^{-5}	2.11	

^(a) Fraction of total neutron source slowing down past lower energy limit of the group.

averaged into 25 broad groups through an intermediate normal-mode fine-group ($\Delta u = 0.25$) calculation. These cross sections were found to be in reasonable agreement with other comparable cross-section sets except for the absorption cross sections of U-238 and U-235 in the resolved resonance range. However, other evidence exists that the resolved resonance calculation in the current version of MC² is in error. The absorption and fission cross sections in this range were therefore calculated using the program ERIC-2,⁽¹¹⁾ and these cross sections were substituted for the MC²-derived cross sections in energy groups 14 through 24. The U-238 resolved parameters used were taken from Table 2 of Ref. 12. The unresolved parameters for U-238 and U-235 used in these calculations are shown in Table 5. For U-238, the p -wave contribution was assumed to be zero in the resolved range (below 3904 eV). Above this range the p -wave contribution was adjusted in magnitude such that the sum of the p - and s -wave contributions at infinite dilution agreed with the infinite-dilution values listed in BNL-325.

For comparison, an alternate cross-section set for the core region was derived from the Argonne set 801. This set is based on an earlier version of the MC² code, which contained a microscopic data library and again per-

TABLE 5. UNRESOLVED PARAMETERS

<i>U-238</i>		
<i>s-wave:</i>	$S_0 = 1 \times 10^{-4}$	
	$\bar{D}_0 = 18.5 \text{ eV}$	
	$\Gamma_\gamma = 0.0246 \text{ eV}$	
<i>p-wave:</i>	$S_1 = 2 \times 10^{-4}$	
	$\bar{D}_{1, 1/2} = 18.5 \text{ eV}$	
	$\bar{D}_{1, 3/2} = 9.25 \text{ eV}$	
$\nu = 1$ Neutron width distribution		
$\mu_0 = 1$		
<i>U-235</i>		
<i>s-wave:</i>	$S_0 = 1 \times 10^{-4}$	
	$\bar{D}_{0, 3} = 1.72 \text{ eV}$	
	$\bar{D}_{0, 4} = 1.34 \text{ eV}$	
	$\Gamma_\gamma = 0.033 \text{ eV}$	
$\nu = 1$ Neutron width distribution,		
$\eta = 2$ Fission width distribution		
<i>p-wave:</i>	$S_1 = 2.5 \times 10^{-4}$	
	$\bar{D}_{1, J} = (12.04)/(2J + 1); J = 2, 3, 4, 5$	
	$\Gamma_\gamma = 0.033 \text{ eV}$	
$\nu = 1 \quad (J = 2, 5); \quad \nu = 2 \quad (J = 3, 4); \quad \eta = 2$		
$\mu_1 = 1 \quad (J = 2, 5); \quad \mu_1 = 2 \quad (J = 3, 4)$		
Group	$E_L \text{ (eV)}$	$\Gamma_f \text{ (eV)}$
11	24790	
12	15030	0.16
13	9119	0.155
14	4307	0.150
15	2613	0.160
16	2035	0.155
17	1234	0.150
18	961	0.160
19	583	0.120
20	275	0.180
21	101.3	0.090
22	29.02	0.080
23	13.71	0.040
24	3.059	0.020

formed a fine-group normal-mode calculation for broad-group weighting but did not include an ELMOE calculation. An ELMOE calculation for the specific composition of the Assembly-4Z test zone was done, however, using these cross sections, and the 801 cross sections were corrected accordingly. This set will be designated as 801-25 and consists of the ELMOE-corrected 801-set cross sections with ERIC-2 U-238 and U-235 capture and fission cross sections in groups 14 through 24.

The 801 set was also used for the driver, and again an ELMOE calculation was done for the specific driver composition, and the appropriate corrections made to

the 801 set for the driver composition. In the buffer and reflector regions the 801 set was used directly.

Table 4 lists the 25-group spectra at the center of Assembly 4Z as calculated by the MC²-25 set and the MACH-1 diffusion code. Also listed, as a Doppler-range index, are the fractions of the total neutron source slowing down past the lower group boundaries of groups in the Doppler range.

4.2. Calculation of the Doppler Changes in Cross Sections for the Doppler Elements

The Doppler samples were assumed isolated and the effective σ_p values were calculated from the expression

$$\sigma_p = \sigma_{p_0} + \frac{\bar{a}}{lN_0},$$

where

$$\sigma_{p_0} = \sum_i \frac{\Sigma_{p_i}}{N_0};$$

Σ_{p_i} = macroscopic potential scattering cross section of isotopes i in the sample;

N_0 = absorber atom density in the sample;

$\bar{a}$ = correction factor for the rational approximation.

We assume $\bar{a} = 1.35$.

Table 6 lists the effective σ_p values for the various samples.

The Doppler changes in cross section were calculated for a series of σ_p values for U-238 and U-235 over the

TABLE 6. EFFECTIVE BACKGROUND SCATTERING OF THE SAMPLES, σ_p , IN BARNS

Sample	$\sigma_{p_0}^{28}$	$\bar{a}/lN_{28}$	$\sigma_{p_{28}}$	$\sigma_{p_0}^{25}$	$\bar{a}/lN_{25}$	$\sigma_{p_{25}}$
N1	17.16	30.66	47.82	2338	4176	6512
N5	17.15	68.42	85.57	2355	9396	11750
N6	15.36	36.87	52.23	6879	16510	23390
N8	17.12	31.55	48.67	2344	4319	6663
N9	17.12	47.37	64.49	2344	6485	8829
N11	10.05	11.89	21.94	2471	2927	5398
LE1	21.03	39.35	60.38	145.2	271.8	417.1
LE2A	21.02	43.19	64.21	145.2	298.3	443.6
LE3	21.05	38.86	59.91	145.4	268.4	413.8
HE1A ₁	769.3	1510	2279	18.04	35.40	53.44
HE1A ₂	768.7	1398	2166	18.05	32.81	50.86
HE2	769.2	1450	2219	18.04	34.01	52.05
HE3	147.2	174.4	321.6	10.73	12.70	23.43
HE4	746.3	2889	3636	17.97	69.56	87.53
HE5	547.9	2046	2594	12.85	47.98	60.85
E33A	28.07	51.22	79.29	55.42	101.1	156.6
E50A	35.62	67.11	102.7	35.16	66.23	101.4
E66A	52.03	99.86	151.9	125.7	49.31	75.00

range covered by the experimental samples, using the ERIC-2 and RP-270 codes. Both of these codes calculate broad-group cross sections from the formulas

$$I_i = \sigma_p \int \frac{\sigma_R}{\sigma_i} dE;$$

$$\bar{\sigma}_{R_i} = \sum_i \frac{I_i}{\Delta E},$$

where the summation extends over all resonances in the broad group of width ΔE .

The resonance parameters used are the same as those used for the reactor calculations. Figure 9 summarizes the results for the various groups for U-238 between room temperature and 1100°K as a function of the equivalent potential scattering cross section.

4.3. Comparison of U-238 Doppler Measurements with Calculations as a Function of σ_p

The measured U-238 Doppler results (1100°K — 293°K) are compared with calculation as a function of σ_p in Fig. 10. The data are corrected for expansion effects, the U-235 Doppler effect has been subtracted, and the data are normalized per kg of U-238. The measured results shown include the 1-in., 3/4-in., and 1/2-in.-diameter, natural UO₂ elements; the 1-in.-diameter with ratios of U-238 to U-235 of 7:1, 2:1, 1:1, and 1:2 UO₂ samples; and the 1-in.-diameter depleted uranium metal and 1/2-in.-diameter UC elements. The expansion corrections were made as outlined above. The U-235 contributions were taken from the U-235 experimental results, corrected for expansion, and adjusted for the difference in σ_p on the basis of calculations to be discussed below.

Calculated results are shown for the MC²-25 and the 801-25 set calculations. To illustrate the effect of uncertainties in the σ_p of U-238 for the core, calculations were also done with cross sections in which the U-238 capture cross section in MC²-25 in the groups 14 through 25 was modified for $\sigma_{p25} = 15$ b, and 35 b. The results are designated on the figure as MC²-15 and MC²-35. In these modifications the value of σ_{p25} remains at 78 b. It is believed that the calculation of the U-238 σ_p in the core is uncertain by about 2 to 5 b. (The homogenized σ_p for U-238 in the test region is 33 b.) The quoted uncertainty is due to the irregular spacing of the U-238 plates in the test region and the uncertainty in the $\bar{a}$ factor. It can be seen that the U-238 Doppler effect is fairly sensitive to the choice of σ_p for the reactor U-238.

The perturbation normalization integral is about 5% higher for the 801-25 set than that for the MC²-25 set. This point is discussed further,⁽¹³⁾ but our conclusion is that the results are unlikely to be in error by as much as

FIG. 9. U-238 Capture Cross Section Doppler Differences (1100°K — 293°K) as a Function of Group.

FIG. 10. Measured and Calculated U-238 Doppler Effect in ZPR-6 Assembly 4Z (1100 — 293°K).

10% due to errors in the perturbation normalization. (Both the 801-25 and MC²-25 values are within 5% of the transport-theory value.)

4.4. Possible Heated Sample—Unheated Environment Interaction Effects

Inspection of Fig. 10 indicates that in the set of U-235/U-238 mixture samples in which the size, density, and chemical form of the elements was constant and the U-238 σ_p variation was due to isotopic variations, the observed Doppler effect decreased with increasing σ_p , in reasonable agreement with the theoretical predictions. In the samples in which the σ_p of U-238 varied due to differences of sample size and chemical composition, however, the observed Doppler effect apparently increases with decreasing σ_p , although the results do show some scatter. This type of behavior as a function of σ_p and the sample type might be expected if the resonances in the unheated U-238 in the core around

the Doppler element were interacting with the heated-sample resonances.

It is possible to show a rough correlation between the sample type and the apparent interaction effect on the basis of simple theory. To a first approximation, the fraction of the neutrons interacting in the sample at a given energy which made their last collision outside the samples is given by the ratio $\sigma_e/(\sigma_e + \sigma_{p0})$, where $\sigma_e = \bar{a}/\bar{N}_0$ in the notation of Sect. 4.2. Figure 11 shows a plot of the percentage deviation of the results from the MC²-25 predictions as a function of the percentage of external source. The choice of MC²-25 is to some extent arbitrary; the comparison would be equally valid with any of the other predicted curves. The measured deviations show a good correlation with this parameter.

A theoretical indication of the expected "hot sample-cold reactor" interaction effect was obtained from a calculation using the RABBLE code, a modified version of the RIFF-RAFF⁽¹⁴⁾ code. The energy region of the calculation was from 1.0 to 1.4 keV, and included the 25 resolved resonances of U-238 in this energy region. The geometry was a three-region cylinder. The inner region comprised a 0.5625-cm-radius U-238 metal rod;

FIG. 11. Correlation of Deviation of Measured U-238 Doppler Effect (1100 – 293°K) from Calculated Value with External Source Fraction.

the second region was a 0.5625-cm annulus which contained either void or various amounts of iron; the outer region had an outer radius of 3.0 cm and was a typical fast reactor composition. Table 7 lists the results for a temperature change from 290 to 750°K. The listed changes in cross sections for the rod and the cell are defined as reaction rates in either region divided by the flux in that region. The listed changes in reaction rates are normalized to the reaction rates at 290°K and are also normalized per U-238 atom in the rod. The sum of the two gives an indication of the expected effect in this energy region. The total amount of steel surrounding the Doppler element in the experimental arrangement is approximately 0.17 in., so that the results should fall somewhere between the calculated result for the 1/8-in. and 1/4-in. steel cases, predicting a result in the neighborhood of a 10% increase in reaction rate. This is, of course, not directly relatable to the reactivity change, and the reactor compositions used for these cases are not identical to the Assembly 4Z compositions, but the general trend and magnitude of the results is probably meaningful.

A measurement to evaluate the interaction effect was done in this series. The U-238 plates in a region around the Doppler element were replaced with a quantity of molybdenum calculated to keep the Doppler-range fluxes and adjoints the same as they were with the U-238 present. The aim was to remove the interacting cold resonances from the vicinity of the Doppler element. The measured U-238 Doppler effect in this configuration did not differ significantly from the result when the U-238 plates were present.

It is tentatively concluded, however, that the interaction effects enter to some degree, and further work, both experimental and theoretical, is proceeding. However, it is believed that the uncertainties from this phenomenon are, at worse, no greater than the uncertainties in both sample and reactor cross sections. Further, the direction of the necessary correction is known and the magnitude is not expected to be much greater than 10 or 20%, so that this effect is not expected to lead to large uncertainties in the interpreta-

TABLE 7. CALCULATED CHANGES IN CROSS SECTIONS AND REACTION RATES IN THE ENERGY REGION FROM 1.0 TO 1.4 KEV FOR HEATED U-238 ROD IN UNHEATED ENVIRONMENT

Region			$\delta\sigma_r$		$\frac{\delta A}{A}$			
1	2	3	Rod	Cell	Rod	Cell	Total	% Diff.
U metal	Void	No U-238 Resonances	0.16793	—	0.17418	—	0.17418	0
U metal	Void	With U-238 Resonances	0.17748	0.001589	0.24627	-0.02034	0.22593	+29.7
U metal	1/8-in. FE	With U-238 Resonances	0.17418	0.001415	0.21363	-0.01462	0.19901	+14.3
U metal	1/4-in. FE	With U-238 Resonances	0.17198	0.001091	0.19742	-0.01135	0.18607	+6.8

tion of the results. It can be seen from Fig. 10 that MC²-25 and the 801-25 cross-section sets, both of which have the same σ_p for the uranium isotopes, predict results about 20% apart. The 801-25 set appears to match the results as shown better, but when the possible interaction effects are considered the MC²-25 set probably gives the better result. It is concluded that there is little basis for choice between the two sets, and that either probably predicts the U-238 Doppler effect satisfactorily for power reactor purposes.

4.5. Comparison of U-235 Doppler Measurements with Calculations as a Function of σ_p

The U-238 Doppler results, 1100°K – 293°K, are shown as a function of σ_p in Fig. 12. Again the data are corrected for expansion effects: the U-238 contribution has been subtracted, and the data are normalized to 1 kg of U-235. The data shown are for the 1-in. metal element, the 1-in.-diameter oxide elements, and the 1/2-in.-diameter oxide elements. Calculated results are shown for the MC²-25 cross section set. The calculated U-235 Doppler effect is not very sensitive to the σ_{p28} in the core or to the choice between MC² and 801 sets. It can be seen that although the variation with σ_p of the measured data is analogous to the predicted variation, the predictions far overestimate the measured Doppler effect. The measured Doppler effect is only about 38% of the predicted, and it is apparent that a serious review of the parameters and theory is necessary.

4.6. Results for U-235-U-238 Mixtures

The results for mixtures, shown in Fig. 7, have been replotted in Fig. 13 to show explicitly the agreement of the results with a simple proration of the results of the two isotopes alone on the basis of the masses of each isotope present. The experimental data are for the 1100°K – 293°K measured reactivity change corrected for expansion effects. The solid line falling almost through the points is a simple proration of the U-235 and U-238 experimental results, as measured with the elements HE-2 and N-1, corrected for the differences in σ_p for the individual isotopes in the various mixtures. Also shown for comparison are the Doppler effects of the individual constituents, again corrected for σ_p differences. It can be seen that the Doppler effect in the mixtures is given quite accurately by prorating the measured Doppler effects of the individual constituents on the basis of the amounts of each constituent present.

4.7. Effect of Sodium Voiding on the Doppler Effect of U-238 and U-235

Figure 14 compares the measured reduction in the U-238 and U-235 Doppler effects, 1100°K – 293°K and

FIG. 12. Measured and Calculated U-235 Doppler Effect in ZPR-6 Assembly 4Z (1100°K – 293°K).

FIG. 13. Doppler Reactivity Changes for 1-in.-diameter Samples of UC₂ Containing Various Ratios of U-235 and U-238.

500°K – 293°K, with the reductions calculated using the 801-25 set. As previously mentioned, the calculations indicated that the spectrum in the vicinity of the Doppler samples was that of a fully voided, full-scale core of the same composition. It can be seen that the calculations considerably overestimate the reductions in the Doppler effect of both U-235 and U-238. The calculations were done for U-238 σ_p values the same as in the full sodium case. Removal of the sodium actually reduces the σ_p of the core U-238 by 1–2 b, which if taken into account would decrease the calculated ratios for U-238 by 1–2%, a trivial amount compared to the ob-

FIG. 14. Calculated and Measured Doppler-effect Reduction on Sodium Voiding.

served discrepancy. It is considered very significant from the point of view of safety that the measured reductions in Doppler effect are very considerably less than those predicted by current theory.

5. Discussion and Conclusions

The results, of course, cannot be interpreted directly as values for the Doppler effect in some "equivalent" full-scale core. The σ_p values for the samples are not the same as the σ_p for the uranium plates in the mockup, or for fuel in a projected power reactor of the same composition. Further, the apparent presence of hot-cold resonance interaction does not allow a straightforward adjustment of the experimental result for the σ_p variations. However, if adjustment is made on the basis of the MC²-predicted variation in Doppler effect with σ_p , to a value of $\sigma_{p28} = 25$ b, the U-238 Doppler coefficient $T(dk/dT)$ for a full-size core of the same composition as the test zone would be $T(dk/dT) = -0.011$, extrapolated from the one-inch U-238 metal result; and $T(dk/dT) = -0.013$, extrapolated from the one-inch oxide result. The difference in these two numbers probably gives a reasonable indication of the current uncertainty in the extrapolation of the heated-sample numbers to a full-size system. Further work is needed on the interaction-effect problem to reduce this uncertainty.

The value of the Doppler-coefficient exponent γ (where $T^\gamma[dk/dT] = \text{constant}$) is of interest in extrapolating the experimental results to the temperature regions of interest to power reactor safety studies. A least-squares fit of the U-238 data gives a value for γ of 0.85 ± 0.06 for U-238. The extrapolation from the temperature range of these measurements to the tem-

perature ranges contemplated for ceramic fuels is so great that relatively small uncertainties in this exponent can lead to large uncertainties in the residual Doppler effect at very high temperatures. Extension of the experimental temperature range is necessary, along with the interaction effect work, to reduce this uncertainty.

The derived value of the exponent for U-235 depends to some degree on which data is fitted, because of the scatter in the data from the various elements. If only the data from FE and NE elements are used, the least-squares value of γ is 1.2 ± 0.1 . If the FE, NE, and NNE elements are considered collectively as a single ensemble, the value for γ becomes 1.5 ± 0.4 , with the larger uncertainties reflecting the scatter of the data between the various elements. The U-235-metal data were not considered in evaluating the exponent because of the very large expansion corrections involved with this element.

Finally, it is concluded that current theory gives a relatively good estimate of the U-238 Doppler coefficient. Despite uncertainties in the exact interaction effect and uncertainties in cross sections, agreement between theory and experiment is probably within $\pm 20\%$.

The method for U-235-expansion-effect corrections outlined in this paper provides a consistent explanation of the various U-235 results. As a function of σ_p^{25} the results for the Doppler effect in U-235 are consistent with the theoretically predicted trend, but are smaller than the predicted magnitude by almost a factor of three.

Experimental results for mixture samples do not indicate measurable effects of interaction between U-235 and U-238.

A point of some importance to safety analyses of reactors of the type studied here is the significant calculated overestimate of the reduction in the Doppler effect upon sodium voiding. The predicted reduction of about 50% is compared to a measured reduction of about 35%.

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Discussion

Mr. Pitterle (APDA): You indicated that there is no buffer zone around the Doppler element as they have used it in some of these experiments.

Mr. Till: That is true. What I did not mention was that at the start of this series of experiments we replaced the U-238 around the element by a quantity of molybdenum calculated to keep the Doppler fluxes and adjoint constant. We didn't, within our statistics, measure any difference between these two conditions.

Mr. Pitterle: What size of sample was present for this?

Mr. Till: It was the one-inch oxide sample.

Mr. Tiren (AB Atomenergi): Would you comment on the heterogeneity effects due to the surrounding core region and structural materials?

Mr. Till: I tried to indicate the influence of these effects with the U-238 data. From the results, one would think that, with the particular amounts of steel that we have around our Doppler element, the uncertainty due to heterogeneity effect may be 25 to 30%. The calculation that I mentioned used a modified version of the RIFF-RAFF code over a limited energy range from 1 to 1.4 keV. The results of this calculation indicated that one would expect an increase in the absorption of about 25% for a half-inch metal element with no intervening steel, about 12% with $\frac{1}{8}$ inch of intervening steel, and about 7% with $\frac{1}{4}$ inch of intervening steel.

Mr. Travelli (ANL): When you remove the sodium, the potential scattering cross section can change considerably. Did you take this into account when you compared the results with theory?

Mr. Till: Yes. We calculated that for our particular arrangement. The change in potential scattering of the U-238 in the core was about 1 or 2 barns, a deviation from the original 25-barn value. This would give a difference in the results of about 1 or 2% as compared to 15 or 20% difference that we found.

Mr. Khairallah (CEN): Did you take into account the interference effect between the U-238 and the U-235 in the mixed samples?

Mr. Till: No. The point that I was trying to demonstrate was that if one takes the measured U-238 Doppler effect and the measured U-235 Doppler effect and makes the proper correction in Σ_p , then one reproduces the experimental results of the mixture almost exactly.

Mr. Khairallah: Does this mean that the interference effect is negligible?

Mr. Till: I think this depends largely on the way that the interference effect is defined.

Presented by T. H. Springer

Dependence of the Doppler Coefficient of Reactivity for Heavy Elements on Chemical Form, Surface-to-mass Ratio, and Neutron Spectrum*

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ABSTRACT

A series of experiments has been carried out in a fast-neutron spectrum, characterized by a median fission energy of 62 keV, in order to measure the Doppler coefficient and other related temperature effects for a variety of materials which are of particular interest in fast power reactor technology. Special emphasis has been placed on the U^{235} isotope which has been investigated in several chemical forms which are amenable to utilization in high-temperature, high-efficiency fuel elements. Changes in the size and chemical composition of samples of this isotope and other heavy-element isotopes have been made in order to evaluate the effects on the Doppler coefficient of changes in the surface-to-mass (S/M) ratio and changes induced by the addition of carbon or oxygen to form carbides and oxides. In addition, the effects of localized spectrum perturbation on the Doppler coefficient of thorium have been studied by surrounding the sample with "blankets" (up to $\frac{1}{2}$ in. thick) consisting of heavy resonance absorbers, structural materials, and several types of scatterers, including sodium.

A series of experiments has been carried out in a fast-neutron spectrum in order to measure the Doppler coefficient and other related temperature effects for a variety of materials which are of particular interest in fast power reactor technology. The measurement technique consisted of oscillating the sample of interest in and out of a 60-cm diameter test region.

Studies of the effects of the surface-to-mass ratio and the chemical form of the sample on the Doppler coefficient of reactivity, on the absolute reactivity worth, and on the expansion coefficient of reactivity have been carried out. In addition, small "blankets" of heavy-element absorbers, scattering materials, etc., have been placed at the center of the reactor test

region in order to investigate Dancoff interactions, resonance interference, and gross spectrum changes on the three aforementioned parameters.

General information regarding the design and operation of the reactor in which these measurements were carried out have been given in References (1), (2), (3), and (4), and the techniques of measurement and data analysis described therein apply also to the data presented in this paper. The particular spectrum in which these studies were made had a median fission energy of 62 keV with an energy distribution as given in Fig. 1. The 60-cm-diameter, fast-spectrum test region consisted of U^{235} , Th, C, Al, and Fe roughly with atom densities of 1.69×10^{21} , 4.32×10^{21} , 2.25×10^{22} , 1.51×10^{22} , and 1.29×10^{22} atoms per cubic centimeter of test region. The U^{238} contained in the 93% enriched uranium fuel plates had an atom density of about 1.23×10^{20} atoms/cm³.

Table 1 lists some of the more pertinent information regarding temperature effects for the U^{235} isotope. The Doppler reactivity change is represented here in terms of a specific reactivity change, i.e., the change in reactivity per unit mass of sample over some arbitrary temperature interval. It is to be noted, however, that, in this spectrum, the variations of the reactivity worth with temperature are decidedly nonlinear; consequently, for purposes of defining a single parameter which describes the coefficient, the temperature interval is taken to be 20 to 600°C. Although any curvature is of considerable importance with regard to theoretical expectations and has been investigated over a very wide range of neutron-energy distributions, this phenomenon will be neglected for present purposes (see Ref. 5).

In the light of present-day calculational methods for fast reactors, it is particularly interesting to compare some of the measured Doppler effects and absolute worths listed in the table with comparable analytical values. The analytical techniques used to obtain the listed results follow, in principle, those described in References 2 and 3, although, in this case, more recent

* Work performed under Contract AT(11-1)-GEN-8 for the U. S. Atomic Energy Commission.

FIG. 1. Neutron-energy Distribution in Core 13-13 (62 keV).

TABLE 1. REACTIVITY WORTH AND DOPPLER-COEFFICIENT MEASUREMENTS IN CORE 13-13 U-235

Material	Total Sample Mass (gm)	Reactivity Worth at 20°C (¢)			Doppler Reactivity Change 20–600°C (¢)		
		Measured ^(a)	Calculated	Normalized Calculations	Measured	Calculated	Normalized Calculations
93% U ²³⁵ (~½ in. metal)	183.3132	+2.08903	3.2155	2.0890	+0.006226	+0.0115	+0.00747
93% U ²³⁵ (¼ in. metal)	59.1139	+0.64493	1.0383	0.6746	+0.00196	+0.0024	+0.00156
93% U ²³⁵ (⅛ in. metal)	15.1292	+0.16310	—	—	—	—	—
93% U ²³⁵ (~½ in. carbide)	103.43	+1.0942	—	—	−0.000772	—	—
93% U ²³⁵ (~ ½ in. oxide)	101.36	+0.954	—	—	+0.00236	—	—
62% U ²³⁵ (Ann. + U ²³⁸ ¼ in.)	177.7317	+1.25840	—	—	+0.00055	—	—
31% U ²³⁵ (¼ in. + U ²³⁸ Ann.)	176.9858	+0.54306	—	—	−0.00602	—	—
22% U ²³⁵ (hom. metal)	181.4599	+0.31216	—	—	−0.01293	−0.0156	−0.0101
10% U ²³⁵ (hom. metal)	180.0753	+0.02374	—	—	−0.01293	−0.0185	−0.0120

^(a) The uncertainties in the measurements are ~0.0005¢.

resonance parameters and slightly more sophisticated codes were employed. Basically the analytical values for the reactivity worths and Doppler coefficients depend upon group-averaged flux and adjoint values obtained from a spherical-geometry, homogeneous-type, diffusion calculation using the CAESAR II code.⁽⁶⁾ This code starts with an infinitely dilute, 18-group cross-section set derived from updated, basic, differential-type data for each of the elements used in the reactor. Some self-shielding is accounted for only in thorium. This flux and adjoint set are then combined with Doppler-broadened group cross sections obtained from the TRIX⁽⁷⁾ code in a more or less standard perturbation calculation which yields the absolute worth and,

when several temperatures are considered, the Doppler coefficient of a given sample. The TRIX code calculates multigroup, Doppler-broadened effective resonance integrals from which the multigroup cross sections are derived. The code uses basic resonance parameters, a geometry factor, a concentration factor, and light-scattering effects in calculating resolved and unresolved resonance integrals in addition to the $1/v$ contribution.

When U²³⁵ is involved some calculational difficulties arise in connection with the single-level formulation used in the code, the sample worth being generally considerably smaller than the measured worth. To overcome this difficulty to some extent, the infinitely dilute

cross sections generated by the *TRIX* code are adjusted to correspond to the infinitely dilute cross-section set derived from the basic differential-cross-section data. This adjustment is then applied to the self-shielded, Doppler-broadened cross sections which are also calculated by the code. In general, this procedure improves the agreement, although often the analytical values become somewhat too large. The calculated worth of the $\sim 1/2$ -in.-diameter U^{235} (93% enriched) metal sample shown in the table becomes 3.216¢ after this adjustment is carried out. This value is roughly 50% too large; consequently, from more or less fundamental data, the calculated worth for the U^{235} sample is not in particularly good agreement with measured results. Calculations for some of the absorber materials which will be discussed below also appear to be somewhat too large; therefore a normalization has been applied by forcing the calculated worth of the $\sim 1/2$ -in.-dia U^{235} metal sample to correspond to that measured. This normalization factor, 0.6497, is then applied to all subsequent calculated values. This procedure produces a somewhat more consistent set (internally, at least) and suggests that the usual integral-over-the-whole-reactor which constitutes the denominator in the perturbation equation is not correctly calculated.

One further comment relating to calculated values should be made: there has been no attempt made to take into account any resonance overlap between U^{238} and U^{235} ; consequently, the Doppler effects calculated for the 10 and 22 w/o U^{235} homogeneously mixed samples consist of a simple algebraic summation of the Doppler coefficients of the two isotopes with each possessing the proper S/M factor. This procedure seems to produce, within the elementary approach used here, reasonably valid results.

Some comments pertaining to the experimental values listed in Table 1 may also be appropriate at this juncture. It is found that the measured U^{235} Doppler-reactivity changes shown in the table cannot, in some instances, be determined directly. In these cases, the total temperature coefficient is first measured simply by observing the change in reactivity which occurs in the sample as it is heated during oscillation in the reactor. This coefficient has been shown to be made up of two main temperature effects, the first being the Doppler broadening of the resonance and the second being the reactivity change associated with changes in physical size of the sample as a result of thermal expansion. The latter quantity, the expansion coefficient of reactivity, is measured separately, and an algebraic subtraction is then carried out to yield the net Doppler effect. This expansion coefficient of reactivity has been discussed at some length in several previous papers (see References 5 and 8) and, basically, one finds that

it consists of negative and positive components—a negative effect arising from an increase in the probability that a fast neutron born in the sample will escape before causing an interaction (self-multiplication), and a positive effect arising from self-shielding of incoming neutrons. In sufficiently hard spectra (the 62-keV core being in this category) the former phenomenon overrides the latter and a net negative expansion coefficient ensues.

Since the expansion coefficient of reactivity varies with the U^{235} dilution as well as with the linear coefficient of expansion, α , of the particular type of fissile sample under investigation, it is clear that for some samples, such as oxides or carbides in which α is relatively small, the expansion coefficient may be a very small part of the total temperature coefficient. In other cases, in which the sample is specially constructed, the expansion can be entirely prevented, and thus the Doppler effect will be obtained directly in a heated-sample reactivity measurement. For U^{235} metal, on the other hand, the expansion coefficient of reactivity may completely override the change due to Doppler broadening of the resonances.

Five independent methods for measuring the expansion phenomenon, all yielding essentially identical results, were devised and carried out about two years ago in other fast spectra. At present only two or three of these methods are routinely used, and one of the most simple and straightforward methods is based upon a determination of the variation in the worth-per-unit-mass with changes in the S/M ratio—or rather the product Nr , which turns out to be the same thing for the infinite cylinder case, but is analytically a more convenient term to use. In the latter notation, N is the number of U^{235} atoms/cm³ and r is the radius of the sample in cm.

The results of a typical experiment of this type are given in Fig. 2. By and large the data are reasonably consistent, indicating that, for this spectrum, the expansion coefficient is negative and roughly linear in Nr . This linearity is significant in that the data points correspond to a wide variety of samples (see Table 1) possessing dilutions from as low as the 22 w/o U^{235} up to the 93% metal samples and also including the oxide and carbide specimens. Thus, the worth per unit mass of U^{235} is not strongly affected by chemical form.

The uppermost solid line shown in Fig. 2 has been calculated by perturbation theory in a manner similar to that described above. In this case, however, instead of using *TRIX*-generated cross sections in the perturbation equation, a set of modified infinitely dilute cross sections were employed. This modification involved the use of escape probabilities for infinite cylinders as derived by Case, deHoffman, and Placzek.⁽⁹⁾ The abso-

lute worth of the sample again turns out to be too large; consequently, for purposes of conveniently displaying the data, the value for $\frac{1}{2}$ -in.-diameter cylinder has been normalized to the calculated curve by adding an appropriate correction factor. This same correction factor has also been added to all other experimental points, but the important factor in determining the expansion coefficient of reactivity is the slope of the line and, from this viewpoint, the agreement between experiment and theory is quite good. Expansion coefficients are derived from the data contained in Fig. 2 by simply multiplying the slope of the line (in terms of the change in the worth-per-unit-mass per unit change in the product Nr) by the product of the sample mass and the change in Nr over some temperature interval. Once the expansion coefficient has been determined in this manner, it is algebraically subtracted from the total measurement coefficient to obtain the values shown in Table 1.

Most of the present measurements—and this generalization includes all of the samples listed in Table 1—involve samples approximating infinite cylinders; however, Fig. 2 contains two other sets of points (given by the diamond and square symbols) obtained from measurements using cylindrical specimens in which the length-to-diameter ratio was less than one. The same normalization was applied to these points as was applied to those represented by the infinite cylinders. The solid line corresponding to the short cylinders was calculated using escape probabilities which were derived by a procedure recommended in a recent paper by Gubbins.⁽¹⁰⁾ The agreement with the calculated values is again seen to be reasonably good, although there are only two points on which a judgment can be based. The point represented in the square was obtained from a sample in which the length-to-diameter ratio was as small as 0.281 and, although it consisted of nearly 100% theoretical density metal, it approaches the infinitely dilute condition.

The analysis of the data of Table 1 in terms of the change in the Doppler effect with S/M is not very clear at the present time. If one calculates the specific Doppler change, it is seen that the results for the metal samples are nearly invariant with changes in S/M , whereas the oxide and carbide samples show a stronger dependence, the coefficient for the carbide being actually nearly zero within statistics. These results imply that oxygen and carbon have a much stronger effect on the specific Doppler coefficient than upon the specific worth. Note, for example, that the $\frac{1}{2}$ - and $\frac{1}{4}$ -in.-diameter metal samples would yield a specific Doppler coefficient of 3.38×10^{-5} and 3.31×10^{-5} $\Delta\rho$ /unit mass, respectively. It will be seen below, however, that the effect of carbon and oxygen on the specific Doppler

FIG. 2. Specific Reactivity vs. Nr . FIG. 3. Specific Reactivity of Several U^{235}/U^{238} Metallic Samples with Nearly Constant Surface-to-mass Ratio. FIG. 4. Change in Reactivity (20-600°C) per Unit Mass as a Function of Percent U^{235} Core 13-13 (62-keV MFE).

TABLE 2. MEASUREMENTS OF REACTIVITY WORTH AND DOPPLER COEFFICIENT IN CORE 13-13

Material	Total Sample Mass (gm)	Reactivity Worth at 20°C (¢)			Doppler Reactivity 20–600°C (¢)			Specific Doppler (¢/gm)
		Measured ^(a)	Calculated	Normalized Calculations	Measured	Calculated	Normalized Calculations	
U ²³⁸	179.1653	–0.18273	–0.15603	–0.1014	–0.0152	–0.02393	–0.01555	-8.50×10^{-5}
U ²³⁸ (Ann)	117.87	–0.11976	—	—	–0.00998	—	—	-8.51×10^{-5}
U ²³⁸ (¼ in.)	59.9856	–0.06026	–0.05900	–0.0383	—	—	—	—
U ²³⁸ O ₂	81.2508	–0.054	–0.06343	–0.0412	–0.00595	–0.00894	–0.00581	-8.38×10^{-5}
U ²³⁸ C	100.0	–0.09386	–0.08553	–0.0556	–0.00905	–0.01197	–0.00780	-9.54×10^{-5}
Th	113.99	–0.20033	–0.39540	–0.2569	–0.0121	–0.01643	–0.01067	-1.06×10^{-4}
Th (¾ in.)	36.238	–0.06334	–0.13200	–0.0858	–0.0038	–0.00508	–0.00330	-1.05×10^{-4}
ThO ₂	74.4219	–0.120	–0.22723	–0.1476	–0.0071	–0.00920	–0.0060	-1.08×10^{-4}
W(Natural)	187.97	–0.28573	–0.35102	–0.2280	–0.0109	–0.0165	–0.0107	-5.80×10^{-5}
W-182 ^(b)	183.2024	–0.25423	–0.29816	–0.1937	–0.0099	–0.0130	–0.0084	—
W-184 ^(b)	176.6789	–0.19925	–0.20157	–0.1310	–0.0063	–0.0088	–0.0057	—
W-186 ^(b)	187.3724	–0.13702	–0.25125	–0.1632	–0.0057	–0.0130	–0.0084	—
Mo	100.10	–0.16999	—	—	–0.0063	—	—	-6.29×10^{-5}
Ta	163.78	–0.79527	—	—	–0.0496	—	—	-3.03×10^{-4}
Nb	84.94	–0.22051	—	—	–0.0153	—	—	-1.80×10^{-4}

^(a) The uncertainties in the measurements are approximately ±0.0005¢.
^(b) Separated tungsten isotopes loaned through the courtesy of NASA-Lewis Research Center.

FIG. 5. Arrangement to Obtain Spectrum Hardening.

of the fertile isotope thorium and of U^{238} is not so strong.

The variation of the Doppler coefficient of U^{235} with neutron spectrum has been dealt with at some length in a previous paper.⁽⁵⁾ We pointed out that many of the data points relating to the spectrum effect were obtained by surrounding the sample with polyethylene "blanket" in order to soften the spectrum. Considerable difficulty in the calculation of the spectrum in these blankets was noted. Since the time of publication of the aforementioned data, some significant improvements in calculational methods have led to the conclusion that the spectrum under the polyethylene is considerably softer than previously supposed. In view of this new information, we now conclude that the point of zero Doppler (i.e., the neutron energy at which the Doppler effect changes from a positive to a negative effect in a $\sim 1/2$ -in.-dia $\times$ 4-in.-long 93% U^{235} metal sample) will be nearer the spectrum characterized by a median fission energy of 500 to 700 eV rather than the 20- to 30-keV region indicated in Ref. 5. Measurements presently underway in a 10-keV "clean" core may provide a more reliable verification of this conclusion.

Another interesting facet of the information contained in Table 1 concerns the variation of the specific Doppler effect and the specific worth with the weight percent of the contained U^{235} in a series of mixed U^{235} - U^{238} metallic samples of approximately constant mass. Some of these samples, such as the 93% U^{235} ($\sim 1/2$ in. in dia), the 22 w/o U^{235} , and the 10 w/o U^{235} , represent homogeneous mixtures of the two isotopes. The heterogeneous samples such as the 62 w/o and 31 w/o specimens are constituted of one zone consisting largely of one isotope (a 0.438-in.-OD $\times$ 0.250-in.-ID $\times$ 4.00-in.-long annulus) and another zone consisting of a second isotope placed inside the first (a 0.250-in.-dia $\times$ 4.00-in.-long cylinder). The results of these studies (see Figures 3 and 4) indicate that, within statistics, resonance interference between U^{235} and U^{238} is not significant. These conclusions are particularly interesting with regard to the suitability of experimental measurement carried out in heterogeneous critical assemblies and calculation based upon homogeneous approximations. The "zero percent U^{235} " point was derived from measurements on a 0.22% U^{235} depleted sample. In the spectrum utilized in these studies, the point of zero specific worth and zero specific Doppler effect lies at 8.6 and 64.4% U^{235} , respectively.

Studies similar to those described above have been carried out with the fertile materials thorium and U^{238} as well as with some refractory metal, pure absorbers (see also Ref. 11). These results are delineated in Table 2. One can see from the data that the specific Dop-

TABLE 3. DOPPLER COEFFICIENT OF THORIUM METAL^(a)
Core 13

Blanket	Reactivity Worth (ϵ)	Reactivity Change (20–800°C) (ϵ)	Average Doppler Coefficient ($^{\circ}\text{C}^{-1}$) ($\times 10^4$) $\left[\frac{1}{\rho} \right] \frac{d\rho}{dT}$	Blanket Effect (%) (± 2.0)	Calculated Median Fission Energy (eV)
$1/2$ -in. B ₄ C	−0.1249	−0.00438	−0.350	−67.1	94,000
$1/2$ -in. Natural U	−0.1949	−0.0128	−0.841	−21.2	—
0.3-in. Natural U	−0.2066	−0.0145	−0.899	−15.8	—
No blanket	−0.2003	−0.0167	−1.068	—	61,700
$1/2$ -in. SS	−0.2064	−0.0174	−1.080	+1.1	—
$1/2$ -in. Na	−0.1965	−0.0174	−1.135	+5.8	—
$1/2$ -in. C	−0.2325	−0.0222	−1.224	+14.5	—
$3/8$ -in. Th	−0.1762	−0.0184	−1.338	+25.2	—
$1/2$ -in. Polyethylene	−0.9924	−0.108	−1.360	+27.2	0.029
$3/8$ -in. Polyethylene	−0.7800	−0.105	−1.682	+57.4	0.031
$1/4$ -in. Polyethylene	−0.6156	−0.096	−1.949	+82.3	0.037
$1/8$ -in. Polyethylene	−0.3849	−0.066	−2.1984	+105	30.7

^(a) The sample dimensions are 0.438-in. diameter $\times$ 4.00-in. length, and it has a mass of 112.5676 gm.

pler effect in thorium and U^{238} is relatively insensitive to the S/M ratio and to the chemical form, although for U^{238} some measurable contribution from carbon may be detectable within statistical uncertainties. The latter phenomenon may provide additional evidence on the seemingly strong importance of carbon in the U^{235} carbide discussed above. The expansion coefficients of reactivity for U^{238} and thorium have been measured, but amount to only about 5% of the total temperature effect.

The U^{235} normalization discussed in a previous section has been applied to the calculated values for these isotopes and the latter one reported in a separate column. The calculated worth and Doppler effect for natural tungsten consist of simple summations of these factors as contributed by each isotope.

Extensive studies of temperature effects in the refractories have been carried out in the 62-keV spectrum and are also shown in Table 2. Note particularly the results obtained for the separated tungsten isotopes, which were provided on a loan basis through the courtesy of NASA-Lewis Research Center. These results and those applying to some of the other refractories are the first such measurements ever performed in a fast-neutron spectrum. All the separated tungsten samples are enriched to more than 93% in the isotope

designated along with the element symbol. It should be noted that the expansion coefficient of reactivity for natural tungsten and tantalum have been measured and found to be reasonably significant, amounting to $2.71 \times 10^{-6} \text{ } \beta / ^\circ\text{C}$ and $6.0 \times 10^{-7} \text{ } \beta / ^\circ\text{C}$ for the tantalum and tungsten samples listed. Corrections for expansion in niobium and molybdenum appear to be negligibly small, however.

The Doppler coefficient of thorium has been measured in several neutron spectra which were obtained by surrounding the oscillator hole at the center of the test region with various scattering and absorbing materials. Thus, selected spectrum modifications can be accomplished in the region in which Doppler coefficients are determined. General softening of the spectrum is obtained by surrounding the sample by moderators ranging from the slightly effective stainless steel to highly effective blankets of polyethylene. The fine structure of the spectrum is changed by use of blankets of resonance absorbers, and gross spectrum hardening is accomplished through the use of strong absorbers such as B_4C . The arrangement of blankets of this type is shown in Fig. 5. The results of measurements of this type are shown in Table 3. One of the most interesting observations concerns the Doppler-coefficient increase and the sample-reactivity decrease under the thorium blanket. The moderators, on the other hand, increase the coefficient as the spectrum softens until the absorption by thermal neutrons begins to predominate over that by resonance neutrons, at which point the trend reverses.

Turning now to some more recent developments, we can report that considerable progress has been made in the direction of attaining measurements at higher temperatures. Previous reports on fast-spectrum Doppler measurements performed at this laboratory over the last $1\frac{1}{2}$ to 2 years were generally based on results obtained with heaters capable of routine operation at approximately 1250°K , and coefficients for thorium up to this point have been reported. A new heater has been developed which has reached 2400°K in laboratory tests, and its construction principles have been applied to the fabrication of a heater which has been used in-pile at temperatures up to 2300°K . The in-pile heater is 2 in. in diameter by 13 in. long, and requires approximately 600 W to attain 2000°K .

Measurements down to 20°K are reported in a separate paper at this conference.

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Discussion

Mr. Kato (ANL): You depend strictly on calculation for your knowledge of spectra. Have you done any other measurements, such as fission ratios or central perturbation measurements, to confirm the spectrum that you calculate? I notice that the experimental values for materials like tungsten and U-235 at room temperature disagree rather markedly with the calculated values. Do you really know what your spectrum is?

Mr. Springer: We have done some of the routine type of fission-counter traverses and source importance measurements. Certainly inside materials like polyethylene blankets we can't claim that we know the spectrum very well. Generally we get good agreement with fission-counter measurements. Part of the problem with calculation, particularly in the absolute worth measurement, involves the integration over the whole reactor. Hence, the normalization to U-235 helps that problem to some extent.

Doppler Coefficient Measurements in the Cryogenic Temperature Region*

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ABSTRACT

The Doppler-coefficient measurement program at Atomics International has been extended into the cryogenic temperature range. The inverse temperature dependence of the coefficient results in relatively larger reactivity changes at low temperature, although crystalline binding effects begin to be significant in this range. The effective-temperature model has been used to correct for binding effects. The method used for the measurements was essentially the same as that for previous work at higher temperatures—the power fluctuations as a sample was oscillated in and out of the reactor were Fourier-analyzed to give the reactivity effect of the sample. Different sample temperatures were obtained using a “Cryo-Tip” (T. M. Air Products and Chemicals, Inc.) refrigerator, which uses Joule-Thomson expansion of nitrogen and hydrogen to cool the sample. Measurements are reported for thorium and tungsten in a 62-keV median-fission-energy spectrum. Also included are measurements at higher temperature to cover the temperature range from 100 to 1000°K. The measured temperature dependence of the Doppler effects agree well with calculated values, with the crystalline binding effects being evident in tungsten.

1. Introduction

The Doppler-coefficient measurement program at Atomics International has been extended into the cryogenic temperature range to take advantage of the relatively large reactivity changes which occur there. The large changes arise from the nature of the temperature dependence of the Doppler coefficient. It is approximately proportional to $T^{-\gamma}$ where γ ranges from $1/2$ for thermal spectra to $1\frac{1}{2}$ for a hard spectrum.⁽¹⁾ The shape of the curve of the Doppler effect (the temperature integral of the Doppler coefficient) which results from this dependence is one in which the greatest

reactivity changes occur at low temperatures, with a flattening at higher temperatures. For example, for a value of γ of 1 (which is typical for a fast reactor spectrum) the Doppler effect goes as $\ln T$ and the reactivity change between effective temperatures of 100 and 300°K equals that from 300°K to 900°K. Viewed as an extension of the temperature range of previous measurements, cooling as low as 100°K gives the same additional information as heating above 2000°K. Measurements in the cryogenic range, then, can contribute greatly to the determination of the magnitude and temperature dependence of the Doppler coefficient.

The relative advantage of measurements at low temperatures is tempered somewhat, however, by the crystalline binding effects which begin to appear at temperatures of the order of the Debye temperature. At higher temperatures the thermal motion of the absorber atoms can be considered to be Maxwellian and the usual free-gas approximation holds for the temperature-dependent, effective absorption cross section. Following Dresner,⁽²⁾ this can be written

$$v\sigma_{\text{eff}}(v) = \int |\mathbf{v} - \mathbf{V}| \sigma(|\mathbf{v} - \mathbf{V}|) P(\mathbf{V}) d^3Z,$$

where $\mathbf{v}$ and $\mathbf{V}$ are the velocities of the neutron and absorber nucleus, respectively, $\sigma(|\mathbf{v} - \mathbf{V}|)$ is the cross section as a function of the relative velocities, $P(\mathbf{V})$ is the Maxwellian velocity distribution, and the integral is over volume. At low temperatures the absorber-atom vibrations are reduced by crystalline binding and the Maxwellian distribution no longer holds; however, it has been shown by Lamb⁽³⁾ that this distribution may still be used if an effective temperature is substituted for the thermodynamic temperature. Using the Debye model of the solid state, the effective temperature is

$$T^* = \frac{3}{2} \theta_D \int_0^1 X^3 \coth\left(\frac{X\theta_D}{2T}\right) dX,$$

where the parameter θ_D is the Debye temperature and

* Work performed under Contract AT(11-1)-GEN-8 for the U. S. Atomic Energy Commission

FIG. 1. Capsule for Doppler-effect Measurements at Cryogenic Temperatures.

X is the energy in terms of resonance half-widths $[(E - E_0)/(\Gamma/2)]$. The relationship of T and T^* derived from this equation⁽⁴⁾ was used to correct for binding effects.

2. Experimental Method

The method used for the measurements in the cryogenic region is essentially the same as that used for measurements at higher temperature previously reported.⁽⁵⁾ Briefly, a sample is placed inside a capsule and cooled or heated to the desired temperature. The capsule is oscillated so that the sample moves between the center of the test region and a position outside the reactor. The resulting power fluctuations are Fourier-analyzed to give the reactivity effect of the sample. The measurements are repeated with different sample temperatures to determine the Doppler effect.

The capsule used for the cryogenic measurements is shown in Fig. 1. Its major component is the "Cryo-Tip" * refrigerator, which uses Joule-Thomson expansion of high-pressure (up to 1500 psig) nitrogen and hydrogen to cool the samples as low as 20°K.

The Cryo-Tip has separate lines for the two gases. In the nitrogen line, the colder, expanded gas is used to precool the incoming high-pressure gas. The system temperature is gradually reduced until liquid nitrogen is formed after the expansion. The incoming hydrogen is cooled below its inversion temperature by the liquid nitrogen, and a counterflow arrangement similar to that of the nitrogen eventually produces liquid hydrogen. In these experiments nitrogen is used in both gas lines for measurements to 100°K to reduce the reactivity effect of the gases and to minimize the potential hazard of the use of hydrogen. Samples have been cooled as low as 21°K, although the lowest measurements reported here are at 100°K.

The sample is placed in an aluminum holder which is in thermal contact with the liquid reservoir of the hydrogen line. It is held in place by Teflon spacers at the end of the holder. The design shown in Fig. 1, with the spacers held in place by spring washers which were compressed by the capsule cover, has been replaced to reduce the heat leakage through the end. In the improved design, the Teflon end piece is screwed to the sample holder and does not touch the capsule cover. The sample-holder assembly is supported by a spacer placed about 6 in. from the end of the Cryo-Tip.

The sample temperature is measured with chromel-constantan thermocouples in the sample holder near the Cryo-Tip and in the opposite end of the sample. A coil of chromel wire is also wound around the sample holder to serve as a heater. This is used to permit more rapid temperature changes and to heat the sample to approximately 100°C to provide overlap with the oven capsule measurements.

The sample holder and the cold tip of the heat exchanger are surrounded by a silver-plated copper heat shield which is in thermal contact with the nitrogen-line cold reservoir. The entire heat exchanger is enclosed in a cover tube which is bolted to the Cryo-Tip head. The heat exchanger section is 3 ft long to place the sample at the core center while keeping the head, with the gas connection and instrument lead-throughs, outside the matrix in which the reactor is built. The region inside the capsule cover is evacuated to provide insulation. A small ion pump is attached to the oscillator bar to provide continuous pumping during the measurements.

The capsule assembly is placed inside a square tube which is bolted to the head. A flow of dry air is maintained through this tube to prevent the buildup of moisture on the outside of the capsule cover. The other

* T. M. Products & Chemicals, Inc.

end of the head is bolted to the flange on a rod connected to the oscillator mechanism.

3. Results

The results of measurements in a 62-keV median-fission-energy spectrum with thorium and tungsten in both the oven and cryogenic capsules are shown in Figures 2 and 3. The measured reactivities were normalized to zero at 300°K and corrected for empty-capsule temperature-dependent effects. The cause of this void effect in the oven capsules is not yet known; however, its magnitude has been measured. The measured value of $(-2.0 \pm 1.0) \times 10^{-6} \text{C}^{-1}$ was used for the corrections to these measurements. The void effect of the cryogenic capsule is attributed to the reactivity effect of the nitrogen and hydrogen used for cooling. The correction was based on measurements of the effect of the gases at different temperatures measured with no sample in the capsule. The effect of the nitro-

gen at 100°K was $+0.00064 \pm 0.0003\text{c}$, resulting in a small correction. The effect of liquid nitrogen was much larger ($\sim 0.003 \pm 0.001\text{c}$, with $\sim 0.01 \text{ in.}^3$ of liquid present). The amount of liquid present was not known with sufficient accuracy to make a good correction; therefore, the measurements were made at a temperature at which there was no liquid. No correction was made for expansion and contraction effects. These are of the order of 10% of the Doppler effect, so their neglect is not significant.

The calculated Doppler effects are also shown in Figures 2 and 3. The method of calculation is the same as described previously.⁽⁶⁾ No normalization has been applied to the thorium calculation. The fit to the measurements would be improved by multiplying the calculation by 0.95. The unnormalized calculation is described by

$$\rho = -0.1419 + 0.2466 T^{-0.1},$$

where the parameter γ is 1.1. The effect of crystalline binding is not shown for the thorium calculation. The effective temperature for a thermodynamic temperature of 110°K is 100°K, so the effect is small.

The calculation for tungsten, shown in Fig. 3, has been multiplied by a normalizing factor of 0.7. Also included in the figure is the calculated value after correction for binding effects, using a Debye temperature of 310°K. The calculation could not be fit to a single equation of the above form since it was the sum of the effects of four isotopes having different values of γ .

Acknowledgments

The author would like to acknowledge the work of R. D. Burch in the design and installation of the cryo-

FIG. 2. Thorium Doppler Effect in a 62-keV Median-fission-energy Spectrum.

FIG. 3. Tungsten Doppler Effect in a 62-keV Median-fission-energy Spectrum.

genic capsule and its supporting equipment. The machining of various components was done by C. D. Hobbs. The author would also like to acknowledge the assistance of R. J. Tuttle in suggesting the Cryo-Tip for use as the refrigerating device, of C. H. Knox in the initial stages of design, and of T. H. Springer and S. G. Carpenter for many helpful discussions.

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Discussion

Mr. Purohit (RPI): I believe that at low temperature, the Debye model would not be sufficient to cal-

culate the effective temperature. For low temperatures we need the exact lattice vibrations in the solid. I think that, in most of the cases, the lattice vibrations are available and they can be used to calculate the effective temperatures. Furthermore, I think we have sufficient information to calculate exact effective temperatures for the oxide case without making use of the Debye temperature. This should improve the situation.

Mr. Mountford: We are going to investigate this. When we get measurements at lower temperatures, we should be able to make some interesting comparisons.

Mr. Persiani (ANL): Particularly at low temperature, you have a shift in the symmetry of the Breit-Wigner line shape, for which you don't account in the Debye model. There has been some work done using more appropriate models. I think that you should use a better model with the lattice vibration.

Mr. Purohit: If you were to use the Debye model for uranium oxide, then I think there exist sufficient data from which one can obtain Debye temperature as a function of temperature. Then, for different temperatures, one can get different Debye temperatures and can calculate this effective temperature exactly.

Results of Recent Doppler Experiments in ZPR-3

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1. Introduction

This paper presents in detail the Doppler experiments performed in two cores at the ZPR-3 facility in Idaho. These cores are of particular interest due to the calculational effort devoted to these measurements: the SEFOR core because of the interest in comparing a small-sample experiment with experiments performed on an entire core; and the ZPR-3 Assembly 48 which is the focus of an international intercomparison study.

2. Doppler Samples and Experimental Procedure

The Doppler samples used in these experiments were those developed by Fischer and Meneley.⁽¹⁾ The basic unit of these samples is an oxide pellet, 1.27 cm in diameter by 1.27 cm long. The pellets have a glass bonding agent and are approximately 70% of theoretical density. The pellets are assembled to form a single element, 15.24 cm long. Two elements are assembled on a single axis with a gap of 12 cm to form a single Doppler sample. The center of the gap is placed at the midplane of the core, and the elements are fixed so that one expands toward the midplane and the other away from the midplane. This provides some cancellation of expansion into flux gradients. The compositions and masses of the samples are given in Table 1, and a schematic view of the assembled experiment is shown in Fig. 1.

The Doppler sample mounted in the Doppler "rod" is oscillated with respect to the reference material, which is basically an identical but unheated sample. The trays are loaded with reactor materials to provide minimum reactivity swing as the rod is oscillated. The Doppler rod is enclosed and cooling air is passed through it to remove the heat lost from the sample. The air is exhausted from both ends of the rod without contact with the reactor. The cooling air is regulated to maintain the Doppler-rod temperature equal to the reactor environment in order to minimize undesirable effects due to temperature variations.

The data were taken using a precision autorod. The

uncertainties quoted are those calculated directly from the autorod data by the techniques described by Bennett.⁽²⁾ The results given are precisely those obtained in the experiments and include every result obtained. No corrections have been made for expansion effects. The only corrections made were small corrections to all of the SEFOR data for a small, experimentally measured effect of the empty cans. It is believed that this arose from a small systematic effect and was not a true reactivity effect of the empty cans. The experimental procedure was slightly modified for Assembly-48 experiments, and no obvious systematic effect was observed.

In both SEFOR and Assembly 48, the plates which form the cell structure were oriented edgewise to the Doppler sample in an attempt to provide an "average" core spectrum. No isolation "anti-Dancoff" region was used in either core. The compositions and plate structures are available in other papers presented at this conference.^(3, 4)

3. Results in the SEFOR Mockup

Measurements were performed with four samples. One of these, the natural uranium sample, was measured in three environments: the normal core, the core voided in sodium, and a boron sleeve. The radius of the sodium-voided region was 19.4 cm, and the thickness of the natural boron carbide sleeve 1.27 cm. The results in lh/kg of the metal components are given for the natural uranium and U/Pu alloy samples in Table 2, and shown graphically in Fig. 2. Ratios of the various experiments relative to natural uranium in the normal core are given in Table 3.

All of the values for natural uranium are within two standard deviations of the weighted average of the three runs made. Unfortunately, the reactor scrammed during two of the experiments, and only one value is available for 1100°K. The ratios given in Table 3 indicate that there are apparently no serious systematic uncertainties in the experiment. Also, if the averaged

results for natural uranium are prorated by mass and subtracted from the mixed uranium-plutonium values, the contribution from the plutonium is zero within the experimental uncertainties (this linear addition of reactivity signals is reasonable if there is no correlation between the resonance structure of the two materials). This result is supported by the experiments with fissile samples reported below.

The experimental results for the fissile samples are given in Table 4 and shown in Fig. 3. Two effects need explanation. One is the misplaced reference for the first plutonium experiment in the normal core. This was caused by improper control of the Doppler-rod cooling

air which caused a perturbation to the reactor. This effect is difficult to measure, but it has been shown experimentally that proper control would move the reference value towards zero. Because of this, the other three data were normalized to the second experiment with a fit indicating good reproducibility.

The other effect was a continuously varying reactivity signal evident in all runs with the fissile samples at a temperature of 1000°K. Two experiments were performed to determine the cause, which is still unexplained. Because of this, all of the fissile data beyond 800°K are suspect and should not be used even though the data are consistent and reproducible.

First, the data were analyzed to see if there were any permanent changes in the worths of the samples with respect to the references. The second plutonium experiment was compared to the normalized results of the first run. For the U-235 sample, a second reference experiment was performed. In both cases, a positive change was measured, but the change was only slightly greater than two standard deviations for the experiment. Therefore, these results were inconclusive. Second, radiographs were made of the plutonium sample to determine if the pellets had changed geometrical shape. The horizontal and vertical diameters were measured as identical to within 0.06 %, indicating no significant geometrical changes. Also, as stated before, the non-fissile samples indicate no obvious systematic effects of such magnitude. Therefore, the cause of this effect is still unexplained.

If the data beyond 800°K are ignored, some conclusions can be drawn. Particularly, preliminary calculations by Till yielded an expansion effect for plutonium in the normal core almost identical to the measured reactivity change, which indicates a plutonium Doppler effect which is near zero for this core. This supports the conclusions drawn from the natural- and mixed-sample results, since the expansion effects of the very low-density plutonium in the mixed sample should be very small.

TABLE 1. MASSES AND COMPOSITIONS OF THE ZPR-3 DOPPLER-EFFECT SAMPLES

Isotope	Atoms/cm ³ × 10 ⁻²⁴			
	Pu-239	8:1 U-Pu	U-238	U-235
U-238	—	0.01327	0.0156	0.0004
U-235	—	0.00010	0.0001	0.0154
Pu-239	0.0147	0.00153	—	—
Pu-241	0.0004	0.00012	—	—
O	0.0302	0.03012	0.0314	0.0316
Total Mass (g)	263.8	261.8	274.4	275.8

FIG. 1. Physical Arrangement of the ZPR-3 Experimental Doppler-effect Equipment.

TABLE 2. MEASURED REACTIVITY CHANGE DUE TO HEATING IN THE SEFOR MOCKUP

Sample	Date	Environment	1h/kg of Metal Component at			
			300°K	500°K	800°K	1100°K
Natural U	4-21	Normal Core	0 ± 0.050	-0.806 ± 0.074	-1.831 ± 0.058	—
"	4-22	"	0 ± 0.025	-0.947 ± 0.040	-1.906 ± 0.066	-2.567 ± 0.050
"	5-9	"	0 ± 0.033	-0.947 ± 0.050	-1.993 ± 0.066	—
"	Average	"		-0.926	-1.903	-2.567
"	4-25	Sodium Void	0 ± 0.017	-0.736 ± 0.050	-1.575 ± 0.050	-2.133 ± 0.033
"	4-28	Boron Sleeve	0 ± 0.033	-0.120 ± 0.041	-0.227 ± 0.050	-0.331 ± 0.058
U/Pu	4-19	Normal Core	0 ± 0.052	-0.797 ± 0.095	-1.645 ± 0.052	-2.264 ± 0.061

FIG. 2. Measured Reactivity Changes in the SEFOR Mockup due to Heating.

TABLE 3. MEASURED CHANGE RELATIVE TO NATURAL URANIUM IN NORMAL CORE

Sample and Environment	Temperature, °K		
	500	800	1100
Natural uranium in sodium void	0.795	0.828	0.831
Natural uranium in boron sleeve	0.130	0.119	0.129
Mixed U/Pu in normal core	0.861	0.864	0.882

4. Results in Assembly 48

In this assembly a minimum program to investigate the Doppler effect was undertaken. Four measurements were performed, all in the normal core environment, including a repeat measurement of the natural uranium sample. The results are given in Table 5 and shown in Fig. 4. The discrepancy in the 500°K datum of the second run was again caused by improper control of the

TABLE 4. RESULTS FOR FISSILE SAMPLES IN THE SEFOR MOCKUP

T(°K)	Ih/kg of Metal Component			
	Pu-239 in Boron Sleeve	Pu-239 in Normal Core (First Run) ^(a)	Pu-239 in Normal Core (Second Run)	U-235 in Normal Core
300	—	—	—	0 ± 0.025
350	0 ± 0.026	-0.160 ± 0.071	—	—
370	—	—	0 ± 0.026	—
500	+0.270 ± 0.039	-0.213 ± 0.077	-0.270 ± 0.052	+0.378 ± 0.025
650	—	—	-0.391 ± 0.045	+0.579 ± 0.043
800	+0.665 ± 0.045	-0.546 ± 0.084	-0.472 ± 0.032	+0.833 ± 0.056
900	—	—	-0.429 ± 0.045	+1.039 ± 0.037
1050	—	—	-0.086 ± 0.052	+1.837 ± 0.049
1100	+1.498 ± 0.039	-0.044 ± 0.077	-0.112 ± 0.045	+1.901 ± 0.043

^(a) The reference run at 350°K was performed in a manner inconsistent with all other experiments. The results were normalized to place the other three data on the curve of the second run.

FIG. 3. Measured Reactivity Changes for Fissile Samples.

TABLE 5. EXPERIMENTAL RESULTS FOR THE
DOPPLER-EFFECT SAMPLE OF NATURAL
URANIUM OXIDE IN ASSEMBLY 48

$\delta k \times 10^6$					
T(°K)	Experimental Results		Fitted to $1/T$ Curve		Average
	1st Run	2nd Run	1st Run	2nd Run	
300	0 ± 0.07	—	-0.08	0	0
322	—	0 ± 0.04	—	-0.20	-0.20
500	-1.67 ± 0.09	$-1.19 \pm 0.06^{(a)}$	-1.75	—	-1.75
800	-3.65 ± 0.10	-3.59 ± 0.05	-3.73	-3.79	-3.76
1100	-4.75 ± 0.13	-4.76 ± 0.10	-4.83	-4.96	-4.89

^a This datum is inconsistent with the remainder and was not included in the analysis or the average. See the text for additional details.

Doppler-rod cooling air. The discrepancy would be reduced if the experiment had been performed properly. Since the reference temperature of the second measurement was higher than the first, both sets of data were fitted to $dk/dT = 1/T$ curves, and the data were adjusted to force the curves to zero at 300°K. If the one inconsistent point is ignored, the agreement is very good.

FIG. 4. Reactivity Effect due to Heating a Natural Uranium Oxide Sample in Assembly 48.

The plutonium and empty-can results are given in Table 6 and Figures 5 and 6. The empty-can data are included to show that there were no apparent systematic effects in the measurements. The plutonium data do not show the inconsistency seen in the SEFOR measurements. It can be seen that a $1/T$ curve fits the data. A straight line, which would be expected if there are only expansion effects and a zero Doppler effect, would fit the data equally well. Indeed, calculations by Till again yield an expansion effect nearly equal to the total

TABLE 6. EXPERIMENTAL RESULTS FOR THE DOPPLER-EFFECT SAMPLE OF PLUTONIUM OXIDE IN ASSEMBLY 48

$\delta k \times 10^6$		
T(°K)	Plutonium Sample	Empty Cans
300	—	0 ± 0.046
355	0 ± 0.04	—
500	-0.29 ± 0.05	$+0.055 \pm 0.052$
650	-0.49 ± 0.06	—
800	-0.87 ± 0.06	-0.023 ± 0.056
900	-1.01 ± 0.06	—
1052	-1.25 ± 0.06	—
1100	-1.16 ± 0.08	-0.015 ± 0.061

FIG. 5. Reactivity Effect due to Heating a Pu-239 Oxide Sample in Assembly 48.

FIG. 6. Reactivity Effect of Heating Empty Cans in Assembly 48.

measured reactivity change, again indicating a near-zero Doppler effect.

The calculated data presented in the Assembly-48 intercomparison study show a large variation. However,

the trend has been to reduce the positive magnitude of the plutonium Doppler effect. This has been supported by the always negative Doppler plus expansion effects measured in the ZPR-3 plutonium-fueled cores.

Acknowledgements

C. E. Till of the Reactor Physics Division, Argonne, performed the expansion calculations for the plutonium sample. R. A. Lewis was most helpful in both providing some calculations and assistance during the initial phases of the SEFOR measurements. Their support is gratefully acknowledged.

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Discussion

Mr. Specht (AI): Were the empty cans for which you measured the Doppler worth the same cans in which the samples were placed?

Mr. Gasidlo: Everything was identical in all cases except that there were no oxide pellets. The loading of the trays obviously was changed to represent a material which is consistent with the empty cans. In the other experiments, the trays were loaded with a material which was used to represent the reference material, be it natural uranium, U-235, or plutonium.

Mr. Specht: How much stainless steel were you using?

Mr. Gasidlo: The empty capsules are approximately 100 grams total. The total amount of oxide is about 270 grams. Tables of compositions and other information are included in the paper for the Proceedings.

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Interpretation of Doppler Coefficient Measurements in Fast Critical Assemblies*

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ABSTRACT

The problem of calculating the Doppler reactivity associated with the heating of a fuel sample in a fast critical assembly is treated. After an initial discussion which shows some of the main features, a perturbation expression for the reactivity change is derived on the basis of integral transport theory, using the narrow-resonance approximation.

The resulting formula can be further evaluated in the special case for which the sample has the same composition as the core. This evaluation leads to a result which was already obtained by Storrer *et al.* The reactivity change is the same as if the whole reactor were heated, aside from a correction term of second order in the temperature difference and for a statistical weight factor.

This statement no longer holds if the sample is different in composition from the core, which is the case in most experiments. An approximate evaluation in this more general case is given, assuming that the source densities are flat in the sample and in the surrounding core. Numerical results for an experiment carried out at ZPR-6 are presented. The calculated values are about 12 % smaller than the measured values.

1. Introduction

This paper is concerned with the interpretation of Doppler experiments in which the reactivity change due to heating of a sample in a fast critical assembly is measured. Experiments of this kind were carried out at Argonne^(1, 2) and Atomics International,⁽³⁾ and are being planned at the critical assembly SNEAK at Karlsruhe.⁽⁴⁾

Probably the major difficulty in the theoretical treatment of such an experiment is that the flux dips because the resonances are different in the sample and in the core, which are at different temperatures. At and near the boundary (very often the sample diameter

is less than one mean free path, so that all of the sample volume is near the boundary) these fluxes overlap, and for the calculation of reaction rates in both regions one has to take account of this overlap. Clearly, this is a more complicated problem than the calculation of the standard Doppler coefficient, for which the core is assumed to have uniform composition and temperature.

The first correct treatment of the problem just outlined is due to Storrer, Khairallah, and Ozeroff.⁽⁵⁾ These authors assume that the sample has the same composition as the surrounding core, and they calculate the temperature-dependent reaction rates in the sample and in the core for this special case.

However, in all the experiments mentioned above, the sample composition is entirely different from that of the core. It is the main purpose of this paper to develop a formalism, valid in this more general case, which allows the calculation of the reactivity change due to heating of the sample. The formulas to be derived will be based on integral transport theory and perturbation theory. In order to illustrate the problem, a special case will be discussed first.

2. Preliminary Discussion of the Case of a Small Sample which has the Same Composition as the Core

The case of a sample which is small compared to one mean free path and which has the same composition as the core is suitable to provide some physical insight, although it is of no practical importance. It is also a useful example to study the question whether a sample measurement is of any use to obtain the Doppler coefficient of a whole reactor. The following discussion is similar to that of Storrer *et al.*,⁽⁵⁾ although a different approach will be taken.

The following notation is used:

- v, s = volume, surface of the sample;
- $\bar{l}$ = $4 v/s$, mean chord length of the sample;
- p_{11} = probability that a neutron born in the sample makes its first collision in the sample ($= \bar{l} \Sigma_a$ for a small sample);

* Work performed within the association in the field of fast reactors between the European Atomic Energy Community and Gesellschaft für Kernforschung mbH., Karlsruhe.

Γ_1 = probability that a neutron impinging on the sample will make a collision in the sample
(= $\bar{l}\Sigma_{t1}$ for a small sample);

S = source density.

Subscript 1 refers to the sample, 2 refers to the core.

If (i,k) denotes the number of neutrons which are produced in region i and absorbed in region k , one can easily write down the absorption rates

$$(1,1) = Sv p_{11} \frac{\Sigma_{a1}}{\Sigma_{t1}} = S \frac{s}{4} \bar{l}^2 \Sigma_{a1};$$

$$(1,2) = Sv(1 - p_{11}) \frac{\Sigma_{a2}}{\Sigma_{t2}} = S \frac{s}{4} \bar{l} \frac{\Sigma_{a2}}{\Sigma_{t2}} - S \frac{s}{4} \bar{l}^2 \Sigma_{t1} \frac{\Sigma_{a2}}{\Sigma_{t2}};$$

$$(2,1) = S \frac{s}{4\Sigma_{t2}} \Gamma_1 \frac{\Sigma_{a1}}{\Sigma_{t1}} = Sv \frac{\Sigma_{a1}}{\Sigma_{t2}};$$

$$(2,2) = S \frac{s}{4\Sigma_{t2}} (1 - \Gamma_1) \frac{\Sigma_{a2}}{\Sigma_{t2}} = S \frac{s}{4} \frac{\Sigma_{a2}}{\Sigma_{t2}^2} - Sv \frac{\Sigma_{t1}\Sigma_{a2}}{\Sigma_{t2}^2}.$$

In the last two equations, $Ss/4\Sigma_{t2}$ is the current of neutrons into the sample.

The terms (1,1) and (1,2) depend on the sample temperature only to the order $\bar{l}^2$, which can be neglected for small samples. Thus, the change in absorption rate when the sample is heated is entirely due to neutrons which were produced in the surrounding core, namely, terms (2,1) and (2,2). Term (2,2) describes the "filter" effect of the sample: the number of neutrons which traverse the sample and are absorbed in the core depends on the sample temperature.

The net change in absorption rate upon heating of the sample is

$$\delta A' = Sv \int dE \left(\frac{\Sigma_{a1}}{\Sigma_{t2}} - \frac{\Sigma_{t1}\Sigma_{a2}}{\Sigma_{t2}^2} \right).$$

The quantity of interest, however, is the change in absorption rate that occurs if the whole reactor is heated, namely,

$$\delta A'' = Sv \int dE \left(\frac{\Sigma_{a1}}{\Sigma_{t1}} - \frac{\Sigma_{a2}}{\Sigma_{t2}} \right).$$

These two quantities shall now be compared, assuming that the temperature change is small; one can then put $\Sigma_1 = \Sigma_2 + \delta\Sigma$, and retain only terms of first order in $\delta\Sigma$. One obtains

$$\begin{aligned} \delta A' &= Sv \int dE \left(\frac{\Sigma_{a2} + \delta\Sigma_a}{\Sigma_{t2}} - \frac{(\Sigma_{t2} + \delta\Sigma_t)\Sigma_{a2}}{\Sigma_{t2}^2} \right) \\ &= Sv \int dE \left(\frac{\delta\Sigma_a}{\Sigma_{t2}} - \frac{\Sigma_{a2}\delta\Sigma_t}{\Sigma_{t2}^2} \right) \end{aligned}$$

and

$$\begin{aligned} \delta A'' &= Sv \int dE \left(\frac{\Sigma_{a2} + \delta\Sigma_a}{\Sigma_{t2} + \delta\Sigma_t} - \frac{\Sigma_{a2}}{\Sigma_{t2}} \right) \\ &= Sv \int dE \left(\frac{\delta\Sigma_a}{\Sigma_{t2}} - \frac{\Sigma_{a2}\delta\Sigma_t}{\Sigma_{t2}^2} \right). \end{aligned}$$

Thus, $\delta A' = \delta A''$, and we have the result: in an experiment in which only a small sample is heated, the changes in absorption rate are due to neutrons which have the resonance structure of the environment ("cold" neutrons). If the whole reactor is heated, the changes are due to neutrons which have the resonance structure of the sample ("hot" neutrons). It has been shown that the two changes are equal for small temperature changes. Therefore, a sample measurement is, in principle, suitable to measure the Doppler coefficient of a fast reactor if the temperature changes are kept small.

3. Derivation of a Perturbation Expression for the Reactivity Change

In this section, an expression for the reactivity change will be derived which is rigorous except for the use of perturbation theory. More specifically, the following equations will be based on integral transport theory, which means that isotopic scattering will be assumed. The narrow-resonance approximation, which is standard in most Doppler calculations in fast spectra, will also be made.

The integral transport equation reads

$$\Sigma_t(E,r)\phi(E,r) = \int_v S(E,r')p(r' \rightarrow r,E) dv'. \quad (1)$$

In this and the following equations, the notation is

$\Sigma(E,r)$ macroscopic cross section, cm^{-1} ;

t,s,f,p,c subscripts that refer to the cross section for total collisions, scattering, fission, potential scattering, formation of the compound nucleus;

$S(E,r)$ source density of neutrons due to scattering and fission, per unit volume, energy and time, $n/\text{cm}^3\text{-eV-sec}$;

S_x rate of reactions of type x in the energy group i ;

$S^{j \rightarrow i}$ rate of (elastic or inelastic) scattering events in group j , with moderation of the neutrons into group i ;

F_j fission rate in the energy group j ;

V total reactor volume;

$\langle \rangle$ average over an energy group;

$\bar{\Sigma}_{p1} = \Sigma_{p1} + (1/\bar{l})$ equivalent potential cross section of the sample.

The function

$$p(r' \rightarrow r, E) = \frac{\Sigma_t(E, r)}{4\pi} \frac{e^{-L(r', r, E)}}{|r' - r|^2} \quad (2)$$

in eq. (1) is the probability that a neutron born at r' will make its next collision in the unit volume at r ; L is the "optical path" between r and r' .

We now multiply eq. (1) by the cross-section ratio Σ_x/Σ_t , and integrate over the energy group i . This gives

$$\begin{aligned} S_{xi}(r) &= \int \Sigma_x(E, r) \phi(E, r) dE \\ &= \int_v dv' S_i(r') \left\langle \frac{\Sigma_x}{\Sigma_t}(E, r) p(r' \rightarrow r, E) \right\rangle_i. \end{aligned} \quad (3)$$

In deriving eq. (3) we have made the narrow-resonance approximation. As a consequence, the source density $S(E, r)$ is a slowly varying function of energy that and can be taken out of the averaging process over energy.

$$-\frac{\delta\nu}{\nu} = \frac{\sum_{i,j \leq i} \int dv' \int dv S_i^+(r) \delta P_i^{j \rightarrow i}(r', r) S_j(r') + \sum_{i,j} \int dv' \int dv \chi_i S_i^+(r) \nu \delta P_{fj}(r', r) S_j(r')}{\sum_{i,j} \int dv' \int dv \chi_i S_i^+(r) \nu P_{fj}(r', r) S_j(r')}. \quad (9)$$

By definition, the group integrated source density which occurs in eq. (3) is the sum of the moderation and fission source:

$$S_i(r) = \sum_{j \leq i} S_j^{j \rightarrow i}(r) + \chi_i \sum_j \nu F_j(r). \quad (4)$$

Obviously, we can express the reaction rates on the right side of eq. (4) through eq. (3), and thus obtain

$$\begin{aligned} S_i(r) &= \sum_{j \leq i} \int dv' S_j(r') \left\langle \frac{\Sigma_j^{j \rightarrow i}}{\Sigma_t}(E, r) p(r' \rightarrow r, E) \right\rangle_j \\ &+ \chi_i \sum_j \int dv' S_j(r') \left\langle \frac{\nu \Sigma_{fj}}{\Sigma_t}(E, r) p(r' \rightarrow r, E) \right\rangle_j. \end{aligned} \quad (5)$$

These are the multigroup transport equations in a form in which the source density S_i is the variable; this form has the advantage that it is obvious which quantities have to be averaged over the resonances in space-dependent problems. The meaning of the variable S_i is made obvious by eq. (4).

The notation can now be simplified somewhat by introducing the following group-averaged probability distributions:

$$P_{xi}(r', r) = \left\langle \frac{\Sigma_x}{\Sigma_t}(E, r) p(r' \rightarrow r, E) \right\rangle_i. \quad (6)$$

Then we can write the equations (5) as

$$\begin{aligned} S_i(r) &= \sum_{j \leq i} \int_v dv' S_j(r') P_s^{j \rightarrow i}(r', r) \\ &+ \chi_i \sum_j \int_v dv' S_j(r') \nu P_{fj}(r', r). \end{aligned} \quad (7)$$

We can now proceed to find the perturbation equations based on (7) and on the following equations which are adjoint to (7):

$$\begin{aligned} S_i^+(r) &= \sum_{j \geq i} \int_v dv' S_j^+(r') P_s^{i \rightarrow j}(r, r') \\ &+ \sum_j \chi_j \int_v dv' S_j^+(r') \nu P_{fj}(r, r'). \end{aligned} \quad (8)$$

We assume that there are small perturbations δP_x in the P_x :

$$P_{xi}(r', r) = P_{xi}^{(0)}(r', r) + \delta P_{xi}(r', r),$$

and that the effect of these δP_x is compensated in first order by a change $\delta\nu$ in ν . Then we obtain, by standard manipulations, the following expression for the first-order reactivity effect of the perturbation:

Within the frame of first-order perturbation theory, this equation gives the reactivity effect of any small-enough perturbation in a reactor which can be described by the multigroup transport equations. We shall apply it to the interpretation of Doppler-effect measurements of the type mentioned above.

4. Application to the Case where the Sample Has the Same Composition as the Core

We consider first the case in which a small central portion of the core is heated, which is that treated by Storrer *et al.*,⁽⁶⁾ where the sample has the same composition as the core. The unperturbed reactor is homogeneous and at uniform temperature.

Let us assume a reactor large enough so that both the S and S^+ can be considered as constant over the region of perturbation, which is obviously the sample volume and the region around it, within two or three mean free paths. Then the double integrals in (9), which have to be extended over the region of perturbation, can be split up into four integrals in the following way:

$$\begin{aligned} &\int_v dv' \int_v dv P_x(r', r) \\ &= \left\langle \frac{\Sigma_{x1}}{\Sigma_{t1}}(E) \int_{v_1} dv' \int_{v_1} dv p(r' \rightarrow r, E) \right\rangle \\ &+ \left\langle \frac{\Sigma_{x1}}{\Sigma_{t1}}(E) \int_{v_2} dv' \int_{v_1} dv p(r' \rightarrow r, E) \right\rangle \end{aligned} \quad (10)$$

$$+ \left\langle \frac{\Sigma_{x2}}{\Sigma_{t2}}(E) \int_{v_1} dv' \int_{v_2} dv p(r' \rightarrow r, E) \right\rangle \\ + \left\langle \frac{\Sigma_{x2}}{\Sigma_{t2}}(E) \int_{v_2} dv' \int_{v_2} dv p(r' \rightarrow r, E) \right\rangle.$$

Obviously, the integral

$$\int_{v_a} dv' \int_{v_b} dv$$

represents the contribution of neutrons which are moderated to energy E in region a and make the next collision in region b , to the rate of reactions of type x . Thus, we have the four terms which were already discussed in the first section.

In order to evaluate the integrals, we introduce the probability

$$P_1 = \frac{1}{v_1} \int_{v_1} dv' \int_{v_1} dv p(r' \rightarrow r) \quad (11)$$

that the first collision of a neutron born in the sample will be in the sample.

Then one can easily evaluate the two integrals:

$$\int_{v_1} dv' \int_{v_1} dv p(r' \rightarrow r) = v_1 P_1; \\ \int_{v_1} dv' \int_{v_2} dv p(r' \rightarrow r) = v_1 (1 - P_1).$$

The other two integrals in (10) can be evaluated if the reciprocity theorem

$$v_1 \Sigma_{t1} P_{1 \rightarrow 2} = v_2 \Sigma_{t2} P_{2 \rightarrow 1} \quad (12)$$

is used.

One obtains by simple manipulations

$$\int_{v_2} dv' \int_{v_1} dv p(r' \rightarrow r) = v_1 \frac{\Sigma_{t1}}{\Sigma_{t2}} (1 - P_1),$$

and further

$$\int_{v_2} dv' \int_{v_2} dv p(r' \rightarrow r) = v_2 - v_1 \frac{\Sigma_{t1}}{\Sigma_{t2}} (1 - P_1) \\ - \int_{v_2} dv' \int_{v_\infty - v_1 - v_2} dv p(r' \rightarrow r).$$

The last term on the right side represents the leakage out of the region of perturbation, which is in good approximation independent of the sample temperature, and so is the first term, v_2 . As we are interested only in the temperature dependence of P_x , we omit these two terms and collect all the other terms. We call the resulting temperature-dependent integral $\tilde{P}_x$ and obtain

$$\tilde{P}_x = v_1 \left\langle \frac{\Sigma_{x1}}{\Sigma_{t1}} + (1 - P_1) \right. \\ \left. \cdot \left(\frac{\Sigma_{x2}}{\Sigma_{t2}} - \frac{\Sigma_{t1}}{\Sigma_{t1}} \right) \left(1 - \frac{\Sigma_{t1}}{\Sigma_{t2}} \right) \right\rangle. \quad (13)$$

Thus, we have found an explicit expression for the integrals in eq. (9) of our special problem. All that remains is to insert the quantities $\delta \tilde{P}_x$, which are changes in the $\tilde{P}_x$ defined by eq. (13) into the general eq. (9). Thus we obtain

$$\frac{\delta v}{v} = -\frac{1}{D} \left[\sum_{i,j} \chi_i S_i^+ \nu \delta \tilde{P}_{fj} S_j - \sum_j S_j^+ \delta \tilde{P}_{aj} S_j \right. \\ \left. + \sum_j (S_{j+1}^+ - S_j^+) \delta \tilde{P}_s^{j \rightarrow j+1} S_j \right], \quad (14)$$

where use has been made of the fact that

$$\Sigma_{tj} = \Sigma_{aj} + \Sigma_s^{j \rightarrow j} + \Sigma_s^{j \rightarrow j+1}$$

in energy groups where the Doppler effect is important. D is the denominator of eq. (9).

Expression (14) is the reactivity change associated with the heating of the sample. The three terms on the right side of eq. (14) are the effects of changes in the fission rates, absorption rates, and moderation rates, respectively, in both the sample and the surrounding core. It can be shown that the third term is small and can be neglected in all cases of interest.

5. Discussion of These Equations

5.1. The perturbation expression (9), which is "exact" in the sense that it was derived with only a few minor approximations mentioned in the introduction, can be further evaluated in the case for which the unperturbed reactor is homogeneous and large enough, so that in the center, where the perturbation occurs, the asymptotic solution of the transport equation is valid. This is exactly the case considered in the paper by Storrer *et al.*⁽⁶⁾ Our eq. (13) contains essentially the same results as those obtained in Ref. 5, and for a detailed discussion and numerical evaluation in special cases we refer to that paper. Equation (14) represents the link between the reaction rates and the reactivity change, which is different from the standard perturbation expressions in that it contains the source densities and not the fluxes.

5.2. The source densities S , which have to be calculated for the unperturbed reactor, are related to standard quantities by eq. (4). From a consideration of neutron balance, it follows that the source density is the sum of the total collision density and the leakage rate. In the unperturbed reactor, this can be loosely designated by $\Sigma_t \phi + DB^2 \phi$. In all homogeneous cores of interest the term DB^2 is much smaller than Σ_t and may be neglected. In the same approximation, S^+ is equal to the ordinary adjoint flux, and we may approximate the terms in eq. (14) in the following way:

$$S^+ \delta \tilde{P} S \approx \phi^+ \delta \tilde{P} \frac{\phi}{\langle 1/\Sigma_t \rangle}, \quad (14a)$$

where ϕ means the group flux. However, though this is a good approximation for homogeneous cores, it might not be good in cases where the sample is different from the core, and strongly absorbing, so that some group fluxes show a strong curvature in the vicinity of the sample.

5.3. Storrer *et al.*⁽⁵⁾ broke up the right side of eq. (13) into two terms, namely,

$$\left\langle \frac{\Sigma_{x1}}{\Sigma_{t1}} \right\rangle \quad (15a)$$

and

$$\left\langle (1 - P_1) \left(\frac{\Sigma_{x2}}{\Sigma_{t2}} - \frac{\Sigma_{x1}}{\Sigma_{t1}} \right) \left(1 - \frac{\Sigma_{t1}}{\Sigma_{t2}} \right) \right\rangle, \quad (15b)$$

and showed that in representative cases, heating of the sample causes a change of second order in the temperature difference in (15b), whereas the change in (15a) is of first order. Therefore, one can consider the term (15b) as a small correction, and from insertion into (14) one can see that the sample measurement gives just the Doppler coefficient of the core, except for the correction (15b) and a statistical weight factor. In this sense, one can say that the Doppler coefficient of the core can be determined directly by a sample measurement, provided that the sample has the same composition as the core. If the sample is different in composition from the core, no such interpretation is possible, because one measures the Doppler coefficient appropriate to the equivalent potential cross section of the sample.

6. Evaluation of the Perturbation Expression in the Two-region Approximation

In the more general case that the sample is different in composition from the core, the evaluation of the reactivity change is much more complicated. In fact, the calculation of the "unperturbed" source densities, with the sample at room temperature, is then already a difficult problem of transport theory. Thus, in this case of practical importance, the theoretical treatment becomes rather involved.

The simplest approach is to assume that the source densities S and S^+ are flat in the sample and in the core region around the sample. Then a two-region approximation can be used to evaluate eq. (9). Let us further assume that the sample contains only one fuel isotope. Then, the same analysis which leads to eq. (13) gives

$$\int dv' \int dv S^+ \delta P_x S = v_1 \delta \left\langle \frac{\Sigma_{x1}}{\Sigma_{t1}} S_1^+ S_1 + (1 - P_1) \cdot \left(\frac{\Sigma_{x2}}{\Sigma_{t2}} S_2^+ - \frac{\Sigma_{x1}}{\Sigma_{t1}} S_1^+ \right) \left(S_1 - \frac{\Sigma_{t1}}{\Sigma_{t2}} S_2 \right) \right\rangle. \quad (16)$$

In this more general case, the second term in the

brackets is no longer a correction of second order in the temperature difference, and, therefore, it appears somewhat artificial to isolate the term Σ_{x1}/Σ_{t1} . Therefore, the equation shall be rewritten in a different form.

If, for simplicity, Wigner's rational approximation

$$P_1 = \frac{l \Sigma_{t1}}{1 + l \Sigma_{t1}}$$

is introduced at this point, one obtains

$$\begin{aligned} \int dv' \int dv S^+ \delta P_x S = v_1 \delta \left\langle \frac{\Sigma_{x1}}{(1/l) + \Sigma_{t1}} S_1^+ S_1 \right. \\ \left. + \frac{1}{l[(1/l) + \Sigma_{t1}]} \left(\frac{\Sigma_{x2}}{\Sigma_{t2}} S_2^+ S_1 + \frac{\Sigma_{x1}}{\Sigma_{t2}} S_1^+ S_2 \right. \right. \\ \left. \left. - \frac{\Sigma_{x2} \Sigma_{t1}}{\Sigma_{t2}^2} S_2^+ S_2 \right) \right\rangle. \end{aligned} \quad (16a)$$

One can easily recognize the four terms (i,k) in the form (16a) of the equation.

Furthermore, it is obvious from eq. (16a) that the resonance neutrons do not really "see" the potential cross section Σ_{p1} of the sample, if Σ_{p1} is different from Σ_{p2} . Rather, they see the equivalent cross section $\bar{\Sigma}_{p1}$ of the sample and, of course, the cross section Σ_{p2} , which appear in the denominators of eq. (16a). Therefore, the term Σ_{x1}/Σ_{t1} of eq. (16) has no physical meaning.

We now split the term (2,1) (absorption in the sample of neutrons produced in the core) into two terms, say (2,1)a and (2,1)b, namely,

$$\begin{aligned} (2,1) = \frac{S_1^+ S_2}{l \Sigma_{p2}} \left[\delta \left\langle \frac{\Sigma_{x1}}{(1/l) + \Sigma_{t1}} \right\rangle \right. \\ \left. - \left\langle \frac{\Sigma_{x2}}{\Sigma_{t2}} \delta \left(\frac{\Sigma_{x1}}{(1/l) + \Sigma_{t1}} \right) \right\rangle \right]. \end{aligned} \quad (17)$$

In eq. (16a), the two terms (1,1) and (2,1)a have the form of resonance integrals, and can be calculated by the usual methods of Doppler-coefficient calculations.

The remaining 3 terms, however, are more complicated. They disappear if there is no resonance absorption in the core; let us call them core-absorption terms.

They can only be evaluated analytically at high neutron energies, where the so called "High Energy Approximation" used in Doppler-coefficient calculations is valid. Indeed, one obtains in this approximation, for example for the term (1,2),

$$\begin{aligned} \left\langle \frac{\Sigma_{x2}}{l \Sigma_{t2}} \delta \left(\frac{1}{(1/l) + \Sigma_{t1}} \right) \right\rangle \\ = - \frac{1}{l \Sigma_{p1}} \sum_s \frac{Q_{xs} (E_x^{(s)} - e^{(s)})}{\sqrt{2\pi} \langle \bar{\sigma}_{p1} + \sigma_{c1} \rangle \langle \sigma_{t2} \rangle} \\ \cdot \left(\frac{1}{\Delta'} - \frac{\sqrt{2}}{\sqrt{\Delta'^2 + \Delta''^2}} \right). \end{aligned} \quad (18)$$

Similar expressions are obtained for the other terms. The notation in (18) is the usual one in Doppler-coefficient calculations. The Doppler widths Δ' and Δ'' refer to the core temperature and sample temperature, respectively. It is interesting to compare eq. (18) with the "High Energy Approximation" of the term (1,1), namely,

$$\delta \left\langle \frac{\Sigma_{x1}}{(1/l) + \Sigma_{x1}} \right\rangle = \sum_s \frac{Q_{xs} (E_x^{(s)} - e^{(s)})}{\sqrt{2\pi} \langle \sigma_{p1} + \sigma_{c1} \rangle^2} \cdot \left(\frac{1}{\Delta'} - \frac{1}{\Delta''} \right). \quad (18a)$$

In eq. (18), the cross sections of both regions appear in the denominator. If they are equal, (18) is, for small temperature changes, just one-half of (18a), because only one of the two regions is heated. It should be mentioned that all the 3 terms: (1,2), (2,1)b, and (2,2), give positive contributions to the change in reaction rates in this approximation, and they are of the same size as the other two terms.

In the spectra typical of fast power breeders, most of the Doppler effect is at low energies where the so-called "isolated resonance approximation" is valid. In this energy region, the core-absorption terms can only be evaluated numerically. Calculations show that they can be of either sign, but are small compared to the terms (1,1) and (2,1)a.

It has been shown in this section that the analysis of a sample experiment, in addition to requiring a lot of numerical work, involves additional uncertainties which do not appear in the usual Doppler-coefficient calculation. The major uncertainty is in the calculation of the group fluxes in the cold sample. It is, therefore, desirable to design experiments such that the perturbation of the group fluxes by the cold sample is small.

The method appropriate to the analysis of sample experiments is based on ratios of reaction rates, rather than on effective group cross sections, as can be seen from (16a). It is, therefore, an extension of the "reaction rate method" used by Froelich and Ott⁽⁶⁾ and by Hummel and Hwang⁽⁷⁾ to space-dependent problems. The overlap correction between resonances of different isotopes, which is small in the method of effective cross sections, may be quite large in the method of reaction rates. Therefore, the analysis is simplified if the sample contains only one fuel isotope.

7. Numerical Results

The numerical work at Karlsruhe to evaluate the above expressions is still in progress. The results to be presented here are for samples of $U(238)O_2$, of the type used in the experiments on ZPR-3⁽¹⁾ and ZPR-6.⁽²⁾ The multigroup diffusion calculations were done with a

26-group cross-section set of Abagyan *et al.*,⁽⁸⁾ which includes self-shielding. The statistical method was used for the Doppler calculations.

A code was written to evaluate the core-absorption terms for isolated resonances, using the asymptotic expression for the ψ -function.

The temperature dependence of both the terms (1,2) and (2,1)b is described by the function

$$\left\langle \frac{\Sigma_{a2}}{\Sigma_{t2}} \delta \left(\frac{\Sigma_{c1}}{\Sigma_{p1} + \Sigma_{c1}} \right) \right\rangle = \frac{\Gamma_\gamma}{\langle s \rangle} \left\langle \int_0^\infty dx \frac{\psi_2}{\beta_2 + \psi_2} \cdot \delta \left(\frac{\psi_1}{\beta_1 + \psi_1} \right) \right\rangle, \quad (19)$$

which is plotted in Fig. 1 for different values of σ_{p2} , the potential cross section per atom in the surrounding core. The sample is $U(238)O_2$, 1 in. in diameter, the energy $E = 1.47$ keV (group 14 of the cross-section set).

For the usual values of σ_{p2} the curves start with a negative tangent, but are strongly curved, and then change sign. The term (2,2) leads to a different function, but shows the same qualitative behavior. Figure 2 shows all the five terms for $\sigma_{p2} = 30.8$ b, which cor-

FIG. 1. The Function

$$\frac{\Gamma_\gamma}{\langle s \rangle} \left\langle \int dx \frac{\psi_2}{\beta_2 + \psi_2} \delta \left(\frac{\psi_1}{\beta_1 + \psi_1} \right) \right\rangle$$

for Different σ_{p2} . Sample: $U(238)O_2$, 1 in., $E = 1.47$ keV.

responds to the core composition of ZPR-6. The core-absorption terms are small in the temperature range of interest.

In order to understand the behavior of the function (19), let us look, for example, at the term (1,2). Then the second factor, $\Sigma_{c1}/[(1/\bar{l}) + \Sigma_a]$ in eq. (19), is the temperature-dependent spectrum of neutrons which leave the sample without a collision. This factor increases with temperature in the wings and decreases in the center of a resonance, leaving a net increase of the integral. The neutrons with this spectrum are subjected to the absorption probability Σ_{a2}/Σ_{t2} in the core, which is largest in the center of the resonance and may, therefore, change the above balance into a net decrease. Thus, the sign of the term is very sensitive to the parameters, and the net balance is small. Only at high temperatures and/or large potential cross sections, is the net effect in (19) always negative, giving a positive contribution to the term (1,2).

Table 1 shows the calculated reactivity change for an experiment with ZPR-6, Assembly 4Z.⁽²⁾ The sample is natural uranium oxide, 1 in. in diameter, with expansion constrained (measurement No. 7 of Ref. 2). The calculated values are for a sample heated to 500°K and 800°K. The unperturbed group fluxes were used; no correction was made for the Debye temperature. The calculated values are 10% and 13% smaller than

FIG. 2. Contribution of the Different Terms to δk . Sample: U(238)O₂, 1 in., $E = 1.47$ keV.

TABLE 1. RESULTS FOR THE ZPR-6 EXPERIMENT, 1-IN. SAMPLE

Term	$-10^6 \delta k/\text{kg U-238}$			
	293–500°K		293–800°K	
	Above 10 keV	Below 10 keV	Above 10 keV	Below 10 keV
(1,1)	0.23	0.44	0.38	0.84
(1,2)	0.08	0.01	0.16	0.05
(2,1)a	0.32	0.59	0.53	1.14
(2,1)b	0.20	0.02	0.39	0.13
(2,2)	0.10	0.00	0.18	0.03
Total calculated	1.99		3.83	
Experimental	2.22 ± 0.09		4.38 ± 0.09	

the measured values. As fast reactor spectra are not well-known, this may be considered a fair agreement. It is important to note that without the core-absorption terms the agreement would be much worse. These terms contribute about 20% in the relatively hard spectrum of Assembly 4Z, where almost 50% of the Doppler coefficient comes from energies above 10 keV.

There is, however, a strong disagreement with a measurement where U-238 was replaced by molybdenum in the environment of the sample. The result should be smaller, because the core-absorption terms are missing. However, the experiment gave a larger result. Possible explanations are that the group fluxes change, or that the calculated spectrum is too hard. More work is required to understand these effects.

Acknowledgment

The author wishes to thank Mr. G. Bruhn for writing the computer code.

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Discussion

Mr. Pitterle (APDA): You commented that you can only make this solution for the change in reaction rates and not for the change in effective cross section. I think that once you make the assumption of the flat flux in both regions you can calculate the same equations, only including the flux normalization.

Mr. Fischer: I have made a brief attempt to work out the general formulation in terms of effective cross sections, and I wasn't able to do it. There is a possibility that somebody can find it.

Mr. Pitterle: Since the Doppler effect which you calculate in this way is not interpretable in terms of the Doppler effect of the cores as a whole, and since you have already assumed that the spectrum on the sample

is the homogeneous-core spectrum, which is doubtful in a plate configuration, do you feel that it is worth the effort to bother with the homogeneous-core spectrum on the sample rather than the buffer region?

Mr. Fischer: The situation is similar for many of the problems connected with measurements in critical facilities. It is known, for example, that mockups in critical facilities don't have the proper isotopic composition of plutonium, and one tries to go through a lot of analytical evaluation in the calculations to make proper corrections for this. In the same way, I believe that if one makes a Doppler measurement and gets reliable experimental results, then it is worthwhile to try to calculate the results for this particular experiment as well as you can.

Presented by J. Codd

Some Theoretical Studies Concerning the Interpretation of the Doppler Loop Measurements in ZEBRA

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ABSTRACT

ZEBRA Core 5 contained an axial heated loop for measuring the Doppler coefficient of fast reactor assemblies mounted at the centre of the core. Subsidiary measurements of reactivity worth for a long steel plate held vertically in the loop showed that the correction to the observed temperature coefficients for radial thermal expansion of the loop walls was much greater than anticipated. Analytical studies are described which suggest that the effect could be a consequence of neutron streaming. Numerical calculations using a two-dimensional transport-theory code are in reasonable agreement with the steel-plate measurements in the empty loop, but the agreement is less good for a case in which an assembly was present in the test zone.

Approximate calculations are also made of the Doppler effect of steel in selected test assemblies, as contributed by the 1.15-keV capture resonance in Fe^{56} . The results imply only a small correction to the large negative Doppler coefficients observed for assemblies containing substantial proportions of U^{238} . For assemblies including Pu^{239} and steel but no U^{238} , and for those containing steel only, the effect amounts to roughly one-third of the observed negative Doppler coefficients.

1. Introduction

Core 5 of the zero-energy fast reactor ZEBRA contained an axial heated loop for measuring the Doppler coefficient of test assemblies, as shown schematically in Fig. 1. Each assembly was mounted centrally in the loop to form an inner zone of the core, and the temperature was then raised by circulating hot nitrogen. The reactivity change was measured by adjustment of a control rod, and after correction for thermal-expansion effects gave the result for the Doppler coefficient of the assembly. An account of the experiments and a comparison of the observed and calculated coefficients has been reported.⁽¹⁾

An initial difficulty in the interpretation of the meas-

urements was that the negative temperature coefficient for the empty loop was several times greater than predicted on the basis of thermal expansion of the steel structure and the nitrogen gas. Subsequent measurements with a long, vertical steel plate showed a strong variation of reactivity worth across the loop. The worth gradient close to the loop walls indicated that the effect of their radial thermal expansion was much larger than had been anticipated and would account for the magnitude of the empty-loop coefficient. The effect was difficult to understand physically since the computed multi-group fluxes were nearly radially constant in the neighbourhood of the loop. Theoretical investigations were accordingly undertaken in an attempt to suggest an explanation. This work is described in Sections 2 and 3 of the present paper. Section 2 contains an approximate one-group analytical study of the reactivity worth of a scattering sample in an empty channel, and of how it is influenced by neutron streaming. Section 3 describes more detailed multigroup transport theory calculations for an idealized model of ZEBRA Core 5. The computed reactivity worth of a long steel plate inserted in the loop is compared with the measurements.

A conclusion from the temperature-coefficient measurements for test assemblies which contained only steel was that this material may have a small negative Doppler coefficient. Section 4 describes preliminary estimates of the steel Doppler coefficient for several test assemblies, including some which contained U^{238} or Pu^{239} as well as steel. The calculations are limited to the contribution from the 1.15-keV capture resonance in Fe^{56} , which is the only presently known level in the constituents of steel capable of giving a significant Doppler effect.

2. Perturbation Sample Worth in an Empty Channel

As explained in Sect. 1, the investigation of the empty-loop coefficient included a radial scan across the loop of the reactivity worth of a long steel plate. This was held parallel to one of the loop walls and extended

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vertically from the centre-plane to above the top of the reactor. The results are included in Fig. 5 and show a strong radial variation, the worth at the centre of the loop being nearly twice that at the surface. The effect was unfamiliar, and the following analysis was carried out to assist in understanding the results. The model assumed is similar to that in Ref. (2), in which the effect of a channel on the diffusion coefficient of a reactor was studied.

Consider an infinite reactor with an empty axial channel of radius a . Take origin at the centre and measure the co-ordinate z along the axis, and r perpendicular to it (see Fig. 2). One-group theory is assumed. It is postulated that the neutron flux in the reactor medium, including the surface of the channel, has a prescribed axial dependence $F(z)$, but is constant radially. It is now required to obtain the flux Φ_P and the angular flux $\Psi_P(\underline{\Omega})$ at any point P in the channel. The neutron direction vector $\underline{\Omega}$ is defined by the angle θ made with the position z axis, and the azimuthal angle ϕ in a plane perpendicular to this axis.

Let the vector $\underline{\Omega}$ at $P(r, z)$ extend backwards to meet the channel surface at $Q(a, z_1)$ say. From transport theory, ⁽³⁾ $\Psi(\underline{\Omega})$ is constant along the line QP . In particular $\Psi_P(\underline{\Omega})$ is equal to $\Psi_Q(\underline{\Omega})$, the angular flux emerging from the reactor medium at Q . We assume for this quantity the value from diffusion theory (P_1 approximation) and obtain

$$4\pi\Psi_P(\underline{\Omega}) = F(z_1) - \lambda \cos \theta F'(z_1), \quad (2.1)$$

where λ denotes the total mean free path in the reactor. The required geometrical relation between the z co-ordinates of P and Q is

$$z_1 = z - s, \quad (2.2)$$

where

FIG. 1. Cylindrical Model for ZEBRA-5I Calculations.

FIG. 2. Co-ordinate System for Calculation of the Angular Neutron Flux $\Psi(\underline{\Omega})$ at Point P in a Channel.

$$s = \{ (a^2 - r^2 \sin^2 \phi)^{1/2} + r \cos \phi \} \cot \theta. \quad (2.3)$$

As a first example, consider the case treated in Ref. 2, in which the reactor flux $F(z)$ is assumed to vary linearly with z , denoting the constant gradient by G . From (2.1) with (2.2) one obtains

$$4\pi\Psi_P(\underline{\Omega}) = F(z) - \lambda G \cos \theta - sG. \quad (2.4)$$

Noting that the element of solid angle $d\Omega$ is $\sin \theta d\theta d\phi$, integration of (2.4) over angle ($\theta = 0$ to π and $\phi = 0$ to 2π) gives the total flux at P . The integrals of the terms in G each vanish, since the contributions from up and down the channel cancel, and thus

$$\Phi_P = F(z). \quad (2.5)$$

Equation (2.5) states that the total flux is radially constant across the channel and into the reactor. The angular flux (2.4), however, has a radial dependence from the term sG , which arises from neutron streaming along the channel. The other two terms have a diffusion-theory form and give the angular distribution in the limit of zero channel radius. The above results were obtained in Ref. 2.

Consider the reactivity worth of a small steel sample placed in the channel. The average one-group capture cross section in a dilute fast reactor is typically of order 10 mb per atom, and so flux depression will be negligible. The capture component of the worth will depend on the square of the total flux and should thus be radially constant across the channel. The scattering component depends, however, on the difference in 'importance' of

neutrons before and after collision. It will thus be a sensitive function of the angular flux distribution incident upon the sample, and according to the above model will have a radial variation across the channel. In a fast reactor the worth of steel is predominantly due to scattering, except close to the core centre, since the capture cross section is so small. The above analysis, although based upon a hypothetical reactor model, thus indicates qualitatively how it could be possible to have a radial variation in steel worth in a region of flat flux.

We consider next a periodic axial flux distribution

$$F(z) = \cos \alpha z, \quad (2.6)$$

where $\alpha = \pi/H$ and H is the core length. $F(z)$ will give a good approximation to the axial flux distribution in a large bare reactor containing a central channel. However, the assumption that $F(z)$ is periodic, i.e., continues beyond the ends of the core, will distort the angular flux near the ends of the core. For this reason the following analysis is intended to refer primarily to regions of the channel away from these boundaries. The flux will again be assumed radially constant in the reactor region adjoining the channel.

Inserting (2.6) into (2.1) and using (2.2), we obtain

$$4\pi\Psi_P = \cos \alpha(z-s) + \alpha\lambda \cos \theta \sin \alpha(z-s). \quad (2.7)$$

Let W_s denote the reactivity worth of a small sample of scattering material having volume V and macroscopic cross section Σ_s . From first-order transport perturbation theory,⁽⁴⁾ one may write

$$W_s M / V \Sigma_s = \Phi_P^2 - 4\pi \iint \Psi_P(\underline{\Omega}) \Psi_P(-\underline{\Omega}) d\Omega, \quad (2.8)$$

where M denotes the total production rate of neutron 'importance' in the reactor, and Φ_P is the integral over angle of $\Psi_P(\underline{\Omega})$, or equally well, of $\Psi_P(-\underline{\Omega})$. To preserve symmetry it is convenient to replace Φ_P^2 by $\iint \Psi_P(\underline{\Omega}) d\Omega \cdot \iint \Psi_P(-\underline{\Omega}) d\Omega$ in (2.8). Insertion of (2.7) into (2.8) gives the following expression for W_s , noting that a number of terms in the working vanish when the integration over θ is performed:

$$W_s M / V \Sigma_s = A(r) \lambda^2 \text{grad}^2 F(z) / 3 - [B(r) F^2(z)], \quad (2.9)$$

where

$$A(r) = 3 \langle f(\alpha s) f(\alpha t) \rangle \lambda^2 \alpha^2; \quad (2.10)$$

$$B(r) = \langle g(\alpha s) g(\alpha t) \rangle - \langle g(\alpha s) \rangle \langle g(\alpha t) \rangle. \quad (2.11)$$

s is defined in (2.3), while t is obtained from s by replacing the term $+r \cos \phi$ by $-r \cos \phi$. The functions f and g are defined by

$$f(x) = \sin x + b \cos x; \quad (2.12)$$

$$g(x) = \cos x - b \sin x, \quad (2.13)$$

with

$$b = \alpha \lambda \cos \theta. \quad (2.14)$$

The angular brackets denote an average over solid angle.

In the limiting case of zero channel radius, $A(r)$ and $B(r)$ tend to unity and zero, respectively, and (2.9) reduces to

$$W_s M / V \Sigma_s = \lambda^2 \text{grad}^2 F(z) / 3 \quad (\text{no channel}). \quad (2.16)$$

Comparison of (2.9) and (2.16) shows that both the magnitude and spatial variation of W_s will in general be markedly influenced by the presence of the channel. At the reactor centre plane ($z = 0$) the axial flux gradient $\text{grad} F(z)$ is zero, from (2.6). Equation (2.9) shows that at this position W_s is negative in sign with a radial variation like $B(r)$. This result is in contrast to the zero value of W_s in the case of no channel, as given by (2.16). Away from the centre plane, $\text{grad} F(z)$ and $F(z)$, respectively, increase and decrease with z . Thus the first term on the right of (2.9) will eventually become dominant, and W_s is then given approximately by

$$W_s M / V \Sigma_s = A(r) \lambda^2 \text{grad}^2 F(z) / 3 \quad (2.17)$$

in a region of the channel far from the centre plane. In this region W_s has a similar z -dependence to the case of no channel (2.16), but is increased by the radially dependent factor $A(r)$.

The analysis thus predicts that W_s is negative in an axial region of the channel close to the core centre plane, and the changes to positive over the remainder of the core length. This conclusion is supported by the computer results to be described in Sect. 3, although no experimental data are available for comparison. The calculations refer to the worth of a small steel sample in the empty loop of ZEBRA Core 5I, and the radial and axial variations of this quantity are shown in Figures 3 and 4. A close comparison is not justified since the analysis refers to a highly idealized system, whereas, the computer results are based on the model of Fig. 1, which attempts to simulate the actual reactor. With this proviso the computer results are in broad agreement with the analysis.

Approximate calculations of the coefficients $A(r)$ and $B(r)$ occurring in (2.9), which are described below, indicate a similar variation of radial worth to that shown in Fig. 3. It should be noted that the analysis refers to a purely scattering sample, and the negative worth predicted near the core centre is a scattering, not a capture, effect. A qualitative explanation of this negative scattering effect may be suggested by reference to the basic perturbation-theory formula (2.8) for the worth W_s . The first term on the right represents the

FIG. 3. Calculated Radial Variation of Steel Worth.

FIG. 4. Calculated Axial Variation of Steel Worth.

average 'importance' of neutrons after being scattered isotropically by the sample; the second term represents their average importance before this event. W_s itself is proportional to the difference between these two quanti-

ties. When the sample is located close to the core centre, neutrons will tend on average to be scattered to points of lower importance down the channel, and W_s will be negative. On the other hand, when the sample is nearer

to the ends of the core, they will tend, on the average, to be scattered along the channel to points of higher importance, and W_s will be positive. This explanation refers to an isotropic scattering sample. It is evident that the behaviour of W_s in general will be sensitive to the scattering law assumed.

As an application of (2.9) consider a pin-shaped sample extending over the core length and parallel to the channel axis, with radial co-ordinate r . Let $W_p(r)$ denote the sample worth, normalised to unity in the limiting case of zero channel radius. Integration of (2.9) over z gives

$$W_p(r) = A(r) - [3B(r)/\alpha^2\lambda^2], \quad (2.19)$$

after making the normalization. The terms on the right denote the contributions from the axial regions of positive and negative worth, respectively. The behaviour of $A(r)$ and $B(r)$ has been studied by expanding (2.10) and (2.11) as power series in the small

quantity α . One finds that $A(r)$ is $O(1)$, while $B(r)$ is $O(\alpha^4)$ at $r = 0$ and decreases to zero at $r = a$. Thus $A(r)$ is the principal term in (2.19), and for simplicity we shall neglect the term in $B(r)$. The expansion method is invalid for long neutron paths (θ close to 0 and π). We therefore study only the contribution to the worth from a limited angular range θ_1 to $\pi - \theta_1$, where θ_1 is arbitrary but not close to zero. Taking the leading term in the series expansion of $A(r)$, one obtains the approximation

$$W_p(r) = 3(a^2 - r^2)\{\log \cot(\theta_1/2) - \cos \theta_1\}/\lambda^2 \\ + 3aE(r/a)\{\pi - 2\theta_1 - \sin 2\theta_1\}/\pi\lambda \quad (2.20) \\ + \cos^3 \theta_1 + O(\alpha^2)$$

for the range θ_1 to $\pi - \theta_1$. E denotes the complete elliptic integral of the second kind.

Numerical values of $W_p(r)$ are given in Table 1 for the case $\lambda = a = 10$ cm, which corresponds roughly to the ZEBRA Doppler loop surrounded by the insulating region. The results refer to $r = 0$ and a , and are given for selected values of θ_1 . In the limit of $\theta_1 = \pi/2$, we find $W_p(0)/W_p(a)$ tends to $4\pi/(\pi + 4)$.

$W_p(r)$ is normalised to unity in the limit of zero channel radius. The magnitude of the tabulated results thus indicates that the sample worth is appreciably increased by the presence of the channel. The worth ratio between the channel axis and surface is roughly 2, as compared with the ratio of 1.8 observed for a long steel plate in the empty Doppler loop (see Fig. 5).

TABLE 1. $W_p(r)$ AT CHANNEL AXIS AND SURFACE:
CONTRIBUTION FROM NEUTRON ANGULAR RANGE
 $\theta = \theta_1$ TO $\pi - \theta_1$

θ_1	Axis ($r = 0$)	Surface ($r = a$)	Axis/Surface Ratio
$\pi/10$	6.4	2.7	2.4
$\pi/6$	3.8	1.8	2.1
$\pi/4$	1.7	0.9	1.9
$\pi/2$	0	0	1.76 (limit)

FIG. 5. Worth of Long Steel Plate across Loop Region in ZEBRA Core 5L.

The variation of $W_p(r)$ across the channel is a consequence of the radial variation of the streaming term s in the angular flux distribution (2.7). Streaming causes the anisotropy of the angular flux to be, on the average, greater at the channel axis than at points nearer to the reactor medium. Since the sample worth is determined by the flux anisotropy, as can be shown from (2.8), it will be a maximum at the axis and decrease towards the surface.

To summarize, the analysis indicates that both the magnitude and spatial variation of the reactivity worth of a predominantly scattering material may be quite different in an empty channel than in a reactor containing no voids, due to neutron-streaming effects. The approximate numerical calculations, while referring to an idealized model, suggest that streaming could account to a large extent for the observed radial variation in steel-plate worth across the empty Doppler loop, and hence for the radial wall-expansion effect.

3. Numerical Calculations of Reactivity Worth of Steel in the ZEBRA Doppler Loop

It would be difficult to apply the analysis of Sect. 2 to a realistic model of ZEBRA. Instead, the calculations were made using a two-dimensional transport-theory code TURTLE Mark IV for the STRETCH computer.^(5, 6) Transport theory is required in order to represent adequately the angular flux in the loop, which is predicted in Sect. 2 to be a principal factor determining the worth of steel. TURTLE is a general-purpose code not specifically designed to treat streaming problems, and the present application is to this extent of an exploratory nature. The code solves the multigroup Boltzmann equations in (r, z) geometry using the Carlson S_N method. For the specified reactor dimensions, k_{eff} , the normal and adjoint angular fluxes, and perturbation-theory functions^(6, 7) were computed. The observed reactivity worth of the steel plate at the axis of the loop was 3.5×10^{-4} in k_{eff} . It would be difficult to compute such a small effect accurately by differencing the k_{eff} values with and without the plate present, and we have therefore used first-order perturbation theory. In a notation similar to that of Ref. (7), the reactivity change resulting from the addition of steel in the mesh region (r_i, z_j) of volume V_{ij} is given by

$$\Delta(1/k_{\text{eff}}) = \frac{V_{ij}}{\Lambda} \sum_g \left\{ 4\pi \Delta \Sigma_{Tg} \iint_{\Omega} \psi_g(r_i, z_j, \underline{\Omega}) \psi_g^*(r_i, z_j, -\underline{\Omega}) d\Omega \right. \\ \left. - \sum_{g'} \Delta \Sigma_{scgg'} \psi_g(r_i, z_j) \psi_{g'}^*(r_i, z_j) \right\}, \quad (3.1)$$

where

$$\Lambda = \sum_R \sum_g \sum_{g'} (\Sigma_{fgg'})_R \int_R \Psi_g(\underline{r}) \Psi_{g'}^*(\underline{r}) d\underline{r}, \quad (3.2)$$

$\Psi_g(r_i, z_j)$ is the scalar flux and $\Psi_g(r_i, z_j, \underline{\Omega})$ the angular flux in group g and direction $\underline{\Omega}$. Here $\Psi_g^*(r_i, z_j, -\underline{\Omega})$ is the adjoint angular flux. Also, Σ_{Tg} denotes the macroscopic total cross section in group g , and $\Sigma_{scgg'}$ the cross section for scattering from group g to g' . $\Sigma_{fgg'}$ is the macroscopic fission cross section in group g multiplied by the fission neutron yield into group g' . The suffix R refers to reactor region.

The main calculations were made for ZEBRA Core 5I with the empty loop, using the simplified cylindrical model of the reactor shown in Fig. 1. This model, with the addition of the appropriate test assemblies, was also assumed in the Doppler-coefficient calculations reported in Ref. (1), for comparison with the measured coefficients. The isotopic concentrations in the several regions, assumed to be homogeneous, are similar to those in Table 2 of Ref. (7). The finite-difference mesh consisted of 39 axial and 29 radial points, using 12 of the latter to cover the region within and close to the loop. The solution was started in the S_4 approximation, and after convergence was continued in S_8 . The nuclear data assumed constituted a 4-group set of cross sections which was obtained by condensation from the current standard Winfrith 33-group set (FD2) for fast-reactor calculations. The reactivity worth was calculated in this manner for a small steel sample as a function of position in the loop, and also for a long steel plate of thickness 0.32 cm, width 17.5 cm, and length 80 cm, extending from the core centre plane to the top of the axial reflector. In the model of Fig. 1 the loop has a circular cross section. To evaluate the worth of the rectangular plate in the actual square loop, it was assumed that the steel worth calculated for a particular (r, z) mesh region was equal to that in the corresponding region of the actual system.

The calculated radial and axial worth variations for the small steel sample in the empty loop are shown in Figures 3 and 4, respectively; no experimental results were available for comparison. The spatial variation is similar to that predicted by the analytical treatment in Sect. 2 (see discussion following Eq. 2.9). Thus the worth is negative in a central region of the loop and positive in the outer regions, with maxima at the core-reflector boundaries. The absolute value of the worth is in general greatest at the loop axis and decreases steadily towards the surface and into the reactor. Figure 4 shows that the worth near the steel location plate is only about half the value at the corresponding point on the other side of the origin. This effect was observed in some steel-worth measurements in the empty loop of another system, Core 5F.⁽⁸⁾

An analysis of the numerical values of the several terms in Eq. (3.1) showed that the sample worth and its spatial variation arise mainly from elastic scattering, i.e., scattering in which the neutron remains in the same energy group after collision. The corresponding terms of Eq. (3.1) represent the multigroup analogue of the one-group theory formula, Eq. (2.8), used in the analytical study. The numerical data showed that the spatial variation of the worth, as illustrated in Figures 3 and 4, arises from the behaviour of the angular neutron distribution, which may be attributed to streaming. The remaining components of the sample worth are those due to scattering in which the neutron is transferred to a group of lower energy and to capture. In general these components amount to only a few percent of the elastic-scattering component, except close to the loop surface near the core centre, where all three components are of comparable magnitude, although small. It should be remarked that the negative sample worth in the central region of the loop, as shown in Figures 3 and 4, is mainly a scattering, not a capture, effect. This was discussed in Sect. 2 and corresponds to the term $-B(r)\Sigma_s$ in Eq. (2.9) for the sample worth. In the same way the regions of large positive worth away from the centre correspond to the term $A(r)\Sigma_s$ in this equation.

The calculated reactivity worth of the long steel plate in the empty loop, extending vertically from the core centre plane to the top of the axial blanket, is shown in Fig. 5 as a function of radial distance from the loop centre. No correction has been made for the fact that in the experiments the plate extended above the top of the axial blanket, but this would be expected to be small. Measured results are also included in Fig. 5, and have an estimated uncertainty of less than $\pm 10\%$.⁽⁸⁾ The comparison shows that theory correctly predicts the strong radial variation in steel worth across the loop, although the magnitude is overestimated by about 25% near the axis and 15% near the surface. This is regarded as reasonable agreement in view of the generally experienced difficulty in predicting streaming phenomena in geometrically complex systems. A noteworthy feature of the results is that the steel worth is very much less in the buffer region, where streaming effects are negligible, than in the loop.

Calculations using the TURTLE code were also made for Core 5J, which contained an assembly in the test region of the loop representative of a fast breeder reactor lattice. The model assumed was similar to that shown in Fig. 1 with the addition of a central homogeneous region of length 68.6 cm having the same average composition as the test assembly. For simplicity, the model was made to be axially symmetric by omitting the steel structure in the 'loop pipes' region of Fig. 1. Steel present at the top and bottom of the test pins was

allowed for approximately by including a thin axial region of this material with reduced density at the corresponding positions in the model. The observed and calculated results for the worth of the long steel plate with its lower end at the core centre plate are shown in Fig. 6. The measurements were made only for positions close to the loop axis and surface. One row of test pins was removed to accommodate the plate. The measurements with the plate close to the loop surface were repeated with a second adjoining row of pins removed. The results for this case are included in Fig. 6, but the data for just one row of pins are preferred for comparison with theory. It will be observed that the calculations overestimated the steel plate worth by about a factor of two. Further calculations were made in which the order of approximation was increased from S_8 to S_{16} , but this made little change in the results. Apart from the discrepancy in the magnitude of the steel worth, the calculations give approximately the correct variation between the loop axis and surface.

To summarize, the calculations for Core 5I are consistent with the analysis of Sect. 2 and show that streaming could account for the previously unexplained reactivity effects associated with thermal expansion of the empty loop. The reactivity worth of steel and its radial variation across the loop are predicted to within 15–25%. This is considered reasonable agreement in view of the difficulties in calculating streaming effects for complex geometrical systems. For Core 5J, which had a reactive assembly in the test region, theory overestimated steel worth in the loop by a factor of two. It is possible that the worse agreement for this core is due to some inadequacy of the method and data, or of the simplified model of the reactor, which causes the predicted angular flux to be too anisotropic in the empty space above a reactive assembly, but which is relatively less important in dealing with the empty loop of Core 5I.

4. Calculations of the Steel Doppler Effect

Several of the ZEBRA Core-5 test Assemblies were constructed of steel pins only. The observed temperature coefficients of these systems, after correction for thermal-expansion effects, were small and negative. This result indicated a Doppler effect in steel, or some unknown expansion effect not allowed for.⁽¹⁾ The purpose of the calculations described in this Section was to estimate the steel Doppler coefficient for selected Core 5 assemblies, including some which contained fuel materials as well as steel.

A study of the limited resolved resonance data available⁽⁹⁾ showed that only the 1.15-keV capture level in Fe^{56} would contribute a significant Doppler effect in steel. This isotope constitutes about 65% of the material

FIG. 6. Worth of Long Steel Plate across the Loop Region in ZEBRA Core 5J.

content. The measured capture and neutron widths of the resonance are 0.673 ± 0.2 eV and 0.052 ± 0.005 eV, respectively. Strong broadening is indicated by the fact that the total width is of the same order as the theoretical Doppler width, namely, 1.4 eV at 1.15 keV and temperature 300°K.

The Doppler coefficient of steel resulting from broadening of this single resonance was estimated by means of multigroup perturbation theory. A group-averaged capture cross section for iron was evaluated at 300 and 600°K, for an energy group 0.75–1.23 keV which includes the resonance concerned. The narrow-resonance approximation was used, assuming constant collision density over the group. The alternative assumptions of an E or $1/E$ dependence of the collision density would respectively increase or decrease the computed Doppler coefficients by about 20%. The group capture cross section depends not only on temperature but also upon σ_b , the total scattering cross section of all other materials present in the assembly, per atom of iron. This cross section determines the degree of self-shielding of the resonance concerned. Table 2 gives some typical values for the iron group cross section $\bar{\sigma}_\gamma(\text{Fe})$ and its Doppler change between 300 and 600°K. The values of σ_b cover the range of present interest. For a typical dilute power reactor lattice (Assembly 5J), $\sigma_b(\text{Fe})$ is about 12 b. The tabulated values are taken from Ref. (9), which includes further details of the method of

calculation, as well as a comparison of the iron and U^{238} Doppler effects in mixtures of these materials.

The required perturbation theory formulae were mostly obtained from already available one- or two-dimensional multigroup calculations for the assemblies concerned. These formulae give the reactivity changes corresponding to prescribed changes in the multigroup cross sections. The steel Doppler effect could thus be calculated from the known change in the iron group cross section with temperature. It is noted that the coefficient will depend on the group flux and adjoint near 1 keV, as well as upon $\sigma_b(\text{Fe})$, and will thus not in general be simply proportional to the iron concentration in each assembly.

The calculated Doppler coefficients for steel are compared in Table 3 with the observed Doppler coefficients

TABLE 2. DOPPLER CHANGE IN THE IRON CAPTURE CROSS SECTION FOR ENERGY GROUP 0.75–1.23 KEV. TEMPERATURE RISE 300–600°K

σ_b (b)	$\bar{\sigma}_\gamma(\text{Fe})$ (b)	$\Delta\bar{\sigma}_\gamma(\text{Fe})$ (b)
0	0.1095	0.0116
5	0.1365	0.0147
10	0.1580	0.0165
15	0.1750	0.0175
20	0.1890	0.0180

TABLE 3. OBSERVED DOPPLER COEFFICIENTS IN SOME ZEBRA CORE-5 ASSEMBLIES WITH CORRECTION FOR THE CALCULATED STEEL DOPPLER EFFECT
Units: $10^{-7}/^{\circ}\text{C}$

Assembly	Content of Test Zone	Fe Atoms/cc in Test Zone ($\times 10^{-24}$)	Calculated Steel Doppler Coefficient	Observed Doppler Coefficient ^(a)	Residual Doppler Coefficient ^(b)
J	PuO ₂ -UO ₂ , Fe, Na	0.009	-0.46	-6.1 ± 0.5	-5.6 ± 0.5
B	PuO ₂ , B, Fe	0.045	-0.51	-0.8 ± 0.3	-0.3 ± 0.3
C	PuO ₂ , B, Fe	0.024	-0.31	-0.8 ± 0.4	-0.5 ± 0.4
K	PuO ₂ , Na, Fe	0.022	-0.68	-2.9 ± 1.0	-2.2 ± 1.0
E	Fe ($\frac{1}{2}$ loading)	0.024	-0.33	-1.3 ± 0.5	-1.0 ± 0.5
L	Fe (full loading)	0.051	-0.62	-1.7 ± 0.2	-1.1 ± 0.2
I	Empty	—	-0.05	-0.4 ± 0.3	-0.3 ± 0.3

(a) From Ref. 1, Table 6, column 4. This is the measured temperature coefficient corrected for thermal expansion effects.

(b) The observed Doppler coefficient corrected for the steel Doppler effect.

of some representative Core-5 test assemblies corrected for thermal-expansion effects. The final column gives the 'residual' Doppler coefficient, which is obtained by correcting the observed coefficient further for the steel Doppler effect. The steel coefficients denote average values for the temperature range from 300 to 600°K, whereas the observed Doppler coefficients refer to a small temperature range in the neighbourhood of 450°K.

Assembly 5J, which represents a sodium-cooled mixed-oxide power reactor lattice, has a large negative Doppler coefficient arising from the U²³⁸ content. The estimated negative steel Doppler effect is relatively unimportant, contributing only about 7% of the observed Doppler coefficient.

The results for Assemblies 5B, C, and K, which contained plutonium but no U²³⁸, suggest that steel contributes significantly to the observed negative Doppler coefficients. The residual coefficient is still negative, however, in contrast to the predicted positive Doppler coefficients for these assemblies reported in Ref. 1 ($+2.8$, $+1.8$, and $+3.0 \times 10^{-7}/^{\circ}\text{C}$, respectively, for the FD2 multigroup constants).

In the case of assemblies E and L, with half- and full loadings of steel pins, respectively, the results indicate that steel could account for about one-third of the observed negative Doppler coefficient. Since no other material than steel was present in the assemblies, the discrepancy can be attributed to a further steel Doppler effect or some expansion mechanism not corrected for. In connexion with the first possibility, it is noted that the present calculations treat only the single iron resonance at 1.15 keV. There may be unresolved resonances in the constituents of steel capable of giving an additional Doppler effect, so that from this point of view the present results would represent a lower estimate of the steel effect. The estimated steel Doppler coefficient shown for Assembly 5I (empty loop) refers

to the loop walls and other structure. The steel coefficients for the other assemblies also include a similar small contribution from this source.

To summarize the conclusions from Table 3, one can say that the residual coefficients for Assemblies J, B, C, and K represent the best values for the Doppler coefficient of the fuel. An idea of the possible systematic errors is given by the residual coefficients for Assemblies E, L and I, which contained no fuel.

To complete this investigation, the same method of calculation and data were used to estimate the steel Doppler coefficient of a typical sodium-cooled fast breeder reactor design, having an Fe/U²³⁸ atomic ratio in the core of about 2.5. The computed steel coefficient, as contributed by the 1.15-keV capture resonance in Fe⁵⁶, was negative and about 5% of the negative U²³⁸ coefficient. This result is consistent with the data shown in Table 3 for Assembly 5J.

5. Conclusions

(i) The investigations of Sections 2 and 3 suggest that the radial wall-expansion effect in the empty ZEBRA Doppler loop can be largely explained as a neutron-streaming phenomenon. The results of the numerical calculations are in reasonable agreement with the relevant measurements of steel reactivity for the case of the empty loop. When an assembly is present in the test zone, however, the agreement is less satisfactory, and shows that some feature of the method and data of calculation, or of the simplified model assumed for the loop region, is causing an overestimate of streaming effects above the assembly.

(ii) The calculations in Sect. 4 indicate that the Doppler effect in steel contributes only a small correction to the large negative Doppler coefficients observed for assemblies containing substantial proportions of U²³⁸. For assemblies including plutonium fuel with no U²³⁸, or for those containing steel pins only, the effect

could account for an appreciable part of the observed negative Doppler coefficients. The calculations refer to broadening of the single 1.15-keV capture resonance in Fe⁵⁶. Since there may be still unresolved resonances in the constituents of steel capable of contributing a significant effect, the results may be regarded as giving a lower estimate of the steel Doppler coefficients.

Acknowledgment

The authors express their thanks to Miss P. B. A. Dickson for performing the computer calculations and for the preparation of the diagrams.

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Discussion

Mr. Specht (AI): Did you make any boron traverse measurements in the empty loop to investigate the possibility of a flat flux?

Mr. Codd: No.

Mr. Gandini (CNEN): One should be cautious in using conventional perturbation theory for calculations of Doppler coefficients. As I showed in the previous session, we found examples where the indirect effects related to variation of the neutron flux not accounted for in the conventional perturbation theory are large. For instance, an increase in $\delta\Sigma$ capture of U-238 in the example we gave for a 4000-liter carbide system increases, on the one hand, the negative Doppler effect, while on the other hand it reduces the spectrum, giving a contribution of the opposite sign, which was about 50 % of the value calculated by conventional perturbation theory.

The use of conventional perturbation methods to calculate the Doppler effect leads to an error because you don't account for the variation in the flux that you introduced when you increased the cross section. This also changes the flux and this is not accounted for in conventional perturbation theory.

Mr. Codd: In this case the Doppler effect from steel is so small that this should not be a particular problem.

Mr. Gandini: I feel that you are comparing small numbers so the relative error may be not small.

Mr. Storrer (CEN): It is not quite correct to say that in first-order perturbation theory one does not take into account at all the flux change.

Mr. Gandini: First-order perturbation theory accounts only for variation of $\Delta\Sigma$. If you want to have a complete first-order perturbation theory, you should have first-order variation of flux and first-order variation of the adjoint flux. In that case you have a complete first-order perturbation theory, and it has been shown that the sizes of the changes due to the variation of the flux are rather relevant, even if it is a small effect, because you always deal with a relative effect.

Mr. Campise (AI): In view of the sensitivity of fast reactors to radial motion of fuel, are temperature gradients in the loop sufficient to move the core itself? From the size of the number involved, it appears that one-mil motion of the core could account for the reactivity change that you are getting.

Mr. Codd: You suggest that the radial expansion of the loop walls could force the fuel radially. In fact, there were several centimeters between the fuel and the loop; there was an insulating zone and also an air gap. I don't think that there was any interaction between the loop wall and the fuel.

Mr. Campise: From an experimental standpoint, have you tried to put some plugs in the loop to decrease the streaming effect to see what kind of reactivity feedback you will get?

Mr. Codd: There are some data on the effect of a plug in a loop. This plug, in fact, was the base plate on which the pins rested, and one finds that this does indeed have an effect on the reactivity worth of the steel. It does affect the streaming.

Mr. Baker (UK-Winfrith): If Mr. Campise had seen the array of dial gauges we had over the loop, he would understand that we don't think anything moved outside the loop. We spent weeks trying to find out why that loop had an effect when it was empty, and we don't think anything moved in the part of the reactor that was supposed to stay cold. Now, as to the second point, about whether one could block off the streaming, the difficulty arises because you must have some way to get the gas in. This limits what you put in above the loop. Secondly, anything put above the pins has to be

supported with something, and if the supports get warm you risk moving a very large quantity of reflector. So putting anything in the region above and below those pins tends to make things worse.

Mr. Meneghetti (ANL): How thick was the face plate, and was the flux anisotropy preferentially radially inward or axially outward?

Mr. Codd: It was more isotropic at the center of the hole than at the surface on the average. As for the thickness of the base plate, this is given in the paper; it is of the order of an inch of steel.

Mr. Specht: Do you plan any additional measurements in the ZEBRA loop?

Mr. Codd: As far as I know, no more are intended in the foreseeable future.

Mr. Khairallah (CEN): Did you make a correction for the radial expansion of the fuel, in regard to the change of the self-multiplication?

Mr. Codd: When there was an assembly in the center of the loop, a correction was made for the radial expansion of the fuel. Our calculations presented here referred to the empty loop with no fuel in the center.

Mr. Snyder (GE): Is there a Doppler effect associated with scattering which could in some strong scattering resonances send an altered number of neutrons out toward the ends of the loop?

Mr. Codd: We have considered this possibility, but for this particular resonance at 1 keV the capture component would be much stronger, and therefore, we were concentrating on the effect we thought would be most important. As for the other scattering resonances, they were far too broad to give any significant Doppler effect. But I think there is still something to be investigated about scattering resonances.

Mr. Storrer: The Russians have actually measured scattering resonances in some cases.

Calculation of the Sodium-void Effect in Large Carbide Cores

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1. Introduction

For safety reasons it is very important to predict the sodium-void effect in the design of large fast power reactors. The sodium-void coefficient was studied in detail with Assemblies 2 and 3 of ZPR-6, which represent in their composition large uranium-fueled and sodium-cooled carbide cores. Measurements were performed in central regions of different sizes and in a large number of off-center regions throughout the cores. Calculations of the sodium-void effect and of its individual components were made for the experimental regions by one- and two-dimensional diffusion theory. Perturbation and k -calculations were used in one dimension, while only perturbation calculations were used in two dimensions. The results given by the various calculational techniques are compared to each other and to the experimental values.

2. Description of the Cores

Assemblies 2 and 3 of ZPR-6 are large carbide cores of identical core composition but with different shapes. Assembly 2 had a core of about equal length and diameter ($L \approx D \approx 92$ cm) and a volume of 620 liters; Assembly 3 had a pancake core with a length-to-diameter ratio of about 1:3 ($L \approx 51$ cm; $D \approx 156$ cm) and a volume of 967 liters. The core composition is given in Table 1. Both assemblies had a depleted uranium reflector of 30-cm thickness. More detailed descriptions of the cores can be found in Refs. (1) and (2).

The two assemblies offer a good opportunity to study the leakage component of the sodium-void effect, which is predominant in both cores and very sensitive to the difference in core shape. The spectral component is slightly negative in the center of the cores and becomes positive near the core blanket boundary.

3. Methods of Calculation

3.1. Cross Sections

The cross sections which were used in the calculations were derived from ANL cross-section set 224.⁽³⁾ The

cross sections used for U^{235} took into account the values measured by White,⁽⁴⁾ and for U^{238} the cross sections reported by Columbia University⁽⁵⁾ were used. The elastic downscattering and transport cross sections of the light materials (C, Na, Fe, Cr, and Ni) were averaged in each group over the detailed flux spectrum calculated by the code ELMOE.⁽⁶⁾ The averaging is done over the spectrum of a bare core and does not take in account the spectral variation as the core-blanket boundary is approached. This, however, should not have a serious effect on the accuracy of the calculations because the leakage effect, which is least sensitive to small cross-section changes, is strongly predominant near the core edge.

As ELMOE calculations supply cross sections for compositions rather than for individual isotopes, changes in sodium concentration were simulated by replacing the original core composition by a new material which was ELMOE corrected with the changed sodium content. This implies that the spectrum in the voided region is essentially the equilibrium spectrum for its changed composition, an assumption which is best justified for large regions. The linear dependence between the Δk calculated by perturbation theory and the sodium concentration when the cross sections are ELMOE averaged was checked by using two different sets of cross sections in some perturbed regions. One was derived by an ELMOE calculation for the core composition without any sodium. All tabulated results were calculated using these cross sections. The other set was found by an ELMOE calculation for the core composition with a change in sodium content of only 1.26% of the original amount. The results obtained with the two sets were very close to each other; the largest relative difference found for the sodium-void effect was 3%, with the second set giving the smaller value. The closeness of the results confirms that the nonlinearities of the ELMOE-averaged cross sections versus the sodium concentration do not constitute a serious problem.

TABLE 1. COMPOSITION OF ASSEMBLIES 2 AND 3

	Core			Blanket		
	Atom Density, atoms cm ⁻³ × 10 ⁻²⁴	Avg Density, g cm ⁻³	Vol Fraction	Atom Density, atoms cm ⁻³ × 10 ⁻²⁴	Avg Density, g cm ⁻³	Vol Fraction
U ²³⁵	0.0022849	0.8915	0.04777	8.104 × 10 ⁻⁵	0.03162	0.001694
U ²³⁸	0.009853	3.892	0.2060	0.03993	15.777	0.8348
C	0.01290	0.2573	0.161	—	—	—
Na	0.007910	0.3019	0.311	—	—	—
Fe	0.01038	0.9621	0.1224	0.004223	0.3916	0.0498
Cr	0.002983	0.2575	0.03627	0.001214	0.1048	0.01477
Ni	0.001390	0.1355	0.01522	0.000566	0.0552	0.0062
SS-304 ^(a)	0.01475	1.355	0.1739	0.006003	0.552	0.07077

^(a) Equal to the sum of individual constituents: Fe, Cr, and Ni.

3.2. One-dimensional Perturbation Calculations

The subroutine DEL of the code MACH-1⁽⁷⁾ was used for one-dimensional perturbation calculations. The calculations can be performed in slab geometry by assuming a Bessel-function flux distribution in the radial direction, or in cylindrical geometry by assuming a cosine distribution in the axial direction. In order to take into account spectral changes near the core boundaries it is important to perform slab calculations for regions near the axial blanket and cylindrical calculations for regions near the radial blanket. However, to allow a comparison of the results, for most regions the calculations were made using both methods.

The perturbation equations used assumed a linear dependence between the sodium concentration and the diffusion operator, the perturbation of the diffusion coefficient being defined as

$$\delta D = \frac{-\delta \Sigma_{tr}}{3(\Sigma_{tr})_{original}^2}.$$

3.3. One-dimensional *k*-Calculations

For a number of experimental regions the sodium-void coefficient was calculated by one-dimensional MACH-1 calculations which were performed in two steps.

First, the effect of spectral change, absorption, and axial leakage was calculated in infinite slab geometry. (The effect of the change in radial leakage was excluded from this part of the calculation by introducing the radial leakage as a fictitious absorbing material whose concentration was left unchanged when the sodium was voided.) The result of this calculation corresponds to the effect of a void which extends radially through the whole reactor. However, the radial dependence of the sodium-void effect, excluding the radial leakage, is proportional to the product of flux and adjoint flux, so that the effect of the actual void could be approxi-

mated by multiplying the calculated result with a factor

$$\frac{\int_{\text{void}} r J_0^2 \left[\frac{2.405 r}{R} \right] dr}{\int_0^{R_0} r J_0^2 \left[\frac{2.405 r}{R} \right] dr},$$

where R_0 is the core radius plus reflector savings.

The effect of the radial leakage was calculated in cylindrical geometry by a similar procedure. The removal of the sodium was simulated by adding a negative concentration of a fictitious material which had the transport cross section of sodium and zero for all other cross sections. Suppression of the transverse (in this case the axial) leakage effect and reduction of the calculated result to the actual dimension of the void were achieved by the same methods as above.

This type of calculation has the advantage that, particularly for large voids, it takes into account part of the nonlinearities in the reactivity change, caused by changes in real and adjoint fluxes and spectra which first-order perturbation theory is not able to describe. The reliability is best near the core axis where the radial leakage component is very small and the spectrum is close to the equilibrium spectrum for an infinite slab. In radially outlying regions the treatment of the radial leakage as an additive term gives the calculation a "perturbation character." Also, the change in spectrum as the radial blanket is approached is not considered in the axial slab calculation.

3.4. Two-dimensional Perturbation Calculations

After calculating fluxes and adjoint fluxes with the two-dimensional diffusion code CANDID 2D,⁽⁸⁾ the sodium-void effect in all regions was calculated using the perturbation code PERC.⁽⁹⁾ As in the one-dimensional calculations, first-order perturbation theory was used. Two-dimensional calculations are the only

way to obtain accurate results for the effect of voiding regions which are both near the axial and radial blanket or for the effect of voiding the total core.

4. Results and Discussion

Figures 1 and 2 show the regions from which the sodium was voided in Assembly 2 and in Assembly 3, respectively. Calculated and measured data are given in Tables 2 and 3. In the central regions the agreement between one- and two-dimensional perturbation calculations is very good. Minor discrepancies may be due to the fact that different techniques and different codes were used in the computations. Near the core edge, the agreement is very good when the one-dimensional calculations were performed in such a way that spectral changes were properly taken into account, that is, for slab calculations in regions near the axial blanket and for cylindrical calculations near the radial blanket. If this rule is not followed, large errors are introduced into the spectral component, while the agreement in the leakage components is not seriously affected. This is illustrated in Fig. 3, where the results of both types of one-dimensional calculations and the results of the two-dimensional calculations are indicated for each region on a map of Assembly 2. The discrepancies in the spectral component cannot be resolved where the spectrum is influenced by both the axial and radial blanket, including the cases of total core voiding. In these cases one-dimensional calculations are not applicable.

FIG. 1. Regions for Sodium-void Measurements in Assembly 2.

FIG. 2. Regions for Sodium-void Measurements in Assembly 3.

The one-dimensional k -calculations yield a sodium-void effect which is about 10% smaller than the perturbation results in small central regions, about equal to the perturbation results in larger central regions and in regions near the core edge, and 5 to 10% larger for voiding the total core. The difference between perturbation and k -calculations for voiding the total core cannot be explained as the sum of the differences in smaller regions. This is due to the fact that the results of the k -calculations for different regions are not additive, because they take into account higher-order perturbation terms.

The difference between the calculated capture-plus-scatter components in the center of Assemblies 2 and 3 is mostly due to the difference in the reactor size which determines the magnitude of the denominator $\int_{\text{reactor}} SS^* dV$ of the perturbation expression (S and S^* are the real and adjoint sources). If one corrects for this difference, a discrepancy of about 10% remains which accounts for the perturbation in the central spectrum of Assembly 3 which is induced by the proximity of the axial blanket.

TABLE 2 CALCULATED SODIUM-VOID COEFFICIENTS IN ASSEMBLY 2 (lh/kg)

Type	Region Boundaries (cm)		Capture + Scatter		Leakage		Total			Total Exp
	Axial	Radial	DFL 1-dim	PERC 2-dim	DEL 1 dim	PERC 2-dim	DEL 1 dim	PERC 2 dim	k (calc) 1-dim	
Core Center	0-7 62	0-9 342	-2 45	-2 46	-0 44	-0 44	-2 89	-2 90	-2 71	-1 6
Slab	0-10 16	0-14 27	-2 32	-2 32	-0 92	-0 94	-3 24	-3 26	-3.07	-1 9
Slab	17 78-27 94	0-14 27	-1 52	-1 52	-3 16	-3 21	-4 68	-4 73	-4 63	-4 0
Slab	35 56-45 72	0-14 27	+0 58	+0 54	-6 38	-6 50	-5 80	-5 96	-5 91	-7 5
Ring	0-10 16 ^(a)	14 27-25 11	-1 74	-1 72	-2 86	-2 84	-4 60	-4 56	—	-4 4
Ring	17 78-27 94	14 27-25 11	-1 19	-1 13	-3 96	-4 04	-5 15	-5 17	-5 14	-4 3
Ring	35 56-45 72	14 27-25 11	+0 45	+0 43	-5 56	-5 71	-5 11	-5 27	-5 22	-5 5
Ring	0-10 16 ^(a)	25 11-35 37	-0 79	-0 81	-5 08	-5 07	-5 87	-5 89	—	-5 96
Ring	17 78-27 94 ^(a)	25 11-35 37	-0 56	-0 52	-4 88	-4 93	-5 44	-5 44	—	-5 52
Ring	35 56-45 72	25 11-35 37	+0 29	+0 35	-4 43	-4 61	-4 14	-4 26	-4 22	-5 02
Ring	0-10 16 ^(a)	35 37-46 50	+0 57	+0 53	-6 53	-6 54	-5 96	-6 01	—	-7 26
Ring	17 78-27 94 ^(a)	35 37-46 50	+0 40	+0 40	-5 23	-5 27	-4 83	-4 87	—	-5 31
Ring	35 56-45 72 ^(a)	35 37-46 50	+0 15	+0 38	-3 25	-3 39	-3 10	-3 01	-3 13	-3 39
Total Core	0-45 72 ^(a)	0-46 50	-0 36	-0 20	-4 68	-4 71	-5 04	-4 91	-5 46	-5 15

^(a) In these regions the one-dimensional perturbation calculations were made in cylindrical geometry.

TABLE 3. SODIUM-VOID COEFFICIENTS IN ASSEMBLY 3 (lh/kg)

Type	Region Boundaries (cm)		Capture + Scatter		Leakage		Total			Total Exp
	Axial	Radial	DEL 1-dim	PERC 2-dim	DEL 1-dim	PERC 2-dim	DFL 1-dim	PERC 2-dim	k (calc) 1-dim	
Central Region	0-7 62	0-9 342	-1 45	-1 49	-0 44	-0 44	-1 89	-1 93	-1 72	-0 0
Central Region	0-15 24	0-14 27	-1 06	-1 10	-1 57	-1 57	-2 63	-2 67	-2 55	-1 23
Central Region	0-20 32	0-20 66	-0 73	-0 71	-2 54	-2 55	-3 27	-3 26	-3 21	-2 29
Central Region	0-25 4	0-24 91	-0 27	-0 28	-3 52	-3 54	-3 79	-3 82	-3 83	-3 23
Slab	0-7 62	0-14 27	-1 42	-1 42	-0 49	-0 49	-1 91	-1 91	-1 77	-0 18
Slab	7 62-15 24	0-14 27	-0 77	-0 79	-2 64	-2 64	-3 41	-3 43	-3 34	-2 33
Slab	15 24-20 32	0-14 27	+0 30	+0 27	-5 48	-5 48	-5 18	-5 21	-5 15	-5 0
Slab	20 32-25 4	0-14 27	+1 54	+1 48	-7 88	-7 99	-6 34	-6 51	-6 40	-5 82
Ring	0-7 62	25 87-36 45	-1 05	-1 13	-1 12	-1 21	-2 17	-2 34	-2 06	-1 06
Ring	0-7 62	46 30-58 18	-0 53	-0 50	-1 85	-1 89	-2 38	-2 39	-2 33	-2 16
Ring	0-7 62	65 10-69 98 ^(a)	-0 07	-0 05	-2 05	-2 07	-2 12	-2 12	—	-2 25
Ring	0-7 62	72 70-77 86 ^(a)	+0 25	+0 24	-1 98	-2 05	-1 73	-1 81	—	-2 13
Total Core	0-25 4	0-77 86	-0 12	-0 06	-2 61	-2 64	-2 73	-2 70	-2 83	-2 83

^(a) In these regions the one-dimensional perturbation calculations were made in cylindrical geometry.

Comparison of calculated and experimental values shows that the effect of central voiding is calculated with poor accuracy. This indicates that either the cross-section set or the homogeneous approximation which has been used in the calculation is not adequate for an accurate calculation of the spectral component. However, in the case of the two reactors which are con-

sidered here, this deficiency is not too significant, because the leakage effect is strongly predominating.

Calculated and experimental results for the sodium-void coefficient agree within 10 to 20% in outlying regions where the leakage component is most important. At all boundaries of Assembly 2 and at the radial boundary of Assembly 3, the leakage appears to be

FIG. 3. Map for Assembly 2 with Results for Different Techniques of Calculations.

underestimated by the calculation. This disagreement is probably due to transport effects which are not properly taken into account by diffusion theory. At the axial boundary of Assembly 3 the calculated effect is larger; however, due to the core shape the voided boundary region is still in a relative high-flux area and the result of the analysis is probably obscured by the inaccuracy in calculating the spectral effect.

In spite of the difficulties in calculating the sodium-void effect in the central regions, the agreement for the cases of voiding the total core is quite good. This may be partly due to the overall predominance of the leakage effect in the core. The experimental values were found by extrapolating from the cylindrical ring measurements in Assembly 2 and from a measurement in a core sector in Assembly 3.

As a conclusion, the general agreement between the results of the experiments and the calculations performed in connection with this work seem to indicate that the basic features of the sodium-void effect were correctly taken into account. Also, the agreement between one- and two-dimensional theories shows that one-dimensional calculations may provide useful information about the sodium-void effect even in reactors in which the L/D ratio is very different from unity.

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Discussion

Mr. Gandini (CNEN): I suggest that you use higher-perturbation techniques in the analysis of your experiments for the sodium-void coefficient. We have found that the second-order terms are calculable and they are of some relevance in several cases. We performed some sensitivity calculations, and for certain cases 30 or 40% changes due to variation of the flux result. We also think that variation of the adjoint flux plays a major role. We have used the Russian approach

instead of the ELMOE code for calculating the sodium-void coefficient, and for two typical cases we did not find a relevant difference. If you use the ABN library and accept the calculated self-shielding effect, which means you do not account for interferences between resonances of the different light elements, the results for the sodium-void coefficient should be different from the one we calculated using the ELMOE code.

Mr. Specht (AI): Were your k difference calculations for your central worth correlations performed using different ELMOE spectra for the voided and unvoided cases?

Mr. Helm: Yes. Calculations were performed by making the ELMOE average over the unvoided case and then over the voided case, and using the cross sections which were derived from the ELMOE calculations.

Mr. Specht: Did you use the voided results for a small region voided of sodium?

Mr. Helm: For the small region, we realize that this is a rather rough approximation. We also derived perturbation cross sections by voiding only a very small fraction of the sodium, but we didn't get any appreciable difference in the sodium-void coefficient.

SESSION V-A

October 12, 1966

SPECIAL EXPERIMENTS AND THEIR ANALYSIS

Chairman: R. D. Smith
UKAEA-Winfrith

Secretaries: A. Travelli
F. H. Helm
Argonne National Laboratory

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Calculational Studies of Sodium-void Reactivity Variations due to Thin Slab Heterogeneities in Fast Critical Assemblies

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ABSTRACT

Fast critical systems at the Argonne Zero-power Reactors ZPR-III, VI, and IX are constructed of drawers containing plates of various basic materials. The plates are placed in some sequence relative to each other within a drawer. These heterogeneities cause variations of sodium-void reactivity values which depend upon the intra-drawer plate configurations. The variations of sodium-void reactivity can be interpreted as arising from differences in spatial variations of multigroup fluxes across the drawer-cell, differences in effective resonance self-shielding of the heavy isotopes in the lower energy regions, and in some cases also anisotropic leakage (streaming).

The results of calculational analyses generally compare favorably for a series of heterogeneous sodium-void experiments reported for ZPR-VI Assembly No. 3. With increasing thickness of unseparated sodium plate-regions there resulted perceptibly less-negative calculated values than experimentally reported. The difference is attributed to anisotropic leakage.

Results are also given of calculations of the excess reactivity, i.e., smaller critical mass, of the heterogeneous ZPR-VI Assembly No. 3 system relative to a homogeneous system.

1. Introduction

Mockups of fast reactor systems on the Argonne Zero Power Reactors ZPR-III⁽¹⁾ VI,⁽²⁾ and IX⁽²⁾ are composed of plates of, for example, enriched uranium, depleted uranium, steel, aluminum, sodium contained in plate-shaped steel cans, and graphite. The number of plates corresponding to the composition of interest are placed in a configuration relative to each other within each of the many core drawers of the two honeycomb arrays which form the two halves of the critical-assembly machine. Detailed descriptions of few-group DSN (Discrete S_n) calculations of critical-mass corrections to homogeneous because of these thin slab-cell heterogeneities have been previously reported.^(3, 4) In

terms of excess reactivities these heterogeneities represented of the order of tenths to a few percent additional k effective. Analogously, recent normal-mode scoping studies with simple heterogeneous cell configurations indicated that the effects of such thin-slab heterogeneities upon the value of nonleakage component of sodium-void reactivity can be increments of the order of tenths of percent in k .⁽⁵⁾ These correction increments, moreover, may represent large fractional corrections to the sodium-void reactivity values.

Rusch and Helm have reported experimental measurements of sodium-void reactivity effects due to different intra-drawer loading configurations of Assembly 3 in ZPR-VI.⁽⁶⁻⁸⁾ The present calculations endeavor to analyze these experimental reactivity variations in order to gain better insight into the component effects responsible for these variations and to test the efficacy of the methods herein employed to calculate these effects.

The analyses include the calculations of the reactivity effects of intracell-flux fine-spatial variations and consideration of the effects of variations of the multigroup cross-section values of fertile and fissile materials due to differing resonance self-shielding effects with sodium-in and sodium-out for the various heterogeneous intracell configurations and for the homogeneous cases. Resonance self-shielding effects become more important as systems having more degraded spectra are studied, since the effective multigroup cross sections are more sensitive to level of potential scattering at the lower energies.

Heterogeneity effects upon sodium-void reactivity due to anisotropic leakages (i.e., streaming) are not included in these calculations. Correlations between calculated and experimental values, however, allow an estimate of the streaming effects to be deduced.

Results of auxiliary calculations of the excess reactivity (smaller critical mass) of the heterogeneous ZPR-VI Assembly No. 3 critical relative to a homogeneous case are given in the Appendix.

2. Theoretical Considerations

The corrected values of the calculated multiplication constants for homogeneous and heterogeneous cases having sodium-in and sodium-out can be expressed in terms of correction terms applied to the reference non-resonance-self-shielded homogeneous values. Thus for the homogeneous cases

$$k_{in}^{home} = k_{in}^{hom} + \Delta k_{in}^{homr}$$

and

$$k_{out}^{home} = k_{out}^{hom} + \Delta k_{out}^{homr},$$

where k_{in}^{hom} and k_{out}^{hom} are the homogeneous multiplication values, assuming invariance of the microscopic cross sections of the fertile and fissile atoms with potential scattering, and Δk_{in}^{homr} and Δk_{out}^{homr} represent the corrections to be applied if the differences in resonance self-shielding effects are included. Similarly for the heterogeneous cases

$$k_{in}^{hetc} = k_{in}^{hom} + \Delta k_{in}^{hetf} + \Delta k_{in}^{hetr} + \Delta k_{in}^{hetl}$$

and

$$k_{out}^{hetc} = k_{out}^{hom} + \Delta k_{out}^{hetf} + \Delta k_{out}^{hetr} + \Delta k_{out}^{hetl},$$

where Δk_{in}^{hetf} and Δk_{out}^{hetf} are the multiplication correction increments due to the effects of the differing fine spatial variation of the intracell fluxes in the various energy groups, Δk_{in}^{hetr} and Δk_{out}^{hetr} are the resonance self-shielding effects in the heterogeneous sodium-in and -out cases, and Δk_{in}^{hetl} and Δk_{out}^{hetl} represent the multiplication effects due to anisotropic leakage (i.e., streaming).

The changes in multiplications due to sodium voiding may then be expressed as

$$k_{out}^{home} - k_{in}^{home} = (k_{out}^{hom} - k_{in}^{hom}) + (\Delta k_{out}^{homr} - \Delta k_{in}^{homr})$$

and

$$\begin{aligned} k_{out}^{hetc} - k_{in}^{hetc} &= (k_{out}^{hom} - k_{in}^{hom}) + (\Delta k_{out}^{hetf} - \Delta k_{in}^{hetf}) \\ &+ (\Delta k_{out}^{hetr} - \Delta k_{in}^{hetr}) + (\Delta k_{out}^{hetl} - \Delta k_{in}^{hetl}) \end{aligned}$$

for homogeneous and heterogeneous cases, respectively.

First consider the term $(\Delta k_{out}^{hetf} - \Delta k_{in}^{hetf})$ occurring in the heterogeneous case. For fast critical assemblies Δk_{in}^{hetf} and Δk_{out}^{hetf} generally are positive. Whether Δk_{out}^{hetf} is greater or less than Δk_{in}^{hetf} depends primarily upon the changes in the values of the $\bar{\phi}_{fuel j}/\bar{\phi}_{diluent j}$ ratios for each energy group j . ($\bar{\phi}_{fuel j}$ and $\bar{\phi}_{diluent j}$ are spatial averages of the neutron flux over fuel and diluent regions, respectively.) Also important in affecting the change in heterogeneity is the importance reweighting of these modified spatial-flux ratios for the groups relative to each other because of the overall

spectral shift of the flux resulting from reduction of sodium content.

For example, in a repetitive binary cell the values of $\bar{\phi}_{fuel j}/\bar{\phi}_{diluent j}$ for a given j tend toward unity primarily because the reduction in diluent density represents reduction in diluent thickness. Values of $\bar{\phi}_{fuel j}/\bar{\phi}_{diluent j}$ closer to unity imply a decrease in heterogeneity, i.e., $\Delta k_{out}^{hetf} < \Delta k_{in}^{hetf}$. On the other hand, the importance reweighting, of the modified flux ratios relative to each other, generally would tend to increase the heterogeneity, i.e., tend to make $\Delta k_{out}^{hetf} > \Delta k_{in}^{hetf}$, since the overall spectral shift is toward higher energies where $\bar{\phi}_{fuel j}/\bar{\phi}_{diluent j}$ have values greater than unity and away from the lower energies where the ratios are less than unity. (The former effect leading to $\Delta k_{out}^{hetf} < \Delta k_{in}^{hetf}$ is predominant in the cases calculated here.)

Consider next the terms $(\Delta k_{out}^{hetr} - \Delta k_{in}^{hetr})$ and $(\Delta k_{out}^{homr} - \Delta k_{in}^{homr})$, representing corrections due to differing resonance self-shielding effects for the heterogeneous and homogeneous situation. The relation between heavy-isotope capture (and fission, if fissionable) cross sections for a given energy group (in a multigroup description) are monotonically increasing functions of potential scattering per absorber atom with asymptotic behavior as potential scattering becomes very large, i.e., infinite-dilution situation. Furthermore, this cross-section variation with potential scattering generally becomes more pronounced the smaller the group energy. Thus for any given energy group j , the effective cross section for capture (or fission) will be smaller for the homogeneous sodium-out case than for the homogeneous sodium-in case. In the heterogeneous cases the presence of sodium between absorber plates decreases the Dancoff interactions so that upon reduction of sodium content the equivalent potential scattering is relatively smaller than the heterogeneous sodium-in case. These effects of sodium removal on potential scattering, and hence on heavy-isotope cross sections, are especially pronounced in the energy group ($j = 15$ in this study) containing the large sodium scattering resonance at 2.85 keV if, in addition, for the heavy isotope being considered the variation of cross section as a function of potential scattering is not negligible in this energy group.

For fertile isotopes the resonance self-shielding effects generally give positive values for the quantities (Δk_{out}^{hetr}) , (Δk_{in}^{hetr}) , and $(\Delta k_{out}^{homr} - \Delta k_{in}^{homr})$. For fissile isotopes the analogous corrections are negative because the reactivity effect of decrease in fission cross section is generally more effective than the decrease in capture cross section.

The quantity $(\Delta k_{out}^{hetl} - \Delta k_{in}^{hetl})$ representing the difference in streaming in the heterogeneous sodium-in and -out cases is a negative number since both terms

are nonpositive and clearly the first term is more negative than the second term. This quantity has not been explicitly calculated in the present analyses.

3. Analyses of ZPR-VI Assembly No. 3 Configurations

Assembly No. 3 in ZPR-VI^(7, 8) had a reflected, pancake-shaped cylindrical core, having an axial dimension of 50.8 cm and a diameter of 155 cm. The composition represented a uranium carbide core of ~950-liter volume. The centrally located core volume experimentally voided of sodium was 30.5 cm in axial dimension and about 28 cm in diameter. The 30-cm-thick axial and radial blankets consisted of about 83 v/o depleted uranium and about 7 v/o stainless steel.

Of the core-drawer plate configurations for which the experiments of sodium-void reactivity dependence upon heterogeneity have been reported,⁽⁶⁻⁸⁾ the following cell configurations have been calculated:

- A. S(0.21), D(0.32), N(0.64), C(0.32), E(0.16), D(0.32), N(0.64), C(0.32), D(0.32), N(0.64), E(0.16), C(0.32), D(0.32), N(0.64), S(0.21)
- B. S(0.21), N(0.64), C(0.32), D(0.32), E(0.16), N(0.64), C(0.32), D(0.32), D(0.32), N(0.64), E(0.16), D(0.32), C(0.32), N(0.64), S(0.21)
- C. S(0.21), N(0.64), C(0.32), E(0.16), D(0.32), N(0.64), D(0.32), C(0.32), D(0.32), N(0.64), C(0.32), E(0.16), D(0.32), N(0.64), S(0.21)
- D. } S(0.21), D(0.32), N(0.64), C(0.32), E(0.16), C(0.32),
E. } N(0.64), C(0.32), D(0.32), N(0.64), D(0.32),
F. } E(0.16), D(0.32), N(0.64), S(0.21)
- G. S(0.21), D(0.32), C(0.32), E(0.16), D(0.32), N(0.64), N(0.64), C(0.32), D(0.32), E(0.16), C(0.32), D(0.32), N(0.64), N(0.64), S(0.21)
- H. S(0.21), N(0.64), N(0.64), N(0.64), N(0.64), D(0.32), C(0.32), E(0.16), D(0.32), C(0.32), D(0.32), E(0.16), C(0.32), D(0.32), S(0.21).

The letters E, D, C, and N denote enriched uranium, depleted uranium, graphite, and sodium regions, respectively. The S-regions at the two ends represent the steel of the side of the tubes and drawers comprising the ZPR-VI honeycomb arrays. The numbers in parenthesis are the regional thicknesses in centimeters.

The normal configuration of plates in a core drawer was that of Case A. (In the sodium-void experiments, with cell configurations other than the configuration A, only the core drawers comprising the cylindrical core region subsequently to be voided and the axial extensions of this region to the axial reflectors were changed.) Configuration B had the sodium regions predominantly next to U-238 and graphite, whereas configuration C had sodium predominantly next to U-238. In the configuration denoted by D, E, and F the right two

of the four sodium regions were surrounded predominantly by U-238, whereas the left two were surrounded predominantly by graphite. For this configuration the sodium regions surrounded predominantly by U-238 and the sodium regions surrounded predominantly by graphite were separately voided (Cases D and E, respectively) and also voided simultaneously (Case F). The effect of arranging sodium regions in various thicknesses was another variation. For example, in configurations G and H, respectively, the sodium was arranged into two 1/2-in. regions (G) and into one 1-in.-thick region (H), respectively. Cell boundary conditions in all cases were periodic.

The atom densities for the homogeneous cell and for the regions of the heterogeneous configurations are listed in Table 1. (The mnemonic labels for the materials of the Argonne cross-section set 224 are also listed.) In terms of plate regions the composition corresponds to two 1/16-in. enriched uranium plates, four 1/8-in. depleted uranium plates, three 1/8-in. graphite plates, and four 1/4-in. sodium plate regions. In the plate regions the listed stainless steel atom densities include 0.00463 to account for the steel of the bottoms and tops of the tubes, and bottoms of the drawers of the honeycomb ZPR-VI arrays.

4. Calculational Procedure

First-order perturbation analyses⁽⁹⁾ employing 22-group real and adjoint fluxes from two-dimensional, *R-Z* geometry, diffusion-theory calculations obtained by the two-dimensional CANDID program⁽¹⁰⁾ are used to determine the correction terms. Direct two-dimensional solutions are obtained for both the reference homogeneous sodium-in and the homogeneous sodium-out cases. The quantity $(k_{out}^{hom} - k_{in}^{hom})$ is therefore not obtained by perturbation analysis.

The quantities Δk_{in}^{hetf} , Δk_{in}^{hetr} , and Δk_{in}^{homr} are obtained by perturbation analyses using real and adjoint fluxes of the reference, homogeneous, sodium-in solutions. Analogously, the quantities Δk_{out}^{hetf} , Δk_{out}^{hetr} , and Δk_{out}^{homr} are obtained using the real and adjoint fluxes of the reference, homogeneous, sodium-out solutions. This procedure circumvents the errors otherwise introduced because of the inaccuracy of perturbation analysis in going from a sodium-in to a sodium-out situation.

4.1. Calculation of Cross-section Perturbations Due to Intracell Spatially Dependent Flux Variations

The incremental changes of the composition of homogenized cross sections required to represent the reactivity effects of the fine-spatial variations of the fluxes in heterogeneous cases were obtained by regional cross-section weighting using the fine-spatial flux

TABLE 1. HOMOGENEOUS AND REGIONAL ATOM DENSITIES ($\times 10^{-24}$) USED FOR ZPR-VI ASSEMBLY 3 SODIUM-VOID HETEROGENEITY CALCULATIONS

Material	Homogeneous	Regional Compositions ^(a)					Cross-section Set 224 Label
		E	D	C	N	S	
U-235	0.002272	0 0390	0.0000838				U235A
U-238	0 009826	0 00294	0.0418				U238C
C	0 01291			0 0745			C-B
Na	0 00790				0.0171		NA-F
SS ^(b)	0.01487	0 00463	0.00463	0.00463	0.01563	0.0726	Fe-C NI-C CR-C

^(a) E, D, C, N, denote enriched U, depleted U, graphite, and sodium plate regions, respectively. S denotes region corresponding to side of drawers and tubes.

^(b) Atomic fractions of Fe, Ni, and Cr used for the stainless steel were 0.72, 0.09, and 0.19, respectively.

variations from cell calculations. These 22-group slab-cell calculations for the composition and configurations of interest were made using the SNARG (Argonne Discrete S_n) Code.⁽¹¹⁾ The cell calculations employed a modified single-Gaussian quadrature⁽¹²⁾ for better utilization of the angular-flux directions in these thin region cells. For these cases fourteen angular-flux directions ($N = 14$) were used to obtain solutions sufficiently insensitive to further increase in N approximations.

For each energy group the effects of the overall leakage out of, or into, the gross core region being voided was simulated in the cell calculations by group-dependent absorptions equal to the ratio of net leakage to integral flux for the gross region of interest. The latter quantities were obtained from the reference 2-dimensional CANDID solutions. For sodium-out cell situations the gross leakage-absorptions were based upon the sodium-out CANDID solution. These leakage-absorption cross sections were introduced uniformly throughout all regions in the cell calculations.

An example of the ratios of spatially flux-averaged macroscopic group cross sections (not including considerations of changes in U-238 or U-235 resonance self-shielding) across a drawer of configuration B to the direct flat flux averages are listed in Table 2 for the central core region being voided. Though not listed, the transfer cross sections also became altered as do also the $\bar{\nu}$, values in energy groups above the U-238 fission threshold. The spatially averaged spectra over the centrally located core region voided are also listed in Table 2 (from CANDID solutions of the sodium-in and sodium-out reference homogeneous situations) for comparison of spectra.

4.2. Calculation of Cross-section Perturbations Due to Differences in Heavy-isotope Resonance Self-shielding

The incremental changes in cross sections to account for the heavy-isotope resonance self-shielding correction

terms, $(\Delta k_{out}^{homr} - \Delta k_{in}^{homr})$ and $(\Delta k_{out}^{h-tr} - \Delta k_{in}^{hetr})$, were derived from calculated curves of the relative changes in capture (and fission) cross section versus potential scattering. In these resonance self-shielding-correction analyses, $\Delta k_{in}^{homr} = 0$, since the reference homogeneous sodium-in cross sections are taken as the resonance-corrected homogeneous sodium-in values.

Alterations of the cross sections to account for resonance self-shielding effects in U-238 and in U-235 are based upon the calculated curves shown in Figs. 1, 2, and 3. The curves illustrate the relative changes in the group capture (and fission) cross sections as functions of potential scattering per absorber atom. For each energy group the curves have been arbitrarily normalized to unity at 80 b for U-238 and at 300 b for U-235. When necessary, extensions to larger values of potential scattering were by extrapolation fittings of the form $[C_1 - (C_2/\sigma_p)]$, where C_1 and C_2 are (group-dependent) fitting constants.

For U-238, the curves for groups 12, 13, and 14 are based on statistical calculation methods,⁽¹³⁾ whereas for groups 15 through 22 resolved parameter data were available. These lower groups were calculated using the recent Columbia University parameters⁽¹⁴⁾ which extend the region to 4 keV. For the unresolved energy region the parameters used were $\langle \Gamma_n^0 \rangle / D = 1. \times 10^{-4}$, $D = 18.5$ eV, and $\Gamma_\gamma = 0.0246$ eV for the s-wave contribution. The p-wave parameters used were $\langle \Gamma_n^0 \rangle / D = 1.5 \times 10^{-4}$ with D values of 18.5 and 9.25 eV for $J = 1/2$ and $J = 3/2$, respectively, and $\Gamma_\gamma = 0.0246$ eV.

For U-235 the curves are based upon statistical calculation. The s-wave parameters used were $\langle \Gamma_n^0 \rangle / D = 1 \times 10^{-4}$ with D values of 1.72 and 1.34 eV for $J = 3$ and $J = 4$, respectively, and $\Gamma_\gamma = 0.033$ eV. The p-wave parameters used were $\langle \Gamma_n^0 \rangle / D = 2.5 \times 10^{-4}$ with D values of 2.40, 1.72, 1.34, and 1.10 eV for $J = 2, 3, 4$, and 5, respectively, and $\Gamma_\gamma = 0.033$ eV.

These correlation curves of relative changes in group cross sections include considerations of the effects of

TABLE 2. RATIOS OF HOMOGENIZED MACROSCOPIC CROSS SECTIONS SHOWING EFFECTS OF SPATIAL FINE-FLUX WEIGHTINGS IN THE HETEROGENEOUS SODIUM-IN AND SODIUM-OUT CONFIGURATIONS B

Group j	E_{lower}	$\bar{\Sigma}_{\text{in}}^{\text{hetf}} / \Sigma_{\text{in}}^{\text{hom}}$			$\Phi_{\text{in}}^{\text{hom}}$	$\bar{\Sigma}_{\text{out}}^{\text{hetf}} / \Sigma_{\text{out}}^{\text{hom}}$			$\Phi_{\text{in}}^{\text{hom}}$
		Transport	Capture	Fission		Transport	Capture	Fission	
1	3.68 MeV	1.0114	0.9867	1.0690	0.0178	1.0106	0.9866	1.0610	0.0186
2	2.23	1.0066	1.0130	1.0570	0.0342	1.0052	1.0105	1.0496	0.0356
3	1.35	1.0020	1.0149	1.0374	0.0568	1.0014	1.0135	1.0341	0.0576
4	0.821	1.0023	1.0134	1.0462	0.0863	1.0019	1.0111	1.0394	0.0904
5	0.498	1.0015	1.0120	1.0244	0.1209	1.0019	1.0108	1.0210	0.1272
6	0.320	1.0002	1.0044	1.0070	0.1487	1.0010	1.0053	1.0071	0.1500
7	0.183	1.0006	1.0017	1.0002	0.1364	1.0012	1.0022	1.0005	0.1374
8	0.111	0.9991	0.9965	0.9922	0.1163	1.0001	0.9980	0.9933	0.1128
9	0.0674	0.9989	0.9931	0.9851	0.0952	0.9989	0.9934	0.9858	0.0932
10	0.0409	0.9994	0.9931	0.9816	0.0651	0.9994	0.9931	0.9825	0.0644
11	0.0248	0.9968	0.9863	0.9731	0.0500	0.9974	0.9875	0.9742	0.0467
12	0.0150	0.9953	0.9801	0.9643	0.0325	0.9948	0.9803	0.9654	0.0305
13	9.12 keV	0.9956	0.9760	0.9564	0.0187	0.9955	0.9765	0.9577	0.0172
14	4.31	0.9958	0.9684	0.9413	0.0138	0.9952	0.9689	0.9434	0.0124
15	2.61	0.9866	0.9635	0.9069	0.0012	0.9895	0.9515	0.9218	0.0022
16	2.03	0.9812	0.9408	0.9206	0.0023	0.9845	0.9492	0.9209	0.0014
17	1.23	0.9824	0.9434	0.9114	0.0021	0.9837	0.9475	0.9148	0.0012
18	0.961	0.9858	0.9510	0.9033	0.0008	0.9859	0.9532	0.9071	0.0005
19	0.583	0.9804	0.9281	0.8910	0.0007	0.9797	0.9296	0.8936	0.0004
20	0.275	0.9793	0.9148	0.8776	0.0002	0.9782	0.9161	0.8801	0.0001
21	0.101	0.9738	0.8940	0.8524	0.0000	0.9732	0.8960	0.8554	0.0000
22	29.0 eV	0.9618	0.8633	0.8121	0.0000	0.9604	0.8661	0.8162	0.0000

FIG. 1. Relative Changes of U^{238} Group-capture Cross Sections as Functions of Potential Scattering.

resonance corrections to group fluxes in the relationship between group cross section and group resonance integral.⁽¹⁵⁾

For heterogeneous cell configurations the equivalence relation for lattices was applied.⁽¹⁶⁾ Since, for example, $U-238$ plate regions are neither equidistant nor contain

FIG. 2. Relative Changes of U^{235} Group-fission Cross Sections as Functions of Potential Scattering.

identical materials interspersed between them, the use of the equivalence principal for binary lattices is not directly applied. The equivalence parameter, σ_e , contains the surface correction term $[1 - 2E_3(T)]$ for binary slab lattices, where T is the thickness in units of scattering mean free path of the material interspersed between equispaced resonance-absorption regions. To circumvent this difficulty the assumption was made that the analogous quantity applicable to a given

absorption region is approximated by $[1 - E_3(T_l) - E_3(T_r)]$, where T_l is the interspersed material thickness on the left of the absorber plate and T_r that on the right, respectively, to the nearest-neighbor absorber plates of the same species.

Representative values of potential scattering per U-238 absorber atom, σ_p , for homogeneous sodium-in and sodium-out situations and of equivalent potential scattering values, σ_e , for the U-238 regions in configuration B are listed in Table 3.

From values of σ_e for each such region relative to the

FIG. 3. Relative Changes of U²³⁵ Group-capture Cross Sections as Functions of Potential Scattering.

TABLE 3. COMPARISON VALUES OF σ_p AND σ_e FOR U-238 IN HOMOGENEOUS AND HETEROGENEOUS (CASE B) SODIUM-IN AND SODIUM-OUT SITUATIONS IN BARNS PER REGIONAL U-238 ATOM

Group j	Homogeneous		Heterogeneous (Configuration B) ^(a)					
	Sodium-in	Sodium-out	Sodium-in			Sodium-out		
			a	b	c	a	b	c
12	29.2	25.6	28.5	15.5	26.0	26.4	15.0	23.7
13	33.2	29.0	30.4	16.7	28.1	28.3	15.5	25.7
14	41.8	34.6	33.7	18.2	31.6	30.9	16.4	28.4
15	192.0	31.3	46.7	28.0	46.6	29.5	15.9	26.9
16	61.4	29.2	39.0	21.9	37.8	28.5	15.6	25.8
17	34.7	30.2	31.1	16.9	28.7	28.9	15.7	26.3
18	34.3	31.2	30.9	16.7	28.5	29.4	15.9	26.8
19	35.2	32.6	31.2	16.8	28.8	30.0	16.1	27.4
20	36.0	33.5	31.5	16.9	29.1	30.4	16.2	27.9
21	36.9	34.4	31.9	17.0	29.5	30.8	16.4	28.3
22	37.0	34.5	31.9	17.0	29.5	30.9	16.4	28.3

^(a) a, b, and c refer to the three depleted U-238 regions, from left to right of configuration B, having regional thicknesses $\frac{1}{8}$, $\frac{1}{4}$, and $\frac{1}{8}$ in., respectively.

TABLE 4. RATIOS OF HOMOGENIZED MACROSCOPIC CAPTURE CROSS SECTIONS SHOWING EFFECTS OF U-238 RESONANCE SELF-SHIELDING IN THE HETEROGENEOUS CONFIGURATIONS B AND THE HOMOGENEOUS SODIUM-OUT CASE

Group j	$\bar{\Sigma}_{in}^{het}/\Sigma_{in}^{hom}$	$\bar{\Sigma}_{out}^{het}/\Sigma_{out}^{hom}$	$\Sigma_{out}^{homr}/\Sigma_{out}^{hom}$
12	0.9890	0.9866	0.9946
13	0.9806	0.9770	0.9938
14	0.9647	0.9590	0.9872
15	0.8758	0.8183	0.8496
16	0.8900	0.8437	0.8823
17	0.9241	0.9105	0.9736
18	0.9411	0.9343	0.9858
19	0.9235	0.9166	0.9842
20	0.9216	0.9155	0.9852
21	0.9277	0.9224	0.9863
22	0.9341	0.9293	0.9873

potential scattering of the reference homogeneous case, for each energy group, were obtained the applicable changes in capture (and fission) cross-section values from the calculated curves relating variation of the group cross sections with the equivalence parameter. The group capture (or fission) cross-section increments of the resonance-absorber regions were then weighted relative to each other and homogenized, using the fine-spatial group fluxes previously obtained from the cell calculations, to obtain the cross-section perturbation due to differences in resonance self-shielding.

Representative ratios of the total macroscopic capture cross sections homogenized over the heterogeneous cell of configuration B with consideration of U-238 resonance self-shielding differences relative to the uncorrected values are listed in Table 4. Ratios illustrating the corresponding changes in the homogeneous situation with sodium-out are also listed.

5. Results and Conclusions

The calculated sodium-void reactivity corrections due to the spatially fine intracell flux variations and to considerations of fertile and fissile resonance self-shielding effects are given in Table 5 for the series of heterogeneous configurations. The values refer to the centrally located volume region of the assembly. Values are given in units of inhours by use of the conversion factor 450 Ih/%k. The corrections given in Table 5 are those applied to the homogeneous sodium-void values based on use of U-238 and U-235 cross sections invariant to homogeneous sodium removal in order to obtain the corrected heterogeneous values.

To obtain the differences between the listed calculated heterogeneous values, ρ_{corr}^{het} , and the corresponding corrected homogeneous value, ρ_{corr}^{hom} , required the correction of the reference homogeneous value for the

effects of changes in the group potential scatterings between the homogeneous sodium-in and sodium-out situations. These corrections are given in Table 6 for the cases of total and one-half homogeneous removal of sodium from the centrally located core region.

The calculated correction increments affect essentially the nonleakage components of the uncorrected homogeneous sodium-void values. Since the value of the nonleakage components of the uncorrected homogeneous sodium-void case is calculated to be only $\sim +0.9$ Ih, the resonance correction of $\sim +2.8$ Ih represents a relatively large correction in the nonleakage component. For the heterogeneous configurations the resultant of the correction increments can be either positive or negative, and are generally smaller values. These are either positive or negative largely because of the relative magnitudes of the fine-flux and the resonance corrections, which are usually opposing in sign. Furthermore, the resonance-correction increments are not as large for the calculation of the heterogeneous cases because the equivalent potential scatterings, σ_e , for heterogeneous sodium-in and sodium-out situations do not differ from each other as greatly as do the σ_p values of the homogeneous sodium-in and sodium-out cases. This is especially the case for the energy groups at and near to the 2.85-keV sodium resonance.

Graphical comparisons of the variations of the sodium-void total reactivity values of the calculated homogeneous and the heterogeneous cases are shown in Fig. 4 together with the reported⁽⁶⁻⁸⁾ experimental

TABLE 6. RESONANCE SELF-SHIELDING CORRECTIONS FOR THE HOMOGENEOUS SODIUM-VOID REACTIVITY

Case	$\Delta k_{out}^{homr} - \Delta k_{in}^{homr}, \text{Ih}$		Sum of Corrections, Ih	$\rho_{corr}^{hom}, \text{Ih}^{(a)}$
	U ²³⁸	U ²³⁵		
Total Sodium Removal	+2.85	-0.02	+2.83	-4.9(5)
One-half Sodium Reduction	+1.1	0.0	+1.1	-2.7

$$^{(a)} \rho_{corr}^{hom} \cong (k_{out}^{hom} - k_{in}^{hom}) + (\Delta k_{out}^{homr} - \Delta k_{in}^{homr})_{U^{238}} + (\Delta k_{out}^{homr} - \Delta k_{in}^{homr})_{U^{235}},$$

$k_{out}^{hom} - k_{in}^{hom} = -7.78$ Ih and -3.8 Ih, respectively, for total-out and one-half-out situations.

FIG. 4. Comparisons of Calculated and Experimental Sodium-void Reactivity Values for Various Intracell Plate Configurations in ZPR-VI, Assembly No. 3.

heterogeneous values. The latter are reported⁽⁸⁾ to be experimentally reproducible within ~ 0.5 Ih. It may be noted, for comparison, that the homogeneous values obtained assuming invariance of heavy-isotope cross sections with potential scattering were calculated to be -7.78 and -3.8 Ih, respectively, for the total and one-half sodium removal, i.e., the uncorrected homogeneous values are closer to the heterogeneous values.

Of importance in this study are the comparisons of the relative correlations of the calculated variations with the experimental variations between configurations. The variations of the calculated reactivities, shown in Fig. 4, for the heterogeneous cases A, B, C, D, and F correlate well with the variations of the experimental values. For cases G and H, the relatively more negative experimental values suggest perceptible heterogeneous leakage (i.e., streaming) effects because

TABLE 5. SODIUM-VOID HETEROGENEITY CORRECTIONS AND HETEROGENEOUS REACTIVITIES FOR VARIOUS CELL CONFIGURATIONS OF ZPR-VI ASSEMBLY 3

Case	$\Delta k_{out}^{hetf} - \Delta k_{in}^{hetf}, \text{Ih}$	$\Delta k_{out}^{hetr} - \Delta k_{in}^{hetr}, \text{Ih}$		Sum of Corrections, Ih	$\rho_{Corrected}^{het}, \text{Ih}^{(a)}$
		U ²³⁸	U ²³⁵		
A	-0.52	+1.00	+0.07	+0.55	-7.2(3)
B	-0.97	+0.32	+0.07	-0.58	-8.3(6)
C	-0.40	+0.55	+0.08	+0.23	-7.5(5)
D ^(b)	-0.2	+0.6	0.0	+0.4	-3.4 ^(b)
E ^(b)	-0.4	-0.1	0.0	-0.5	-4.3 ^(b)
F	-0.60	+0.51	+0.08	-0.01	-7.7(9)
G	-0.34	+0.53	+0.08	+0.27	-7.5(1)
H	-0.89	-0.11	+0.08	-0.92	-8.7(0)

$$^{(a)} \rho_{Corrected}^{het} \cong (k_{out}^{hom} - k_{in}^{hom}) + (\Delta k_{out}^{hetf} - \Delta k_{in}^{hetf}) + (\Delta k_{out}^{hetr} - \Delta k_{in}^{hetr})_{U^{238}} + (\Delta k_{out}^{hetr} - \Delta k_{in}^{hetr})_{U^{235}};$$

$$k_{in}^{hom} = 1.000470;$$

$$k_{out}^{hom} = 1.000643, \text{ i.e., } (k_{out}^{hom} - k_{in}^{hom}) = -7.78 \text{ Ih.}$$

^(b) For sodium half-out cases the mean value of perturbations with sodium-in and with sodium-out real and adjoint fluxes used. Also $(k_{out}^{hom} - k_{in}^{hom}) = -3.8$ Ih.

of these particular heterogeneous configurations. These configurations contained 0.5-in.-thick and 1-in.-thick sodium regions, whereas all the other configurations contained $\frac{1}{4}$ -in.-thick sodium regions.

The reason for the comparatively less satisfactory correlation in Case E is not known. In Cases D and E only two of the four sodium regions of the cell were simultaneously voided in each case. (Perusal of the reported experimental data, however, suggests the possibility of error in the reported experimental value. The sum of the reported experimental reactivity values for Cases D and E differ from the experimental value for Case F by 1.4 lh. On the other hand, there were reported experimental values for identical cell-voiding configurations but with a larger region of the core being voided. The experimental value for the sum of cases analogous to D and E is very close to the case analogous to F.)

In conclusion it should be noted that the calculated correction increments represent sizable fractional corrections because of the basically small sodium-void reactivity value. The correction increments, however, are small.

APPENDIX

Calculation of Reactivity of Heterogeneous Relative to Homogeneous ZPR-VI Assembly No. 3

Effects of thin-slab heterogeneities upon the critical masses of ZPR critical assemblies have often been estimated by use of infinite, periodic, slab-cell calculations using group absorptions, derived from normal-mode analysis, to simulate overall leakages out of the system.^(1, 3, 4) The calculations therefore considered only the effects of the fine spatial variations of the flux across the cell and the computed reactivities applied to the case of an equivalent bare core, i.e., a simulated heterogeneous bare-core multiplication is compared with a homogeneous bare-core multiplication. Further-

more, the fine spatial variations of flux were assumed not to affect significantly the average transport cross sections, i.e., the homogeneous leakage-absorptions were retained also in the heterogeneous cell calculations.

For the case of ZPR-VI Assembly No. 3 the calculated bare-core reactivity estimate has been compared with a calculated reflected-core value. Since the spectrum of this carbide-simulating assembly contains non-negligible flux in some of the energy groups wherein resonance self-shielding effects, especially in U-238, can be significant, estimates of these effects have also been calculated.

Analyses in the reflected case employed the same cross-section perturbations previously described in this paper for the regional computation of Δk_{in}^{heff} , Δk_{in}^{hetr} (U-238), and Δk_{in}^{hetr} (U-235). These macroscopic cross-section perturbations were applied instead to the entire core. The reference sodium-in, two-dimensional real and adjoint fluxes were used. Possible lesser additional spectral perturbing effects upon the cross-section perturbations for core regions close to the reflector are therefore not included. (This more detailed consideration may be important for analyses of the sodium-void heterogeneity effects in peripheral regions.)

Results are given in Table 7 for the heterogeneous reference loading Case A and for assumed entire core loadings represented by configurations B, C, F, G, and H, respectively.

The values for the normal-mode (bare-core) analysis method differ by $\sim 10\%$ from the corresponding reflected values for Δk_{in}^{heff} , except for Cases G and H, which had the thicker sodium regions. The latter differ because the fine spatial averagings of the group transport cross sections differ more from the homogeneous values in these cases. For Case H, about $+0.07\%$ of the $+0.89\%$ value of Δk_{in}^{heff} represents this transport perturbation. Consideration of the resonance self-shielding increases the heterogeneity corrections by about $+0.2\%$.

Acknowledgment

The author wishes to thank Dr. H. H. Hummel for valuable comments on various aspects of this study, and Dr. R. N. Hwang for numerous helpful discussions on the problem of calculation of the relation of group cross sections to potential scattering. The author also wishes to acknowledge the machine programming effort of Miss JoAnn Zeman, and the computational assistance of Mrs. Barbara Chandler, Miss Lois Meyer, and Mr. K. E. Phillips.

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TABLE 7. HETEROGENEITY REACTIVITY CORRECTIONS FOR ZPR-VI ASSEMBLY NO. 3 CRITICAL MASSES

Case	Normal Mode $\Delta k_{in}^{hetf}, \%k$	Reflected			Total Correction (Reflected), $\%k$
		$\Delta k_{in}^{hetf},$ $\%k$	$\Delta k_{in}^{hetr}, \%k$		
			U ²³⁸	U ²³⁵	
A	+0.48	+0.42	+0.17	−0.01	+0.58
B	+0.55	+0.49	+0.22	−0.01	+0.70
C	+0.49	+0.45	+0.20	−0.01	+0.64
F	+0.58	+0.53	+0.20	−0.01	+0.72
G	+0.66	+0.62	+0.18	−0.01	+0.79
H	+0.91	+0.89	+0.22	−0.01	+1.10

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Discussion

Mr. Reynolds (GE): Will you review and give us a feeling for what would happen if we did not include the high-energy heterogeneity effect but did include all of the resonance heterogeneity effects that you mentioned? That is, if we make a first-order approximation of what happens when you lose sodium and take into account only the low-energy heterogeneities, how close would we come to the right answer?

Mr. Meneghetti: To what system do you refer? What are the materials?

Mr. Reynolds: I'm just trying to get a general feeling about how far off we would be if we just included all the low-energy heterogeneity effects but completely neglected the high-energy heterogeneity effects. About what percentage error would we make in the spectral-hardening component of the sodium-loss effect? Can you give any feeling for this?

Mr. Meneghetti: In the case of the sodium-void effect, it is rather hard to say; you really have to have a knowledge of the spectrum, because there are various compensating effects that occur here. For example, if you void a region which is relatively small, you may get one value; if you void a region which is somewhat larger, you get another value because the ambient spectrum changes depending on how big the voided region is. So it is rather hard to say. But the high-energy heterogeneity effect certainly should be calculated—it wouldn't surprise me a bit if it was at least half the effect.

Mr. Okrent (ANL): Does your calculation of the change in critical mass allow, in any way, for changes in leakage due to a streaming effect?

Mr. Meneghetti: No. It doesn't; however, in this particular aspect, I don't think it's too serious. I think that if we are going to worry about the streaming effect, we should worry about it relative to the sodium-void effect, rather than to the critical mass, because in the sodium-void effect it should show up more severely. Even here it shows up in relatively small values, and I suppose that if one were to stick to sodium regions of $\frac{1}{4}$ in. or perhaps even $\frac{1}{2}$ in. it actually shouldn't give too much trouble.

Mr. Okrent: I agree that it would be still more sensitive to sodium void. I am still bothered by the apparent lack of agreement between some old theoretical work on streaming, which indicates that there may be reactivity effects associated with heterogeneities without complete voiding, and no sign of such effects apparently when experiments are done. Can you or somebody else comment on the earlier work, or is this still an uncertainty?

Mr. Meneghetti: In my opinion the calculation of the streaming effect in both these instances is not yet in a good state.

Mr. Tirén (AB Atomenergi): I would like to comment on the problem of streaming effects. We have made some measurements of this kind in the FRO reactor where $\frac{1}{4}$ -in.-thick channels were made throughout the reactor, including top and bottom reflectors. We found that streaming in such channels was of the order of 10^{-4} in $\delta k/k$.

R. B. Nicholson

A Preliminary Investigation of the Effect of Heterogeneity on Activation Doppler Experiments

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ABSTRACT

A method has been developed for approximate calculation of the Doppler coefficient of activation in small-sample experiments when the experimental environment consists of a heterogeneous arrangement of plates. The activation is calculated in cylindrical geometry on the axis. The surrounding material is divided into any number of $(\Delta r, \Delta\varphi)$ zones with uniform composition within the zone. The activation is given by

$$A(T_c, T_s) = \sum_{i,g} \frac{S_i^g \Delta\varphi_i}{2\pi} \int_g \frac{dE \sigma_\gamma(E, T_s)}{\Sigma_{t,i}(E, T_c)} \cdot [Ki_2(P_i(E, T_c)) - Ki_2(P'_i(E, T_c))],$$

where i stands for a zone; g for an energy group; T_c is the core temperature; T_s the sample temperature; S_i^g the smoothed neutron source in zone i and group g (proportional to the scattering cross section if one uses the flat-flux approximation); $\Delta\varphi_i$ the angular width of zone i ; σ_γ the activation cross section; P_i the optical path from the axis to the inner radius of zone i ; and P'_i the optical path to the outer radius of zone i . The function $Ki_2(P)$ is

$$Ki_2(P) = \int_1^\infty \frac{dx \exp(-Px)}{x^2 \sqrt{x^2 - 1}}.$$

Preliminary analysis indicates that it is very difficult to attain a flux spectrum at the sample characteristic of a homogeneous medium with the plate thicknesses normally available in critical assemblies. It is further shown that if one wishes to attain the best approach to homogeneity, the plates should be oriented so that the sample views them broadside rather than end on. Some rough guidelines have been established for selecting the best plate arrangement.

The goal of the present work is mainly an understanding of the effects that the heterogeneity inherent in ZPR-3 critical experiments have on the Doppler coefficient of activation in the Davey-Amundson type

of activation experiment.⁽¹⁾ From this understanding it is hoped that one will better be able to interpret the experiments and will better be able to design future experiments in such a way as to minimize the effects of heterogeneity. The procedure of surrounding the experiment by an anti-Dancoff region that is sometimes used in reactivity Doppler experiments is not useful here, because in the small-sample activation experiment the flux depressions are due entirely to the incident flux. An anti-Dancoff zone would merely wipe out the total effect.

Although the motivating interest of this work was the activation experiments, the conclusions from the numerical calculations seem to apply equally well to small-sample reactivity Doppler experiments if the goal of the experimenter is to provide sample environment characteristic of the homogenized core.

In activation Doppler experiments Davey and Amundson⁽¹⁾ have observed that a change in the orientation of the plates in the drawers immediately adjoining the test position can change the Doppler coefficient of activation by a factor of 2. We wish to investigate this effect analytically. To solve the problem for the actual critical experiment geometry is too difficult at the present time. Instead, we take a more simple cylindrical geometry. To simplify the presentation of the method we assume there are just two materials, U-238 and the mixture of the remaining core materials. It will be evident that the method can be generalized to more materials.

The activation sample of U-238 is assumed to be a thin sample and is located at the center of a cylindrical void surrounded by the heterogeneous core. We require that each particular zone of the core be bounded by cylindrical surfaces and plane surfaces that include the axis of symmetry. We might then have the idealized experimental geometry shown in Fig. 1, in which U-238 plates are shown as black and the remainder of the core as white. For demonstration purposes we show only four U-238 plates. In Fig. 2 the orientation of the fuel plates is changed.

Even in this simple geometry it is necessary to resort to a computer. Once that step is taken, it is just as easy to study more complicated arrangements of fuel plates, provided only that each region is bounded by the normal surfaces of cylindrical geometry. For example, we could study the configuration of Fig. 3.

In actual application one would expect that only the heterogeneities in the neighborhood of the sample are important to the activation experiment and would therefore homogenize the balance of the core beyond a certain radius.

The simplicity of the method rests largely on the assumption that the sample is located exactly on the axis of the cylinder. There will be some variation of the spectrum throughout the void region. However, we anticipate on physical grounds that the spectrum will be stationary with respect to small displacements from the center, so that the activation of the sample should be fairly uniform and approximately equal to the value calculated on the axis, provided that the sample is fairly small and nearly centered on the axis. It is difficult to check quantitatively the error in this assumption.

From the final equations a result is seen that might at first seem paradoxical. With the further approximations discussed below, equations (10) and (11) predict that the configuration of Fig. 2 will give identically the same flux at the sample position as a similar configuration in which each of the four plates is split into several thinner plates spaced in a like manner around the ring as many spokes of a wheel. Clearly as one further and further subdivides the plates, the flux must approach the homogeneous result. Our equations state that this is not true for the precise central point, and they are indeed correct for that point.

The resolution of the paradox is the following. The stationary property of the solution in the neighborhood of the axis when the plates radiate as spokes of a wheel is limited to a region of the order of the thickness of a plate. If one goes outside this region, the flux spectrum can change significantly. In the case where the spokes

FIG. 2. Spoke Arrangement.

FIG. 3. Alternative Spoke Arrangement.

are subdivided to infinitesimal thickness, one need only move from the axis an infinitesimal distance to go into a flux characteristic of a homogeneous system.

The configuration of Fig. 2 would then result in a flux that depends fairly strongly on the position of the sample, and our calculation method is not capable of providing that information. Further, one can well imagine that calculations in this highly idealized geometry will not compare accurately with the results of a normal critical experiment. Nevertheless, we believe that the qualitative conclusions regarding the changes that occur when one changes from an orientation of the type in Fig. 1 to that of Fig. 2 can be relied upon. Also, we believe that configurations like Fig. 1 can be used as approximate quantitative models for critical experiments which have plate orientations of that type.

From the above discussion one should already begin to wonder whether perhaps plate configurations of the type of Fig. 2 should be avoided. Even in the less symmetrical geometry of a real experiment it seems likely that, if one arranges the plates to point more or less at the experiment, the flux spectrum in the experiment would vary with position of the sample—an undesirable condition. We believe that our calculations

FIG. 1. Idealized Experimental Geometry.

below give further and rather strong indications that configurations like Fig. 2 should be avoided if one wishes to produce a flux characteristic of a homogenized system.

We employ two more important approximations. The source of neutrons of energy E due to scattering from higher energies is assumed spatially uniform through each region. It may differ from one region to another due to differences in the material. Secondly, the source is assumed to be smooth in energy through the resonances. It will have the usual coarse variation with energy, following the neutron flux spectrum calculated for the homogenized system. These two assumptions are equivalent to the narrow-resonance approximation combined with the assumption that the off-resonance flux is spatially flat. We will further neglect effects of resonance overlap.

Under the above approximations the flux of neutrons of energy E at the sample position ($r = 0$) due to the scattering source $S_i(E)$ in region i is given by

$$\phi_{0i}(E) = \frac{S_i(E)}{4\pi} \int_{\text{region } i} \int_{-\infty}^{\infty} \frac{dz \exp\left[\frac{-P(r, \varphi, E)}{\cos \theta}\right]}{r^2 + z^2}, \quad (1)$$

where $P(r, \varphi, E)$ is the total-cross-section optical path to the axis in a direction perpendicular to the axis, and θ is the angle between the perpendicular and the actual path from the origin to the point (r, φ, z) . To our approximation $S_i(E)$ is simply proportional to the total smoothed scattering cross section of region i .

We require (and it is always possible to do so) that each region be subdivided if necessary into smaller regions such that $P(r, \varphi)$ is independent of φ within each region i . Then the φ integration is trivial, and we have

$$\begin{aligned} \phi_{0i}(E, T_c) &= \frac{S_i(E)\Delta\varphi_i}{4\pi} \int_{r_i}^{r_i+\Delta r_i} dr r \\ &\quad \cdot \int_{-\infty}^{\infty} \frac{dz \exp\left[-P_i(r, E, T_c) \frac{\sqrt{r^2 + z^2}}{r}\right]}{r^2 + z^2} \\ &= \frac{S_i(E)\Delta\varphi_i}{2\pi} \int_{r_i}^{r_i+\Delta r_i} dr \\ &\quad \cdot \int_1^{\infty} \frac{dx \exp[-xP_i(r, E, T_c)]}{x\sqrt{x^2 - 1}}, \end{aligned} \quad (2)$$

the second step having been accomplished by a simple change of variable in the integration over z . The radius r_i is the inner radius of region i .

The optical path is

$$P_i(r, E, T_c) = \sum_j \Sigma_{tj}(E, T_c) \Delta r_j + \Sigma_{ti}(E, T_c)(r - r_i), \quad (3)$$

where the sum is over all regions j between region i

and the sample. Define also the optical paths to the inner and outer edges of region i :

$$\begin{aligned} P_i(E, T_c) &= \sum_j \Sigma_{tj}(E, T_c) \Delta r_j; \\ P'_i(E, T_c) &= P_i(E, T_c) + \Sigma_{ti}(E, T_c) \Delta r_i, \end{aligned} \quad (4)$$

where again the sum is over all regions interior to region i .

Substitute in equation (2) and interchange orders of integration over r and x :

$$\begin{aligned} \phi_{0i}(E, T_c) &= \frac{S_i(E)\Delta\varphi_i}{2\pi} \int_1^{\infty} \frac{dx \exp\{-xP_i(E, T_c)\}}{x\sqrt{x^2 - 1}} \\ &\quad \cdot \int_{r_i}^{r_i+\Delta r_i} dr \exp\{-x\Sigma_{ti}(E, T_c)(r - r_i)\} \\ &= \frac{S_i(E)\Delta\varphi_i}{2\pi\Sigma_{ti}(E, T_c)} \\ &\quad \cdot \int_1^{\infty} \frac{dx [\exp\{-xP_i(E, T_c)\} - \exp\{-xP'_i(E, T_c)\}]}{x^2 \sqrt{x^2 - 1}} \\ &= \frac{S_i(E)\Delta\varphi_i}{2\pi\Sigma_{ti}(E, T_c)} [Ki_2(P_i(E, T_c)) - Ki_2(P'_i(E, T_c))], \end{aligned} \quad (5)$$

where Ki_2 is one of the class of functions

$$Ki_n(X) = \int_0^{\infty} \frac{\exp(-X \cosh u) du}{(\cosh u)^n} \quad (6)$$

discussed and tabulated by Bickley and Nayler.⁽²⁾ These functions are merely multiple integrals of the K_0 Bessel Function. Table 1 gives a few additional values of Ki_2 calculated to fill out and extend the table of Bickley and Nayler.

The values in Table 1 are believed to be all accurate to the last significant figure given. The tabular values can also be fit by two relatively simple analytical expressions to an accuracy of about 1%:

$$Ki_2(P) \approx \frac{\exp\left(\frac{-3P}{4}\right)}{1 + \frac{3P}{4}} \quad \text{for } P \leq 1.5; \quad (7)$$

$$Ki_2(P) \approx \sqrt{\frac{\pi}{2(P+2)}} e^{-P} \quad \text{for } P \geq 1.5. \quad (8)$$

The formula for $P \leq 1.5$ is an empirical fit. The formula for $P \geq 1.5$ is the asymptotic formula for $P \gg 1$. It is obtained in the following manner. By a simple transformation, $Ki_2(P)$ can be put in the form

$$Ki_2(P) = \int_1^{\infty} \frac{e^{-xP} dx}{x^2 \sqrt{x^2 - 1}}. \quad (9)$$

We notice that for large P the contributions to the integrand in equation (9) come mainly from the neigh-

TABLE 1. THE FUNCTION $Ki_2(P)$

P	Ki_2	P	Ki_2	P	Ki_2	P	Ki_2
0	1	0.35	0.614	1.4	0.170	3.2	0.0224
0.01	0.9846	0.43	0.554	1.5	0.152	3.4	0.018
0.022	0.9668	0.51	0.500	1.6	0.135	3.6	0.0145
0.030	0.9552	0.59	0.452	1.7	0.120	3.8	0.0116
0.038	0.9438	0.67	0.408	1.8	0.107	4.0	0.0093
0.062	0.9110	0.75	0.370	1.9	0.096	4.2	0.0075
0.102	0.8600	0.85	0.327	2.0	0.084	4.4	0.0061
0.150	0.8039	0.95	0.290	2.2	0.068	4.6	0.0049
0.190	0.7608	1.05	0.257	2.4	0.054	4.8	0.0040
0.23	0.7205	1.15	0.282	2.6	0.043	5.0	0.0032
0.27	0.6828	1.20	0.215	2.8	0.035		
0.31	0.6475	1.30	0.191	3.0	0.028		

borhood of $x = 1$. Then in the limit of large P it is permissible to expand $1/x^2$ in the form

$$\frac{1}{x^2} = \frac{1}{1 + (x^2 - 1)} \approx \exp [-(x^2 - 1)] \quad (10)$$

$$\approx \exp [-2(x - 1)].$$

Also,

$$\sqrt{x^2 - 1} \approx \sqrt{2(x - 1)}. \quad (11)$$

When (11) and (10) are inserted in (9) there results

$$\begin{aligned} \psi(P) &\approx \frac{e^2}{\sqrt{2}} \int_1^\infty \frac{dx \exp [-x(P + 2)]}{\sqrt{x - 1}} \\ &= \frac{e^{-P}}{\sqrt{2}} \int_0^\infty \frac{dy \exp (-y(P + 2))}{\sqrt{y}} \\ &= \sqrt{\frac{\pi}{2(P + 2)}} e^{-P} \quad \text{for } P \gg 1, \end{aligned} \quad (12)$$

which is the asymptotic formula given in (8).

Although (8) is only an asymptotic formula, it is already fairly good at $P = 1$ (about 3% error) and is less than 1.5% in error for $P > 1.5$. Bickley and Nayler give the complete asymptotic expansion. However, it is not very useful for moderate values of P ($P < 3$ for example) where our equation (8) is good enough for our purposes.

With Ki_2 available as a tabulated or approximate analytic function, the activation can be expressed as

$$A = \sum_i \frac{\Delta\varphi_i}{2\pi} \int dE \frac{S_i(E)\sigma_\gamma(E, T_s)}{\Sigma_{ti}(E, T_c)} \cdot [Ki_2(P_i(E, T_c)) - Ki_2(P'(E, T_c))]. \quad (13)$$

For convenience in numerical calculation one would normally divide the E integration into an integration over energy groups g and a sum over g :

$$A = \sum_{i,g} \frac{\Delta\varphi_i S_i^g}{2\pi} \int_g dE \frac{\sigma_\gamma(E, T_s)}{\Sigma_{ti}(E, T_c)} \cdot [Ki_2(P_i(E, T_c)) - Ki_2(P'(E, T_c))]. \quad (14)$$

Some Sample Calculations

Because of the complicated energy dependence of the U-238 cross section, it is difficult to evaluate equation (14) without the aid of a computer. The computer program has not yet been completed. However, it is illuminating to calculate the flux at the sample in a typical resonance relative to its off-resonance value. This can be obtained easily from equation (5), using the values of Ki_2 from Table 1. We can then calculate the flux depression for a homogeneous system and various heterogeneous mockups, and determine which mockups give a flux nearest to the homogeneous system.

As a byproduct of the calculation, we can also see how the contributions to ϕ fall off as we go farther from the sample. It will be seen that to a fair approximation one need only be concerned with the plate arrangements in the eight drawers of a critical experiment immediately surrounding the sample drawer.

We choose a highly simplified and arbitrary system for the sample calculations. We ignore capture and assume we have a single resonant material (1) and a uniform mixture of diluent scattering material (2). The resonant material is characterized by a resonant cross section Σ_{1r} , a potential cross section Σ_{1p} , and a mean total effective scattering cross section Σ_{1t}^{eff} . The diluent scatterer is characterized by Σ_{2t} , its total scattering cross section. The volume fractions of the two materials are F_1 and F_2 , respectively. The values used in sample calculations are given in Table 2.

Although the cross sections have been chosen arbitrarily, Cases A-C can be related crudely to real reactor situations. Case A corresponds to a U-238 resonance material at room temperature and a neutron energy of about 2 keV. Case B is the same but with a neutron energy of about 3.5 keV. Case C is again the same but the neutron energy is about 20 keV. Since in the narrow-resonance and flat-flux approximations the diluent enters only as a total scattering cross section, the diluent mixture can be anything provided it adds up to $\Sigma_{2t} \approx 0.3 \text{ cm}^{-1}$, which is roughly typical of most fast reactors. In retrospect the value $\Sigma_{1t}^{eff} = 0.75 \text{ cm}^{-1}$ is a bit high for U-238, but that will not unduly upset the results.

In addition to the variations in resonance cross sections we consider variations in geometrical arrangements as given in Figures 4-8.

Figure 4 is the case of a 2-in. test hole surrounded

TABLE 2. PARAMETERS FOR SAMPLE CALCULATIONS

	Σ_{1p}	Σ_{1r}	Σ_{1t}^{eff}	Σ_{2t}	F_1	F_2
Case A	0.5	4.0	0.75	0.3	1/6	5/6
Case B	0.5	3.0	0.75	0.3	1/6	5/6
Case C	0.5	.40	0.75	0.3	1/6	5/6

by a homogeneous core of the given cross sections. In Fig. 5 all of the material within a cylinder of 3-in. diameter has been segregated into resonance material and diluent, with the resonance material placed at the mean radius of 2 in. Outside the 3-in. cylinder the material is left homogeneous.

In Fig. 6 the resonance material is put in four radial plates (actually as noted earlier the flux at the center is independent of the number of radial plates into which the resonance material is divided).

In Fig. 7 we have subdivided the resonance material into two rings. All of the resonance material within a

FIG. 4. Case I—Homogeneous Core.

FIG. 5. Case II—A Single Circumferential Resonance Plate.

FIG. 6. Case III—Radial Resonance Plates.

FIG. 7. Case IV—Two Circumferential Resonance Plates.

FIG. 8. Case V—Eight Quarter-plates.

2-in. radius was placed in a ring at the mean radius of $1\frac{1}{2}$ in. and between the 2- and 3-in. radius it was placed in a ring at the mean radius of $2\frac{1}{2}$ in. This is aimed, of course, at attempting a step towards homogenization as compared to Fig. 5.

Supposing one is limited in how thin the plates can be and nevertheless wishes to attain a better degree of homogenization than is attained in Fig. 7. We suggest as a possibility (and the results bear this out) that the configuration of Fig. 8 might be a closer approach to the homogeneous case. Here each of the rings of Fig. 7 is divided into four sectors, and the rings are placed at different radii in each sector such that the resonance material more or less samples all the positions within the 3-in. circle. In order to simplify the calculation we have distorted the original sector when moving it from one position to the next such that it always occupies

a quarter sector, and always contains the same amount of material. Admittedly, one cannot do this in quite the same way in the critical assembly, but I think the calculation will still demonstrate the effect reasonably well.

The results of the calculations described above are given in Table 3. In that table, Case II-B, for example, means the configuration Case II and the cross-section Case B.

Discussion of Numerical Results

The results shown in Table 3 are believed to be of considerable significance in the performance of Doppler experiments in fast critical assemblies. Recall that Cases A and B correspond to U-238 resonances at a few keV. It is evident from the large deviations of the flux depression from that in a homogeneous system that for the available plate thicknesses, there is no reasonable distribution of the plates that closely approximates the homogeneous condition. In some cases the deviations are very great. In cases V-A and V-B the deviation is just about tolerable.

It should be noted that while we have considered the average resonance at a few keV, there is in fact a broad distribution of resonance peaks and some are much stronger than this. Then when one averages over all resonances the situation may be even worse than it appears from Table 3.

In all the heterogeneous calculations in Table 3, the cylinder of 3-in. radius was divided into cylindrical shell cells and the resonance material allocated to the cell was positioned at the center of the cell. It is evident that if one were to displace the material slightly toward the axis rather than in the center of the cell, a position could be found that would, for a particular value of the resonance cross section, bring the flux depressions for heterogeneous and homogeneous systems into agreement. This might be worth exploring further, but it would clearly not be possible to accomplish this for all resonances at the same time. It also has the disadvantage that the experimentalist must rely on a separate theoretical calculation for the specification of his plate arrangements in each separate experiment. It seems more satisfactory to have a simple rule such as locating the resonance material at the center of each cell, as we have done in the present calculations.

It is also worth noting that the present procedure of putting the resonance material at the center of the cell consistently results in a positive deviation from the homogeneous result. We have not determined whether this will continue to hold true for all possible variations of the cross-section parameters.

We have considered a uniform mixture of materials as the diluent, whereas the experimentalist is usually

TABLE 3. RESONANCE FLUX DEPRESSION

Case	ϕ_{nr}	ϕ_r	ϕ_r/ϕ_{nr}	% Deviation from Homogeneous
I-A	1.125	0.375	0.333	—
I-B	1.125	0.450	0.400	—
I-C	1.125	0.938	0.833	—
II-A	1.092	0.64	0.59	+ 79
II-B	1.092	0.68	0.62	+ 55
II-C	1.092	0.952	0.872	+ 3.5
III-A	1.095	0.784	0.716	+115
III-B	1.095	0.801	0.734	+ 84
III-C	1.095	0.987	0.884	+ 6.1
IV-A	1.117	0.494	0.441	+ 32
IV-B	1.117	0.542	0.487	+ 22
IV-C	1.117	0.943	0.844	+ 1.3
V-A	1.133	0.457	0.403	+ 21
V-B	1.133	0.507	0.447	+ 12

restricted to using a mixture of plates perhaps $\frac{1}{8}$ in. thick. If it is supposed that the "cell" consists of a U-238 plate, a stainless steel plate, a Pu-239 plate, and an aluminum plate, it is obviously impossible to locate the U-238 plate at the center of this "cell." One must do the best he can. In the case of a number of diluent materials rather than a homogeneous mixture, the center is to be interpreted, not as geometrical center, but rather as a center with respect to optical path and with respect to scattering source. Ideally, one would put half of the optical path and half of the scattering source of the diluent on each side of the resonance material. Since this cannot be accomplished exactly with plates of finite thickness it seems best (because of the consistent positive deviation discussed above) to err in the direction of putting less optical path and less scattering source on the side of the resonance material towards the experiment, and more on the far side. In the particular case of plates of equal thickness of Pu-239, stainless steel, and aluminum, this would suggest putting the steel and aluminum together on the far side and the Pu-239 plate on the near side. In this case it is about a toss up whether to put the aluminum with the plutonium or the steel.

The result of the calculations of perhaps the most immediate interest is that the Case-III geometry gives significantly poorer agreement with the homogeneous case than any of the others considered. This we interpret to be a clear mandate against turning the plates surrounding an experiment such that they all point towards or nearly towards the experiment. In fact, in retrospect (and admitting that on first hearing that this was a popular procedure, it seemed a good one), we state that it is obvious that this is just about the worst thing that could be done if one wishes to attain a nearness to homogeneity. The experiment, after all,

sees the surroundings in considerable depth rather than seeing merely the surface of the plates. What is desired to approach homogeneity is that *each* neutron beam in the material surrounding the experiment traveling in a direction that can potentially intercept the experiment, should see the experiment through an optical path length that is as nearly as possible equal to the optical path in the homogeneous material. If the plates are turned to point in the direction of the experiment, then the neutrons are divided into two classes: those that have to look almost entirely through the resonance material, and those that look through the diluent.

The cases V-A and V-B suggest that with a particular set of available plate sizes, the experimentalist can perhaps do better than to make each of the surrounding drawers identical. By making them each different he can effectively sweep the resonance material through the drawer in four successive steps and thereby effect an artificial homogenization. He cannot, of course, do this in quite as precise a manner as we have done in the calculations, but he can very likely attain a substantial improvement over the procedure of making each drawer the same. The recipe here is again to jump in increments of equal optical paths as nearly as possible.

The experimentalist might also consider doing experiments in which the region immediately surrounding the sample is not homogenized but is in some other way simplified with respect to calculation of the activation by our method. We suggest, for example, that the plates be oriented perpendicular to the path to the sample, that the innermost plates be of the resonance material, and that the next few plates be of some simple scattering material such as graphite. Any other resonance material should then be placed about a mean free path or more from the sample, so that it will have little influence on the experiment. In this case the experiment could be well approximated by a configuration similar to that in Fig. 5 and our method would be reliable.

Finally, we note that the function $Ki_2(P_i)$ is a rapidly decreasing function for $P_i > 1$. If the optical path through the first drawers surrounding the experiment is greater than unity (preferably greater than 1.5), then, except for very accurate experiments, it is permissible to ignore the heterogeneities except in the eight nearest drawers. Within these eight drawers one should attempt to homogenize the material to the best of his ability. While this may seem a trivial statement, we believe the above calculations provide it with at least a semiquantitative significance.

Summary of Conclusions

1. To best approach homogenization with plates in the eight drawers surrounding an experiment, they

should be oriented such that the experiment sees them in a direction as nearly as possible perpendicular to the plane of the plates.

2. A really accurate homogenization is not possible with the plate thicknesses normally used in critical assemblies. Serious thought should be given to providing limited amounts of finer material to fabricate more nearly homogeneous mixtures in the region immediately surrounding experiments of this type.
3. Some rough guidelines have been outlined for attaining the best possible degree of homogenization with the available plates.

Acknowledgment

Discussions with W. G. Davey, R. G. Palmer, P. I. Amundson, and R. W. Goin have been most helpful in motivating this work.

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Discussion

Mr. Kato (ANL): Would you repeat exactly what the experiments were for which you said that there was a factor of two between the results?

Mr. Nicholson: The experiments were done by Davey and Amundson in the SEFOR assembly. In this assembly they irradiated a U²³⁸ foil at low temperature in a furnace; then they irradiated a comparable foil (or the same one—I'm not sure) at a high temperature. There was a change in activation of 3.5% when the plates were all oriented toward the foil. When they turned the plates in these drawers the other way, the change in activation between the two temperatures was 7%. However, they do not claim high accuracy for these measurements and so there is still some uncertainty here.

Mr. Lantz (NASA): What materials did you activate? Why didn't you use *xy* geometry in your calculations?

Mr. Nicholson: The reason why we use radial geometry is that the calculation is extremely simple in the cylindrical geometry because at the center point of the cylindrical geometry every neutron which arrives there has traveled along a radial path and therefore does not cross sideways through the boundaries of the geometry; the calculation is simplified. You asked what materials were irradiated. I think that in the Davey-Amundson experiment only U²³⁸ was irradiated. The Swedish group has done U²³⁵ in addition.

Mr. Lantz: How much of this is a fast effect due to

uncollided flux rather than resonance? If you're irradiating U^{238} you have a certain number of uncollided neutrons incident upon the U^{238} . In one case it might be different than in the other.

Mr. Nicholson: I don't understand the effect, so I can't answer the question, but I think that the total effect that we are getting here is a resonance effect.

Mr. Specht (AI): What do you think would be the influence on the activation Doppler effect of orienting the diluents away from one end of the box as they tried in several earlier experiments?

Mr. Nicholson: Do you mean placing all of the resonance plates together and all the diluents together? Well, then you would have a substantially increased

activation effect with the resonance plates close to the sample, that is, the effect on the activation would tend to be increased when the resonance material was near the sample and it would be decreased quite appreciably when it was the other way around. There could be, I think, easily a factor of two between the results, or perhaps considerably more than that.

Mr. Snyder (GE): In making the Doppler activation measurements, how is the flux measured? How is the situation at one temperature compared with the situation at the other?

Mr. Nicholson: One doesn't measure flux; one measures only the activation of the sample at the two different temperatures.

Presented by A. Khairallah

Calculation of the Sodium-void and Doppler Reactivity Coefficients in Fast Reactors and Critical Assemblies, with Heterogeneity Taken into Account*

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ABSTRACT

A perturbation method, based on the use of collision probabilities in the transport operator, is presented for the investigation of the Doppler and sodium-loss effects in a heterogeneous medium. The effectiveness of the method is first illustrated by its application to the homogeneous case, where it is shown to be superior to the conventional perturbation theory and to give results very close to those of exact calculations, even for large perturbations. Thus it is believed to give also correct results for the heterogeneous case.

It is shown that heterogeneity decreases both the positive component of the sodium-loss effect and the additional positive reactivity effect due to the change in the resonant-fuel "effective dilution" with sodium loss, especially in metal-fueled critical assemblies where heterogeneity is pronounced. In contrast, heterogeneity seems not to change appreciably the total Doppler effect, even though it may increase the individual contributions of the fertile and fissile isotopes.

The possibility is mentioned of applying this method to other reactivity effects which are proper to a heterogeneous medium (i.e., which have no equivalent in a homogeneous medium), such as the effect of radial expansion of the fuel on spatial self-shielding and self-multiplication.

1. Introduction

Except for some attempts to account for heterogeneity,^(1, 2) Doppler and sodium-loss coefficients in fast reactors are usually calculated under the assumption that the reactor is homogeneous.† It is generally accepted that this is a reasonable approximation for power reactors; however, this may not be the case for some critical assemblies where heterogeneity is more

pronounced. A method is presented here for investigating the influence of heterogeneity on these coefficients, with essentially no more fundamental assumptions than for the usual homogeneous calculations. This method has been outlined in a preceding paper⁽³⁾ and since been refined, particularly in view of calculating large perturbations and of taking into account the change in the dilution of the resonant fuel with sodium loss. Its efficiency and the validity of the approximations made are discussed here.

In a previous paper⁽⁴⁾ a perturbation method was presented for calculating the effect of heterogeneity on k_{eff} and on the spatial fine structure of the flux and the adjoint flux, without accounting for the change, with heterogeneity, of the effective dilution of the resonant-fuel cross sections. The core was replaced by an equivalent infinite medium (by the use of fictitious capture cross sections for the leakage) that was divided into a certain number of homogeneous regions, generally two: the "fuel region" and the "diluent region." The collision-probability formalism was used in the integral transport equation, and the effects of heterogeneity were calculated by a perturbation method, the homogeneous core being the unperturbed reference and the perturbation being due to the change in the collision probabilities when going from the homogeneous core to the heterogeneous one. Numerical results, obtained with the code PERHET, were presented.

The investigation of the heterogeneous Doppler and sodium-loss effects that is presented here is based on an extension of the method of Ref. 4. The model used is that of the infinite equivalent core, and the formalism is the same, except for two main differences:

a) The reference (unperturbed) core is the actual heterogeneous one (before the change in the fuel temperature of the sodium loss), the k_{eff} , flux, and adjoint flux of which are calculated by the perturbation method of Ref. 4.

b) The perturbation formulae for the Doppler and sodium-loss reactivity changes contain not only terms proportional to the change in the collision probabilities,

* Work performed under the EURATOM-C.E.A. Association for Fast Reactors.

† It has become usual, however, to account partly for heterogeneity by correcting the effective cross sections of the resonant fuel according to the "equivalence theorems" between heterogeneous and homogeneous media.

but also terms proportional to the change in the cross sections of the fuel region (Doppler) or the diluent region (sodium loss) or both regions (change in the effective dilution of the resonant-fuel isotopes and change in the elastic removal cross sections, in case of large sodium loss).

It should be noted, concerning the sodium-loss effect, that the present investigation is restricted to the generally positive component of this effect (absorption and spectrum hardening) and ignores the negative leakage component. Heterogeneity is known to increase the latter component, because of streaming. An increase of a few percent was found⁽⁵⁾ by a method based on the work of Benoist⁽⁶⁾ on the diffusion coefficient in a lattice.

2. Principle of the Method

As in Ref. 4, the basic equations are the nN multi-group transport equations for an infinite core consisting of a lattice of n regions, N being the number of energy groups, of the form

$$\mu_{ij}^g \phi_j^g - \sum_{k,i} P_{ij}^g \mu_{ik}^{k \rightarrow g} \phi_i^k = 0, \quad (1)$$

where

- ϕ_j^g is the average group- g flux in region j of the heterogeneous core;
- P_{ij}^g is the collision probability in region j of group- g neutrons born in region i ;
- μ_{ij}^g is the total cross section for group- g neutrons in region j ;
- $\mu_{ik}^{k \rightarrow g} = \mu_{ik}^{k \rightarrow g} + (\chi^g/k_{\text{eff}})\nu\mu_{ik}^k$ is the transfer cross section (by scattering and possibly fission) from group k to group g , in region i ;

The μ are macroscopic effective group cross sections, the definition of which in the heterogeneous case is discussed in the appendix, *per unit volume of the whole core*. Lower indices refer to regions and upper indices to energy groups.

The adjoint flux is defined by the set of equations adjoint to Eq. (1), of the form

$$\mu_{ij}^g \phi_j^{*g} - \sum_{k,i} P_{ij}^k \mu_{ik}^{g \rightarrow k} \phi_i^{*k} = 0. \quad (2)$$

It was shown in Ref. 4 that the adjoint flux defined by Eq. (2) does not have the physical meaning of the conventional importance function, and that ϕ_j^{*g} should be interpreted as the importance of a neutron of group g that will suffer its next collision in region j .

The change in reactivity that results from a temperature change or a sodium loss is given by the following first-order perturbation equation that is easily derived from Eqs. (1) and (2):

$$\frac{\delta k_{\text{eff}}}{k_{\text{eff}}} = \frac{\sum_{g,j} \phi_j^g \{ -\delta \mu_{ij}^g \phi_i^{*g} + \sum_{k,i} \delta (\mu_{ij}^{g \rightarrow k} P_{ij}^k) \phi_i^{*k} \}}{\frac{1}{k_{\text{eff}}} \sum_{g,j} \phi_j^g \nu \mu_{ij}^g \sum_{k,i} \chi^k P_{ij}^k \phi_i^{*k}}, \quad (3)$$

where δx indicates the change in x that results from the Doppler broadening or the sodium loss. The denominator of Eq. (3) is the so-called importance of fission neutrons and will be designated by I_f .

The physical meaning of Eq. (3) is clear enough. For the subsequent discussion, however, it will be convenient to expand

$$\delta (\mu_{ij}^{g \rightarrow k} P_{ij}^k) = \delta \mu_{ij}^{g \rightarrow k} P_{ij}^k + \delta P_{ij}^k (\mu_{ij}^{g \rightarrow k} + \delta \mu_{ij}^{g \rightarrow k}),$$

and to introduce a new quantity $\bar{\phi}^*$, defined by

$$\bar{\phi}_i^{*g} = \sum_j P_{ji}^g \phi_j^{*g},$$

that has the same meaning as the conventional importance function. Then Eq. (3) may be written as

$$\begin{aligned} \frac{\delta k_{\text{eff}}}{k_{\text{eff}}} = \frac{1}{I_f} \{ & - \sum_{g,j} \phi_j^g \delta \mu_{ij}^g \bar{\phi}_i^{*g} + \sum_{g,j} \phi_j^g \sum_k \delta \mu_{ij}^{g \rightarrow k} \bar{\phi}_i^{*k} \\ & + \sum_{g,j} \phi_j^g [\delta \mu_{ij}^g \sum_i P_{ji}^g (\phi_i^{*g} - \bar{\phi}_i^{*g}) \\ & + \sum_{k,i} \delta P_{ji}^k (\mu_{ij}^{g \rightarrow k} + \delta \mu_{ij}^{g \rightarrow k}) \phi_i^{*k}] \}. \end{aligned} \quad (4)$$

2.1. Remarks

1. Eq. (3) is the same as Eq. (1) of Ref. 3, except that in the latter the term $\delta \mu_{ij}^{g \rightarrow k} \delta P_{ij}^k$ was neglected.

2. Eq. (3) is also an extension of Eq. (8a) of Ref. 4, where only the change in the collision probabilities was taken into account in the calculation of the effect of heterogeneity on k_{eff} (PERHET code). Improvement of the latter calculation according to Eq. (3) is intended, in order to account for the change, with heterogeneity, of the resonant-fuel effective cross sections.

2.2. Comparison with conventional perturbation theory in the homogeneous case

The perturbation Eq. (4) can be compared to what conventional perturbation theory would give. In effect, Eq. (4) is applicable to the particular case of the homogeneous core, by putting the appropriate values of the changes in the effective group cross sections and collision probabilities and putting ψ and ψ^* for ϕ , and ϕ_j^* , respectively, ψ and ψ^* being the classical flux and importance function in the homogeneous core. Conventional first-order perturbation theory would give, for this infinite homogeneous core, the following perturbation equation:

$$\frac{\delta k_{\text{eff}}}{k_{\text{eff}}} = \frac{1}{I_f} \{ - \sum_{g,j} \psi_j^g \delta \mu_{ij}^g \psi_i^{*g} + \sum_{g,j} \psi_j^g \sum_k \delta \mu_{ij}^{g \rightarrow k} \psi_i^{*k} \}. \quad (5)$$

The first two terms of Eq. (4) correspond to the two

terms of Eq. (5), and have the same meaning, i.e., they represent the effect due to the change in the absorption and slowing-down rates of neutrons. The third term of Eq. (4) represents the effect of the spatial shift of neutrons, and does not appear in the classical perturbation Eq. (5). It can be shown that for the homogeneous case this term is of second order in the cross-section perturbation. Thus Eq. (4) applied to the homogeneous case and Eq. (5) do not give the same result for large cross-section perturbations. Actually, calculations show that for arbitrarily large perturbations in a homogeneous core the results of Eq. (4) are very close to those of direct calculations; the results of Eq. (5) are correct only for small perturbations. This is related to the fact that, in contrast with conventional first-order perturbation theory, the perturbation of the operator in this method is not linear in the cross-section perturbations but contains terms of higher order. This explains why, in spite of being a first-order perturbation method, this method gives correctly not only the effect of small perturbations, but also that of large ones.

Finally, the effectiveness of this method in the homogeneous case gives some confidence that it yields also for heterogeneous cores correct results even for large perturbations and that it is not necessary to carry it, as was mentioned in Ref. 3, to higher orders.

3. Application to the Calculation of the Sodium-loss Effect

The calculations are performed by a subroutine that uses the perturbation Eq. (4), which was added to the code PERHET. Provision is made for taking into account, in addition to the sodium removal, the effect of the resulting changes in the dilution of the resonant isotopes and possibly in the elastic slowing down

TABLE 1. SODIUM-LOSS EFFECT^(a) ($\delta k/k \times 10^2$)

	Sodium loss		
	20%	50%	100%
Homogeneous direct calculation	+0.144	+0.383	+0.881
	+0.173	+0.449	+1.060
Homogeneous classical perturbation theory	+0.141	+0.354	+0.707
	+0.171	+0.432	+0.923
Homogeneous, this method	+0.146	+0.391	+0.884
	+0.176	+0.460	+1.090
Power reactor, this method	+0.137	+0.368	+0.833
	+0.165	+0.429	+0.998
Critical mockup, this method	+0.096	+0.263	+0.622
	+0.110	+0.294	+0.700

^(a) Upper lines: without change in the fuel effective dilution; lower lines: with change in the fuel effective dilution.

FIG. 1. Sodium-loss Effect without Change in the Resonant-fuel Dilution.

FIG. 2. Sodium-loss Effect with Change in the Resonant-fuel Dilution.

(change in the inside-group weighting flux). Actually, however, the latter effect was not included in the calculations, because a code of the ELMOE type applicable to a heterogeneous lattice was not available.

The numerical results presented here are relative to a reference homogeneous core containing about 35 v/o PuO_2 - UO_2 (isotopic ratio about 5), 14 v/o stainless steel, and 51 v/o sodium. Two 2-region heterogeneous versions of this core were considered, a power reactor (oxide fuel, 6-mm diameter) and its critical mockup (metal fuel, containing fertile and fissile materials together, $\phi = 12$ mm, oxygen mixed with the diluent). The ABBN⁽⁷⁾ cross-section set was used, with resonance self-shielding corrections corresponding to the homogeneous core and to its two heterogeneous versions, and to 0, 20, 50, and 100 % sodium loss.

Table 1 and Figures 1 and 2 summarize the results

obtained for 20, 50, and 100% sodium losses. In order to test the effectiveness of this method, three kinds of calculations were made for the homogeneous case: a direct calculation of k_{eff} , a conventional perturbation calculation, and a calculation by the method of this paper. It is seen that the latter gives very close results (better than 3%) to those of direct calculations, much closer than those of conventional perturbation theory. Two other main conclusions appear, concerning the influence of heterogeneity on the sodium-loss effect:

a) Figures 1 and 2 show that heterogeneity tends to decrease the positive component of the sodium-loss effect. This decrease is relatively small in the case of a power reactor (of the order of 6 to 10% of the positive component), but may become appreciable in the corresponding critical mockup (of the order of 30 to 40%). This conclusion appears to be valid for any sodium-loss percentage, in particular to the sodium coefficient (i.e., the tangents to the curves of Figs. 1 and 2). The effect of heterogeneity should thus be taken into account in the interpretation of the measurements of this coefficient in critical assemblies.

That heterogeneity should decrease the positive sodium-loss effect is illustrated by the first diagram of Fig. 3. The full curve, corresponding to the homogeneous core with a positive sodium coefficient, is concave upward because of sodium self-shielding. The dashed line corresponds to the heterogeneous core. For sodium-in the heterogeneous core is more reactive than the homogeneous one, as is usually the case for fast systems. For sodium-out, however, the homogeneous and heterogeneous cores should be equivalent, since empty channels have no effect in an infinite medium. From these assumptions, it follows directly that the positive sodium component must be smaller in the heterogeneous core than in the homogeneous one, as is confirmed by our calculations. The reactivity increase resulting from total sodium loss should be, in this idealized situation, reduced by the amount of reactivity gained by heterogeneity in the sodium-in case.

The second diagram of Fig. 3 illustrates the (negative) leakage component of the sodium-loss effect, with the nonleakage probability $P = k_{\text{eff}}/k_{\infty}$ as ordinate. The heterogeneity reduces P because of the streaming effect, which is certainly more important in the sodium-out situation. It was estimated⁽⁴⁾ that this reduction results in an increase of a few percent in the negative leakage component.

b) It is known from homogeneous calculations that the change in the resonant-fuel effective dilution with sodium loss results in an additional positive reactivity effect. Table 1 shows, as can be expected, that this remains true in the heterogeneous case, but that this unfavorable effect is also decreased by heterogeneity especially in the critical mockup.

FIG. 3. Heterogeneous Effects on Multiplication Factor.

Thus the net sodium-loss effect is certainly less positive in the heterogeneous case than in the homogeneous one. For power reactors, the influence of heterogeneity seems to be relatively small. In some critical assemblies, however, heterogeneity could more likely turn a positive effect into a negative one, particularly in view of the fact that the net sodium-loss effect, being a small difference of two large components, is very sensitive to corrections on these components. It seems thus important that a correction for heterogeneity should be made for the interpretation of the measurements of sodium-loss coefficients in critical assemblies.

4. Application to Doppler Calculations

Here spatially constant temperatures and effective cross sections are assumed inside the fuel elements, while the diluent cross sections are independent of temperature. No account is taken of possible interference effects between the resonances of different isotopes. With these assumptions, the main problem in the calculation of the isothermal heterogeneous Doppler effect consists in finding the proper changes in the effective group cross sections of the fuel and in the group collision probabilities to be used in the perturbation eq. (4).

The effective cross sections of the fuel depend on the energy fine structure of the average fuel flux, and consequently on that of the cross sections and that of the collision probabilities. It is clear that if more than two regions are considered or if as "exact" values as possible for the collision probabilities are used, the energy fine

structure of the fuel flux can only be known by means of numerical calculations.

The problem is greatly simplified if only two regions are considered: the fuel region and the diluent region. We use as far as possible results of homogeneous calculations and take advantage of the success of the Wigner-type rational approximation in the calculations of effective resonance integrals; rational expressions of this type are used for the collision probabilities. For instance, the fuel-fuel collision probability at energy E is given by

$$P_{uu}(E) = \frac{\mu_{tu}(E)}{\mu_{tu}(E) + \mu_F}, \quad (6)$$

where index u refers to the fuel region, and μ_F , which will be defined later, is considered as constant over many resonances; the three other collision probabilities at energy E are derived from $P_{uu}(E)$ according to the reciprocity and complementarity relations.

Under the above assumptions and the usual ones that the neutron slowing-down sources and the diluent cross sections are slowly varying functions of energy (or at least do not have energy fine structures correlated to that of the fuel resonant cross sections), it is a simple and classical matter to show (see Appendix) that the average fuel flux is proportional to

$$\phi_u(E) \sim \frac{1}{\mu_{tu}(E) + \mu_F}, \quad (7)$$

and that the effective cross sections of the fuel can be calculated as in a homogeneous medium, but with a different "effective dilution," μ_F replacing μ_{tm} , the diluent total cross section. This results in an increase of the change with temperature of the effective cross sections of the fuel in most of the Doppler energy range, and thus an increase of the Doppler effect is to be expected.

It is also shown in the Appendix that the group collision probabilities derived from Eq. (6) are simple

FIG. 4. Bell Factor for a Lattice of Cylindrical Rods (vs the Optical Thickness of a Rod).

functions of the fuel group effective total cross section, of μ_{tm} , and of μ_F . Thus the temperature changes of the effective group cross sections and of the group collision probabilities are easily obtained from temperature- and dilution-dependent group cross sections calculated for the homogeneous case, provided μ_F is properly calculated.

The most familiar approximation is Wigner's expression:

$$\mu_F = \frac{V_u}{V} \frac{(1-C)}{\bar{l}_u}, \quad (8)$$

where $\bar{l}_u$ is the average chord (practically the diameter) of the fuel pins and $(1-C)$ the Dancoff factor, which depends only on the geometry of the lattice and on the diluent total cross section μ_{tm} . Calculations of effective resonance integrals have proven Eq. (8) to be a poor approximation, and many improvements have been proposed, following Bell's suggestion⁽⁸⁾ of replacing it by

$$\mu_F = \frac{V_u}{V} \frac{a(1-C)}{\bar{l}_u}, \quad (9)$$

where a is a constant known as the "Bell factor."

Strictly speaking, a should be a function of the fuel cross sections (i.e., of energy), implicitly defined by Eqs. (9) and (6) where $P_{uu}(E)$ is the exact fuel-fuel collision probability. Figure 4 represents this function for various practical values of $(1-C)$, and shows that a may change at most by about 40% when the total cross section of fuel goes from very large values to very small ones, e.g., across a resonance. However, it is usual (and much more practical) to use an average, energy-independent value of a . The original suggestion of Bell was to use $a = 1.3$ for any lattice of cylindrical rods. At the opposite extreme, Otter⁽⁹⁾ and Kelber⁽¹⁰⁾ have recently proposed an iterative numerical averaging of the energy-dependent a such that it preserves the effective resonance integral. Although the latter procedure is exact in principle, it seems too tedious for the purpose of this paper. A mean term was preferred; we used the approximation of Leslie *et al.*,⁽¹¹⁾ i.e., a Bell factor that is independent of the fuel cross sections but depends on the Dancoff correction (i.e., on the geometry of the lattice and on the diluent cross sections). In Ref. 11 the following expression for a is recommended, that has the desired property that a tends to 1, so that μ_F tends to the diluent total cross section μ_{tm} , as it should, in the limiting homogeneous case:

$$a = \frac{1}{1 - \left(\frac{\alpha}{\alpha + \sqrt{1-C}} \right)^2 \frac{1-C}{3}}, \quad (10)$$

where $\alpha \simeq 2.37$.

Although Eq. (10) seems to give effective resonance

integrals that are close to the results of exact numerical calculations,⁽¹¹⁾ it is not certain, *a priori*, that it should also be a good approximation for Doppler calculations. This is because in a resonance the maximum contribution to the resonance integral comes from the central region (high cross section) while the Doppler effect arises mainly from the wings (low cross section). This and Fig. 4 suggest that the average a to be used for Doppler calculations should be larger than that given by Eq. (10). In the "white-limit" (optical thickness of the fuel rods $\Sigma_{tu} \bar{l}_u \ll 1$), it may be easily found that the exact a is given by

$$a = \frac{1}{1 - \frac{1-C}{3}}. \quad (11)$$

The Bell factors given by Eqs. (10) and (11) are represented by curves I and II of Fig. 5, respectively, together with exact values of a , as a function of $(1-C)$. The appropriate a for Doppler calculations should lie between curves I and II. However, numerical calculations using either Eq. (10) or Eq. (11) have shown that the choice between these two expressions has very little bearing on the calculated Doppler coefficient, and thus Eq. (10) was adopted.

4.1. Numerical calculations

The Doppler calculations were performed for the same reference core and its two heterogeneous versions for which the sodium-loss effect was calculated. The temperature- and dilution-dependent Fischer's cross sections⁽¹²⁾ were interpolated and used. The effective cross sections for each isotope are calculated as

$$\bar{\sigma}_x = \int \frac{\sigma_x}{\sigma_t + \sigma_p} dE \bigg/ \int \frac{1}{\sigma_t + \sigma_p} dE,$$

where σ_t contains the resonances of that isotope only, the resonances of the other isotopes being replaced by an effective total cross section. The difference between

FIG. 5. Bell Factor for a Lattice of Cylindrical Rods (vs the Dancoff Factor).

TABLE 2. DOPPLER COEFFICIENT $T(dk/dT)$ ($\times 10^2$)

	Contribution of		
	U ²³⁸	Pu ²³⁹	Total
Homogeneous, 1200°K	-0.565	+0.111	-0.454
Power reactor, 1200°K	-0.570	+0.113	-0.457
	-0.574	+0.116	-0.458
Homogeneous, 300°K	-0.545	+0.137	-0.408
Critical mock up	-0.597	+0.179	-0.418
	-0.608	+0.181	-0.427

the homogeneous and heterogeneous effective cross sections is thus due primarily to the difference in σ_p between the two cases. The results obtained for the heterogeneous cases were compared to those obtained by the method in Ref. 1, which uses the homogeneous conventional perturbation equation but with the heterogeneous changes in the effective cross sections.

Table 2 compares the values of $T(\partial k_{eff}/\partial T)$ for the homogeneous core and the power reactor at 1200°K, and for the homogeneous core and the critical mockup at 300°K. The lines labelled "exact" refer to the method of this paper; the label "approximate" refers to the method of Ref. 1. No appreciable difference was found between this method and conventional perturbation theory for the homogeneous case. The contribution due to spatial shift of neutrons was found negligible in all cases, less than 1% of the calculated Doppler effect. Some interesting conclusions appear from Table 2:

a) Heterogeneity seems to have a negligible effect on the total Doppler effect, as well for the oxide-fuel power reactor as for the metal-fuel critical mockup.

b) However, in the latter case, the various contributions, and especially those of the fissile isotope, are increased. This may have some safety implications. It may be worth recalling that the critical mockup is assumed to be loaded with cylindrical fuel elements (the type of fuel for the critical assembly Masurca) with fissile and fertile materials mixed together. No calculation was made for a plate-loaded assembly with separated fissile and fertile materials.

c) The fact that the contribution of the spatial shift of neutrons is negligible indicates that the heterogeneous Doppler effect can be computed as a local effect, i.e., as depending only on the flux and adjoint flux at the fuel location, provided the diluent region is taken into account for the computation of the effective dilution.

d) This last result justifies the approximation of Ref. 1, since the spatial fine structure of the flux and

adjoint flux is not so pronounced as to influence the Doppler effect appreciably.

5. Application of this Method for the Calculation of other "Heterogeneous" Reactivity Effects

The method that has been presented and illustrated by the calculation of the Doppler and sodium-loss effects is actually applicable to the calculation of any reactivity effect that involves a change in the cross sections or in the spatial fine structure (change in the collision probabilities) of a heterogeneous medium.

When a reactor or a critical assembly is brought up in power or in temperature, various expansions, displacements, and changes in the densities take place. Some of the resulting reactivity effects can be estimated, as is usually done, by homogeneous calculations; however, as was shown for the sodium-loss coefficient, the method of this paper makes possible a more accurate calculation of these effects. In addition, some reactivity effects are proper to the heterogeneous structure of the medium, i.e., they have no equivalent in a homogeneous medium. They cannot be even roughly estimated by a homogeneous method, and this method provides a useful means for calculating them.

Two examples will be discussed.

5.1. Radial expansion of the structures results mainly in a spreading of the fuel elements. In a power reactor, the outward displacement of the fuel is accompanied by an inward displacement of the sodium. The resulting reactivity effect can be partly estimated by a homogeneous calculation, but the additional effect due to the modification of the lattice structure should be accounted for by a "heterogeneous" method, such as that presented in this paper. A similar problem arises in a heated critical assembly.

5.2. Radial expansion of the fuel elements may cause, in a power reactor, some sodium expulsion which can again be estimated by a homogeneous calculation (or more accurately with the "heterogeneous" method of this paper). But the effect on reactivity of the decrease in the self-multiplication and spatial self-shielding of the fuel elements is proper to the heterogeneous structure of the reactor and can only be accounted for by a "heterogeneous" method. This last effect seems to be particularly important in Doppler experiments in critical assemblies, where it may in some cases be of the same order of magnitude as the effect to be measured.

APPENDIX

Group Effective Cross Sections and Group Collision Probabilities

The basic equations are the following coupled energy-transfer and space-transfer equations for a lattice of n regions:

$$\frac{V_i}{V} S_i(E) = \int \mu_i(E' \rightarrow E) \phi_i(E') dE'; \quad (A1)$$

$$\mu_{ti}(E) \phi_j(E) = \sum_{i=1}^n P_{ij}(E) \frac{V_i}{V} S_i(E), \quad (A2)$$

where

$\phi_j(E)$ is the average flux at energy E in region j ;

$S_i(E)$ is the neutron source, by scattering or fission per unit energy and unit volume in region i ;

$\mu_i(E' \rightarrow E) = \mu_{si}(E' \rightarrow E) + (\chi(E)/k_{\text{eff}}) \nu \mu_{fi}(E')$ is the transfer cross section in region i , including elastic and inelastic transfer as well as fission;

$\mu_{ti}(E)$ is the total cross section in region i ;

$P_{ij}(E)$ is the collision probability in region j of neutrons born at energy E in region i ;

V_i is the volume of region i , and V the total volume.

The group effective cross sections in region j are defined as in the homogeneous case, by weighting the energy-dependent cross sections by the inside-group flux in that region. For group g of width ΔE^g , the effective cross section for process x in region j is thus

$$\mu_{xj}^g = \frac{\int_{\Delta E^g} \mu_{xj}(E) \phi_j(E) dE}{\int_{\Delta E^g} \phi_j(E) dE} \quad (A3)$$

and the transfer cross section from group g to group k is

$$\mu_j^{g \rightarrow k} = \frac{\int_{\Delta E^k} dE' \int_{\Delta E^g} \mu_j(E \rightarrow E') \phi_j(E) dE}{\int_{\Delta E^g} \phi_j(E) dE}, \quad (A4)$$

where

$$\int_{\Delta E^g} \phi_j(E) dE = \phi_j^g$$

is group- g flux in region j .

Finally, the group collision probability P_{ij}^g is defined as

$$P_{ij}^g = \frac{\int_{\Delta E^g} P_{ij}(E) S_i(E) dE}{\int_{\Delta E^g} S_i(E) dE}. \quad (A5)$$

According to the above definitions, Eq. (A1) may be written as

$$\frac{V_i}{V} S_i(E) = \sum_{k=1}^N \int_{\Delta E^k} \mu_i(E' \rightarrow E) \phi_i(E') dE', \quad (A6)$$

and the integration of equations (A6) and (A2) over the group- g width gives

$$\frac{V_i}{V} S_i^g = \sum_{k=1}^N \mu_i^{k \rightarrow g} \phi_i^k \quad (\text{A7})$$

and

$$\mu_{ij}^g \phi_j^g = \sum_{i=1}^N P_{ij}^g \frac{V_i}{V} S_i^g. \quad (\text{A8})$$

When $(V_i/V) S_i^g$ is eliminated between Eqs. (A7) and (A8), Eq. (1) of the text is obtained:

$$\mu_{ij}^g \phi_j^g - \sum_{k,i} P_{ij}^g \mu_i^{k \rightarrow g} \phi_i^k = 0. \quad (\text{A9})$$

The way of obtaining Eq. (A9) shows that while the group effective cross sections should be weighted by the flux, the group collision probabilities should be weighted by the neutron slowing-down sources. In practice, however, it is usual to calculate the effective cross sections in the narrow-resonance approximation by taking into account only the energy fine structure of the flux. Moreover, since in this approximation the neutron sources are slowly varying functions of energy, P_{ij}^g given by Eq. (A5) may be taken to be

$$P_{ij}^g = \frac{1}{\Delta E_g} \int_{\Delta E_g} P_{ij}(E) dE, \quad (\text{A10})$$

the latter definition having the advantage of preserving the reciprocity and complementarity relations.

Particular Case of Two Regions, with a Wigner-type Rational Approximation

Equation (A2) may be written, for the flux in the fuel region, as

$$\begin{aligned} \mu_{tu}(E) \phi_u(E) &= \frac{V_u}{V} P_{uu}(E) S_u(E) \\ &+ \frac{V_m}{V} P_{mu}(E) S_m(E), \end{aligned} \quad (\text{A11})$$

where index u refers to the fuel region and index m to the diluent region. The fuel-fuel collision probability $P_{uu}(E)$ is approximated by the following rational expression:

$$P_{uu}(E) = \frac{\mu_{tu}(E)}{\mu_{tu}(E) + \mu_F}. \quad (\text{A12})$$

$P_{mu}(E)$ is derived from $P_{uu}(E)$ according to the reciprocity and complementarity relations:

$$P_{mu}(E) = \frac{\mu_{tu}(E) \mu_F}{\mu_{tm}(E) (\mu_{tu}(E) + \mu_F)}. \quad (\text{A13})$$

Eq. (A11) then gives

$$\phi_u(E) = \frac{\frac{V_u}{V} S_u(E) + \frac{V_m}{V} \frac{\mu_F}{\mu_{tm}(E)} S_m(E)}{\mu_{tu}(E) + \mu_F}. \quad (\text{A14})$$

In the narrow-resonance approximation, $S_u(E)$ and $S_m(E)$ are slowly varying functions of energy. $\mu_{tm}(E)$ is also assumed to vary slowly with energy, so that the numerator of the right-hand side of Eq. (A14) may be considered as constant over many fuel resonances. Eq. (A14) shows thus that the fine structure of the fuel flux is proportional to $1/(\mu_{tu}(E) + \mu_F)$, whereas in the homogeneous case it would be proportional to $1/(\mu_{tu}(E) + \mu_{tm}(E))$. This means, as is known in effective resonance integral calculations, that the effective cross sections of the fuel can be calculated as in the homogeneous case, but with a different dilution, μ_F replacing μ_{tm} . It is worthwhile noting that since

$$\begin{aligned} P_{uu}(E) &= \frac{\mu_{tu}(E)}{\mu_{tu}(E) + \mu_F} \\ &> P_{uu}(E)_{\text{homog}} = \frac{\mu_{tu}(E)}{\mu_{tu}(E) + \mu_{tm}}, \end{aligned}$$

μ_F should be smaller than μ_{tm} , that is, the fuel is less diluted in the heterogeneous case than in the homogeneous one. This results in a decrease in the effective cross sections, but generally in an increase in their temperature-change in most of the Doppler energy range.

With this particular fine structure of the flux calculation of P_{uu}^g according to Eq. (A10) and μ_{tu}^g according to Eq. (A3) shows readily that

$$P_{uu}^g = \frac{\mu_{tu}^g}{\mu_{tu}^g + \mu_F}. \quad (\text{A15})$$

Since Eq. (A10) preserves the reciprocity and complementarity relations, the three other group collision probabilities can be derived from Eq. (A15) according to these relations.

Thus, the group collision probabilities, and their change with temperature, can be calculated in terms of the group effective cross sections and of their change with temperature, respectively.

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Discussion

Mr. Gandini (CNEN): Is this method related to the British work in which the perturbation is obtained from the expansion of the diffusion kernel by stopping at the first term?

Mr. Khairallah: Yes, the basic method on the effect of heterogeneity on k_{eff} is similar to the perturbation method used by James and Matthews. The only difference between the two is that the method of James and Matthews was restricted to plane lattices whereas here we may have in principle any lattice form, but practically we should have a regular lattice. Mainly we treat two regions in this method. In the fuel region we have in mind the fuel pins either of a power reactor or of a critical assembly such as MASURCA.

Mr. Gandini: Your adjoint function appears to have the same physical meaning as the conventional one, since it refers to the same physical quantity, namely, the power level, which is a real function of the neutron flux. Here you used a different transport equation based on collision probability. Consequently you have a different adjoint function from the mathematical point of view, but physically it seems to be closely related to the traditional one.

Mr. Khairallah: The adjoint function which we have when we use the collision probability in the integral equation is neither mathematically nor physically identical to the usual importance function, but they are very closely related as follows:

$$\bar{\phi}_j^* = \sum_i P_{ji} \phi_i^*.$$

ϕ_i^* is the importance function derived from the equations that I have presented. $\bar{\phi}_j^*$ is the importance function that has the same meaning as the usual importance function, and this equation gives the relation between the two. When you put a neutron in region j it will have a collision somewhere in one of the other regions, or maybe in region j also, with a probability P_{ji} . If it has a collision in region i , the importance of the neutron for this collision is ϕ_i^* .

Mr. Gandini: Have you used the Monte Carlo code for P_{ji} ?

Mr. Khairallah: No, we haven't. For the calculation of the collision probability until now we have only used the Suer approximation for two-region cells.

Mr. Meneghetti (ANL): I don't suppose that when you say that, in cases for which the heterogeneity correction in both sodium-in and sodium-out situations is positive, the correction in the case of sodium-in is necessarily greater than in the case of sodium-out. This is generally true only for the case of a binary cell, because in cases that you don't have a binary cell it will go one way or the other depending upon the other diluents.

Mr. Khairallah: This is true. The other diluents are assumed to be homogeneous in this two-region cell. The conclusion may apply more exactly to power reactors when you have almost always two regions than to a critical assembly.

Experimental Verification of Reactor Physics Parameters of Zoned Cores

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1. Introduction

Core zoning is a technique whereby the total fuel inventory required to study a large fast reactor can be substantially reduced. This technique usually comprises a test zone, a buffer region, a driver region, and a reflector. The composition of the test zone is usually identical to the composition of the large core whose spectrum is to be duplicated at the center of the zoned core. The driver region must necessarily have greater concentration per unit volume of the fissile material than in the test zone if the reduction in fuel inventory is to be realized. The buffer region, between the test and driver zones, must be tailored as to composition and thickness in such a way that the neutron energy and importance spectra at the center of each of the test zone and the full-size core are the same. The composition of the reflector can be the same for both the zoned and full-size cores.

The choice of diluent materials and their concentrations in the driver region is, by and large, a matter of taste, although there may be a particular composition which tends to minimize the critical mass. The choice of materials for the buffer region, to a large measure, depends on the neutron spectrum in the driver. For example, if the neutron spectrum in the driver is harder in the entire energy range than in the test zone, then the buffer region should generally have an inelastic scatterer for degrading the high-energy neutrons and an elastic scatterer for the low-energy neutrons. The desirable composition is usually arrived at by adjusting the concentration of one component at the expense of the other until the spectral matching in the two systems is achieved. The thickness of the buffer region is usually chosen according to the ease with which a shell can be constructed, and this depends on the physical properties of the facility involved.

This concept of zoned cores is being experimentally verified at Argonne National Laboratory, and this paper should be considered as a progress report rather than a finished piece of work, because the results are still slowly "coming" in. The experimental program involves

three different assemblies, two zoned and one full-size cores. The two zoned assemblies consist of one with a large test zone (radius = 41.53 cm) and another with a rather small test zone (radius = 20.89 cm). The large zoned core is designated as Assembly No. 4Z of ZPR-6 and the smaller core is designated as Assembly No. 11 of ZPR-9. The test zones of both cores have the composition of a 2600-liter reactor, the full-size system, which is designated as Assembly No. 5 of ZPR-6.

The major objective associated with the zoned cores is to test, in conjunction with the full-scale system, how well reactor physics parameters for Assembly No. 5 can be predicted from the zoned-core measurements. Another important aspect of zoned cores is how small can they be built and still be useful?

The types of physics parameters that can be studied with zoned cores are the all-important reactivity coefficients such as the sodium-void and Doppler coefficients, and reactivity worths of samples of various materials. The accuracy with which such reactivity coefficients can be predicted for the full-size core from measurements performed in the zoned cores depends on: (1) the accuracy of the calculated importance-weighted integral of the source of fission neutrons in the entire volume of each of the assemblies, and (2) the accuracy of the spectral matching at the center of the systems. The following relation is used to predict the values of the coefficients for the full-size system:

$$W_F = W_Z(S_Z/S_F), \quad (1)$$

where

W_F = the predicted worth of a sample in the full-size core;

W_Z = the measured worth of the same sample in the zoned core;

$$S = \int_V \left[\sum_i (\nu \Sigma_f)_i \phi_i \right] \left[\sum_i \chi_i \phi_i^+ \right] dV,$$

where V is the volume of the reactor, and the subscripts F and Z refer to the full-size and zoned cores, respectively. Equation (1) results from perturbation theory,

and it is valid provided the measurements of reactivity worth are done at a position in the zoned core where the neutron energy as well as the spatial distributions of the real and adjoint fluxes are the same as those in the full-scale system. The spatial distributions of the fluxes are important particularly in nonzero gradients of the fluxes where the leakage contribution to the perturbation can be significant.

For successful extrapolation of results from the zoned to full-size systems, it is necessary to obtain accurate value of the quantity S_Z/S_F . At present, S_Z/S_F can be obtained by calculations only. (Later in this paper, a possible method for experimentally determining S_Z and/or S_F will be discussed.) Since there will always be some uncertainties in the calculation of S_Z/S_F , the magnitude of such uncertainties may be estimated with the proper types of measurements. The measurements that are pertinent to the evaluation of zoned cores are: (1) the verification of the spectral matching as obtained from fission ratio measurements; (2) the verification of the spatial distribution of the real fluxes, especially around boundaries; (3) the spatial distribution of the importance of fission neutrons; and (4) the reactivity worths of various materials measured at the center of the three assemblies. The last measurement should provide the acid test as to the limitations of

TABLE 2. COMPARISON OF FLUXES AND ADJOINTS
AT 30 CM FROM THE CENTER OF ASSEMBLIES
NO. 5 AND 4Z^(a)

Group	$\phi_{30 \text{ cm}},$ Assembly No. 5	$\phi_{30 \text{ cm}},$ Assembly No. 4Z	$\phi_{30 \text{ cm}}^*,$ Assembly No. 5	$\phi_{30 \text{ cm}}^*,$ Assembly No. 4Z	$\phi\phi_{No. 5-}^*$ $\phi\phi_{No. 4Z}^*$
1	1.476	1.470	5.411	5.530	-0.142
2	2.838	2.800	5.130	5.219	-0.054
3	4.775	4.681	4.676	4.737	+0.154
4	7.362	7.242	4.054	4.077	+0.320
5	10.478	10.490	4.028	4.048	-0.254
6	12.993	13.042	4.069	4.089	+0.540
7	13.194	13.240	4.064	4.068	-0.240
8	11.443	11.480	4.046	4.033	-0.001
9	9.822	9.874	4.035	4.008	+0.057
10	7.801	7.850	4.037	4.000	+0.093
11	5.953	5.985	4.064	4.019	+0.139
12	4.744	4.741	4.137	4.086	+0.254
13	2.979	2.963	4.252	4.203	+0.214
14	2.332	2.319	4.445	4.409	+0.142
15	0.251	0.250	4.621	4.600	+0.010
16	0.518	0.515	4.637	4.611	+0.027
17	0.489	0.481	4.630	4.607	+0.048
18	0.257	0.249	4.636	4.616	+0.042
19	0.244	0.232	5.025	5.010	+0.064
20	0.0819	0.0775	5.284	5.280	+0.024
21	0.0256	0.0243	5.392	5.400	+0.007
22	0.00165	0.00158	5.323	5.340	+0.003

^(a) Calculations were done in spherical geometry (MAIM-6, cross-section set 224⁽²⁾).

TABLE 1. COMPARISON OF FLUXES AND ADJOINTS
IN ASSEMBLIES NO. 5 AND 4Z^(a)

Group/ E_L ^(b) (keV)	$\phi_{\text{Center}},$ Assembly No. 5	$\phi_{\text{Center}},$ Assembly No. 4Z	$\phi_{\text{Center}}^*,$ Assembly No. 5	$\phi_{\text{Center}}^*,$ Assembly No. 4Z	$\phi\phi_{No. 5-}^*$ $\phi\phi_{No. 4Z}^*$
1/3680	1.462	1.475	5.411	5.425	-0.091
2/2230	2.837	2.835	5.130	5.137	-0.009
3/1350	4.774	4.763	4.676	4.678	+0.042
4/821	7.301	7.282	4.054	4.051	+0.099
5/498	10.479	10.459	4.029	4.024	+0.133
6/302	12.995	12.981	4.070	4.061	+0.174
7/183	13.197	13.193	4.064	4.052	+0.175
8/111	11.445	11.448	4.046	4.034	+0.205
9/67.4	9.823	9.832	4.034	4.024	+0.062
10/40.9	7.801	7.814	4.037	4.029	+0.010
11/24.8	5.952	5.965	4.063	4.059	-0.029
12/15.0	4.742	4.756	4.137	4.135	-0.048
13/9.11	2.977	2.986	4.252	4.254	-0.044
14/4.31	2.330	2.336	4.445	4.448	-0.034
15/2.61	0.251	0.251	4.621	4.625	-0.001
16/2.03	0.518	0.519	4.637	4.641	-0.007
17/1.23	0.488	0.489	4.630	4.634	-0.007
18/0.961	0.256	0.256	4.635	4.640	-0.001
19/0.583	0.243	0.243	5.025	5.029	-0.001
20/0.275	0.082	0.082	5.284	5.289	-0.001
21/0.101	0.0255	0.0255	5.393	5.397	-0.000
22/0.029	0.0016	0.0016	5.323	5.327	-0.000

^(a) Calculations were done in spherical geometry (MAIM-6, cross-section set 224⁽²⁾).

^(b) Lower energy limit of group.

cross-section sets and calculational methods which are used for the predictions of reactivity worths in the full-size core.

2. Design of Assemblies 4Z and 5 of ZPR-6 and 11 of ZPR-9

The material concentrations as well as the radii of the various regions of Assemblies 4Z, 5, and 11 are given in Ref. 1. The test zone in Assembly 4Z is rather large ($r = 41.53$ cm); this size was chosen in order to insure spectral matching with Assembly 5 over a large region in the test zone. The buffer region consisted of graphite and U^{238} (elastic and inelastic scatterers) with a ratio of atomic concentrations of 1.39/1, respectively. The driver zone had the same material concentrations as the test zone with the U^{235} concentration increased at the expense of the U^{238} . The calculated spectral matching at the center of each of Assemblies 4Z and 5 is shown in Table 1. The fluxes and adjoints shown in Table 1 were normalized to 100 neutrons at the center. Although the fluxes and adjoints were calculated in spherical geometry, a similar agreement was obtained in cylindrical geometry. Table 2 shows the comparison between the fluxes and adjoints in the two assemblies at 30 cm from the center of each core. The last columns of Tables 1 and 2 are an indication of the "goodness" of

spectral matching. It is seen that the spectral matching at 30 cm from the center is quite good although it is not as good as that at the center.

The test zone in Assembly 11 was rather small ($r = 20.89$ cm); this size was chosen in order to reduce the critical mass. The critical mass of Assembly 11 was about 500 kg U^{235} , about $\frac{1}{3}$ of the critical mass in Assembly 5. The driver region of this assembly was composed of graphite, U^{235} , and some structural stainless steel. The neutron spectrum in this region was harder in the high-energy groups and softer in the low-energy groups than that in Assembly 5. This necessitated the introduction of natural boron in the buffer region to eliminate the low-energy neutrons. The rest of the materials in the buffer region were the same as in the test zone with the exception of U^{235} , which was replaced with U^{238} . The reflector region in Assembly 11 was steel instead of the depleted uranium (0.2% enrichment), which was in the reflector regions of Assemblies 5 and 4Z. Table 3 compares the calculated spectra in Assemblies 5 and 11. It is seen from Table 3 that the spectral matching is not as good as in Assembly 4Z; however, the last column of Table 3 shows that for an absorption type of perturbation (the reactivity worth of a pure absorber is proportional to the product of real and adjoint fluxes) the spectra are matched to within 3%. The scattering contribution to the perturbation depends on the difference in importance between the group from which the neutron is scattered to the group into which it is scattered. Further examination of the data in Tables 1 and 3 reveals that the difference in importance between adjacent groups in Assembly 4Z is almost identical to that in Assembly 5. The same is not true for Assembly 11. The criterion, then, that should be followed to insure adequate spectral matching, is not only comparison of the real and adjoint fluxes, but also calculation of the reactivity worths of many samples of different nuclear properties in both the zoned and reference systems. A good spectral matching will be insured if the ratio of the worth of a sample in the zoned core to that in the full-size system is the constant for all samples

3. Measurement of Neutron Spectrum, Fission Ratios, Resonance Self-shielding in Mn^{55} , and Fission Traverses

3.1. Spectrum Measurements

E. Bennett measured the neutron spectrum in Assembly 4Z in the energy range from 1 keV to 1 MeV with a proton-recoil counter. The results are reported in Ref. 6 together with spectrum measurements obtained with Li^6F solid-state spectrometer. The neutron spectrum in Assembly 11 was also measured by the

TABLE 3. COMPARISON OF REAL AND ADJOINT FLUXES AT CENTER OF ASSEMBLIES 5 AND 11^(a)

Group	$\phi_{\text{Center, Assembly 5}}$	$\phi_{\text{Center, Assembly 11}}$	$\phi_{\text{Center, Assembly 5}}^*$	$\phi_{\text{Center, Assembly 11}}^*$	$\frac{\phi_{\text{No. 5}}^*}{\phi_{\text{No. 11}}^*}$
1	1.48	1.45	4.85	4.88	+0.10
2	2.81	2.76	4.61	4.66	+0.09
3	4.82	4.77	4.23	4.31	-0.17
4	7.42	7.47	3.66	3.75	-0.86
5	10.33	10.50	3.64	3.73	-1.56
6	12.72	12.91	3.67	3.75	-1.73
7	12.75	12.04	3.68	3.71	-0.72
8	11.36	11.38	3.66	3.65	-0.04
9	9.67	9.65	3.65	3.61	+0.36
10	7.84	7.80	3.64	3.58	+0.61
11	5.94	5.90	3.66	3.58	+0.62
12	4.68	4.61	3.71	3.62	+0.67
13	3.27	3.20	3.75	3.67	+0.52
14	2.38	2.31	3.78	3.73	+0.38
15	0.449	0.435	3.83	3.78	+0.075
16	0.534	0.517	3.88	3.83	+0.092
17	0.803	0.776	3.96	3.91	+0.146
18	0.251	0.234	4.01	3.97	+0.078
19	0.277	0.271	4.32	4.29	+0.034
20	0.166	0.165	4.52	4.51	+0.006
21	0.0446	0.0454	4.19	4.19	-0.003
22	0.00410	0.00421	4.09	4.12	-0.001

^(a) Calculations were done in cylindrical geometry (MACH-1⁽³⁾, Cross-section set 801⁽⁴⁾).

same method and the results were reported in Ref. 7. The results from both assemblies show general agreement. It is questionable, however, that this method of spectrometry would show any fine differences, since the resolution of the instrument is of the order of 15-20%.

3.2. Fission Ratios

Fission ratios of U^{234} , U^{238} , and Pu^{239} with respect to U^{235} were measured in Assembly 4Z at the midplane as a function of the axial direction and at $1\frac{3}{4}$ in. from the midplane in the radial direction. These measurements were obtained with a surface-barrier solid-state detector. Figure 1 shows the counter arrangement that was used. The counter was driven with a stepping motor attached to a lead screw with practically no backlash. Position indication was reproducible to within 0.002 cm.

The axial traverse of the fission ratios was performed in Drawer 23-23 at the center of Assembly 4Z. (The numbers after a drawer refer to row and column, respectively, in the matrix.) The two $\frac{1}{2}$ -in. sodium cans normally loaded in Drawer 23-23 were replaced with two $\frac{1}{4}$ -in. cans to provide space for the counter traverse mechanism. Figure 2 shows the loading of this drawer. The result of these measurements are shown in Table 4.

The fission ratios in the radial direction were done in the same manner as the traverse in the axial direction

except that the traverse was through Drawers 23-23-13-23 in the vertical direction. Figure 3 shows the modification in the core materials that was made to provide space for the counter. Note that $\frac{1}{2}$ in. of depleted uranium was used to hold the $\frac{1}{4}$ -in. sodium cans in place at the front of each drawer. The results of these measurements as well as the calculated values with transport (S_4 approximation) and diffusion codes, using the same cross-section set (224), are given in Table 5.

FIG. 1. Small Surface-barrier Solid-state Counter Used for Fission-ratio Traverses.

IN VOIDED CENTRAL REGION SODIUM CANS WERE REPLACED WITH 1/4 in. FRAMES

FIG. 2. Drawer for Axial Traverse.

Comparison of the results in Table 4 at $Z = 0$, $r = 0$ and those in Table 5 at $r = 0.0$, $Z = 1\frac{3}{4}$ in., reveals that the $\frac{1}{2}$ -in. depleted uranium spacer used in the radial traverse affected the measurements. It is seen from the fission ratios of U^{234} and U^{238} to U^{235} that the upper portion of the energy spectrum became softer because of the spacer. It is further noticed from the fission ratio $\text{Pu}^{239}/\text{U}^{235}$ that the spacer hardened the lower-energy portion of the spectrum. When the measurements at $r = 0$, $Z = 0$ in Table 4 are compared with the calculations in Table 5, it is found that the calculated $\text{U}^{238}/\text{U}^{235}$ fission ratio is in good agreement with measurements.

The $\text{U}^{234}/\text{U}^{235}$ fission ratio indicates that the calculated spectrum that is pertinent to this ratio is harder than measurements indicate. The same is true of the $\text{Pu}^{239}/\text{U}^{235}$ fission ratio. However, since the introduction of a small, depleted uranium sample affects the measurements, it is clear that the heterogeneous arrangement of the core material within a drawer can produce effects as large as the discrepancies between the calculations and the measurements or even larger.

It is seen from Table 5 that the transport calculations give a harder spectrum in the driver ($r = 50$ cm) than diffusion calculations. This is reasonable because the leakage term in diffusion theory is overestimated and it affects the higher portion of the energy spectrum. The large discrepancy in the fission ratio at $r = 50$ cm is due to the substantial perturbation introduced in the driver to allow for the insertion of the probe. Figure 4 shows how the driver material was rearranged. That the discrepancy between measurement and calculation of the fission ratios at $r = 50$ cm is not real can be seen from

TABLE 4. FISSION RATIOS AS FUNCTION OF THE AXIAL DISTANCE IN ASSEMBLY 4Z

Axial Position, cm from Center ($r = 0$)	Measured Fission Ratio			
	U^{234}/U^{235}	U^{238}/U^{235}	Pu^{239}/U^{235}	U^{238}/U^{234}
0.0	0.187 ± 0.005	0.0256 ± 0.0008	0.901 ± 0.024	0.136 ± 0.006
10.0	0.179 ± 0.005	0.0261 ± 0.0008	0.876 ± 0.023	0.146 ± 0.006
20.0	0.178 ± 0.005	0.0263 ± 0.0008	0.854 ± 0.023	0.147 ± 0.006
30.0	0.173 ± 0.005	0.0266 ± 0.0009	0.875 ± 0.025	0.154 ± 0.006
40.0	0.168 ± 0.005	0.0262 ± 0.0009	0.863 ± 0.026	0.155 ± 0.006
50.0	0.165 ± 0.005	0.0235 ± 0.0009	0.850 ± 0.026	0.142 ± 0.006
56.3	0.160 ± 0.005	0.0231 ± 0.0008	0.857 ± 0.027	0.144 ± 0.006

ALL DIMENSIONS IN inches

A - ALUMINUM TRAVERSE TUBE 9/16 o.d. x 0.020 WALL

IN VOIDED CORE ALL Na CANS WERE REPLACED WITH 1/4 in. FRAMES

FIG. 3. Drawer for Radial (Vertical) Traverse.

Fig. 5 in which the normalized absolute fission ratio is compared to the normalized relative fission ratio as obtained from activation of U^{238} and U^{235} foils. Also included in Fig. 5 is the calculated fission ratio by transport and diffusion codes with cross-section set 224.

The two principal sources of error in the fission-ratio measurements are in:

- (1) determining the amount of material present in a counter, and
- (2) measuring the fission rates to a desired degree of accuracy.

Except for those materials with an extremely low fission rate, the first factor is the predominant source of error.

The amount of material present in a counter is obtained from the disintegration rate of the sample. The disc containing the fissionable material is counted in a low-geometry alpha counter whose accuracy is such that the disintegration rate can be determined to within $\pm 0.1\%$. An isotopic analysis of the material with an overall accuracy of better than 0.02% is obtained from a mass spectrometer. The analyses for the materials used are shown in Table 6. By use of the known disintegration rates for each isotope, a disintegration rate per unit weight can be calculated for the material. Division of the measured disintegration rate by the calculated value gives the amount of material

TABLE 5. RADIAL FISSION RATIOS

Radial Position, cm from center ($Z = 1\frac{3}{4}$ in.)	Measured Fission Ratios				Calculated Fission Ratios ^(a)			
	U^{234}/U^{235}	U^{238}/U^{235}	Pu^{239}/U^{235}	U^{238}/U^{234}	U^{234}/U^{235}	U^{238}/U^{235}	Pu^{239}/U^{235}	U^{238}/U^{234}
0.0	0.172 ± 0.004	0.0238 ± 0.0007	0.967 ± 0.025	0.128 ± 0.004	0.222 (0.221)	0.0267 (0.0266) ^(b)	0.943 (0.941)	0.120 (0.120)
15.0	0.168 ± 0.004	0.0230 ± 0.0008	0.964 ± 0.025	0.137 ± 0.004	0.222 (0.021)	0.0266 (0.0265)	0.942 (0.941)	0.120 (0.120)
30.0	0.163 ± 0.004	0.0232 ± 0.0008	0.956 ± 0.025	0.142 ± 0.005	0.220 (0.219)	0.0263 (0.0261)	0.940 (0.939)	0.119 (0.119)
40.0	0.174 ± 0.004	0.0228 ± 0.0008	0.934 ± 0.025	0.131 ± 0.005	0.223 (0.213)	0.0242 (0.0249)	0.930 (0.933)	0.108 (0.116)
50.0	0.241 ± 0.007	0.0377 ± 0.0012	1.020 ± 0.027	0.156 ± 0.008	0.324 (0.314)	0.0491 (0.0439)	1.052 (1.040)	0.151 (0.139)
55.0	—	0.0395 ± 0.0015	1.047 ± 0.030	—	—	0.0418 (0.0397)	1.062 (1.050)	—

^(a) Values in parenthesis were obtained by diffusion theory with Set 224; the other values were obtained with the same cross-section set by transport S_4 calculations). The U^{235} group cross sections used in the calculations did not include the White⁽⁸⁾ data. The fission ratio (U^{235} non-White/ U^{235} White) was 1.042.

^(b) This ratio was also obtained with the 801 set and diffusion theory.

ALL DIMENSIONS IN inches

TOP VIEW

A-ALUMINUM TRAVERSE TUBE 9/16 o.d. x 0.020 WALL

FIG. 4. Arrangement of Driver Drawer to Allow Space for Fission Counter.

on the disc. The principal source of error in this method arises from the half-lives which are used to calculate the disintegration rates, shown in Table 7. The relative error in the mass is then determined from the formula

$$\sigma_m = \sqrt{\sigma_1^2 + \sigma_2^2}$$

where σ_1 is the standard deviation in the calculated disintegration rate, and σ_2 is the standard deviation in

the experimental disintegration rate. The errors quoted in Tables 4 and 5 are the total propagated errors.

3.3. Resonance Detector (Mn^{55})

Manganese has several resonances in the energy range from 350 eV to about 100 keV. Its use as a detector depends on the self-shielding that occurs when the resonance neutrons are absorbed in the outer foils of a

stack of foils. This technique was used in Assembly 3 of ZPR-6⁽⁹⁾ and repeated in Assembly 4Z. This detector will also be used in Assembly 5 for the purpose of verifying whether or not the low-energy spectrum in Assemblies 4Z and 5 are the same.

The experiment consisted of stacking 39 manganese foils of 98% purity (1% copper and 1% nickel) and $\frac{5}{8}$ -in. diameter. This stack comprised a central section consisting of nineteen 10-mil foils and two outer layers, one on each side of the central section, consisting of ten 1-mil foils. The stack was irradiated at the center of Assembly 4Z in the space that was provided for the fission counter (see Fig. 3). This stack was placed against the drawer material, 2 in. from the front of Drawer 23-23. After irradiation, the foils were reduced in size from $\frac{5}{8}$ -in. to $\frac{3}{8}$ -in. diameter and then counted. The reduction in size was done in order to eliminate edge effects.

The specific activity of the foils is plotted as a function of depth from one end in Fig. 6. The depth of the valley in this plot is a measure of the spectrum softness.

FIG. 5. Comparison of Calculated and Measured Fission Ratio of U^{238}/U^{235} .

TABLE 6. ISOTOPIC CONCENTRATIONS (W/O) OF FISSIONABLE MATERIALS

Isotope	Weight per cent				
	U^{234}	U^{235}	U^{238}	Pu^{239}	Pu^{240}
U^{234}	79.45	15.16	5.39	—	—
U^{235}	0.93	93.14	5.91	—	—
U^{238}	—	0.014	99.986	—	—
Pu^{239}	—	—	—	99.922	0.028

TABLE 7. HALF-LIVES USED IN ANALYSIS

Material	Half-life, yr
U^{234}	$(2.475 \pm 0.016) \times 10^5$
U^{235}	$(7.13 \pm 0.16) \times 10^8$
U^{238}	$(4.50 \pm 0.01) \times 10^7$
Pu^{239}	$(2.435 \pm 0.005) \times 10^4$

FIG. 6. Manganese Activation in a Stack of Foils.

If the depth is measured by taking the ratio of the specific activity at the edge to that at the center, it is found that this ratio is 1.82. This ratio for Assembly 3 of ZPR-6 was 1.66, indicating the spectrum was harder in Assembly 3. It is noted from Fig. 6 that the specific activity of the foil next to the core material is lower than that on the other side. This is due to the fact that there are fewer resonance neutrons in this region.

3.4. Fission Rate Traverses with U^{235} and U^{238}

The accuracy of the calculated spatial distribution of the real fluxes in Assembly 4Z was examined by measuring fission traverses axially and radially with U^{235} and U^{238} foils. One-dimensional homogeneous calculations in cylindrical geometry were made using the diffusion-theory code MACH-1⁽³⁾ with cross-section sets 224⁽²⁾ and 801,⁽⁴⁾ and the transport-theory code SNARG⁽¹⁰⁾ with cross-section set 224.

All the foils used in the measurements were 0.0125-cm (5-mil) thick and had a diameter of 1.27 cm ($\frac{1}{2}$ in.) and 0.95 cm ($\frac{3}{8}$ in.), respectively, for U^{238} and U^{235} . They were placed vertically between adjacent material plates inside the core drawers and horizontally on top of U^{238} blocks in the reflector. The foils were located in the stationary half of the assembly; for the radial

FIG. 7. Fine Structure in the Fission Rate of U^{238} in Central Region.

traverses they were placed at a distance of 7.62 cm (3 in.) from the interface in order to eliminate the influence of the front walls of the drawers and of the residual gap between the halves.

Axial traverses were taken at the core center, at the boundary of the central zone, and in the middle of the driver. Radial traverses were taken in horizontal, vertical, and diagonal directions (at 45° to the horizontal diameter). This was done in order to detect any distortion caused by the orientation of the material plates to the core radius and by the irregular shape of the boundaries between the various zones.

In order to study the influence of boundaries on the flux distribution, most foils were placed at the outer

edge of the central zone and in the buffer and driver regions, where the flux gradient was expected to be highest.

The fine structure of the U^{238} fission rates was measured across a sequence of three drawers (23-21 to 23-23) in the core center. The same was done across six consecutive drawers (23-28 to 23-33) in order to cover the outer edge of the central zone and the buffer and driver regions. The results are shown in Figs. 7 and 8.

The fine structure of the U^{235} fission rate (see Fig. 9) was measured in a sequence of three drawers (23-25 to 23-27). A detailed mapping of the radial traverse for U^{235} is shown in Fig. 10. Included in Fig. 10 are the calculated fission rates by transport and diffusion theory using two cross-section sets (224 and 801). It is seen that the transport as well as the diffusion calculations with set 224 are in better agreement with the measurements than set 801. Cross-section set 801,

FIG. 9. Fine Structure of Fission Rate for U^{235} Foils Placed between Plates in Assembly 4Z of ZPR-6.

FIG. 8. Fission Rate of U^{238} Foils Placed between Plates in Assembly 4Z of ZPR-6.

FIG. 10. Radial Traverse of U^{235} Fission Rates in ZPR-6 Assembly 4Z.

even though Elmoe-averaged for Assembly 4Z, does not predict the fission rates across the buffer and driver regions as well as set 224. This is probably due to the inapplicable Elmoe averaging in the narrow driver region.

The fission rate of U^{235} as a function of radius is shown in Fig. 11. It is seen that the results of transport calculations give better agreement with measurements. In obtaining the measured fission rates it was necessary to average over the fine-structure measurements in order to make meaningful comparison with the homogeneous calculations.

The axial fission rate traverses of U^{235} and U^{238} are given, respectively, in Figs. 12 and 13. In both cases the experimental results follow very closely a cosine distribution and yield a reflector savings of 16.1 cm when extrapolated to zero flux. It is significant to note that a reflector savings of 16.1 cm is independent of the region in which the measurements were made.

Reaction rates of U^{235} and U^{238} in Assemblies 11 and 5 are yet to be measured.

4. The Spatial Distribution of the Importance of Fission Neutrons

4.1. Theory

The reactivity worth of a sample is usually calculated by perturbation theory in which adjoint fluxes play as important a role as the real fluxes. However, the calculated adjoint fluxes are usually taken for granted, and all the experimental work is directed toward verifying the calculations of the real fluxes. In this work it was decided to check on the relative spatial distributions of the importance of fission neutrons by using a Cf^{252} source. The measurements are based on the following principles.

The change in reactivity, $\Delta k/k$, according to perturbation theory⁽¹¹⁾ can be written as

$$\Delta k/k = \frac{\int_v (\phi^+) (\delta P - \delta L) (\phi) dv}{\int_v (\phi^+) (P) (\phi) dV}, \quad (4.1.1)$$

Fig. 11. Fission Rate of U^{238} as a Function of Radius.

where

ϕ^+ is a row matrix representing the adjoint fluxes;

ϕ is a column matrix representing the real fluxes;

P is a square matrix representing the production of neutrons;

δP represents the change in P as a function of space;

δL represents the change in the losses of neutrons as a function of space;

v is the volume of the perturbed region;

V is the volume of the entire reactor.

If δL is ignored and the numerator of Eq. (4.1.1) is expanded into energy groups, and if it is assumed that the change in P is a Dirac δ function, then the numerator of Eq. (4.1.1) reduces to

$$\text{Numerator} = \int_v \left\{ (\phi_1^+ \phi_2^+ \cdots \phi_n^+) \begin{bmatrix} \chi_1 \delta(\nu \Sigma_f)_1 \delta(r \rightarrow r') & \chi_1 \delta(\nu \Sigma_f)_2 \delta(r \rightarrow r') & \cdots & \chi_1 \delta(\nu \Sigma_f)_n \delta(r \rightarrow r') \\ \chi_2 \delta(\nu \Sigma_f)_1 \delta(r \rightarrow r') & \chi_2 \delta(\nu \Sigma_f)_2 \delta(r \rightarrow r') & \cdots & \chi_2 \delta(\nu \Sigma_f)_n \delta(r \rightarrow r') \\ \vdots & & & \\ \chi_n \delta(\nu \Sigma_f)_1 \delta(r \rightarrow r') & \chi_n \delta(\nu \Sigma_f)_2 \delta(r \rightarrow r') & \cdots & \chi_n \delta(\nu \Sigma_f)_n \delta(r \rightarrow r') \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_n \end{bmatrix} \right\} dv \quad (4.1.2)$$

$$= \int_v \left\{ \sum_{i=1}^n \chi_i \phi_i^+ \right\} \left\{ \delta(r \rightarrow r') \left[\sum_{i=1}^n \delta(\nu \Sigma_f)_i \phi_i \right] \right\} dv.$$

Suppose the reactor system is just critical; then Eq. (4.1.2) is zero and the power level is steady in time. Also suppose that a point neutron source that does not absorb any neutrons is introduced into the system at position r' . Assume that the neutron source emits S neutrons per unit time and that the energy grouping of the spectrum is such that

$$S = \sum_{i=1}^n S_i. \quad (4.1.3)$$

Upon the introduction of the source into the system the power level increases linearly in time. The increase in power level may be equated to a pseudoreactivity whose value depends on the power level at which the source

FIG. 12. Axial Traverse of U^{235} Fission Rates in ZPR-6 Assembly 4Z. FIG. 13. Axial Traverse of U^{238} Fission Rates in ZPR-6 Assembly 4Z.

FIG. 14. Holder for Californium Sample.

was introduced. Similarly, the source neutrons in group i can be assumed to be resulting from a pseudo-change in the fission cross sections, i.e.,

$$S_i \delta(r \rightarrow r') = \chi_i \left[\sum_{i=1}^n \delta(\nu \Sigma_f)_i \phi_i \right] \delta(r \rightarrow r'). \quad (4.1.4)$$

The right-hand side of Eq. (4.1.4) represents the net increase in the neutron population of group i at r' due to a pseudo-change in the fission cross sections. Combining Eqs. (4.1.3) and (4.1.4), one gets

$$S \delta(r \rightarrow r') = \left[\sum_{i=1}^n \delta(\nu \Sigma_f)_i \phi_i \right] \delta(r \rightarrow r'). \quad (4.1.5)$$

It is implied in Eq. (4.1.5) that $\sum_{i=1}^n \chi_i = 1$. Substituting Eq. (4.1.5) into Eq. (4.1.2), one obtains

$$(\Delta k/k)' = \frac{\left(\sum_{i=1}^n \chi_i \phi_i^+ \right) (S)}{\text{N.I.}}, \quad (4.1.6)$$

where $(\Delta k/k)'$ designates the pseudo-reactivity change, and N.I. is the normalization integral that appears in the denominator of Eq. (4.1.1). It is apparent from Eq. (4.1.6) that the importance of the fission neutrons at position r' is simply

$$\left[\sum_{i=1}^n \chi_i \phi_i^+ \right]_{r'} = (\Delta k/k)'_{r'} \frac{\text{N.I.}}{S_{r'}}, \quad (4.1.7)$$

provided the neutron source has the same spectrum as the fissile materials in the system. It is apparent from Eq. (4.1.7) that the relative distributions of $\sum_{i=1}^n \chi_i \phi_i^+$ as a function of space can be determined by measuring $(\Delta k/k)'$ at the same power level for the corresponding position of S and normalizing the results to some point in the reactor.

Another important feature of Eq. (4.1.7) is that if $(\Delta k/k)'_{r'}$ is measured at a known absolute fission source rate,

$$\left[\sum_{i=1}^n (\nu \Sigma_f)_i \phi_i \right],$$

then the normalized quantity, $(\text{N.I.})'$ can be obtained experimentally:

$$(\text{N.I.})' = [\text{N.I.}/S_r] = 1/(\Delta k/k)'_{r'},$$

where $\sum_{i=1}^n \chi_i \phi_i^+$ has been normalized to 1. Note that N.I. is independent of where the source is placed. It is relatively simple to experimentally obtain the absolute fission rate, $\sum_{i=1}^n (\Sigma_f)_i \phi_i$, but the quantity $\sum_{i=1}^n (\nu \Sigma_f)_i \phi_i$ is generally not measurable inside the system. However, some average ν can be obtained from calculated group-dependent fluxes and from the empirically known dependence of ν on the neutron energy. Equations (4.1.7) and (4.1.8) are valid provided the system is closely near critical so that perturbation theory can be applied.

4.2. Measurements

The measurements were performed in Assembly 4Z with a microgram quantity of Cf^{252} . The Cf^{252} source was housed in a container (see Fig. 14) which was attached to the drive mechanism that was used with the fission-ratio measurements. The core materials were rearranged, to allow the introduction of the container, in the same manner as was done for the fission ratios (see Figs. 2, 3, and 4). The effect of the container without the source but with a glass ampule similar to the source ampule on the system was checked by first noting the critical position of one control rod at a power level where the power coefficient was zero. (The power coefficient is defined here as the amount of pseudo-reactivity associated with the spontaneous fissions occurring in U^{238} and U^{235} , and any neutron emission resulting from cosmic-ray interactions.) Later the power level was decreased to the level at which the

measurements with Cf^{252} source were conducted, and the control rod was used to establish steady state. The difference between the two positions of the control rod was noted. The container was then introduced into the system at the center of the core and the power level studied as a function of time. The container had no measurable effect on the system. The container was then moved in increments from the center toward the core boundary, in both the radial and axial directions, and again the container had no measurable effect.

The same type of procedures were repeated with the Cf^{252} source. After the difference between the two control-rod positions was determined for the two power levels without the Cf^{252} source ($\sim 5 \times 10^{-5} \Delta k/k$), the power level was decreased by withdrawing the control rod to such a level that after inserting the Cf^{252} source into a specified location, a period in time of no less than 5 min would result before reaching the power level at which measurements were made. This procedure allowed enough time for any transients to die out. The linear rise in the power level was monitored by a fission chamber located at the driver-reflector interface (drawer location 14-27). The shortest distance between the Cf^{252} source and the chamber was in the radial traverse and that was 23 cm. A typical set of data of the linear rise is shown in Fig. 15. The slope of the curve shown in Fig. 15 is proportional to the pseudo-reactivity, $(\Delta k/k)'$, at a certain power level. If the ratio of the slope of the linear rise obtained at position x to the slope obtained at the center of the core is plotted as a function of position, the result is a curve of the normalized $\sum_{i=1}^n \chi_i \phi_i^+$ versus position.

The radial traverse of $\sum_{i=1}^n \chi_i \phi_i^+$ is shown in Fig. 16. For comparison, the calculated importance of the fission neutrons by transport and diffusion, one-dimensional codes with Sets 224 and 801 are included. The calcula-

FIG. 15. Linear Rise in Power due to Cf^{252} Source (Radial Traverse at $r = 20.00$ cm).

FIG. 16. Radial Traverse of Cf^{252} Neutron Importance.

FIG. 17. Axial Traverse of Cf^{252} Neutron Importance.

tions were done for a cylindrical configuration. The results from diffusion calculations with Set 224 are in better agreement with measurements than those from transport calculations. This is contrary to what was found with the reaction rate of U^{238} (see Fig. 11). The uncertainties in the measured points are within the circles.

The axial traverse of $\sum_{i=1}^n \chi_i \phi_i^+$ is shown in Fig. 17. Included in this diagram are the results of diffusion calculations with Set 224 and a cosine distribution. It is

TABLE 8. VALUES OF χ_i 'S FOR Cf^{252} AND U^{235}

Energy Group	E_L , ^(a) MeV	$(\chi_i$'s) Cf^{252}	$(\chi_i$'s) U^{235}	$(\chi_i$'s) Set 224
1	3.6788	0.1783	0.1270	0.13035
2	2.2313	0.2065	0.1990	0.21089
3	1.3534	0.2198	0.2266	0.23223
4	0.82085	0.1678	0.1834	0.17941
5	0.49787	0.1056	0.1200	0.11487
6	0.30197	0.0605	0.0704	0.064840
7	0.18316	0.0310	0.0374	0.034097
8	0.11189	0.0152	0.0181	0.017125
9	0.06738	0.0073	0.0089	0.0083959
10	0.04087	0.0038	0.0042	0.0040697
11	0.02478	0.0019	0.0022	0.0019532
12	0.01503	0.0010	0.0013	9.3328×10^{-4}
13	0.0091188	0.0005	0.0008	4.3952×10^{-4}
14	0.0043073	0.0003	0.0005	2.6677×10^{-4}
15	0.0026126	} = 0.0005	} = 0.0002	6.8147×10^{-5}
16	0.0020347			1.8459×10^{-5}
17	0.0012341			2.2379×10^{-5}
18	9.611×10^{-4}			6.1189×10^{-6}
19	5.8295×10^{-4}			7.1596×10^{-6}
20	2.7536×10^{-4}			4.4062×10^{-6}
21	1.0130×10^{-4}			1.5641×10^{-6}
22	2.902×10^{-5}			3.9265×10^{-7}

^(a) E_L is the lower boundary of the energy grouping.

seen that the measured points are higher than theory predicts.

The validity of the assumption that the Cf^{252} fission neutrons have the same energy distribution as U^{235} was checked by calculating the χ_i 's that are characteristic of Cf^{252} , multiplying these χ_i 's with the calculated adjoint fluxes obtained with the transport code, and comparing the results with those obtained both with the transport code and with those obtained with the χ_i 's of U^{235} . The spectrum of Cf^{252} is reported⁽¹²⁾ to obey the following empirical relation:

$$N(E) dE = 2.410 E^{1/2} e^{-E/1.466} dE, \quad (4.2.1)$$

where E is in MeV. The fraction of neutrons that are emitted in energy group i is

$$\chi_i = \frac{\int_{E_i + (1/1.466)}^{E_i/1.466} (E')^{1/2} e^{-E'} dE'}{\int_0^\infty (E')^{1/2} e^{-E'} dE'}. \quad (4.2.2)$$

The expression for χ_i is in the form of incomplete Γ -functions, and it can be evaluated directly from the tabulations of Pearson⁽¹³⁾ Table 8 lists the values of χ_i 's for Cf^{252} as obtained from Eq. (4.2.2) and the data from Ref. 12. The values of the χ_i 's for U^{235} obtained by the same technique are also shown in Table 8. The values of the χ_i 's that are used in Set 224 are given for

comparison. It was found that the shape of $\sum_{i=1}^n \chi_i \phi_i^+$ as a function of radius is not particularly sensitive to the set of χ_i 's used. The largest deviation among the sets was a fraction of a percent. Thus, it seems that the assumption of the similarity between the fission neutron spectra of U^{235} and Cf^{252} is not a bad one.

Throughout the analysis the number of fissions that occurred due to the interaction of neutrons with Cf^{252} was ignored because it was considered that one microgram of Cf^{252} was too small to influence the system.

This technique of measuring $\sum \chi_i \phi_i^+$ will be performed with Assemblies 11 and 5, with particular emphasis in obtaining the denominator of the perturbation expression experimentally.

5. Prediction of S_Z/S_F

5.1. Comments on the Calculations

We have tried whenever possible to verify the calculational techniques and to a certain degree the accuracy of the group-dependent cross sections with the experimental results. Two cross-section sets were used in the analysis: (1) Set 224 uses 22 groups and the cross sections of the isotopes used in this set have been Elmoe-averaged for typically large carbide core; and (2) Set 801 uses 25 groups and the cross sections in this set have been Elmoe-averaged for Assembly 4Z in the central zone and in the driver region. Both sets were used with a one-dimensional diffusion code, MACH-1. Set 224 was also used with a one-dimensional transport code, SNARG, in the S_4 approximation. The following remarks about the theoretical results are made on the basis of comparison with experimental results:

(1) The calculated spectra with Sets 801 and 224 are in reasonable agreement with measurements.^(6, 7) Reasonable agreement between the calculated and measured fission ratios was also obtained in the central zone of Assembly 4Z.

(2) The fission-rate traverses with U^{238} and U^{235} showed that the application of diffusion theory to zoned cores with narrow regions results in underestimation of the spatial distribution of the high-energy groups. Use of transport theory is more appropriate (see Fig. 11). It is seen from Fig. 12 that the Elmoe averaging for a core with the composition of a narrow zone and the use of this averaging in a multizone reactor does not improve the agreement with measurements; it makes it worse. Elmoe averaging would be very appropriate when such technique could be applied to a multiregion core instead of a one-region core.

(3) Whereas the spatial distribution of the real fluxes is better predicted with transport than with diffusion calculations, the opposite is true for the importance of

fission neutrons, $\sum_i^n \chi_i \phi_i^+$ (see Fig. 16). The reasons for this phenomenon are presently under investigation.

5.2. Prediction of S_Z/S_F

The denominator of the perturbation expression for Assembly 4Z was calculated by the MACH-1 code with Sets 801 and 224, by the SNARG code with Set 224, and by using the experimental results for the importance of fission neutrons (see Fig. 16) and the calculated sources of fission neutrons $\left[\sum_{i=1}^n (\nu \Sigma_f)_i \phi_i \right]$ from the SNARG code. The results are given in Table 9.

It is seen that the difference in S_Z between the results from MACH-1, Set 224, and SNARG with the same set is only 2.2 %, even though there is a considerable difference between the quantity $\sum \chi_i \phi_i^+$ obtained from the two codes (see Fig. 16). The reason for this is that diffusion theory underestimates the quantity, $\sum_i (\nu \Sigma_f)_i \phi_i$, but gives reasonably accurate $\sum_i \chi_i \phi_i^+$. The opposite is true with transport theory (see Figs. 11 and 16). For this reason, the measured importance of the fission neutrons and the calculated source of fission neutrons from SNARG were used to obtain S_Z .

The calculated value of S_F , the denominator of the perturbation expression for the full-size core, is 6.500×10^5 . This value was obtained from MACH-1 and Set 224. Thus, the quantity $S_Z/S_F = 6.772 \times 10^5 / 6.500 \times 10^5 = 1.041$. In other words, the central worth measurements in Assembly 5, the full-size core, will be 4 % greater than those in Assembly 4Z. In arriving at this ratio it was assumed that the difference in S_Z as obtained from the semi-empirical method and from MACH-1 is due to the presence of various zones in Assembly 4Z. This assumption is justified since for large cores diffusion and transport theories give the same critical mass.⁽¹⁴⁾

The S_Z for Assembly 11 has not been determined properly because much of the experimental data are not yet available.

6. Central Worth Measurements

6.1. Material Reactivity Worth at Center of Assemblies 4Z, 5, and 11

The central worths of different samples were measured at the center of each assembly by the following technique:

6.1.1. Criticality was achieved with dual-purpose rods and with the sample changer in the "in" position. The sample changer consisted of a drawer having the same composition as the central zone of the core with a voided front section (2 x 2 x 1 in.). Using an attached drive mechanism, the operator can insert or remove this

TABLE 9. CALCULATED VALUES OF $S_Z^{(a)}$ FOR ASSEMBLY 4Z CODE AND CROSS-SECTION SET

	MACH 1, Set 224	MACH 1, Set 801	SNARG, Set 224	$\sum_i (\nu \Sigma_f)_i \phi_i$ from SNARG and Meas- ured $\sum \chi_i \phi_i^+$
S_Z	6.588×10^5	6.126×10^5	6.448×10^5	6.772×10^5

^(a) S_Z is defined as

$$\int_V \left[\sum_i \chi_i \phi_i^+ \right] \left[\sum_i (\nu \Sigma_f)_i \phi_i \right] dV.$$

The values listed are normalized so that $\sum_i \chi_i \phi_i^+ = 1$ and $\sum_i (\nu \Sigma_f)_i \phi_i = 1$ at the center of the core.

drawer from the core remotely. The voided front section of this drawer was used for the sample whose worth was to be measured.

6.1.2. After criticality was achieved at the proper power level (that level which had zero power coefficient), the sample changer was withdrawn from the core, thus causing the neutron density to decrease. The drawer was worth about 30 lh. By means of the non-scrammable rod, the equivalent of about 10 lh was added to the system. The nonscrammable rod was equipped with a precision position indicator (0.002 cm). The power level was decreased to a point where, upon insertion of the sample changer to its original position, a waiting period of not less than 5 min resulted before that power level at which the reactor-period measurement was initiated, was reached. The reactor period was measured "on line" by feeding the data directly to the DDP-24 computer⁽¹⁴⁾ for an interval of about 5 min.

6.1.3. This procedure was repeated several times. The differences in reactivity between the runs with and without a sample gave the worth of a sample.

It should be noted that the 10-lh excess reactivity added could have been obtained through a dual-purpose rod. However, if at any time during the series of runs it were found that a sample was worth more than the 10 lh, it would have been necessary to move the dual-purpose rod. Since there was some "backlash" associated with these rods it was decided to use the nonscrammable rod.

At some times during the runs and especially after a weekend, a systematic drift in the reference runs was observed, of the order of 0.01 lh. Whenever such drift was observed, the samples whose worths were 1.0 lh or greater were measured. The uncertainty in these measurements was considered to be the same as the size of the drift. The sample whose worth was small, especially the sodium, was measured when the drift was

TABLE 10. CENTRAL WORTH MEASUREMENTS IN ZPR-6 ASSEMBLIES 5 AND 4Z

Sample and Sample Wt, g	Nature and Wt of Can, g	Gross Worth in Assembly 5, 1h ^(a)	Net Worth in Assembly 5, 1h ^(b)	Net Worth in Assembly 4Z, 1h ^(c)	Measured	Calc by Set 224
					Worth in 5 Worth in 4Z	Worth in 5 Worth in 4Z
Be (114.308)	Bare	+1.130 ± 0.048	1.130 ± 0.048	+1.107 ± 0.010	1.021	0.987
C (103.0)	Bare	+0.183 ± 0.022	+0.183 ± 0.022	+0.197 ± 0.010	0.943	1.080
Ni (546.0)	Bare	-1.134 ± 0.015	-1.134 ± 0.015	-1.148 ± 0.010	0.988	1.011
V (183.67)	55.30 (SS)	-0.246 ± 0.010	-0.136 ± 0.014	-0.133 ± 0.010	1.022	
Li (28.30)	57.03 (SS)	-0.790 ± 0.010	-0.677 ± 0.015	-0.595 ± 0.010	1.137	
Zr (406.0)	Bare	-0.671 ± 0.020	-0.671 ± 0.020	-0.650 ± 0.005	1.032	
Mo (599.0)	Bare	-3.416 ± 0.015	-3.416 ± 0.015	-3.415 ± 0.010	1.000	
Al (166.90)	Bare	-0.300 ± 0.017	-0.300 ± 0.017	-0.297 ± 0.013	1.010	
B ¹⁰ (29.29)	55.73 (SS)	-23.298 ± 0.030	-23.400 ± 0.033	-23.339 ± 0.200	1.003	1.012
B ₄ C (59.62)	64.70 (SS)	-6.266 ± 0.010	-6.137 ± 0.015	-6.410 ± 0.025	0.960	
Polythene (58.98)	Bare	+19.122 ± 0.010	+19.122 ± 0.010	+19.696 ± 0.010	0.970	
Nb (481.2)	30.10 (SS)	-3.888 ± 0.010	-3.939 ± 0.015	-3.791 ± 0.010	1.039	
W (1108.0)	Bare	-5.792 ± 0.020	-5.792 ± 0.020	-5.742 ± 0.010	1.010	
Ta (924.70)	27.54 (SS)	-11.955 ± 0.010	-12.001 ± 0.015	-11.907 ± 0.200	1.008	
U ²³⁸ (1153.80)	Bare	-3.869 ± 0.010	-3.869 ± 0.010	-3.857 ± 0.010	1.003	1.011
U ²³⁵ (13.869)	48.968 (SS)	+0.545 ± 0.010	+0.648 ± 0.012	+0.611 ± 0.020	1.060	1.013
U ²³⁵ (105.771)	Bare	+4.943 ± 0.010	+4.943 ± 0.010	+4.855 ± 0.010	1.018	
U ²³⁵ (27.719)	SS can (47.30)	+1.174 ± 0.010	+1.278 ± 0.012	+1.246 ± 0.030	1.030	
U ²³⁵ (64.183)	SS can (47.30)	+2.951 ± 0.010	+3.055 ± 0.012	+2.890 ± 0.030	1.057	
Na (57.38)	56.42 (SS)	-0.144 ± 0.008	-0.041 ± 0.010	-0.049 ± 0.005	0.840	1.003
SS-304 (499.6)	Bare	-0.851 ± 0.020	-0.851 ± 0.020	-0.849 ± 0.010	1.002	
Fe (488.0)	Bare	-0.737 ± 0.005	-0.737 ± 0.005	-0.739 ± 0.007	0.997	
U ²³⁸ (146.708)	Al (5.286)	+12.183 ± 0.200	+12.217 ± 0.200	+11.643 ± 0.200	1.049	
	SS (19.977)					
Pu ²³⁹ (117.157)	Al (7.093)	+8.539 ± 0.200	+8.575 ± 0.200	8.446 ± 0.200	1.015	
	SS (20.060)					

^(a) Some samples were incased with SS cans.

^(b) 1% $\Delta k/k = 476$ 1h.

^(c) 1% $\Delta k/k = 468$ 1h.

absent, and this measurement was repeated at least ten times. The uncertainty in these measurements was based on the square root of the variance,

$$\sigma = \left[\sum_{i=1}^n (\chi_i - \bar{\chi})^2 / (n - 1) \right]^{1/2},$$

in the data, and it was propagated according to the theorem of propagation of error to account for the fact that some samples were in stainless steel cans.

The results are shown in Tables 10 and 11. It is seen from Table 10 that the ratio of the worth of a sample in Assembly 5 to that in Assembly 4Z is essentially constant for all samples. The average for all the measured ratios in Table 10 (a weighting of unity was used for all samples) is 1.01 ± 0.05 . The agreement between the predicted values for the worths of the samples in Assembly 5 from the measurements in Assembly 4Z is just good. The analysis for S_z of Assembly 11 has not been made yet, but it appears that the ratio of this quantity in Assemblies 11 and 4Z is not properly predicted with simple diffusion-theory calculations. This discrepancy can be seen in the last two columns in Table 11 for essentially pure absorbers, such as the

B-10 sample. It can also be seen from Table 11 that the reactivity worth of scattering materials such as Be, C, and Na are considerably different from those in Assembly 4Z. This indicates that the spectral matching in Assembly 11 is just not adequate.

6.2. Worth Measurements at 16.5 Cm Radially from the Center at the Midplane in Assemblies 4Z and 5

The same type of measurements described above were performed for a few samples at 16.5 cm radially from the center in the midplane of Assemblies 4Z and 5. These measurements were done in order to verify the spectral matching in both assemblies at some distance from the center of each assembly. This verification is valid provided the spatial distribution of the fluxes in both assemblies is the same. The results are given in Table 12. It is seen from this table that the worth of a sample is essentially the same in both assemblies.

7. Conclusions

The following conclusions may be made on the basis of the experimental results:

1. The measured neutron-energy spectrum is in

TABLE 11. CENTRAL WORTH MEASUREMENTS IN ZPR-9 ASSEMBLY NO. 11 AND THEIR RATIOS TO THOSE IN ASSEMBLY 4Z

Sample and Sample Wt, g	Nature and Wt of Can, g	Gross Worth, $I_h^{(a)}$	Net Worth in Assembly 11, I_h	Net Worth in Assembly 4Z, I_h	Measured	Calc by Set 801
					Worth in 11 Worth in 4Z	Worth in 11 Worth in 4Z
Be (114.308)	Bare	$+0.671 \pm 0.057$	0.671 ± 0.057	$+1.107 \pm 0.010$	$+0.606$	$+0.090$
C (103.0)	Bare	-0.220 ± 0.057	-0.220 ± 0.057	$+0.197 \pm 0.010$	-1.116	$+9.36$
Ni (546.0)	Bare	-1.973 ± 0.057	-1.973 ± 0.057	-1.148 ± 0.010	$+1.718$	$+1.90$
V (183.67)	55.30 (SS)	-0.498 ± 0.057	-0.334 ± 0.060	-0.133 ± 0.010	2.511	$+2.00$
Li (28.30)	57.03 (SS)	-1.478 ± 0.023	-1.208 ± 0.029	-0.595 ± 0.010	2.030	—
Zr (406.0)	Bare	-1.136 ± 0.023	-1.136 ± 0.023	-0.650 ± 0.005	1.748	1.93
Mo (599.0)	Bare	-5.293 ± 0.023	-5.243 ± 0.023	-3.415 ± 0.010	1.550	1.89
Al (166.90)	Bare	-0.518 ± 0.023	-0.518 ± 0.023	-0.297 ± 0.013	1.744	2.08
B ¹⁰ (29.29)	55.73 (SS)	-35.676 ± 0.030	-35.520 ± 0.035	-23.339 ± 0.200	1.522	1.88
B ₄ C (59.62)	64.70 (SS)	-9.876 ± 0.028	-9.684 ± 0.034	-6.410 ± 0.025	1.511	—
Polythene (58.98)	Bare	$+28.652 \pm 0.028$	28.652 ± 0.028	$+19.696 \pm 0.010$	1.454	—
Nb (481.2)	30.10 (SS)	-5.900 ± 0.048	-5.811 ± 0.051	-3.791 ± 0.010	1.533	1.88
W (1108.0)	—	-8.928 ± 0.030	-8.928 ± 0.030	-5.742 ± 0.010	1.555	1.90
Ta (924.70)	27.54 (SS)	-18.30 ± 0.10	-18.22 ± 0.10	-11.907 ± 0.200	1.530	1.88
U ²³⁸ (1153.80)	Bare	-5.908 ± 0.010	-5.908 ± 0.010	-3.857 ± 0.010	1.532	1.88
U ²³⁵ (27.719)	SS can (47.30)	$+1.946 \pm 0.025$	1.833 ± 0.028	$+1.246 \pm 0.030$	1.471	1.94
U ²³⁵ (64.183)	SS can (47.30)	$+4.827 \pm 0.050$	$+4.714 \pm 0.053$	$+2.890 \pm 0.030$	1.631	—
Na (51.38)	56.42 (SS)	-0.282 ± 0.015	-0.114 ± 0.035	-0.049 ± 0.005	2.326	2.41
SS-304 (499.6)	Bare	-1.401 ± 0.040	-1.401 ± 0.040	-0.849 ± 0.010	1.650	—
Pu (117.157)	Al (7.093)	$+13.714 \pm 0.050$	$+13.795 \pm 0.055$	$+8.44 \pm 0.200$	1.634	1.93
	SS (20.06)					
U ²³⁸ (146.708)	Al (5.286)	20.987 ± 0.200	$+21.067 \pm 0.200$	11.643 ± 0.200	1.809	—
	SS (19.977)					

^(a) 1% $\Delta k/k = 423$ Ih.

TABLE 12. WORTH MEASUREMENTS AT 16.5 CM FROM CENTER IN ASSEMBLIES 4Z AND 5

Sample	Gross Worth in Assembly 5	Net Worth in	
		Assembly 5	Assembly 4Z
Na	-0.127 ± 0.008	0.035 ± 0.010	-0.032 ± 0.007
Be	$+1.104 \pm 0.010$	$+1.104 \pm 0.010$	$+1.098 \pm 0.040$
Fe	-0.645 ± 0.010	-0.645 ± 0.010	-0.663 ± 0.014
Zr	-0.565 ± 0.010	-0.565 ± 0.010	-0.594 ± 0.002
C	$+0.212 \pm 0.010$	$+0.012 \pm 0.010$	$+0.181 \pm 0.051$
U ²³⁵ (13.869)	$+0.520 \pm 0.005$	$+0.622 \pm 0.010$	$+0.568 \pm 0.030$
U ²³⁵ (27.719)	$+1.098 \pm 0.005$	$+1.200 \pm 0.010$	$+1.164 \pm 0.030$
U ²³⁵ (64.183)	$+2.699 \pm 0.005$	$+2.801 \pm 0.010$	—
Nb	-3.512 ± 0.012	-3.542 ± 0.015	-3.543 ± 0.015
W	-5.289 ± 0.010	-5.289 ± 0.010	-5.268 ± 0.004
U ²³⁸	-3.533 ± 0.010	-3.533 ± 0.010	$-4.212 \pm 0.014^{(a)}$
B ¹⁰	-21.390 ± 0.015	-21.300 ± 0.015	-20.600 ± 0.100
Mo	-3.120 ± 0.015	-3.120 ± 0.015	-3.062 ± 0.020
B ₄ C	-5.464 ± 0.010	-5.374 ± 0.015	

^(a) This sample of U²³⁸ was different from the sample used in Assembly 5.

general agreement with the calculated spectrum. Spectral indices obtained with fission ratios of different materials also support this conclusion.

2. Fission-rate traverses showed that the application of diffusion theory to zoned cores with narrow regions results in underestimation of the spatial distribution of the high-energy groups. Results from a

transport code, SNARG, were in better agreement with measurements.

3. Elmoe averaging for a core with the composition of a narrow region and the subsequent use of this averaging in a multizone core does not improve the agreement with measurements; it makes it worse. Elmoe averaging would be very appropriate when such

technique could be applied to a multiregion core instead of a one-region core.

4. Whereas the spatial distribution of the real fluxes was better predicted with transport than with diffusion calculations, the opposite is true for the importance of fission neutrons, $\sum_i \chi_i \phi_i^+$. The reasons for this are presently under investigation.

5. The predicted values for the reactivity worths of various samples in Assembly 5 from measurements in Assembly 4Z are in agreement with the actual values measured in Assembly 5. These predictions were made well in advance of the measurements in Assembly 5.⁽¹⁵⁾

6. Conceptually there is no reason to limit the size of the test zone to that of Assembly 4Z. The difficulty with Assembly 11 originates with the poor spectral matching and nothing else. It must be conceded, however, that the spectral matching in a large test zone may be easier to achieve than in a small zone.

7. A useful criterion for whether or not the spectral matching is adequate is to calculate the worth of several samples with different nuclear properties in the zoned and full-size systems and obtain the ratio of the worth of a sample in one system to that in another. If this ratio is constant for all the samples, then the matching is adequate.

Acknowledgments

We thank D. Meneley for his generous help and unlimited patience in shifting the SNARG fluxes to make possible their use with the diffusion perturbation subroutine of MACH-1. We also thank W. C. Redman for lending us his Cf²⁵² source. Thanks are due W. R. Robinson for helping with the counting of the manganese, U²³⁵, and U²³⁸ foils. R. A. Lewis and R. B. Pond designed Assembly 11 of ZPR-9. The diligence of the operating crews of ZPR-6 and -9 is gratefully acknowledged.

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Discussion

Mr. Gandini (CNEN): I have two questions. I note that you are concerned about matching and adjusting real and adjoint fluxes by sigma-inelastic for high-energy neutrons and sigma-elastic for low-energy neutrons. How does this interfere with the fine structure of the spatial distribution of the neutrons, which seems to play a major role in this experiment? The second question is this: The effect of fine-flux structure seems to me to be important, especially with respect to pancake reactors. How may this experiment give information on these border effects, which might be important?

Mr. Karam: The fine structure does exist and we believe it does have an effect. The reaction-rate traverses shown were actually made by averaging the fluxes over this fine structure, especially for U²³⁸. The fine structure in U²³⁵ does not exist.

Mr. Gandini: I am concerned about the temperature coefficient rather than the sodium-void coefficient. I consider this effect as very important. Is the sodium-void coefficient sensitive to experiments of this kind?

Mr. Karam: Yes, I believe so.

Mr. Tuttle (AI): In your measurement technique, do you use the method of linear rate of rise at each radial point to determine the radial variation of importance?

Mr. Karam: I use the slope and then take the ratio of the slope from one point to a reference point; in other words, I normalize it and treat it as a measure of reactivity.

Mr. Tuttle: We use a dynamic reactivity method similar to that used by C. E. Cohn for reactivity measurements. If you have a strong enough source, your method is, I would guess, much faster and at least of equivalent accuracy.

Studies of Analytical Problems in Interpretation of Zone-loading Experiments, and Basic Considerations in Zone-loaded Core Design

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1. Introduction

This paper contains the results of a study to explore the importance of some known computational difficulties in the analysis of zone loadings, and to predict major physical properties of a single-zone core (ZPR-6 Assembly 5) from an analysis of its zone-loaded equivalent (ZPR-6 Assembly 4Z). This prediction was made before the start of loading of Assembly 5 and represented a test of present analytical techniques. The authors of the preceding paper very kindly supplied all of the experimental data needed in the analysis.

One of the authors was also responsible for the design of three fast reactor zone loadings of ZPR-3. Assembly 42⁽¹⁾ represented a zone-loaded equivalent of a previous Assembly 34.⁽²⁾ Assembly 43⁽³⁾ represented a zone-loading mockup of a U-235/U-238 monocarbide fast breeder of about 3000-l core volume. Assembly 45A⁽⁴⁾ was a plutonium-fueled zone loading of a fast power breeder, with a U-238/Pu-239 fuel ratio close to 7/1. Assemblies 43A and 45 were created by replacing 40 % of the sodium in the central zones of Assemblies 43 and 45A, respectively. This variation provided second bases for comparing theory with experiment, particularly for central sodium-void and Doppler-effect measurements.

A significant part of this paper treats the less obvious aspects of zone-loading design, of precautions necessary if misleading results are to be avoided, and of procedures or applications of zone-loading methods which could lead to attractive measurements. Much of this information was gained during the course of the design and analysis of the above assemblies.

2. Zone-loaded Core-design Procedure

To review very quickly the basic structure of the ANL zone-loading design procedure, it can be said that these zone loadings had a cylindrical rather than spherical geometry. This choice of geometry, with buffer and driver located only radially, makes it possible to have along the axis of the zoned reactor a situation which is very close to what happens along the axis of

the corresponding full reactor. Thus, it can yield some additional physics information, such as a close approximation to the leakage component of the sodium-void coefficient in the axial direction. There is also the practical advantage of a considerable simplification of the experimentalists' problem in loading the reactor. The problems of computational analysis are not significantly more complicated.

Before embarking on the description of more detailed design considerations, it is convenient to give a general description of a typical zone-loaded assembly configuration. Figure 1 represents a cross-sectional view of the loading design of Assemblies 45 and 45A of ZPR-3. The squarish outlines result from the procedure of mounting core materials into drawers which then slide into square matrix tubes with approximately a 5.5 x 5.5-cm cross section. The central zone is effectively about 3.8 drawers (21 cm) in radius; the filter is only a half-drawer thick. The difference between Assemblies 45 and 45A in the figure is due to the fact that in Assembly 45 the inner radial reflector replaced part of the U-238 blanket of Assembly 45A in order to help preserve the spectrum of neutrons in the few-kilovolt range, which is critical for sodium-void and Doppler-effect measurements. Figure 2 demonstrates the quality of the radial dependence of the spectral flux for the spectral region of 0.55 to 9.1 keV for Assembly 45, as calculated by use of ANL set 224, ELMOE averaging of light-element scattering properties, and diffusion theory only.

Assembly 45A was a relatively successful zone loading, particularly as an early design with restricted dimensions, in providing valuable physics information on Pu-239/U-238-fueled large fast breeders. Only 56 kg of Pu-239 were needed for this assembly.

Zone-loading design begins with a careful calculation of the real and adjoint spectrum at the center of the reactor which is to be matched in the zone loading. These spectra can be computed by means of fundamental-mode calculations using the ELMOE⁽⁵⁾ code. The MC² code,⁽⁶⁾ described in Session I of this meeting,

FIG. 1. Cross Section of Zone-loading Areas for ZPR-3 Assemblies 45 and 45A. Except that Assembly 45A did not employ the radial reflector of the figure, the dimensions of the two assemblies are identical.

FIG. 2. Spatial Dependence of Normalized (except for 6P and 18P), Real Neutron-energy Spectrum Segments of ZPR-3 Assembly 45. Curves 6P and 18P show the actual non-normalized spatial dependence of an upper energy group (0.498–0.825 MeV) and a Doppler interval group (0.960–1.23 keV).

is now replacing ELMOE at ANL. The next and a major problem is to find a composition to be used in the driver, which will have a much greater specific reactivity than the central zone while matching its real and adjoint spectra as closely as possible.

The reason for the high specific reactivity that is required of the driver material is due to the fact that the driver must provide a major part of the total reactivity of the assembly, from an annular geometry. This means, for most laboratories, that the driver fuel must be concentrated in order to add to the neutron production without adding too much to the neutron leakage, and in order to reach criticality with the fissile inventory available.

The high specific reactivity of the driver is convenient also because it provides a suitable location for the

control and safety rods. The control rods require a relatively high fuel density to be able to provide the necessary shutdown margin on scram. If the driver is relatively dense in fissile loading, these rods can be loaded with the same composition and inserted in the driver, thus causing very little local perturbation of the flux.

Having chosen a desired density of fissile fuel per drawer of driver, the designer tries to match the spectrum of the core, using several guidelines. In general, it is more convenient for the designer of the driver to concentrate on matching the macroscopic properties of the spectrum, like the fractions of neutrons present over broad energy ranges, rather than to concern himself about the microscopic properties, like the oscillations due to sharp scattering resonances. The reason for this is that the resonances that cause the microscopic features of the spectrum have very large cross sections, and that therefore those features can be established over a very narrow zone of the reactor. The macroscopic features of the spectrum, on the other hand, require a larger space to be established. It follows from this that it is more convenient to concentrate on the agreement of the macroscopic properties of the spectrum over the driver, and to correct the microscopic properties of the spectrum over the buffer, which is closer to the core.

Materials having strong inelastic scattering are very useful for downward adjustment of the spectrum in the energy range above 200 keV. Some of them, however, like U-238, have unfortunate tendencies to absorb neutrons in the keV region. From this point of view, nickel is an example of a possible alternative inelastic scatterer.

Elastic scatterers, like carbon, are often useful, but introduce serious heterogeneity and streaming problems if they are not used with care. It is useful to include at least minor concentrations of resonance scatterers present in the core, particularly if one is interested in sodium-void or Doppler studies. More than a small amount of sodium may heighten heterogeneity and streaming problems.

It is essential to realize that zone-loaded drivers are usually so narrow in radius that the spectrum of a fundamental-mode calculation for this material is a very crude approximation to the actual spectrum. D. Meneghetti, A. Rago, and K. Phillips of ANL have developed a preliminary version of a space-dependent ELMOE calculation which should be of considerable utility for this and for a number of other equally important purposes. A reasonable alternative to this procedure would be to develop a carefully structured cross-section set of about 90 groups. Modern computers could perform 1-dimensional calculation with such a set in a few minutes.

A buffer, or filter, region is usually interposed between the central zone and the driver in order to help smooth

FIG. 3. Calculated Spatial Profile of Percent Spectral Deviation of Real Flux of ZPR-6 Assembly 4Z from the Central Real Flux of ZPR-6 Assembly 5.

out the worst inequalities in spectral match with special consideration for the microscopic properties of the spectrum. Several comments might be helpful here. One is that the designer should be careful to include resonance scattering or absorbing materials of the central zone in the buffer. Again, sodium is particularly important, especially if sodium-void or Doppler experiments are to be performed. A second is that there is no reason why a buffer should not contain fissile material, especially toward the central zone, if it would appear to be of advantage. A third is that there may be a significant difference between spectral behavior across the buffer as calculated by diffusion and transport theory methods. DS_8 calculations frequently are valuable. A fourth is that the designer must keep the experimentalist's problems closely in mind. It is often most convenient to design buffers in terms of unit drawers, or half-unit drawers, recognizing the difference between the actual jagged rectangular experimental x - y geomet-

try and the analyst's smooth equivalent cylinders. And finally, the designer should in general always attempt to achieve the simplest possible assembly. The complexities of zone-loading experiments add undesirable additional sources of uncertainty to any analysis, and these should be reduced whenever possible.

Figure 3 is an example of a spectral-matching difficulty which can occur if resonance-scattering materials of the central zone are not present in the filter zone. For this zone loading (Assembly 4Z of ZPR-6) the buffer did not contain sodium. The failure to match in the buffer the strong resonance removal of neutrons by sodium in the region about 2.85 keV, as it occurs in the central zone, is the source of the surplus of neutrons in this energy region at the edge of the central zone. The deficiency of neutrons just below this energy at the same core-edge region represents a second trouble, due to the lack of the same resonance which would strongly scatter neutrons *into* this energy interval. The excess of

neutrons at a few hundred eV is produced by an excess of graphite in the buffer. If two plates of ZPR-6 graphite, $\frac{1}{8}$ in. thick, had been replaced by one $\frac{1}{4}$ -in.-thick can of sodium, all three troubles would have been significantly reduced.

Radial blankets have tended to be constructed of depleted uranium. This is largely because of historical precedent and partly in the interest of simplicity. Designers of zone loadings who are especially concerned with good spectral agreement over the kilovolt region of neutron energy might consider iron as a possible alternative to U-238. For particularly soft-spectrum cases a special inner region, as used in Assembly 45, can significantly ease the design problem.

3. Advantages and Disadvantages of Zone-loading Procedures

This section will treat only briefly the more obvious advantages and disadvantages of the zone-loading method, and will concentrate on some of the less well-known aspects of the subject.

It is generally accepted that critical assembly reactors of comparatively small dimensions can be employed, using zone-loading procedures and relatively few kilograms of material in a central zone, to make a number of very satisfactory measurements on the physics of the large fast breeder reactor. Measurements of the central spectrum of the type described by E. Bennett of ANL at this meeting are an example. Other measurements, such as central, channeled, or axially dependent measurements of sodium-void worth, Doppler effect, and material worth involve some additional complications. One wants to be able to relate the result of a worth measurement made in a zone loading to the worth that the same measurement would yield if made in a full-scale reactor. Results of a study of some of the problems involved in this relation are discussed in the next section.

One parameter which zone-loading experiments cannot give directly is the critical mass of the full-zone reactor. However, this limitation is not as serious as it might appear. Although the reactor designer must be able to predict the critical mass of a proposed reactor with a reasonable degree of confidence, the use of several zones of enrichment for large cores, for which this prediction is particularly difficult, helps to reduce the practical effect of this uncertainty. The critical mass is a useful check point for the analyst. D. Meneghetti (ANL) feels that much of the present uncertainty in this problem lies in the spatial dependence of the cross sections and of the flux spectra in the core as the radial and axial blanket interfaces are approached. The previously mentioned spatially dependent ELMOE program should help the analyst to overcome this difficulty. Cal-

culations of this effect can be tested on zone loadings when they are mounted in cylindrical geometry according to the general concept set forward in Sect. 2. It is experimentally possible and easy to make spectrum, fission traverse, and other measurements toward the axial blankets on both ends of the central zone. These axial blankets of the zoned reactor could have the composition of either the radial or of the axial blanket of the full-zone reactor. A central column of blanket material of suitable radius could be placed in the geometrical center of the central zone as another simple procedure for studying this problem.

Zone loadings have a less-recognized advantage for experimentalists who must measure very small reactivity changes, as in Doppler-effect measurements. The heat due to the spontaneous decay of Pu-239 and of its higher isotopes is a serious source of reactor drift. In a zoned reactor, since only a fraction of a full core of plutonium fuel is needed, this difficulty is proportionately reduced.

As a precaution, it is very important to recognize that the zone-loading procedure considerably multiplies the number of sources of calculational error, and that the analyst must be much more careful of the validity of his procedures and of his cross sections. Although there has been a considerable improvement in basic cross-section data over the last few years, this information frequently has not yet been included in cross-section sets. Cross-section sets, also, are seldom free of accidental errors.

A particularly dangerous trap of zone-loading experimentation is to attempt to study the physics of a large core, fully voided of sodium, by simply removing all of the sodium in the central zone. For zones of modest radii, this procedure would result in a poor spectral matching at the center of the reactor. A successful experiment of this type usually will require redesign of the filter and possibly some change in the driver as well.

A slight extrapolation of the zone-loading concept could lead to a study of the physics of the modular fast reactor, which itself deserves experimental examination. Many interesting physics properties of the modular concept could be tested simply on a highly simplified modular representation involving U-235 fuel only. The problems concerning the design of a fuel-irradiation facility are frequently close to those concerning the design of a zone loading. Communication between the groups involved in the design of the two types of facility might be mutually beneficial.

There are some additional practical advantages to be found in the operation of a critical assembly with plutonium in a limited zone rather than in a full core. Because of simple difficulties, such as the need to check

for surface contamination before using each plutonium rod or plate, loading of a plutonium-fueled critical usually proceeds very slowly. A zone loading can significantly reduce the amount of plutonium which must be handled, relative to the equivalent single-zone core, and therefore speed the approach to critical. Another consideration is that technicians for a plutonium-fueled reactor tend to receive undesirable irradiation exposures because of the activity of the plutonium fuel. Zone loadings tend to relieve this problem. At the present time, when the need for plutonium-fueled-critical data suggests survey experiments and therefore much loading and unloading, these considerations are important.

The problem of safety of operation of a critical reactor must usually be discussed by a reactor manager for each loading; this is especially true for an unusual loading, as a zone loading might be considered to be. It is not possible to say that a zone loading is inherently more safe or less safe than the equivalent single-zone core, if that core could be built. In a few severe accident cases analyzed at ANL it has been shown that a zone loading can be designed so that early material motions in the driver produce a slightly autocatalytic accident behavior. It has also been shown that relatively small outward displacements of the driver can produce very significant degrees of shutdown, compared to displacements required in the single-zone core. Because of the relatively high fuel density in the driver zone, the "trigger" for disassembly, in the sense of Bethe-Tait analysis, occurs (very desirably) early in the accident. Also, the usually large gradient in fission density in the driver is closely related to the pressure gradients generated during disassembly. It is this gradient in pressure which induces a highly desirable outward acceleration.

As a typical precaution, a radial blanket surrounding the driver and composed of beryllium or carbon, rather than of iron or U-238, could greatly change the shape of the fission distribution across the driver, and therefore the profile of the pressure gradient during a severe accident.

If the fuel of the driver zone is composed of separated U-235 rather than U-235 mixed with U-238, there will be no (or possibly a slightly positive) Doppler effect to contribute to a prompt shutdown response during a mild accident. Driver spectra are often rather hard, however, in such a way that there would be very small Doppler contribution even if mixed fuel were to be used. The mixing would reduce the reactivity loss available through expansion of the driver fuel on heating since the U-235 would have to share the fission heat with the U-238. Because of the large radial components of neutron leakage from the driver, axial expansion of

FIG. 4. Cross Section of Zone Loading of ZPR-6 Assembly 4Z. The light lines define mesh spaces used in a x - y transport-theory calculation for this assembly.

the U-235 plates (or rods) should induce a relatively large shutdown effect.

A final, hardly cheering, note on the safety of zone loadings is obtained by considering that a severe accident with a zone loading could potentially release at most some fraction of the ~ 50 to 200 kg of plutonium of the zone loading, while a full-zone reactor could release perhaps a similar fraction of perhaps ~ 2000 kg of plutonium.

4. Study of Zone-loading Analytical Uncertainties, and Prediction for ZPR-6 Assembly 5

In the late fall of 1965 the authors were given the assignment to predict as closely as possible the results of experimental measurements to be made on ZPR-6 Assembly 5, well before that reactor was constructed, from an analysis of its zone-loaded equivalent, Assembly 4Z. A quadrant of the cross section of the 4Z core is shown in Fig. 4. Calculations showed that because of the relatively large radius of the central zone of this core, and because of the generally satisfactory choice of filter and driver, it was apparent that such quantities as the central neutron-energy spectrum and central fission ratios would be essentially identical for the two reactors, with essentially no theoretical effort needed to establish this information.

The value of the critical mass of Assembly 5 would not come readily from a study of 4Z, but it would be calculated by using some of the experience gained during the calculations made for Assembly 4Z.

It was decided to give much more attention to the

question of how well a central worth measurement for Assembly 5 could be predicted from a knowledge of this worth in 4Z. Success here would increase confidence in the usefulness of central sodium-void, Doppler-effect, control-rod, and other worth measurements made in zone-loaded reactors.

From the point of view of perturbation theory, since the central region of 4Z was relatively very large and the filter and driver reasonably well designed, the real and adjoint fluxes of 4Z and 5 could be assumed to be essentially the same over the region of a perturbation measurement. Therefore, the numerators of perturbation calculations for the two assemblies would be essentially identical if a proper normalization was used. This was checked by means of a series of preliminary one-dimensional diffusion-perturbation calculations in which the numerator of the perturbation expression for the central worth of most materials of interest was computed in both reactors. The calculations showed that the relative difference of the numerators in the expressions for the two reactors was usually smaller than 0.5%, and that the effect of the imperfect matching of the real and adjoint spectra to the central worths was always smaller than half of the experimental error. The focus of analysis, therefore, was concentrated on the prediction of the denominator terms.

The thinness of the driver, which resulted from the high reactivity given to the central zone through the choice of a large radius, and the great dissimilarity between its composition and the composition of the filter region (5.5 cm of alternating plates of U-238 and graphite), made the use of diffusion-theory calculations for the analysis somewhat inappropriate. The use of relatively large amounts of sodium in the driver led to a heterogeneity difficulty. Besides, the jagged profile of the cross section of 4Z caused sharp local variations in the thickness of both the buffer and the driver, and it was of interest to explore the importance of this effect relative to the analytic approximation as a smooth cylinder. Finally, since at least early design calculations are routinely done in one-dimensional cylindrical geometry, it was of interest to determine the importance of two-dimensional rz -geometry calculations relative to the common one-dimensional calculations.

If the integral over the denominator term, labeled NI here, is normalized as in the equation

$$NI = \int_{\text{Vol}} \left[\int_{\mathbf{E}} \chi \phi^+ dE \int_{\mathbf{E}} \nu \Sigma_f \phi dE \right] d \text{Vol} / \left[\int_{\mathbf{E}} \chi \phi^+ dE \int_{\mathbf{E}} \nu \Sigma_f \phi dE \right]_{r=0},$$

then a desirable calculation of NI might involve a good transport approximation, such as the DS₈ approximation, in xyz geometry with 22 energy groups. This calculation could be labeled NI(xyz , 22G, DS₈). The

more common calculation NI(r , 22G, D) uses radial (cylindrical) geometry and diffusion theory.

These two calculations can be related by the algebraic identity

$$NI(xyz, 22G, DS_8) \equiv NI(r, 22G, D) \frac{NI(r, 22G, DS_8)}{NI(r, 22G, D)} \cdot \frac{NI(xy, 22G, DS_8)}{NI(r, 22G, DS_8)} \frac{NI(xyz, 22G, DS_8)}{NI(xy, 22G, DS_8)}.$$

The first ratio of this expression represents a correction factor due to the difference between diffusion and transport theory, in one-dimensional geometry. The second ratio represents a correction factor due to the difference between the true jagged cross section of Fig. 4 and the smooth cylinder used for the one-dimensional computation. The third ratio represents a correction factor due to the inclusion in the calculation of the correct z dependence of the flux in Assembly 4Z.

As a practical expedient, with little loss, we chose to approximate the second fraction as

$$\frac{NI(xy, 22G, DS_8)}{NI(r, 22G, D)} \approx \frac{NI(xy, 5G, DS_4)}{NI(r, 5G, D)}.$$

Similarly, the factor that takes into account the z -dependent aspect of the problem can be represented reasonably well by

$$\frac{NI(xyz, 22G, DS_8)}{NI(xy, 22G, DS_8)} \approx \frac{NI(rz, 5G, D)}{NI(r, 5G, D)}.$$

Calculations of this type were performed and yielded the result that

$$NI(4Z) = 0.70093 \times \underset{\substack{\text{(Transp.} \\ \text{vs.} \\ \text{Diff.}}}{1.0155}} \times \underset{\substack{\text{(xy vs.} \\ \text{Cyl.} \\ \text{Geom.)}}}{1.0066}} \times \underset{\substack{\text{(rz vs.} \\ \text{1-D Cyl.} \\ \text{Geom.)}}}{0.9834} \text{ m}^3,$$

$$NI(4Z) = 0.70460 \text{ m}^3.$$

Since the geometry of Assembly 5 was to be a simple large cylinder, the approximation

$$\begin{aligned} NI(5) &= NI(r, 22G, D) \times \frac{NI(r, 22G, DS_8)}{NI(r, 22G, D)} \\ &= 0.70564 \times 0.9960 \\ &= 0.7028 \text{ m}^3 \end{aligned}$$

was chosen. From these numbers, the ratio of worth measurements to be made in Assembly 5 to those made in 4Z was predicted to be

$$\frac{\text{WORTH}(5)}{\text{WORTH}(4Z)} = \frac{NI(5)}{NI(4Z)} = 1.003.$$

Discussions with Meneghetti (ANL) led to his rough estimation that heterogeneity effects for the driver used might increase this ratio by as much as +0.01. Thus heterogeneity uncertainties of the driver chosen for 4Z are an important concern.

TABLE 1. EXPERIMENTAL DETERMINATION OF WORTH RATIOS, ASSEMBLY 5/ASSEMBLY 4Z, ZPR-6

	Assy 5, 1h	Assy 4Z, 1h	Ratio ^(a)	Weight = 1/ μ^2
Be	+1.130 $\pm$ 0.048	+1.107 $\pm$ 0.010	1.021 $\pm$ 0.044	510
C	+0.183 $\pm$ 0.022	+0.197 $\pm$ 0.010	0.929 $\pm$ 0.121	68
Ni	-1.134 $\pm$ 0.015	-1.148 $\pm$ 0.010	0.988 $\pm$ 0.016	4,109
V	-0.136 $\pm$ 0.014	-0.133 $\pm$ 0.010	1.023 $\pm$ 0.130	59
Li	-0.677 $\pm$ 0.015	-0.595 $\pm$ 0.010	1.138 $\pm$ 0.032	1,001*
Zr	-0.671 $\pm$ 0.020	-0.650 $\pm$ 0.005	1.032 $\pm$ 0.032	989
Mo	-3.416 $\pm$ 0.015	-3.415 $\pm$ 0.010	1.000 $\pm$ 0.005	35,600
Al	-0.300 $\pm$ 0.017	-0.297 $\pm$ 0.013	1.010 $\pm$ 0.072	191
B ¹⁰	-23.400 $\pm$ 0.033	-23.339 $\pm$ 0.200	1.003 $\pm$ 0.009	13,212
B ₄ C	-6.137 $\pm$ 0.015	-6.410 $\pm$ 0.025	0.957 $\pm$ 0.004	51,653*
Poly	+19.122 $\pm$ 0.010	+19.696 $\pm$ 0.200	0.971 $\pm$ 0.010	9,612*
Nb	-3.939 $\pm$ 0.015	-3.791 $\pm$ 0.010	1.039 $\pm$ 0.005	43,403*
W	-5.792 $\pm$ 0.020	-5.742 $\pm$ 0.010	1.009 $\pm$ 0.004	65,746
Ta	-12.001 $\pm$ 0.015	-11.907 $\pm$ 0.200	1.008 $\pm$ 0.017	3,543
U ²³⁸	-3.869 $\pm$ 0.010	-3.857 $\pm$ 0.010	1.003 $\pm$ 0.004	73,046
U ²³⁵ ₍₁₎	+0.648 $\pm$ 0.012	+0.611 $\pm$ 0.020	1.061 $\pm$ 0.040	631
U ²³⁵ ₍₂₎	+1.278 $\pm$ 0.012	+1.246 $\pm$ 0.030	1.026 $\pm$ 0.027	1,413
U ²³⁵ ₍₃₎	+3.055 $\pm$ 0.012	+2.890 $\pm$ 0.030	1.057 $\pm$ 0.012	7,305*
Na	-0.041 $\pm$ 0.010	-0.049 $\pm$ 0.005	0.837 $\pm$ 0.221	21
SS	-0.851 $\pm$ 0.020	-0.849 $\pm$ 0.010	1.002 $\pm$ 0.026	1,435
Pu	+8.577 $\pm$ 0.200	+8.446 $\pm$ 0.200	1.016 $\pm$ 0.034	967
U ²³³	+12.217 $\pm$ 0.200	+11.643 $\pm$ 0.200	1.049 $\pm$ 0.025	1,765
Fe	-0.737 $\pm$ 0.005	-0.731 $\pm$ 0.007	1.008 $\pm$ 0.012	7,182

$$^{(a)}\text{Ratio} = \frac{\sum_i W_i (\text{Ratio})_i}{\sum_i W_i} = 1.003 \pm 0.002 = (1.005 \pm 0.002)^*.$$

A summary of the experimental worth measurements made by the ZPR-6 staff on 4Z, and on 5 after the above prediction, is given in Table 1 along with the ratio of the worths of each material for the two assemblies. In order to treat the reported experimental errors in a consistent manner, it was decided to assume that they were all of statistic nature. If the ratios are weighted accordingly, as shown in Table 1, the result is that

$$\text{Effective Ratio} = \frac{\sum_i \text{Weight}_i \times \text{Ratio}_i}{\sum_i \text{Weight}_i} = 1.003 \pm 0.002$$

if no ratios are dropped. The result becomes

$$\text{Effective ratio} = 1.005 \pm 0.002$$

if the ratios with asterisks are dropped from the averaging procedure because the ratios for those isotopes differ from the average by more than three times their standard deviations. The difference is thus negligible.

The agreement between the theoretical prediction of worth measurements to be observed with ZPR-6 Assembly 5 and the actual experimental value thus turned out to be very good, except for the scarcely treated correction for the heterogeneities of the driver.

By a similar prescription it was predicted that the critical mass of Assembly 5 would be

$$\text{CM}(5) = \text{CM}(r, 22\text{G}, \text{D}) \frac{\text{CM}(r, 22\text{G}, \text{DS}_4)}{\text{CM}(r, 22\text{G}, \text{D})}$$

— heterogeneity correction (of Assembly 5 core composition).

A heterogeneity correction of +0.6% was estimated through reference to ANL-6218,⁽⁷⁾ and a mass correction factor $(\delta M/M)/(\delta k/k) \equiv 10$ was obtained from the DS₄ calculations. Use of the above procedure with ANL set 224 and ELMOE gave the results

$$\begin{aligned} \text{CM}(5) &= 1,786 \times 0.991 - 106.2 \text{ kg} \\ &= 1,664 \text{ kg, predicted.} \end{aligned}$$

The experimental critical mass of Assembly 5 was found to be 1,590 kg. Several experimental corrections should reduce this experimental value downward, perhaps toward 1580 kg. The difference of 84 kg represents an approximately 0.5% uncertainty in the calculated k_{eff} of the theoretical prediction. In the prediction, the estimated critical mass was expected to be correct within 150 kg, which corresponds to a relatively liberal $\pm 1\%$ in $\delta k/k$ because of inaccuracies in the cross-section set and in procedures used. Thus, also the prediction of the critical mass was reasonably well verified by the experiments.

Acknowledgment

Much of the understanding of the physics of zone loading possessed by the senior author is owed entirely

to what he learned as a result of a close and fruitful collaboration with the staff of the ZPR-3 reactor. Joe Gasidlo was most closely associated with these assemblies, and especially valuable. Other close associates were Paul Amundson and Leo McVean. John Long and Fred Thalgott, as administrators, were exceptionally cooperative.

Many important insights into zone-loading theory were gained as a result of a very generous sharing of ideas on the subject by Dave Meneghetti. Dan Meneley provided much of the design calculations for Assembly 45A. He was regularly consulted on cross sections, computing methods, and the use of the MACH-1 diffusion-theory code. Harry Hummel and Robert Avery strongly supported this work and frequently made valuable contributions to the physics.

Most of the physics and calculations of the analysis of Assembly 4Z and prediction for Assembly 5 was performed by A. Travelli.

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Discussion

Mr. Smith (UK-Winfrith): I would like to comment on one remark. You suggested that in very large reactors with multizone loading you are not interested in calculating the reactivities so accurately because you can interchange enrichments in these zones. In practice surely this may require a large number of subassemblies of various enrichments which represents a big investment in money, and the designer will still want to know accurately his critical values.

Mr. Fischer: I was not trying to demean the problem by any means. I only suggest that if you have two or three zones of different enrichments then simply by removing low-enrichment subassemblies and substituting high-enrichment subassemblies at the interface

between two enrichments one can make a large correction on total critical mass.

Mr. Okrent (ANL): Can you comment on whether there are difficult corrections to measurements of sodium-void coefficient in zones because of changes in the leakage component in the zoned core compared to the uniform core?

Mr. Fischer: I think that one must be very careful in designing zone-loading experiments and also in analyzing them. What I was suggesting here was that, since the zone loading represents a cylindrical mockup, one can study sodium voiding in the axial direction rather nicely, avoiding all the hazards and pitfalls of spectral equilibration in the radial direction, particularly if the zone has a small radius. I think that one should investigate further what leakage studies can be made in the axial direction by means of zone loadings. Here the problems and the theoretical uncertainties are much smaller; however, of course, one does not assume that there is an immediate and obvious correspondence to what happens in the fast breeder. One may have to do two-dimensional calculations as well. The problem is not easy but the potential is there, and I would be inclined to recommend that sodium-voiding studies which involve leakage components be performed in the axial direction rather than toward the filter and driver regions in order to really have confidence in the measurement.

Mr. Specht (AI): Did you calculate the multigroup constants for each region with fundamental-mode ELMOE calculations or did you use space-dependent ELMOE calculations?

Mr. Fischer: Here one is in rather serious trouble for the driver, in particular; the central zone is not bad. Some progress is being made in this area. Toppel, for instance, says that in time he will have a space-dependent MC² code which will handle the situation properly. In the meantime Meneghetti and Rago have been working on a rather simple space-dependent ELMOE calculation which will give space-dependent cross sections using the ELMOE formalism rather than the MC² formalism.

Mr. Specht: What did you use in the driver?

Mr. Fischer: For these calculations we used ELMOE in the driver. We also did some studies which I haven't mentioned, where we tried using the old MC² code developed by O'Shea. In that MC² code, one can impose a basic flux. I divided the driver into three regions for group-collapsing purposes and estimated a fine-group flux spectrum for each region on the basis of a previous diffusion-theory calculation. The three sets of MC² cross sections were then used with a new diffusion-theory calculation, which in turn was the source of a new set of fine-flux estimates. By this iteration procedure I arrived at cross sections for the three intervals into which the driver had been separated.

Progressive Substitution Method in Fast Multizone Experiments*

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ABSTRACT

The progressive substitution method has yielded good results when applied to the determination of material buckling in thermal reactors. It has been thought useful to investigate if this method could also be applied to fast reactors, in particular to the interpretation of fast multizone experiments. Preliminary results presented here show that material-buckling differences between the reference and the substitution media can be obtained with good accuracy when the reflector saving does not vary during the substitution. If this is not the case, the interpretation of such experiments requires further work.

LIST OF SYMBOLS

- β^2 material buckling of reference medium
- $-\beta_1^2$ second eigenvalue (in 2-group formalism) of reference medium
- (S'_1) first eigenvector in 2-group formalism
- (S'_2) second eigenvector in 2-group formalism
- primed notations are for substitution medium—
- R_0 critical radius of reference medium alone
- R substitution radius
- R_e critical radius with substitution
- $\delta R_e = R_e - R_0$

1. Introduction

Due to the uncertainties which still affect the calculations of fast reactors, the design of a plutonium-fueled prototype, the power of which would be in the 250-MWe range, will require numerous reactor physics experiments, in particular critical experiments. As the quantity of plutonium available for these experiments is rather limited, the use of multizone assemblies is mandatory. In the classical configuration of such assemblies the medium under study is in the central zone, with a driver zone around it to achieve criticality.

* Work performed under the EURATOM-C.E.A. Association for Fast Reactors.

† On loan from Electricité de France (EDF).

One tries to obtain that the neutron spectrum and the importance function be independent of position in the largest possible part of the central zone; in order to achieve it a buffer zone may be added.^(1, 2)

Such multizone experiments yield reaction rates and relative danger coefficients in the central zone⁽³⁾ and should be useful to predict the critical mass, or other critical parameters, for the medium under study.⁽⁴⁾

The analysis of such assemblies is most often based on the use of multigroup cross-section sets and diffusion theory. If such a method is of interest when planning an experiment, it is hardly tractable for extrapolating experimental results obtained from the multizone assembly; the multigroup formulation makes use of too many parameters, and it is impossible to relate a difference between experiment and calculations to the inaccuracy of this or that parameter, which makes the extrapolation to real reactors difficult or even problematical.

It is therefore essential to describe these experiments by few integral quantities; their choice depends of course on which parameters one is interested in.

The characteristic chosen here as the one of primary interest is the critical mass, and the parameter chosen is the material buckling of the medium. In one-group theory, critical mass and material buckling are indeed very simply related and their variations are proportional; for more sophisticated models the relationship is less simple, but variations of critical mass are still mainly due to variations of the material buckling (see Sect. 7).

As the progressive substitution method used for the material-buckling determination has yielded good results for thermal reactors, it seems worthwhile to check if it can also be applied to fast reactors. This is the aim of this paper.

2. Physical Description of the Progressive Substitution Method

The progressive substitution method has been developed for thermal reactors⁽⁵⁻⁸⁾ in order to measure

FIG. 1. Fundamental-mode Spectrum. Normalization:
 $\sum_i \Delta u_i \phi_i = 1.$

differences between the physical properties (in particular, material-buckling differences) of two similar media.

In the reference medium, whose properties are assumed to be known (for instance, via measurements of critical mass, reaction rates, and flux mapping), one progressively replaces in the central zone the reference medium by the medium under study; criticality is maintained for each partial replacement by varying, in spherical geometry for instance, the external radius R_e of the reference medium. The experimental information thus obtained is the variation δR_e of the external radius versus the radius R of the zone in which the substitution has been done.

For interpreting this information, it is helpful to note that two phenomena contribute to the reactivity variation:

- the difference between the material bucklings;
- the differences between the spectra of both media, giving rise to fast variations of the flux around the interface (space and energy transients).

For thermal reactors, the two-group diffusion theory shows that the variation of reactivity (or of the external critical radius) is due to:

- the difference between the material bucklings, difference corrected by a "weight" expressing the importance of the substituted zone;
- interface transients which decrease approximately as $1/R$, where R is the radius of the substituted zone.

From measurements for different radii, it is thus possible to distinguish between the contributions "material buckling" and "interface," and to obtain the difference between the material bucklings of the two media. The transposition of this technique to fast multi-zone experiments will be possible only if one can define a sufficiently simple model describing these experiments. A numerical example will show in the next section how a multigroup formalism can be reduced to

such a simple model. From now on, spherical geometry is assumed for all calculations.

3. Choice of a Simplified Formalism

In multigroup diffusion theory, the flux variation in a given medium obeys the equation

$$\nabla^2 \bar{\phi}(r) + M \bar{\phi}(r) = 0, \quad (1)$$

where $\bar{\phi}(r)$ is a vector the components of which are the fluxes in the different energy groups and M is a square matrix depending on the cross sections of the medium. $\bar{\phi}(r)$ can be expressed through the eigenvalues B_i^2 and the corresponding eigenvectors $\bar{\phi}_i$ of the matrix M and the eigenfunctions f_i of the Laplacian:

$$\bar{\phi}(r) = \sum_i A_i f_i(r) \bar{\phi}_i \quad \text{with} \quad \Delta f_i(r) + B_i^2 f_i(r) = 0.$$

In this expression the coefficients A_i are determined by the limit conditions. The largest eigenvalue, B_1^2 , is by definition the material buckling of the medium in diffusion theory. This eigenvalue is positive for a medium with $k_\infty > 1$; the associated eigenvector, with all components positive, is the asymptotic spectrum.

To the material buckling, which will be denoted in the following by $B_1^2 = \beta^2 > 0$, is associated the fundamental spatial mode, of the form $f_1(r) = \sin \beta r / r$ in spherical geometry.

In the expression of $\bar{\phi}(r)$ the fundamental term may thus be explicitly written as

$$\begin{aligned} \bar{\phi}(r) &= A_1 \bar{\phi}_1 \frac{\sin \beta r}{r} + \sum_{i>1} A_i \bar{\phi}_i f_i(r) \\ &= A_1 \bar{\phi}_1 \frac{\sin \beta r}{r} + \bar{\epsilon}(r), \end{aligned} \quad (2)$$

where $\bar{\epsilon}(r)$ is the contribution of the transient modes.

With the help of a numerical example the possibility of expressing it in a simpler form is now investigated. The reference medium which has been chosen is a UO_2 -fueled fast reactor; three substitute media, to be used in the central zone, having fuel ratios for Pu-239 to U-238 close to that of the prototype PuO_2 -fueled reactor, are considered for this analysis. The material bucklings of these three media differ respectively by 0, -0.87, and -5.33 % from the material buckling of the reference medium. The compositions of these media are given in Appendix I; the asymptotic direct and adjoint spectra are shown in Figures 1 and 2 for the reference medium and for the substitution media (the direct and adjoint spectra are about the same for the three plutonium media; the very small differences between them cannot be shown on the figures).

For a substitution of a given radius, $\bar{\phi}(r)$ can be obtained by a standard multigroup diffusion code; $\bar{\phi}_1$ and

the material buckling β^2 are determined by a fundamental-mode calculation. Then $\bar{\epsilon}(r)$ is easily found by difference [from eq. (2)] as soon as the coefficient A_1 is known. This coefficient is calculated, for instance at the center where the best accuracy is obtained, by elimination of the transients through scalar multiplication of the flux by the fundamental adjoint flux $\bar{\phi}_1^*$, solution of $(M^* - \beta^2 I)\bar{\phi}_1^* = 0$ (where M^* is the operator adjoint to M).

The expression for $\bar{\epsilon}(r)$ is then

$$\bar{\epsilon}(r) = \bar{\phi}(r) - \frac{\langle \bar{\varphi}_1^*, \bar{\phi}(0) \rangle}{\langle \bar{\varphi}_1^*, \bar{\varphi}_1 \rangle} \frac{\sin \beta r}{\beta r} \bar{\varphi}_1. \quad (3)$$

For the S_3 medium substituted in a 25-cm-radius zone (see Appendix I), the components of $\vec{\epsilon}(r)$ are indicated in Table 1 and drawn in Fig. 3 as a function of position in the central zone. One can note the change of sign in the components of $\vec{\epsilon}(r)$ occurring between groups 5 and 6, which corresponds to a change of sign in the difference between the fundamental modes of the reference and the substitution media (see Fig. 1).

FIG. 2. Adjoint Fundamental-mode Spectrum. Normalization: $\sum_i \phi_i^* = 1$.

FIG. 3. Variation of Components of $\epsilon(r)$ (16 groups, Hansen and Roach set).

Flux transients also influence integral quantities like numbers of fission neutrons or absorbed neutrons; the variations of these quantities with distance are given on Fig. 4, where the ratios

$$\frac{\text{fission or absorbed neutrons due to flux transients } \bar{\epsilon}}{\text{fission or absorbed neutrons due to fundamental flux } \bar{\phi}_1}$$

are given.

It was found that these transients can be described with an excellent accuracy by expressions like $\sinh \beta'_1 r/r$ in the central zone and $[\cosh \beta_1 (r - a)]/r$ in the outer zone; it has moreover been checked that the quantities β'_1 and β_1 are independent of the radius R of the substituted zone.

3.1. Remark

The calculation of the influence of a transient is a little more complicated in the outer zone (the reference medium) than in the central zone because the contribu-

TABLE 1. $\phi(r)$ AND $\varepsilon(r)$ AS FUNCTIONS OF POSITION r INSIDE THE CENTRAL ZONE

[illegible]

FIG. 4. Variation of $\epsilon(r)$ versus r .

tion of the fundamental mode is of the form $[A \sin \beta(r - b)]/r$ in the reference medium; it is necessary to calculate the fundamental-mode contribution for at least two radii in order to deduce A and b and obtain again the transients by difference.

These results show that the transients may be described in each medium by only one eigenfunction solution of $\nabla^2 f + B^2 f = 0$; a two-group theory is therefore an excellent approximation to the classical multigroup calculations and can be used instead. It is the reason why the analytical study has been done with a two-group formalism.

4. Analytical Study of the Progressive Substitution

As mentioned earlier, only spherical assemblies are considered. In order to simplify the equations, it is assumed that the blanket influence is well described by the hypothesis of constant reflector saving, whatever the dimensions of the central zone.

The substituted medium, with material buckling β'^2 , extends from 0 to R ; the reference medium, with material buckling β^2 , extends from R to R_e (where R_e is the extrapolated radius).

As a first step, one gives the results of a one-group theory, in which only the material-buckling difference plays a role.

4.1. One-group Model

The flux in the central zone is

$$\phi(r) = \frac{\sin \beta' r}{r};$$

that in the outer zone is

$$\phi(r) = \frac{A \sin \beta(R_e - r)}{r}.$$

By assuming that the diffusion coefficients are equal for the two media, the critical equation can be written as

$$\frac{\tan \beta' R}{\beta'} = \frac{\tan \beta(R - R_e)}{\beta}, \quad (4)$$

which is obtained through the elimination of the constant A between the two continuity conditions at the interface.

By introducing buckling and critical-radius differences $\Delta\beta = \beta' - \beta$ and $\delta R_e = R_e - R_0$, where R_0 is the critical radius of reference medium alone, and by taking only first-order terms in δR_e and $\Delta\beta$ into account, eq. (4) can be written as

$$\frac{\delta R_e}{R_0} = -\frac{\Delta\beta}{\beta} \frac{1}{2\pi} (2\beta R - \sin^2 \beta R) \quad (5)$$

or, using $\delta R_e/R_0 = -\delta\beta/\beta$, where $\delta\beta$ is the geometric buckling variation of the two-region core,

$$\delta\beta = \Delta\beta W(R). \quad (6)$$

Here $W(R) = (1/2\pi)(2\beta R - \sin^2 \beta R)$ is in fact the ratio

$$\int_0^R \phi^2 dV / \int_0^{R_0} \phi^2 dV$$

obtained by first-order perturbation theory; it is the "weight" of the substituted region.

4.2. Two-group Model

In the outer region, the fluxes are of the form

$$\begin{aligned} \phi_1 &= \frac{\sin \beta r}{r} + \epsilon \frac{\cos \beta r}{r} + m \left[\frac{\sinh \beta_1 r}{r} + \epsilon' \frac{\cosh \beta_1 r}{r} \right]; \\ \phi_2 &= S_1 \left[\frac{\sin \beta r}{r} + \epsilon \frac{\cos \beta r}{r} \right] \\ &\quad + m S_2 \left[\frac{\sinh \beta_1 r}{r} + \epsilon' \frac{\cosh \beta_1 r}{r} \right], \end{aligned}$$

and in the central region

$$\begin{aligned} \phi_1 &= E \frac{\sin \beta' r}{r} + F \frac{\sinh \beta'_1 r}{r}; \\ \phi_2 &= S'_1 E \frac{\sin \beta' r}{r} + S'_2 F \frac{\sinh \beta'_1 r}{r}. \end{aligned}$$

The eigenvectors (S'_1) and (S'_2) are associated with the eigenvalues β^2 (fundamental) and $-\beta_1^2$ (transient) of the outer medium. Primed notations are used for the central medium.

In order to simplify the formulae, diffusion coefficients are assumed to be equal for the two media.

The quantities ϵ , ϵ' , m , E , and F are eliminated by the conditions at the interface and at the outer boundary, using the method followed by R. Naudet.⁽⁶⁾

One obtains the criticality equation

$$\frac{[\sin \Lambda] + \epsilon[\cos \Lambda]}{[\sin \Lambda'] + \epsilon'[\cos \Lambda']} = \frac{S}{S'} \frac{[\sinh \Lambda] + \epsilon'[\cosh \Lambda]}{[\sinh \Lambda'] + \epsilon'[\cosh \Lambda']}, \quad (7)$$

with the notations

$$[\sin \Lambda] = \sin \beta R + \beta \Lambda \cos \beta R;$$

$$[\cos \Lambda] = \cos \beta R - \beta \Lambda \sin \beta R;$$

$$[\sinh \Lambda] = \sinh \beta_1 R + \beta_1 \Lambda \cosh \beta_1 R;$$

$$[\cosh \Lambda] = \cosh \beta_1 R + \beta_1 \Lambda \sinh \beta_1 R;$$

$$[\sin \Lambda'] = \sin \beta R + \beta \Lambda' \cos \beta R.$$

The quantities

$$\Lambda = -\frac{1}{\beta'} \tan \beta' R_e;$$

$$\Lambda' = -\frac{1}{\beta'_1} \tanh \beta'_1 R$$

are calculated at the interface; the values

$$\epsilon = -\tan \beta R_e$$

and

$$\epsilon' = -\tanh \beta_1 R_e$$

are functions of the outer radius; also

$$S = \frac{S_2 - S'_2}{S_1 - S'_2};$$

$$S' = \frac{S_2 - S'_1}{S_1 - S'_1}$$

are combinations of the coupling coefficients.

4.3. Note

In the critical equation (7), the substitution radius R is present in each term in $[\]$, whereas the extrapolated radius R_e appears only in ϵ and ϵ' .

It is also to be noted that the coupling terms S appear only in the ratio S/S' , a simplification resulting from our assumption that the diffusion coefficients are equal. This assumption is well justified for the media we are considering here (see Fig. 4A); moreover, rejecting this assumption would make the equations more intricate, without altering our conclusions.

Equation (7) is significantly simplified if one notes that $\epsilon' = -\tanh \beta_1 R_e$ is very close to -1 (with an error of 10^{-3} in our numerical example). This simplification is due to the large value of the ratio β_1/β . Equation (7) can thus be written

FIG. 4A. Variation of Diffusion Coefficients with Energy

$$\frac{[\sin \Lambda] + \epsilon[\cos \Lambda]}{[\sin \Lambda'] + \epsilon'[\cos \Lambda']} = \alpha \quad \text{with} \quad \alpha = \frac{S}{S'} \frac{1 - \beta_1 \Lambda}{1 - \beta'_1 \Lambda'}. \quad (8)$$

In this formula the variations of transients with radius do not appear any more.

4.3.1 Approximate Equation

By means of some simplifications explained in Appendix II, the critical equation (8) becomes

$$\frac{\delta R_e}{R_0} = -\frac{\Delta \beta}{\beta'} W(R)(1 - A), \quad (9)$$

where $A = \alpha[\sin \Lambda']/[\sin \Lambda]$ goes to zero as $1/R$; one thus finds again the limit the one-group formula.

We shall see in the numerical applications (Sect. 5) that, as opposed to the situation in thermal reactors, the correction term A does not decrease very fast as R grows.

We cannot then get $\Delta \beta$ from a curve of variation of $\delta R_e/W$ versus $1/R$, based on the use of the approximate equation. The complete equation (8) will therefore be used.

4.3.2 Complete Equation

From (8) one gets

$$\epsilon = -\tan \beta \delta R_e = \frac{\alpha[\sin \Lambda'] - [\sin \Lambda]}{[\cos \Lambda] - \alpha[\cos \Lambda']}.$$

As $R_e = R_0 + \delta R_e$, we obtain a relation between the variation δR_e of the critical radius and the substitution radius R :

$$\tan \beta \delta R_e = \frac{[\sin \Lambda] - \alpha[\sin \Lambda']}{[\cos \Lambda] - \alpha[\cos \Lambda']}. \quad (10)$$

In certain substitution cases (see Sect. 5) the curve of δR_e versus R is linear for a large range of the substitution radius. This led us to check whether the derivative of δR_e with respect to R does not show special proper-

FIG. 5. Variation of $\delta R/W$ versus $W(R)$ (2-group analytical expression).

FIG. 6. Substitution by Medium S_3 in bare reactor.

ties. The expression of this derivative is rather easy to obtain at $R = \pi/2\beta'$, where Λ becomes infinite (this point, when $\Delta\beta$ is small, is close to $\frac{1}{2}R_0$, half the critical radius of the reference medium).

The calculation of the derivative at the point $R = \pi/2\beta'$ is explained in Appendix II. One obtains with good approximation

$$\left. \frac{d}{dR} (\delta R_e) \right|_{\Lambda=\infty} = \frac{\beta^2 - \beta'^2}{\beta^2}, \quad (11)$$

and for β' close to β ,

$$\left. \frac{d}{dR} (\delta R_e) \right|_{\Lambda=\infty} = -2 \frac{\Delta\beta}{\beta} \quad \text{with} \quad \Delta\beta = \beta' - \beta. \quad (12)$$

The handling of the curve δR_e versus R will be done by means of (11) and (12).

5. Numerical Applications

Different substitutions have been considered for assemblies with reflector saving assumed to be constant,

in order to verify the analytical study. The reference medium is the one containing U-235; the substitution media are the three media containing plutonium (see Appendix I).

5.1. Substitution with Medium S_3

Calculations have been performed with the 16-group Hansen-Roach cross sections and with 2-group cross sections; the latter have been obtained for each medium by reduction of the 16-group cross sections weighted by the fundamental flux of the considered medium; the limit between the two groups has been chosen at 100 keV, which keeps the fission sources and the inelastic scattering in the fast group. This weighting does not change the material buckling, but does not yield necessarily the argument β_1 as the second eigenvalue of the transients, which has been determined by the method of Sect. 3. The two-group cross sections can easily be altered so that both the material buckling and the argument of the transient are not changed.

5.1.1 Simplified Equation

Figure 5 gives the curve of $\delta R_e/W(R)$ obtained from eq. (9) (simplified equation) and from eq. (10) (complete equation) with the two-group cross sections. It shows that the simplifications made to get eq. (9) are valid.

However, according to (9), $\delta R_e/W(R)$ should approach a limit if A decreases rapidly enough for increasing R [or $W(R)$]. This condition is not satisfied; A goes to zero but for values of the radius which are larger than the critical radius.

This method of interpretation based on the use of $\delta R_e/W(R)$, valid for thermal reactors, cannot be applied to fast reactors, because the transients do not decay rapidly enough. For the same reasons it is not possible to extrapolate the variation of A versus $1/R$ as it does not reach any asymptotic value either.

5.1.2 Complete Equation: δR_e versus R

Figure 6 gives the variation of δR_e versus R obtained in three ways:

- diffusion code with 16-group constants;
- diffusion code with 2-group constants;
- analytical expression with 2-group constants [eq. (10)].

The shift between the 2 curves obtained with the two-group constants shows the influence of the non-identity of the diffusion coefficients (assumed to be the same for the two media in the analytical expression). These curves are quite insensitive to even large variations of β_1 and β'_1 with β and β' kept constant; they are shifted when S/S' is varied, with β and β' kept constant.

The slopes of these curves for $35 \text{ cm} \leq R \leq 45 \text{ cm}$

are strictly identical and correspond exactly to the material-buckling difference between the reference medium and the substituted medium [see eq. (12)].

Spectral effects are responsible for the shift of the curve of δR_e obtained via 16-group or 2-group constants; these effects are also noticeable for small substitution radii. Since the plutonium medium is less reactive than the uranium medium, one would expect an increase of δR_e with R ; on the contrary, δR_e begins to decrease (16-group calculation). This occurs because the replacement of a small quantity of the reference medium by the plutonium medium increases the reactivity, as long as the spectrum is not altered (reference-medium spectrum even in the substituted zone). In the same way, in a complete plutonium assembly, the replacement of a small quantity of the plutonium medium by the uranium medium has a positive reactivity effect. This is a phenomenon analogous to resonance self-shielding, but occurring here for the macroscopic flux; it can be best shown when the substitution with medium S_1 is performed.

5.2. Substitution with Media S_1 and S_2

Medium S_1 has exactly the same material buckling as the reference medium.

Due to the spectral effects mentioned in the preceding paragraph, δR_e is negative, whereas in a one-group theory it should be identically equal to zero (see Fig. 7).

The interpretation of the curve δR_e is difficult because even if the slope is effectively equal to zero at one point, that slope changes quickly around that point, which is not *a priori* known.

Medium S_2 gives results intermediate between those of S_1 and S_3 (see Fig. 7). The spectral effect still disturbs the δR_e curve; the value of the slope at the inflexion point around 45 cm would lead to $|\Delta\beta^2/\beta^2| \simeq 2.5\%$, a result which is three times the correct value. In order to get a correct evaluation of $|\Delta\beta^2/\beta^2|$, it is necessary

FIG. 8. Substitution by Medium S_3 for Reflected Reactor (16-group diffusion code).

that the influence of the material-buckling difference be larger than the spectral effects, as it was the case for the S_3 medium.

6. Some Results for Reflected Reactors

For thermal reactors the reflector influence can be treated as a small correction to the constant reflector saving calculation.

For fast reactors some 16-group calculations have been performed to see if it is also true.

Figure 8 gives the curve of δR_e versus R when medium S_3 is substituted in the reference medium; δR_e is the variation of the outer radius of the reference medium, the thickness of the blanket being kept unaltered (blanket composition—see Appendix I). This curve differs in two ways from the one relative to the bare case:

- no linear part;
- a much larger slope for $R > 20$ cm.

Up to now no interpretation of such a curve has been available. However, some other calculations with much larger buckling differences are more encouraging. For a uranium-fueled version of the PHENIX reactor,⁽⁹⁾ the substitution of the low-enrichment core 1 into the high-enrichment core 2, with criticality maintained by variation of the outer radius of core 2, yields the result drawn in Fig. 9. This curve has again a linear part, the slope of which is equal to 0.74, while the relative buckling difference between the two media is 0.724, a result in complete agreement with the two-group analytical study. The reference medium used in the previous sections is in fact the high-enrichment core of the latter example; that the substitution by a uranium medium with a quite different material buckling yields better results than the substitution by a plutonium medium (like S_3) might be due to:

- the large buckling difference;
- the spectrum similarity for the 2 uranium media.

Analytical study of these reflected-reactor problems

FIG. 7. Variation of δR_e versus Substitution Radius R for Bare Reactor (16-group diffusion code).

FIG. 9. Substitution of PHENIX Core 1 into Core 2.

must still be done, probably with the help of more elaborate methods⁽¹⁰⁾ than those used up to now.

7. Relationship between the Critical Mass and the Material Buckling

7.1. One-group Theory

In cylindrical geometry for a one-region reactor, the criticality equation can be written as

$$\beta^2 = \left(\frac{2.405}{R_c + S} \right)^2 + \left(\frac{\pi}{H_e} \right)^2, \quad (13)$$

with β^2 the material buckling, R_c the critical radius, S the reflector saving, H_e extrapolated height, and M the critical mass (proportional to R_c^2). If criticality is reached by adjustment of the reactor radius, differentiation of (13) with respect to R_c gives a relationship between $\Delta\beta^2$ and ΔR_c (or ΔM):

$$\Delta\beta^2 = - \left(\frac{2.405}{R_c + S} \right)^2 \frac{R_c}{R_c + S} \frac{\Delta M}{M}. \quad (14)$$

If $\Delta\beta^2$ represents the difference between the experimental and the calculated values of the material buckling, the accuracy for the critical mass can be enhanced by deducing ΔM from (14); $M + \Delta M$ is the corrected critical mass.

This procedure can be extended to multizone reactors. For a two-zone reactor, for example, (14) will be replaced by

$$\Delta M = a\Delta\beta^2 + b\Delta\beta'^2, \quad (15)$$

where $\Delta\beta^2$ and $\Delta\beta'^2$ are the differences between the measured and the calculated values of the material buckling of each zone; a and b are coefficients depending on the neutronic characteristics and on the geometric dimensions. For complicated geometries, the corrected critical

mass could be obtained by using one-group diffusion codes with the corrected material bucklings.

7.2. Two-group Theory

Upon differentiation of the critical equation given in Sect. 4, the variation of the critical mass will be expressed as a function of the variations of the different parameters:

$$\Delta M = a_1\Delta\beta^2 + a_2\Delta\beta'^2 + a_3\Delta\beta_1^2 + a_4\Delta\beta_1'^2 + a_5\Delta(S/S').$$

To get orders of magnitude, relative variations have been imposed on each parameter, the others being kept constant; there is a corresponding variation $\delta R_e/R_e$, and the "importance" of a parameter X can be characterized by $(\delta R_e/R_e)/(\delta X/X)$. Values of this latter quantity for the different two-group parameters are (two-zone loading of Sect. 5.1, with $R = R_0/2$):

X	β^2	β'^2	β_1^2	$\beta_1'^2$	S/S'
$\frac{\delta R_e}{R_e} / \frac{\delta X}{X}$	-24.71	-34.95	0.02842	0.03572	-0.06355

These results show that the critical radius, and therefore the critical mass, is very sensitive to buckling variations and is effectively independent of the transient eigenvalues. The results of relative variations have also been included, but one should not forget that relative variations do not make sense for S/S' ; absolute variations should be used instead as this parameter can assume positive or negative values.

From these results one could conclude that for critical-mass determination accurate knowledge of β^2 , β'^2 , and probably S/S' is essential, and that β_1^2 and $\beta_1'^2$ need not be known with high accuracy.

The same reasoning could be applied to the multi-group formalism, but in that case the large number of parameters makes the method cumbersome.

8. Conclusions

The application of the progressive substitution method to fast multizone assemblies is not straightforward because the transients propagate to large distances from the interfaces; in contrast to thermal reactors, no asymptotic behavior with distance can be relied upon in fast reactors.

However, with the constant-reflector-saving assumption a simplified model with a limited number of parameters can be used; these parameters are related to the fundamental-mode spatial variation and to an overall spatial variation of transient modes in the different media. From the measurement of the critical radius, one can deduce with good accuracy the relative differ-

ence $\Delta\beta^2/\beta^2$ between the material bucklings of the reference and the substituted media. This is true as soon as the spectral effect is negligible with respect to the effect of the difference in material bucklings, i.e., as soon as $\Delta\beta^2/\beta^2$ is larger than a few percent. The measurements should be performed on a zone extending up to a radius a little larger than half the extrapolated critical radius.

The interpretation of the method applied to reflected reactors is not done yet and further work will be pursued on this subject.

APPENDIX I

Composition of the Different Media

The atomic densities are given in atom numbers per barn-cm.

Elements	Reference Medium R	Substitution Medium S_1	Substitution Medium S_2	Substitution Medium S_3	Blanket
U-235	0.00251688	—	—	—	—
U-238	0.00674415	0.0067441	0.0067441	0.0067441	0.0108114
Pu-239	—	0.00157536	0.00156706	0.00152653	—
Pu-240	—	0.0004193	0.0004171	0.0004063	—
Pu-241	—	0.00005704	0.00005674	0.00005527	—
Fe	0.0127146	0.0127146	0.0127146	0.0127146	0.007651
Cr	0.0036410	0.0036410	0.0036410	0.0036410	0.002191
Ni	0.0021501	0.0021501	0.0021501	0.0021501	0.001294
Mo	0.0002741	0.0002741	0.0002741	0.0002741	0.000165
O	0.0175719	0.0175719	0.0175719	0.01746635	0.021780
Na	0.0066194	0.0066194	0.0066194	0.0066194	0.007746
Material buckling, cm^{-2}	0.00252095	0.00252095	0.0024992	0.0023933	—
Relative differences for the material buckling $(S_i - R)/R$, %	—	0.00	-0.87	-5.33	—

APPENDIX II

1. Simplified Equation

In the denominator of eq. (8), $\epsilon[\cos \Lambda']$ is small with respect to $[\sin \Lambda']$, at least for small R . Replacement of ϵ in eq. (8) by its approximate expression $-[\sin \Lambda]/[\cos \Lambda]$ yields

$$\epsilon \simeq -\frac{[\sin \Lambda]}{[\cos \Lambda]} \left[1 - \alpha \left(\frac{[\sin \Lambda']}{[\sin \Lambda]} - \frac{[\cos \Lambda']}{[\cos \Lambda]} \right) \right]$$

or, as the ratio $[\cos \Lambda']/[\cos \Lambda]$ is negligible compared to $[\sin \Lambda']/[\sin \Lambda]$,

$$\epsilon \simeq -\frac{[\sin \Lambda]}{[\cos \Lambda]} \left(1 - \alpha \frac{[\sin \Lambda']}{[\sin \Lambda]} \right).$$

To the first order in $\delta\beta$,

$$\frac{[\sin \Lambda]}{[\cos \Lambda]} = -\frac{\delta\beta}{\beta} (\beta R - \frac{1}{2} \sin 2\beta R),$$

and the preceding equation becomes

$$\frac{\delta R_e}{R_0} = -\frac{\Delta\beta}{\beta} W(R)(1 - A),$$

where

$$A = \alpha \frac{[\sin \Lambda']}{[\sin \Lambda]}.$$

2. Expression of the Derivative of ϵ at $R = \pi/2\beta'$

ϵ is given by

$$\epsilon = \frac{-[\sin \Lambda] + \alpha[\sin \Lambda']}{[\cos \Lambda] - \alpha[\cos \Lambda']},$$

where

$$\alpha = \frac{S}{S'} \frac{1 - \beta_1 \Lambda}{1 - \beta_1 \Lambda'};$$

$$\Lambda = -\frac{1}{\beta'} \tan \beta' R;$$

$$\Lambda' = -\frac{1}{\beta_1} \tanh \beta_1' R.$$

By differentiation with respect to R , one gets

$$\frac{d\epsilon}{dR} = \frac{N}{D},$$

where

$$N = \beta[-(1 + \beta^2 \Lambda^2) + 2\alpha(1 + \beta^2 \Lambda \Lambda') - \alpha^2(1 + \beta^2 \Lambda'^2) + \{1 - (S/S')\}(1 + \Lambda^2 \beta'^2)];$$

$$D = [(1 - \alpha) \cos \beta R + \beta(\alpha \Lambda' - \Lambda) \sin \beta R]^2.$$

When Λ goes to infinity, the main terms in these equa-

tions are the ones in Λ^2 ; hence, the approximate equations are

$$N \simeq \beta \Lambda^2 [\beta'^2 - \beta^2 + (2\gamma - \gamma^2)\beta^2 - (S/S')\beta'^2 - \gamma^2\beta'^2];$$

$$D \simeq \Lambda^2 [\beta \sin \beta R (1 - \gamma) - \gamma \beta_1' \cos \beta R]^2,$$

where

$$\gamma = -\frac{S}{S'} \frac{\beta_1 \Lambda'}{1 - \beta' \Lambda'} \simeq \frac{S}{S'} \frac{\beta_1}{\beta_1 + \beta_1'}.$$

The numerical example relative to the substitution with medium S_3 shows that, in N , the term $\beta'^2 - \beta^2$ is much larger than the others.

For this case, indeed, $\gamma = (2.7)10^{-3}(S/S') = (4.9)10^{-3}$; hence $\beta'^2 - \beta^2 = (-1.28)10^{-4}$, while the sum of the remaining terms is only $(+1.7)10^{-6}$. Therefore,

$$N \sim \beta \Lambda^2 (\beta'^2 - \beta^2)$$

and

$$D \sim \Lambda^2 \beta^2 \sin^2 \beta R \text{ (around } R = \pi/2\beta').$$

As $\epsilon = -\tan \beta \delta R_e$, one obtains finally

$$\left. \frac{d(\delta R_e)}{dR} \right|_{\Lambda \rightarrow \infty} = \frac{\beta^2 - \beta'^2}{\beta^2}.$$

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The Application of Space-Energy Synthesis to the Interpretation of Fast Multizone Critical Experiments*

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ABSTRACT

Information gained from multizoned critical experiments is rather difficult to interpret and to extrapolate with a multigroup theoretical model.

A much simpler model involving only few parameters has been tried out which would still give a good representation of the physical processes. This simple model is based on a two-group formalism, in which the groups overlap, derived from the diffusion equation by a variational procedure.

Comparison between conventional multigroup and two-overlapping-group formalisms gives excellent agreement. It is hoped that with the help of this method the interpretation of multizoned critical experiments might be greatly eased. It is also suggested that synthesis methods in general be used to solve some special problems in fast reactor physics.

1. Introduction

Because of lack of plutonium in sufficient quantity it is not yet possible to perform full-size critical mock-ups of fast power reactors in the 200- to 1000-MWe range. One is thus led to perform experiments requiring less plutonium, such as multizone criticals and exponential assemblies. Methods have to be developed to interpret these experiments and to extrapolate their results to obtain the desired characteristics of the power reactors. If these methods prove successful, it is possible that multizone criticals will continue to be performed on a routine basis, even when enough plutonium is available, in order to reduce the inventory of simulation elements, the loading time, and the hazards of handling large quantities of plutonium. When designing a critical facility, its size may be considerably reduced if one has confidence in the success of multizone experiments.

This point was made at the 1961 Vienna Conference by Okrent⁽¹⁾ and has been troubling the designers of critical facilities for many years. Even though

considerable work has been done on the interpretation of multizone experiments, we believe that a more vigorous effort is needed, possibly along new lines, before a satisfactory confidence level is reached. This is what we plan to discuss in this paper.

What is the present status? Thanks primarily to the excellent work which has been done at Atomic International and at Argonne National Laboratory, we think it is well-established that multizone experiments have proven adequate for the determination of asymptotic properties of a medium: the asymptotic spectrum and associated spectral indices, relative reaction rates and danger coefficients of various materials, other relative reactivity coefficients, such as the Doppler coefficient and the sodium-void coefficient, and the asymptotic internal conversion ratio. This is certainly an asset but it is by no means sufficient. For an optimal design of a power reactor much more information is needed: critical mass or enrichment for a given size, power distribution throughout core and blanket, absolute reactivity coefficients and their spatial distribution, distributed conversion ratio, effective lifetime, and delayed-neutron fractions, etc. Thus somehow this kind of information has to be extrapolated from multizone criticals to large power reactors. The task is very difficult mainly because fast assemblies are calculated by multigroup theory, which is an excellent tool for the production of accurate numerical results, but which is a very poor tool for extrapolation of integral data.

Furthermore, it is difficult to single out from integral experiments the cross sections which are in error, because compensating errors have to be untangled. Interesting work along these lines has been done by Meneghetti and Ishikawa,⁽²⁾ Edwards,⁽³⁾ Helm,⁽⁴⁾ Hemment and Pendlebury,⁽⁵⁾ Cecchini *et al.*,^(6,7) and Baker.⁽⁸⁾ Nevertheless we believe that more appropriate tools should be developed.

We quote here two paragraphs from Spinrad's introductory lecture at the 1961 Vienna Conference.⁽⁹⁾

"The universal method used for reactor statics and reactor design is multigroup theory, generally

* Work performed under the Euratom-C.E.A. Association for fast reactors

with many groups and as detailed a set of group-transfer matrices as the cross sections will justify and the computing device will admit. This procedure is proper and useful, but the use of complex computations for reactor surveys has a good many drawbacks. One of them is cost. Another is that *we cannot make direct use of integral data obtained from critical and exponential experiments, threshold detector responses, or reactor substitutions*. Such data can only be used as check points on our cross-section set. Perhaps the most important objection to the technique is that it tends to put a barrier between the design engineer and the reactor physicist who must collaborate on the reactor design. It is difficult for the physicist to explain his ideas when his only tool of communication is a multigroup calculation.

"Therefore, although it would not improve the precision of fast- and intermediate-reactor theory, I think we need a heuristic description of the neutronic process in these reactors. The analogue in thermal reactor theory is the sort of description permitted by the four-factor formula, age-diffusion theory, and the perturbed Maxwellian spectrum. The practice of collapsing many-group into few-group calculations is not the proper answer, because it is primarily a computational technique which does not contribute to the reasoning process."

He goes on to suggest a simplified formalism in which the spectrum of any fast assembly could be synthesized through a linear combination of elementary spectra. His ambition is not to obtain a precision comparable to that of multigroup methods, but to obtain a simple, qualitative description, provided only that it applies to a usefully large set of cases. This is certainly a worthwhile goal and some work has been done along those lines. Blasser and Diana⁽¹⁰⁾ proposed three years ago a simple representation of any fast reactor spectrum involving only five parameters. Driscoll and Kaplan⁽¹¹⁾ recently derived a simple spectral shape function containing only two parameters. Also entering in this category is recent work of Cadilhac and Pujol⁽¹²⁾ in which slowing down is described not by a 2-dimensional array of multigroup constants, but by a 1-dimensional array. It is our opinion that such work should be pursued to a point where the objective set forth by Spinrad five years ago would be reached. It would be highly desirable that these methods yield a good description, not only of the asymptotic properties of a medium, but of the neutron behavior near interfaces as well.

The purpose of this paper is to seek a simple formalism involving as few parameters as possible which could describe various assemblies that are made up of a limited number of *given* media but in various geometrical arrangements. Let us be more precise:

Assume that we want to determine important char-

acteristics of a plutonium-fueled power reactor with a core comprising two concentric zones of different enrichments in order to achieve some power flattening. Such a reactor consists essentially of three media: the two core zones and the blanket. Thus one would perform experiments involving these media plus possibly a driver medium. Such an experiment might be a three-zoned critical: the same blanket as that of the power reactor, a uranium-fueled driver zone, and a central experimental zone which would be one or the other core zone of the power reactor. Another experiment might be a multizoned critical or exponential experiment in which the interface between the two core zones or between the outer core zone and the blanket would be represented. It is clear that the experimental determination of important characteristics of the power reactor such as critical mass and power distribution requires that some experiments be performed where such interfaces are mocked up, possibly with a different geometry.

In other words, it is not sufficient to determine asymptotic properties of the media involved, but also those of the interfaces which couple two media.

Thus one will be led to perform a series of experiments involving four media: the three power reactor media and the experimental driver. A progressive substitution experiment is a useful sequence in such a series of experiments, since by varying the ratio of interface surface to volume of the experimental zone, one can to some extent untangle the effects on reactivity of the asymptotic properties of the experimental medium and of the interface. Also, the effects on reactivity of uncertainties in the characteristics of, respectively, the reference and the substitution media can to some extent be untangled by this method.

Another paper presented at This Conference⁽¹³⁾ aims primarily at the determination of the material buckling, an important asymptotic property, of the experimental medium; the interface effect is eliminated.

Our aim here is to obtain a formalism involving few parameters which would describe adequately all the experiments performed with the above-mentioned four media, and the power reactor as well. It is less ambitious than that of Spinrad because we limit ourselves to given media; for other media the formalism would be the same but the parameters would be different. The formalism presented here is primarily a computational—or rather an extrapolation—technique, and it does not, unfortunately, contribute to the reasoning process. It is based on few-group calculations, with overlapping rather than conventional groups.

2. Specifications for the Desired Formalism

As an example, consider a very simple case: a bare, two-zone assembly with a central plutonium-fueled

experimental zone surrounded by a uranium-fueled driver with a not-too-different material buckling. The size of the central zone can be varied upward from zero. A simple formalism, involving few parameters, is desired to describe this "progressive substitution" experiment, i.e., that would give the critical extrapolated radius R_c as a function of the substitution radius R , the space and energy distribution of flux and adjoint flux, the distributed danger coefficients, etc.

Assume for the purpose of this discussion, that a 16-group diffusion-theory model is perfectly accurate. The flux in a region, say in the central region, would be given as the sum of 16 separable solutions. The first would satisfy the equation

$$\nabla^2 \bar{\varphi}^{(1)} + B^2 \bar{\varphi}^{(1)} = 0,$$

where B^2 is the material buckling or first eigenvalue of the multigroup matrix and $\bar{\varphi}^{(1)}$, the asymptotic spectrum, is the associated eigenvector. The 15 other terms satisfy the equations

$$\nabla^2 \bar{\varphi}^{(i)} = \mu^{2(i)} \bar{\varphi}^{(i)},$$

where $-\mu^{2(i)}$ is one of the 15 other eigenvalues and $\bar{\varphi}^{(i)}$ is again the associated eigenvector. Here $\mu^{(i)}$ is the inverse of the relaxation length for the i -th harmonic. Each of the 16 modes will be excited to some amplitude. The amplitude of each mode will be dictated primarily by the nature of the other medium across the interface. Clearly one group is not sufficient to approximate the flux behavior. Thus we try with two groups. In other words, we try to express the flux as the sum of only two separable solutions. The first would describe the asymptotic flux and the second would lump all the harmonics together, i.e., would contain a spectrum $\bar{\varphi}_H$ and a relaxation length $1/\mu_H$ which would be representative of the overall harmonics behavior. A conventional two-group formalism was successfully used to analyse progressive substitution experiments for thermal-neutron systems for which the moderator was the same in both zones but for which the fuel lattices differed.⁽¹⁴⁻¹⁷⁾

It was decided to try out the same formalism for the analysis of similar experiments with fast-neutron assemblies.⁽¹³⁾ Some interesting conclusions were reached, but the main objective was not attained; the conventional two-group formalism did not provide a good description of this experiment, as can be seen from Fig. 6 of Ref. 13, which is reproduced as Fig. 1 of the present paper.

This is not surprising since conventional two-group theory gives for μ_H^2 and φ_H , the respective eigenvalue and the eigenvector of the single harmonic, values which depend only on the properties of the medium under consideration while, as was said above, the transient behavior of the flux near an interface is very much affected by the medium across the interface. Further-

FIG. 1. Variation of δR_c versus Substitution Radius R for Bare Reactor.

more, in conventional two-group theory, the choice of the energy separating the two groups has an immediate, and artificial, consequence: the spectrum of the harmonic will change sign at that energy. (For thermal-neutron systems the conventional groups—a fast and a thermal group—are more “natural” since the substitution of a different lattice in a moderator leads essentially, except for the effect of resonance capture, to a disruption of the balance of the thermal and fast fluxes).

What should then be done for fast-neutron assemblies? Clearly one should find a simple formalism which would give a reasonably good description, not only of the asymptotic solution in each medium, but also of the overall harmonics behavior. If the asymptotic spectra in media 1 and 2 are respectively $\bar{\varphi}_1$ and $\bar{\varphi}_2$, physical intuition tells us that the harmonics excited by the interface in the two media will provide for a more or less smooth transition between $\bar{\varphi}_1$ and $\bar{\varphi}_2$. In the improved two-group formalism that we are looking for, i.e., with one harmonic approximating the overall harmonics behavior, it is reasonable to expect that the spectrum of the single harmonic would be a linear combination of $\bar{\varphi}_1$ and $\bar{\varphi}_2$, namely, $\bar{\varphi}_2 - Q_1 \bar{\varphi}_1$ in medium 1, and $\bar{\varphi}_1 - Q_2 \bar{\varphi}_2$ in medium 2.

3. Synthesis Method: Overlapping Groups

The preceding discussion, based essentially on physical reasoning, clearly hints that a synthesis method, and more precisely a particular kind of space-energy synthesis known as “overlapping group method” is what is needed here.

Synthesis methods have been widely used in reactor physics for the reduction of the number of dimensions in multidimensional problems (3 space dimensions, energy, time), as, for instance, given by Selengut⁽¹⁸⁾ and

Kaplan.^(19, 20) The method of overlapping groups is a synthesis method in which the energy variable is integrated out. Calame and Federighi⁽²¹⁾ have applied it to an inhomogeneous thermal-neutron problem (with source), while we will apply it to a homogeneous fast-neutron problem.

The overlapping-group formalism will be presented here in a very straightforward manner, as it applies to the very simple problem (bare two-zone assembly) which is treated as an example. The (supposedly exact) continuous energy formalism leads to an equation

$$D_i(E)\nabla\varphi(E, \bar{r}) - \Sigma_{t,i}(E)\varphi(E, \bar{r}) + \int \sigma_i(E' \rightarrow E)\varphi(E', \bar{r}) dE' = 0, \quad (1)$$

where $i = 1$ or 2 refers to zone i . The third term represents the slowing-down source and the fission source as well. The boundary conditions are that at the interface $\varphi(E, \bar{r})$ and $D(E)\nabla\varphi(E, \bar{r})$ are continuous, and that $\varphi(E, \bar{r}) = 0$ at the extrapolated boundary.

A solution is tried of the form

$$\varphi(E, \bar{r}) = g_1(\bar{r})\varphi_1(E) + g_2(\bar{r})\varphi_2(E), \quad (2)$$

where $\varphi_i(E)$ is the asymptotic spectrum for zone i , or in other words,

$$-D_i(E)B_i^2\varphi_i(E) - \Sigma_{t,i}(E)\varphi_i(E) + \int \sigma_i(E' \rightarrow E)\varphi_i(E') dE' = 0, \quad (3)$$

where B_i^2 is the material buckling of zone i . Similarly, the adjoint flux will be approximated by

$$\varphi^*(E, \bar{r}) = g_1^*(\bar{r})\varphi_1^*(E) + g_2^*(\bar{r})\varphi_2^*(E), \quad (4)$$

where φ_i^* is the asymptotic adjoint spectrum for zone i :

$$-D_i(E)B_i^2\varphi_i^*(E) - \Sigma_{t,i}(E)\varphi_i^*(E) + \int \sigma_i(E \rightarrow E')\varphi_i^*(E') dE' = 0. \quad (5)$$

If the exact solution $\varphi(E, \bar{r})$ is inserted in (1), one obtains an identity in E that can be written symbolically as

$$\mathfrak{F}_i(E, \bar{r}) \equiv 0. \quad (6)$$

For an arbitrary weighting function $W(E)$ one thus has also

$$\int W(E)\mathfrak{F}_i(E, \bar{r}) dE = 0. \quad (7)$$

If a trial solution (2) is substituted in (7), the energy variable is integrated out, and one obtains one equation in $g_1(\bar{r})$ and $g_2(\bar{r})$. The boundary conditions become

- (i) the fluxes $g_1(\bar{r})$ and $g_2(\bar{r})$ must both be continuous across the interface;
- (ii) continuity of the current: the continuity of $D(E)\nabla\varphi(E, \bar{r})$ at the interface for any E , now becomes the continuity of

$$\int W(E)D(E)\nabla\varphi(E, \bar{r}) dE \quad \text{across the interface.}$$

If two different (but otherwise arbitrary) weighting functions $W_i(E)$ are chosen, two equations are thus obtained that allow, with the corresponding boundary conditions, the determination of $g_1(\bar{r})$ and of $g_2(\bar{r})$. These are the overlapping-group equations.

If the *exact* solution could be written as (2), it would be immaterial which weighting functions were chosen. Many suggestions can be found in the literature⁽²⁰⁾ for the choice of weighting functions. One can use, for instance, the trial functions themselves (Galerkin's method), δ -functions, nonoverlapping step functions, or the asymptotic adjoint functions φ_1^* and φ_2^* (variational method). There are good variational arguments^(18, 21) and practical reasons (see below) for choosing the variational method.

The set of two overlapping-group equations then becomes, for zone i ,

$$\begin{cases} D_{(11)i}\nabla g_1(\bar{r}) + D_{(12)i}\nabla g_2(\bar{r}) + A_{(11)i}g_1(\bar{r}) \\ + A_{(12)i}g_2(\bar{r}) = 0; \end{cases} \quad (8)$$

$$\begin{cases} D_{(21)i}\nabla g_1(\bar{r}) + D_{(22)i}\nabla g_2(\bar{r}) + A_{(21)i}g_1(\bar{r}) \\ + A_{(22)i}g_2(\bar{r}) = 0, \end{cases} \quad (9)$$

with boundary conditions at the interface $\bar{r} = \bar{r}_I$:

$$g_1(\bar{r}_{I-}) = g_1(\bar{r}_{I+}); \quad (10)$$

$$g_2(\bar{r}_{I-}) = g_2(\bar{r}_{I+}); \quad (11)$$

$$\begin{aligned} D_{(11)1}\nabla g_1(\bar{r}) + D_{(12)1}\nabla g_2(\bar{r}) \big|_{\bar{r}=\bar{r}_I-} \\ = D_{(11)2}\nabla g_1(\bar{r}) + D_{(12)2}\nabla g_2(\bar{r}) \big|_{\bar{r}=\bar{r}_I+}; \end{aligned} \quad (12)$$

$$\begin{aligned} D_{(21)1}\nabla g_1(\bar{r}) + D_{(22)1}\nabla g_2(\bar{r}) \big|_{\bar{r}=\bar{r}_I-} \\ = D_{(21)2}\nabla g_1(\bar{r}) + D_{(22)2}\nabla g_2(\bar{r}) \big|_{\bar{r}=\bar{r}_I+}, \end{aligned} \quad (13)$$

with

$$D_{(k,l)i} = \int \phi_k^*(E)D_i(E)\phi_l(E) dE; \quad (14)$$

$$\begin{aligned} A_{(k,l)i} = \int dE \left\{ \phi_k^*(E) \left[-\sigma_{t,i}(E)\phi_l(E) \right. \right. \\ \left. \left. + \int \sigma_i(E' \rightarrow E)\phi_l(E') dE' \right] \right\}. \end{aligned} \quad (15)$$

There are thus, for two groups, four "effective" diffusion coefficients $D_{(k,l)i}$, and four "effective" cross sections $A_{(k,l)i}$ per zone.

In practice, these "effective" coefficients are to be determined from the best multigroup cross-section set available, and from the calculated asymptotic spectra ϕ_k^* and ϕ_l . As will be seen later, some of these "effective" coefficients can be experimentally determined as well.

These "overlapping" group equations are only slightly more complicated than conventional two-group equations. They can be solved analytically for simple geometries, and numerically for more complicated geometries. Thus one can obtain the criticality condition and the flux distribution in space and energy.

A similar procedure can be used for the adjoint flux, approximated by (4). In this case it is natural to use $\phi_1(E)$ and $\phi_2(E)$ as weighting functions; the adjoint equations are then the transposes of the direct equations, and the same effective coefficients may be used.

Equations for $g_1^*(\bar{r})$ and $g_2^*(\bar{r})$ are

$$D_{(11)1}\nabla g_1^* + D_{(21)1}\nabla g_2^* + A_{(11)1}g_1^* + A_{(21)1}g_2^* = 0; \quad (16)$$

$$D_{(12)1}\nabla g_1^* + D_{(22)1}\nabla g_2^* + A_{(12)1}g_1^* + A_{(22)1}g_2^* = 0, \quad (17)$$

with boundary conditions at the interface $\bar{r} = \bar{r}_I$:

$$g_1^*(\bar{r}_{I-}) = g_1^*(\bar{r}_{I+}); \quad (18)$$

$$g_2^*(\bar{r}_{I-}) = g_2^*(\bar{r}_{I+}); \quad (19)$$

and

$$\begin{aligned} D_{(11)1}\nabla g_1^*(\bar{r}) + D_{(21)1}\nabla g_2^*(\bar{r}) \big|_{\bar{r}=\bar{r}_I-} \\ = D_{(11)2}\nabla g_1^*(\bar{r}) + D_{(21)2}\nabla g_2^*(\bar{r}) \big|_{\bar{r}=\bar{r}_I+}; \end{aligned} \quad (20)$$

$$\begin{aligned} D_{(12)1}\nabla g_1^*(\bar{r}) + D_{(22)1}\nabla g_2^*(\bar{r}) \big|_{\bar{r}=\bar{r}_I-} \\ = D_{(12)2}\nabla g_1^*(\bar{r}) + D_{(22)2}\nabla g_2^*(\bar{r}) \big|_{\bar{r}=\bar{r}_I+}. \end{aligned} \quad (21)$$

3.1. Eigenvalues and Eigenfunctions

In matrix notation (8) and (9) can be written for medium 1 as

$$\begin{vmatrix} D_{(11)1}\nabla + A_{(11)1} & D_{(12)1}\nabla + A_{(12)1} \\ D_{(21)1}\nabla + A_{(21)1} & D_{(22)1}\nabla + A_{(22)1} \end{vmatrix} \begin{vmatrix} g_1(\bar{r}) \\ g_2(\bar{r}) \end{vmatrix} = 0. \quad (22)$$

The buckling eigenvalues B^2 are solutions of

$$\begin{vmatrix} A_{(11)1} - D_{(11)1}B^2 & A_{(12)1} - D_{(12)1}B^2 \\ A_{(21)1} - D_{(21)1}B^2 & A_{(22)1} - D_{(22)1}B^2 \end{vmatrix} = 0. \quad (23)$$

However, as indicated by (3) with $i = 1$, $\phi_1(E)$ is the asymptotic spectrum of medium 1, with material buckling B_1^2 . Multiplying (3), with $i = 1$, successively by ϕ_1^* and ϕ_2^* , and integrating over energy, one obtains

$$-D_{(11)1}B_1^2 + A_{(11)1} = 0;$$

$$-D_{(21)1}B_1^2 + A_{(21)1} = 0.$$

Similarly, from (5) one obtains, by multiplying successively by $\phi_1(E)$ and $\phi_2(E)$, and integrating

$$-D_{(11)1}B_1^2 + A_{(11)1} = 0;$$

$$-D_{(12)1}B_1^2 + A_{(12)1} = 0.$$

Thus

$$\frac{A_{(11)1}}{D_{(11)1}} = \frac{A_{(21)1}}{D_{(21)1}} = \frac{A_{(12)1}}{D_{(12)1}} = B_1^2, \quad (24)$$

and (23) can be written as

$$\begin{vmatrix} A_{(11)1} \left(1 - \frac{B^2}{B_1^2}\right) & A_{(12)1} \left(1 - \frac{B^2}{B_1^2}\right) \\ A_{(21)1} \left(1 - \frac{B^2}{B_1^2}\right) & A_{(22)1} - D_{(22)1}B^2 \end{vmatrix} = 0 \quad (25)$$

or

$$\begin{aligned} \left(1 - \frac{B^2}{B_1^2}\right) \left\{ B^2 \left[\frac{A_{(21)1}A_{(12)1}}{B_1^2} - A_{(11)1}D_{(22)1} \right] \right. \\ \left. + A_{(11)1}A_{(22)1} - A_{(12)1}A_{(21)1} \right\} = 0. \end{aligned} \quad (26)$$

Thus the two roots are

$$B^2 = B_1^2 \text{ as could be expected (fundamental mode)}$$

and

$$B^2 = - \frac{A_{(11)1}A_{(22)1} - A_{(12)1}A_{(21)1}}{A_{(21)1}A_{(12)1}B_1^2 - A_{(11)1}D_{(22)1}} = -\mu_1^2 \quad (27)$$

(harmonic);

μ_1^2 can be expected to be positive, since it corresponds to a harmonic, which should decrease away from the interface, with a relaxation length μ_1^{-1} . The fundamental eigenvector is (1, 0), confirming that the fundamental spectrum is given by $\phi_1(E)$.

The other eigenvector is $(-Q_1, 1)$ where $Q_1 = A_{(12)1}/A_{(11)1}$, thus leading to an eigenspectrum

$$\phi_2(E) = \frac{A_{(12)1}}{A_{(11)1}} \phi_1(E). \quad (28)$$

Similar expressions hold in zone 2.

3.2. Number of Parameters

It is interesting to note how many parameters are necessary to represent the two-medium problem: B_1^2 , B_2^2 , μ_1^2 , μ_2^2 , the eight coefficients $A_{(kl)i}$, and the eight coefficients $D_{(kl)i}$. However, the parameters D are linked with the others through either (24) or (27). Thus they can be dropped. Further, the eight parameters $A_{(kl)i}$ can be reduced to 5, i.e.,

TABLE 1. PARAMETERS IN TWO-MEDIUM PROBLEM

Depending only on Medium 1 (asymptotic parameter)	Depending on both Media (coupling or interface parameter)	Depending only on Medium 2 (asymptotic parameter)
B_1	$\mu_1, \mu_2, \lambda_{11}, \lambda_{22}, \lambda_{12}, \lambda_{21}, \theta$	B_2

TABLE 2. COMPOSITION (NUMBER OF NUCLEI PER $\text{cm}^3 \times 10^{-24}$)

	Medium 1	Medium 2
Fe	0.012 715	0.012 715
Cr	0.003 641	0.003 641
Ni	0.002 150	0.002 150
Mo	0.000 274	0.000 274
O	0.017 466	0.017 572
Na	0.006 619	0.006 619
U ²³⁸	0.006 744	0.006 744
U ²³⁵	—	0.002 517
Pu ²³⁹	0.001 526	—
Pu ²⁴⁰	0.000 406	—
Pu ²⁴¹	0.000 055	—

$$\lambda_{kl} = \frac{A_{(kl)1}}{A_{(kl)2}}; \quad k = 1, 2; \quad l = 1, 2,$$

and

$$\theta = \frac{A_{(11)1}A_{(22)1} - A_{(12)1}A_{(21)1}}{A_{(11)2}A_{(22)2} - A_{(12)2}A_{(21)2}},$$

because the eight parameters $A_{(kl)l}$, are to some extent dependent on the arbitrary normalization of the 4 trial and weight functions, so that only their combinations λ_{kl} and θ appear in the criticality equation. Thus there remain 9 parameters, which are shown in Table 1.

The possibility of deriving these parameters directly from the experiments will be discussed in Sect. 5. It is remarkable that the ratios $A_{(11)1}/A_{(11)2}$ and $A_{(22)1}/A_{(22)2}$ can be measured directly. As can be seen from expression (15), $A_{(11)1}/A_{(11)2} = \lambda_{11}$ is the ratio of the central (and asymptotic) danger coefficients in medium 1 of unit volumes having respectively the composition of medium 1 and of medium 2. Similarly, $A_{(22)1}/A_{(22)2} = \lambda_{22}$ is the ratio of similar danger coefficients in medium 2.

4. Numerical Applications

We have considered the progressive substitution in a U-235-fueled reference bare core of a plutonium-fueled substitution medium (medium S_3 of Ref. 13).

The composition of both media is given in Table 2. Note that the material buckling (obtained with the 16-group Hansen and Roach cross sections) of the substitution medium 1 is about 5% smaller (less reactive) than that of the reference medium 2.

The following parameters have been calculated:

medium 1	medium 2
$B_1 = 0.048921 \text{ cm}^{-1}$	$B_2 = 0.050209 \text{ cm}^{-1}$
$\mu_1 = 0.124552 \text{ cm}^{-1}$	$\mu_2 = 0.163509 \text{ cm}^{-1}$
$A_{(11)1} = 0.36204$	$A_{(11)2} = 0.42123$
$A_{(12)1} = 0.33139$	$A_{(12)2} = 0.34269$
$A_{(21)1} = 0.39832$	$A_{(21)2} = 0.41148$
$A_{(22)1} = 0.39016$	$A_{(22)2} = 0.37288$

With these parameters as input, a machine program was written in order to solve the analytical expression giving implicitly the increase δR_e of the extrapolated radius as a function of the radius R of the substitution zone.

On Fig. 1 are plotted

the 16-group result, which is taken as the "exact" reference;

a 2-group result, with conventional 2 groups;⁽¹³⁾

the 2-overlapping-group result (5-sec computing time for the whole curve on an IBM 360-50), which is exactly the same as the 16-group result.

Thus it seems that the overlapping-group method gives excellent results, at least if the neighbouring media have not-too-different material bucklings and spectra.

It is interesting to note that, even though the substitution medium is less reactive (smaller buckling) than the reference medium, δR_e is negative for a small substitution radius R . This is a self-shielding phenomenon for the macroscopic flux (self-shielding of the narrow resonances was not treated in these calculations).

The overlapping-group model perfectly reproduces this effect, because the danger coefficient of medium 1 in medium 2 = $A_{(22)1} = 0.390$ is greater than $0.373 = A_{(22)2}$ = danger coefficient of medium 2 in medium 2, even though $B_2 = 0.050 \text{ cm}^{-1}$ is greater than $0.049 \text{ cm}^{-1} = B_1$.

From the input parameters one can also compute very easily the transient spectra characteristics Q_1 and Q_2 [see expression (28)]:

$$Q_1 = \frac{A_{(12)1}}{A_{(11)1}} = 0.914; \quad Q_2 = \frac{A_{(21)2}}{A_{(22)2}} = 1.104.$$

The relaxation lengths of the harmonic in medium 1 and 2 are, respectively,

$$\begin{aligned} \mu_1^{-1} &= 8.03 \text{ cm}; \\ \mu_2^{-1} &= 6.12 \text{ cm}. \end{aligned}$$

5. Fitting of the Parameters and Extrapolation

Consider this somewhat artificial problem. In the preceding example assume that enough U-235 is available to achieve criticality with the reference medium alone. One would like to determine δR_e for, say, $R = 55$

cm, but the available quantity of plutonium limits the substitution radius to, say, $R = 35$ cm.

As was said above, 9 parameters are necessary for a correct extrapolation. It would be illusory, though, to try to determine them exactly from only the portion $R < 35$ cm of the curve of Fig. 1.

However, one can measure directly

$$\lambda_{22} = \frac{A_{(22)1}}{A_{(22)2}}$$

by a simple substitution of plutonium for U-235 at the center;

$$\lambda_{11} = \frac{A_{(11)1}}{A_{(11)2}}$$

by a substitution of U-235 for plutonium at the center of the substitution zone, if it is large enough for the asymptotic spectrum to be established there;

$$-B_2^2,$$

from an all-reference critical. Thus there remain $9 - 3 = 6$ parameters.

One can calculate, by using the formalism presented in this paper, the sensitivity (partial derivatives) of δR_e to the 6 parameters and to the radius R . It will likely be found that δR_e at 55 cm and at 35 cm is very sensitive to one (or two) parameter(s)—say B_1^2 —and quite insensitive to the others. One can then use calculated values for the latter and use the measurements between $R = 0$ and 35 cm essentially to determine B_1^2 . The curve for small R is probably more sensitive to the “coupling” or interface parameters (see Table 1) and can be used to determine them or to check their calculated values. One can also use the property (see Ref. 13) that over the linear portion of the curve of Fig. 1, one has

$$\frac{d(\partial R_e)}{dR} = \frac{B_2^2 - B_1^2}{B_2^2}.$$

One can, of course, use other experimental information besides criticality and central danger coefficients: spatial distribution of (energy-dependent) flux and adjoint flux, and of danger coefficients can also help to correlate experiments with some parameters, particularly the buckling eigenvalues B^2 and μ^2 .

This brief discussion was intended to show the usefulness of having only a few parameters, with a physical meaning, rather than a multigroup description involving hundreds of parameters.

6. Applications of Perturbation Theory

Perturbation theory often greatly simplifies the mathematics.

In the preceding section it was shown that partial derivatives with respect to the various parameters

provided useful guidelines. It may be particularly useful to treat the difference between calculated and measured quantities as perturbations.

Numerical difficulties may arise in the overlapping-group formalism if the asymptotic properties of both media are very similar. It may then be better to treat $\sigma_2 - \sigma_1$, $\varphi_2(E) - \varphi_1(E)$, and $\varphi_2^*(E) - \varphi_1^*(E)$ as perturbations. The cross sections are then $\sigma_1(E)$ and $\sigma_1(E) + \epsilon \delta \sigma(E)$, the trial functions are $\varphi_1(E)$ and $\epsilon \delta \varphi(E)$, and the weighting functions $\varphi_1^*(E)$ and $\epsilon \delta \varphi^*(E)$. It will be interesting to find what the formulae of this paper become when $\epsilon \rightarrow 0$, i.e., when the two media are very similar. Terms of higher order could be dropped, thus considerably simplifying the formalism and reducing the number of parameters.

7. Experimental Program

A good interpretation of a progressive substitution experiment in a reflected facility should provide for the description of 3 media. If the asymptotic properties of these media vary widely, three overlapping groups should probably be used. We are not sure, though, whether the transients at a core-blanket interface would be represented as well by the overlapping-group formalism as the transients at a plutonium core-U-235 core interface.

The experimental program for PHENIX, a two-zoned plutonium-fueled core, should involve at least 4 media: the two core media, the blanket, and a driver medium. It might thus be necessary to use as much as 4 overlapping groups, but probably—and hopefully—no more.

If this formalism is used, and if one wants to derive information not only from the asymptotic characteristics of the experimental zone, but also from the criticality of the multizone assembly, one should probably avoid buffer zones and thermal-neutron drivers, which make the interpretation much more difficult.

The experimental program should probably not be limited to critical experiments. The core-blanket coupling and the power distribution in the blanket could be investigated in an exponential experiment. As was suggested in the introduction, it is probably also necessary to perform experiments where the interface between the two (power reactor) core zones is mocked up, possibly with a different geometry.

The overlapping-group formalism should also be very adequate to analyse special experiments, such as the sector-loaded zoned critical, used to measure radial distribution of fluxes and danger coefficients.

8. Conclusion

The overlapping-group formalism seems to be very adequate—much more so than the conventional multi-

group method—for the purpose set forth in the introduction, i.e., to reduce the number of parameters while still giving a good description of the zoned loadings that were considered. Actually it is not surprising that a reasonably simple curve such as that given in Fig. 1 could be described accurately with nine parameters! The hundreds or thousands of parameters used in conventional multigroup theory are obviously redundant to describe this curve.

The efficiency of the overlapping-group formalism comes from the fact that:

- (a) the asymptotic properties of the various media are perfectly described, by definition;
- (b) the transients excited by the interfaces are well enough described by assuming, on physical grounds, that they provide a smooth transition between the asymptotic spectra.

We believe that this formalism will prove very useful, not only when the number of parameters has to be minimized in order to allow extrapolation, but also in some straightforward calculations.

There are now very sophisticated multi-multigroup codes that calculate very detailed spectra, but always for the fundamental mode. Close to the interface, one always resorts to coarser groups, which cannot describe the fine-spectrum transients. The overlapping-group method could improve the treatment of the detailed spectra near interfaces. We know of no other method, except Monte Carlo, that could do this.

Synthesis methods in general are routinely used in thermal reactor calculations in order to decrease the number of dimensions. We think they should also be used for fast-neutron systems. The following possibilities exist:

- (a) the energy is eliminated (overlapping groups);
- (b) one or more spatial dimensions are eliminated;
- (c) the time is eliminated in kinetic or burnup problems;
- (d) the angular dependence is eliminated.

In static problems, diffusion or asymptotic transport theory, but with a fine-energy resolution, is often used sufficiently distant from the interfaces, where slowing down and absorption are predominant. Closer to the interfaces, leakage becomes more important, thus requiring transport theory, possibly with a poorer energy resolution.

A synthesis method could again prove very adequate to satisfy these apparently conflicting requirements, since it can handle many dimensions (in this case, energy and angle in addition to space).

As a general conclusion, we may say that synthesis methods certainly provide a good tool for the extrapolation of experimental information, and that they should

also be used as a computational tool for some problems which are not adequately treated by other methods.

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Discussion

Mr. Gandini (CNEN): I believe that integral parameters such as temperature and sodium-void coefficients would be affected by burnup conditions. During its life a reactor gets poison, changes of composition, is no longer homogeneous, and experiences a redistribution—this is a very big problem. You have to be concerned with the safety of the reactor during its entire life.

Mr. Storrer: Indeed I have simplified my PHENIX representation here—three clean zones. We, of course, want to use similar methods and similar experiments to find the effect of fission products and other changes.

Mr. Long (ANL): Have you looked at the pertinent ZPR-3 experiments in connection with this method?

Mr. Storrer: Not as yet. To test the method we prefer to take a 20-group calculation as artificial experimental data because we then have all the required information. We have the flux, the adjoint, and so on, so it is just easier to test this method first on the calculation. But we certainly intend to test it later on the ZPR-3 assemblies that have been described here.

Mr. Persiani (ANL): Why do you confine yourself to two groups? This technique could certainly be expanded to, maybe, 22 groups.

Mr. Storrer: I limited myself to two groups only because of the limited time available for this presentation. Our general philosophy is that for a reactor with three regions you would use, of course, three groups with the asymptotic fluxes in each region. It is the same for our zoned experiments; you take as many fundamental spectral shapes as you have media that have asymptotic spectra.

Mr. Persiani: This technique could be used in 22 groups as a method of weighting for finding average cross sections.

Mr. Storrer: Yes, this is certainly possible, but I was complaining that MC² was a little too complicated, and if you take the same number of overlapping groups it can be even more complicated. It might be a better description but it is certainly more complicated. The matrices are all full, and the boundary conditions are more difficult to express, and it would not be a good tool for extrapolating or getting simple parameters.

Mr. Travelli (ANL): Your method of overlapping spectra bears a considerable resemblance to a method that was developed by Calame and Federighi a few years ago. I wonder whether you have made a comparison between the two methods.

Mr. Storrer: The synthesis method that we have used was not invented by us. For instance, this method has been used considerably to treat three-dimensional problems in thermal reactors; it has been used also to calculate transient spectra when you have different temperatures across an interface. It is a very similar method that we use; we just apply it to fast systems. You asked me about the difference between the two. Basically it is the same. We just use it for this experiment and for fast problems.

Mr. Travelli: I thought that you used different weighting functions, and I was wondering whether you have done calculations with both methods to find out whether one was better than the other.

Mr. Storrer: No, we haven't. I think that we used different weighting spectra. People have been using all kinds of weighting spectra and it is still an unresolved situation.

H. Häggblom

A P_N Approximation Using Integral Transforms and Its Application to Control-rod Calculations for the Zero-energy Fast Reactor FRO

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ABSTRACT

A new method for the solution of the P_N equations is described. The conventional P_N equations in spherical geometry are transformed to coupled integral equations. Hereby, a "spherical" Hankel transform is used, i.e., the transformation kernel is a spherical Bessel function. This can be interpreted as an expansion of the flux into a continuous spectrum, where the new independent variable is the buckling.

No inner boundary conditions are required, and the outer boundary conditions are fulfilled by assuming a surrounding medium having only the capture cross section different from zero. The equations are solved numerically by Lanczos' method. A program, HATTEN, has been developed which solves the equations in P_3 approximation for two media. Criticality calculations in 1, 3, and 6 energy groups have been compared to corresponding solutions in diffusion and S_4 approximations. We find that P_3 and S_4 calculations for a reflected system give comparable accuracy in critical mass, and the fluxes are nearly identical except at the outer boundary. The computation times are of the same order. We also find that there is no sharp peak in the flux amplitude as a function of buckling, not even for bare systems. It is also possible to calculate higher eigenvalues, and it is shown that the first eigenvalues are very close together.

The scalar flux together with the Legendre moments are used in perturbation-theory calculations to obtain the reactivity of control rods. Thus, the streaming effect is included in the calculation. The reactivity is obtained as a function of the axial and radial position of the control rod in a spherical system.

1. Introduction

Our interest in the P_N approximation for criticality calculations is due mainly to two properties, namely, that one can obtain Legendre coefficients of the flux in a straightforward way and that it is easy to obtain solutions which include higher Legendre moments of the

scattering matrix. Some drawbacks of the conventional P_N method are:

- i. The boundary conditions at an interface between different media are necessary for mathematical rather than for physical reasons and will probably introduce errors.
- ii. The boundary condition at a free surface cannot accurately describe the fact that the neutron flux in an inward direction is zero.
- iii. For even-order approximations, a solution is possible only for special boundary conditions of a more or less artificial type.
- iv. Multigroup P_N calculations are very time-consuming.

Below is given a method in which drawbacks i and iii are eliminated. Further, the boundary condition at a free surface is substituted by the boundary condition at infinity. This is done by transforming the P_N equations to a system of integral equations and by assuming that the space outside of the system is filled by a purely absorbing medium. The time spent for a P_3 calculation is somewhat longer than for a corresponding DS_4 calculation, but then the adjoint flux is obtained simultaneously, and the computing time can probably still be reduced.

The procedure is applied to P_3 calculations in spherical geometry. In this case, a suitable transform is a spherical Hankel transform. The program is made for two material zones and up to 16 energy groups.

The transformed equations are solved numerically by Lanczos' method of expanding the eigenfunction in an orthogonal series of trial functions.⁽¹⁾ The adjoint solution is then obtained simultaneously with the ordinary one. It is also possible to obtain higher eigenfunctions. It is shown how the time-dependent equation can be expanded in terms of these eigenfunctions and how the corresponding decay constants are obtained.

Solutions of the time-independent P_3 equations are used to calculate reactivity values of control rods in different positions. The reactivity is expressed in the approximation of first-order perturbation theory. The

influence of channels can then be taken care of in a straightforward way.

2. The Integral Transform of the P_N Equations in Spherical Geometry

We adopt from the beginning the formalism of discrete energy groups and spherical geometry. The homogenous P_N equations can then be written

$$\begin{aligned}
 -\frac{1}{v_g} \frac{\partial \phi_{g,n}}{\partial t} = & \Sigma_g \phi_{g,n} - \sum_{g'} [(f_n \Sigma)_{g',g} \phi_{g',n} \\
 & + \chi_g (\nu \Sigma_f)_{g',g} \delta_{0,n}] \\
 & + \frac{n}{2n+1} \left(\frac{\partial \phi_{g,n-1}}{\partial r} - \frac{n-1}{r} \phi_{g,n-1} \right) \\
 & + \frac{n+1}{2n+1} \left(\frac{\partial \phi_{g,n+1}}{\partial r} + \frac{n+2}{r} \phi_{g,n+1} \right),
 \end{aligned} \quad (1)$$

where

- t = time;
- r = radius;
- $\phi_{g,n}$ = n th Legendre moment of the neutron flux in group g ;
- v_g = neutron velocity in group g ;
- Σ_g = total cross section in group g ;
- $(f_n \Sigma)_{g',g}$ = n th Legendre moment of the scattering cross section which brings the neutrons from group g' to group g ;
- χ_g = the partial number of fission neutrons in group g ;
- $(\nu \Sigma_f)_{g'}$ = number of fission neutrons per cubic centimeter caused by unit flux in group g' .

In eq. (1) it is assumed that the fission cross section is isotropic. We also assume that the cross sections are constant within a zone, m . In the subsequence, the cross sections are therefore given a corresponding upper index, for example, $\Sigma_g^{(m)}$.

We assume that the time dependence can be accounted for by the usual exponential expansion:

$$\phi_{g,n} = \sum_i a_i \psi_{g,n} e^{\alpha_i t}. \quad (2)$$

It is then possible to separate each exponential mode.

We now introduce a Hankel transform of the variable $\psi_{g,n}$, defined by the "spherical" Fourier-Bessel integral:

$$\begin{aligned}
 \psi_{g,n}(r) = & \frac{2}{\pi} \int_0^\infty J_n(kr) k^2 dk \\
 & \cdot \int_0^\infty \psi_{g,n}(r') J_n(kr') r'^2 dr' \\
 = & \frac{2}{\pi} \int_0^\infty \bar{\psi}_{g,n}(k) J_n(kr) k^2 dk.
 \end{aligned} \quad (3)$$

The expression (3) is synonymous with the ordinary Fourier-Bessel integral for Bessel function of order $n + \frac{1}{2}$ given, for example, by Morse and Feshbach.⁽²⁾ The upper limit for the integral over r is infinity, and it is convenient, although not necessary, to assume a pure absorption cross section, Σ_g^{M+1} , beyond the outermost zone, M . The reactor consists of M material zones. The transform of each exponential mode in eq. (1) can then be shown⁽³⁾ to be

$$\begin{aligned}
 \frac{\alpha_i}{v_g} \bar{\psi}_{g,n}(k) = & -\Sigma_g^{M+1} \bar{\psi}_{g,n}(k) + \frac{2}{\pi} \sum_{m=1}^M \sum_{g'} S_{n,g,g'}^{(m)} \\
 & \cdot \int_0^\infty K_{n,m}(k, k_1) \bar{\psi}_{g',n}(k_1) k_1^2 dk_1 \\
 & - \frac{k}{2n+1} [(n+1) \bar{\psi}_{g,n+1}(k) \\
 & - n \bar{\psi}_{g,n-1}(k)],
 \end{aligned} \quad (4)$$

where

$$\begin{aligned}
 S_{n,g,g'}^{(m)} = & -\Sigma_g^{(m)} \delta_{g,g'} + (f_n \Sigma)_{g,g'}^{(m)} + \Sigma_g^{(m+1)} \delta_{g,g'} \\
 & - (f_n \Sigma)_{g,g'}^{(m+1)} + \delta_{0,n} [\chi_g^{(m)} (\nu \Sigma_f)_{g'}^{(m)} \\
 & - \chi_g^{(m+1)} (\nu \Sigma_f)_{g'}^{(m+1)}]; \\
 K_{n,m}(k, k_1) = & \int_0^{r_m} J_n(kr) J_n(k_1 r) r^2 dr \\
 = & \begin{cases} \frac{r_m^2}{k^2 - k_1^2} [k_1 J_n(kr_m) J_{n-1}(k_1 r_m) \\ - k J_{n-1}(kr_m) J_n(k_1 r_m)] \\ \text{for } k \neq k_1 \\ \frac{r_m^3}{2} [J_n^2(kr_m) - J_{n+1}(kr_m) J_{n-1}(kr_m)] \\ \text{for } k = k_1. \end{cases}
 \end{aligned} \quad (6)$$

Here, r_m is the outer radius of zone m .

If we consider the functions $\bar{\psi}_{g,n-1}(k)$ and $\bar{\psi}_{g,n+1}(k)$ as source functions, eq. (4) can be solved for $\bar{\psi}_{g,n}(k)$ [Refs. (3) and (4)]:

$$\begin{aligned}
 \bar{\psi}_{g,n}(k) = & \frac{\lambda_{g,1}^{(M+1)} k}{2n+1} [n \bar{\psi}_{g,n-1}(k) \\
 & - (n+1) \bar{\psi}_{g,n+1}(k)] \\
 & + \frac{2}{\pi} \sum_{m=1}^M \int_0^\infty K_{n,m}(k, k_1) \\
 & \cdot \left\{ \frac{(\lambda_{g,n}^{(m)} - \lambda_{g,n}^{(m+1)}) k_1}{2n+1} [n \bar{\psi}_{g,n-1}(k_1) \right. \\
 & - (n+1) \bar{\psi}_{g,n+1}(k_1)] \\
 & + \sum_{g' \neq g} [\lambda_{g,n}^{(m)} (f_n \Sigma)_{g,g'}^{(m)} \\
 & \left. - \lambda_{g,n}^{(m+1)} (f_n \Sigma)_{g,g'}^{(m+1)}] \bar{\psi}_{g',n}(k_1) \right\} k_1^2 dk_1,
 \end{aligned} \quad (7)$$

where

$$\lambda_{g,n}^{(m)} = \left[\Sigma_g^{(m)} - (f_n \Sigma)_{g,g}^{(m)} + \frac{\alpha_i}{v_g} \right]^{-1}. \quad (8)$$

Now $\Sigma_g^{(M+1)}$ is an arbitrary absorption cross section which we put equal to the transport cross section for the outermost zone:

$$\Sigma_g^{(M+1)} = \Sigma_g^{(M)} - (f_1 \Sigma)_{g,g}^{(M)} + \frac{\alpha_i}{v_g} = \Sigma_{g,1}^{(M)}. \quad (9)$$

Thus, $\lambda_{g,1}^{(M+1)} = \lambda_{g,1}^{(M)}$. This makes eq. (7) simpler, especially if $f_n = 0$ for $n > 1$, which is a common assumption. Further, if the scattering matrix for $g' \neq g$ is isotropic, the last square bracket in eq. (7) is cancelled. Otherwise, a first approximation to eq. (7) must be obtained by neglecting this bracket. A better approximation can then be obtained by using the approximative value of $\tilde{\psi}_{g',n}$ on the right-hand side of eq. (7).

For $n = 0$, we must use eq. (4). It is then convenient to introduce the following functions:

$$y_{g,0}(k) = \left[1 + \frac{(\lambda_{g,1}^{(M)} k)^2}{3} \right]^{\frac{1}{2}} k \tilde{\psi}_{g,0}(k); \quad (10)$$

$$y_{g,n}(k) = k^2 \tilde{\psi}_{g,n}(k), \quad n = 1, 3 \dots; \quad (11)$$

$$y_{g,n}(k) = k \tilde{\psi}_{g,2}(k), \quad n = 2, 4 \dots. \quad (12)$$

Further, we introduce a number of operators, defined by the following expressions:

$$P_0^{(0)} y_{g,0}(k) = \left(\frac{2}{\pi} \right) \sum_{m=1}^M \lambda_{g,1}^{(M)} k \cdot \int_0^\infty \left[\sum_{g'} S_{o,g,g'}^{(m)} K_{0,m}(k, k_1) k_1 y_{g',0}(k_1) dk_1 \right]; \quad (13)$$

$$Q_0^{(l)} y_{g,n}(k) = \left(\frac{2}{\pi} \right) \frac{\lambda_{g,1}^{(M)}}{2l+1} \sum_{m=1}^M (\lambda_{g,l}^{(m+1)} - \lambda_{g,l}^{(m)}) k^{2-i} \cdot \int_0^\infty K_{l,m}(k, k_1) y_{g,n}(k_1) k_1^{2-i} dk_1, \quad (14)$$

where, when $l > 0$,

$$i = \begin{cases} 0 & \text{for } l = 1, 3 \dots \\ 1 & \text{for } l = 2, 4 \dots \end{cases}$$

$$P_0^{(l)} y_{g,n}(k) = \left[\frac{1}{2l+1} \lambda_{g,1}^{(M)} k^{2-j} - \frac{Q_0^{(l)}}{\lambda_{g,1}^{(M)}} \right] y_{g,n}(k); \quad (15)$$

and

$$j = \begin{cases} 0 & \text{for } l = 1, 3 \dots \\ 2 & \text{for } l = 2, 4 \dots \end{cases}$$

$$P_1^{(l)} y_{g,l}(k) = \left(\frac{2}{\pi} \right) k^{2-i} \sum_{m=1}^M \sum_{g' \neq g} [\lambda_{g,1}^{(m)} (f_1 \Sigma)_{g,g'}^{(m)} - \lambda_{g,1}^{(m+1)} (f_1 \Sigma)_{g,g'}^{(m+1)}] \int_0^\infty K_{l,m}(k, k_1) y_{g',l}(k_1) k_1^i dk_1; \quad (16)$$

$$O y_{g,n}(k) = \left[1 + \frac{(\lambda_{g,1}^{(M)} k)^2}{3} \right]^{-1} y_{g,n}(k).$$

The operator $P_0^{(0)}$ corresponds to production and absorption; the other operators correspond to leakage.

From eq. (4) and (7) we then obtain

$$y_{g,0}(k) = [O(P_0^{(0)} + Q_1^{(1)})O] y_{g,0}(k) - OP_1^{(1)} y_{g,1}(k) - 2OP_0^{(1)} y_{g,2}(k). \quad (17)$$

The operators within the square bracket of eq. (17) correspond to the P_1 approximation if the nondiagonal elements of the scattering matrix are isotropic. The next term corresponds to the first Legendre moment of the mentioned matrix elements. The last term corresponds to the higher Legendre moments of the neutron flux, $\psi_{g,n}(r)$, where $n > 1$.

The functions $y_{g,n}(k)$ for $n > 0$ can be calculated from eq. (7) and expressed in the following form:

$$y_{g,1}(k) = P_0^{(1)} O y_{g,0}(k) - 2P_0^{(1)} y_{g,2}(k); \quad (18)$$

$$y_{g,n}(k) = nP_0^{(n)} y_{g,n-1}(k) - (n+1)P_0^{(n)} y_{g,n+1}(k) \quad (19)$$

for $n > 1$.

Higher Legendre functions can then be successively calculated from eq. (18) and (19) by neglecting the last term of the right-hand side. When the N th moment is obtained, better approximations to the lower moments are calculated from the full expressions in eq. (18) and (19).

For simplicity, we assume in the following that the nondiagonal elements of the scattering matrix are isotropic. The final eigenvalue equation in P_3 approximation is then

$$y_{g,0}(k) = O[P_0^{(0)} + Q_0^{(1)} + 4\lambda_{g,1}^{(M)} P_0^{(1)} (P_0^{(2)} - 9P_0^{(2)} P_0^{(3)} P_0^{(2)}) P_0^{(1)}] O y_{g,0}(k). \quad (20)$$

The eigenvalue is inherent in $P_0^{(0)}$.

It is important to note the symmetry of the operator multiplications and the self-adjointness in the k, k_1 -space of all operators in eq. (20). Thus, when calculating the adjoint function to $y_{g,0}(k)$ one has only to transpose the scattering matrix. Unfortunately, this is not the case when the non-diagonal elements of the scattering matrix are nonisotropic.

The scalar flux is obtained from the inverse transform:

$$\psi_{0,g}(r) = \frac{2}{\pi} \int_0^\infty \frac{J_0(kr) y_{0,g}(k) k dk}{\left[1 + \frac{(\lambda_{g,1}^{(M)} k)^2}{3} \right]^{1/2}}, \quad (21)$$

or

$$\psi = B y. \quad (21a)$$

If the original eigenvalue equation corresponds to the operator F such that

$$F\psi = \psi \quad (22)$$

and if we write eq. (20) in the form

$$Ay = y, \quad (23)$$

we can obtain the adjoint flux by using the defining equation

$$F^+\psi^+ = \psi^+, \quad (24)$$

where F^+ is the adjoint of the untransformed P_N equations. This is obtained by changing the sign of the Legendre moments of odd order and transposing the scattering matrix. The first operation has no influence upon the eigenvalue equation for the scalar flux. The adjoint flux is therefore obtained from the solution of the equation

$$A^+y^+ = y^+, \quad (25)$$

where A^+ is the adjoint of A and is obtained for isotropic removal cross sections just by transposing the scattering matrix.

This results in the following expression:

$$\psi_{0,g}^+(r) = \int_0^\infty \frac{J_0(kr) y_{0,g}^+(k) k dk}{\left[1 + \frac{(\lambda_{g,1}^{(M)} k)^2}{3}\right]^{1/2}}. \quad (26)$$

Not all of the kernel functions which we have considered are quadratically integrable when the upper limit is infinity. But for physical reasons, we do not need to go to very high k -values. If the smallest thickness of any zone is t , it is sufficient that $tk_{\max} = 10-15$, where $k_{\max}$ is the highest k -value considered. For finite k -values, the integrals have no singular points.

3. The Method of Solution of the Eigenvalue Equation

In the stationary problem, the eigenvalue is a constant which is multiplied by $\nu\Sigma_f$. In the time-dependent problem, the α_1 can be considered as eigenvalues. In both cases, we can write the equation in the following form:

$$(A + \lambda_n B)y_n = y_n. \quad (27)$$

Instead of eq. (27), we consider a similar equation, namely,

$$(A + e_1 B)u_i = \mu_i u_i. \quad (28)$$

Here, e_1 is a first guess to the highest eigenvalue, λ_n , of eq. (27). The eigenvalue of eq. (28) is μ_i .

We expand the y_n into terms of u -functions:

$$y_n = \sum_i a_i^{(n)} u_i. \quad (29)$$

In order to obtain the $a_i^{(n)}$, we also need the solution of the following equation:

$$Av_j = \nu_j v_j. \quad (30)$$

We also assume that we have the solutions to the corresponding adjoint equations. These are orthogonal to the ordinary solutions and can be normalized according to the following expression for scalar products:

$$(u_i, u_j) = (v_i, v_j) = \delta_{i,j}. \quad (31)$$

Using eq. (27)–(31), one can obtain the following system of equations for the $a_i^{(n)}$,

$$\sum_i a_i^{(n)} b_j^{(i)} \left[\left(1 - \frac{\lambda_n}{e_1}\right) \nu_j + \left(\frac{\mu_i \lambda_n}{e_1} - 1\right) \right] = 0, \quad (32)$$

where

$$b_j^{(i)} = (u_i, v_j). \quad (33)$$

Equation (32) is an eigenvalue equation for the λ_n and has as many solutions as we have solutions of u_i and v_j . If we want only the highest eigenvalue, we can assume as a first approximation:

$$b_j^{(1)} \approx \delta_{j,1}. \quad (34)$$

This means that we assume that $v_1 = u_1$. Then, we must also neglect the $a_i^{(n)}$ for $i > 1$, and eq. (32) gives, after some algebra,

$$\lambda_1 \approx e_1 + \frac{1 - \mu_1}{(u_1, B u_1)}. \quad (35)$$

Thus, to this approximation, we do not need to calculate the eigenvalue of eq. (30). The value of λ_1 obtained from eq. (35) can then be inserted in eq. (28) instead of e_1 , and the procedure is repeated until the change in λ_1 is sufficiently small.

This iterative procedure is used for solving the stationary equation. The procedure for finding μ_1 and u_1 is a modification of the method of Lanczos.⁽¹⁾ A detailed description is given in Ref. (2).

The eigenvalues λ_n can also correspond to the time-dependent equations. In this case, B is the diagonal matrix of inverse velocities. It is easy to use Lanczos' method to obtain a large number of the eigenfunctions u_i and v_j . Then, eq. (32) can be used to obtain the same number of eigenvalues λ_n and the coefficients $a_i^{(n)}$. The time-dependent eigenfunctions are finally obtained from eq. (29).

4. Application to Perturbation Theory for Control-rod Calculations

The P_N expansion of the neutron flux is very convenient for application to perturbation theory when empty channels are included in the perturbed zones. Diffusion theory causes difficulties here because the diffusion constant is infinite in the channels. But

TABLE 1. GEOMETRICAL QUANTITIES, NUMBER OF ENERGY GROUPS, AND CALCULATED k_{eff}

Problem No.	Core Radius, cm	Reflector Thickness, cm	Number of Groups	k_{eff} , P ₁	k_{eff} , P ₃	k_{eff} , DS ₄
1	25.0	0	1	1.0062	1.0367	1.0337
2	15.0	30.0	1	0.9699	0.9830	0.9760
3	17.0	30.0	3	0.9879	1.0132	—
4	17.0	35.0	6	0.9705	0.9969	0.9957
5	17.0	35.0	14	0.9790	1.0060	1.0017

we can write the first-order perturbation equation in P_N approximation in the following way:

$$\frac{\Delta\nu}{\nu} = N^{-1} \left\{ - \sum_g \sum_n \int_{\Delta V} \Delta[\Sigma_g - (f_n \Sigma)_{gg}] (2n+1) \cdot \psi_{g,n}^+(r) \psi_{g,n}(r) dV + \sum_g \sum_{g'} \int_{\Delta V} [\chi_g \Delta(\nu \Sigma_f)_{g'} + \Delta \Sigma_{g'g} (1 - \delta_{g'g})] \psi_{g,0}^+(r) \psi_{g',0}(r) dV, \right. \quad (36)$$

where

$$N = \sum_g \sum_{g'} \int_V \chi_g (\nu \Sigma_f)_{g'} \psi_{g,0}^+(r) \bar{\psi}_{g',0}(r) dV. \quad (37)$$

In eq. (36), all terms are finite also in the presence of channels. A program has been written which solves eq. (36) in P_3 approximation for spherical geometry of the unperturbed system. The perturbed volume, ΔV , consists of a channel with rectangular cross sections. In an axial direction, the channel can have different compositions. Therefore, it can correspond to a control rod which is more or less withdrawn.

The neutron flux and its adjoint which are given in the input data at a number of radial points are assumed to vary quadratically over the perturbed cross sections. The first and second derivatives are found from flux differences at the surrounding points, given in the input data. If the rectangle has the side lengths s_1 and s_2 , the result of the integration over the cross section is

$$\int_{-s_1/2}^{s_1/2} dx \int_{-s_1/2}^{s_1/2} \psi^+(x,y) \psi(x,y) dy = s_1 s_2 \left[\psi(0,0) \psi^+(0,0) + \frac{1}{12} (s_1^2 \alpha_2 + s_2^2 \beta_2) + \frac{1}{80} (s_1^4 \alpha_4 + s_2^4 \beta_4) + \frac{1}{144} s_1^2 s_2^2 \gamma_4 \right], \quad (38)$$

where

$$\alpha_2 = \frac{\partial \psi}{\partial x} \frac{\partial \psi^+}{\partial x} + \frac{1}{2} \left(\frac{\partial^2 \psi}{\partial x^2} \psi^+ + \frac{\partial^2 \psi^+}{\partial x^2} \psi \right); \quad (39)$$

$$\beta_2 = \frac{\partial \psi}{\partial y} \frac{\partial \psi^+}{\partial y} + \frac{1}{2} \left(\frac{\partial^2 \psi}{\partial y^2} \psi^+ + \frac{\partial^2 \psi^+}{\partial y^2} \psi \right); \quad (40)$$

$$\alpha_4 = \frac{1}{4} \frac{\partial^2 \psi}{\partial x^2} \frac{\partial^2 \psi^+}{\partial x^2}; \quad (41)$$

$$\beta_4 = \frac{1}{4} \frac{\partial^2 \psi}{\partial y^2} \frac{\partial^2 \psi^+}{\partial y^2}; \quad (42)$$

$$\gamma_4 = \frac{\partial^2 \psi}{\partial x \partial y} \frac{\partial^2 \psi^+}{\partial x \partial y} + \frac{1}{4} \left(\frac{\partial^2 \psi}{\partial x^2} \frac{\partial^2 \psi^+}{\partial y^2} + \frac{\partial^2 \psi^+}{\partial x^2} \frac{\partial^2 \psi}{\partial y^2} \right). \quad (43)$$

The integration in the axial direction is done numerically.

5. Results of Criticality Calculations

The P_3 programme has been used for calculations for compact systems of essentially the same composition. The fuel consists of 20% enriched uranium and is the single constituent of the core in problems No. 1 and 2 (see Table 1). In problems 3–5, the core is diluted with 6.5% stainless steel. Problem 1 represents a bare reactor. The reflector in problem 2 consists of pure copper, whereas the other reflectors contain 6.5% stainless steel.

A comparison between k_{eff} calculated by different methods can be made on the basis of Table 1. The P_1 values are obtained as a first approximation to the P_3 values, so that the method of integral transform is used in both cases. It is seen, that the results of P_3 calculations are comparable with those of DS_4 calculations.

The bare reactor is a purely theoretical study. According to diffusion theory, the flux is represented by the function

$$\psi_D(r) = J_0 \left(\frac{\pi r}{R_x} \right),$$

where R_x is the extrapolated radius, which in this case is 27.6 cm. The corresponding spectral representation would then be a δ -function with the peak at $k = \pi/R_x$. The corresponding function for the integral transform is given in Fig. 1. Its maximum value is at $kR_0 = 3.25$, or $kR_x = 3.59$, thus somewhat higher than π . But more important is that the curve has no sharp peak; in fact, the distance between the first two zero-points is $R_x \Delta k \approx 2\pi$. The explanation of this is the absence of finite boundaries. Also this corresponds more closely to the physical picture of the neutron flux.

For the bare reactor, we have calculated eight orthogonal eigenfunctions and eigenvalues to the transformed equation, in abbreviated notation given by eq. (28). For one energy group we can write:

$$\kappa_i^{-1} = \frac{1}{\nu \Sigma_f} \left(\frac{\Sigma_s + \nu \Sigma_f}{\mu_i} - \Sigma_s \right), \quad (44)$$

where κ_1 is the multiplication constant and the μ_i are the eigenvalues of eq. (28). The series obtained for these were: 1.0006, 0.8667, 0.7310, 0.7056, 0.5547, 0.2847, 0.01739, and 0.0002268.

Problems 1–3 represent earlier calculations than prob-

lems 4 and 5. The macroscopic cross sections are calculated from two corresponding sets of differential cross sections, and the results from the two groups are therefore not quite comparable. This explains why k_{eff} for problem 3 is higher than for problems 4 and 5.

6. Results of Control-rod Calculations

Problems 3–5 of Table 1 correspond to assembly 2 of the Swedish fast zero-power reactor FRO. The control rods of this reactor are loaded with core and reflector material in a pattern appropriate to their positions. Reactivity measurements on these rods are reported in Ref. (5).

Two control rods, at positions 10.2 and 16.4 cm from the center, have been objects for calculations. In addition, calculations have been made for a fictitious control rod going through the center. The fluxes which we have used in the perturbation-theory calculations are obtained from the P_3 calculations given as problems 2–4 in Table 1. In problem 2, the geometrical dimensions are somewhat different from FRO/2, but these results are included because they can give us an estimation of the difference between results from 1- and 3-group calculations. Some of the results are given in Table 2.

The calculations on a central rod were made in order to obtain the average reactivity worth at a sufficient number of distances from the centre so that an interpolation can give us reactivity worths of a control rod at an arbitrary position. Also, a central control rod can for comparison be dealt with by a 2-dimensional Carlson calculation. This was made for a cylindrical system, assembly 1, which had the same composition as assembly 2.⁽⁶⁾ The axial position of the rod was 20 cm from the innermost position. A TDC calculation gave then the reactivity worth 1.34%. For assembly 2, a corresponding calculation by perturbation theory gave 0.80%. But the calculated quotient between the reactivity for a central rod and a rod 10.2 cm from the centre was 1.40. Use of this value by transforming the TDC value for assembly 1 to a corresponding value at the distance 10.2 cm from the centre in assembly 2 gave the reactivity worth 0.96%. The experimental value was 0.78% and the corresponding value from perturbation theory 0.59%. Thus, the “reduced” TDC value gives just as large an error as that obtained by perturbation theory. But the transformation factor may, of course, contribute by a large part to the error in the TDC value. For example, we earlier used the factor 1.54 instead of 1.40 to reduce the reactivity worth at the centre to a corresponding worth at the distance 10.2 cm. This is the relative reactivity coefficient for copper in the centre and at the actual radial point. The “reduced” TDC value is then 0.87%.

FIG. 1. Spectral Representation of the Flux in a Bare Reactor.

TABLE 2. TOTAL REACTIVITY WORTH OF CONTROL RODS IN FRO/2

Distance of Axis from Centre (cm)	Number of Energy Groups	Theoretical Reactivity Worth (%)	Experimental Reactivity Worth (%)
0	1	2.78	—
0	3	2.12	—
0	6	2.47	—
10.0	3	1.40	—
10.2	6	1.76	1.53
10.0	6 ^(a)	1.53	—
16.4	6	0.470	0.271

^(a) Old cross-section set.

In Figs. 2 and 3 are shown reactivity worths of a control rod whose axis is 10.2 cm from the centre. The curves give the reactivity as a function of the position along the axis, obtained by 1-, 3-, and 6-group calculations and the experimental values. The best agreement with experiments is obtained by 6-group calculations using an old cross-section set. But, unfortunately, we know that this set is erroneous for the reflector, so the agreement is only fortuitous. But it is interesting to note the strong dependence of the cross sections on the form of the curve. Generally, for a smaller number of groups, the reactivity change is faster at the beginning of the curve than is the corresponding change for a

FIG. 2. Reactivity Worth of a Start Rod in FRO/2 as a Function of Position along the Axis.

FIG. 3. Reactivity Worth of a Start Rod in FRO/2 as a Function of Position along the Axis.

higher number of groups. The reason for this may be that the weighting spectrum used for the reflector material is too soft when this material is drawn into the core. Thus, for one- and few-group calculations, the cross sections of a material should depend upon the position. The one-group calculation gives in addition too high a total reactivity worth, but this error may be caused by the neglect of stainless steel in this calculation.

In Fig. 4 are shown reactivity worths of the fine control rod, whose axis is 16.4 cm from the centre. The discrepancy between theory and experiment is larger here, at least when the rod is far from the inner position.

This can perhaps be explained by the fact that the flux varies very strongly over the rod cross section and cannot be described accurately by a polynomial of second degree. Also, the reflector surface in the experiment was cylindrical and in the calculation spherical. The error introduced thereby increases when the distance of the rod from the centre increases.

Another assembly for which extensive experiments have been made is FRO/3. This is a cylindrical assembly the core of which consists of 58.5% uranium (20% enriched), 29.2% graphite, and 6.5% steel. No P_3 calculations have been performed on this assembly, but we can report results from DS_4 calculations and subsequent perturbation calculations for a start rod. Here, only Legendre moments of zero'th and first order are considered in the perturbation calculation.

FIG. 4. Reactivity Worth of Fine Control Rod in FRO/2 as a Function of Position along the Axis.

FIG. 5. Reactivity Worth of a Start Rod in FRO/3 as a Function of Position along the Axis.

Six energy groups were used. They were obtained from the same microscopic cross sections as in problem 4 of Table 1, which led to the results given by the full-line graph in Fig. 3. A corresponding graph for assembly 3 is given in Fig. 5. The distance of the rod from the centre is 12.8 cm. The experimental values are given as points. The agreement between theory and experiment is very good, but this is probably fortuitous because of the coarse approximation we make in the perturbation theory.

The errors caused by perturbations in the neutron flux are not considered here at all. Of course they can be large, but normally the trend will be that the first-order reactivity worth is too large. For our latest 6-group calculations this would mean that the error for the total reactivity worth would decrease but for intermediate rod positions the error would increase. This result strengthens the suspicion that the reflector cross sections have bad accuracy.

7. Conclusions

We have shown how the difficulties caused by the boundaries in P_N^* calculations can be avoided by transforming the ordinary P_N equations to a system of integral equations. This principle is applied to the time-independent P_3 approximation in spherical geometry, and it is found that the accuracy in multiplication constant is of the same order as in DS_4 calculations. The fluxes obtained are then used in connection with first-order perturbation theory to obtain the reactivity values of control rods at different radial and axial positions. The maximum discrepancy between theory and experiment is then dependent upon the distance of the rod from the centre, but when the rod is well inside the core, the maximum error is of the order of 30% for 6-group calculations. There are indications that this error depends upon errors in cross sections, but more definite conclusions cannot be made without introducing flux perturbations in the control rods.

Control-rod calculations based on perturbation theory can also start from DS_n calculations. This has been made for one assembly, and the results are similar to those starting from P_3 calculations.

Another application of the integral transform of the P_N equations is the solution of the time-dependent problem. Here, a set of decay constants and corresponding orthogonal functions can be obtained by solving two stationary equations which are determined by a zero decay constant and a guessed one. The stationary solutions must be obtained for the same number of eigenfunctions as the number of wanted decay constants. This can be accomplished by using the method of Lanczos.⁽¹⁾ Such calculations are as yet only in the beginning, but a series of stationary eigenvalues for a 1-group problem is obtained. Some of these eigenvalues are very close together, but this does not necessarily mean that the decay constants are equally closely spaced.

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Discussion

Mr. Adkins (Carnegie Inst. of Tech.): One point is very interesting in the curves that you showed. The S_N approximation predicts the right S-shape of the control-rod worth, in contrast with the results for the P_N approximations. Why is this?

Mr. Häggblom: The latter gives the right shape of the curve in some cases too. It depends on the choice of cross-sections sets. For some cases we used cross sections which gave the S form of the curve.

Presented by L. Caldarola

The Balanced Oscillator Tests in SEFOR*

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ABSTRACT

In SEFOR, balanced oscillator tests will be performed to measure reactivity coefficients. Two types of balanced oscillator tests are planned:

- 1st *Balanced Oscillator Test* (1st B.O.T.) in which, while the coolant temperature is kept constant, the transfer functions between reactivity and power and between power and coolant flow are measured;
- 2nd *Balanced Oscillator Test* (2nd B.O.T.) in which, while the reactor power is kept constant, the transfer function between reactivity and coolant temperature is measured.

Particular effort has been devoted to studying possible ways of obtaining balancing conditions in the reactor plant. Two different types of control are being considered:

1. *Open Loop Control* in which predetermined input signals will be given to the reactor plant in order to hold constant the coolant temperature (for the 1st B.O.T.) and the power (for the 2nd B.O.T.).
2. *Closed Loop Control* in which the balance condition (constancy of the coolant temperature for the 1st B.O.T. and power for the 2nd B.O.T.) is reached with the aid of closed loop control.

Both control methods are described in this paper and their performance characteristics are also analysed.

1. Introduction

In SEFOR (Southwest Experimental Fast Oxide Reactor), "Balanced Oscillator Tests" will be performed to measure reactivity coefficients and thermal param-

eters of fuel rods. Two types of "Balanced Oscillator Tests" are planned:

- 1st *Balanced Oscillator Test* (1st B.O.T.) in which, while the coolant temperature, θ , is kept constant, the transfer functions between reactivity and power and between power and primary coolant flow are measured;
- 2nd *Balanced Oscillator Test* (2nd B.O.T.) in which, while the reactor power is kept constant, the transfer function between reactivity and coolant temperature is measured.

The description of these "Balanced Oscillator Tests" and the analysis of the obtainable results have already been previously described.⁽¹⁻⁵⁾ Here we want to examine these tests from the point of view of the possible ways of performing them in SEFOR. Figure 1 shows the cooling system of the reactor plant. The heat produced in the core is removed by the primary coolant, sodium, and is then transferred to the secondary sodium circuit. From here the heat is rejected to the atmosphere by open-circuit forced-air cooling. Figure 2 shows a schematic block diagram of reactor transfer functions.

Two different types of balancing may be considered (see Fig. 2).

1. To choose the input oscillating signals of reactivity Δk , the primary coolant flow Δw_p and the secondary coolant flow Δw_s are selected in such a way that inlet and outlet coolant temperatures remain constant ($\Delta\theta_i = \Delta\theta_{out} = 0$). This test is called 1st B.O.T. and the condition $\Delta\theta_i = \Delta\theta_{out} = 0$ is called "balance condition for the 1st B.O.T."

2. To choose the input oscillating signal Δk , select Δw_p and Δw_s in such a way that $\Delta P = 0$ or at least that $P(t)$ does not contain oscillations at the fundamental frequency. This test is called 2nd B.O.T. and the condition $P = 0$ [or $P(t)$ not containing the fundamental harmonic] is called "balance condition for the B.O.T."

2. Methods of Performing the Balanced Oscillator Tests

Two different methods have been thought of which can be used to perform the Balanced Oscillator Tests

* This work was performed under the SEFOR pre-operational research and development program and the Fast Breeder Project Association Karlsruhe—Germany.

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FIG. 1. Schematic Diagram of Reactor Flow.

in SEFOR:

1. Open Loop Control System, and
2. Closed Loop Control System.

With the "Open Loop Control System" (see Fig. 2) the operator sets predetermined input control values which produce balanced conditions in the plant. These required input signals are found by an analytical procedure carried out before the test is performed. This analytical procedure is described in Sect. 2.1. In the "Closed Loop System," instead, the balanced condition is reached with the aid of controllers and feedback loops.

At the present time it is intended to provide SEFOR at least with the open-loop control system. The "Closed

FIG. 2. Schematic Block Diagram of Reactor Transfer Functions.

Loop Control System" is being studied at Karlsruhe. The type of "Closed Loop Control System" shown in this paper can be basically applied to any reactor, but the numerical evaluations refer to SEFOR. Open- and closed-loop control systems are described respectively in Sections 2.1 and 2.2.

2.1. Open Loop Control System

2.1.1 Test Variables

In the balanced oscillator tests, there are three dependent variables which must be either oscillated or held constant, depending on the desired results:

- 1) reactor power P ;
- 2) reactor coolant temperature rise, $\theta_{out} - \theta_i$;
- 3) reactor coolant inlet temperature, θ_i .

Control of these variables is accomplished by oscillation of three independent variables:

- 1) reactivity Δk ;
- 2) primary coolant flow Δw_p ,
- 3) secondary coolant flow Δw_s .

2.1.2 Effects of Oscillation

The ability to hold one or two variables constant while the others are oscillated is subject to the effects of oscillation on the dependent variables. Studies of these effects made on the analog model of the SEFOR reactor showed the following results:

- 1) When an independent variable is oscillated sinusoidally about its average value, the resulting oscillations of the dependent variables are also sinusoidal at the fundamental frequency, but with different phase and amplitude for each variable.
- 2) Oscillations at harmonic frequencies may also be present, but their amplitudes are small enough to be disregarded. The one exception to this is the oscillation of coolant temperature rise caused by oscillation of primary coolant flow. This causes a problem in the second B.O.T., which will be discussed later.
- 3) The amplitude of oscillation for a dependent variable is proportional to the amplitude of oscillation for the independent variable for the amplitudes of interest in the B.O.T.
- 4) The phase relationship between the two variables does not change with amplitude for a given set of test conditions.
- 5) The effects of oscillating more than one independent variable can be found by vector addition of the individual effects on a given dependent variable, using vector magnitude to represent amplitude and vector direction to represent phase angle.

These results lead to the conclusion that a balanced oscillator test can be set up for the SEFOR reactor

using an open-loop control system, and this was verified on the analog model.

2.1.3 Method of Control

In the open-loop control system for the SEFOR plant, each of the three independent variables is controlled from the rod-oscillator device, thus maintaining identical frequencies for each variable. This device converts the rotating motion of a constant-speed drive to vertical reciprocating motion through a Scotch-yoke mechanism. The speed and amplitude of this device are adjustable over a wide range, but will remain fixed during a test. The reciprocating motion is applied directly to a poison rod in the center of the core, thus achieving the desired sinusoidal oscillation of reactivity.

The motion of the rod oscillator is also used to generate control signals for the primary and secondary coolant pumps. The phase and amplitude of these signals can be independently adjusted so as to achieve the desired balanced conditions. Thus, all three independent variables will be oscillated at identical frequencies with independently adjustable phase and amplitude.

2.1.4 First Balanced Oscillator Test

Simulated tests of the open-loop system on an analog computer have shown that, for the first balanced oscillator test, coolant temperatures can be balanced with an accuracy of $\pm 1\frac{1}{2}^\circ\text{F}$ at 20 MW when the proper phase and amplitude are set for each independent variable. The method used to determine these phase and amplitude values is first to measure the effects of oscillating one independent variable at a time. Typical values for these effects are shown by the curves in Appendix I. Determination of these effects can be done experimentally in a minimum amount of time, since analog results have shown that less than 3 cycles are required to achieve stable conditions after starting or changing an oscillation. Once the individual effects are known, vector diagrams can be drawn as shown in Figures 3-5.

In Figure 3 the vector sums for inlet temperature and temperature rise have been set to zero by varying the primary and secondary coolant-flow vectors. This can be done quite easily by using primary coolant flow to balance temperature rise and secondary coolant flow to balance inlet temperature. A computer code has been written to solve the required vector equations. A similar procedure could be used experimentally. However the analytical procedure is obviously less costly, and analog results have indicated that analysis will yield balanced conditions.

FIG. 3. SEFOR Vector Diagrams for the First Balanced Oscillator Test.

2.1.5 Second Balanced Oscillator Test

The independent vectors for the second B.O.T., in which power is held constant while reactivity is oscillated, can be found by an analytical procedure similar to that used in the first B.O.T., using the same data for effects of individual oscillators. Since only one parameter, reactor power, is required to be balanced, it is only necessary to oscillate one additional parameter, primary coolant flow. However, oscillation of the secondary coolant flow may also be used to reduce the control sensitivity required or to reduce the flow amplitude required for a given reactivity amplitude.

Figure 4 shows a typical vector diagram for the second B.O.T. in which the secondary coolant flow is oscillated so as to hold the reactor inlet temperature constant. However, this requires a rather large amplitude for the secondary flow oscillation, which is undesirable. Figure 5 shows the vector diagram for the same experiment, but with the inlet temperature oscillating. In this case the secondary coolant flow vector was arbitrarily chosen to illustrate one of the many possible choices. The final choice for this vector will be based primarily on the ease of reaching a balanced condition if the initial vector choice is slightly in error.

FIG. 4. SEFOR Vector Diagrams for the Second Balanced Oscillator Test with Inlet Temperature Balanced.

FIG. 5. SEFOR Vector Diagrams for the Second Balanced Oscillator Test with Primary Cold-leg Temperature Oscillating.

2.1.6 Requirements for Control Sensitivity

In order to reduce the reactivity effect due to the reactor power within the limits of $\pm 0.15\%$ it is necessary to balance the reactor power within ± 0.02 MW in the second B.O.T. Comparison of this requirement to the requirement of balancing temperatures to approximately 1% of the temperature rise ($1\frac{1}{2}^\circ\text{F}$) in the first

FIG. 6a. Effect of Change in Amplitude at Balanced Condition.

$$\Delta E = A_0 \cdot C \cdot \frac{\sin \Delta \theta}{\sin \beta} \approx A_0 \cdot C \cdot \sin \Delta \theta \quad \text{for } \Delta \theta < 10^\circ$$

FIG. 6b. Effect of Change in Phase Angle at Balanced Condition.

B.O.T. indicates that the second B.O.T. will be more difficult to balance than the first. An estimate of the control sensitivity required to balance a parameter can be obtained by calculating the effect of small changes in the phase or amplitude of an independent variable. As shown by the vector diagrams in Figures 6a and 6b, these effects can be calculated for a balanced variable from the equations

$$\Delta E = A_0 C (\Delta A / A_0) \text{ for amplitude changes, and}$$

$$\Delta P = A_0 C \sin \Delta \theta \text{ for phase angle changes less than } 10^\circ,$$

where

ΔE = change in amplitude of balanced variable;

ΔA = change in amplitude of independent variable;

A_0 = initial amplitude of independent variable;

C = ratio of dependent variable amplitude to independent variable amplitude;

$\Delta \theta$ = change in phase angle of the independent variable.

Comparison of these effects to the allowable limits of unbalance will then give an indication of the control sensitivity required to obtain a balanced condition. Table 1 shows these values for the vector diagrams shown in Figures 3–5. Changes of 10% in amplitude or 5.7° in phase angle were chosen for the table values because the resulting changes in amplitude of the dependent variables are equal.

2.1.7 Harmonics in the Second B.O.T.

It has been observed that in the second B.O.T., sinusoidal oscillation of the primary coolant flow introduces a harmonic of significant amplitude into the reactivity

feedback. Since the reactivity feedback is balanced against the fundamental sine wave of the reactivity input, this results in a power oscillation at twice the fundamental frequency, with an amplitude approximately equal to or slightly greater than the allowable limits for amplitude of the fundamental.²

The source of the harmonic can be found in the equation relating coolant flow and temperature rise at constant power:

$$\theta_{\text{out}} - \theta_i = P_0 / w_p. \quad (1)$$

For sinusoidal flow oscillations,

$$w_p = w_0 + A \sin \omega t; \quad (2)$$

$$\theta_{\text{out}} - \theta_i = (\theta_{\text{out}} - \theta_i)_0 + f(t); \quad (3)$$

$$P_0 = w_0 (\theta_{\text{out}} - \theta_i)_0; \quad (4)$$

Solution of these equations for the temperature oscillation, $f(t)$, yields

$$f(t) = \frac{-A \sin \omega t}{w_0 + A \sin \omega t} (\theta_{\text{out}} - \theta_i)_0. \quad (5)$$

This function contains many harmonics of generally small amplitudes, but the first harmonic ($\cos 2\omega t$) has a significant amplitude. Mathematically, this problem can be eliminated by using a function generator to

TABLE 1. OSCILLATION AMPLITUDE OF A BALANCED VARIABLE CAUSED BY CHANGING AN INDEPENDENT VARIABLE BY 10% OF ITS AMPLITUDE OR BY 5.7° IN PHASE ANGLE

B.O.T.	Independent Variable	Initial Amplitude	Balanced Variable		
			Temperature Rise, °F	Inlet Temperature, °F	Power, MW
I	w_p	5.8%	0.9	0.5	—
	w_s	9.5%	0.1	0.4	—
	$\Delta k/k$	10¢	0.9	0.2	—
II	w_p	8.5%	—	0.7	0.22
	w_s	19.1% ^(a)	—	0.7	0.11
	$\Delta k/k$	3¢	—	0.1	0.14
II	w_p	4.1%	—	(Oscillating	0.10
	w_s	6.0%	—	Inlet Tem-	0.03
	$\Delta k/k$	3¢	—	perature)	0.13

Notes: (1) Average reactor power = 20 MW

(2) Average coolant flow rate = 80% of design value (4310 gpm)

(3) Maximum allowable amplitude of:
Balanced Temperature = $1\frac{1}{2}^\circ\text{F}$;
Balanced Power = 0.02 MW.

^(a) Maximum allowable flow amplitude is 20%.

correct the coolant flow oscillation so that it is

$$w = w_0 + \frac{B \sin \omega t}{1 - B \sin \omega t} w_0.$$

This function was found by arbitrarily choosing a temperature rise function of

$$\theta_{\text{out}} - \theta_i = (\theta_{\text{out}} - \theta_i)_0 (1 - B \sin \omega t).$$

Tests with the analog model have indicated that correction of the primary flow oscillation in the above manner does reduce the amplitude of the first harmonic. However, an increase in the amplitude of the second harmonic was also noted, which may be due to transient effects in the IHX.² Thus it appears that final elimination of the harmonics in the power oscillation will have to be done by analytical methods in the data-reduction process.

2.1.8 Time Required to Achieve a Balanced Condition

The time required to obtain a balanced condition in either B.O.T. will depend on the oscillator frequency, since two to three cycles are required to achieve stable conditions after the oscillator is started or a change is made. Setting of the desired phase and amplitude should be accomplished in a minimum amount of time through proper calibration and test procedures. The oscillator settings determined by vector diagrams for the first B.O.T. should produce the required balanced conditions with no further adjustments required. However, the second B.O.T. requires more accurate control settings, and is expected to require an average of 10 cycles per test to obtain the final balanced condition.

2.1.9 Conclusions for the Open-loop Control System

- 1) Analog studies indicate that it will be possible to perform balanced oscillator tests on SEFOR through controlled oscillations of reactivity, primary coolant flow, and secondary coolant flow.
- 2) An open-loop control system in which the phase and amplitude of the independent variable oscillations are manually adjusted will provide adequate control to obtain balanced conditions within reasonable limits.
 - (a) Balanced parameter limits of $\pm 1\frac{1}{2}^\circ\text{F}$ in the first B.O.T. appear to be reasonable.
 - (b) The balanced parameter limit of $\pm 0.1\%$ of reactor power in the second B.O.T. is more difficult to achieve, but analog studies indicate that there is reasonable expectation of approaching this limit.
- 3) A vector analysis method of combining the effects of oscillating each independent variable can be used to determine the oscillator phase and ampli-

tude requirements for each balanced oscillator test.

- 4) It is expected that 10 cycles or less will be required to achieve balanced conditions after the precalculated amplitudes and phase angles are set.
- 5) The open-loop control system provides a simple and effective means of running a balanced oscillator test and eliminates control problems normally associated with large phase lags and parameter interactions.

2.2. Closed-loop Control System

2.2.1 General

Figure 7 shows a schematic block diagram of the connections of the closed-loop control system to the plant in the case of the 1st B.O.T. The reactor is fed with a sinusoidal reactivity signal of frequency f_0 , which is produced by the "Frequency and Sine Function Generator." Then,

$$\Delta k = \Delta k_m \sin 2\pi f_0 t. \quad (1)$$

The input signal to the "Controller No. 1" is the difference between the signal of outlet and inlet reactor coolant temperatures, $\theta_{\text{out}} - \theta_i$. The output signal acts on the "Primary Pump," which will tend to change the primary sodium flow in such a way that

$$\theta_{\text{out}} - \theta_i = \text{constant}. \quad (2)$$

The input signal to the "Controller No. 2" is the outlet primary heat-exchanger sodium temperature, T_{out} . Its output signal acts on the "Secondary Pump" which will tend to change the secondary coolant flow in such a way that

$$T_{\text{out}} = \text{constant}. \quad (3)$$

The "Transfer Function Analyser" (T.F.A.) measures the transfer functions, respectively, between power P and reactivity Δk , and between primary coolant flow w_p and power P .

Figure 8 shows a schematic block diagram of the connections of the closed-loop control system to the plant for the 2nd B.O.T. The plant is fed with a sinusoidal signal at frequency f_0 on the primary pump, and

$$\Delta w_p = \Delta w_{pm} \sin (2\pi f_0 t). \quad (4)$$

The secondary pump can either be controlled to keep T_{out} constant (as in the 1st B.O.T.) or be fed with a sinusoidal signal:

$$\Delta w_s = \Delta w_{sm} \sin (2\pi f_0 t + \alpha), \quad (5)$$

with Δw_{pm} and α chosen in such a way that the maximum possible change of the reactor average coolant

FIG. 7. First Balanced Oscillator Experiment—Schematic Block Diagram of the Connections of the Closed-loop Control System to the Plant.

FIG. 8. Second Balanced Oscillator Experiment—Schematic Block Diagram of the Connections of the Closed-loop Control System to the Plant—First Philosophy Design.

FIG. 9. Second Balanced Oscillator Experiment—Schematic Block Diagram of the Connections of the Closed-loop Control System to the Plant—Second Philosophy Design.

temperature, $\Delta\bar{\theta}$, compatible with the safety and the limitations of the plant, is produced. Both possible control schemes of the secondary pump have not been shown in Fig. 8.

The input signal to the "Controller" is the power P , which is measured by a flux detector. Its output signal acts on a control rod to produce a change of reactivity, Δk , which will tend to keep the power constant:

$$P = \text{const.} \quad (6)$$

The T.F.A. measures the transfer function between the reactivity Δk and the average coolant temperature $\bar{\theta}$, defined as

$$\bar{\theta} = \frac{\theta_i + \theta_{out}}{2}. \quad (7)$$

An alternative to the scheme of Fig. 8 is shown in Fig. 9, in which the power P is kept constant by acting on the primary coolant flow.

The two control loops of Fig. 7 and those of Figures 8 and 9 are schematically described by the diagram shown in Fig. 10. In Fig. 10, U is the controlled variable which is to be kept constant. When an input signal I is introduced in the plant, this will produce a change U_1 of U through the transfer function $P_1(s)$. The controlled variable U is measured by the feedback circuit, which has the transfer function $H(s)$. The output signal Y from this circuit is compared to the reference R and the difference ϵ feeds the "Controller." The output signal γ from the Controller acts on the plant and produces a

FIG. 10. Schematic Block Diagram of the Closed Control Loop.

change U_2 of U which tends to compensate for the previous change U_1 due to the input signal I . The plant transfer function between U_2 and γ is indicated by $P_2(s)$.

The Controller consists of two parts: a "Regulator" and "High Gain Unit" (H.G.U.), having, respectively, transfer functions $G(s)$ and $M(s)$. The function of the "Regulator" is to amplify the input signal ϵ and to correct it (by means of a corrective network) to get phase advance of ϵ and stability. The function of the "High Gain Unit" is to suppress the oscillations of the controlled variable U at the frequency f_0 at which the B.O.T. is performed. This means that any disturbance U_1 of U at the frequency f_0 is compensated by an oscillation U_2 having the same amplitude as U_1 . The way in which the H.G.U. operates will be described in the following paragraph. The H.G.U. can be connected or disconnected from the loop operating the switch S_w

(see Fig. 10) without affecting the stability of the system.

2.2.2 Basic Analytical Considerations

By examination of Fig. 10, we can write the following equations in the Laplace domain:

$$U = U_1 + U_2; \quad (1)$$

$$U_2 = \gamma P_2(s); \quad (2)$$

$$\gamma = \eta + \lambda; \quad (3)$$

$$\lambda = \gamma M(s); \quad (4)$$

$$\eta = \epsilon G(s); \quad (5)$$

$$\epsilon = R - Y; \quad (6)$$

$$Y = U H(s). \quad (7)$$

From eqs. 2 to 7 we get (for R a constant):

$$U_2/U = -K(s), \quad (8)$$

where

$$K(s) = W(s) \frac{1}{1 - M(s)} \quad (9)$$

and

$$W(s) = P_2(s)H(s)G(s). \quad (10)$$

From eqs. 1 and 8, we get

$$\frac{U}{U_1} = \frac{1}{1 + K(s)}. \quad (11)$$

The transfer function of eq. 11 is called the closed-loop transfer function; that of eq. 8 is called the open-loop transfer function. This terminology is due to the fact that eq. 8 would represent the transfer function of the loop supposed to be ideally cut at the point where U_1 and U_2 are added.

From eqs. 8 and 11 it is clear that the properties of the closed-loop transfer function U/U_1 can be derived by analysing the open-loop transfer function $K(s)$. The second control loop of the 1st B.O.T. is the worst from the point of view of stability because of the time constants involved. We shall therefore discuss this loop here.

In this case, in Fig. 10 U would represent the primary heat-exchanger outlet coolant temperature T_{out} , and γ the signal to the secondary pump. $P_2(s)$ includes the transfer function of the secondary pump $[1/(1 + 2s)]$, that between primary heat-exchanger outlet coolant temperature and secondary coolant flow and the 1.5-sec time constant of the lower mixing of the heat exchanger.

The feedback transfer function $H(s)$ includes the time constant of the thermocouple (supposed to be 1

sec) and the 1-sec time lag between the place at which the thermocouple is mounted and the outlet of the primary heat exchanger. This situation would correspond to a coolant flow equal to 80 % of its rated value.

Curve No. 1 of Fig. 11 shows the polar diagram of the frequency response of the function

$$W(j2\pi f) = P_2(j2\pi f)H(j2\pi f)G(j2\pi f), \quad (12)$$

where

$$G(s) = G_0 \frac{1 + 4s}{1 + 0.2s} \quad (13)$$

with G_0 chosen in such a way that

$$W(0) = 1. \quad (14)$$

Let us suppose for the moment that the H.G.U. is a low-pass filter (L.P.F.). If the input signal γ is at low frequency, it will pass through the filter and the output λ will be added to the signal η from the regulator. If, instead, γ is at higher frequency, it will be attenuated and shifted by the filter, and therefore the output signal λ will have practically no regulating action. This means that although the controller would be able to give a precise control at low frequencies, at higher frequencies the control becomes less accurate. This loss of accuracy is due to two causes:

- The higher the frequency, the more delayed in phase is the signal λ in respect to η ;
- The higher the frequency, the bigger is the attenuation between λ and γ .

The first cause can be eliminated by making a phase correction with a device which changes the phase without changing the amplitude, for example a pure time delay.

If we incorporate in our H.G.U. a memory in cascade to the low-pass filter, the frequency response of the transfer function $M(s)$ would be of the type

$$M(j2\pi f) = A \frac{\exp(-j\psi f/f_0)}{(1 + jf/f_m)^n}, \quad (15)$$

where f_m is the cut-off frequency of the filter. The angle ψ must be chosen in such a way that, at the frequency f_0 of the experiment, the sum of ψ and of the phase shift φ_0 of the filter gives 2π , that is,

$$\psi + \varphi_0 = 2\pi. \quad (16)$$

The curve of Fig. 12 shows the polar diagram of the frequency response of

$$\frac{1}{1 - M(jf/f_0)} \quad (17)$$

as function f/f_0 , where $M(s)$ is given by eq. 15 with

$$n = 4; \quad f_0/f_m = 0.2; \quad A = 1. \quad (18)$$

FIG 11 Frequency Response for Open Loop

This curve shows high gain at the frequency f_0 . At the higher harmonics $2f_0$, $3f_0$, etc., the gain also presents a maximum value which is becoming smaller as the order of the harmonic increases.

If we now introduce this modified H.G.U. in our control loop, the frequency response of the open-loop transfer function $K(j2\pi f)$ becomes the curve No. 2 of Fig. 11 if the system is set at the frequency

$$f_0 = 0.02 \text{ cps.} \quad (19)$$

The gain at frequency f_0 is about 12.5, which means

that the amplitude of the oscillations of the controlled variable U at this frequency is reduced to $1/12.5 = 8\%$ of that of the disturbance U_1 . Curve No. 2 of Fig. 11 shows (according to the Nyquist criterion) that the closed loop will be stable because the open-loop transfer function does not encircle the point -1 . This conclusion can be easily drawn if one thinks that stability means that the characteristic transcendental equation of the closed loop:

$$W(s) \frac{1}{1 - M(s)} + 1 = 0, \quad (20)$$

FIG. 12. Frequency Response of the H.G.U. with L.P.F. and Memory ($A = 1$)

must not have any root with real part positive. If one puts in eq. 20

$$s = \alpha + j2\pi f \quad (21)$$

for $\alpha > 0$, one realizes that for the same value of f the function $W(s)$ becomes smaller in modulus and phase shift. This means that for a given frequency the corresponding point on the curve (No. 1 of Fig. 11) tends to move from the left to the right (as indicated by the small arrow), while the curve tends to squeeze itself towards the origin. At the same time the function $1/[1 - M(s)]$ of Fig. 12 tends for $\alpha > 0$ to squeeze it-

self towards the point 1. Because of all these effects, the envelope of the lobes of curve No. 2 in Fig. 11 for $\alpha > 0$ tends to squeeze itself towards the origin. This behaviour of the function $W(s)/[1 - M(s)]$ seen on the Nyquist diagram (curve No. 2 of Fig. 11) ensures us that the characteristic equation 20 is always for $\alpha > 0$ different from -1 :

$$W(s) \frac{1}{1 - M(s)} \neq -1, \quad (22)$$

which means that the system is stable. In order to im-

FIG 13 Frequency Response of the H G U closed loop with L P F and Memory ($A = 0.94$, $C = 0.98$)

prove the accuracy of the system, one can use a L.P.F. with a damping factor ζ different from 1. The frequency response of the transfer function $M(s)$ of the H.G.U. would be of the type

$$M(j2\pi f) = A \frac{\exp(-j\psi f/f_0)}{[1 + (2j\zeta f/f_m) - (f/f_m)^2]^n} \quad (23)$$

The damping factor ζ must be chosen in such a way that the modulus of $M(j2\pi f)$ has its maximum value at $f = f_0$. Then A is chosen so that the modulus C of $M(j2\pi f)$ at $f = f_0$ is as close as possible to 1.

The curve of Fig. 13 shows the polar diagram of eq. 23 as function of f/f_0 with

$$n = 1, \quad f_0/f_m = 0.5; \quad \zeta = 0.6; \quad C = 0.98. \quad (24)$$

The gain A of H.G.U. at frequency f_0 would be

$$A = C \left[1 - 2(1 - 2\zeta^2) \left(\frac{f_0}{f_m} \right)^2 + \left(\frac{f_0}{f_m} \right)^4 \right]^{n/2} = 0.94. \quad (25)$$

Curve No. 2 of Fig. 14 is the polar diagram of the

FIG. 14. 1. Open-loop Frequency Response without H.G.U. 2. Open-loop Frequency Response with an H.G.U. having a memory ($A = 0.94$; $C = 0.98$).

open-loop frequency response $K(j2\pi f)$ at $f_0 = 0.02$ cps in the case in which $M(j2\pi f)$ is defined by eqs. 23 and 24. The system also in this case is stable, and the gain at $f = f_0$ has become about 50, which means a precision of about 2%.

Preliminary work has been carried out on the analog computer just to see if this type of control system would have major troubles or not. The model used was an old model of the SEFOR plant and the control system was not optimized.

The 1st B.O.T. was carried out at a frequency of

0.0025 cps (see Fig. 15). The amplitude of the input reactivity signal was 10¢. The trace of the average coolant temperature θ is shown: its behaviour is in agreement with the results of the simplified analysis developed in Appendix 2. The control scheme adopted in Fig. 15 is slightly different from the control scheme of Fig. 7. In Fig. 15 the input signal to the first controller is the reactor outlet coolant temperature and not the difference between reactor outlet and inlet coolant temperature.

The 2nd B.O.T. was also carried out on the analog

FIG. 16. Control System and Power Trace.

FIG. 17. Error E/E_{id} as Function of the Error $\delta\psi$ of Phase Setting.

(see Fig. 13)

$$|1 + W(j2\pi 0.02)| = 1.85, \quad (3)$$

which means that the amplitude of the temperature oscillation will be reduced in 6 cycles to about 2.5 % of its initial value.

From Fig. 13 we see that the modulus of the functions $K(j2\pi f)$ and $W(j2\pi f)$ can be increased by a factor of 1.5 without having problems of stability. If this is done, the reduction factor becomes 2.4 instead of 1.5 (as given by eq. 3), and the same reduction of amplitude to 2.5 % will be reached in only 4 cycles.

4. It is very interesting to notice that, after few cycles, the memory has already stored the right corrective signal so that the control loop can even be open while the tape continues to feed the plant with the right corrective signal. In this case it is more convenient to have a second reading head (at 360° from the writing head) which gives the signal to the writing head in such a way that the signal always remains unchanged on the tape. The first reading head will continue to feed the plant. This feature seems valuable if one plans to repeat a B.O.T. In this case the right corrective signal already exists in the tape, which can feed the plant directly. An additional control loop able only to cope with the drift of the plant could be added to the system. For a better precision it would be convenient to have this second loop working in parallel with the tape, even when the tape is recording the corrective signal.

5. For safety purposes the control system must have

the following two features:

- a) The controller must be designed in such a way that the output signal cannot in any case exceed some extreme values fixed by safety considerations.
 - b) A device must be incorporated which switches the controlling variable (coolant flow or reactivity) to a fixed constant value before the signal reaches one of the above-mentioned extreme values. This device also gives an alarm signal.
6. Other types of closed-loop control systems are under investigation at Karlsruhe. We mention here particularly systems having a bandpass filter either directly on the main control line or as a positive feedback on it.
7. The closed-loop control system has the following properties:

- a) The balance condition may be obtained with very good precision.
- b) The closed-loop control system releases the operator of the tediousness of carrying out the experiments and of making all the analysis which precedes them, as shown in Sect. 2.1. Reactor operating time will be reduced in the second B.O.T. where the balanced power requires greater precision. The balance condition is obtained in a lower number of cycles than that of the open loop. The time saved is not only that needed to reach the balance condition, but also that needed to evaluate the individual transfer functions whose knowledge is necessary to carry out the vector analysis shown in Sect. 2.1.

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APPENDIX 1

Oscillator Effects

The curves in this appendix show the effect on each of three dependent variables produced by individual oscillation of each of three independent variables.

These data are used to construct vector diagrams and thus determine control settings required for balanced conditions.

It should be noted that phase angle is defined as the lag angle (time) between maximum values of the dependent and independent variables. This definition of phase angle was chosen to avoid confusion in the use of the data for vector diagrams.

The primary cold-leg temperature refers to the temperature at the IHX outlet as measured with a thermocouple having a 5-sec time constant. The core-temperature rise refers to the actual value of core outlet temperature minus core inlet temperature. Reactor power refers to neutron flux.

APPENDIX 1, Fig. 1. Oscillator Effects on Neutron Flux at 80% Average Coolant Flow for $\pm 10\%$ and $\pm 10\%$ Oscillator Amplitude.

APPENDIX 1, FIG. 2. Oscillator Effects on Neutron Flux at 80% Average Coolant Flow.

APPENDIX 1, FIG. 3. Oscillator Effects on Primary Cold-leg Temperature at 80% Average Coolant Flow for $\pm 10\%$ and $\pm 10\%$ Oscillator Amplitude.

APPENDIX 1, FIG. 4. Oscillator Effects on Primary Cold-leg Temperature at 80% Average Coolant Flow.

APPENDIX 1, FIG. 5. Oscillator Effects on Core Coolant ΔT at 80% Average Coolant Flow for $\pm 10\%$ and $\pm 10\%$ Oscillator Amplitude.

APPENDIX 1, FIG. 6. Oscillator Effects on Core Coolant ΔT at 80% Average Coolant Flow.

APPENDIX 2

Evaluation of the Behaviour of the Controlled Variable in the Time Domain

Let us consider the closed-loop control system of Fig. 10. We write eq. 11 of Sect. 2.2.2:

$$\frac{U}{U_1} = \frac{1}{1 + K(s)}, \quad (1)$$

where

$$K(s) = W(s) \frac{1}{1 - M(s)} \quad (2)$$

and

$$W(s) = P_2(s)H(s)G(s). \quad (3)$$

We suppose that $U_1(t)$ is a sinusoidal function in the time domain. We have

$$U_1(t) = U_0 \sin 2\pi f_0 t. \quad (4)$$

In the Laplace domain we have

$$U_1 = U_0 \frac{2\pi f_0}{s^2 + (2\pi f_0)^2}. \quad (5)$$

Eq. 1 becomes

$$U = U_0 \frac{2\pi f_0}{s^2 + (2\pi f_0)^2} \frac{1}{1 + W(s) \frac{1}{1 - M(s)}}. \quad (6)$$

From eq. 6 in the time domain we get

$$U(t) = U_0 L^{-1} \left\{ \frac{2\pi f_0}{s^2 + (2\pi f_0)^2} \frac{1}{1 + W(s) \frac{1}{1 - M(s)}} \right\}, \quad (7)$$

where the symbol L^{-1} indicates antitransformation. We shall solve eq. 7 in the case

$$M(s) = AF(s) \exp(-\psi s/2\pi f_0), \quad (8)$$

where $F(s)$ is the transfer function of the L.P.F.

By taking into account eq. 8, eq. 6 becomes

$$U = U_0 \frac{2\pi f_0}{s^2 + (2\pi f_0)^2} \frac{1}{1 + W(s)} \cdot \frac{1 - AF(s) \exp(-\psi s/2\pi f_0)}{1 - [AF(s) \exp(-\psi s/2\pi f_0)]/[1 + W(s)]}, \quad (9)$$

which can be written as follows

$$U = U_0 \frac{2\pi f_0}{s^2 + (2\pi f_0)^2} \cdot [1 - AF(s) \exp(-\psi s/2\pi f_0)] \cdot \sum_{n=1}^{\infty} \left\{ (-1)^n \frac{[AF(s)]^{n-1}}{[1 + W(s)]^n} \exp \left[- (n-1) \frac{\psi s}{2\pi f_0} \right] \right\} \quad (10)$$

or

$$U = U_0 \frac{2\pi f_0}{s^2 + (2\pi f_0)^2} \sum_{n=1}^{\infty} \left\{ (-1)^{n-1} \frac{[AF(s)]^{n-1}}{[1 + W(s)]^n} \cdot \left[\exp \left[- (n-1) \frac{\psi s}{2\pi f_0} \right] - AF(s) \exp \left(-n \frac{\psi s}{2\pi f_0} \right) \right] \right\}. \quad (11)$$

We shall antitransform eq. 11 in the particular case

$$AF(s) = 1; \quad (12)$$

$$W(s) = W_0 = \text{constant}; \quad (13)$$

$$\psi = 2\pi. \quad (14)$$

Then eq. 11 becomes

$$U = U_0 \frac{2\pi f_0}{s^2 + (2\pi f_0)^2} \sum_{n=1}^{\infty} \left\{ (-1)^{n-1} \frac{1}{(1 + W_0)^n} \cdot \left[\exp \left[- (n-1) \frac{s}{f_0} \right] - \exp \left(-n \frac{s}{f_0} \right) \right] \right\}. \quad (15)$$

The antitransform of eq. 15 is

$$U(t) = U_0 \sin(2\pi f_0 t) \sum_{n=1}^{\infty} \left\{ \frac{1}{(1 + W_0)^n} \cdot \left[1 \left(t - \frac{n-1}{f_0} \right) - 1 \left(t - \frac{n}{f_0} \right) \right] \right\}, \quad (16)$$

APPENDIX 2, FIG. 1. Behavior of the Controlled Variable U in the Ideal Case.

where

$$n = \text{nth oscillation}$$

and

$1(t)$ indicates a step function.

Eq. 16 is shown in Fig. 1 of this appendix. The controlled variable $U(t)$ oscillates with an amplitude which decreases with time and takes the following values:

$$\text{1st oscillation: } \frac{U_0}{1 + W_0};$$

$$\text{2nd oscillation: } \frac{U_0}{(1 + W_0)^2};$$

$$\text{"n"th oscillation: } \frac{U_0}{(1 + W_0)^n}.$$

At any cycle, therefore, the oscillation decreases by a factor $(1 + W_0)$.

In the real case this factor will be approximately

$$|1 + W(j2\pi f_0)|$$

if the phase shift of $W(j2\pi f_0)$ is not too large.

APPENDIX 3

Dependence of the Precision upon the Error $\delta\psi$ in Setting the Phase

The error E of the controlled variable U is given by eq. 11 of Sect. 2.2.2:

$$E = \left| \frac{U}{U_1} \right| = \left| \frac{1}{1 + K(j2\pi f_0)} \right| \cong \left| \frac{1}{K(j2\pi f_0)} \right| = \left| \frac{1}{W(j2\pi f_0)} \right| |1 - Ce^{j\delta\psi}|, \quad (1)$$

where C is defined by eq. 25 of Sect. 2.2.2. Eq. 1 can be written as follows:

$$\begin{aligned}
 E &= \left| \frac{1}{W(j2\pi f_0)} \right| |1 - C \cos \delta\psi + jC \sin \delta\psi| \\
 &= \left| \frac{1}{W(j2\pi f_0)} \right| \sqrt{(1 - C \cos \delta\psi)^2 + (C \sin \delta\psi)^2}.
 \end{aligned} \tag{8}$$

Since

$$\cos \delta\psi \cong 1 \tag{3}$$

and

$$\sin \delta\psi \cong \delta\psi, \tag{4}$$

eq. 2 becomes

$$E \cong \frac{1 - C}{|W(j2\pi f_0)|} \sqrt{1 + \left(\frac{C}{1 - C} \delta\psi \right)^2}. \tag{5}$$

Figure 17 shows the ratio

$$\frac{E}{E_{1d}} = \frac{E |W(j2\pi f_0)|}{1 - C} = \sqrt{1 + \left(\frac{C}{1 - C} \delta\psi \right)^2} \tag{6}$$

as a function of $\delta\psi$ for different values of C .

Discussion

Mr. Kohler (Texas A&M): If you want to keep the inlet and outlet temperatures constant and you oscillate the reactivity with an amplitude of approximately 10 cents, what amplitude of oscillation in the flow of secondary coolant would be required?

Mr. Caldarola: That depends on the frequency. Just to give an order of magnitude, for an oscillation of 10 cents one can get an oscillation of the temperature of the order of 10–15 degrees and one wants to keep it within 1 degree.

Mr. Kohler: Then what does the oscillation in the flow of the secondary coolant have to be?

Mr. Caldarola: About 7 to 8 %.

Mr. Lantz (NASA): Is there a significant reactivity coefficient due to the expansion of the whole core with an increase in temperature?

Mr. Caldarola: One can associate some amount of reactivity with the average coolant temperature and the coefficient would be about 0.4 cent/°F. This would include the structure expansion, the change in density of sodium, and part of the axial expansion of the clad.

SESSION V-B

October 12, 1966

SPECTRUM MEASUREMENTS

Chairman: H. Meister
Kernforschungszentrum Karlsruhe

Secretaries: R. Gold
C. E. Till
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Spectrum Measurements in a Large Dilute Plutonium-fueled Fast Reactor*

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Presented by E. F. Bennett

ABSTRACT

Spectrum measurements have been made at the center of the ZPR-3 Assembly 48, a large, dilute, plutonium-fueled fast reactor. The energy distribution of fragments from the $\text{Li}^6(n,\alpha)t$ reaction and of protons recoiling in a hydrogen proportional counter can be interpreted in terms of the neutron-energy spectrum. The results of measurements with the two techniques are compared, and the agreement is within estimated errors.

1. Introduction

Assembly 48 of ZPR-3 is a plutonium-fueled critical assembly which simulates a class of large power reactors. The scheduling of experiments for this assembly allowed for neutron-spectroscopy measurements to commence early in the program. In this paper the results of two of these measurements will be described. The techniques are by proton-recoil proportional counting and by the $\text{Li}(n,\alpha)t$ reaction. Both have been used previously for in-core measurements at Argonne, and are typical of in-core spectrometers in use and under development at Argonne and elsewhere.

2. Proton-recoil Result

In-core measurements with proton-recoil counters have been made previously in ZPR-6 and ZPR-9. Cores that have been measured were fueled with U^{235} and had a fast-neutron spectrum not greatly different from that reported for the plutonium-fueled Assembly 48 of the ZPR-3 (see Table 1).

Problems are encountered in measurements made with plutonium-fueled assemblies that are different from those encountered with U^{235} as the fissionable material. The large spontaneous-fission neutron source in plutonium will lead to high neutron-flux levels as criticality is approached, and since electronic count-

rate limitations exist, it is not possible to operate a plutonium assembly near critical using proton-recoil detectors of fixed sensitivity. In addition to neutron sensitivity, the background of gamma radiation will produce a substantial counting rate which cannot be assumed to be fixed in passing from uranium to plutonium. The purpose of these measurements was to arrive at a better understanding of the response of the detectors to the radiation field; information which can serve as the basis for an improved counting arrangement for spectroscopy in plutonium critical experiments.

Two counters were used for the measurements reported here. The first contained a predominantly hydrogen gas filling (about 5 atm total pressure) and was used below about 150 keV. The diameter of this counter was 1.59 cm and the effective length (as defined by needles having an OD of 0.3 mm) was 3.2 cm. The walls of the counter were of stainless steel and were 0.8 mm thick. A sleeve of lead, 6 mm thick, surrounded the effective counting length and provided a slight decrease in gamma sensitivity with no significant effect upon the neutron spectrum. The second counter (of 2.5-cm diameter) was filled with methane to a pressure of about 5 atm and was used in the energy region above 100 keV. The additional stopping power of the more dense methane filling limited wall-and-end effect distortion to a relatively small, but not insignificant, amount below 1 MeV; and 1 MeV represents the present useful upper energy limit of these detectors. The methane counter contained, over its effective length, 7.2 times the number of hydrogen atoms contained in the hydrogen gas-filled counter. It also had a 0.8-mm-thick stainless steel wall and was surrounded by a 0.6-cm-thick lead cylinder.

Measurements were made at the center of the ZPR-3 core, and in order to maintain the maximum counting rates at about 3000 c/s, it was found that the reactor could not be brought closer than about 10% to critical. The spectrum measured was, consequently, the central spectrum for a subcritical assembly lacking about half

* Work performed under the auspices of the U. S. Atomic Energy Commission.

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TABLE 1. COMPOSITION OF ASSEMBLY 48 OF ZPR-3

Material	Composition, 10^{24} Nuclei/cm ³	
	Core	Blanket
U ²³⁵	0.000016	0.000084
U ²³⁸	0.007406	0.039976
Pu ²³⁹	0.001644	—
Pu ²⁴⁰	0.000106	—
Pu ²⁴¹	0.000010	—
C	0.020765	—
Na	0.006230	—
Fe	0.009899	0.004453
Cr	0.002658	0.001196
Ni	0.002658	0.001196
Mo	0.000206	—
Al	0.000110	—

its critical mass. Nevertheless, it appears that the spectrum existing at the time of the measurement was close to the spectrum at critical. A calculation of the subcritical spectrum is not, at present, available.

The background of gamma radiation appeared to be of about the same intensity as in measurements on uranium-fueled assemblies. It does not follow, however, that the "dirtier" mixture of plutonium that may be used for future critical experiments will provide an equally favorable gamma background.

Counting was done for about 6 hr for the spectrum

reported here. The neutron flux incident upon the detectors was held to a value of $1.1 \times 10^6/\text{cm}^2\text{-sec}$ integrated over all energies from 1 keV and to 1 MeV. Resolution depends upon energy but is close to 20% (F.W.H.M.) except at the lowest energies.

The measured spectrum is shown in Fig. 1. The structure in the spectrum can be mostly attributed to large scattering resonances in the lighter materials listed in Table 1. Systematic errors in the spectrum are expected to be less than statistics except near the energy extremes. Uncertainty in W , the energy lost per ion pair created by protons in stopping, introduced an increasingly large error into the spectrum as energy decreased below about 2 keV.

The spectrum above 500 keV was perturbed by wall-and-end effects in which the recoiling-proton track was truncated due to the finite stopping power of the gas, as well as to the finite size of the detectors. Wall-and-end effect corrections are needed in order to extend results to higher energies; the spectrum at 1 MeV may be subject to systematic errors as large as 20%, the error diminishing as energy decreases.

In summary, the counters used in this experiment and described above are much too sensitive to permit measurements to be made with plutonium-fueled assemblies close to critical. The large spontaneous-fission source restricts neutron multiplication to a value of about 10; higher multiplications produce unacceptably

FIG. 1. Central Neutron Spectrum in Assembly 48.

high counting rates. Counters can be desensitized by decreasing their size. It appears to be feasible to build workable counters smaller than those described here. An increase in the rate at which counting can be done without distortion is also probably possible.

3. Results with Li^6 Fast-neutron Spectrometer

Measurements were made in the core center of ZPR-3 Assembly 48 with the Li^6 -semiconductor sandwich fast-neutron spectrometer.⁽²⁾ The device used is composed of two silicon surface-barrier detectors mounted with sensitive areas facing each other and separated by a distance approximately equal to the diameter of the sensitive area. Midway between the diodes is a thin organic film, less than $10 \mu\text{g}/\text{sq cm}$, which carries a layer of LiF . The two products of the $\text{Li}^6(n, \alpha)$ reaction are detected by the surface-barrier diodes and the output pulses summed to give a pulse proportional in height to the total energy, reaction "Q" plus neutron energy, of the reaction. Fast coincidence requirements are used to eliminate unrelated events in each detector.

The general response of the instrument to a neutron flux is given by

$$C_i = \int_{E_n=0}^{E_{\max}} \phi(E_n) \sigma(E_n) K_i(E_n) dE_n, \quad (1)$$

where C_i is the number of events observed in Channel i ; $\phi(E_n)$ is the neutron-energy spectrum; $\sigma(E_n)$ is the $\text{Li}^6(n, \alpha)$ cross section; and $K_i(E_n) dE_n$ gives the probability that an event due to a neutron having an energy between E_n and $E_n + dE_n$ results in a count in Channel i .

For the practical case, this equation must be approximated by a matrix equation:

$$C_i = \sum_j K_{ij} \phi_j \sigma_j. \quad (2)$$

The matrix K_{ij} represents the response of the spectrometer. Each column of the matrix describes the spectrometer response to neutrons in energy channel j , that is, each column describes the response to monoenergetic neutrons. The complete experimental determination of the spectrometer response has not been carried out due to lack of a suitable neutron source; however, response to thermal neutrons has been measured. For neutrons with energies between 0 and about $\frac{1}{2}$ MeV, the total energy of the reaction products covers a range between 4.8 and 5.3 MeV. This is a range of about 10% in energy. It is assumed that the shape of the device response to monoenergetic neutrons of any energy in this range is the same as the thermal-neutron response. There is some experimental justification for this.³ It was also assumed that the probability of detecting an (n, α) event once it occurred was not a function of neutron

FIG. 2. Comparison of Spectra as Measured by $\text{Li}^6(n, \alpha)t$ and by Proton Recoil.

energy. This is an approximation which, over this limited range of neutron energy, should not cause a serious error.

The measurements were made at the center of the core about one inch from the midplane. This is very near the position of the proton-recoil measurements. The reactor was loaded to critical and the measurements were made at a power of about 2.5 W. Following the core runs, the Li^6 layer was removed and the background response of the same two diodes measured. It was completely negligible.

The pulse-height distribution in the "sandwich" spectrometer was not "unfolded" to give an estimate of the spectrum. Rather, the comparison with the proton-recoil data was made by calculating from the experimental spectrum an expected pulse-height distribution. This calculated distribution is the one expected if the "sandwich" spectrometer were placed in that spectrum. Experience has shown that the difference between these two curves at the low-energy end is particularly sensitive to the portion of the neutron spectrum below about 20 keV.

The expected distribution was calculated from eq. (2). The response matrix K_{ij} was constructed from the thermal-neutron response. The full width at half-maximum of the monoenergetic response for this experiment was 72 keV. The $\text{Li}^6(n, \alpha)\text{H}^3$ cross sections

were based on recent measurements,⁽⁴⁾ which show considerable difference from older published data.⁽⁵⁾

Figure 2 shows the two distributions. The agreement between the two is within counting statistics (about 400 counts/channel maximum) except at the peak corresponding to very low-energy neutrons. This small discrepancy, however, corresponds to a very small difference in total low-energy flux in the core because of the very rapid increase in $\text{Li}^6(n,\alpha)t$ cross section for low energy. To make the calculation of the expected distribution, the flux per unit lethargy as given by the proton-recoil experiment was extrapolated linearly to zero at thermal energy. The extrapolation was linear on an energy scale. A more rapidly decreasing estimate would have reduced the discrepancy in the low-energy region of the pulse-height distribution.

In general, the results from the Li^6 spectrometer are

consistent with those from the proton-recoil measurements.

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A Comment on the Comparison of Theoretical with Experimental Neutron Spectra in Fast Critical Assemblies

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1. Introduction

The experimental data obtained by Bennett in the measurement of central neutron spectra in fast critical assemblies⁽¹⁾ have a resolution of 15–20% in energy over the energy range, which extends approximately from 1 keV to 1 MeV. The high resolution, combined with the small statistical error of the data, makes it possible to measure, in addition to the macroscopic behavior of the neutron spectrum, some of the microscopic variations of the spectrum at those energies for which resonances of the light elements cause sharp variations of the transport cross section.

The greater resolution of the experimental spectra raises the question as to how accurately the present analytical methods can predict the microscopic variations of the spectrum, and adds considerable interest to the comparison of experiment with theory. It is the purpose of this note to show the results of two such comparisons, in which some detailed experimental spectra were matched to corresponding high-resolution calculations.

2. Experiments and Calculations

The experimental spectra used in the comparison were measured by Bennett^(1, 2) by means of his proton-recoil counter in two different reactors. One was Assembly 48 of ZPR-3, which has been extensively analyzed in connection with the intercomparison calculations.^(3, 4) The other reactor was Assembly 4Z of ZPR-6, which also has been the object of some papers at this conference.^(5, 6) The core compositions of the two reactors are given in Table 1. The reactors are by no means similar, because Assembly 48 is a plutonium-fueled reactor, while Assembly 4Z is fueled with U-235. Also the concentrations of the other core constituents are not the same in the two reactors, with the result that their spectra are different, the 4Z spectrum being softer. This means that the comparisons made for the two reactors are, to a certain extent, independent of each other.

The theoretical spectra were obtained by use of the ELMOE Code⁽⁷⁾ in connection with the Argonne Cross Section Set 224. The Pu-239 and U-235 cross sections took into account the recent measurements of White⁽⁸⁾ and the Columbia University Parameters were used for U-238.⁽⁹⁾

The ELMOE Code performs a fundamental-mode calculation of the spectrum, in which the elastic-scattering resonances of the light elements may be taken into account with great resolution. The resolution of the capture and inelastic resonances is limited to the resolution of the 22-group Set 224, but those resonances should affect the details of the spectrum only to a limited extent over the energy range that is considered in the comparison.

The high resolution for the elastic-scattering resonances was obtained by means of a 603-group calculation. The groups over the energy range of interest had a uniform lethargy width approximately equal to 0.0174, small enough to provide the shape of the theoretical spectra with a much greater resolution than can be expected from an experimental measurement. Besides, some calculations run with much greater resolution ($\sim 2,000$ groups) do not indicate that appreciable changes in the details of the theoretical spectra can be expected from the use of smaller lethargy widths.

The fundamental-mode calculation was made in the P-1 approximation. Both reactors are representative of large carbide cores, with small material bucklings (approximately 10^{-3} cm^{-2} and $2.5 \times 10^{-3} \text{ cm}^{-2}$, respectively, in Assembly 4Z and in Assembly 48) which depend only slightly on transport effects and for which the P-1 approximation is quite adequate.

The large size of the cores makes it possible to neglect also the effect that the presence of the reflector may have on the central neutron spectrum. This was checked by means of a 22-group S_4 calculation for Assembly 48 in spherical geometry with the SNARG-1D Program.⁽¹⁰⁾ The calculations showed that the

TABLE 1. CORE COMPOSITIONS, ATOMS CM⁻³ × 10⁻²⁴

Material	Assembly 48	Assembly 4Z
Pu-239	0.001644	—
Pu-240	0.000106	—
Pu-241	0.000010	—
U-235	0.000016	0.001531
U-238	0.007406	0.010560
C	0.020765	0.012920
Na	0.006230	0.009258
Fe	0.009899	0.009042
Cr	0.002658	0.002386
Ni	0.001308	0.001130
Mo	0.000206	—
Al	0.000110	—

normalized spectrum at the center of the reflected core is slightly softer than the corresponding fundamental-mode spectrum. However, the difference at any energy is never larger than 0.5% of the maximum value of the fundamental-mode spectrum.

At the time the experiments were made, the reactors were slightly subcritical. This might have caused some contamination of the fundamental mode at the center of the cores, because some higher modes could also have been excited by the uniform distribution of the external sources. In order to evaluate the extent to which this effect might impair the validity of a fundamental-mode calculation, the spectrum of a spherical reflected reactor with the core composition of Assembly 48 and an effective multiplication constant equal to 0.9, driven by a fission source distributed uniformly over the whole core, was calculated in 22-group diffusion theory with the MACH-1 Program.⁽¹¹⁾ The result of the calculation showed that the perturbation of the central neutron spectrum due to higher spatial modes makes the spectrum harder at the center of the reactor. The effect is somewhat stronger than the effect due to the presence of the reflector, but of opposite sign. When the two effects are combined, the changes of the spectrum from the fundamental-mode spectrum over the whole energy range of interest is never appreciably larger than 1% of the maximum value of the fundamental-mode spectrum.

The other limitations of the theory are due to the fact that neither heterogeneity effects, nor the perturbation of the neutron spectrum which is induced by the presence of the instrument itself, are taken to account.

3. The Comparison

Figures 1 and 2 show, respectively, the results of the comparison between theory and experiment for Assembly 48 and for Assembly 4Z.

The theoretical spectra are given in the form of histograms, which point out the resolution of the cal-

culations. Both theoretical spectra were obtained well in advance of the experimental data, and the Assembly 48 theoretical spectrum is the one that was predicted for ANL in connection with the intercomparison calculations.⁽⁴⁾ Thus, the spectra are representative of typical neutron spectra evaluated in connection with current reactor calculations. The histograms show many wide fluctuations which correspond to the scattering resonances of the light elements. The isotopes that contribute these resonances may be easily identified, and they are marked in both figures for some resonances of particular interest. Most of the finer oscillations that can be seen in the histograms at energies larger than 70 keV, and which are not identified in the figures, are due to iron resonances.

The experimental points are marked in the figures as vertical segments, whose length indicates their statistical errors. They are determined but for an arbitrary constant, and they have been normalized to give approximately the best possible fit with the theoretical histograms.

The experimental points are in very satisfactory agreement with theory over the general shape of the spectrum. The energy at which the maximum flux is found and the average energy of the neutrons seem to be given quite consistently by theory and by experiment in both reactors. The microscopic variations of the flux are also found with a striking correspondence both in theory and experiment. The oscillations of the experimental flux take place almost at the same energies as the theoretical oscillations, and their behavior is also very similar; however, the experimental oscillations have smaller amplitudes than the theoretical oscillations and extend over a wider energy range. This effect is probably due to the finite energy resolution of the proton-recoil counter.

In order to take the resolution of the instrument into account for a more significant comparison, the theoretical histogram was smoothed by means of a Gaussian window with a lethargy width of 0.15. This means that, if $\phi(u)$ is the flux represented by the histogram, the function

$$f(u) = \frac{\sqrt{2}}{w\sqrt{\pi}} \int_0^\infty \phi(u') \exp \left\{ -2 \left[\frac{u - u'}{w} \right]^2 \right\} \cdot du'; \quad w = 0.15$$

was calculated at any lethargy u and superimposed as a continuous curve to the theoretical histogram. The curve $f(u)$ corresponds to the response that would be obtained if an instrument with lethargic Gaussian resolution of 0.15 width was used to scan the theoretical spectrum. The shape of the window was chosen to be Gaussian because this should be fairly close to the

FIG. 1. Comparison of Theoretical with Experimental Neutron Spectra in Assembly 48. The histogram gives the result of the ELMOE calculation; the continuous curve shows the same results through a Gaussian window.

FIG. 2. Comparison of Theoretical with Experimental Neutron Spectra in Assembly 4Z. The histogram gives the result of the ELMOE calculation; the continuous curve shows the same results through a Gaussian window.

actual shape of the resolution of the proton recoil counter. The width of the window, w , was chosen in such a way that the amplitude of the oscillations of the function $f(u)$ was close to the amplitude of the experimental oscillations. The value obtained in this way (0.15) is close to the value estimated by Bennett⁽¹⁾ for the relative energy resolution of his proton-recoil counter (0.20). The shape of the window, its width, and their constancy with energy are somewhat arbitrary at this time; however, the values that have been chosen seem to be reasonable enough to provide a convenient means of comparison.

The agreement between the experiments and the smoothed theoretical curves is quite satisfactory over most of the flux oscillations. For Assembly 48, the smoothing reduces considerably the amplitude of the oscillation due to the sodium resonance at 2.85 keV, and moves the energy at which its minimum occurs toward a higher value which is in very good agreement with the experiment. The agreement between the experimental points and the smoother theoretical curve is almost perfect also over the two wide oscillations which, around 29.2 keV and around 51.0 keV, are due, respectively, to iron and to chromium resonances. Several other oscillations of the experimental points are also found in the smoothed theoretical curve with a consistency that indicates a clear correlation. The same excellent general agreement is found for Assembly 4Z. Some small discrepancies which are found for Assembly 48 around the energies of 7, 18, 90, and 400 keV are also consistently reflected in the comparison of the spectra of Assembly 4Z, and may point to some inaccuracy of the cross sections.

4. Conclusion

The agreement that has been found in the two cases which have been considered in this comparison seems to indicate that the methods used both in the theory and in the experiment are fundamentally correct. This reflects one of the advantages that spectrum measurements have over most other integral experiments in nuclear reactor physics, because the large number of details that must independently agree with each other in a spectral comparison reduces greatly the chance of compensating errors.

Some additional interest in the good agreement between theoretical and experimental spectra is brought about if one considers the possibility of using measured spectra to extract modified cross-section sets from the measurements, rather than using the experiment as a check of assumed cross-section sets. Several methods have been presented at this conference for obtaining this kind of information from integral experiments.⁽¹²⁻¹⁴⁾ All of them share three basic require-

ments for a useful application. First, the measured quantity must be very sensitive to changes of the cross sections of interest. Second, the result of the measurement must be reliable and accurate. Third, the cross sections should not be too much in error, because their modifications should be considered as perturbations of the assumed cross-section set rather than as drastic changes.

The first requirement, which is concerned with the sensitivity of the experiment to the details of the cross sections, is probably satisfied by a spectrum measurement better than by any other measurement performed in a nuclear reactor. This is due to the fact that a disagreement between theory and experiment around a given energy can be pinpointed to a cross-section inadequacy within a restricted energy range, rather than to an integral inadequacy as happens in most other types of nuclear reactor measurements. The remaining two requirements, which are concerned with experimental and theoretical accuracy, could be considered to be satisfied if an extensive and systematic comparison of theory with experiment would show a consistent pattern of agreement of the type that has been found in this comparison.

Acknowledgment

The author wishes to thank D. A. Meneley and G. J. Fischer for many useful discussions on the cross sections and on the final results. D. T. Woodruff and his DROPLET code were very helpful in plotting the fluxes.

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Comment on Spectrum Measurements in a Large, Dilute Plutonium-fueled Fast Reactor*

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General Electric has been working on a proton-recoil spectrometer very similar to that developed by E. Bennett of ANL. The preamplifier is identical to that described by Bennett,⁽¹⁾ but the remainder of the electronics consists of equipment normally used with solid-state detectors. Discrimination in pulse rise time is used to separate gamma-background effects. Pulse height is fed to the y axis of a 4096-channel two-parameter analyzer, and the derivative of the pulse rise is fed to the x axis, whereas in Bennett's system the ratio of the derivative to the pulse height is fed to the x axis. The neutron events fall on the diagonal, from lower left to upper right, of a two-parameter array, and the gamma events fall to the left of the diagonal. Excellent discrimination between gammas and neutrons has been achieved. A cobalt-60 source is used to provide a gamma background which is normalized to the gamma-only region of the two-parameter array, thereby permitting a correction for the few gamma events which fall along the diagonal. The chambers are stainless steel tubes, one inch in diameter by 7 in. long, with an effective length of 3 in. They are filled with either 3 atm of hydrogen gas or 4 atm of methane, and contain some nitrogen for calibration, which is done by use of protons with an effective energy of 615 keV produced by the thermal-neutron reaction $N^{14}(n,p)C^{14}$. The hydrogen tubes contain a small amount of methane for quenching.

The spectrometer has been used to measure the neutron spectrum at the centers of the core and the U-238 blanket of ZPR-3 Assembly 48. Although the blanket measurement was performed without difficulty, high detector count rates were encountered in the core, due to the appreciable Pu-240 spontaneous-fission source. Had smaller, less sensitive chambers been used, the count rate would have been sufficiently low. Distortion in the proton-recoil data made it necessary to separate the table halves and withdraw all control

rods (i.e., fuel) in order to reduce the count rate to an acceptable level for the core measurements. Even then, some distortion remained in the data, consisting of a small fraction of the events falling to the right of the diagonal, where no events should occur. Subsequent analysis revealed that these events were caused by neutrons, and that count-rate-sensitive component, apparently in the rate of rise channel of the electronics, was affecting the differentiated pulses going to the x axis of the analyzer. No evidence could be found that pulse heights (i.e., proton energies) were being affected. Hence, it was assumed that the events to the right of the diagonal could be included with the proton peak at the diagonal. This assumption was apparently valid since the parts of the spectrum measured at different high-voltage settings and with different chambers agreed at their end points.

The neutron spectrum at the core center at a table separation of 3 in. is given in Fig. 1 and compared with a spectrum calculated using 60-group one-dimensional diffusion theory. To determine the effects on the measurement of separating the table halves, the measurement was repeated at a separation of 4 in. With the use of only the methane chambers, the spectrum above 140 keV was measured with the table halves closed and all control rods inserted. Comparison of the three measurements showed that the spectra softened with increased table separation because of the enhanced high-energy leakage, but that the degree of softening was approximately equal to the statistical accuracy of the measurement, namely, about 5%. Hence, it is believed that the measurements with the table halves separated are fairly representative of the asymptotic neutron spectrum. The measured spectrum is in general agreement with that calculated. The lower limit of measurement was 14 keV because count rates below that energy were too high for the electronics. Resonance depressions are evident in the measured spectra, occurring at energies about 15% higher than nearby calculated depressions. This disagreement is considerably higher than the estimated uncertainty in the

* Work done under AEC Contract AT(04-3)-189, P.A. 10, Sodium Cooled Reactors, Fast Ceramic Reactor Development Program.

FIG. 1. Neutron Spectrum at Core Center of ZPR-3 Assembly No. 48.

FIG. 2. Neutron Spectrum in Blanket of ZPR-3 Assembly No. 48.

energy calibration and may indicate a systematic error in the method. The measured depressions are wider and not as deep as those calculated because of the finite energy resolution of the measurement.

The neutron spectrum measured in the blanket, again with a lower limit of 4 keV, is given in Fig. 2. The gaps in the spectrum arise because insufficient data were taken. The measured spectrum is considerably softer than that calculated with 60-group one-dimensional diffusion theory. Bennett⁽²⁾ has observed similar disagreement between theory and experiment in a U-238 Snell Block. The agreement between the measured and calculated resonance depressions is about the same as that in the core.

Acknowledgments

Thanks are expressed to F. W. Thalgott, J. K. Long, R. J. Huber, and the staff of ZPR-3 for their excellent cooperation during these measurements. The efforts of J. J. Del Cotto and H. F. Johnston in the solution of the electronics problems have contributed appreciably to the work.

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Discussion

Mr. Tuttle (AI): Have you checked the perturbation of the spectrum caused by the lead shield around the counter? Have you applied gamma discrimination over the entire energy range? If you have, have you checked the effect of this? Is there a significant difference above, say 50 keV, between the gamma-discriminated spectrum and the untreated spectrum?

Mr. Bennett: Yes, I have checked the effect of the 6 mm of lead. I only did this above 100 keV because I felt that inelastic scattering in the lead would be most significant here. I found no significant difference. The shape discrimination was applied everywhere except in the methane counter above about 300 or 400 keV. The gamma-initiated events increase markedly at the lower energies, and it would be impossible to analyze these data in the presence of the amount of gamma radiation that one ordinarily gets in the reactor.

Mr. Stegemann (Karlsruhe): Having had the experience with the plutonium-fueled assemblies and using the gamma-discrimination technique, what is your thinking about the lower energy limit of the spectrometer?

Mr. Bennett: As I mentioned previously, the pur-

pose of these measurements was more to determine what could be done than to get accurate spectrum measurements. These measurements are not good from the point of view of resolution or statistics. The reason that they were not carried to higher accuracy was because of the possibility of systematic errors. I mentioned this for the higher energy region. I don't think, for instance, that any comparisons with calculations above 500 keV are very meaningful because of wall- and end-effect corrections. At low energies, it would be very useful to have a better experimental understanding of several things, such as the energy required per ion pair formed, in order to say definitely what kind of systematic errors are present. I don't think that they are very large above about 1.5 keV.

Mr. Amundson (ANL): Travelli's calculations were for a homogeneous system. The measured spectrum in Assembly 48 is an average over a very heterogeneous cell. Consequently these large deviations between the dips that were calculated and the ones that were measured are to be expected.

Mr. Baker (UK-Winfrith): I would like to know, in view of the good agreement in the core, whether there are any reasons why the blanket measurements might be inferior?

Mr. Bennett: I have one more slide which bears upon this question. This is a measurement of an "asymptotic" uranium block spectrum of neutrons 53.3 cm from the face of a slab of depleted uranium (see Fig. 1 of this discussion). Thermal neutrons incident upon the face of the slab are converted and at a sufficient distance inside the slab the spectrum will cease to change shape. A 22-group diffusion calculation is also shown for comparison. The shape-discrepancy is very large. The data are subject to numerous systematic errors. For example, large flux depressions due to resonances in the stainless steel environment of the detector are clearly discernible.

As far as I know, the same cross sections that Travelli used for the ELMOE calculations were used here. It is a little surprising, as U-238 is a major component for all these systems. I can't really understand why one gets poor agreement for the case of an almost pure U-238 environment and a good agreement for Assembly 48, for example. Travelli mentioned previously that he may have been using different U-238 cross sections than I used for the Snell block.

Mr. Russell (General Atomic): There have been at least two or three measurements in U-238. They all show that the calculated spectra do not have enough neutrons at the lower energies. For the calculated spectra in an in-core situation, one has a lot of light material, which is the primary means of getting neu-

FIG. 1. Spectrum at 53.3 cm in a Snell Block.

trons into the low-energy groups in the large cores we are discussing. The comparison here is much better. The implication is that the inelastically scattered neutron spectrum in U-238 is being handled incorrectly.

Mr. Butler (ANL): The Li-6 measurements which you showed appear to be very inferior to the proton-recoil measurements. I'd like to know whether this means that Li-6 does not look promising, or is this just because these are just preliminary results?

Mr. Huber (ANL): I agree with you that the proton-recoil results look much better. It's rather obvious that the neutron-energy resolution, $\delta E/E$, goes to ∞ , if you have a fixed resolution on the total energy as measured by the two reaction products. There is some possibility of achieving a better resolution with the Li-6 sandwich method at the higher energy end. It will

not compete favorably with the proton-recoil method at low energy.

Mr. Weale (UK-Aldermaston): What was the thickness of the foil that you used in the Li-6 measurement?

Mr. Huber: Of the order of $20 \mu\text{g}/\text{cm}^2$.

Mr. Davey (ANL): In Table 11 of my paper in Session II there is approximately a 50% discrepancy between the Argonne calculations by Travelli and the measured spectrum. It would seem, however, that this discrepancy is not a real one. The difficulty may be that the pulse-height spectrum that was averaged to obtain the experimental data may be displaced due to the experimental resolution effects. The agreement is indeed much better at low energies than was apparent from these coarse tables.

Presented by N. L. Snidow

Wall-effect Corrections in Proportional-counter Spectrometers

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ABSTRACT

The measurement of neutron spectra with proton-recoil spectrometers sometimes requires wall-effect corrections. This article presents an accurate method of calculating the wall effect, when measuring an isotropic neutron flux, for a spherical counter without a dead region and for a finite-length cylindrical counter, either with or without a dead region at each end. Analytical expressions are obtained for four path-length probability functions, in which the geometric aspects are handled in an exact manner. The path-length probability functions and range data are applied to proton-recoil spectrometers to determine the energy lost in the sensitive region by protons starting in either the dead region or the sensitive region and stopping in the gas or hitting the wall.

To verify the calculated wall effect experimentally, two cylindrical proportional counters, one containing helium-3 in methane and the other helium-3 in hydrogen, were subjected to thermal neutrons. The observed spectra are compared with the calculated spectra. Results are given for cases in which most of the recoil particles lose some energy in collisions with the wall.

1. Introduction

If an isotropic neutron flux is incident on a proportional counter containing hydrogen, the proton recoils according to scattering theory are isotropic and have an energy distribution given by

$$P(E) = \int_E^\infty \frac{\Sigma(E')\phi(E')}{E'} dE', \quad (1)$$

where $\Sigma(E')$ is the macroscopic scattering cross section of hydrogen at energy E' , and $\phi(E')$ is the neutron flux per unit energy. Thus $P(E) dE$ is the number of recoil protons produced per unit volume per unit time having energy between E and $E + dE$. If all protons are stopped in the counter gas and if the ionization produced is proportional to the proton energy, then eq. 1 also gives the distribution of pulse heights from the counter.

In addition to the contribution of recoil protons to the pulse spectrum, there is a gamma contribution, par-

ticularly at low energy, from Compton-scattered electrons. The usefulness of proton-recoil spectrometers at low energy depends largely on how well the gamma contribution can be eliminated. Benjamin⁽¹⁾ has found that the maximum energy of the gamma response can be made lower than the energy of the recoil protons under observation by reducing the gas pressure. In this case, however, all of the protons are not stopped, and the distribution of pulse heights from the counter is no longer given by eq. 1. Benjamin has used a Monte Carlo calculation to correct for this. Bennett⁽²⁾ has developed a gamma discriminator which distinguishes between counts induced by electrons and protons and, therefore, allows operation at higher pressure. If, however, the range of the recoil protons of highest energy produced is comparable to the counter dimensions, then some corrections due to a wall effect are necessary.

If the range of some of the protons is appreciable and if the counter has a dead region (usually present at both ends of cylindrical counters), then four different events may be distinguished. Recoil protons may:

1. start in the sensitive region and be stopped in the sensitive region;
2. start in the sensitive region and hit the wall or enter the dead region after losing some energy in the sensitive region;
3. start in the dead region and be stopped in the sensitive region;
4. start in the dead region, enter the sensitive region after losing some energy in the dead region, and then hit the wall or enter another dead region after losing some energy in the sensitive region.

In each case, the count obtained is determined by the energy lost by the proton in the sensitive region. In this article, analytical expressions are obtained for four path-length probability functions, which are then used to determine the probability of each of the events listed above. The geometric aspects are handled in an exact manner assuming that it is equally likely that a path length starts at any point in a given volume and that any direction from the starting point is equally likely. In applying the path-length probability functions to

counters, additional assumptions are made. The general derivation of the path-length probability functions is in Sect. 2. The specific derivation for a sphere with no dead region is in Sect. 3. The specific derivation for a cylinder with a dead region is in Sect. 4. Applications to proportional spectrometers are made in Sect. 5 and 6.

2. General Derivation of Path-length Probability Functions

In arbitrary volume V_s shown in Fig. 1, it is assumed that one path length per unit volume starts everywhere in the volume and that any direction is equally likely. The total number of paths being considered is then V_s . The volume is translated a distance l in a direction opposite to that defined by a unit vector $\bar{\Omega}$. $V_c(l, \theta, \phi)$ is defined as the volume common to both the original volume and the translated volume. If the volume is translated an additional distance dl , then

$$-\frac{\partial V_c}{\partial l} dl$$

defines a volume element where paths, starting in directions within a solid angle $d\Omega$ around $\bar{\Omega}$, have lengths to the surface of the original volume between l and $l + dl$. Since one path starts per unit volume, the number starting in the volume is just equal to the volume element, and the fraction of these in $d\Omega$ is

$$\frac{\sin \theta d\theta d\phi}{4\pi}.$$

The total number of path lengths between l and $l + dl$ is obtained by integration over the range of θ and ϕ for which l is possible. $N(l) dl$, the probability that a path length is between l and $l + dl$, is obtained by dividing by the total number of paths, V_s . Therefore, after cancelling dl on both sides,

$$N(l) = \frac{1}{4\pi V_s} \iint \left(-\frac{\partial V_c}{\partial l} \right) \sin \theta d\theta d\phi.$$

Only shapes with ϕ symmetry are considered in this paper; therefore,

$$N(l) = \frac{1}{2V_s} \int -\frac{\partial V_c}{\partial l} \sin \theta d\theta, \quad (2)$$

where the integration extends over the range of θ in the interval from 0 to π for which l is possible.

Those paths which start in V_c and have a direction in $d\Omega$ about $\bar{\Omega}$ must have a length greater than l to the surface of the original volume. The number starting in this common volume is just V_c , and the fraction of these in $d\Omega$ is $(\sin \theta / 4\pi) d\theta d\phi$. For shapes with ϕ symmetry, the probability that a path length is greater than some value l is

$$F(l) = \frac{1}{2V_s} \int V_c(l, \theta) \sin \theta d\theta, \quad (3)$$

where the integration extends over the range of θ in the interval from 0 to π for which l is possible. $F(l)$ may also be obtained from

$$F(l) = \int_l^{l_{\max}} N(l') dl',$$

where $l_{\max}$ is the maximum chord length of the volume. For $l > l_{\max}$, $F(l) = N(l) = 0$.

In preparation for treating dead regions, a circular cylinder divided into two circular cylinders with volumes V_s and V_d is considered in Figure 2. In later application to counters, V_s will be a sensitive region and V_d a dead region. Only those paths which start in V_d and enter V_s will be considered. The entire volume is translated a distance l in a direction opposite to that defined by the unit vector $\bar{\Omega}$. In addition, volume V_d is translated a distance l' in the same direction opposite to $\bar{\Omega}$. V_{cc} is defined as the volume common to the entire volume before and after translation and also common to volume V_d before and after translation. Paths starting in the volume element

$$\frac{\partial^2 V_{cc}}{\partial l \partial l'} dl' dl$$

and traveling in the direction $d\Omega$ around $\bar{\Omega}$ have a path length in V_d between l' and $l' + dl'$ and a total path length to the cylindrical surface between l and $l + dl$. If the probability of paths starting in V_d and meeting

FIG. 1. Volume V_c Used to Determine $F(l)$ and Volume Element $-(\partial V_c / \partial l) dl$ Used to Determine $N(l)$.

FIG. 2. Volume Element $-(\partial V_{cc}/\partial l') dl'$ Used to Determine $G(l', l)$ and Volume Element $(\partial^2 V_{cc}/\partial l \partial l') dl' dl$ Used to Determine $H(l', l)$.

this specification is $H(l', l) dl' dl$, then by the same arguments given above

$$H(l', l) = \frac{1}{2V_d} \int \frac{\partial^2 V_{cc}}{\partial l' \partial l} \sin \theta d\theta. \quad (4)$$

Since only paths to the right need be considered, the integration extends over the range of θ in the interval from 0 to $\pi/2$ for which the combination of l' in V_d and total path l to the cylindrical surface is possible.

If both l' and l are specified for paths which start in V_d and leave V_s through the far end, then at most only one value of θ , namely, $\theta = \cos^{-1} [L/(l - l')]$, is possible. Paths starting in the volume element defined by $-(\partial V_{cc}/\partial l') dl'$ evaluated at $\theta = \cos^{-1} [L/(l - l')]$ and traveling in the direction defined by $d\theta$ about θ have a path length in V_d between l' and $l' + dl'$. If $d\theta = (1/l)(\cot \theta) dl$, then they also have a total path length to the far end between l and $l + dl$. If $E(l', l) dl' dl$ is the probability that paths starting in V_d have a path length in V_d between l' and $l' + dl'$ and a total path length to the far end between l and $l + dl$, then

$$E(l', l) = -\frac{L}{2V_d(l - l')} \frac{\partial V_{cc}}{\partial l'} \left(\theta = \cos^{-1} \frac{L}{l - l'} \right). \quad (5)$$

If the combination of l' and l is impossible, $E(l', l) = 0$.

$M(l', l) dl' dl$ is defined to be the probability that paths starting in V_d enter V_s after having a path length in V_d between l' and $l' + dl'$ and have a total path length to the surface between l and $l + dl$. Therefore,

$$M(l', l) = H(l', l) + E(l', l). \quad (6)$$

Considering Fig. 2 again, $-(\partial V_{cc}/\partial l') dl'$ defines a volume element where paths, starting in directions within $d\Omega$ around $\bar{\Omega}$, have lengths in V_d between l' and $l' + dl'$ and total lengths greater than l to the surface. If the probability of paths starting in V_d and meeting this specification is $G(l', l) dl'$, then by the same arguments given previously

$$G(l', l) = \frac{1}{2V_d} \int -\frac{\partial V_{cc}}{\partial l'} \sin \theta d\theta, \quad (7)$$

where the integration extends over the range of θ in the interval from 0 to $\pi/2$ for which the combination of l' and l is possible.

3. $N(l)$ and $F(l)$ for a Sphere with No Dead Region

If the volume in Fig. 1 is considered to be a sphere with radius a , then

$$V_c = \pi a^3 \left(\frac{4}{3} - \frac{l}{a} + \frac{l^3}{12a^3} \right) \quad (8)$$

and

$$-\frac{\partial V_c}{\partial l} = \pi \left(a^2 - \frac{l^2}{4} \right). \quad (9)$$

Substitution of eq. 9 into eq. 2 and of eq. 8 into eq. 3 gives

$$N(l) = \frac{\pi}{2V_s} \left(a^2 - \frac{l^2}{4} \right) \int \sin \theta d\theta$$

and

$$F(l) = \frac{1}{2V_s} \left(\frac{4}{3} \pi a^3 - \pi a^2 l + \frac{\pi l^3}{12} \right) \int \sin \theta d\theta,$$

where the integration extends over the range of θ in the interval from 0 to π for which l is possible. If $l > 2a$, no value of θ is possible and $N(l) = F(l) = 0$. For $l < 2a$, θ may extend over its entire range from 0 to π , so that

$$N(l) = \frac{3}{4a} \left(1 - \frac{l^2}{4a^2} \right) \quad (10)$$

and

$$F(l) = 1 - \frac{3l}{4a} + \frac{1}{16} \frac{l^3}{a^3}, \quad (11)$$

where $4\pi a^3/3$ has been substituted for V_s . Plots of $N(l)$ and $F(l)$ are given in Fig. 3.

4. $N(l)$, $F(l)$, $M(l', l)$, and $G(l', l)$ for a Finite-length Cylinder with a Dead Region

Consider volume V_s in Fig. 1 to be a circular cylinder of length L and radius a with its axis aligned with the z

FIG. 3. $F(l)$ and $aN(l)$ for a Sphere with Radius a .

axis. Then V_c is given by

$$V_c = (L - l \cos \theta) \left(\pi a^2 - l \sin \theta \sqrt{a^2 - \frac{l^2 \sin^2 \theta}{4}} - 2a^2 \sin^{-1} \frac{l \sin \theta}{2a} \right) \quad (12)$$

and

$$-\frac{\partial V_c}{\partial l} = \sqrt{1 - \frac{l^2}{4a^2} \sin^2 \theta} (2aL \sin \theta - 3al \sin \theta \cos \theta) + \pi a^2 \cos \theta - 2a^2 \cos \theta \sin^{-1} \frac{l \sin \theta}{2a}. \quad (13)$$

Substituting eq. 13 into eq. 2, replacing V_c with $\pi a^2 L$, and defining k as $l/2a$ and α as $L/2a$, we obtain

$$N(l) = \frac{2}{\pi a} \int_{\cos^{-1} m}^{\sin^{-1} n} \sin^2 \theta \sqrt{1 - k^2 \sin^2 \theta} d\theta - \frac{3k}{\pi a \alpha} \int_{\cos^{-1} m}^{\sin^{-1} n} \sin^2 \theta \cos \theta \sqrt{1 - k^2 \sin^2 \theta} d\theta + \frac{1}{2a\alpha} \int_{\cos^{-1} m}^{\sin^{-1} n} \sin \theta \cos \theta d\theta - \frac{1}{\pi a \alpha} \int_{\cos^{-1} m}^{\sin^{-1} n} \cos \theta \sin \theta \sin^{-1}(k \sin \theta) d\theta. \quad (14)$$

Because of symmetry, eq. 14 has been multiplied on the right by 2, and the limits of integration have been changed to the range of θ in the interval from 0 to $\pi/2$ for which l is possible. These limits are $\cos^{-1} m < \theta < \sin^{-1} n$, where m is the smaller of L/l and 1, and n is the smaller of $1/k$ and 1.

$$N(l) = 0 \quad \text{if } m^2 + n^2 < 1, \quad (15)$$

i.e., if

$$l > \sqrt{L^2 + 4a^2}.$$

The first integral in eq. 14 is given by Gröbner and Hofreiter⁽³⁾ in terms of elliptic integrals, and the other integrals are straightforward. The solution for $N(l)$ for $l < 2a$, i.e., $k < 1$, is

$$N(l) = \frac{2(1 - k^2)}{3\pi a k^2} [K(k) - F(\cos^{-1} m, k)] - \frac{2(1 - 2k^2)}{3\pi a k} [E(k) - E(\cos^{-1} m, k)] - \frac{1 + 4k^2}{8\pi a k^2} \sin^{-1} k + \frac{[1 + 4k^2(1 - m^2)]}{8\pi a k^2} \sin^{-1} k \sqrt{1 - m^2} + \frac{m^2}{4\alpha a} + \left[\frac{2m}{3\pi a} + \frac{6k^2(1 - m^2) - 1}{8\pi a k} \right] \cdot \sqrt{(1 - m^2)[1 - k^2(1 - m^2)]} + \frac{1 - 6k^2}{8\pi a k} \sqrt{1 - k^2}. \quad (16)$$

For $l > 2a$, i.e., $k > 1$, the solution is

$$N(l) = \frac{2(2k^2 - 1)}{3\pi a k} \left[E\left(\frac{1}{k}\right) - E(\sin^{-1} k \sqrt{1 - m^2}, 1/k) \right] - \frac{4(k^2 - 1)}{3\pi a k} K\left(\frac{1}{k}\right) + \frac{4(k^2 - 1)}{3\pi a k} F\left(\sin^{-1} k \sqrt{1 - m^2}, \frac{1}{k}\right) + \left[\frac{2m}{3\pi a} + \frac{6k^2(1 - m^2) - 1}{8\pi a k} \right] \quad (17)$$

$$\begin{aligned} & \cdot \sqrt{(1-m^2)[1-k^2(1-m^2)]} \\ & + \frac{4k^2(1-m^2)+1}{8\pi\alpha k^2} \sin^{-1}k\sqrt{1-m^2} \\ & - \frac{1+4k^2(1-m^2)}{16\alpha k^2}, \end{aligned}$$

where $F(\phi, k)$ and $E(\phi, k)$ are incomplete elliptic integrals of the first and second kind, respectively:

$$\begin{aligned} F(\phi, k) &= \int_0^\phi \frac{d\theta}{\sqrt{1-k^2\sin^2\theta}}; \\ E(\phi, k) &= \int_0^\phi \sqrt{1-k^2\sin^2\theta} d\theta. \end{aligned}$$

$K(k)$ and $E(k)$ are complete elliptic integrals given by

$$K(k) = F(\pi/2, k); \quad E(k) = E(\pi/2, k).$$

$F(l)$ is obtained by substituting eq. 12 into eq. 3 and performing the integration. The limits of integration are again from $\cos^{-1}m$ to $\sin^{-1}n$, where m is the smaller of L/l and 1. The solution for $l < 2a$, i.e., $k < 1$, is

$$\begin{aligned} F(l) &= \frac{4(1-k^2)}{3\pi k} [K(k) - F(\sin^{-1}\sqrt{1-m^2}, k)] \\ & - \frac{4(1+k^2)}{3\pi k} [E(k) \\ & - E(\sin^{-1}\sqrt{1-m^2}, k)] \\ & + \frac{4k^2-1}{4\pi\alpha k} \sin^{-1}k \\ & - \left[\frac{2m}{\pi} + \frac{4k^2(1-m^2)-1}{4\pi\alpha k} \right] \\ & \cdot \sin^{-1}k\sqrt{1-m^2} + m - \frac{km^2}{2\alpha} \\ & - \left[\frac{2km}{3\pi} + \frac{2k^2(1-m^2)+1}{4\pi\alpha} \right] \\ & \cdot \sqrt{(1-m^2)[1-k^2(1-m^2)]} \\ & + \frac{1+2k^2}{4\pi\alpha} \sqrt{1-k^2}. \end{aligned} \quad (18)$$

For $l > 2a$, i.e., $k > 1$, the solution is

$$\begin{aligned} F(l) &= \frac{4(k^2-1)}{3\pi} \left[K\left(\frac{1}{k}\right) \right. \\ & - F\left(\sin^{-1}k\sqrt{1-m^2}, \frac{1}{k}\right) \Big] \\ & - \frac{4(k^2+1)}{3\pi} \left[E\left(\frac{1}{k}\right) \right. \\ & - E\left(\sin^{-1}k\sqrt{1-m^2}, \frac{1}{k}\right) \Big] \end{aligned}$$

$$\begin{aligned} & - \left[\frac{2m}{\pi} + \frac{4k^2(1-m^2)-1}{4\pi\alpha k} \right] \\ & \cdot \sin^{-1}k\sqrt{1-m^2} + m - \frac{1}{8\alpha k} \\ & + \frac{k}{2\alpha}(1-m^2) \\ & - \left[\frac{2km}{3\pi} + \frac{2k^2(1-m^2)+1}{4\pi\alpha} \right] \\ & \cdot \sqrt{(1-m^2)[1-k^2(1-m^2)]}. \end{aligned} \quad (19)$$

Plots of $N(l)$ and $F(l)$ are given in Fig. 4 for a cylinder with a length-to-diameter ratio of three ($\alpha = 3$).

For the cylinder in Fig. 2,

$$\begin{aligned} V_{cc} &= (L' - l' \cos \theta) \left(\pi a^2 - la \sin \theta \right. \\ & \cdot \sqrt{1 - \frac{l^2}{4a^2} \sin^2 \theta} - 2a^2 \sin^{-1} \frac{l \sin \theta}{2a} \Big); \\ - \frac{\partial V_{cc}}{\partial l'} &= \cos \theta \left(\pi a^2 - la \sin \theta \right. \\ & \cdot \sqrt{1 - \frac{l^2}{4a^2} \sin^2 \theta} - 2a^2 \sin^{-1} \frac{l \sin \theta}{2a} \Big) \end{aligned} \quad (20)$$

and

$$\frac{\partial^2 V_{cc}}{\partial l \partial l'} = 2a \cos \theta \sin \theta \sqrt{1 - \frac{l^2}{4a^2} \sin^2 \theta}. \quad (21)$$

$H(l', l)$ is obtained by substituting eq. 21 into eq. 4 and performing the integration. The integration extends over the range of θ in the interval from 0 to $\pi/2$ for which the combination of l' and l is possible. This is accomplished by letting the limits of integration be from $\cos^{-1}m$ to $\sin^{-1}n$, where m and n are given by

$$\begin{aligned} m &= \frac{L'/l'}{1} \Bigg\} \text{the smallest;} \\ n &= \frac{1/k}{1} \Bigg\} \text{the smaller.} \end{aligned} \quad (22)$$

The resulting equation, where $k = l/2a$, is

$$\begin{aligned} H(l', l) &= \frac{a}{8V_d k^2} \left\{ n(2n^2 k^2 - 1)(1 - n^2 k^2)^{1/2} \right. \\ & + [1 - 2k^2(1 - m^2)] \\ & \cdot \sqrt{(1 - m^2)[1 - k^2(1 - m^2)]} \\ & \left. + \frac{1}{k} (\sin^{-1}kn - \sin^{-1}k\sqrt{1 - m^2}) \right\}, \end{aligned} \quad (23)$$

where m and n are determined from eq. 22.

$E(l', l)$ is obtained by substituting eq. 20 into eq. 5 to

FIG. 4. $F(l)$ and $aN(l)$ for a Cylinder with Radius a and Length-to-diameter Ratio of 3.

provide the following result, where m is determined by eq. 22:

$$E(l', l) = \frac{am^2}{4V_d k} [\pi - 2k \sqrt{(1-m^2)[1-k^2(1-m^2)]} - 2 \sin^{-1} k \sqrt{1-m^2}] ; \quad (24)$$

$$E(l', l) = 0 \quad \text{if} \quad m \neq \frac{L}{l-l'}.$$

$M(l', l)$ is given by

$$M(l', l) = H(l', l) + E(l', l);$$

$$M(l', l) = 0 \quad \text{if} \quad l' > l \quad \text{or if} \quad m^2 + n^2 < 1.$$

$G(l', l)$ is obtained by substituting eq. 20 into eq. 7 and performing the integration. The limits of integration are from $\cos^{-1} m$ to $\sin^{-1} n$, where m and n are given by

eq. 22. This gives

$$G(l', l) = \frac{a^2}{4V_d} \left\{ \pi(n^2 + m^2 - 1) + 2 \left(\frac{1}{4k^2} - n^2 \right) \cdot \sin^{-1} kn + 2 \left(1 - m^2 - \frac{1}{4k^2} \right) \sin^{-1} k \cdot \sqrt{1-m^2} - n \left(\frac{1}{2} + k^2 n^2 \right) \left(\frac{1}{k^2} - n^2 \right)^{1/2} + \left[\frac{1}{2} + k^2(1-m^2) \right] \cdot \sqrt{(1-m^2) \left(\frac{1}{k^2} - 1 + m^2 \right)} \right\}. \quad (25)$$

When $l = 0$ or $2a$, the equations for $N(l)$ and $F(l)$ give indeterminates, as do the equations for $M(l', l)$

and $G(l', l)$ when $l' = l = 0$. The limiting values as $l \rightarrow 0$ or $l = 2a$ are found to be

$$\begin{aligned} N(0) &= \frac{1}{2a} + \frac{1}{4\alpha a} = \frac{S}{4V}; \\ N(2a) &= \frac{2}{3\pi a} - \frac{1}{16\alpha a} \quad \text{for } \alpha > 1; \\ F(0) &= 1; \\ F(2a) &= 1 - \frac{8}{3\pi} - \frac{1}{8\alpha} \quad \text{for } \alpha > 1; \\ M(0,0) &= \frac{a}{3V_d}; \\ G(0,0) &= \frac{\pi a^2}{4V_d}. \end{aligned}$$

5. Application of Path-length Probability Functions to Counters

The expressions for $N(l)$, $F(l)$, $M(l', l)$, and $G(l', l)$ just derived may be applied to proton-recoil proportional spectrometers under certain conditions. The generation of recoil protons must be isotropic and uniform throughout the counter. The range $R(E)$ of protons in the counter gas must be known, and each recoil proton must travel in a straight line. Under these conditions $N(l) dl$ is the probability that protons generated in the sensitive volume will follow a path with length between l and $l + dl$ to the surface of the sensitive volume. Among these protons generated in sensitive volume $F(l)$ is the fraction of protons that follow paths with length greater than l to the surface of the sensitive volume. If we let $l = R(E)$, then $F(R) = F(E)$ gives the fraction of protons stopped in the sensitive volume after generation in the sensitive volume at energy E . $M(l', l) dl' dl$ is the probability that protons generated in a dead region will follow a path with length between l and $l + dl$ to the surface of the sensitive volume and that part of this path, a distance between l' and $l' + dl'$, will be in the dead region. $G(l', l) dl'$ is the probability that recoil protons generated in a dead region with range l will follow a path with length in the dead region between l' and $l' + dl'$ and then be stopped in the sensitive volume.

As an example, consider 100-keV recoil protons generated uniformly in a 2.0-cm-radius sphere filled with one atmosphere of hydrogen. If l is equal to the range of the protons, 0.362 cm for this case, then l/a is equal to 0.181. From Fig. 3 or eq. 11, F is equal to 0.864, that is, 86.4 % of the 100-keV protons are stopped in the hydrogen, and 13.6 % hit the wall.

Next, consider 400-keV protons, which have a range of 2.13 cm. Again using Fig. 3 or eq. 11, we find that

27.7 % are stopped in the hydrogen and 72.3 % hit the wall.

As another example, consider a 2-cm-radius cylindrical counter with a 6-cm-long sensitive region filled with one atmosphere of hydrogen. We find, from Fig. 4 or eq. 18, that 89.6 % of the 100-keV protons and 43.5 % of the 400-keV protons are stopped in the sensitive volume and the remainder hit the wall.

Let $P(E') dE'$ be the number of recoil protons produced per unit volume per unit time having energy between E' and $E' + dE'$ in the cylindrical counter in Fig. 2. An expression can now be written for $C(E) dE$, the number of recoil protons that lose energy between E and $E + dE$ in the sensitive volume per unit time. This will give the observed count-rate spectrum, provided a count is obtained between E and $E + dE$ for each proton that loses energy between E and $E + dE$ in the sensitive volume. $C(E)$ is given by

$$\begin{aligned} C(E) &= F(E)P(E)V_s + V_s \int_E^{E'_{\max}} \frac{dR}{dE} (E' - E) \\ &\quad N[R(E') - R(E' - E)] P(E') dE' \\ &\quad + V_d \frac{dR(E)}{dE} \int_E^{E'_{\max}} G[R(E')] \\ &\quad - R(E), R(E')] P(E') dE' + V_d \int_{E=E}^{E'_{\max}} \\ &\quad \cdot \int_{E''=0}^{E'-E} \frac{dR}{dE} (E' - E'' - E) \frac{dR}{dE} \\ &\quad \cdot (E' - E'') M[R(E')] \\ &\quad - R(E' - E''), R(E')] \\ &\quad - R(E' - E'' - E)] P(E') dE' dE'', \end{aligned} \quad (26)$$

where F , N , G , and M are the appropriate functions from Sect. 4, evaluated at values of the arguments given in eq. 26, and where $R(E)$ is the range of protons having energy E . The four terms on the right, in order, give the contribution to $C(E)$ from the four types of events itemized in Sect. 1. $E'_{\max}$, the upper limit of the integral over E' , is the maximum energy of recoil protons generated. There is a maximum energy for a proton that gives a count at E for each mechanism represented by an integral in eq. 26. If this is less than $E'_{\max}$, then the probability function in the integrand will go to zero at the appropriate energy, and there will be no contribution to the integral from that energy to $E'_{\max}$.

If a cylindrical counter has identical dead region volumes at both ends, then the terms in eq. 26 containing V_d are multiplied by two.

$C(E)$ for a sphere with no dead region is given by eq. 26 using $G = M = 0$, and F and N as given in Sect. 3.

TABLE 1. THE CONTRIBUTIONS OF THE VARIOUS TERMS OF EQUATION 26 TO THE TOTAL COUNTS

Term No.	Location of Reaction	Proton Stopped in	Triton Stopped in	Fraction of Total Counts	Energy Distribution, keV
Methane Counter					
1	Sensitive region	Sensitive region	Sensitive region	0.341	764
2P	Sensitive region	Wall	Sensitive region	0.477	190 to 764
2T	Sensitive region	Sensitive region	Wall	0.097	574 to 764
None	Sensitive region	Wall	Wall	0.026	0 to 764
3P	Dead region	Sensitive region	Dead region	0.034	0 to 574
4P	Dead region	Wall of sensitive region	Dead region	0.012	0 to 574
3T	Dead region	Dead region	Sensitive region	0.013	0 to 190
4T	Dead region	Dead region	Wall of sensitive region	0.001	0 to 190
Hydrogen Counter					
1	Sensitive region	Sensitive region	Sensitive region	0.343	764
2P	Sensitive region	Wall	Sensitive region	0.509	190 to 764
2T	Sensitive region	Sensitive region	Wall	0.096	574 to 764
None	Sensitive region	Wall	Wall	0.022	0 to 764
3P	Dead region	Sensitive region	Dead region	0.017	0 to 574
4P	Dead region	Wall of sensitive region	Dead region	0.007	0 to 574
3T	Dead region	Dead region	Sensitive region	0.006	0 to 190
4T	Dead region	Dead region	Wall of sensitive region	0.0003	0 to 190

$C(E)$ for a cylinder with no dead region is given by eq. 26 using $G = M = 0$, and F and N as given in Sect. 4.

If monoenergetic neutrons are considered, then $P(E')$ in eq. 26 is constant for all values of E' from zero to $E'_{\max}$ and can be taken outside the integrals. This simplifies the expression to terms containing only path-length probability functions and particle reciprocal specific energy losses.

6. The $\text{He}^3(n,p)\text{H}^3$ Thermal-neutron Spectrum in Cylindrical Counters

As an illustration of how eq. 26 is applied and as a check of its accuracy, the spectrum resulting from the interaction of He-3 and thermal neutrons is analyzed in this section for two counters with the following specifications:

Designation	Methane Counter	Hydrogen Counter
Gas composition	99% CH_4 , 1% He-3	94% H_2 , 5% CH_4 , 1% He-3
Pressure, atm	0.4	3
Radius, cm	2.50	1.25
Sensitive length, cm	20.2	21.0
Dead volume, %	11.0	12.0

The calibrating gas, He-3, has a large cross section for thermal-neutron absorption (≈ 5400 b). Only a trace of the gas is needed to establish an energy point, and this trace can be ignored in determining particle ranges in the counter gas. However, interpretation of the

“thermal spectrum” is complicated somewhat by the presence of the recoiling triton. The Q -value for the reaction is +764 keV, of which 574 keV appears as kinetic energy of the proton and 190 keV as kinetic energy of the triton. Thus, the second, third, and fourth terms on the right in eq. 26 become two terms each, one for the proton, another for the triton. The counts from these terms occur in the range from 0 to 764 keV. The first term on the right is just the number of counts occurring at 764 keV and has a single value. The two particles can be treated individually since in all but about 2% of the cases, if one of them hits the wall, the other is captured with its full energy. Table 1 lists the contribution of each term in eq. 26 to the total counts. For example, term 2P means the second term on the right in eq. 26 evaluated for the proton. Values are given for both the methane and the hydrogen counters. Column 2 in the table gives the location of the reaction of the He-3 nucleus with the thermal neutron. Columns 3 and 4 give the respective points where the proton and triton are stopped. Column 5 contains the fraction of total counts attributable to each term of eq. 26, and column 6 gives the energy limits between which the counts are distributed. With one exception all values in the table were calculated using the appropriate path-length probability functions and range data. The exception is the case for which both the proton and the triton strike the wall of the sensitive volume after being born in that volume. Equation 26 contains no term accounting for

FIG. 5. Calculated Thermal Spectrum for the Methane Counter.

such an event. However, it can easily be shown that the fraction of total counts in this case is just

$$f_{PT} = 1 - F(l_P) - F(l_T) + F(l_{PT}), \quad (27)$$

where $F(l_i)$ is given by either eq. 18 or 19, and P and T refer, respectively, to proton and triton. Path length l_{PT} is the sum of the individual path lengths l_P and l_T . The counts are distributed between 0 and 764 keV. Although no distribution equation is given here, one could be obtained with some extra effort.

Table 1 shows that, for both counters, more than 91 % of all counts are due to terms 1, 2P, and 2T of eq. 26. The explicit forms of these terms are as follows:

Term 1

Both the triton and the proton lose all their energies in the sensitive region. Then,

$$C(764 \text{ keV}) = \Sigma(\text{th})\phi(\text{th})V_s \cdot \left[\frac{1}{\sigma\sqrt{2\pi}} e^{-(E-764)^2/2\sigma^2} \right] F(764 \text{ keV}), \quad (28)$$

where the term in brackets defines a Gaussian distribution about 764 keV, and σ is the standard deviation, determined in this case experimentally from the upper half-width of the peak.

Term 2P

The triton loses 190 keV, and the proton loses energy E in the sensitive region. Then,

$$C(190 \text{ keV} + E) = \Sigma(\text{th})\phi(\text{th})V_s \cdot \left[\frac{dR}{dE}(574 \text{ keV} - E) \right]_P N[R(574 \text{ keV}) - R(574 \text{ keV} - E)]_P. \quad (29)$$

Term 2T

The proton loses 574 keV, and the triton loses energy E in the sensitive region. Then,

$$C(574 \text{ keV} + E) = \Sigma(\text{th})\phi(\text{th})V_s \cdot \left[\frac{dR}{dE}(190 \text{ keV} - E) \right]_T N[R(190 \text{ keV}) - R(190 \text{ keV} - E)]_T. \quad (30)$$

In the case of monoenergetic particles, which we are concerned with here, the integration over E' in the second term of eq. 26 gives only one value at the energy in question. The Gaussian distribution added in term 1 is an attempt to account in part for the resolution of a counter.

FIG. 6. $\text{He}^3(n,p)\text{H}^3$ Spectrum for the Methane Counter at 0.4 Atm

Equations 28, 29, and 30 are plotted in Fig. 5 for the methane counter. The calculated thermal spectrum is then the sum of the three curves, as shown in Fig. 6 along with the experimental spectrum obtained with the counter. The normalization is to the total number of counts within the thermal spectrum. The agreement between the calculated and experimental spectra is considered very good. Most disagreement can be attributed to the lack of complete accounting for energy resolution in the calculated spectrum and to omission of the third and fourth terms of eq. 26.

Figure 7 shows the calculated and experimental spectra for the hydrogen counter. The agreement between spectra is not as good as that for the methane counter, principally because of the considerably slower response of the hydrogen counter. The particle tracks extending along radii of the counter produced slowly rising pulses, and pulse-height linearity was not maintained in the particular electronics used. These counts always occurred at a lower energy than that expected because of the premature clipping of the pulses by the multichannel analyzer. However, the agreement is still considered acceptable. The nonlinearity can be overcome with different electronics and is in no way a limiting factor in the use of the hydrogen counter as a fast-neutron spectrometer.

FIG. 7. $\text{He}^3(n,p)\text{H}^3$ Spectrum for the Hydrogen Counter at 3.0 Atm.

It is concluded that the wall-effect correction presented here in the form of eq. 26 is accurate.

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Discussion

Mr. Weale (UK-Aldermaston): Did I understand correctly that calculations were done for an isotropic distribution? If so, is it true that the spherical counter would work satisfactorily in a nonisotropic system, but the cylindrical one would not, at least not with this analysis?

Mr. Snidow: This analysis was for an isotropic neutron flux. We assume then that we would have an isotropic proton spectrum. The analysis would not be appropriate for a case in which a cylindrical counter was in a nonisotropic situation. I think it would still be applicable for a spherical counter that was in a non-

isotropic flux. I think the equations for the sphere would be the same, for example, for a beam of neutrons.

Mr. Meister (Karlsruhe): Is the assumption of straight-line recoil paths a good one?

Mr. Snidow: It's probably not important except at very low energy right at the end of the path. The agreement that we did get for the calibrating experiments indicates that it would not be too important.

Mr. Tuttle (AI): Was there any normalization used between the calibrating experiments and the calculations?

Mr. Snidow: Yes. We do not have a measurement of the absolute value of thermal flux to which the counter was exposed. We could not calculate an absolute value. I neglected to mention that the two curves were normalized by making the area under the two curves equal.

Mr. Weale: I would like to add that we used to correct wall effects by using a Monte Carlo calculation instead of an analytical treatment, and we have found by testing these two methods that the analysis and Monte Carlo methods give very close agreement.

A Comparison of Measured and Calculated Energy Spectra in Two VERA Cores

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ABSTRACT

Spectra have been measured over the whole energy range in two VERA fast reactor assemblies, designated VERA 16Z5A and VERA 17Z5J. In particular, the low-energy spectra have been determined by the time-of-flight technique, and this has been checked in one of the assemblies against a proportional counter using pulse-shape discrimination. The measured spectra are compared with spectra calculated by means of the FD1 and FD2 data sets, and these are shown to be in error in predicting too low a neutron flux in the low-energy range. The experimental assemblies are similar to two ZEBRA assemblies in which the Doppler temperature coefficient has been measured.

1. Introduction

There is a fair amount of indirect evidence from measurements of reaction rates and perturbation effects that the low-energy neutron fluxes in fast reactors are more intense than predicted by present methods of calculation. This is shown, for example, by a comparison of the measured and calculated properties of the ZEBRA fast reactor assemblies.⁽¹⁾ Recently, because of improved spectrometry techniques, it has become possible to check the calculated spectra directly by measurements over the whole energy range, and two of the ZEBRA assemblies have been re-built on the VERA reactor for this purpose. The measured spectra and the corresponding calculated spectra are presented in this paper.

2. Description of the Assemblies

The assemblies were re-built versions of the systems ZEBRA 5A and ZEBRA 5J which had previously been studied as part of a series of measurements of Doppler temperature coefficient of the ZEBRA reactor.⁽²⁾ The assemblies were constructed in the form of three-zone cores, the central zone being a square lattice of fuel and diluent rods. The rods were of mixed Pu/U_{nat} oxide pellets clad in steel, sodium clad in steel,

and plain steel, arranged on a lattice pitch of 0.453 in. The rods were 27 in. long and 0.400 in. in outer diameter; the central zone contained 256 of them. The VERA assemblies used the same types of rod as the ZEBRA assemblies, but there were no insulator zones and there were some differences in the general structure. The VERA systems were therefore given different reference numbers. The compositions of the central zones are listed in Table 1.

The central zone of VERA 17Z5J represents the composition of a typical oxide-fuelled, sodium-cooled fast reactor. The central zone of VERA 16Z5A has a similar fuel content but contains a much larger amount of steel and no sodium.

3. Spectrum Measurements

The spectra were determined by proton-recoil measurements in photographic emulsions, proton-recoil measurements in proportional counters, and time-of-flight analysis of extracted beams. The proportional-counter measurements on VERA 17Z5J were extended to the 1-keV energy region by using pulse-shape discrimination to eliminate γ -ray effects. The time-of-flight measurements were made on systems which were 7% subcritical, driven by a pulsed source of 14-MeV neutrons. The proton-recoil counters were placed in cavities at the core centres and the time-of-flight beams were extracted from cavities of the same size. Details of the experimental techniques and assessments of the accuracies at present obtainable by these methods have been published.⁽³⁾ The assessments, based on comparison of results on an absolute intensity scale obtained by a variety of experimental techniques, lead to the conclusion that the group fluxes are measured to a relative accuracy of about 10% (standard deviation).

The results of the measurements are shown in Figures 1, 2, and 3. Each experimental technique is referred independently to a standard reactor power, and discrepancies in the regions of overlap in Figures 1 and 2 therefore indicate errors in standardisation or

TABLE 1. COMPOSITIONS OF THE CENTRAL ZONES OF VERA 16Z5A AND VERA 17Z5J

	Atoms per cm ³ × 10 ⁻²⁰								
	Pu ²³⁹	Pu ²⁴⁰	O	Fe	Cr	Ni	Na	U ²³⁵	U ²³⁸
VERA 16Z5A	9.86	0.558	137	229	61.6	30.3	—	0.239	54.7
VERA 17Z5J	7.39	0.419	103	86.0	23.1	11.4	76.9	0.180	41.1

errors in the spectrum shape given by the detectors. Similarly, the time-of-flight and proportional-counter results in Fig. 3 show, for example, that the proportional counter indicates less flux than the time-of-flight technique at energies around 30 keV.

4. Calculations

The spectra were calculated by diffusion theory using the CRAM code⁽⁴⁾ with the data sets FD1 and FD2. The data set FD1⁽⁵⁾ consists essentially of the data of Yiftah, Okrent, and Moldauer⁽⁶⁾ with the

addition of extra data to cover the low-energy range. The data set FD2⁽⁷⁾ is a 33-energy-group set derived from the UK Data File. In the calculations the central core zone was taken to be a homogeneous mixture and self-shielding of resonances was allowed for as described by Rowlands and Wardleworth.⁽⁵⁾

The calculated spectra are shown in Figures 1, 2, and 3. They are referred independently to the reactor power scale used for the experimental results.

5. Comparison of Measured and Calculated Spectra

Before attempting to derive any conclusions from the comparison of the measured and calculated spectra it should be noted that there are two possible sources of error in addition to the inherent errors in the experimental techniques. The first of these is the presence of 0.75-in.-thick steel plates above and below the cavity in which the measurements were made. The effect of these steel plates was not measured, but calculations for an earlier assembly, VERA 7A,⁽³⁾ showed that surrounding the measuring cavity with 0.1-in. steel had no appreciable effect on the observed spectrum between the energies of 100 eV and 1.2 MeV, and

FIG. 1. Neutron-energy Spectrum at Core Centre of VERA 16Z5A.

Fig. 2. Neutron-energy Spectrum at Core Centre of VERA 17Z5J.

Fig. 3. Comparison of Proportional-counter and Time-of-flight Spectra in VERA 17Z5J.

TABLE 2. COMPARISON OF MEASURED AND CALCULATED SPECTRA IN THE LOW-ENERGY RANGE

	Energy Range 0.2 to 2 keV		Energy Range 2 to 10 keV		Energy Range 10 to 50 keV	
	VERA 16Z5A	VERA 17Z5J	VERA 16Z5A	VERA 17Z5J	VERA 16Z5A	VERA 17Z5J
Measured Flux	251	319	543	437	1127	1229
Calculated Flux using FD1	74	153	303	273	1206	1317
Calculated Flux using FD2	164	265	486	366	1228	1336
FD1 Error	-70%	-52%	-44%	-38%	+7%	+7%
FD2 Error	-35%	-17%	-11%	-16%	+9%	+8%

measurements of reaction rates in the same assembly showed results which were reasonably consistent with the calculations. Although the steel plates above and below the measuring cavities in VERA 16Z5A and VERA 17Z5J are much thicker, they account for only 7% of the solid angle subtended at the core centre by the cavity surroundings, and their effect is therefore considered negligible. The second possible source of error is the use of a homogeneous composition for the calculation, whereas the actual cores were made of steel-clad fuel pins. The importance of this effect has not been assessed.

The main features of the comparison between calculation and experiment can be seen in Figs. 1 and 2. In order to show more clearly the differences in the lower part of the energy range, the measured and calculated fluxes in this region are also displayed in tabular form in Table 2. The figures in the table have been obtained by integrating the curves from Figures 1 and 2 over three broad energy groups from 0.2 to 50 keV.

The differences between the measured spectra and the spectra calculated using the FD1 data in the range from 0.5 to 10 keV and in the range from 100 to 800 keV are large in comparison with the experimental errors of the spectrometry techniques. It can therefore be concluded with confidence that calculations using the FD1 data are wrong in that they predict too much flux in the middle range and too little flux in the low-energy range.

The spectra calculated using the FD2 data are in much better agreement with the measured values, and the discrepancies are generally comparable with the experimental errors. However, these discrepancies are in the same direction as those noted for the FD1 calculations, and in particular there is a definite tendency for the FD2 calculations to predict too low a flux in the energy range below 10 keV. This is shown clearly

by the trend of the figures in Table 2. Although the proportional-counter measurements shown in Fig. 3 confirm this trend for the energy groups from 2 to 50 keV, they show a markedly smaller neutron flux in the energy groups from 0.2 to 2 keV. However, because the proportional counter fails to reproduce the expected shape of the sodium-resonance dip in this region, we are at present inclined to prefer the time-of-flight result. Since the whole energy range below 50 keV is covered by the time-of-flight technique, the shape of the measured curve is known more accurately than the absolute flux because only the shape of the $\text{Li}^6(n, \alpha)$ excitation curve is involved and in this respect the experimental accuracy may be rather better than 10%. It might be argued that too much reliance is being placed on time-of-flight measurements in subcritical systems, since it has not yet been shown conclusively that these spectra are the same as those in the critical systems. The available experimental evidence on this point, summarised by Weale *et al.*,⁽³⁾ indicates that reactivity dependence of the spectrum does not make a large contribution to the experimental error. Some recent calculations (W. B. McCormick, unpublished) using diffusion theory indicate that there is an appreciable effect. For a 7½% subcritical VERA 17Z5J core the following ratios of critical to subcritical group fluxes were found:

41 to 67 keV	1.00 (normalisation point)
9 to 15 keV	1.013
1.2 to 2.0 keV	1.030
167 to 275 keV	1.057

These figures show that measurements in a subcritical system are likely if anything to yield spectra which fall off towards the lower-energy groups rather more rapidly than the critical-system spectra. If these corrections were applied to the results of Table 2, the discrepancies between calculation and measurement would become slightly greater at low energies. The conclusion of the present comparison therefore is that neutron fluxes calculated with the FD2 data are too low in the range from 0.2 to 10 keV, the discrepancies being between 10% and 30% for VERA 16Z5A, and between 10% and 20% for VERA 17Z5J.

6. Conclusions

Measurements of spectra in core compositions of interest in fast power reactor development show that the FD2 data give considerably more accurate predictions of spectra than the FD1 data. However, the FD2 data are probably still predicting a flux which is between 10% and 20% too low in the range from 0.2 to 10 keV in a sodium-cooled Pu/U_{nat} oxide core.

7. Acknowledgments

We would like to acknowledge the assistance of the other members of the VERA reactor team and to thank Mr. M. J. Stevenson for providing the calculated spectra.

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Discussion

Mr. Butler (ANL): In the time-of-flight measurements that you reported there is a problem with the multiplying assembly due to the lengthening of the pulse. Did you have to unfold the pulse formed in the assembly, in obtaining the time-of-flight information, or could you consider this instantaneous?

Mr. Weale: No. We did use an unfolding method which becomes appreciable at energies of about 15 keV and above for the system which we had. We had a 50-m flight path, and a multiplication of 14.

Mr. Butler: Are you fairly confident about this unfolding technique, or do you believe that you need to look into it also before you have complete confidence in it?

Mr. Weale: We are fairly confident about it. What we would like to do is to get agreement between the time-of-flight method and the proportional-counter method, in that region where we have applied the unfolding technique. I think that is the emphasis that

ought to be put on spectrometry measurements now. The accuracy that we would like to have is of the order of 5% standard deviation. In order to achieve that and be really sure that we've got it, I think we have to get the same result by two independent methods, if possible.

Mr. Hungerford (Purdue): Do you have any reasonable explanation for the fact that your proportional counter didn't see the effect of the sodium resonance?

Mr. Weale: No. I have no reasons other than the ones that Bennett suggested in his paper—the W value is not too well-known, the background of gamma rays is high, and the counting rate is high. This also is a plutonium system, of course. We made these measurements at about two or three percent subcritical. The counting rates went up to 5000/sec at the low end of the range. Thus, the explanation may be pulse pile-up, or overloading of the amplifier, or the W value.

Mr. Hungerford: What is the cause of the double peak at the peak of your spectrum? It showed up on all your measurements as well as in your calculations. Is there any particular reason for that in your particular assembly?

Mr. Weale: You would think that it would be a resonance in steel perhaps, but I haven't identified it with any particular resonance.

Mr. Bennett (ANL): Your results seem to indicate that cavities of fairly sizeable extent of stainless steel in these reactors do not bother significantly the spectra over most of the range at which measurements were reported. Possibly this would not be the case if one did a similar calculation down into the region of resonances in the iron or other materials. I wonder if this is something that is intended or might be done with the same techniques.

Mr. Weale: I agree that it would be a good idea. Large dips were shown in Travelli's calculations. You might think that an ELMOE calculation of that type with a cavity lined with steel would show a marked effect in those resonance regions. We haven't done that. We've only done these 33-group calculations which don't show the resonance structure in detail.

Presented by M. Kuchle

Measurements of Neutron Spectra and Decay Constants with the Fast Subcritical Facility SUAK*

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ABSTRACT

SUAK is a small, unreflected fast subcritical facility fuelled with thin plates of 20% enriched uranium metal. With a 200-keV, pulsed neutron generator, measurements of the prompt-neutron decay constant and of the neutron spectrum were performed. The first assembly consisted of pure uranium only; then different amounts of polyethylene were added to shift the neutron spectrum. The neutron spectra at the center and at the surface of the pure uranium assembly have been measured by the time-of-flight method and with a series of spherical proton-recoil proportional counters.

The experimental results were compared with calculations in which different approximations and cross sections were used. The experimental neutron lifetime was always longer than the calculated one. The deviation amounts to only 4% for the pure uranium assembly and to about 30% for the hydrogen-moderated assemblies. Situations with good time-eigenvalues, with quasi-modes and where no exponential could be found, were observed in the SUAK assemblies in agreement with theoretical predictions. Different types of pulsed reactivity determinations were used. In addition, a modulated source technique was tried for the determination of $(1 - k_{\text{eff}})/l$.

In connection with the measurement of the spectra by the time-of-flight method the problems of background, detector efficiency, and resolution correction are discussed.

The calibration and the evaluation procedure of data obtained with the proton-recoil detector are described. The measured spectra of the first assembly obtained by both methods agree rather well with each other in the common measured energy range from 25 keV to 1.5 MeV, but the measured spectra are softer than the calculated spectra.

* Work performed within the association in the field of fast reactors between the European Atomic Energy Community and Gesellschaft für Kernforschung m.b.H., Karlsruhe.

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1. Introduction

The experimental reactor physics program of the Karlsruhe Fast Breeder Project is essentially supported by the versatile critical facility SNEAK. However, because the construction of this facility took some time two additional smaller facilities were built which have now been in operation for about two years. These are the coupled fast-thermal reactor STARK and the subcritical facility SUAK. The main objective of the use of these facilities is the development of experimental techniques, but some preliminary information about cross-section sets and calculational methods were also obtained.

SUAK is primarily used for pulsed neutron-decay measurements and for spectrum determinations by the time-of-flight technique. For easier interpretation of measurements only bare assemblies were constructed which also contained as few materials as possible. To minimize the influence of backscattered neutrons the assembly is located 5 m above the ground in the centre of a thin-walled building of dimensions 10 m x 15 m x 10 m. A 200-keV Cockcroft-Walton accelerator and auxiliary equipment is placed on a grid 3.5 m above the ground.

The assembly is essentially composed of 2 x 2 x 1/8-in. plates of 20% enriched uranium, which are held in aluminum square tubes of 1-mm wall thickness. 25 or 36 tubes were closely packed together on an aluminium base plate of 3-cm thickness to form a parallelepiped with side lengths of 27.0 or 32.3 cm, respectively. The height of the assembly can be varied by changing the filling height of the tubes. By the admixture of plates of other materials the composition can be varied. The tubes are closed on the top with a plug of 3-cm aluminium and at the bottom with 3.5-cm stainless steel. Two central tubes act as safety rods and drop out of the assembly once a given multiplication is exceeded.

Figure 1 shows the assembly and the accelerator. On the left-hand side the end of the flight tube can be

FIG. 1. The SUAK Facility.

seen. A description of the assembly and the first preliminary measurements are given elsewhere.⁽¹⁾

So far three assemblies have been investigated. The first contained 20% enriched uranium only. This had the advantage that a minimum of materials was involved, and discrepancies between measurements and calculation could primarily be attributed to uranium data. Moreover, due to the hard neutron spectrum the decay time, and thus the neutron burst width for time-of-flight measurements, was short (about 200 nsec) and allowed spectrum measurements over a wide energy range. On the other hand, the spectrum was very different from that of a large dilute breeder reactor and contained very few neutrons in the energy range below 50 keV, where most of the discrepancies seem to occur. Therefore, two additional assemblies, which contained different amounts of polyethylene, were built to shift the neutron spectrum to lower energies. Hydrogen moderation was chosen since this was the only possibility to achieve a sufficiently high multiplication with the limited amount of fuel available

(120 kg ^{235}U). The influence of hydrogen is also of interest in connection with steam-cooled fast reactors. But due to the softer spectrum the neutron lifetime now increased so much that time-of-flight measurements were restricted to the low-energy end of the spectrum.

The calculated neutron spectra of the three assemblies are illustrated in Fig. 2, together with the spectrum of a 1000-MWe breeder reactor. The spectrum-softening effect of hydrogen is remarkable, but it is not possible to simulate a breeder spectrum very well.

2. Investigations on the Decaying Neutron Field

2.1. Measured Parameters

The quantity which primarily can be determined by a pulsed-source measurement is the prompt-neutron decay constant α . This can either be compared directly with a calculated α value, or the relation

$$\alpha = \frac{1 - k_{\text{eff}}(1 - \beta)}{l} \quad (1)$$

FIG. 2. Comparison of Calculated Spectra.

can be used to determine either k_{eff} or the neutron lifetime l , provided that the other quantity is known.

Moreover, with the additional use of the delayed-neutron tail in a repetitively pulsed experiment, reactivity and lifetime can be determined simultaneously by the methods proposed by Sjöstrand, Gozani, and Garelis.⁽²⁻⁴⁾ However, since the applicability of these methods to fast systems and to far-subcritical conditions is not obvious, we must consider this more as a check of the methods than as a reactivity determination. Equation (1) is also valid only in near-critical situations and, therefore, a direct comparison of α values was

preferred. In order to see whether discrepancies between measurements and calculations were mainly due to lifetime or reactivity deviations, an independent comparison of measured and calculated values of k_{eff} was made. The value of k_{eff} was determined by a multiplication measurement.

Since the accuracy of measurements for $k_{eff} \approx 0.9$ is rather poor, a check was desired as to whether significant results can be obtained. Therefore, the influence of uncertainties in nuclear data on the calculated values of α was investigated for different assemblies. Zero-dimensional diffusion calculations were performed with

TABLE 1 INFLUENCE OF UNCERTAINTIES IN NUCLEAR DATA ON THE PROMPT-NEUTRON DECAY CONSTANT α OF SUAK ASSEMBLIES

Composition	Increase in Cross Section	Resultant — $(\delta\alpha/\alpha)$
20% enriched uranium	$\frac{\Delta\sigma_{el}}{\sigma_{el}} (^{238}\text{U}) = +10\%$	+31.3%
	$\frac{\Delta\mu}{\mu} (^{238}\text{U}) = \begin{cases} +10\% & E > 1 \text{ MeV} \\ +5\% & E < 1 \text{ MeV} \end{cases}$	
	$\frac{\Delta\sigma_f}{\sigma_f} (^{235}\text{U}) = \begin{cases} +10\% & E > 250 \text{ keV} \\ +20\% & E < 250 \text{ keV} \end{cases}$	+30.8%
	$\frac{\Delta\sigma_{in}}{\sigma_{in}} (^{238}\text{U}) = +10\%$	-1.03%
	$\frac{\Delta\sigma_{el}}{\sigma_{el}} (^{235}\text{U}) = +10\%$	+8.3%
13% enriched uranium	$\frac{\Delta\mu}{\mu} (^{235}\text{U}) = +10\%$	
	$\frac{\Delta\sigma_{el}}{\sigma_{el}} (^{238}\text{U}) = +10\%$	+32.9%
	$\frac{\Delta\mu}{\mu} (^{238}\text{U}) = \begin{cases} +10\% & E > 1 \text{ MeV} \\ +5\% & E < 1 \text{ MeV} \end{cases}$	
20% enriched uranium mixed with polyethylene to an atomic ratio of hydrogen/uranium = 0.45	$\frac{\Delta\sigma_f}{\sigma_f} (^{235}\text{U}) = \begin{cases} +10\% & E > 250 \text{ keV} \\ +20\% & E < 250 \text{ keV} \end{cases}$	+38.8%
	$\frac{\Delta\sigma_{el}}{\sigma_{el}} (^{238}\text{U}) = +10\%$	+10.5%
	$\frac{\Delta\mu}{\mu} (^{238}\text{U}) = \begin{cases} +10\% & E > 1 \text{ MeV} \\ +5\% & E < 1 \text{ MeV} \end{cases}$	
	$\frac{\Delta\sigma_f}{\sigma_f} (^{235}\text{U}) = \begin{cases} +10\% & E > 250 \text{ keV} \\ +20\% & E < 250 \text{ keV} \end{cases}$	+1.8%
	$\frac{\Delta\sigma_f}{\sigma_f} (^{235}\text{U}) = \begin{cases} +10\% & E > 250 \text{ keV} \\ +20\% & 250 \text{ keV} > E > 20 \text{ eV} \\ 0\% & 20 \text{ eV} > E > 0 \end{cases}$	

Notations: σ_{el} = σ -elastic; σ_f = σ -fission; σ_{in} = σ -inelastic; μ = mean cosine of scattering angle.

the 26-group set of Abagian *et al.*⁽⁵⁾ Then, one cross section was increased by an amount that roughly corresponds to the present uncertainties in nuclear data,⁽⁶⁾ and the calculation was repeated. In order to see if a fortuitous cancelling of errors that may occur at one assembly would also appear at another assembly, three compositions were investigated which could be realized in SUAK with the amount of fuel available. The value of k_{eff} was kept at about 0.86. The results are given in Table 1.

For the change in the transport cross section it was assumed that $\Delta\sigma_{el}$ and $\Delta\mu$ add geometrically. The following conclusions can be drawn:

1. Since α can be determined to better than 2%, the deviations due to uncertainties in nuclear data are large enough for the measurements to give significant results.
2. If a cancelling of errors would appear at the assembly with 20% enrichment, it would probably also appear at the assembly with 13% enrichment, but not, however, in the case where hydrogen is added to the fuel.
3. α is very insensitive to the inelastic cross section; this is due to the rather weak dependence of the adjoint flux on energy.

The last lines of Table 1 indicate that the change in

$\delta\alpha/\alpha$ strongly depends on the special assumptions made in the fission cross-section changes, and one must be careful in generalizing the results.

Though a measurement of the decay constant α looks attractive, it is not clear that it always can be performed. In fast systems an exponentially decaying neutron field with a time-independent spectrum can only be established via the multiplication process. If the multiplication is not high enough, low-energy neutrons decay more slowly than the bulk of the other neutrons, thus forming a nonexponential tail of the neutron time distribution which can be described as a continuum of eigenfunctions. Sometimes, however, the fraction of neutrons in the parasitic modes is sufficiently small so that an almost exponential decay of the other neutrons can be observed, and a decay constant is deduced and interpreted theoretically with reasonable accuracy.

This problem of quasi-modes in the continuum has extensively been studied theoretically by French and Belgian groups.⁽⁷⁻¹⁰⁾ In Karlsruhe a number of zero-dimensional multigroup diffusion codes were developed by Borgwaldt, which are essentially based on the $k(p)$ formalism proposed by Storrer and Stievenart.⁽⁷⁾ They allow

1. performance of survey studies and investigation of the mode situation,
2. determination of α if it exists as a true eigenvalue or quasi-eigenvalue,
3. explicit calculation of the time-dependent neutron flux following a source burst of finite size and of arbitrary, but given, neutron spectrum.

Once a fairly good estimate of α exists, other codes as, for example, one-dimensional multigroup diffusion or S_N codes were used to determine α more precisely by the α -iteration method.⁽¹¹⁾ This method is based on the fact that a subcritical system with an exponentially decaying neutron field $\phi \sim e^{-\alpha t}$ is rigorously equivalent to a system which has been made critical by homogeneously subtracting $1/v$ -absorber to an amount $\delta\Sigma_a = \alpha/v$. For the unreflected rectangular assemblies, separability of the space distribution is a very good approximation, and one-dimensional calculations with buckling representation for the other dimensions are sufficient. Because of the base plate and the top and bottom part of the fuel elements, the assembly is reflected somewhat in the vertical direction, and a normally space-dependent calculation in this direction was used.

In order to evaluate measurements even in the case that no exponential decay could be observed, a modulated source formalism proposed by Borgwaldt was used.⁽¹²⁾ To understand this, the basic ideas of the $k(p)$ calculation will first be indicated.

Normally, two types of treatment of subcritical reactors are used. Either the fission-source term is multi-

plied by a factor $1/k_{\text{eff}}$ to make the reactor just critical, or the time-dependent solution is considered in, which the flux decays exponentially with $\phi(t) \sim e^{-\alpha t}$ and α is an eigenvalue of the system. But we can also consider situations which are somewhere in between the two and search for a value $1/k(p)$ by which the fission-source term has to be multiplied to fulfil the balance equation, if some arbitrary time dependence $\phi \sim e^{pt}$ is imposed on the system. $k(p)$ is an obvious generalization of the normal k_{eff} formalism; it is the transfer function linking successive generations of fission neutrons. If delayed neutrons are neglected,

$$k(0) = k_{\text{eff}};$$

$$k(-\alpha) = 1.$$

The connection between source intensity $s(p)$ and flux $\phi(p)$ is analogous to the stationary case given by

$$\phi(p) = A \frac{s(p)k(p)}{1 - k(p)}, \quad (2)$$

provided that the source distribution is described by the fundamental mode

If we now use a Taylor expansion for $1/k(p)$,

$$\frac{1}{k(p)} = \frac{1}{k(0)} [1 + l_1 p + l_2 p^2 + l_3 p^3 + \dots], \quad (3)$$

where l_1 is the prompt-neutron generation time, we get for the special case of a sinusoidally modulated source of frequency ω ,

$$\begin{aligned} \frac{s(i\omega)}{\phi(i\omega)} &= \frac{1}{A} \left[\frac{1}{k(i\omega)} - 1 \right] \\ &= \frac{1 - k_{\text{eff}}}{A k_{\text{eff}}} \left[1 + \frac{i\omega l_1}{1 - k_{\text{eff}}} - \frac{\omega^2 l_2}{1 - k_{\text{eff}}} \right. \\ &\quad \left. - \frac{i\omega^3 l_3}{1 - k_{\text{eff}}} + \dots \right] \end{aligned} \quad (4)$$

or

$$\begin{aligned} \frac{\phi(0)}{s(0)} \frac{s(i\omega)}{\phi(i\omega)} &= 1 - \frac{\omega^2 l_2}{1 - k_{\text{eff}}} + \dots \\ &\quad + i \left[\frac{\omega l_1}{1 - k_{\text{eff}}} - \frac{\omega^3 l_3}{1 - k_{\text{eff}}} + \dots \right]. \end{aligned} \quad (5)$$

If amplitude as well as phase of source and flux distribution are measured, a plot of the imaginary part of the left-hand side of Eq. (5) as a function of ω gives a curve with a slope at $\omega = 0$ of $l_1/(1 - k_{\text{eff}})$.

It is not necessary, however, really to use a sinusoidally modulated source. It is sufficient to Fourier-transform the results of a periodically pulsed experiment made with an arbitrary but known source shape. If we let $s(t)$ and $\phi(t)$ be the measured source and flux distribution, we get

$$\frac{\phi_0}{s_0} \frac{bc - ad}{a^2 - b^2} = \omega \frac{l_1}{1 - k_{\text{eff}}} - \omega^3 \frac{l_3}{1 - k_{\text{eff}}} + \dots, \quad (6)$$

with the notations

$$\begin{aligned} \int \phi(t) \cos \omega t \, dt &= a; & \int s(t) \cos \omega t \, dt &= c; \\ \int \phi(t) \sin \omega t \, dt &= b; & \int s(t) \sin \omega t \, dt &= d; \\ \int \phi(t) \, dt &= \phi_0; & \int s(t) \, dt &= s_0. \end{aligned}$$

When the left-hand side of Eq. (6) is calculated as a function of ω , $[l_1/(1 - k_{\text{eff}})]$ can be obtained.

The formalism was extended by Borgwaldt to include an arbitrary source spectrum and detector response. The method has the advantage of being applicable to situations for which no exponential decay can be observed, but breaks down when spatial harmonics become important and when the spectrum is too different from that of the critical reactor. The quantity $l_1/(1 - k_{\text{eff}})$ refers to the stationary reactor and, therefore, differs from $1/\alpha$. Measured and calculated values of $l_1/(1 - k_{\text{eff}})$ were compared.

In addition to the decay measurements, fission-rate ratios were determined in repetitively pulsed experiments. When the measurements were integrated over time, the results refer to the spectrum of the subcritical assembly with a continuously running source. They are strongly influenced by the primary 14-MeV source neutrons and, therefore, are not of direct relevance to reactor situations. If the measurements were time gated and a time interval is selected for which the asymptotic mode prevails, the fission-rate ratios refer to the spectrum of the asymptotic mode. In view of the much lower intensity and the high accuracy of timing required, these measurements are less accurate, however. If no persistent mode exists, the fission-rate ratios remain time-dependent, and no meaningful time-gated measurements are possible.

2.2. Experimental Results and Comparison with Calculations

2.2.1 The Multiplication Measurement

For the multiplication measurements a 1-Ci Po-Be source—which had to be removed for the pulsed neutron experiments—was inserted from below in the bottom part of the central fuel tube. With three long counters placed on different sides of the assembly at a distance of about 1 m, the counting rate as a function of the number of fuel elements, N , was measured. To minimize scattering and shadowing effects due to irregular tube arrangements, for each configuration a second measurement was performed with the fuel tubes filled with lead.

After subtraction of this counting rate, the inverse multiplication $1/M$ was plotted against $1/\sqrt{N}$. For $M > 5$ an almost straight line was obtained. The multiplication as a function of the number of fuel elements also was calculated using several hundred harmonic modes. Under the assumption that the calculation gives at least the shape of the dependence correctly, the experimental $1/M$ curve was extrapolated to $1/M = 0$. From the deviation of the calculated k_{eff} at that point from unity, and from the calculated k_{eff} for the number of fuel elements in the assembly, the value of k_{eff} of the assembly was determined. The accuracy of this procedure was checked by using different plots and extrapolations, and turned out to be about $\Delta k = \pm 1\%$.

2.2.2 Pulsed Decay Measurements

The pulsed decay measurements are straightforward in principle. The 200-keV ion accelerator with a combined beam-extraction and beam-deflection pulsing device delivered pulses of about 10^4 neutrons in 0.1- μsec bursts at a repetition rate of up to $40 \cdot 10^3$ cycles per second. The neutrons were produced by the $T(d,n)^4\text{He}$ reaction, and the target was located at the centre of one surface of the assembly. Different detectors and detector positions were used to provide a better indication of the spatial and energy harmonics. ^{235}U -, ^{238}U -, and ^{237}Np -fission counters with cross-over—pick-off discrimination were applied. The time resolution was determined with 1-nsec pulses from a Van de Graaff accelerator and turned out to be about 40 nsec. For the hydrogen-containing assemblies the neutron decay time was long enough to make the requirements in timing accuracy less stringent, and a BF_3 counter also could be used. A stilbene scintillator for proton-recoil detection with a fast-slow system and pulse-shape discrimination turned out to be inconvenient, due to the high sensitivity and large deadtime corrections necessary. Normally the counting rate was low enough to keep the deadtime losses below 1%.

To check the importance of γ discrimination, in one case the neutron decay time also was measured with a γ -ray detector. The result agreed with those of other measurements; the constant background was much higher, however.

The measurements were done with a TMC 256-channel time analyzer with a model 219 plug-in unit capable of channel widths down to 10 nsec.

The background was obtained from some pre-burst channels, and from channels that correspond to a time interval after the burst where the prompt-neutron intensity was negligible. Due to the high repetition rate the background was practically time-independent. It can be attributed to delayed neutrons and room-scattered neutrons. The influence of room-scattered

TABLE 2. GEOMETRICAL DIMENSIONS AND ATOMIC NUMBERS OF SUAK ASSEMBLIES

Geometrical Dimensions	U1A	U1B	UH1A	UH1B	UH2A	UH2B
Side length (cm)	26.9	32.3	26.9	32.3	26.9	32.3
Top thickness (cm)	3.0	3.0	3.0	3.0	3.0	3.0
Core height (cm)	35.1	35.1	33.05	33.05	35.05	35.05
Bottom (cm)	3.5	3.5	3.5	3.5	3.5	3.5
Base plate (cm)	3.0	3.0	3.0	3.0	3.0	3.0
Atomic Numbers (10^{-2} /barn-cm)						
Core	Al	0.4277	0.4277		0.4277	
	H	—	1.4844		0.8227	
	C	—	0.7422		0.4113	
	^{235}U	0.8150	0.6502		0.7216	
	^{238}U	3.2897	2.6237		2.9118	
	Ni	0.0660	0.0529		0.0588	
Top	Al	4.38	4.38		4.38	
	Fe	—	0.54		—	
Bottom	Fe	8.44	8.44		8.44	
Base plate	Al	6.00	6.00		6.00	

neutrons was determined in the following way: The counting rate of a detector at the surface of the assembly was compared with the counting rate measured under the same conditions but with the assembly removed. When normalized to the same monitor counting rate, the ratio of outgoing to ingoing neutron current was obtained, from which a corrected extrapolation distance was determined. The correction was always negligible except for the delayed-neutron tail, where 6% to 18% of the flux was due to backscattered neutrons.

In cases where an exponential decay existed, it was established after about two decay times and could be followed for an interval of about two decades. For each measurement nearly 10^6 counts were accumulated for this time interval, and the accuracy of the decay constant obtained was $\pm 1\%$.

As mentioned above, so far three fuel compositions have been investigated. The first (U 1) contained 19.9% enriched uranium only, the second (UH1) was obtained by insertion of two polyethylene plates of 0.4-mm thickness between each uranium plate of $\frac{1}{8}$ -in. thickness, and the third (UH2) contained only one polyethylene plate per uranium plate.

For each composition measurements were made with 25 boxes (type A) and with 36 boxes (type B) built up in a quadratic 5×5 or 6×6 arrangement.

In addition to the real core zone, the top and bottom part of the fuel boxes and the base plate have to be taken into account. The bottom part of the fuel boxes

consists of stainless steel, which we simplified in the calculation as being only of iron. The top parts and the base plate are made out of aluminum. The openings in the top part were homogenized, and the fact that the base plate is a little bit larger than the core region was neglected in our calculation. When the aluminum of the boxes was homogenized in the core region and the nickel cladding of the uranium plates was taken into account, the atomic numbers as given in Table 2 were obtained. In case of the not completely filled boxes (assemblies UH1), a spring on top of the fuel plates was inserted to hold the plates down. This was taken into account by adding iron to the top part composition.

The various measured time decays are illustrated in Figures 3, 4, and 5. A true eigenvalue was predicted for assembly UH1B; reasonably accurate quasi-modes should exist for the assemblies U1A, U1B, and UH1A, and a nonexponential behaviour was predicted for UH2A and UH2B. This is in agreement with the experimental results except for assembly U1A, where no exponential decay could be observed.

A comparison of experimental results with calculated values is given in Table 3. For the smaller assemblies (A) the multiplication was not high enough to determine k_{eff} . To apply the pulsed reactivity determinations as proposed by Garelis and Gozani a small modification was necessary to take into account the almost triangular burst shape of our accelerator. For the very fast decay ing system U1B, the accuracy was limited by the precision with which the time zero and the burst width could be determined. To convert ρ/β_{eff} into k_{eff} , β_{eff} was calculated using the results of a $^{235}\text{U}/^{238}\text{U}$ fission rate ratio measurement.

The decay times given in Table 3 are average values from measurements made with different detectors. For the case of the hydrogen-containing assemblies one must be careful when using ^{235}U - or BF_3 -detectors, since the spectra of these assemblies contain an appreciable fraction of thermal neutrons, for which the flight time to the detector can be much longer than the decay time of the neutron field, even if the detector is immediately at the surface. This leads to serious errors in the measured decay constant. For assembly UH1B a cadmium cover around the detector solved the problem; this was not quite certain, however, for assembly UH1A, where even epicadmium neutrons could cause trouble.

The values of $l_1/(1 - k_{\text{eff}})$ given in Table 3 were obtained from the modulated source evaluation technique mentioned above. About 10 ω points in the range $0 < \omega < 2\alpha$ were used to determine the slope. The errors indicated are due to the inaccuracy in the determination of the slope, the uncertainty of the accelerator burst shape, and the deviations obtained from measurements with different detectors.

FIG. 3. Time Decay of the Neutron Field in Assembly U1.

In view of the good separability of the space coordinates of unreflected parallelepipeds, one-dimensional S_4 calculations were performed. A buckling representation for the other directions was used, which was equivalent to S_4 calculations in these directions as far as k_{eff} is concerned. It was checked that the difference between S_4 and S_8 calculations was negligible. Different cross-section sets were applied, but only the results obtained with the 26-group set of Abagjan *et al.*⁽⁶⁾ are included in Table 3. The difference with the results from the KFK set is very small. The values in the last two columns of Table 3 were obtained by multiplying the values in the two preceding columns by $1 - k_{\text{eff}}$ with k_{eff} taken from columns two and five, respectively.

Measurements and calculations agree well for the pure uranium assembly U1B, but a difference of 3.5% between measured and calculated k_{eff} values is found for the hydrogen-containing systems. The large differences in $1/\alpha$ and $l_1/(1 - k_{\text{eff}})$ for these systems are essentially due to the k_{eff} deviations. The reason for the large discrepancy is not yet clear, but may be due to the use of an S_N code with isotropic scattering.

The results of the fission-rate ratio measurements will not be given here. In view of the limited time for which the fuel was available, only preliminary fission-counter

calibrations were performed, and accurate calibrations were postponed.

3. Measurement of Neutron Spectra

Measurements of neutron spectra have been performed for the pure uranium assembly U1B by the time-of-flight method and with a series of spherical proton-recoil proportional counters.

The purposes of these experiments were three: to develop the measuring techniques, to check the reliability of both methods, and to compare the measured spectra with calculations. Due to the long decay time of the system, the time-of-flight technique cannot be applied at the large critical facility SNEAK, and differential spectrum measurements will mainly be made with the proton-recoil counter. Therefore, a check of this counter against the time-of-flight method is important.

3.1. The Time-of-flight Method

In the performance of spectra measurements by the time-of-flight method the time for a neutron to cover a definite flight path is measured. For a finite pulse width Δt the energy resolution is proportional to $\Delta t/t$, where t represents the time of flight.

FIG. 4 Measured Time Decay of the Neutron Field in Assembly UH1 ($H/U = 0.45$).

If the buildup and decay of the neutron field in a subcritical assembly after an injection of a neutron burst is short compared with the flight time, what is measured is essentially the stationary spectrum of the subcritical assembly with a continuously running source. The buildup and decay of the neutron field determines the distribution in neutron starting times. This time-resolution function, which is essentially determined by the decay constant $1/\alpha$, has to be taken into account if $t \lesssim 10/\alpha$.

Up to now all measurements have been performed as if all neutrons escaped at the mean emission time. To correct for the resolution effect due to the time-resolution function, a code has been written at Karlsruhe by K. Bohnel.

The calculation of the resolution correction relies on an iterative procedure defined by the following recursion formula:

$$\frac{f_n}{Ff_n} = \frac{f_{n+1}}{M}, \quad \text{with } f_0 = M, \quad (7)$$

where M is the measured time distribution and F is the folding operator. The principle of the correction procedure assumes that the ratio defined in (7) remains the

same for similar functions. The code operates with a resolution function that is unique for all energies, but its shape may be polynomial, exponential, or a combination of these. The measured time distribution and all subsequent iterates are accurately fitted by one or a sequence of well-overlapping polynomials up to the 5th order. The time moments of the resolution function are calculated once for each time channel at the beginning of a run. Thus, each subsequent folding of a polynomial can be quickly performed by the proper addition of the time moments corresponding to the powers of the polynomial. In practice, convergence is achieved after 5 to 6 iterations, which take less than 3 min for the IBM 7074 computer.

Measurements of the time-dependent neutron flux performed at the assembly U1B, with a series of fission chambers with various thresholds, gave the same decay constant, $\alpha^{-1} = 230$ nsec, and indicated that the asymptotic spectrum was reached in 150 nsec after the burst. If the length of the flight path, s , and the channel width were optimized, the counting rate C per channel is given by

$$C(t) = \text{const.} \cdot \eta(E) \phi(E) \alpha^2 \left(\frac{\Delta E}{E} \right)^4, \quad (8)$$

FIG. 5. Measured Time Decay of the Neutron Field in Assembly UH2 ($H/U = 0.22$).

TABLE 3. RESULTS OF PULSED DECAY MEASUREMENTS AND COMPARISON WITH CALCULATIONS

Assem- bly	hydro- gen uranium	k_{eff} ($1/M$)	k_{eff} (Garelis)	k_{eff} (Gozani)	k_{eff} calc	$1/\alpha$ measured (μ sec)	$1/\alpha$ calc (μ sec)	$\frac{l_1}{1 - k_{eff}}$ measured (μ sec)	$\frac{l_1}{1 - k_{eff}}$ calc (μ sec)	l_1 meas- ured (nsec)	l_1 calc (nsec)
U1B	0	0.86 ± 0.01	0.845 ± 0.015	0.860 ± 0.010	0.869	0.230 ± 0.003	0.223	0.202 ± 0.012	0.208	28.3	27.25
UH1A	0.453	—	—	—	0.8258	2.45 ± 0.03	1.72	1.53 ± 0.03	0.744	—	129.5
UH1B	0.453	0.945 ± 0.010	0.967 ± 0.005	0.947 ± 0.005	0.907	3.80 ± 0.03	2.29	3.17 ± 0.03	1.456	174	135.3
UH2A	0.226	—	no expon. ^(a)	no expon. ^(a)	0.811	no expon. ^(a)	no expon. ^(a)	0.758 ± 0.023	0.3615	—	68.3
UH2B	0.226	0.932 ± 0.010	no expon. ^(a)	no expon. ^(a)	0.896	no expon. ^(a)	no expon. ^(a)	1.45 ± 0.04	0.700	98.6	72.9

(a) No stationary, but continuous, eigenvalues.

where $\eta(E)$ is the sensitivity of the detector, $\theta(E)$ is the neutron spectrum, and

$$\frac{\Delta E}{E} = \frac{2\Delta t}{t} = \text{const.} \frac{\sqrt{E}}{s\alpha}. \quad (9)$$

Though the energy resolution becomes extremely good at low energy with a flight path of 53 m, the counting rate becomes too small. Therefore, the spectrum has been successively measured with flight paths of 53 m, 21.5 m, and 11.5 m. Each measured spectrum was normalized to the preceding one in an energy range where reliable results from the two measurements overlapped.

The approximate energy resolution obtainable at various energies for the flight path s and the energy ranges, in which the spectra were normalized, is given in the following table:

	53 m	21.5 m	11.5 m
$\Delta E/E = 10\%$	1.5 MeV 240 keV — 165 keV	300 keV 70 keV — 45 keV	86 keV —

The experimental setup has already been described.⁽¹⁾ The evacuated flight tube has a maximum length of 53

m and is sealed on both sides by "Melinex"-(Mylar)-foils of 15.0- and 12.5-cm diameter, respectively, and 0.12-mm thickness. Three steel collimators of 20-cm thickness were placed in the flight tube. Their inner part consisted of replaceable annular cylinders of Cunifer 30 with a cylindrical or square hole to adjust the opening of the collimator to the requirements of the experiment. For the measurements of the leakage spectrum from the surface of the assembly all collimators had an inner diameter of 12.5 cm, which selected a circular area of 123 cm² at the center of one side of the assembly. The background determination by the transmission of black resonances has already been described and corresponds at the transmission dips with the signal obtained with a Cunifer 30 plug of 20-cm thickness inserted in the first collimator. Thus, the latter method has been used to determine the background for all subsequent measurements of the leakage spectra from the surface.

To measure the spectrum from the center of the assembly a difference technique was applied which allows the most reliable background determination. For these measurements, the central row of fuel elements was replaced by a horizontally stacked row which left the dimensions of the assembly unchanged. The fuel of the central element of this row could be removed to generate a horizontal through-channel of 5 cm by 5 cm. This channel could be half loaded from the side opposite to the detector. For spectrum measurements from the center, all collimators except the first one had an inner diameter of 12.5 cm. The latter selects a square area of 25 cm² centered at the beam-hold axis. The spectrum from the central region of the assembly is obtained by subtracting the time-of-flight distribution with the through-channel from the time-of-flight distribution obtained with the channel loaded with fuel up to the center. Since the withdrawal of the fuel plug from the central channel reduces the multiplication of the assembly by $\Delta k_{\text{eff}} = 0.014$, it also reduces the background. Therefore, the measurements were normalized to the same counting rate of a long counter. For all the spectra measurements, an additional hole of the same size was required to have the accelerator target at the center of the assembly.

The time-of-flight measurements are made with an accelerator burst of 100 nsec at the high repetition frequency of 25 kc. The ion current pulse at the target is used to trigger a TMC-256 channel-time-analyzer with a channel width of 125 nsec. The detector pulses are delayed so that the pre-burst background could also be measured. The time-scale calibration is obtained from the gamma-ray signal of the assembly (corrected for time of flight) and from the flux depressions at the known aluminium resonances.

All the spectra are measured with a ⁶Li-glass scintil-

lator. Primarily to check the efficiency calibration of this detector, an identical measurement with a ¹⁰B slab and NaI crystal was made that covered the energy range from 40 keV to 2 MeV. The efficiency of the detectors is illustrated in Fig. 6.

The ⁶Li-glass detector is made of two identical NE 905 ⁶Li-glasses, each of 4 $\frac{3}{8}$ -in. diameter and $\frac{1}{2}$ -in. thickness. Both discs are optically sealed together and to a 6364 DuMont photomultiplier. The pulse-height resolution is only 46 % and can be improved to 35 % by a light-pipe. Though the light-pipe did improve the pulse-height resolution, it was not used for the final spectra measurements because of the neutron scattering in the light-pipe. The efficiency of the ⁶Li detector has been simply calculated from the known ⁶Li content and the recommended ⁶Li(*n*, α) cross section.⁽¹³⁾ Furthermore, it has been measured with mono-energetic neutrons obtained with a Van de Graaff accelerator from the ⁷Li(*p*, *n*) reaction in the energy range from 60 to 320 keV. The neutron energy was varied by changing the energy of the proton and by selecting different angles. The neutron flux was determined from the counting rate of a long counter at a fixed angle with the calculated yield $d\sigma(E, \vartheta)/d\vartheta$. By appropriate choice of the angles, the variation of neutron energy at the long counter was appreciably smaller than the energy variation at the ⁶Li detector. The energy sensitivity of the long counter was taken from Marion and Fowler.⁽¹⁴⁾ More energy-calibration points of the ⁶Li glass are required, but no other mono-energetic neutron sources were available at the time of the measurement.

The boron slab detector is made of 92 % ¹⁰B powder, $\rho = 1.018$ g/cm³, enclosed in an aluminium box, 129 mm in diameter, 120 mm in length, and with a 1-mm wall thickness. Front and back sides are made of 0.1-mm polyester foil. One NaI crystal with a 4-in. diameter and a 5-in. length is mounted on a XP 1000.40 photomultiplier, with a pulse-height resolution of 9 % for the 481-keV capture gamma ray. The crystal axis is perpendicular to the flight-path axis and shielded from the direct beam. The calculated efficiency takes into account only the ¹⁰B(*n*, α)⁷Li* reaction, gamma-ray absorption, a geometrical factor, and the gamma-ray detection efficiency.

The energy spectrum deduced from a measurement with the ¹⁰B slab agrees with the corresponding ⁶Li measurement up to 500 keV. Above this energy, the assumed efficiency of the boron slab is much too small due to competing reactions, which also appear if the boron slab is replaced by a graphite slab which is equally thick for elastic scattering. The ⁶Li(*n*, α)³H reaction has a large resonance at 250 keV. Thus, a small energy shift should produce a large error in the calculation of the energy spectrum from the measured time distribution. The rather smooth energy spectra obtained with the ⁶Li

FIG. 6. Calculated Efficiency of Neutron Detectors in the Energy Range 1 keV to 2 MeV.

detector and the agreement with the boron slab measurement show that the resonance is not a very serious drawback for the use of the ^6Li detector with the obtainable resolution at the 53-m detection station.

3.2. Spectra Determination with the Spherical Proton-recoil Proportional Counters

Hydrogen- and methane-filled, spherical proportional

TABLE 4. FILLING GAS, PRESSURE, AND USEFUL ENERGY RANGE FOR THE 4-CM SPHERICAL PROPORTIONAL COUNTERS

Counter No.	Filling Gas	Pressure (atm)	Energy Range (keV)
1	hydrogen	1	40-180
2	hydrogen	2	80-300
3	hydrogen	4	200-550
4	methane	2.5	350-900

FIG. 7. Leakage Spectra from the Surface of Assembly U1B.

counters (4 cm in diameter)* were used to measure the neutron spectrum over the energy interval from about 40 keV to 1 MeV. There is only a restricted useful energy range for each counter (see Table 4), because the sensitivity to gamma rays sets a limit at the low-energy and the wall effects at the high-energy side, so four counters with different filling gases and pressure had to be used.

The leakage spectrum was measured at the time-of-flight tube side of the assembly with a distance of a few mm between detector and surface. To measure the

* Fabricated by 20th Century Electronics Ltd. of Great Britain.

spectrum in the center of the assembly, the central element of the horizontal stacked row was half loaded to have the same conditions as in the time-of-flight measurements. Measurements at each position were of approximately 8-hr duration. Counting rates were below about 1500 c/s; deadtime losses of 15 %, at most, were corrected for by a live timer in the analyzer.

The well-known relationship

$$\phi(E) = \frac{1}{NT} \frac{E^2}{\sigma(E)} \frac{dD(E)}{dE} \quad (10)$$

was used to determine the neutron flux (per unit

FIG 8 Leakage Spectra from the Surface of Assembly U1B.

lethargy), $\phi(E)$, from the measured distribution of recoil protons (per unit energy), $D(E)$.

$\sigma(E)$ is the n,p -scattering cross section and NT is the product of the number of hydrogen atoms in the sensitive region of the detector by the live counting time.

Wall effects were corrected for as described by Benjamin⁽¹⁵⁾ by subtracting first from the measured proton-recoil distribution the contribution of neutrons with energy greater than the upper limit of the detector, and then multiplying the slope of the resultant proton-recoil distribution with correction factors for the wall effect.

The high-energy neutron contribution of the proton-recoil spectrum and the energy-dependent correction factors for wall effects were calculated with a Monte Carlo Code.⁽¹⁶⁾

The computation was done by the following relation-

ship, which accounts for the wall-effect correction:

$$\phi(E) = \frac{1}{NT} \frac{E^2}{\sigma(E)} WF(E) \frac{d}{dE} [D(E) - M(E)]. \quad (11)$$

Here M is a straight-line fit to the high-energy contribution, normalized to the measured proton-recoil spectrum, and $WF(E)$ is a polynomial least-square fit to the wall-effect correction factors. The n,p -scattering cross section is approximated (from thermal to 20 MeV within 4%) by⁽¹⁷⁾

$$\sigma(E) = \pi \cdot \left(\frac{3}{1.206E + (-1.86 + 0.09415E + 0.0001306E^2)^2} + \frac{1}{1.206E + (0.4223 + 0.13E)^2} \right). \quad (12)$$

FIG. 9. Leakage Spectra from the Surface of Assembly U1B.

Differentiation of $[D(E) - M(E)]$ consists of determination on a logarithmic scale of a slope and associated error to sections of the corrected proton-recoil spectrum. The slope-taking interval can be chosen as a fixed percentage of the energy and is normally set at 25 %, which is about twice the resolution of the detectors (10–15 %).

Monoenergetic neutrons from a Van de Graaff were used to make the energy calibration and to check the calculated correction factors for wall effect.

It should be noticed that normalizing of the four parts of each spectrum is done by using the counting

rate of a long-counter monitor, the live counting time, and the hydrogen content of the counts. No arbitrary constant is involved in fitting the different parts of the spectra to each other, as measured by the four counters. From the detailed spectra of Fig. 8 it can be seen that for each counter there is an increase in the spectrum at the low-energy limit because of γ pulses, and a decrease at the high-energy limit because the fit to the wall correction factors does not account for the steep increase of the wall effect above the high-energy limit.

The measured spectrum obtained in the energy range from 25 to 825 keV has been used to evaluate the abso-

FIG. 10. Spectrum of the Center of Assembly U1B.

lute fission rate of a calibrated ^{235}U -fission chamber. To account for the lacking part of the spectrum, the theoretical spectrum has been normalized to the measurements between 25 and 40 keV and between 500 and 825 keV. The calculated and measured absolute fission rates are, respectively, 0.0359 and 0.0363 fission-chamber counts per monitor count.

3.3. The Comparison of the Measured and Calculated Spectra of Assembly U1B

The measured spectra obtained with both methods and the corresponding theoretical spectra are illustrated in Figures 7, 8, 9, and 10, in units of flux per lethargy interval. Figures 9 and 10 give the measured spectra fitted by a smooth curve. The measured spectra

as well as the theoretical spectra are normalized to equal areas between 25 keV and 1.350 MeV.

The theoretical spectra are calculated with the 20-groups YOM set in S_8 approximation for a sphere with the geometrical buckling of the assembly and an isotropic spherical source at the center. The source spectrum is the first group of the 20-group set, which corresponds well with the 14-MeV D,T -neutron source. The leakage spectrum perpendicular to the surface, which is to be compared with the time-of-flight measurement, is obtained by plotting for each energy group the nine components of the vector flux as a function of $\cos \vartheta$ and extrapolating to $\cos \vartheta = +1$. The leakage spectrum from the surface, which is to be compared with the spectra deduced from the measurements with

the proportional counters, is a mean value of all escape components of the vector fluxes.

The spectrum at the center is the isotropic flux as obtained by S_8 calculation.

The flux depressions due to large scattering resonances of aluminium are larger in the spectra obtained by the time-of-flight method since the energy resolution is better. If we disregard the energy-resolution effect of both methods, the measurements agree rather well between 40 keV and 1 MeV within the expected accuracy.

The slight difference between the measured leakage spectra is expected from the calculated spectra. The difference in the measured spectra from the center above 600 keV could be due to various weightings of the 14-MeV neutron source. To measure the central spectrum the spherical counters were located in front of the target at the center of the core. But to perform the time-of-flight measurements the target was shadowed by collimators, since collimation is done on a 25-cm² area at the center in front of the target.

Anyway, the spectra obtained up to now show that the calculated spectra are too hard. To explain this disagreement the effect of inaccuracies in cross-section data and fission spectrum will be investigated. To improve the accuracy of the time-of-flight measurements the efficiency of the ⁶Li detector should be determined in detail over a larger energy range, and the resolution correction will be applied to extend the reliable energy range to energies above 1 MeV. To improve the proportional-counter method a detailed study of the wall-effect correction procedure is in progress and a γ, n -pulse-shape discrimination system⁽¹⁸⁾ will be used to extend the measurable energy range down to about 1 keV.

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Discussion

Mr. Stievenart (Belgo-Nucleaire): Can you comment on the validity of this method when it is presumed that no decay constant exists for this assembly?

Mr. Küchle: This method can only be applied where a fundamental-mode decay constant exists.

Mr. Hungerford (Purdue): How did you get the neutrons from the center of the assembly for the time-of-flight measurements?

Mr. Küchle: We made a 2-in. by 2-in. re-entrant hole into the center of this assembly.

Mr. Hungerford: Were you able to exclude the outside leakage?

Mr. Küchle: Yes. This was made by the background measurement I mentioned.

Mr. Hiraoka (JAERI): I want to ask about the decay curve. Is the agreement between experiment and the analysis using the Storrer-Stievenart theory good in all energy regions?

Mr. Kuchle: No, you have different situations. I did not go into details here. In all assemblies: assemblies that have the two fundamental modes, those which have a reasonably good pseudo-mode, and those which do not show any exponential decay, we have observed different results. And if there was some reasonably good pseudo-mode we could evaluate reactivities with normal methods. In cases where there was no mode we used the technique I mentioned with a sinusoidally modulated source.

Mr. Purohit (RPI): Do I understand you correctly that if there is no pseudo-mode in an assembly then the neutron beam method is better than the pulse method?

Mr. Kuchle: One must be careful. It is not really a neutron-wave method. We assume the neutrons to have a spatial mode and also the source to have a spatial fundamental mode. We assume sinusoidal modulation in the amplitude.

Mr. Purohit: Because you are using a modulated source, it seems to me that you are using a certain frequency, and I think the limitations that apply in the case of the pulsed method will also apply in this case too. If there are small frequencies that are related with long times then they would be connected with the asymptotic.

Mr. Kuchle: The essential point of this technique is just to emphasize an energy region which is not very

far in time. If there exists a buildup and decay of the neutron field, we are normally looking for the asymptotic mode at long times, and here we have distortions due to the low-energy neutrons. If you use a sinusoidally modulated source, you have a weighting function over this range, and you use much more of this part; therefore, the contribution of the low-energy neutrons is less important.

Mr. Stievenart: I would like to ask if anybody here is aware of further developments of the methods that we did with Mr. Storrer, using a space-, time-, and energy-dependent model? You said that you wrote a few codes at Karlsruhe.

Mr. Kuchle: Yes, but they were essentially on the same basis. We tried to consider time-dependent flux and to locate the poles of this distribution, just as you've done at Cadarache.

Mr. Stievenart: We have studied the space-dependent problem. What we have observed is that k_{eff} disappears and that you obtain from a multi-point model a vector k_{eff} . What still exists in such a model, for small k , is an eigenvalue. You no longer have k_{eff} ; you have to define it.

Mr. Weale (UK-Aldermaston): How did you make the comparison between the time-of-flight method and the proportional-counter method?

Mr. Kuchle: They are normalized to the same total area.

Presented by J. L. Russell

Time-of-flight Measurements of Neutron Spectra in U^{235} and Tungsten*

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ABSTRACT

The pulsed-source, time-of-flight method was used to measure neutron angular flux spectra to 15 MeV from a spherical U^{235} -metal assembly and spheres of tungsten. The object of the U^{235} experiment was to test calculational methods for fast reactor spectra and the model for neutron inelastic scattering. The tungsten spectra were measured at 300°K and 800°K as part of a current program to measure the effect of Doppler broadening on spectra. The principal features of the experimental technique are discussed, particularly time resolution, beam extraction, and detectors. The U^{235} -sphere measurements are presented and compared with GAPLSN calculations. The spectrum is very close to that in a critical reactor, but there is a discrepancy between theory and experiment. The tungsten results show a clear difference between hot and cold, even to high energies. The difference cannot be explained by the density change, and it is apparently the result of Doppler-broadening scattering resonances. Calculations do not take this into account. Measurements in a larger tungsten sphere, which has a softer spectrum, are now in progress.

1. Introduction

The pulsed-source, time-of-flight method was used to measure neutron angular flux spectra in bulk media from thermal to 15 MeV with good resolution and accuracy. Both multiplying and nonmultiplying materials were studied, with the object of testing and improving methods of computing the nuclear properties of reactors and shields. In this paper we shall discuss the time-of-flight method as developed at the General Atomic Linear Accelerator Facility, some recent measurements of the neutron spectrum in a U^{235} -fueled fast assembly, and partial results of a current program to measure the Doppler effect on the neutron spectrum in a nonmultiplying sphere of tungsten.

* This work was supported by the U. S. Atomic Energy Commission under Contract AT(04-3)-167, P. A. Nos. 2 and 14.

2. The Time-of-flight Method

2.1. Emission Time and Energy Resolution

The pulsed-source, time-of-flight method has been used for many years in measurements of thermal-neutron spectra, and its principles are well-known. However, it may be worth emphasizing that the spectrum measured with an impulse source (and no chopper) is identical to the spectrum which exists in the experimental assembly when the same source is operated continuously. It is not a time-dependent spectrum; in particular, it is not a time-asymptotic spectrum. This is intuitively obvious when we consider that a continuous source is really an emitter of single-neutron pulses, and in the time-of-flight method we integrate over the pulse width of neutrons of a particular energy.

Ideally, the pulse width is very short compared to the flight time of the fastest neutrons detected. Short pulses normally imply a small number of neutrons; however, with the electron linear accelerator we have more than enough intensity. The instantaneous yield of 30-MeV electrons on a uranium target is some 10^{17} n/sec, and we can get 10^9 neutrons in a 10-nsec pulse. Even in a nonmultiplying assembly, however, the width of the burst of neutrons which enters the flight path is increased because of the time required to slow down and migrate to the point of the measurement. In the experiments we correct the measured time interval between the electron burst and detection for the mean emission time. The mean emission time is defined as the first time moment, $\langle t \rangle$, a function of energy and to some extent a function of position and angle. The variance in emission time, $\sigma^2 = \langle t^2 \rangle - \langle t \rangle^2$, where $\langle t^2 \rangle$ is the second time-moment, contributes to the time resolution (along with the detector and electronics jitter and time-analyzer channel width, both of which can be made small). The energy resolution, $\Delta E/E$, is twice the time resolution, $\Delta t/t$, and

$$\frac{\Delta t}{t} = \frac{\sigma v}{L}, \quad (1)$$

if σ is the main contribution to the timing uncertainty, and L is the flight distance.

The methods for computing the time moments have been described in an earlier paper.⁽¹⁾ It is interesting to note the approximate times involved in heavy-metal and multiplying assemblies of interest in fast reactor research. In an infinite medium, with constant elastic scattering, negligible absorption, and $A \gg 1$,

$$\langle t \rangle = \frac{A\lambda_s}{v}; \quad (2)$$

$$\sigma = \sqrt{\frac{2}{3A}} \langle t \rangle. \quad (3)$$

For tungsten, $A = 184$ and the scattering mean free path $\lambda_s \simeq 1.6$ cm in the keV region. Then $\langle t \rangle = 294/v$ and $\sigma = 17.6/v$. The fast-spectrum measurements in tungsten were made on our 50-m flight path, $L = 5 \times 10^3$ cm; hence $\Delta t/t = 17.6/L = 0.3\%$, independent of velocity, and $\langle t \rangle = 213 \mu\text{sec}/\sqrt{E(eV)}$.

These calculations are only approximate. In order to correct for $\langle t \rangle$, accurate calculations or measurements must be made, taking into account nonelastic interactions and the variation of the elastic-scattering cross section with energy. Furthermore, in practice, the measurements are made in relatively small geometries with non-negligible leakage. Fortunately, inelastic scattering, capture, and leakage all reduce the size of the correction.

The emission time in a multiplying assembly is more significant. To a first approximation in assemblies which are not too far subcritical, the neutron pulse is very nearly a single decaying exponential, $e^{-\alpha t}$. Then $\langle t \rangle = \sigma = 1/\alpha$. In order to make the mean emission-time correction small and to achieve good energy resolution, $1/\alpha$ has to be small compared to the flight time of the fastest neutrons detected. In a subcritical assembly the decay constant of the prompt neutrons in the fundamental mode, which we are assuming is predominant, is

$$\alpha = \frac{1 - k(1 - \beta)}{l}. \quad (4)$$

In order to increase α it is necessary to decrease k for a given delayed-neutron fraction β and prompt-neutron lifetime l . A small k also reduces the background due to delayed neutrons.

The spectrum in the subcritical assembly is the same as that which would exist with a continuously operating source, but it is no longer exactly equal to the critical reactor spectrum. However, the difference in fast-neutron spectra is not very great with $k \geq 0.9$, especially since the uranium target photofission-photoneutron spectrum is similar to the neutron-induced fission spectrum. At $k = 0.9$, and for $l = 10^{-7}$ sec, the resolution is

better than 10% up to 0.5 MeV, using the 200-m flight path. Unfolding the pulse-decay shape can extend this limit. Without unfolding, the energy resolution would be 31% at 5 MeV. In many fast critical assemblies the lifetime is less than 10^{-7} sec, which improves the resolution attainable.

Regardless of the value of k , the experiment is designed so that the source can be included explicitly in the calculations. We measure the spectrum of the source by the time-of-flight method. Usually we have quite clean, frequently one-dimensional, assemblies which can be analyzed with the rigorous one-dimensional codes, GAPLSN⁽²⁾ and its successor DTF-IV.⁽³⁾ Multi-group cross sections are obtained from GAM-II,⁽⁴⁾ which includes the Nordheim resonance-absorption theory. Monte Carlo codes such as FMC-N⁽⁵⁾ and O5R⁽⁶⁾ are also used. Some of these codes have already been tested against time-of-flight spectrum measurements in CH_2 ⁽⁷⁾ and in depleted uranium.⁽⁸⁾

2.2. Beam Extraction and Collimation

The spectrum emerging from the surface of the experimental assembly is observed by the detector through a collimation system which defines the area and solid angle contributing to the counting rate. For internal measurements, a small-diameter hole has to be introduced to extract the beam. Figure 1 shows the collimation system for a particular measurement in a CH_2 sphere, but essentially the same system was used in the U^{235} assembly experiments.

The experiment measures $C(t)$, the counting rate per time channel vs flight time. The counting rate per unit energy is given by

$$C(E) = C(t) \frac{dt}{dE}. \quad (5)$$

Since the angular flux, $\psi(E)$, is defined in terms of the area perpendicular to the neutron path,

$$C(E) = \eta A_s \frac{A_d}{L^2} G(E) T(E) \psi(E), \quad (6)$$

where

A_s is the emitting source area (perpendicular to the collimator beam);

η is the efficiency of the collimator system for A_s ;

ηA_s is the effective area viewed by the collimator;

A_d is the area of the detector;

L is the flight distance;

A_d/L^2 is the solid angle subtended by the detector;

$G(E)$ is the intrinsic efficiency of the detector in parallel beam geometry;

$T(E)$ is the flight-path transmission.

The factor ηA_s is obtained approximately by the ray-tracing analysis in Fig. 1, and more precisely by the

CAP code.⁽⁹⁾ The solid angle A_d/L^2 is easily computed. For our present detectors at 50 m it is of the order of 10^{-5} steradian. The function $G(E)$, the detection efficiency, is discussed in the following section, and $T(E)$ is calculated from the total cross section. The transmission includes the Mylar windows and residual air in the evacuated drift tubes, the air spaces between drift tubes, and any filters in the path.

In internal spectrum measurements, the neutron-beam-extraction hole (probe hole) represents a perturbation which tends to decrease the flux and may affect the spectrum shape as well. However, the time-of-flight method does not require insertion of foreign material (detector) inside the assembly. Also, the perturbation problem can be minimized in the time-of-flight measurement with a high-intensity Linac, because the very large pulsed neutron source intensity available permits the use of small reentrant holes. In closely spaced, thermal-spectrum lattices, where probe-hole perturbation would be more serious than in a relatively homogeneous fast reactor, a diameter of only 0.25 cm is used. The electron beam tube and target represent another possible perturbation, if the target is located internally. The diameter of the focused electron beam is about 1 cm, and in the tungsten measurements we used a 7.6-cm-diameter target of depleted uranium. In the U^{235} fast subcritical assembly, the target was the uranium itself, and an electron beam was admitted to the center by a 2.5-cm-diameter hole.

The time-of-flight experiment measures the angular flux, whereas the scalar flux is sometimes of more direct interest in reactors. The angular flux is usually measured at several angles, from which the scalar flux and other angular moments can be derived. In the interior of a reactor, where the flux is nearly isotropic, measurements at 0° and 90° to the flux gradient (or even a single measurement) sometimes suffice.

2.3. Neutron Detectors

We used two detectors to cover the energy range from thermal to about 15 MeV. The high-energy detector was an NE211 or NE213 liquid scintillator on a fast 14-stage multiplier phototube. Two sizes were used, one 5.1 cm in diameter by 5.1 cm high, and the other 12.7 cm in diameter by 12.7 cm thick. A great deal of effort has gone into the development of these detectors and their efficiency calibration. Efficiencies have been calculated by Verbinski *et al.*,⁽¹⁰⁾ using the O5R Monte Carlo code and experimental data. The pulse-height scale is related to the half-way down point on the Compton edge of the pulse-height distribution from Co^{60} (not extrapolated endpoint as in the reference); this is referred to as the cobalt unit. We chose to

FIG. 2. Efficiency of the 5-in. (12.7-cm) NE213 Detector.

bias our detectors at the total absorption peak of the Am^{241} gamma ray, which turns out to be 0.046 cobalt unit. To check the calculated efficiency of neutron detection near the bias energy, we calibrated the detectors with monoenergetic neutrons from the Texas Nuclear Corporation Van de Graaff accelerator. The neutron flux was measured by a long counter which had just previously been calibrated against a proton-recoil telescope. Results are shown in Fig. 2 for the 12.7-cm detector. The agreement is good.

For measurements below 1.5 MeV we have used a moderated boron capture gamma-ray detector. Neutrons are incident on the face of a disk of 75 w/o B_4C and 25 w/o CH_2 , 19.1 cm in diameter and 10.2 cm thick. Mounted coaxially and in contact with the opposite face is a 12.7-cm-diameter by 7.6-cm-deep NE226 (hexafluorobenzene base) scintillator. The neutrons are moderated, mostly by hydrogen, and captured by the boron. The spatial distribution of the neutron-capture probability has been calculated by the O5R code. The 480-keV boron capture gamma ray is detected by the scintillator, which is insensitive to neutrons. The gamma-ray detection efficiency is measured with a small calibrated Be^7 source as a function of position within a duplicate disk. The efficiency curve is given in Fig. 3. The dip at 0.4 MeV is due to a boron resonance. The efficiency is almost constant at lower energies because of a fortunate cancellation of two opposing effects. The neutrons of lower energy are captured more efficiently, but they are captured nearer the front face, away from the scintillator, decreasing the gamma-ray detection efficiency. The mean time to capture is about 150 nsec and σ is about 180 nsec. The timing uncertainty can be reduced by enriching the boron or decreasing the H/B ratio. However, there are two serious problems with this detector; the low boron

FIG. 3. Efficiency of the Boron Capture Detector.

FIG. 4. Cross Section through APFA.

capture gamma ray detection efficiency, and sensitivity to γ rays in the beam. Another type of detector is being built which will alleviate these problems.

3. Spectrum from the Accelerator Pulsed Fast Assembly

The leakage neutron spectrum from the Accelerator Pulsed Fast Assembly (APFA) has been measured by the pulsed-source, time-of-flight technique, and compared with S_n transport calculations. The APFA is an unreflected sphere of 93% enriched uranium metal, similar to the GODIVA reactor. Its primary purpose is to provide short, intense neutron bursts by multiplication of photoneutrons generated by the electron linear accelerator.

Figure 4 is a cross section of the reactor as assembled for the spectrum measurements. The upper segment is spaced 0.5 in. from the remainder of the core. This reduces the reactivity below critical, giving a prompt multiplication of about 18 and a dieaway time constant of 112 nsec. The flight time is corrected for the mean time of emission, but the spread in emission time, as well as other sources of timing uncertainty, result in an

energy resolution of 20% at 10 MeV for our 50-m flight distance. The resolution improves at lower energies, and no unfolding is necessary. The reentrant hole for admission of the electron beam is shown in the figure. The electrons strike the bottom of the hole, generating bremsstrahlung and in turn neutrons by (γ, n) and (γ, f) processes. Beam conditions were a peak current of a few mA at 29 MeV, with a pulse width of 20 nsec and a repetition rate of 360 pps

We measured the angular flux spectrum by viewing four different positions on the surface of the sphere with a 0.87-in.-diameter collimator. Figure 5 shows the positions and the corresponding angles to the radius vector: 0° , 16.6° , 34.8° , and 59.3° . The last position was displaced $2\frac{1}{4}$ in. forward from the vertical centerline to avoid the support socket for the lower uranium segment.

The measured spectra are plotted in Fig. 6. The absolute efficiency of the detectors as used in these experiments was known to $\pm 15\%$ or better. Statistical errors were small, $\pm 2\%$ at several MeV to at most $\pm 10\%$ in the ten-keV region. Background for the 5-cm fast-neutron detector was obtained from the average counting rate at times corresponding to the arrival of neutrons below the bias of the detector; this background was constant and known to a percent or so. The background for the boron detector was obtained also, in this particular experiment, from late channels. This is not strictly correct since the detector is sensitive to neutrons of all energies. However, the APFA spectrum drops off rapidly at lower energies, and since the late-channel background was essentially equal to ambient, the error is not large at 10 keV and above. The runs

FIG. 5. Positions for Spectrum Measurements.

were normalized by referring all spectra to a standard monitor count from a sulfur pellet irradiated in a standard position. The relative normalization for the series of measurements was accurate to an estimated error of $\pm 2\%$. The boron-detector spectra were corrected to the 5-cm-detector spectra by multiplying by the ratio of the detector areas (0.091) since the flight distances were the same and the absolute intrinsic efficiency had already been included in the data-reduction procedure.

When the angular flux spectra were compared, we found the angular distribution in the MeV range dropped off somewhat faster than a cosine function, and the spectrum softened with increasing angle. However, between 0° and 16.6° the change was quite small.

In Fig. 7 the 0° measurement is compared with a P_2 , S_{16} , 17-group GAPLSN calculation of the spectrum at a mean angle of 20° . GAPLSN is a one-dimensional code; hence the calculation was made for a 7.95-

FIG. 6. Measured Angular Flux Spectra in U^{235} Reactor.

FIG. 7. Zero-degree Measurement and Calculation.

FIG. 8. Isometric Cutaway View of the Depleted Uranium Target.

FIG. 9. Drawing of the 25 cm diameter Tungsten Sphere Assembly.

cm-radius sphere, i.e., one which had the same k as the APFA. Cross sections were obtained from a GAM-II calculation. The measured and calculated spectra were arbitrarily normalized at 3.7 MeV. There is some question about the best place to normalize; nevertheless, there is clearly a discrepancy outside of experimental error.

The source spectrum for the GAPLSN calculation was obtained from a measurement with a thin target of U^{235} . However, the photoneutron source was rather unimportant since the multiplication was high. This was confirmed by the GAPLSN calculation for the critical reactor, indicated by the squares on the graph. The spectrum of the subcritical assembly was essentially the same as the spectrum of the critical reactor, except at the highest energies.

FIG. 10. Room-temperature Leakage Spectrum at 47.5°.

4. Spectra in Tungsten

Measurements are underway of the angular flux spectrum in two different spheres of tungsten, one of 25-cm and the other of 50-cm diameter, with our 7.6-cm-diameter, water-cooled, depleted-uranium target at the center. The results will be used to check calculated spectra with different amounts of fast leakage. The smaller sphere is intended to emphasize inelastic scattering, whereas the larger sphere is intended for investigations of resonance absorption. Both are pro-

vided with electrical resistance heaters and the spectrum can be measured as a function of temperature up to about 800°K.

An isometric cutaway sketch of the target is shown in Fig. 8. (In the tungsten experiments the support rod shown was removed.) The target was designed to have isotropic emission relative to the electron beam. The angular flux spectrum at the surface has been measured for input to calculations.

The tungsten is incorporated as vibratory-compacted

FIG. 11. Comparison of Theory and Experiment for Tungsten Sphere.

powder between stainless steel spheres (0.3-cm wall, 15.2-cm ID, 25- or 50-cm OD). A drawing of the smaller sphere is given in Fig. 9. The packed density was 6.98 g/cm^3 for the small sphere. A different powder size was used for the large sphere to obtain a higher density ($\sim 14 \text{ g/cm}^3$).

Figure 10 shows the spectrum leaking from the surface of the 25-cm-diameter sphere at an angle of 47.5° to the normal to the surface. Also shown is the result of a DTF-IV calculation using cross sections generated by GAM-II. The normalization between theory and experiment is arbitrary. In order to see the discrepancies more clearly, a "smooth" curve was drawn through the data, and the ratio of theory to experiment was calculated at each energy group point in the calculation. This ratio is plotted in Fig. 11.

To investigate the effect of temperature on the spectrum, the tungsten sphere was heated to 800°K while holding the uranium source at room temperature. Figure 11 shows the results of that measurement plotted in the same manner as are the room-temperature data. The interesting part about this curve is that theory predicts no observable difference between the hot and cold spectra in this energy interval. Therefore, the difference

seen here is entirely due to the change in the measured spectra with temperature. To see this change, a plot of the ratio of the cold spectrum to the hot spectrum for these same spectra is provided in Fig. 12. In addition to the experimental matter, there is a definite indication that the neutron leakage from the 25-cm sphere is some 15% to 20% greater for the cold sphere than the hot sphere in the energy range near 50 keV. This effect cannot be explained by expansion, since the density change from cold to hot was only 1.8%. This result is contrary to the predictions of all reactor computational methods now in use. The effect involved here is not included in any methods so far developed for calculating reactor leakage. The effect is believed to be simply the Doppler broadening of scattering resonances.

This effect is too complicated to calculate analytically, and no computer methods are yet available to give results for comparison with Fig. 12. However, it is possible to make a very crude estimate of the difference in leakage one might expect. This can be done by estimating the effect of temperature on neutron transmission. If the leakage from the reactor is temperature-dependent, then there must be a similar temperature dependence to the transmission cross section. Consider

FIG. 12. Ratio of Cold Leakage Flux to Hot Leakage Flux for a Tungsten Sphere.

FIG. 13. Square Resonance Model.

the number of neutrons transmitted through a sample which is a few mean free paths thick, at an energy so high that the resolution for the measurement is poor, i.e., much greater than the average resonance spacing D . Also assume that resonances are square in shape with width equal to the Doppler width Δ , and area S equal to the average area of an S-wave resonance at energy E_0 . The effect of Doppler broadening is illustrated in Fig. 13. The area of the resonance is unchanged by temperature, but the height is reduced and widened. With this simple model,

$$\frac{T_c}{T_h} = \frac{(D - \Delta_c)e^{-\sigma_p n t} + \Delta_c \exp \left[-\left(\sigma_p + \frac{S}{\Delta_c} \right) n t \right]}{(D - \Delta_h)e^{-\sigma_p n t} + \Delta_h \exp \left[-\left(\sigma_p + \frac{S}{\Delta_h} \right) n t \right]},$$

where

c, h are subscripts denoting cold and hot;

$$\Delta = 2 \sqrt{\frac{EkT}{A}};$$

$$S = 2\pi^2 \lambda_0^2 g \sqrt{E_0} \Gamma_n^0.$$

Tungsten consists of several isotopes and it is difficult to choose a meaningful average reduced neutron width, Γ_n^0 . However, U^{238} has nearly identical absorption Doppler effects for similar sample sizes, and also its Γ_n^0 is fairly well known to be $\sim 1.6 \times 10^{-3} \text{ eV}^{1/2}$, so the effect will be calculated for uranium instead of tungsten. The potential scattering cross section is assumed to be 10 b.

The ratio $T_{300^\circ\text{K}}$ to $T_{800^\circ\text{K}}$ is calculated for several different thicknesses and plotted in Fig. 12. This should not be construed as a prediction of the effect, but merely a plausibility argument supporting the existence of the effect. Neglected in the calculation are such items as (1) the true resonance shape, (2) statistical variation of resonance areas and spacing, (3) P-wave and D-wave resonances, and (4) relationship between leakage spectrum and transmission cross section. It is interesting to note that inclusion of both items (3) and (4) will markedly enhance the effect at higher energies. It is interesting to speculate about the consequences of this effect on the prompt reactivity coefficients of reactors consisting largely of U^{238} or tungsten.

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Discussion

Mr. Pendlebury (UK-Aldermaston): Have any calculations been done to show the sensitivity of the spectrum to changes in the cross-section data?

Mr. Russell: Only a few some time ago.

Mr. Pendlebury: Do you think there will be any advantage in carrying out these experiments with a range of sphere sizes? In the limit you could imagine, in fact, determining the surface which gives you the spectrum as a function of core radius, by sort of unfolding this spectrum in some way in terms of the cross section and thus determining the cross sections.

Mr. Russell: That's an interesting idea. It might be easier to measure the cross sections rather than make all the spectrum measurements, but I do think it is very worthwhile to make the spectrum measurements for at least two sphere sizes. The rationale for this is as follows: for a small sphere, or thin shell, the spectrum at low energy is determined, primarily by the spectrum of inelastically scattered neutrons that have suffered only one or two collisions. So the ability to calculate this spectrum is a very good test of any model which predicts this inelastically scattered neutron spectrum.

The large sphere has very little leakage, by comparison, and the spectrum is determined not only by inelastic scattering but also by the capture processes. Now, in this case you have good sensitivity to how well you are handling the capture resonances, and how good are your nuclear cross-section models which predict the appropriately self-shielded capture cross sections in the unresolved resonance region.

Mr. Hiraoka (JAERI): You said that you can apply the multigroup analysis to a uranium system. I think it is too small to apply multigroup analysis. There should be some continuous eigenvalue. What do you think about this?

Mr. Russell: This is a point which I should have emphasized at the beginning. The spectrum which you observe in this type of measurement is the same as you would observe with a steady-state source, so that the existence of an eigenvalue is not essential to the experiment, either for its understanding or its performance.

Mr. Gaertner (RPI): Would you comment on the effect of the wall return on the measurements that you have made? Is this an important consideration and, if so, how is it taken into account?

Mr. Russell: Wall effects were not investigated for this particular experiment. You would expect them to be important possibly in the thermal energy range for this type of system because the fast neutrons are dominant here. There are practically no thermal neutrons in this system; if any arrived, they would be by a wall-return effect. The measurements for that particular system did not go down that far. We did have some associated evidence, so that it wasn't all arm waving. We have done measurements of neutrons produced by a source traveling through a slab of material and then into a collimator. Then, of course, the wall effects would be important. By putting a slab of material here we find that it amplifies the wall effect and we found that we had a total of 30% change down to about 10 keV, so the effect should be much higher.

Mr. Kuchle (Karlsruhe): Have you estimated the time resolution you would get from the detector which you first mentioned?

Mr. Russell: That particular detector has a time resolution of about 15 nsec. This is determined primarily by the electronic circuitry. I believe liquid scintillators and phototubes can go to much shorter time.

Mr. Hungerford (Purdue): Did you take into account the effect of the water passage on the outside of this sphere?

Mr. Russell: Yes, we did, by calculation.

Mr. Hungerford: What kind of effect does this have on your results?

Mr. Russell: It has almost no effect on the spectrum, except at energies below a kilovolt, and there it affects the results by some 20 to 30 %.

Mr. Hungerford: How thick was that water passage?

Mr. Russell: 60 mils.

Mr. Gold (ANL): You mentioned, in the GODIVA experiments, a localization of source to approximately one centimeter. Did you use a target for that experiment?

Mr. Russell: No. The electron beam struck the uranium of the assembly directly.

Mr. Gold: Your source arises from photofission presumably?

Mr. Russell: Yes.

Mr. Gold: One would, therefore, have to be cognizant of the fact that it could arise over a much more extended region than one centimeter, depending on the photofission cross section.

Mr. Russell: We have measured the spatial distribution of photoneutron production in uranium metal by taking foils of uranium metal and investigating a photon distribution in this assembly, and also looking at the total neutron yield as a function of target thickness. The distribution that you see is exactly what you say that it is. It is the distribution of high-energy photons in the uranium metal; it's just that they don't go very far at these energies. The pair-production cross section for a 14-MeV gamma ray is enormous.

Measurement of Epithermal Spectra in Fast Assemblies Using Resonance Sandwich Detectors*

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ABSTRACT

Resonance detectors, in general, are not suited for spectrum measurements in fast assemblies because of the contributions of $1/v$ captures and captures in resonances of higher energy. The sandwich-foil technique can be used to eliminate, for the most part, these interfering activities.

Sandwich-foil measurements were made in the fast core of the Mixed Spectrum Critical Assembly (MSCA) and in the ZPR-3 Assembly 48. The detector foil sandwiches had been previously calibrated in a known $1/E$ spectrum. The results reported in this paper are of a preliminary nature only. They show that the sandwich-foil technique does provide a measure of the fast-reactor spectra below 3 keV. However, the present state of the art only gives a qualitative picture of the spectrum.

1. Introduction

Resonance detectors, in general, have more than one resonance and a non-resonance $1/v$ cross section which cannot be neglected. In thermal-reactor spectra the contributions of the higher resonances and the $1/v$ cross sections are small compared with the main resonance, and may be minimized by using very dilute concentrations of resonance materials.⁽¹⁾ In a fast-reactor spectrum, however, the contributions of the $1/v$ captures and captures in resonances of higher energy are larger, and it is difficult to correct for these contributions to obtain the activity caused by the main resonance.

The sandwich method can be used to eliminate, for the most part, contributions to the activity from small cross sections, and the measured activity is a sum of contributions from large resonances only. The measurements reported in this paper are of a preliminary nature only. They show that the sandwich-foil technique does provide a measure of the fast-reactor spectra below 3 keV, but the present state of the art only gives a quali-

tative picture of the spectrum. Further improvements in both the experimental and analytical aspects of the problem should make the sandwich-foil measurements more quantitative. The method offers a low-energy complement to proton-recoil spectrometry which presently has a lower energy cutoff of about 1.5 keV.

2. Theory of the Measurement

The theory of the sandwich measurements has been given by Judd⁽²⁾ and Ehret.⁽³⁾ In essence, the reaction rate of a foil may be divided into two parts: one due to the strong resonances, and the other due to very weak resonances and the $1/v$ cross section. The foil thickness is chosen so that the self-shielding is small away from the strong resonances. Following Judd and assuming that the introduction of the foil does not perturb the spectrum, we may write for the foil reaction rate

$$R(t) = \int_{\text{non-res}} t \Sigma_a(u) \Phi(u) du + \sum_i R_{\text{res } i}, \quad (1)$$

where $\Phi(u)$ is the flux per unit lethargy, i denotes a particular resonance, t is the foil thickness, and R_{res} is the resonance reaction rate. The integral is taken over the lethargies where the flux depression is small, i.e., where the reaction rate is linear with t . The sum represents the strongly self-shielded resonance contributions. To calculate R_{res} for a single resonance, Judd used the Breit-Wigner line shape for the cross section. However, some detectors such as gold, indium, or tungsten have several higher-energy resonances which become important in a fast-reactor spectrum, and Doppler broadening of the line shape should be considered. Thus, assuming that a neutron is removed from a resonance by either absorption or scattering and that the fraction $\sigma_a/(\sigma_a + \sigma_s)$ results in the production of an active nuclide, one obtains for the activation rate

$$R_{\text{res } i} = \frac{1}{2} \Phi_i(u) \Gamma_i \int_0^\infty [1 - 2E_3(Z, \Psi(\theta, x))] dx, \quad (2)$$

where $\Phi_i(u)$ is the flux per unit lethargy at the reso-

* Work done under AEC Contract AT(04-3)-189, P.A. 10, Sodium Cooled Reactors, Fast Ceramic Reactor Development Program.

nance energy, Γ_γ is the gamma-radiation width, $E_3(x)$ is the exponential integral $\int_1^\infty (e^{-xt}/t^3) dt$, Z is the foil thickness in total mean free paths at the peak of the resonance, and $\Psi(\theta, x)$ is the usual Doppler-broadened Breit-Wigner line shape. The assumption that neutrons are removed from a resonance by one elastic scattering is valid for most foil materials under consideration since the total widths, Γ , are small relative to the average energy loss per collision. If a sandwich of three identical foils is irradiated, the reaction rates of the outer and inner foils of the sandwich are, respectively,

$$C_0 = \frac{1}{2}R(t) - \frac{1}{2}R(2t) + \frac{1}{2}R(3t); \quad (3)$$

$$C_i = R(2t) - R(t),$$

and therefore the difference in activity is

$$C_0 - C_i = \frac{1}{2}[3R(t) - 3R(2t) + R(3t)]. \quad (4)$$

Upon substituting Eq. (1) in Eq. (4) it is seen that the integral term vanishes because it is linear with t . The remaining difference activity, C , is due to the strong resonance contribution and may be expressed as

$$C = C_0 - C_i = \alpha \sum_{i=1}^n K_i \Phi_i(u), \quad (5)$$

where α is a proportionality constant typical of each material. It includes the detector efficiency, constant gamma width (Γ_γ), and the amount of material, as well as a flux-normalization factor. Also, $K_i = [3I(t) - 3I(2t) + I(3t)]$, where the I 's are the integrals in Eq. 2 taken for one, two, and three foil thicknesses.

By irradiating the sandwich foils in a flux that is known to be $1/E$ -dependent, the constant α is determined from

$$\alpha = \frac{C(1/E)}{\Phi_{1/E} \sum_{i=1}^n K_i}, \quad (6)$$

where $C(1/E)$ is the $1/E$ difference activity. Thus an unknown flux at the lowest-energy resonance, $\Phi_{1X}(u)$, may be expressed as

$$\Phi_{1X}(u) = \frac{1}{K_1} \left\{ \frac{C(X)}{C(1/E)} \Phi_{1/E} \sum_{i=1}^n K_i - \sum_{i=2}^n K_i \Phi_{iX}(u) \right\}, \quad (7)$$

where $C(X)$ is the measured difference activity in the unknown flux. If there is only one resonance, as in La-139, Cd-114, and Na-23, the activity difference between the outer and inner foil is due only to this resonance and no corrections are necessary, i.e.,

$$\Phi_{1X}(u) = \frac{C(X)}{C(1/E)} \Phi_{1/E}. \quad (8)$$

The data from the single-resonance detectors are used as bases for computing the higher-resonance contributions for the other sandwich materials.

3. Description of the Measurements

3.1. MSCA Fast Core

Sandwich-foil measurements were made at Vallecitos Atomic Laboratory as part of the Mixed Spectrum Superheater Critical Experiments.⁽⁴⁾ The Mixed Spectrum Critical Assembly (MSCA) is a full-scale mockup of a 75- to 150-MWe power plant reactor core. The reactor contains four distinct fuel zones: a fast core, a fast buffer, a thermal buffer, and a thermal core. The fast core of approximately 407 liters contains about 2000 kg of UO_2 with an average enrichment of 23.1 % U-235, and 1100 kg of Inconel. The fast buffer zone contains 1.3 w/o enriched UO_2 fuel rods clad in stainless steel and effectively absorbs most of the thermal-neutron current emanating from the thermal region. This arrangement results in an asymptotic spectrum in the fast core which approximates that of a large, dilute, fast ceramic reactor.

Indium, gold, tungsten, lanthanum, cadmium, sodium, and manganese were found to be suitable for the sandwich measurements. The foil thicknesses were chosen so that the center foils were substantially self-shielded at the dominant resonance cross section but had much smaller or negligible self-shielding at the other resonance and non-resonance cross sections. The foil thicknesses and materials used are given in Table 1.

All foils were 0.5 in. in diameter and were enveloped by a ring foil to position the foils and to prevent edge effects. The ring foils were three times the thickness of the single detector foils and were 0.75 in. in diameter. Each sandwich was taped together with small pieces of Teflon tape. To prevent oxidation of the lanthanum foils during the experiment, a thin coating of silicon vacuum grease was used. All sandwiches, including those of cadmium, were irradiated in small boxes of 20-mil-thick cadmium. This prevented a thermal-flux depression in the sandwiches during the $1/E$ calibration. To verify the cadmium measurements, cadmium sandwiches were also irradiated in covers of enriched B-10.

The foils were first irradiated in the Nuclear Test Reactor at Vallecitos, which has a $1/E$ -dependent epithermal spectrum. The gamma activities were measured with a 3 x 3-in. NaI detector and a 256-channel analyzer. In this manner the foil response and detector efficiencies were calibrated for the known spectrum. In the $1/E$ spectrum, greater than 97 % of the difference activity measured for each material was due to its lowest resonance. The measured differences varied from

about 0.5 % of a single-foil activity for sodium to about 50 % for indium. Considerable care was exercised to obtain good statistics for the smaller difference activities.

Identical sandwiches were subsequently irradiated at the center of the fast core of the MSCA, and the foils were counted in the same manner as before. The relative fluxes at the lanthanum, cadmium, and sodium resonances were determined by means of Eq. (8). Based on these results, an iterative procedure was used to apportion the measured difference activities among the several contributing resonances for the tungsten, gold, and indium foils. For the iteration it was assumed that the flux $\Phi(u)$ had the form aE^b , i.e., a straight line on a log-log plot of $\Phi(u)$ vs E .

Since the manganese resonances are wide compared with the average energy loss per elastic collision, the broad higher resonances should not contribute to the measured difference activity. For this reason the manganese activity was assumed to be due only to the contribution of the lowest resonance. A more detailed treatment of the manganese data is required, however, to assure its validity.

3.2. ZPR-3 Assembly 48

A similar sandwich-foil measurement was made at the center of the ZPR-3 Assembly 48.⁽⁵⁾ Since the neutron spectrum was expected to be harder, the indium-foil measurements were omitted. Three sets of foils were irradiated at the core center and two sets in the blanket. The manganese foils were counted at ZPR-3 using a Co-58 standard to intercalibrate with the Vallecitos counting equipment. The other foils were counted at Vallecitos, and the data were treated in the above manner.

4. Results and Discussion

The results of the MSCA measurements are shown in Fig. 1, and compared with a calculated 24-group spectrum by a least-squares normalization. Several of the calculated higher-resonance points are included together

FIG. 1. Spectrum in Fast Core of MSCA.

with the percentage of the measured activity contributed by the resonance. It is seen that only 3.3 % of the gold activity is due to the 4.9-eV resonance, accounting for the large uncertainty in that point. The higher resonances, containing more of the activity, have smaller uncertainties and so may be used as flux indicators. The cadmium point is an average of the cadmium- and boron-covered sandwich measurements, which agreed to within 10 %. The large error bars on all of the data were primarily due to the fact that anisotropies in the flux often caused the two outer foils to have different activities. Theoretical considerations indicate that small flux gradients across the sandwiches may be neglected and the activities of two outer foils averaged. However, it was decided at this state of the measurement to treat the differences as errors. By neglecting this uncertainty the error bars would have been reduced to well under ± 20 % in most cases.

Except for the sodium point, which appears to be low, and the cadmium point, which appears to be a little high, the measured spectrum is in general agreement with the calculation.

The results of the measurements at the center of ZPR-3 Assembly 48 are shown in Fig. 2, and compared with two preliminary calculated spectra: a 24-group calculation by ANL and a 60-group calculation by General Electric. The blanket data did not yield significant results because of low count rates.

In Fig. 2 it is seen that the nuclides that have higher resonance corrections are not in agreement with the other nuclides. It is probable that two factors contribute to this. One is that there are very few neutrons

TABLE 1. SANDWICH-FOIL MATERIALS

Nuclide	Energy of Principal Resonance, eV	Material		Thickness (mils)
In-115	1.46	In-Pb	50% alloy	6
Au-197	4.9	Au	Metal	1.5
W-186	18.8	W	Metal	2
La-139	73.5	La	Metal	10
Cd-114	120	Cd	Metal	20
Mn-55	335	Mn-Cu	80%-20% alloy	2
Na-23	2950	NaCl	Crystal	20

FIG. 2. Spectrum in ZPR-3 Assembly 48.

t the energies of the lower gold and tungsten resonances, as predicted by the 24-group calculation. The other is that the contributions of the higher resonances are overestimated. This effect would become more sig-

nificant in a spectrum with a smaller low-energy tail such as is indicated for Assembly 48.

The method of correcting for the higher resonances was further investigated by irradiating 7-foil sandwiches of gold and tungsten in a $1/E$ spectrum. The normalized measured difference activities are shown in Fig. 3, together with values calculated in a manner analogous to Equations 2 through 5. The agreement appears to be quite good. However, the data were normalized at the outer foils and, in the $1/E$ spectrum, the difference activity of the outer foil is dominated by the lowest-energy resonance. This has the effect of masking the contributions of the higher-energy resonances. It is seen that the experimental points for foils (2 and 6) and (3 and 5) fall somewhat below the analytical values, particularly for the tungsten. Even a slight overestimation of the higher-energy resonance contributions would become quite significant in the fast spectra of the MSCA and Assembly 48. For this reason, additional effort, both analytical and experimental, is still required in order to obtain a more consistent picture of the resonance self-shielding in the sandwich foils. Measurements with 7-foil sandwiches in the fast spectra should be particularly useful.

It should be noted in both Figures 1 and 2 that the

FIG. 3. Flux Depression in Seven-foil Sandwiches in a $1/E$ Spectrum.

cadmium data points appear high in each case. A possible cause is that flux depressions due to the other cadmium isotopes cause measurable perturbation in the Cd-114 difference activities. This is not unreasonable since, while the $1/v$ capture cross section is low at the higher energies, the relative flux is much greater. This perturbation effect should be evaluated for all of the foil materials containing more than one nuclide.

The results presented at this time are of a preliminary nature only. There is still a need for considerable further work, particularly in the analysis of the resonance self-shielding effects that give rise to the measured difference activities. However, it is believed that the experimental results to date, even using the simplest analytical approach, are encouraging. With additional effort the sandwich-foil technique should offer a quantitative measure of fast reactor spectra below 3 keV.

Acknowledgments

The author wishes to acknowledge the work of W. A. Mayer and P. S. Brown, who initiated the sandwich experiments at Vallecitos. The author also wishes to express his thanks to F. W. Thalgott, J. K. Long, and the Staff of ZPR-3 for their excellent cooperation during the Assembly-48 measurements.

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Discussion

Mr. Weale (UK-Aldermaston): I would like to comment really to point out something you didn't mention but no doubt realize, that this is a potentially valuable technique as compared with the proportional counter and time-of-flight methods. It is an in-core technique and it can be put into a very small cavity in the core without such problems as disturbing the material around the sample. We have tried to use manganese. It seems that if one assumed that the flux was known, one could get the slope of the spectrum in the region from 330 eV to the next higher resonance in manganese at 1 keV. Alternatively, if you know the

slope of the spectrum you could get its absolute value, but in both cases the results don't seem very reliable. We found errors of about 30 % or so in this.

Mr. Weitzberg: An additional difficulty with manganese is that the high resonances are quite broad, and the assumption that you can remove a neutron from a resonance just by single scattering is not valid. Inherent in Judd's first formulation was the assumption that a neutron is removed either by absorption or scatter, and then one takes the ratio Σ_a/Σ_t . Now I think in broad resonances you get a contribution from multiple scattering and this should be taken into account. In fact, simple analysis obviously doesn't work, and it's a question of going perhaps to ELMOE-type calculations, not only for manganese, but for the others as well.

Mr. Weale: McCracken at Harwell did some interesting work on this recently, involving measurements in the ZEBRA blanket. This was reported at the International Conference on Radiation Measurements in Nuclear Power, held at Berkeley, England, in September 1966. It would be interesting to compare that work.

Mr. Pearlstein (BNL): One of the comprehensive experimental papers on the sandwich-foil technique is that by Ehret of Germany. In his paper he showed that he could not get agreement between calculated and experimental ratios of the outer foil to the inner foil. Now at Brookhaven, what we did was a calculation including Doppler broadening and scattering, and we produced agreement. Unless you calibrate your foils in a flux that you know to be $1/E$, you are in trouble if you rely on calculations that don't include these sophisticated corrections.

Mr. Weitzberg: I agree that $1/E$ calibration is the only absolute standardization that you can get, but unfortunately such a spectrum deviates drastically from a fast spectrum. You would be shifting emphasis from where your activity is perhaps 97 % in the lowest energy resonance to where it is less than 1 %. It would be nice if there were some higher-energy spectrum that could be used but this is not the case.

Mr. Weale: Could you comment on the possibility of calibrating the foil method using time-of-flight techniques?

Mr. Weitzberg: Yes, this would be valuable.

Mr. Kulian (AEG—Germany): At the 1965 Conference here (ANL-7120), some Doppler activation measurements in the MSCA were reported. The comparison with theory indicated that the experimental spectrum might be softer than calculated. Now, you show good agreement with the calculated spectrum within the limits of error. I would like to know if you checked against those old Doppler measurements?

Mr. Weitzberg: Yes. In the work which you men-

tioned the Doppler results were in fairly good agreement with this spectrum. This was the so-called soft spectrum. One of the possible reasons for the fact that the gold and indium sandwich foils gave quite high results was the possibility of streaming in the assembly. When these measurements were repeated, we took great care to do the measurements in a little central void. Now the sandwich-foil measurements are fairly consistent with the calculations, which are in turn consistent with the calculated Doppler effect and that measured by Pflasterer. The reason for the large error bars is because there is very small activity in the gold. The error bars are still quite large, and the reason for this is that in doing these measurements I have decided that, when anisotropies in the flux result in different outer foil activities, I would treat them as errors rather than averaging them. The theory indicates that for small flux gradients you could just average the outer

two foils, but I think that in the present state of the technique larger error bars are more honest.

Mr. Stegemann (Karlsruhe): I have a comment on the sodium-activation measurements. There are three points which have to be considered. The first one was mentioned by Weitzberg—the interference with chlorine. You can get rid of this if you use sodium fluoride instead of sodium chloride, which we did with measurements in STARK. The second thing is that for intensity reasons you have to use thicker foils of sodium fluoride. You have to take into account multiple scattering. This has been done, at our Laboratory, by a numerical procedure which will be published later. The third thing I would like to mention is that if you measure the sodium activity absolutely by gamma-gamma coincidence techniques, you have to be careful because there are angular correlation effects, requiring a correction factor of up to 10 %.

SESSION VI

October 12, 1966

REACTOR DESIGN AND PERFORMANCE MEASUREMENTS

Chairman: H. A. Morewitz
Atomics International

Secretaries: G. K. Rusch
R. A. Karam
Argonne National Laboratory

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The Utilization of Fast Critical Assemblies for Fast Reactor Design

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ABSTRACT

The role of the fast critical assembly for fast reactor design is reviewed primarily on the basis of past experience and projected ultimate application. Certain integral data which have not received much recent experimental emphasis but are important to confident reactor design are described. It is judged that engineering-mockup-related critical experiments will be needed for some time, at least until analytical methods and differential nuclear data may be confidently used for conventional fast reactor design. It is, however, necessary to project the development of relatively clean critical experiment data beyond the time when engineering mockups will be required for reactor design. Such problem oriented experiments will continue to give insight on more confident fast reactor design by narrowing the gaps between predicted and observed high-power LMFBR performance.

1. Introduction

There is probably no unique set of procedures for utilizing data from fast critical assemblies for reactor design. A variety of approaches have, can, and probably will be used depending upon the nature of the particular system and the particular point in time when data will need to be developed. The timing aspect will depend upon the evolving degree of reliability pertaining to appropriate analytical techniques along with microscopic nuclear-input data.

A number of assumptions are implied here in order to develop some ideas of how fast-critical-assembly data can or may be generated. These are neither assumed to be unique nor optimum for all points in time. It is hoped that they are at least provocative. Furthermore, specialized studies of coolant and Doppler reactivity are currently heavily stressed elsewhere. Such integral experiments are here generally implied for all major experimental investigations and are not discussed further. This discussion will attempt to focus on the problems of neutronic simulation as well as need for the performance of applied experiments which have not recently received a great deal of emphasis.

The latter will be concerned with obtaining a few analytical anchor points for specific calculations as well as the development of general integral data for reactor design.

The role of the fast critical assembly for fast reactor design is changing as design emphasis is changing from the relatively small, low-burnup, uranium-fueled systems (such as EBR-I, EBR-II, Fermi, and Dounreay) to the larger, high-burnup, plutonium-fueled systems (such as FFTF, a several hundred megawatt demonstration system, and the 1000-MWe or larger LMFBR systems). All of the small U-235-fueled systems were studied with reasonably detailed engineering mockups of the reactor neutronics with a zero-power facility. They were also studied via relatively clean mockups facilitating comparison with reactor analysis in relatively simplified geometry. Another type of basic fast critical experiment with extremely simple geometry and composition was also used for comparison with predictions of reactor performance and extrapolated to the engineering system. All three approaches have advantages as well as disadvantages with respect to application to reactor design.

The amount of flexibility available to perform detailed engineering-mockup experiments seems to be somewhat less as one contemplates the current projected plutonium-fueled designs of interest. This is not considered very limiting since analytical and nuclear-data developments are constantly improving.

The nature of some problems facing the designers of smaller, earlier reactors are also somewhat different from those facing the current and future designers. It is judged that this will ultimately be reflected in a major reconsideration about the desirability to perhaps even dispense with the detailed engineering mockup in favor of a series of related critical assemblies and/or zoned systems. These may, in fact, provide better information for confident reactor design. Useful data for a reactor design can be generated as a function of system configuration, gross composition, and isotopic composition, within a parametric but realistic framework.

The role of the critical experiment in the reactor itself must also be considered. The potential of the dry and wet critical experiments cannot be neglected with at least the first loading of the reactor. Significant long-term reactivity data can be generated by the reactor itself through periodic zero-power tests. These can be correlated with small-sample replacement experiments in critical assemblies to isolate nuclear from mechanical burnup effects. The latter pertain to fuel growth and/or redistribution.

Three major items are discussed below. These refer to the past history of generating nuclear data for operating fast reactors, the projected ultimate utilization of fast critical experiments for fast reactor design, and the performance of some integral experiments which would lend confidence to current design prognostications.

2. Experience

To date, the experience of utilizing fast-critical-assembly data for reactor design has been uniformly good. The three most interesting operating systems for such correlations are the EBR-II,⁽¹⁾ Fermi,⁽¹⁾ and Dounreay⁽²⁾ reactors. Here the zero-power critical-assembly data were translated into engineering data for operating high-power-density reactors. The correlations between experimental data and operating reactors have been better than the applied analytical techniques and nuclear data might warrant. It should be noted that in two of these three cases, mainly those of Fermi⁽³⁾ and Dounreay,⁽⁴⁾ the zero-power systems did not simulate all details of the final operating system. There were, however, detailed simulations of similar core compositions. The EBR-II neutronic simulation^(5, 6) was also somewhat different from that of the actual system. This differed more in reactor size than in composition from what was actually desired.

The experience with EBR-II points up the fact that one can consider close neutronic simulation of an engineered reactor at too early a point in time. Discussions considered the simulation of EBR-II with ZPR-III as early as 1955. If this had been prematurely carried out, the reactor simulation would not have been the one for the first loading. The early EBR-II reactor design featured a central blanket of fertile material. This has not been included in the reactor to date. Similarly, the Fermi core composition was slightly altered between the neutronic simulation of ZPR-III and the final design of the reactor core. Changes of the Fermi composition were dictated by engineering considerations. Dounreay redesign probably came through accommodation for a device to prevent fuel-element bowing. Both Fermi and EBR-II neutronics were also studied on what may be called a clean-core

version of the engineering mockup^(3, 6) which did not treat all of the geometrical asymmetries of the reactor design. These experiments were, in a sense, much more useful for testing calculational methods and nuclear data. The analytical methods at that time were generally believed more reliable and appropriate in clean geometry. With reasonably reliable methods, one had confidence that the correlation between theory and experiment was primarily an indication of cross-section deficiencies. However, all of these systems seem to have been reasonably well neutron engineered with what are now known to be strong limitations on knowledge of neutron cross-section data.

All three of the operating fast breeder reactors had extremely good correlations between predictions, extrapolations, and actual observations. Tables 1 and 2 give a good idea of what the situation is for EBR-II and Fermi. Table 1, dealing with EBR-II, cites correlations for critical mass, various control mechanisms, isothermal temperature coefficient of reactivity, neutronic power calibration, and various substitution experiments. "Original Prediction" refers to that before mockup experiments, whereas "Final Prediction" was influenced by mockup data. It should also be noted that the ZPR-III Assemblies 2 and 5⁽⁷⁾ were actually perhaps more useful for neutronic interpretation than the engineering mockups. These assemblies were generally used for most analytical correlations, while the engineering mockup was primarily an empirical anchor point.

The measured power coefficient of reactivity ($4.2 \times 10^{-5} \Delta k/\text{MW}$),⁽¹⁾ at constant flow, was reasonably consistent with predictions ranging from 3.7 to $4.9 \times 10^{-5}/\text{MW}$,⁽⁸⁾ depending upon assumptions regarding fuel-element bowing and relative thermal motion of control rods relative to core. In all cases, the critical-assembly data improved performance predictions for the operating system. The predictions relied upon correlations with material-replacement data. The predicted constancy with power level at constant flow was confirmed, suggesting a trivial influence of fuel-element bowing.

Table 2, dealing with Fermi data, generally shows the same trends as EBR-II data.

The correlation between predicted and measured power distribution in EBR-II surpassed expectations. Figure 1 shows the results of prediction, zero-power measurement, and high-power burnup data.⁽¹⁾ However, it was found that the distribution for the currently perturbed driver configuration is not in close agreement with what was measured.⁽⁹⁾ These perturbations are due to the large number of long-term in-core prototype fuel, clad, and structural specimens.

Burnup reactivity was found to be in reasonable

TABLE 1. PREDICTED AND MEASURED EBR-II NUCLEAR PARAMETERS⁽¹⁾

	Original Prediction	ZPR-III Mockup Measurement	Dry Critical Measurement	Final Prediction	Wet Critical Measurement
Critical mass, kg U-235	172	165	228	176-184	181.2
Reactivity worth, % $\Delta k/k$					
Control rod (avg)	0.45	0.37	0.35	0.34	0.345
Control rods (12-banked)	—	—	—	—	4.4
Poison ($B^{10}C$) ^(a)	—	0.55	0.58	0.55	0.57
Safety rods (2)	1.5	1.36	1.0	1.3	1.2
Sodium					
Total	—	6.9	—	6.0 $\pm$ 0.5	5.8
Core (% $\Delta k/k$)kg	0.14	0.116	—	0.116	—
Isothermal temp coef. [$(\Delta k/k)/^{\circ}C$] $\times 10^{-5}$					
Structure (including fuel)	-1.9	—	-2.6	-2.2 $\pm$ 0.4	—
Coolant	-1.7	—	—	-1.8	—
Total	-3.6	—	—	-4.0 $\pm$ 0.4	-4.3 ^(b)
Power calibration, counts/W-sec	—	730	—	500	390
Subassembly substitution, % $\Delta k/k$					
Core center	1.43 ^(c)	1.53 ^(c)	—	1.26 ^(d)	1.2 ^(d)
Row 5 (Radius = 21 cm)	—	0.77 ^(c)	—	0.64 ^(d)	0.56 ^(d)
Row 6 (Radius = 27 cm)	—	—	0.35 ^(d)	—	0.35 ^(d)

^(a) Alternate control rod containing 253 gm of 87% enriched $B^{10}C$ in upper 7 in. of control-rod follower.^(b) Measured at 237-315°C.^(c) Enriched uranium subassembly versus sodium.^(d) Enriched uranium subassembly versus natural uranium subassembly of otherwise identical composition.TABLE 2. PREDICTED AND MEASURED FERMI NUCLEAR PARAMETERS⁽¹⁾

	Calculated ^(a)	ZPR-III Mockup	Final Prediction ^(b)	Measured ^(b)
Critical mass (at 205°C), kg U ²³⁵ :	416.9 (17°C)	439.9	467.9	465.6
Reactivity worth, % $\Delta k/k$:				
Shim rod	0.304	0.310	0.317	0.309
Regulating rod	0.304	0.310	0.317	0.290
Safety rod (avg)	0.888	0.914	0.974	0.849
Safety rods (7-ganged)	4.93	5.06	5.40	5.30
Sodium (avg) [$(\Delta k/k)/kg$] $\times 10^{-5}$	+22.5	+13.2	+15.9	+13.2
Isothermal temp coef. [$(\Delta k/k)/^{\circ}C$] $\times 10^{-5}$:				
At 205°C	-2.4 (avg)	-3.37 (avg)	-3.21	-3.12
At 316°C	-2.4 (avg)	-3.37 (avg)	-3.41	-3.22
Power calibration, counts/(W)(sec): ^(c)	18.3	—	13.3	14.4
Subassembly substitution, % $\Delta k/k$:				
Core center	0.992	0.875	0.870	0.774
Core edge (Radius = 39.9 cm) ^(d)	0.166	0.203	0.193	0.225

^(a) 367-liter core, including ten control and safety rod channels.^(b) 392-liter core, including ten control and safety rod channels.^(c) 200- μ g absolute fission counter in core.^(d) Core-blanket interface is at 40.3 cm.

FIG. 1. EBR-II ^{235}U Radial Fission Rate Distribution.

agreement with prediction and material-replacement data from ZPR-III ($\sim 10\%$).

Even some transmutation reactivity effects due to three-day neptunium data were in qualitative agreement with predictions.

No evidence of significant phase changes as based on material-replacement data was observed, though a small effect was predicted.

3. Future Prospects

Both ZEUS, the zero-power reactor simulating the Dounreay system, and ZPR-III, the zero-power reactor originally conceived in support of the EBR-II reactor, were designed to simulate a fairly specific reactor configuration. It was thus possible to design the zero-power machine so that it could best simulate the reactor of interest. This is in stark contrast with the present situation, in which zero-power facilities are being and have been designed with only general ideas as to the nature and composition as well as size of the systems that may ultimately have to be simulated. Thus, there has been a real premium on flexibility of zero-power reactor design, so that as many of the design variations that can be dreamt up can, in fact, be accommodated. This required flexibility in facility design does leave some interpretive margin between the

zero-power simulation and the actual reactor system of interest. This margin will have to be covered by analytical techniques and nuclear data which are constantly improving and significantly better than those that were used to design the three operating high-power-producing machines.

There is an important intermediate step between the detailed design of systems with which we now have power experience and the longer-range high-power-producing systems that fall into both the demonstration and full-scale systems. Two examples in the United States include the SEFOR and the FTR. SEFOR has already been simulated in ZPR-III with plutonium fuel. The experimental data gave reasonably good correlation with theory. However, provisions for flexibility in the facility introduced some compositional asymmetries into the mockup which raise questions on proper analytical interpretation and their ultimate effect on the reactor.

The large power-producing system simulations, several years hence, cannot avoid the question of isotopic content for the zero-power fuels. The relative contents of Pu-239, Pu-240, and Pu-241 will generally not be the same as those of the projected systems. The extent to which the engineering configuration must simulate the actual isotopic content is a matter for discussion. However, practical considerations require that the slack be taken up by analysis and small experiments. As such, it may not be necessary or perhaps even desirable to perform detailed engineering mock-ups. It may be much more desirable for reliable analysis of the critical and translation to the engineering design to perform critical experiments for these large breeder systems in relatively clean geometry. The analytical techniques may be used to handle the geometric asymmetries. The latter can be verified by related geometric experiments. Such an approach is believed to be ultimately the most useful. However, during the transition period, when confidence in analytical techniques may be limited and lack complete verification, such an approach might produce some extremely tense moments, particularly about the time when one fixes the fuel configuration. It will also be necessary for the translational analysis to handle problems related to fission products. The projected systems will operate with an equilibrium configuration which probably cannot be properly simulated. No way has been devised properly to introduce fission products commensurate with three or five percent average fission product concentrations through the core in currently projected fast-spectrum zero-power reactors. However, it appears that fission product effects are probably not dominant in unmoderated applications compared

to fissile burnup, higher isotope effects, and possible poisoning for excess fuel-cycle reactivity.

It will be necessary to establish whether the engineering mockup or related experiments should simulate the initial configuration of the reactor or simulate the operating equilibrium configuration. The difference could well represent reactivity of the order of a percent or more, depending on the ability to simulate properly either of the two configurations. If the startup isotopic content is different from that ultimately in the reactor, these considerations are further complicated.

Proper study of the variation in power shift from core to blanket, as well as changes in peak blanket power density and peak core power density as a function of blanket residence time, are items requiring experimental verification. One approach might be simply the replacement of depleted by natural uranium to simulate the growth of plutonium in the blanket in either heterogeneous or homogeneous array. It is expected that modular-core performance and absolute flux-monitoring considerations would be most sensitive to such effects.

The utilization of fast criticals for fast reactor design after certain questions of validity for certain analytical techniques and cross-section measurements and assumptions are verified can be more easily developed. It is expected that there will be no need to perform detailed engineering mockups. It is, however, expected that there will be need for integral data. Whether these will be obtained from full-scale critical experiments or zoned configurations, which develop the spectra of interest, is not clear.

Current emphasis on testing various types of zoned experiments and concepts may have strong bearing on the ultimate course. For example, experiments such as those performed on ZEBRA in support of PFR development may be quite attractive in this context. As the range of validity of certain methods and input data are developed, it is expected that the role of some sort of zoned experiment will increase and the role of the large critical will decrease. Until then, it is expected that large, clean-geometry critical experiments will be needed as a fairly important ingredient to reactor-design programs. The emphasis for detailed engineering mockups will decrease and, to some extent, vanish. Perhaps the only time a detailed engineering mockup will be needed will be in support of a radically different design. It is not likely that many such radically different core and reflector designs will evolve once there is a commercial industry. There will be improvements and small deviations from then existing designs, more on the lines of an evolutionary process. However, should some radically different core composition look economically attractive, at some point in

time, it is expected that some sort of engineering mockup, perhaps in clean geometry, may be required. This experiment will, if nothing else, instill confidence in analytical techniques at an early date.

Critical experiments in the reactor during the startup program and following extended fuel burnup will provide much information that cannot be generated efficiently by a zero-power facility. It would be extremely informative to have a basis for measurement of actual reactivity lifetime which would be affected by transmutation as well as mechanical aspects of fuel growth and relocation. Such experiments will also have useful demonstration aspects for confirming expected reactor-performance data. A dry critical experiment can be used simply to confirm certain assertions regarding the reactivity worth of sodium coolant. It is judged that the dry critical experiment will not be intrinsically useful for general reactor physics other than for such demonstrations.

The need for certain types of critical and integral data will, to some extent, be determined by allowable uncertainties of criticality, power distribution, and operative reactivity coefficients and effects. If these margins are extremely narrow, as perhaps dictated by economics, the need for simulation types of experiments will persist longer than if these margins can be rather broad initially. Some technical-design flexibility can usually be provided to accommodate some such uncertainties. This may, however, have economic implications.

The isotopic content of critical-assembly plutonium must be considered in terms of desired integral data. Much of the zero-power information will be generated with material of relatively low Pu-240 content. In virtually every case it is judged that, at least beyond the initial stages, the Pu-240 content of the reactor design will be somewhat higher than that used in the U.S. zero-power machines. It is intended that the critical assembly inventories will be augmented by smaller inventories of plutonium of high Pu-240 and reasonable Pu-241 content. This material will afford a basis for experimental transition from the large, low-Pu-240 critical assembly to the operating reactor. This will be affected via perturbation and zoned-core measurements. These can be studied either near the center of the core or near the core boundary in zones that are large enough so that some sort of equilibrium can be achieved. The latter may pose some interesting interpretive aspects.

4. Relatively Current Problems

Some types of design-related neutronic data should probably be developed as analytical reference points for the reactor designer. As such, these reference points

should not be peculiar to a reactor design but should serve to provide experimental data for particular problems. (Some of these have already been referred to above.) With the exception of data in support of FARET, there are virtually no interesting data to test calculations on the spatial variation of the radial core power distribution, and related reactivity effects, of a radially power-flattened configuration. Such data may flag the need for more accurate cross-section-averaging techniques within an engineered core. (To date, there are also no routine analytical or iterative techniques to facilitate power flattening.) Experimental studies to confirm attenuation of blanket and reflector neutrons are also needed, including some indication of blanket-buildup effects. These studies will also give data for comparison with predicted out-of-core instrument response as a function of buildup. The latter problem is probably most pronounced for modular arrays. This type of investigation is also oriented toward optimizing blanket inventory by providing experimental data for realistic estimates on blanket leakage.

Another type of experiment which has received little recent attention and may be quite important for reactor design pertains to the measurement of γ heating in structural, clad, and fertile materials.* This problem is particularly pronounced in non- or low-fission-density regions adjacent to high-fission-density regions. Absolute values, related to fission density, are as important as the spatial attenuation of the γ heating. This type of experiment can be done with zero-power facilities; however, some ingenious instrument development may be required for routine measurements.

The required disposition and worth of control mechanisms requires emphasized experimental effort. Simple material-replacement experiments will continue to provide first-order effects. However, weak coupling in large cores and analytical assurance of reliable holdown require experimental confirmation on some general basis.

5. Conclusion

Some aspects of these projections on utilizing critical-assembly data are necessarily vague. This is due to current uncertainties pertaining to the degree of reliability of various analytical methods as a function of time. However, within these projections it was possible to define a number of topics of interest to the reactor designer. These are neither unique nor all inclusive. They are only intended to be typical.

Experience to date in using critical-assembly data

* Measurements on heating graphite in ZEBRA were reported at this conference (See Paper No. 6 in Session III-A).

for reactor design has been good. This experience is probably not unrelated to the ability to separate vital from peripheral problems as they arise. Current activities are an indication that problems are being sized up in some depth so that such good experience with neutronic engineering can continue.

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Discussion

Mr. Hofmann (Battelle-Northwest): We are involved in the design of a fast test reactor which has an about 800-liter core—highly unsymmetric, full of closed and open loops, and full of interfaces. What is your view of the utility of engineering-mockup experiments for this kind of project? Would you suggest that we rely only on calculations and extrapolations from clean critical experiments, or do you think that we might be better off with some kind of engineering-mockup experiments? We don't have a very large core and we don't have these very small clean cores.

Mr. Loewenstein: What I've tried to indicate here is that ultimately the need for the detailed engineering mockup may in fact vanish. I have indicated that there will be a need for engineering mockups until that time. Certainly you are not in the ultimate situation unless your schedule slips, so you are going to

need some sort of mockup experiments. There is a possibility that you will want an engineering mockup too early. Within Argonne National Laboratory we have experienced pressure for the performance of an engineering mockup too early. If we had done this, it would have resulted in a certain amount of wasted effort. We held off for two or three years before we did the mockup, and so our simulation was much closer to what actually went into the reactor.

Mr. Smith (UK-Winfrith): In the tasks for future effort, you didn't mention the effects of fission products. I have in mind particularly their effect on reactivity and on sodium and Doppler coefficients. Do you think there is a role here for the criticals or do you think the sort of results we have obtained previously using physicum or other strange mixtures are satisfactory?

Mr. Loewenstein: We undertook a calculational experiment a few years ago in which we tried to assess the effect of fission products on reactivity coefficients and compared it with effects like fuel loss and Pu-240 content. Certainly there was an effect, but it did not seem that this was an overriding effect. In all critical facilities as now designed we would have a difficult time making measurements with actual fission products. This is probably a reflection of the fact that the people designing the facilities share the view that I just expressed that fission products in themselves are not an overriding effect here, because fission products are accompanied by the destruction of a fissile atom.

Mr. Engelmann (Karlsruhe): I would like to add something to the answer to Mr. Smith's question. We have studied the effects of fission products with special emphasis for steam-cooled fast reactors. We found that the effect of fission products is extremely large. Therefore, we have investigated how we could simulate the fission products in a critical facility. I will comment on this in the final session.

Mr. Loewenstein: Nothing that I have said pertained to steam-cooled reactors. My remarks were related to liquid-metal-cooled reactors.

Mr. Campise (AI): You stated that the predictions for EBR-I and Fermi were about 5% off before any critical experiments were performed. Now many years have passed, and a great deal of new data has been acquired and far more elaborate calculational techniques are used. Yet there are discrepancies of 30% in calculating ZPR-3 Assembly 48. Would you comment on this?

Mr. Loewenstein: The difference is primarily due to the relative hardness of the spectra.

Mr. Thalgott (ANL): There was some related experimental work preceding the EBR-I and Fermi criticals in ZPR-3, and the information gained from this work was factored into the EBR-I and Fermi calculations.

Mr. Smith: Early calculations for the Dounreay reactor, performed at Los Alamos, deviated by 15-20% from the final critical mass. Calculations carried out subsequent to the ZEUS experiments were very good.

Presented by P. J. Persiani

The Critical Experiments and Preliminary Analysis of Multifueled, Nonuniform Core Loadings for the FARET Program

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ABSTRACT

A series of small, Pu-plus-U-235-fueled fast reactor cores with steel radial and axial reflectors were constructed in Argonne's Zero Power Reactor-3 as part of the design program for the FARET reactor. These studies, designated ZPR-3 Assembly 46, were essentially mockups of possible loadings of the FARET core. The primary objective of the studies was to confirm the physics analysis of multifueled, nonuniform core loadings as were envisaged for the FARET system.

A principal interest in the studies was to establish experimentally the predicted reactivity control afforded by the control-rod designs for FARET. The agreement obtained between calculations and the results of experiments for control-rod worth was sufficient to establish the range of control possible in the FARET reactor. Of equal importance was the substantiation of the neutronic behavior of a mixed core in FARET when subassemblies of different types of fuels were interchanged. Analytical calculations were done for all experiments, and the methods for analysis that were adopted will be discussed in the paper. The agreement of the analyses with the experimental results is found to be generally good. A study of the mixed-core systems and the comparisons of calculations with results of fuel-exchange experiments in three cores indicate that the behavior of a mixed-core loading can be understood as well as that of a uniform core loading.

1. Introduction

Assembly 46 in the ZPR-3 was a series of studies of small plutonium-uranium-fueled fast reactor cores which simulated possible loadings of the FARET reactor.^(1, 2) Experiments with this assembly were planned in conjunction with the design development for the fuel loadings of FARET. Of principal importance in the studies was to evaluate the control-rod

system designed for FARET and to assess the difficulties of predicting reactivity changes due to exchange of different types of fuel subassemblies within a core. Basically, the results from the critical experiment have been used to verify the calculation procedures involved in the FARET design and to substantiate the philosophy of the then-planned FARET loadings.

In the course of the Assembly-46 program, seven different cores were constructed. Extensive studies were made with three: cores B, C, and F, as simulations of possible FARET loadings. Core 46B was the simulation of a 68-liter, carbide-fuel loading, and core 46C simulated a 93-liter oxide system. In the 46F core were four radial fuel zones: an oxide, a carbide, an oxide, and a mixed oxide-alloy simulation zone. In all three cores, the worths of fuel interchanges were measured and scans were made with reactivity samples and neutron detectors. Also, the worths of possible FARET B₄C control rods were determined with these cores.

The compositions of the three cores, the control rods, replacement subassemblies, and the reflector regions are listed in Appendices I-IV.

2. FARET Critical Experiment 46B

The basic design of Assembly-46 was a geometric representation of the proposed FARET design. Figure 1 is a cross-sectional view of the ZPR-3 reactor, at the interface, showing the loading with core B. Around the core are two radial reflector regions, "fine" and "coarse," which simulated the designed FARET reflector of steel subassemblies. The "Plenum" shown in Fig. 1 was composed of nickel and aluminum to simulate a radial plenum region of sodium in FARET. The basic reflector composition represented 82 v/o steel plus 18 v/o sodium.

In all of the drawer loadings in the core of Assembly 46, the compositions were chosen to represent FARET subassembly specifications of 36 v/o fuel, 44 v/o coolant, and 20 v/o structure. The fuel contents were fairly closely simulated in ZPR-3, with deviations how-

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FIG. 1 Interface View of FARET Mockup with Carbide Core.

ever on the sodium and steel compositions. The fuels simulated include oxide ($\text{PuO}_2\text{-UO}_2$), carbide (PuC-UC), and an alloy (Pu-U-Zr), with various average enrichments in the uranium. For all of the cores, the axial reflectors contained the same sodium content as the cores, amounting to 38 v/o, with steel replacing the 36 v/o for fuel, giving about 56% total.

The basic cross-section set used in the preliminary analysis of the critical experiments was the 22-group set designated as ANL set 224. This set includes isotopic cross sections which have been averaged in typical core composition in order to account for the effect of resonance interaction in the low-energy region. These averaged cross sections were used for the range of fuel compositions being planned in the FARET program. This is not a bad approximation since it has been found that the resonance-averaged cross sections are relatively insensitive to moderate composition changes and therefore are usually applicable for a large class of reactor systems.

In the reflector regions, however, reaction rates as well as wirth traverses for the heavy elements would not be predicted by use of the core-averaged cross-section set. The cross section averaged for these regions were obtained from fundamental-mode calculations using an ultra-fine-group cross-section structure (1400 energy points) and then averaging over the 22-

group coarse structure. The computations for the reflector regions were carried out for zero buckling. The reaction-rate profiles computed from the above cross-section structure are compared with the measured radial traverses for the carbide Assembly 46B in Figs. 2 and 3.

The peaking in the reflector region for Pu-239 and U-235 fission rates would not be produced with the core-averaged cross sections. In fact, with the core-averaged set only a slight curve inflection was perceptible at the core-reflector interface. The neutron spectrum in the reflector is softened to an extent such that the fine structure in the resonance region becomes a more important factor in the averaging process. In the case of the U-238 fission rate, as should be expected, no difference was observed between the core-averaged and the reflector-averaged cross section. A more sensitive test on this reflector-averaging technique would be the capture rates of the fertile isotope.

The computed and measured boron (n,α) capture rates are compared in Fig. 3. Since the FARET facility was designed for a poisoned control-rod system located in the reflector region, it was most important to be able to establish the boron capture profile in good agreement with experiment. The curves agree well to within 10% of each other. In the reflector region, where the reaction rate peaks, the experimental values

FIG. 2. U-238, U-235, and Pu-239 Fission-rate Radial Traverses in FARET Core 46B.

FIG. 3. Boron (n, α) Capture Rate Radial Traverse in FARET Core 46B.

could be somewhat higher if corrections were made for the averaging effect due to the finite extent of the neutron detector.

The experimental program that had been planned for the FARET system involved frequent replacement and redistribution of experimental fuel subassemblies.

Each fueled assembly would contain prototype reactor fuel compositions and fuel-element design. In order to understand the reactivity effects associated with sub-assembly replacements and/or redistributions, it becomes necessary to develop a two-dimensional computational program. This need is further emphasized since small high-power-density core test facilities are systems of high neutron leakage. The preliminary attempt at the two-dimensional computation was made by developing a five-group cross-section set for use with transport codes.

In constructing the group structure attempts were made to satisfy several conditions: (1) The coarse group set should reproduce the worth of materials, having spectral- and leakage-dependent components within some acceptable margin of error. A measure of this margin could be based on the economics of computing. (2) The general energy dependence of the 22-group adjoint function should be reasonably simulated by the coarser 5-group set. The material worths of immediate interest would be those used in the design of fast power reactors, namely, sodium, iron, nickel, chromium, carbon, and oxygen, as well as uranium-238, uranium-235, and plutonium-239 and -240. This is in effect adjoint weighting the averaged cross section but is accomplished by doing the direct k_{eff} calculation.

The final group structure was selected by the comparison of the nonfissile material worths as computed in spherical and cylindrical geometry with the 22-group and the 5-group cross-section sets. The comparisons were made for the central-region worths where the spectral-component contribution is significant and then over the whole of the core regions where the leakage effects are expected to contribute significantly. The 5-group set adopted for this preliminary analysis appeared to reproduce the 22-group material worths to better than 40% for the central region, but for the overall core worths, which includes the leakage components, the values were in agreement to within 8%. Some typical values are presented in Table 1 for the study made with Assembly 46B. In this particular case the 22-group material worths for the center drawer are in fair agreement with experimental values except for stainless steel. The values for the material worths obtained with the 5-group set were in poor agreement with experimental data in the central P-17 drawer. However, when used in a two-dimensional transport analysis for the reactor system without the central subassembly, the prediction for the worths of sodium and stainless steel were in better agreement with the experimental values. The 5-group set in the (x, y) and (r, z) geometry appears to account for the leakage-dominated effects. A computational scheme that this begins to suggest is to use a spherical geometry and multigroup

cross-section set calculational scheme for those reactivity changes with a significant spectral component. In those regions where leakage effects become significant a two-dimensional geometry computational scheme with a tailored coarse-group cross-section set may prove to be adequate and economical. A significant improvement in predicting the nonleakage component is expected with a slightly less coarse group than the 5-group set used in this analysis.

The computed worths of fertile and fissile materials are compared with experimental data in Table 2. The comparisons made are for the experimentally determined radial and axial worth distributions. The calculation for the central zone in spherical geometry is compared with the central-zone measurements of both the radial and axial distributions. The calculation for the central subassembly in cylindrical geometry is compared with the integrated value of the axial distribution over the core height. The comparison is good for the fissile material since these worths are comprised mainly from the changes in the sources and absorption reaction rates and have a negligible leakage component. The fertile material has a leakage factor which is not correctly computed in cylindrical geometry. However, this is still a small effect and introduces a tolerable inaccuracy in the experimental fuel-exchange program.

The fuel-exchange experiments in core 46B were conducted by replacing the carbide-fuel drawers with oxide-fuel simulated drawers in the positions indicated in Fig. 1. Single-drawer, oxide for carbide, fuel exchanges were made at four radii in the four successive rings of drawers from the center in one-half of assembly. In addition, an annular section of the carbide fuel in the "3rd ring," composed of four full and two half-drawers in each assembly half, was replaced by the oxide fuel. The reactivity changes computed for this sequence of experiments are included in Table 3. The 4-group structure used in the two-dimensional diffusion calculation was not developed in as detailed a manner, as the 5-group structure was designed for the transport computation. The agreement between all methods and group structure is good and well within ± 10 lh for the singly placed fuel assemblies. This again results from the fact that the reactivity component is predominantly nonleakage dependent.

A perturbation computation (see Table 4) indicates that in the replacement of the carbide-fuel by the oxide-fuel assembly the major reactivity effect (200 lh) results from the change in the source and absorption component of Pu-239. The next two large components in the reactivity change are the scattering and leakage effects of the loss of carbon (-29 lh) and the gain of oxygen atoms ($+33$ lh) in the fuel assemblies. The

TABLE 1. DISTRIBUTED WORTHS OF MATERIALS IN THE FARET CRITICAL EXPERIMENT 46B

Carbide Core (68 Liters)				545 Inhours = 1% <i>k</i>		
Material	Inhours/kilogram					
	Central Drawer (P-17)			System Except Central Drawer		
	Experiment	Calculation in Spherical Geometry		Experiment	S ₂ 5-group	
		22- group	5- group		xy- Geom- etry	yz- Geom- etry
Sodium	38.6±1.3	47	31	55.9±0.7	47	52
Stainless Steel	-1.25±0.22	-2.7	-1	9.9±0.1	9.1	
Carbon	114±3	123	92	132±1		
U ₃ O ₈	16.2±0.3			20.3±0.2		
Depleted Uranium	6.6±0.2	5		7.4±0.1		
Oxygen	70±2 ^(a)	78	43	92.3±1.5 ^(a)		

^(a) Coefficients derived from U₃O₈ and depleted uranium.

TABLE 2. URANIUM AND PLUTONIUM WORTHS IN THE FARET CRITICAL EXPERIMENT 46B

	Inhours/kilogram			
	Spherical Geometry Calculation, at Center	Experimental Data, Central Average	Cylindrical Calculation, Center Assembly	Experiment Integration of Axial Distribution over Core Height
U-235	330.5	354	260	254
U-238	3.9	3.0	12	7.2
Pu-239	539	616	417	440

magnitudes are comparable but of opposite sign in this replacement sequence, so that the net contribution in the exchange of carbon for oxygen atoms is an increase in reactivity of almost 4 lh.

Referring again to Table 3, when the replacement experiment is performed with the 5-assembly bank, the diffusion computation results in a positive 27% interaction whereas the two-dimensional transport computation tends to give a negative interaction effect amounting to 13%.

3. FARET Critical Experiment 46C

The 46C assembly design was a simulation of a 20-in.-high, 87-liter, oxide-fueled core. The composition was essentially equivalent to 36 v/o fuel (PuO₂ + UO₂), with a uranium-to-plutonium ratio of 6.1:1 and

TABLE 3. SUBASSEMBLY REPLACEMENTS IN THE FARET CRITICAL ASSEMBLY 46B

Carbide Core (68 liters)		545 Inhours = 1% k				
Oxide for Carbide Replacements (1.87-liter Assembly)	Position	Reactivity Change, Δh				
		Experiment	Calculation			
			PDQ(xy) 4-group	$S_2(xy)$ 5-group	Diffusion Calc. for Cylinder Geometry	
					22 group	5-group
	P-17	-173.4 ± 1.8	-164.4	-183	-167	-174
	Q-17	-161.6 ± 1.6			-162	-170
	R-17	-128.2 ± 1.2		-106	-127	-132
	S-17	-85.0 ± 1.0	-87.0		-88	-89
5-assembly Annulus	P-15 $\frac{1}{2}$ Q-15; $\frac{1}{2}$ Q-16 R-18 R-17 R-16	-618 ± 6		-530	-773	-800

TABLE 4. ESTIMATES OF REACTIVITY COMPONENTS $\delta k/k$ IN THE SUBASSEMBLY-REPLACEMENT EXPERIMENT 46B

Oxide for Center Carbide P-17 Assembly						
Material	Net Weight Change, g	$\delta k/k \times 10^3$				Inhours
		Source	Capture Plus Fission	Leakage	Scattering	
U-238	-1681	-1.431	+1.431	-0.058	-0.070	-7.1
Pu-239	-388	-5.926	+2.254	-0.013	-0.011	-206.2
Pu-240	-18	-0.098	+0.052	-0.0006	-0.0005	-2.7
U-235	-3	-0.052	+0.026	-0.0002	-0.0004	-1.5
Na	-17		+0.0002	-0.0006	-0.0027	-0.18
C	-267			-0.069	-0.445	-28.7
SS	+92		-0.012	+0.0046	+0.0031	-0.22
O	+447		-0.059	+0.086	+0.556	+32.5
Sum of Individual Components		-7.506	+3.692	-0.051	+0.028	-214.0
Combined Perturbation Calculations		-7.506	+3.692	-0.026	+0.028	-212.6

an average uranium enrichment of 49 a/o U-235. In Fig. 4 is an interface view of the Assembly 46C.

A series of measurements of control-rod worths were made with four mockup rods denoted as rods 1, 2, 3, and 4 in a clockwise direction. Each designated rod channel was constructed as partial loadings of two or four adjoining matrix units. In this manner, the average rod radii from the core axis were close to the simulated rod positions in FARET.

Two series of mockup rod experiments were performed using a 20-in.-length active-section B₄C loading of two B¹⁰ enrichments: a low enrichment of 32 a/o B¹⁰ and a high enrichment of 69 a/o B¹⁰. The rod-worth experiments with the low-enrichment rods included the worths of four rods replacing reflector, worth

variations with axial position, and shadowing effects between rods. The sequence of experiments consisted essentially of inserting rod No. 1 from an "up-position" to "full-in," followed by a similar insertion of rod No. 4, with the final step in the sequence being the insertion of rods Nos. 2 and 3 in a bank. The results of the analysis for this sequence of experiments are included in Table 5. In order to make a valid comparison it should be noted that the computations performed relate to a bare-core height whereas the experiment was performed with rods whose active sections range only over the physical height of the core proper. From the experimental data, which measured the worth of the rods as a function of distance above the "up position," the integrated worth over a distance above the core

FIG. 4. Interface View of FARET Mockup with Oxide Core.

equal to the reflector savings was added to the experimental value. This amounted to an increase of about 15 to 20%, and it is this value of the rod worth that is compared with the computed value. The $S_2(x,y)$ -computed estimates are found to be in a reasonably good agreement with experiment. The slight discrepancy in the banked set of rods from the expected shadowing effect results primarily from the meshing not coinciding exactly with the actual placement of the rods. In the computation, the banked rods were closer to the core, with rods 1 and 4 being slightly further away from the core. An improved representative meshing would result in closer agreement.

As had been expected, in the stainless steel-reflected FARET cores, there existed a substantial amount of neutronic worth in the "cylindrical-edge" regions, the major contribution coming from a region extending about 15 to 20 cm from the core height. This result clearly points out that a more effective design for a control system is that in which the control region extends some distance beyond the physical dimensions of the core. If the control system is the usual vertical-rod scheme, then the shutdown margin with rods "fully in" can be increased for the same number rods as previously but shorter. Similar design considerations should be applied to other fuel-test facilities, since these are usually designed with reflectors of stainless steel or high-nickel content.

TABLE 5. WORTHS OF B_4C CONTROL RODS IN THE FARET CRITICAL EXPERIMENT 46C

Rod Locations	Oxide Core (87 Liters) 517 Inhours = 1% k $\beta = 0.00569$		
	Calculated from Experiment ^(a)	Two-dimensional xy Calculations	
		PDQ 4-group	S_2 5-group
32% Enriched			
1	-288	-248	-276
1 Plus 4	-574	-515	-560
All 4 Rods	-1046	-886	-1080
69% Enriched			
1	-462	-437	-415
1 Plus 4	-900	-825	-822
2 Plus 3			-847
All 4 Rods	-1619	-1320	-1530

(a) Includes estimated worth of control rod in "up" position.

The fuel exchanges within core 46C were done with two types of fuel simulations replacing the initial oxide-core drawers. A carbide-fuel drawer and an alloy-fuel drawer were substituted separately for the oxide-fuel drawers at various locations. In Table 6 are presented the experimental and computed data for this replacement scheme. Examination of Table 7 shows that the combined positive addition of Pu-239 and

TABLE 6. SUBASSEMBLY REPLACEMENTS IN THE FARET CRITICAL EXPERIMENT 46C

Oxide Core (87 Liters)		517 Inhours = 1% k		
Replacement (1.56-liter Assembly)	Position	Reactivity Change, Δk		
		Experiment	Two-dimensional xy Calculations	
			PDQ 4-group	S ₂ 5-group
Carbide for Oxide	P-17	264	306	287
Alloy for Oxide	P-17	89		
Carbide for Oxide	R-17	211	246	236
Alloy for Oxide	R-17	66		
Carbide for Oxide	T-17	114	126	
Alloy for Oxide	T-17	24		20
Carbide for Reflector	S-20	354	345	
Carbide for Reflector	U-17	260	250	260
Alloy for Reflector	U-17	192		
Oxide for Reflector	U-17	184	171	

U-235 is partially compensated by the negative scattering and leakage effect due to the decrease in the carbon and oxygen concentrations. The agreement between the measured values and computed values even for the 4-group two-dimensional diffusion computation reveals that the gross effects are accounted for.

4. FARET Critical Experiment 46F

The final experiment in the Assembly 46 program was the multifueled zoned-core 46F. This assembly, shown in Fig. 5, was the simulation of a 61-assembly loading of FARET in which the first two rings of hexagonal subassemblies were of oxide-fuel, the third ring of carbide, the fourth ring of oxide, and the fifth ring contained alternate loadings of oxide- and alloy-fuel subassemblies. In the oxide simulation the loading was equivalent to 36 v/o PuO₂-UO₂ with the atom ratio of uranium to plutonium being 6.1 and a 32.8% U-235 enrichment. For the carbide drawer, the fuel param-

TABLE 7. ESTIMATES OF REACTIVITY COMPONENTS $\delta k/k$ IN THE SUBASSEMBLY-REPLACEMENT EXPERIMENT 46C: CARBIDE FOR CENTER OXIDE P-17 ASSEMBLY

Material	Net Weight Change, g	$\delta k/k \times 10^3$				Inhours
		Source	Capture Plus Fission	Leakage	Scattering	
U-238	+708	+0.541	-0.561	+0.013	+0.063	+2.9
Pu-239	+359	+5.042	-1.968	+0.0073	+0.019	+160.3
Pu-240	+16	+0.081	-0.044	+0.00032	+0.00089	+1.96
U-235	+756	+7.870	-4.117	+0.015	+0.088	+199.4
Na	+11		-0.00044	+0.0012	+0.0094	+0.52
C	-128			-0.022	-0.253	-14.3
SS	-90		+0.0061	-0.0011	-0.0033	+ 0.09
O	-375		+0.042	-0.062	-0.429	-23.2
Sum of Individual Components		+13.534	-6.642	-0.048	-0.505	+327.67
Combined Perturbation Calculation		+13.005	-6.100	-0.055	-0.559	+325.3

FIG. 5. Interface View of FARET Mockup with Multifueled Core.

ters were 36 v/o PuC-UC, atom ratio of uranium to plutonium of 5.5:1, and 45.5% U-235 enrichment. The alloy represented was a mixture of 20 w/o Pu, 71 w/o U, and 9 w/o Zr, with the uranium enrichment of 11.5% in U-235.

In Fig. 6 are presented the experimental and computed fission-rate distributions throughout the core and reflector regions. The characteristic peaking in the reflector section is again evident. The flattening effect of the U-238 fission rate near the edge of the oxide region shows a locally hardened spectrum arising from the high-enrichment carbide region. The experimental curve is fairly well predicted by the computation.

A comparison of the average worths determined experimentally with the values computed by a variety

of calculational methods is presented in Table 8. The results in this table indicate that when the leakage components begin to dominate, the two-dimensional S_4 transport approximation with the 5-group cross-section set seems to agree within 10% of the observed values.

An extensive study was carried out in core 46F of the reactivity effects from fuel interchanges and displacements. The ability to predict these effects is of great importance to the operation of a fuel-test facility where such core mixtures and interchanges would be the general plan of the operational program. Consequently, replacements were made of fuel in each of

the four radial zones of Assembly 46F with different fuel-type core assemblies. Presented in Table 9 are the results of these measurements along with the computed predictions. It can be seen that the agreement is well within a $\pm 10\%$ margin of error between the measured value and the S_2 transport computations. The replacement technique is understood enough such that a reasonable approach to the computational models should result in a good capability to predict reactivity changes in these small mixed cores.

The worths of B_4C rods in the reflector region were studied with 4 high-enrichment rods, located in the positions shown in Fig. 5. A similar sequence of rod insertions was followed in this experiment as had been done in FARET Assembly 46C. The estimated capability of the control worth in the mixed-core system is experimentally established and the extent of agreement is presented in Table 10. In these computations the two-dimensional meshing was more representative of the control-rod lattice positions.

5. Summary

The program for Assembly 46 essentially established that the operational program planned for a fuel-test facility, such as FARET, would involve no major physics uncertainties. The reactivity change associated with the mixed fuel-assembly interchanges could be computed to well within the normal operational limits of uncertainty. Although some predictions for material worth were not as close as desired, the fact that these reactivity coefficients were small, and further that the net change in concentration of the material was also small, indicated that the errors introduced in the computations were negligible. The major component resulted from the reaction-rate changes in the fissile material and from the scattering and leakage com-

FIG. 6. U-238, U-235, and Pu-239 Fission-rate Radial Traverses in FARET Core 46F.

TABLE 8. AVERAGE WORTHS OF MATERIALS IN CORE ZONES IN THE FARET CRITICAL EXPERIMENT 46F

Materials	Reactivity Coefficients in Zone (lh/kg)									
	Oxide I			Carbide II		Oxide III		Mixed IV		
	Exp	Spherical Geometry, 22-group	PDQ(rz)	Exp	PDQ(rz)	Exp	PDQ(rz)	Oxide		Alloy
								Exp	PDQ(rz)	
Na										
0-4 in.	24.6		44	44.6	45	62.5	45	63	39	63
0-10 in.	41.5	39	94	52.8	91	53.3	83	52.5	70	52.5
Stainless Steel 304	2.26	1	2.7	4.83	3.9	9.71	11	13.6	12	10.6
Depleted Uranium	3.24	2.5		6.01		4.88		5.51		5.51
Graphite	107	93		116		108.8				
Zirconium										17.2
Oxygen	66	56				76		84		14

ponents in the exchange of carbon and oxygen atoms present in the fuels.

Of equal importance in the irradiation program is the verification of the range of control capability that is available in a stainless steel-reflected, small high-power-density core system. Control worths from 9% to 10% k can be expected for these high-neutron-leakage cores, and these can be computed with a good degree of confidence. The experiments also indicated that a substantial improvement in control can be obtained with control-rod designs having the active sections extending beyond the physical dimensions of the core.

TABLE 9. SUBASSEMBLY REPLACEMENTS IN THE FARET CRITICAL EXPERIMENT 46F

Mixed Core (93 Liters)		559 Inhours = 1% k		
Replacement (1.56-liter Assembly)	Position	Reactivity Change, Ih		
		Experiment	Two-dimensional xy Calculations	
			PDQ 4-group	S ₂ 5-group
Carbide for Oxide	P-17	+396	+375	+423
Alloy for Oxide	P-17	+214	+195	+212
Carbide for Oxide	Q-17	+382	+370	+410
Alloy for Carbide	R-17	-150	-154	-180
Carbide for Oxide	S-17	+264	+237	+284
Alloy for Oxide	S-17	+140	+114	+141
Oxide for Alloy	T-17	-84	-66	-91
Alloy for Oxide	T-18	+82	+63	+84
Oxide for Reflector	U-17	+150	+148	+143
Carbide for Reflector	U-17	+270	+248	+267
Reflector for ½ Oxide	S-14	-212	-234	-208

TABLE 10. WORTHS OF B₄C CONTROL ROD (69% B¹⁰ ENRICHMENT) IN THE FARET CRITICAL EXPERIMENT 46F

Mixed Core (93 Liters)		559 Inhours = 1% <i>k</i>	
$\beta = 0.00523$			
Rod Locations	Calculated from Experiment ^(a)	Two-dimensional xy Calculations	
		PDQ 4-group	S ₂ 5-group
P-22	− 500	− 529	− 533
J-17			− 337
P-22 Plus J-17	− 825	− 850	− 790
K-19/20 Plus M-21/22			− 765
All 4 rods	− 1509	− 1375	− 1411

^(a) Includes estimated worth of control rod in "up" position.

The significance that may be attached to these preliminary analyses and critical experiments in the FARET Assembly 46 program is that the behavior of a multifueled nonuniform core loading can be understood as well as that of a uniform core loading.

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Appendix I

AVERAGE ATOMIC DENSITIES, 10²² ATOMS/CC, IN ASSEMBLY 46B

	Pu-239	Pu-240 + Pu-242	Pu-241	U-235	U-238	U-234 + U-236	C	Na	Fe	Cr	Ni	O
Core	0.1558	0.0074	0.0007	0.3346	0.5598	0.0048	1.1096	0.8469	1.0576	0.2840	0.1397	—
Substitution Oxide Fuel	0.1062	0.0051	0.0005	0.3340	0.3406	0.0048	0.5044	0.8531	1.0908	0.2929	0.1441	0.8586

	B ¹⁰	B ¹¹	C	Na	Fe	Cr	Ni
Natural B ₄ C Control Rod	0.562	2.363	0.788	0.846	1.539	0.413	0.203
53% Enriched B ₄ C Control Rod	1.436	1.300	0.868	0.846	1.539	0.413	0.203
Control-rod Follower	—	—	—	0.848	3.440	0.924	0.454

Appendix 2AVERAGE ATOMIC DENSITIES, 10^{22} ATOMS/cc, IN ASSEMBLY 46C

	Pu 239	Pu-240 + Pu-242	Pu-241	U-235	U-238	U-234 + U-236	Na	C	C	Fe	Cr	Ni
Core	0.1062	0.0051	0.0005	0.3343	0.3432	0.0048	0.8242	0.8582	0.5104	1.1270	0.3026	0.1489
Substitution Carbide Fuel	0.1481	0.0071	0.0006	0.4451	0.4465	0.0065	0.8573	—	1.1061	1.0631	0.2854	0.1404
Substitution Alloy Fuel	0.2620	0.0125	0.0011	0.1309	0.8268	0.0019	0.8688	—	—	1.4802	0.4284	0.2109

	B ¹⁰	B ¹¹	C	Fe	Cr	Ni	Na
32% Enriched B ₄ C Control Rod	1.533	3.216	1.369	1.272	0.341	0.168	0.422
69% Enriched B ₄ C Control Rod	3.337	1.493	1.665	1.052	0.282	0.138	0.422
Rod Follower	—	—	—	4.482	1.203	0.592	0.418

Appendix 3AVERAGE ATOMIC DENSITIES, 10^{22} ATOMS/cc, IN ASSEMBLY 46F

	Pu-239	Pu-240	Pu-241 + Pu-242	U-235	U-238	U-234 + U-236	Na	O	C	Fe	Ni	Cr	Zr
Core Zone I	0.1062	0.0051	0.0005	0.2243	0.4575	0.0032	0.8525	0.8582	0.5042	1.0927	0.1444	0.2933	—
Core Zone II	0.1558	0.0074	0.0007	0.4103	0.4819	0.0059	0.8577	—	1.1061	1.0593	0.1399	0.2844	—
Core Zone III	0.1062	0.0051	0.0005	0.2243	0.4575	0.0032	0.8295	0.8582	0.5042	1.1602	0.1533	0.3115	—
Core Zone IV	0.1841	0.0088	0.0008	0.1682	0.6206	0.0024	0.8519	0.4291	0.2522	1.1594	0.1532	0.3113	0.1497
Substitution Oxide Fuel	0.1062	0.0051	0.0005	0.2243	0.4575	0.0032	0.8525	0.8582	0.5042	1.0927	0.1444	0.2933	—
Substitution Carbide Fuel	0.1557	0.0074	0.0007	0.4103	0.4819	0.0059	0.8577	—	1.1061	1.0593	0.1400	0.2844	—
Substitution Alloy Fuel	0.2620	0.0124	0.0012	0.1121	0.8474	0.0016	0.8688	—	—	1.1417	0.1509	0.3065	0.3170

69% Enriched B₄C Active Control Rod and Rod Follower as in Assembly 46C.**Appendix 4**

AVERAGE COMPOSITIONS OF ASSEMBLY 46 REFLECTOR REGIONS

Reflector Region	Material Densities, 10^{22} atoms/cc						
	Fe	Cr	Ni	Mn	C	Na	Al
Axial Reflector	3.425	0.920	0.452	—	—	0.849	—
Fine Radial Reflector	4.690	1.259	0.620	—	—	0.212	0.306
Coarse Radial Reflector I	7.413	0.119	0.058	0.057	0.065	—	—
Coarse Radial Reflector II	6.579	0.830	0.409	0.028	0.035	—	—
Plenum	0.445	0.119	1.923	—	—	—	1.384
Spring Gap ^(a)	1.654	0.444	0.218	—	—	—	—

^(a) Between 53.42 and 54.08 cm from interface in axial reflector regions of both assembly halves.

Presented by R. L. McVean

Critical Assembly Mockup of the SEFOR Reactor in ZPR-3

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ABSTRACT

The critical assembly for the SEFOR reactor, which has been built in ZPR-3, is the first large, plutonium-fueled critical assembly to be built in the fast breeder program. Beryllium oxide has been included in the design of the 528-liter core to soften the neutron spectrum to match more nearly that of a large power reactor and to enhance the Doppler effect.

Metal fuel plates are used in the core. Sodium carbonate, along with some aluminum, is used to represent the sodium coolant and the oxygen in the SEFOR oxide fuel. There is a region of sodium and stainless steel between the core and the radial reflector. The reflector is made of nickel and is used for control of the SEFOR reactor.

Because of the SEFOR design requirement to minimize the effects of axial thermal expansion, measurements were made in three different core mockups. The basic measurements were made in a one-segment, or reference, core, and several of these measurements were repeated in mockups of a two-segment fuel element core and a three-segment fuel element core.

1. Introduction

In 1964 the United State Atomic Energy Commission, South West Atomic Energy Associates, the Karlsruhe Laboratory of West Germany, EURATOM, and the General Electric Company embarked on a program to construct and operate a 20-MW ceramic-fueled, sodium-cooled fast flux reactor. A test program is planned for the SEFOR reactor⁽¹⁾ to verify calculations which indicate that fast ceramic power reactors can be designed with stable and desirable operating and safety characteristics. The SEFOR reactor will use a mixed fuel of plutonium and uranium oxides. Also, beryllium oxide will be added to the core to soften the spectrum to match more nearly that of a

large fast oxide power reactor. The SEFOR reactor will be particularly designed for the systematic determination of the Doppler coefficient of reactivity up to and beyond the fuel conditions corresponding to normal operation of a power reactor.

A six-month program of critical experiments to facilitate the physics design of the SEFOR reactor was run with ZPR-3.⁽²⁾ The Zero Power Reactor-3 is the fast critical facility operated by the Idaho Division of Argonne National Laboratory. Some of the major objectives of this program were the determination of the plutonium enrichment of the fuel, the evaluation of the SEFOR control rods, and the evaluation of segmented fuel-element designs.

The ZPR-3 machine is shown in Fig. 1. It is a conventional split-half critical assembly. Since it has been in use for over 10 years, it is not necessary to go into the details of the machine.

2. SEFOR Mockup

An innerface view of the SEFOR mockup in ZPR-3 is shown in Fig. 2. Two different core regions are shown outlined on the innerface. Two core regions were used because one of the operating requirements for the ZPR-3 machine states that the calculated power coefficient of reactivity must be negative whenever plutonium is used in the reactor; therefore, the new plutonium fuel which was obtained for this SEFOR experiment was ordered in the form of a plutonium-uranium-molybdenum alloy. This new fuel was used in the inner core region to make the best use of the negative Doppler coefficient of reactivity associated with the depleted uranium in the plutonium-uranium fuel alloy. In the outer region the solid plutonium fuel plates which were already on hand at the ZPR-3 reactor were used.

The core has a radius of 17 in. and a height of 34 in.; the inner core region has a radius of 12 in. and a height of 28 in. The total volume of the core is 528 liters. Surrounding the core is a sodium and stainless steel region. Beyond this region is a 6-in.-thick reflector

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FIG. 1. ZPR-3 Machine.

region of nickel with a height of 40 in. The control of the SEFOR reactor is accomplished by movement of the reflector, which is divided into ten segments. Two of the SEFOR control-rod mockups are shown in the innerface view. The top SEFOR control-rod mockup was not used because of the ZPR-3 instrumentation located on top of the reactor; rather, the equivalent positions on either side of the reactor were used. Also, at the bottom of the matrix can be seen the boron carbide neutron shield which is mocked up behind the lower control rod mockup.

Figure 3 shows a typical core drawer which was loaded in the central core region. In order to mockup the SEFOR core composition, it required the use of five different drawer loadings to obtain the correct composition. This five-drawer unit cell was then repeated throughout the core. The new type of fuel plates which were obtained for this experiment were used in the first 14 in. of the core. In the last 3 in. the

existing plutonium plates which were on hand were used. The remaining 4 in. in the drawer is part of the axial reflector. The materials used in the core mockup are metal fuel plates, sodium carbonate, aluminum, beryllium oxide, and stainless steel.

Figure 4 shows a curve indicating the approach to critical, plotted from data obtained from one of the two proportional counters which were located in the nickel reflector. For the specified composition of the SEFOR reactor mockup in ZPR-3, a value of 305 kg of Pu-239 + Pu-241 for the critical mass in the 528-liter core was obtained from the General Electric Company prior to loading the assembly. The initial fuel loading was 40% of this estimate of critical mass, with subsequent fuel loading steps of 10% or less. Subcritical multiplication count rates were taken between fuel loadings with the reactor halves closed and all control and safety rods inserted.

Figure 5 shows the curve on an enlarged scale for ap-

FIG. 2. Innerface View of the SEFOR Mockup in ZPR-3.

FIG. 3. Typical Loading of Core Drawer.

FIG. 4. Approach to Critical in Assembly 47 (SEFOR).

FIG. 5. Extrapolated Critical Mass for Assembly 47 (SEFOR).

proach to critical. The core had been actually loaded beyond the specified size. With the core loaded to 548 liters, the fuel loading was 312 kg of Pu-239 + Pu-241. At this point the reactor was still subcritical. The ap-

proach-to-critical curve extrapolates to an estimated critical mass of 345^{+10}_{-5} kg of fuel if the core were enlarged radially. This estimate is $13^{+3.3}_{-1.5}\%$ above the originally calculated mass of 305 kg.

At this point the core size was reduced to the original 528 liters. The fuel enrichment was then increased by replacing columns of depleted uranium with columns of plutonium in several of the drawers throughout the core. The loading of one of the five drawers in the unit cell had been planned for this substitution should it be found that the plutonium enrichment needed to be increased to achieve criticality. Fifteen drawers in the inner zone were spiked with extra plutonium, while seven drawers in the outer zone were spiked with extra fuel. A column of fuel in the outer core region contained approximately twice as much plutonium as a column of fuel in the inner core region. After spiking the 22 drawers, the reactor was critical and contained 314.3 kg of Pu-239 + Pu-241. If the ZPR-3 control rod was completely inserted, the multiplication constant k for this loading would be 1.0003. This number is not corrected for any nonuniform distribution of

spiked drawers, or for any heterogeneity effects which might be present in the core.

The core composition for the initial loading⁽³⁾ of the SEFOR mockup in ZPR-3 is given in Table 1. Since the SEFOR fuel is an oxide fuel and the plutonium at ZPR-3 is in the form of a metal, oxygen was introduced in the core mockup by the use of sodium carbonate. The sodium was used to represent the sodium coolant. The carbon in the sodium carbonate along with some added aluminum was used to represent oxygen. This oxygen substitution was made on the following atomic basis: 0.33 carbon + 0.76 aluminum = 1.0 oxygen. Also, in the inner-zone mockup, the molybdenum in the fuel alloy was added to the stainless steel to represent the stainless steel of the SEFOR reactor.

Table 2 gives the compositions for the remaining SEFOR regions in the mockup in ZPR-3. In the axial reflector and the axial sodium and stainless steel regions, aluminum and carbon were used to mockup sodium. In the radial sodium and stainless steel region, sodium in stainless steel cans was used. Here it should be noted that the volume fraction of sodium in this region as loaded in ZPR-3 was based on a lower number given by the General Electric Company. After the first loading, a revised number of 0.42 was obtained for the sodium. On a subsequent loading the sodium mockup was changed to match more nearly that of the actual SEFOR reactor.

3. Experimental Results

A Pu-239 fission-rate traverse in the radial direction through the SEFOR reactor mockup is shown in Fig. 6. The solid line gives the calculated fission rate. It should be noted that in the sodium and stainless steel radial-reflector region the calculated peak is much lower than the experimental peak. The calculations were made with diffusion theory and ANL cross-section Set 224. This cross-section set goes down to only 29 eV. It was found that about 30% of this calculated peak was due to the energy region from 29 to 275 eV. Therefore, it is believed that a cross-section set which incorporates energy groups closer to thermal would give better agreement in the radial reflector.

It should also be noted that the flux on the left-hand side appears to be higher than that on the right-hand side. This was also noted when the SEFOR control-rod measurements were made. Upon examination of the actual fuel loading in the core, it was found that the part of the core on the left-hand side was, indeed, loaded heavier than that on the right side. This was due to the way the boundary between the inner and outer zones broke up the 5-drawer unit cell. The dip in the experimental number at the very center and on

TABLE 1. CORE COMPOSITIONS FOR SEFOR
MOCKUP IN ZPR-3 (g/cc)

Material	SEFOR	Loading 47-15 (1 Segment)	
		Inner Zone (203.1 l)	Outer Zone (324.5 l)
Pu + U	3.703	3.706	3.755
Pu-239 + Pu-241		0.5875	0.6007
Pu-240		0.0525	0.0287
U-238		3.060	3.120
U-235		0.006	0.006
Be	0.084	0.085	0.081
O	0.646	0.416	0.406
Mo		0.082	—
C		0.067	0.067
Na	0.277	0.253	0.252
Al		0.313	0.346
SS	1.45	1.31	1.43

TABLE 2. COMPOSITIONS FOR SEFOR MOCKUP IN ZPR-3
(VOLUME FRACTION)

Material	Region							
	Axial Reflector		Axial Na and SS		Radial Na and SS		Radial Reflector	
	SEFOR	ZPR-3	SEFOR	ZPR-3	SEFOR	ZPR-3	SEFOR	ZPR-3
Ni	0.48	0.479					0.74	0.715
SS	0.22	0.202	0.40	0.446	0.23	0.269		0.075
Na	0.30		0.60		0.42	0.259		
Al		0.089		0.179		0.008	0.16	0.102
C		0.025		0.053

FIG. 6. Radial Fission Traverse with Pu-239 Counter (ZPR-3 Assembly 47, SEFOR).

FIG. 7. Radial Fission Traverse with U-238 Counter (ZPR-3 Assembly 47, SEFOR).

either side, midway from the core center line to the radial sodium and stainless steel, is due to the fact that only four of the five drawers in the unit cell contained beryllium oxide. The point where the fission rate dips is in the drawers which did not contain beryllium oxide.

This indicates that the neutron spectrum was not softened as much in these drawers as in the other four drawers.

Figure 7 gives the results of a radial fission traverse using a U-238 counter. Again the flux unbalance can

TABLE 3. WORTH OF 40 IN. OF NICKEL
IN SEFOR CONTROL-ROD MOCKUP

Core Loading	Reactivity (ϵ) (B_4C and Polyethylene behind rods)			
	Rod I	Rod II	Rod III	Average of Rods II and III
1-Segment	107	121	88 ± 4	105 ± 4
2-Segment	—	114	115	115

be seen. The two points on either side of the core centerline which fall considerably below the others are the drawers in which provision was made to add an additional column of plutonium. In the drawers in which the traverse went through, this spiking had not taken place; therefore, there was an additional amount of depleted uranium at these points, which decreased the higher end of the neutron spectrum.

The worth of the nickel in the SEFOR control-rod mockups is given in Table 3. Besides the nickel, the worths of aluminum and stainless steel were also measured, so that corrections could be made for the actual SEFOR control-rod design. As seen in a previous figure, SEFOR rod No. I is the rod at the very bottom of the matrix in the innerface view of the mockup. Rod No. II was on the side with the higher flux. Rod No. III was on the side with the lower flux. The three rods have an average worth of \$1.05 for the one-segment core mockup. In the two-segment core mockup

with a 2-in. gap placed 5 in. from the innerface in half No. 2 of the assembly, control rods No. II and No. III were again measured. It was found that the flux balance had been improved by a change in fuel loading. Here the average rod worth was found to be \$1.15. This indicates that the gap in the fuel increased the neutron leakage, thereby increasing the worth of the control rods by approximately 10%.

An axial fission traverse using a Pu-239 counter through the three-segment core mockup is shown in Fig. 8. Each of the gaps contained beryllium oxide, so that the spectrum was softened in these positions. Also, the peak can be seen in the axial reflector. The solid line here only connects the experimental points. No calculations were made for the axial traverses.

Figure 9 is an axial fission traverse of the same three-segment core mockup using a U-238 counter. Here the high-energy flux was depressed through the gap due to the absence of fissionable material.

Figure 10 shows the method used to expand the fuel columns in the three-segment fuel loading. In the normal fuel column at the top can be seen the position of the $\frac{1}{8}$ -in. aluminum spacers and also the two gaps in the fuel. For the expanded condition, the $\frac{1}{8}$ -in. aluminum shims were moved to the positions shown in the expanded fuel column. Assuming the base line is fixed, the lower 10-in. segment expands $\frac{1}{8}$ in. into the first gap. The 12-in. middle section expands $\frac{1}{4}$ in. into the second gap. The top 12-in. segment then expands $\frac{1}{8}$

FIG. 8. Axial Traverse with Pu-239 Counter (ZPR-3 Assembly 47A, SEFOR).

FIG. 9. Axial Traverse with U-238 Counter (ZPR-3 Assembly 47A, SEFOR).

FIG. 10. Simulated Expansion of Fuel Column (three segments) for SEFOR Mockup in ZPR-3 Assembly 47A.

TABLE 4. RESULTS OF FUEL-EXPANSION EXPERIMENTS IN SEFOR MOCKUP

Core Loading	Reactivity Change (ϵ)		
	Old Fuel	New Fuel	Total
1-Segment	—	—	-138
2-Segment	-9	-37	-46
3-Segment	-5	-15	-20

TABLE 5. COMPARISON OF EXPERIMENTAL AND CALCULATED PARAMETERS (ZPR-3 ASSEMBLY 47, SEFOR)

Parameter	Measured	Calculated
Central Fission Ratios		
σ_{28}/σ_{25} (Counter)	0.0264 ± 0.0005	0.0234
σ_{23}/σ_{25} (Counter)	1.45 ± 0.02	1.39
σ_{49}/σ_{25} (Counter)	0.916 ± 0.015	0.907
σ_{40}/σ_{25} (Counter)	0.201 ± 0.003	0.177
σ_{28}/σ_{25} (Radiochemical)	0.0242 ± 0.0021	0.0234
Capture to Fission Ratio, $\sigma_c^{28}/\sigma_f^{25}$ (Radiochemical)	0.137 ± 0.010	0.134
β/l (Noise Analysis) (sec^{-1})	4967 ± 94	5650
(Pulsed Neutron Source) (sec^{-1})	4880 ± 220	5650
U-238 Doppler Capture Increase	$3.5 \pm 0.5\%$	3.6%
Manganese Activation, $\sigma_c^{\text{Mn}}/\sigma_f^{25}$	0.0358 ± 0.0008	0.0442

in. out of the core. This, therefore, increased the overall core length by only $1/8$ in., the remaining $3/8$ -in. expansion is taken up in the gaps. A similar type of expansion was also done for the two-segment core.

Table 4 shows the results of the fuel expansion for the three different core loadings which were made. A one-segment expansion increased the overall length of

the core by $1/2$ in., losing \$1.38 of reactivity in this expansion. When expanding with the two-segment fuel-element design, the loss was only 46¢. In going to the three-segment fuel-element design, the expansion lost only 20¢.

Measurements were made of central fission ratios, U^{238} Doppler capture increase,⁽⁴⁾ manganese activation,⁽⁵⁾ and β/l . These results together with results of calculations are given in Table 5. The diffusion-theory calculations were made with ANL cross-section set 224, spherical models, and a 6-group, two-dimensional cylindrical model.

These results indicate that the calculated spectrum is too soft and that a more sophisticated analysis, including heterogeneous-cell calculations, resonance self-shielding, and a better cross-section treatment is required.

A complete report of the SEFOR critical experimental program in ZPR-3 will be published in the near future in the form of an Argonne National Laboratory report.

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Application of the SEFOR Critical Experiments at ZPR-3 to SEFOR

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ABSTRACT

A series of critical experiments was performed with a mockup of SEFOR at ZPR-3. Analyses of these experiments and the application of the results to the SEFOR design are discussed.

Values of the critical mass were determined for 1-, 2-, and 3-segment SEFOR fuel designs in order to help establish the plutonium atom fraction in the SEFOR fuel.

Reactivity effects of axial fuel expansion were measured which led to selection of the 2-segment design for SEFOR.

Measurements of the reactivity worth of the radial reflector established the adequacy of the SEFOR reflector-control system.

The Doppler coefficient was measured. The calculated U^{238} Doppler coefficient was in agreement with the experimental value; the measured Pu^{239} contribution to the SEFOR Doppler coefficient was near zero. It was demonstrated that the SEFOR Doppler coefficient [$T(dk/dT) = -0.0085$] is significantly more negative than the conservative value assumed for safeguards analysis.

The maximum positive reactivity due to loss of sodium was measured; the measured reactivity was small (+6¢) and close to the calculated value.

The ratio of prompt-neutron lifetime to delayed-neutron fraction was measured both by the pulsed neutron technique and by noise analysis. The values measured by the two techniques were in agreement, and, for a calculated β_{eff} of 0.0033, gave $l = 0.68 \mu\text{sec}$.

Fission ratios, fission and boron traverses, and plutonium worth distributions were measured and compared with calculations.

1. Introduction

SEFOR (Southwest Experimental Fast Oxide Reactor) is a 20-MW(t) fast-spectrum reactor fueled with PuO_2 - UO_2 and cooled with sodium. SEFOR will have characteristics similar to the large, soft-spectrum fast breeder reactors fueled with mixed PuO_2 - UO_2 .

SEFOR will be used to obtain physics and engineering data at fuel compositions, temperatures, and crystal-line states characteristic of operating conditions of power reactors. SEFOR is particularly designed for the systematic determination of the Doppler coefficient of reactivity at temperatures up to the vicinity of fuel melting.

The SEFOR Project consists of two major parts: the design and construction of the reactor, and a related research and development program. Funds for the design and construction of the facility are being provided by the Southwest Atomic Energy Associates (a group of seventeen investor-owned utility companies located in the South and Southwest part of the United States), together with the Karlsruhe Laboratory of the Federal Republic of Germany, EURATOM, and the General Electric Company. The United States Atomic Energy Commission is supporting the Research and Development Program.

This report describes work performed as part of the Research and Development Program. A series of experiments was conducted on a mockup of SEFOR in the ZPR-3 facility in a joint program by the General Electric Company and Argonne National Laboratory. Criticality of the mockup was achieved November 17, 1965; the experimental program was conducted from October 1965 to April 1966. Reference 1 provides a description of the experiments and experimental results. The present report gives an analysis of these results and describes how they are applied to the SEFOR design. Further details of the analysis are presented in Ref. 2.

The following experiments were performed on the SEFOR mockup:

1. critical mass for 1-, 2-, and 3-segment core designs;
2. reactivity of axial fuel expansion for 1-, 2-, and 3-segment designs;
3. worth of reflector control;
4. Doppler coefficient;
5. sodium-loss reactivity;

6. ratio of prompt-neutron lifetime to effective delayed-neutron fraction, l/β_{eff} ;
7. fission ratios and reaction-rate distributions.

2. Description of the Mockup

2.1. Characteristics of SEFOR of Significance to the Mockup Design

A brief description of SEFOR will provide a basis for discussion of the mockup. A more complete description is given in Ref. 3. The SEFOR fuel rods are composed of two fuel segments with a gap between the segments into which fuel can expand axially. The purpose of this segmented fuel design is to reduce the reactivity effect due to axial expansion, as described further in Sect. 4. The active fuel height is 85.9 cm and the effective core diameter 88.2 cm.

The reactor is controlled by movement of the 6-in.-thick radial nickel reflector; the reflector is divided into 10 segments which can be withdrawn, leaving a nearly completely voided reflector region. Behind the SEFOR reflector is a 2-in. neutron shield region containing boron carbide, followed by a serpentine neutron shield.

Between the core and the radial reflector are a sodium downcomer region, the steel reactor vessel, and a steel shroud. Above and below the core are axial reflectors in which nickel replaces fuel.

After the ZPR-3 experimental program was completed, an additional change was made in the SEFOR core design—in particular, the BeO volume fraction was reduced. The volume fractions (not including the gap) are given in Table 1 for both the final SEFOR design and the design represented by the mockup.

2.2. Geometry and Composition of the Mockup

Core dimensions and compositions were chosen to mock up the SEFOR core at 350°F, which is approximately the temperature for refueling and for the initial wet critical for SEFOR.

One purpose of the mockup was to facilitate the choice of the number of fuel segments for the SEFOR design. Therefore, experiments to measure critical mass and axial expansion were made for 1-, 2-, and 3-segment core mockups.

TABLE 1. SEFOR VOLUME FRACTIONS

Material	Final Design	Mockup Design
Fuel	0.432	0.429
Sodium	0.295	0.303
Steel	0.216	0.182
BeO	0.057	0.086

FIG. 1. Interface View of SEFOR Mockup Assembly 47A, Loading 15.

A cross-sectional view and an elevation view of the 1-segment mockup are shown in Figures 1 and 2. For the 2- and 3-segment mockups, the radius of the inner core was increased to 33.8 cm. The increase in core heights for the 2- and 3-segment mockups and location of the gaps are described in Sect. 4 (see Figures 6 and 7). The average compositions of each region for the 1-, 2-, and 3-segment mockups are given in References 1 and 2. Details of the drawer loadings are presented in Ref. 1.

Criticality was achieved first with 1-segment Assembly 47, Loading 15. This assembly is described in Ref. 4 and was chosen for analysis in Ref. 5 since it had the cleanest (i.e., simplest and most uniform) geometry. The 1-segment experiments analyzed in the present report were made with Assembly 47A, Loading 15 in which additional sodium was placed between the core and reflector, the nickel from two simulated reflector control rods was removed, and the ZPR-3 control rods were spiked further to add to the operating reactivity margin.

2.3. Oxygen Mockup

The SEFOR fuel is $\text{UO}_2\text{-PuO}_2$. The ZPR-3 fuel is metallic plutonium, uranium, and Pu-U alloy. Therefore it was necessary to mock up in ZPR-3 the oxygen in the SEFOR fuel.

The oxygen was mocked up by a combination of oxygen in sodium carbonate (Na_2CO_3), carbon in

FIG. 2. Elevation View of SEFOR Mockup Assembly 47A, Loading 15.

Na_2CO_3 , and aluminum.* The mockup contained 0.525 atom of oxygen, 0.175 of carbon, and 0.388 of aluminum per atom of oxygen in the SEFOR fuel.

The oxygen affects the core design primarily through its influence on leakage and neutron spectrum. In order to assess the effectiveness of the oxygen mockup, pre-experimental leakage and spectrum calculations were made for the SEFOR core at 350°F (hence, with the correct amount of oxygen) and for the mockup in ZPR-3 with the oxygen mocked up by oxygen, carbon, and aluminum. These results are given in Table 2.

3. Plutonium Atom Fraction

The plutonium atom fraction in the SEFOR fuel was based on calculations normalized to critical mass calculations of the SEFOR mockup. The procedure for establishing the plutonium atom fraction included the

TABLE 2. EFFECTIVENESS OF OXYGEN MOCKUP

	Actual Oxygen	Oxygen Mockup
Criticality factor	1.0041	1.0043
Leakage Probability		
Radial	0.234	0.230
Axial	0.092	0.090
Fraction of Fissions below 9 keV	0.203	0.198

following steps:

1. calculation of criticality factors for the SEFOR mockup and for SEFOR, using identical methods;
2. normalization of the calculation of the SEFOR criticality factor by the ratio of the calculated and experimental criticality factors for the SEFOR mockup;
3. establishment of the design criticality factor for 20-MW operation for SEFOR;
4. adjustment of the SEFOR plutonium atom fraction to obtain the design criticality factor.

* This mockup for oxygen was proposed by the Karlsruhe Laboratory of the Federal Republic of Germany which is participating in the SEFOR Project.

TABLE 3. CALCULATED CRITICALITY FACTORS FOR MOCKUP
(Calculated Values Are to be Compared with Experimental
Values of Unity)

Loading	Calculational Model	Calculated Criticality Factor
1-segment	2-dimensional (r, z), 4-group	0.985
2-segment	2-dimensional (r, z), 4-group	0.977
3-segment	2-dimensional (r, z), 4-group	0.974

3.1. Criticality Calculations for SEFOR Mockup

Criticality calculations were made for the 1-segment, 2-segment, and 3-segment SEFOR mockups. Diffusion theory was used throughout. Calculations included 4-group, 2-dimensional (r, z) and 18-group, 1-dimensional (both radial and axial) calculations. Four-group* and 18-group cross sections were condensed from 60-group cross sections in each important region, using 60-group, 1-dimensional calculations in both the radial and axial directions. The PDQ code was used for 2-dimensional calculations; the DEMON code was used for 1-dimensional calculations. The 60-group cross-section set used was based on the 1965 60-group General Electric set.^(4, 7)

The results of the 2-dimensional calculations for each of the critical loadings are given in Table 3. It was learned during the early analysis of the segmented core mockups that the use of 18-group, 1-dimensional axial calculations introduced considerable uncertainty in the radial leakage at the gaps. Therefore, greater reliance was placed on the use of the 2-dimensional calculations, together with space-dependent condensation of 60-group cross sections to four groups.

Small corrections to the criticality calculations were necessary since all the nickel in the reflector and all the ZPR-3 control rods were assumed to be inserted for the computer calculations, whereas some of the nickel and control rods were removed in the actual critical loadings for the mockups in ZPR-3. The worth of the removed nickel and control rods in the 1-segment loading was $\Delta k/k = 0.0073$; the worth in the 2-segment loading was $\Delta k/k = 0.0042$; the worth in the 3-segment loading was $\Delta k/k = 0.0007$. These corrections have been taken into account in the results listed in Table 3 so that the values in Table 3 should be compared with the experimental values of unity.

Shelf-shielding at low energy was accounted for by Bell's technique⁽⁸⁾ for applying the rational approximation to a repeating lattice. No corrections were made for high-energy heterogeneity effects in either the

mockup or the SEFOR criticality calculations. Additional corrections due to the gap between the two halves of the ZPR-3, the noncylindrical surface of the core, and streaming parallel to the plates of materials in the mockup are of little significance.⁽⁶⁾

The value of 0.985 for the criticality factor of the 1-segment core is consistent with those calculations of the initial SEFOR mockup loading reported in Ref. 5 which were based on the same cross-section data used in the SEFOR program. A new set of cross sections is recommended in Ref. 5 which gives much closer agreement with the initial loading.

It is noted from Table 3 that the reactivity effect of the gap is overestimated. From comparison of the critical masses of the 1-, 2-, and 3-segment mockups, it is estimated that the reactivity effect of the gap in the 2-segment mockup was $-2.25\% \Delta k/k$ and that the combined reactivity of the gaps in the 3-segment mockup was $-4.5\% \Delta k/k$. The 2-dimensional results in Table 3 indicate -3.0% and -5.6% reactivities for the gaps for the 2- and 3-segment mockups, respectively.

Several sources for the uncertainty in the treatment of the gap are possible. Diffusion theory may be inadequate for the accurate calculation of leakage at the gaps; transport approximations may provide better agreement. The use of additional neutron groups in a diffusion theory code may provide better agreement. For the results reported in Table 3, all materials in the gap were homogenized to obtain atom densities in the gap. Slightly different criticality factors are obtained for different gap models—for example, a gap model in which the gap is separated into three axial regions or a model in which a small section of the core is homogenized with the gap.

3.2. Design Criticality Factor for SEFOR

SEFOR is designed to reach a power level of 20 MW with a fixed core diameter with fuel rods of a single atom fraction of plutonium. A reasonable margin of error must be allowed for the extrapolation from the mockup in ZPR-3 to SEFOR at 20 MW.

Moreover, a second core (Core II) will replace the first core (Core I) later in the SEFOR experimental program. Although the specific design for Core II is not fixed, various designs being considered involve removal of the BeO (to harden the spectrum) with an accompanying loss in reactivity in the range of 1.5% to 2.5% in $\Delta k/k$.

These considerations led to the choice of a design criticality factor of $k = 1.035$. This allows a 1.0 to 2.0% margin for overestimating the criticality factor at 20 MW of Core I together with a margin of 1.5 to 2.5% reactivity loss for the change in Core II.

* The lower energies of groups 1, 2, and 3 for the 4-group structure were 1.35 MeV, 180 keV, and 9.1 keV, respectively.

3.3. Calculations of SEFOR Plutonium Atom Fraction

The 2-segment design was chosen for SEFOR. In order to select the fuel atom fraction of plutonium for the SEFOR fuel, 4-group, 2-dimensional, and 18-group axial calculations of SEFOR are being made using the same techniques used for the ZPR-3 criticality calculations. The calculated SEFOR criticality factors are normalized by the ratio of the experimental to calculated values given in Table 3.

Uranium-238 cross sections in SEFOR were calculated for the average fuel temperature at 20 MW and 1400°K, using the same techniques used for the room-temperature ZPR-3 mockup. For Pu²³⁹, room-temperature cross sections are used in the 20-MW SEFOR calculations since the ZPR-3 experiments, in verification of earlier experiments, showed almost zero Doppler contribution from Pu²³⁹.

4. Reactivity Effect of Axial Fuel Expansion

The quantities that will be measured in the SEFOR Doppler experiments will be power and energy coefficients of the fuel, from which will be calculated fuel temperature coefficients. The fuel temperature coefficient will include two components—the Doppler effect and the axial fuel-expansion effect. In order to obtain the Doppler effect, it is necessary to subtract the calculated value of the effect of axial fuel expansion from the total fuel-reactivity effect. Therefore, the accuracy of the measured Doppler coefficient is strongly influenced by the accuracy with which the effect of axial fuel expansion can be calculated.

With no special provisions in the fuel design to reduce the axial expansion effect, this effect in SEFOR would be almost as large as the Doppler effect. Therefore, two techniques are being incorporated in the SEFOR fuel design to reduce the axial expansion effect—dishing the ends of the fuel pellets, and segmenting the fuel.

To help in the selection of the SEFOR fuel design and to check calculational techniques, axial-expansion reactivity was measured for designs with 1, 2, and 3 segments.

4.1. Design of ZPR-3 Experiments

4.1.1 Selection of Experiments

For segmented fuel there exist two principal components to the effect of axial fuel expansion: a large negative component due to increased radial leakage in the fuel regions when the fuel expands, and a large positive component due to decreased radial leakage in the gap regions as the gap thickness is decreased. It was important to measure each of these two components independently to check the calculational method

for each effect and to measure them together to see if the effects are superposable.

Measurement of the effect of axial fuel expansion in the 1-segment core provided an independent measure of the first component. Decreasing the thickness of a gap while not expanding the fuel provided an independent measure of the second component; this was carried out for the larger gap in the 3-segment mockup. In addition, the effects of simultaneous expansion of fuel and compaction of the gaps were measured for both the 2- and the 3-segment mockups. Total fuel expansion in all cases was 1/2 inch. The gap in the 2-segment mockup was closed 3/8 inch. The upper gap in the 3-segment mockup was closed 1/4 inch; the lower gap was closed 1/8 inch.

The gap thickness was the same at all radial positions in the mockup. In SEFOR there will be a radial distribution both of the gap thickness and of the change in gap thickness with a change in power due to the radial distribution in fuel temperature. Hence, in some of the ZPR-3 expansion experiments, the inner (new) fuel zone was completely expanded before any of the outer (old) fuel zone in order to obtain information on the radial dependence of the axial-expansion effect that could be compared with a calculational model.

4.1.2 Gap Mockup

It was recognized early that the gap mockup in ZPR-3 must accurately represent the important geometrical characteristics of the gap in SEFOR. This meant that the axial distribution of the void and other materials in the gap between the fuel segments in SEFOR must be mocked up in ZPR-3 in order to assure that the streaming effects were measured correctly.

Schematic drawings of the gap in the 2-segment SEFOR design and the gap mockup in ZPR-3 are shown in Figures 3 and 4. Figure 3 shows the gap with the fuel unexpanded; Figure 4 shows the same gap with fuel expanded. This gap mockup describes well the geometrical distribution of material in the SEFOR gap. The material in the gap mockup was located in adjacent drawers in such a way that no straight-through streaming paths were present, thereby mocking up the correct situation for SEFOR.

4.1.3 Mockup of Expansion

To mock up expansion of the fuel, 1/8-in. aluminum spacers were located between fuel plates. All experiments mocked up an expansion of 1/2 in.; hence, four spacers were required. Later, the reactivity effect of the aluminum spacers was measured. This method of expansion differs from that in SEFOR in that the expansion occurs throughout the length of the fuel rod

in SEFOR, the expansion being proportional to the change in fuel temperature in the axial direction. However, the mockup calculations account for the discrete placement of the spacers, and the error due to this

FIG. 3. Gap Mockup in ZPR-3 (Fuel Unexpanded).

FIG. 4. Gap Mockup in ZPR-3 (Fuel Expanded).

difference between the mockup and SEFOR is not considered to be significant.

The locations of the aluminum spacers for the 1-, 2-, and 3-segment experiments are given in Figures 5, 6, and 7, together with dimensions before and after expansion.

Treatment of the expansion of the fuel into the gap was shown in Fig. 4. It is noted that aluminum spacers are placed in the spaces at the ends of the non-fuel columns adjacent to the gap. To reduce the uncertainty introduced by this aluminum, a very nearly equal amount of aluminum was removed from the center part of the gap to accommodate the fuel expansion. Thus the total aluminum in and immediately around the gap remained nearly constant before and after the expansion.

FIG. 5. Expansion Experiment in One-segment Mockup (Dimensions in Inches).

FIG. 6. Expansion Experiment in 2-segment Mockup (Dimensions in Inches).

Fig. 7. Expansion Experiment in 3-segment Mockup (Dimensions in Inches).

4.2. Results

Experimental and calculated results are compared in Table 4.

The experimental results show clearly the efficacy of segmenting the fuel. Observation of the results of the 1-segment experiment and the 3-segment experiment in which only the gap thickness was changed shows the compensating effects that take place during a power change with segmented fuel. The two compensating components appear to be approximately superposable. For example, it is reasonable to estimate that if the lower gap of the 3-segment core had been closed $\frac{1}{8}$ in. in addition to closing the upper gap $\frac{1}{4}$ in., the combined reactivity effect of changing the gap thicknesses only would be $\sim +\$1.1$. Combined with the $-\$1.4$ value for the effect of fuel expansion alone gives $-\$0.3$ for the net effect, which compares well the measured value of $-\$0.2$.

It is also reasonable (as indicated by the experiments) that the effect of the $\frac{3}{8}$ -in. decrease in gap thickness for the 2-segment case is less than the combined decrease in the gap thicknesses for the 3-segment case. The reason is that the gap in the 2-segment case skews the flux toward the bottom part of the core, thus decreasing the importance of the gap location in the 2-segment case relative to the 3-segment case.

The calculated results differ significantly with the experimental values. The fuel-expansion component (1-segment case) was calculated to be 21% larger than that experimentally observed. The gap-compaction component (3-segment, gap-only case) was calculated to be 36% higher than observed in the experiment. These errors combine in a fortuitous way to give close

agreement for the 2-segment case. Normalization of the present diffusion-theory calculational method to the ZPR-3 results provides a technique which is sufficiently accurate for the SEFOR experimental program, however, since the magnitude of the axial-expansion effect is reduced so much by the combined techniques of dishing the pellets and using 2-segment fuel.

The results in Table 4 have been corrected for removal of the aluminum spacers used to expand the fuel in the mockup. The effect of removal of the aluminum was measured experimentally. The size of this correction and comparisons with calculations are shown in Table 5.

4.3. Calculational Model

The calculations were made by 2-dimensional, 4-group diffusion theory using PDQ. Four-group cross sections were condensed from the SEFOR-mockup 60-group set for each important region using 1-dimensional radial and axial problems. All materials in the gap were homogenized to obtain atom densities in the gap. Homogenizing a small section of the core with the gap gave results slightly closer to experiment; separating the gap into three axial regions gave results which disagreed further from the experiment. Care was taken in mocking up exact compositions and dimensions, and accounting for the presence of control drawers, variations in fuel plate locations, and details of the fuel plate design at the gap-fuel interface. For each calculation, material balances were checked before and after expansion to assure that material was not inadvertently added or lost; this was important since some problems contained as many as 50 regions.

TABLE 4. REACTIVITY DUEL TO AXIAL FUEL EXPANSION

Seg-ments		Experimental (\$)	Calculated (\$)
1	Fuel expanded	-1.39 ± 0.04	-1.68
2	Fuel expanded	-0.46 ± 0.02	-0.49
	Fuel expanded in inner fuel region only	-0.37 ± 0.02	-0.36
3	Fuel expanded	-0.20 ± 0.02	-0.04
	Upper gap thickness decreased only (no accompanying fuel expansion)	$+0.74 \pm 0.02$	+1.01

TABLE 5. EFFECT OF ALUMINUM SPACERS
($\Delta k/k$ for Removal of Aluminum Spacers)

Case	Experimental, ¢	Calculated, ¢
1-segment	-5	-7
2-segment	-7	-6
3-segment	-4	Not calculated

5. Reflector Control Worth

The reactivity effect of the reflector of the SEFOR mockup was measured in order to evaluate the worth of the SEFOR control system. The value obtained for the SEFOR control worth was $\$(11 \pm 2)$. This value compares with the predicted operating control requirement for SEFOR of \$5.5. The large difference between the \$11 control worth and the \$5.5 operating requirement insures adequate availability of shutdown reactivity.

In this section the experimental procedures and results are described, together with two methods used to obtain the SEFOR reflector worth from the ZPR-3 measurements. It was not feasible to measure the worth of a segment of the reflector larger than one quadrant. Therefore, calculations were necessary to provide an extrapolation to the total SEFOR reflector worth.

5.1. Experimental Procedure

The reactivity effects of removing both one-tenth and one-fourth of the nickel from the ZPR-3 reflector were measured in order to simulate approximately the withdrawal of one of the ten SEFOR reflector control rods and a quadrant of the SEFOR reflector.

The single reflector-rod measurement was made by the rod-swap technique. The nickel from 21 of the 236 drawers in the reflector region of Fig. 1 was removed in small increments. The reactivity effect of

each increment was measured by compensation with a calibrated ZPR-3 control rod. The nickel removed in each increment was placed in a reflector position on a different side of the core before removal of the next increment.

The worth of one quadrant was determined both by subcritical multiplication and by pulsed neutron techniques.

There were important differences between the mock-up and the SEFOR reflector which had to be accounted for by both special measurements and calculations. In SEFOR removal of a rod leaves a large void. In the mockup, removal of the nickel plates left behind the steel ZPR-3 drawers and matrix. The reactivity effect of this steel was estimated by measuring the worth per unit weight of steel in the reflector after removal of the nickel.

A second important difference was the difference in materials outside the reflector. In SEFOR an aluminum structure followed by 2 in. of rods filled with B_4C and then by rods of serpentine lie beyond the reflector. In the mockup, only two drawers were available beyond the reflector at the position where the reflector worth was measured. The single-rod worth was measured first with no material outside the reflector except the steel matrix forming the two drawer locations (hence, effectively about 4 in. of 10% steel). A second measurement was then made with $\frac{1}{4}$ in. of $B_4^{10}C$ and one inch of polyethylene outside the reflector. The quadrant worth was measured only with the steel matrix outside the reflector. The aluminum structure outside the SEFOR reflector, together with structural aluminum webs between the SEFOR rods, was mocked up by aluminum plates in the reflector region of the mockup.

Other small differences in composition and geometry in the reflector, in the regions between the core and reflector, and in the core were present, but it is expected that these differences are treated adequately by calculation.

5.2. Experimental and Calculated Results of ZPR-3 Reflector Experiments

The experimental results for the 1-segment loading are given as the starred values in Table 6. The rod positions I, II, and III are indicated in Fig. 1. Rod positions II and III correspond to the positions for which calculations were made. Therefore, the results for rods II and III were averaged to obtain the experimental values that were compared with calculation. The large difference between rods II and III was caused by an asymmetric loading of plutonium in the 1-segment core, a situation that was corrected before the 2-segment reflector measurements were made.

TABLE 6. RESULTS OF MEASUREMENTS OF 1-SEGMENT REFLECTOR WORTH

	Rod I	Rod II	Rod III	Average of Rods II and III
Nickel:				
total worth	$100 \pm 3\epsilon^*$	$127 \pm 4\epsilon^*$	$93 \pm 5\epsilon^*$	110¢
Aluminum:				
worth/kg		0.82¢/kg*		
mass		17.9 kg		
total worth		14.7¢		13¢
Steel:				
worth/kg		0.46¢/kg*		
mass		39.2 kg		
total worth		18.0¢		16¢
Change in rod worth due to placing B_4C and polyethylene behind reflector		-6.2¢*		-5.4¢

* All starred values represent experimental results as reported in Ref. 1.

TABLE 7. COMPARISON OF CALCULATED AND EXPERIMENTAL VALUES FOR NICKEL IN REFLECTOR OF MOCKUP

	Calculated	Experimental
Worth of nickel in one rod		\$1.10
Ratio: $\frac{\text{Worth of reflector quadrant}}{\text{Worth of one rod}}$	2.13	2.29
Ratio: $\frac{\text{Worth of total reflector}}{\text{Worth of quadrant}}$	3.58	
Calculated worth of nickel in total reflector	\$9.45	
Extrapolated worth of nickel in total reflector = $(\$1.10)(2.29)(3.58)$		\$9.02

For the 2-segment core, the measured nickel worths in rods II and III were 120¢ and 121¢, respectively.

Calculated and experimental values for the nickel worth are compared in Table 7. The calculated values were based on the use of diffusion theory throughout the mockup, including the reflector region after removal of the nickel. The use of diffusion theory in the reflector region of the mockup was considered satisfactory for this purpose since a relatively large amount of steel and aluminum remained in the region after the nickel had been removed. The total nickel worth was calculated using 18 groups and one dimension (cylindrical). The ratios of one rod and one quadrant to the total worth were obtained by 4-group PDQ x,y calculations.

5.3. Methods of Obtaining SEFOR Reflector Worth from the ZPR-3 Results

The following two methods were used to evaluate the SEFOR reflector worth from the ZPR-3 experimental results:

- (1) Direct Extrapolation Method—The experimental results were extrapolated to SEFOR, accounting for the differences between ZPR-3 and SEFOR compositions and dimensions. Calculated reflector worths enter this extrapolation only in the form of ratios, thereby reducing the uncertainties introduced by calculations.
- (2) Boundary Condition Method—The reflector worth was calculated by replacing the void left by the withdrawn reflector and the shield outside the reflector with a calculated boundary condition. Calculations were made for the complete removal of the reflectors from both SEFOR and the ZPR-3. An approximate experimental value for this condition in the mockup was obtained by extrapolation of the ZPR-3 experimental measurements to the case for complete removal of reflector from the mockup. Comparison of this extrapolated experimental re-

sult and the calculation for ZPR-3 provided the required normalization for the SEFOR calculation.

5.3.1 Direct Extrapolation Method

The direct extrapolation method gave the following control worths for the 1- and 2-segment SEFOR designs:

Worth for 1-segment design = \$10.2;

Worth for 2-segment design = \$11.2.

Details of the extrapolation are given in Ref. 2.

5.3.2 Boundary Condition Method

The effect of the void left by the withdrawn SEFOR reflector rods was calculated by replacing the void and all regions outside the reflector with boundary conditions at the inside of the void region. Four-group one-dimensional (cylindrical) diffusion theory was used. The technique involved the calculation of leakage probabilities in a manner similar to that used by Oliver and Norman.⁽⁹⁾

The steps in calculating the boundary conditions were:

- (1) The effective leakage probabilities across the void and escape probabilities from the void were calculated.
- (2) A 4-group matrix albedo at the outer void surface was calculated.
- (3) Ratios of exit to entering current densities at the inside of the void were calculated.
- (4) Boundary conditions were calculated from the current densities using conventional diffusion theory.

The boundary condition method was applied both to SEFOR and to the mockup. The reactivity effect of removing all material from the mockup was extrapolated from the experimental results of Table 6.

The calculated values for the 1-segment core for both SEFOR and the mockup are given in Table 8.

TABLE 8. COMPARISON OF BOUNDARY CONDITION REFLECTOR CALCULATIONS WITH EXTRAPOLATED ZPR-3 MEASUREMENTS FOR THE 1-SEGMENT CORE

Description	Calculated, \$	Extrapolated from Measurements, \$ (All material removed from reflector region)
SEFOR	10.2	—
Mockup with B ₄ C and polyethylene behind reflector	9.9	10.9
Mockup with 10 cm of 10% steel behind reflector	10.7	11.4

The values for the mockup extrapolated from the experimental data are also given in the table.

Normalization of the calculated results of Table 8 to the experimental values gives a predicted value of \$11.1 for the 1-segment SEFOR core. This value is 8% higher than the value of \$10.2 obtained from the direct extrapolation method.

An approximate calculation of the 2-segment reflector worth in the mockup was made by homogenizing the gap material with the fuel for the radial calculation. This resulted in increased radial leakage and a 5% increase in reflector worth, which compares with a measured increase of 9%. The 5% calculated increase gives a reflector worth of \$11.6 for the 2-segment SEFOR core, which is 4% higher than the value obtained from the direct extrapolation method.

The effect on reflector strength of placing an absorber and homogeneous moderator behind the reflector is predicted qualitatively. The B_4C and polyethylene behind the reflector in the mockup reduced the reflector worth by 5%, which compares to the calculated reduction of 8%. The use of the 10 cm of 10% steel behind the reflector to mock up the ZPR-3 matrix may be inaccurate; to check the sensitivity of this assumption the reflector worth was also calculated with 20 cm of 10% steel. This gave a worth of \$10.2, to be compared with the \$10.7 value in Table 8.

The details of the method for calculating the boundary conditions are given in Ref. 2.

6. Doppler Coefficient

Experiments were performed in the mockup to help determine the magnitude of the SEFOR Doppler coefficient. The results showed close agreement between calculation and experiment for the U^{238} Doppler coefficient and a near-zero contribution by Pu^{239} , results which are consistent with earlier ZPR-3 measurements.⁽¹⁰⁾ Applying the same techniques for the U^{238} Doppler calculation for SEFOR and assuming a zero contribution from Pu^{239} , the calculated Doppler coefficient for the SEFOR 2-segment design is $T(dk/dT) = -0.0085$. This value is well above the value of -0.004 used for SEFOR safeguard analyses.

6.1. Experimental Procedure and Results

The ZPR-3 measurements were performed by the "hot sample-cold reactor" technique⁽¹⁰⁾ in which a heated sample is exchanged with an equivalent cold sample and the difference in steady-state reactivities is measured. Measurements were made for $U^{238}O_2$ samples and for 12.5% PuO_2 -87.5% $U^{238}O_2$ samples. Measurements for the $U^{238}O_2$ sample were later repeated with sodium removed from the regions surrounding

the samples. Measurements were made for sample temperatures of 300°K, 500°K, 800°K, and 1100°K.

The results of the measurements with sodium present are shown in Fig. 8 with reactivity per kilogram U^{238} plotted against $\ln(T/T_0)$. The $U^{238}O_2$ results are plotted as measured. The 12.5% PuO_2 - UO_2 measured values are shown also plotted on a $\delta k/k$ per kilogram U^{238} basis, assuming a zero contribution from the plutonium. The fact that the results are indistinguishable from the results for the pure UO_2 samples forms the basis for the assumption that the plutonium contribution to the SEFOR Doppler coefficient is negligible.

A straight line through the experimental points in Fig. 8 represents a T^{-1} variation of the Doppler coefficient, dk/dT (i.e., $T(dk/dT) = \text{constant}$). The solid line in Fig. 8 represents the T^{-1} variation drawn through the high-temperature points which correspond to the fuel temperatures of most interest to the SEFOR experimental program. The dashed line represents a $T^{-0.9}$ variation. Although it is possible to construct a T^{-1} line that lies within the experimental error at each temperature, the $T^{-0.9}$ curve provides a better fit. Although the data are insufficient to establish the temperature variation accurately, it is interesting to note that the calculated temperature variation that best fits the calculated SEFOR isothermal Doppler coefficient from 300°K to 5000°K is $T^{-0.9}$.*

6.2. Comparison of Calculated and Experimental Values

The calculated Doppler effect of the heated sample was based on a U^{238} temperature rise from 700°K to 1400°K. From Fig. 8, the experimental value corresponding to this temperature change is $-1.5 \times 10^{-5} \delta k/k/\text{kg } U^{238}$.

The calculated reactivity effect for a U^{238} temperature change from 700°K to 1400°K was also $-1.5 \times 10^{-5} \delta k/k/\text{kg } U^{238}$. The calculated value for the Doppler effect of the heated sample was obtained by perturbation theory using a two-dimensional 16-group CRAM problem. The change in U^{238} absorption cross section ($\delta\sigma_a$) below 4 keV was calculated by the

*The calculated temperature variation of the U^{238} Doppler coefficient in SEFOR was obtained from the following 18-group spherical calculations in which the U^{238} fuel temperature was assumed constant throughout the core and represented the only change between problems:

U^{238} Temperature, °K	Criticality Factor
300	1.04804
700	1.03983
1400	1.03245
3000	1.02389
5000	1.01804

FIG. 8. U^{238} Doppler Measurements in SEFOR Mockup in ZPR-3. (Temperatures in $^{\circ}K$)

RAPTURE code⁽¹¹⁾ using recent Columbia resonance parameters.⁽¹²⁾ For the unresolved resonances (>4 keV), the results of Greebler and Goldman⁽¹³⁾ were used (which were also based on the use of RAPTURE). Values of $\delta\sigma_a$ were calculated for the case in which both the absorber and the surrounding medium are at the same temperature. The SEFOR potential cross sections, σ_p , which were nearly equal to the ZPR-3 values, were used for the initial, uncorrected calculation.

A correction was made in the calculation to account for the fact that only the sample was heated while the surrounding medium remained at room temperature. A rough estimate by Greebler for an effective potential scattering cross section for U^{238} to account for this effect was 54 b, compared to 37 b calculated for SEFOR. Using the 54-b value for the heated sample increases the calculated U^{238} Doppler effect by about 1%, indicating that the net small sample correction is low.*

* The net correction is small because the magnitude of the Doppler effect is decreased by the larger value of σ_p above 1 keV and increased below 1 keV.

The agreement between theory and experiment is tempered by the recognition that uncertainty remains concerning the accurate interpretation of the experiment, and that all known refinements have not been included in the Doppler calculations. For example, interference effects were not included, which may be a correction of the order of 10%. Hence, it is not our purpose to emphasize the close agreement between calculation and experiment. It is our purpose, however, to show sufficient agreement between calculation and experiment to assure the validity of the conclusion stated in the initial paragraph of this section, namely, that the calculated SEFOR Doppler coefficient is significantly larger in magnitude than the value used for SEFOR safeguards analyses.

6.3. Effect of Loss of Sodium on Doppler Coefficient

Loss of sodium results in hardening of the spectrum and an accompanying reduction in the Doppler coefficient. The U^{238} Doppler effect was measured in the mockup with sodium removed from a large enough area

TABLE 9. REDUCTION IN DOPPLER COEFFICIENT FROM LOSS OF SODIUM

Sodium Volume Fraction	Fractional Reduction, %	
	Calculated	Experimental
0.244	9.6	17.5
0.280	12.4	
0.303		

around the sample to obtain a spectrum characteristic of the reactor with sodium removed.

As shown in Table 1, the sodium volume fraction mocked up in ZPR-3 was 30.3%. The reduction in Doppler coefficient due to loss of sodium was calculated for earlier SEFOR designs containing sodium volume fractions of 24.4% and 28.0%.

The experimental and calculated results for the reduction in Doppler coefficient from loss of sodium are listed in Table 9. Use of the measured 17.5% value would reduce the SEFOR Doppler coefficient [$T(dk/dT)$] from -0.0085 to -0.0070 .

7. Sodium-loss Reactivity

7.1. Sodium Loss from Center of Core

The calculated value for the maximum positive reactivity from loss of sodium from the center of the

FIG. 9. Measured and Calculated Sodium-loss Reactivity.

SEFOR core was $+3\%$. This value was low enough to indicate that this reactivity effect does not pose a safety problem for SEFOR. To verify this expectation, the maximum positive reactivity due to sodium loss was measured in the mockup.

The reactivity effects of complete removal of sodium from the central 5, 10, and 15 v/o of the 1-segment core were measured. The maximum measured positive effect was $+6\%$, which occurred with sodium removed from the central 10% of the core. The sodium-loss reactivity became decidedly negative with removal from the central 15% of the core.

The calculated values together with the three experimental points are plotted in Fig. 9.

In the ZPR-3 measurements, the voided zone was distributed uniformly around the center of the core only in the 10% measurement; the voids were slightly displaced for the 5% and 15% measurements. Had they also surrounded the core center uniformly, it is expected that the experimental results for these points would have been slightly more positive. However, it is concluded from the experiments that the $+6\%$ at 10% sodium loss is close to the maximum positive effect and that the effect becomes strongly negative when more than 13% of the volume is voided.

7.2. Sodium Loss from the Gap

Since the 2-segment design has been selected for SEFOR, it was useful to establish experimentally that the loss of sodium from the gap would not cause an additional large positive reactivity. This was done by adding sodium to the gaps of 63 central drawers (which is one-third of the drawers in that half of the mockup). The resulting reactivity was positive, $+1.1\%$ /kg of sodium added. Hence, loss of sodium from the gap would be negative.

7.3. Calculational Method

Spherical geometry was used for sodium-loss calculations in order to maximize the positive effect and to avoid the use of transverse bucklings. The sodium-loss effect has two principal components—a large positive component due to hardening of the spectrum, and a large negative component due to increased leakage. The former component has its maximum influence and the latter component its minimum influence when sodium is lost from the center of the core. Hence, the maximum positive net effect should occur when sodium is lost from a small spherical region around the center of the core. In addition, there is a smaller positive effect caused by the absence of capture by the removed sodium.

The calculated reactivities for sodium loss as a function of fraction of the core voided are listed in Table

10. In addition, an approximate breakdown of the components which make up this reactivity effect are listed. It is observed from Table 10 that the small maximum positive effect is the sum of large positive and negative components. For this reason, the agreement between theory and experiment shown in Fig. 9 is gratifying.

8. Ratio of Prompt-neutron Lifetime to Effective Delayed-neutron Fraction

8.1. Experimental Results

The ratio l/β_{eff} was measured by the pulsed neutron technique and by noise analysis using cross correlation. The pulsed neutron measurements were reported by Brunson.⁽¹⁴⁾ The noise analysis was reported by Morrell.⁽¹⁵⁾ Extrapolating the results in References 14 and 15 to the critical reactor gives for the two methods:

Pulsed neutron: $l/\beta_{\text{eff}} = 2.05 \times 10^{-4}$ sec;

Noise analysis: $l/\beta_{\text{eff}} = 2.05 \times 10^{-4}$ sec.

The experimental error in these numbers is of the order of 5%. For the calculated value of β_{eff} of 0.0033 (see below) and the experimental value of l/β_{eff} , the lifetime is 0.68 μsec .

This noise measurement represented the first application of the cross-correlation technique to a fast reactor. The agreement with the pulsed neutron result has led to the recommended use of noise analysis to measure l/β_{eff} in SEFOR.

8.2. Calculated Value of β_{eff}

The calculated value of β_{eff} for the SEFOR mockup is 0.0033. Delayed-neutron parameters for fast fission recommended by Keepin⁽¹⁶⁾ were used. β_{eff} is dependent both on the relative number of fissions between U^{238} and Pu^{239} and on the relative effectiveness of delayed versus prompt neutrons. The relative number of fissions was calculated using cross sections which gave reasonably good agreement with the measured $\text{U}^{238}/\text{Pu}^{239}$ fission ratio (see Sect. 9 below). Approximately 12% of the fissions in the mockup occur in U^{238} . The relative effectiveness of delayed neutrons to prompt neutrons, β_{eff}/β , is 0.91, and this result is essentially the same for each delayed-neutron group.

8.3. Calculated Value of Prompt-neutron Lifetime

The lifetime calculation for the SEFOR mockup is reported in Ref. 4. Using a two-dimensional 16-group CRAM calculation and using cross sections almost equivalent to ours, Greebler *et al.* obtained $l = 0.65$ μsec . Using the SEFOR mockup 60-group cross sections and one-dimensional radial calculations, we obtained $l = 0.54$ μsec ; however, it is known that the

TABLE 10. CALCULATION OF SODIUM-LOSS REACTIVITY

Voided Zone		Reactivity (β)			
Relative Radius (r/R_0) to which Sodium is Voided	Fraction of Volume Voided, %	Total	Leakage Component	Spectrum Component	Sodium Capture Component
0.4	6.4	+0.03	-0.43	+0.41	+0.05
0.5	12.5	-0.05	-0.92	+0.78	+0.09
0.6	21.6	-0.28	-1.70	+1.26	+0.16
0.7	34.3	-0.76	-2.86	+1.87	+0.23
1.0	100	-4.4	-8.7	+3.7	+0.6
Vessel	—	-10.1	-15.6	+4.9	+0.6

one-dimensional result is sensitive to the transverse bucklings used. Calculated values were obtained from the calculated reactivity effect of a $1/v$ absorber.

For $l = 0.65$ μsec , the calculated value of l/β_{eff} is 1.96×10^{-4} sec, which is close to the experimental values. The 0.54- μsec lifetime gives an l/β_{eff} value 20% below the experiment.

9. Reaction Parameters

9.1. Reaction Ratios

Relative fission rates of U^{235} , U^{238} , Pu^{239} , and Pu^{240} were measured at the core center by means of fission counters. In addition, U^{235} and U^{238} foils were irradiated at the center and analyzed radiochemically at Argonne to determine the ratios of the U^{238} capture and fission rates to the U^{235} fission rate.

The measured reaction rates relative to the Pu^{239} fission rate are given in Table 11. The radiochemical results were normalized to the Pu^{239} fission rate by multiplying by the fission-counter-measured $\text{U}^{235}/\text{Pu}^{239}$ fission ratio. Values listed are ratios of reaction rate per atom; hence, they can be compared to the calculated ratios of the quantity $\int \sigma(E) \Phi(E) dE$. Calculated values are also listed in Table 11. SEFOR-mockup 18-group cross sections were used for the calculations; hence, the Pu^{239} cross sections were self-shielded and the U^{235} were infinitely dilute.

TABLE 11. CENTRAL REACTION RATIOS

Ratio	Experimental Technique	Measured	Calculated
$\sigma_f(\text{U}^{235})/\sigma_f(\text{Pu}^{239})$	fission counter	1.09	1.15
$\sigma_f(\text{Pu}^{240})/\sigma_f(\text{Pu}^{239})$	fission counter	0.219	0.183
$\sigma_f(\text{U}^{238})/\sigma_f(\text{Pu}^{239})$	fission counter	0.029	0.028
$\sigma_f(\text{U}^{238})/\sigma_f(\text{Pu}^{239})$	foil irradiation (radiochemical)	0.026	
$\sigma_c(\text{U}^{238})/\sigma_f(\text{Pu}^{239})$	foil irradiation (radiochemical)	0.150	0.143

The U^{238}/Pu^{239} ratios are in agreement; the U^{235} and Pu^{240} values are not as close. Better agreement is obtained for the U^{235}/Pu^{239} fission ratio by using infinitely dilute Pu^{239} cross sections (1.08 instead of 1.15), but the self-shielded value is probably more appropriate and its use leads to better agreement elsewhere.

9.2. Reaction-rate Traverses

Fission-rate traverses were made using U^{238} and Pu^{239} fission counters. A boron absorption-rate traverse

FIG. 10. Radial Pu^{239} Fission Traverse.

was made with a BF_3 chamber. The shape of the traverses in the x direction (radial or axial) were compared with calculated traverses using eighteen energy groups.

The measured and calculated radial Pu^{239} fission traverses at the midplane of the 2-segment loading are plotted in Fig. 10. Axial Pu^{239} and U^{238} fission traverses and the axial boron-absorption traverse for the 2-segment mockup are shown in Figures 11–13. In each case the curves were normalized so that the spatial integrals $\int \sigma \Phi dV$ over the core were equal for the experimental and calculated curves. For the Pu^{239} traverses, self-shielded Pu^{239} cross sections were used in the fuel regions; infinitely dilute cross sections were used in nonfuel regions.

Use of the region-dependent cross sections provided good agreement for the radial Pu^{239} traverse. Peaking in and near the gap in the axial traverses was underestimated. It is likely that this underestimate results from the use of diffusion theory; in fact, Helm *et al.*⁽¹⁷⁾ reported similar underestimates of peaking near boundaries for traverses in ZPR-6 using diffusing theory and agreement using transport theory.

9.3. Pu^{239} Worth Traverse

An axial Pu^{239} reactivity worth traverse was made in the 2-segment core with a 24.09-g sample of Pu^{239} . The relative calculated and measured distributions are

FIG. 11. Axial Pu^{239} Fission Traverse.

FIG. 12. Axial U-238 Fission Traverse.

FIG. 13. Axial Boron-absorption Traverse.

FIG. 14. Axial Pu-239 Worth Traverse.

shown in Figure 14 normalized to equal areas under the curves in the core.

The calculated distribution was obtained from the expression

$$\sum_j [\sum_i \nu_i \sigma_{f,i}(z) \phi_i(z)] \chi_j \phi_j^*(z) - \sum_i \sigma_{a,i}(z) \phi_i(z) \phi_i^*(z),$$

where the summations are over the eighteen groups.

As was the case for the fission traverses, the agreement is fair, but the worth in the gap is underestimated.

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Discussion

Mr. Gandini (CNEN): You mentioned that if you remove sodium in the vicinity of the sample you lose

about 17% of your negative Doppler coefficient. Is the amount of sodium which you remove large or small? Removal of a small quantity will give different results from removal of a large quantity.

Mr. Reynolds: That's right. This is the effect of removing all of the sodium from the center of the core for the volume mentioned.

Mr. Kohler (Texas A&M) : On the basis of the measurements for the control rods in the reflector, did you decide that you don't have to have in-core control rods in your final design?

Mr. Reynolds: Yes, we are sure now that we shall not have to make provisions for that because we have sufficient reactivity control margins.

Presented by G. Kistner

Critical Experiments with a Mockup of the Repetitively Pulsed Reactor SORA*

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ABSTRACT

A series of critical experiments has been performed on a close mockup of the SORA reactor. SORA is a NaK-cooled, repetitively pulsed fast reactor which will be used as a high-intensity neutron source for time-of-flight experiments. The reactivity of this reactor is periodically varied by means of a movable reflector. Those parameters which are related to the kinetics of the reactor have been investigated thoroughly in the critical experiments.

The total reactivity value of the movable reflector determines the ratio of the peak power to that between pulses. The reactivity of the movable reflector as a function of its position and the prompt-neutron decay constant determine the half-width of the neutron pulses. These parameters have been measured for beryllium and for iron reflectors of several sizes, and for various core and reflector configurations. It has been shown, for example, that a 24-cm-high, 7-cm-thick, and 21-cm-wide beryllium reflector operating under reactor design conditions will produce pulses about 50 μ sec wide with a peak-to-minimum power ratio of approximately 4×10^3 .

1. Introduction

A program of static critical experiments has been completed at the Oak Ridge National Laboratory Critical Experiments Facility in cooperation with EURATOM supporting the design of SORA (Sorgente Rapida).

SORA⁽¹⁾ is a fast, liquid-metal-cooled, repetitively pulsed reactor, which will be used as a high-intensity neutron source for time-of-flight experiments. The core volume is about 5 liters, the fuel is highly enriched uranium, and the mean power is planned to be 1 MW ultimately. The reactivity is periodically varied by a movable reflector which passes by one side of the core

and increases the reactivity above prompt critical for $\sim 100 \mu$ sec with a repetition rate of 50 pulses per second.

This report emphasizes the measurements related to the kinetics of SORA.

2. Description of the Assembly

The mockup of SORA, shown in Figs. 1 and 2, closely resembled the reactor design. It differed principally in the use of iron to represent NaK and of uranium instead of uranium-molybdenum alloy as fuel. The core of the mockup was a hexagonal-shaped iron matrix containing unclad uranium-metal rods with a ²³⁵U enrichment of 93.18 w/o. The rods were 1.379 cm in diameter and 24 cm long, and were arranged in a triangular pattern with a 1.630-cm pitch. With this geometry the core volume fractions were 0.649 U, 0.344 Fe, and 0.007 void. The core was mounted between a 17.6-cm-thick bottom reflector and a 16.3-cm-thick top reflector, both of iron.

A 28-cm-thick stationary iron reflector, containing six control rods and two 14.1-cm-diameter scatterers (used for moderating neutrons leaking out the horizontal beam holes) enclosed five sides of the core. Adjacent to the sixth side of the core a movable beryllium or iron reflector, with a height of 24 cm, a thickness of 7 cm, and various widths ranging between 11 and 26 cm, could be positioned. The reflector block was mounted on an arm which could be moved remotely through $\pm 45^\circ$ from its central position for purposes of reactivity calibration. The core was mounted on the movable platform of the ORNL Critical Testing Unit (CTU) and was raised into the stationary reflector by a pneumatic-hydraulic mechanism in the approach to criticality. Figure 3 shows the core mounted on the CTU in a position withdrawn from the reflector.

Eighty-six rods, containing 56 kg of uranium, were required for criticality.

3. Discussion and Results

One parameter related to reactor kinetics is the total reactivity value of the movable reflector. This

* Research sponsored by the U. S. Atomic Energy Commission under contract with the Union Carbide Corporation.

† On assignment to Oak Ridge National Laboratory from EURATOM, Ispra, Italy.

FIG. 1. Mockup of SORA Critical Assembly.

value determines the ratio of the power in the pulse to that between pulses⁽²⁾ from the following formula:

$$\frac{P_p}{P_b} = \frac{|\rho| - 1}{\theta f},$$

where

P_p = pulse peak power;

P_b = average power between pulses;

ρ = total reactivity value of the movable reflector, dollars;

θ = half-width of neutron pulse, sec;

f = pulse repetition rate, sec⁻¹.

With a reflector reactivity of \$5, which has been set as a lower acceptable design limit, about 80% of the power is developed within the pulses. The reactivities measured* in these experiments ranged between \$6 and \$12

* The reactivity measurements were performed with an analog computer⁽³⁾ which has as input the current from a BF₃ ionization chamber responding to power changes in the assembly produced by reactivity changes.

FIG. 2. Cross Section of the Mockup of the SORA Critical Assembly.

FIG. 3. SORA Mockup Mounted on Critical Testing Unit.

for beryllium reflectors and between \$4 and \$7 for iron reflectors of various sizes.

A second parameter α_x related to the kinetics describes the reactivity of the movable reflector as a function of its position. This dependence on position is parabolic and is characterized by the constant in the equation for the parabola. A third parameter is the prompt-neutron decay constant. These two parameters determine the neutron pulse shape and, in particular, the half-width of the pulse. The pulse width of SORA has been estimated by use of an expression obtained

from one-group, space-independent kinetics with the approximation used by Bondarenko and Stavisskii:⁽⁴⁾

$$\theta = \frac{1.67}{\sqrt[4]{\alpha_x (\alpha_1 v)^2 \epsilon}},$$

where

- α_x = constant of the parabola describing the reactivity of the movable reflector, dollars/cm²;
- α_1 = prompt-neutron decay constant, sec⁻¹;
- v = linear speed of the movable reflector, cm/sec;
- ϵ = superprompt critical reactivity, dollars

The value of ϵ is obtained as a function of ρ , α_x , α_1 , v , and f from a condition⁽⁴⁾ which states that the mean power during repetitively pulsed operation remains constant in time:

$$\frac{1.57}{v\sqrt{\alpha_x\epsilon}} f \exp\left(1.33 \frac{\alpha_1}{v} \sqrt{\frac{\epsilon^3}{\alpha_x}}\right) = 1 - \frac{1}{|\rho|}.$$

In order to obtain α_x the reactivity of the movable reflector has been measured as a function of the linear displacement (arc length) from the position of maximum reactivity. The data were fitted to a parabola in the range important to pulsed operation. If $\rho(0)$ is the maximum reactivity, the reactivity at position x is

$$\rho(x) = \rho(0) - \alpha_x x^2.$$

The measured values of α_x were between 8 and 10 cents/cm² for beryllium reflectors, and between 3 and 6 cents/cm² for iron reflectors.

The prompt-neutron decay constants have been obtained by the Rossi- α technique using instrumentation described previously.⁽⁵⁾ For assemblies with beryllium reflectors a decay constant of $0.3 \times 10^6 \text{ sec}^{-1}$, which corresponds to a lifetime of $2.2 \times 10^{-8} \text{ sec}$, was obtained. The decay constant of assemblies with iron reflectors was $0.5 \times 10^6 \text{ sec}^{-1}$, corresponding to a lifetime of $1.3 \times 10^{-8} \text{ sec}$.

The results, listed in Table 1, show the feasibility of the SORA design and indicate that pulse widths will be between 40 and 60 μsec . They also show that the reactivity values of movable reflectors will be larger than \$5 in a variety of configurations.

In addition to the measurements related to the kinetics, the reactivity of various parts, the reactivity coefficients of several materials in the core, and the reactivity temperature coefficient of the core have been measured and will be reported elsewhere.

Acknowledgment

The authors wish to acknowledge the assistance of the staff of the ORNL Critical Experiments, in par-

TABLE 1. PARAMETERS DETERMINING THE PULSE CHARACTERISTICS OF SORA

Material	Movable Reflector		α_x (cents/ cm ²)	Prompt Fast Neutron Decay Constant α_1 (μsec^{-1})	Predicted Neutron Pulse Half- width ^(b) θ ($\mu\text{sec.}$)	Mass of Uranium in Core (kg)
	Width ^(a) (cm)	Reac- tivity (\$)				
Be	11	5.9	8.6	0.32	54	57.2
Be	16	8.8	9.5	0.34	52	54.0
Be	21	11.2	9.7	0.30	50	51.2
Fe ^(c)	11	3.7	5.3	0.55	52	57.7
Fe ^(c)	16	5.2	4.2	0.48	60	56.4
Fe ^(c)	21	6.5	4.0	0.49	60	55.2

^(a) The height and thickness of the movable reflector were fixed at 24 and 7 cm, respectively.

^(b) These values assume a pulse repetition rate of 100 sec^{-1} and a reflector speed of $2.83 \times 10^4 \text{ cm/sec}$.

^(c) For these experiments the polyethylene scatterers and the beryllium control rods in the reflector were replaced by iron.

ticular the work of J. J. Lynn and J. R. Taylor in the performance of this experiment.

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Discussion

Mr. Stevens (GA): Do you have some way of correlating the worth of the wheel obtained during the static measurement with the dynamic worth in the same position? There are transients in the spectrum that you don't observe in the static measurements.

Mr. Kistner: This may be right. We are planning a kinetic experiment to determine the answer to this question.

Presented by J. C. Smith

Theoretical and Experimental Analysis of Reaction Rate Distribution in the Dounreay Fast Reactor

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ABSTRACT

During initial commissioning of the Dounreay Fast Reactor (DFR) an extensive programme of physics measurements was carried out at low power, substantially confirming the design predictions. Since that time the reactor has been modified to improve its performance as an irradiation facility, principally by increasing the enrichment of the fuel and by substituting nickel for natural uranium in the blanket adjacent to the core. A short programme of flux and reaction-rate measurements early in 1965 showed serious discrepancies between theoretical predictions and experimental observation.

The success of earlier theoretical treatment suggested that the cause of the failure to deal satisfactorily with the modified reactor must lie in incorrect data for nickel. An extensive programme of calculations was undertaken using more up-to-date cross-section sets, notably the FD2, a thirty-three-group set based on the Aldermaston/Winfrith data file and the Russian 26-group set of Abagyan *et al.* These showed considerably better agreement than had been obtained with the YOM 16-group set, but discrepancies between theory and experiment still remained.

At this stage it could not be clearly seen to what extent the discrepancies between theory and experiment were due to inadequacies of nuclear data and the use of a diffusion theory code or to the necessary geometrical simplifications made in adapting a two-dimensional code to the problem. It was therefore decided to construct on the VERA assembly machine at A.W.R.E. Aldermaston an assembly which would approximate to DFR in size, critical mass, and spectrum, but would be free of the detailed geometrical complexities of DFR and would at the same time allow

access for experimental measurement to regions where the corresponding positions in DFR were only accessible for relatively inaccurate activation techniques.

An account of the measurements made in DFR and VERA is presented and the theoretical interpretation is discussed.

1. Introduction

The Dounreay Fast Reactor (DFR) began as a demonstration of the feasibility of the concept of liquid-metal-cooled fast reactors. By 1963 it had reached its design power of 60 MWt and had shown that its uranium-molybdenum alloy fuel was capable of useful sustained operation. Its primary purpose having been satisfactorily demonstrated, the emphasis on its use changed to that of a materials testing reactor and by 1966 it contained about 50 irradiation rigs in the core and a similar number in the innermost rows of the blanket.

The irradiations cover a range of fields of interest, including the development of fast reactor ceramic fuels in single-pin specimens and multiple-pin subassemblies, the irradiation effects on fast reactor fuel cladding, irradiation damage in thermal-reactor graphite and zirconium where accelerated life testing is possible, and others in support of various British and European projects. The extent of the experimental loading requires knowledge of reactor parameters at all points of the reactor core and in the inner rows of the radial blanket, the requirement varying from that of fissile reaction rates for fuel experiments to spectrally defined fluxes for damage studies on clad or graphite.

Unlike the majority of materials testing thermal reactors which were expressly designed for this function, DFR has little provision for instrumentation of

experiments; to date not more than four instrumented experiments have been loaded in any one irradiation period. Consequently it is necessary to predict irradiation conditions with some confidence before loading the experiment. This is particularly the case for fuel irradiations for which a considerable emphasis is placed on determining the operating limits of the fuel either in rating or burnup. Sponsors of experiments would like data on reaction rates to an accuracy better than 5%, and it is against this background that the success or failure of the predictive techniques must be judged.

The attack on the problem has two aspects, experimental and theoretical. Experimentally, limitations are encountered in two ways. Firstly, the difficulties which prevent instrumentation of irradiation rigs also present problems in acquiring experimental data and, secondly, good techniques of spectral investigation are scarce and difficult to apply in a power reactor. The range of data and the wide distribution of points in the reactor at which data are required make it necessary to rely primarily on calculation; experiment is used mainly as a check on calculation rather than as a prime source of data.

2. Description of DFR

DFR (see Figs. 1 and 2) is a uranium-metal-fuelled fast reactor cooled with liquid sodium-potassium alloy. Only those features directly involved in the data-acquisition problem, either in modelling for calculation or in determining experimental technique, are described here. A fuller description may be obtained in Ref. 1. The reactor core takes the form of a right hexagonal prism, 21 in. across flats and 21 in. high. A central hexagonal region, 6 in. across flats, can accommodate special irradiation rigs such as multipin fuel sub-assemblies. The remainder of the core is loaded with fuel elements (or experimental rigs identical externally to fuel elements) in 324 locations of which twelve groups of ten are contained in vertically movable carriers at the core edge for reactor control. The fuel of the standard elements is 75% enriched uranium-molybdenum alloy containing 7 w/o molybdenum. A two-zone core loading is employed with elements containing more fissile material in the outer zone to flatten power and temperature distribution, and provide maximum reactivity for irradiation-rig loading. Variation of mass of inner and outer zone elements is achieved by variation of the annular fuel-slug ID and inner-can diameter. The OD of the outer can is the same for all elements (or element-type rigs) in order to mate with a nesting set of tubes at the vertical mid-plane of the core, which constrains elements from bowing in thermal gradients. Stainless steel (SS) end-

pieces on the elements locate in holes in stainless steel core top and base plates.

Radially outside the core there is a blanket, 86 in. high and 26 in. thick. Originally this blanket consisted of SS-clad natural uranium rods forming a breeder. When the emphasis of DFR usage changed from that of a prototype power plant to a materials testing reactor, the three rows of the blanket nearest the core were loaded with unclad nickel rods to raise flux levels near the core edge and provide additional reactivity. Several hundred natural uranium elements were removed from the middle region of the blanket after inadequate cooling was detected, this space now being occupied by coolant.

Above the reactor, access to the core and blanket is provided by eccentric rotating shields. Two pluggable charge holes in the inner shield allow elements to be loaded or removed from the reactor, and in the parked position of the shields give possible access to the central axis of the reactor and to a position near the blanket boundary. There are a further eighteen penetrations through the shield. Twelve of these are vertically above control rods and are provided to allow control-rod followers to penetrate to sensors above the reactor. The remaining six are above the innermost row of the blanket to accommodate, in the original design, four safety shutoff rods, a neutron-source machine, and a reactivity oscillator. The twelve control-rod-follower penetrations are above nominally identical lattice positions, as are the six blanket penetrations. Figure 2 shows a simplified plan cross section through the reactor, illustrating the position of these access holes. At low power (<1 MW) it is possible to insert stainless steel thimbles into any of these penetrations to obtain experimental access to the reactor for fission-chamber or activation measurements. At any other position or at high power when a higher standard of containment is required (with the argon gas blanket above the coolant pressurised to permit full flow on the EM pumps), no direct access to the reactor is obtainable and data can only be obtained by activation techniques. The penetrations may also be used to instrument irradiation rigs in the core or blanket locations beneath them. Naturally, not all can be made available simultaneously, since reactor operational requirements necessitate that some fulfil their original design function.

3. Early Operation

During initial commissioning of the reactor, the accent of the programme of physics measurements was to demonstrate the adequacy of the theoretical predictions and measurements on the zero-energy reactor

FIG. 1 Simplified Cutaway View of DFR.

ZEUS for the safe operation of DFR as a power reactor. Of greater importance was the determination of core fuel ratings, and for this agreement between experiment and prediction was obtained to within a few per cent.⁽²⁻⁵⁾ Some disagreement was found, particularly outside the core, for low-energy flux-sensitive detectors, but these discrepancies had little significance

for power operation. When the reactor's function changed from that of a power reactor demonstration to a materials testing reactor, the requirements for accurate data became more stringent, since data were now required at positions in the reactor which were less important in its power function, and greater detail in spectral information was demanded for irradiations

FIG. 2. Cross Section of Reactor Core and Breeder Showing Positions of Experimental Thimbles.

material than natural uranium in the first few rows of the blanket was suggested as a way out of both these difficulties, and nickel was chosen. Unclad nickel rods were substituted for SS-clad uranium elements in the three rows of the blanket nearest the core, giving a nickel content of $\sim 65\%$ by volume, but as the irradiation programme developed the innermost-row nickel elements have largely been replaced by experimental irradiation rigs. The changeover from an all-uranium blanket to one containing nickel took place in stages as material and reactor time became available. During this period reliance was placed on calculation to predict reactor conditions, since adequate accuracy had been obtained for the uranium blanket.

By January 1965 the reactor had run for three

FIG. 3. (r,z) Model of Reactor.

months with the programme of nickel substitution completed, and time was allocated for a low-power physics programme to check the theoretical predictions. Within the core the calculations were substantially in agreement with the experimental results, justifying the decision to rely on calculation for irradiation data since, at that stage, the nickel blanket was not being fully utilised as an irradiation facility and core data were of primary importance. Near the edge of the core and in the reflector regions immediately surrounding it, the situation was rather different. Discrepancies of a factor of two between calculated and measured U-235 fission rates occurred, due presumably to flux softening in the nickel. Since the calculations using the 16-group YOM data were satisfactory for the uranium-reflected reactor, it was natural to suspect the relatively high lower-energy bound of the set and possibly nickel data in particular as reasons for the discrepancy. The remainder of the paper describes the results of our investigations of the predictions of reactor flux and reaction-rate distribution using other data sets which have since become available, and the comparison of these results with experimental data obtained from DFR and a zero-energy assembly on the VERA machine at A.W.R.E., Aldermaston.

4. Theoretical Derivation of Reactor Data

All theoretical calculations performed recently to investigate the statics of DFR have used the multi-

group diffusion theory code CRAM⁽⁶⁾ written for the IBM 7090 computer. This code is restricted to one- or two-dimensional problems in slab, cylindrical, or spherical geometry, and this imposes limitations on the accuracy to which the DFR hexagonal-prism geometry can be represented.

For derivation of reactor data for the early stages of the irradiation programme, the 16-group nuclear data set of Yiftah *et al.*⁽⁷⁾ was used. When it became clear that appreciable flux at energies lower than the 500-eV limit of this set was present near the core boundaries of DFR, sets extending to lower energies were employed. The first to become available was FD1,⁽⁸⁾ an extension of YOM to lower energies by the addition of further groups to form a 33-group set extending to 0.4 eV. Later, FD2,⁽⁹⁾ a set with similar group structure to FD1 and based on the Aldermaston Winfrith nuclear data file, was investigated, using both the original nickel data of the set derived from Yiftah and Sieger and as revised by Ravier.⁽¹⁰⁾ When using FD1 or FD2 sets the CRAM code computes zone cross sections allowing for resonance shielding. Finally, the Russian 26-group set of Abayan *et al.*⁽¹¹⁾ has also been used. The cross-section shielding data of this set are not compatible with the CRAM cross-section processing routine and the data have only been used unshielded. Limitations of computer storage and the running time of a problem have generally limited the actual number of energy groups employed. The 33-group sets FD1 and FD2 have been condensed to 18 groups or less in most cases. The Russian set has not been used extensively and only in the full 26 groups.

5. Calculations in r,z Geometry

The greatest effort of calculation has been devoted to (r,z) models of the reactor in which the geometrical properties have been simplified to represent all hexagonal boundaries as circles of equal area and to average out circumferential variation of structures such as control rods and their guides into annular zones of uniform composition. Even with this degree of simplification an adequate representation involves considerable geometrical detail to take account of radial and vertical variation in composition as can be seen in Fig. 3, where a typical (r,z) model is shown. The considerable differences in support structure above and below the core, and the loading of the central hexagon have been shown experimentally to produce an asymmetrical flux pattern, and the model must be full length and cannot assume the simplification of midplane symmetry in the z -dimension. The principal deficiency of this model is the failure to represent correctly the control-rod region of the core and the boundary of the core and blanket, a region

where there are notable spectrum changes affecting experimental irradiations. The innermost row of the blanket, in particular, is an attractive site for irradiations, since it combines useful fissile reaction rates with a low reactivity penalty for substitution of a rig in place of the normal nickel blanket element, and there is a corresponding demand for accurate data in this region, which cannot be guaranteed from (r,z) calculations.

6. Calculations in r,θ Geometry

An (r,θ) representation, as in Fig. 4, can overcome this problem to a considerable extent if leakage in the vertical direction is simulated by fictitious absorption cross sections derived from bucklings given by the (r,z) calculation. Near the core-blanket boundary the spectral changes entail considerable variation in buckling with energy group and radius. It is therefore necessary to input buckling corrections for each zone and energy group of the model. The version of CRAM available imposes restrictions on the amount of such additional absorption data which can be included in the input to a calculation, and the running time involved in handling the energy and geometry detail further limits the accuracy to which simulation can be extended. The results of the one calculation run to date indicate up to 20% variation in reaction rate at a constant radius near the core-blanket boundary and give a measure of the errors consequent on the simplifications adopted in the (r,z) calculations. The general character of the differences between the (r,θ) and (r,z) calculations is in reasonable agreement with physical intuition, e. g., the rise in U-235 (n,f) reaction rate in the nickel radial blanket tends to follow the core-blanket boundary, and equal reaction-rate contours in this region are more nearly hexagonal than circular. When core fuel is present in the (r,θ) model at radii greater than in the (r,z) model, the (r,θ) model leads to a prediction of a lower reaction rate, since the presence of fuel prevents the build-up of low-energy flux occurring in the nickel which is at the corresponding point of the (r,z) model. A corresponding opposite effect occurs where the fuel is present in the (r,z) model at radii greater than in the (r,θ) model. Variations of high-energy flux between the models can be similarly explained. Since the buckling corrections differ significantly near the core-blanket boundary for the different nuclear-data sets used and the (r,θ) calculation was expensive in computer time, full application of the (r,θ) model has been delayed pending a decision on the set of nuclear data which, on the basis of the VERA experiments, gives best overall agreement with experimental evidence on reaction rates near the core-blanket interface.

FIG. 4. (r,θ) Model of Reactor.

A not inconsiderable problem is how to present the combined results of (r,z) and (r,θ) calculations in a manner which can be readily interpreted by users of the reactor for irradiation purposes who cannot be expected to have full understanding of the approximations involved. Ideally, the requirement is to provide an (r,θ,z) documentation of the reactor for the important reaction rates.

7. Experimental Investigation of Reaction Rates

Experimental data on reaction rates have been obtained in two ways. Firstly, with the reactor at low power and the gas blanket unpressurised, thimbles have been inserted into the reactor through penetrations in the top shields along the central axis, through control rods and in the innermost row of the radial blanket, allowing vertical scans to be performed with fission chambers. Latterly, measurements have concentrated on the use of Pu²³⁹, U²³⁵, and U²³⁸ chambers.⁽¹²⁾ Attempts to assess the lower-energy flux with BF₃ proportional chambers have been unsuccessful owing to the high gamma field present even at low power, due to activation of the core structure by prolonged high-power operation.

Secondly, at the normal operating power of 60 MW, activation measurements using fissile samples and threshold detectors loaded in experimental rigs have been carried out. The choice of materials for activation has been limited to those with reasonably long half-lives for their activity, as recovery of the specimens requires breakdown of the carrier in the high-active cave facilities after reactor shutdown and a fuel-element discharge operation. The delay involved after shutdown is not less than several days and may be determined by other demands on these services.

The accuracy obtainable from the high-power techniques is generally poor. Examination of fissile samples by fission product analysis is limited in absolute accuracy by the knowledge of fission product yields in the DFR spectrum, and even relative reaction rates of one sample to another do not consistently give re-

sults to better than $\pm 10\%$. Threshold monitors, of which the most satisfactory that have been used are based on the $\text{Fe}^{54}(n,p)\text{Mn}^{54}$ reaction, produce similar accuracies. A main problem is not that of shortcomings in the basic counting techniques, but the practical difficulties of applying them. After long-term irradiation at fluxes $\sim 10^{15}$ n/cm²-sec, large samples must be counted in shielded facilities not specifically designed for accurate activation analysis. If samples small

enough to have activities suitable for counting outside thick shielding are used, it is difficult to ensure the absence of contamination picked up during recovery in the highly active environment of the caves.

Information on reaction-rate distributions can also be obtained from post-irradiation examination of driver charge elements and experimental oxide pins. Gamma scans along these should give a record of the integrated reaction rate provided the fission product

FIG. 5. Comparison of Calculated and Measured Reaction-rate Scan.

FIG. 6. Comparison of Experimental and Calculated Radial U^{235} Fission-rate Distribution.

whose activity is being measured does not migrate in the fuel and the fuel remains stable. Zr^{95}/Nb^{95} activity in driver charge fuel subject to less than 2% burnup has given good agreement with other methods. Less satisfactory results have been obtained from oxide fuels for which irradiation effects on the fuel were greater.

For accurate data to compare with the results of calculation, reliance has been placed mainly on the fission-chamber investigations. It is relatively simple to obtain scans with individual points for a single isotope accurate with respect to one another to 1%. Comparison of reaction rates between isotopes depends on the absolute calibration of the chambers. Such absolute calibration has been carried out in the case of U^{235} and Pu^{239} by α -counting the chamber foil before assembly, by intercalibration of the foil with a standard in a demountable chamber in the NESTOR reactor, and final intercalibration after assembly. Agreement between the methods was better than 2%. With allowance made for setting-up errors, the experimental inaccuracy in $\sigma_f^{235}/\sigma_f^{239}$ should not exceed $\sim 3\%$. U^{238} chambers available at present have been less satisfactorily calibrated on the basis of their U^{235} content

in a thermal flux, but it is intended that they should be calibrated in a 14-MeV beam to give better results.

In addition to the basic experimental programme to check the accuracy of calculation, fission-chamber scans have been used to assess the distortions in flux pattern near partially inserted control rods. This basically three-dimensional problem is not amenable to calculation with the present computer codes nor can it be tackled by activation measurements during power operation during which the configuration necessarily alters. The results of these experiments are of direct interest to sponsors of experiments located near moving control rods.

8. Discussion of Experiment and Calculation

Figures 5 through 9 show some comparisons of the results of experiment and calculation. The most obvious feature of the comparison is the inadequacy of the YOM and FD1 sets to treat conditions just outside the core boundary for U^{235} and Pu^{239} fission rates. Neither set correctly predicts the rise in fission rate in the nickel region of the radial blanket or above and below the core in the control-rod region. Also to be noted are the variations between data obtained experi-

FIG. 7. Comparison of Experimental $\text{Fe}^{54} (n,p) \text{Mn}^{54}$ Reaction Rate and Theoretical High-energy (>3.68 MeV) Radial Flux Distribution.

FIG. 8. Comparison of Calculated and Experimental Vertical Scan of $\text{U}^{238} (n,f)$ Reaction Rate through Control-rod Region.

mentally in 1965 and 1966, caused partly by alterations in the experimental rig loading in the innermost blanket row and differences in the structure of the control rods through which the scans were made.

The model for (r,z) calculations assumed an average composition of the reactor for each annular zone and did not take account of such local variations from one

experimental period to another. Qualitatively the results obtained using FD2, FD2 with Ravier nickel data, and the Russian set represent an improvement in that the secondary peaking observed experimentally is reproduced; in no case, however, is the agreement between theory and experiment sufficiently close to identify one of the sets as definitely superior. The

FIG. 9. Comparison of Fission-rate Distributions in DFR Fuel Elements Derived Theoretically and from Fission Product Gamma Activity.

Russian set can probably be excluded as it consistently overestimates the peaking on vertical scans, and fair agreement on the radial scan secondary peak is at the expense of an underestimate in the outer regions of the core. The choice between FD2 and FD2 with Ravier nickel data is less clear. For vertical scans of $U^{235}(n,f)$, FD2 is in better agreement; for the radial scan FD2/Ravier seems superior, although in this latter case the experimental data are less reliable and the theoretical model which ignores hexagonality introduces further difficulties in interpretation.

By comparison threshold reaction rates, such as those of $Fe^{54}(n,p)Mn^{54}$ and $U^{238}(n,f)$, are better predicted. Figure 7 shows the experimental radial variation for $Fe^{54}(n,p)Mn^{54}$ compared to the FD2 calculation. Agreement is as good as can be expected

from the accuracy of the experimental data. Similarly, for a vertical $U^{238}(n,f)$ scan through a control rod, calculated and experimental data are in fair agreement, as may be seen in Fig. 8. Discrepancies near the ends of the scan may be due in part to errors in the corrections applied to the experimental data for contributions from $U^{235}(n,f)$, which account for $\sim 50\%$ of fission-chamber counts when the $U^{238}(n,f)$ reaction rate is small. No significant differences from these results occur in the predictions by FD2/Ravier or the Russian set.

9. VERA Assembly

The variations amongst predictions of fissile and low-energy-sensitive reaction rates in the radial nickel blanket using the various data sets presents a problem

which is difficult to resolve in DFR. This region is inaccessible to other than activation detectors whose accuracy is barely adequate to decide which theoretical estimate is the most accurate. At the same time the reaction rates obtainable in the nickel are clearly at least as high as in much of the core, and the lower reactivity penalty of blanket sites make them attractive for irradiation purposes. If the nickel zone were thickened, even more attractive conditions might be realised, but greater confidence than has been obtained from comparison of calculation and experiment in DFR would be necessary before reliance could be placed on theoretical prediction of the potential gains.

The possibility of a significant improvement in experimental reactor data in this inaccessible region of DFR being slight, it was decided to make measurements with a zero-energy assembly. Better access could lead to superior experimental information which might be of assistance in deciding the relative merits of the data sets. At the same time a core-reflector boundary region of simpler geometry might reduce the approximations required in modelling the reactor for calculation.

In the Spring of 1966 a short period was available

with the VERA assembly machine⁽¹³⁾ at Aldermaston, and it was decided to construct a mockup which would, ideally, reproduce the essential features of DFR, i.e., the assembly should have core and blanket compositions similar to DFR so that dimensions, critical mass, and spectrum should be comparable to ensure no major problems in interpreting the DFR situation in the light of results obtained on VERA.

These requirements could be met only approximately. The U^{235} inventory of VERA was insufficient to construct a core of DFR composition and had to be supplemented with VERA plutonium and U^{235} borrowed from the ZEBRA project. Sodium-potassium had to be simulated with reduced-density aluminium. The nickel-rich blanket region comprised DFR nickel rods and aluminium to simulate NaK. The VERA lattice pitch prevented the achievement of the full nickel density of DFR. As the overall height of the assembly could not exceed 43 in., neither the full height of the blanket nor the NaK pools above and below the core could be included. The assembly constructed, VERA 18A, had one quadrant in which an approximation to the ideal requirement was achieved, as shown in plan in Fig. 10. A variant of this assembly,

FIG. 10. Horizontal Section through Core of VERA Assembly 18A.

FIG. 11. Comparison of Experimental and Calculated Variation of U^{238} Fission Rate in VERA Assembly 18A.

VERA 18B, was constructed with a thicker nickel region in its blanket to accentuate the reaction-rate peaking and help to decide between the predictions of the rival data sets, as well as supplying information of direct interest to DFR since a five-row loading of nickel was under consideration.

The experimental data which could be obtained from these assemblies was limited by the time available, and comprised radial and axial scans with U^{235} and U^{238} fission chambers and a BF_3 proportional counter supplemented with some $U^{238}(n,\gamma)$ and $Mn(n,\gamma)$ radial scans in the quadrant of near-correct composition.

10. Discussion of Results

Theoretical analysis of these assemblies is not complete, and only (r,z) finite cylindrical calculations are so far available.

The composition of the models chosen for analysis followed the composition of the quadrant which approximated to the ideal assembly. They consisted of a core of U^{235} , stainless steel, and aluminium, surrounded by an inner blanket of nickel, stainless steel, and aluminium, either 3 or 5 VERA elements thick, and an outer blanket of natural uranium and steel. The analyses were carried out using FD2, FD2 with Ravier nickel, and the Russian data sets. Although the core cross-section area was made equal to that of the experimental assembly, the geometrical differences between the circular model and the near-rectangular

experimental assembly prevent exact comparison between experiment and calculation of radial variation of reaction rates. A more valid comparison will be possible when the results of (x,y) calculations corrected for axial leakage and using the correct variable core and blanket composition are completed.

At this stage some general conclusions can be drawn from the comparison of calculation and experiment, which is presented in Figures 11 through 15. In these figures experimental and calculated data have been drawn so as to align the core-blanket boundary of the experimental assembly and the calculated model.

- (1) All data sets produced good agreement with experimental data for the threshold $U^{238}(n,f)$ reaction (see Fig. 11).
- (2) For the low-energy-sensitive $B^{10}(n,\alpha)$ reaction, FD2 and the Russian set overestimate the rise in reaction rate in the nickel region of assembly 18A to a considerably greater degree than does FD2 with Ravier nickel (see Fig. 12), although in the case of the thicker nickel zone this latter set is also considerably in error (see Fig. 13), but to a much lesser extent than FD2 or the Russian data.
- (3) For U^{235} fission, FD2/Ravier nickel gives the best results (see Figures 14 and 15). For the thinner nickel blanket assembly, the calculation lies entirely below the experimental scan, both in the core and blanket, and this may in part be due to deficiencies in the model, but it is the

FIG. 12. Comparison of Experimental and Calculated Radial Variation of $B^{10}(n,\alpha)$ Reaction Rate in VERA Assembly 18A.

FIG. 13. Comparison of Experimental and Calculated Radial Variation of $B^{10}(n,\alpha)$ Reaction Rate in VERA Assembly 18B.

FIG. 14. Comparison of Experimental and Calculated Radial Variation of U^{235} Fission Rate in VERA Assembly 18A.

FIG. 15. Comparison of Experimental and Calculated Radial Variation of U^{235} Fission Rate in VERA Assembly 18B.

only one of the three data sets to predict correctly the absence of a noticeable peak in the nickel region. For the thicker nickel blanket the peak has developed both experimentally and by calculation. FD2/Ravier nickel overestimates it by 10%, but is considerably better than FD2 or the Russian data, which are in error by 50% and 30%, respectively.

The progressive discrepancy from threshold reactions, through $U^{235}(n,f)$ to $B^{10}(n,\alpha)$, indicates that these three data sets predict excessive low-energy flux in the nickel region. FD2 and FD2 with Ravier nickel differ, of course, only in their nickel data, and examination shows that Ravier nickel absorption cross sections are of the order of five times as large as the Yiftah-Sieger cross sections of the FD2 set below 1 keV, corresponding to the greater buildup of low-energy flux using FD2. No such simple explanation can be offered for the differences between FD2/Ravier nickel and the Russian set, as their nickel absorption data are in good agreement at low energies, as are absorption cross sections for other constituents of the nickel zone; but the absence of shielding corrections to the cross-section data probably contributes. Further work is required to suggest which parts of the data

sets require modification to reconcile calculation with experiment. At present the best results are achieved with the FD2 library combined with Ravier's revision of the nickel data.

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Static Physics Measurements in the Fermi Reactor

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ABSTRACT

This paper reviews static physics measurements which have been made to date in the Fermi reactor. Both low-power (<1 MWt) and high-power (1 MWt–200 MWt) measurements are discussed. In the case of the low-power measurements, emphasis is placed primarily on those experiments which can be directly compared with ZPR-3 critical experiment data. In particular, results of measurements made to determine the reactor critical mass, reactivity worths of fuel and blanket materials, sodium worth, fission distributions, and control and safety rod worths are discussed. Although the ZPR-3 mockup was not an exact representation of the actual reactor, thus necessitating in some cases considerable extrapolation of the critical experiment data, it has been found that in general the measured data obtained agree very well with extrapolations from critical experiment results. However, in the cases of the measured sodium worth and ganged safety-rod worth, agreement was poor. This led to a modification in the original interpretation of the critical experiment data. An anomalous reactivity increase with time which was observed at low power is also discussed. In the absence of burnup this has been related to temperature effects in the reactor.

The static physics measurements which have been made at successively increasing power levels in the high-power tests (up to 100 MWt to date) cannot be directly related to critical experiment data. These consist of power- and flow-coefficient measurements, absolute power-calibration data, and an observed reactivity decrease during power operation which cannot be accounted for by burnup. The measured power-coefficient value is about 30% smaller than had been predicted for the reactor. This has been attributed primarily to errors in the assumed worth distributions used in the predictions. The preliminary flow-coefficient data seems to indicate a smaller proportion of the total feedback than expected and is due to the prompt, flow-independent, component. However, it is believed that

this difference is due to limitations in the experimental data. The absolute power-calibration data which have been obtained from heat-balance measurements and from nuclear calibration are in good agreement. The anomalous reactivity loss during power operation has not yet been explained. However, since it is of opposite sign than the reactivity gain observed during low-power operation, it quite possibly could be due to a reversal of the earlier effect.

1. Introduction

As one of the few fast power reactors operating today, and the largest built to date, the Enrico Fermi Fast Breeder Reactor (EFFBR) located at Monroe, Michigan, provides a unique opportunity to compare directly critical experiment results with measurements in an operating reactor. This paper describes some of the static physics measurements which have been made to date for the Fermi Reactor with emphasis placed primarily on those measurements which can be compared with critical experiment results. Both low-power (<1 MWt) and high-power (1 MWt–200 MWt) test results are discussed.

The Fermi reactor is a sodium-cooled fast breeder reactor which utilizes U-235 as the fissionable isotope in the form of metallic fuel elements in its first core loading. The Fermi plant has a maximum capability of 430 MWt, although the initial core (Core A) is designed for maximum 200-MWt operation. The critical loading of the Fermi reactor began in late July 1963, and criticality was achieved in August 1963. The low-power nuclear testing at powers below 1 MWt began immediately and continued actively through December 1963. Further low-power testing was done on an intermittent basis because of other plant modifications and tests but was completed in May 1965. Modification and testing of steam generators, and mechanical difficulties with the fuel-transfer cask car, were the primary reasons for not completing this program in early 1964. An AEC license for high-power

operation was received in December 1965. The first step in the approach to the 200-MWt power level was made in late 1965. The program has continued actively through September 1966, proceeding satisfactorily through the first six steps of the total eight-step power-escalation program. This involves tests conducted up to 100 MWt. Testing still remains to be done at the 133-MWt and 200-MWt power levels. The present schedule calls for completion of the 200-MWt nuclear testing in November 1966. Following completion of this program, the Fermi reactor will be modified for extended operation for use in fast fuel-element irradiation. This operation will be at 110 MWt and reduced temperatures for the purpose of extending the operating life of the 200 Core-A-type subassemblies (two core loadings) now on hand.

The construction of the Fermi reactor was preceded by detailed critical assembly mockup experiments in ZPR-3. The critical assembly experiments were of great use in establishing the fundamental physics parameters of the reactor. However, the critical experiment data required interpretation in order to correct for composition defects which existed between the mockup and the reactor, subsequent changes of reactor design, and to establish characteristics of physics design, such as fuel enrichment, control-rod composition, power distribution, breeding ratios, and temperature and power coefficients of reactivity. In view of the data interpretation required, the experimental data obtained in the reactor measurements agrees well with extrapolated results of critical experiments. However, in some cases, such as the data for sodium worth and ganged-safety-rod worths, a modification in the original interpretation of the critical data was required in view of the results of the reactor measurements. This paper outlines the extrapolations and interpretations which were required of the critical experiment data.

A large portion of the nuclear testing on Fermi has been concerned with the verification of engineering-design parameters by means of nuclear-reactivity measurements. Included in these investigations were items such as gas entrainment, pressure drop across the control rods, core reproducibility, and calibration measurements of rod drop times and source reactivity. The results of these tests have been reported previously,^(1, 2) and since they do not involve basic physics parameters, they are not discussed here. Instead, this paper will be primarily concerned with those measurements which can be directly or indirectly compared to critical experiment data. In particular, in the case of the low-power test program, the results of measurements made of the reactor critical mass, reactivity worths of fuel and blanket materials, sodium worth,

isothermal temperature coefficient, fission distributions, and control- and safety-rod worths are discussed. An observed anomalous reactivity increase with temperature at low power is also discussed.

The static physics measurements under high-power test can be compared only indirectly with critical experiment results. These consist of power-coefficient and flow-coefficient measurements, absolute power-calibration data, and an observed reactivity loss during power operation which cannot be attributed to burnup effects. These data can be compared with calculations which in part are based on critical experiment measurements of material reactivity worths, etc.

2. Brief Description of the Fermi Reactor

The Enrico Fermi reactor and its associated structures are shown in perspective in Figure 1. The reactor is contained in a stainless steel reactor vessel sealed at the top by a rotating shield plug which supports the control mechanisms, the hold-down mechanism, and the fuel-handling mechanism. The reactor vessel is surrounded by borated and plain graphite neutron shielding material which is contained inside the primary shield tank. The reactor is of the fast-breeder type, cooled by sodium, and operated at essentially atmospheric pressure. The maximum reactor power with the first core loading (Core A) is 200 MWt.

The core and blanket, located in the lower reactor vessel, consist of square subassemblies containing the fuel pins and blanket rods arranged to approximate a cylinder about 80 in. in diameter and 70 in. high. The core, contained in the central portion of the core subassemblies, approximates a right cylinder, 31 in. in diameter and 31 in. high; it is axially and radially surrounded by breeder blankets. The fuel in Core A consists of zirconium-clad pins containing U-10 w/o Mo alloy with the uranium enriched to 25.6 w/o U-235. Each fuel subassembly in the core contains 140 fuel pins for a total mass of approximately 4.75 kg U-235 per subassembly. The blanket is depleted U-3 w/o Mo alloy. The core and blanket compositions and dimensions are shown in Table 1.

The reactor cross section, shown in Fig. 2, indicates the placement of individual components within the lower reactor vessel. A total of 149 central lattice positions are occupied by core and inner radial blanket subassemblies, the neutron source, and the 10 operating control- and safety-rod channels. All these positions are supplied with sodium coolant flowing upward from a high-pressure plenum which is connected to the discharge lines of the three primary sodium pumps. The coolant flows upward through the individual core and inner radial blanket subassemblies into a large upper plenum, from there by gravity to

FIG. 1. Perspective View of Fermi Reactor.

the three intermediate heat exchangers, and then to the suction side of the primary pumps. Sodium also is used in the secondary cooling system.

The lattice positions surrounding the inner radial blanket comprise the outer blanket area, and when filled with outer radial blanket subassemblies they form an annular region whose top and bottom are at the same elevation as the top and bottom of the inner radial blanket. Surrounding the outer radial blanket are lattice positions used for stainless steel subassemblies to provide thermal and neutron shielding. The outer radial blanket and shielding lattice positions are supplied with sodium coolant from the low-pressure plenum. The sodium from the low-pressure plenum, after cooling the blanket and thermal shield subassemblies, is mixed in the upper plenum with the sodium

from the core before flowing to the intermediate heat exchangers.

The reactor is controlled by two operating control rods and seven safety rods. Provisions have been included in the design for an eighth safety rod. However, during the nuclear test program the eighth safety rod position (PO3-POO, see Fig. 2) has been filled with either an oscillator rod, temporary instrument thimble, or a temporary retractable neutron source. The rods are of the poison-type, containing boron carbide (B_4C) in which the boron is enriched in boron-10. One operating control rod is for regulating purposes and the other for shimmiing; the average rates of reactivity insertion of these rods are approximately one cent per second and one cent per minute, respectively. Both rods have approximate reactivity worths of 46 cents.

The seven safety rods, which are spaced around the center of the core, provide shutdown reactivity. During operation of the reactor, the safety rods are held just

TABLE 1. ENRICO FERMI REACTOR COMPOSITION AND DIMENSIONS

Dimensions			
Core Height	30.5 in.		
Core Diameter	30.5 in.		
Core Volume	400 liters		
Radial Blanket Thickness	24 in.		
Axial Blanket Height	14 in.		

Composition (v/o)			
Material	Core	Axial Blanket	Radial Blanket
Fuel	33.8	27.0	42.3
Structure	21.0	16.7	18.6
Coolant	45.2	56.3	39.1

above the axial blanket section of the core so that they can be rapidly inserted into the core if a scram becomes necessary. Each of the safety rods has a reactivity worth in excess of \$1.00. The movements of the control and safety rods are actuated from the top. During fuel reloading, they are detached from their drive extensions to allow them to remain in the core.

The neutron detectors (fission chambers and ion chambers) for normal reactor operation at power are located in six neutron-counter tubes (NCT) embedded in the graphite neutron shield surrounding the reactor vessel. There are eleven channels of nuclear instrumentation distributed throughout the six neutron-counter tubes in a manner which will cover the full power range during reactor operation. These detectors located in these channels are designed to operate at temperatures up to 500°F.

An antimony-beryllium (Sb-Be) neutron source is located in the reactor at the core-blanket interface (see Fig. 2) to provide a neutron flux at the neutron detec-

FIG. 2. Cross Section of Fermi Reactor.

tors during reactor startup and to maintain a flux when the reactor is shut down. The radioactive antimony portion of the source is made as a separate piece for easy replacement and is in the form of a rod approximately 0.7 in. in diameter and 25 in. long. The radioantimony rod fits inside a beryllium assembly which is in the form of a hollow cylinder inside a stainless steel can having the external dimensions of a normal core subassembly fuel can. The handling head on the antimony section allows it to be inserted and withdrawn from the beryllium by means of the normal fuel-handling mechanism. The beryllium portion must be installed and removed from the reactor through a special access port in the reactor vessel shield plug.

Additional information concerning the reactor design may be found in the Enrico Fermi Hazards Summary Report.⁽³⁾

3. Description of the Fermi Reactor Test Program

The Enrico Fermi nuclear test program was based technically to a large degree upon the results of the ZPR-3 critical experiment work conducted for the reactor.⁽⁴⁾ The critical experiment work established the basic physics parameters of the reactor, and the primary purposes of the test program were to verify these data and to assure the operational and safety characteristics of the facility. Because the Fermi reactor was the first privately constructed and licensed fast reactor, the test program that evolved resulted in a more careful and extensive nuclear investigation than might have otherwise been undertaken. In particular, studies relating to reactor safety and stability represented a significant portion of the program. Furthermore, as a result of the fact that the critical experiment studies with ZPR-3 were limited to the use of a simulant material for a majority of the sodium coolant, it was evident that a detailed study in the reactor of reactivity worths of fuel, reflector, and coolant materials was desirable. Other detailed physics measurements, such as fission distributions and spectral measurements, were also included.

The low-power test designation refers to tests which were performed at reactor power levels less than 1 MWt. There were 26 low-power tests in all. The high-power test designation refers to tests which were conducted at reactor power levels above 1 MWt. There were 11 of these tests. The arbitrary separation of low- and high-power tests at 1 MWt was based primarily on the ability to obtain satisfactory neutron signals in conjunction with the lack of sensible heating up to 1 MWt. The basic parameter utilized throughout the low-power portion of the test program was therefore the measurement of the reactivity change associated

with perturbations made in the system. On the other hand, the high-power portion of the program utilized reactivity measurements as well as temperature and electronic system responses to transients. The majority of the nuclear tests were conducted at power levels below 1 MWt. This was done in order to minimize the activation of core components and experimental equipment, and to facilitate accurate reactivity measurements. The only tests planned for high-power operation were therefore those in which either high neutron fluxes or sensible heat were required to obtain the desired data.

The tests conducted in the low-power portion of the nuclear test program for Fermi Core A can be considered to fall into one of four general categories. The main category was the *Verification of Basic Nuclear Characteristics*. This involved measurement of physics parameters such as critical mass, the reactivity worth of control and safety rods, worths of fuel and blanket material, sodium-coolant worth, and uranium fission and capture distributions. These characteristics were all measured in the critical experiment studies for the Fermi reactor conducted with ZPR-3, and they are the measurements primarily discussed in this paper. The other categories were entitled *Confirmation of Nuclear Related Mechanical and Hydraulic Phenomena*, *Calibration Measurements*, and *Special Tests*. The tests in these latter categories did not involve measurement of basic nuclear parameters as such and are not reported here. For information concerning these tests, the reader is referred to References 1 and 2.

The high-power nuclear test program consisted of a series of measurements at each step in an eight-step power-escalation program from 1 to 200 MWt. The power-escalation program consisted of alternate reactor operation using two and three-loop flow at power levels of 1.3, 2.0, 13.3, 20.0, 66.7, 100, 133, and 200 MWt. To date, the measurements through 100 MWt have been completed. The program consists of performing, at each step, transfer-function measurements with oscillator equipment, static power- and flow-coefficient measurements, power-level calibrations, and several tests to check control-system functions. In addition, pulse and noise experiments, safety-rod drop tests, and measurement of chronological reactivity changes were also made at various stages in the program. The static power- and flow-coefficient measurements, power-calibration data, and chronological reactivity data are discussed in this paper. None of the data obtained can be directly compared to critical experiment data although some indirect comparisons are possible. The high-power oscillator and noise measurements are discussed in the following paper by A. E. Klickman *et al.*

4. Summary of Results of Static Physics Measurements

In general, the program of static physics measurements for the Fermi reactor has been conducted successfully with results which are either in satisfactory or excellent agreement with predicted behavior. The good correlation has served to lend greater confidence in the understanding of fast reactor design through the use of critical experiments. As a consequence, in future fast reactor commissioning programs attention should be given to the possibility of abbreviating the physics measurements programs. Certainly, in the commissioning of reactors which closely parallel a design which has been thoroughly tested in critical experiments, and whose operational characteristics have been proven, there is little question that minimum programs will suffice. However, it is obvious this is true only for those characteristics relating to static physics parameters. It is not possible to fully verify either the dynamic or transient behaviour of a reactor in a zero-power critical experiment and these measurements cannot be eliminated from a fast reactor commissioning program. However, some abbreviations and time-saving steps could be taken. For example, a reduction in operating time requirements for reactor test could come from the development of techniques, alternate to the oscillator method, of determining reactor stability and dynamic characteristics, i.e., the use of noise, pulse, or rod-drop techniques. These methods are being investigated in the Fermi program and are briefly discussed in the following paper by A. E. Klickman *et al.*

The results of the low-power test series showed that the critical assembly data generally predicted the reactor statics physics parameters for Fermi satisfactorily. However, some of the low-power tests did yield differences which, even though fairly small, were significant. The best agreement by far was between the critical mass predicted from critical experiment data and that measured in the reactor. The agreement was within several kilograms ($1\frac{1}{2}\%$) in spite of the large extrapolations required in the critical experiment data to fit the as-built reactor design.

On the other hand, it was found that the unshadowed control- and safety-rod worths were smaller than had been estimated from critical assembly data by an average of 8% and 15%, respectively. In the case of the ganged safety-rod worths, the initial discrepancy found was 100% in the shadowing effect of one rod on another, based on the original interpretation of the critical experiment data. However, after making the measurements in the reactor it was discovered that the critical experiment data had been incorrectly interpreted. This correction resulted in good shadowing agreement.

The measured core-average reactivity worths of fuel and blanket material were in good agreement with the critical experiment data, but the distributed worths were found to be smaller than predicted near the center of the core, and larger than predicted at the core-blanket interface. The overall agreement was still quite good, however. The results of distributed sodium-worth measurements in the reactor were also quite different than those predicted from critical experiments, but this has been attributed to experimental deficiencies in the critical experiment data and our interpretation of it. As a consequence of the discrepancies in distributed fuel and sodium worths, some important components of the isothermal temperature coefficient have had to be re-estimated.

The measured temperature coefficient was found to be 7% less than originally calculated, based on the critical experiment distributed worths. The fission-distribution measurements were found to be in good agreement with critical experiment predictions in the core and, although the agreement is not quite so good at the core-blanket interface and in the blanket itself, the overall agreement is still very satisfactory. Lastly, the ability to measure reactivity accurately in the low-power tests has made it possible to detect and monitor a small anomalous reactivity increase with time that has been related to the reactor temperature.

The high-power static physics measurements of the power and flow coefficients in the reactor cannot be directly compared with critical experiment data. However, the measured power coefficients are approximately 30% smaller than had been predicted based on the original calculations. These used the distributed fuel and coolant reactivity worths determined in the critical experiment, calculated temperature distributions, and calculated temperature variations with power. By reviewing and re-estimating the important power-coefficient components with use of the reactivity worths measured in the reactor, and also using a better calculational model in some cases, the agreement between calculation and measurement is much improved. The absolute power calibration from heat-balance measurements made during the high-power portion of the test program resulted in a power calibration 12% smaller than that obtained by nuclear measurements using absolute fission counters in the low-power portion of the test program. This difference is well within the uncertainty limits of either type of calibration.

Lastly, a small reactivity decrease has been found during reactor operation at power that cannot be accounted for by normal burnup effects. At present no quantitative explanation of this has been made. However, since the reactivity variation observed is of op-

posite sign than the earlier anomalous increase measured during the low-power tests, it is quite possible that a reversal of the earlier effect is being seen.

5. Low-power Static Physics Measurements

5.1. Loading to Critical

The loading to criticality⁽⁵⁾ was carried out in a number of steps, starting with an initial loading of 50 fuel subassemblies located about the core axis, followed by a second loading of 16 fuel subassemblies and subsequent loadings progressing radially outward from the core center with the fuel replacing sodium-filled dummy subassemblies. The depleted uranium blanket subassemblies were also in place prior to the fuel loading, as were the two enriched boron carbide operating control rods, seven safety rods, temporary instrument thimble, and a 350-Ci Sb-Be neutron source which maintained a measurable neutron flux at the detectors throughout the loading. Count rates were obtained from both in-core and out-of-core neutron detectors prior to and following the insertion of each fuel subassembly. The actual fuel-loading increments, beyond the second loading of 16 subassemblies, were determined after considering the extrapolation of the

inverse-count-rate curves to obtain a predicted critical mass after each loading.

The loading of fuel into the reactor began on July 24, 1963; criticality was achieved on August 23, with 99 fuel subassemblies loaded into the reactor. Figure 3 illustrates the inverse-count-rate curves obtained during the loading both for the in-core fission-counter channel and a typical out-of-core BF₃ proportional-counter channel. The large difference in apparent neutron multiplication seen by the in-core detector compared to the out-of-core detector located in the graphite shield could not be anticipated from critical experiment results, since in the critical experiment detectors were located only in the core and blanket regions. Also, the loading sequences were performed in a different manner. However, the observed difference was predicted by calculation prior to the actual loading.⁽⁶⁾ The explanation for the difference can be related to differences in the neutron spectrum at each detector location as the loading progressed. In the case of the in-core detector, the relatively soft initial Sb-Be-source neutron spectrum in the core was hardened because of source multiplication as fuel was loaded radially outward from the core center. This tended to offset the increase in source multiplication resulting

FIG. 3. Inverse-count-rate Curves for Critical Approach.

from fuel loading seen by the detector and accounted for the relatively small increase in count rate observed on this channel during the loading period. However, at the location of the out-of-core detectors in the graphite shield the neutron flux was in spectral equilibrium essentially throughout the whole loading. Therefore, the detector efficiency remained approximately constant. The hardening of the core spectrum did cause an increase in the relative number of U-238 fast fissions in the blanket, however, and because of this tended to increase the out-of-core detector response. Consequently, whereas spectral hardening in the core tended to offset the apparent increase of source multiplication seen by the in-core channel, it had the opposite effect on the out-of-core channels.

Based upon criticality being attained with 99 core subassemblies loaded into the reactor and critical rod-withdrawal positions of 8.00 in. for the shim rod and 9.25 in. for the regulating rod, the estimated critical mass at 400°F and all rods withdrawn was 465.55 kg U-235. The predicted value was 467.88 kg U-235 based upon an extrapolation from the critical experiment results in ZPR-3. The excellent agreement obtained is considered in a large part to be fortuitous because, although the critical experiment was an engineered critical, many extrapolations were required to arrive at the actual reactor design.⁽⁴⁾ Material-worth corrections were required to account for differences in uranium enrichment between the mockup and actual reactor and also to account for differences in structural-material compositions in the actual reactor and the critical experiment. In some cases these were fairly large. Also fuel-pin-swaging corrections had to be made, as well as bunching corrections to account for the heterogeneity effect of the fuel plates used in the critical experiment, core-temperature corrections, and corrections to account for various geometrical differences between the actual reactor and the critical experiment. Similar corrections were required for the blankets. Finally, the agreement between the actual and predicted critical mass is considered to be especially close in view of the rather significant changes in the design of the core fuel element which were made a number of years subsequent to the Core-A critical experiment studies in ZPR-3. These resulted in almost an 8% increase in core size which had to be accounted for using fuel and material danger coefficient worths obtained in the critical experiment with small-size sample substitutions.

5.2. Reactivity Worths of Fuel and Blanket Materials

Measurements were made to determine the worths of fuel and blanket materials averaged over the core height as a function of radial core or blanket subassem-

bly position to permit accurate future core-loading adjustments and to check calculational and critical experiment worth data.⁽⁷⁾ Worth measurements of columnar fuel material were made to obtain data which would describe the reactivity coefficients of U-235 and U-238 as a function of radial position in the reactor core. Measurements of core-subassembly worth in the inner radial blanket (IRB) were made to determine the net reactivity effect of changing the fuel loading at the core edge. Reactivity worths of blanket subassembly were measured, relative to a sodium-filled dummy subassembly, to obtain the worths of the depleted uranium pins within the subassembly as a function of subassembly position in the inner radial blanket and in the first several rows of the outer radial blanket.

All reactivity determinations were made using both critical rod-position and positive-period measurements. Reactivity corrections were made for temperature deviations from 517°F, the nominal zero-power temperature.

Typical results are given in Figures 4 and 5, which summarize the U-235 versus U-238 worths in the core, and the core- and blanket-subassembly substitution worths in the radial blanket, respectively. Each figure includes a plot of the predicted worths for comparison purposes. Although the U-238 versus sodium worth in the core was also measured in the reactor experiments, the results are not reported here because similar measurements in the critical experiment for comparison were not made.

Two sets of values were predicted for the reactivity worths of U-235 versus U-238. The first was based upon the worth curves for U-235 versus U-238 obtained from the Core-A critical assembly mockup of ZPR-3.⁽⁴⁾ Since the Core-A critical mass and core size are greater than the critical assembly mockup, due to the subassembly design changes subsequent to the critical studies, these predictions were regarded more as useful experimental guides than as accurate predictions. The second set of predicted worths was based upon a two-dimensional, cylindrical, CRAM⁽⁸⁾ code, diffusion-theory calculation using a 24-group cross-section set collapsed to 8 energy groups flux-weighted by region-averaged spectra. Columnar worths were derived from the basic calculational parameters by use of first-order perturbation theory. These data appear in Fig. 4.

The results of the worth measurements of fuel versus IRB subassemblies were predicted only approximately, using radially extrapolated U-235 versus U-238 worth data from the ZPR-3 critical studies. Worths of minor structural materials such as SS, Zr, and Mo were ignored since they appeared to be insignificant.

FIG. 4. Measured and Predicted Columnar Reactivity Worths of U-235 Versus U-238 in the Core.

A radial profile of these worth predictions appears in Fig. 5.

Approximate predictions of the relative worths of blanket and blanket dummy subassemblies were made on the basis of Core-A critical studies with ZPR-3. However, due to the absence of sodium-worth data in the blanket, aluminum-worth data were used to estimate the sodium-worth distribution. Aluminum density and columnar-height extrapolations for the worths of depleted uranium and aluminum also were required. After completion of the blanket worth measurements, two-dimensional, first-order perturbation-theory calculations were made of the constituent worths. A radial plot of the predicted and calculated relative worth values appears in Fig. 5.

As seen in Figures 4 and 5, the predictions of reactivity worth based upon the ZPR-3 clean critical assembly mockup did not adequately describe the worths in the larger Fermi core. However, the predicted results were sufficiently accurate to allow the test measurements to be planned and conducted in a generally efficient manner. The average slope of the radial distribution of U-235 worth, predicted by the critical assembly data, was about 30% steeper than measured. This was partially attributed to the size difference between the reactor and critical assembly; however, to a greater extent, it was caused by the fact that the

clean critical mockup did not simulate the control- and safety-rod channels. This overestimated U-235 worth gradient contributed to an overestimate of the reactivity decrease due to radial thermal expansion.

Two-dimensional perturbation-theory analyses provided fairly close agreement with measured values except in the case of the subassembly worths in the outer radial blanket. The uniform overestimate of the magnitude of columnar U-235 and U-238 worths by about 10% indicates that the basic multigroup cross sections are in need of refinement. Apparently, calculations overestimated the sodium worth in the blanket, which led to a substantial underestimate of blanket versus dummy subassembly worths.

The validity of the multidimensional application of first-order perturbation theory is supported by the close agreement between the calculated and measured reactivity worth profiles in the vicinity of compositional irregularities, particularly near control- and safety-rod channels.

5.3. Measurements of Sodium Worth

The reactivity worth of the sodium in a core subassembly was measured as a function of the subassembly radial position in the core lattice.⁽⁹⁾ Specially designed core subassemblies for determination of sodium worth, in which the core section was either com-

FIG. 5. Measured and Predicted Subassembly Worths in the Radial Blanket.

pletely voided of sodium, or filled as in a normal subassembly, were used. The axially averaged, or columnar, core worth of sodium was obtained at seven core radii by measuring the reactivity of seven pairs of core loadings, each pair of loadings containing four of the special subassemblies in symmetrical locations at an equal radius from the core center. The special subassemblies were voided of sodium in the first loading of each pair and filled for the second. Four subassemblies were used to ensure that the reactivity difference obtained at each radius allowed evaluation of the columnar core sodium worth at that radius. The effect of "bunching" the four special subassemblies into adjacent rather than separated locations also was investigated by a further loading.

The results of the sodium-worth measurements are illustrated in Fig. 6. These results were based upon 7.60 liters or 6.75 kg of sodium in the core sections of the four special subassemblies. The following values for columnar sodium worth were obtained:

Minimum, near core center = +1.82 cents/kg;

Maximum, near core edge = +2.19 cents/kg;

Average, over core = +2.01 cents/kg.

All results have probable errors of less than $\pm 5\%$ as determined by a statistical analysis of measurement uncertainties and reloading uncertainties. The results were all obtained by averaging several reactivity measurements, thereby reducing the probable errors due to measurement.

Comparison of the sodium worth for the "bunched" measurement with the isolated, individually measured worths in the same subassembly positions indicated that if there had been appreciable neutron-spectrum effects from self-shielding of the sodium resonances, this effect was not enhanced by the consolidation of four small channels into one large channel. The results for four subassembly substitutions in each case are 2.16 ± 0.06 cents/kg and 2.12 ± 0.06 cents/kg, respectively.

Two separate predictions were made for the columnar sodium worths. Results from the Core-A critical assembly mockup on ZPR-3 were used for the first

FIG. 6. Measured and Calculated Sodium Worths.

set of predictions and two-dimensional CRAM diffusion-theory calculations were used for the second set. The predicted values are also shown in Figure 6. As can be seen, the agreement between the critical experiment predictions and the measurements is not very good, the predicted values being quite a bit smaller; however, accurate predictions from the critical experiment data were difficult for four reasons.⁽¹⁰⁾

The sodium worth in the critical assembly was measured only for insertion from zero to one-half the full-core sodium density due to a lack of sufficient canned sodium. Therefore, the required extrapolation of the results to obtain the incremental sodium worth at full sodium density, believed to correspond to the situation in the Fermi experimental measurements, was an uncertain procedure. In the light of more recent work,⁽¹¹⁾ it now appears that the assumed extrapolated reduction of sodium worth with increased core sodium density was too strong, thus leading to predicted incremental worths at full density lower than they should have been and lower than those observed. However, the extrapolation used apparently does not greatly affect the core-averaged sodium worth. Thus, whereas the earlier extrapolation resulted in a full-density incremental sodium worth considerably smaller than the core-averaged sodium worth, it now appears that the full-

density incremental worth is only slightly less than the average worth.

The sodium deficit in the mockup was made up by aluminum, which could significantly perturb the neutron spectrum and the variation of the observed worth results from zero to one-half the full-core sodium density, as well as the extrapolation from that case. Also, corrections for aluminum worth further complicated the extrapolation.

The spatial distribution of sodium worth over the core, needed for the predictions, was not measured in the critical assembly, but was assumed to follow the aluminum worth profiles.

The core of the ZPR-3 assembly was smaller than the final Fermi Core-A design, which could also lead to an error in the predictions.

The combined effect of these uncertainties led to predicted values shown in Fig. 6, which underestimated the observed distributed incremental sodium worth by about 20% near the core edge and by over 100% near the core center. However, the measured core-averaged sodium worth, 2.0 cents/kg, agreed well with the value of 1.9 cents/kg estimated from the ZPR-3 critical experiment.

A two-dimensional, cylindrical, CRAM diffusion-theory calculation in eight energy groups, followed by a first-order multigroup perturbation-theory treatment,

was also used to obtain the second, more complete set of predictions.⁽¹²⁾ Cross sections used were from a 24-group set condensed to 8 groups using one-dimensional calculated core and blanket spectra. A further two-dimensional calculation, similar to the above but in a rectangular geometry at the reactor midplane, also was performed. Combining the two sets of results slightly improved the agreement with the measured values. The calculated and measured values finally obtained are in fairly good agreement and are illustrated in Figure 6.

In summary, the columnar worth of sodium was observed to be positive throughout the core lattice, thus confirming that the sodium temperature and void coefficients of reactivity are negative. The measured core-averaged sodium worth, 2.0 cents/kg, agreed well with the value of 1.9 cents/kg estimated from the ZPR-3 critical experiment, and with the value of 2.4 cents/kg calculated with the GRAM diffusion-theory code. The measured results have probable errors of less than $\pm 5\%$, even though the reactivity changes being measured were very small. Also as mentioned previously, a reinterpretation of the full-density incremental sodium-worth data from the critical experiment would now give considerably higher values such that the core-averaged incremental worth and core-averaged worth were essentially the same and in good agreement with the measured values.

5.4. Fission-distribution Measurements

Detailed fission-distribution measurements for U-235 and U-238 have been made in the core and blanket.^(13, 14) The majority of these measurements utilized foil-irradiation techniques since cable access to the core and blanket is limited. However, axial traverses using U-235 and U-238 absolute fission counters were made at the one core position where such access was available, i.e., the temporary instrument thimble position (see Fig. 2).

The foil measurements were accomplished by use of special foil subassemblies that incorporated a hollow steel tube in place of the central fuel pins. Foils were loaded into the tubes, and the subassemblies were then loaded and removed by the normal fuel-handling procedures. Measurements were made at 11 of the 99 core subassembly positions and at 18 of the 499 outer radial blanket positions with foils at 11 axial elevations. The U-235 foils consisted of 18 w/o highly enriched uranium-aluminum alloy pin-type foils, $\frac{3}{4}$ in. long. The U-238 foils consisted of depleted pin-type foils, one inch long.

Relative fission rates were obtained by comparison of the fission product activity in the foils with that of a reference foil. There were no on-site facilities for absolute counting. An absolute determination of the number

of fissions occurring for a given energy output of the reactor was obtained by selecting a limited number of foils for radiochemical analysis at Argonne National Laboratory.

The absolute fission-counter measurements were performed by inserting the appropriate detector into the reactor inside the 2-in.-diameter steel thimble which extended from the operating floor downward 25 ft to the reactor core. To minimize flux distortion caused by the void in the thimble, the fission counters were located at the bottom of the thimble, and the void volume immediately above the counter was filled with an aluminum plug designed to simulate sodium. An axial traverse was accomplished by raising the thimble.

Figures 7 and 8 show the resulting axial U-235 and U-238 fission distributions, respectively, obtained using the fission counters, together with the corresponding ZPR-3 critical experiment predictions and results of multigroup two-dimensional diffusion-theory calculation with the CRAM code. A midplane, radial U-235 fission distribution obtained from foil measurements is given in Figure 9, together with the corresponding critical experiment and calculated values. Also, although the data are not explicitly shown, the U-235 to U-238 fission ratio in the core obtained from the absolute fission-counter measurements was found to agree to within 5% of both the critical experiment and calculated fission ratios, i.e., the U-235 to U-238 fission ratio measured at the core midplane in the temporary instrument thimble was 26.4, whereas multigroup diffusion-theory calculations give 25.3 and critical experiment data gave 26.3.

Note that in all three traverses there is good agreement between measurement, calculation, and critical experiment in the core, but that for locations in the blanket both the critical experiment and calculated values fall off more sharply than those obtained from the measurements. Furthermore, note that the discrepancy becomes larger the further the penetration into the blanket. On the other hand, in the blanket regions the critical experiment and calculated distributions are seen to be in surprisingly good agreement. This is especially true in the case of the axial distributions, and even in the case of the radial distributions the agreement is not bad.

The axial-distribution figures also show a difference in symmetry of the measured versus computed values about the core center. This seems to indicate that perhaps a wrong assumption was made in determining the nuclear center of the core when making the experimental measurements. If this correction were applied to the measured data in the reactor, the agreement between measurement and prediction would be somewhat better.

It is believed that a partial explanation of the dis-

FIG. 7. Axial Fission Distribution for U-235 in Core A.

FIG. 8. Axial Fission Distribution for U-238 in Core A.

FIG. 9. Radial Fission Distribution for U-235.

crepancy between measurement and prediction for the axial blanket region in the axial traverses is due to the fact that the geometry of the reactor exterior to the core in the axial direction is quite complicated. This made an accurate homogeneous mockup difficult in both the critical experiment and calculational models. It is not thought that any appreciable discrepancy between the predicted and measured neutron spectra is an explanation, because if this were the case one would expect the discrepancy in the case of one isotope to be larger than the other and from the data this does not appear to be so.

In the radial U-235 fission distribution traverses made using foils, the discrepancy between critical experiment and measurement in the radial blanket can probably be partially accounted for by the fact that the blanket mockup in the critical experiment was only approximate. Also, it included no sodium. This theory is supported by the fact that the agreement between calculation and measurement in this case, for which an accurate calculational model could be used, is better than for the axial distributions. This holds true except at the outer boundary of the radial blanket, where it might be expected that the calculated deep-penetration neutron fluxes might be in error using diffusion theory. Critical experiment data is lacking in this region because low flux levels did not allow measurements.

The data obtained for U-238 radial fission distribution using foils in the reactor are not given. This is because at deep penetrations in the blanket poor data were obtained. This was due to the large corrections which were required in the measurements to account for the U-235 component in the foil. However, it can be noted that the measurements of U-238 radial fission

distribution in the core were also in good agreement with calculational and critical experiment predictions.

The excellent agreement between fission-distribution measurements and predictions in the core adds confidence to the direct use of either computed or critical-experiment-determined flux distributions in calculating static parameters for the core. The blanket discrepancy would not effect breeding determinations to a large degree because most of the breeding occurs in the inner blanket regions where the agreement is still fairly good. The most significant effect of the discrepancies exterior to the core is in the predicted values of heat generation in the blanket.

5.5. Rod Calibrations

5.5.1. Control-rod Calibrations

The purpose of this test was to obtain an accurate calibration of the reactivity worths, with respect to position, of the shim and regulating control rods for various degrees of shadowing of each rod by the other. The two control rods containing enriched boron carbide are identical in design and composition but have different speeds of travel. The average reactivity insertion rates of the regulating and shim rods are approximately 1 cent/sec and 1 cent/min, respectively.

To determine the reactivity worths of the shim and regulating control rods over their full range of travel, a combination of positive and negative period reactivity measurements, subcritical count rate reactivity measurements, and intercalibration rod measurements was made.⁽¹⁵⁾ Data were obtained for both rods for the fully shadowed and fully unshadowed conditions, as well as for intermediate conditions of shadowing in the case of the regulating rod.

The measured calibration curve for the regulating rod appears in Fig. 10. A similar curve was obtained for the shim rod. Over the first 20 in. of travel, the measured worth of the unshadowed regulating rod was 44.8 cents and that of the fully shadowed rod worth measured was 43.7 cents. The measured shadowing effect was therefore 2.5%. For the shim rod, the unshadowed and shadowed worths were found to be 47.6 cents and 46.7 cents, respectively, for the first 20 in. of rod travel. The measured shadowing in this case was therefore about 2.0%.

An interesting result of the shadowing measurements made at intermediate degrees of shadowing of the regulating rod by the shim rod was that the minimum shadowing effect occurred at an intermediate withdrawal position (12 in.) of the adjacent shim rod rather than with the shim rod fully withdrawn. The effect was small, however, and the regulating rod worth only decreased slightly with further shim rod withdrawal.

Since both control rods are identical in design, the predicted shim and regulating rod worths from the critical experiment were the same. However, the predicted

FIG. 10. Calibration Curve for Regulating Rod.

values did not consider the perturbing effect on the neutron spectrum of the beryllium can (3.4 kg beryllium) of a retractable neutron source which was in place in lattice position PO3-POO (see Fig. 2) in the vicinity of the shim rod during the calibration measurements in the reactor. Neglecting this, the predicted unshadowed and fully shadowed rod worths for both rods were 49.0 cents and 47.8 cents, respectively, for the first 20 in. of travel. The shape of the rod-calibration curves in each case from the critical experiment was essentially the same as that measured in the reactor and shown in Fig. 10. The 2½% shadowing effect of the shim rod on the regulating rod measured in this test was therefore in agreement with critical experiment predictions, while the measured 2% shadowing effect of the regulating rod on the shim rod was slightly less than predicted. The unshadowed rod worths are considered to be in satisfactory agreement with predictions considering the effect of the retractable source in the measurements and other differences (see below).

Extrapolation of measurements obtained by withdrawing the regulating rod the maximum distance of 30 in. indicated a total unshadowed regulating rod

worth of 47.1 cents if the rod could be withdrawn completely from the core. The similar value for the shim rod was 49.9 cents. The predicted value in both cases was 52.0 cents.

The agreement between the measured and predicted control rod worths, a discrepancy of less than 8½%, is considered very good in view of control-rod design modifications which were made after the original Core-A critical experiment studies upon which the predictions were based. The boron-10 enrichment of the rod poison has been doubled since that time, with a corresponding reduction in the volume of boron carbide. The resulting increase in neutron capture self-shielding appears to account for the smaller measured values of rod worth. Similarly, the shim-rod calibration was made with the beryllium can of the retractable source in a nearby position. The resultant local spectral softening apparently provided an 8% enhancement of the shim-rod worth not considered in the predictions. This has now been verified by measurements with the retractable source removed. Also, although it was not anticipated that the regulating-rod worth would attain its maximum value at an intermediate condition of shadowing,

TABLE 2. SAFETY-ROD WORTHS

	Safety Rod No.	Predicted Worth, \$	Measured Worth, \$
Unshadowed Worthy	1	1.45	1.28
	2	1.65	1.46
	3	1.45	1.20
	4	1.34	1.20
	5	— ^(a)	— ^(a)
	6	1.45	1.25
	7	1.65	1.38
	8	1.45	1.16
Total Unshadowed Worth		10.44	8.93
Shadowing Effect, Original		22%	18–20%
Shadowing Effect, Revised		11%	9–10%
Ganged Worth (7 rods), Original		8.45	8.00
Ganged Worth (7 rods), Revised		9.45	

^(a) Temporary instrument thimble present during these measurements.

the effect is quite small and probably can be explained by the complex geometrical situation which exists in the reactor because the poison sections of the control rods are only 10 in. long, whereas the total travel of each rod is in excess of 20 in.

5.5.2. Safety-rod Calibration

The purpose of this test was to obtain the total reactivity worth of each of the enriched B_4C safety rods under unshadowed and shadowed conditions. In addition, data were obtained to determine the reactivity worth of the ganged safety rods as a function of position during the simultaneous withdrawal of all seven available rods. Information about the individual unshadowed safety-rod worth was needed to confirm the design calculations of rod worth which gave a rod worth value of one dollar or more. This is important from the viewpoint of reactor safety. The shadowed data made it possible to predict the net shadowed worth for ganged insertion or withdrawal. The data about ganged safety-rod withdrawal were needed to establish accurately the two subcritical stop points used for criticality prediction. Count rates taken at these stop points are used to predict the degree of subcriticality with the rods fully withdrawn. This is important in meeting the operating requirement that criticality not be achieved during safety-rod withdrawal.

Because of safety-rod-design limitations, and because of excess reactivity limitations of the reactor, several calibration methods had to be used to obtain complete rod calibrations.⁽¹⁵⁾ The rod design precluded the use of period measurements for any of the calibrations. Furthermore, reactor excess reactivity limitations al-

lowed determination of only the unshadowed rod worths by intercalibration with the previously calibrated operating control rods. The individual shadowed rod worths, obtained by withdrawing a rod with the two adjacent rods on either side fully inserted, had to be found from measurements of subcritical count-rate reactivity. Lastly, the ganged safety-rod worth had to be obtained from rod-drop kinetic measurements because the large degree of reactor subcriticality with all rods inserted precluded use of either of the other two methods.

The calibration data for the safety rods obtained in the reactor measurements are summarized in Table 2. The predicted reactivity worths of the unshadowed, shadowed, and ganged safety rods, based upon critical experiment data, are also given. As can be seen, the measured individual unshadowed safety rod worths varied from \$1.16 to \$1.46, depending upon the radial position in the core. Similarly, the predicted reactivity worths of the unshadowed rods varied from \$1.34 to \$1.65. Thus, it is evident that the unshadowed safety rods are worth somewhat less than predicted. The discrepancy varied from rod to rod but, on the average, their worth was about 15% less than predicted. However, the unshadowed rod worths still satisfied the design specifications. The observed differences in the measured and predicted rod worths, and between individual rod worths, can be attributed partially to known differences in the as-built boron-10 loadings of the rods.

The shadowing effect measured in the reactor varied from 18 to 20% for a safety rod having the adjacent rods fully inserted on both sides, whereas the predicted shadowing based on critical experiment data was 22% for all safety rods. The slightly less than predicted shadowing effect on a safety rod situated between two adjacent rods, is consistent with the 15% less than anticipated unshadowed rod worths measured.

The comparison of ganged safety-rod worths shown in Table 2 requires some clarification. From measurements of the ganged rod worth made in the reactor, in which all seven rods were dropped together and their worth measured by rod-drop kinetic measurements, a worth of \$8.00 was obtained.^(16, 17) However, in the Core-A critical experiment a calibration of the ganged-safety-rod worth could not be made by means of simultaneous withdrawal of all safety rods, because of mockup difficulties and because of the large negative reactivity involved. Instead, each individual unshadowed safety-rod worth was obtained together with the shadowing effect on each rod of the two adjacent rods in the array (see Fig. 2). The latter measurements were made by withdrawing a safety rod with the two adjacent rods in place. The unshadowed and shadowed data found are those given in Table 2. To obtain a

prediction of the ganged rod worth from these data, it was then noted that five of the seven rods in the array during ganged-rod withdrawal are fully shadowed by two adjacent rods and that the two rods on either side of position PO3-POO are half-shadowed by one adjacent rod. It was therefore reasoned that the ganged-rod worth would be the sum of the five appropriate fully shadowed (22% shadowing) individual rod worths and two half-shadowed (11% shadowing) individual rod worths. The results obtained by this method of analysis gave a predicted ganged-rod worth of \$8.45 as summarized in Table 2, where this is referred to as the "original" prediction of the ganged-rod worth.

After the calibration measurements of the ganged safety rods in the reactor were completed, it was clear that an unexplained inconsistency in the data existed. The inconsistency was the fact that the measured \$8.00 seven-rod ganged worth agreed very well with the original predictions (only ~5% smaller), whereas the measured individual unshadowed rod worths were quite a bit less than predicted (~15% less on the average). The measured shadowing of one rod on another was in close agreement with predictions. On this basis, it would have been thought instead that the measured ganged-rod worth would have also been about 15% less than predicted. The inconsistency was also shown by the fact that if the measured unshadowed rod worths in the reactor and the measured shadowing effect were used to calculate a ganged-rod worth in the reactor, using the same analysis technique as used for the critical experiment described above, that a ganged-safety-rod worth of \$7.20 was obtained rather than the measured \$8.00.

For this reason, the critical experiment data were re-evaluated, and it was found that the original data interpretation used for determination of the ganged-rod withdrawal was in error. In particular, it was realized that the original interpretation had assumed that the shadowing or coupling effect measured in the critical experiment between adjacent rods was a measure of the shadowing occurring in one rod only, i.e., the rod being withdrawn, and that there was no mutual interaction between it and the adjacent rods. Part of the confusion arose because this is the type of shadowing interpretation which was correct for the two operating control rods where the shadowed-rod worth desired was the reactivity effect as one rod is completely withdrawn with the adjacent rod remaining in place. However, for ganged-safety-rod withdrawals, it became clear that the average shadowing effect on any one rod in the array would actually be only half the value as measured in the critical experiment, or 11%. This is also true for the shadowing measurements made in the reactor since they were made in the same way.

With this reinterpretation for the shadowing data ob-

tained in both the critical experiment and reactor measurements in the ganged-rod-withdrawal analysis, the previous inconsistencies discussed above were eliminated. The revised seven-rod ganged worth predicted from the critical experiment data becomes \$9.45 (see Table 2). The \$8.00 measured worth is then 15% less than predicted as expected based on the difference between the measured and predicted unshadowed and shadowed individual rod worths. Also, if the measured unshadowed and shadowed individual rod worths are used to obtain a calculated ganged-rod worth using the revised method of interpretation, a calculated worth of \$8.07 is obtained, which is consistent also with the actual ganged measurement.

5.6. Isothermal Temperature Coefficient

The isothermal temperature coefficient was measured in the reactor over a range of temperatures from 400 to 600°F.⁽¹⁸⁾ The reactivity determinations were made by means of critical control-rod position and positive period reactivity measurements at 40°F temperature intervals, starting at 600°F. The temperature was lowered by means of cold-trap operation with the primary sodium pumps at reduced flow.

The temperature coefficient was also measured in the critical facility at room temperature with the engineered core mockup, but these results cannot be compared directly with the measurements in the reactor since integral fuel subassemblies were not used in the mockup, i.e., the major components of the temperature coefficient result from fuel and core structural-expansion effects, and the fuel plates, drawers, etc., used in the mockup cannot be expected to duplicate the thermal-expansion effects seen in the actual reactor. For this reason, the predicted isothermal temperature coefficient had to be found indirectly from thermal-expansion calculations which used the reactivity worths of fuel and coolant materials measured in the critical experiment.

Nevertheless, for the measurements of the isothermal temperature coefficient from 400 to 600°F, close agreement existed between the measured and predicted values; the measured value was 8% smaller, on the average, than predicted.⁽¹⁹⁾ The measurements made in this test, illustrated in Fig. 11, showed an average temperature coefficient of -0.85 Ih/F^* over the temperature range from 400 to 600°F, whereas the predicted value from calculations was -0.91 Ih/F . The value measured in the critical facility at room temperature was -0.82 Ih/F but, as mentioned previously, no particular significance can be attached to this value.

The agreement here is considered to be satisfactory

* For the Fermi reactor, $319 \text{ Ih} = \$1$ and $\beta_{eff} = 0.0066$.

FIG. 11. Isothermal Temperature Coefficient of Reactivity Versus Temperature.

in view of the uncertainties involved in the calculations of the predicted value. Furthermore, since the measured value was smaller than the predicted value, a preliminary conclusion was formed that the requirements of excess reactivity for temperature override in going to power are probably less than expected. The high-power results appear to verify this conclusion. Thus, no difficulty was expected to result from the limitation of 92 cents on allowable excess reactivity.

From the data obtained, the temperature coefficient at 400°F was found to be -0.83 Ih/F, increasing in a linear fashion to a value of -0.87 Ih/F at 600°F. The observed increase with temperature, approximately 5% over this temperature range, was somewhat smaller than the predicted 9.5% increase. However, this is a measurement of a small, second-order effect which cannot be determined very accurately.

The predicted values for each of the components of the isothermal temperature coefficient were based on calculations described in detail in Ref. 3. As mentioned previously, the material reactivity worths measured in the critical facility were used. However, as pointed out earlier, the average slope of the U-235 radial worth distribution predicted by the critical assembly data was about 30% steeper than measured in the reactor. This overestimated worth gradient for U-235 contributed to

an overestimate of the reactivity decrease due to radial thermal expansion in the original predictions, and is one of the major reasons attributed to the smaller measured value of the isothermal temperature coefficient than predicted. As a matter of fact, on the basis of this and other low-power test results, such as the sodium-worth measurements, revised predictions of the individual components of the temperature coefficient have been made and it has been found that the revised predicted temperature coefficient is in very good agreement with the measured temperature coefficient.⁽²⁾

5.7. Anomalous Reactivity Increase with Time

Early in the course of the low-power nuclear test program, it became apparent that there was a gradual increase with time in the reactivity state of the nominal reference core loading, with sharp increases in reactivity appearing after intervals of high-temperature operation of the primary system.⁽¹⁾ Upon detection of this unexpected phenomenon, a history of the previously measured reactivity states of the reactor over the test period was assembled, and a detailed set of measurements was made during the remainder of the low-power tests, a period in excess of 400 days. The purpose of these reference reactivity measurements for each individual test was to provide a valid time-dependent reactivity base for comparing with the reactivity states of perturbed loadings. The purpose of measuring the entire detailed reactivity history of the system was primarily to provide sufficient data to construct correlations with hypothesized temperature-related causes of the reactivity increase. A second purpose was to maintain daily knowledge of the total excess reactivity available for a given loading for operational and safety reasons.

The data obtained to define the long-term trend of the reactivity state of the reference loading are illustrated in Fig. 12, along with the temperature history of the primary sodium system. An obvious correlation exists between brief periods of high isothermal temperature operation of the primary system and relatively large increases in reactivity.

Two hypotheses were formed to explain the apparent reactivity response to the intervals of high-temperature operation. The first involved contractions of the fuel pins due to time- and temperature-dependent phase-transformation shrinkage of the uranium-10 w/o molybdenum alloy. The second hypothesis was that the zirconium cladding on the fuel pins was picking up or gettering hydrogen from the sodium coolant at an increased rate during high-temperature operation.

The support for the two hypotheses that has been found to date includes measurement of the fuel-pin lengths in a sodium-worth subassembly, removed from

Fig. 12. Reactivity Gain—Temperature History.

the reactor during July 1965, and metallographic measurements of the hydrogen content of the zirconium cladding on several pins. The fuel pins of the sodium-worth subassembly appear to be representative of the reactor fuel as a whole in terms of temperature history. These measurements indicate an average fuel-pin shrinkage of $0.177\% \pm 0.057\%$, based upon examination of 20 pins. Metallographic measurements of 6 pins indicate an average hydrogen content of approximately 100 ppm in the zirconium cladding, experimental accuracy not being established.

The reactivity change due to the measured fuel-pin shrinkage has been calculated, based upon a multigroup diffusion-theory analysis of an overall core-height reduction, and corresponds to a reactivity increase of 27 Ih. The reactivity change attributable to an average concentration of 100 ppm hydrogen in the zirconium cladding throughout the core, based upon polyethylene reactivity coefficient data from the Core-A critical assembly mockup on ZPR-3, is 22 Ih. Therefore, the calculated reactivity change due to both measured effects, is approximately 49 Ih. This compares closely with the measured overall reactivity increase of 47 Ih, as of June 21, 1965, shown in Fig. 12.

Following completion of the low-power tests, the reactor was shut down from June 21, 1965 through December 27, 1965 prior to the start of high-power testing. One

additional reference reactivity measurement, not shown in Fig. 12, was made in December just prior to the start of high-power testing. It showed an additional reactivity increase of approximately 1.0 Ih during this period, which also can be related to a period of high-temperature operation of the primary system at 650°F for 7 days during the shutdown period. The total anomalous reactivity increase measured is therefore 48 Ih.

6. High-power Static Physics Measurements

6.1. Reactivity Feedback Measurements

6.1.1. Power Coefficient

The power coefficient of reactivity has been determined by means of static physics measurements in four separate tests to date. With two-thirds of Core-A flow, it has been determined for the nominal range of powers from zero power to 13.3 MWt and zero power to 67 MWt; with full Core-A flow it has been determined for the nominal range of powers from zero power to 20 MWt and from zero power to 100 MWt. For each range, critical-rod-position reactivity measurements were made after power stabilization at power levels of 500 kW, in the middle of the range, and at the end of the power range. This procedure was performed both during

FIG. 13. Power Coefficient.

TABLE 3. POWER-COEFFICIENT VALUES

Test	Measured Power Coefficient, Original Data, ¢/MWt	Corrected Power Coefficient, ¢/MWt	Predicted Power Coefficient, ¢/MWt
0-13 MWt, Two Loops	$-0.269 \pm 25\%$	$-0.247 \pm 10\%$	-0.346
0-20 MWt, Three Loops	$-0.200 \pm 25\%$	$-0.204 \pm 10\%$	-0.246
0-67 MWt, Two Loops	$-0.283 \pm 25\%$	$-0.264 \pm 10\%$	-0.346
0-100 MWt, Three Loops	$-0.162 \pm 25\%$	$-0.174 \pm 10\%$	-0.246

power-ascent as well as power-descent modes of operation. The reactivity differences measured at the different power levels were corrected for burnup effects and for changes in the core-coolant inlet temperature, and the plot of power versus reactivity was fitted to a straight line by least squares.

It should be pointed out that at the time of the origi-

nal measurements, accurate power and flow calibrations for the reactor from heat-balance data did not exist. Therefore, the nuclear-power calibration obtained during the low-power test program was used together with the flowmeter calibrations obtained during pre-operational testing. Because of this, rather large uncertainties existed in the power-coefficient values obtained from the original data analysis.

Typical results are shown in Fig. 13. This figure shows the experimental reactivity versus power data originally obtained in the 13.3-MWt, two-loop test (two-thirds flow) and in the 20-MWt, three-loop test (full Core-A flow). Similar data were obtained in the 67-MWt, two-loop and 100-MWt, three-loop tests. The power coefficients obtained in all four tests, with no power or flow corrections, are shown in Table 3.

It will be noted in Fig. 13 that the power coefficient in the range from zero power to 13.3 MWt with two-thirds flow had an apparent 8% difference in the power-ascent and power-descent modes of operation. Although this difference was well within the experimental

error, estimated to be $\pm 25\%$ in these uncorrected measurements, this result did indicate the possibility of a small hysteresis effect in the variation of reactivity as a function of power. This had been noted earlier in EBR-II operation.⁽¹⁾ However, the 20-MWt test did not show any indication of hysteresis, and neither did the later measurements at 67 and 100 MWt, in which more accurate inlet-temperature-monitoring measurement devices were in operation. Data now available indicate that the earlier apparent hysteresis was actually due to inaccurate inlet-temperature monitoring in the early measurements and that no variation in reactivity feedback as a function of power in Fermi exists.

Since these measurements, analysis of detailed heat-balance data obtained during the power-coefficient tests has been completed. From these data correction factors have been obtained for the power and flow-meter calibrations used in the original power-coefficient analysis (see below). By use of these results the power-coefficient values for all tests have been corrected for the power and flow uncertainties in the original analysis. The corrected values are also shown in Table 3. By making the corrections, the estimated overall error in the measurements has been reduced to $\pm 10\%$ from the original $\pm 25\%$.

The corrected power-coefficient data from all four tests appear to be consistent within the experimental error except for the 20-MWt, three-loop results. No explanation for the discrepancy between the latter and the other results exists at present. However, power-coefficient measurements still remain to be made at 133-MWt, two-loop operation and at 200-MWt, three-loop operation. The accuracy of the latter measurements will improve considerably as the power is increased, and it is hoped that this will help resolve the inconsistency.

From analysis of the power-coefficient data obtained thus far, it appears that the feedback reactivity is, on the average, approximately 30% smaller than was predicted by the original calculations described in Ref. 3. This is true for both two-loop and three-loop operation (excluding the 20-MWt test results). This conclusion is also supported by the dynamic feedback analysis. The original predicted power coefficient values for two-loop and three-loop operation are shown in Table 3. Since the experimental accuracy of the measurements is $\pm 10\%$ at present, the exact difference between measurement and calculation is still somewhat uncertain. However, a smaller power coefficient is not completely unexpected since the measured isothermal temperature coefficient was found to be about 8% less than the originally predicted value. The original predictions of power coefficient were based on material reactivity

coefficient worths measured in the critical experiment, and they also involved complex thermal heat-transfer calculations and structural-deflection calculations. Therefore, it is not too surprising that the agreement between measurement and prediction is not better. Agreement to within 30% is considered adequate for this quantity, which is a combination of several complex affects.

Because of the observed discrepancy, a review of the original calculations of power coefficient is presently being made. A basis for doing this is available from the worth measurements of U-235, U-238, and sodium that were obtained in the low-power test program. As discussed earlier, the radial (and assumed axial) U-235 worth distribution measured is flatter than that obtained in the critical experiment. This results in an appreciable reduction in the power-coefficient components due to both radial core expansion and axial fuel-pin expansion. An opposing effect is the fact that the measured incremental sodium worth is quite a bit larger than was predicted in the critical experiment. This results in larger negative values in the new predictions for those components related to the sodium worth. In addition, more recent Doppler-coefficient data, and a correction for a primary flow reduction made in the standard operating conditions since the time of the Ref. 3 calculations, have been included in the revisions. The above modifications in the original calculations for the power coefficient appear, from preliminary analysis, to explain adequately the observed difference between the measured and predicted power coefficients.

6.1.2. Flow Coefficient

The flow coefficient is obtained from measurements of the variation of power coefficient with flow. Detailed measurements of flow coefficients in Fermi are scheduled to be made over a wide range of flows at a later date in the testing planned for 133 and 200 MWt. However, preliminary flow-coefficient data have been obtained from analysis of the two-loop and three-loop power-coefficient measurements already made. The data have been fitted to an equation in which the power coefficient is represented as the sum of a constant term, which doesn't vary with flow, and a second term that varies inversely as flow, i.e.,

$$P.C. = A + \frac{B_0 F_0}{F}.$$

In this equation, the constant term A represents the feedback due to the temperature rise in the fuel and is essentially flow independent. The rest of the feedback is lumped into the last term. It represents the flow-dependent portion of the feedback resulting from

the rise of coolant temperature. The arbitrary constant B_0 in the second term is the coolant-related feedback existing at the arbitrary reference flow $F = F_0$. For the purpose of the analysis, the reference flow was defined as the three-loop, full Core-A flow, 8.85×10^6 lb/hr.

By use of the corrected two-loop and three-loop power-coefficient values given in Table 3, the best values of A and B_0 were found to be

$$A = -0.0056 \text{¢/MWt};$$

$$B_0 = -0.1680 \text{¢/MWt}$$

or

$$P.C. = -0.0056 - \frac{0.1680 F_0}{F} \text{¢/MWt}$$

where

$$F_0 = 8.85 \times 10^6 \text{ lb/hr.}$$

This result is rather surprising since it shows very little feedback independent of flow, i.e., only about 3 to 4%. Based on calculations, the predicted flow-independent feedback constitutes about 20% of the total three-loop feedback. The experimental results are only preliminary, however. The 10% uncertainty in the measured two-loop and three-loop power-coefficient values is sufficiently large to account for the difference seen. The accuracy of the measurements will be improved when the detailed flow-coefficient measurements are made at higher power levels. It is expected that the agreement with predictions will then improve.

6.2. Absolute Power-calibration Measurements

Two methods have been used to measure the power level of the EFFBR. A nuclear calibration was performed early in the low-power test program in 1963, and heat-balance measurements were performed this year. The nuclear calibration employed temporary in-core U-235 and U-238 absolute fission counters located in the temporary instrument thimble at the core midplane. The midplane responses, f/sec/gm, for each isotope were related to integrated power throughout the whole reactor by considering the uranium loadings of the subassemblies and by means of one-dimensional and two-dimensional calculated power distributions. Comparison at a later date of calculated with measured fission distributions substantiated the analytical procedures.

Performance of the heat-balance tests followed achievement of stable operating power levels in the high-power test program high enough to produce superheated steam. The feedwater enthalpy changes in the steam generators could then be derived from measure-

ments of the temperature change and feedwater flow rate. Only minor corrections were necessary to relate the steam-generator output power to reactor power.

The results of the two power-calibration tests were compared by using reactor neutron-leakage detectors located in the graphite shield which had been power calibrated during both tests. A 12% difference was found, the nuclear calibration yielding the higher power value. The estimated experimental error involved in the nuclear power calibration and its maintenance over the two-year low-power test period is $\pm 13\%$. The estimated error in the heat-balance calibration is $\pm 5\%$. The heat-balance data could also be used to obtain a check on the primary flowmeter calibrations which had been obtained during pre-operational non-nuclear testing. It was found that the original flowmeter calibrations for each loop gave $\sim 12\%$ higher flows, on the average, than indicated by the heat-balance data.

Nonlinearities in the response of the intermediate and power-range ion chambers with power are also of concern in the absolute power calibrations. Reactor neutron-leakage changes with isothermal reactor temperature were indicated by measurements made during the low-power tests. This effect was estimated to produce a nonlinearity in response of the reactor neutron power detectors of up to 8% in going to full power. This potential error therefore enters into the comparisons of absolute power obtained by the two calibration methods, since the two power calibrations were performed at 100 W and 100 MW, respectively. Tests to measure any ion-chamber nonlinearities which may exist are presently being made. Although no definitive results have been obtained to date, it appears that any nonlinearity if it exists is small.

6.3. Anomalous Reactivity Decrease with Burnup

During low-power nuclear testing reactivity measurements were made periodically, and it was discovered that an anomalous gain in reactivity was occurring. As explained earlier, this was later attributed to intervals of high-temperature operation in which contractions of the fuel pins due to fuel-alloy phase transformation is thought to have occurred and in which also the zirconium cladding on the fuel pins is believed to have gathered hydrogen from the sodium coolant.

Reactivity changes during high-power nuclear testing have also been monitored by making reference critical rod-position measurements at zero power before and after periods of substantial high-power operation. Analysis of these measurements indicate that in contrast to the low-power measurements, a gradual loss in reactor reactivity is occurring which cannot be

attributed to burnup effects. The data are quite erratic, however, and the reactivity loss does not appear to be a function of the integrated power history of the reactor. The net unexplained loss to date is -11.4 Ih.

In analyzing the reference reactivity measurements made during the high-power tests, the effect of burnup reactivity losses had to be taken into account. The anomalous reactivity loss was then found by subtracting the burnup loss from the total reactivity change measured. This procedure results in some uncertainty in the anomalous reactivity data because to make this correction calculated burnup losses as a function of integrated reactor power had to be used. These calculations were further complicated by the fact that at the time of some of the reference reactivity measurements, full plutonium formation resulting from neptunium decay did exist. However, experimental determinations were made of the burnup reactivity loss rate during a time in which the reactor was operated continuously at a power level of 100 MWt for three days, and they showed that the measured and predicted values were in fairly good agreement. The measured value was -0.014¢/MWd , compared to a predicted value of -0.011¢/MWd . The measured value in this case is adjusted to equilibrium plutonium formation for comparison with predictions.

Table 4 summarizes the results of the reference reactivity measurements made at power. As seen, the unexplained loss is erratic and cannot be correlated with the previous reactor power history in any systematic manner. One of the measurements even shows a reactivity gain. At present no adequate explanation has been found for this behavior. Although the measured rate of burnup reactivity loss is apparently somewhat larger than predicted, the difference is not nearly enough to account for the anomalous reactivity loss measured.

Since the sign of the unexplained reactivity effect at power is opposite to that seen at low power, it is possible that the effect could be due to a reversal of the previous effect. In particular, it could be due to fuel-phase retransformation resulting from reactor operation at high fission density and relatively low temperature. There is also the possibility that cold-trap operation during the high-power tests could have resulted in hydrogen being removed from the primary sodium in the core. This would also cause a negative reactivity effect. Further analysis and measurements are being made in an attempt to identify the phenomena and verify it quantitatively.

7. Conclusions

The Fermi reactor nuclear test program has provided a unique opportunity to make direct compar-

TABLE 4. REACTIVITY LOSS AT POWER

Dates of Reference Reactivity Measurements	Power History between Measurements		Measured Reactivity Loss, Ih	Predicted Reactivity Loss, Ih	Unexplained Loss, Ih
	Integrated Power, MWd	Maximum Power, MW			
12-27-65 to 3-26-66	104	48	-9.80	-3.70	-6.10
3-26-66 to 4-26-66	82	67	-3.30	-2.90	-0.40
4-26-66 to 7-1-66	79	70	-5.14	-3.16	-1.98
7-1-66 to 8-1-66	137	118	-2.31	-4.82	+2.51
8-1-66 to 8-9-66	275	110	-16.65	-11.20	-5.45
Total					-11.42 Ih

isons between basic physics parameters obtained in critical experiments and actual reactor design. In general, it has been found that the agreement between measurements in the two cases is very good. This has been found to be true even though in some instances large extrapolations of the critical experimental data were needed. In the case of physics parameters such as critical mass, reactivity worths of fuel and blanket material, control-rod calibrations, and fission distributions, the agreement was found to be particularly good. In the case of other parameters, such as sodium worth and safety-rod calibrations, the agreement was not as good. However, this was due more to deficiencies in the experimental mockup and inaccurate data interpretation than in basic deficiencies in the critical experiment itself. Certainly, if an adequately detailed mockup of the reactor had been made in the critical facility, it appears that all static physics parameters would have been measured with very good accuracy. Even in the case of dynamic physics parameters such as temperature and power coefficients, which could be found only indirectly from critical experiment data, the agreement between experiment and measurement was satisfactory considering the unknown factors involved.

It may be concluded, therefore, that in the case of Fermi the detailed critical experiments which were made prior to reactor construction certainly appear

to be justified. It is true that there were varying degrees of agreement between experiment and reactor design, but in all cases the critical experiment at least gave data of sufficient accuracy for use in the engineering design of the reactor. Thus, even though the same may not necessarily hold true in all criticals for the types of advanced fast reactors now being studied, and even though not all physics parameters of interest for reactor design can be measured in zero-power critical experiments, it appears as if the use of these criticals in predicting reactor performance will continue to play an important role in fast reactor development.

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Discussion

Mr. Hungerford (Purdue): Can some of the discrepancy between the measured and calculated results in your radial and axial traverses be attributed to poor cross-section data or an inadequate number of groups?

Mr. Mueller: It's true that the calculated U-235 fission distribution does not agree very well that measured for deep penetrations in the radial blanket. We attribute this to the fact that diffusion theory underpredicts the deep penetration flux from the core.

Mr. Hungerford: Did you do any two-dimensional calculations for comparison?

Mr. Mueller: We have recently done quite extensive two-dimensional calculations; the results do not differ much from those obtained in the radial or axial direction using one-dimensional calculations. We get excellent agreement in the core and this is important for predicting power distributions. The discrepancies in the blanket are unimportant, since good agreement is obtained in the inner blanket, and the only thing affected in the outer blanket would be heating, which is underpredicted.

Reactor Dynamics Measurements on the Enrico Fermi Reactor

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1. Introduction

The dynamics measurements program for the Enrico Fermi Reactor was established and conducted with the goal that the dynamic characteristics of the plant must be adequately measured and understood in order to permit safe operation of the reactor. The characteristics of the reactor are measured by obtaining the reactor transfer function. Several methods have been used to do this. One consisted of the analysis of the response to a sinusoidal reactivity perturbation produced by a rotating oscillator rod; another consisted of the analysis of the response to both single and multiple trapezoidal reactivity pulses produced by the motion of a regulating rod. Experiments were done with the reactor at various power levels as well as with the reactor under both manual and automatic control. This permitted the determination of the zero-power transfer function, the stability of the reactor, the feedback transfer function, and the control-system transfer function.

The noise of selected signals such as ion chambers, thermocouples, and flowmeters have been recorded and analyses have been initiated. The auto-spectral density and cross-spectral density for two ion chambers have been determined as well as the frequency spectrum of the noise source.

Certain important plant dynamics characteristics have been determined by measurement of the response of the plant to scheduled transients. The selected transients were a power-setback transient, a scram transient, an automatic approach to power, and shut-down of a single pump.

Analysis of the results of these tests has led to an

understanding of the reactor and plant characteristics, and has pointed out areas where further work is needed in order to make precise prediction.

2. Zero-power Transfer Function

2.1. Measurement

The zero-power transfer function has been measured by determining the power perturbation produced by the rotation of a known amount of poison in the form of an oscillator rod. The oscillator rod is a set of seven boron carbide slugs loaded into the core in a safety-rod channel eccentrically with respect to the axial centerline of the channel. Rotation of the rod through the flux at that position produces a sinusoidal reactivity perturbation with a peak-to-peak amplitude of 1.7 cents. The rod is driven by a variable-speed drive mechanism with a frequency range from 0.001 to 10 cps.

The analysis equipment is a synchronous transfer-function analyzer similar to that used at the Dounreay Fast Reactor. A block diagram is shown in Fig. 1. The principle of operation is the following. The oscillator rod, rotating at a set frequency, will provide a perturbation in the reactor which is detected by an ion chamber. The ion-chamber current, which is proportional to reactor power, is converted to a voltage signal of approximately 1 V by an amplifier and feedback circuit. The steady dc component is suppressed by an adjustable dc supply. The signal is then modulated by a transistorized chopper at a frequency of 210 cps and put through an ac amplifier. The output is fed to a synchro-resolver which is mechanically coupled to the oscillator rod. The resolver multiplies the modulated signal by

FIG. 1. Block Diagram of Synchronous Method of Transfer-function Analysis.

$\sin \omega t$ and $\cos \omega t$, where ωt is the time-dependent attitude of the oscillator rod. The outputs are the in-phase and quadrature components of the reactor power oscillations. These two outputs of the resolver are separately demodulated and then detected by amplifier/integrators which have digital-voltmeter readouts. The outputs are integrated over an integral number of cycles to provide signals which are proportional to $B/2 \cos \theta$ and $B/2 \sin \theta$, where B is the amplitude of the transfer function and θ is the phase angle. Thus, from the two readouts the amplitude and phase angle can be calculated. Calibration of this instrument with a standard known signal simulating an ion-chamber signal during oscillation has shown that the response of the analyzer is flat over the entire frequency spectrum and that the instrument has a precision of giving the correct result to within one per cent for the amplitude and 0.5° for the phase angle.

The zero-power transfer function has been measured several times by the oscillator method, and consistent results have been obtained. They are shown in Figures 2 and 3. The experimental data for 16 passes through the spectrum have been normalized and averaged. The

normalization on the amplitude was to 1.000 s^{-1} over the range from 2 to 5 cps, and the phase-angle normalization was to -1.0° at 5 cps.

Also shown on these figures are the theoretical values of the transfer function. The phase-angle agreement appears to be good; however, the differences in the amplitude are appreciable, being as large as about 10% at the low frequencies. Discrepancies of this same nature and magnitude have been reported for oscillator tests conducted on some of the ZEUS critical assemblies.⁽¹⁾ Before the experimental data are normalized, the amplitude data serve as a measure of the power perturbation, $\delta P/P$. When the frequency-dependent values of the power perturbation are compared with the theoretical values of the transfer function, a frequency-dependent value of the amount of reactivity being oscillated is obtained. When this was done it was found that the dynamic rod worth agrees with the static rod worth at low frequencies, but is lower by about 10% at high frequencies.

The cause for this discrepancy has been investigated and no clear reason can be advanced. Measurements with different ion chambers, and with gamma compensa-

FIG. 2. Amplitude of Zero-power Transfer Function.

tion, and re-calibration of the analyzer have been done; none of these showed significant differences from previous measurements. Two approaches to the theoretical transfer function have been considered. One involves changes in the delayed-neutron data used in the calculation of the transfer function. It was found that to get agreement between theory and experiment it is necessary to adjust several of the relative precursor concentrations of U-235 by 10 to 25%. This is outside the range of uncertainty given by Keepin.⁽²⁾ However, such an adjustment would also produce agreement between the static and dynamic oscillator-rod worths by causing an adjustment in the number of inhours per dollar. The other theoretical approach has been to consider the spatial dependence of the transfer function, using some new techniques which are discussed in the next section. The conclusion of this study was that except for a detector very close to the oscillator rod there should be no spatial effects and the point kinetics equations should be applicable.

2.2. Theory

To show the validity of the use of a point kinetics model in the analysis of the oscillator tests on the Enrico Fermi Reactor, a multipoint, two-energy-group

model using the CRAM code is developed. The method is a numerical technique based on the source transfer-function method using a static multigroup code as suggested by Cohn⁽³⁾ and had been used in the analysis of the Nora reactor.⁽⁴⁾

The Enrico Fermi Reactor is approximated by a one-dimensional cylinder with three regions corresponding to the core, the blanket, and the reflector with a known axial buckling of the system from the criticality. The two-group telegrapher's equations for the flux at a point $\bar{X}$ in the reactor were written:

Fast group:

$$D_1 \nabla^2 \phi_1(\bar{X}, t) - \Sigma_{r1} \phi_1(\bar{X}, t) + S_{p1}(\bar{X}, t) + S_1(\bar{X}, t) = \left[\frac{1 + 3D_1 \Sigma_{r1}}{v_1} \right] \frac{\delta \phi_1(\bar{X}, t)}{\delta t} + \frac{3D_1}{v_1^2} \frac{\delta^2 \phi_1(\bar{X}, t)}{\delta t^2}; \quad (1)$$

Slow group:

$$D_2 \nabla^2 \phi_2(\bar{X}, t) - \Sigma_{r2} \phi_2(\bar{X}, t) + \Sigma_{1 \rightarrow 2} \phi_1(\bar{X}, t) + S_{p2}(\bar{X}, t) + S_D(\bar{X}, t) + S_2(\bar{X}, t) = \left[\frac{1 + 3D_2 \Sigma_{r2}}{v_2} \right] \frac{\delta \phi_2(\bar{X}, t)}{\delta t} + \frac{3D_2}{v_2^2} \frac{\delta^2 \phi_2(\bar{X}, t)}{\delta t^2}, \quad (2)$$

FIG. 3. Phase Angle of Zero-power Transfer Function.

where Σ_{rk} and D_k are the removal cross section and the diffusion coefficient in group k , respectively. The parameter $\Sigma_{1 \rightarrow 2}$ is the downscattering cross section, and the other symbols in Eqs. (1) and (2) have their usual meanings.

The group structure used in the calculation was such that delayed neutrons in the core and blanket were born in the slow group, and fissions in U-238 occurred in the fast group, with the lower limit of the neutron energy being 500 keV.

The prompt-neutron source, $S_{pk}(\bar{X}, t)$ in the core was expressed by

$$S_{pk}(\bar{X}, t) = X_k(1 - \beta_1)\nu\Sigma_{f1}(\bar{X})\phi_1(\bar{X}, t) + X_k(1 - \beta_2)\nu\Sigma_{f2}(\bar{X})\phi_2(\bar{X}, t), \quad (3)$$

where X_k is the function of the prompt-neutron fission spectrum and β_k is the total delayed-neutron fraction in the group k . The number of neutrons created per fission in the group k , $\nu\Sigma_{fk}(\bar{X})$, is zero in the reflector, but constant in the core region and the blanket region with different magnitudes.

The delayed-neutron source in the core and the blanket was expressed by

$$S_D(\bar{X}, t) = \sum_i \lambda_i^{25} C_i^{25}(\bar{X}, t) + \sum_i \lambda_i^{28} C_i^{28}(\bar{X}, t), \quad (4)$$

where the precursor concentrations in the core and blanket, C_i^{25} and C_i^{28} , are to be obtained from

$$\frac{\delta C_i^{25}(\bar{X}, t)}{\delta t} = \gamma_i^{25} \beta_i^{25} \nu_1 \Sigma_{f1} \phi_1(\bar{X}, t) + \beta_i^{25} \nu_2 \Sigma_{f2} \phi_2(\bar{X}, t) - \lambda_i^{25} C_i^{25}(\bar{X}, t); \quad (5)$$

$$\frac{\delta C_i^{28}(\bar{X}, t)}{\delta t} = \gamma_i^{28} \beta_i^{28} \nu_1 \Sigma_{f1} \phi_1(\bar{X}, t) - \lambda_i^{28} C_i^{28}(\bar{X}, t), \quad (6)$$

where γ_i^{25} and γ_i^{28} are the fractions of U-235 and U-238 productions in the core or in the blanket, depending on the use of Eqs. (5) and (6) in the core or in the blanket.

An external positive neutron source that was proportional to the real slow flux, $\phi_{2R}(\bar{X}, t)$, was introduced at the position $\bar{X}_0$ in the calculation:

$$S_1(\bar{X}, t) = 0; \quad S_2(\bar{X}, t) = \Sigma_f^0(\omega, \bar{X}_0) \phi_{2R}(\bar{X}_0, t), \quad (7)$$

where $\Sigma_f^0(\omega, \bar{X}_0)$ is the unknown fission cross section introduced either over the whole core or locally near the center of the core.

Assuming the same time dependences of $\phi_k(\bar{X}, t)$, $S_{pk}(\bar{X}, t)$, $S_D(\bar{X}, t)$, and $S_k(\bar{X}, t)$ of the form $e^{j\omega t}$, the non-steady-state equations were obtained from Eqs. (1) and (2) by substitution of Eqs. (3) through (7) and neglecting the steady-state part:

$$[A(\omega, \bar{X})][\phi(\omega, \bar{X})] = 0, \quad (8)$$

where

$$[\phi] = \begin{bmatrix} \phi_{1R} \\ \phi_{1I} \\ \phi_{2R} \\ \phi_{2I} \end{bmatrix}, \quad (9)$$

ϕ_{1R} and ϕ_{1I} are the real and imaginary parts of the oscillating part of the fast-group flux; ϕ_{2R} and ϕ_{2I} those of the slow-group flux, respectively, and

$$\begin{aligned} A_{11} &= A_{22} = D_1 \nabla^2 - (\Sigma_{r1} - a_1) \\ &\quad + X_1(1 - \beta_1)\nu\Sigma_{f1}; \\ A_{12} &= -A_{21} = b_1; \\ A_{13} &= A_{24} = X_1(1 - \beta_2)\nu\Sigma_{f2}^*; \\ A_{14} &= A_{23} = 0; \\ A_{31} &= A_{42} = \nu\Sigma_{f1}(\gamma_5 M_5^* + \gamma_8 M_8^*) \\ &\quad + X_2(1 - \beta_1)\nu\Sigma_{f1} + \Sigma_{1 \rightarrow 2}; \\ A_{32} &= -A_{41} = \nu\Sigma_{f1}(\gamma_5 W_5^* + \gamma_8 W_8^*); \\ A_{33} - \Sigma_f^0 &= A_{44} = D_2 \nabla^2 - (\Sigma_{r2} - a_2) + \nu\Sigma_{f2} M_5^* \\ &\quad + X_2(1 - \beta_2)\nu\Sigma_{f2}; \\ A_{34} &= -A_{43} = b_2 + \nu\Sigma_{f2} W_5^*, \end{aligned} \quad (10)$$

where the parameters in Eq. (10) are region-dependent constants, except the parameters M_n^* and W_n^* . The frequency-dependent parameters a_k , b_k , M_n^* , and W_n^* are defined by

$$a_k = \frac{3D_k}{V_k^2} \omega^2; \quad (11)$$

$$b_k = \frac{1 + 3D_k \Sigma_{rk}}{V_k}; \quad (12)$$

$$M_n^* = \sum_{i=1}^6 \frac{\omega \beta_{ni} \lambda_{ni}}{\omega^2 + \lambda_{ni}^2}; \quad (13)$$

$$W_n^* = \sum_{i=1}^6 \frac{\beta_{ni} \lambda_{ni}^2}{\omega^2 + \lambda_{ni}^2}, \quad (14)$$

and subscript n denotes the type of the fissionable isotopes, namely, U-235 for $n = 5$ and U-238 for $n = 8$. The delayed-neutron data given by Keepin⁽²⁾ were used in the calculation.

The parameters in Eq. (10), with the exception of $\Sigma_f^0(\omega, \bar{X}_0)$, are known for a given angular frequency ω . In the limit of zero frequency, the axial buckling of the system was calculated from the criticality relations, which were given by the first and the third equations of the matrix equation given by Eq. (8). With a known axial buckling, Eq. (8) was solved for $[\phi(\omega, \bar{X})]$ and $\Sigma_f^0(\omega, \bar{X}_0)$ with various frequency-dependent parameters of $b_k(\omega)$, $a_k(\omega)$, $M_n^*(\omega)$, and $W_n^*(\omega)$. From the definition of the source transfer function, the transfer function at the position X and for the group k is given by

$$T_k(\omega, X) = \frac{\phi_{kR}(\omega, X) + j\phi_{kI}(\omega, X)}{\Sigma_f^0(\omega, X_0)\phi_{2R}(\omega, X_0)}. \quad (15)$$

From Eq. (15) the gain G and the phase θ are obtained:

$$G_k(\omega, X) = \frac{\sqrt{\phi_{kR}^2(\omega, X) + \phi_{kI}^2(\omega, X)}}{\Sigma_f^0(\omega, X_0)\phi_{2R}(\omega, X_0)} \quad (16)$$

and

$$\theta_k(\omega, X) = \arctan \left[\frac{\phi_{kI}(\omega, X)}{\phi_{kR}(\omega, X)} \right]. \quad (17)$$

Using these prescriptions, proper input to simulate the transfer function at a number of frequencies was prepared for the CRAM diffusion-theory code. Two basic cases were considered with the CRAM method. In one the fictitious source was considered to be spread uniformly over the entire core, and in the other it was considered to be localized in a central cylinder with a radius of 4.227 cm. In Table 1, the transfer function of the slow group calculated by this method and by the point kinetics method are given. The gains shown in Table 1 are the gains of the transfer function normalized to those at 5 cps. At the frequency of 5 cps the gain of the transfer function is equal to the inverse dollar of the system. In fact, the value of the effective total delayed-neutron fraction deduced from the case of the CRAM method with constant Σ_f^0 over the core was found to be 0.0071, which is in good agreement with an independent calculation of the parameter β_{eff} .

The gains and phases of the transfer function in the reflector far away from the oscillator rod are shown to be independent of the method used or the spatiality of the source. The results of the CRAM method with the uniform source over the core are found to be almost identical to the results of a point kinetics method. The results of the CRAM method with the localized source, given in the last column in Table 1, are only about a percent different in gain and in phase at the low frequencies. Therefore it was concluded that the experimental results of the oscillator tests measured in the reflector should agree with those predicted by a point kinetics method within a few per cent.

Space dependence of the transfer function with a

TABLE 1. ZERO-POWER TRANSFER FUNCTIONS IN THE REFLECTOR

Frequency (cps)	Point Kinetics Method		New CRAM Method			
			Uniform Source		Local Source	
	Gain	Phase, deg	Gain	Phase, deg	Gain	Phase, deg
5.0	1.0	-1.0	1.0	-1.0	1.0	-0.95
0.5	1.05	-7.4	1.045	-7.4	1.045	-7.5
0.05	1.48	-28	1.48	-28	1.49	-28.2
0.01	2.89	-45	2.90	-44	2.93	-44.3
0.007	3.42	-49	3.42	-48.4	3.45	-48.5

TABLE 2. ZERO-POWER TRANSFER FUNCTION NEAR THE OSCILLATOR ROD

Frequency (cps)	Transfer Function	
	Gain	Phase, deg
5.0	1.0	-0.87
0.5	1.045	-6.9
0.05	1.43	-26.5
0.01	2.75	-43
0.007	3.21	-47.4

localized oscillator rod is found to be strong near the source point in only the slow group. In Table 2 the gain and phase of the transfer function at the source point are given. In comparison with the results of the transfer function in the reflector, the normalized gain of the transfer function near the source point is shown to be different by as much as 7% at the low frequency of 0.007 cps. The phase of the transfer function near the localized source is found to be different by about one degree. No space dependence was found in the fast group even when the neutron source introduced in the slow group was localized near the center. This shows that the fundamental mode is dominant in high and low frequencies except in the source group. In the source group there seem to be higher spatial modes near the source in the high-frequency region. Far away from the source only the fundamental modes predominate in all frequencies because higher modes attenuate fast, as pointed out by Weinberg and Schweinler.⁽⁵⁾ The attenuation of the fundamental mode, in the frequency near 5 cps, between the source point and the detector position is given by the difference of gains in the transfer functions for uniform and local sources, which is shown to be less than a few per cent in Table 1.

3. Measurements of At-power Transfer Function

3.1. Introduction

The reactor transfer function has been measured at several powers at which there is feedback reactivity

by using two methods and with two modes of reactor control. The nominal power levels at which measurements were made were 6, 13, 20, 30, 48, 67, and 100 MW. Two coolant loops were used at 13 and 67 MW; three coolant loops were used on all other tests. At all power levels except 6 MW, tests were performed with the reactor under manual as well as under automatic control. The two methods of measurement were rod-oscillator tests, similar to those described above, and reactivity-pulse tests. The oscillator tests were performed to determine reactor stability and reactor-feedback information. The pulse tests were performed to develop the technique as an alternative to the oscillator tests. Since the total reactivity feedback is substantially different for the two modes of control, it is appropriate to discuss the results of the tests performed under the two sections below. The pulse tests are discussed in a third section.

3.2. Oscillator Tests—Manual Control

The data from the tests performed under manual control at low powers were used to predict reactor stability at higher powers, and this could be accomplished quite easily. The basic stability criterion used was that the ratio of the amplitude of the at-power transfer function to the amplitude of the zero-power transfer function should not exceed a specified value at a given power level in order to justify stable operation at some higher power. This criterion is developed in Ref. 6, and the numerical limits established for the amplitude ratio took into account the self-imposed limitation that if a resonance instability should occur, the next power increment would never be more than one-half the increment to completely unstable behavior. This appeared to be a reasonable limitation on the basis of EBR-I data. The data obtained on the amplitude ratio from the rod oscillator tests indicated that the reactor is very stable. The ratio never exceeded 1.0, within the accuracy of the measurements, and in the frequency range of prime interest (<1 cps) the amplitude ratio was substantially less than 1.0 and smooth, giving a preliminary indication that the total feedback was negative over the entire frequency range. Figure 4 shows the behavior of the amplitude ratio for powers below 67 MW.

However, at powers below 67 MW, the temperature rise across the core was not sufficient to introduce enough feedback into the transfer function to permit separation of the feedback term by analytical methods.

The most interesting information on reactor stability and reactivity feedback were obtained for the recent tests at 67 and 100 MW. The amplitude and phase angle for the 67-MW and zero-power transfer functions are shown in Figures 5 and 6, respectively. From Figure 5 it can be seen that the amplitude

FIG. 4. Transfer-function Amplitude Ratio $|G_p|/|G_0|$.

at 67 MW is clearly below that for the zero-power transfer function at all frequencies below 1 cps, and above 1 cps the two amplitudes are the same to within the experimental accuracy of one percent. This shows the system to be stable to higher powers if the feedback is not power-dependent. The phase-angle data in Fig. 6 likewise show good stability characteristics. At high frequencies the two phase-angle curves become asymptotic, and they do not have any point of intersection at lower frequencies.

Similar information was obtained from the tests at 100 MW. Figure 7 shows the amplitude of the transfer function at 100 MW and Figure 8 the phase angle. Both curves demonstrate the good stability characteristics of the reactor.

At powers below 67 MW the presence of feedback reactivity is observable in the transfer functions. However, it is not strong enough to permit its extraction with accuracy. Consequently, the only feedback data available are from the tests conducted at nominal powers of 67 and 100 MW. These data have been adjusted to remove the transfer function of the fission product decay heat,⁽⁷⁾ and the Argand plots of the

feedback transfer function are given in Figures 9 and 10. Two generalizations can be made. The first is that the absence of any loops in the frequency domain above 0.005 cps demonstrates that the dominant feedback mechanisms must be time-constant mechanisms rather than transport mechanisms. The second has to do with the shape of the transfer function: the shape is much flatter than anticipated, indicating that the time constants of the feedback mechanisms are much more separated than anticipated.

For further information, detailed analyses must be performed. These have been begun, but only preliminary results of certain aspects are available. Storrer has shown that if the ratios of the phase angle of the feedback transfer function to the frequency, and the tangent of the phase angle to the frequency, are plotted as functions of frequency and extrapolated to zero frequency, one can obtain the low-frequency time constant for feedback response.⁽⁸⁾ This time constant is the sum of the time constants of all of the feedback mechanisms, each being weighted by its relative contribution to the total feedback. This time constant was found to be about 1.0 sec for both the 67-MW

FIG. 5. Amplitude of Reactor Transfer Function at Zero Power and 67 MW.

FIG. 6. Phase Angle of Reactor Transfer Function at Zero Power and 67 MW.

FIG. 7. Amplitude of Reactor Transfer Function at Zero Power and 100 MW.

and the 100-MW feedbacks. This time constant characterizes the low-frequency behavior of the reactor and is often useful in the analysis of slow accidents. If the product of the frequency and the imaginary component of the feedback transfer function are plotted and extrapolated, the energy coefficient of reactivity is obtained. These were found to be -0.16¢/MW-sec for the 67-MW test and -0.22¢/MW-sec for the 100-MW test. These values are often useful in the analysis of fast accidents in which the reactor is intact.

In order to learn more about the feedback mechanisms, attempts were made to fit simple models to the data. No unique model was found which fits the data with an accuracy superior to other fits. However, if certain time constants are assigned their predicted values, the gains associated with these time constants can be determined by fitting. This was done with the 100-MW data with the time constant for one term also being allowed to be adjustable to obtain the best fit for a model of this type. It was found that about 70% of the feedback has a time constant of 0.3 sec, 10% a time constant of 0.8 sec, and 20% a time constant of 4 sec. This fit assigns far more feedback to a

long time constant than anticipated, and perhaps has implications on the time and frequency response of the various feedback mechanisms.

3.3. Oscillator Tests—Automatic Control

The automatic control system for the reactor is shown in a block diagram in Fig. 11. During oscillator tests the mode of control is through the dn/dt signal, the time rate of change of the neutron level. This signal, divided by the neutron level, provides a signal to a servomechanism which controls the regulating-rod velocity, the action being one to drive the rod into the core when the reactor period is positive and out when the period is negative. The differentiating circuit has a time constant of about 10 sec, and the servomechanism has a double time constant of about 0.4 sec.

The reactor transfer function with the reactor under automatic control was measured at 13.3, 20, 30, 48, 67, and 100 MW. The measured amplitudes are shown in Fig. 12. From the curves, it is clear that in the frequency range from about 0.4 to 0.05 cps at low powers and at 0.1 cps at high powers, the at-power amplitude is larger than the zero-power amplitude. Because this might be taken as a sign of instability induced by the

FIG. 8. Phase Angle of Reactor Transfer Function at Zero Power and 100 MW.

FIG. 9. Feedback Transfer Function H for 60 MW.

automatic control system, a more exact stability analysis was made. The complex quantity G_0H was plotted for a Nyquist analysis and the reactor was found to be stable. The plot of G_0H for 100 MW is shown in Fig. 13. From this it can be seen that the

reactor does not circle the point $(+1.0, 0.0)$ nor will it do so at any higher power, using a linear extrapolation. The phase margin for stability is 66° at 67 MW and 72° at 100 MW. This increase denotes increasing stability at higher powers.

FIG. 10. Feedback Transfer Function H for 100 MW.

FIG. 11. Reactor Control System.

In order to provide some insight into the operational characteristics, the feedback due to the automatic control systems was extracted by taking the difference of the inverses of the transfer functions measured under manual and automatic control. The temperature feedback cancels out in this analysis, leaving only the feedback due to the automatic control system. The data from the tests at 30, 48, and 67 MW are shown in Fig. 14, and smooth curves were fitted to the amplitude and phase-angle data. The smooth curves were fitted to a model of the form

$$H_{\text{auto}} = \frac{G}{(1 + i\omega\tau_1)(1 + i\omega\tau_2)^2},$$

where G is the gain of the system, τ_1 is the time constant of the differentiating circuit, and τ_2 is the time constant of the servomechanism. It was found that the gain of the system is $-\$2.78$, the time constant

of the differentiating circuit is 8.6 sec, and the time constant of the servomechanism is 0.6 sec. The measured gain is less than the predicted value of $-\$3.96$, but it is still very satisfactory. The measured value of τ_1 is very close to the predicted value of 10 sec, and the measured value of τ_2 is higher than the predicted value of 0.4 sec. Neither of these differences are causes for concern, since it was found that the automatic control system controls the reactor well by compensating for low- and medium-frequency power perturbations. It is anticipated that tests will be made at higher powers to substantiate the behavior of the automatic control system at those power levels.

3.4. Pulse Tests

The oscillator-rod tests described above are the most straightforward method for measuring reactivity transfer functions. However, the apparatus occupies one

FIG. 12. Reactor Transfer-function Amplitude with Automatic Reactor Control.

centrally located lattice position, a position highly desirable for irradiation of test samples. Investigations have been made in which the regulating rod was pulsed while tape recordings were made of power and temperature. Hopefully, the transfer function measured by this method will be sufficiently accurate that the oscillator assembly can be removed.

Two types of regulating-rod motion have been studied: (1) a single pulse during which the rod is withdrawn a desired amount (usually $\frac{1}{4}$ in.), held there for a period of time (5 to 100 sec), and then reinserted to its critical position. The rod motion is a single trapezoidal pulse. (2) A repetitive pulse during which the rod oscillates between two positions symmetrical about the critical position. The rod motion is a trapezoidal wave.

Located at the Fermi plant site is a 130-amplifier analog computer originally used for plant simulation and operator training. This computer was used in these tests for the generation of the signals which controlled the motion of the regulating rod and for the subsequent analysis of the pulse tests.

During these tests the automatic control circuit was modified such that the regulating-rod velocity demand was generated from the analog computer rather than the normal reactor power/temperature circuits. At the start of a test, the operator switched into automatic mode and the analog computer generated a withdrawal-velocity demand until the rod had moved the desired amount (usually $\frac{1}{4}$ in.). After a pre-set time, the computer circuit generated an insertion-velocity demand until the rod returned to its initial critical position (or to a symmetrical subcritical position for the repetitive pulse tests). The operator was able to take control of the reactor at any time simply by switching back into manual mode.

The parameters of the pulse are rod velocity, time during which rod motion is permitted, and time interval during which no rod motion is permitted. For the tests conducted, the values of the first two were kept at 10 in./min and 1.5 sec, respectively; the third was varied between 3 and 97 sec. This gave trapezoidal pulses with an amplitude of about one cent and with total durations between 6 and 100 sec.

FIG. 13. Nyquist Plot of $G_o H$ at 100 MWt, 3 Loops.

In order to obtain as much information as possible about the reactor behavior during a transient, it has been found to be advantageous to record the following signals:

1. two ion chambers;
2. core outlet temperature;
3. sodium flow;
4. control-rod position.

The length of the recording for a single pulse was five pulse-lengths in order to include all necessary information on the transient.

Initial tests were conducted at zero power (0.1 MW) using both the single-pulse and repetitive-pulse modes of pulsing. The results are shown in Fig. 15. The agree-

ment between the two modes of pulsing is satisfactory if only the first three harmonics are considered for a repetitive pulse and if the frequency range for a single pulse is restricted to the range from $0.2/D$ to $2/D$, where D is the pulse duration. From the standpoint of the analysis the single-pulse data are easier to analyze and contain more frequency information than the repetitive-pulse data. Therefore the single-pulse mode was chosen for the tests at higher powers.

Pulse tests were conducted at powers of 63 and 100 MW. For these tests only single pulses having durations of 6, 25, and 100 sec were used; these pulses permitted coverage of the frequency range from 0.002 to 0.2 cps.

FIG. 14. Automatic Control-system Transfer Function.

A Fourier analysis is performed on each signal on the analog computer. This analysis consists of integrating the products of the signal with a sine function and a cosine function of the desired frequency. A "gating" signal is recorded on the tape prior to analysis. This signal starts shortly before the pulse and extends two pulse-lengths. The gate starts a sine/cosine generator circuit and switches the recorded signal into multipliers. The circuit is designed so that the integration is performed over the nearest number of whole cycles which exceeds the gate time. Each pulse is analyzed over frequencies $0.2/D$ to $2/D$, where D is the pulse-length or duration.

To perform a Fourier analysis of a pulse starting at arbitrary time, it is necessary that the dc component of the pulse be zero. The most convenient method we found for satisfying this requirement was to differentiate the signal and perform the analysis on the first derivative rather than the signal itself.

This analysis yields in-phase and quadrature components of the signals at each frequency. The vector

sum of the components is the amplitude, and the arc tangent of the component ratios is the vector direction. The amplitude of the transfer function is computed from the ratio of the amplitudes of reactor power (output) to rod position (input). The phase of the transfer function is simply the difference between the two vector directions.

The Fourier analysis of the repetitive pulse does not require the differentiation. However, only odd harmonics are present in oscillations of this sort. It was difficult to obtain on the analog computer the desirable precision in setting up the gating signal and generator frequency for harmonics greater than the third.

The single-pulse data taken at 63 and 100 MW have been analyzed, with the results shown in Fig. 16. For all points shown on the graph the amplitude of the two power signals agreed within 7%; their directions agreed within several degrees.

These results are compared with the oscillator measurements in Fig. 17. The consistency of the differences probably indicates either a fundamental dif-

FIG. 15. Reactivity to Power-rate Transfer Function.

FIG. 16. Reactivity-Power Transfer Function for Regulating-rod Pulsing Results.

FIG. 17. Comparison of Oscillator and Pulse Measurements of Transfer Function.

ference between the two types of measurements or a systematic error in one. Further study will be given to both methods in an attempt to resolve the differences.

4. Noise Analysis

4.1. Introduction

Power reactors have many more aspects of dynamic behavior than those described by the neutron-kinetics equations alone. The dynamic behavior of equipment coupled to the reactor can produce intrinsic reactivity-noise sources since random perturbing variations associated with this equipment can be reflected into the reactor by virtue of various coefficients of reactivity. Various transfer functions relating equipment parameters to reactor behavior may be defined and analyzed even in stable systems. Thus there are a variety of applications for noise analysis in power reactors. In order to investigate some of these characteristics for the Enrico Fermi power reactor it was decided to run a preliminary test of several instrumented variables and process the noise of these variables to ascertain information regarding reactor-transfer functions and any oscillatory tendencies present.

4.2. Enrico Fermi Power Reactor Instrumentation

From a preliminary investigation of the noise characteristics for the Enrico Fermi reactor, it was concluded

that the frequency range of interest was from approximately 0.001 to 10 radians per second (far below the cutoff frequency determined by the time constant l/β for this reactor). This frequency range was selected since it appeared that some of the heat-removal equipment could conceivably produce oscillatory tendencies in the reactor near these frequencies. A preliminary test was run at a reactor power of 67 MW and 6 instrumentation signals were recorded in fm fashion on magnetic tape:

1. reactor power as indicated on a Keithley Picoammeter from an out-of-core ionization chamber;
2. reactor power as indicated on the deviation amplifier in the Transfer Function Analyzer as measured by a second out-of-core ionization chamber;
3. reactor inlet temperature, loop 1, TE 110-1, as measured by a thermocouple;
4. core outlet temperature, TE 115-26, as measured by a thermocouple;
5. reactor outlet temperature, loop 1, TE 217-1, as measured by a thermocouple;
6. sodium flow, loop 1, 14-in. line, FE 201-1, as measured by a flowmeter.

The measurements, resulting in data from two out-of-core ionization chambers, three temperatures (one core inlet and two core outlet), and one flowmeter, were recorded on a Precision Instruments, 8-channel,

FIG. 18. Analog Computer Network for Power Spectral Density.

fm tape recorder after proper amplification. The recording speed was 0.375 in./sec, and the time for recording at a stable reactor power was in excess of 100 min. Processing of these data is now in progress, and the program plan calls for a direct analog calculation of power spectral density for each of the signals covering a frequency range from $\omega = 0.01$ to 10 radians per second. This will be accomplished by playing back the signals at 100 times the recording speed, or 37.5 in./sec, and re-recording the information at 7.5 in./sec. on the VCO fm tape-recording equipment developed by the Chrysler Corporation.⁽⁹⁾ The result of this re-recording is a signal duration of approximately one minute and a tape record approximately 45 in. in length. This tape record is then formed into a tape loop which is used for direct computation of the power spectral density using an analog computer. The details of the computation equipment are described in Ref. 9, and Fig. 18 shows the analog computer network used for computing power spectral density. The initial portion of the circuit is a high-pass filter which has a 3 db per octave cutoff below 0.1 radian per second (0.001 radian per second real time). This is included to attenuate any reactor or instrumentation drift below this frequency which is not of interest in this noise analysis. The second section of the computer power spectral density circuit is a constant- Q band-pass filter which has a Q of 10.

This value for Q was chosen in order to yield the desired resolution in the frequency domain. The frequency of this band-pass filter is determined by the two potentiometers of the servo-multiplier. As the tape loop is continuously repeated, the servo-multiplier is used to change slowly the center frequency of the band-pass filter in such a manner that the mean-square value of the filtered function can be plotted vs. frequency directly on an X-Y recorder. The signal, after being filtered, is squared and averaged, and then passed through a logarithmic converter in order to make a direct log-log plot of the desired function. A one-cycle, antilog, function generator is used to scan the band-pass filter in such a manner that the speed of sweep is proportional to the bandwidth of the filter. After scanning through one decade, the frequency-sweep circuit is reset, and the gain of the band-pass filter is increased to allow scanning of the subsequent decade. In addition, a gain adjustment, which is not shown in Fig. 18, was incorporated into the circuit such that at any time the signal out of the filter decreases by a factor of 10, and the gain increases to allow analog computation near the 100-V range and reduce the signal-to-noise ratio in the analog computing equipment.

The power spectral density obtained from the signal from one of the ion chambers is given in Fig. 19. This

FIG. 19. Power Spectral Density.

plot is preliminary and detailed analysis has not been completed at this time. It appears, however, that a resonance phenomenon occurs at approximately 0.004 radian per second, and harmonics of this peak can be seen in this plot at 0.008, 0.012, 0.016, and 0.024 radian per second. These resonances might be real; however, they may be attributable to the use of a tape loop, to simulate infinitely long operation, made from a finite length of tape. The period of the 0.004-radian-per-second resonance corresponds to one-fourth the loop time.

Plans for future work include the computation of the autocorrelation function directly from the signals using analog computer techniques. The time lag for this computation will be 0 to 0.1 min (10 min real time). This time lag is accomplished by using two tape heads in playback of the signals, one of which is movable with respect to the other. Details of this technique are described in Ref. 9. An analog-to-digital converter will be used to convert the autocorrelation data into digital

form, and a digital computer will be used to compute the Fourier transform of the autocorrelation to yield a second measure of the power spectral density. This technique will be used to validate the digital Fourier transform program and the analog methods of analysis. Once the techniques have been validated, cross-correlations will be computed between five pairs of signals directly on the analog computer. This information will be processed through an analog or digital converter, and the Fourier transform will be used to give measures of cross spectral density. The five pairs of signals which have been selected for cross-correlation studies are:

1. the two ionization chambers;
2. the core input thermocouple and one ionization chamber;
3. the core outlet thermocouple and one ionization chamber;
4. the flowmeter and one ionization chamber;
5. the inlet and outlet thermocouples.

FIG. 20. Two-chamber Cross Spectral Power Density.

The time delay for the cross-correlation plots will be the same as used for autocorrelation, namely, $\tau = 0$ to 10 min real time.

If the data look favorable at this power level, it will probably be desirable to perform the same data analysis at other power levels. This noise analysis is just beginning at the present time, and future results of this program will be reported in the literature.

Recently it has been shown that the cross-power spectral density of a reactor can be informative since the noise inherent in the detectors is removed by the cross-spectrum technique.⁽¹⁰⁾ From two-hour recordings of the noise levels from two similar ion chambers located in the graphite shield, the cross spectral density was obtained. The cross spectral analysis was performed on an analog computer. Each signal taken from the tape recorder was first amplified and broad-band filtered before being fed into a high- Q filter circuit. From the high- Q filter, the signal from the first ion chamber went directly into two quarter-square multipliers; the signal from the second ionization chamber went directly to one of these multipliers and to a 90° phase-shifting circuit, which in turn went to the other multiplier. The multiplier outputs were integrated over measured time intervals. The sum of the absolute values of the integrators divided by the bandwidth of the filter is the cross spectral power; the vector sum of the square roots of their amplitudes is their relative phase. (For the

cases reported here the relative phase was always nearly zero, since both signals were reactor power.) An eleven-cup servo-multiplier was used to tune simultaneously all circuits to the desired frequency.

Suitable data were taken at power levels of 63 and 100 MW. At the former, only two primary coolant loops were operating, while at the latter all three primary loops were operating. The results of the analysis are shown in Fig. 20. Although the absolute units attached to the results have no meaning, the data have been corrected for total signal level, so that the relative value between the two curves is meaningful. The analysis shows a sizeable decrease in noise going from two-loop to three-loop operation, as would be expected. Over the frequency range for which there is coincident information for the cross-spectrum and auto-spectrum, the slope of the cross-spectrum at 63 MW is the same as the slope of the auto-spectrum at 67 MW, indicating that there is little inherent noise in the ion chambers at the frequencies of interest in this study and verifying the techniques of the two methods. This study was not done with a tape loop and the analysis was not carried down to frequencies where the accuracy might become questionable.

Since the information presented by the cross-spectrum is essentially the product of the noise source squared and the transfer function squared, observations can be made on the spectrum of the noise source by

FIG. 21. Noise-driving Function.

removing the transfer function. This was done and the results are shown in Fig. 21. From this it can be deduced that the spectrum of the noise source is flow-dependent. For the three-loop operation it varies inversely with the frequency, and for the two-loop operation inversely with the three-halves power of the frequency. It has not been possible to associate any significance to this information yet. More experiments at other flow rates and with different pumps operating should be done to complete this portion of the study.

5. Transient Tests

Test procedures have been developed to measure some of the power- and flow-dependent parameters both within the reactor vessel and in the heat-removal systems external to the reactor, and to determine the efficiency of the reactor control systems and the feed-water steam-control systems.

The tests were conducted at low power ($\sim 10\%$) for two primary reasons: first, to check out the techniques of conducting the tests and recording the data and, second, to have an early warning of any large temperature or power perturbations that might develop at higher powers. It was believed that little quantitative data would be obtained because of the small temperature differentials at 10% power. The second power level for testing is $\sim 33\text{--}50\%$ power. To date, all of these tests have not been completed.

Prior to conducting the tests on the actual plant, the procedures were tested on the plant simulator to prove the method and to obtain predicted data.

The first test presented is the setback transient. The setback mode of operation is an intermediate excursion control that will act before a scram action; this concept will reduce the number of scrams and their attendant thermal shocks, and improve the "on-the-line" time. When the neutron flux reaches 110% of full power level or the reactor outlet temperature reaches 825°F , the setback system inserts negative reactivity. This is accomplished by removing the reactor control channel signal, whether on automatic or manual control, from the regulating-rod drive and substituting in its place a fixed ac voltage (variable only as a calibration adjustment) of such a phase as to drive the regulating rod into the reactor at 5.85 in./min for 20 sec. After this time, the initiating relay is reset and, if the alarm condition still exists, the system is started through another cycle.

The setback system contains a total of six channels of information:

- the three safety-system power-range nuclear channels;
- The three reactor sodium outlet temperature channels.

Signals from the bistable trip units of the reactor sodium outlet temperature channels are fed to a two-out-of-three logic network. Outputs from the logic network feed into an OR gate (setback mixer). Similarly, outputs from the flux-channel bistable trip units are fed to a two-out-of-three logic network which feeds into the OR gate. Thus, when two or three temperature signals exceed 825°F or when two or three neutron-flux signals exceed the equivalent of 220 MW, each of the bistable trip units fed by these channels produces an UNSAFE signal of zero volt dc. These UNSAFE signals cause the logic network to send a signal to the OR gate, and the OR gate energizes the relay which initiates setback. Besides producing the setback action, this energizes an alarm and starts a counter providing the operator with information on the number of setback cycles that have occurred. A pushbutton is mounted on the console for the operator to start a setback manually if he deems it appropriate.

When the setback condition disappears, the regulating-rod drive motor is disconnected from the setback system and connected to the reactor manual control system; thus, the regulating-rod and shim-rod positions are fixed, except for operator or further setback action.

The purpose of the setback transient test is to determine:

- That the settings of the temperature setback and temperature scrams have been properly coordinated. The temperature scram signal is obtained from core outlet temperatures, whereas the setback signal is obtained from reactor outlet temperatures with several

seconds delay from the core outlet temperatures. This delay would make it possible for a temperature scram to occur before a setback if the setback and scram temperatures are too close together.

2. That the setback rate and duration have been coordinated with the system dynamics.

The test at each power level was conducted in three steps: (1) a manual setback to confirm the setback rate and duration, (2) a relatively fast power-ramp manual control to approximate the effect of a runaway control rod, and (3) a maximum power ramp in automatic reactor control to simulate a control-system failure. The transients of step 2 were checked to determine if there was a power overshoot above the setback level, if the deadband in the power-range units resulted in an excessive number of setbacks, and if the reactor outlet temperature would rise to a value to give a delayed setback. Since the reactor may operate on either two or three coolant loops, the tests are conducted under both conditions.

The results of the tests to date are the following:

Condition I

Reactor Power = 50 MW.

Total Reactor Coolant Flow = 5.42×10^6 lb/hr using only loops 1 and 2.

	Measured Results	Plant Simulator Results
Power reduction/setback	17.1 MW	10.8 MW
Regulating rod insertion/setback	1.74 in.	1.91 in.
Power rate of change during fast power ramp	21 MW/min	47.5 MW/min
Number of setbacks from fast power ramp	2 (none from high temperature)	4 (at least one from high temp)
Maximum power ramp in automatic control	1.8 MW/min	2.75 MW/min
Number of setbacks	3	3

Condition II

Power = 75 MW

Reactor Coolant Flow = 7.93×10^6 lb/hr on all 3 loops.

	Measured Results	Plant Simulator Results
Power reduction/setback	26 MW	16.2 MW
Rod insertion/setback	1.78 in.	1.91 in.
Power rate of change during fast power ramp	33.8 MW/min	72 MW/min
Number of setbacks from fast ramp	2	5
Maximum power ramp in automatic control	1.58 MW/min	3 MW/min
Number of setbacks	4	3

These results are acceptable, and much of the dis-

parity between measured and simulated values can be explained with the smaller power coefficient and the longer sodium transport and mixing delays in the upper pool.

The next transient test discussed is the scram transient. In the Fermi reactor, the safety (scram) system is designed to insert over -8.00 of reactivity in less than a second, to initiate a series of interlock actions that will bring the remainder of the heat-removal systems down to a shutdown condition without excessive thermal transients, and to reset systems such as the automatic programmer.

The purposes of the scram transient test are to determine the sodium mixing factors of the upper pool, to observe and record and analyze the thermal transients that occur throughout the heat-transport systems, and to confirm interlock operation.

The scram is initiated manually, and the data are obtained on tape recorders and high-speed multi-channel recorders. The highest power from which scram data have been obtained to date is 67 MWt with two primary loops operating at flows of 2.88×10^6 lb/hr and 3.14×10^6 lb/hr in loops No. 1 and No. 2, respectively. At this power a series of fluctuations were observed in the recordings of reactor outlet temperature that would indicate that complete mixing is not taking place in the upper pool, and thermal transients with small amplitude and high rate are apparent at the reactor outlets. These fluctuations were not observed when a 20-MW, 3-primary-loop scram test was conducted; the absence of the fluctuations could be attributed to better mixing with 3-loop operation and/or the total temperature change was too small to cause the fluctuations. Detailed analyses of the data have not been made; however, preliminary results indicate that sodium mixing and transport times are longer than predicted.

The purpose of the "Automatic Approach to Power" test is to confirm the operation of the automatic plant-loading programmer. The purpose of the plant-loading programmer is to furnish a reliable automatic program for power-level changes, and in particular to provide an automatic start-up program in the power range. It is designed to raise or lower reactor power automatically at two different speeds during normal start-up or shutdown. The water-steam temperature, TE-601, is used to index the speed at which the programmer will travel over a given range. The slow speed corresponds to a 30°F/hr change in reactor outlet temperature.

The control signal for this system is the demand setpoint signal T_u received from the hand-set point. It is sent to one input of a comparator unit. The other input to this unit is the actual setpoint signal T_I . It is received from a proportional-type operational amplifier,

described below. The demand setpoint is compared to the actual setpoint by the comparator and switching magnetic amplifier unit. The difference between T_u and T_I is the error signal, $T_u - T_I$, whose polarity determines the direction that the programmer motor will rotate.

The output of the timer assembly is the actual setpoint T_I . The timer assembly consists of a two-speed motor, which is used to drive a cam that positions a 50-V movable-core transformer. The direction and speed of motor rotation determine whether the actual setpoint T_I is being increased or decreased, and whether it is at the fast or slow rate.

The actual setpoint signal T_I is calibrated and its relative signal strength adjusted in a proportional amplifier. It is then sent to the previously mentioned comparator and to a summing-type operational amplifier that compares the actual setpoint signal T_I to the actual temperature signal T_A . The difference between these two signals is the error signal, $T_I - T_A$, which is the input to the reactor control system.

The error signal $T_I - T_A$ from this summing-type operational amplifier is also a command signal for the timer-reversing magnetic amplifier. This unit reverses the direction of the programmer motor at any power level should the actual temperature signal T_A be less than the actual setpoint signal T_I by 5°F or more. Once an error signal of this type is corrected, the programmer motor will continue to run in its normal direction as determined by the difference $T_u - T_I$. This reversal feature prevents the setpoint signal T_I from getting too far ahead of the actual temperature during a programmed power increase.

The timer assembly also receives a signal from the water-steam temperature detectors TE-601 at each steam-generator outlet. The signals from loops 1, 2, and 3 are auctioneered by both high- and low-auctioneer circuitry. When the high-auctioneered signal is above its trip point of 550°F and when the low-auctioneered signal is below its trip point of 638°F, pressure switches are actuated which result in the timer motor running at the slow speed. Thus, the timer runs at its slow speed whenever one of the three loop steam temperatures is in between the two setpoints. The following is a tabulation of the speed of the timer motor versus the TE-601 temperature range:

Water-Steam Temperature, °F	Timer Motor Speed
517 to 550	Fast
550 to 638	Slow
638 to Max	Fast

The action of the programmer during an automatic approach to power is best demonstrated with an example. Assume that the proper prestart conditions have

been met, the reactor outlet temperature TE-201 and the water-steam temperature TE-601 are both at 517°F, and it is desired to increase the power to full power. The operator manually sets the hand setpoint TE-201 to 658°F. The programmer motor will increase the setpoint because the demand setpoint is greater than the actual setpoint. Because TE-601 is between the temperature range of 517°F to 550°F, the motor will travel at the fast rate corresponding to 100°F/hr change in TE-201.

When the highest of the three TE-601 temperature signals as selected by the high-auctioneer circuitry reaches 550°F, the programmer motor speed is changed to the slow rate, corresponding to 30°F/hr change in TE-201. It will continue to rotate at this rate until the lowest TE-601 temperature signal reaches 638°F, or the actual setpoint T_I reaches the hand setpoint, the latter condition probably being realized first.

The operator now changes the feedwater and steam-cycle pressure from 950 to 600 psia. This decrease in pressure results in a TE-601 temperature increase to 638°F. At this point, the turbine is warmed up and loaded to the capacity of the available steam, approximately 33 MWe. The turbine pressure governor is placed on automatic pressure control.

In order to continue with an increase in power, the TE-201 hand set-point must be changed to the new desired level and one or more FSV-609 valves must be open. This will enable the programmer motor to move at the fast rate until the new setpoint is reached, because the demand setpoint is greater than the actual setpoint. Once the new power level is reached, the programmer motor will stop and no further increase in power will take place. When the programmer is on the setpoint, an indicating lamp indicates that the setpoint has been reached.

The procedure for a normal shutdown is essentially the reverse of the above start-up procedure. The one important difference is that the timer cannot continue below a specified temperature unless all FSV-609 valves to the main steam line are closed. Closure of the FSV-609 valves transfers steam flow to the bypass line, preventing wet steam from entering the main steam line.

If the reactor is scrammed, relay contacts are actuated to de-energize the automatic programmer motor.

A control switch is available to the operator to adjust the actual set-point up or down; this adjustment is used to facilitate the transfer from manual to automatic reactor control. If this switch is operated, the reactor control is tripped to the manual mode, and the automatic mode is locked out of operation if the actual setpoint T_I is more than 5°F greater than actual temperature T_A . In general, the programmer has

functioned very well. All interlocks have operated as required. The temperature-loading rate is 85°F/hr instead of 100°F/hr during the fast rate; however, with the feedwater control in manual, the rate of decrease of superheated steam temperature was approximately 195°F/hr for small increments when the programmer was reducing reactor outlet sodium temperature at the rate of 30°F/hr. Subsequent operation indicates that the rate of loss of superheat will be much less when the automatic mode of feedwater flow control is used.

The single-circuit shutdown transient is the final test presented here. The systems for reactor and plant control are designed so that operation can be carried out using only two of the three cooling circuits. This feature provides the advantage that upon a malfunction of a component or system in one coolant circuit, the circuit can be shutdown without the overall plant being shutdown, thus minimizing thermal shocks to components in the plant. Also, the plant can continue to operate at a reduced power level if one coolant circuit is shutdown for a prolonged period of time.

During operation, a cooling circuit will be shutdown automatically (single-circuit shutdown) from any one of seven causes:

1. a reduction of sodium flow in a primary cooling loop to 25 % of nominal, or 6.4×10^6 lb/hr on the 14-in. flowmeters;
2. a reduction of sodium flow in a secondary cooling loop to 60 % of nominal, or 1.77×10^6 lb/hr;
3. a reduction of a steam-generator sodium outlet temperature to 485°F or less;
4. an increase in a steam-generator sodium outlet temperature to 550°F or more;
5. a feedwater demand that exceeds actual feedwater flow by 15 % of nominal, or 3.2×10^4 lb/hr;
6. a feedwater flow that exceeds feedwater demand by 10 % of nominal, or 2.13×10^4 lb/hr;
7. a steam-generator isolation due to a sodium-water reaction.

Upon the automatic initiation of a single-circuit shutdown, the following actions occur:

1. The main pump motors for the primary and secondary coolant circuit are de-energized.
2. The pony motors in the primary and secondary coolant circuits will start, unless the single-circuit shutdown was caused by a steam-generator sodium-water reaction. Even though the pony motor starts, the discharge pressure of the primary coolant circuit pump is not sufficient to open the check valve on the discharge side of the pump when the two remaining pumps are running at normal speeds.
3. Feedwater flow-control valves and the shut-off valve will close.

4. If the circuit that is shut down is circuit No. 2, the reactor outlet temperature signal that is used to program feedwater flow is removed from the feedwater programmer, and the signal from circuit No. 1 reactor outlet temperature is transmitted to the programmer.
5. The steam-temperature signal from the circuit that is shutdown is removed from the steam-temperature auctioneering circuit.
6. The inputs from the primary and secondary flow channels to the safety system are tripped; thus, the two circuits that continue to operate are in the auctioneering mode for reactor scram.

Upon a shutdown of one coolant circuit, the sodium flows in each of the two operating primary coolant circuits increase from 2.95×10^6 to 3.68×10^6 lb/hr. The operator, by manual control, reduces the flow in each coolant circuit to its nominal value of 2.95×10^6 lb/hr. Since reactor power is controlled from reactor outlet temperature, and reactor outlet temperature is maintained at 800°F, reactor power output is proportional to sodium flow through the reactor. Thus, the plant will operate at a power output of 133 MWt. The sodium flows in the operating secondary coolant circuits are not affected, since each of the three secondary coolant circuits is independent.

The feedwater-steam system controls will continue to function as during three-circuit operation. The reactor control will operate in a manner similar to that during three-circuit operation; however, the response of the reactor outlet temperature will be slower because of the reduced sodium flow rate in the reactor.

The purpose of the test is to observe and record the hydraulic changes and the control systems responses when one circuit is shutdown.

The shutdown of one loop was initiated by tripping a breaker that supplies power to the main motors of the primary and secondary pumps in one circuit. This shutdown has been initiated twice: once at approximately 20 MWt with the reactor control in manual, and again with the reactor control in automatic.

While the low-power level limits the amount of quantitative data, certain values were obtained from the recordings and compared with the data obtained from the plant simulator. These comparisons are listed in Tables 3 and 4.

The sodium flows in the two operating primary loops increased to 115 % of normal flow when the third primary pump motor was tripped; the predicted value was 125 %.

The single-circuit shutdown test must be conducted at higher powers to obtain better quantitative data.

General conclusions of the transient tests conducted to date are that control actions have been about as

TABLE 3. SINGLE-CIRCUIT SHUTDOWN WITH REACTOR CONTROL IN MANUAL

	Initial Temp, °F	Maximum Temp, °F	Final Temp, °F
Reactor outlet temperature—actual	543	546	542.5
Reactor outlet temperature—simulated	545	546	543
Reactor inlet temperature—actual	523	528	525
Reactor inlet temperature—simulated	520.5	523	522.5

TABLE 4. SINGLE-CIRCUIT SHUTDOWN WITH REACTOR CONTROL IN AUTOMATIC

	Initial Temp, °F	Maximum Temp, °F	Final Temp, °F
Reactor outlet temperature—actual	531	534	531
Reactor outlet temperature—simulated	545	552	545
Reactor inlet temperature—actual	512	516	515
Reactor inlet temperature—simulated	520.5	526.5	526.5

expected. The transport and mixing times in the sodium circuit are larger than expected, but this may be due, in part, to errors in the measured sodium flows; these could be of the order of 10% and better values will be obtained from detailed heat-balance measurements.

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Discussion

Mr. Becker (RPI): You noted that the amplitude of the transfer function obtained by oscillator tests disagreed with calculations, and that the results from oscillator tests disagreed with those obtained from the pulse test. How would the pulse-test results compare with calculated results?

Mr. Klickman: The pulse-test results would perhaps be a little lower than the calculated values. Incidentally, we do have zero-power test data on tape. Therefore, we can pursue this problem and hope to obtain a solution.

Mr. Noble (GE): You mentioned the discrepancy in your zero-power transfer function and indicated that you could possibly account for this by an increase in the effective delayed-neutron fraction for some of the delayed groups. Did you attempt to calculate an effectiveness for each group using delayed-neutron spectra such as Keepin gives?

Mr. Klickman: We did.

Mr. Perez (Univ. of Florida): This disagreement between the amplitude and phase shift is somewhat puzzling. The amplitude and phase shift are related, one being a transform of the other. Therefore, you can devise a formula which could check on the internal consistency of the experiment. Have you tried this?

Mr. Klickman: We are puzzled by the agreement on phase angle but not on amplitude. Perhaps when you shift from a U-235 transfer function to a U-238 transfer function you really don't get much of a shift in the phase angle corresponding to the shift in the amplitude. Perhaps an amplitude discrepancy such as was seen here doesn't really mean such a large phase-shift discrepancy.

Mr. Perez: We have also made some measurements of amplitude and phase shift, although not in fast reactors, and the phase shift always comes out much better than the amplitude. When you measured the transfer function by the pile oscillator method, did you perform a Fourier analysis of the wave form?

Mr. Klickman: The instrument we have is an analog Fourier Analyzer. We hope to digitize the data we have on tape and do additional Fourier analyses.

Mr. Perez: I think this would be very useful. You might have some cancellation in the phase shift which you don't have in the amplitude.

Mr. Hungerford (Purdue): What exactly was your experimental procedure?

Mr. Klickman: We have an analog computer circuit

which generates a signal which withdraws the regulating rod, for example, for $1\frac{1}{2}$ sec. This results in about a 1-cent withdrawal. The regulating rod is then kept fixed for a predetermined number of seconds—anywhere between 5 and 96 sec—after which the rod will be reinserted to its original position.

Mr. Hungerford: Was this done at the normal rod-drive rates?

Mr. Klickman: Yes. We used an automatic circuit.

Mr. Uhrig (Univ. of Florida): In your noise measurements, you removed the influence of the transfer function to achieve the driving function. Was that a calculated transfer function or an experimentally measured transfer function that you removed?

Mr. Klickman: It was measured.

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SESSION VII

October 13, 1966

EXPERIMENTAL TECHNIQUES AND EQUIPMENT

Chairman: J. D. Orndoff
Los Alamos Scientific Laboratory

Secretaries: J. M. Gasidlo
W. P. Keeney
Argonne National Laboratory

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Some Recent Experimental Techniques Developed in Idaho Division of Argonne National Laboratory

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ABSTRACT

The improvement of existing experimental techniques and the development of new methods have been undertaken in the Idaho Division of Argonne National Laboratory with the principal aim of understanding the physics of large, dilute, plutonium-fueled fast reactors. The techniques described are (1) improvement of fission-ratio measurements, (2) an activation method for measuring the U-238 capture cross section, (3) a measurement of the change in activation of a thin U-238 foil due to the Doppler effect, (4) the measurement of the boron-10 ($n\alpha$) and helium (np) cross sections relative to the fission cross section of U-235, (5) the measurement of the manganese and sodium capture cross sections relative to the fission cross section of U-235, and (6) measurements of the effect of inelastic scattering in shells of reactor materials.

1. Introduction

The improvement of existing experimental techniques and the development of new methods have been undertaken in the Idaho Division of Argonne National Laboratory with the principal aim of understanding the physics of large, dilute, plutonium-fueled, fast reactors. To a large degree, the techniques being developed are directed towards the very large critical assemblies to be constructed in the Zero Power Plutonium Reactor (ZPPR) beginning in 1968, but the experimental methods are largely developed and used in the investigation of the smaller systems constructed in Zero Power Reactor 3 (ZPR-3).

Because of the increased importance of the low-energy tail of the neutron spectrum in large, fast reactors, several low-energy indices have been investigated. These include the measurement of the Na($n\gamma$), Mn($n\gamma$), B¹⁰($n\alpha$), and He³(np) cross sections relative to the U-235(nf) cross section. In addition, the U-238($n\gamma$) Doppler effect has been investigated directly by measuring the change in the activation of a U-238 foil when it is heated in a fast reactor spectrum. This

latter technique utilizes equipment and methods developed for the measurement of the ratio of the U-238($n\gamma$) and U-235(nf) cross sections in a fast reactor spectrum. The accuracy and reliability of the higher-energy indices given by fission-ratio measurements have also been steadily improved. Greatly improved confidence in the fission-ratio measurements has been obtained, and some results of an intercomparison of the Idaho Division fission chambers with those used by groups elsewhere is given in another paper in this conference.⁽¹⁾ The vastly improved repeatability of fission-ratio measurements by these techniques has permitted experiments in ZPR-3 in which the inelastic-scattering cross sections of reactor materials are investigated.

In some cases, development of these techniques has essentially been completed while in others development is still underway. Here we give a status report of these techniques.

2. Fission-chamber Development

Although interest in fission ratios as spectral indices has lessened with the improvement of neutron spectrometers, there is still a strong need to obtain accurate ratios of the fission cross sections of reactor materials in order to evaluate correctly the neutron economy in fast reactors. For example, knowledge of the relative fission cross sections of the isotopes Pu-239, Pu-240, Pu-241, and Pu-242 will aid in the evaluation of the effect of change of plutonium isotopic composition with time in a fast reactor. The bulk of fission-ratio measurements in ZPR-3 have been made with sealed fission chambers having moderately thick steel walls and containing approximately 500 micrograms of fission material. Difficulties have been encountered through in-leakage of air, inelastic scattering in the chamber walls, and due to the pile-up of alpha-particle pulses in the relatively thick fissile deposits. The ratios were usually measured by determining the integral count rate above some pulse-height level.

A combination of fairly standard techniques has been used to improve these fission-ratio measurements. These are (a) the use of continuous gas-flow chambers, (b) the use of thin, aluminum-walled, chamber construction, (c) thin, low-mass, foils in back-to-back geometry, and (d) multichannel analyzers for data accumulation. In addition, thermal irradiation has been used to check electroplated mass calibrations and to permit the rechecking of calibrations simply and quickly.⁽¹⁾

The use of gas-flow chambers is clearly not essential since it is practicable to build reliable sealed chambers of long life; however, we have found this method to be exceptionally reliable and to permit the design of chambers from which foils can be easily removed and easily reinserted. Scattering in the fission-chamber walls of the older ZPR-3 chambers has been found to be a moderately severe problem, leading to corrections of up to 8% in the U-238 fission-ratio measurement. We currently use aluminum for chamber construction since this material has an appreciably lower inelastic-scattering cross section than steel and the wall thicknesses have been reduced to approximately 0.030 in. in thickness. From both measurements and calculations we estimate that scattering effects by the wall in these new chambers will introduce errors only of the order of 0.2% in measured fission ratios and, as such, these corrections are negligible. To avoid the problems of alpha-particle pile-up, we have adopted the straightforward procedure of reducing the fissile plated mass, approximately 100 micrograms being used in the new chambers. Foils of U-233, U-234, U-235, U-236, U-238, Pu-239, and Pu-240 have been constructed of this mass, and in general less than 1% of the fission pulses are believed to be lost in the electronic and alpha-particle noise low-energy portion of the pulse-height distribution, and even in the case of Pu-240, only 1½% of the fission pulses are estimated to be in the noise region. The chambers are back to back, i.e., there are two chambers within a single shell, the plated deposits of fissile material being back to back inside of this shell so that they are essentially in identical portions of the reactor spectrum. One-half of each shell always contains a U-235 foil, so that ratios are always made relative to the U-235 fission cross section.

Measurements of fission ratios are now always made by storing the pulses from both halves of the back-to-back chambers in the memory of a pulse-height analyzer. Special routing circuitry has been designed to avoid errors which might arise due to accidental coincidence of pulses from both halves of the chambers. In this manner, a detailed pulse-height distribution is immediately obtained and displayed on an oscilloscope screen which clearly shows the discrimination between

fission-particle pulses and alpha-particle or noise pulses. We believe that this immediate display of a differential pulse-height spectrum is of great value in identifying clearly and quickly whether or not the chamber, the foils, and the electronics are in good working condition. The vastly greater detail obtainable in such differential spectra clearly shows malfunctions which we believe could not be found when using the older, integral-discriminator techniques.

Finally, in all plated foils has been incorporated a certain known fraction of thermally fissile material. For example, the U-238 chamber contains a natural uranium foil, and accurate isotopic analyses are available for all foils. The foils are manufactured by electroplating from a solution of known content of fissile material, and by analysis of the residual electrolyte an accurate knowledge of the plated deposit is obtained. However, errors in plating can sometimes occur, and we adopt the procedure of intercalibration of foils by irradiation in a thermal-neutron flux. The thermal cross sections of U-233, U-235, Pu-239, and Pu-241 are very well-known, and clearly the relative fission rates in a thermal flux give an excellent measure of the relative masses of thermally fissile materials present in the chamber. Use of the isotopic analysis gives the relative total masses which can be compared with those deduced from the electroplating process. A further advantage of thermal intercalibration is that the effective mass is measured, that is, the mass as determined by a measured fission rate, so that the geometrical efficiency, effects of absorption in the foil, and similar effects are automatically included. This technique we have found to be invaluable in identifying small discrepancies in plated masses, and in general agreement to better than 2% has been obtained between the relative thermal calibration masses and the electroplated masses. The masses deduced from the thermal irradiation are used in calculating final fission ratios since they incorporate the geometrical efficiency of the fission chambers. Not the least of the advantages of the thermal irradiation technique is that it is a simple and straightforward procedure to recheck the calibration of any fission chamber by repeating the thermal irradiation. This we believe to be of great importance when one is conducting a series of experiments on related fast reactor cores extending over a considerable period of time.

3. The U-238($n\gamma$) Cross Section

An activation method of measuring the ratio of the U-238($n\gamma$) cross section and the U-235(nf) cross section which does not involve radiochemical analysis was developed to investigate a long-standing discrepancy between radiochemically measured and calculated

values of this ratio. The activation measurements were found to be in good agreement with the calculated data, and later an error in the calibration of one of the detectors used in the radiochemical method was discovered. When the radiochemical data were suitably corrected, the activation and the radiochemical measurements were both in good agreement with calculation so that the discrepancy was resolved. Since the technique and results have been published fully elsewhere,⁽²⁾ no detailed discussion of the method will be given here. Basically, the method is the very simple one of comparing count-rate ratios in a fast spectrum and in a thermal-neutron spectrum, and using the well-known ratios of thermal cross sections to determine the ratios of fast cross sections. The innovations introduced in this technique were (a) the measurement of the activity of 23-min half-life U-239 rather than that of 2.4-day Np-239 so that saturation could be obtained in irradiation of the order of an hour, (b) the use of a thin-crystal scintillation detector with high efficiency for detection of the 74-keV γ ray of U-239 and low efficiency for other gamma rays. This scintillation detector eased the problems of measuring the U-239 activity in the presence of the products of fast fission in the U-238 foil. Considerable improvement in the discrimination between the U-238 γ ray and γ -rays of U-238 fission products appears to be offered by the use of solid-state γ -ray detectors rather than sodium iodide scintillation counters. This technique is being investigated.

4. U-238 Doppler Measurements

The scintillation detector discussed in Sect. 3 has been used to measure the change in the U-239 activity of a depleted uranium foil when it is heated in a fast-neutron spectrum. This activation Doppler measurement is made by placing a thin (0.005 in.) foil of depleted uranium inside a small furnace. This furnace, less than 2 in. in diameter and approximately 6 in. long, may be placed in any of the matrix tubes of the ZPR-3 structure. Foils may be heated up to approximately 600°C.

The experiment consists simply of measuring the activity of a foil placed within this furnace when the furnace is both at room temperature and then heated to approximately 500 to 600°C. The activity of the 74-keV gamma ray of U-239 is measured.

This technique is still under development, but measurements made in ZPR-3 Assembly 47,⁽³⁾ the SEFOR mockup, have indicated that adequate accuracy is obtainable. Repeatability of activation count rates of the order of a few tenths of one percent was obtained, and the change in activity was found to be several percent, which indicates that the method is practicable.

However, measurements made with two different arrangements of the local fuel and diluent plates in the reactor core gave results differing by a factor of approximately two. This indicates that, as expected, the low-energy spectrum is strongly affected by the local environment. The effect is presumably due to the change in the magnitude of the flux depressions at the U-238 resonance energies in the furnace location with different local plate arrangements. This would arise by the diluent plates between U-238 plates in the core and the furnace location filling the flux depression to different extents. We would most strongly emphasize that these measurements only indicate the ambiguities in this particular type of thin-foil measurement. There is no reason to expect that Doppler measurements made with thick heated samples show effects of this magnitude, and currently we are considering the development of the furnace in which the foil to be activated is sandwiched inside a cylindrical uranium or uranium oxide rod.

In addition, we would emphasize that uncertainties of this large magnitude would also not apply to the measurements of the Doppler effect by reactivity methods, since these use thick heated samples.

5. B¹⁰ and He³ Measurements

The boron-10($n\alpha$) and the helium-3(np) cross sections both show a $1/v$ energy dependence at low energies. Consequently, the ratio of these cross sections to that of a cross section which varies less rapidly with neutron energy is a valuable low-energy spectrum index. We are in the process of developing counters in which the ratio of the B¹⁰($n\alpha$) and He³(np) cross sections to the U-235(nf) cross section is measured. To avoid extensive instrument development, commercially available B¹⁰ and He³ gas proportional counters are used for this work. These are cylindrical in shape and each is enclosed inside a cylindrical U-235 fission chamber in which the plating of fissile material extends over the active length of the gas counter. Again, the ratio of the fast cross sections is measured in terms of the well-known thermal cross sections by irradiating the double counters in both a thermal flux and a fast-neutron spectrum. The reaction rates in the proportional chambers and in the fission chambers are measured by means of pulse-height analyzers. In principle, the method is very simple, but in practice, ambiguity in interpretation of results is found due both to change in the shape of the pulse-height spectrum of the proportional chambers in going from a thermal flux to a fast flux and in the presence of the high gamma flux in the fast spectrum. Currently we believe that an uncertainty of the order of 20% must be assigned to our measurements, but it appears to be practicable

to reduce this uncertainty to an acceptable 5% to 10%. In addition, these two spectral indices are closely similar, and comparison of the experimental results will give some indication of where experimental uncertainties lie. It may also be possible to infer if the assumed energy dependences of the $B^{10}(n\alpha)$ and $He^3(np)$ reactions are indeed correct.

In this regard it is interesting to note that the value of this B^{10} counter measurement may largely lie in its use in conjunction with B^{10} reactivity measurements, since there appears to be a possible difference between the charged-particle cross section and the absorption cross section.⁽⁴⁾

6. Manganese Capture Measurements^(5, 6)

The neutron capture cross section in Mn-55 shows a prominent resonance in the region of 300 eV, and measurement of the ratio of the $Mn(n\gamma)$ and the $U-235(nf)$ cross sections is an index which is sensitive to the fraction of the flux which lies in the region of 300 eV. The capture cross section in manganese at thermal-neutron energies is well-known and, consequently, we have again adopted the procedure of measuring the ratio of the manganese and U-235 fast cross sections in terms of the thermal cross sections. A very thin manganese deposit is used in order to avoid any depression of the neutron flux at the peak of the manganese 300-eV resonance. The manganese is electroplated on an aluminum backing foil. This foil is then irradiated together with a U-235 fission chamber in both a thermal-neutron spectrum and in the ZPR-3 core under investigation. The measurement in itself is simple, the only difficulties being the simple ones of low manganese activity and activation of the backing foil, but the interpretation of the results is subject to some uncertainty.

Clearly, interpretation of the reaction rate ratio as a spectral index is only possible if the reaction cross sections are known accurately as a function of neutron energy. As in so many other cases, it appears that the manganese capture cross section is not very well-known, and a major portion of the effort in development of this technique was spent in deriving a manganese cross-section set. Where the resonances are well-resolved, the cross sections are measured with moderate accuracy, and at high energies where resonances overlap strongly, other activation measurements are available. However, in the region between, cross sections are very poorly known and semi-empirical interpolation between the high-energy and low-energy ends is necessary.

Some check upon reliability of this set has been made by performing these index measurements in a fairly hard spectrum in which the low-energy end of

the neutron cross section plays an unimportant part. Good agreement with calculation was obtained, thus indicating that the interpolated region of the manganese capture curve was not grossly in error. Measurements in ZPR-3 Assembly 47, the SEFOR mockup, gave an experimental result approximately 20% lower than that calculated using Argonne Set 224. This difference is believed to be experimentally significant, and it does indicate that this technique does give valuable measurements. Interestingly, it indicates that the calculated spectrum is somewhat too soft, as has been the general indication in the past with other measurements.

7. Capture Measurements in Sodium

The curve of sodium capture cross section versus energy shows a well-known, well-defined resonance at approximately 3 keV, and thus possesses at least some of the properties of a valuable low-energy spectral index. An experiment utilizing this resonance has been developed.

The experimental approach taken is that of measuring the ratio of the effective sodium($n\gamma$) and U-235(nf) cross sections. The present approach is somewhat similar to that adopted for the spectral index measurement with manganese; however, the experimental procedures do not involve thermal calibration.

As with the manganese foil experiment, the foils are irradiated adjacent to a standard U-235 fission chamber. The position of the sodium foil and the U-235 deposit in the chamber is such that they see the same cell-averaged flux.

The foils are metallic sodium canned in thin aluminum. The sodium foils are prepared by rolling to a thickness of approximately 0.010 to 0.020 in., then cut in a die to give a resulting diameter of 0.350 in. The foils are placed in a circular depression formed in a 0.010-in.-thick aluminum sheet and covered with a sheet of aluminum of equal thickness. The aluminum sheets are pressure-welded together in a die which also cuts the encapsulation to a 0.400-in. diameter. The total mass of sodium is approximately 10–20 mg and that of aluminum about 40 mg.

To achieve cell averaging, the sodium foils are placed parallel and adjacent to the U-235 deposit in the standard chamber and perpendicular to the plane created in the assembly by the fuel and diluent plates. Pulses from the standard fission chamber are collected and recorded over the period of the irradiation to establish the base number of the index.

The irradiated sodium foils are placed between two closely spaced sodium iodide detectors with sufficient shielding to eliminate the Na-24 high-energy beta particle. A cesium-137 source is used to set the lower-

level discriminators of both channels to approximately 670 keV. The integral counts of both channels are recorded along with the output of a fast coincidence circuit. For sufficiently active samples, the coincidence technique directly yields the absolute disintegration rate. For less active samples, the efficiencies of the two channels can be accurately determined from a separate high-level irradiation. One convenient method used for this determination utilizes aluminum samples activated in the center of AFSR. The reaction utilized, $\text{Al}(n\alpha)\text{Na-24}$, can yield enough sodium-24 activity to give a very accurate determination of yield by the coincidence method. The coincidence experiment generates a source-averaged activity which can then be converted to individual counter efficiencies by use of the integral counts collected at the same time as the coincidence experiment was performed.

The primary purpose of this activation measurement is, of course, to determine the relative flux at the large, 3-keV sodium resonance. The absorption component of the resonance (approximately 1 barn) is shielded by a very large scattering component (600 barns), however. If single collision removal theory is used, self-shielding of approximately 20% is calculated for a 0.010-in.-thick metallic sodium foil. The resonance is so wide, however, that a single scatter does not necessarily remove the neutron from the resonance. The self-shielding will therefore be determined by a very fine group structure in a multigroup transport-theory calculation.

Problem areas presently under investigation include methods of making foil uniformity more consistent. This arises in the present measurements because metallic sodium at room temperatures can be distorted with a very small amount of force. Sodium compounds that can be fabricated into thin foils having good structural strength at room temperature are under consideration.

The measured index averaged over a rather heterogeneous cell is only a crude approximation to the measurement made in a truly homogeneous system. Preliminary calculations indicate that self-shielding within a foil is relatively insensitive to depressions in the incoming flux and varies slowly as the depression is increased. However, the resonance activation is very sensitive to both the magnitude and shape of such resonance-flux depressions. Since the core sodium cans are approximately one-quarter inch thick, the depressions across the cell are severe wherever there are sodium cans, and the depression in the average flux is less than if the sodium were distributed homogeneously through the cell. Hence, the experiment will ultimately utilize analytical techniques which are not yet fully developed.

8. Shell Measurements

The measurement of nonelastic neutron cross sections by observing the counting rate of an energy-dependent detector with and without a spherical shell of scattering material around the detector has been used for several decades. The work of Bethe, Beyster, and Carter⁽⁸⁾ on the analysis of sphere transmission experiments indicated that this technique could give reliable nonelastic cross sections to an accuracy of a few percent. Shell measurements with mono-energetic neutrons have largely been superseded by neutron time-of-flight and gamma-ray-excitation techniques, but we have applied this technique to measurements in the broad spectrum in the core of an assembly in ZPR-3.

Briefly, the experiment consists of measuring the changes in U-238/U-235, U-236/U-235, and U-234/U-235 fission ratios caused by placing shells of graphite, sodium, aluminum, iron, stainless steel, lead, and depleted uranium around the fission chambers. Clearly the changes in the reaction rates in the threshold detectors U-234, U-236, and U-238 are measures of the cross sections for inelastic scattering from above to below the threshold energy. The U-235 reaction is essentially a flux monitor.

Space limitations restricted the shell thickness to approximately 0.25 in., and consequently the *maximum* measured change in fission ratio (in the U-238/U-235 ratio with a U-238 shell) was only 6½%. Great care had to be exercised to measure such small changes accurately, and the back-to-back fission chambers discussed in Sect. 2 were used for this purpose. The experimental repeatability was found to be excellent and the relative accuracy of measuring two fission ratios, e.g., with and without a shell, was about 0.2%.

The principal analysis of the data was by comparison of the measured changes in fission ratios with values calculated using a spherical model of the reactor and the transport-theory code SNARG. Two ANL cross-section sets, Set 635 and the later improved Set 224, were used. It was found that, except in the case of aluminum, Set 224 showed a considerable improvement over Set 635, which probably reflects the large number of good microscopic inelastic-scattering measurements which have been made in recent years. With aluminum, Set 224 was grossly in error; Set 635 gave better, though not good, agreement with experiments.

These experiments obviously are only supplements to the microscopic inelastic-scattering measurements, but the shell technique does give sufficiently accurate results to aid in evaluating the correctness of a cross-section set.

This work will be published in full in the near future.⁽⁷⁾

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Discussion

Mr. Kato (ANL): You have investigated the effect of the wall thickness on your fission-detector data. Have you considered how to correct for the steel in the matrix and drawer which surrounds the detector in order to get a better comparison with theory?

Mr. Davey: The wall-effect measurements were straightforward. We have concentrated on correcting

the older published ratios because it is impractical to repeat those measurements with thin-walled detectors. The adjacent steel matrix tubes combine to give a thickness of the order of 60 mils. Obviously this does affect the spectrum significantly. First, we must find suitable analytical codes to describe the reaction rates within and between the plates of what might be termed an infinite parallel-plate reactor. This, of course, does not describe the square matrix or the square-cell effect. Then we may be able to extend the analysis, but I'd like to turn the question back largely to the analysts. We would like them to realize that the geometry is complex. The matrix is a very necessary part of the core for a critical facility which, by its very nature, must mock up a very large variety of cores. We intend to make more detailed measurements to give data which can indeed be compared with the calculations. One of the things we are considering is replacement of a number of drawers and matrix tubes around the measuring region with pin-type fuel elements which would be more homogeneous. These would be only a few experiments rather than experiments which would be performed in many cores.

Special Experimental Techniques Developed Recently for Application in Fast Zero-power Assemblies*

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ABSTRACT

A description and discussion of three different experimental methods are given. These were developed particularly for application in fast zero-power assemblies.

The first method deals with the absolute determination of ^{238}U capture rates. The principle is based upon the fact that ^{243}Am and ^{239}U decay via ^{239}Np into ^{239}Pu . The α -disintegration rate of ^{243}Am is determined absolutely by low-geometry α -particle counting. From this the absolute ^{239}Np disintegration rate follows directly, which is measured by the 106-keV γ -X-ray coincidence technique, so avoiding radiochemical separation. The γ -X-ray self-attenuation within thick uranium foils is treated quantitatively and an optimum foil thickness is derived. Special attention has been given to the reduction and correction of background due to fission product γ activity. A detailed error analysis shows an overall accuracy of about 1% for the absolute ^{239}Np disintegration rate including γ -attenuation effects within the foils.

The second method is concerned with the application of resonance activation foils in the keV region, where, besides ^{23}Na , the application of ^{19}F and ^{27}Al was found to be interesting. Due to the short half-lives of 11.5-sec ^{20}F and 2.3-min ^{28}Al , an automatic-rabbit irradiation system was developed by which the irradiated foils were transported immediately between two detectors so that counting could be started 3 sec after the end of irradiation. The calibration procedure for foils and detectors is given. An error analysis is also presented which showed that the accuracy is limited by the cross-section data available at the present time.

The third method is used to determine the prompt-neutron decay constant, the reactivity, and the absolute reactor power by investigation of the neutron noise. The probability distribution of detector counts in given time intervals is measured by a "probability

distribution analyzer." The specific feature of this technique is that the complete probability distribution of interest can be measured at once. The experimental set-up of the analyzer is described in detail. The derivation of reactor parameters from the measured distributions is discussed, as is the applicability of this technique to plutonium-fuelled fast reactor assemblies.

1. Method for Absolute Determination of ^{238}U -Capture Rates

1.1. Introduction

The special feature of the method is the use of the α decay of ^{243}Am into ^{239}Np , which in turn decays to ^{239}Pu . The disintegration rate of ^{239}Np is measured by the well-known 106-keV γ -X-ray coincidence technique.⁽¹⁻³⁾ The ^{243}Am disintegration rate is absolutely measured by low-geometry α counting. The ^{243}Am - ^{239}Np - ^{239}Pu decay chain enables one, therefore, to obtain the absolute ^{239}Np disintegration rate.

Comparing this method to others applied so far to ^{238}U capture rate measurements in fast and thermal systems, we found certain characteristics. The first is the obtainable accuracy of ca. 1% for the determination of absolute ^{239}Np disintegration rates, including the effect of γ -X-ray self-attenuation within the uranium foils. For this accuracy achieved, the easy control of long-time electronics stability by the constant ^{243}Am source is a necessity. The second characteristic is concerned with the suppression of γ background due to fission-fragment activity. This is of considerable importance in fast reactor assemblies because of the significant fraction of fast fission in ^{238}U . The application of the γ -X-ray coincidence technique allows a successful reduction of fission- γ background.⁽¹⁾ Experimental results obtained from foil irradiations in fast-neutron spectra showed very clearly the superiority of the coincidence experiment over the single-detector method in this respect. The third characteristic is that radiochemical separation of ^{239}Np with all its difficulties can be avoided.

A simultaneous determination of capture rates

* Work performed within the association in the field of fast reactors between the European Atomic Energy Community and Gesellschaft für Kernforschung m.b.H., Karlsruhe.

FIG. 1. Combined Decay Schemes.

(^{238}U) and fission rates (^{239}Pu , ^{235}U , ^{238}U , etc., according to type of foils) is also possible, if the fission product γ rays of energy greater than 660 keV and the coincident 106-keV γ -X-rays are counted in the same experimental run. The counting rate due to fission product γ rays can be related to the absolute fission rate, if parallel-plate fission chambers⁽⁴⁾ and foils are closely attached and irradiated in the same manner. Care has to be given in this case to the particular time dependence of fission product γ -ray activity.

1.2. Description of Method

The principle of the method is based upon the fact that ^{243}Am as well as ^{239}U decays via ^{239}Np into ^{239}Pu . Thus ^{239}Np is common to both decay chains and can be used to interrelate them quantitatively. The decay schemes are shown in Fig. 1. The α -disintegration rate of ^{243}Am is best suited for an absolute measurement. Because of the long half-life, the detection and spectrometry of the emitted α -particles can easily be performed by use of semiconductor detectors, and suitable sources can be fabricated. Therefore, it was decided to apply low-geometry α -particle counting to the absolute

calibration of the ^{243}Am reference sources. This is described in detail below.

The half-lives of ^{243}Am and ^{239}Np are $T_{1/2}^{53} = 7950 \pm 50$ yr and $T_{1/2}^{39} = 2.35 \pm 0.01$ d, respectively. Due to the extremely long-lived parent, we have the state known as secular radioactive equilibrium. Thus the activity A^{39} of the daughter product ^{239}Np is given by

$$A^{39} = A^{53}, \quad (1.1)$$

where

$$A^{39} = ^{239}\text{Np disintegration rate (sec}^{-1}\text{);}$$

$$A^{53} = ^{243}\text{Am disintegration rate (sec}^{-1}\text{).}$$

To measure the ^{239}Np disintegration rate the 106-keV γ -X-ray coincidence-counting method was chosen. By use of the ^{243}Am reference source the γ -X-ray coincidence-counting equipment can be calibrated precisely. For thin sources and foils, where γ self-attenuation can be neglected, the γ -X-ray coincidence-counting rate $C_{\gamma 0}^{39}$ of ^{239}Np , is related to the disintegration rates by

$$C_{\gamma 0}^{39} = EA^{39} = EA^{53}, \quad (1.2)$$

where E is the calibration factor.

The quantities and physical phenomena included in the calibration factor are discussed separately below. In the irradiation experiments performed, thick uranium foils were used to obtain reasonable intensities.

The measured absolute disintegration rate A^{39} of ^{239}Np is connected to the ^{238}U capture rate CR^{28} by the following equation:

$$A^{39} = CR^{28} \left\{ \frac{\lambda^{29}}{\lambda^{29} - \lambda^{39}} (1 - e^{-\lambda^{39}t_i}) e^{-\lambda^{39}t} + \frac{\lambda^{39}}{\lambda^{39} - \lambda^{29}} (1 - e^{-\lambda^{29}t_i}) e^{-\lambda^{29}t} \right\}, \quad (1.3)$$

where

$\lambda^{39}, \lambda^{29}$ = decay constants of ^{239}Np and ^{239}U , respectively;

t_i = irradiation time;

t = time after irradiation.

1.3. Calibration of ^{243}Am Reference Source

The very thin americium source was fabricated by electroplating. The available ^{243}Am contained impurities of ^{241}Am and ^{244}Cm . By use of a high-resolution semiconductor α detector, the α peaks of ^{241}Am , ^{243}Am , and ^{244}Cm could be well separated so that the correction due to these impurities was less than 1%. To make sure that the ^{243}Am reference source was properly calibrated, two independent methods of absolute calibration were compared, namely, the low-geometry α counting and the α - γ coincidence technique. Because

of the impurities, the ^{243}Am source itself was not used for this comparison, but a very pure ^{241}Am source prepared in the same way. Because the comparison is described elsewhere,⁽⁵⁾ only the results are given here. The absolute disintegration rates measured by low-geometry α counting and by coincidence counting agreed within 0.5 %. The total error in both determinations was estimated to be less than ± 1 %.

The apparatus for the low-geometry measurements is shown in Fig. 2. The distance between source and surface-barrier detector could be changed in discrete steps by distance holders. The aperture in front of the detector determined the solid angle. To prevent scattering from the chamber walls other apertures were inserted. Backscattering from the source backing can be neglected in this geometry.⁽⁶⁾ The absolute disintegration rate A^{51} of ^{241}Am is given by the equation

$$A^{51} = \frac{C_\alpha}{G_s}, \quad (1.4)$$

where

C_α = measured α -counting rate;
 G_s = geometry factor.

In the definition (1.4), it is assumed that all α particles impinging onto the counter are detected, which is the case if semiconductor detectors are used. According to Jaffey,⁽⁷⁾ the geometry factor G_s for a uniformly spread source coaxial with aperture is given by

$$G_s = \frac{1}{2} \left[1 - \frac{z}{\sqrt{a^2 + z^2}} \right] \cdot \left\{ 1 - \frac{3}{8} b^2 \left[\frac{z(z + \sqrt{a^2 + z^2})}{(a^2 + z^2)^2} \right] \right\}, \quad (1.5)$$

with

a = radius of aperture;
 b = radius of source;
 z = coaxial distance between source and aperture.

The second term in the curved bracket of (1.5) turned out to be negligible because of very small ratios a/z and b/z . In this approximation the relative error of G_s due to inaccuracies in a and z , given by $\Delta a/a$ and $\Delta z/z$, can be found from the Gaussian error propagation law by

$$\frac{\Delta G_s}{G_s} = 2 \sqrt{\left(\frac{\Delta z}{z} \right)^2 + \left(\frac{\Delta a}{a} \right)^2}. \quad (1.6)$$

The relative error in geometry has been estimated to be 0.007 from the relative errors $\Delta z/z = 0.002$ and $\Delta a/a = 0.003$. Other errors from α -self-absorption in the source, backscattering effects, and electronic deadtime can be

FIG. 2. Apparatus for α and α - γ Calibration.

neglected. The relative statistical accuracy was always better than 0.6 %. Thus an overall error in the absolute disintegration rate of less than 1 % is reliable.

1.4. Determination of Calibration Factor E for Thin Sources and Foils

The calibration factor E , defined in (1.2), connects the 106-keV coincidence-counting rate $C_{\gamma 0}^{39}$ to the absolute ^{239}Np disintegration rate A^{39} . The relation (1.2) is restricted to very thin foils in which the γ -X-ray self-attenuation can be neglected. The influence of attenuation in thick foils will be considered below.

The simplicity of this calibration is evident because a detailed knowledge of all physical phenomena entering into E is not necessary. Those phenomena are due to: (a) nuclear parameters, i.e., complex decay scheme, branching ratios, electron conversion coefficients, angular correlations (γ - γ and γ -X-ray), fluorescent

yield, internal bremsstrahlung, etc.; (b) special experimental features particular to the detecting equipment used, i.e., detector geometry, detector efficiency, source geometry, backscattering (spurious coincidences), sum coincidences, external bremsstrahlung, etc.; (c) setting of differential analyzer (window width and threshold).

Attempts to take into account all these phenomena separately fail because they are very complex and partly depend on each other. To assure that all effects are fully contained in the factor E of the relation (1.2), only three boundary conditions must be fulfilled for the ^{243}Am source and the uranium foil. Those are:

(a) The layer thickness must be sufficiently small so that no particles are lost due to absorption or scattering within the material. (b) The source and foil dimensions should be equal. (c) The same position between the detectors and the same setting of the electronic equipment has to be used if the source or the foil is measured.

1.5. Consideration of γ -X-Ray Self-attenuation Effects in Thick Foils

For the actual determination of ^{238}U capture rates in fast zero-power assemblies, thick uranium foils have to be used for intensity reasons because only a power density of several tens of milliwatts per liter is available in these systems. Appropriate foil dimensions are around 0.1 to 0.5 mm for thickness and 25 mm for diameter. In foils of this thickness γ attenuation can no longer be neglected. The attenuation of the ^{139}Np 106-keV γ -X-rays affects the single as well as the coincidence-counting rate considerably. The effect upon the coincidence count rate is of primary interest in this context. This will be taken into account by introducing a γ -X-ray self-attenuation factor S . According to its experimental determination, S is defined by

$$S = \frac{C_{\gamma}^{39}}{C_{\gamma 0}^{39}} \frac{W_0}{W}, \quad (1.7)$$

where

$C_{\gamma 0}^{39}$ = γ -X-ray coincidence-counting rate of a thin foil;

C_{γ}^{39} = γ -X-ray coincidence-counting rate of a thick foil;

W_0 = weight of a thin foil;

W = weight of a thick foil.

The experimental determination of S for a foil of thickness h is simply performed by comparing the γ -X-ray coincidence-counting rate of a thin reference foil to that of the thick one of equal diameter, both irradiated under the same conditions. The advantage of this procedure is that all effects depending on the foil

thickness, i.e., absorption, scattering, coincidences due to backscattering, etc., are fully included in S .

A simple model has been chosen to calculate the influence of γ -X-ray self-attenuation on the coincidence-counting rate. The model is based on the assumption that γ -ray attenuation obeys an exponential law and that γ - and X-ray emission are isotropic. Furthermore, an infinite-slab geometry was adopted for the foil, which is a good approximation for the practical ratio of diameter to thickness. The analytical expression for S is then given by

$$S = \frac{1}{h} \int_0^h E_2(E \cdot \chi) \cdot E_2[\Sigma(h - \chi)] d\chi, \quad (1.8)$$

where

h = thickness of foil;

E_2 = second Placzek integral;

Σ = total macroscopic γ -scattering cross section (16.8 cm^{-1} in this case).

The factor S has been calculated as a function of h using relation (1.8). The curve is given in Fig. 3. The results of experiments performed with three different foils of thicknesses 1.03, 14.6, and 86.0 microns are also indicated. The preparation of these uranium foils is described in Ref. 5. It follows from examination of Fig. 3 that the agreement between experiment and calculation is quite satisfactory. As can be seen from the data, even for the 1.03-micron foil, S is equal to 0.985, which means that a correction of 1.5% due to γ -X-ray attenuation has to be applied. The error of S , however, is the limiting factor for the overall accuracy of about 1%. From the data a criterion can be obtained for optimum foil thickness. Assuming equal neutron irradiation dose and uniform activation of foils with different thickness h , the coincidence counting rate C_{γ}^{39} is proportional to Sh . For this reason Sh versus h has been plotted in Fig. 4, showing the dependence of coincidence-counting rate on foil thickness. The optimum for metal foils lies around 500 microns and should be chosen if only moderate neutron doses are available.

1.6. Correction due to Fission Product γ Activity

Due to fast-neutron fissions in ^{238}U , in addition to fissions in ^{235}U , a considerable γ background in hard reactor spectra results from fission product activity. Corrections have to be applied for fission γ rays falling into the 106-keV γ -X-ray peak. This contribution can amount to 26% and more⁽⁸⁾ if only single-detector counting rates are investigated. This background is considerably reduced by the γ -X-ray coincidence method. Results of quantitative investigations concerning the number of fission product γ - γ coincidence

FIG. 3. γ -X-ray Self-attenuation Factor S versus Foil Thickness h .

counts within the 106-keV peak per fission product γ count above 1.3 MeV are given in Ref. 1. For a ^{235}U metal foil this number is around $5 \cdot 10^{-3}$. It is assumed that this number is also true for ^{238}U .

The quantitative correction can be carried out by an experiment in which two foils of different enrichment in ^{235}U are irradiated and counted under the same conditions, simultaneously measuring the 106-keV γ -X-ray activity and fission product γ activity above 660 keV. The arithmetical procedure is given in Ref. 5. For foils (0.2% ^{235}U) irradiated in a fairly hard fast reactor spectrum (Fast-Thermal Argonaut Reactor STARK, assembly 4,⁽⁹⁾), the measured correction to the ^{239}Np γ -X-ray activity was less than 1%.

1.7. Set-up of γ -X-Ray Coincidence Equipment

The experimental set-up and the equipment used are shown in Fig. 5. Except for the stabilizing electronic circuitry, the set-up and the detector design are very similar to those of Tunnicliffe *et al.*⁽¹⁾ Uranium metal foils of 25.4-mm diameter and 100-micron thickness, as well as the ^{243}Am source, were inserted into special foil holders (anti-Compton shields) which minimized unwanted coincidence counts due to Compton- γ -scattering from one NaI crystal into the other. The crystal was 4 in. in diameter by 2 in. RCA 7046 photo-

multipliers were attached to the crystals. Commercially available amplifiers analyzers, and stabilizers (omitted in Fig. 5 for simplicity) were chosen together with a coincidence unit (80-nsec time resolution) for the experiments. The complete apparatus including automatic sample changer and data-acquisition system is described in detail elsewhere.⁽¹⁰⁾

1.8. Discussion and Conclusions

Several conclusions can be drawn when the ^{239}Np γ -X-ray coincidence method with ^{243}Am calibration is compared with other relevant methods. The potentialities of coincidence counting for ^{238}U capture rate measurements should be fully utilized to reduce the background of fission product γ rays. In single-detector γ -ray spectrometry this background is particularly cumbersome and affects the accuracy of ^{238}U capture rate measurement appreciably if foils are irradiated in fast reactor spectra. This problem has been discussed in Ref. 11 for ^{239}U disintegration-rate measurements, and in Ref. 8 for ^{239}Np activity determinations by single-detector set-up.

Furthermore, it can be concluded that the use of ^{243}Am is essential to obtain an overall accuracy of 1% for the absolute ^{239}Np disintegration rate. This is based on the reliability of low-geometry α calibration

FIG. 4. $S\hbar$ versus h .

by which any difficulty of quantitative radiochemical processing of ^{239}Np is avoided. In addition, the constant ^{243}Am source allows a simple stability control of electronic circuitry. The main interest in precise determination of the absolute ^{238}U capture rate is its use as a reference rate for reaction-rate ratios in reactor systems and in connection with ^{238}U capture cross-section measurements.

2. Application of Resonance Activation Foils for Investigation of Neutron Spectra in the keV-region

2.1. Introduction

The measurement of a fast reactor neutron spectrum in the low energy range from about 0.1 to about 100 keV is still a difficult experimental task. Although time-of-flight technique and proton-recoil proportional counters

have been applied in this region to determine the differential neutron spectrum, the accuracy is still insufficient for the comparison of theoretical and experimental results. In order to support the possibility for intercomparing measuring techniques in this low energy range—for the elimination of systematic errors, etc.—resonance-absorption detectors have been investigated. Their applicability depends on various conditions:

There should be only one main resonance for (n,γ) -activation in the keV-region; the (n,γ) -capture cross section, $\sigma_{n\gamma}(E)$, should be well known and large enough in the resonance and low elsewhere; the half-life of the resulting radioactive nuclei must be sufficiently long for the counting procedure, but not too long in view of the disintegration rate. Further, interfering activities from (n,p) and (n,α) reactions should allow either energy or half-life discrimination. Finally,

FIG. 5. Set-up of γ -X-ray Coincidence Equipment.

the material should be suited for foil fabrication. Taking these conditions into account the nuclides ^{19}F , ^{23}Na , and ^{27}Al were chosen for investigation.

Fluorine foils were made of polytetrafluorethylene $(\text{C}_2\text{F}_4)_n$ as discs, 18 mm in diameter by 1.5 mm thick. Sodium foils were fabricated from sodium fluoride crystals of 18-mm diameter by 1.0 mm thick. Aluminium was used in metallic form (18 mm in diameter and 0.5 mm thick). An activation analysis performed with thermal neutrons did not show any activities from impurities of the materials used.

2.2. Resonance Parameters and Cross Sections

The most important known resonance parameters of the three nuclides selected are given in Table 1. The resonances of ^{19}F at energies of 15.3 and 27.3 keV found in measurements of total and capture cross sections are given in Refs. 12 and 13, respectively. The experimental data, particularly around the first resonance, show rather large inaccuracies, so that a determination of accurate resonance parameters is not

TABLE 1. RESONANCE PARAMETERS FOR ^{19}F , ^{23}Na , AND ^{27}Al

Nucleus	I^π	E_0, keV	Γ, keV	Γ_n, keV	Γ_γ, eV	l	J^π	$\Gamma_{\text{Doppler}}, \text{eV}$
^{19}F	$\frac{1}{2}^+$	15.3						9
		27.3	0.3		0.7 ± 0.2	1	2^-	12
^{23}Na	$\frac{3}{2}^+$	2.95	0.2204	0.22	0.4	0	2	3.6
		54.1		0.75		1	3	15.5
		55		0.20		0	2	
^{27}Al	$\frac{5}{2}^+$	5.906	0.02			1		4.7
		35.04				0	3	11.8

possible. Capture-cross-section data for ^{19}F down to 10 keV were taken from BNL-325.^(14, 15) Between 10 keV and thermal energy no measurements have been performed so far. Although in this energy region there exists a superposition of contributions, the one proportional to $1/v$ (s-resonance) and the other proportional to v (p-resonance), a linear interpolation between thermal and 10 keV has been taken as an upper limit.

TABLE 2. GROUP CROSS SECTIONS FOR ^{19}F , ^{23}Na , AND ^{27}Al , IN BARNS

i	E_n	U	^{19}F		^{23}Na			^{27}Al		
			$\sigma_{n\gamma}$	$\sigma_{n\gamma_{\text{eff}}}$	σ_t	$\sigma_{n\gamma}$	$\sigma_{n\gamma_{\text{eff}}}$	σ_t	$\sigma_{n\gamma}$	$\sigma_{n\gamma_{\text{eff}}}$
1	6.5–10.5 MeV	0.48	0.00002		2.20	0.00015 ^(a)		1.90	0.0002 ^(a)	
2	4.0–6.5 MeV	0.48	0.00002		2.30	0.00015 ^(a)		2.20	0.0002 ^(a)	
3	2.5–4.0 MeV	0.48	0.00004		2.60	0.0001 ^(a)		2.70	0.0004 ^(a)	
4	1.4–2.5 MeV	0.57	0.00012		3.00	0.00015 ^(a)		3.00	0.0004	
5	0.8–1.4 MeV	0.57	0.00018		3.80	0.0002		3.20	0.0004	
6	0.4–0.8 MeV	0.69	0.00045		4.50	0.0003		4.00	0.0007	
7	0.2–0.4 MeV	0.69	0.0005		4.00	0.0006		3.90	0.001	
8	0.1–0.2 MeV	0.69	0.0006		3.80	0.0012		5.20	0.003	
9	46.5–100 keV	0.77	0.0020	0.0015	5.30	0.0016		5.00	0.002	
10	21.5–46.5 keV	0.77	0.0075	0.0051	4.30	0.0026		7.40	0.006	0.0055
11	10.0–21.5 keV	0.77	0.0050	0.0047	5.00	0.001		1.00	0.001	
12	4.65–10.0 keV	0.77	0.0035		8.00	0.001		2.60	0.060	0.052
13	2.15–4.65 keV	0.77	0.0036		100.1	0.060	0.043	1.40	0.0007	
14	1.0–2.15 keV	0.77	0.0038		6.21	0.010		1.40	0.0010	
15	465–1000 eV	0.77	0.004		3.30	0.005		1.40	0.0015	
16	215–465 eV	0.77	0.0042		3.11	0.006		1.40	0.0021	
17	100–215 eV	0.77	0.0045		3.11	0.007		1.40	0.0031	
18	46.5–100 eV	0.77	0.005		3.11	0.010		1.40	0.0046	
19	21.5–46.5 eV	0.77	0.005		3.12	0.015		1.41	0.0067	
20	10.0–21.5 eV	0.77	0.0055		3.12	0.022		1.41	0.010	
21	4.65–10 eV	0.77	0.006		3.13	0.032		1.42	0.015	
22	2.15–4.65 eV	0.77	0.006		3.15	0.046		1.42	0.021	
23	1.0–2.15 eV	0.77	0.0065		3.17	0.068		1.43	0.031	
24	0.465–1.0 eV	0.77	0.007		3.20	0.101		1.45	0.046	
25	0.215–0.465 eV	0.77	0.0075		3.25	0.147		1.47	0.067	
T	0.0252		0.009		3.63	0.525		1.64	0.241	

^(a) Corrected values: In the original work the $\sigma_{n\alpha}$ and σ_{np} cross sections are enclosed.

For ^{23}Na only measurements of total cross section have been made around the main resonance at 2.95 keV. Therefore, the capture cross section has been calculated by Schmidt⁽¹⁶⁾ from known resonance parameters. The calculated data have been compared to those of experiments performed above 20 keV⁽¹⁷⁾ and showed good agreement.

Measurements of the capture cross section for ^{27}Al have been made only at thermal energy and above 20 keV, so that estimations similar to ^{19}F were necessary in the intermediate range and around the 5.9-keV resonance.

For comparison of experimental and theoretical results a 26-group cross-section set has been chosen for ^{19}F , ^{23}Na , and ^{27}Al . For ^{23}Na and ^{27}Al the set derived from the above-mentioned microscopic cross sections is in agreement with the set of Abagjan *et al.*⁽¹⁸⁾ For ^{19}F the group cross sections were made in the same way. This group cross-section set for the three materials is given in Table 2.

2.3. Activation of Resonance Foils

A thorough treatment of foil activation by neutrons under several conditions is given in Ref. 19. The results have been used to calculate corrections for the activa-

tion due to self-shielding and scattering in the resonance probes. In foils of finite thickness both effects reduce the activation by neutrons of resonance energy. In calculating the corrections, care has to be taken if the mean energy loss per collision at resonance energy is equal or smaller than the resonance width. This is the case with sodium. Then, besides the probability for first collisions, also the probabilities for second, third, and fourth collisions have to be estimated and taken into account. In the example of the given sodium foil, it was found that in the 2.95-keV resonance region 100 first collisions are followed by 57 to 61 further collisions (summed up to the fourth collision).

The correction factors have been applied to the group cross sections around resonance energies. For the foil dimensions, given above, the resulting group capture cross sections, $\sigma_{n\gamma_{\text{eff}}}$, computed in this way are included in Table 2.

2.4. Decay Schemes and Counting Method

The decay and level schemes of ^{20}F , ^{24}Na , and ^{28}Al are shown in Fig. 6a–c. In all cases β transitions are followed by γ deexcitations or γ cascades, respectively. Therefore, β - γ and γ - γ coincidence-counting methods are applicable to determine absolute disintegration

rates. Due to the low neutron density in fast zero-power assemblies, relatively thick foils have to be used; this leads to an appreciable β -self-absorption. It was decided to count γ rays for these reasons.

In order to calibrate the γ -counting equipment, the foils were irradiated in a thermal column and their absolute disintegration rates were measured by coincidence technique. Whereas for ^{24}Na the foils of 1-mm thickness were measured directly by the γ - γ coincidence method, for ^{28}Al thin foils were measured by 4π β - γ coincidence method. Because difficulties arose with ^{20}F in the β - γ coincidence technique due to bremsstrahlung, the nuclide ^{52}V (see Fig. 6d), which was also measured by 4π β - γ coincidence method, was selected additionally for calibration. By use of ^{28}Al and ^{52}V it was possible to determine the efficiency of the γ counter at 1.78 (^{28}Al) and 1.44 MeV (^{52}V). From this the efficiency for the 1.63-MeV γ rays of ^{20}F could be interpolated

The estimation of the total calibration error was about 2% for ^{28}Al and ^{20}F , taking into account bremsstrahlung and background, and 4% for ^{24}Na , due to angular correlation, inaccuracies in the data of the complex decay schemes, sum coincidences, and background.

In fast reactor irradiations, primarily (n,p) and (n,α) reactions within the foil materials give rise to interfering activities. Reactions of the ($n,2n$) type can be neglected because of their high threshold.

In case of ^{23}Na the radioactive nuclides resulting from (n,p) and (n,α) reactions are ^{23}Ne ($T_{1/2} = 38$ sec) and ^{20}F ($T_{1/2} = 11.46$ sec) with comparable short half-lives, so that after a few minutes waiting time only ^{24}Na ($T_{1/2} = 15$ hr) is present.

For ^{27}Al the disturbing activities result from ^{27}Mg ($T_{1/2} = 9.4$ min) and ^{24}Na . The γ energies of ^{27}Mg range from 0.834 to 1.015 MeV and can be eliminated by energy discrimination. Due to its relatively long half-life, ^{24}Na is counted and subtracted together with the background.

Finally, in fluorine, ^{19}O ($T_{1/2} = 29$ sec) and ^{16}N ($T_{1/2} = 7.15$ sec) is produced besides ^{20}F . The γ rays of ^{19}O are eliminated by energy discrimination, and the upper end of the bremsstrahlung spectrum reaching onto the photopeak of the ^{20}F 1.63-MeV γ -line can be neglected. This is different for ^{16}N because γ rays as well as bremsstrahlung contribute to the photopeak of ^{20}F . A correction is possible because the half-lives of ^{16}N and ^{20}F differ by a factor of 1.6.

2.5. Counting Equipment

The irradiated foils were transported between two scintillation counters, facing each other, in order to achieve maximum efficiency. The scintillation counters

FIG. 6. Decay Schemes of ^{20}F , ^{24}Na , ^{28}Al , and ^{52}V .

themselves—consisting of 3-in.-diameter by 3-in. NaI crystals (Harshaw integral line) connected to photomultipliers (RCA 8054)—were shielded by 3 in. of lead to reduce the background from the nearby reactor. Conventional amplifiers, discriminators, and scalars were used in connection with a multichannel pulse-height analyzer.

The linearity between γ energy and pulse height was checked in the range from 0.51 to 2.75 MeV using the isotopes ^{22}Na (0.51 MeV), $^{137\text{m}}\text{Ba}$ (0.66 MeV), ^{54}Mn (0.84 MeV), ^{65}Zn (1.12 MeV), ^{60}Co (1.33 MeV), ^{52}V (1.44 MeV), ^{28}Al (1.78 MeV), and ^{24}Na (2.75 MeV). No deviations from linearity could be observed within $\pm 1\%$. The energy resolution of the two scintillation counters at 0.66 MeV was found to be 8% and 7.8%, respectively.

2.6. Rabbit System

The short half-lives of ^{20}F ($T_{1/2} = 11.46$ sec) and ^{28}Al ($T_{1/2} = 2.28$ min) require a fast transportation of the foils from the reactor. For this reason a pneumatic rabbit system was developed by which the irradiated

FIG. 7. Set-up of Rabbit System.

foils were automatically placed between the detectors within 3 sec after irradiation.

The schematic set-up of the rabbit system is shown in Fig. 7. The construction of the rabbit allows an automatic mechanical opening when entering the deloading position after irradiation, so that the foil is falling directly between the two detectors. The rabbit is fabricated of stainless steel with a minimum amount of material around the foil to avoid distortion of the neutron spectrum. Within the reactor the rabbit is guided by an experimental tube of 42-mm diameter fixed in an empty rectangular subassembly (2 in. by 2 in.). Foil irradiations can be performed in different axial and radial positions of the core. Outside the reactor core the guiding tube is made of plastic for maximum flexibility of the system.

The electronic control of the system is coupled to the γ -counting equipment in a way that each irradiation cycle runs preprogrammed. After loading into the rabbit the foil will be shot into the reactor. During the preset irradiation time, of the order of two or three half-lives (40 sec for F and 5 min for Al) the γ background is measured and stored. At the end of irradiation time the rabbit is shot back, the foil is deloaded into the detector station, and counting is started. The time between departure of the foil from the reactor core and start of counting is also measured in order to correct for decay. During the counting period a new foil is loaded into the rabbit, and at the end of counting the cycle starts again. The cycles were repeated until a statistical accuracy of about 1% was achieved.

Details of the mechanical construction and the electronic control system are given in Ref. 20.

2.7. Conclusions

Special results of measurements performed with F, Al, and Na in the fast core of the fast-thermal coupled Argonaut Reactor STARK are described elsewhere⁽⁹⁾ so that here more general aspects will be discussed

concerning the applicability of resonance activation detectors in fast zero-power assemblies. The main conclusion from the experiments conducted so far is the verification that the foil materials investigated are fairly sensitive to changes in the low-energy part of a fast reactor neutron spectrum. However, to take full advantage of this sensitivity to measure reaction rates or ratios and compare the results to those of other methods, several further improvements are necessary. This is especially important in view of resonance parameters and cross sections for ^{19}F and ^{27}Al . The partial lack of these data is the main disadvantage for the application of resonance activation detectors. Further, the perturbation of foil activation by the rabbit and its empty guiding tube inside the core have to be investigated more thoroughly. The influence of scattering and capture resonances in structural materials (Fe, Al, Na, etc.) have also to be taken into account. In combination with foils of fissile and fertile materials, additional ratios of reaction rates can be measured (i.e., $^{24}\text{Na}/^{235}\text{U}$, $^{24}\text{Na}/^{238}\text{U}$, $^{10}\text{F}/^{235}\text{U}$, $^{10}\text{F}/^{28}\text{Al}$, etc.), which are a helpful tool for the investigation of spectral and heterogeneity effects in sodium- and steam-cooled fast reactor systems.

3. Determination of Reactor Parameters from Neutronic Noise by Measurement of Probability Distributions

3.1 Introduction.

To determine the prompt-neutron decay constant, the reactivity, and the absolute fission rate in fast zero-power assemblies, several experimental techniques for reactor noise analysis have been investigated and developed in parallel. This was thought to be necessary because of the great importance of β/l as integral fast reactor parameter. In large plutonium-fuelled systems special consideration has to be given to the large neutron source strength from spontaneous

fissions and (α, n) reactions. For the well-known Rossi- α technique, for example, problems arise from this because the ratio of correlated to uncorrelated detector counts decreases with increasing fission rate in the system.

The contribution of uncorrelated detector events, which is always the background in the signal, is eliminated by a new two-detector cross-correlation method.⁽²¹⁾ The detector output currents are passed through band filters, and are multiplied and averaged; the output signal, the cross-power spectral density, is free from the uncorrelated background noise. This technique has been successfully applied to measurements with STARK and also recently during the SEFOR critical experiments in ZPR-3.

The method described here is based on the direct determination of probability distributions. The probabilities p_n are investigated that n processes occur in a fixed time interval of length T . A classical example is the Poisson distribution. This distribution law is valid for completely uncorrelated events as is the case, for instance, in radioactive decay. The existence of correlated events leads to probability distributions which deviate from the Poisson law. The deviation is the stronger the more correlated processes are present. Reaction chains in nuclear systems and their observation by detectors are responsible for the occurrence of correlated events in the detector output signals. The time behaviour of the correlated detector events is closely related to the time behaviour of the reaction chains. This is the basic reason why reactor parameters can be derived from this as well as from other methods of noise analysis.

3.2. Theory

A common theoretical basis for neutronic noise-analysis experiments⁽²²⁾ has been used to derive formulas for the experimental technique described here. Only the results important for experimental application will be mentioned. There are two possibilities to characterize a probability distribution: (a) by its moments, i.e., mean value, variance, etc.; (b) by its probabilities p_n . Both ways will be discussed.

From the theoretical treatment⁽²²⁾ only the calculation of moments is possible. In the point reactor model approximation, neglecting delayed neutrons, the equation to calculate the reactor physics parameters is given by

$$\frac{\overline{n^2}(T) - \bar{n}^2(T) - \bar{n}(T)}{T^2} = \frac{\bar{n}(T)}{T} \frac{W\chi_2 k^2}{\alpha l^2} \frac{e^{-\alpha T} - 1 + \alpha T}{(\alpha T)^2}, \quad (3.1)$$

with the following symbols:

n = number of counts registered in a time interval of length T ;

T = length of a time interval;

W = detector efficiency;

k = multiplication constant;

l = prompt-neutron lifetime;

$$\chi_2 = \frac{\nu(\nu - 1)}{\bar{\nu}^2} = \text{nuclear parameter};$$

ν = number of neutrons per fission;

β = effective fraction of delayed neutrons;

$$\alpha = \frac{1 - k_p}{l} = \frac{1 - k(1 - \beta)}{l} = \text{prompt-neutron decay constant}.$$

The numerator on the left-hand side of (3.1) contains the difference between the measured variance $(\overline{n^2} - \bar{n}^2)$ and the variance of the Poisson distribution, which is equal to $\bar{n}$. A rearrangement of (3.1) leads to the equivalent formulation

$$\frac{\overline{n(n-1)}(T) - \bar{n}^2(T)}{\bar{n}(T)} = W\chi_2 \frac{k^2}{(1 - k_p)^2} \cdot \left[1 - \frac{1 - e^{-\alpha T}}{\alpha T} \right], \quad (3.2)$$

which has been used for the evaluation of the experimental results. This expression gives the deviation from the reduced variance (normal variance divided by mean value) of the Poisson distribution and results solely from correlated detector events. The uncorrelated background noise is, therefore, already eliminated. Equation (3.2) consists of two distinct terms on the right-hand side. The second one within the brackets represents the mean time behaviour of correlated processes. It approaches unity as $T \rightarrow \infty$. In this case the time interval of observation is so long that all correlations of detector events have been taken into account. The first term contains the dependence on detector efficiency, reactivity, and nuclear constants. Near the delayed critical state it is

$$\begin{aligned} \left(\frac{\overline{n(n-1)} - \bar{n}^2}{\bar{n}} \right)_{T \rightarrow \infty} &= W\chi_2 \frac{k^2}{(1 - k_p)^2} \\ &= W \frac{\chi_2}{\beta^2} \left[\frac{\alpha_c}{\alpha} \right]^2. \end{aligned} \quad (3.3)$$

The efficiency W can be calculated from (3.3) if $\alpha_c = \beta/l$ and α have been determined from the measurements and if β is known from reactor calculations. The absolute fission rate F in the system is given by

$$F = \frac{C}{W}, \quad (3.4)$$

where C is the mean counting rate in the experiment. The reactivity follows from the relation

$$-\rho[\mathcal{F}] = \frac{\alpha}{\alpha_c} - 1. \quad (3.5)$$

The second way, the direct calculation of probabilities $p_n(T)$, has been investigated by Zolotukhin and Mogilner.⁽²³⁾ They used the technique of probability-generating functions to calculate the distributions. The probability-generating function $E(T, u)$ is defined by

$$E(T, u) = \sum_{n=0}^{\infty} p_n(T) u^n, \quad (3.6)$$

where

T = length of a time interval;

u = auxiliary variable.

The main purpose for the introduction of a probability-generating function is to obtain an easy computation of probabilities and moments. From (3.6) it follows that

$$p_n = \frac{1}{n!} \left. \frac{d^n E(T, u)}{du^n} \right|_{u=0}; \quad (3.7)$$

$$\text{variance} = \left. \frac{d^2 E(T, u)}{du^2} \right|_{u=1} + \left. \frac{dE(T, u)}{du} \right|_{u=1} - \left\{ \left. \frac{dE(T, u)}{du} \right|_{u=1} \right\}^2. \quad (3.8)$$

In the point reactor model approximation and neglecting delayed neutrons the probability-generating function is given by

$$\ln E(T, u) = \bar{n}(T) (u - 1) \left\{ \frac{1 - \varphi}{\hat{z}(u - 1)} - \frac{2}{\alpha T \hat{z}(u - 1)} \ln \left[1 + \frac{(\varphi - 1)^2}{4\varphi} (1 - e^{-\alpha T \varphi}) \right] \right\}, \quad (3.9)$$

where

$$\varphi = \sqrt{1 - 2\hat{z}(u - 1)}; \quad (3.9a)$$

$$\hat{z} = \frac{W\chi_2 k^2}{(1 - k_p)^2}, \quad (3.9b)$$

with other symbols as already defined. The probabilities p_n can principally be derived from (3.9) using relation (3.7). The probability p_0 , that nothing is registered during the time interval T , follows as

$$p_0(T) = \exp \left\{ -\bar{n}(T) \frac{2}{\varphi_0 + 1} \cdot \left[1 + \frac{2}{(\varphi_0 - 1)\alpha T} \ln \left(1 + \frac{(\varphi_0 - 1)^2}{4\varphi_0} (1 - e^{-\alpha T \varphi_0}) \right) \right] \right\}, \quad (3.10)$$

with

$$\varphi(\hat{z}, u = 0) = \varphi_0 = \sqrt{1 + 2\hat{z}}.$$

If p_0 and $\bar{n}$ are measured for different values of T , the parameters α , k , and W can be found. The computation of probabilities with higher values of n , however, is almost impossible in this manner. For the limiting case $\alpha T \rightarrow \infty$, the probability-generating function (3.9) reduces to

$$E_{\infty}(u) = \exp \left[\frac{\bar{n}}{\hat{z}} \left(1 - \sqrt{1 - 2\hat{z}(u - 1)} \right) \right]. \quad (3.11)$$

By use of (3.7) and repeated differentiation, the following recursion formula can be set up:

$$p_{n\infty} = p_{m+i} = \frac{\bar{n}}{(1+m)\sqrt{1+2\hat{z}}} \sum_{j=0}^m p_{m-j} \cdot \frac{(2j-1)!}{j!} \left[\frac{\hat{z}}{1+2\hat{z}} \right]^j. \quad (3.12)$$

The reduced variance was also computed by use of (3.8) to compare the results of both theoretical ways. The expression found was identical to (3.1).

The connection between the distribution of probabilities p_n and the mean value, variance, etc., is given by the following relations:

$$\bar{n}(T) = \sum_{n=1}^{\infty} p_n(T) n; \quad (3.13)$$

$$\bar{n}^2(T) = \sum_{n=1}^{\infty} p_n(T) n^2; \quad (3.14)$$

$$\overline{n(n-1)}(T) = \bar{n}^2(T) = \left[\sum_{n=2}^{\infty} p_n(T) n(n-1) \right] - \left[\sum_{n=1}^{\infty} p_n(T) n^2 \right]. \quad (3.15)$$

In our method the probabilities $p_n(T)$ are determined for different values of T by the "probability distribution analyzer," described below. From the p_n the left-hand side of (3.2) is computed by use of relations (3.13) to (3.15). By fitting the expression (3.2) to the experimental curves the reactor parameters can be derived.

3.3. Probability Distribution Analyzer

An experimental set-up was built for the direct measurement of the probabilities $p_n(T)$, where n can range from 0 to 127. This probability distribution analyzer is shown schematically in Fig. 8. The operating principle is as follows: Pulses from one or more detectors are fed into the input A of a fast-acting electronic step switch, which proceeds by one step for each incoming pulse. Pulses from a time marker terminating the length T of the time interval are fed into input B. According to the position of the electronic switch,

a gate is opened for the time pulse, which gives a signal to the scaler connected to this output. If, for instance, 4 counts came in during the interval, output 4 of the switch opens gate 4 so that the pulse from the time

FIG. 8. Schematic Diagram of Probability Distribution Analyzer.

FIG. 9. Block Diagram of Data-acquisition and -reduction System.

FIG. 10. Measured Probability Distributions — and Equivalent Poisson Distributions ----.

marker delivers a signal into scaler 4. That means, that 4 counts came in during this time interval. The step switch is reset and the probability distribution analyzer is opened for a new cycle. After a sufficiently long time, the run for one T -value is finished and the probabilities $p_n(T)$ are obtained directly by dividing the number R_n of counts in scaler n by the number R_T of counts in the time interval scaler, giving

$$p_n = \frac{R_n}{R_T}. \quad (3.16)$$

The performance of the analyzer is checked by the monitor scaler, because the relation

$$R_D = \sum_{n=1}^{127} n R_n \quad (3.17)$$

must be satisfied for the number R_D of counts in the monitor scaler. An additional check is given by

FIG. 11. Deviation from the Reduced Variance of the Poisson Distribution as Function of Interval Length.

$$R_T = \sum_{n=0}^{127} R_n. \quad (3.18)$$

The analyzer was designed for high counting rates (20 Mc per sec), a necessity for experiments in plutonium-fuelled fast assemblies. Therefore, a large number of detector counts can be handled in relatively short measuring times according to the statistical accuracy required. Because many measurements have to be performed in a variety of reactor assemblies, automatic components are built into the experimental set-up. The complete data-acquisition and -reduction system is shown in Fig. 9. By means of a tape reader a sequence of T -values, repetitive or successive, is given into the time marker. Within the range from 10 μ sec to 640 msec, 51 time-interval lengths may be chosen in discrete steps. This is sufficient for measurements in fast and thermal systems.

An experiment is usually run in the following way: According to the assembly investigated and its state of reactivity, the sequence, magnitudes, and number of time-interval lengths are programmed. Usually one T -value is repeated several times in order to see and to exclude drift effects of the reactor or electronics. The measuring time per interval length depends on the counting rate and the statistical accuracy required. For each value of T (and its repetitions) the complete probability distribution is measured. If the run is finished the scaler outputs are punched on tape and written out for control before the next run is started. All data of the experiment are collected and fed into an electronic computer for evaluation according to rela-

tions (3.13) to (3.15) and (3.2). A least-squares fit is used to derive the prompt-neutron decay constant and the quantity $\hat{z}$ given by (3.3) and (3.9b), respectively.

3.4. Results

Measurements have been performed with the fast-thermal Argonaut Reactor STARK⁽⁹⁾ from which a few typical results shall be discussed here for further explanation of the technique. In Fig. 10 measured probability distributions (full line) are shown for five different values of the time interval length T . In addition, equivalent Poisson distributions (dashed line) are included for comparison. The distributions are taken from an experiment for which high detector efficiency was available. The quantity $\hat{z}$, which gives the maximum ratio of correlated to uncorrelated detector events, was 1.64 and α was 139 sec⁻¹. Due to the large $\hat{z}$ values, the deviation from the Poisson law is clearly visible, increasing with larger values of T . For very large T ($T \rightarrow \infty$) the asymptotic probability distribution was measured during another experiment and evaluated by use of relation (3.12). The reactor parameters derived agreed very well to those found by other means.

The normal way of evaluation, however, is done by use of (3.2). An example for the experimentally determined deviation from the reduced variance of the Poisson distribution as function of interval length T is shown in Fig. 11. In this experiment the detector efficiency was much lower and, therefore, $\hat{z}$ was 0.34, which means that only 34% (maximum) of correlated

TABLE 3. ESTIMATED DATA FOR APPLICATION OF PROBABILITY DISTRIBUTION ANALYSIS IN FAST PLUTONIUM-FUELLED REACTOR ASSEMBLIES

$\bar{\rho}$ (\$)	F (sec ⁻¹)	$P(W)$	W	ξ (sec ⁻¹)	$T = 1/\alpha$ (sec)	$\bar{n}$
0.1	$3.12 \cdot 10^{10}$	$9.8 \cdot 10^{-1}$	$6.5 \cdot 10^{-6}$	$2.0 \cdot 10^5$	$1.35 \cdot 10^{-4}$	27.3
0.2	$1.56 \cdot 10^{10}$	$4.9 \cdot 10^{-1}$	$7.7 \cdot 10^{-6}$	$1.2 \cdot 10^5$	$1.24 \cdot 10^{-4}$	14.9
0.3	$1.04 \cdot 10^{10}$	$3.3 \cdot 10^{-1}$	$9.1 \cdot 10^{-6}$	$9.4 \cdot 10^4$	$1.15 \cdot 10^{-4}$	10.8
0.4	$7.80 \cdot 10^9$	$2.4 \cdot 10^{-1}$	$1.1 \cdot 10^{-5}$	$8.2 \cdot 10^4$	$1.07 \cdot 10^{-4}$	8.7
0.5	$6.25 \cdot 10^9$	$2.0 \cdot 10^{-1}$	$1.2 \cdot 10^{-5}$	$7.5 \cdot 10^4$	$9.95 \cdot 10^{-5}$	7.5
0.7	$4.45 \cdot 10^9$	$1.4 \cdot 10^{-1}$	$1.6 \cdot 10^{-5}$	$6.9 \cdot 10^4$	$8.78 \cdot 10^{-5}$	6.1
1.0	$3.12 \cdot 10^9$	$9.8 \cdot 10^{-2}$	$2.1 \cdot 10^{-5}$	$6.7 \cdot 10^4$	$7.45 \cdot 10^{-5}$	5.0
1.5	$2.08 \cdot 10^9$	$6.5 \cdot 10^{-2}$	$3.3 \cdot 10^{-5}$	$6.9 \cdot 10^4$	$5.97 \cdot 10^{-5}$	4.1
2.0	$1.56 \cdot 10^9$	$4.9 \cdot 10^{-2}$	$4.8 \cdot 10^{-5}$	$7.5 \cdot 10^4$	$4.97 \cdot 10^{-5}$	3.7
2.5	$1.25 \cdot 10^9$	$3.9 \cdot 10^{-2}$	$6.5 \cdot 10^{-5}$	$8.1 \cdot 10^4$	$4.28 \cdot 10^{-5}$	3.5
3.0	$1.04 \cdot 10^9$	$3.3 \cdot 10^{-2}$	$8.6 \cdot 10^{-5}$	$8.9 \cdot 10^4$	$3.73 \cdot 10^{-5}$	3.3
3.5	$8.91 \cdot 10^8$	$2.8 \cdot 10^{-2}$	$1.1 \cdot 10^{-4}$	$9.6 \cdot 10^4$	$3.02 \cdot 10^{-5}$	2.9
4.0	$7.80 \cdot 10^8$	$2.4 \cdot 10^{-2}$	$1.3 \cdot 10^{-4}$	$1.0 \cdot 10^5$	$2.98 \cdot 10^{-5}$	3.0

Values used for calculations: $\hat{z} = 0.35$; $\chi_2 = 0.8$; $\beta = 3.5 \cdot 10^{-3}$; $s_0 = 3 \cdot 10^7$ sec⁻¹; $\alpha_c = 6.7 \cdot 10^3$ sec⁻¹.

events were present in the detector output signals. At least 10^5 detector counts were analyzed per T value indicated in the curve given in Fig. 11. Other experiments were conducted with STARK under various conditions to determine the range in which the method of probability distribution analysis can be applied successfully.

3.5. Conclusions

The most important conclusion from the experiments performed with STARK is that even values of $\hat{z}$ around 0.3 allow an accurate (ca. 2–3 %) determination of the prompt-neutron decay constant α by this method. On the basis of this result all necessary data for the application in plutonium-fuelled fast assemblies can be estimated. This has been done, and the results are listed in Table 3. A fast reactor assembly with 300 kg plutonium (92 % ²³⁹Pu and 8 % ²⁴⁰Pu) was chosen. The data used for computation are: neutron source strength due to spontaneous fissions and (α, n) reactions, $s_0 = 3 \cdot 10^7$ neutrons per sec; prompt-neutron decay constant at delayed critical, $\alpha_c = 6.7 \cdot 10^3$ sec⁻¹; effective fraction of delayed neutron, $\beta = 3.5 \cdot 10^{-3}$; the nuclear parameter $\chi_2 = 0.8$. With the constant value $\hat{z} = 0.35$, the following quantities have been calculated in the reactivity range from 0.1 to 4 dollars: fission rate F in the system, reactor power P , minimum detector efficiency W , mean detector counting rate ξ , order of magnitude for time interval length T , and mean value of detector counts per interval, $\bar{n}$. As can be seen from the estimated results, measurements of α can be conducted between reactivities of ca. 0.2 and 3 dollars. No measurement is possible at delayed critical due to the relatively large power. The prompt-

neutron decay constant at delayed critical has, therefore, to be determined by extrapolation. Because of the high counting rates the time necessary for the measurement of one α -value should not exceed 20 min. Finally, it can be concluded that the technique of probability distribution analysis is applicable to determine reactor physics parameters in fast uranium- and plutonium-fuelled reactor assemblies.

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Discussion

Mr. Cohn (ANL): Would your probability method of noise measurement be capable of yielding significant results with lower values of detector efficiency than are usable for the more conventional type of noise measurements; if so, about what gain could be realized in this region?

Mr. Stegemann: The technique of calculation here is similar to the actual experiment used in the two detector cross-correlation experiment described elsewhere, and what has been concluded from those results is also applicable as a general conclusion for these results. The main quantity of interest is the minimum detector

efficiency, derived from the minimum required ratio of correlated to noncorrelated detector events. This minimum ratio depends not only on efficiency, but also on the effective delayed-neutron fraction of the system, so you must calculate it for the specific assembly to be analyzed. We have found from this technique that the minimum ratio of correlated to noncorrelated detector events must be at least 30%. The efficiencies we calculated for this last plutonium assembly are in the range from 10^{-6} to 10^{-5} detector count per fission in the system, which can be achieved.

Mr. Karam (ANL): Did you investigate neutron self-shielding in the activation of thick foils of U-238?

Mr. Stegemann: We used a simple model to calculate the gamma-X-ray self-attenuation factor. The neutron-shielding results have been included in this one factor, and we didn't separate the effects. In our opinion the neutron self-shielding is negligible, although it's already contained in this factor, because there are many more captures in the higher energy region than in the resonance region.

Mr. Karam: What time-gate width did you use in your coincidence circuit in the gamma-X-ray technique? Did you vary the time-gate width and, if so, what were your findings?

Mr. Stegemann: We changed the time-gate width from 20 nanoseconds up to 1 microsecond, and we found no difference in the results.

An International Comparison of Fission Detector Standards

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Presented by P. I. Amundson

ABSTRACT

Parallel-plate fission chambers presently in use in Idaho Division of Argonne National Laboratory for measurements of fission ratios have been compared with selected chambers from the Zebra reactor group at Winfrith, the fast reactor group at Cadarache, and the critical experiments group at Los Alamos. An attempt has been made to tie these measurements in with foils used for measurements of differential cross sections at Los Alamos and Aldermaston.

At present, comparisons with Winfrith have been made using U-235, U-233, and Pu-239 chambers. Measurements were made both in the Zebra reactor at Winfrith and in Idaho Division's Fast Source Reactor. Results show agreement among fission ratios of better than 1%.

Comparisons of Winfrith U-233, U-235, U-238, and Pu-239 chambers with similar chambers used in the Vera reactor at Aldermaston have been made in Zebra. The mass calibrations are in agreement within the measurement errors of 1-2%.

The comparison with Cadarache has been limited to a direct U-235 chamber comparison. The initial work showed a 25% discrepancy. A French re-appraisal of calibration techniques resulted in a quoted mass only a few percent different from that estimated from a comparison with the Idaho Division and Winfrith chambers.

The comparison with Los Alamos values involves fission-ratio comparisons in the Flattop reactor and a U-235 thermal comparison in the thermal column of the Water Boiler reactor with standards used by various groups measuring differential cross sections.

1. Introduction

This paper describes some of the initial measurements which are part of a continuing program to es-

tablish standards of comparison between fission ratios reported by fast reactor critical facility groups throughout the world. The program in progress involves comparisons of fissionable materials on foils used in parallel-plate fission chambers to measure fission ratios in fast critical assemblies.

The measurement of ratios of fission cross sections in fast critical assemblies provides an important check on the accuracy of measurements of microscopic cross sections. The experimental accuracy of such measurements is typically 1-2%. Since the masses of fissile coatings in chambers used by different laboratories are often determined by different methods, it is essential that inter-laboratory consistency of measured fission ratios should be established by exchange of chambers. The demonstration of consistency provides the analyst with confidence in the measurements in different reactors.

Although the field is rapidly expanding, the majority of fast critical assemblies in the United States has been constructed in Argonne's ZPR-3, -6, and -9, in the United Kingdom in Zebra at Winfrith and in Vera at Aldermaston. It is for this reason that the initial comparison measurements were made between fission chambers used in ZPR-3 and Zebra. Other comparisons have been made between chambers from Winfrith and Aldermaston, between Idaho chambers and the Fast Critical Assemblies Group at Los Alamos, the fast reactor group at Cadarache, and a general tie-in has been attempted with several groups measuring differential cross sections through a thermal inter-comparison technique used by Dr. Monson Thorpe at Los Alamos.

2. Winfrith-Argonne, Idaho Comparisons

The completed phase of the Winfrith-Argonne comparisons involved the isotopes U-235, U-233, and Pu-239. The initial comparisons were limited to these isotopes to eliminate the uncertainties introduced by the comparison of threshold fissile isotopes in chambers with different wall thicknesses and configuration.^(1, 2)

Ideally, a chamber comparison involves the direct measurement of the response of two similar chambers.

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This has been done. However, since there are differences in construction, plating techniques, and plated masses of the Winfrith and Argonne, Idaho fission chambers, the data have also been presented in the form of measured fission ratios. This form of data presentation tends to cancel out many of the intrinsic errors of a direct comparison technique. It also should be pointed out that a comparison of fission ratios measured by these foils in the chambers in which they are normally used is the most precise method of generating a comparison between the data published in the ZPR-3 and Zebra reports.

Comparisons were made in the Zebra reactor at the same time as a duplicate set of chambers were being

compared in the Argonne Fast Source Reactor in Idaho. The Argonne Fast Source Reactor is a versatile neutron source not plagued with the busy programming schedule of ZPR-3, thus allowing a considerably longer period of time to make measurements of the required precision.⁽³⁾ The comparisons at Winfrith were made in Cores 4A, 5A, and 5B.

The fission chambers used in Idaho Division and at Winfrith are of parallel-plate geometry. The Winfrith chambers are sealed and filled with 98% argon and 2% nitrogen at one atmosphere (0.5 atm in the case of Pu-239). The chambers sent to England for comparison were an unsealed type operating as a flow chamber using a mixture of 90% argon and 10%

FIG. 1. Absolute Fission Chambers.

FIG. 2. Zebra Parallel-plate Fission Chamber.

methane. The Idaho chambers used for the comparison in the Argonne Fast Source Reactor are of a heavy-wall, sealed construction using a filling of 90% argon and 10% methane. The differences in the design of the chambers are indicated in Figures 1 and 2.

In Zebra the fission chambers were compared in back-to-back geometry at the core midplane. The measurements in Core 4A were made at the core center, whereas those in the mixed critical cores 5A and 5B were made in the buffer zone. A special light brass frame was designed to create a cavity, on the core center-line, which rigidly located the chambers and provided support for the core plates above the cavity. The Idaho chamber was placed in the top position in each case and was close to the upturned base of the Winfrith chamber although not actually resting on it. An axial hole through the plates allowed connection of cables to external electronics and a gas-flow tube to a cylinder of 90% argon and 10% methane for the Idaho chamber. The relative fission rate in pairs of chambers containing the same principal isotope was measured using fast integral counting systems. The power level during each run was monitored by taking counts from a set of pulse fission chambers used on the instrumentation channels. This measurement of power level was necessary to allow conversion of the results from the direct comparison of similar fission chambers to fission ratios typical of, say, Pu-239 and U-235 chambers from one of the groups. The operating bias level for each fission chamber was chosen by examination of the differential bias curve displayed on a pulse-height analyzer. The selected level was transferred to the scaler discriminator by using a pulse generator with an output typical of that from the pre-amplifier. As a check on this transfer a bias curve was taken on the scaler discriminator to confirm the chosen operating point.

The comparison conducted in the Argonne Fast Source Reactor (AFSR) in Idaho utilized the inner

end of the beam-hole facility. Previous experiments have shown that the count rate in this position for non-threshold fission chambers is independent of the wall thickness of the chamber within the range of wall thicknesses normally used.⁽⁴⁾ Although all chambers compared fall into the thickness range satisfying the above criterion, a steel closed-end sleeve was constructed to position accurately the Winfrith chambers relative to the Idaho chambers and also to equalize the chamber wall thickness. A measurement of the axial flux gradient at this position using a U-235 chamber gave a value of approximately 5% per centimeter. The accuracy of re-positioning these chambers against the end stop in this facility is estimated to be 0.05 cm.

Since very fast electronic equipment, such as that used at the Zebra facility, was not available, the data were all collected on a multichannel analyzer to allow for a more reliable extrapolation to the zero bias for the more heavily loaded Winfrith chambers.

A monitor channel was utilized in order to compensate for drifts in reactor power. This monitor channel counted pulses from a U-235 fission chamber placed in the bottom of a vertical access hole above the AFSR core. The position of this monitor was fixed so that during the conduct of the experiments its sensitivity to reactor flux did not vary. The electronic circuitry of the monitor was similar to that used for the collection of data from the chambers in the beam hole except that a scaler replaced the multichannel analyzer. Prior to each day's operation an integral bias curve experiment was performed on the monitor channel and the bias (discriminator) point set to allow maximum monitor stability. For each run the AFSR linear power trace and monitor count were compared to determine if the two measuring systems showed good relative agreement. In order to eliminate dead-time corrections, the reactor was operated at a low power (usually 1 watt), resulting in count rates that varied

between 12,000 and 20,000 counts per minute. The analyzer collected data during 5-min live-time periods while the monitor was run for 5 min of clock time. This procedure usually resulted in the monitor being turned off for a few seconds before the analyzer due to blanked-out memory storage time in the analyzer. Since the reactor was operated at a nearly constant power, this procedure introduced a random experimental error small enough to be ignored.

The measured relative fission rates must be corrected for (1) fission fragment pulses lost below a bias level if an integral count is taken, (2) alpha-particle pulses recorded in the lower channels of a differential count, and (3) fission fragments completely absorbed in the fissile coating. To sum precisely the pulses due to fission fragment variation, either below the bias point or in the multichannel-analyzer data, an energy distribution of the degraded fission fragments must be assumed. The number of fission fragments with energies comparable with the alpha-particle background was relatively higher for the Winfrith chambers, due to the heavier plating (2 mg compared with 400 μg for the Idaho chambers) which also contained more alpha activity. The extrapolation to zero fission fragment energy for the Zebra chambers was based on a careful examination of the pulse-height distribution from a U-238 fission chamber, which is not significantly complicated by an alpha-particle background.

The corrections to the count rates for fission pulses which were not detected, i.e., the extrapolations to zero pulse height, were typically 2% for the Zebra chambers and 1% for the ZPR-3 chambers.

Corrections for fission fragments completely absorbed in the foils were made for both sets of chambers using values calculated from the results of work by White and Perkin⁽⁵⁾ at Aldermaston, and include a linear absorption term based on a mean range for fission fragments of $7.5 \pm 0.5 \text{ mg/cm}^2$ in U_3O_8 and a and a nonuniformity factor of 0.8–1.8. The nonuni-

TABLE 1. DIRECT COMPARISON OF CHAMBERS CONTAINING THE SAME PRINCIPAL ISOTOPE

Principal Isotope	Reactor Used	Fission-rate Ratio, Winfrith/Idaho	Calibrated Mass Ratio, Winfrith/Idaho	Fission Ratio/Mass Ratio
U-233	Zebra	2.320 ± 0.030	2.32 ± 0.03	1.000 ± 0.018
	AFSR	4.090 ± 0.060	4.08 ± 0.05	1.002 ± 0.018
U-235	Zebra	4.835 ± 0.050	4.75 ± 0.07	1.018 ± 0.017
	AFSR	2.665 ± 0.040	2.65 ± 0.04	1.005 ± 0.021
Pu-239	Zebra	3.740 ± 0.040	3.70 ± 0.04	1.010 ± 0.014
	AFSR	4.664 ± 0.070	4.69 ± 0.05	0.994 ± 0.018

TABLE 2. FISSION-RATE RATIOS CALCULATED FROM WINFRITH AND IDAHO CHAMBERS IN ZEBRA CORES 4A AND 5A, AND IN AFSR

Core	Ratio	Chambers Used	Count-rate Ratios	Corrected Fission Ratios
Zebra 4A	$\frac{\text{U-233}}{\text{U-235}}$	Winfrith $\frac{2}{8}$	1.631 ± 0.004	1.445 ± 0.021
		Idaho $\frac{23}{13B}$	3.357 ± 0.008	1.455 ± 0.020
Zebra 4A	$\frac{\text{Pu-239}}{\text{U-235}}$	Winfrith $\frac{14}{8}$	0.909 ± 0.002	0.974 ± 0.015
		Idaho $\frac{33}{13B}$	1.174 ± 0.003	0.981 ± 0.014
Zebra 5A	$\frac{\text{Pu-239}}{\text{U-235}}$	Winfrith $\frac{14}{8}$	0.922 ± 0.002	0.988 ± 0.015
		Idaho $\frac{33}{13B}$	1.203 ± 0.003	1.006 ± 0.014
AFSR	$\frac{\text{U-233}}{\text{U-235}}$	Winfrith $\frac{15}{6}$	1.606 ± 0.006	1.587 ± 0.017
		Idaho $\frac{\text{K16}}{\text{K5}}$	1.043 ± 0.005	1.583 ± 0.024
AFSR	$\frac{\text{Pu-239}}{\text{U-235}}$	Winfrith $\frac{11}{6}$	1.678 ± 0.005	1.494 ± 0.016
		Idaho $\frac{\text{K21}}{\text{K5}}$	0.962 ± 0.004	1.501 ± 0.022

formity factor is defined as σ/t from the formula $(\sigma^2 + t^2)/2Rt$, where R is a mean range of fission fragments, t is a foil thickness, and σ is the normal variation of thickness about the value t . The corrections applied to the Idaho and Winfrith chambers for fission fragments absorbed were typically 1–2% and 4.5%, respectively. Table 1 gives the results of a direct mass comparison between the Winfrith and Idaho chambers. The results for U-233 and Pu-239 are all within the experimental error limits; however, there is a possible small discrepancy between the measurements at Winfrith and those at Argonne, Idaho, for the U-235 chambers. The results of prime interest in this comparison are the fission ratios measured relative to U-235, which are given in Table 2. The measured ratios all lie within the limits of experimental error.

3. Winfrith-A.W.R.E. Foil Comparisons

The Winfrith fission chambers have also been compared with A.W.R.E. foils of the same principal isotope in Vera fission chambers by measuring relative fission rates in Zebra.⁽⁶⁾ The A.W.R.E. foils are from

a set used by White at Aldermaston to measure differential microscopic fission cross sections. Both sets of foils were painted from the same batches of fissile material by the same technique. The A.W.R.E. foils were, however, approximately 1.7 times as thick as the Winfrith foils. The comparison procedures were as described in the preceding paragraphs for the Idaho-Winfrith comparison in Zebra. Both sets of foils were also calibrated for mass by the use of low-geometry alpha assay. The results are given in Table 3. Again the results are within the limits of experimental error. Destructive techniques for mass assay were used on some foils, yielding results in good agreement with the low-geometry alpha assay.

4. Winfrith Painted vs Electrodeposited Foils

A new set of electrodeposited foils has recently been made by the Zebra group at Winfrith.⁽⁶⁾ These new foils have a more uniform coating and a lighter mass of fissile material (900 micrograms) than the earlier painted foils. The final mass of fissile material on each of the Winfrith and A.W.R.E. foils is determined by alpha assay, although for the electrodeposited foils the estimate of the mass of uranium deposited, based on the strength of the electrolyte and the amount left in solution after the deposition, was within the statistics of the alpha assay. The mass determined by low-geometry alpha assay was used to maintain consistency with quoted masses for the painted foils. Direct comparisons of fission ratios were made in Zebra Assembly 5J between the pair of new U-235 chambers and a U-235 chamber containing a painted foil. The results are given in Table 4 and are compared with the alpha-assay ratio. The agreement for the painted foil in chamber No. 8 and the more uniform electrodeposited foil in chamber No. 18 gives confidence in the thickness corrections applied to the painted foil.

5. Cadarache-Idaho Comparison

A single U-235 chamber from the Fast Reactor physics group in Cadarache (see Fig. 3) was compared in AFSR during the period of the Winfrith-Idaho cham-

TABLE 4. COMPARISON OF FISSION AND MASS RATIOS OF WINFRITH U-235 ELECTRODEPOSITED FOILS WITH PAINTED FOIL No. 8

Isotope	Chamber Number	Fission-rate ratio	Mass Ratio (α Assay)	Fission-rate Ratio/Mass Ratio
U-235	19/18	1.007 ± 0.011	1.005 ± 0.006	1.002 ± 0.011
U-235	8/18	2.476 ± 0.025	2.476 ± 0.010	1.000 ± 0.011

FIG. 3. Ionization Fission Chamber, Cadarache.

ber comparisons. The techniques used for this chamber are as described in the preceding text. Analysis of the data relative to the Winfrith and Idaho U-235 chambers indicated a discrepancy of $25\% \pm 4\%$. Further work at Cadarache indicated that the original calibration was in error. A reassessment of chamber mass by three independent techniques now yields a calibration of 222 micrograms of total uranium as opposed to the AFSR estimate, using the Winfrith and Idaho chambers as standards, of 230 micrograms.

6. Flattop-Idaho Comparison

Several of the Los Alamos critical systems have spectra of a fundamental nature⁽⁷⁾ in that the cores and reflectors are of simple composition and geometry. A large number of spectral indices have been measured in these critical assemblies. For these reasons an attempt was made to correlate measurements in ZPR-3 with measurements in the U-235-fueled version of the Flattop reactor.

Flattop has a spherical, highly enriched uranium core, approximately 6 in. in diameter, with a surrounding 8-in.-thick depleted uranium blanket. Penetration to the core center is available through a machined 1-in.-diameter access hole through both the blanket and core.

Since none of the available fission chambers in Idaho Division would fit the Flattop penetration, special foils and chambers were designed for this intercomparison. The foils were made by electrodepositing the

TABLE 3. COMPARISON OF A.W.R.E. FOILS IN VERA CHAMBERS WITH WINFRITH (ZEBRA) CHAMBERS

Isotope	A.W.R.E. Foil Number/Zebra Chamber Number	Fission-rate Ratios	Mass Ratios (α Assay)	Fission-rate Ratio/Mass Ratio
U-238	3.4/10	1.71 ± 0.03	1.73 ± 0.03	0.988 ± 0.020
U-235	1.7/8	1.77 ± 0.02	1.783 ± 0.009	0.993 ± 0.014
Pu-239	6 7/3	0.794 ± 0.004	0.794 ± 0.004	1.000 ± 0.007
U-233	IV/2	0.403 ± 0.003	0.0400 ± 0.002	1.008 ± 0.009

TABLE 6. THERMAL COMPARISON OF IDAHO AND LOS ALAMOS U-235 FOILS IN WATER BOILER THERMAL COLUMN

Foil	Reported Mass Ratio	Relative Correction to Zero Pulse Height	Correction for Totally Absorbed Fragments	Corrected Count-rate Ratio	Count Ratio/Reported Mass Ratio
Idaho 7 Los Alamos Standard	1.207 ± 0.012	+0.3%	+0.12%	1.1928 ± 0.006	0.988 ± 0.012
Idaho 38 Los Alamos Standard	1.215 ± 0.012	+2.0%	+0.78%	1.199 ± 0.006	0.987 ± 0.012

bers allows close examination of the low-energy tail of the fission distribution. This examination in turn allows some empirical insight into the amount of fission fragments which lose some or all of their energy within the foil deposit. The data from this comparison is given in Table 6 along with the corrections indicated by Thorpe's empirical evaluation of the low-energy tail of the fission distribution.

The data tend to indicate that the Idaho Division U-235 foils are, on the average, 1% low in mass relative to Thorpe's foil. Previous comparisons between Thorpe's foil and foils used by White, Diven, and Stein showed that the latter were also an average 1% low in mass relative to Thorpe's foil. These results suggest close agreement between the Idaho Division foils and the foils used by the cross-section-measurement groups concerned. The closeness of this indirect comparison should not be overemphasized, however, because of the difficulty in interpreting accurately the number of fission fragments partially or completely absorbed in the foils used for measurements of differential cross sections. The analysis of the low-energy tail for these foils is difficult because of the rather high plating mass necessary to achieve reasonable count rates when measuring in a mono-energetic beam of low intensity.

8. Errors

The published error values for the plated masses of the Winfrith U-233 and Pu-239 chambers are $\pm 0.3\%$, whereas those for the U-235 chamber and all of the Idaho chambers are given as $\pm 1\%$. These values were used wherever applicable in the data analysis for Tables 1 and 2. Although in Table 1 the assumed mass ratio of the principal isotope (fourth column) utilizes these published errors, the real error is probably less than this since the Idaho chambers were chosen as being representative of the average of all of the Idaho fission chambers as determined in a separate experiment.

In all cases the total number of counts analyzed was large enough so that one standard deviation amounted to no more than 0.2% of the total counts.

In the AFSR measurements a positioning error of approximately 0.1% for similar counters and of 0.2% for the direct comparison is taken into account. The higher error limit for the direct comparison is indicated due to possible positioning errors in the use of the steel sleeve with the Winfrith counters.

Although for the AFSR measurements no specific errors have been assigned to the estimated correction to zero pulse height or the estimated foil absorption, an error limit of $\pm 1\%$ has been assigned to the composite of these corrections in the experimental-ratio column of the direct comparison table, Table 1. These errors are diminished somewhat in the fission-ratio calculations, since the errors associated with these corrections will, for the most part, be systematic errors inherent in the method of correction. For the direct comparison measurements in Zebra, the errors for the estimated correction to zero pulse height and the error associated with foil absorption are given as 1% in each case.

The errors quoted for the comparison of the Cadarache U-235 chamber with those of Winfrith and Idaho Division ($\pm 4\%$) are dictated almost entirely by systematic positioning errors. In a preliminary run, a rather large discrepancy between the chamber calibrations was noted, and consequently no attempt was made to achieve high reproducibility of positioning in these comparisons.

For the measurements in Flattop, the quoted errors are largely a composite of chamber and monitor statistics as well as probable calculational errors for the correction factors applied. No positioning errors are indicated because of the rather accurate fit in the Flattop penetration and because the chambers were virtually packed in place by the enriched uranium shim pieces around the chamber stem.

The errors associated with the thermal comparison by Thorpe are a composite of his quoted $\pm 1\%$ uncertainty in the mass of his standard foil, counting statistics, an assigned error for the determination of the fission statistics, and an assigned error for the determination of the fission fragments partially and completely absorbed in the foil.

TABLE 7. SUMMARY OF U-235 COMPARISONS

Foil and Measurement Identification	Winfrith Foils, Zebra and AFSR Comparison	Idaho Foils, Zebra and AFSR Comparison	M. Thorpe Case-25 Foil, Water Boiler Thermal Column	A.W.R.E. Foils, Water Boiler Thermal Column Comparison	Winfrith Foils, Zebra Comparison with A.W.R.E. Foil (1.7)
Relative Mass Numbers	1.0000 ^(a)	0.9886	1.0015	0.9886 ^(b)	0.9960
Discrepancy from 1.000		-1.14%	+0.15%	-1.14%	-0.4% ^(c)

^(a) Winfrith foils No. 8 and 6 have arbitrarily been taken as a base and have been assigned a value of 1.000 for the Winfrith-Idaho comparison.

^(b) Data obtained from M. Thorpe.

^(c) This discrepancy can be called the error of closure.

9. Discussion of Results

The results of the comparison measurements in Zebra at Winfrith and in AFSR at Idaho Division indicate that, provided the chambers used by each group are consistent with those actually exchanged, the fission ratios calculated from the fission-rate data for U-235, U-233, and Pu-239 chambers as measured by a set of Winfrith chambers will agree with the equivalent fission ratios measured using a set of Idaho chambers to an accuracy of about 1%.

A comparison of Winfrith chambers with those used in Vera, which contained foils prepared by White at Aldermaston for his measurements of fission cross section, showed that the calibration of the two sets of chambers agreed to within 1-2%.

Comparison of the ratios measured in Flattop show generally good agreement. The ratio $\bar{\sigma}_f \text{U-238} / \bar{\sigma}_f \text{U-235}$ measured with the Idaho chambers is compared with a recent measurement reported by Grundl utilizing miniature fission chambers calibrated in a 2.5-meV monoenergetic beam of neutrons. This measurement thus has a direct tie-in with the ratio measurements recently reported by Stein. The void correction is an estimate based on calculations of the flux perturbation due to the void necessary for measuring with the Los Alamos quad-type fission chambers. The void is slightly larger and has a different height-to-diameter ratio for a present measurement; consequently, the assigned error to this correction is quite high.

The $\bar{\sigma}_f \text{Pu-239} / \bar{\sigma}_f \text{U-235}$ and $\bar{\sigma}_f \text{U-233} / \bar{\sigma}_f \text{U-235}$ ratios have only moderately good agreement. The worth of this comparison is also in doubt since the reported Los Alamos ratios were measured prior to 1954 and represent a selected average of measurements made by several techniques. No attempt has been made to investigate the discrepancy between these ratios because the original data are not available.

The comparisons performed by Thorpe in the Water Boiler thermal column indicate a 1% discrepancy between two representative Idaho Division foils and his standard (case 25) foil. The consistency of the two

measurements tends to indicate that this discrepancy is real.

A survey of the available data in this paper shows that there is a closed loop of intercomparisons that can be analyzed for the case of U-235. This loop utilizes the measurements in Zebra, AFSR, the thermal comparisons in the Water Boiler reactor, and back to other comparisons in Zebra. The foils used in this circuit are the Winfrith painted foils, the Idaho Division electrodeposited foils, Thorpe's vacuum-deposited foil, White's painted foils, and again, the Winfrith painted foils. Table 7 is a summary of this closed group of measurements. For this type of comparison a relative starting point is necessary, and here the Winfrith foils have been taken as an arbitrary unit of one to give in each case the fractional deviation from the Winfrith foils. It is immediately obvious from this table that the error of closure in this loop is very small. This small error does tend to indicate that all measurements were made with care.

10. Conclusions

Sets of U²³³, U²³⁵, and Pu²³⁹ fission chambers used for measurements of cross-section ratios in the zero-energy fast reactors Vera, Zebra, and ZPR-3 have been compared. The results indicate that fission-ratio measurements made in Vera, Zebra, and ZPR-3 assemblies for U²³³, U²³⁵, and Pu²³⁹ should be consistent to an accuracy of about 1%.

Comparison of some fission-chamber standards used in fast reactor assemblies with those of the fission cross-section-measurement groups also reveals consistency of mass calibration to an accuracy of about 1-2%.

Acknowledgments

The authors wish to gratefully acknowledge the able assistance of the following in performing these experiments: Dave Barton of the critical assemblies group in Los Alamos who assisted in the measurements in Flattop; Monson Thorpe who performed the intercali-

bration of U-235 foils in the graphite column of the Water Boiler reactor at Los Alamos; and M. H. McTaggart of the Vera group who supplied the Vera chambers for the comparisons in Zebra.

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Errors and Optimization of Neutron Importance Measurements*

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ABSTRACT

The methods of measuring neutron importance are briefly discussed, and the errors arising in the use of the reactivity oscillator method are discussed in detail. Relative measurements with an accuracy of $\pm 0.5\%$ are shown to be feasible. Optimum reactivity conditions are derived. The application of this method to measurements of neutron importance in polyethylene-moderated test cells in a fast reactor test region is described and the results compared with theory. This comparison indicates faults in the calculational technique used.

1. Introduction

Neutron importance measurements provide a check for reactor calculations that is complementary to measurements of the flux spectrum. The flux spectrum is determined by the nuclear properties of the reactor at energies above the measurement energy, whereas the importance is determined by properties at energies below the measurement energy. Neutron importance does not exhibit the large variation with energy that the flux spectrum does, however, and so importance measurements must achieve relatively high accuracy in order to produce significant differences between the experimental and the calculated values.

Several methods have been used to measure neutron importance. The earliest measurements used subcritical multiplication.^(1, 2) Later investigators have used reactivity oscillators with isotopic neutron sources^(3, 4) and absorbers,⁽⁵⁾ and the dynamic reactivity technique to obtain the spatial variation of importance.^(3, 4) Differences in the reactivity of fissile samples have also been used to determine neutron importance.⁽⁶⁾ Of these methods, the reactivity oscillator provides the greatest accuracy, and so the sources of error in this method will be examined in detail.

In the reactivity oscillator method, a neutron source (or absorber) is oscillated between the center of the

reactor and an exterior position, and the reactor response to this square-wave perturbation is analyzed to determine the importance of the neutrons emitted (or absorbed) by the sample. The use of absorbers, while involving problems outside the scope of this paper, offers considerable promise in extending the energy range and usefulness of importance measurements.

2. Sources of Error in Reactivity Oscillator Method

The sources of error involved in the use of the reactivity oscillator method are listed in Table 1, with the magnitudes achieved in current experiments, and the values to which these errors may be reduced in the near future. The sources of error are:

2.1. Determination of Power Oscillation Amplitude

Proper design of the reactivity oscillator system can minimize or eliminate the effect of sample transfer time and drift on reactor power level. Source-oscillation measurements are usually performed at much lower power levels than normal reactivity measurements, and the characteristics of the system may differ significantly. The effect of the variation of phase shift with subcriticality should be checked, as should be the time response and gamma compensation of the power monitor.

The accuracy of the determination of the power-oscillation amplitude is governed primarily by the source strength, detector sensitivity, and measurement time. Considering the range of these variables, an accuracy of 0.1% appears feasible.

2.2. Determination of Relative Reactor Power

A relative measure of the reactor power must be provided for correction of the reactivity of the source material. The major cause of error in this measurement lies in the gamma sensitivity of the power monitor, since the gamma-to-neutron ratio in the reactor may be different at very low power than at the normal operating power. However, with the proper detector this error may be made negligible.

* This work was supported by the U. S. Atomic Energy Commission.

2.3 Correction for Variation of the Transfer Function with Subcriticality

In addition to the change in phase shift mentioned above, which usually is a small effect, the amplitude of the reactor transfer function decreases with increasing subcriticality. The effect of this correction is shown in Fig. 1, where the amplitudes of the power oscillations produced by oscillating a Po-Be neutron source are plotted against the average reactor power level. (Additional measurements were made at 14 W to provide for correction for the reactivity of the source material.) Extrapolation to zero power yields the amplitude due to the source neutrons above. The amplitude determined by the uncorrected points is about 6% lower than that determined by the corrected points. The subcritical reactivity must be measured, and this is the major source of error here; the potential error is probably about 0.1%.

2.4. Correction for Material Reactivity

Part of the power oscillation is due to reactivity changes caused by the source material. Although this effect can be reduced by use of inactive sources to compensate for the material reactivity change,⁽³⁾ some

TABLE 1. ERRORS IN OSCILLATOR METHOD

Source of Error	Present Range	Potential
	(Percentage Error in Final Result)	
a. Oscillation Amplitude	0.5-1.0	0.1
b. Relative Reactor Power	0.1-0.5	0.01
c. Transfer Function	0.5-1.0	0.1
d. Reactivity Correction	0.1-0.5	0.01 (or 0)
e. External Source	0.7-1.5	0.1 (or 0)
f. Relative Source Strength	1.0-2.0	0.5
g. Source Decay	0.1-0.5	0.01
Resultant for Relative Measurements	1.4-3.0	0.5
h. Absolute Source Strength	1.0-7.0	0.5-2.0
i. Absolute Reactor Power	10-20	7.0
j. Delayed-neutron Fraction	1.0	0.5
Resultant for Absolute Measurements	10-21	7.0-7.5

FIG. 1. Variation of Oscillation Amplitude with Reactor Power Level, Showing Effect of Transfer-function Correction.

difference may remain. After correction for the transfer-function variation, the observed amplitude follows the relation

$$A_o = A_s + a_m P,$$

where

A_o = observed amplitude;

A_s = amplitude due to source neutrons;

a_m = amplitude factor due to source material;

P = reactor power.

By making measurements over a wide power range, we can determine the source amplitude A_s . This requires that the response of the reactivity oscillator system be identical over this power range. If the material-reactivity difference is small, and measurements are made over a wide power range, this error can be made negligible. It is of interest to note that this correction can be made zero by use of an accelerator neutron source which is pulsed rather than mechanically oscillated.

2.5. Correction for the Effect of Neutrons from the External Source

Since the reactivity oscillator measures the difference in importance between the source at the center and at the exterior position, a correction must be made for the importance at the exterior position. This effect can be made small by moving the source far from the reactor and interposing shielding material. If an accelerator source is used, this correction can be zero.

2.6. Determination of Relative Source Strength

Accurate determinations of the relative source strengths are essential for intercomparison of importance values measured with different sources. Several calibration methods are suitable, with the manganese sulfate total absorption bath providing the greatest accuracy. However, some care must be exercised when using sources containing beryllium or fissionable isotopes to account for multiplication in the source of source neutrons returned from the surrounding medium. In beryllium, the $(n,2n)$ reaction may increase the effective emission rate of a source in a heavy scatterer, whereas the strong moderation provided by hydrogen prevents this effect from being a problem in the manganese sulfate solution. An approximate calculation of this effect, based on singly scattered return neutrons, indicates that the emission rate of a 2-cm-diameter beryllium (α,n) source may be increased by 2% over its free-space value when placed in a large volume of uranium.

Similar effects must be considered for large spontaneous fission sources. These effects must be calcu-

lated for the specific source and surrounding materials under consideration.

An additional problem with a beryllium source is the production of photoneutrons by gamma rays from long-lived fission products, Ba^{140} - La^{140} in particular, remaining from previous high-power operation of the reactor. This effect may be investigated experimentally by measuring the reactivity of a beryllium sample over the same power range used in the source measurements.

2.7. Correction for Decay of the Source

During the course of measurements, the active isotope in an isotopic neutron source will decay, resulting in a decrease in the source emission rate. For (α,n) sources this is a relatively small change during the measurement time, but for photoneutron sources this decay can be quite large. If the power-oscillation amplitude consists of both a source-neutron component and a source-material component, then this correction cannot be applied directly, but must be applied to the source-neutron component only. This can be done iteratively so that the error due to this correction is limited to the uncertainty in the decay of the active isotope.

In order to permit direct comparison of the experimental results with calculations, without any intermediary standard, it is also necessary to know the absolute source strength, the absolute reactor neutron production rate, and the delayed-neutron fraction. The error in reactor power calibration, however, is so large as to obscure the variation in the measurements. Perhaps 7% suggested as the potential accuracy is overly optimistic, since most power calibration involves conversion from neutron flux to fission rate to neutron production rate, or heat production rate to fission rate to neutron production rate. Noise analysis provides a more direct measure and this method may be capable of higher accuracy.

3. Optimization of Neutron importance measurements

The variation of the signal-to-noise ratio with subcritical reactivity has been investigated theoretically. The neutron noise in a reactor is proportional to the square root of the power, while the signal, the source-neutron component of the oscillation, is independent of power. If the reactor is subcritical, the power is determined by the source strength (in the absence of other sources) and the subcritical reactivity, and so a relation can be derived for the signal-to-noise ratio as a function of subcritical reactivity:

$$s/n \sim S_e |A(\omega, k)| / \sqrt{S_e / (1 - k)},$$

where

Fig. 2. Variation of Signal-to-Noise Ratio with Subcriticality.

s/n = signal-to-noise ratio;

S_e = effective source strength;

$|A(\omega, k)|$ = amplitude of the transfer function as a function of frequency ω and criticality constant k .

For unit source strength this becomes

$$s/n > \sim |A(\omega, k)| \sqrt{1 - k},$$

which is plotted in Figure 2. This was calculated using Littler's equation⁽⁷⁾ with $\beta_e = 0.071 \times 10^{-2}$, $l = 27.8 \times 10^{-6}$ sec, and the U^{235} delayed-neutron parameters of Keepin,⁽⁸⁾ for an oscillation period of 60 sec. Variation of these values is not expected to alter significantly the conclusion that it is unnecessary to operate further subcritical than $\Delta k = -1 \times 10^{-3}$, but that the reactor should be more than -1×10^{-4} subcritical for the bulk of the measurements. This also insures that, with a multiplication of 1000 or more, the fundamental fission distribution is well established and the measurements are not biased by the properties of the region immediately surrounding the source.

4. Measurements in Polyethylene-moderated Test Cells

The reactivity oscillator method was used in conjunction with four isotopic neutron sources (Po-Be,

Po-B¹¹, Po-F, and Po-Li⁷) to measure the variation of neutron importance with energy in a fast reactor test region perturbed by a series of polyethylene-moderated test cells in the multiregion critical assembly at the Epithermal Critical Experimental Laboratory (ECEL).⁽⁴⁾ These test cells were constructed by surrounding the central sample position with polyethylene slabs of varying thicknesses, in an arrangement approaching cylindrical geometry. Due to limitations on the available time and manpower, it was necessary to calculate these perturbed test regions using a diffusion-theory program with a spherical model of the polyethylene blanket. The results of measurements in the unperturbed test region and in test cells with 0.125-, 0.250-, and 0.500-in-thick polyethylene are compared with the calculations in Fig. 3. The sources have not been calibrated accurately, and so the results for the unperturbed core are normalized to the calculated values, thus making all other measurements relative to the unperturbed calculations.

The marked differences between the experimental variation of importance with polyethylene thickness and that calculated can be understood in view of the approximations involved in the calculations. A neutron undergoing a first scattering in the blanket can have a longer path in polyethylene in the real blanket, be-

FIG. 3. Comparison of Experimental and Calculated Source-importance Values in Polyethylene-moderated Test Cells.

cause of the flat slabs used, than in the spherical shell used in the calculations, thus increasing the probability of a second scattering and increasing moderation. This is probably the reason for the greater initial increase in importance shown by the measurements. The calculated importance values become increasingly greater with polyethylene thickness whereas the experimental values exhibit a saturation effect. This is due to the fact that while some neutrons may reach thermal equilibrium in the real blanket, the calculation does not permit up-scattering, and so the probability that the neutron leaves the blanket in the lowest energy group, below 0.009 eV, is greater than in reality. Although the uncertainties are too large for the difference to be significant, there is a suggestion in the Po-Be and Po-B¹¹ results that the Po-Be spectrum is softer than that used in the calculations. Recent measurements by Geiger⁽⁹⁾ indicate a group of neutrons peaking at 0.3 MeV that could account for this difference.

Acknowledgments

The author wishes to acknowledge the assistance of the staff of the ECEL and particularly thanks T. H.

Springer, S. G. Carpenter, and L. A. Mountford for helpful discussions and encouragement.

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Discussion

Mr. Engelmann (Karlsruhe): Do you think you could achieve the same accuracy in plutonium-fueled systems where there is a rather strong inherent neutron source from spontaneous fission?

Mr. Tuttle: Perhaps so. It's a problem of the comparative strength of the source used. The inherent source will shift my optimization curve. Essentially, for the same subcriticality, one will have a smaller signal-to-noise ratio because the noise is greater.

Mr. Engelmann: What was the strength of the sources that you used?

Mr. Tuttle: These were very weak, of the order of 2×10^5 down to 2×10^4 neutrons per second. This is much smaller than I would like to use, since something like 5×10^6 is about optimum for our reactor. Of course, one solves considerable problems by going to higher source strengths.

Mr. Gandini (CNEN): The relative neutron-source ratios may be correlated linearly to any assigned perturbation which makes the system critical. In this case one has the advantage that no strength calibra-

tion is needed. Only the source reproductibility is important, besides, of course, the neutron distribution of the source. Can you comment on this procedure in respect to the distribution of the source neutrons?

Mr. Tuttle: The problem is particularly severe for a Po-Be source. The spectra are fairly well determined for the other three sources. Good measurements have been done particularly for Po-F and the Po-Li⁷ sources. They agree well with the calculations performed for these sources. Po-Be is a difficult one to measure, however. The low-energy group I mentioned is thought to come from inelastic scattering of the alpha particles producing an excited state in Be⁹. Neutron emission from this excited state is not very well understood, and there are several mechanisms by which the emission can take place. I think this technique will be very interesting in the future.

Presented by J. Kockum

Pulsed-source, Rossi- α and Variance-to-mean Measurements Performed at the FRO Reactor*

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ABSTRACT

Pulsed-source and Rossi- α measurements have been performed with three different FRO assemblies, two with copper and one with natural uranium reflectors. The results have been compared with calculations made with a time-dependent S_N code and with applications of a point-reactor and a two-region model. The experiments and calculations show that the time necessary to reach equilibrium in the low-energy part of the neutron spectrum is comparable to the total decay time of the neutron pulse when the reflector consists of light material with low absorption.

The possibility of using the pulsed-source technique for reactivity determinations in this type of reactors has been demonstrated.

β_{eff} separated from other neutron parameters has been measured in one assembly using the variance-to-mean method and an absolute efficiency calibration of a He^3 gas scintillator. The accuracy obtained is about 7%. Means of improving this to about 3% are given. The measured value agrees with theory within the uncertainties.

1. Introduction

In fast reactors with their broad energy spectrum it cannot be taken for granted that the decay of a short pulse of neutrons introduced into the reactor can be described by a superposition of different modes with well-separated decay constants.⁽¹⁾ This is particularly true for systems reflected with comparatively light materials which enhance the number of low-energy neutrons and more or less give rise to two neutron populations of different mean energies that interchange with each other. Since most FRO assemblies have been of this type, it has been thought of value to investigate theoretically the kinetic behaviour of this type of reactor and compare the results with experiments.

Two theoretical approaches have been tried. One

* Experimental work performed at AB Atomenergi, Studsvik, Nyköping, Sweden.

starts from first principles and involves a program for solving the time-dependent Boltzmann equation in the S_N formalism. The other, proposed by Cohn⁽²⁾ and further developed by others,⁽³⁻⁵⁾ is more phenomenologic and divides the neutron population into one part for the core and one part for the reflector; time-independent coupling coefficients between the two populations are postulated.

Experiments have been made mostly by using the pulsed-source technique, in some cases complemented with measurements of the Rossi- α type. The variance-to-mean method has been used in an attempt to measure β_{eff} separated from other neutron parameters in one of the assemblies.

2. Theory

2.1. Time-dependent S_N Calculations

The programme starts with the one-dimensional Boltzmann equation in multigroup formalism:

$$\begin{aligned} \frac{1}{v_k} \frac{d}{dt} N_k(t, r, \mu) + \mu \frac{d}{dr} N_k(t, r, \mu) \\ + \frac{1 - \mu^2}{r} \frac{d}{d\mu} N_k(t, r, \mu) + \sigma_k N_k(t, r, \mu) \\ = S_k(t, r, \mu), \end{aligned} \quad (2.1)$$

where

- v_k = neutron velocity in group k ;
- r = room coordinate;
- μ = cosine of angle between flux and radius vector;
- $N_k(t, r, \mu)$ = neutron flux in group k ;
- σ_k = transport cross section;
- $S_k(t, r, \mu)$ = source term.

The source term is given by the expression

$$S_k = S_k^* + B_k(t, r),$$

where

$$S_k^i = \chi_k \sum_j \frac{1}{2} \int_{-1}^1 (\nu \sigma_f)_j N_j d\mu + \sum_j \frac{1}{2} \int_{-1}^1 \sigma_{j \rightarrow k} N_j d\mu \quad (2.2)$$

and B_k is an external source. The cross sections are assumed spherically symmetric and the source terms isotropic.

The fluxes $N_k(t, r, \mu)$ are calculated assuming that all $N_k(t - \Delta t, r, \mu)$, $S_k^i(t - 2\Delta t, r)$, and $B_k(t - \frac{1}{2}\Delta t, r)$ are known. First, $S_k^i(t - \Delta t, r)$ is calculated by numerical integration of (2.2), using $N_k(t - \Delta t, r, \mu)$. By linear extrapolation, the term $S_k^i(t - \frac{1}{2}\Delta t, r)$ is then derived from $S_k^i(t - 2\Delta t, r)$ and $S_k^i(t - \Delta t, r)$. Equation (2.1) can now be treated separately for each group, and the group index is dropped in the following expressions. The equation is changed to a difference equation by replacing the derivatives by central differences. When evaluating the flux at time t , the scheme proposed by Carlson⁽⁶⁾ is followed, starting with the integration of the inward fluxes at the outer boundary, where they are zero, and going inwards. These fluxes are then used as initial values for the integration of the outgoing fluxes.

In order to be able to follow this scheme, the following numerical functions are defined:

$$\begin{aligned} A_{im} &= N(t_l, r_{i-1/2}, \mu_{m-1/2}); \\ B_{im} &= N(t_{l-1}, r_{i-1/2}, \mu_{m-1/2}); \\ C_{im} &= N(t_{l-1/2}, r_i, \mu_{m-1/2}); \\ D_{im} &= N(t_{l-1/2}, r_{i-1/2}, \mu_m). \end{aligned}$$

Here $\Delta r_i = r_i - r_{i-1}$ and $r_{i-1/2} = r_i - \frac{1}{2}\Delta r_i$, and similarly for the other variables. With the use of these functions the difference version of eq. (2.1) can be written in the vicinity of $(t_{l-1/2}, r_{i-1/2}, \mu_{m-1/2})$ as

$$\begin{aligned} &\frac{1}{v\Delta t} (A_{im} - B_{im}) + \frac{\mu}{\Delta r} (C_{im} - C_{i-1,m}) \\ &+ \frac{1 - \mu^2}{r\Delta\mu} (D_{im} - D_{i,m-1}) \\ &+ \frac{1}{2}\sigma(A_{im} + B_{im}) \\ &= S(t_{l-1/2}, r_{i-1/2}). \end{aligned} \quad (2.3)$$

The following relations of locally linear character have to be assumed:

$$\begin{aligned} \frac{A_{im} + B_{im}}{2} &= \frac{C_{im} + C_{i-1,m}}{2} \\ &= \frac{D_{im} + D_{i,m-1}}{2} \\ &= N(t_{l-1/2}, r_{i-1/2}, \mu_{m-1/2}). \end{aligned}$$

With these approximations the integrations necessary to determine A_{im} as a function of earlier calculated quantities can be made, and it is possible stepwise to calculate the neutron flux as a function of time, energy, radius, and direction.

The programme has been written for an IBM-7090 computer with a capacity large enough to deal with, for instance, 15 energy, 45 radial, and 5 angle intervals. Different time steps have been tried, a typical choice being 10^{-9} sec during the transients in the beginning of the pulse, gradually increasing to $2 \cdot 10^{-8}$ sec when the equilibrium spectrum is being approached. A typical computing time is about one hour.

In the experiments the external source has consisted of 14-MeV neutrons produced at the centre of the core. In the calculations it has been approximated by a source term in the highest energy group spread over a few of the innermost radial intervals. It is believed that this approximation only affects a very short time interval after the shut off of the source, since spectrum equilibrium is rapidly reached for the higher energy groups (cf. Fig. 6).

2.2. Two-region Model

The calculations related in Sect. 2.1 are rather cumbersome and the results not too easy to systematize. We have therefore tried to apply the extension of the point-reactor model proposed by Cohn⁽²⁾ and further developed by others.⁽³⁻⁵⁾

The kinetic behaviour of a reactor with a nonmultiplying reflector is described by the following coupled equations:

$$\begin{aligned} \frac{dN_c}{dt} &= \frac{k_c(1 - \beta) - 1}{l_c} N_c + \frac{k_2}{l_r} N_r \\ &\quad + \sum_1^N \lambda_i C_i + S_c; \\ \frac{dN_r}{dt} &= \frac{k_1}{l_c} N_c - \frac{1}{l_r} N_r; \\ \frac{dC_i}{dt} &= \frac{k_c \beta_i}{l_c} N_c - \lambda_i C_i, \end{aligned} \quad (2.4)$$

with the definitions:

- N_c = number of core neutrons at time t ;
- N_r = number of reflector neutrons at time t ;
- k_c = probability that a disappearing core neutron will give rise to a new fission neutron in the core;
- k_1 = probability that a disappearing core neutron will reappear in the reflector and there be slowed down considerably;

- k_2 = probability that a disappearing reflector neutron will reappear in the core;
 l_c = neutron life time in the core;
 l_r = neutron life time in the reflector;
 C_i = number of delayed-neutron precursors of type i at time t ;
 β_i = effective fraction of fission neutrons appearing as delayed neutrons of type i ;
 $\beta = \sum_i^N \beta_i$, total effective fraction of delayed neutrons;
 λ_i = decay constant of neutron precursors of type i ;
 S_c = number of external source neutrons at time t .

The model presupposes that the spectrum does not change considerably during the time interval of interest so that the lumped parameters (l and k) can be regarded constant. Moreover, the separation between core and reflector neutrons is rather arbitrary. A neutron which returns to the core after only a few elastic collisions in the reflector is regarded as a core neutron all the time since it does not change its energy and the life time considerably.

2.2.1 Application to Pulsed-source Experiments

If the source is supposed rectangular,

$$S_c = \begin{cases} S_0 & 0 \leq t \leq \tau \\ 0 & t > \tau, \end{cases}$$

one can, by Laplace transform technique, show that the prompt part of the solution to (2.4) for $t > \tau$ is given by the following expressions:

$$\begin{aligned}
 N_c(t) &= \frac{S_0 \left(\frac{1}{l_r} - \alpha_1 \right)}{\alpha_1 (\alpha_1 - \alpha_2)} (1 - e^{\alpha_1 \tau}) e^{-\alpha_1 t} \\
 &\quad + \frac{S_0 \left(\frac{1}{l_r} - \alpha_2 \right)}{\alpha_2 (\alpha_2 - \alpha_1)} (1 - e^{\alpha_2 \tau}) e^{-\alpha_2 t}; \quad (2.5) \\
 N_r(t) &= \frac{S_0}{\alpha_1 (\alpha_1 - \alpha_2)} (1 - e^{\alpha_1 \tau}) e^{-\alpha_1 t} \\
 &\quad + \frac{S_0}{\alpha_2 (\alpha_2 - \alpha_1)} (1 - e^{\alpha_2 \tau}) e^{-\alpha_2 t}.
 \end{aligned}$$

Here the α 's are roots of

$$\left(\alpha + \frac{1}{l_r} \right) \left(\alpha - \frac{k_c(1 - \beta) - 1}{l_c} \right) - \frac{k_1 k_2}{l_r l_c} = 0. \quad (2.6)$$

2.2.2. Reactivity Determination

The method proposed by Sjöstrand⁽⁷⁾ has been applied to the model in order to investigate the possibilities of use of pulsed-source experiments for reactivity measurements in actual reactors. It is first to be ob-

served that the experiments are made with a repetitive source of period T . Let the number of prompt and delayed neutrons detected by a counter placed in the reactor be A_1 and A_2 , respectively. Now A_1 is proportional to $\int_0^T S(t) dt$ and A_2 proportional to $\int_0^T \sum_i^N \lambda_i C_i dt$.

From use of eqs. (2.4) one gets

$$\begin{aligned}
 A_2 &= C_2 \int_0^T \sum \lambda_i C_i dt = C_2 \frac{\beta}{l_c} k_c \int_0^T N_c dt; \\
 A_1 &= C_1 \int_0^T S(t) dt \\
 &= C_1 \int_0^T N_c dt \left(\frac{1 - k_c - k_1 k_2}{l_c} \right). \quad (2.7)
 \end{aligned}$$

If $1/\alpha_i \ll T \ll \lambda_i$, the prompt-neutron population has time to die out during T while the delayed neutrons give rise to a constant background, and it is easy to separate A_1 and A_2 . It is seen that

$$\frac{A_1}{A_2} = \frac{C_1}{C_2} \frac{1 - k_c - k_1 k_2}{\beta k_c}. \quad (2.8)$$

Waltar and Ruby⁽⁴⁾ have pointed out that

$$\rho = (k_c + k_1 k_2 - 1)/k_c \quad (2.9)$$

is a consistent definition of reactivity for this model since it reduces to the ordinary reactivity for disappearing reflector ($k_1 k_2 = 0$) and leads to $\alpha_1 = 0$ for a prompt critical reactor ($\rho = \beta$). If $C_1 = C_2$, that is, if the prompt and delayed modes have the same amplitude where the detector is placed, eqs. (2.8) and (2.9) give

$$\rho = -\beta \frac{A_1}{A_2}. \quad (2.10)$$

The model does not say anything concerning the last condition, which must be established experimentally.

2.2.3 Evaluation of Lumped Parameters

In a pulsed experiment it is possible to determine τ , α_1 , α_2 , ρ , and the ratio R between the amplitudes of the two exponentials of $N_c(t)$ (see eq. 2.4).

Some algebra (compare Ref. 5) gives the following expressions for the different parameters:

$$\begin{aligned}
 \lambda_r &= \frac{1}{l_r} = \frac{R(1 - e^{\alpha_2 \tau}) + (1 - e^{\alpha_1 \tau})}{R \frac{1 - e^{\alpha_2 \tau}}{\alpha_2} + \frac{1 - e^{\alpha_1 \tau}}{\alpha_1}}; \\
 \lambda_c &= \frac{k_c(1 - \beta) - 1}{l_c} = \alpha_1 + \alpha_2 - \frac{1}{l_r}; \\
 k_c &= \frac{\frac{\alpha_1 \alpha_2}{\lambda_r \lambda_c}}{\rho + \beta + \frac{\alpha_1 \alpha_2}{\lambda_r \lambda_c} (1 - \beta)};
 \end{aligned}$$

$$l_c = \frac{1 - k_c(1 - \beta)}{\lambda_c};$$

$$k_2 k_1 = l_c l_r (\lambda_c \lambda_r - \alpha_1 \alpha_2).$$

Finally, Ref. (5) gives the following expression for the weighted overall life time:

$$l_0 = \frac{l_c + k_1 k_2 l_r}{1 + k_1 k_2}.$$

2.3. Variance-to-mean Ratio

Bennett⁽⁸⁾ has given the following formula for the variance-to-mean ratio in the point-reactor model:

$$\sigma = \frac{\bar{c}^2 - \bar{c}^2}{\bar{c}} \quad (2.11)$$

$$= 1 + 2Y \sum_1^{N+1} \frac{A_p T(s_p)}{s_p} \left[1 - \frac{1 - e^{-s_p \tau}}{s_p \tau} \right],$$

where $Y = \epsilon[(\bar{\nu}_p^2 - \bar{\nu}_p)/\bar{\nu}_p^2]$ and ϵ is the efficiency of a counter in the reactor which registers c counts during an interval τ . The efficiency is defined as the number of counts per fission in the system. ν_p is the number of prompt neutrons per fission, and the bars denote mean values. Further, s_p are the poles of the zero-power bare-reactor transfer function:

$$T(s) = \sum_1^{N+1} \frac{A_p}{s + s_p}.$$

Bennett also shows that $s_1 = \alpha_1$, the decay constant of prompt neutrons, and that the corresponding term in the sum reduces to

$$Y \frac{(1 - \beta_{\text{eff}})^2}{(-\rho + \beta_{\text{eff}})^2}.$$

The other terms can be made small by choosing τ small.

2.3.1 Two-region Model

In order to take the reflector into consideration, the transfer functions for core and reflector were derived starting from eqs. (2.4) with $S = 0$. In the usual way a varying perturbation δk_c around the steady state $\Delta k_c = 1 - k_c$ is introduced, causing variations $(N_{c0} + \delta N_c)$, $(N_{r0} + \delta N_r)$, and $(C_{i0} + \delta C_i)$. By Laplace transform technique it can be shown that the transfer functions are given by

$$T_c(s) = \frac{\delta N_c(s)}{N_{c0} \delta k_c}$$

$$= \frac{1 - s \sum_1^N \frac{\beta_{i \text{ eff}}}{s + \lambda_i}}{s \left[l_c + \sum_{i=1}^N \frac{\beta_{i \text{ eff}} (1 - \Delta k_c)}{s + \lambda_i} \right] - \frac{\lambda_r}{s + \lambda_r} k_1 k_2 + \Delta k_c} \quad (2.12)$$

$$T_r(s) = \frac{\delta N_r(s)}{N_{r0} \delta k_c} = \frac{\lambda_r}{s + \lambda_r} T_c(s)$$

$$= \sum_{p=1}^{N+2} \frac{A_{rp}}{s + s_p}.$$

Two of the poles to the equations (called s_1 and s_2) are of the order of $\lambda_r \gg \lambda_i$, and by neglecting λ_i compared to these it can be shown that

$$\frac{A_{r1} + T(s_1)}{s_1} + \frac{A_{r2} T_r(s_2)}{s_2} \quad (2.13)$$

$$= \frac{(1 - \beta_{\text{eff}})^2}{[\beta_{\text{eff}} (1 - \Delta k_c) + \Delta]^2}.$$

If $\Delta = 1 - k_c - k_1 k_2 \ll 1$, the corresponding factor for the smallest pole s_{N+2} is

$$\frac{A_{r,N+2} T_r(s_{N+2})}{s_{N+2}} = \frac{1}{\Delta} \frac{1}{(1 - \Delta k_c)} \sum_1^N \frac{\beta_i}{\lambda_i} \frac{1}{2s_{N+2}}. \quad (2.14)$$

Since

$$1 - \frac{1 - e^{-s_p \tau}}{s_p \tau} = \begin{cases} 1 & \text{if } s_p \tau \gg 1; \\ \frac{s_p \tau}{2} & \text{if } s_p \tau \ll 1, \end{cases}$$

equations (2.11, 2.13 and 2.14) give

$$\sigma = 1 + Y \frac{(1 - \beta_{\text{eff}})^2}{[\beta_{\text{eff}} (1 - \Delta k_c) + \Delta]^2}$$

$$+ \frac{Y}{2} \frac{\tau}{(1 - \Delta k_c)} \sum_2^N \frac{\beta_{i \text{ eff}}}{\lambda_i} \frac{1}{\Delta} \quad (2.15)$$

$$+ 2Y \sum_3^{N+1} \frac{A_{rp} T_r(s_p)}{s_p} \left[1 - \frac{1 - e^{-s_p \tau}}{s_p \tau} \right].$$

If the variance-to-mean method is going to be used for determination of β_{eff} , terms which are dependent on other delayed-neutron parameters have to be made small. This can be done for the last two terms by choosing a small τ and a suitable value of Δ .

2.3.2 Dead-time Effects

Equation (2.11) was derived under the assumption that the counter used is free from dead time. Srinivasan and Sahni⁽⁹⁾ have shown that a finite dead time d can be taken into account by adding a term $A(d)$ (if correlated effects are present or not) and by multiplying the correlation terms by factors $B(s_p, d)$. The corrections are given by the expressions

$$\begin{aligned}
 A(d) &= \bar{c} \left[\left(1 - \frac{d}{\tau} \right)^2 (1 + P_{\text{loss}}) e^{-2P_{\text{loss}}} \right. \\
 &\quad \left. - \frac{1}{1 + P_{\text{loss}}} \right]; \\
 B(s_p, d) &= e^{-s_p d} \left(1 - \frac{d}{\tau} \right) [e^{-2P_{\text{loss}}} (1 + P_{\text{loss}})]; \\
 P_{\text{loss}} &= \int_0^d P(t) dt,
 \end{aligned} \tag{2.16}$$

where $P(t)$ is the probability of getting a pulse in the counter in the time interval $(t, t + dt)$ following a pulse at time $t = 0$ in the case of no dead time.

3. Pulsed-source and Rossi- α Measurements

Pulsed-source measurements made with three FRO assemblies are reported in this paper, namely, for Assembly 2 (spherical, 20% enriched, undiluted uranium core with copper reflector), Assembly 4 (cylindrical, undiluted core with natural uranium reflector), and Assembly 3 (cylindrical, 20% enriched uranium core diluted with 29% graphite and provided with a copper reflector). The assemblies are further described in Ref. (10). Measurements of a more preliminary nature were described in Ref. (11). The reactivity at which the measurements were made was changed from -0.7 to -6% either by pulling out control rods or by reloading the cores or reducing the reflectors so as to facilitate comparisons with S_N calculations. The pulsed-source measurements were complemented with Rossi- α -type experiments closer to criticality.

3.1. Experimental Equipment

The neutron source used was a Texas Nuclear Model 9505 Neutron Generator capable of producing down to 50-ns-wide, triangular pulses and down to 1- μ s-wide square pulses with variable frequency. In order to have as symmetrical a geometry as possible, the neutron source was introduced to the centre of the core through a narrow (2.7 cm square) channel. The time-measuring equipment was based on a time-to-pulse height converter whose starting pulse came from a detector placed

at various positions in the assembly and stopping pulse from the pulsing system of the neutron source. Five different detectors were used: a stilbene scintillator with pulse-form γ discrimination, a He^3 gas scintillator, a He^3 -filled proportional counter, a H_2 -filled spherical proportional counter, and a U^{235} fission ionization chamber. The He^3 scintillator consisted of a 1-in.-diameter, 2-in.-high cylindrical Kovar vessel with a plane glass window. The walls were internally coated with MgO and the window with quaterphenyl as a wavelength shifter. The counter was filled with a mixture of 80% He^3 and 20% Xe to a total pressure of 3 atm. The counter was coupled to a 1-in. photomultiplier. The obtained energy resolution was poor ($\sim 50\%$), but it was easy to discriminate between neutrons and gamma rays.

The characteristics of the counters are given in Table 1.

The Rossi- α measurements were made with the same equipment, with the start and stop pulses coming either from the same or from two different detectors. The experimental equipment and the data-handling procedure are further described in Ref. (11).

3.2. Results and Comparison with Theory

Some typical results of the pulsed measurements are shown in Figs. 1-4.

Figure 1 gives results from Assembly 2 when the reactivity was reduced to -3.6% by changing the core diameter from 17.2 to 16.3 cm. The detector was a He^3 scintillator placed in the core and in two positions in the reflector, respectively. The pulse length of the source was in all three cases 1 μ s. The curves have been normalized to the same height at the end of the measured time interval. It is seen that the maximum neutron density (which essentially is the measured quantity for this $1/v$ detector) is reached at the end of the source pulse in the core but at later times in the reflector. It is also obvious that the decay is much faster in the core than in the reflector during the measured time interval. Table 2 shows the results when two exponentials are fitted to the curves and also the reactivities according to eq. (2.10). It was not possible to approximate the core curve with two exponentials at this rather low reactivity, probably due to the existence of higher spatial modes close to the source.

Figures 2-4 show results of in-core measurements with different detectors in the three assemblies. In all cases the results were dependent on the energy response of the detectors. The high-energy recoil detectors give a steeper curve than the low-energy detectors. The effect is less pronounced in the uranium-reflected core (Assembly 4). The results indicate that the spectrum is changing even at long times after the source

TABLE 1. CHARACTERISTICS OF COUNTERS USED

Counter	Spectral Sensitivity	Time Resolution
Stilbene scintillator	0 below 1 MeV	better than 50 ns
He^3 gas scintillator	$1/v$ below 0.5 MeV	better than 50 ns
He^3 proportional counter	$1/v$ below 0.5 MeV	$\sim 2 \mu$ s
H_2 proportional counter	0 below 0.1 MeV	$\sim 2 \mu$ s
U^{235} fission chamber	$1/v$ below 0.2 eV	$\sim 1 \mu$ s

pulse and that at least the low-energy detectors see the returning reflector neutrons before they have caused fissions in the core and lost their identity. The discrepancies at the beginning of the curves are partly due to different time resolutions of the detectors and different source pulse lengths. In each figure the curves are normalized at a suitable time. The results of the fitting of two exponentials are given in Tables 3 and 4 for Assemblies 2 and 3. For Assembly 4 a fit with two exponentials gave only a slight improvement compared to the one-exponential fit, and the uncertainty in α_2 becomes about 50%.

3.2.1 Comparison with Time-dependent S_N Calculations

In Fig. 5 are shown the results of measurements on two versions of core 3 and pertaining calculations. The counter was in both cases a stilbene scintillator placed at the core-reflector boundary. Although the measurements were made in cylindrical cores, the calculations were made for with spheres. When calculating the appropriate sphere radius from the known cylinder

FIG. 1. Results of Pulsed-source Experiments with Assembly 2 (core radius of 16.3 cm), Obtained with a He^3 Gas Scintillator in Different Positions. Source pulse length was 1 μs .

FIG. 2. Results of Pulsed-source Experiments with Assembly 2, Obtained with Different Detectors Placed Within the Core. The reactivity reduced to -3.6% by withdrawing two control rods.

mass, a form factor of 0.961 was applied, obtained from comparison of experimental critical masses. The calculations were made with a 14-group cross-section set (Set 2B of Ref. 10), in which the lowest energy considered is 0.12 keV. This energy cut-off, although appropriate for calculations of critical mass, might explain the fact that the calculated curves have already reached a constant slope at about 8 μs , whereas the experimental curve is slightly concave, even at the end of the time interval shown in Fig. 5.

Figure 6 gives further evidence that the calculations ought to be extended to lower energies. It illustrates the shift in calculated neutron spectrum in the centre of a core with a radius of 20.5 cm ($\rho \approx -7\%$). The ratio of the flux in group i to the flux in group 1 (2.5–10 MeV), normalized to the same quantity calculated for a steady-state critical core, is plotted as a function of time. At higher energies equilibrium is reached about 0.5 μs after the end of the source pulse; at the lowest energy taken into account, this does not occur until after 10 μs . The difference in time behaviour of the two

FIG. 3. Results of Pulsed-source Experiments with Assembly 3, Obtained with Different Detectors Placed within the Core. The reactivity reduced to -0.7% by partly withdrawing one of the start rods.

low-energy groups is rather large, indicating that the time at which the equilibrium spectrum is reached will be delayed further if the calculations are extended to lower energies.

It might also be observed that the equilibrium spectrum differs from the critical spectrum in the low-energy part. Attempts have been made to include lower neutron energies in the calculations, but so far without success, probably due to the fact that it has been done by doubling the group widths instead of increasing the number of groups. In this way the elastic scattering of neutrons out of the groups will be depressed. Since the capacity of the computer memory is fully exploited, the next step will be to increase the number of energy groups at the expense of the number of radial points.

3.2.2 Comparison with the Point-reactor and the Two-region Models

The lifetime has been determined as a function of reactivity for the different cores, and the results are shown in Figs. 7–9. In case of pulsed measurements

with Assemblies 2 and 3, the two-region model has been used (lifetime = l_0 in Sect. 2.2.3); the point-reactor model has been applied to the Rossi- α measurements

FIG. 4. Results of Pulsed-source Experiments with Assembly 4, Obtained with Different Detectors Placed within the Core. The reactivity reduced to -1.2% by partly withdrawing one of the start rods.

TABLE 2. ASSEMBLY 2; CORE RADIUS, 16.3 CM

Detector Position, cm	$\alpha_1 \times 10^{-5}$	$\alpha_2 \times 10^{-5}$	R	$-\rho, \%$
10.4				3.64 ± 0.04
23.5	4.02 ± 0.04	0.721 ± 0.015	15.5	2.88 ± 0.02
44.5	4.68 ± 0.14	0.646 ± 0.003	7.26	2.52 ± 0.02

TABLE 3. ASSEMBLY 2; TWO RODS FULLY OUT: $\rho_{inv} = -3.59\%$

Detector	$\alpha_1 \times 10^{-5}$	$\alpha_2 \times 10^{-5}$	R	$-\rho, \%$
He ³ scintillator	4.77 ± 0.07	1.47 ± 0.06	23.9	3.56 ± 0.15
U ²³⁵ fission chamber	4.25 ± 0.07	1.38 ± 0.08	23.8	3.53 ± 0.04
H ₂ proportional counter	4.16 ± 0.13	1.93 ± 0.20	26.7	

and the measurements in Assembly 4. The reason for this different treatment of similar data is that in the Rossi- α measurements one has to cope with a rather large background of delayed neutrons, making it difficult to make measurements over a time interval large enough to establish two exponential decays with any accuracy. In Assembly 4 the contribution from the second decay is very small and its parameters accordingly difficult to determine. Since the two α values do not differ by more than about a factor of 5 (cf. Tables

TABLE 4. ASSEMBLY 3; ONE ROD PARTLY OUT: $\rho_{inv} = -0.697$

Detector	$\alpha_1 \times 10^{-5}$	$\alpha_2 \times 10^{-5}$	R	$-\rho, \%$
He ³ proportional counter	1.13 ± 0.07	0.55 ± 0.04	4.5	0.707 ± 0.007
U ²³⁵ fission chamber	1.25 ± 0.23	0.74 ± 0.8	1.5	0.665 ± 0.012
H ₂ proportional counter	1.15 ± 0.01	0.34 ± 0.03	88.9	0.705 ± 0.007
Stilbene scintillator	1.22 ± 0.04	0.41 ± 0.11	32.7	0.719 ± 0.004

FIG. 5. Comparison between Experiments (crosses and circles) and Time-dependent S_N Calculations (full and dashed line) Made for Assembly 3 at Two Different Subcritical States.

FIG. 6. Theoretical Investigation of the Approach to Equilibrium Neutron Spectrum in Different Energy Groups. The flux in group i is divided by the flux in group 1 at each time and normalized with the same quantity taken from calculation for a critical steady-state spectrum.

FIG. 7. Lifetime Determinations in Assembly 2 as a Function of Shutdown Reactivity.

2-4), their determination is rather unprecise, which is reflected in the large error bars on the l_0 values. A theoretical value calculated by perturbation theory for cores 2 and 3 is included in the figures.

In Fig. 7 it is shown that the l_0 values increase some-

FIG. 8. Lifetime Determination in Assembly 3 as a Function of Shutdown Reactivity.

FIG. 9. Lifetime Determination in Assembly 4 as a Function of Shutdown Reactivity.

FIG. 10. Block Diagram of the Experimental Setup for the Variance-to-mean Measurements of β_{eff} .

what with decreasing reactivity and that even if the Rossi- α result seems to agree with the pulsed source result at criticality, it is far off already at $\rho = -0.7\%$, indicating that the point-reactor model is inadequate for this assembly. The theoretical value is low, but on the other hand it is calculated with a low energy cut-off at 0.12 keV and the contribution from group 14 is still 6%.

The results for Assembly 3, shown in Fig. 8, indicate

that the point-reactor model is more appropriate for this assembly, for which the core and reflector spectra are more similar than in Assembly 2. On the other hand, if the simple model is going to be used when analyzing the pulsed-source results, the main mode has to be identified not with the slowest, but with the second slowest exponential decay.

In Assembly 4, finally, for which both core and reflector spectra are hard, the point-reactor model seems adequate.

As a conclusion of the pulsed-source measurements reported here, one might say that the point-reactor model gives an approximately true picture of the experiments if the core and reflector spectra are similar and that the two-region model can extend this type of description also to assemblies with large differences in core and reflector spectra. On the other hand, if one wants to take into account fully the shift in spectrum which extends to long times after the start of the pulse, one has to use more sophisticated calculations. The reason that equilibrium is slowly approached in Assemblies 2 and 3 seems to be that neutrons that are slowed down in the reflector take a long time to reach the core and still are of importance when they do so.

3.2.3 Reactivity Determination

Equation (2.10) has been applied to the pulsed-source measurements and compared with multiplication measurements in some cases. From Tables 3 and 4 it is seen that the pulsed-source results agree with the reciprocal count rate results within 5% for in-core detectors. On the other hand, the reactivity according to eq. (2.10) decreases considerably if the detector is placed in the reflector, as is seen in Table 2. These results are confirmed by other similar measurements.

4. Variance-to-mean Measurements

4.1. Equipment

Variance-to-mean type of measurements have been made in an attempt to determine β_{eff} in the FRO Assembly 2 (20.1% enriched uranium core with a 35-cm-thick copper reflector). Similar measurements are reported in Ref. 12. In order to get a variance-to-mean ratio σ significantly differing from 1, the detector efficiency has to be at least of the order of β_{eff}^2 (cf. eq. 2.11). In order to achieve this without introducing large dead-time corrections, the He^3 gas scintillator described in Sect. 3.1 was used.

A simple block diagram showing the experimental set up is given in Fig. 10. The amplifier was double delay-line clipped (50 ns). Because a fast discriminator was lacking, the intrinsic bias of the fast gate was used to discriminate between gammas and neu-

TABLE 5. VARIANCE-TO-MEAN DETERMINATIONS

Run	Power level (W)	τ	$\bar{c} \times 10^2$	$\frac{N}{N-1} \sigma$	$Y \times 10^5$	$A(d)$	$B(s_1d)$	$2V \sum f(s_p)$	$\frac{Y}{2} \frac{\tau}{\sum \frac{\beta_{\text{eff}}}{\lambda_i}} \frac{1}{\Delta}$	$\beta_{\text{eff}} \times 10^3$		Systematic error (%)
											Mean	
1	0.008	0.02	2.76	1.891 ± 0.045	3.83 ± 0.05	-0.003	0.949	0.011	0.188 ± 0.075	7.42 ± 0.32	7.60 ± 0.21	6.5
2	0.008	0.02	2.65	1.807 ± 0.051	3.82 ± 0.05	-0.003	0.949	0.011	0.181 ± 0.072	7.78 ± 0.29		
3	0.002	0.05	1.19	1.717 ± 0.038	3.90 ± 0.07	-0.002	0.954	0.024	0.081 ± 0.032	7.86 ± 0.31		
4	0.002	0.05	1.25	1.763 ± 0.056	3.90 ± 0.07	-0.002	0.954	0.024	0.085 ± 0.034	7.60 ± 0.39	7.73 ± 0.25	3.5
5	uncorr.	1.0	1.16	1.027 ± 0.023	—	10^{-8}	1.0	—	—	—		
									Theoretical		7.84	

trons. The gate, scaler (capable of recording 30 Mc/s), and puncher were controlled by the RAMSES⁽¹³⁾ data-collecting system. RAMSES is controlled by a crystal oscillator giving a negligible uncertainty in the gate-time τ . The recording time of the puncher is about 0.2 sec, determining the maximum speed of data collection. The dead time of the entire system was determined by using it in a positive reactor period measurement and fitting a dead time to the data so as to restore an exponential rise. The obtained value was $(4.0 \pm 0.4) \cdot 10^{-7}$ sec.

4.2. Variance-to-mean Determination

The scintillation counter was placed in the reflector 18 cm from the core surface and was surrounded by 45 g of plexiglass in order to increase the detector efficiency. The measurements were made at two different power levels ($8 \cdot 10^{-3}$ and $2 \cdot 10^{-3}$ W) and with two different gate lengths. Due to the spontaneous fission source present, this means that the assembly was slightly subcritical. In each run the counts in between 2000 and 4000 time intervals τ were recorded. In order to check the equipment a run was made with the counter close to a thermalized Po-Be neutron source. The results of the measurements are summarized in Table 5.

4.3. Efficiency Calibration

The number of fissions in the core per count in the He³ detector was determined with the aid of an absolutely calibrated miniature fission detector with the same percentage of fissionable isotopes as the fuel (20.1% enriched uranium). The detector and calibration procedure is described in Ref. (10).

The fission counter was successively placed at six positions in a narrow channel (6 mm squared) along a radius in the spherical core, and the number of fissions per count in the He³ counter was determined. In order to bridge the large gap in efficiency of the two detectors without making prohibitively long counting times or large dead-time corrections necessary, an intermediate

detector (fission chamber) was used as monitor and placed in the reflector. By a curve-fitting procedure the mean number of fissions in the miniature chamber was determined, and from the known ratio of fissionable atoms in core and counter the efficiency of the He³ counter was calculated. The uncertainty in the calibration is estimated to be $\pm 3\%$, including uncertainties in the integration procedure and in number of atoms in the fission chamber. Since the electronics drift can be expected to be larger for the He³ counter than the monitor chamber, their efficiencies were compared before and after each variance determination.

4.4. Data Evaluation

The recorded data from each run were transferred to an IBM-360 computer and there grouped for determination of the variance-to-mean ratio. From the variance of these ratios the uncertainty in the mean σ_N was derived. It can be shown that

$$\sigma_{N=\infty} = \frac{N}{N-1} \sigma_N,$$

and this correction has been applied.

4.5. Dispersion in $\bar{v}$

The ratio $(\bar{v}^2 - \bar{v})/\bar{v}^2$ was averaged over a theoretical neutron spectrum for Assembly 2. Experimental data, obtained from Hopkins and Diven⁽¹⁴⁾ and from Terrel⁽¹⁵⁾ for U-235, and from Condé and Holmberg⁽¹⁶⁾ for U-238, are summarized in Fig. 11. (In drawing the U-235 line a measured point at 15 MeV not shown in the figure has been taken into account.) The quoted experimental error is about 1% for each point for U-235 and 3% for U-238, resulting in an overall uncertainty of about 2%.

4.6. Dead-time Corrections

In order to determine the probability $P(t)$ in eq. (2.16), Rossi- α measurements were made at the two power levels used. The He³ counter in its reflector posi-

FIG. 11. Experimental Values⁽¹⁴⁻¹⁶⁾ of $(\bar{v}^2 - \bar{v})/\bar{v}^2$ for U^{235} and U^{238} as a Function of Energy.

tion served both as start and stop counter. By extrapolating the decaying curve back to zero time, an approximate value of $P_{\text{loss}} = dP_0$ was obtained and used in the corrections. As seen in Table 5, these are rather insignificant.

4.7. Corrections Due to Delayed Neutrons and Subcriticality

The last term in eq. (2.15) was determined by a numerical solution of the transfer function (eq. 2.12) using delayed-neutron parameters calculated for this core.⁽¹⁷⁾ The parameters l_c , λ_r , and k_1k_2 were obtained from pulsed-source measurements and Δk_c from reactivity measurements to be described. The correction term under discussion is very insensitive to these parameters. As can be seen in Table 5, this correction is rather small and an uncertainty of more than 10% can be tolerated.

The factor Δk enters both the second and third terms in the left member of eq. (2.15). Unfortunately, its magnitude effects the uncertainty in the final value of β_{eff} in the opposite directions through the two terms. By setting the derivatives of eq. (2.4) equal to zero and observing the relationship between neutron population in reflector, counter efficiency, and average counting rate in the detector, one easily derives

$$\Delta k = \frac{Sk_1\epsilon\tau}{\bar{v}\bar{c}}. \quad (4.1)$$

Since the configuration of the reactor is not changed when going from one power level to the other except for a slight movement of the fine control rod, Sk_1 can be regarded as constant. $Sk_1/\bar{v}$ was determined by introducing eq. (4.1) into eq. (2.15) and using mean values of the measured quantities for the two power levels while solving the equations by successive approximation of the second term. The value of the source term

obtained in this way has a rather large uncertainty ($\sim 40\%$) due to the subtraction of the $(\sigma - 1)$ values for the two power levels. This uncertainty is regarded as systematic in the final evaluation of β_{eff} .

4.8. Results and Discussion

The results of the measurements are summarized in Table 5. It is seen that they are consistent with the theoretical value.⁽¹⁷⁾ The largest contribution to the errors comes from the correction term proportional to $1/\Delta k$. The best way of increasing the accuracy seems to be to make a more efficient counter, for instance, by increasing the amount of homogeneous material around it. In this way it should be possible to reduce both the statistical and systematic uncertainties in the final values of β_{eff} to about 2%, where one is helped by the fact that the measured quantity essentially is β_{eff}^2 . The difference in using the two-region or the point-reactor model is not very large, but a somewhat less efficient reflector would be preferable to diminish the slight uncertainties introduced by the factors $B(s_p, d)$ and $(1 - \Delta k_c)$.

Acknowledgments

The authors want to express their gratitude for the technical assistance given by the operating crew and for many fruitful discussions with all members of the physics group at the FRO reactor. Thanks are also due to the numerical group at AB Atomenergi for excellent help in writing auxiliary programmes.

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Discussion

Mr. Becker (RPI): In measurements of reactivity you used the Sjöstrand area method, which tacitly assumes that the prompt and delayed distributions in space are the same. But you found that the lifetime varied with subcriticality. This means that the lifetime between the prompt distribution and the delayed distribution must be different, and so the prompt and delayed space-energy distributions must be different. We found this to be quite important in thermal systems, although probably not as important as in fast systems. It's not clear that it can be taken into

account in as simple a model as you get with a two-region, two-mode model. Have you looked into this system in detail?

Mr. Kockum: We found that the experimental reactivity agrees with this simple model and with the other types of measurements of the reactivity.

Mr. Becker: I thought you said that the reactivity that you calculated in the center of the core was low and that it varied as you went out of the reflector.

Mr. Kockum: Out of the reflector it doesn't agree with other types of measurements, but it is within 5% in the reflector.

Presented by R. J. Huber

Precision Pulsed Measurements with Reflected Fast Critical Assemblies

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ABSTRACT

Pulsed neutron-die-away experiments were performed with three fast critical assemblies having reflectors of relatively low-density material. In all three cases the descriptions of the neutron-die-away curves required two exponentials plus a background term. The data were analyzed in terms of a simple two-region model which considers delayed neutrons as part of the background. The equations were solved by transfer-function techniques. By this analysis it was possible to assign a prompt-neutron lifetime to core neutrons and to the system as a whole. Further, the model used predicts that the die-away curves in the reflector will contain two exponential terms having the same two exponential decay constants as were found in the core, and that the coefficients of the faster will be negative. This was found to be the case.

1. Introduction

The experimental program with the Argonne Zero Power Reactor-3 (ZPR-3) has included the determination of the prompt-neutron lifetime for each of the different assemblies by the Rossi- α technique.⁽¹⁾ For most fast assemblies having uranium-235 as fuel and heavy-element reflectors, this technique works tolerably well. For assemblies with light-element reflectors, interpretation of the data in terms of a single exponential function was not adequate and a more detailed analysis was used. With the advent of large plutonium assemblies, however, the Rossi- α measurement becomes impracticable due to the combined effect of a large spontaneous neutron source and a relatively long prompt-neutron lifetime. Similar measurements, which are described here, for three such assemblies were made by the pulsed neutron technique. This experiment can be carried out in the presence of a large neutron background and it yields better neutron-die-away data than does the Rossi- α experiment.

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2. Equipment and Procedure

The Argonne Zero Power Reactor-3 (ZPR-3) has been described in detail in various references.^(2, 3) Briefly, it is a versatile split-table critical-assembly machine for fast reactor studies. Each half consists of a matrix of square stainless steel tubes into which are placed drawers containing plates of the proper material to simulate whatever composition is being studied. The machine is brought critical remotely by driving one half against the other and inserting control drawers which are essentially the same in composition as drawers occupying adjacent matrix positions.

The particular systems in which the pulsing experiments were performed are identified as ZPR-3 Assembly 46-B,⁽⁴⁾ Assembly 46-C,⁽⁴⁾ and Assembly 47 (SEFOR mockup).⁽⁶⁾ The first two of these were versions of a proposed fast test reactor (FARET). Assembly 46-B represented Pu-U-C fuel having the approximate atom ratio 1:2:3:7 for plutonium, uranium-235, uranium-238, and carbon, respectively. The L/D (length/diameter) ratio was ~ 1.6 . Assembly 46-C represented Pu-U-O fuel having the approximate atom ratio 1:3:3:8 for plutonium, uranium-235, uranium-238, and oxygen, respectively. The L/D ratio was ~ 1.1 .

The general characteristics of the FARET cores are summarized in Table 1. Detailed compositions of the cores and reflectors are given in Table 2. Reflector composition was the same for both assemblies.

The SEFOR mockup was a much larger system, and the simulated fuel contained plutonium, uranium-238, beryllium, carbon, and oxygen. The reflector included simulated control elements of nickel. The general characteristics are given in Table 1 and detailed compositions in Table 3.

All three systems represented tentative power-reactor designs having coolant plenums and support structure. As a result, the reflectors were quite complex and not conducive to detailed calculation. All three differed from most earlier ZPR-3 systems in that the reflectors were of relatively low-density material, such as steel,

TABLE 1. SUMMARY OF CORE CHARACTERISTICS

	46B	46C	SEFOR
Physical size:			
Length (cm)	61.1	50.95	86.5
Diameter (cm)	37.6	46.60	88.2
Volume (liters)	67.8	86.9	528.0
Critical mass (kg):			
Plutonium-239 + 241	42.1	36.8	314.3
Plutonium-240	2.0	1.8	20.0
Total Plutonium	44.1	38.6	334.2
Uranium-235	88.5	113.4	3.0
Uranium-238	150.0	117.8	1633.0
Effective delayed-neutron fraction:	5.55×10^{-3}	5.87×10^{-3}	3.30×10^{-3}
Inhours/\$	300	295	320
Inhours/%	541	503	970

nickel, and sodium, rather than depleted uranium. The SEFOR reflector was particularly "leaky."

All pulsing experiments were conducted in subcritical systems, normally achieved by withdrawing the previously mentioned control drawers. Reactivities were measured in the conventional way by combined use of calibrated control rods (calibrated by positive asymptotic periods) and subcritical neutron multiplication. The reactivities quoted in the results represent Δk_p (not $\Delta k_p/k_p$) and are believed accurate to $\pm 1\frac{1}{2}\%$ relative to each other. However, the reactivities quoted for Assemblies 46-B and 46-C have a common uncertainty of around $\pm 6\%$ due to the delayed-neutron data⁽⁶⁾

on which the control rod calibration is based. For SEFOR, the common uncertainty is $\sim 8\%$.

The neutron bursts for these experiments were obtained from a small commercially manufactured machine (Texas Nuclear Corporation Model 9405). Neutrons are produced by the $T(d,n)\text{He}$ reaction. An important feature of this particular machine is a double beam-deflection system. As a very low duty cycle was used, typically less than $\frac{1}{2}\%$, the ratio of the "off" beam to the "on" beam was kept low to reduce the background neutron level. This ratio here was about 10^{-5} . The neutron detectors used in most measurements were small traverse chambers normally used for the ZPR-3 flux-mapping experiments. The physical parameters of these chambers are summarized in Table 4. Two different time analyzers were employed. Both versions of Assembly 46 used an analyzer having 128 channels of either 0.91- or 1.82- μsec width, and the SEFOR assembly used an analyzer having 512 or 1024 channels of either $\frac{1}{2}$ - or 1- μsec width.

3. Theory and Analysis

These pulsing experiments were analyzed in the framework of a simple two-region model. Among the various investigators who have treated this type of system are Avery,⁽⁷⁾ Cohn,⁽⁸⁾ Kistner,⁽⁹⁾ Borgwaldt *et al.*,⁽¹⁰⁾ and Hosino *et al.*⁽¹¹⁾ There is no essential difference between the formulations found in the first three references and ours except: (1) here we treat an experiment conducted at such a high repetition rate that delayed neutrons can be treated as background, and (2) the differential equations are represented in transfer-function form. Avery's very general equation can be

TABLE 2. COMPOSITION OF FARET SYSTEMS

	Core 46B		Core 46C		Axial Reflector		Fine Radial Reflector		Coarse Radial Reflector I		Coarse Radial Reflector II		Plenum	
	¹⁰²² Atoms/cc	Vol. %	¹⁰²² Atoms/cc	Vol %	¹⁰²² Atoms/cc	Vol %	¹⁰²² Atoms/cc	Vol %	¹⁰²² Atoms/cc	Vol %	¹⁰²² Atoms/cc	Vol %	¹⁰²² Atoms/cc	Vol %
Pu ²³⁹	0.1558	4.3	0.1062	2.9	0.849	33.7	0.212	8.4	0.065	0.7	0.035	0.4	0.445	5.25
Pu ²⁴⁰⁺²⁴²	0.0074		0.0051											
Pu ²⁴¹	0.0007		0.0005											
U ²³⁵	0.3346	7.0	0.3343	7.0										
U ²³⁸	0.5598	11.9	0.3432	7.3										
U ²³⁴⁺²³⁶	0.0048		0.0048											
Na	0.8469	33.6	0.8242	32.8										
O			0.8582	—										
C	1.1096	10.8	0.5104	5.0										
Fe	1.0908	12.9	1.1270	13.3	3.425	40.5	4.690	55.5	7.413	87.5	6.579	77.8		
Cr	0.2929	3.5	0.3026	3.6	0.920	11.0	1.259	15.1	0.119	1.4	0.830	10.0	0.119	1.4
Ni	0.1441	1.6	0.1489	1.6	0.452	5.0	0.620	6.8	0.058	0.6	0.409	4.5	1.923	21.1
Al							0.306	5.0					1.384	23.1
Mn									0.057	0.7	0.028	0.4		

TABLE 3. COMPOSITION OF SEFOR CRITICAL

	Core				Reflector							
	Region 1A		Region 1B		Region 2		Region 3		Region 4		Region 5	
	10^{22} Atoms/cc	Vol %	10^{22} Atoms/cc	Vol %	10^{22} Atoms/cc	Vol %	10^{22} Atoms/cc	Vol %	10^{22} Atoms/cc	Vol %	10^{22} Atoms/cc	Vol %
Pu ²³⁹⁺²⁴¹	0.1480	4.3	0.1513	4.2								
Pu ²⁴⁰	0.0132		0.0072									
U ²³⁸	0.7740	16.3	0.7892	16.6								
U ²³⁵	0.0015	0.03	0.0016	0.03								
Be	0.569	4.6	0.542	4.4								
O	1.566	—	1.528	—								
Mo	0.051	0.8										
C	0.336	3.3	0.336	3.3					0.176	1.7	0.376	3.7
Na	0.663	26.1	0.660	26.0	1.003	39.5						
Al	0.698	11.6	0.772	12.8	0.049	0.8	0.616	10.2	0.533	8.9	1.074	17.8
Ni	0.133	1.5	0.143	1.6	0.226	2.5	6.585	72.2	4.521	49.5	0.369	4.0
Fe	1.014	12.0	1.122	13.2	1.704	20.1	0.464	5.5	1.262	14.9	2.782	32.8
Cr	0.278	3.3	0.301	3.6	0.463	5.5	0.127	1.5	0.335	4.0	0.740	8.9

TABLE 4. COUNTERS USED IN PULSING EXPERIMENTS

Nominal Detector Material	Approximate Isotopic Composition	Counter Number	Approximate Effective Loading (mg)	Approximate Physical Size, Diameter × Length (in.)
U ²³⁵	93.2% U ²³⁵	G	1.5	1/2 × 3
	6.0% U ²³⁸	I	1	1/2 × 3
	1.0% U ²³⁴	Ga	15	
U ²³⁴	89.3% U ²³⁴	K	2.5	1/2 × 3
	7.7% U ²³⁵	N	3	1/2 × 3
	2.9% U ²³⁸			
Pu ²³⁹	94.92% Pu ²³⁹	Q	5	1/2 × 3
	4.75% Pu ²⁴⁰	R	6	1/2 × 3
	0.315% Pu ²⁴¹	S	2	1/2 × 3
	0.010% Pu ²⁴²			
U ²³⁸	<80 ppm of other isotopes	24-5	5	1/2 × 3
		1-in. R-S	100	1 × 6
		2-in. R-S	1000	2 × 10
B ¹⁰ (in BF ₃)	No competing isotopes	Gb	50 cm Hg	3/8 × 3
		R-S	20 cm Hg	1 × 6

reduced to our form when adjustment is made for a fission-free reflector.

The two-region transfer-function model is shown in Fig. 1. The core region is characterized by the time constant λ_c which, in turn, is given by the expression $\lambda_c = \Delta k_c / l_c$, where Δk_c is the reactivity of the core region alone and l_c is the neutron lifetime for the core. The parameter K_1 is the probability that a neutron

FIG. 1. Two-region Model.

which disappears from the core region will enter into the reflector region, and K_2 is the similar parameter for the reflector. The reflector is characterized by the single time constant λ_r .

The transfer function relating input (source neutrons per unit time) to output (core population as a function of time) is

$$G(s) = \frac{1}{s + \lambda_c} \cdot \frac{1}{1 - \frac{1}{s + \lambda_c} \frac{1}{s + \lambda_r} \frac{K_1 K_2}{l_c l_r}} \quad (1)$$

Simplifying and setting $\Gamma = K_1 K_2 / l_c l_r$, we have

$$G(s) = \frac{s + \lambda_r}{(s + \lambda_c)(s + \lambda_r) - \Gamma} \quad (2)$$

which can be expressed in an equivalent form

$$G(s) = \frac{s + \lambda_r}{(s + \alpha_1)(s + \alpha_2)} \quad (3)$$

The response of this system to a delta function input is, neglecting absolute amplitudes,

$$F(t) = Re^{-\alpha_1 t} + e^{-\alpha_2 t}, \quad (4)$$

where

$$R = \frac{\lambda_r - \alpha_1}{\alpha_2 - \lambda_r}. \quad (5)$$

The parameters measured by the experiment are α_1 , α_2 , and the ratio of amplitudes, R . Relationships derivable among the model parameters are

$$\lambda_c = \alpha_1 + \alpha_2 - \lambda_r \quad (6)$$

and

$$\alpha_{1,2} = \frac{\lambda_r + \lambda_c \pm \sqrt{(\lambda_r - \lambda_c)^2 + 4\Gamma}}{2} \quad (7)$$

where

$$\Gamma = \lambda_c \lambda_r - \alpha_1 \alpha_2. \quad (8)$$

An experimental quantity which is usually available in a pulsing experiment is the overall prompt reactivity, Δk_p . This gives the prompt multiplication $1/\Delta k_p$, which is the system response to a steady-state input.

The transfer function $G(s)$ describes the core neutron population as a function of time. The transfer function of the total core-loss rate (multiplication) is

$$\frac{G(s)}{l_c} = \frac{1}{l_c} \frac{s + \lambda_r}{(s + \lambda_r)(s + \lambda_c) - \Gamma}. \quad (9)$$

The equilibrium, or long-time, response of the system to a steady-state input can be calculated from this ex-

FIG. 2. Least-squares Fit to Typical Core-neutron Die-away.

pression. It is

$$\frac{1}{\Delta k_p} = \frac{1}{\bar{l}_c} \frac{1}{\lambda_c - (\Gamma/\lambda_r)}. \quad (10)$$

The experimental value of the multiplication provides a fourth experimentally determined quantity. From these, the model parameters λ_c and λ_r , the product $K_1 K_2$, the core reactivity Δk_c , or various combinations of them may be determined.

Still another quantity of interest that can be determined from the model is the average neutron lifetime for the system. This is the weighted average of core and reflector neutron lifetimes. For this calculation a core neutron is counted when it disappears from the core and the reflector neutrons are counted if they return to the core and affect the kinetic behavior. The overall

lifetime thus calculated is

$$l_0 = \frac{l_c + K_1 K_2 l_r}{1 + K_1 K_2}. \quad (11)$$

The transfer function of the reflector population as a function of time after a δ -function input is

$$G_r(s) = \frac{\left(\frac{1}{s + \lambda_c}\right)\left(\frac{1}{\bar{l}_c}\right)(K_1)\left(\frac{1}{s + \lambda_r}\right)}{1 - \left(\frac{1}{s + \lambda_c}\right)\left(\frac{1}{\bar{l}_c}\right)(K_1)\left(\frac{1}{s + \lambda_r}\right)(K_2)}. \quad (12)$$

Simplification gives

$$G_r(s) = \frac{K_1/\bar{l}_c}{(s + \lambda_c)(s + \lambda_r) - \Gamma}. \quad (13)$$

The two decay constants observed in the reflector

FIG. 3. Least-squares Fit to Typical Reflector-neutron Die-away Observed with BF₃ Chamber.

FIG. 4. Decay Constants Obtained by Fitting $N(t) = A_1 e^{-\alpha_1 t} + A_2 e^{-\alpha_2 t} + \text{Background}$ to Neutron Pulse Die-away Data in Assembly 46B.

FIG. 5. Decay Constants Obtained by Fitting $N(t) = A_1 e^{-\alpha_1 t} + A_2 e^{-\alpha_2 t} + \text{Background}$ to Neutron Pulse Die-away Data in Assembly 46C.

FIG. 6. Decay Constants Obtained by Fitting $N(t) = A_1 e^{-\alpha_1 t} + A_2 e^{-\alpha_2 t} + \text{Background}$ to Neutron Pulse Die-away Data in SEFOR Mockup

should be the same as those obtained with a counter in the core. The impulse response of the reflector population is therefore

$$F_r(t) = -e^{-\alpha_1 t} + e^{-\alpha_2 t}. \quad (14)$$

A counter in the reflector, depending on its nature and location, will detect neutrons belonging to the core population as well as those associated with the reflector. Consequently, the response actually observed in the reflector should be a mix of Eq. 14 with the core response, Eq. 4.

4. Results

Each experiment proceeded in the following manner. A neutron pulse, typically of about 3- μ sec width, was injected into the assembly and the subsequent neutron counting rate measured as a function of time until it decayed into the background rate. Functions of the form

$$F(t) = \sum_{i=1}^N A_i e^{\alpha_i t} + B$$

were fit to the experimental die-away data by a non-linear least-squares iterative technique⁽¹²⁾ (Gauss method). Fits with $N = 1$ (single exponential) were inadequate in all cases. Two exponential fits to data taken in the core and the "round-shouldered" reflector data were satisfactory. Meaningful parameters for a third exponential could not be extracted. Figures 2 and 3 show the results of this least-squares analysis for typical runs in the core and reflector of Assembly 46-C. Both sets of data were taken at the same reactivity. As predicted from the model, the exponential coefficients are the same. They are:

Position	$\alpha_1 \times 10^4$	$\alpha_2 \times 10^4$
Core	8.73 ± 0.69	2.515 ± 0.079
Reflector	9.10 ± 0.34	2.430 ± 0.020

However, only the BF_3 detector in the reflector yielded values for α_1 consistent with the associated run in the core. U^{235} and plutonium counters in the reflector gave large values of α_1 , and large uncertainties in both α_1 and the amplitude of that component of the decay. This is reasonable in view of the original formulation in which we used transfer functions to describe the time behavior of the *populations* of the core and reflector. The isotope B^{10} , with its $1/v$ cross section, can sample the population nearly independently of the changing spectrum during the transients at the beginning of the pulse; the others cannot.

The experimental decay constants (α 's) are shown graphically in Figures 4, 5, and 6.

We have the following values for lifetimes (in sec $\times 10^{-7}$) in the three systems:

	Core Lifetime	Lifetime ($1/\lambda_r$)	Overall Lifetime	Theoretical Lifetime
46-B ($6 \times 10^{-3} < \Delta k_p < 2.7 \times 10^{-2}$)	2.65 ± 0.20	140 ± 20	3.50 ± 0.20	$2.71^{(13)}$
46-C ($6 \times 10^{-3} < \Delta k_p < 2.6 \times 10^{-2}$)	2.95 ± 0.15	150 ± 20	3.90 ± 0.20	$2.15^{(13)}$
SEFOR ($4 \times 10^{-3} < \Delta k_p < 2.25 \times 10^{-2}$)	6.0 ± 0.3	220 ± 30	7.0 ± 0.4	$6.0^{(14)}$
SEFOR ($\Delta k_p = 1.4 \times 10^{-2}$ by reflector removal)	5.2 ± 0.4	150 ± 30	5.9 ± 0.5	—

5. Conclusions

The model presented is, undoubtedly, a great simplification of reality. It ignores effects of spectral changes, spatial harmonics, etc. The reflector neutrons are described by a single time constant. It does, however, describe the experimental data quite well and can, therefore, be used to extrapolate the general kinetic behaviour of cores of this type from areas supported by experiment into areas not supported by experiment.

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Discussion

Mr. Perez (University of Florida): I was very interested in seeing the pulse propagation that you actually obtained in going from the core to the reflector. You could see that the peak of the pulse was displaced, and this is what I shall discuss in the following paper. I am very satisfied that you actually could see it experimentally.

Mr. Huber: The B-10 reaction rate, as a function of position across the core, had a large peak in the reflector. Our detectors for the reflector measurements were placed as near to this peak in the B-10 reaction rate as the structure would allow.

Presented by R. B. Perez

The Theory of Fast Reactor Integral Measurements by Propagation Methods

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ABSTRACT

The application of pulse propagation techniques to integral measurements for the determination of removal cross sections and the anisotropic moments of the scattering kernel in the few-MeV neutron-energy range is described. A new technique (the Operational-B_N method), which extends the B_N method to finite geometries, is introduced. The method is used to obtain the dispersion law for the complex inverse relaxation length ($K = \alpha + i\xi$) of monoenergetic neutron waves in a nonmultiplying and nonmoderating parallelepiped.

Theoretical dispersion laws for 0.84- and 1.24-MeV neutrons in a lead assembly and for 1.28-MeV neutrons in an iron assembly are given. The removal cross section and moments of the scattering kernel are related to the coefficients of a power-series expansion of K^2 in terms of $i\omega$, where ω is the angular frequency. The sensitivity of these coefficients to the cross sections is shown.

The advantages of measuring a complex eigenvalue K , rather than a real eigenvalue as in the case of the die-away or static experiment, is discussed. Finally, some results from a preliminary pulse propagation experiment for 2.14-MeV neutrons in lead are shown to demonstrate that the technology to perform such experiments is presently available.

SYMBOLS

- a half-width of assembly for x -coordinate
- b half-width of assembly for y -coordinate
- ω angular frequency (rad/sec)
- v neutron speed
- Ω unit vector in direction of neutron travel
- $\Omega_1 \sin \theta \cos \phi$
- $\Omega_2 \sin \theta \sin \phi$
- $\Omega_3 \cos \theta = \mu$
- $\mathbf{r} \ x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$
- Σ_T total macroscopic cross section
- Σ_{S0} zeroth moment in Legendre expansion of scattering cross section
- Σ_{S1} first moment in Legendre expansion of scattering cross section

Σ_R removal cross section

Φ Fourier-transformed angular flux

R resolvent operator

Ψ scalar flux

$\mathbf{J}$ current = $J_1\mathbf{i} + J_2\mathbf{j} + J_3\mathbf{k}$

γ ($i\omega/v$) + Σ_T

K Laplace transform of z -coordinate

k_x, k_y transverse eigenvalues

$\partial_1 \partial/\partial x$

$\partial_2 \partial/\partial y$

$\partial_3 \partial/\partial z$

1. Introduction

Much of the physics effort supporting the fast reactor program is related to integral experiments. The objective of this paper is to introduce and discuss the application of pulse propagation techniques to integral measurements for the determination of removal cross sections and the anisotropic moments of the scattering kernel, in the few-MeV neutron-energy range. The crux of the method is the study of the propagation of disturbances of the neutron field excited by a very short burst of neutrons at one of the boundaries of the system. Analysis of the neutron pulse into its Fourier components describes the neutron field as a superposition of damped neutron waves. The damping coefficient and phase shift of the neutron wave are the real and imaginary parts, respectively, of the complex inverse relaxation length:

$$K(\omega) = \alpha(\omega) + i\xi(\omega).$$

This equation, relating $\alpha(\omega)$ and $\xi(\omega)$, with frequency as a parameter, is termed the dispersion law of the assembly, in analogy to optics.

The present work is primarily concerned with the derivation of the dispersion law of monoenergetic neutrons propagating in a nonmultiplying and nonmoderating medium.

In this case, only transport phenomena are observed and accurate solutions of the Boltzmann equation may be obtained. The first section of the paper is devoted to

presenting the resolvent-operator technique as applied to the solution of the neutron-transport problem.

This leads to the formulation of the Operational- B_N method which is applied, in Sect. 2, to the derivation of the dispersion law. The remainder of the paper outlines methods to obtain neutron cross sections and shows some preliminary experiments performed at M.I.T. by Beghian, Wilensky, and Hofmann.⁽¹⁾

2. Theory

2.1. The Operational- B_N Method

For linearly anisotropic scattering and a nonmultiplying medium, the Fourier transform of the time-dependent neutron flux satisfies the following monoenergetic Boltzmann equation:

$$\left\{ \frac{i\omega}{v} + \Sigma_T + \mathbf{\Omega} \cdot \nabla \right\} \phi(\mathbf{\Omega}, \mathbf{r}, \omega) = \frac{1}{4\pi} \int_{\Omega'} d\Omega' [\Sigma_{s0} + 3\Sigma_{s1}\mathbf{\Omega} \cdot \mathbf{\Omega}'] \phi(\mathbf{\Omega}', \mathbf{r}, \omega). \quad (1)$$

For the case of an infinite medium, the usual procedure for solving Eq. (1) is to take a Fourier transform with respect to the space variable and then to apply the methodology of the B_N technique.⁽²⁾ In a finite medium such an approach will introduce complicated surface integrals. We have, instead, followed a formalism based on the use of the resolvent operator⁽³⁾ which allows the utilization of the B_N method in its original simplicity and efficiency.

To this end multiplication of Eq. (1) by the resolvent operator

$$R(\mathbf{\Omega}, \mathbf{r}, \omega) = \frac{1}{[\gamma(\omega) + \mathbf{\Omega} \cdot \nabla]} \quad (2)$$

yields

$$\phi(\mathbf{\Omega}, \mathbf{r}, \omega) = R(\mathbf{\Omega}, \mathbf{r}, \omega) \cdot \left\{ \frac{1}{4\pi} \int_{\Omega'} d\Omega' [\Sigma_{s0} + 3\Sigma_{s1}\mathbf{\Omega} \cdot \mathbf{\Omega}'] \phi(\mathbf{\Omega}', \mathbf{r}, \omega) \right\}, \quad (3)$$

where

$$\gamma(\omega) = \Sigma_T + \frac{i\omega}{v}. \quad (4)$$

The result given by Eq. (3) is the basis of the Operational- B_N formalism. We define the angular flux moments

$$\bar{\phi}(\mathbf{r}, \omega) = \int_{\Omega} d\Omega \phi(\mathbf{\Omega}, \mathbf{r}, \omega) \quad (\text{scalar flux}); \quad (5)$$

$$J_i(\mathbf{r}, \omega) = \int_{\Omega} d\Omega \Omega_i \phi(\mathbf{\Omega}, \mathbf{r}, \omega) \quad (i\text{-th component of the vector current}); \quad (6)$$

and in general the n -rank tensor

$$\phi_{ij\dots k}(\mathbf{r}, \omega) = \int d\Omega \Omega_i \Omega_j \dots \Omega_k \phi(\mathbf{\Omega}, \mathbf{r}, \omega), \quad (7)$$

from which the angular flux $\phi(\mathbf{\Omega}, \mathbf{r}, \omega)$ can be reconstructed, i.e.,

$$\begin{aligned} \phi(\mathbf{\Omega}, \mathbf{r}, \omega) = & N_1 \Psi(\mathbf{r}, \omega) + N_2 \sum_i \Omega_i J_i(\mathbf{r}, \omega) \\ & + \dots N_n \sum_{ij\dots k} \Omega_i \Omega_j \dots \Omega_k \phi_{ij\dots k}(\mathbf{r}, \omega), \end{aligned} \quad (8)$$

where the coefficients N_i are determined from the definitions (5) through (7); successive multiplication of Equation (3) by the operators

$$\int_{\Omega} d\Omega; \quad \int_{\Omega} d\Omega \Omega_i$$

and in general by

$$\int_{\Omega} d\Omega \Omega_i \dots \Omega_k$$

yields

$$\begin{aligned} \Psi(\mathbf{r}, \omega) = & \frac{1}{4\pi} \int_{\Omega} d\Omega \left\{ \Sigma_{s0} R(\mathbf{\Omega}, \mathbf{r}, \omega) \Psi(\mathbf{r}, \omega) \right. \\ & \left. + 3\Sigma_{s1} R(\mathbf{\Omega}, \mathbf{r}, \omega) \left(\sum_i \Omega_i J_i(\mathbf{r}, \omega) \right) \right\}; \end{aligned} \quad (9)$$

$$\begin{aligned} J_i(\mathbf{r}, \omega) = & \frac{1}{4\pi} \int_{\Omega} d\Omega \left\{ \Sigma_{s0} R(\mathbf{\Omega}, \mathbf{r}, \omega) \Omega_i \Psi(\mathbf{r}, \omega) \right. \\ & \left. + 3\Sigma_{s1} R(\mathbf{\Omega}, \mathbf{r}, \omega) \right. \\ & \left. \cdot \left(\sum_j \Omega_j J_j(\mathbf{r}, \omega) \right) \right\}; \end{aligned} \quad (10)$$

$$\begin{aligned} \phi_{ij\dots k}(\mathbf{r}, \omega) = & \frac{1}{4\pi} \int_{\Omega} d\Omega \left\{ \Sigma_{s0} \Omega_i \Omega_j \dots \Omega_k \Psi(\mathbf{r}, \omega) \right. \\ & \cdot \phi(\mathbf{r}, \omega) + 3\Sigma_{s1} R(\mathbf{\Omega}, \mathbf{r}, \omega) \\ & \left. \cdot \left(\sum_m \Omega_m \dots \Omega_k \Omega_m J_m(\mathbf{r}, \omega) \right) \right\}. \end{aligned} \quad (11)$$

The analogy with the B_N method results is obvious. There is, however, a fundamental departure from the usual B_N method: in the latter, Fourier transformation of Equations (9), (10), and (11) allows exact solutions for both the scalar flux and the vector current. In this case, due to the finiteness of the system, the above magnitudes can only be determined, to any desired order of approximation, by a perturbation technique based on the iteration of the resolvent operator $R(\mathbf{\Omega}, \mathbf{r}, \omega)$. This is accomplished as follows: the resolvent is written as

$$\begin{aligned} R(\mathbf{\Omega}, \mathbf{r}, \omega) = & \frac{1}{\gamma(\omega)} [\gamma(\omega) + V(\mathbf{\Omega}, \mathbf{r}) \\ & - V(\mathbf{\Omega}, \mathbf{r})] \left[\frac{1}{\gamma(\omega) + V(\mathbf{\Omega}, \mathbf{r})} \right], \end{aligned} \quad (12)$$

where

$$V(\mathbf{\Omega}, \mathbf{r}) = \mathbf{\Omega} \cdot \nabla = \sum_i \Omega_i \partial_i \quad (13)$$

or

$$R(\mathbf{\Omega}, \mathbf{r}, \omega) = R_0(\omega)[1 - V(\mathbf{\Omega}, \mathbf{r})R(\mathbf{\Omega}, \mathbf{r}, \omega)] \quad (14)$$

with

$$R_0(\omega) = \frac{1}{\gamma(\omega)}. \quad (15)$$

Then Equation (14) is iterated to obtain the series

$$R^{(N)}(\mathbf{\Omega}, \mathbf{r}, \omega) = \sum_{n=0}^N (-1)^n R_0 (VR_0)^n + (-1)^{N+1} (VR_0)^{N+1} R, \quad (16)$$

where the term $(-1)^{N+1} (VR_0)^{N+1} R$ is the remainder of the series. Introduction of the iterated resolvent into Equations (9) and (10) leads to

$$\begin{aligned} \Psi^{(N)}(\mathbf{r}, \omega) = & \sum_{n=0}^N (-1)^n \left\{ \frac{\Sigma_{S0}}{4\pi} \int_{\mathbf{\Omega}} d\mathbf{\Omega} R_0 (VR_0)^n \right. \\ & \cdot \Psi(\mathbf{r}, \omega) + \frac{3\Sigma_{S1}}{4\pi} \int_{\mathbf{\Omega}} d\mathbf{\Omega} R_0 (VR_0)^n \\ & \cdot (\mathbf{\Omega} \cdot \mathbf{J}(\mathbf{r}, \omega)) \left. \right\} + \frac{(-1)^{N+1}}{4\pi} \int_{\mathbf{\Omega}} d\mathbf{\Omega} \\ & \cdot (VR_0)^{N+1} R[\Sigma_{S0} \Psi(\mathbf{r}, \omega) \\ & + 3\Sigma_{S1} \mathbf{\Omega} \cdot \mathbf{J}(\mathbf{r}, \omega)] \end{aligned} \quad (17)$$

and

$$\begin{aligned} J_i(\mathbf{r}, \omega) = & \sum_{n=0}^N (-1)^n \left\{ \frac{\Sigma_{S0}}{4\pi} \int_{\mathbf{\Omega}} d\mathbf{\Omega} R_0 (VR_0)^n \Omega_i \right. \\ & \cdot \Psi(\mathbf{r}, \omega) + \frac{3\Sigma_{S1}}{4\pi} \int_{\mathbf{\Omega}} d\mathbf{\Omega} R_0 (VR_0)^n \\ & \cdot \left(\sum_j \Omega_i \Omega_j J_j(\mathbf{r}, \omega) \right) \left. \right\} + \frac{(-1)^{N+1}}{4\pi} \\ & \cdot \int_{\mathbf{\Omega}} d\mathbf{\Omega} (VR_0)^{N+1} R \left[\Sigma_{S0} \Omega_i \Psi(\mathbf{r}, \omega) \right. \\ & \left. + \Sigma_{S1} \left(\sum_j \Omega_i \Omega_j J_j(\mathbf{r}, \omega) \right) \right]. \end{aligned} \quad (18)$$

The results given by Equations (17) and (18) are still exact. Upon cancellation of the remainder terms, one obtains the N -th approximation in the Operational- B_N method.

Before writing down the general form, and in view of the commutation properties of the R_0 , $\mathbf{\Omega}$, and V operators, it is to be noticed that operators of the form $(\mathbf{\Omega} \cdot \nabla)$ are double dot products of the n -rank tensors:

$$T_{ij \dots kl} = \Omega_i \Omega_j \dots \Omega_k \Omega_l$$

and

$$\partial_{ij \dots kl} = \partial_i \partial_j \dots \partial_k \partial_l;$$

hence,

$$(\mathbf{\Omega} \cdot \nabla)^n = \sum_{ij \dots kl} T_{ij \dots kl} \partial_{ij \dots kl}. \quad (19)$$

This leads to the following expressions for the N -th approximation of the scalar flux and current:

$$\begin{aligned} \Psi^{(N)}(\mathbf{r}, \omega) = & \frac{\Sigma_{S0}}{4\pi} \sum_{n=0}^N \sum_{ij \dots kl} (-1)^n R_0^{n+1} \langle T_{ij \dots kl} \rangle \\ & \cdot \partial_{ij \dots kl} \Psi^{(N)}(\mathbf{r}, \omega) \\ & + \frac{3\Sigma_{S1}}{4\pi} \sum_{n=0}^N \sum_{ij \dots kl m} (-1)^n \\ & \cdot R_0^{n+1} \langle T_{ij \dots kl, m} \rangle \partial_{ij \dots kl} J_m^{(N)}(\mathbf{r}, \omega); \end{aligned} \quad (20)$$

$$\begin{aligned} J_m^{(N)}(\mathbf{r}, \omega) = & \frac{\Sigma_{S0}}{4\pi} \sum_{n=0}^N \sum_{ij \dots kl} (-1)^n R_0^{n+1} \langle T_{ij \dots kl, m} \rangle \\ & \cdot \partial_{ij \dots kl} \Psi^{(N)}(\mathbf{r}, \omega) \\ & + \frac{3\Sigma_{S1}}{4\pi} \sum_{n=0}^N \sum_{ij \dots kl p} \langle T_{ijkl, m, p} \rangle \\ & \cdot \partial_{ij \dots kl} J_p^{(N)}(\mathbf{r}, \omega), \end{aligned} \quad (21)$$

where the bracket notation indicates angular integration, i.e.,

$$\begin{aligned} \langle T_{ij \dots kl} \rangle = & \int_0^{2\pi} d\phi \int_{\pi}^0 d(\cos \theta) \Omega_{ij \dots kl} \\ = & \int_0^{2\pi} d\phi \int_{\pi}^0 d(\cos \theta) \cos^{\alpha} p \sin^{\beta} p \\ & \cdot \cos^{\gamma} \theta \sin^{\delta} \theta. \end{aligned} \quad (22)$$

The symbols α , β , γ indicate the multiplicity of sub-indices 1, 2, and 3, respectively, and δ is equal to the sum of the multiplicities of the subindices 1 and 2. It is immediately seen that if the multiplicity of any sub-index is odd the integral vanishes; hence, the brackets of odd-rank tensors cancel out.

Inspection of Equations (20) and (21) reveals that this technique offers the same advantages as the usual B_N method over the P_N approximation, viz., for a given degree of anisotropy the equations for the higher angular flux moments are uncoupled. Thus, for linearly anisotropic scattering only the scalar flux and current are coupled and, therefore, their magnitudes may be obtained to any desired degree of accuracy independent of the higher moments. The main difference between the Operational- B_N and the conventional B_N method is that in the former a set of coupled partial differential equations are obtained, whereas the latter results in a set of coupled algebraic equations.

2.2. Application to Neutron-wave Propagation

The Operational-B_N technique will be used to study fast neutron-wave propagation in the longitudinal direction of a long parallelepiped. The assembly with the associated coordinate system is shown in Fig. 1. The assembly is assumed to be infinitely long so that "end effects" may be neglected. Since the object is to determine the complex inverse relaxation length, this simplification is justified. The finite size in the transverse dimension will be treated as a perturbation to the infinite case.

Laplace transforming Equations (9) and (10), with respect to the z -coordinate and specializing to the present case, gives

$$\begin{aligned} \Psi(K, x, y, \omega) &= \frac{\Sigma_{s0}}{4\pi} \int_{\Omega} d\Omega R \Psi(K, x, y, \omega) \\ &+ \frac{3\Sigma_{s1}}{4\pi} \int_{\Omega} d\Omega R \sum_{j=1}^3 \Omega_j J_j(K, x, y, \omega) \quad (23) \\ &+ \text{inhomogeneous terms;} \end{aligned}$$

$$\begin{aligned} J_i(K, x, y, \omega) &= \frac{\Sigma_{s0}}{4\pi} \int_{\Omega} d\Omega \Omega_i R \Psi(K, x, y, \omega) \\ &+ \frac{3\Sigma_{s1}}{4\pi} \int_{\Omega} d\Omega \Omega_i R \sum_{j=1}^3 \\ &\cdot \Omega_j J_j(K, x, y, \omega) \\ &+ \text{inhomogeneous terms;} \\ i &= 1, 2, 3, \end{aligned} \quad (24)$$

where

$$\begin{aligned} R &= \frac{1}{\gamma + \mu K + \Omega_1 \partial_1 + \Omega_2 \partial_2} \\ &= R_0 \sum_{n=0}^N (-1)^n V^n R_0^n + (-1)^{N+1} (V R_0)^{N+1} R, \end{aligned} \quad (25)$$

with

$$R_0 = \frac{1}{\gamma + \mu K}; \quad (26)$$

$$V = \Omega_1 \partial_1 + \Omega_2 \partial_2. \quad (27)$$

$K = \alpha + i\xi$ is the complex inverse relaxation length, and the inhomogeneous term is due to the source and boundary condition at $z = 0$. In the truncated-series approximation it is assumed that, for sufficiently large values of N , the remainder term may be neglected. Hence,

$$R = R_0 \sum_{n=0}^N (-1)^n V^n R_0^n. \quad (28)$$

At this point the ansatz is made that the solutions to

Equations (23) and (24) have the form

$$\begin{aligned} \Psi(K, x, y, \omega) &= \psi(K, k_x, k_y, \omega) \cos k_x x \cos k_y y; \\ J_1(K, x, y, \omega) &= j_1(K, k_x, k_y, \omega) \sin k_x x \cos k_y y; \\ J_2(K, x, y, \omega) &= j_2(K, k_x, k_y, \omega) \cos k_x x \sin k_y y; \\ J_3(K, x, y, \omega) &= j_3(K, k_x, k_y, \omega) \cos k_x x \cos k_y y. \end{aligned} \quad (29)$$

When these expressions are substituted into Equations (23) and (24), the proper differential operations associated with V performed and the integrals calculated, it is found that in Equation (23) the only nonzero terms are those which prefix the term $(\cos k_x x \cos k_y y)$. Similarly for Equation (24) with $i = 1, 2, 3$, the only nonzero terms prefix $(\sin k_x x \cos k_y y)$, $(\cos k_x x \sin k_y y)$ and $(\cos k_x x \cos k_y y)$, respectively. Thus the dependence on x and y can be cancelled from these equations. After eliminating j_1 , j_2 , and j_3 in favor of ψ , the equations reduce to

$$\begin{aligned} \left\{ 1 - \frac{\Sigma_{s0}}{4\pi} S_0 - \frac{3\Sigma_{s1}}{4\pi} \left[S_1 \frac{\Delta_1}{\Delta_0} + S_2 \frac{\Delta_2}{\Delta_0} + S_3 \frac{\Delta_3}{\Delta_0} \right] \right\} \\ \cdot \psi(K, k_x, k_y, \omega) = \text{inhomogeneous terms.} \end{aligned} \quad (30)$$

The S_i 's and Δ_i 's in the bracketed term multiplying ψ are known functions of K , k_x , k_y , and ω , and are defined in the appendix. For given k_x and k_y , the non-trivial solutions to the homogeneous part of Eq. (30) yield the discrete eigenvalues, K , or the dispersion law of the system.

FIG. 1. Sketch of Assembly.

The next step is to determine the transverse eigenvalues k_x and k_y from the boundary conditions. For simplicity, let $a = b$; $k_x = k_y$. The no-return-current condition has the form

$$\int_{1/2 \text{ space}} d\Omega \, \Omega \cdot \mathbf{n} \phi(\Omega, K, \pm a, y, \omega) = 0. \quad (31)$$

After proceeding in the same manner as above, this

FIG. 2. Dispersion Law for 1.24-MeV Neutrons in Lead.

FIG. 3. Dispersion Law for Transverse Eigenvalue.

condition yields the equation

$$\begin{aligned} \frac{\Sigma_{s0}}{4\pi} (U_{00} \cot k_x a - U_{10}) + \frac{3\Sigma_{s1} \Delta_1}{4\pi \Delta_0} \\ \cdot (U_{11} \cot k_x a - U_{01}) + \frac{3\Sigma_{s1} \Delta_2}{4\pi \Delta_0} \\ \cdot (T_{12} \cot k_x a - U_{02}) + \frac{3\Sigma_{s1} \Delta_3}{4\pi \Delta_0} \\ \cdot (U_{03} \cot k_x a - U_{13}) = 0. \end{aligned} \quad (32)$$

The U_{ij} 's and Δ_i 's are known function of K , k_x , and ω (see appendix for definitions). The simultaneous solution of equations (30) and (23) determine K and k_x as functions of ω . A computer code has been written which solves these equations using the Newton-Raphson technique.

2.3. The Dispersion Law

In Fig. 2 the dispersion law is plotted for 1.24-MeV neutrons in a lead assembly. The B_N approximation ($N = 8$) is compared with the P_1 and P_3 approximations,⁽⁴⁾ and with the infinite-medium case, which is known exactly.^(5, 6)

At high frequencies, the effect of transverse leakage becomes less important, and the B_8 dispersion law approaches the infinite-medium curve as expected. The P_3 curve is in close agreement with the B_8 curve for frequencies up to 20 Mc/sec but above this frequency rapidly diverges. Although the P_3 approximation shows marked improvement over P_1 , it is not practical to go to higher-order P_N as an alternate to the B_N approach because of the extensive theoretical manipulation required. Even the derivation of the P_5 approximation would be formidable.

The region where the dispersion law crosses the line $\alpha = \Sigma_T$ is of interest because the source neutrons propagate with attenuation Σ_T . For frequencies near and above this crossover point it may not be experimentally possible to eliminate the contamination by source neutrons.

Included in the functional form of the dispersion law are $\ln [(\gamma + K)/(\gamma - K)]$ terms which arise from integrals over the angles. Associated with these terms are a set of branch cuts in the K -plane for

$$|\operatorname{Re} K| \geq \Sigma_T; \quad |\operatorname{Im} K| \leq \frac{\omega}{v},$$

corresponding to a continuum of eigenvalues. Because of the expansion used in the resolvent operator, the continua associated with the transverse eigenvalues cannot be described by the present formalism.

In Fig. 3 the dispersion law of the transverse eigen-

FIG. 4. Convergence for B_N Approximations.

value k_x is shown. The fact that k_x is complex indicates that the wave propagates in three dimensions. The wave properties are associated with the imaginary part of the eigenvalue. In the frequency range of interest the imaginary part of the longitudinal eigenvalue is larger by two orders of magnitude than the imaginary part of the transverse eigenvalue. Although there is wave propagation toward the sides of the assembly, it is not as pronounced as the longitudinal propagation. There will be little phase difference between points on a transverse plane.

FIG. 5. Dispersion Law for 0.84- and 1.24-MeV Neutrons in Lead.

2.4. Convergence of the Method

The convergence of the B_N method is a function of the transverse dimension of the assembly. Figure 4 shows the convergence of the approximation for a fairly large assembly ($a = 34$ cm). $N = 8$ represents convergence to four decimal places. For an assembly approximately half as wide, similar convergence requires $N = 14$. Since for large N the solution time required is proportional to N^2 , there is some advantage in using wide systems. There is no need to go to small assemblies in the neutron-wave experiment since measurements are made as a function of frequency and not size as, for example, in the die-away experiment.

3. The Utilization of Pulse Propagation Techniques for Cross-section Measurements

From the frequency spectrum of the neutron pulses measured at various points in the assembly, amplitude and phase plots are obtained; these in turn yield the real and imaginary parts of the complex inverse relaxation length as a function of frequency. The experimental dispersion law is then compared with the theoretical model, which includes assumptions as to the degree of anisotropy and as to the values of the removal cross section, as well as the moments of the scattering kernel.

In Figs. 5 and 6, the dispersion laws for 0.84-MeV neutrons and 1.24-MeV neutrons in a lead assembly and for 1.28-MeV neutrons in an iron assembly are shown. Figure 5 shows the sensitivity of the dispersion law to changes in cross section and neutron speed.

The direct comparison method has some disadvantages. It is difficult to determine what changes of cross sections will improve the match between theory and experiment. There are also the questions of what criterion to use for "goodness of fit" and of internal consistency of the assumed set of parameters.

FIG. 6. Dispersion Law for 1.28-MeV Neutrons in Iron.

Another method is suggested by the die-away experiment.⁽⁷⁾ In that experiment the decay constant (the time eigenvalue) λ is often expanded in terms of the geometric buckling. In this case, we expand K^2 in powers of $i\omega$:

$$K^2 = \sum_n a_n (i\omega)^n, \quad (33)$$

where

$$K^2 = (\alpha^2 - \xi^2) + 2i\alpha\xi. \quad (34)$$

Equating real and imaginary parts gives

$$\alpha^2 - \xi^2 = \sum_n a_n \omega^n; \quad (35)$$

$$2\alpha\xi = \sum_n a_n \omega^n. \quad (36)$$

In Fig. 7, $\alpha^2 - \xi^2$ is plotted vs. ω^2 , and $2\alpha\xi$ is plotted vs. ω . At low frequencies K^2 is well represented by an expansion truncated at $n = 2$.

The coefficients $a_0, a_1 \dots a_n$, obtained in practice by least-square fitting the power expansions (35) and (36) to the experimental results, form a set of n relations which represent constraints to be satisfied by the assumed set of cross sections.

FIG. 7. Real and Imaginary Parts of K^2 .

TABLE 1. SENSITIVITY OF COEFFICIENTS TO CROSS-SECTION CHANGE
(Fractional change in a_i per fractional change in cross section, 1.24 MeV, Pb)

$\frac{\Sigma_R}{a_0} \frac{\delta a_0}{\delta \Sigma_R} = 0.7$	$\frac{\Sigma_R}{a_1} \frac{\delta a_1}{\delta \Sigma_R} = 0.05$	$\frac{\Sigma_R}{a_2} \frac{\delta a_2}{\delta \Sigma_R} = 0.1$
$\frac{\Sigma_{s0}}{a_0} \frac{\delta a_0}{\delta \Sigma_{s0}} = 0.7$	$\frac{\Sigma_{s0}}{a_1} \frac{\delta a_1}{\delta \Sigma_{s0}} = 0.1$	$\frac{\Sigma_{s0}}{a_2} \frac{\delta a_2}{\delta \Sigma_{s0}} = -0.3$
$\frac{\Sigma_{s1}}{a_0} \frac{\delta a_0}{\delta \Sigma_{s1}} = -0.1$	$\frac{\Sigma_{s1}}{a_1} \frac{\delta a_1}{\delta \Sigma_{s1}} = -0.2$	$\frac{\Sigma_{s1}}{a_2} \frac{\delta a_2}{\delta \Sigma_{s1}} = 0.4$
$\frac{B_{\perp}^2}{a_0} \frac{\delta a_0}{\delta B_{\perp}^2} = 0.4$	$\frac{B_{\perp}^2}{a_1} \frac{\delta a_1}{\delta B_{\perp}^2} = 0.005$	$\frac{B_{\perp}^2}{a_2} \frac{\delta a_2}{\delta B_{\perp}^2} = -0.04$

The power-expansion method also affords the possibility of studying the influence of the various parameters on the dispersion law as shown in Table 1. Inspection of these results reveals that in order to measure the higher moments of anisotropy, one has to be able to determine at least the terms (in the dispersion law) corresponding to second powers in the frequency. The zero-frequency case corresponds to the static experiment.⁽⁸⁾ Hence, static experiments will always lead to very scant information on anisotropic scattering.

The influence of assembly size on the measurement is illustrated in the last row of values in Table 1. For thermal systems, the leakage effects are minimized in view of the relative magnitudes of physical sizes and neutron mean free paths. The contrary holds true for fast assemblies where leakage effects become very important unless one goes to sizeable inventory costs. Thus, one advantage of dynamic techniques is evident, since useful information is obtained at high frequencies where leakage effects are down by several orders of magnitude.

3.1. Comparison with the Die-away Method

An alternative technique to measure cross sections by "integral" methods is the die-away technique,⁽⁷⁾ in which the decay constant is measured as a function of the size of the system. Both the die-away and pulse propagation techniques use the same experimental tools and general set-up as developed by the M.I.T. group.^(1, 7)

The difference lies in the fact that the latter utilizes all the information in the pulse while the former extracts its information from the tail of the pulse only. Thus, instead of a real time eigenvalue λ , one obtains a complex space eigenvalue K , whose real and imaginary parts are related through the dispersion law. This poses a much more stringent test on the theoretical model and facilitates parameter search procedures.

3.2. Comments on Fast Pulse Propagation Experiments and Preliminary Results

The relationship between neutron cross sections and the dispersion law in a fast nonmultiplying assembly has been discussed in previous sections. There are two different, although equivalent, ways of performing the experiment. From the theoretical viewpoint, it is much easier to work in the frequency domain. Here the Fourier components of the neutron field represent propagating waves of the neutron population, whose amplitude and phase shift yield the dispersion law as was shown above. In order to obtain the Fourier components experimentally, sinusoidally modulated sources of fast neutrons can be used. Although the technical problems are not insurmountable, it is at best difficult to modulate charged-particle beams at frequencies of tens of megacycles.

Experimentally it is much easier to introduce nanosecond-long bursts of neutrons, a technique which has been well developed in connection with time-of-flight methods presently used in many experimental facilities. Subsequent Fourier transformation of the response into the frequency domain yields results directly comparable with theory. This technique has worked out quite successfully for thermal systems.⁽⁹⁾ One of the difficulties in fast systems, however, arises from the generally large size of the neutron detectors. Nevertheless, recent developments in small photomultiplier tubes and the ability to introduce discrimination in 10-dynode tubes⁽⁷⁾ will ease the problem.

The feasibility of the technique has been proven by some preliminary experiments, performed at M.I.T. in collaboration with our group, by Beghian *et al.*⁽¹⁾ Figure 8 shows two neutron pulses at distances of 6 and 10 in. from the source respectively. The input pulse was 4 ns long. A stilbene neutron detector was placed at several positions in the upper face of a prism of lead of dimensions 12 x 12 x 20 in. Proper precautions were taken to shield the detector from first-flight neutrons and for selecting a given neutron energy (2.14 MeV) from the output of a He^3 reaction at the M.I.T. Rockefeller Van de Graaff machine.

The propagation of the pulse is clearly seen by the displacement of its maximum (6.15 ns in a span of 10.16 cm). The frequency spectra of both pulses are shown in Fig. 9. If a δ -function input is assumed, these spectra describe the space-dependent transfer function of the system.

Presently a nanosecond pulsing facility is being installed at the University of Florida (4 MeV) Van de Graaff accelerator, to start an experimental program in fast pulse propagation studies in both nonmultiplying and multiplying media.

FIG. 8. Pulse of 2.14-MeV Neutrons in Lead (MIT Propagating Pulse Experiment).

FIG. 9. Fourier Transform of Pulses (MIT Propagating Pulse Experiment).

4. Conclusions

In this work, the propagation of fast monoenergetic neutrons in finite, nonmultiplying and nonmoderating media has been studied with the emphasis on the deter-

mination of the theoretical dispersion law in lead and iron assemblies. It is then possible to relate the dispersion law to integral parameters of interest. In particular, for nonmultiplying media it is possible to obtain removal cross sections as well as the higher anisotropic moments of the scattering kernel. The resolvent-operator technique has been introduced to obtain solutions to the Boltzmann equation. This technique leads to the Operational-B_N method, which is an extension of the B_N method to finite geometries and possesses the same advantages as the latter.

There is a wide range of frequencies over which an asymptotic behavior may be observed before the dispersion law approaches the region of the continuum of eigenvalues.

The influence of neutron energy as well as the sensitivity of the dispersion law to the value of the various cross sections have been analyzed.

The experimental feasibility of the pulse propagation method was established by the work of Beghian and coworkers,⁽⁷⁾ as illustrated in the discussion in Sect. 3.

Pulse propagation techniques offer the following advantages over static and die-away experiments:

- The experiment is performed without changing the size of the system.
- The higher-order frequency terms of the dispersion law are very sensitive to the anisotropy of the scattering kernel, and quite independent of leakage effects.
- Two quantities, amplitude and phase shift, are obtained as opposed to the die-away technique, in which only the decay constant is available for parameter fitting.

Appendix

$$(1) S_0 = \sum_{p=1}^{N+1} \sum_{\substack{q=1 \\ \text{odd}}}^p (-1)^{(p-1)/2} \frac{(p-1)!}{(q-1)!(p-q)!} A_{pq} k_x^{p-1}$$

$$(2) S_1 = \sum_{p=2}^{N+1} \sum_{\substack{q=1 \\ \text{even}}}^p (-1)^{p/2} \frac{(p-1)!}{(q-1)!(p-q)!} B_{pq}^1 k_x^{p-1}$$

$$(3) S_2 = \sum_{p=2}^{N+1} \sum_{\substack{q=2 \\ \text{even}}}^p (-1)^{p/2} \frac{(p-1)!}{(q-1)!(p-q)!} B_{pq}^2 k_x^{p-1}$$

$$(4) S_3 = \sum_{p=1}^{N+1} \sum_{\substack{q=1 \\ \text{odd}}}^p (-1)^{(p-1)/2} \frac{(p-1)!}{(q-1)!(p-q)!} B_{pq}^3 k_x^{p-1}$$

$$(5) P_{10} = -S_1$$

$$(6) P_{20} = -S_2$$

$$(7) P_{30} = S_3$$

$$(8) P_{ii} = \sum_{p=1}^{N+1} \sum_{\substack{q=1 \\ \text{odd}}}^p (-1)^{(p-1)/2} \frac{(p-1)!}{(q-1)!(p-q)!} C_{pq}^{ii} k_x^{p-1}; \quad i = 1, 2, 3$$

$$(9) P_{12} = \sum_{p=1}^{N+1} \sum_{\substack{q=2 \\ \text{odd}}}^p (-1)^{(p-1)/2} \frac{(p-1)!}{(q-1)!(p-q)!} C_{pq}^{12} k_x^{p-1}$$

$$(10) P_{12} = P_{21}$$

$$(11) P_{13} = \sum_{p=2}^{N+1} \sum_{\substack{q=1 \\ \text{even}}}^p (-1)^{(p/2)+1} \frac{(p-1)!}{(q-1)!(p-q)!} C_{pq}^{13} k_x^{p-1}$$

$$(12) P_{13} = P_{31}$$

$$(13) P_{23} = \sum_{p=2}^{N+1} \sum_{\substack{q=2 \\ \text{even}}}^p (-1)^{(p/2)+1} \frac{(p-1)!}{(q-1)!(p-q)!} C_{pq}^{23} k_x^{p-1}$$

$$(14) P_{23} = P_{32}$$

$$(15) A_{pq} = \int_{\Omega} d\Omega R_0^p \Omega_1^{p-q} \Omega_2^{q-1}$$

$$(16) B_{pq}^i = \int_{\Omega} d\Omega R_0^p \Omega_i \Omega_1^{p-q} \Omega_2^{q-1}; \quad i = 1, 2, 3$$

$$(17) C_{pq}^{ii} = \int_{\Omega} d\Omega R_0^p \Omega_i \Omega_j \Omega_1^{p-q} \Omega_2^{q-1}; \quad i = 1, 2, 3; \\ j = 1, 2, 3$$

$$(18) \begin{bmatrix} \frac{3\Sigma_{s1}}{4\pi} P_{11} - 1 & \frac{3\Sigma_{s1}}{4\pi} P_{12} & \frac{3\Sigma_{s1}}{4\pi} P_{13} \\ \frac{3\Sigma_{s1}}{4\pi} P_{21} & \frac{3\Sigma_{s1}}{4\pi} P_{22} - 1 & \frac{3\Sigma_{s1}}{4\pi} P_{23} \\ \frac{3\Sigma_{s1}}{4\pi} P_{31} & \frac{3\Sigma_{s1}}{4\pi} P_{32} & \frac{3\Sigma_{s1}}{4\pi} P_{33} - 1 \end{bmatrix}$$

$$\begin{bmatrix} j_1 \\ j_2 \\ j_3 \end{bmatrix} = \Psi \begin{bmatrix} -\frac{\Sigma_{s0}}{4\pi} P_{10} \\ -\frac{\Sigma_{s0}}{4\pi} P_{20} \\ -\frac{\Sigma_{s0}}{4\pi} P_{30} \end{bmatrix} + \text{inhomogeneous terms}$$

$$(19) \Delta_0 = \text{determinant of square matrix}$$

$$(20) \Delta_i = \text{determinant of matrix with column "i" replaced by the vector multiplying } \psi \text{ for } i = 1, 2, 3$$

$$(21) U_{00} = \sum_{p=1}^{N+1} \sum_{\substack{q=1 \\ \text{odd}}}^p (-1)^{(p-1)/2} \frac{(p-1)!}{(q-1)!(p-q)!} D_{pq} k_x^{p-1}$$

$$(22) U_{10} = - \sum_{p=2}^{N+1} \sum_{\substack{q=1 \\ \text{even}}}^p (-1)^{p/2} \frac{(p-1)!}{(q-1)!(p-q)!} D_{pq} k_k^{p-1}$$

$$(23) U_{01} = \sum_{p=1}^{N+1} \sum_{\substack{q=1 \\ \text{odd}}}^p (-1)^{(p-1)/2} \frac{(p-1)!}{(q-1)!(p-q)!} E_{pq}^1 k_x^{p-1}$$

$$(24) U_{11} = - \sum_{p=2}^{N+1} \sum_{\substack{q=1 \\ \text{even}}}^p (-1)^{(p/2)-1} \frac{(p-1)!}{(q-1)!(p-q)!} E_{pq}^1 k_x^{p-1}$$

$$(25) U_{02} = \sum_{p=1}^{N+1} \sum_{\substack{q=2 \\ \text{odd}}}^p (-1)^{(p-1)/2} \frac{(p-1)!}{(q-1)!(p-q)!} E_{pq}^2 k_x^{p-1}$$

$$(26) U_{12} = - \sum_{p=2}^{N+1} \sum_{\substack{q=2 \\ \text{even}}}^p (-1)^{(p/2)-1} \frac{(p-1)!}{(q-1)!(p-q)!} E_{pq}^2 k_x^{p-1}$$

$$(27) U_{03} = \sum_{p=1}^{N+1} \sum_{\substack{q=1 \\ \text{odd}}}^p (-1)^{(p-1)/2} \frac{(p-1)!}{(q-1)!(p-q)!} E_{pq}^3 k_x^{p-1}$$

$$(28) U_{13} = - \sum_{p=2}^{N+1} \sum_{\substack{q=1 \\ \text{even}}}^p (-1)^{(p/2)-1} \frac{(p-1)!}{(q-1)!(p-q)!} E_{pq}^3 k_x^{p-1}$$

$$(29) D_{pq} = \int_{\frac{1}{2} \text{ space}} d\Omega R_0^p \Omega_1^{p-q+1} \Omega_2^{q-1}$$

$$(30) E_{pq}^i = \int_{\frac{1}{2} \text{ space}} d\Omega R_0^p \Omega_1^{p-q+1} \Omega_2^{q-1}; \quad i = 1, 2, 3$$

Acknowledgments

The authors are indebted to Drs. Beghian, Wilensky, and Hofmann for permitting them to use some experimental results before publication.

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Discussion

Mr. Travelli (ANL): Your continuum begins at Σ_t for the real part of complex inverse relaxation length and this is true in an infinite medium. If one assumes a cosine distribution in the finite directions, the limit should be changed. This is important because the beginning of the continuum defines the highest frequency

at which the experiments can be performed. Probably in fast systems it is going to be a more limiting factor than in thermal systems.

Mr. Perez: We were concerned about this. However, when this is done in a finite medium we have to use the perturbation approximation. As soon as you do a perturbation approximation for the transverse leakage you lose the continuum. In order to get a continuum in the transverse direction, you have to know the spectral values of that resolvent operator, and this is a very complex mathematical problem. What are you saying—that you might find a continuum in the transverse directions before you find a continuum in the axial direction?

Mr. Travelli: No, I have in mind also that the continuum in the longitudinal direction, the Z direction, might not be the same as before.

Mr. Perez: Yes; however, this is like, for example, when you do a P_n expansion of the Boltzmann equation; you have lost the continuum immediately. It is not embodied in the theory. This happens anytime you use an expansion.

Mr. Travelli: I believe that basically you have considered a buckling in the transverse direction and that you have expanded its square root into the components K_x and K_y .

Mr. Perez: Yes, this buckling is $K_x^2 + K_y^2$. It is just a carry-over from diffusion theory.

Mr. Travelli: I find it very interesting that you have found complex values for the transverse buckling while in the usual analysis one assumes real values for it. Have you compared the results that you have found in this way with the results that one could find with the usual model, assuming a real value for the buckling, assuming an extrapolation length in the X and Y directions, and then finding the eigenvalue in the Z direction?

Mr. Perez: Then one is assuming that the problem is solvable in the transverse direction from a longitudinal direction, and it is not obvious that is the case. We have to solve it from the beginning.

Mr. Travelli: Your method is much, much better, but have you made any calculations to estimate what is the error of the other method?

Mr. Perez: We haven't, but we shall do it soon.

Mr. Travelli: I note that the terms K_x , K_y , and K_z are basically components of some vector K in space. Could they be related in a simple way to the same K that is found in an infinite medium? The sum of the squares, $K_x^2 + K_y^2 + K_z^2$, should give in complex numbers the square of the inverse relaxation length in an infinite medium. That would be a very simple relation.

Mr. Perez: In discussions with Moore at Brookhaven

the same question was considered. We could perhaps try to solve for the modulus of the vector, K^2 , which would be much easier.

Mr. Paik (APDA): I have a question on the continuum. I don't think anybody has proven where the continuum is in three-dimensional assemblies. Do you really know whether the continuum exists?

Mr. Perez: If we were able to find the spectral distribution of the resolvent operator mentioned previously, we could answer that question. I don't think that anybody has done it yet. It is a very difficult problem. In our case we have solved it exactly in one direction and only by an approximation method in the other direction. Of course, as soon as you do it, you don't find a continuum for a three-dimensional case.

Mr. Paik: Did you obtain the complex transverse buckling from measurements or do you calculate it?

Mr. Perez: We compute it from the coupled transcendental equations. In a three-dimensional system, you have a way of providing three directions, and we have to find the wave number in the three directions.

Mr. Paik: I did some analysis of the Beghian measurements of the fundamental decay constant, assuming that a fundamental decay exists in a three-dimensional

block. It turns out that at least down to about 1 mean free path in the axial direction and five mean free paths in the X and Y directions, results of the fundamental decay in three-dimensional transport solutions were considerably different from one-dimensional solutions. In other words, the assumption of the extrapolation distance is questionable. There is also a question of the three-dimensional fundamental decay constant. Experimentally it seems to me that there exists some kind of fundamental decay in various positions, which can be proven to be very well-predicted by the theory. Do you have any opinions on the experimental measurements? Do you think that some kind of fundamental decay is measurable, or do you think that it might possibly be nonexistent?

Mr. Perez: Well, I think that for very small sizes or for very high frequencies, if you do a die-away experiment, eventually you will find a region in which there is no discrete mode. The continuum that we found is only approximate, however, because we couldn't solve it in the X and Y directions. But I am convinced that one will get a continuum of eigenvalues. The die-away or the static experiments then will not be meaningful unless you develop a theory to better the continuum.

Construction and Experimental Equipment of the Karlsruhe Fast Critical Facility, SNEAK*

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ABSTRACT

Construction of the Karlsruhe Fast Critical Facility SNEAK was completed in the summer of 1966. A brief description of its main design features, its experimental equipment, and the methods used for precise reactivity measurements are given.

SNEAK is a fixed vertical assembly with fuel elements suspended from a top grid. Horizontal and vertical channels may be formed in the reactor. The two horizontal channels will be used for a material replacement drawer with automatic sample changer, a detector or material traverse device, and a pulsed neutron source. Four vertical channels close to the core center are available for installation of a pile oscillator. A gas-heated loop is being constructed for Doppler experiments.

Four methods used in SNEAK for precise reactivity measurements are compared: the asymptotic period method, the pile-oscillator method, the inverse kinetics method, and the autorod method. Some details are given about the equipment and data treatment, in particular about measurements in the presence of a spontaneous neutron source. Signals from linear pulse or ionization chamber channels may be collected either by a four-channel data-acquisition system or a 256-channel analyzer. A special technique for storing periodic signals enables elimination of linear drift.

1. Introduction

The first ideas for a fast critical facility in Germany date back to the end of 1961. At that time, substantial uncertainties existed in basic nuclear data. Many design parameters of large fast power reactors, such as critical mass, breeding ratio, and the Doppler reactivity coefficient, could not be predicted with sufficient

accuracy. It was believed that a fast zero-power reactor could provide appreciable support for the fast power reactor development work in Germany.

In 1962 the basic design objectives of SNEAK were fixed: SNEAK should allow the investigation of large, dilute fast systems with plutonium fuel; it should provide accessibility for experiments inside and outside of core and blanket; reactor safeguards should be stressed because SNEAK would be Germany's first fast reactor with plutonium fuel. In order to provide adequate safety, any possibility of large accidental reactivity additions and fast uncontrolled reactivity additions had to be excluded. The reactor containment should protect the surrounding area even in the unlikely case of a large fire inside the building.

The design began in 1963. Siemens (SSW) was chosen as architect-engineer and primary contractor. Construction at the site started by the end of 1963. The buildings were basically completed by the end of 1965, and in 1965 the major reactor parts were supplied. Installation and functional tests continued during 1966.

SNEAK was constructed and will be operated as part of the association EURATOM-GfK in the field of fast reactors.

In Part 2 of this paper a brief description of the facility will be given. Part 3 describes the experimental equipment which is directly associated with the reactor. Part 4 deals with reactivity measurement techniques.

2. Description of SNEAK

2.1. General Arrangement of Buildings

The general arrangement of SNEAK is shown in Figs. 1, 2, and 3. The reactor is housed in an airtight and pressure-resistant building. In order to withstand even a very severe metal fire, the design pressure was chosen as 2 atm gauge. Two steel shells, each with a leak rate smaller than 1% per day, guarantee an extremely small overall leakage. By pressure regulation

* Work performed within the association in the field of fast reactors between the European Atomic Energy Community and Gesellschaft für Kernforschung mbH., Karlsruhe.

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in the interspace, the leakage can be controlled so that all air leaving the reactor building is given to the atmosphere through filters.

The reactor building is connected with the operations and storage building by a personnel air lock and with the loading area by the fuel-element-transit lock.

2.2. Reactor Construction

SNEAK is a fixed vertical assembly with fuel elements suspended from a grid. A major point in select-

FIG. 1. SNEAK and Institut für Angewandte Reaktorphysik.

ing this type was that the large reactivity addition in closing the halves of a split-table machine could be avoided. The loading is performed from below in order to exclude accidents resulting from dropping of fuel rods. The fixed vertical assembly type furthermore has the advantage of smaller void and steel fractions (6.5% and 7.5%, respectively) as well as convenient cooling and loop installation. These advantages and the increased safety had to be paid for by a rather high building and by the need for a number of transport and loading devices.

The reactor has a maximum diameter of 3.2 m and a maximum height of 2.7 m. It is assembled out of square elements which are suspended from a top grid plate, and, after completion of loading operations, are fixed in their positions by clamping devices at the bottom. There are two loading machines: a lower loading machine which transports elements between the reactor and the fuel-element-transit lock; an upper loading machine which hoists the elements into the reactor.

The fuel elements in the core section are square stainless steel tubes of 51 x 51-mm inner dimensions; the elements in the blanket section are square tubes of 153 x 153-mm inner dimensions. The lattice pitch is

FIG. 2. Section Through Reactor Building and Loading Area.

FIG. 3. Floor Plan of Building.

54.4 mm in the core and 163.2 mm in the blanket. At present, the core section contains a 24 x 24 array of elements (see Fig. 4); it can later be extended by replacing blanket elements with bundles of 9 fuel elements.

The core elements have a specially shaped cross section so that cooling channels are created between adjacent elements. Air taken from the reactor hall flows downward through these channels at a flow rate sufficient to hold material temperatures close to ambient. As the coolant air flows outside the fuel-element tubes, there is no direct contact between it and fuel pieces and the air can not be contaminated.

2.3. Safety System and Instrumentation

The inherent safety of a flexible system such as SNEAK is relatively poor. Calculations show⁽¹⁾ that only 0.3 to 0.4% $\Delta k/k$ can be compensated by the negative temperature coefficients before dangerous temperatures are reached. Therefore the safety system must be very reliable. Neutron flux, core temperature, γ flux, uranium and plutonium concentrations, β activity, and air pressure inside the reactor building are

measured by fixed instrumentation. Reactor power is controlled from the beginning of fuel loading by means of a logarithmic BF_3 -counter channel. At higher power a logarithmic ionization-chamber channel takes over automatically. The safety channels are built in 3 parallel units, working for greater reliability in a 2-out-of-3 system. Incorporated in the control system is a circuit which checks the digital part of the system by periodically supplying test pulses. Thus, the maximum time interval of undetected failures in this part of the instrumentation is kept very small (< 2 sec). The analog part of the system is checked by comparing the output signals of any 2 parallel units. Upon a failure in one unit, the 2-out-of-3 system is automatically switched to a 1-out-of-2 system.

At scram the safety rods are released from their holding magnets and accelerated downward by an electromagnetic impulse generated by discharging a high-voltage capacitor (3000 V and 500 μF) through a coil. The rods reach a speed of 1 m/sec within 1 msec after the scram signal.

There are up to 20 shim rods and up to 10 safety rods. All control rods may be used either as fuel or as

	■	■	□
PILE OSCILLATOR	×		
SAFETY RODS		×	
SHIM RODS	×	×	
REGULATING RODS	×	×	
CORE ELEMENTS WITH VERT EXP CHANNELS	×	×	×
CORE ELEMENTS	×	×	×

Fig. 4. Arrangement of Experimental Channels and Loading Plan.

fuel-poison rods, thus providing sufficient shutdown reactivity even in very large assemblies. Control rods may be placed at any 9th core element position (see Fig. 4). All parts of the rods, including the drives of the shim rods, are contained within a guide tube of normal fuel-element cross section. The moveable parts are tubes of somewhat smaller cross section (47 x 47 mm square). Shim and safety rods add reactivity in moving upward; therefore the motor speeds are restricted in order to keep reactivity insertion rates below \$0.1/sec. After a scram all shim rods are driven out of the core. During loading operations the safety rods are in and the shim rods are out.

2.4. Core and Blanket Materials

The core materials have been fabricated into square (50.75 x 50.75 mm) plates, varying in thickness between 1.57 and 25.37 mm. Most pieces are 3.15 mm thick; the steel-canned sodium and $\text{PuO}_2\text{-UO}_2$ plates are 6.30 mm thick. The metallic uranium plates are nickel plated to prevent corrosion. At present, about

560 kg U-235 are available, most of it in 20% or 35% enrichment. 175 kg of plutonium are being fabricated into $\text{PuO}_2\text{-UO}_2$ pellets with a ratio of plutonium to uranium of 1:3 and will be available early next year. This ratio was chosen in order to provide a negative Doppler coefficient in the assemblies. Each fuel plate contains 9 $\text{PuO}_2\text{-UO}_2$ pellets in a stainless steel can.

Materials for blanket and reflector simulation, i.e., depleted uranium, steel, aluminium, and graphite, are also available as rods of 17 x 17 x 305 mm (see Fig. 5). Bundles of 9 of these rods fit in a core element, 81 in a blanket element.

The materials are stored in a series of cells in the storage building. The fissile material containers are mounted in birdcages in order to prevent accidental criticality during storage and handling.

2.5. Loading of Elements

Normally, core elements are loaded and unloaded by ZEBRA-type automatic loading and unloading machines. The machines can identify the various types of plates by means of two identification slots located at one edge of the plate. The loading machine fills the core tubes automatically with a stack of plates in a sequence programmed on tape. The unloading machine takes the plates out of the core elements and sorts them back into their storage containers. The machines guarantee correct loading, and reduce manual handling and the radiation exposure of the loading personnel. The loading operations are made under a hood whose exhaust air is monitored for alpha activity due to uranium and plutonium, and then fed through filters into the exhaust system.

For the loading of special elements there is a hand-loading station. A special device exists for the loading of the heavy blanket tubes.

Filled elements are either stored in a storage area

Fig. 5. Core and Blanket Materials.

in the loading room, or they are taken to the element-transit lock through which they are brought into the reactor building, taken over by the lower loading machine, and hoisted into the reactor by the upper loading machine (see Figs. 2 and 3).

3. Experimental Equipment

3.1. Linear Neutron-flux Channels

In addition to the logarithmic channels, which are connected to the reactor safety system, two linear ionization-chamber channels are installed for experimental purposes. Various BF_3 - and He^3 -filled ion chambers of high neutron sensitivity serve as detectors for the linear channels. Two kinds of amplifiers are used: the electrometer type and that based on voltage-dependent capacities. A deviation meter with display on the control desk facilitates accurate reactor operation. It shows the deviation of reactor power from a set point in percent. For easy data acquisition the signals are digitized by voltage-to-frequency conversion. Several BF_3 - and He^3 -filled proportional counters are available for linear pulse channels. Small U-235, U-238, and Pu-239 fission chambers will be used for power calibrations of the SNEAK assemblies.

3.2. Autorod

In addition to the standard control rods, a high-precision automatic rod^(2, 3) has been built which accurately compensates reactor perturbations and measure their magnitude in terms of rod travel. The rod is moved automatically until the output of a linear channel equals a highly stabilized reference voltage. The reactor power is thus kept constant, and a change in rod position is a direct measure of the reactivity worth of the perturbation in the reactor. The autorod assembly is mounted on the top grid plate and can be located above any fuel-element position. It is driven by a printed-circuit motor because of its favorable torque-inertia ratio. The vertical stroke of the rod is limited to 300 mm. The time constant of the rod is smaller than 0.1 sec, and its maximum velocity is approximately 75 mm/sec. Digital position indication is obtained by optical decoding of the angular position of two discs, one of which is directly fixed to the lead screw of the autorod. The resolution of this system is one part in 10^5 . The part of the rod which travels in the reactor core contains natural boron carbide in a tube. Other loadings are prepared in identical tubes so that the boron carbide can easily be replaced by a different material.

3.3. Reactivity Meter

It is possible to obtain continuous reactivity information with the aid of a small on-line analog

computer that solves the inverse reactor kinetic equations.^(4, 5) Although very high precision cannot be expected, the fact that the information is instantaneous facilitates experimental reactor operation. For this reason a reactivity meter has been incorporated into the SNEAK instrumentation. Its output is displayed on a recorder at the control console and on one in the experiment-control room. Four operational amplifiers and one servo-multiplier are incorporated in the fixed wired circuit of the meter; the delayed neutron terms are generated by passive networks. Automatic switching of the source-term simulator occurs simultaneously with range switching of the linear amplifier in the ionization-chamber channel.

3.4. Vertical and Horizontal Core Channels (see Fig. 4)

The SNEAK design with fuel elements suspended from a top grid makes it relatively easy to generate vertical channels at any desired location. A number of fuel and blanket elements can be equipped with a hollow suspension rod with an inside diameter of 18 mm. This rod is followed by a tube with equal inside diameter protruding through the fuel element. The vertical channel which is formed in this way from the top of the reactor allows the insertion of a neutron source, detectors, or material samples. The space between the cylindrical tube and the sides of the fuel element can be filled with core materials.

Larger vertical channels can be formed at four locations near the core center where the top grid has square holes with sides of 55 mm. These large channels will, e.g., be utilized for pile-oscillator experiments.

A horizontal channel in SNEAK has to penetrate a row of core and blanket elements. To allow this, these elements are provided with windows which give access to a rectangular tube with sides of 31 and 58 mm. One horizontal channel can be inserted at midcore level in the north-south direction, another one 30 cm lower in the east-west direction. The horizontal channels will be used for detector and material traverses as well as for insertion of a pulsed neutron source. A pneumatic system enables rapid transport of activated foils with short halflife from the reactor core to a counting device.⁽⁶⁾

3.5. Horizontal Drawer with Sample Changer

As the reactor hall is not accessible during operation, a mechanism is constructed which can move and exchange various sample materials in a special drawer through a horizontal reactor channel. The drawer is subdivided into 30 compartments of 54.4-mm length. Each can hold a container with a sample of dimensions $25.5 \times 51 \times 51$ mm. A sample may consist of a stack of standard SNEAK material plates. A sample

a) Horizontal Channel with Drawer

b) Sample Changer in Front of Reactor

FIG. 6. Horizontal Drawer with Sample Changer.

container can be inserted manually or by an automatic sample changer which has storage capacity for 24 samples. The reactor does not have to be shut down between measurements if the sample changer is used. The sample drawer can be moved with speeds of 1.3 or 10 cm/sec by a high-precision lead screw. Digital position indication with a resolution of one part in 10^5 is obtained from a coded disc which rotates when the drawer moves along its 2-m travel path (see Fig. 6).

An electronic control unit enables automatic operation of sample changer and drawer. Entire sequences of operations, consisting of repeated sample changing, positioning to an accuracy of 0.2 mm, and removal of the sample after a certain time interval, may be read into the electronic control unit from punched paper tape. The automatic operation can be interrupted at any time and continued by manual instructions to the electronic unit which is located in the experiment-control room. The mechanism can also be used for detector traverses.

3.6. Mechanism for Vertical Traverse

Traverses in the vertical direction in any one vertical channel can be made by a mechanism that can be operated automatically from the experiment-control room. For example, a set of fission chambers can be moved stepwise into the reactor core by means of a program punched on paper tape. Although the mechanism is primarily intended for detector traverses, lightweight samples may also be traversed. Exchange of samples must be done manually in the reactor hall.

The traverse mechanism has been developed at STARK by Stegemann.⁽⁶⁾

3.7. Pile Oscillator

A pneumatic pile oscillator has been built for measurement of very small reactivity effects. Two specific examples are the effect of temperature rise in a sample and that of voiding a core material sample of sodium.

The part of the oscillator which moves through the reactor core is a square tube with the same cross section as a control rod. In addition to holding samples, the tube can be filled with material plates over its full length. The tube is much longer than a normal fuel element, so that, even in the extreme oscillating positions, the part of the oscillator channel which is located in the reactor core is filled with material. If the oscillator is filled with core material, the reactor core is perturbed by the sample only. The tube is oscillated pneumatically with a stroke of 80 cm and a transition time of less than 2 sec. The whole mechanism is mounted on a motor-driven carriage so that the oscillator can be remotely repositioned vertically over a length of 70 cm. This is of interest when space-dependent sample worths are measured: if the oscillator could not be repositioned, it would be necessary to relocate the sample inside the oscillator tube several times in order to measure space-dependent worths. Exchange of samples is done manually because the oscillator is not meant for routine measurements of large series of samples.

The oscillator will be utilized for Doppler-reactivity

FIG. 7. Special Element with Doppler Sample.

measurements by oscillation of a heated fuel sample. The sample is electrically heated up to 1000°C in an oven which is insulated from the reactor by a thermal shield and a vacuum. The cylindrical fuel samples have a diameter of 3.5 cm and lengths of 9 and 15 cm.

The pile oscillator and Doppler furnace have been developed at STARK by L. Barleon.⁽⁷⁾

3.8. Doppler Loop

The Doppler effect will be measured not only by exchange of hot and cold fuel samples, but also by periodic heating of a geometrically fixed amount of fuel.⁽⁸⁾ This Doppler sample consists of pins which are located in the core center in special fuel elements. The size of the samples can be varied up to 4.5 l by addition or removal of these special elements. The composition of the sample can be altered by mixing pins of $\text{PuO}_2\text{-UO}_2$, pure UO_2 , a sodium alloy, zirconium hydride, and various structural materials. Except for a narrow gas-flow channel, the unheated volume of the Doppler elements is completely filled with standard SNEAK material plates (see Fig. 7).

The sample is periodically heated and cooled by a stream of CO_2 through a temperature interval between 50 and 350°C . In order to avoid thermal instabilities, two separate loops are used for cooling and heating. In each of these loops the gas flows continuously either through the sample or through a dummy which has equal heat capacity and flow resistance. The sample is thermally insulated from the reactor core by double walls and a flow of air.

4. Techniques of Reactivity Measurement

4.1. Methods for Precise Reactivity Measurements

Among the methods to determine reactivity there are some which enable the measurement of very small effects with high accuracy. Each of the methods has its specific merits, and therefore several will be applied at SNEAK, the choice depending on the requirements for various experiments. The methods differ in their sensitivity to noise, drift, and transients, and in the amount of equipment needed for control of the experiment and analysis of the data. Furthermore the length of time required for a measurement and for its analysis varies widely.

Measurement of asymptotic periods certainly is the most popular method to determine reactivities. It has the advantage of simplicity and therefore of reliability. Special equipment is not required, neither for the measurement nor for its analysis, although digital output of the varying neutron flux over a selected time interval and subsequent treatment of the data by a digital computer considerably facilitates the experiment. Disadvantages of the method are: 1) an asymptotic period is established only after transients have died out; 2) it is difficult to eliminate the influence of reactor drift.

In a plutonium reactor the presence of a large spontaneous neutron source prevents the establishment of an asymptotic period at low power. A computer program that follows the procedure of the commonly used graphical analysis of asymptotic periods would be of

little use at SNEAK. Therefore, two other methods of analysis which take the influence of a source into account were programmed. Each method calculates in a different way the magnitude of the spontaneous neutron source and extrapolates the measured reactor periods to an asymptotic one.

The first method was developed at the Swiss reactor SAPHIR⁽⁹⁾ as a graphical extrapolation procedure by which the stable reactor period can be found from measured positive or negative reactor periods. The reciprocal values of instantaneous reactor periods are plotted as a function of the reciprocal of reactor power. The points lie on a straight line which intersects the ordinate at the reciprocal stable reactor period.

A different graphical method was used at the Halden Boiling Water Reactor in Norway.⁽¹⁰⁾ The influence of a photoneutron source on period measurements was reduced by an iterative procedure. We have programmed this method in order to allow more iterative cycles.⁽¹¹⁾ For a supercritical reactor a logarithmic plot of count rates versus time deviates from a straight line if a spontaneous neutron source is present. The deviation disappears if a constant source term is added to each count rate. In order to determine the value of this source term, a straight line is fitted to the last part of the data, where high count rates occur and the source term has little influence. The source term is subsequently obtained from the first part of the data by determining the difference between the fitted line and these data. With this term added to the measured count rates, a logarithmic plot now approaches the expected straight line more closely than the original data points. The procedure is repeated with larger portions of the data. Iteration continues until all experimental data are utilized and the slope of a fitted straight line does not deviate significantly from the previous one. This slope gives the reciprocal asymptotic period. It can be shown that this procedure leads to a higher accuracy than given by the Swiss method.

Another precise method for measurement of reactivities which has been in use for many years is the pile-oscillator method. Advantages over the asymptotic period method are: 1) the influence of reactor drift can be effectively eliminated and 2) data can be collected continuously during an arbitrary length of time. This is accomplished at the cost of a relatively complicated mechanism that accurately oscillates the sample. Also required is computing equipment that calculates the amplitude of the fundamental harmonic of the reactor power signal. At SNEAK this is done with a digital computer. The program is based on cross correlation of a cosine and the experimental signal; it removes the influence of linear drift. The periodic reactivity signal is not necessarily generated

by a moving sample; e.g., in the SNEAK Doppler loop, periodic heating and cooling of a sample causes a reactivity signal.

A third method of determining reactivity is to solve the inverse reactor kinetic equations with the aid of an analog or digital computer.^(4, 12) The input is a signal proportional to the time-varying neutron flux of the reactor; output is the reactivity as a function of time. In addition to the effective delayed-neutron constants, the effective strength of a spontaneous neutron source is needed as a constant in the equations. The source term can be determined by solving the equations without this term after a step change in reactivity at low reactor power. The main advantage of the method is the time saving resulting from the fact that one does not have to discard the data taken during flux transients, since their effect is taken into account properly in the reactor kinetic equations. A disadvantage is its sensitivity to noise and reactor drift. It is possible, however, to reduce their influence by repeating the measurements. Data collection need not be interrupted between measurements. The inverse kinetics method is applied at SNEAK by off-line digital analysis for precision measurements and by on-line analog computing (reactivity meter) for ease of reactor operation. The digital code will be used for rapid rod calibrations and sample reactivity measurements.

The fourth method utilizes an automatic regulating rod.^(2, 3) Each reactor perturbation is immediately compensated by this rod and no transients of reactor power occur (only the spatial distribution fluctuates slightly). Reactivity is derived from differences in control-rod position, which makes a separate rod calibration necessary. In contrast to the other methods, this method makes only relative reactivity measurements. The influence of drift and noise can be reduced by careful planning of measurement and analysis. Very good results with the autorod method have been reported at the ARMF.⁽¹³⁾

4.2. Data Acquisition from Periodic and Nonperiodic Signals

A four-channel data-acquisition system is available for general use, in particular for collecting nonperiodic signals. The system consists of a timer, four counters, a printer, and a punch unit. The eight-channel paper tape can be manually or automatically prepared for computer input. Both alternating or continuous modes of operation are possible. In the alternating mode two of the counters are read out while the other two collect counts. In the continuous mode all four count independently, so that four signals can be accepted.

A 256-channel analyzer will be utilized for data

collection of periodic signals from one of the linear channels. The analyzer has been modified slightly to permit direct access to any one channel. This has been done for use with a special method for elimination of linear drift; this method is as follows. Assume that $4n$ counts have to be collected during each period. The first n counts, which are collected during the first quarter-period, are stored in a separate set of analyzer channels $5n + 1$ through $6n$. The remaining $3n$ counts of the first period are stored in the channels $n + 1$ through $4n$. The $4n$ counts of each following period are stored in the channels 1 through $4n$, which results in a great reduction of data. If the measurement must be interrupted due to drifting of the signal out of the linear-amplifier range, the data storage is not simply stopped at the end of a full period, but the counts arriving during the first quarter of the next period are also stored separately in the channels $4n + 1$ through $5n$. This storage method offers the usual periodical data storage with arbitrary initial phase, with the added possibility of shifting this phase up to a quarter-period. The correlation function of the signal with a cosine can now be calculated with a correlation time varying between zero and a quarter of an oscillation period. It should be noted that a signal drift which is linear during each single period does not contribute to this function. The amplitude of the fundamental harmonic of the signal is determined by means of two cosine cross correlations with correlation times zero and a quarter-period, respectively.

In the case of Doppler-loop experiments, not only the periodic neutron-flux signal, but also temperature signals, have to be stored. Up to four signals may be stored on a time-sharing basis in the multichannel analyzer. Because temperature signals have a very large signal-to-noise ratio, they can be recorded with sufficient accuracy in a short time. Therefore, most of the analyzer time is available for the collection of the neutron-flux signal.

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Discussion

Mr. Uhrig (University of Florida): What is the total cost of the SNEAK facility, exclusive of fuel?

Mr. Engelmann: Approximately 6 million dollars.

Mr. Uhrig: I wonder in that size of system why you would not go to a complete digital-computer data-acquisition system?

Mr. Engelmann: We were thinking of this and maybe later we will. But for the moment with start-up imminent, we shall use only off-line computing.

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SESSION VIII

October 13, 1966

FUTURE PROGRAMS

Chairman: W. C. Redman
Argonne National Laboratory

Secretaries: C. E. Cohn
E. F. Bennett
Argonne National Laboratory

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The Future United States Fast Reactor Physics Program

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I. F. Zartman

1. Introduction

In introducing my topic this afternoon, I would like to quote from Mr. Shaw's* paper on the fast breeder reactor program in the U. S. which he delivered at the British Nuclear Energy Society Conference on Fast Breeder Reactors⁽¹⁾ in May 1966. He stated:

"The primary objective of the civilian power reactor development program in the United States is to achieve widespread commercial use of nuclear energy for the production of heat and electricity while fully exploiting the energy available in our resources of uranium and thorium. The Commission's objective also includes fostering the development of a self-sufficient and competitive nuclear industry."

My discussion will not concern itself with the overall power reactor development program but only those parts in which reactor physics is most directly concerned. The reactor physics program hopes to provide an adequate and sound basis of technical knowledge, a design capability, and an understanding of fundamentals with which the Commission and industry can proceed in their efforts to design the nuclear reactor portion of safe, reliable, and economical power plants. The reactor physics program is concerned with the neutronic portions of reactor cores, the fissioning and control of rate of fissioning, interactions with other areas such as shielding, instrumentation, heat generation and removal, fuel cycles, and perhaps most importantly, safety.

The Future U. S. Fast Reactor Physics Program must be defensible in terms of the ultimate objective which it is supporting, namely, the development of safe, reliable, and economic fast breeder reactors, and the development of a self-sufficient industry. To accomplish this, the Reactor Physics program (a) must contribute directly to the objective, (b) must contribute on an orderly and a timely basis so that the

program output is available when needed, and (c) the costs must not be excessive in relation to the economic benefits to be achieved by the objective.

As you are aware, the Commission has given one of the highest priorities to the concept of a sodium-cooled fast breeder reactor and has made a commitment to construct a Fast Flux Test Facility in support of that concept. As a result the major emphasis in the fast reactor physics program is given to the Liquid Metal Fast Breeder Reactor (LMFBR) concept and the Fast Flux Test Facility (FFTF) design. However, the Physics Program does not exclude consideration of coolants other than sodium, is not limited as to fuel cycle, nor is it restricted to a specific breeder reactor type.

2. Formal Review of Fast Breeder Reactor Program

Last year as a part of an overall review of the Atomic Energy Commission (AEC) LMFBR Program, the physics efforts which will be required in support of fast breeder reactor development were examined. This examination compared the needed efforts with the existing reactor physics programs, both here and abroad. Consistent with other portions of the LMFBR program review, the reactor physics study was concerned primarily with information that will be needed in the development of sodium-cooled breeders using the Pu-U-238 fuel cycle. The results of the study were published in a report.⁽²⁾ The report includes:

- A. a summary of physics efforts currently in progress, in the U. S. and abroad, which are pertinent to fast breeder reactor development (Part A summarizes U. S. Efforts and Part B outlines the European and Soviet Activities);
- B. a discussion of the technical problems that must be solved, both in the near-term and long-term development of fast breeder reactors (Part C);
- C. a recommended program of research in fast reactor physics (Part D).

I will not discuss this report in detail but will con-

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cern myself with some of the more important facts, what has been accomplished since the report was approved by the Commission as a part of the LMFBR Program, and some of our plans for the future. First of all, the LMFBR Program Office was recently established at Argonne National Laboratory by the Commission to serve as an extension of the Division of Reactor Development and Technology (DRDT) and assist the Reactor Division in program analysis, detailed planning, coordination, and evaluation of the various technical activities of the LMFBR Program. The Program Office is continuing the physics review and extending it by factoring in the technical input and requirements from industry, and detailing the interdisciplinary activities such as with safety, control and instrumentation, plant systems, fuel cycles, fuels, and materials.

2.1. Areas Needing Attention

As a result of this review it was concluded that although the current U. S. effort in fast reactor physics was well founded, there was need for augmentation in certain areas. It was recognized that there were significant gaps in our knowledge and understanding, that many difficult problems remained to be solved, and that the design methods used in developing the thermal reactors were inadequate for the fast breeder reactors and required extensive development. Some of the areas needing attention were:

- (1) nuclear data in the energy regions applicable to the LMFBR concepts under consideration;
- (2) the neutron spectrum in these reactors as well as proven measurement techniques;
- (3) proven design methods and calculational techniques for large fast breeder reactor design;
- (4) confirmation of proposed reactor designs.

These areas in which there were deficiencies could be placed into three general groups, namely,

- (1) basic nuclear data;
- (2) integral and critical experiments, experimental techniques;
- (3) theoretical and analytical efforts.

2.2. Recommendations

Taking into consideration the long lead times required in providing facilities, the report recommended the procurement of a fast-neutron generator for the Argonne National Laboratory (ANL) so that, with the Atomic Energy Commission (AEC) Linac operating at Rensselaer Polytechnic Institute (RPI), the Linac authorized for construction at the Oak Ridge National Laboratory (ORNL), and other cross-section-measuring facilities, the AEC would have available adequate cross-section-measuring facilities to cover the

energy range from 1 keV to 10 MeV in which the major lack of nuclear cross-section data exists. Greater utilization of the ANL critical assemblies, the ZPR-3, -6, and -9, was recommended, as well as modifications to ZPR-6 and -9 to permit their use with larger loadings of plutonium.

The early completion of the construction of the ZPPR with its capacity for 6000-liter-core mockups was recommended in the report, together with an immediate determination of the fuel specifications (isotopic composition and fuel fabrication specifications), so that the lengthy and costly fuel procurement for this facility would not delay its availability for engineering mockups and for the large core studies which might be found necessary.

2.3. Emphasis

From these recommendations, it would appear that facilities or tools were the major outcome of the review. This was not the case, however. The tools require time and funds to procure and hence must come first in the program—assuming, of course, one knows for what purpose they will be needed. The design methods or techniques must be capable of designing large reactor cores, since plant and system designers predict plant sizes of 1000 MWe and larger, and there must be an assurance that they will operate safely. Furthermore, the reactors must be capable of high-gain breeding and of operation with economical fuel cycles. The existing design methods are meeting their first challenges in the design of the FFTF and the first round of demonstration power plants. However, for the design methods to be applicable, accurate nuclear data are required. To be accepted with confidence by the reactor designer, the design methods must be experimentally verified, i.e., the ZPR's must confirm the computations.

Thus, the first emphasis was to provide the tools to procure the basic nuclear data for the reactor designer and to perform the integral and critical experiments necessary to confirm design methods. Confirmation would be obtained initially through basic critical experiments, followed by engineering mockups of the reactor design. These topics will be discussed further, but now let me inject a timing element.

Making use of the nuclear physicist's "cloudy crystal ball" in an attempt to find the research and development path and schedule along which the development of the safe, reliable, and economical fast breeder power plant will follow, the reactor physicist finds a multi-level structure of problems outside the neutronic area which add a fuzziness to each path he believes worth following. There are engineering problems, costs must be kept within bounds, components have their limita-

tions and influence the core design, and only a selected few materials are compatible.

However, our crystal ball clearly indicates that the research and development path will be somewhat like that experienced in developing the thermal reactors. So we must resort to this past experience and try to profit by the successes and avoid the pitfalls.

There seems to be ample confidence that a Liquid Metal Fast Breeder Reactor in the lower power range (up to 200-300 MWe) can be designed that will operate safely and will breed; indeed, this has been done. However, plant economics and fuel-cycle performance and costs are only vaguely known, and will remain so until further operating experience is available from the demonstration power reactor plants. However, the path is less clear for the larger power reactors. The many conceptual studies for large breeder reactors, here and abroad, consider cores of various geometrical shapes, are not in unanimous agreement on the methods for satisfying the safety requirements, and vary widely on the predicted economics and breeding gain.

2.4. Physics Program Parts

As in the case for the thermal reactors, the fast breeder reactor concepts approach the large-sized reactors which are expected to meet the Commission's goals by way of smaller-sized demonstration reactors. Thus, in planning the fast breeder reactor physics program, we find it necessary to divide the program into at least two parts. These parts are not necessarily separated in time at all stages, but they have different missions.

There is the *short-term* portion whose physics output must meet the early demands for the first-of-its-kind demonstration LMFBR's and the program's facilities such as the FFTF. And then there is the *long-term* portion which must provide for the large-sized reactor design requirements on a more relaxed schedule.

FTTF design and construction plans, and conceptual design studies relating to the demonstration reactors, indicate that the short-term output must be available to the reactor designers within the next 2 or 3 years, whereas the large-sized-reactor designers must have their design information at later times during the coming 10-year period.

The design of the large-sized reactors, following the demonstration plants by this time interval, permits us to improve our design methods, develop more precise cross sections and other nuclear data, and perhaps factor into the design some operating experience from the demonstration reactors.

This program schedule requires that immediate, intensive efforts be devoted primarily to questions of safety for the FFTF and the demonstration reactors,

while accepting the best breeding gain and fuel economics possible at this time. Likewise, the time-consuming portions of the program must be started early so that detailed information and data will be available when final core designs, fuel cycles, and breeding gains are ready for optimization. As the more immediately required results are obtained, the Research and Development effort will be shifted from that required for the demonstration reactors to that for the large-sized reactors.

Peering into the "How Long" region of the "cloudy crystal ball," I can't see an end to the path. After the earlier stated objectives of the fast breeder reactor program are attained to a limited degree, one would expect a gradual reduction possible in the physics efforts. However, the process of optimization followed to obtain these objectives proceeds slowly; furthermore, one is not certain when the objectives are reached. Each reactor plant put into operation will feed back information which can contribute to achieving the objective—it will also feed back many problems. The technical problems confronting the fast breeder program are serious and far from solutions. Research and development in reactor physics and its interdisciplinary areas will be necessary for a long time; it will continue to challenge the AEC-supported reactor physics groups as well as those supported by industry.

As I mentioned earlier, I will now discuss portions of the reactor physics program which we consider of major importance, particularly for the long term. In doing this I will confine my remarks to some of the steps we have taken to implement the recommendations of the report, to the identification of areas which we believe will require considerable study and effort in the future if the detailed design required for the optimization of safe, reliable, and economic reactor cores is to be attained, and to pose some questions for future consideration as well as problems which, if workable solutions are found, will speed up and improve the data and design methods we are seeking.

3. Basic Nuclear Data

3.1. Cross-section Measurements

A key portion of both the short-term and the long-term physics programs is the measurement of neutron cross sections. The need for such data has been recognized for a number of years. However, data especially required by the LMFBR concepts are in scarce supply. Furthermore, progress in obtaining the necessary accuracy over the energy ranges of interest unfortunately has been rather slow. I need not devote any time in justifying this need, for other papers at this conference

have discussed this topic and we are continually being reminded of the need for these data by the reactor designers. Therefore, let me discuss what we are doing to supply this deficiency.

Earlier, I referred to WASH-1066⁽²⁾ and to the additions being made to the reactor cross-section-measuring facilities supported by the AEC. These efforts to measure cross sections are concentrated principally at RPI, ANL, and ORNL, where advanced neutron sources are available or planned for construction. The combination of facilities should provide coverage of the entire range of neutron energy of interest to fast breeder designs.

The program of measurements for these facilities is largely determined by the results of studies and requests made by the users of cross-section data. Request lists have been extensively studied in this country and abroad. Current lists covering these countries, for example, are given in WASH-1057 and in a publication of the European American Nuclear Data Committee, EANDC 55U.⁽³⁾

The general worldwide agreement on a wide number of requests should be noted even though there is not worldwide agreement on the needed accuracy in all cases. More recently we have updated the U. S. reactor cross-section request listing, and this listing will become available in the near future.

More quantitative studies of fast breeder cross sections, including desired accuracies, will be carried out over the next few years to establish relative priorities among the various competing needs. These studies are being made to determine the sensitivity of reactor-design parameters to cross-section uncertainties. Examples of such studies which have proved helpful to date are those of Greebler of the General Electric Company and Hummel of ANL, which were reported.⁽⁴⁾

In general, we would hope to give the greatest support to measurements which would bring about the greatest improvements in reactor-design calculations at the earliest date. Of course, at any stage in time we must reach some compromises as to the accuracies that are practically obtainable at that time and the accuracies which are ultimately desired in fast breeder design technology.

Although the data needs vary from reactor concept to concept, there are many data needs which are common to several breeder types, and our cross-section efforts in the near future will concentrate to a great extent on those measurements of general value. As our program focuses on a specific concept, you can expect a corresponding focusing on the cross-section measurements needed in the design of that concept.

We do not expect many changes in the listed needs

for fast breeder reactor cross-section data in the United States over the next several years. However, the priority for measurements will change somewhat from time to time. I would emphasize that the order in which measurements are made will depend not only on recommendations by the Advisory Committee on Reactor Physics and the recommendations of the LMFBR program office, but also on the results of the quantitative studies to be carried out from time to time by the national laboratories and industry, the availability of suitable measurement facilities and instrumentation, and, more generally, upon the solution of samples and standards problems.

The problems of samples and standards deserve further comment. As you are aware, the accuracy requirements placed upon cross-section measurers is quite high compared to the state of the art. Where there are no data, almost any measurement is helpful, but as discrepancies arise between similar measurements and as design methods and computational techniques improve, the pressure increases for more accurate data. To obtain the greater accuracies required by the detailed calculations used to optimize the design of the large-sized commercial reactors, improvements will have to be made in the purity of the materials used as samples for the measurements and in the techniques of sample fabrication. Considerable efforts will have to be devoted to this problem, starting now, to assure the availability of the enriched isotopic material, to assure the high precision in the analysis, and to assure that techniques are available to produce samples of high uniformity in thickness.

The need to intercompare cross-section measurements made with facilities both in the U. S. and abroad requires mutually agreed upon standards of reference. In addition to the sample requirements previously discussed, radiation standards, such as standardized detectors and neutron fields, are required. This problem of standards is not unique to reactor cross sections; the intercomparison of critical experiment results, integral measurements, and neutron spectra likewise require appropriate standards.

3.2. Compilation and Evaluation of Nuclear Data

A problem created for the cross-section measurers by the improved machines, detectors, and data-accumulation methods is the processing and analysis of the large quantities of data. The time gap between the cross-section measurement as represented by "counts per channel" and the "barns as a function of energy" is too great.

At the present time, this time lag may be 2-3 years from the time of measurement to the actual availability of the data for reactor calculations. The suggestion is

that much greater use can be made of computer codes and man-machine interface techniques in the analysis and processing of the machine-output data into usable cross sections. By encouraging such activity, I hasten to say, we are not trying to put any nuclear experts out of a job. Rather, we are hoping to utilize them more effectively. Some notable starts have already been made in machine processing of data, such as with the total cross-section measurements at ANL and with the capture cross-section analysis code developed at General Atomics (GA).

The enormous quantities of experimental cross-section data which have been and will be generated in the United States and abroad requires that automated methods be used for storage, retrieval, and evaluation of the data. Several computer systems have been developed for this purpose. Experimental data are being stored on SCISRS, an acronym for Sigma Center Information Storage and Retrieval System, which is a magnetic-tape file developed at Brookhaven National Laboratory (BNL). These data can then be evaluated and placed on the Evaluated Nuclear Data File, Version A (ENDF/A) and on the Evaluated Nuclear Data File, Version B (ENDF/B) systems developed at BNL. The difference between these systems is that ENDF/B contains a single complete set of data for an isotope in a form suitable for further processing into multigroup sets or Monte Carlo calculations, whereas ENDF/A contains many alternate evaluations of the individual cross sections and is incomplete in the sense that not all reaction types or full energy ranges may be covered.

The processing, evaluation, and conversion from one file to another involve automatic-computer manipulations and considerable human judgment. Computer codes for this purpose are under development, but it will take considerable research to achieve the proper balance between automation and human judgment.

In an effort to develop a single set of evaluated data usable by all reactor design groups, a Cross Section Evaluation Working Group (CSEWG) consisting of evaluators from national laboratories and industry was formed by the Atomic Energy Commission, charged with collecting the best sets of evaluated data which presently exist, putting them into the ENDF/B format, and preparing computer codes to process the resulting tapes into group cross sections. This effort involving about 20 people will be completed this year. Next year the CSEWG will test the data and processing codes. In subsequent years the data will be re-evaluated, new experimental data included, and the entire system of formats and codes will be refined. This iteration process hopefully will converge to a set of data and processing codes usable by all reactor de-

sign groups and in whose development all groups have participated.

4. Theoretical and Analytical Methods

4.1. General Trends in Techniques and Computation

The calculation methods used today were, for the most part, developed for the analysis and design of thermal reactors. This situation will continue for the next few years, although a few calculational methods specifically developed for fast breeder reactors are starting to appear. These methods combined with a modest experimental program will undoubtedly be adequate for the design of a demonstration fast breeder reactor. However, as I mentioned before, we will be relying more and more heavily on the calculational methods to design and refine the design of economical and safe large-scale fast breeder reactors. Our present methods are not adequate for this purpose, and we plan to carry out a strong research program to develop these methods.

Our design methods are actually digital-computer codes. Ten years ago this statement was only partially true, today it is largely true, and ten years from now it will be entirely true. The development of calculation methods must be tied in with the development of computers. Enormous advances in computer speed, capacity, and cost per calculation should be achieved in the next decade, and it will then be practical to include much more accuracy, complexity, and sophistication in our design calculations. However, these advances in computers will not solve our problems any more than the advances in computers in the last decade have solved them. The reactor physicist and designer have always been and will continue to be confronted by problems well beyond the capacity of the computer.

It is then necessary to improve our numerical methods, especially in the area of multidimensional calculations. Even with the most optimistic view of the computer of the future, two- or three-dimensional, multigroup kinetics calculations will not be practical with the methods of solution available today. This research in numerical analysis will take two different directions. In one case the emphasis will be toward putting more physics and geometrical complexity into the calculation as a substitute for doing an experiment, and in the other case the emphasis will be toward reducing calculation costs of the actual design of a reactor while still retaining an adequate description of the physics. I would like to discuss these points in more detail.

4.2. Nuclear Data Reduction and Utilization

I have mentioned previously that considerable effort is being devoted to the establishment of data files

containing the best and most up-to-date nuclear data. These data will be point cross sections, resonance parameters, angular distributions, and thermal-scattering laws. In addition, there will be gamma-ray-production data and cross sections.

The detail is required so as to have all the physics data at hand and applicable to a wide variety of calculational methods. Additional processing of the data is required before they can be used in design calculations. We will have to develop methods and codes to accomplish this data reduction in an accurate and reproducible manner. Let me illustrate this procedure and some of the problems.

The main input to neutronics calculations is a set of multigroup cross sections, that is, cross sections which have been averaged over some energy band by use of an approximate neutron spectrum as a weighting function. The averaging procedure is straightforward; the difficulty is in choosing the proper neutron spectrum. In thermal reactors it turned out that the results of a neutronic calculation were quite insensitive to the choice of a weighting spectrum, so a multigroup set could be produced and used in a wide variety of reactors of varying composition. Unfortunately, this is not the case for the class of fast reactors which concerns us, and it is questionable whether a multigroup set can be obtained which has widespread applicability. Thus, the whole question of cross-section utilization must be re-examined. The Russian method of including composition-correction (self-shielding) tables along with the multigroup set is a start in this direction. The method of overlapping groups used successfully for thermal reactors is another possibility.

It is difficult enough to generate accurate multigroup sets suitable for diffusion-theory calculations in near-asymptotic regions, but we must also generate sets useful for transport-theory calculations in nonasymptotic regions. Shielding calculations are the outstanding example here.

A final problem area in the use of cross-section data is the generation of few-group sets. In order to do a two-dimensional statics calculation, a one-dimensional kinetics or burnup calculation, or a more complex calculation, the reactor designer always collapses the energy structure to a few groups. The method of collapsing and its validity need reexamination.

It may be argued that the next generation of computers will permit retention of a multigroup structure and eliminate the problem. It is more probable that the designer will use the increased speed and storage of new computers to describe geometrical complexity more fully and still use few-group methods.

4.3. *Transport Theory and Monte Carlo Methods*

If we are to rely heavily on calculational methods to refine the design of fast breeder reactors, the methods must be fully developed and tested. As our knowledge of cross sections improves, we can place more confidence in the numerical methods and rely less on an experimental program. We have not yet progressed to this stage. We will need increased efforts to develop the numerical and statistical methods for solving the transport equation. Research is also needed on methods for combining the numerical and statistical approaches.

One-dimensional multigroup and two-dimensional few-group transport-theory calculations are presently being used on a routine basis. Unfortunately, they are still quite costly in computer time, the adequacy of the angular mesh is frequently not verified, and the cross-section input generally does not adequately describe anisotropic scattering. We need to improve the existing numerical methods or develop new methods for solving the transport equation so that a more accurate description of the physical and geometrical complexities can be economically incorporated.

Monte Carlo methods are inherently capable of treating cross-section and geometric complexity. Speed, capacity, and cost of existing computers have limited the usefulness of the Monte Carlo method in the past. With the new computers predicted for the next decade, Monte Carlo will undoubtedly emerge as one of our most important analysis tools, provided that the development of Monte Carlo techniques proceeds at a rate comparable to the development of computers. The main development areas are the prediction of small perturbations caused by changes in temperature and composition, the accurate calculation of competitive effects where strong cancellation occurs, the ability to increase efficiency by focusing the calculation on the desired quantities, and the ability to calculate a variety of configurations with the same set of particle histories.

The applications of Monte Carlo methods to fast breeder reactors are many. The most important applications in the near future will be the analysis in depth of heterogeneity effects in plate-type critical experiments, prediction of these same effects in the practical geometries of the fast breeder reactor, studies of streaming in voided and unvoided regions, and verification of more approximate calculational methods.

4.4. *Design Methods and Codes*

Design codes generally sacrifice detail in the treatment of neutron energy and transport in favor of higher speed and more geometrical complexity. Thus, few-group diffusion theory is the basis for most design

codes. It will be necessary to determine the range of validity of these methods for the reactors of interest and under what circumstances it is necessary to use a more accurate energy treatment and approximation to the transport equation.

The key to better design codes is a fast and efficient method for solving multidimensional problems. Considering three space dimensions, a neutron-energy dimension, and a time dimension, the designer is faced with a problem in five dimensions. With our present computers and methods we can solve problems in two space dimensions with a few energy groups for a total of about two and a half dimensions. The next generation of computer may give us one more dimension, so that, unless we obtain major breakthroughs in numerical methods, we will still be well short of having the desired multidimensional methods.

Let me illustrate these remarks by listing some of the multidimensional calculations we will be developing for the fast breeder reactor program:

- A. three-dimensional flux map of a reactor which includes control-rod perturbations and reactivity changes caused by coolant voiding in one or more channels or portions of channels;
- B. combined thermal-hydraulic-neutronic kinetic calculation of a typical channel which accurately describes neutron-spectrum changes and streaming effects;
- C. two- or three-dimensional burnup studies which include control-rod perturbations;
- D. core meltdown, fuel slumping, and other accident studies which involve movement of part of the reactor.

4.5. Important Applications

In the previous sections, I have dealt mainly with the methods that must be developed to enable us to predict the behavior of a fast breeder reactor. In this section, I would like to mention briefly the more important applications of these methods which are essential to the development of the fast breeder reactor. Our physics program is directed toward the development of those techniques which will solve the most pressing problems.

The outstanding problem in the design of fast breeder reactors is safety. To guarantee the safety of a reactor we must understand the kinetic behavior of the coupled thermal-hydraulic-neutronic system. This requires a sophisticated treatment of the energy spectrum, reaction rates, and the changes of these quantities with temperature, change of state, and material changes. It also requires a detailed knowledge of sodium-boiling phenomena and good numerical

methods for solving time-dependent problems. In order to perform calculations of this complexity, it will be necessary to sacrifice other detail, such as nonuniformity of the core and control rods.

Another problem, again concerned with safety, arises from the large physical size of proposed fast breeder reactors. The neutronic coupling between distant regions of the core may be weak and control difficult. This problem is more severe in thermal reactors but will be present in large fast reactors. Two-dimensional kinetics methods are required to tackle this problem and provide data required to design an adequate control system.

Once a fast breeder reactor is demonstrated to be safe and its dynamic characteristics well understood, attention can be turned to optimization of the design. For this purpose we must be able to predict three-dimensional power distributions as a function of the burnup of the core. Studies of this type will permit power flattening, higher operating power levels, and improved zoning of the core. The burnup studies will be quite important to insure that the reactor is safe throughout its life.

5. Integral and Critical Experiments

5.1. Introduction

I shall not devote much time to the discussion of integral and critical experiments, and their analysis, since this is the subject of the conference. The papers presented and discussed the past few days have done this extensively.

The AEC is supporting a vigorous program of experiments and is adding facilities as I mentioned earlier. These facilities must serve for concept design studies, evaluation of new computational techniques, testing cross-section sets, making the numerous kinds of reactivity measurements, and for engineering mockups of reactor cores.

If we are to obtain sufficient information to test our design methods from a limited experimental program, we must insure that we are measuring the important quantities with adequate precision. The biggest weakness in the experiments at the present time is our lack of knowledge of the neutron spectrum, particularly in the region from 100 eV to 100 keV. Several techniques have been used to measure fast-neutron spectra, notably proton-recoil and time-of-flight. Each has its limitations and needs considerable development. Another area that will need further development is the interpretation of pulsed experiments, either for the purpose of measuring the spectrum by time-of-flight, or reactivity and lifetime measurements.

5.2. Zoned Experiments

A zoned experiment consists of a central region containing fuel materials to be studied surrounded by a buffer and driver region. The advantage is that only a small amount of the fuel to be studied is required in the central zone compared to the amount needed to construct a large core. The difficulty lies in relating the results measured in a zone to those which would be measured in a large core. The central zone should be sufficiently large for the flux and adjoint spectra to be the same as in the large core, but small enough so that little fuel is needed.

Experiments are presently being performed to investigate the design of zoned experiments and answer questions about how large the zone need be, how close it is necessary to match flux and adjoint spectra, and how to design a good buffer and driver zone. Additional analysis of this problem is planned. Zoned experiments have also been done with a fast central zone and a thermal driver, and several other facilities are available and being used for this purpose. If the theory of zoned experiments were put on a firm basis, we would be able to make much greater use of existing facilities and greatly reduce the cost for large criticals and the thousands of kilograms of fuel they require.

5.3. Experiments in the Demonstration Plants

The operation of demonstration reactor plants should provide important information on the success in design and construction of safe and efficient fast breeder reactors. The experimental programs planned for the demonstration plants should provide data indicating how well the design methods work. With some forethought and planning, we should also obtain information on *why* they work as well (or as poorly) as is found. The demonstration reactors provide a unique opportunity for obtaining physics data which cannot be obtained (or only at great difficulty) by any other method. Let me cite two examples.

Only in an operating power reactor does one have the proper combination of thermal-hydraulic-neutronic conditions to study kinetic behavior. The power coefficient of reactivity is a composite of several competing effects. Experiments must be designed to unscramble these effects so that we may gain confidence in our ability to describe each mechanism that influences reactivity. This detailed understanding is necessary before extrapolations to potential accident conditions can be made with confidence.

A second example concerns the absorption and reactivity characteristics of fission products. At the present time we have little or no knowledge of the equilibrium concentration of fission products in a degraded

fast spectrum, or of their cross sections and Doppler coefficients. Since fission product cross sections and Doppler coefficients may have a strong influence on safety and economics, they must be well understood prior to the design of a large fast breeder reactor.

5.4. Analysis of the Experiments

The purpose of analysis is to obtain and interpret reactor physics parameters from the observable quantities measured. This is a highly complex procedure, and requires considerable insight and sophistication on the part of the analyst. The most up-to-date methods and cross-section data must be used, and the analysis must be repeated by independent groups using a variety of methods. Let me explore this further by subdividing analysis into two phases which I will identify as survey and correction.

The survey analysis attempts to discover the accuracy and range of validity of reactor design methods and cross-section data by using the design methods to predict quantities obtained from a wide variety of experiments. For such a survey to be meaningful, the experimentally derived quantities must have been corrected, in the sense that effects peculiar to the particular experimental setup have been removed. The analysis of these corrections is extremely difficult since much of the geometrical complexity of the experiment must be included. Monte Carlo methods will find their most important application here.

At the present time the methods for correcting experimental data accurately are not at hand, available methods are not being fully exploited, and not enough independent groups are involved in the analysis effort. This situation will have to improve if we are to make maximum use of a limited experimental program. This is an area in which further work is planned and in which participation by industry is very desirable.

6. Areas of Exchanges and Cooperation

At various times I have alluded to items which are not unique to the U. S. reactor physicists; they are equally important to those in other countries. Questions regarding "best" neutron cross-section values, calculational techniques, spectral measurements, and safety considerations are suitable topics for consideration as a part of the agreements on exchange of information and cooperation existing between the U. S. and European programs. Additional topics which can be mutually profitable and be of considerable assistance to the fast breeder reactor program are the preparation and testing of evaluated nuclear-data libraries, the performance of jointly planned experiments or measurements aimed at settling uncertainties in nu-

clear data, reactivity coefficients, the results from critical experiments, and the comparison of computations and experimental data.

7. Summary

In summary, the objectives of the Reactor Physics Program are the provision of an adequate and sound basis of knowledge, proven design capabilities, and a complete understanding of fundamentals whereby the Atomic Energy Commission and industry can design safe, reliable, and economical fast breeder reactors and test facilities. The major facilities required for the program are either available or are being provided. The attainment of these objectives will require the procurement of a large quantity of more accurate nuclear data over a wide neutron-energy range, improved knowledge of reactor spectra, an increased de-

sign capability including new design methods, and confirmation of proposed design methods and reactor designs.

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Presented by L. Noble

Recent Developments in the SEFOR Experimental Program

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ABSTRACT

The objective of the SEFOR experimental program is to obtain reactivity-coefficient information that can be used to predict with confidence the kinetic behavior of large, fast, ceramic reactors. In order to obtain this information, three types of experiments will be performed: (1) static tests, (2) oscillator tests, and (3) transient (including superprompt critical) tests.

In the static tests, the power, flow, and inlet temperature can be changed in such a way that the reactivity effect associated with a change in fuel temperature due to power only, the coolant temperature coefficient, the combined power and temperature coefficient, and the flow coefficient can be determined.

Three types of oscillator tests have been analyzed:

1. The conventional oscillator tests in which the reactivity-power transfer function due to the oscillation of a poison rod is determined.
2. Balanced oscillator tests in which the reactivity-power transfer function is determined for the case in which the relative phase and amplitude of a reactivity oscillation and a flow oscillation are adjusted so as to keep the coolant temperature constant.
3. Balanced oscillator tests in which the reactivity-coolant temperature transfer function is determined for the case in which the reactivity and flow oscillations are adjusted so as to keep the power constant.

Transient tests have been analyzed for various total reactivity worths and ejection rates at different power levels. Such tests allow the reactivity-energy coefficient to be determined over a wide range of fuel temperatures.

A description of each test and of the power-tempera-

ture domain over which each test can be performed based on current SEFOR design capabilities is presented. Analytical models for the various feedback effects, together with recently calculated coefficients consistent with the SEFOR mockup tests in ZPR-3, are given. Methods developed for the evaluation of the feedback coefficients in the analytical model from the results of the static, oscillator, and transient tests are described. The proposed instrumentation, data-acquisition system, and data-processing techniques are also described.

1. Introduction

SEFOR (Southwest Experimental Fast Oxide Reactor) is a 20-MW(*t*), fast-spectrum reactor fueled with $\text{PuO}_2\text{-UO}_2$ and cooled with sodium. SEFOR will be used to obtain physics and engineering data for fuel compositions, and temperatures characteristic of large, fast, oxide power reactors.

The SEFOR Project consists of two major parts: the design and construction of the reactor, and a related research and development program. Funds for the design and construction of the facility are being provided by the Southwest Atomic Energy Associates (a group of seventeen investor-owned utility companies located in the southern and southwestern United States), together with the Karlsruhe Laboratory of West Germany, Euratom, and the General Electric Company. The United States Atomic Energy Commission is supporting the research and development program.

This report describes the present status of the pre-operational planning and analysis of the physics experiments to be performed to obtain SEFOR reactivity characteristics. Earlier descriptions of the experimental program appear in References 1 to 4.

* EURATOM, Brussels, delegated to the Karlsruhe Fast Reactor Project.

2. Reactivity Model and Coefficients

One of the feedback mechanisms of primary interest in SEFOR is the Doppler coefficient of the fuel. Determination of the Doppler coefficient, and most other reactivity-feedback mechanisms, in an operating reactor is complicated by the fact that most practical experiments involve simultaneous changes in more than one reactivity-feedback mechanism. Therefore, an analytical description of reactivity feedback must be composed of individual reactivity-feedback models.

The separation of reactivity-feedback effects is not exact due to errors in experimental data and lack of fundamental understanding of all feedback effects. Nonetheless, it is expected that the feedback effects described below will adequately describe the SEFOR reactivity behavior, and that this model can be used to obtain a fit to the experimental data.

2.1. Individual Reactivity Effects

2.1.1 Doppler Effect

Calculations have shown that for SEFOR the Doppler effect, Δk_D , may be closely approximated by:

$$\Delta k_D = \frac{\int_{V_{\text{Core}}} P^2(r) f(r) dV}{\int_{V_{\text{Core}}} P^2(r) dV}, \quad (2-1)$$

where

P = power;

$f(r)$ = a function of the change in local fuel temperature, $T_f(r)$.

A local Doppler coefficient, Γ , may be defined such that

$$\begin{aligned} f(r) &= \int_{\Delta T(r)} \Gamma dT_f \\ &= \int_{\Delta T(r)} \left(\frac{dk}{dT_f} \right) dT_f = \Delta k_D(r). \end{aligned} \quad (2-2)$$

Analysis has shown that the local Doppler coefficient may be represented as

$$\Gamma = \frac{A}{T_f^m}, \quad (2-3)$$

where A and m are constants.

Substitution and integration over a change in fuel temperature gives

$$\Delta k_D = \frac{A}{1-m} \frac{\int_{V_{\text{Core}}} P^2(r) \Delta [T_f(r)^{1-m}] dV}{\int_{V_{\text{Core}}} P^2(r) dV}, \quad (2-4a)$$

$(m \neq 1),$

where the symbol “ Δ ” denotes a change, or

$$\Delta k_D = \frac{A \int_{V_{\text{Core}}} P^2(r) \Delta [\ln T_f(r)] dV}{\int_{V_{\text{Core}}} P^2(r) dV} \quad (2-4b)$$

$(m = 1).$

It is also planned to represent the Doppler effect by a less complicated form that is more useful for reactor operating purposes. Calculations indicate that one of the following forms may be useful in this regard:

$$\Delta k_D = \frac{A}{1-m} \Delta [T_{\text{eff}}^{1-m}] \quad (m \neq 1); \quad (2-5)$$

$$\Delta k_D = A \Delta [\ln T_{\text{eff}}] \quad (m = 1), \quad (2-6)$$

where

T_{eff} = effective fuel temperature = $\bar{T}_c + R_f P$;

$\bar{T}_c$ = average coolant temperature;

R_f = change in effective fuel temperature with respect to power.

2.1.2 Effect of Axial Expansion of Fuel

The axial dimension of the fuel is related to the fuel temperature. Analysis of experimental SEFOR data will be based on precalculation of the reactivity effect of the axial expansion, Δk_e , of fuel.

2.1.3 Coolant Temperature Effect

It is planned to represent the primary coolant effect by

$$\Delta k_c = C_q \Delta \bar{T}_c, \quad (2-7)$$

where the coefficient C_q includes the direct coolant-temperature coefficient C_c as well as temperature coefficients associated with the expansion of core structure, C_m , and a portion of the clad expansion, C_t .

In the event that the coolant effect is not adequately represented by the average coolant-temperature coefficient above, a second coolant-temperature coefficient associated with one of the other three measures of coolant temperature (inlet temperature, outlet temperature, or coolant-temperature rise) will be included as a secondary reactivity effect, e.g., for the inlet temperature $\bar{T}_i$,

$$\Delta k_i = C_i \Delta \bar{T}_i. \quad (2-8)$$

2.1.4 Effect of Fuel Cladding

The temperature of the clad depends on the surface temperature $\bar{T}_{fs}$ of the fuel as well as upon the coolant temperature. The effect of axial expansion of the cladding may be represented by

$$\Delta k_t = C_t \Delta \bar{T}_t = c C_t \Delta \bar{T}_{fs} + d C_t \Delta \bar{T}_c, \quad (2-9)$$

where

$$c + d = 1.$$

For experimental purposes dC_t is included in C_q .

2.1.5 Effect of Coolant Flow

The change of reactivity, Δk_w , for a change in coolant flow at constant coolant temperature is expected to be very small. This effect should be of the form

$$\Delta k_w = C_w \Delta w, \quad (2-10)$$

where C_w is the coolant-flow coefficient.

2.1.6 Effect of Reflector Temperature

The change of reactivity, Δk_r , for a change in the average temperature of the reflector, should be of the form

$$\Delta k_r = C_r \Delta \bar{T}_r, \quad (2-11)$$

where C_r is the reflector temperature coefficient.

2.2. Composite Reactivity Model

The reactivity model planned for the description of the total feedback reactivity change (Δk) between two reactor conditions is a combination of the individual effects. Only those effects presently thought to be important are included. Thus,

$$\Delta k = \frac{A}{1-m} \frac{\int_{V_{\text{Core}}} P^2(r) \Delta [T_f(r)^{1-m}] dV}{\int_{V_{\text{Core}}} P^2(r) dV} + \Delta k_e + C_q \Delta \bar{T}_c + C_r \Delta \bar{T}_r + c C_t \Delta \bar{T}_{fs}, \quad (2-12)$$

or, using equation (2-5), the model may also be used in a simplified form when the Doppler term is of the form

$$\frac{A}{1-m} \Delta [(T_c + R_f P)^{1-m}].$$

2.3. Reactivity Model for Oscillator Tests

Results of conventional oscillator tests are analyzed to determine the overall power-reactivity frequency response. Figure 1 shows the analytical model. The nomenclature used is as follows:

- Δk_i input reactivity (oscillator) (cents);
- Δk_f feedback reactivity (cents);

FIG. 1. Simple Analytical Model for Oscillator Tests.

Δk_n net reactivity (cents);

$\Delta P/P_0$ percent change in reactor power;

$G_0(j\omega)$ zero-power neutron kinetics frequency response $[(\% \Delta P/P)/\epsilon]$;

$H(j\omega)$ overall feedback frequency response $[\epsilon/(\% \Delta P/P)]$.

Measured quantities are

$G_0(j\omega)$ — measured at zero power (0.01 MW);

$$G_1(j\omega) = \frac{G_0}{1 + G_0 H}$$

— measured at power ($> \simeq 0.1$ MW).

Figure 2 shows a block diagram of a typical analytical model for SEFOR. For clarity, the model is shown for a "point reactor." Actually, the reactivity effects are distributed over the whole reactor. Possible direct effects of inlet and outlet coolant temperature on Δk have been omitted. The fuel coefficient C_f includes both Doppler and axial fuel-expansion effects.

The nomenclature of Figure 2 is as follows:

$G_0(j\omega)$ = zero-power frequency response of neutron kinetics;

$F_{av}(j\omega)$ = average fuel-temperature response to power oscillation;

$F_s(j\omega)$ = average fuel-surface-temperature response to power oscillation (or average fuel-temperature response to coolant temperature oscillation);

$F_n(j\omega)$ = average fuel-surface-temperature response to coolant temperature oscillation;

$F_w(j\omega)$ = core-temperature-rise response to fuel-surface-temperature oscillation (essentially constant in the range 0.001 to 0.1 cps);

$F_t(j\omega)$ = core-inlet-temperature response to fuel-surface-temperature oscillation;

$F_m(j\omega)$ = core-structure-temperature response to average sodium-temperature oscillation;

$F_r(j\omega)$ = reflector-temperature response to core power oscillation (guide-structure time constant, ~ 1.5 min; segment time constant, ~ 18 min).

The $F(j\omega)$ functions described above are normalized so that the zero-frequency values are all 1.0. The "K" values associated with each $F(j\omega)$ will vary with power levels, coolant flow rate, and the method of plant control.

At a reactor power of 20 MW and conditions of full coolant flow, additional parameters are

$$a = 0.5;$$

$$\Delta \bar{T}_c = b \Delta \bar{T}_i + a \Delta \theta;$$

$$b = 1.0$$

$$c = 0.15$$

$$\Delta \bar{T}_t = c \Delta \bar{T}_{fs} + d \Delta \bar{T}_c$$

$$d = 0.85$$

2.4. Calculated Reactivity Coefficients

Calculations for SEFOR indicate that the Doppler power (but not temperature) coefficient will be considerably larger than the other effects. The following values have been calculated or estimated from ZPR-III critical experiments⁽⁵⁾ for the constants that appear in the reactivity-feedback model for 2-segment fuel with dished pellets.

Doppler:

$$m = 1.0 \quad \text{corresponds to} \quad T \frac{dK_D}{dT} = -0.0085$$

$$A = -\$2.58 \quad \text{or at 20 MW} \quad \frac{dK_D}{dT} = -0.109 \text{ } \$/^\circ\text{F}$$

Average coolant-temperature effect:

$$\text{core-structure radial expansion, } C_m = -0.183 \text{ } \$/^\circ\text{F}$$

$$\text{coolant density, } C_c = -0.108 \text{ } \$/^\circ\text{F}$$

$$\text{axial clad expansion, } dC_t = -0.066 \text{ } \$/^\circ\text{F}$$

$$C_q = -0.357 \text{ } \$/^\circ\text{F}$$

Fuel-surface-temperature effect:

$$\text{axial clad expansion, } cC_t = -0.0117 \text{ } \$/^\circ\text{F}$$

Reflector-temperature effect:

$$\text{reflector coefficient } C_r = -0.04 \text{ } \$/^\circ\text{F}$$

Other typical values at a reactor power of 20 MW and conditions of full coolant flow are:

$$K_{av} = 11.4^\circ\text{F}/\% \text{ power (average fuel-temperature response to power change);}$$

$$K_s = 1.9^\circ\text{F}/\% \text{ power (average fuel-surface-temperature response to power change);}$$

$$K_w = 0.63^\circ\text{F}/^\circ\text{F (core } \Delta T \text{ response to fuel-surface-temperature change);}$$

$$K_t = 0.71^\circ\text{F}/^\circ\text{F (core-inlet-temperature response to fuel-surface-temperature change);}$$

$$K_r = 0.7^\circ\text{F}/\% \text{ power (reflector-temperature response to power change).}$$

3. Description of Tests

The following paragraphs describe the types of tests and the experimental domain of interest for the SEFOR reactor. The initial planning required for the specification of equipment and the description of the tests have been completed.^(3, 4) The detailed selection of tests and the specifications of test conditions are now underway.

The tests have been divided into the following categories: critical, static, oscillator, subprompt transients, and superprompt critical transients. In general, the

FIG. 2. Expanded Analytical Model for Oscillator Tests.

tests will be performed chronologically as listed; however, checks of previous results may be performed throughout the program in order to detect changes in plant characteristics.

3.1. Critical Tests

3.1.1 Dry Critical Test

The dry (no sodium) critical mass of the core at room temperature will be determined. This will be compared with the wet (sodium-filled) critical mass to obtain the reactivity worth of complete sodium loss from core and vessel, which is calculated to be approximately $-\$10$.

3.1.2 Wet Critical Tests

In addition to obtaining the wet critical mass at a sodium temperature of 400°F, it is planned that material worths (relative to sodium) of such materials as fuel, B_4C , steel, and nickel will be measured. Measurements of the fission-rate distribution of Pu^{239} along the core axis at different radial locations are being considered.

Analysis of tests performed with the SEFOR critical mockup in ZPR-3 indicates that values of l/β as determined by the pulsed neutron source and noise-analysis methods agree quite well.⁽⁵⁾ It is planned that the noise-analysis method will be used in SEFOR. The expected value of l/β is 2×10^{-4} sec.

3.1.2.1 External Heating

The core and coolant can be heated from $\sim 400^\circ\text{F}$ to $\sim 1000^\circ\text{F}$ by use of an external heat source. This capability will allow thermal equilibrium to be attained at several different temperatures. From measurements of the equilibrium fuel, coolant, and reflector temperature, in addition to control-rod position and neutron-flux level, "isothermal" reactivity coefficients can be obtained.

In order to correct later heat balances at high power, the heat losses from the reactor vessel may be determined at various temperatures by measuring sodium flow and the temperature difference between vessel inlet and outlet.

3.1.2.2 Reactivity Worth of Sodium Flow

The flow-reactivity effect is expected to be small. This expectation can be checked by changing the primary sodium flow and measuring the reactivity effect by a calibrated rod. The flow can be varied over the range between 20 and 100 % of full flow. In addition, the flow can be oscillated at different frequencies in the range from 0.0012 to 0.03 cps.

3.2. Static Tests

The primary purpose of the Static Tests is to determine the change in reactivity associated with

changes in reactor power and coolant temperature between two steady-state conditions. These tests can be performed by changing only one of the variables: power, average coolant temperature, or inlet temperature, while the other two variables are held constant. The resultant change in reactivity between specified steady-state conditions is measured by a calibrated rod. By proper sequencing of the variable which is to be changed, it is possible to cover a wide range of power, average coolant temperatures, and inlet temperatures.

The reflector temperature coefficient can be measured by changing the reflector coolant flow rate while the reactor is at power.

The procedures to be used in performing the static tests are summarized below.

Reactivity Coefficient	Variable Changed	Variables Held Constant
(fuel + clad) power coefficient $K_{av}C_f + cC_iK_s$	P	$T_r, \bar{T}_c, T_i$
(coolant + fuel + clad) temperature coefficient $C + C_f + cC_i$	$\bar{T}_c$	T_r, T_i, P
inlet-temperature coefficient C_i	T_i	$T_r, P, \bar{T}_c$
reflector-temperature coefficient C_r	T_r	$P, \bar{T}_c, T_i$

3.3. Oscillator Tests

3.3.1 Conventional Oscillator Tests

In the conventional oscillator tests the reactivity is oscillated at fixed frequency and amplitude, and corresponding changes in plant variables are recorded.

The SEFOR reactor has one-inch-diameter fuel rods. This allows SEFOR to attain the high temperatures of interest for large fast oxide reactors while operating at the relatively low power level of 20 MW. This also means that SEFOR will have a large (~ 30 sec) fuel time constant and that the frequency range of interest for the oscillator tests will be in the range from 0.0012 to 0.5 cps. The amplitude of the oscillating reactivity can be varied with frequency and power level so that the power oscillation ($\Delta P/P_0$) is 5 to 10 %.

3.3.1.1 Zero-power Tests

Due to an estimated spontaneous fission plus (α, n) source of $\sim 8.0 \times 10^{-4}$ W, the reactor will be 0.3¢ subcritical at 100 W. This implied that the zero-power frequency response should be measured at a power level in the range from 100 W to 1 kW.

It is estimated that for SEFOR (beryllium mass fraction = 0.0089) the delayed photoneutrons will cause a change in the zero-power transfer function of less than 0.2 % in amplitude and 0.2° in phase at 0.0012 cps.

3.3.1.2 Power Tests

Reactor frequency-response measurements in the range of average power levels between 0.1 and 20 MW

are of interest. For a frequency range between 0.0012 and 0.5 cps, the reactor steady-state core-temperature rise for such tests would range up to 120°F. The average fuel-temperature amplitude would vary with frequency and power level between $\pm 10^\circ\text{F}$ and $\pm 100^\circ\text{F}$.

3.3.2 Balanced Oscillator Tests

Balanced oscillator tests have been described by Caldarola.⁽⁶⁾ These tests involve oscillating the input reactivity, the primary coolant flow, and the secondary coolant flow in such a manner that either: (1) the core coolant temperature is held constant (first balanced oscillator experiment), or (2) the power is held constant (second balanced oscillator experiment). The first balanced oscillator experiment allows measurement of the fuel plus clad power coefficient, while the second balanced oscillator test allows measurement of the fuel temperature coefficient (see Sect. 6).

In these tests the temperature and power reactivity effects are separated so that a frequency range of 0.0012 cps to 0.03 cps is adequate (the average fuel-element time constant is ~ 30 sec).

Practical methods for achieving balanced conditions are described in Ref. (7).

3.4. Subprompt Critical Transient Tests

The subprompt critical tests involve rapid insertion of less than \$1.0 positive reactivity. The initial fuel-temperature rise is rapid so that heat conduction in the fuel is negligible during the first 1 to 2 sec (i.e., for times small compared to the ~ 30 -sec fuel time constant).

Tests currently being analyzed cover the following range of conditions:

Initial power	—from 0.1 to 14 MW;
Initial sodium temperature	—500 to 760°F;
Δk insertion	—\$0.1 to $\sim$ \$0.95;
Δk insertion time	—approximately 0.1 sec.

The actual values of initial power and Δk insertion will be limited so that fuel does not melt.

The subprompt critical transients of interest can result in peak power levels of 10 to 20 times the initial power and a maximum temperature rise in the average fuel of 100 to 150°F at rates up to 200°F/sec. At full coolant flow the maximum average temperature rises of clad and sodium are approximately 20°F and 10°F, respectively. The cladding and sodium temperature rises during the first few tenths of a second are due primarily to direct gamma and neutron heating.

3.5. Superprompt Critical Transient Tests

The superprompt critical tests involve rapid insertion of more than \$1.0 positive reactivity. Maximum

transients of this type will result in peak power levels of 10^4 to 10^5 times the initial power and a maximum temperature rise in the average fuel of less than 1000°F at rates approaching 10,000°F/sec. The reactivity effects due to cladding and sodium temperature rise may be 15% of the fuel reactivity effect.

Tests using Δk insertion in the range from \$1.0 to \$1.5 in approximately 0.1 sec are of primary interest, since these result in significant fuel-temperature rise independent of heat-conduction effects. An error analysis indicates that little increase in accuracy can be obtained by using Δk insertions $> \$1.5$.

Tests currently being analyzed cover the following range of conditions:

Initial power	—from 0.1 to 12 MW;
Peak power	— 10^3 to 10^5 MW;
Period range	—0.1 to 0.001 sec;
Δk insertion	—\$1.0 to \$1.5;
Δk insertion time	—approximately 0.1 sec;
Initial sodium temperature	—500 to 760°F.

The actual values of initial power and Δk insertion will be limited so that fuel does not melt.

4. Experimental Domain

It is desired to evaluate and determine the validity of the reactivity model over a wide range of reactor operating conditions. The operating characteristics of SEFOR have been determined using 1) recently calculated fuel temperatures, 2) the coefficient values listed in Sect. 2.4 (excluding C_r), and 3) an axial fuel expansion model, $\Delta k_e(\bar{\epsilon}) = -0.0149 \Delta \bar{T}_e - 0.44 \Delta P$. The resulting SEFOR characteristics are shown in Fig. 3 as the change of k versus the average temperature of the coolant, along lines of constant power. In Fig. 4 the characteristics are shown in terms of the average temperature of the fuel rather than the average temperature of the coolant.

A desirable range of operating conditions over which test data may be obtained must be consistent with reactor capabilities and safety considerations. The range, or experimental region, that is being used for planning the SEFOR tests was obtained using the reactor characteristics shown in Figures 3 and 4, and the following considerations:

- flows in the primary and secondary coolant systems are equal;
- minimum coolant temperature of 400°F in secondary coolant loop;
- maximum core outlet temperature of 1000°F;
- average temperature of the fuel not to exceed the value at 20 MW and full-flow operation (i.e., the maximum fuel temperature remains below the melting point).

FIG. 3. SEFOR Static Characteristics and Experimental Region in Terms of Coolant Temperature.

The resulting experimental region is shown in Figures 3 and 4 as the area enclosed in the dotted lines. The static and oscillator tests, as well as the initial conditions for the transient tests, lie within this region.

5. Experimental Equipment

The instrumented fuel assemblies provide data on fuel and coolant temperatures and coolant flow for both analysis and test-operations purposes. Each assembly contains fuel and coolant thermocouples as well as a coolant flowmeter. There are six positions in the SEFOR core in which the instrumented fuel assemblies can be placed. These positions provide for two assemblies at each of three radial locations.

Each instrumented fuel assembly will contain two special fuel rods with two tungsten-rhenium fuel thermocouples in each rod. Approximately 12% of the normal fuel volume has been removed from the 6-fuel-rod fuel assembly to provide space for the thermocouples and lead wires. One thermocouple in each rod will be located at the core midplane, while the other will be at the approximate position of average axial power. These fuel thermocouples will be located at the radial center of the fuel rod, and inserted 3 in. into the top of a fuel column so that axial heat-conduction effects are negligible. The thermocouples are not intended to follow the most rapid of the planned transient tests; however, they should indicate the maximum transient center fuel temperature. An extensive series of in-pile and out-of-pile tests have been performed in the process

of developing a suitable fuel-thermocouple design. These tests have included: (1) transient tests, (2) steady-state calibration tests, (3) tests to determine compatibility of sheath material and mixed oxide fuel, and (4) determination of gamma- and neutron-irradiation effects on thermocouple and insulator materials.

Each instrumented fuel assembly will contain one chromel-alumel coolant thermocouple at the inlet and two at the exit. The exit thermocouples are located 6 in. above the fuel-channel exit, where calculations indicate a 10° F variation in the coolant temperature due to "channeling" effects. The coolant-thermocouple time constant is estimated to be 0.1 sec in the flowing sodium environment.

Coolant flow in each channel will be measured by a permanent-magnet flowmeter mounted at the channel inlet. The flowmeters replace the normal channel orifice and develop approximately 10 mV at full flow. Flowmeter calibration has been performed in a simulated reactor sodium flow configuration.

5.1. Transient-power Detectors

The purpose of the transient-power detectors is to provide a record of the reactor power transient which can be used for later determination of (1) instantaneous power, (2) transient energy release, and (3) instantaneous net reactivity.

A combination of neutron- and gamma-sensitive detectors together with fast-response logarithmic/linear amplifiers will be used to cover the transient power range from 10^{-1} to 10^6 MW. For design purposes, a minimum period of 1.0 msec has been used to establish the overall time response of the power-recording channel. Using the rule-of-thumb established by the SPERT Project,⁽⁸⁾ the resulting channel bandwidth is 16,000 cps. Prototypes of the ion chambers and amplifier have been built, and extensive tests are scheduled to begin in the near future.

A large (3-in. OD x 16 in. long), gamma-compensated, U^{235} fission chamber located in the inner shield region has been chosen for the power range from 10^{-1} to 6×10^3 MW. The maximum measurable power is determined by the maximum chamber current of 10^{-2} A; the minimum measurable power is set by the amplifier frequency response characteristics at low current. The lower limit inherent in the chamber itself is believed to be approximately 10^{-3} MW as determined by cable leakage and noise considerations. Assuming 95% gamma compensation, calculations show that 98% of the signal at steady state is due to components which should follow the instantaneous transient power.

For the power range from 10^2 to 10^6 MW, a small (0.25-in. OD x 3 in. long) gamma-sensitive chamber located at the inner reflector surface has been selected. The maximum measurable power is determined by the

FIG. 4. SEFOR Static Characteristics and Experimental Region in Terms of Fuel Temperature.

maximum chamber current of 3×10^{-2} A. The minimum measurable power depends on the allowable delayed-gamma contribution. Assuming that the delayed-gamma current can be no more than 2% of the total current, then the minimum measurable power is calculated to be approximately 5 times the initial power on the rising part of a transient, and 1,000 MW on the decreasing part of a typical transient.

The reactor design provides 3 holes around the periphery for the large U^{238} chambers and 3 holes at the inner reflector surface for the small gamma chambers. Initially, it is planned to install two U^{238} chambers in diametrically opposite positions, and two gamma chambers in corresponding reflector positions. Coupled with the log/linear amplifiers, this will provide two power-recording channels for the complete range of transient power. The gamma-sensitive chambers will be calibrated by overlapping the transient records of the neutron- and gamma-sensitive detectors.

Although other transient reactor test projects (KEWB, SPERT, and KUKLA) have consistently used linear amplifiers for power recording, it was decided to pursue the development of a wide-band logarithmic amplifier in order to reduce the number of recording channels and make the data-handling process easier. A fast-response, feedback amplifier was de-

signed to convert the ion-chamber currents to a voltage signal suitable for recording. The selection of either linear or logarithmic feedback is made by use of remotely operated reed switches. There are three linear ranges covering one decade each, and one logarithmic range which covers 4.5 decades. The output rise time for a step input current is less than 10 μ sec. Current measurements of the order of 10^{-7} A are practical on the lowest range. The amplifier uses an input buffer stage to reduce the input impedance so that the combination of cable capacitance and input impedance does not limit the response of the amplifier.

It is recognized that a given fractional error in the logarithmic recording causes a larger fractional error in the linear power which is inferred from the logarithmic record. However, it is believed that by calibrating the recording channels before each test and using direct digital recording, the overall recording error in the linear power can be made less than 5% (this includes time response and noise, as well as drift and calibration errors), but does not include errors in heat balance.

5.2. Plant Instrumentation

In addition to the instrumented fuel assemblies and transient-power detectors discussed previously, some reactor-plant instrumentation will provide necessary

test information. The operational measurements of neutron flux, control-rod position, reactor inlet and outlet coolant temperature, reactor coolant flow, and reflector temperature will be recorded as part of the test data.

The operational neutron monitors will provide a linear signal for recording at power levels above 10 W. In addition, a visual, differential neutron-flux display will be provided with a resolution of 0.1% of the full-scale range at power levels above 10 W.

The fine reflector control rods (used for shimming) will have a visual position indicator with a resolution of 0.01 in., which corresponds to approximately 0.1¢ at the maximum incremental worth of a rod. The control-rod positions will be recorded manually as needed for each test.

The coolant temperatures indicated by the resistance temperature detectors at the vessel inlet and exit will be recorded. These data will be used in conjunction with flow data in the core to determine the reactor power level and to check the in-core coolant thermocouples. The temperature detectors should indicate the reactor coolant ΔT with less than $\pm 3^\circ\text{F}$ error. This results in a calculated reactor power error of less than $\pm 5\%$. The temperature detectors are estimated to have a 5- to 8-sec time constant, so that they are expected to be used only for static test measurements and not in the oscillation or transient tests.

A Venturi flowmeter will be installed in the primary system along with an electromagnetic flowmeter for the purpose of measuring reactor coolant flow. The Venturi meter will be calibrated in water prior to installation, and it is planned to measure full reactor flow with $\pm 2\%$ error.

It is planned to record temperatures at several points in the reflector structure and movable segments. This data will be used in determining the effect of reflector-temperature changes on reactivity.

5.3. Data-acquisition System

The purpose of the data-acquisition system is to provide accurate records of the various tests which can be conveniently used for on-site test guidance and later extensive analysis. The system is planned to be capable of performing limited on-site data processing as well as the recording function. In general, this on-site processing will include:

- conversion of raw data to engineering units for all tests;
- Fourier analysis to determine the fundamental frequency components of two channels during the frequency-response tests;
- synthesis of complete transient records for fast transient tests;

- preparation of data in a form suitable for more extensive analysis at the GE-San Jose site.

Rather than accomplish these tasks by a variety of special-purpose systems, it seemed attractive to consider a general-purpose digital system. It was determined that commercially available components could be combined in a computer-controlled system to perform all the desired on-site functions. The computer controls the scanning, digitizing, and data-storage operations during tests, and is used after the tests to perform limited data processing.

Basically, the data-acquisition system consists of the following components:

- solid-state scanner;
- analog-to-digital converter (11 bits plus sign);
- computer with 8,000-word core storage (16 bits per word);
- teletypewriter with paper-tape punch and reader;
- two digital magnetic-tape units;
- calibration unit;
- digital incremental graph plotter;
- analog recorders for on-line use (pen-type and light-beam oscillographs).

The basic system operates at a rate of 30,000 samples/sec in the analog-to-digital conversion unit. The digitized data are written on one digital magnetic-tape unit at 40,000 samples/sec. It is planned to scan and record 72 channels of low-speed data (thermocouples and flowmeters) at 140 samples per second per channel. Only two of the four fast-response transient-power detector channels (ion chamber-log amplifier system) will be recorded at a rate of 10,000 samples per second per channel. The scanner will switch from the neutron-sensitive chamber to a gamma-sensitive chamber (and vice versa) when a pre-set input signal level is reached. The system makes one scan through a total of 74 variables in 7.4 msec, with time information included in each group of 74 data words. This format is fixed for all tests and the effective sampling rate per channel is changed by changing the time interval between successive scans of all the variables. Thus, for the static tests all variables are scanned and recorded every minute, or on command of the operator. The scanning rate is increased as needed for the frequency-response and transient tests. In order to provide guidance for the frequency-response tests, the computer will perform a Fourier analysis of two channels during the tests and print out Fourier components evaluated for the frequency of oscillation.

Continuous recording will be available for selected channels by use of a 6-channel light-beam oscillograph and a 12-channel pen-type oscillograph. A 100-channel multipoint recorder will be available for trend recording as a check on the long-term behavior of the sensors.

The digital plotter will be used to check the stored data after the frequency-response and transient tests. Static-test data will be printed out on the teletype-writer after conversion to engineering units.

The channel sampling rates have been chosen⁽⁹⁾ by considering the dynamic characteristics of the data, and the data analysis which is to be performed after the tests. It is recognized that unwanted "noise" in a data channel at frequencies higher than one-half the sampling rate can cause errors in reconstructing the data from the samples. Pre-sampling filters are included in all channels to eliminate high-frequency noise, and if necessary the basic sampling rate can be increased from 33,000 to 60,000 samples/sec by adding a second analog-to-digital converter. The most likely noise frequency is 60 cps, so that the maximum sampling rate of 140 samples/sec/channel for the slow-transient data (thermocouples) and 10,000 samples/sec/channel for the fast-transient data (ion chambers) should be adequate. However, electronic filtering of the signals to eliminate unwanted high frequencies may be necessary if it is desired to take data at sampling rates slower than the maximum rates.

5.4. Reactivity Control Equipment

Two special items of experimental reactor equipment have been designed for producing the reactivity changes which are necessary for the oscillation and transient tests. Both the rod oscillator and fast reactivity excursion device are designed to be placed (not simultaneously) in the central dry well in the SEFOR core. Both devices are mounted on the same positioning drive which allows the initial position of the poison material to be varied over the upper half of the core with approximately 0.1-in. increments.

The rod oscillator has a sinusoidal position versus time characteristic which is produced by a variable-amplitude scotch-yoke mechanism. The oscillation amplitude and frequency are continuously variable over a range of ± 0 to ± 4.5 in. and of 0.00125 to 0.5 cps, respectively. A maximum reactivity change of $\pm 10\%$ can be achieved with a poison rod of 30¢ total worth. The rod position is indicated and recorded by the data-acquisition system using a variable-inductance linear transducer with a resolution of 0.01 in. In addition, a selsyn transmitter is driven by the rotating scotch-yoke mechanism for the purpose of balanced oscillator test control. It is possible to position the oscillator rod statically at any point over the stroke in order to measure the reactivity versus position characteristic.

The fast reactivity excursion device is a gas-actuated device capable of ejecting a poison rod from any axial position in the upper half of the core in a time interval

which is variable from 0.050 to 0.150 sec. The poison rod is allowed to return to its initial position approximately 1 to 5 sec after ejection. Tests have shown that the rod-ejection time is reproducible within $\pm 2.5\%$ of the ejection time. The poison-device position is indicated and recorded by the data-acquisition system using 3 magnetic switches indicating lift-off, midpoint of in-core travel, and core exit. In addition, a photoelectric device indicates the motion of the device by registering the passage of alternate dark and light bands past the photocell. The fast reactivity excursion device uses different poison worths and has the capacity to provide reactivity increments over the range from \$0.10 to \$1.50.

6. Data Analysis

Pre-operational analysis of the SEFOR reactor indicates that reactivity changes will be primarily dependent upon the Doppler effect and the average coolant and structure temperatures. If measurements show that the effects of inlet, clad, and reflector temperature changes are indeed small, it is planned that these effects will (when necessary) be applied as corrections to the measured feedback. The "corrected reactivity" will then be used to determine the dependence of the Doppler effect (ΔK_D) and the average coolant-temperature coefficient C_q upon power (or energy) and average coolant temperature. It is planned that the necessary correction for the effect of axial fuel expansion (which should be small because of SEFOR fuel design) will be calculated rather than measured.

The Doppler effect may be determined by using the simplified model given in (2-5) or (2-6), and/or by using the more complicated model described by (2-1). If the simple model is used, calculations can be performed to determine the expected dependence of the parameter R_f upon the coolant temperature, power, and transient energy release. These calculations can then be compared with the actual experimental results of the various tests.

If the latter procedure is used, the spatial power and fuel-temperature distribution, as well as an assumed form for $f(r)$ must be used in equation (2-1). The spatial power and temperature distributions can be obtained from calculations based on measurements of spatial fission distributions and fuel temperatures in the instrumented fuel assemblies. The parameters in the assumed form for $f(r)$ (such as A and m in 2-2) can then be adjusted so as to obtain the best least-squares fit to the measured dependence of ΔK_D upon power and coolant temperature.

6.1. Static Tests

The power coefficient $C_f K_{av} + c C_t K_s$ can be obtained from the static tests in which the power is changed and

the coolant temperatures are held constant. After correction for secondary effects the "static" change in reactivity due to a change in average coolant temperature at constant power will involve the temperature coefficients $C_f + cC_t$, and C_q . A least-squares fit of the reactivity model to these experimental data can be performed to determine the "best" values of the model parameters.

6.2. Oscillator Tests

It is planned that the reactivity transfer functions will be obtained by treating the oscillating signal outputs in the manner described by R. Persson.⁽¹⁰⁾ This technique is similar to a Fourier analysis except that the signal is multiplied by a square wave whose amplitude during successive half-cycles varies as the binomial coefficients. This technique helps to eliminate "drift," and our analysis has shown that any "white" noise contribution decreases as 2^{N-1} rather than as $1/N$ in the "conventional" method.

6.2.1 Conventional Oscillator Tests

It is planned that the feedback response, $H(j\omega)$, will be least-squares fitted to the model described in Sect. 2.3, and/or to a simplified form of it. For SEFOR, $H(j\omega)$, in the frequency range from 0.001 to 0.1 cps, is approximately the sum of the fuel, clad, structure, and sodium power coefficients multiplied by the average fuel-temperature response to power oscillation $F_{av}(j\omega) \simeq 1/(1 + j\omega\tau_f)$, where $\tau_f = 30$ sec is the fuel time constant. At higher frequencies $G_0(j\omega)$ and $G_1(j\omega)$ are nearly identical. This implies that it may be difficult to separate the various reactivity coefficients using only conventional oscillator techniques.

6.2.2 Balanced Oscillator Tests

The first balanced oscillator test allows measurement of two transfer functions. One transfer function, that between primary coolant, W , and power, P , has been shown⁽⁶⁾ to be equal to that between fuel surface temperature, $\bar{T}_{fs}$, and power, that is,

$$\frac{\Delta W(j\omega)/W_0}{\Delta P(j\omega)/P_0} = F_s(j\omega).$$

The other transfer function is that between power and reactivity:

$$H(j\omega)|_{T_c} = C_f K_{av} F_{av}(j\omega) + cC_t K_s F_s(j\omega).$$

An additional transfer function which can be measured, using the fuel thermocouples, is that between power and central fuel temperature, $F_c(j\omega)$. Since F_c is related to $F_{av}(j\omega)$ and $F_s(j\omega)$ by the fuel frequency-response model,⁽⁶⁾ this measurement provides another means of checking the validity of the model.

In the second balanced oscillator experiment the net reactivity is zero, and one thus obtains (after any necessary corrections due to an imperfect balance)

$$\begin{aligned} \frac{\Delta K_i(j\omega)}{\Delta T_c(j\omega)} &= C_c + dC_t + C_m F_m(j\omega) \\ &\quad + cC_t F_n(j\omega) + C_f F_s(j\omega). \end{aligned}$$

For SEFOR, $F_m(j\omega)$ is nearly equal to 1.0 in the frequency range from 0.0012 to 0.1 cps. Thus, a least-squares fit to the experimental values of $\Delta K_i(j\omega)/\Delta T_c(j\omega)$ yields the constant term $C_q = C_c + dC_t + C_m$ and the coefficient of the frequency-dependent term $C_f + cC_t$. If the frequency dependence of F_n and F_s are significantly different, it may be possible to determine separately the coefficients C_f and cC_t , since $F_s(j\omega)$ should be known from the first balanced oscillator experiment.

The values of C_f , cC_t , C_q , and $C_f K_{av} + cC_t K_s$ can be obtained at a number of power levels and average coolant temperatures, and the dependence of these parameters on power and temperature can thus be experimentally determined. In particular, the data can be least-square fit in order to determine the "best" values of the parameters (such as A and m) in the Doppler models described above.

6.3. Transient Tests

As in the oscillator tests, the transient analysis is based on the assumption that the "point reactor" kinetic equations are valid and that the delayed-neutron parameters and l/β are known.

In order to investigate transient spatial effects, the ramp ejection in 50 msec of poison slugs worth ~ 0.95 have been simulated with the VARI-QUIR computer code. The results indicate that the maximum change in spatial power shape during the transients is approximately 1% in a region immediately adjacent to the poison. The change in the group-1 flux ($E_n \geq 0.5$ MeV) divided by the average power, was $\lesssim 0.2\%$ at the U^{238} detector location. Any spatial changes in the gamma flux at the gamma-detector location should be small, since the fraction of the gamma flux arising from low-energy neutron capture is small. Thus, spatial effects should not invalidate the "point reactor" model.

Calculations have shown that the effectiveness of the individual delayed-neutron groups is (within 1%) constant. The validity of the assumed values for the delayed-neutron parameters, as well as possible effects due to reflected neutrons, can be analyzed by comparing calculated and measured transients during the time interval before appreciable feedback occurs. This comparison can be made for transients resulting from the

ejection of various poison worths at low initial power level.

In view of the above discussion, it is therefore planned that the net reactivity will be calculated from the flux transient by using the "point" kinetic inversion method. After smoothing the power data in a manner similar to that used at the SPERT Project, the dependence of the reactivity feedback upon the energy (time-integrated power change) release will be determined.

6.3.1 Subprompt Critical Tests

In these tests, the input reactivity $\Delta K_{in}(t)$ is completely inserted in a time $t_1 \approx 0.1$ sec. Therefore, for times greater than t_1 , the only change $[\delta K_{net}(t) \equiv \Delta K_{net}(t) - \Delta K_{net}(t_1)]$ in the net reactivity $\Delta K_{net} = \Delta K_{in} + \Delta K_{fb}$ will be due to changes in the feedback reactivity ΔK_{fb} . In SEFOR most of the feedback occurs during times greater than t_1 . Thus, analysis of δK_{net} , rather than of ΔK_{net} , eliminates the large error in ΔK_{fb} ($0 \leq |\Delta K_{fb}| \lesssim 10\epsilon$) which could occur as a result of a small error in ΔK_{in} ($0 \leq \Delta K_{in} \lesssim 95\epsilon$).

For the subprompt transients the dependence of feedback upon the energy release, $\Delta E(t)$, should be nearly linear, i.e., $\Delta K_{fb} = \gamma \Delta E$. The dependence of the energy coefficient γ upon the initial power P_0 can be determined by performing the transients from several different initial power levels.

With the use of calculated values of heat capacities, and of the fuel and clad expansion effects, a comparison can be made with the Doppler temperature coefficient obtained from the static and oscillator experiments.

6.3.2 Superprompt Critical Tests

The superprompt transients will be analyzed in a manner similar to the subprompt transients, and the energy dependence of the reactivity feedback will be determined for various initial power levels and for poison-slug worths up to approximately \$1.5. In addition, approximate methods such as those described by Häfele⁽¹¹⁾ can be used for some of the transients, since the power peak should occur on the linear portion of the time-dependent S-shaped reactivity-input curve.

Acknowledgments

We would like to acknowledge the work of the SEFOR project staff and other supporting personnel, which provided the background for the summary of the reactor experimental equipment development reported in this paper. In particular, we wish to acknowledge the work of E. R. Craig (instrumented fuel assemblies); J. T. Cochran (coolant flowmeters); E. E. Goodale, J. P. Neissel, and M. B. Wittry (ion chambers and amplifiers); P. E. Novak (fuel thermocouples); and

E. R. McKechan (rod oscillator and fast reactivity excursion devices).

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Discussion

Mr. Till (ANL): What sort of precision do you expect in your determination of the various coefficients and in particular for the Doppler effect?

Mr. Noble: It depends somewhat on the test. We haven't completed the analyses of all the tests. For the transient tests we have done two analyses, and there is some uncertainty of course in estimating some of the errors. The Doppler portion of the energy coefficient will have 10 to 20% uncertainty, and I would expect it would be in that range for the other tests also.

Mr. Hummel (ANL): Beryllium has a very important influence on the Doppler effect. The biggest uncertainty in the Doppler effect now probably involves the low-energy flux. If we could get the effect without the beryllium, that would be a very valuable measurement. Is it going to be possible to run the experiments without beryllium?

Mr. Noble: After analyzing the ZPR-3 results, the small beryllium side rods were removed. We still have the central beryllium holddown rod and tightener rod. I know of no plans to remove them at this time.

Mr. Hummel: What is the calculated contribution of the beryllium to the Doppler effect relative to what it would be without any beryllium?

Mr. Reynolds (GE): If we remove the 5.7 % beryllium

oxide, we would have a change in Doppler effect, with $T(dk/dT)$ reduced by 0.002. It would go from 0.0085 down to about 0.0065. We do have plans for a second core in SEFOR. We have not decided exactly what our plans are for the second core, but one of the possibilities would be to remove the beryllium oxide that is in the center tightener rod and to replace it with some other material to change the spectrum.

Initial Experiments in the Karlsruhe Fast Critical Facility, SNEAK*

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ABSTRACT

In Karlsruhe work is continuing on the design of sodium- and steam-cooled fast power reactors. In support of this work, reactor physics experiments in the SNEAK facility are planned for a system with a composition typical of a sodium-cooled fast prototype reactor ("SNEAK 2") and a system with a composition typical for a medium sized steam-cooled fast reactor ("SNEAK 3"). The experimental program concentrates on measurements of coolant reactivity effects and Doppler effects.

The shortage of plutonium fuel has led to the design of a number of zoned cores. Most neutron physics parameters of a full plutonium system can be measured with sufficient accuracy in these cores. However, the reactivity effect of coolant loss from a large zone in the core is influenced notably by the uranium driver. In the steam-cooled system the reactivity effect of changes in steam density depends markedly upon the isotopic composition of the plutonium and upon fission product poisoning. Theoretical investigations show that the steam reactivity effect will also be influenced by the plate structure of the SNEAK assembly. Special experiments are planned for the determination of heterogeneity, plutonium-240, and fission product effects. For the latter, a special fission product mockup has been designed.

1. Introduction

The experimental program of SNEAK is starting with measurements with a well-known fast uranium core, a repeat of the ZPR-3 Assembly 41. This assembly has been chosen because all materials necessary for its simulation are already available at SNEAK. The purpose of "SNEAK 1" is to test the SNEAK facility

and its experimental equipment. The data obtained will be compared with those of ZPR-3.

In support of the fast reactor development work at Karlsruhe, the next two assemblies are designed to investigate special features of sodium- and steam-cooled fast plutonium oxide reactors. Preparations for the sodium assembly "SNEAK 2" began in 1964, and for the steam assembly "SNEAK 3" in 1965.

In the meantime a number of experiments with dilute sodium-cooled systems have been performed in the USA and in the UK, partly also with plutonium fuel. However, information on steam-cooled fast reactor systems with plutonium fuel is still scarce. Therefore the SNEAK program has been modified so that "SNEAK 3" will be built before "SNEAK 2." It is planned that mockups of sodium- and steam-cooled fast prototype reactors will follow these initial assemblies.

In this paper we discuss some problems arising in the simulation of fast power reactors, preparations for sodium-void experiments, measurements of steam reactivity effect, Doppler experiments, and a proposed mockup for simulating the fission products in SNEAK. Other experiments such as measurements of spectra, breeding ratios, noise, and material worth are not discussed.

2. Problems in Simulating Power Reactors

Problems of simulation in SNEAK arise from the following facts:

1. At present only 175 kg of plutonium are available.
2. The plutonium used in SNEAK has a far lower Pu-240 content (8%) than is characteristic for fast power reactor fuel, namely, 20–30%.
3. The SNEAK fuel contains no fission products.
4. Some limitations exist in the availability of non-fissile materials.
5. All core materials are in the form of plates.
6. SNEAK will operate at room temperature.

Each of these problems is discussed briefly below.

* Work performed within the association in the field of fast reactors between the European Atomic Energy Community and Gesellschaft für Kernforschung, mbH., Karlsruhe.

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1. The shortage of plutonium fuel leads to the necessity that all experiments must be carried out in U-Pu-fueled assemblies. For the assemblies 2 and 3 we have chosen to separate the uranium and plutonium in order to obtain a plutonium zone with the appropriate composition. The remaining part of the core will be fueled by U-235. To obtain "local correspondence"⁽¹⁾ and good spectral matching with the plutonium cores of interest, the uranium driver zones are made of almost the same size and composition as the plutonium zones they replace. Criticality is achieved by adjusting the enrichment of the driver. For each measurement the shape of the plutonium zone has to be chosen in a way which minimizes the error introduced by the uranium driver zone. For central-reactivity, Doppler, and spectrum measurements, a central plutonium zone will be surrounded by the uranium driver. The linear dimensions and the statistical weight of the plutonium zone are greatest in this geometry. For other experiments such as local void measurements, where the plutonium zone must be representative of the whole core, it will have the shape of a half-cylinder or a sector. Details of the arrangements will be discussed in the following.

2. An extreme case of zone loading will be used to study the influence of Pu-240 on the reactivity change caused by loss of coolant or by flooding of the core of a steam-cooled fast reactor. Only very little plutonium containing the appropriate amount of Pu-240 will be available. It will be used to build a small test zone with the correct Pu-240 concentration and surrounded by a zone with a lower Pu-240 content. To limit the size of the test zone a differential method will be used for the determination of the steam density coefficient of reactivity (SDC).

3. A third limitation is the lack of fission products in the SNEAK fuel. The effects of fission products are most pronounced in a soft spectrum; in a steam-cooled fast reactor of average burnup, the presence of fission products may add more than 1% Δk to the reactivity change due to loss of coolant. A fission product mock-up without highly radioactive isotopes has been designed. It allows the determination of fission product effects on various reactor parameters with relatively good accuracy. Composition and properties of the mockup will be discussed in Sect. 6.

4. In addition to the problems arising from the fuel and fission products, there are also problems in the simulation of other reactor materials. Several of them, e.g., sodium and oxygen, are not available at SNEAK in sufficient quantities to simulate an entire power reactor core. Outside the test zones these materials will be replaced by other materials with similar properties. Our calculations show that spectrum deviations be-

tween SNEAK and the simulated power reactor core can be minimized by simulating a single element by a mixture of suitably chosen elements. If, for example, two elements are chosen, the composition of the mixture is determined by two independent criteria. The errors introduced in the neutron spectrum by one element can to some extent be compensated by the other element. We took as a first condition that the average macroscopic degradation cross sections of the reactor element and the simulating mixture be equal. As a second condition we required the equality of the degradation cross sections in that part of the spectrum of special importance for the measurements. Alternatively, one may impose as a second condition the equality of the average transport cross sections.

5. Several steps have been taken to reduce the heterogeneity effects resulting from the plate structure of the core materials, e.g., diluting the U-235 and Pu-239 with U-238, and using the plutonium as mixed Pu-U oxide. Nevertheless, in some experiments, heterogeneity effects will become important. For instance, the experiments to determine reactivity effects due to changes in coolant densities at various Pu-240 concentrations must be corrected for heterogeneity effects. In some cases the heterogeneity corrections are of the same order of magnitude as the effects to be measured. From a phenomenological point of view, one can divide the heterogeneity effects into three types: a) effects due to the high-energy flux peaking in the fuel plates, b) effects due to increased resonance self-shielding in the eV and keV region, and c) increase of the mean diffusion constant due to differences in the transport cross sections in the various plates. The effects of type b) are not very important in sodium-cooled fast reactors, but they play a major role in steam-cooled systems. In the latter, the net heterogeneity effect depends on steam density and leads to large corrections to the void coefficient. In the flooded reactor, effects a) and b) lead to reactivity gains of the order of 5 to 10% in k . For this reason, heterogeneity investigations will receive more emphasis in SNEAK 3. The primary purpose of the investigations is to obtain experimental information about each of the three types of heterogeneity effects in order to test our calculational techniques.

6. The difference between the temperature of the zero-power experiment and a power reactor leads to changes in material densities and changes in the resonance line shapes. The former effect does not cause problems because the density can be adjusted. The change of reactor parameters due to the latter effect can be estimated from measurements of the Doppler effect.⁽²⁾

3. Sodium-void Experiments

The objective of the assembly SNEAK 2 is to check reactor physics calculations in a system similar to a 300-MWe sodium-cooled prototype reactor. Emphasized are measurements concerning sodium safety coefficients.⁽⁸⁾

3.1. Calculational Reactor Model

A number of one- and two-dimensional diffusion calculations have been carried out in order to determine the restrictions imposed on the experiments by some of the limitations mentioned in Sect. 2. The characteristics of the reactor model are shown in Table 1.

The fuel enrichment in the driver is adjusted so that the critical size of the reactor is not changed if part of the plutonium core is replaced by the uranium driver. In the driver zone 50% of the sodium and all of the oxygen are simulated by a suitable mixture of graphite and aluminium. This is possible without notably affecting measurements in the reactor center; the neutron fluxes change by less than 1%, the sodium danger coefficient by 2%, and the Doppler coefficient by only 1%.

Two shielded cross-section sets were used for the calculations: the Russian ABN set⁽⁴⁾ and the German KFK set,⁽⁵⁾ each with the same 26-group structure. A comparison with experimental results of ZPR-6 showed that the KFK set gives somewhat better agreement with experimental critical masses and spectral indices, but the ABN set gives better values for sodium coefficients.

3.2. Restricted Size of Experimental Zone

Measurements of the sodium-void effect in the entire reactor core require large amounts of materials, a large excess reactivity, and much time for complete reloading of the core. However, in order to obtain values for sodium coefficients in the entire reactor, it is not necessary to extend the measurements over the whole reactor. One may restrict such measurements to a 90° sector of half-core height, as may be seen from the following results in Table 2 of some two-dimensional calculations in r - θ geometry.

The error made by restricting an experiment to a core sector of an all-plutonium reactor, even if the sector measures only $1/8$ of the core, is practically constant and much smaller than the discrepancy between the results of the two cross-section sets. This is caused by the fact that the flux variations in the boundary region have already reached their asymptotic form in such a small sector (see Fig. 1). By performing two measurements, one with $1/8$ of the core and the other with $1/4$ of the core, one can eliminate the boundary error experimentally. The difference of the results of

TABLE 1. REACTOR MODEL

Core diameter and height: 128.64 cm
Reflector thickness: 40 cm
Core volume: 1670 l

	Composition, %:		
	Core	Driver	Reflector
Na	45	45	30
Fe	25	25	22
Fuel	30	30	48
	(UO ₂ /PuO ₂ = 5/1)	(21.3% enrichment)	(UO ₂)

TABLE 2. REACTIVITY DUE TO SODIUM VOIDING IN ALL-PLUTONIUM CORE; ERROR OF SECTOR EXPERIMENTS

Voiding of Entire Reactor Core	$\Delta k_{\text{total}} =$	Cross-section Set	
		ABN	KFK
		$-1752 \cdot 10^{-5}$	$-652 \cdot 10^{-5}$
Error in Δk_{eff} sodium void for:			
Voiding of $1/8$ core	$1/8 \Delta k_{\text{total}} - (\Delta k)_{1/8}$	$14 \cdot 10^{-5}$	$18 \cdot 10^{-5}$
Voiding of $1/4$ core	$1/4 \Delta k_{\text{total}} - (\Delta k)_{1/4}$	$13 \cdot 10^{-5}$	$22 \cdot 10^{-5}$
Voiding of $1/2$ core	$1/2 \Delta k_{\text{total}} - (\Delta k)_{1/2}$	$10 \cdot 10^{-5}$	$22 \cdot 10^{-5}$

Fig. 1. Sector Experiments in All-plutonium Core. Neutron flux at $r = 41.2$ cm as a function of the angular ordinate θ when sectors of different magnitude are voided of sodium.

the two measurements gives exactly $1/8$ th of the sodium-void effect of the entire reactor core. Calculations in slab geometry show that one may reduce the experimental zone in axial direction to $1/2$ of the core height with negligible error ($\approx 2 \cdot 10^{-5}$).

3.3. Influence of the Uranium Driver

If the entire plutonium inventory of 175 kg is placed in a central zone with a radius of 42 cm (355-l volume)

FIG. 2. Mean Sodium Danger Coefficient at the Core Axis as a Function of the Radius of the Plutonium Zone.

FIG. 3. Configurations to Study the Influence of the Uranium Driver.

of the 1670-l reactor (see Table 1), the central sodium danger coefficient differs by only 2% from the central coefficient measured in the full plutonium reactor. If the radius of the central voided zone is increased to

31 cm (142-l volume) the difference becomes 14%. The maximum positive reactivity change due to sodium voiding is achieved in a central zone with a radius less than 30 cm. Therefore it is possible, even with the small amount of plutonium fuel available, to measure the maximum positive void reactivity with sufficient accuracy.

Figure 2 shows the mean value of the axial sodium danger coefficient as a function of the radius of the plutonium test zone. It is seen that the radius should be about 40 cm or larger in order that the influence of the driver on the axial sodium coefficient be negligible.

The error in sodium-void measurements in extended core zones, which is introduced by a uranium driver in the largest part of the core, has been studied with one- and two-dimensional diffusion calculations (see Fig. 3). A sodium-void experiment is assumed to be performed in a plutonium zone consisting of a core sector comprising $\frac{9}{39}$ of the core. The rest of the core consists of the uranium driver and a variable plutonium zone adjacent to the experimental sector. The sodium-void effects in the experimental sector were predicted by calculations in r - θ geometry with the KFK set condensed into 5 energy groups for the cases a through d of Fig. 3. Results are shown in Table 3.

An analogous study has been performed in the axial direction, assuming that the experimental zone is limited to the top half of the core. The bottom half contains a plutonium zone of variable thickness (see Fig. 3, cases e through h). The 40-cm-thick reflector is not shown in the diagrams. Results are given in Table 4.

With the amount of plutonium available, a combination of cases d and g appears to be a reasonable configuration for determining the sodium-void effect of the all-plutonium core.

3.4. Influence of Reflector Composition

The reflector composition has practically no influence on the central sodium coefficient; it changes by only 1.3% if the breeder reflector of Table 1 is replaced by a predominantly U-238 reflector.

The influence of the axial-reflector composition on

TABLE 3. SECTOR EXPERIMENTS IN Pu-U CORE

Case of Fig. 3	Amount of Pu (kg) in Variable Pu Zone	Sodium-void Effect in Fixed Experimental Sector ($10^{-3} \Delta k/k$)	Error (%)
a	587	-1.73	(reference)
b	78	-2.42	40
c	156	-2.14	24
d	144	-2.07	20

TABLE 4. EFFECT OF URANIUM DRIVER IN LOWER HALF OF CORE

Case of Fig. 3	Amount of Pu (kg) in Variable Pu Zone	KFK set (5 groups)		ABN set (5 groups)	
		Sodium Void in Upper Half of Core ($10^{-3} \Delta k/k$)	Error (%)	Sodium Void in Upper Half of Core ($10^{-3} \Delta k/k$)	Error (%)
e	381.5	-3.94	—	-9.58	(reference)
f	91.5	-4.40	11.6	-10.41	8.7
g	46	-4.83	22.6	-10.81	12.7
h	0	-5.73	45.5	-11.46	19.6

FIG. 4. Influence of Axial-reflector Composition on Sodium Coefficient.

the sodium coefficient in a core zone of 10-cm thickness adjacent to the reflector is shown in Fig. 4. A gradually increasing amount of reflector, starting from the outside, has been replaced by U-238. It is seen that the reflector composition has a rather large influence on the neighbouring core zone. One should exactly simulate the reflector over its full thickness in the vicinity of the reactor axis.

In the case of sector experiments the reflector should have breeder composition in the region adjacent to the experimental sector. Replacement of the breeder reflector by U-238 elsewhere introduces a relative error of only 2% in the measured sodium-void effect.

3.5. Sodium-void Experiments in Assembly 2

The assembly SNEAK 2 has been designed on the basis of the foregoing considerations.

The 1380-liter assembly (see Figs. 5 and 6) has the following composition: 38 v/o fuel ($\text{UO}_2\text{-PuO}_2$), 40 v/o Na, and 22 v/o SS.

The central 326-liter zone contains 165 kg plutonium with 8% Pu-240 and 0.8% Pu-241. The equivalent enrichment (ratio of fissile plutonium in this zone to all plutonium and uranium) is 0.15. The rest of the core consists of a uranium driver zone with the same volumetric compositions as the plutonium zone. The fuel enrichment in this zone is 0.19 (620 kg U-235).

Central sodium-void measurements include: 1) danger-coefficient measurements and 2) measurements in central zones of increasing size in order to determine the maximum positive sodium-void effect. In addition, axial sodium-void measurements are planned with the use of the pile oscillator. In order to perform these latter measurements in the upper core-half, the fuel elements containing plutonium are reloaded in such a way that the plutonium zone is located at the upper part of the core.

A measurement in a $1/8$ -core sector is proposed for determination of the sodium-void effect in an entire plutonium core (see Fig. 6). This sector is far enough away from the uranium driver that no spectrum distur-

FIG. 5. Horizontal and Vertical Sections through Assembly SNEAK 2. Configuration for central and axial experiments.

FIG. 6. Horizontal and Vertical Sections through Assembly SNEAK 2. Configuration for sodium-void experiments in an angular core sector.

tions occur at its surface. Calculations show, however, that a correction of 30% must be applied in order to account for the difference between the sodium-void effect in the selected $\frac{1}{8}$ SNEAK core sector and $\frac{1}{8}$ of the sodium-void effect in a complete plutonium core. This difference is due to the influence of the driver on the spectrum and its adjoint in the whole volume of the experimental plutonium sector (see Sect. 4). Despite the relatively large corrections it seems possible, even with the small amount of plutonium fuel available at present, to determine the sodium-void reactivity effects with an accuracy that is sufficient for the safety evaluation of the prototype reactor.

4. Measurements of Steam-density Coefficients of Reactivity in SNEAK 3

4.1. Experimental Program

In order to investigate the characteristics of a fast steam-cooled reactor, a critical assembly is proposed with a core volume of about 700 l and oxide fuel with a ratio of PuO_2 to UO_2 of $\frac{1}{6}$.

Table 5 shows the composition of SNEAK 3. A unit

cell of the core, built up of SNEAK plates and nickel tubes, is shown in Fig. 7. This arrangement of tubes was chosen in order to facilitate measurements of the steam-density coefficient of reactivity (SDC), i.e., the reactivity change per unit density change throughout the entire core. The nickel tubes run axially through the whole core and the upper axial blanket. Since no steam or water can be used in SNEAK for safety reasons, polyethylene will be used to simulate the steam. Polyethylene strings of varying diameter fill the central channels of the nickel tubes. These strings can easily be inserted or removed without unloading the plates.

A part of the reflector of the experimental reactor will be similar to a low-density breeder blanket. How-

TABLE 5. COMPOSITION OF SNEAK 3
(Atom densities in units of 10^{20} cm^{-3})

Isotope	Plutonium Zone	Uranium Zone	Breeder Blanket
Pu-239	13.41		
Pu-240	1.20		
Pu-241	0.12		
Pu-242	0.01		
U-235	0.58	19.15	0.7
U-238	81.54	81.42	100.0
Cr	19.05	46.99	11.5
Fe	76.89	160.9	41.1
Ni	85.52	23.05	64.3
Al		71.20	71.2
C	0.28	47.89	47.9
O {fuel	133.3	106.8	107.0
steam	8.6	8.6	8.6
H	17.23	17.23	17.23

FIG. 7. Unit Cell of SNEAK 3.

ever, most of the outer surface of the core will be reflected by a high-density blanket of depleted uranium.

One possible version of assembly SNEAK 3 is shown in Fig. 8. The cylindrical core will have a height of 90 cm and a diameter of 100 cm. The inner zone, surrounded by a uranium driver, has a height of 65 cm and a diameter of 72 cm, and is plutonium fueled. The mass of fissile material in the two zones is 142 kg Pu-239 and 341 kg U-235, respectively. The core will be surrounded by a blanket of 30-cm thickness.

A comparison of the neutron-energy spectra of the plutonium and uranium reactors (see Fig. 9) indicates that the spectrum in the uranium zone would be somewhat harder than the spectrum of the plutonium zone. More striking are the discrepancies in the adjoint spectra (see Fig. 10). This will be of major importance for the SDC experiments.

The experimental program of SNEAK 3 is aimed primarily towards the investigation of:

- a) the SDC and the reactivity due to coolant loss;
- b) the influence of poisons, e.g., Pu-240 and fission products, on the SDC;
- c) the Doppler coefficient;
- d) material-replacement coefficients, e.g., those of oxygen and structural materials;
- e) heterogeneity effects on the neutron-energy spectrum, the reactivity, and the SDC.

4.2. The Steam-density Coefficient

The reactivity change due to the uniform insertion of H_2O in the core of SNEAK 3 is caused primarily by additional moderation, and partially by reduced leakage. For the assembly 3 without fission products, the dependence of the multiplication constant k on the steam density in the coolant volume fraction of the core (35%) is shown in Fig. 11. The normal hydrogen concentration in the coolant region of assembly 3 is chosen to correspond to that of a typical fast steam-cooled power reactor, i.e., $\rho_N = 0.07 \text{ g/cm}^3$.⁽⁶⁾

Of main interest is the range from $\rho = 0$ (complete coolant loss) to $\rho = 0.1 \text{ g/cm}^3$. From a safety point of view, flooding of the reactor is considered a less serious problem than loss of coolant, since it will not occur as suddenly and can be controlled by some shutdown mechanism. The contributions to reactivity due to neutrons of energy E which are moderated to lower energies by collisions with hydrogen are shown in Fig. 12 for three different steam densities. The net reactivity effect is the sum of large positive and negative contributions, and therefore it is hard to predict the change in the SDC caused by a small change in reactor composition. As soon as there are additional poisons such as Pu-240 or fission products in the core, the SDC

Fig. 8. Horizontal and Vertical Sections through Assembly SNEAK 3. Volume of the plutonium zone 268 l, enrichment 1/7. Volume of the uranium zone 445 l, enrichment 1/5.

changes considerably; instead of an importance gain for degraded neutrons there may be a loss, and the multiplication factor k shown in Fig. 11 does not reach such large values at high steam density.⁽⁶⁾

The neutron-energy spectra and the importance spectra in the center of SNEAK 3 are shown for several steam densities in Figs. 13 and 14. The hardening of the neutron spectrum due to coolant loss is obvious.

The experimental procedure of measuring the SDC appears difficult. Steam-density changes throughout the entire core, blanket, or in zones of the reactor can be simulated by varying the amount of polyethylene in the nickel tubes. This procedure is time-consuming and, furthermore, the accuracy of such measurements depends upon the reproducibility associated with the loading procedure. Therefore, only a few points on the curve $k(\rho)$ will be measured. At the selected steam densities, the slope $\Delta k(\rho)/\Delta \rho$ will also be determined by material-replacement measurements with the pile oscillator or sample changer. Danger coefficients for H_2O will be measured at certain steam densities at several locations in the core; the slope $\Delta k(\rho)/\Delta \rho$ for

FIG. 9. Comparison of Neutron-energy Spectra (Flux per Unit Lethargy).

FIG. 10. Comparison of Adjoint Spectra.

the entire core will then be found by spatial integration. Measurements at 3 or 4 steam densities in the region of main interest from $\rho = 0$ to $\rho = 0.1 \text{ g/cm}^3$ should be sufficient.

The SDC experiments will be made inside the plutonium zone of Assembly 3. SDC experiments at least 10 cm inside the boundary of the plutonium zone will

yield information about an equivalent SDC experiment in an all-plutonium core with a test zone of equal size. It may then be feasible to extrapolate to the SDC of a plutonium core by increasing gradually the size of the test zone. Calculations have been made for 2 kinds of arrangements: 1) concentric spheres and 2) sectors of increasing size.

In the concentric-spheres experiment, the zone of maximum statistical weight (the center of the reactor) is used for the determination of moderation and absorption effects. The second experiment, using sectors, allows the measurement of leakage effects due to steam-density changes. Reactivity effects caused by steam-density changes in 1) concentric spheres of radius r_T inside a reactor with uranium driver and 2) in an equivalent all-plutonium reactor are shown in Table 6. Reactivity changes for corresponding experiments differ up to 20%, although statistical weights, fluxes, and adjoint fluxes in these zones agree within 2%. The difference is caused mainly by the moderation effect. The reactivity worth of neutrons of energy E which are moderated to the energy E' is proportional to $\phi(E) \{ \phi^+(E') - \phi^+(E) \}$. The value $\phi^+(E') - \phi^+(E)$ is of the order of $\frac{1}{100} \phi^+(E)$ to $\frac{1}{10} \phi^+(E)$. A few percent deviation in ϕ^+ in the U-Pu reactor relative to the all-plutonium reactor may account for the quite different moderation effects. The net reactivity effect, which is the sum of large positive and negative contributions of neutrons in the energy ranges from 100 eV to 10 keV and from 100 keV to 10 MeV, can easily differ by as much as 20%.

The sector experiments will be used to determine the radial dependence of the local SDC. It should also be possible to measure leakage effects accurately in a sector and to study the SDC due to steam-density variations in the blanket. However, moderation differences between the U-Pu reactor and the all-plutonium reactor are more pronounced in this experiment.

FIG. 11. Dependence of Multiplication Factor on Steam Density in Assembly 3 (without Fission Products). Spherical core with 53.2-cm radius and depleted uranium blanket.

FIG. 12. Relative Contributions of Flux Moderation to the SDC: $k_i \sim \sum_j \sigma_{i \rightarrow j} \phi_j (\phi_i^+ - \phi_j^+)$ (i = Energy-group Number).

The geometrical arrangement of the experiment is illustrated in Fig. 15.

The multiplication constants for sectors of increasing size for the U-Pu reactor (curves B and C) and the all-plutonium reactor (curves A and D) with corresponding test zones are shown in Fig. 16. Curves A and B give the reactivity change associated with an increase of steam density from ρ_N to $\rho = 0.3 \text{ g/cm}^3$, curves C and D that for a change from ρ_N to $\rho = 0$.

The reactivity values for steam-density changes in the all-plutonium core and/or the blanket can be determined with approximately 30% accuracy by simple extrapolation from sector experiments. By using more sophisticated extrapolation techniques, e.g., by correcting the statistical weight of the test zone for

FIG. 13. Neutron Spectrum at the Center of Assembly 3 at Three Different Steam Densities (without Fission Products); Flux per Unit Lethargy.

FIG. 14. Adjoint Spectrum at the Center of Assembly 3 at Three Different Steam Densities (without Fission Products).

its deviations from linearity, the accuracy can be improved.

4.3. The Effect of Pu-240 on the SDC

The effects of Pu-240 on the SDC will be studied in a multizone version of SNEAK 3. A small test zone within the plutonium zone will be fueled with plutonium of the isotopic composition of interest. A search

for the minimum allowable size of the test zone was carried out at Karlsruhe by W. J. Oosterkamp using first-order perturbation theory in connection with 26-group diffusion theory. The reactivity effects due to the oscillation of a polyethylene sample either in the test zone of the multizone reactor or in the same location of a reactor having the correct Pu-240 content throughout were compared. Differences are caused by devia-

tions of fluxes and adjoint fluxes in the two cases as well as by the different statistical weight of the perturbed region. Table 7 shows the error in the H_2O danger coefficients for three sizes of the test zone and for two Pu-240 concentrations. The error caused by differences in statistical weight may be calculated rather accurately, thus reducing the total error.

It was concluded that about 5 kg of plutonium

TABLE 6. SPHERICAL-ZONE EXPERIMENT IN SNEAK 3; PERCENT CHANGE IN k FOR STEAM DENSITY VARIATIONS IN A TEST ZONE OF RADIUS $r_T^{(a)}$

r_T , cm	All-plutonium Core with Depleted Uranium Blanket; $r_{core} = 53.2$ cm	Plutonium Zone ($r \leq 39$ cm) with Uranium Driver and Depleted Uranium Blanket; $r_{core} = 53.2$ cm
53.2	-1.47 +5.73	
30	-0.66 +2.71	-0.52 +2.39
12	-0.06 +0.27	-0.05 +0.24
0.78 (calculated with perturbation theory)	$-2.08 \cdot 10^{-5}$ $+6.51 \cdot 10^{-5}$	$-1.77 \cdot 10^{-5}$ $+5.58 \cdot 10^{-5}$

^(a) Steam density change from $\rho = 0.0735$ g/cm³ to $\rho = 0$, upper left number; change from $\rho = 0.0735$ g/cm³ to $\rho = 0.3$ g/cm³, lower right number.

FIG. 15. Arrangement for SDC Sector Experiments in Assembly 3.

FIG. 16. Calculated Reactivity Changes for Sector Experiments. Curves A, C: all-plutonium reactor; B, D: U-Pu reactor. Curves A, B: steam-density change in the test zone from $\rho = 0.07$ g/cm³ to $\rho = 0.3$ g/cm³. Curves C, D: steam-density change in the test zone from $\rho = 0.07$ g/cm³ to $\rho = 0$.

TABLE 7. ERROR OF STEAM DANGER COEFFICIENT EVALUATED AT NORMAL STEAM DENSITY

	Atom Ratio Pu-239/Pu-240/ Pu-241/Pu-242				
	80/17/2.5/0.5			63/30/5/2	
Radius of the Test Zone (mm) =	60	120	180	60	120
Error due to Difference of Fluxes and Adjoint Fluxes (%)	23.0	9.2	5.0	20.6	-9.4
Error due to Differences of Statistical Weights (%)	1.8	1.9	2.1	6.7	7.2

having a Pu-240 isotopic content of 30% should be available in addition to the plutonium normally used. This amount will be sufficient to form a spherical zone of 135-mm radius (volume ≈ 10 l), if a 30% Pu-240 isotopic content is desired. Dilution of this material in a 1:1 ratio with the standard SNEAK fuel will lead to a spherical zone of 170-mm radius (volume ≈ 20 l) and a Pu-240 content of 19%.

There will be an additional error introduced by the measurement technique. The polyethylene sample to be oscillated should be large enough to make the reac-

FIG. 17. The Effect of Heterogeneity on the Multiplication Constant in Assembly 3 (with Fission Products). Cell Calculations: $B^2 = 2 \cdot 10^{-3} \text{ cm}^{-2}$. A Unit Cell Consists of Two Slabs with 0.625-cm Thickness: slab 1 contains UO_2/PuO_2 , Fe, Ni, and fission products; slab 2 contains H_2O , Fe, and Ni.

TABLE 8. HETEROGENEITY EFFECTS AT DIFFERENT STEAM DENSITIES

$\rho_{\text{H}_2\text{O}}$ (g/cm ³)	$\frac{\delta k_\infty}{k_\infty}$ (10^{-3})	$\delta k_D^{(a)}$ (10^{-3})	$\frac{\delta k_{\text{eff}}}{k_{\text{eff}}}$ (10^{-3})
0	+2.6	-8.2	-5.6
0.0735	+6.2	-4.7	+1.5
0.3	+15.9	-2.2	+13.7

$$^{(a)} \delta k_D = \frac{\delta k_{\text{eff}}}{k_{\text{eff}}} \frac{\delta k_\infty}{k_\infty}$$

tivity change easily detectable but not so large that it will severely perturb the flux spectrum. A cubic sample of 25-mm edge will produce a reactivity change of about $3 \cdot 10^{-6} \Delta k$. If the precision of a pile oscillator measurement is $\pm 5 \cdot 10^{-8} \Delta k$, such a measurement will have a relative error of about 2%. The estimated error of a danger-coefficient measurement including both the experimental and systematic error due to spectral effects will amount to about 10% for a 135-mm zone of 30% Pu-240 content, and to about 8% for a 170-mm zone of 19% Pu-240 content.

4.4. The Effect of Burnup on the SDC

To study the effect of burnup on the SDC in a fast steam-cooled reactor of medium size, experiments in assembly 3 are planned in which the core will be poisoned by fission products. It is foreseen that the plutonium zone will have a poison content which corresponds to an average burnup of 23000 MWd/t using the fission product mockup described in Sect. 6.

4.5. The Effect of Heterogeneity on the SDC

Figure 17 shows the calculated difference between k_{eff} values of homogeneous and heterogeneous cores at

various steam densities. The curves are based on cell calculations for a geometrically simplified model of assembly 3 using cross sections from the 26-group ABN set. The main features of the calculational method are:

- Collision probability methods are used to treat the transport problems.
- Equivalence relations are used for heterogeneity corrections of self-shielded cross sections.
- Heterogeneity corrections for the group diffusion constants are based on an approximate formula given by Benoist.⁽⁷⁾

Figure 17 shows the large positive effects of heterogeneity at high water densities and the negative effect in the voided reactor.

Table 8 lists the components of the heterogeneity effect at various steam densities. The contribution due to changed diffusion constants, δk_D , is the dominant effect in the voided reactor, whereas for $\rho_{\text{H}_2\text{O}} = 0.3 \text{ g/cm}^3$ the greatest part of the heterogeneity effect can be interpreted as a change in the infinite multiplication factor k_∞ .

In a core without hydrogen, reactivity measurements with test zones of increased heterogeneity are planned. By performing such experiments with the test zones at different radial positions, we expect to be able to separate the effects of δk_D and δk_∞ .

Similar experiments are planned in a core in which a steam density of approximately 0.3 g/cm³ is simulated. In a central test zone of such a core, the δk_D component is negligible. Fine-structure reaction rates will be measured with foils in order to test the calculated fine structure of the neutron flux in different energy regions.

5. Doppler Experiments

A broad program of Doppler measurements is planned for the SNEAK assemblies. These measurements will be directed not only toward the accurate experimental determination of temperature coefficients in typical fast reactor systems, but also toward the verification of the adequacy of resonance cross sections and calculational techniques. In particular, the experimental approach is intended to be very flexible so that the temperature effects can be measured over a wide range of temperatures, sample volumes, sample compositions, and core compositions.

As a basis for the design of the experiments, a series of Doppler-coefficient calculations was carried out for SNEAK 2 and SNEAK 3. The primary goals of these calculations were, in addition to predicting the magnitude of the reactivity effects, 1) a study of the energy dependence of the Doppler effect, 2) a study of the

Doppler coefficient as a function of coolant volume fraction or density, and 3) studies of the effectiveness of buffer zones surrounding the Doppler samples. The calculations were carried out with spherical reactor models, the 26-group ABN cross-section set, diffusion theory, the Doppler program of Froelich,⁽⁸⁾ and first-order perturbation theory. Some of the results for the steam-cooled system are shown in Figs. 18 and 19. The Doppler coefficient is approximately twice as large as for SNEAK 2; however, upon complete loss of coolant, the coefficient is reduced to less than one-third of its normal value. The studies also indicate that the dependence of the coefficient upon coolant density may be experimentally determined by making the appropriate density changes in a small 15-liter volume at the center of the critical facility.

Figure 19 shows that, under normal conditions, the coefficient has a maximum at approximately 500 eV, and that the major part of the effect occurs between 100 eV and 1 keV, i.e., at energies considerably lower than in the sodium system. Therefore greater care must be taken to account properly for heterogeneity effects in the Doppler samples.

Two parallel experimental approaches are planned for the measurements in SNEAK:

- 1) pile oscillator or autorod measurements with relatively small fuel samples (volume ≈ 150 cm³) heated electrically up to 1200°K;
- 2) a Doppler "loop" in which hot gas is used to heat up, over a more limited temperature range, a center portion (as much as 4.5 liters) of the SNEAK assembly.

5.1. Oscillator Measurements

In this type of measurement, two essentially identical samples, one hot and one cold, are oscillated in and out of the reactor. Their reactivity difference is determined by application of either the pile oscillator or autorod techniques.⁽⁹⁾ The samples are cylindrical in form, 15 cm long, and 3.5 cm in diameter. At present, four different sets of Doppler samples are planned: 1) U²³⁸O₂-PuO₂ samples with various fractions of PuO₂; 2) UO₂ samples with various enrichment of U-235; 3) U²³⁵O₂-Al₂O₃ samples with U²³⁵O₂ volume fractions ranging from 0.05 to 0.5; 4) PuO₂-Al₂O₃ samples with PuO₂ volume fractions ranging from 0.05 through 0.5. Samples of types 1) and 2) will cover ratios of fissile to fertile material which are plausible in fast reactor systems. It is hoped that Doppler measurements with types 3) and 4) will shed some light on the uncertainty surrounding the Doppler effect of fissile materials.^(10, 11) These latter samples will cover a broad range of values of σ_p (potential scattering per absorber atom), and therefore should provide an opportunity to

FIG. 18. Total Doppler Coefficient of the SNEAK-3 Plutonium Reactor at 900°K.

FIG. 19. Calculated Energy-group Contributions to the Doppler Effect of the SNEAK-3 Plutonium Reactor at 900°K.

study the adequacy of some of the calculation techniques used for lumped samples, e.g., the flat-flux approximation and the sample-core interaction effect.⁽¹²⁾

5.2. Doppler-loop Measurements

The oscillator measurements described above, although having the advantage of covering a broad temperature range with high-precision reactivity meas-

urements, do not yield Doppler coefficients of the actual core composition. For this purpose, a test zone consisting of 1, 4, or 9 modified SNEAK fuel elements can be inserted into the core and oscillated in temperature between 300 and 600°K. In contrast to the normal platelet construction of the SNEAK elements, the Doppler elements contain a quadratic 49-pin matrix with 0.6-cm-diameter pins. These clad pins are filled with the material necessary to achieve the desired overall composition of the test zone. Three basic pin compositions are planned: 1) $U^{238}O_2$ pins; 2) pins with a ratio of mass of $U^{238}O_2$ to mass of PuO_2 of 3; 3) pins containing structural materials or coolant simulants. The proper combination of types 1) and 2) can produce a test zone with fertile- to fissile-material ratios ranging from three to infinity. Sodium will be simulated by pins containing a lead-sodium alloy; the steam coolant will be simulated by pins containing zirconium hydride.

Doppler-reactivity effects of the order of $10^{-4} \Delta k$ are predicted for a ΔT of 300°K. Calculations made by collision probability techniques have shown that the pin diameter is small enough to require only a small correction for heterogeneity effects.

6. Fission Product Mockup

6.1. Introduction

The radiation level of many fission products prevents their use in a critical assembly. Therefore, a search was carried out for a mixture of stable elements which would simulate the reactivity effects due to the fission products. This mixture contains both fission product nuclides and mockup materials.

In contrast to a former approach to this problem,⁽¹³⁾

the energy-dependent cross sections as given in Refs. 14, 15, and 16 were used for calculating the composition of the mixture, even though rather large uncertainties are associated with many of the cross sections. Our simulation procedure aims at minimizing that part of the effect considered (e.g., the reactivity change due to loss of coolant) that is contributed by mockup materials.

Fission products with half-lives of less than one year were replaced by their decay products, because the anticipated life of a fuel element will reasonably exceed one year. Secondary fission products, i.e., nuclides that are produced by either neutron capture or radioactive decay of directly produced fission products, were taken into account. The total number of fission products considered amounts to 87.

6.2. Basis of Simulation

We introduce the following classification of fission products in order to simulate them by a mixture of stable elements (see Table 9).

The fission products in groups 1 and 5 do not necessitate any simulation. Those in group 2, producing more than half of the reactivity effect, have to be simulated in part only. The natural elements contain all fission products of this group, but their composition is not the one desired. This implies that the mockup will contain some of the nuclides in this group in larger and some in smaller amounts than the original mixture. The nuclides which are not present in adequate amounts will be simulated by the surplus ones; the appropriate composition is determined from use of the cross-section information.

All the fission products of groups 3 and 4 have to be simulated completely. It was aspired to substitute a

TABLE 9. CLASSIFICATION OF FISSION PRODUCTS

The last column shows the relative contribution to the fission product effect on the loss of coolant reactivity

Group	Characterized by			Remarks	Fission Products Concerned	Relative Contribution (%)
	No Simulation	Partial Simulation	Total Simulation			
1	×			Elements with only one stable isotope	Rh^{103} , I^{127} , Cs^{133} , La^{139} , Pr^{141}	15.5
2		×		Isotopes of elements with more than one stable isotope	All that are stable isotopes of Zr, Mo, Ru, Pd, Ag, Cd, Ce, Nd, Sm, Eu, and Gd	54.5
3			×	Gaseous at room temperature	Kr^{85} , all isotopes of Xe	3.5
4			×	Radioactive	Zr^{93} , Tc^{99} , Pd^{107} , I^{129} , Cs^{135} , Pm^{147} , Sm^{151} , Eu^{154} , Eu^{155}	26.0
5	×			Contribute too little to be considered	Se^{82} , Kr^{83} , Kr^{84} , Kr^{86} , Rb^{85} , Rb^{87} , Sr^{88} , Sr^{90} , Y^{89} , In^{115} , Te^{126} , Te^{128} , Te^{130} , Cs^{137} , Ba^{134} , Ba^{136} , Ba^{137} , Ba^{138} , Tb^{159}	0.5

fission product by a material whose absorption cross section has a similar energy behavior, at least with respect to the energy regions in which the main resonances occur. First-order perturbation theory was used in the calculations, since the flux perturbation due to the exchange of a fission product with its substitute is rather small. All nonabsorption effects were neglected. Replacement of a certain fission product by its substitute may result in a small deviation of reactivity for the case of normal coolant density:

$$\Delta k_1 = \frac{\int [\Sigma_a^0(E) - \Sigma_a^*(E)] \Phi_1(E) \Phi_1^*(E) dE}{\int \chi_1(E') \Phi_1^*(E') \int \nu(E) \Sigma_f(E) \Phi_1(E) dE' dE},$$

where

$\Sigma_a^0(E)$ = macroscopic absorption cross section of the fission product;

$\Sigma_a^*(E)$ = macroscopic absorption cross section of the substitute.

Subscripts 1 and 2 refer to normal and zero coolant density, respectively, and all the other symbols have their usual meanings.

Another deviation may result if the same replacement is done at zero coolant density. If we require that these two deviations be equal ($\Delta k_1 = \Delta k_2$), we obtain the correct reactivity change for loss of coolant. This leads to the expression

$$\frac{\frac{\int \Sigma_a^0(E) \Phi_1(E) \Phi_1^*(E) dE}{\int \chi_1(E') \Phi_1^*(E') \int \nu(E) \Sigma_f(E) \Phi_1(E) dE' dE}}{\frac{\int \Sigma_a^*(E) \Phi_1(E) \Phi_1^*(E) dE}{\int \chi_1(E') \Phi_1^*(E') \int \nu(E) \Sigma_f(E) \Phi_1(E) dE' dE}} - \frac{\frac{\int \Sigma_a^0(E) \Phi_2(E) \Phi_2^*(E) dE}{\int \chi_2(E') \Phi_2^*(E') \int \nu(E) \Sigma_f(E) \Phi_2(E) dE' dE}}{\frac{\int \Sigma_a^*(E) \Phi_2(E) \Phi_2^*(E) dE}{\int \chi_2(E') \Phi_2^*(E') \int \nu(E) \Sigma_f(E) \Phi_2(E) dE' dE}} = 1.$$

This equation can be satisfied by multiplying each $\Sigma_a^*(E)$ by a constant factor, i.e., by adjusting the amount of the simulating material.

6.3. Estimation of Error

Examination was made to find what part of the reactivity effect is due to the correct fission product nuclides present in the correct amounts. In order to do so, the reactivity contribution of each fission product nuclide was calculated. This value then was compared to the reactivity contribution of the same nuclide of the mockup. The smaller of the two values was termed the correct reactivity contribution. It can be shown that this method favors the simulation of the fission products in groups 3 and 4 by elements of group 2.

One may also use this method to find the "best"

TABLE 10. COMPOSITION OF THE MOCKUP

Material	Amount (kg)	Material	Amount (kg)
Molybdenum	10.405	Cesium	1.294
Ruthenium	5.750	Lanthanum	2.302
Rhodium	1.663	Cerium	4.761
Palladium	4.680	Praseodymium	2.144
Silver	1.608	Neodymium	8.146
Cadmium	0.212	Samarium	3.338
Iodine	0.371	Europium	0.290
		Gadolinium	0.138

Note: Rhodium, palladium, and ruthenium, which are rather expensive, will partly be substituted by other materials on an experimental basis.

composition of different elements simulating those fission products they contain as isotopes. It is usually of advantage to determine in one calculation the simulating mixture for isotopes that belong to two or three elements. By variation of the amounts to be used of each element, the simulation error may be decreased.

The reactivity contributions were calculated by first-order perturbation theory. Though some of the fission products will produce a non-negligible flux perturbation if added to the reactor, this method appears to be sufficient for an error analysis.

A similar comparison as for the reactivity contributions was made for the mass contributions.

6.4. Application

Table 10 shows the composition of the mockup simulating the fission products for a typical steam-cooled fast reactor of 300-MWt power for an average burnup of about 23000 MWd/t. The isotopic quantity of the elements krypton, xenon, and cesium was reduced by 50% to account for their escape from the core.

The reactivity effect due to fission product nuclides present in the correct quantity amounts to 58.3% of the fission product-induced reactivity change due to loss of coolant. The error introduced by simulating a certain nuclide by a different one was estimated to amount to between 30 and 40%. This will result in a 13 to 17% total uncertainty in the reactivity change due to loss of coolant. In the mixture, 57.5 w/o is con-

TABLE 11. VERSATILITY OF THE MOCKUP

Effect of Interest: Reactivity Change Due to	Reactor Type	% Systematic Deviation in δk
Coolant loss	steam-cooled	0 (reference)
Coolant loss	steam-cooled softer spectrum ^(b)	+5
Accumulation of fission products ^(a)	steam-cooled	-4
Coolant loss	sodium-cooled	-8
Accumulation of fission products ^(a)	sodium-cooled	-6

^(a) Average burnup of 23000 MWd/t.

^(b) Flux below 2 keV artificially increased by factors up to 10 in the lowest groups.

tributed by fission products present in the correct amounts.

In practice most of the materials will be used in the form of oxides (with the exception of rhodium, ruthenium, palladium, silver, and cadmium, which will be present as metals); also iodide and cesium will be used in the form of AgI and Cs₂CO₃.

Because of lack of space, all zirconium fission products were simulated by molybdenum. This does not change the simulation error notably.

6.5. Versatility of the Mockup

The fission product mockup was calculated to give best agreement for the fission product contribution to the reactivity change due to loss of coolant in a steam-cooled reactor. The effect this particular mockup would have for other conditions is shown in Table 11. It gives the systematic deviation (in percent of the correct value) one would obtain by using the mockup that was calculated for coolant loss of a steam-cooled reactor for other situations. The results imply that the mockup is much more versatile than one might have expected.

All calculations were done by means of a FORTRAN program written for an IBM 7074 computer. The fission yields were taken from Ref. 17. The spectra and importance spectra were obtained by a 26-group diffusion-theory calculation. A more detailed treatment of the fission product mockup will be published elsewhere.

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Discussion

Mr. Baker (UK-Winfrith): Would you describe in more detail how you intend to use the mock fission product material?

Mr. Engelmann: We have calculated the mockup material specifically to give the correct reactivity change for the loss of coolant in a poisoned steam-cooled reactor at 23,000-megawatt-days per ton burnup in the spectrum of the SNEAK-3 assembly. We calculated it so that the reactivity change would agree, within the error limits of the cross sections used, with that of the real fission products. Then we asked whether the same mockup material could also be used for other purposes, and we did a study for a reference reactor. There it agreed—there was no deviation. Next, we looked at another steam-cooled design for which we artificially softened the spectrum, and we found it to give the correct effect within 5%. When we looked at the reactivity change due to buildup of the fission products in this steam-cooled assembly SNEAK-3, we

found that it gives the correct answer within 4%. Finally, we asked whether we could use the same mockup also for sodium systems which have a different spectrum. We found that we get deviations of the order of 6% for the accumulation and 8% for the coolant-loss effect. These errors are within the limits of the uncertainty of the fission product mockup, so we might use the same mockup for many purposes.

Mr. Baker: Do you intend to make a series of plates of this material and distribute it throughout your mockup?

Mr. Engelmann: Yes. Most of the materials are in oxide powder form, but some are metals. We will make a mixture and place this mixture either in plates or in bent tubes.

Mr. Stievenart (Belgium): Do you intend to mockup the loss of coolant by this mockup material? I refer to the moderation effect of hydrogen.

Mr. Engelmann: The coolant will be simulated by polyethylene. Loss of coolant will be simulated in the critical by removing the polyethylene foils. We will do the same experiment with and without the fission product mockup in the test region so we can measure the coolant-loss reactivity in the clean reactor and in the poisoned reactor.

Mr. Hummel (ANL): Did the heterogeneity effect that simulates a flooding situation in a steam-cooled reactor refer to a power reactor with 1/4-in. oxide pins?

Mr. Engelmann: No, this was for the critical assembly with the plate structure. In the power reactor the heterogeneity effect would be smaller.

Mr. Hummel: Was the intent to determine the heterogeneity effect on the critical assembly?

Mr. Engelmann: Yes.

Mr. Hummel: What will the experiment be? Will you make the channels twice or three times as large?

Mr. Engelmann: We plan to do many kinds of heterogeneity experiments, including bunching experiments, fine-flux structure measurements, and geometrical forms other than plates. In some regions we will use pins and will simulate the correct steam density

by a low-density plastic material. The main problem in the steam assembly is the heterogeneity effect, but our exact experimental program has not yet been established.

Mr. Mahana (Trombay): You mentioned the adverse effect of core zoning on the sodium-void coefficient. Does this core zoning influence the sodium-void effect in both zones or only in the central zone?

Mr. Engelmann: We have not yet done any experiments, so all information now available is based on calculations. If we perform experiments in the central zone, there we will have the largest dimensions of the plutonium zone so that the perturbation effects from the driver will be small. If we do experiments in large zones with the limited amount of plutonium available, the deviations in the spectrum and the adjoint spectrum will be larger.

Mr. Mahana: The way I understand it is that you calculate a certain value for the sodium-void effect in the uniformly fueled core and then, after you zone it, you recalculate the sodium-void effect for individual zones. If you imagine your uniform core zone divided into zones of the same volume, then is the sodium-void effect unfavorable in either zone or in both zones?

Mr. Engelmann: We made zone studies also for a complete plutonium reactor. There are some deviations in extrapolating, for instance, for a 1/8-sector to the full core. You don't get quite the same effect, as the deviations in the uranium-plutonium zoned system are larger. You can easily correct for the deviations in sector experiments in a clean all-plutonium system. We propose to do the experiment once with a 1/8-sector and once with a 1/4-sector. The difference between these experiments will be multiplied by 8. This comes very close to the effect of voiding the whole core because it eliminates the boundary effects. You cannot eliminate the deviations in the two-zone core.

Mr. Lewis (ANL): Do you intend to do heated-zone as well as single-sample Doppler measurements?

Mr. Engelmann: Yes. I mentioned this in the previous session.

Presented by S. Nomoto

The JAERI Fast Critical Facility

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ABSTRACT

The fast critical facility, called the "FCA," is now almost completed at the Japan Atomic Energy Research Institute. The main features of the facility and the experiment program are presented in this paper.

The machine is the split-table type of horizontal matrix structure. The matrix consists of 35 rows by 35 columns, but it is designed so as to be easily extended to 51 rows by 51 columns in which large cores up to 3000 l with a 50-cm-thick blanket can be built. The facility is provided with a pulsed neutron source of 200-keV, Cockcroft-Walton type of accelerator which generates neutron bursts from 0.05- to 10- μ s width at the center of the assembly.

The facility will go critical in the spring of 1967, with 20% enriched uranium. The experimental program will be mainly concerned with the sodium-void and the Doppler effects in large fast reactors. Accordingly, the heterogeneity effect in the sodium-void experiment is now being theoretically studied. Regarding the Doppler experiment, an intermediate-energy system suitably moderated in order to enhance the Doppler effect is proposed.

1. Introduction

The basic research on fast reactor physics was started at the Japan Atomic Energy Research Institute (JAERI) in 1958; an exponential experiment with a natural uranium system was carried out in 1961. A series of diffusion experiments with a nanosecond-order pulsed neutron source has been made since 1962. The code ESELEM, able to treat up to 1500 fine groups, was developed to obtain accurate neutron spectra and effective cross sections.

A fast breeder reactor development program was prepared by JAERI in 1963. As a part of this program, the construction of a horizontal, split-table-type critical facility, called the "FCA," was started in 1964, at the site of the Tokai Research Establishment. In parallel with the construction, the nuclear characteristics of zoned systems were theoretically studied, since these

systems are necessary for the investigation of large fast cores by limited amounts of fuel. Other activities were the improvement of experimental techniques, especially for spectrum measurements, and the development of noise techniques for the investigation of the space-dependent kinetics.

The critical facility is now almost completed and will go critical in the spring of 1967, with 20% enriched uranium. This will make possible the study of the reactor physics of large fast reactors for an extended period. The above-mentioned activities⁽¹⁾ and some other studies which were performed more recently will be incorporated into the fast critical experiment.

2. The Critical Facility

The main design features of the facility were aimed at achievement of a high flexibility and of a sufficient reproducibility to permit high-accuracy measurements, with a maximum of safety. After careful consideration, the split-table type of horizontal matrix structure was employed (see Fig. 1). On each of the two tables are stacked horizontally 55.2-mm-square, 1324.0-mm-long, stainless steel tubes of 1.0-mm thickness, constrained in position by side and upper frames, forming a half-matrix.

2.1. Matrix and Drawers

In order to assemble the many tubes with high accuracy, each matrix consists of units of 7 rows by 7 columns; in each unit is bundled 25 tubes in 5 rows by 5 columns with high accuracy by spot welding. Thus, the matrix consists of 35 rows by 35 columns for the moment, although the facility is designed so as to be easily extended to 51 rows by 51 columns. The final adjustment of the unit is obtained with high accuracy by means of the many screws installed on the upper and one side frames as shown in Fig. 2. The flatness of the front plane of the matrix is ensured within ± 0.05 mm.

The loading drawer is a stainless steel box of which

Fig. 1. Fast Critical Facility.

the front wall has many small holes for checking the loading pattern. The side walls also have holes separated by 2 in. from each other, through which radial experimental sleeves can be penetrated. The dimensions and their accuracy of the drawer are shown in Fig. 3.

2.2. Bed and Table

One table is supported by recirculating roller bearings and driven on the raceways of the bed by means of an electric motor, which turns the worm, engaged with a rack attached to the lower side of the table. As shown in Fig. 4, the bed and table is of welded structure, 7.5 m in length, 4 m in width, and 1 m in height. The bed is designed so as to be able to hold a total weight of about 250 tons. The reproducibility of the contact between the half-assemblies is very accurate, not exceeding 0.02 mm in the axial direction. The reproducibility in the horizontal and vertical directions is ± 0.05 mm. To adjust finely and keep the level of the table, 60 special leveling blocks are used at the bottom of the bed.

As the table approaches and comes finally in contact with the other table, the speed of the table is changed in three stages: 314, 159, and 15.5 mm/min. At emergency, the table is driven back immediately

Fig. 2. Sectional View of Matrix Tube Assembly.

more than 100 mm within 10 sec from the fixed table, and is finally separated up to 2000 mm. A DC motor

FIG. 3. Loading Drawer

FIG. 4. Bed and Table.

is provided as the standby in the case of failure of AC power. It may be mentioned here that the whole facility can withstand violent earthquakes of 0.6 G.

2.3. Control-Safety Rods and Instrumentation

Each half of the assembly contains four or five control-safety dual-purpose rods of a fuel-removal type.

Usually, one of the rods is used as the control rod for each half, the rest serving as safety rods. Controllability of the rod position is ± 0.1 mm, and accuracy in the position indication is ± 0.05 mm. The total reactivity worths of the control rods and of the safety rods are required to be more than $0.5\% \Delta k/k$ and $2\% \Delta k/k$, respectively. The excess reactivity of the system must

1 Drawer	4 Air Cylinder
2 Rod Shaft	5 Supporter of Mechanism
3 Input of Compressed Air	6 Piston
M_g : Magnet	B: Electro Magnetic Blade
M_a : Armature	A/D: Analog-Digital Converter
L_s : Screw	V_a : Safety Valve
M : Drive Motor	D_u : Shock Absorber

FIG. 5. Structure of Control-safety Rod Drive Mechanism.

not exceed $0.6\% \Delta k/k$. On scram, all the rods are immediately driven out of the core by compressed air; the rods start to move within 20 msec after the neutron flux overrides the trip setting in safety channels and fly within 200 msec over 80% of the full stroke, 600 mm long. The structure of the drive mechanism is shown in Fig. 5.

Each half of the assembly is provided with an americium-beryllium neutron source of about 5 Ci, attached to the top of flexible wire, which can be driven into or out of the assembly by means of a motor. The power level of the assembly is monitored by two fission chambers and six uncompensated $B^{10}F_3$ ionization chambers. Two fission chambers are connected to pulse circuits and used as startup channels. Two of the six ionization chambers are connected to the linear channels, the log N and period channels, and the safety channels, respectively. The block diagram is shown in Fig. 6 and the control console in Fig. 7. Startup-interlock and scram-interlock systems are provided.

2.4. Core Materials

The core materials are mainly of the dimensions of 50.77 mm square, with a thickness of 1.63 or 3.18 mm. The fuel used in the facility is 20% enriched uranium for the time being, although plutonium fuel plates will be used in a few years. The dimensions of the uranium fuel plates under fabrication are shown in Table 1. The main specifications are as follows:

The weight of an individual fuel plate of each type shall not deviate by more than $\pm 0.30\%$ from the mean weight of all fuel plates of the same type. The minimum enrichment of individual fuel plates shall be 19.80 w/o, and the variation of the enrichment from

the minimum to the maximum over the entire individual fuel plate shall be limited to 0.12 w/o. Linear thermal-expansion coefficients shall be more than $14 \times 10^{-6}/^{\circ}\text{C}$ for all directions in the temperature range from 0 to 300°C .

Other materials used are natural uranium, canned sodium, aluminum, stainless steel, oxides of iron or aluminum, and graphite. In addition, polyethylene or beryllium may be used for a special purpose as described later.

As to the plutonium fuel, U-30 a/o Pu-6 a/o Fe alloy plate with jacket of stainless steel is considered as the first choice at this moment. However, safety considerations may limit the amount of the alloy fuel to be used in the facility. Furthermore, a mockup test of the experimental fast reactor which is now proposed in Japan will need the use of oxide fuel. Under these circumstances, most of the plutonium fuel used in the facility for several years will probably be $\text{PuO}_2\text{-UO}_2$.

2.5. Pulsed Neutron Generator

As seen in Fig. 8, the facility is provided with a pulsed neutron source provided by a 200-keV Cockcroft-Walton-type accelerator which generates neutron bursts from 0.05- to 10- μsec width, and from 1- to 200-ke repetition, with a maximum duty cycle of 5%, at the center of the assembly. The pulsing is done before the ion acceleration to minimize the neutron background due to D-D reaction. The main characteristics of this generator are shown in Table 2. This pulsed neutron source will be used for obtaining information concerning the kinetic behaviors of the assembly, and for measuring the neutron spectrum by the time-of-flight method. The flight path is about 11 m for the

FIG. 6. Block Diagram of Instrumentation.

time being, although it is possible to be extended almost twice this length.

2.6. Container and Buildings

The critical assembly is housed in a double container, as shown in Fig. 9. The hypothetical accident which is now assumed for the facility is a meltdown accident of the 700-l uranium core; the whole core is melted and the molten materials flow down to fill the void into the core center, resulting in a nuclear explosion. The energy release was evaluated by the code AX-1. The inner cell is a reinforced concrete structure, a 16-m cube, with a wall thickness of 1.1 m. In order to decide on the design of this inner cell, a series of model experiments was performed by the committee on anti-explosion structure. The inner cell can withstand an internal explosion of 100 kg TNT with only small cracks. The outer cell is a reinforced concrete dome, 25 m in diameter and 21 m high, with a wall thickness of 0.4 m. The inner side is lined with 6-mm-thick steel

plate for leaktightness. The outer cell can withstand possible static pressure rises up to 1.6 kg/cm²(g) due to the hypothetical accident, with the maximum leak rate being 2%/day. During critical operation, ventilation valves are closed.

Near the double container, there are installed four buildings: a fuel-handling building with fuel storages, a control and laboratory building, an auxiliary building, and an accelerator building, in which another 300-keV pulsed neutron source is installed for the pulsed experiments of nanosecond order. Two fuel-transport tubes connect the inner cell and the fuel-handling building, as is seen in Fig. 9.

3. The Experimental Program

A central purpose of the experimental program is to predict accurately the sodium-void and the Doppler effects in large fast reactors. Although considerable effort is being devoted both theoretically and experimentally to the understanding of these effects in many

FIG. 7. Control Console.

laboratories, a lot remains to be done, because of such complexities as the heterogeneity effects and uncertainties in the resonance parameters. More work is required to explain the discrepancy between the theory and experiment at the low-energy flux.

According to the fuel-procurement program, the amount of fuel is increased stepwise, and 500 kg U^{235} and 500 kg plutonium are expected to be available by the end of 1972. The outline of the experimental program as envisaged for the next several years is as follows:

During the first stage of experiments, to be investigated are small systems, fast-thermal coupled systems, and intermediate-energy systems fueled with U^{235} . In these systems, the neutron spectrum, the heterogeneity effect, and the cross section in the resonance-energy region will be studied. In the second stage, with the increased amount of U^{235} and plutonium fuel available, diluted systems and zone systems will be investigated to obtain more direct information about the sodium-void and Doppler effects in large fast reactors. In addition, mockup experiments of the experimental and

TABLE 1. URANIUM FUEL PLATE

Material	20% enriched uranium metal
Density	$\geq 18.7 \text{ g/cm}^3$
Total impurities other than carbon	$<0.1 \text{ w/o}$
Type and dimension before coating.	
Type	Dimension (mm)
2-2-1/8	$(50.75 \pm 0.05) \times (50.75 \pm 0.05) \times (3.10 \pm 0.02)$
2-1-1/8	$(50.75 \pm 0.05) \times (25.35 \pm 0.05) \times (3.10 \pm 0.02)$
1-1-1/8	$(25.35 \pm 0.05) \times (25.35 \pm 0.05) \times (3.10 \pm 0.02)$
2-2-1/16	$(50.75 \pm 0.05) \times (50.75 \pm 0.05) \times (1.55 \pm 0.02)$
Flatness	$\leq 0.03 \text{ mm}$
Roundness of angle	$(0.5 \pm 0.2)\text{-mm}$ radius
Experimental Hole	$(20.0 \pm 0.1)\text{-mm}$ diameter
Coating	$(0.010 \pm 0.005)\text{-mm}$ KEL-lack

the prototype fast reactors which are now proposed will also be done intensively.

3.1. The First Core

The first core is the cylindrical system of 20% enriched uranium fuel with a natural uranium blanket; the volume percentages of the fuel, structural material,

FIG. 8. Pulsed Neutron Generator.

TABLE 2. CHARACTERISTICS OF PULSED NEUTRON GENERATOR

Max Accelerating Voltage	200 keV	
Max Beam Current (Continuous)	450 μ A	
Duty Cycle	up to 5%	
Characteristics of the pulse at 1% duty cycle		
<u>Pulse Width</u> (μ sec)	<u>Repetition</u> Rate (kc)	<u>Max Target</u> Current (μ A)
0.05	200	5.3
0.1	100	5.7
0.2	50	5.7
0.5	20	6.2
1.0	10	6.0
2.0	5	6.0
5.0	2	6.0
10.0	1	6.0
One-shot Pulse Width	10 msec	

and void of the core are 82.42%, 10.71%, and 6.87%, respectively. The blanket also has the same percentage of the respective component. The critical mass calculated by TWENTY GRAND is 100.6 kg U^{235} with a 30-cm-thick blanket. With this core, basic experiments regarding critical and kinetic parameters will be done and results compared with those calculated.

One of the interesting problems is concerned with the neutron spectrum near the vicinity of the core-blanket boundary. As to the reactivity determination in a sub-critical state with the pulsed neutron source, applicability of the Garelis method will be checked in comparison with other methods. The continuous-eigenvalue problem is also interesting. In regard to the Rossi- α method, basic studies will be done to explain the discrepancy between the theory and experiment in the amplitude of correlated term.

3.2. Fast-Thermal Coupled System or Intermediate-energy System

In the fast-thermal coupled system, the fast core of 50-cm diameter with the composition of a large fast reactor will be surrounded by a buffer region and a thermal driver region with polyethylene. The core provides a neutron spectrum similar to that of a large fast reactor, and it will be possible to compare the theoretical and experimental results of various parameters and to obtain informations on the sodium-void and the Doppler effects. One of the important problems to be investigated is the heterogeneity effect in the sodium-void experiment. For the Doppler experiment, the oscillator technique will be used.

FIG. 9. Plan View of Building.

The above system, however, may not be effective to evaluate the cross-section set in the resonance-energy region. Thus, an intermediate-energy system moderated by beryllium is now proposed, in which the low-energy tail enhances the reactivity effect, especially the Doppler effect. With this system, it may be possible to obtain more accurate information on cross sections in the resonance-energy region, through the evaluation of the criticality, neutron spectrum, and Doppler effect.

In these experiments, accurate determination of the neutron spectrum is the most important outstanding problem. Various techniques will be developed for measuring the low-energy neutron spectrum. Other problems peculiar to the coupled system, such as the reactivity share and coupling, will also be studied theoretically and experimentally.

3.3. Diluted System and Zoned System

Studies of the dilution effect of U^{238} , sodium, and graphite will be done to some extent in the diluted system. As mentioned previously, accurate prediction of the detailed neutron spectrum is one of the most important problems to be solved. The pulsed neutron source and the time-of-flight technique will be used to measure the neutron spectrum at low energy in the diluted subcritical system. The result will be incorporated with the neutron spectrum at high energy obtained by a gas-filled proton-recoil detector and compared with the theoretical result. Since the prompt-neutron lifetime is very sensitive to the fraction of the neutron flux at low energy, analysis of the prompt-neutron lifetime is also very interesting.

Nuclear characteristics of a diluted plutonium core will be studied by the zoned system: a plutonium core will be surrounded by buffer and U^{235} driver regions. Some efforts will be needed to obtain a good matching of the real and adjoint fluxes between the zoned core and the full-scale system. The sodium-void and the Doppler effects will be investigated in the zoned core with special attention to the heterogeneity effects. Mockup tests of the experimental fast reactor and the prototype fast reactor will also be started with zoned systems.

4. Supplemental Remark

A lot of problems are due to the imperfect simulation of large fast reactors in critical assemblies. The heterogeneity due to the plate structure of the assemblies may produce a self-shielding effect and strongly perturb the sodium-void experiment. To investigate this heterogeneity effect, especially in the case of a plutonium assembly, calculations are being made with the IBM-7044 for the following six codes linked in series. LTKFK prepares the microscopic group cross-section library tape. CROSS computes the macroscopic group cross section for the input of FASTOS,⁽²⁾ which calculates the spatial variation of flux across a cell by the multigroup integral transport theory. With this flux as the weighting flux, MABN computes the effective cross sections. Then, EXPR 2 or TWFAST calculates the effective multiplication factor by one-dimensional multigroup or two-dimensional six-group diffusion approximations. The Russian ABN set and group cross sections obtained by ESELEM are used in the present study.

A central problem is also the interpretation of Doppler-coefficient measurements. Besides measurements made in a homogeneously loaded, full-size power reactor, only measurements with very small samples can reliably be evaluated. But the accuracy of these

measurements seems to be very poor. In this respect, the Doppler effect in beryllium-moderated systems fueled with 20% and fully enriched uranium or plutonium has been surveyed⁽³⁾ by using the Russian ABN set and EXPR 2. The result indicates that the Doppler-reactivity change in the suitably moderated system is about 20 times larger than that in a typical large fast reactor. This also means that the correction for thermal expansion of the Doppler sample will be relatively small. The reactivity change with the sample, especially a U^{235} or plutonium sample, inserted into the center of the system was evaluated by the perturbation theory with the heterogeneity correction. It appears that the heterogeneity effect is not so serious, although more work is needed.

Thanks are due to Dr. M. Nozawa for his contribution to the program in the early stage, and to Messrs. T. Suzuki, H. Tomioka, M. Nakano, and T. Mukaiyama for their cooperation in the preparation of this report.

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Discussion

Mr. Helm (ANL): What neutron intensity do you think is necessary in order to make useful time-of-flight measurements on a critical facility?

Mr. Nomoto: As an indication of the intensity available in our pulsed neutron source, the maximum beam current on a continuous basis is 450 microamperes.

Fast Test Reactor: Critical Experiments Program*

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ABSTRACT

A conceptual design of the Fast Test Reactor (FTR) is being formulated by Pacific Northwest Laboratory (PNL). The nuclear aspects of the FTR are sufficiently complex that a critical-experiment program is essential to the development of the final design. To assess the current uncertainties in calculational methods and nuclear data, pertinent fast critical experiments spanning a wide range of neutron spectra have been evaluated. Results of calculation of spectral indices, reactivity coefficients, and effective multiplication constants are presented, and their effects on the planned FTR Critical Experiments Program are discussed.

Five major objectives established for this program are to (1) confirm the flux capability of the FTR, (2) confirm the control static characteristics of the FTR, (3) confirm the nuclear safety features of the FTR, (4) contribute information for FTR shield and instrument design, and (5) provide design information for the Proposed Nuclear Proof Test Facility (zero-power support assembly for the FTR).

Time schedules and a brief summary of the Critical Experiments Program dictated by these objectives are presented.

1. Introduction

Interest in a Fast Flux Test Facility (FFTF) at Hanford dates back to 1963. The purpose of such a facility is to provide comprehensive irradiation testing to support fuel and material research and development necessary for the development of fast breeder power reactors.

During the early part of 1964 a very preliminary conceptual design for such a facility was evolved at Hanford Laboratory. Starting in 1965, conceptual design efforts on this facility were intensified in order to arrive at a reasonably firm budget-stage cost estimate.

* Based on work performed under U. S. Atomic Energy Commission Contract AT(45-1)-1830

A brief description of the early conceptual design is given in Ref (1).

At the present time, a full-fledged design effort for the facility and particularly for the Fast Test Reactor (FTR) is underway at Pacific Northwest Laboratory (PNL). Preliminary schedules have been formulated and submitted to the AEC. On a tentative basis, it is now expected that the reactor would achieve criticality by mid-1972 and reach full power at the end of 1972. To attain these objectives, it will be necessary to obtain preliminary safeguards approval by mid-1968; fuel fabrication should start by mid-1969, and final safeguards approval should be available by the end of 1971, so that fuel loading could start at the beginning of 1972.

The complexity of the facility and the difficulties and costliness of late design changes make it necessary to assure that a sound basis is provided for the nuclear design. Experimental verifications of some of the nuclear-design calculations must be obtained early enough to be incorporated into the preliminary safeguards analysis and into the fuel specification. In view of the proposed schedule, it becomes obvious that critical experiments should be conducted soon—during the FY 1968-9 period at the latest. Some very preliminary critical experiments are planned as early as FY-1967.

The overall critical experiment program has been divided into four phases as follows:

Phase I —Scoping of Critical Experiment Program;

Phase II —Planning of Critical Experiments;

Phase III—Performance of Critical Experiments;

Phase IV—Analysis of Critical Experiments.

Although some overlap in time among these phases might occur, they are still sufficiently distinct to be described separately. During Phase I, the broad outlines of the experimental program are to be established. During Phase II, detailed planning for the experiments will be carried out. The actual experimental work will

be conducted during Phase III and analyzed during Phase IV.

At the present time we have completed the scoping phase and are starting on the detailed planning for the experiments. The present paper is primarily concerned with the results of our scoping effort and the preliminary plans for the critical experiment program.

2. Status of FTR Nuclear Design

To understand better the context in which the FTR critical-experiment program is to be carried out, it is useful to give a brief review of the status of the FTR nuclear design, and to point out some of its unique design features and problems.

The early FTR design efforts led to a nominal, 400-MW, PuO₂-SS cermet-fueled, sodium-cooled, tubed design. The overall layout of the reactor and associated plant is shown in Fig. 1. The rationale for this core design is summarized in Ref. (1). Plutonium was selected as the fissile material in order to achieve a low inventory of fissile fuel and obtain a high flux-to-power ratio. Fast flux levels of the order of 10^{16} are a design objective of the FTR. The use of stainless steel cermet rather than mixed-oxide ceramic fuel tends to enhance this high-flux potential. In addition, cermet fuel is also expected to provide a fast-acting, negative shutdown mechanism brought about by the expansion of the reactor fuel. The absence of fertile fuel has the further advantage of assuring the presence of a sizeable, negative sodium-coolant temperature coefficient.

However, the greater availability of information on the performance characteristics of mixed-oxide fuels has led, at present, to the selection of an interim or first-core loading of mixed PuO₂-UO₂ fuel clad in stainless steel. It is recognized that the use of mixed-oxide fuel reduces the flux-to-power ratio of the core, and adversely affects the sodium and the fuel-expansion temperature coefficients. On the other hand, the presence of U²³⁸ introduces a prompt negative Doppler coefficient and increases the effective delayed-neutron fraction. In addition, U²³⁸ has a beneficial effect on the long-term reactivity-time response of the reactor.

Nevertheless, because of the inherently lower flux capabilities of the oxide core, development of a high-flux, full-performance core remains an ultimate objective in the overall FTR program.

While the exact design parameters of the oxide core have as yet not been established, the approximate features of this core are available. The gross neutronics characteristics are shown in Table 1.

The present core concept differs in many important respects from the existing, relatively small, U²³⁵ metal-fueled cores such as FERMI and EBR-II. The purpose of the FTR is, of course, to serve as a test reactor—

hence the provision of open and closed test loops, and the requirements for a high-intensity flux machine. To interpret irradiation results correctly will require a detailed knowledge of both spatial and spectral flux distributions in the test samples. The analytical determination of these quantities is made more difficult by the highly heterogeneous nature of test loops, and the asymmetries they introduce into the core design.

The requirement for substantial in-core test volume has an important effect on the selection of a control scheme. Although the exact control features of the FTR design have as yet not been established, the presently favored arrangement consists of in-core scram rods, with the shim rods located on the core periphery. This arrangement would assist in providing sufficient in-core test volume. On the other hand, reflector-control margins for an 800-liter core are not unduly large. Furthermore, the analytical determination of reflector-rod worths presents difficulties since fairly rapid changes of the spectral flux occur at the core-reflector interface.

Safety is a particularly important topic in a test reactor, where the test fuel cannot, in fact, be considered to be of a "proven" nature. Failure of test fuel may in turn trigger a reactor incident which could bring about driver-fuel failures as well as failures in other test locations. The difficulty of treating these interaction problems analytically is self-evident. The solution of these problems requires an accurate knowledge of the reactor kinetics characteristics and a realistic physical model for the accident.

It should be apparent from the above discussion that detailed experimental verification of the FTR nuclear design, via extensive critical experiments, is essential. Furthermore, the need for critical experiments which simulate the FTR design features with fair precision is also evident.

3. Survey of Available Critical Experiments

Before proceeding with detailed plans for the critical-experiment program, we considered it useful to examine already available experimental information which might be germane to the FTR program. The objectives of such a survey would be to avoid duplication of efforts, and to supply an early opportunity to test calculational routines and basic nuclear parameters used in the present FTR nuclear-design effort. In addition, such a survey would help to identify those areas where additional experimental work is required.

A survey of this nature is presently underway. Sources of pertinent neutronics data are the experiments carried out in Argonne's ZPR facilities and UK's ZEBRA facility. For our initial survey, we have selected a series of experiments spanning a rather wide range of neutron spectra. Specifically, we have carried

FIG. 1. Fast Flux Test Facility.

out calculations for ZPR-3 assemblies numbered 14, 29, 34, 35, 38, 39, 44, 45, 45A, 46B, 47, and 48, ZPR-6 core 2, and ZEBRA core 2. These assemblies range from the very-hard-spectrum assembly 44 (87% flux > 0.1 MeV) to the rather soft-spectrum assembly 45 (59% flux > 0.1 MeV). The characteristics of these assemblies are summarized in Table 2. Calculated flux spectra are shown in Table 3, and selected spectra are displayed graphically in Fig. 2. The FTR spectrum is also shown for comparison. This wide range of spectra and core compositions should provide an integral test of the basic nuclear data and cross-section-averaging routines used in the FTR nuclear-design calculations.

In general, three quantities have been calculated and

compared with available experimental data:

- cross-section ratios;
- k_{eff} of the assemblies;
- central reactivity coefficients.

The cross-section data used for the calculations are based on a modification of the Russian Cross Section Set.⁽²⁾ The modifications in the fission cross sections of the fissile isotopes reflect the recent measurements by White.⁽³⁾ The iron capture cross sections were revised upwards (see Appendix).

Multiplication factors were determined with two-dimensional transport theory (2DF); central reactivity coefficients were obtained from one-dimensional transport calculations (DTF-IV⁽⁴⁾).

The results of the calculations of cross-section ratios, and comparison with available experimental values are shown in Tables 4-7. Similar calculations carried out by ANL⁽⁵⁻⁷⁾ and the U.K.⁽⁸⁾ show comparable error bands. The differences in the cross-section ratios are approximately 5% for Pu²³⁹, while the ratios for the fertile materials show discrepancies in the neighborhood of 15%.

Multiplication constants were only computed for a few assemblies. The results are given in Table 8. The errors in k_{eff} are about 1 to 2%. This error range is

TABLE 1. DESCRIPTION OF PuO₂-UO₂ FTR DESIGN

Core Volume, liters	800
L/D	0.8
Sodium, v/o	45
Stainless Steel, v/o	25
Oxide Fuel, v/o	30
Initial Fuel Composition (includes test fuel)	20.3 v/o Pu ²³⁹ O ₂ 2.3 v/o Pu ²⁴⁰ O ₂ 77.4 v/o U ²³⁸ O ₂
Initial Fissile Inventory	412 kg Pu ²³⁹
Side Reflector	30 cm thick 80 v/o Inconel 20 v/o Sodium
End Reflectors	30 cm thick 30 v/o steel 70 v/o sodium
Control	B ₄ C Rods
Test Position	10-13 closed and open loops
β_{eff}	0.0030
Doppler, $T dk/dT$	-3.9×10^{-3} (initial, clean core)
Sodium coefficient	$-3.5 \times 10^{-6}/^{\circ}\text{K}$
Ratio of Flux to Power (Avg)	1.03×10^{16} (n/sec-cm ²)/ (MW/liter)
Flux Spectrum	67% above 0.1 MeV

TABLE 3. NEUTRON FLUX SPECTRUM

	% of Flux above Indicated Energy		
	0.01 MeV	0.1 MeV	1.0 MeV
ZPR-3-14	94.4	74.8	30.0
29	97.2	73.4	17.5
34	97.2	74.7	17.8
35 FERMI B	97.0	76.3	15.7
38	99.9	85.0	14.0
39	99.3	83.5	21.0
44 RAPSODIE	99.5	87.5	31.5
45	86.8	59.0	14.6
45A	93.3	67.5	14.8
46B FARET	99.0	85.5	28.3
47 SEFOR	89.5	61.2	15.0
48	92.1	67.5	18.3
ZPR-6-2	96.7	73.6	17.0
ZEBRA-Core 2	92.8	66.5	17.1
FTR Oxide	93.6	67.0	14.5

typical for calculations of this kind. For example, k_{eff} for a large number of ZPR's are tabulated in Ref. (6), showing differences up to 2½%. Another example is Ref. (7), where ZPR calculations are described giving discrepancies in k_{eff} from 0.1 to 1.5%.

Calculations of central reactivity coefficients were done for the SEFOR assembly⁽⁹⁾ and the UK ZEBRA core 2. The results are given in Tables 9 and 10. The values for fissile materials agree well with experiment, whereas the values for U²³⁸ and for the lighter elements differ considerably from the experimental results.

Two other areas of major importance to fast reactor neutronics concern the determination of Doppler coefficients and control-element worth. To assess the uncertainties involved in these areas, reference is made

TABLE 2. CENTRAL CORE COMPOSITIONS

Atoms/cm³ × 10²²

Core	U ²³⁵	U ²³⁸	Pu ²³⁹	Pu ²⁴⁰	Be	C	O	Na	Al	Cr	Fe	Ni	Mo
ZPR-3													
14	0.4500	0.0350	—	—	—	5.3000	—	—	—	—	0.790	—	—
29	0.2390	0.4800	—	—	—	—	1.3900	—	1.470	—	2.110	—	—
34	0.2250	0.4950	—	—	—	0.7600	—	—	1.540	—	2.110	—	—
35 FERMI B	0.1954	0.0141	—	—	—	—	0.3994	0.7825	—	0.7556	3.180	0.2975	—
38	0.3638	3.5052	—	—	—	—	—	—	—	0.1507	0.572	0.0716	—
39	0.2792	0.4376	—	—	—	—	—	—	1.410	0.3969	1.507	0.1887	—
44 RAPSODIE	0.5034	0.3365	0.1558	0.0074	—	—	1.9570	—	1.031	0.1831	1.439	0.0908	—
45	0.0015	0.7230	0.1070	0.0051	—	2.4970	—	0.6350	—	0.2490	0.928	0.1230	—
45A	0.0015	0.7230	0.1070	0.0051	—	0.8330	—	1.0580	—	0.2030	0.757	0.1000	—
46B FARET	0.3392	0.5599	0.1565	0.0074	—	1.1080	—	0.8421	—	0.2856	1.064	0.1407	—
47 SEFOR	0.0015	0.7744	0.1480	0.0132	0.568	0.3360	1.5660	0.6630	0.699	0.2731	1.045	0.1080	0.0515
48	0.0016	0.7406	0.1644	0.0106	—	2.0765	—	0.6230	0.011	0.2658	0.990	0.1308	0.0206
ZPR-6-2	0.2285	0.9853	—	—	—	1.2900	—	0.7910	—	0.2983	1.038	0.1390	—
ZEBRA Core-2	0.2400	1.5000	—	—	—	3.8000	—	—	—	0.1000	0.380	0.0390	—
FTR Oxide	—	0.4728	0.1296	0.0144	—	—	1.2335	1.0280	—	0.4127	1.580	0.1625	—

to Fischer's paper on Doppler coefficients⁽¹⁰⁾ and Persiani's paper on FARET.⁽¹¹⁾ As indicated in these ref-

FIG. 2. Central Core Spectra (Fundamental-mode Calculation).

TABLE 4. REACTION-RATE RATIOS AT CORE CENTER
 $\bar{\sigma}_f \text{Pu}^{239} / \bar{\sigma}_f \text{U}^{235}$

Assembly	Experimental	Calculated PNL	E/C ^(a)
ZPR-3-14 ⁽⁶⁾	1.050 ± 2%	1.100	0.940
29 ⁽⁶⁾	1.060 ± 2%	1.080	0.981
34 ⁽⁶⁾	1.067 ± 2%	1.090	0.978
35 FERMI B ⁽¹²⁾	1.090 ± 2%	1.090	1.000
38 ⁽¹⁴⁾	1.170 ± 2%	1.190	0.983
39 ⁽¹⁴⁾	1.180 ± 2%	1.180	1.000
44 RAPSODIE ⁽¹²⁾	1.186 ± 2%	1.230	0.967
45	—	—	—
45A ⁽¹⁵⁾	1.047 ± 2%	0.980	1.068
46 FARET B	—	1.210	—
47 SEFOR ⁽⁹⁾	0.916 ± 2%	0.920	0.996
ZPR-6-2 ⁽¹²⁾	1.090 ± 2%	1.080	1.010
ZEBRA Core-2 ⁽⁸⁾	0.987 ± 0.010	1.003	0.958
FTR Oxide	—	0.976	—

^(a) E/C = Ratio of experimental value to calculated value.

TABLE 5. REACTION-RATE RATIOS AT CORE CENTER
 $\bar{\sigma}_f \text{Pu}^{240} / \bar{\sigma}_f \text{U}^{235}$

Assembly	Experimental	Calculated PNL	E/C ^(a)
ZPR-3-14	—	0.395	—
29 ⁽⁶⁾	0.301 ± 2%	0.297	1.014
34 ⁽⁶⁾	0.282 ± 2%	0.310	0.910
35 FERMI B ⁽¹²⁾	0.250 ± 2%	0.290	0.896
38	—	0.330	—
39 ⁽¹⁴⁾	0.326 ± 2%	0.396	0.823
44 RAPSODIE ⁽¹²⁾	0.490 ± 2%	0.510	0.960
45	—	—	—
45A ⁽¹⁵⁾	0.218 ± 2%	0.221	0.986
46B FARET	—	0.480	—
47 SEFOR ⁽⁹⁾	0.200 ± 2%	0.190	1.053
ZPR-6-2 ⁽¹⁴⁾	0.254 ± 2%	0.290	0.876
ZEBRA Core-2 ⁽⁸⁾	0.237 ± 0.003	0.248	0.955
FTR Oxide	—	0.217	—

^(a) E/C = Ratio of experimental value to calculated value.

TABLE 6. REACTION-RATE RATIOS AT CORE CENTER
 $\bar{\sigma}_f \text{U}^{238} / \bar{\sigma}_f \text{U}^{235}$

Assembly	Experimental	Calculated PNL	E/C ^(a)
ZPR-3-14 ⁽¹³⁾	0.0600 ± 2%	0.071	0.838
29 ⁽¹³⁾	0.0380 ± 2%	0.040	0.960
34 ⁽¹³⁾	0.0366 ± 2%	0.039	0.926
35 FERMI B ⁽¹²⁾	0.0310 ± 2%	0.034	0.912
38 ⁽¹⁴⁾	0.0330 ± 2%	0.035	0.944
39 ⁽¹⁴⁾	0.0480 ± 2%	0.050	0.960
44 RAPSODIE ⁽¹²⁾	0.0820 ± 2%	0.086	0.945
45	—	—	—
45A ⁽¹⁵⁾	0.0279 ± 2%	0.029	0.962
46B FARET	—	0.078	—
47 SEFOR ⁽⁹⁾	0.0264 ± 2%	0.027	0.989
ZPR-6-2 ⁽¹²⁾	0.0360 ± 2%	0.038	—
ZEBRA Core 2 ⁽¹⁸⁾	0.0320 ± 0.0005	0.035	0.912
FTR Oxide	—	0.029	—

^(a) E/C = Ratio of experimental value to calculated value.

erences, the discrepancy between calculated and experimental Doppler coefficients for a 7/1: 238/239 sample is as large as 50%; calculations of reflector-control-rod worths for FARET differ from experimental values by about 10 to 20%.

Although the above discussion certainly does not represent a comprehensive review of available pertinent neutronics data, it does give an indication of the present "state of the art." The implications of these results for the FTR nuclear design are quite important.

If we assume an error of 5% in the Pu²³⁹ fission cross sections, this could cause an error of roughly 5% in the flux-to-power ratio of the FTR. The actual error may

TABLE 7. REACTION-RATE RATIOS AT
CORE CENTER
 $\bar{\sigma}_f U^{238}/\bar{\sigma}_f U^{235}$

Assembly	Experimental	Calculated PNL	E/C ^(a)
ZPR-3-14 ⁽⁶⁾	0.134 ± 2%	0.141	0.950
29 ⁽⁶⁾	0.141 ± 2%	0.136	1.037
34 ⁽⁶⁾	0.150 ± 2%	0.133	1.127
35 FERMI B ⁽¹⁴⁾	0.136 ± 2%	0.142	0.958
38 ⁽¹⁴⁾	0.113 ± 2%	0.125	0.904
39	—	0.123	—
44 RAPSODIE ⁽⁷⁾	0.108 ± 0.003	0.110	0.982
45 ⁽⁷⁾	0.189 ± 0.010	0.130	1.453
45A ⁽⁷⁾	0.152 ± 0.007	0.133	1.142
46B FARET ⁽⁷⁾	0.111 ± 0.005	0.115	0.965
47 SEFOR ⁽⁹⁾	0.144 ± 2%	0.148	0.969
ZPR-6-2 ⁽¹⁴⁾	0.130 ± 2%	0.133	0.977
ZEBRA Core 2 ⁽⁸⁾	0.136 ± 0.007	0.133	1.020
FTR Oxide	—	0.148	—

^(a) E/C = Ratio of experimental value to calculated value.

TABLE 8. CALCULATED EFFECTIVE MULTIPLICATION, k_{eff}

	k_{eff}
ZPR-3	
35 FERMI B	0.988
44 RAPSODIE	1.008
46B FARET	0.985
47 SEFOR	1.014
ZEBRA	
Core 2	0.976

TABLE 9. COMPARISONS OF SEFOR CENTRAL
REACTIVITY COEFFICIENTS

Sample		Reactivity Worth (mk)	
Material	Mass (g)	Experimental ⁽⁹⁾	Calculated
Pu-SS-Al	69.1	0.297	0.300
EnUAl	72.1	0.223	0.220
Depleted U	2460.4	-0.535	-0.722
B ₄ C (Enriched)	32.4	-1.445	-1.420
Na	91.2	-0.031	-0.043
SS	1026.1	-0.114	-0.152
Fe	1028.0	-0.099	-0.147
Be	240.7	0.082	0.045

in fact be larger due to additional uncertainties in the capture cross sections, and due to the expected difficulty of treating local flux perturbations within the test loops.

The present ceramic FTR design has a temperature defect of about 0.01 Δk , and a predicted 0.015 Δk decrement for the desired 42 days of operation at a nominal 400 MW. Consequently, a minimum excess k of 0.025 is required. Errors in k_{eff} of 1 to 2% would

therefore be detrimental to efficient reactor operation, and large design margins would have to be provided.

One control scheme presently envisioned consists of 3-4 in-core safety rods, supplemented by a number of reflector rods to cover the reactor shim, power regulation, and normal shutdown requirements. It appears that 12 B₄C reflector rods can supply about 0.04 Δk , which would produce an initial shutdown margin of 0.015 Δk . Again, if control requirements have substantial uncertainties (due to uncertainties in the sodium worth and Doppler coefficient) and, in addition, the determinations of reflector rod worth is also uncertain [e.g., 10-20% as indicated in Ref. (11)], the requirement for substantial design margins obviously exists. In other words, provision for more control rods may have to be made, since enrichment of reflector rods with B-10 does not increase rod worth rapidly.

Finally, the mode of reactor operation and the type of test specimens that can be accommodated in the FTR are intimately related to the overall reactor kinetics and safety characteristics—specifically, the coolant, structural and fuel-temperature coefficients. Relatively large uncertainties in these coefficients would also require the adoption of substantial design and operational margins.

It is clear, therefore, that the reduction of uncertainties in the reactor neutronics would have the concomitant effect of arriving at a more optimum design and at a more efficient operating procedure. The main function of performing critical experiments on reactor configurations closely resembling those of the FTR is precisely to achieve these objectives.

4. Objectives of FTR Critical Experiments

The proposed FTR Critical Experiment Program has essentially five major objectives:

- (1) confirm the flux capability of the FTR;
- (2) confirm the control static characteristics of the FTR;
- (3) confirm the nuclear safety features of the FTR;

TABLE 10. ZEBRA CORE 2 SMALL-SAMPLE
PERTURBATION CROSS SECTIONS
 $\bar{\sigma}_p$ (barns)

Sample	Experimental ⁽⁸⁾	Calculated
Pu ²³⁹	3.236	3.384
U ²³⁵	2.282	2.291
U ²³⁸	-0.177	-0.170
B ¹⁰	-3.136	-2.179
C	0.010	0.006
Na	0.005	0.004
Al	-0.009	-0.013
Fe	-0.019	-0.029
Ta	-0.790	-0.863

- (4) contribute information for FTR shield and instrument design;
- (5) provide design information for the proposed Nuclear Proof Test Facility (NPTF).*

The fulfillment of the first objective requires verification of initial fuel loading and the measurement of effective cross-section ratios for different materials throughout the core. Special attention will have to be paid to the effect of large loops on reactor reactivity, spatial flux distributions, and flux spectra. Preparations of reactivity-coefficient maps of potential test materials will also be necessary.

The second objective involves detailed measurements of control-element worths, particularly of control rods located in the reflector. Single-rod worth, rod-bank worth, and rod-shadowing effects will be considered.

The confirmation of the nuclear safety features of the reactor requires the determination of reactor temperature coefficients as a function of reactor condition. In addition, the simulation of possible reactor or test-element meltdown configurations could also be of interest.

To provide information of value to the shield and instrument design, it is intended that mockups of the FTR include as much reflector and structural material in both the radial and axial directions as is possible. Neutron and gamma distributions throughout these regions must be known to evaluate heat-generation and radiation-damage rates in various components, and for determining the radiation environment for the sensing elements required to instrument the reactor system. Special shielding materials could be used in the outer regions of the reflector to determine the modifications that can be effected in the neutron-gamma ratio to improve the environment for the sensing elements of the instrument system.

* The NPTF is proposed as a companion facility to the FTR to be used in conjunction with FTR operation.

To assist in the design of the zero-power NPTF, valuable information could be obtained from critical experiments. An important question concerning the design of the NPTF refers to the degree with which this facility might simulate the actual FTR design. Critical experiments, comparing various geometric configurations of the FTR fuel and structural elements, could help resolve this question.

Although the exact details of the FTR Critical Experiment Program have as yet not been established, either a two-part or three-part experimental program is being considered.

Part A of both programs will probably involve the present Assembly 48 in ZPR-3. Although this assembly does not simulate the FTR characteristics in detail, it nevertheless provides an early opportunity to obtain experimental information relating to certain FTR nuclear-design problems. Specifically, since one of the important uncertainties lies in the area of control-rod worth, it has been proposed that rod-worth measurements be made in the reflector and core of Assembly 48. To enhance the worth of the reflector rods and provide a more reasonable simulation of FTR conditions, it is necessary to replace at least a part of the present U²³⁸ blanket in Assembly 48 with Inconel or a suitable nickel-steel mixture.

In the two-part program, it is proposed to proceed directly from Phase A to early, detailed engineering mockup experiments (Phase C) in order to verify some of the nuclear-design data for the preliminary safeguards report. The alternative three-part program would have the engineering mockup experiments just precede the preparation of fuel specifications, and would include an interim, abbreviated Part B program with experiments conducted on a more idealized FTR critical-assembly configuration. At the present time, the relative merits of these alternatives are being considered.

Appendix

Russian Cross-section Modifications

Group	Pu ²³⁹				U ²³⁵				Fe	
	σ_f		σ_c		σ_f		σ_c		σ_c	
	A ^(a)	B ^(a)	A	B	A	B	A	B	A	B
1	2.20	2.20	0.010	0.01	1.75	1.75	0.020	0.02	0.036	0.036
2	1.90	1.85	0.021	0.02	1.15	1.15	0.030	0.03	0.005	0.005
3	1.97	1.97	0.030	0.03	1.25	1.25	0.040	0.04	0.002	0.002
4	2.00	1.95	0.041	0.04	1.31	1.28	0.062	0.06	0.003	0.003
5	1.75	1.80	0.058	0.06	1.25	1.25	0.120	0.12	0.004	0.004
6	1.64	1.68	0.107	0.11	1.20	1.23	0.166	0.17	0.005	0.005
7	1.63	1.66	0.167	0.17	1.35	1.41	0.240	0.25	0.006	0.006
8	1.68	1.68	0.240	0.24	1.525	1.70	0.359	0.40	0.12	0.006
9	1.70	1.85	0.289	0.40	1.85	2.10	0.528	0.60	0.020	0.007
10	1.75	2.15	0.561	0.71	2.30	2.65	0.869	1.00	0.0425	0.017
11	1.80	2.40	0.810	1.08	2.85	3.40	1.256	1.50	0.020	0.005
12	2.50	2.50	1.25	1.25	4.00	4.40	1.910	2.10	0.060	0.004

^(a) A: Modified data.* B: Original Russian data.

* The data in some of the first 12 groups were modified; the data in groups 13–26 have not been modified.

Acknowledgment

The authors acknowledge the contributions of R. W. Hardie and R. A. Harris.

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Discussion

Mr. Okrent (ANL): I understood that a flux of 10^{16} was stated as a target for the FTR. You indicated that, with the mixed-oxide core, one would get 10^{16} neutrons per sec per cm² per megawatt per liter. How many megawatts per liter do you think you can achieve with the mixed-oxide core?

Mr. Bennett: One-half megawatt per liter was the design objective, giving a flux of 5×10^{15} . This is one of the penalties associated with a mixed-oxide core.

Mr. Hofmann (Battelle-Northwest): I would like to make another comment about the flux level in the FTR. The number 5×10^{15} is an average number. The peak flux is higher and, in fact, is close to 10^{16} .

Mr. Okrent: Do you place some limit on the amount of fissile material that you may put in a test loop as a consequence of the β_{eff} ? Do you think that when you

go from cermet to mixed oxide and increase β by 50% you can increase the amount of fissile material that you put in the loop by 50%? Do you have some such relation?

Mr. Little (Battelle-Northwest): We have not set a

definite upper bound on how much fissile fuel we will put in the test loops in the FTR. Of course, one would not want to put as much fissile fuel in a test loop as one would put in a driver loop, although we have not fixed this bound yet.

Comments on Other Planned Critical Assembly Programs

FRO: L. TIRÉN, *AB Atomenergi, Studsvik*

I shall try to deal with a few aspects of the program at FRO for the next one or two years. First, we have a rather severe problem of fuel inventory. At present this is only 120 kg of U^{235} and that restricts us a great deal as to the scope of the program. We hope, however, to obtain some more of this material in the near future. On the other hand, there are no immediate prospects of getting plutonium.

As project work on large fast reactors in Sweden progresses and becomes more concrete, we will try to support this work more directly by making certain experiments in FRO. For example, we will make measurements of polythene-void reactivities, which might give indirect information on coolant coefficients in a steam-cooled fast reactor and measurements of flux distributions in polythene slabs, which could provide information about flux peaking in flooded coolant channels.

Going over to detailed experiments which we plan to do in the nearest period of time, I will just mention a few items. Spectral-index measurements have been described in our summary paper at this conference, and will continue at the reactor. Differential spectrum measurements with hydrogen-filled proportional counters will be made, including the use of gamma-ray discrimination techniques. We have made a series of Doppler activation measurements at 600°C, and this

work will continue. We hope to achieve higher temperatures and are at present designing a furnace for 1500°C. Furthermore, the effect of heterogeneity will be subject to systematic studies using very thin plates of the various core materials.

We feel that there is a case for diluting the FRO reactor with deuterium in some form. Current considerations of coolants for fast power reactors in Sweden include heavy water steam cooling and this provides us with one argument for putting heavy water into the reactor (deuterated polythene being too expensive). I think there are other arguments from the point of view of reactor physics. Heavy water is a more effective moderator than graphite, but will cause a much smaller reactivity loss than graphite. This aspect is important when one is faced with a limited fuel inventory. On the other hand, although deuterium is less effective than hydrogen, the problems of heterogeneity and anisotropic scattering would be less severe with deuterium than with hydrogen.

Finally, I'd like to mention that we are supported as regards cross-section problems by work at the 5-MeV Van de Graaff generator. For example, measurements have been made of inelastic cross sections in copper and, more recently, of elastic scattering cross sections in iron, nickel, and copper.

MASURCA: F. STORRER, *C.E.N., Cadarache*

I will first say a brief word on the other Cadarache facilities.

HARMONIE is a fast-neutron-source reactor which first achieved criticality in August 1965 and has since been performing very well. It is extensively used for testing and calibrating various detectors and spectrometers, and as a source for shielding experiments. Next month it will be used for the first time to drive an exponential experiment.

RAPSODIE, which is not primarily a physics facility, but a 20-MW experimental reactor and an irradiation facility, will nevertheless provide some useful

physics information. According to schedule it should become critical in December 1966. It will be quite interesting to compare the critical mass of this plutonium oxide-fueled reactor with that predicted on the basis of the critical experiment performed with ZPR-3.

MASURCA is a critical facility quite similar to SNEAK. It also has vertically hanging subassemblies, but with a 4 by 4-in. cross section. It can accommodate cores up to 6000 liters in volume. The simulation pieces are square cross-section ($\frac{1}{2} \times \frac{1}{2}$ in.) rodlets, either 4 or 8 in. long, which will also be put vertically in the

subassemblies. The fuel-simulation pieces fit in the same lattice but have a circular cross section (of 1/2-in. diameter), consisting of 30% enriched metallic uranium for U-235, and a 25% Pu-1% Fe-74% U alloy for plutonium. Available materials include 175 kg plutonium of two different Pu-240 concentrations, and 760 kg of U-235. Sodium dioxide will be used to simulate part of the sodium and the oxygen of the oxide fuel.

MASURCA should become critical around the end of 1966. The first loading (core I-A) will consist of a 220-liter plutonium core—a square array of plutonium rods in graphite, with 3 graphite rods per plutonium rod—surrounded by a 20-cm-thick depleted uranium blanket.

The early part of 1967 will be devoted to a thorough testing of the facility and of the experimental techniques. During the remaining part of that year the progressive substitution technique (see papers 6A and 6B, session V-A, of this conference) will be tested. Core I-B—same lattice but U-235 rods replacing the plutonium rods—will be used as the second medium. Three other experiments will be performed concurrently with the composition of core I-B:

- an exponential experiment driven by HARMONIE;
- a coupled fast-thermal experiment driven by the thermal-neutron facility MINERVE at Fontenay-aux-Roses (mainly to develop and test the experimental techniques to be used in MASURCA);
- a pulsed experiment (to measure the spectrum by time-of-flight and the decay constant α), which will probably be performed on the SUAK facility in Karlsruhe.

It is hoped that each experiment will contribute to the understanding of the medium involved and that their interpretation will yield useful indications as to the best experimental program that should be undertaken for the understanding of the prototype reactor PHENIX.

All of 1968 and 1969 will be devoted to PHENIX. Multizone experiments—and particularly progressive substitution experiments—will be performed with MASURCA, and the other facilities will be used as well. It will be quite interesting to see how much can be learned by a combination of experiments, on a reactor with a plutonium critical mass of more than 700 kg, with only 175 kg available.

VERA: J. W. WEALE, UKAEA-Aldermaston

The VERA program has the object of improving our understanding of the basic physics of small fast critical assemblies fueled with U²³⁵ and plutonium. We envisage a set of about ten critical assemblies of simple geometry and simple composition, which contain only two or three main core materials. The energy spectra in these assemblies are varied by adjusting the amount of moderator, which is graphite or polythene, in the core. We have studied in fair detail four uranium systems and one plutonium system so far, but we have not yet made satisfactory spectrum measurements with them. During the next year we shall continue our efforts to perfect the neutron-spectrometry techniques

and shall apply them to the first five assemblies, that is, the ones we have already done. During the following year we shall study another three or four assemblies. We want to include two plutonium systems with fairly soft spectra and probably an assembly containing beryllium and one containing nickel. These experimental results will be used in the DUNDEE-PENICUIK Data Adjustment System, which was described by Pendlebury at this meeting, to provide a set of nuclear data adjusted to give good agreement in both k -value and spectrum over a wide range of neutron energies.

ZEBRA: R. D. SMITH, UKAEA-Winfrith

The ZEBRA reactor is at present being used for a series of measurements with a mockup, which we call Assembly 7, of the prototype fast reactor now being

built at Dounreay. We thought we should build a mockup at this time because of the lack of experience with large plutonium assemblies of this size, and the

present assembly is being used as a sort of combination of an engineering mockup and a general study. Insufficient plutonium plates are available to fuel the whole core with plutonium; there are, in fact, 450 kg of U^{235} and 350 kg of plutonium in the assembly. We are building several different versions, in which the plutonium is concentrated either in the central zone or a radial sector on rather the same principle as P. Engelmann was talking about for SNEAK (Session VIII, Paper 3). We will make, I think, the usual series of measurements we have been hearing about during this week. We are paying particular attention from the mockup point of view to control-rod worths, sodium coefficients, and gamma and neutron fluxes at the instrument positions. These measurements are expected to be completed in the middle of 1967. We will then build a series of plutonium-fuelled cores of various plutonium-to-uranium ratios, probably zoned assemblies, which are intended to be idealized versions of large power reactors. The details of these cores have not yet been settled. In our program we then have planned a second mockup, a real engineering mockup of the PFR. I think it is unlikely that this will be built since I think that sufficiently accurate calculations will be available from the more clean physics experiments, but this is not yet decided.

An aspect of our programme of particular interest is that, like other people, we believe very much that the new spectrometry techniques which have become available are of great importance in improving our knowledge of fast reactor physics. In the next series of assemblies, therefore, we are putting a lot of emphasis on spectrum measurements. We shall use three techniques: Li^6 spectrometers from 200 keV upwards, hydrogen-recoil proportional counters from 10 KeV to 1 MeV, and time-of-flight measurements from 0.1 to

200 keV. Since we have already heard a lot about the other two methods I thought it might be useful just to say a little about our plans for the time-of-flight. We are installing a linear accelerator and it should be in operation towards the fall of 1967. It has a 10-MW klystron accelerator supplied by Vickers Radiation, which will produce 200-mA pulses, 3 μ sec in length at an energy of 14 MeV, with a maximum pulse repetition frequency of 200 a second. We shall have a water-cooled uranium block as a target at the end of the drift tube, and we can put this either in the core or the reflector, displaced from the core central plane. When the accelerator is running, the mean power in the reactor will be somewhere near 200 W. Then we shall take an extracted beam from an area of about 10 sq cm down the flight tube. This is 60 cm in diameter, and has counting stations at 50, 100, and 200 m with both detectors and collimators inside the tube. We propose to use a Li^6 glass detector to cover the range from 100 eV to 100 keV, and hope also to develop boron-loaded liquid scintillation detectors, with pulse-shape discrimination, and these should cover the range up to 400 keV. The reactor will be operated at a k_{eff} of about 0.8 to 0.95. We shall achieve this by the removal of peripheral fuel elements, and we are presently engaged in developing calculational methods to allow for the difference in spectrum between subcritical and critical systems. We are hopeful that the detailed knowledge of the reactor spectrum, which is now becoming available, can in some way be used rigorously and mathematically to improve the microscopic data that we use in calculations. We are making a number of studies to this end. Though I cannot see at the present time any method of doing this, I am hopeful that by next year we will have developed a suitable method.

ZPR-3 AND ZPPR: F. W. THALGOTT, *ANL-Idaho*

I would like to talk a bit about our general program. You have heard about our more detailed experiments during the course of this conference. About a year ago we looked at the program that we had been pursuing to try to decide what should be done in the future. We found that in a period of two years we had done a mockup for RAPSODIE, a mockup for FARET, a mockup for SEFOR, and we had about six months in which we built a zoned plutonium core to study Doppler effects. In these two years we had no full-size plutonium-fueled reactor with a soft spectrum.

We also looked forward to ZPPR, and our thoughts for the two facilities were somewhat different.

We believed that in ZPR-3 we should build a series of full-size plutonium cores with soft spectra even if it meant the compositions were not the compositions of power reactors. Assembly 48 is the first of these. We hope to build six assemblies, all with the same plutonium concentration but with two different U^{238} concentrations—one, as high as we can build with our inventory of fuel; the other, as low as we can build and still stay within the safety limitation of a calculated

non-positive power coefficient. We are building these in simple geometry with simple drawer loadings. The spectra will be varied from the spectrum of Assembly 48 to still softer spectra, by replacing the sodium and uranium with graphite. So we hope to have a series of full-scale soft-spectra plutonium cores. We will also do critical mockup work for FTR, and you have heard some of the ideas along these lines. I expect we will do some control-rod experiments for FTR about the first of the year, although this matter is still being negotiated. I fully expect in the future that more detailed mockups will be made.

On ZPPR we think we have another objective. There we expect to build cores with typical power reactor compositions in sizes that will be of interest.

These will not be mockups. We hope to make them simple cores in simple geometries, uniform cores with uniform reflectors.

In more detail, and these are just plans, our first core will be a 2,000-liter core (oxide simulation) typical of a prototype. The next core will probably be an oxide simulation, if oxide seems to be the preferred fuel. We will build it as large as our fuel supply will allow. We think this might be 3,500 liters, followed by (provided that we have the fuel) a 5,000-liter oxide simulation. We expect these to be cores with a length-to-diameter ratio of 1 and with uniform blankets. These cores can be check points for power reactor designers.

ZPR-6 AND -9: W. Y. KATO, *ANL-Illinois*

The Argonne ZPR-6 and -9 critical facilities are currently being used to investigate the following three

major areas: (1) a study of the sodium-void coefficient as a function of core geometry and composition; (2)

FIG. 1. Aerial Photograph of Argonne's Reactor Physics Laboratory.

FIG. 2. Proposed Confinement Shell for the West Wing of the Reactor Physics Laboratory.

Doppler temperature coefficient studies; (3) investigation of the areas of validity of zoned cores. In addition, preparations are in progress for the conversion of the facilities to the use of plutonium and U^{233} fuels. Some of the work which has been accomplished during the past year in these major areas has been reported at this Conference. At ZPR-6 at the present time, we are continuing our physics investigations of the 2600-liter uranium-carbide system. At ZPR-9 we are currently constructing a small, 25-cm-radius, zoned core having the same central composition as the 2600-liter system for investigations of Doppler effect and sodium-void coefficient in the central region. The investigation of the Doppler effect will be continued, looking at environmental effects and the effects of clustered, high-temperature, plutonium, and high Pu^{240} elements. Measurements of sodium-void coefficient, with special emphasis on the effects of heterogeneity, will also be continued in these facilities. With each of these assemblies that are constructed on ZPR-6 and -9 we plan to do standard measurements such as spectral indices, spectrum measurements with a proton-recoil counter, central perturbation, critical mass, and Rossi-alpha measurements. A small pulsed neutron source has recently been purchased for pulsing assemblies on these facilities for decay-constant meas-

urements. Following the completion of the studies on the 2600-liter carbide system, we will build the largest dilute uranium oxide assembly that can be studied within the limitations of the fuel inventory and the facility size. It is anticipated that these oxide studies will commence in the spring of 1967.

A great deal of effort is being expended in the conversion of our facilities for the use of plutonium and U^{233} . In August of 1966, a preliminary Safety Analysis Report for the utilization of up to 1000 kg of plutonium in each of the facilities (ZPR-6 and -9) was completed. This report was submitted to the U.S. Atomic Energy Commission and is currently being reviewed. Since we have found the 4-ft-thick concrete walls of the reactor cell for the ZPR-6 and -9 facilities to be porous, there is leakage of air when the cell is under pressure. Therefore it has been decided to construct a steel confinement shell around the two concrete reactor cells.

An aerial photograph of the reactor cells housing ZPR-6 and -9 is shown in Fig. 1. A conceptual exterior view of the confinement shell is shown in Fig. 2. A conceptual view of the proposed internal modifications is shown in Fig. 3.

It is proposed to maintain the volume between the confinement shell and the reactor cells at about 1 in.

FIG 3 Proposed Internal Modifications for the West Wing

of water negative with respect to the atmosphere. The effluent from the volume enclosed by the confinement shell as well as from the reactor cells would be filtered and passed up a stack. We are currently in the process of selecting an architect-engineer for the detailed design of these modifications. Our present plans are to start the construction about mid-1967 with the completion of the conversion about mid-1968. It is our intention to use the ZPR-6 and -9 facilities for primarily zoned-core studies with plutonium fuel.

Discussion

Mr Reynolds (GE) In ZEBRA, how is the oxygen introduced and in what form is the plutonium?

Mr Smith (UK-Winfrith) The oxygen put into ZEBRA is in the form of alumina plates. Some of the

plutonium is metal, some of it is mixed with uranium oxide. We have a variety of plates with different compositions of Pu^{240} .

Mr Persiani (ANL) In FARET we used uranium oxide, U_3O_8 , with any deficiency in oxygen being mocked up by carbon. It proved very satisfactory.

Mr Smith We also have uranium oxide plates and we also use graphite for simulation of oxygen so that we have alumina, uranium oxide, mixed plutonium oxide, uranium, and graphite. This gives us a large amount of flexibility.

Mr Broudo (GA) Will the first RAPSODIE loading be a dry loading?

Mr Storrer (Cadarache) Initially we intended to perform a dry critical and then a wet critical, but plans for the first have been dropped.

Chauncey Starr

APPENDIX A

The Fast Reactor Development Program

CHAUNCEY STARR

*North American Aviation, Inc.
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The present intensive interest in the development of fast breeder power reactors is a direct consequence of the successful demonstration of competitive atomic power from thermal-spectrum nuclear reactors, in particular the light water reactor. The emergence of the light water reactor as an economically desirable power generator in the short span of a few years represents the fulfillment of an important goal of atomic power. The literature of the past twenty years on the ability of nuclear fission to meet world power needs has shown a spread of optimism and pessimism on the eventual role of atomic power. It runs the gamut from predicting atomic power to be a total waste of technological resources to the most amazingly optimistic predictions concerning its value as an energy resource. Most of this diffuse debate has now been settled by the recent commercial successes.

However, we have completed only half of our task. We have demonstrated that atomic power can provide an economically acceptable and useful energy resource. We have not as yet demonstrated that atomic power is a relatively inexhaustible supply of energy for the world. Looking forward to the very distant future when the fusion process may be our ultimate energy resource, we have to show that the fission process can bridge the gap from now until then. The fast breeder reactor represents a very attractive concept for this purpose. Its successful development, therefore, represents an important, exciting and perhaps essential next goal of atomic power technology.

A possible competitor to the fast reactor is the thorium-cycle thermal breeder under study at the Oak Ridge National Laboratory. This program faces many engineering-development problems, as well as the pragmatic issues associated with its possible integration into existing light water reactor power systems. It will be interesting to watch the initial development of this system for better evaluation of its performance potential.

A purely experimental demonstration of the feasibility of fast reactors is merely a first step in achieving

a successful fast reactor power economy. I would like to recall for you the fact that this was done by EBR-1, which produced electrical power in 1951. This success of the Argonne National Laboratory, under the leadership of its director, Dr. Walter Zinn, involved some of you in this audience. It should have been the precursor to a major engineering development leading to central station-sized power plants. The fact that this development did not take place provides an interesting area of discussion—a pleasure I will forego at this time. However, this does raise the question as to what further has to be achieved in a fast reactor development to make such a development an industrial success.

Our past experience in bringing atomic power to its present commercial position indicates very forceably that the single over-riding criterion used throughout the world in measuring the achievement of a new power source is economics. Under this very broad measure one can cover all the ancillary criteria, such as reliability, availability of fuel, and even political constraints. In more detail, any operating power system must consider a new power source in the light of initial investment cost, fuel and operating cost, reliability of performance, and such long-term factors as technological obsolescence, changes in production costs, long-term availability of fuel, and long-term maintenance characteristics.

The large investment in an individual power station can only be justified if that investment can produce for a very long period, usually at least 30 years. The 30-year lifetime factors involve a considerable amount of "crystal ball" speculation. It is for this reason that the operating power systems of the world tend to be conservative in their acceptance, on a large scale, of any new developments. The power-generation industry has exhibited a considerable degree of boldness in its present acceptance of thermal-spectrum reactors. The role of the various national governments in stimulating the initial development and supporting the present large-scale use of atomic power has been a crucial factor in the success of this program. I believe that

this government role of stimulating large-scale advanced development will be important for the future development of the fast breeder.

The question now at issue in the fast breeder program is not whether the fast breeder should be developed, but rather in what manner and with what emphasis. I would like to develop the point that several circumstances justify an enlarged, rapid, bold, and comprehensive effort to develop an economically acceptable fast breeder central station. I am using the term "economically acceptable" to include not only the cost of power from an individual power station, but also the long-term national power costs as they are influenced by cumulative uranium ore requirements. For example, in such a national sense, I do not consider that the light water reactor is "economically acceptable" on the very long term—that is, fifty to a hundred years—because of its inefficient utilization of ore resources. This is not the same consideration as the individual economic acceptability of the light water reactor to an operating utility in the immediate or near future.

Let us examine several of the general circumstances which will determine the direction of fast breeder reactor development. The first circumstance is uranium ore reserves. I will not bore you with the detailed numbers concerning the speculations as to what the world may have in the way of uranium ore. Many of these projections have been made, including my own. The one basic fact is that the known proven low-cost reserves are insufficient for more than the initial growth of atomic power. By analogy with other mineral resources, it is reasonable that much more uranium ore can be made available. Uranium is known to exist as a very low percentage constituent of other known ore bodies, and cost estimates for separating the uranium have been made—and of course there is always uranium from seawater. Uranium ore can be made available for a long time—at increasing unit cost.

In relation to ore availability, the fast breeder reactor plays two roles. First, because of its excellent utilization of the fertile isotope U^{238} , it is less sensitive than any other reactor concept to unit ore cost. Second, because of its breeding capabilities, it places a practical ceiling on uranium ore required for any growing power economy. It is a characteristic of the cumulative ore requirements of combined nuclear systems [such combinations as light water reactor (LWR)–fast breeder reactor (FBR); or light water reactor (LWR)–heavy water reactor (HWR)–fast breeder reactor (FBR)] that as the fast reactors begin to breed enough plutonium to supply the inventory needs for power expansion, then the cumulative ore

requirements begin to level off and finally reach a peak.

As a typical example, a light water reactor "only" power system might require during the next fifty years about five million tons of U_3O_8 at a total ore cost in excess of \$100 billion. If high-performance fast breeder reactors, that is, those having doubling times of seven years or less, are fully introduced operationally by 1985, a maximum of about one-third as much is needed for the combination LWR-FBR system, at a total ore cost of about \$20 billion, about one-fifth the cost of the LWR only. If in addition, heavy water reactors are added before 1980, the system LWR-HWR-FBR requires a maximum of one-fifth as much ore as the LWR alone at a total ore cost of about one-tenth the cost of the LWR only (about \$10 billion). These savings are very dependent on the performance of the FBR and the time of its introduction, but are typical of what may reasonably be expected. It is interesting that low-performance FBR's (doubling time about 15 years or more) have very little value for reducing ore requirements.

As illustrated in this example, the advanced converters, such as the heavy water reactor, can play a major role in extending the utilization of low-cost uranium ore. For this reason, such reactor concepts have an important place in our national planning. Specifically, the heavy water reactor serves both as a relatively efficient utilizer of uranium ore (between the LWR and the FBR) and as an efficient producer of plutonium for fast reactor inventory. A power system in which natural uranium-fueled heavy water reactors are plutonium producers for fast reactor inventories not only requires substantially less cumulative ore, but also has a fuel-cycle cost that is much less sensitive to increasing uranium cost. It is for such reasons that my colleague, Dr. W. B. Lewis of Canada, is so enthusiastic concerning the near-term importance of this reactor concept. However, since this meeting is primarily devoted to consideration of the fast reactor, I will confine my further remarks to that subject.

Another boundary condition on the planning of a fast reactor program is the rapid growth of light water reactors. In this regard, most official projections of the growth of nuclear power in the U.S. have been overly conservative. One cannot blame those in public life for being modest in setting goals which are likely to be measured against actual accomplishment. My own opinion is that the latest official statements concerning the growth of nuclear power are low by a factor of two. For example, the latest AEC projection of nuclear power capacity in the U.S. indicates about 100,000

MWe for the year 1980. This appears to be the "conventional wisdom" of most official soothsayers. My own estimate for the year 1980 is closer to 200,000 MWe. This is based on the assumption that between 1970 and 1980, 75 percent of the new capacity going into operation will be nuclear. I would also guess that after 1980, 90 percent of the new power capacity will be nuclear. I think it is worth noting that about ten years ago Ken Davis and Lou Roddis predicted about 200,000 MWe for 1980—a number similar to my own.

It is interesting to look at the effect of such differences in optimism on the nuclear capacity of the U.S. in the year 2000. The modest USAEC projection would indicate about 50 percent of all the generating capacity in the U.S. in the year 2000 will be nuclear. My own bold projection is about 70 percent.

The significance of such a rapid growth of nuclear power on fast breeder development arises from the economics associated with plutonium production from these power stations. Assuming my growth projection as a basis, in 1970 light water reactors in the U.S. will be producing about 2.5 metric tons/year of plutonium, in 1975 about 18 metric tons/year, and in 1980 about 40 metric tons/year. The economic pressure of such vast plutonium production can best be realized in terms of the monetary value of plutonium. If used to recycle in light water reactors, depending on the recycling system, plutonium may be worth about \$5 to \$8 per gram. So, one metric ton/year is equivalent to about \$5–8 million in value. If the same plutonium is used as inventory for fast breeder reactors, it could easily be worth two to three times as much. Thus in the U.S. alone the value to the power producers in having a fast breeder reactor to absorb plutonium production in the 1970s would be measured in the hundreds of millions of dollars per year, and in the 1980s might well be approaching the billion-dollar per year level.

We see then that from the near-term point of view of the power-generation industry, a successful fast breeder reactor may be a requirement to achieve maximum economy of power production from light water reactors. From the long-term national and international point of view, such a reactor is needed to provide a pragmatically inexhaustible energy resource through the efficient utilization of uranium.

In order to meet the conditions imposed by both our national goals and the shorter-term pressures of the power industry, the fast breeder reactor must be able to meet minimum conditions of technical performance. From the point of view of the operating utility, the economics of power production over the lifetime of any given plant is a first consideration. As a rough rule of thumb, the capital costs of all large nuclear power

plants are likely to be the same. The power-cost issue will then center about the operating costs and, most particularly, the fuel cycle. The considerations relevant to a low-cost fuel cycle for a fast reactor are probably familiar to most of you. The achievement of very long-lived fuel (100,000 MW days/ton) appears to be the most significant objective for fuel development. I have no doubt that it will be achieved. Because the subject has been so well-explored elsewhere, I will not discuss it further now.

A more subtle consideration, which affects both the economics of the fast reactor and the eventual utilization of ore resources, is the doubling time of the fast reactor. Although the importance of doubling time is probably understood in this audience, its significance is not well-comprehended generally. In very broad terms, all the near-term projections of the nuclear power growth in the U.S. indicate a doubling in nuclear capacity every 5–7 years. If one assumes that the rate at which fast reactors can be placed in power production will be determined by the availability of plutonium for inventory, it becomes evident that a fast reactor with a very short doubling time will be needed in order to minimize the continued growth of light water reactors, and consequently increased ore requirements.

For example, consider a neutron economy consisting of light water reactors producing plutonium to inventory fast reactors, and assume that the fast reactors are built as rapidly as plutonium production permits. Further, let us assume that fast reactors are available for routine use in 1980. In such a system the cumulative total of uranium ore which must be made available by the year 2000 will be increased about 50 percent if the fast reactor doubling time were to go from 7 to 15 years. In fact, with a 15-year doubling time, the ore consumption is only slightly less than with LWR's only. In a specific example with a 7-year doubling time, such a system might require for the U.S. about \$15 billion worth of U_3O_8 by the year 2000, whereas with a 15-year doubling time about \$30 billion worth of uranium ore will be needed. This takes into account not only the increased ore required, but also the increasing unit price of such ore as the cumulative demand goes up. Thus the cost of research and development for improving a fast reactor so that its doubling time may be reduced from 15 to 7 years can be compared with a saving benefit in national ore costs of about \$15 billion in the next 30 years.

The same example can be used to illustrate the effect of doubling time on the peak amount of uranium ore which must be mined, prior to the leveling off due to the self-sustaining production of plutonium for expansion of the power system. With the 7-year doubling

time, such a peak would be achieved about 50 years from now and would require about 1.25 million tons of U_3O_8 . In the case of the 15-year doubling time, such a peak would be achieved in about 80 years and would require about 7.5 million tons—five times as much.

The compounded doubling time of a fast reactor is given approximately by the ratio of the specific inventory to the breeding gain. The specific inventory is the total kilograms of plutonium invested, both in-core and ex-core, per megawatt electrical output of the fast reactor system. The breeding gain has the customary definition of net grams of fissile material produced per gram of fissile material destroyed.

Based upon reasonable extrapolations of present experience with sodium coolant and oxide fuel, a specific inventory of 2.5 to 3.5 kg Pu/MWe is reasonable to expect in a 1000-MW-sized fast reactor in operation about 1980. This range of specific inventory is being prototyped by the U.K. PFR and the Russian BN-350. The eventual development of sodium-cooled and bonded carbide or nitride fuel may permit a specific inventory of 2 or less.

When one considers coolants other than sodium, such as, for example, gases of various types, the reduced heat-transfer capability results in increased specific inventory. For example, steam- or helium-cooled fast reactors typically have specific inventories of about 4 to 5.

The breeding gain which can be expected from fast reactors depends upon the safety compromises made relative to neutron losses and reactor transient coefficients. Typically, a well-designed, square-core, sodium-cooled oxide-fueled fast reactor will have a breeding gain of about 0.4. The same reactor, partially moderated and "pancaked" to increase neutron leakage for hypothesized safety transient reasons, will have the breeding gain reduced to about 0.2. Conversely, if the square-core, sodium-cooled fast reactor were to utilize carbide fuel most effectively, the breeding gain might be raised to about 0.5.

If one considers alternative coolants to sodium, a steam-cooled, oxide-fueled fast reactor would have a breeding gain about 0.1 to 0.2. On the other hand, a helium-cooled fast reactor could have a breeding gain as high as 0.5.

If one combines these factors to estimate the probable performance of these various reactor combinations, it appears that the sodium-cooled, square-core fast reactor using oxide fuel might be expected to produce doubling times in the range from 6 to 12 years, and could approach a doubling time of 4 years with the use of carbide fuel. On the other hand, if for hypothetical safety considerations the reactor is either

moderated or pancaked or both, the doubling time may be in the range from 15 to 20 years.

Considering other coolants, a steam-cooled fast reactor probably would have a doubling time of 20 to 40 years, and a helium-cooled fast reactor might have a doubling time in the range from 8 to 12 years.

For any given fast reactor doubling time, the specific inventory has an almost linear effect on the peak cumulative ore requirement for a combination converter-breeder economy. Thus, for example, assuming a common 10-year doubling time, if one compares a helium-cooled fast reactor with a specific inventory of 4 kg Pu/MWe to a sodium-cooled reactor utilizing carbide fuel with a specific inventory of 2, then the peak cumulative ore requirement is less by a factor of 2 with the sodium-cooled reactor. In addition, the requirement at any given year, as for example, by the year 2000, is also proportionately reduced by the sodium-cooled reactor.

It is interesting to examine the effect of an early introduction of the fast reactor on the cumulative ore requirements. When one calculates the total requirements for several combined reactor systems, such as light water reactor-fast breeder reactor, or a combination of light water reactor-heavy water reactor-fast breeder reactor, it becomes evident that the earlier the year of introduction of the fast reactor the less the total ore that has to be mined at any time. In a qualitative sense, this conclusion obviously arises from the difference in ore utilization of the fast reactor as compared with the thermal converters. There is also a corresponding reduction in the peak ore requirement for the national power system (roughly about 30–100% per decade of acceleration depending on the system).

A more complete survey of the effect of combined systems of various thermal converters and fast breeder reactors gives more explicitly the effect of individual parameters on the performance of the nuclear economy. I will try to summarize some of the more general conclusions to which such a study leads.

The first principal conclusion is the necessity of achieving short doubling times in order to reduce substantially the total national investment in uranium ore in the growth of our national power system. The second is the importance of low specific inventory for the same objective. A third conclusion is that of all the various coolants that have been considered for fast reactors, sodium is the only one that appears to provide an opportunity for really efficient utilization of ore reserves on a long-term basis. The fourth conclusion is the importance of including in power systems at an early date high-performance fast breeder reactors, that is, those whose configuration optimizes specific inventory and breeding gain.

All of these conclusions point to the fact that there is a tremendous national resource and economic incentive to the early development of a very high-performance fast breeder reactor. Further, the "present worth" economic values to the nation are of the magnitude of billions of dollars. In the light of such potential benefits, our national program on fast breeders is now too small, too slow, and too modest technically.

I have discussed the fast reactor program primarily from the point of view of the broad future value of our national power economy. From the point of view of individual operating power utilities, the same economic argument can be shown to apply. For example, in a typical central-station fast reactor, a reduction of specific inventory in the amount of 1 kg Pu/MWe reduces the cost of the fuel cycle by 0.14 mill/kWh. In a like manner, an increase of breeding gain by the amount of 0.1 will reduce the cost of power by 0.1 mill/kWh. Thus the incentives in any fast reactor power station to move in a direction of low specific inventory and high breeding gain are just as great as for the national economy as a whole. It is also quite clear that the broad considerations involving the total national demand for uranium ore, and the increased magnitude of expenditure for such ore over any given period (as for example over the next 30 years), must eventually show up in the cost of power generation, since such ore is the raw material for the nuclear power system. In sum, therefore, the individual interest of any power-generating utility directly parallels the national interest in achieving an early development of high-performance fast reactors.

I have emphasized the necessity for the rapid development of high-performance fast breeders without compromising such performance for hypothetical reasons of nuclear safety. I do not believe that the absence of such compromise involves, at any point, increased hazard to the public. It is my opinion that any fast reactor power station can be designed with adequate containment to protect the public in the event of any nuclear-core failure. I do not believe that the capital cost of such containment need be excessive. For this reason, I am sure that in any such installation the public will be completely protected.

There is, of course, a major incentive to reduce the probability of a plant failure which may cause it to be unavailable as a power producer for any period of

time. Such unavailability would be of serious economic consequence to any operating utility. In considering this aspect of the problem, there does exist at the present time a considerable variety of opinion as to the possibilities of various types of nuclear-core failures and for their consequences. Because of these uncertainties, there should be undertaken at once a major program in experimental and analytical verification of the validity of hypothesized modes of nuclear-core failure, and for their result. Such a program may clarify the choice between an intensive development of high-performance fast reactors, and a near-term emphasis on low-performance, compromised cores.

As I have mentioned previously, the cumulative economic difference of such a performance compromise is so great over the next several decades that a large-scale program to provide the technical information on safety characteristics is urgently justified now. We should learn from our experience with light water reactors that the time to carry on large-scale integral experiments is during the early development phases, not after many such plants have been built. SPERT was a timely series of experiments. LOFT would have been of much greater value had it preceded the 29,000 MW of water reactors sold in the past several years.

With regard to this issue of transient characteristics of nuclear cores and the choice of compromised versus advanced core configurations, it is my opinion, based on the present studies which have been made in this area, that there does not now appear to exist a valid reason for choosing a compromised performance approach at this time. This conclusion seems to have been reached in most of the program planning outside the U.S. Further, the economic penalty associated with compromised performance is so large that this issue deserves intensive technical attention.

I will end my comments with the reiteration of the general statement that a "bold and vigorous," large-scale development program for advanced-performance, sodium-cooled fast reactors should be undertaken now to fulfill the original objective of an inexhaustible supply of low-cost energy for the world. Further, I will again emphasize that the economic value in placing high-performance fast reactors into our power systems at an early date is so large that a program of much greater magnitude than is now underway can be justified on a purely cost-benefit basis.

APPENDIX B

Preliminary Report on Fuel Damage in the Fermi Reactor*

Presented by E. L. Alexanderson

It is believed that significant damage has occurred to some of the fuel in the Enrico Fermi reactor. This report summarizes the operations during which this damage appears to have occurred and presents the status of knowledge as of 8 A.M., Monday, October 10, 1966. A more detailed report is being prepared.

On Tuesday, October 4, 1966, reactor operations at the Fermi plant were resumed after a lengthy outage due to the necessity for repairs to the No. 1 and No. 3 steam generators. The reactor was kept at 1 MW during the night of October 4-5, during which time reactor heat was used to raise the reactor temperature to the normal level for power start-up.

At 1:45 P.M. on Wednesday, October 5, following a number of minor delays due to adjustments in the feedwater-control systems, an increase of the reactor power level was initiated at the normal rate. Two primary and secondary coolant loops were in operation. By 3 P.M., the power level had been raised to 34 MW(th), a level which had been surpassed many times in the past. At about this time the staff member present observed that the control rods seemed to be somewhat further out of the core than would be normal under the circumstances. He went to the back of the control panel to observe the outlet temperatures from those core subassemblies which have thermocouples at their outlets. He saw that two core-subassembly outlet temperatures were higher than normal. At about the same time the fission product detector indicated a substantial increase in the primary cover-gas activity, and the reactor building radiation monitors indicated a rise in the radiation level in the building large enough so that the reactor building was automatically isolated. The radiation monitor charts on the waste-gas stack show that essentially no radioactivity was released to the atmosphere.

The reactor power was reduced manually from 34 to 29 MW upon the rise in the cover-gas activity, and

the reactor was scrammed shortly after the building isolation occurred. The scram rods operated normally in shutting the reactor down. The radiation levels in the reactor building were not in themselves dangerously high and no personnel overexposure occurred. All health physics procedures were carried out according to the prepared written instructions.

A meeting was held at the plant shortly after the occurrence to determine whether any measures needed to be taken either for public or personnel safety or for protection of plant equipment. No such measures were required, but operations were initiated to obtain samples of the primary sodium and the primary cover gas. The AEC, Division of Compliance, was notified of the occurrence by telephone immediately following the meeting.

A number of facts have been developed since the reactor was shut down on October 5. From the standpoint of significance in indicating the nature and extent of fuel damage the following are the most important:

1. Analysis of the primary cover gas indicates concentrations of fission gases, notably Xe-133 and Xe-135, which would be characteristic of concentrations resulting from the release of fission products from an amount of fuel about equal to that in a core subassembly.
2. Analysis of the reactor-building atmosphere indicates the presence of the same fission product gases in about the concentrations which would be expected due to the known leak rate from the primary cover-gas system to the reactor building. No isotopes of iodine were determinable above background in the gamma-ray spectrometer used for the analysis.
3. A subcritical reactivity check was made on October 7, which indicated that the core reactivity was about 22 cents lower than when the reactor was brought critical on October 4.
4. Correlation of reactor control-rod position, power level, and time indicates that reactivity began to

* This is a summary of the status report on the Fermi Reactor incident, presented at the start of Session IV by E. L. Alexanderson of the Power Reactor Development Company.

be lost on October 5 when the power level reached about 15 MW(th). Loss of reactivity was roughly linear with power thereafter.

5. Allowing for the 13-min overall time constant of the fission product detector, the first indication of fission product release agrees closely with the first loss of reactivity.
6. The readings on the two core-subassembly outlet thermocouples indicated that the temperature difference across these subassemblies was considerably greater than normal.

Taken together these facts indicate that fuel damage has taken place and that some fuel redistribution has taken place. There is a possibility that the damage is limited to the two core subassemblies which indicated high outlet temperatures, but since only one-fourth of the subassemblies have thermocouples this would be fortuitous.

The reactor is presently shut down and the activity in the reactor building atmosphere is low enough to allow accessibility. No surface activity has been found anywhere in the reactor building. All fission products except some of the noble gases and their daughters appear to have been retained in the primary system.

Discussion

Mr. Campise (AI): Were your cold traps running throughout this period?

Mr. Alexanderson: Yes. Our normal operating procedure calls for cold traps to be running in the primary system during all reactor operation. The reason for this is that we thought that this would help us take sodium hydride out of the system if there were any tendency for hydride to form.

Mr. Kohler (Texas A&M): When you made your original interchange of subassemblies, did you check the filters for possible debris in them?

Mr. Alexanderson: This is not possible because the interchange is made within the reactor. The subassembly is raised into a pool of sodium above the core, then moved laterally into a storage area. When the subassembly is interchanged, it never is removed from the reactor vessel. Thus there is no chance to look at it nor to see it in the reactor.

Mr. Lewis (ANL): Do you have any theories on how the postulated plugging occurred? For example, what were the plugging temperatures, and is there any source of carbon?

Mr. Alexanderson: The plugging temperature was kept low at all times, so that we had no indication that there was anything wrong in the sodium. We have had very little difficulty and have had to run our cold traps only occasionally to keep the primary system down to the plugging temperature of around 220 to

240°F. Contrary to that, however, there is some evidence that there could have been contaminants in the system because we have had them before. A long time ago, when we had dummy assemblies in the reactor, we took the whole primary system up to 1000°F. When that was done, there was some material entrapment in the plug. There was some off-gassing from the plug itself, which is made largely of graphite. The graphite that is in the plug had not been out-gassed thoroughly, and it had a binder of some sort in it. When it went up to 1000°F, there was off-gassing from this graphite that went into the reactor cover gas and formed some carbonates. These were cleaned up very thoroughly before we loaded the reactor with the real fuel. However, some of this could have remained in the plug area and eventually reached the sodium, perhaps in chunks of some sort. I don't know whether this actually occurred, but there is that potential.

Mr. Hutter (ANL): How high were your excessive outlet temperatures?

Mr. Alexanderson: The hottest subassembly here was 715°F on the thermocouple. We must remember that the thermocouple is well above the subassembly in the sodium, and it can be influenced by many other things. Also, we are not certain that the temperature as read by this thermocouple is due to this one subassembly. If this channel, for example, had boiling sodium in it, it could very well influence the thermocouple above the subassembly depending upon the flow pattern in the core, and we were on two loops at the time. There is some difference of opinion as to the flow pattern. The outlet nozzles were over in this direction, so if there was some tendency for flow to sweep toward those outlet nozzles, there are some deflectors which would interfere. To my mind, there is a little tendency for boiling sodium in high temperatures here to influence this thermocouple.

Mr. Hutter: What is the normal temperature reading?

Mr. Alexanderson: At that power, it should be around 600°F.

Mr. Baker (UK-Winfrith): Do you think that the troubles with the steam generators could have had anything to do with this incident?

Mr. Alexanderson: I don't believe that the steam generators had anything whatever to do with it. I think they were completely independent. There is a secondary loop in between. The plugging temperature even in the secondary loop was very low. It was just that the hydrogen level in the cover gas was quite high, which indicated we had a leak starting. It hadn't occurred for a long-enough period of time to really influence the plugging temperature yet. So I think that's completely isolated.

Mr. Baker: Can you explain why you think the problems with the safety rods might have been connected with these subassemblies which I understand were two spaces away?

Mr. Alexanderson: We felt that the problem with safety rods was quite isolated too, although this is not necessarily so. We have taken out this one safety rod. We lowered it into a glove box, and one of our engineers at the plant made a very careful visual inspection of the safety rod. He could see no signs of wear. This rod, after all, has been scrammed almost 1000 times now. He could see no signs of wear on the dash ram at the bottom and also no signs of wear where it is gripped at the top and where the fingers come over and latch onto it. There was no sign of oxide forma-

tion on the rod. The rod looked perfectly normal, and as far as I am concerned, it can probably be reinserted. I suspect that the trouble may be more with the extension and lining between the extension and the rod than it was with the rod. When we did move the rod from one position to another, the trouble that we had in the initial position was not the same as when we moved it. It was slightly different, indicating that it could be either the rod or the extension. It may very well be the extension. So I think that our steam generators and safety-rod difficulties were completely isolated. I mention them mainly to alert you to how much fuel movement we had made between the time we last ran and the time we had the difficulty.

APPENDIX C

Non-ANL Attendees

Adkins, C. R., Carnegie Institute of Technology
Adler, F. T., University of Illinois
Alexanderson, E. L., Power Reactor Development Co.
Alter, H., Atomics International
Anderson, C. A., Jr., Los Alamos Scientific Laboratory
Ancarani, A., Euratom, Brussels
Azziz, N., Westinghouse Electric Corp.
Baker, A. R., UKAEA, Winfrith
Barberger, M., Electricité de France
Barnett, A. G., Mound Laboratory
Batch, M. L., Atomic Power Development Associates
Battat, M. E., Los Alamos Scientific Laboratory
Baxter, A., General Atomic
Becker, M., Rensselaer Polytechnic Institute
Bell, V., ENEA, Ispra
Bennett, L. L., Oak Ridge National Laboratory
Bennett, R. A., Battelle-Northwest Laboratory
Best, G. H., Los Alamos Scientific Laboratory
Broido, J. H., General Atomic
Broomfield, A., UKAEA, Winfrith
Brown, P. S., General Electric Co.
Caldarola, L., Euratom, Karlsruhe
Callen, R. C., Atomic Power Development Associates
Campbell, C. G., UKAEA, Winfrith
Campise, A. V., Atomics International
Carlsmith, R. S., Oak Ridge National Laboratory
Carpenter, S. G., Atomics International
Cherry, B., United Nuclear Corp.
Codd, J., UKAEA, Winfrith
Cohen, S. C., General Atomic
Cremer, C. C., Los Alamos Scientific Laboratory
Critoph, E., Atomic Energy of Canada, Ltd.
Cybulskis, P., Battelle-Columbus Laboratory
Daavettila, D. A., Michigan Technological University
D'Ardenne, W. H., The Pennsylvania State University
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Delene, J. G., Oak Ridge National Laboratory
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