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TO: W. R. Gall

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I. Design Specifications for HRT Charcoal Adsorbers

The important design specifications for the HRT fission product adsorption system are outlined as follows:

The Gas Stream. The adsorption system has been designed to process 500 cc (std)/min of gases composed chiefly of the oxygen from the core and an equal amount from the blanket but containing the krypton and xenon isotopes from the core produced at a 10 Mw power level. The flow time through the lines and equipment between the core and adsorber pit has been estimated by Van Winkle⁽⁷⁾ at 13,600 sec. With this delay time the gas at the entrance to the adsorber pit has the composition shown in Table I:

TABLE I
FISSION PRODUCT GAS COMPOSITION
AT ENTRANCE TO ADSORPTION BED

<u>Mass</u>	<u>Half Life</u>	<u>(Atoms/sec)(10⁻¹⁵)</u>	<u>Partial Pressure out of 827 mm Hg total</u>
		sec ⁻¹	mm Hg
Kryptions			
83m	11.4m	0.40	0.001385
83	Stable	1.58	0.005475
84	Stable	2.95	0.010215
85m	4.36h	1.56	0.00541
85	10.27y	1.29	0.00447
86	Stable	5.64	0.019535
87	78m	1.21	0.00419
88	2.77h	4.75	0.01645
Xenons			
131m ₂	12.0d	0.097	0.00035
131	Stable	10.04	0.0348
132	Stable	15.05	0.05215
133m	2.3d	0.525	0.00180
133	5.27d	22.0	0.00762
134	Stable	26.4	0.09145
135	9.13h	14.8	0.0513
136	--	21.70	0.0752
Total		131.0	0.4191

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The activity of the exit gas from the adsorbers is reduced to essentially 6.45 curies/day of $^{10.27\text{y}}\text{Kr } 85$.

Additional Delay Time. In addition to the 13,600 sec delay before the pit, it is necessary to provide an additional decay period to prevent excessive temperatures in the charcoal bed. This is to be done by placing 160 ft of 3 inch empty pipe in the adsorber pit just before the charcoal adsorption units. The most important effect of this additional delay is to reduce the concentration of $\text{Kr } 88$ since its 17.8m Rb daughter is the greatest single contributor to the heat load in the initial portion of the bed.

Charcoal Units. After leaving the 3 inch empty pipe, the 500 cc/min gas stream is split into two streams of 250 cc/min. Each of these 250 cc/min streams enters an identical charcoal adsorption unit. These units were designed for activated charcoal pellets of approximately 8-14 mesh packed in standard pipe with about 80 % voids. The diameters and lengths of packed piping in the two charcoal units are shown in Figure 9.

Temperatures. The arrangement of diameters and lengths has been chosen in the order shown above to prevent the packed beds from reaching temperatures which would cause excessive oxidation of the charcoal. To remove heat, the 160 ft of empty pipe and the two charcoal units are to be immersed in cooling water maintained at an estimated temperature of 85°F . The temperature in any portion of the packed sections should not at any time exceed a maximum of 100°C . As a precaution, thermocouples should be installed to read the temperatures at the center of the pipe just after the beginning of the 1/2", 1", 2" and 6" sections in each unit.

Safety Factors. The degree of overdesign is indicated roughly by the following safety factors based on the ratio of the charcoal volume in the

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specified design to the charcoal volume in units having the limiting dimensions calculated to prevent excessive activity in the exit gas and keep the charcoal temperature just below the maximum allowable level:

TABLE II

COMPARISON OF ACTUAL SIZE OF CHARCOAL BED
WITH CALCULATED REQUIREMENT

	<u>Calculated</u> <u>Limiting Design</u>		<u>Actual</u> <u>Design Specifications</u>		<u>Approx. Safety Factor</u>
	<u>Length</u> <u>ft</u>	<u>Pipe Size</u> <u>in</u>	<u>Length</u> <u>ft</u>	<u>Pipe Size</u> <u>in</u>	
Empty Pipe 500 cc/min	106	2	160	3	3
<hr/>					
Charcoal Unit					
250 cc/min	20	1/2	40	1/2	2
	10	1	40	1	4
	10	2	40	2	4
	10	6	60	6	6

II. Summary of Calculations

The calculations upon which the design is based solve approximate equations for (1) the depletion of the isotopes from the gas stream produced by isothermal increments of the adsorption bed after steady state conditions have been attained; and (2) the radial temperature distribution in the bed at steady state conditions. The principal assumptions made in deriving these calculations are:

- 1) The concentration of each radioactive isotope in the gas stream is low enough so that its depletion has a negligible effect on the total gas

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flow rate and total pressure.

2) Diffusion and mixing in the gas stream are sufficient to eliminate radial concentration gradients in the gas at any fixed cross section of the bed.

3) Pressure drop along the length of the bed due to fluid friction is negligible.

4) All beta particles released in the bed have energies constant at one-third the maximum energy in the beta emission spectrum for the isotope. All beta particles emitted in the bed are adsorbed at the point of their emission while all gamma energy is assumed to escape from the bed entirely.

5) Heat transferred along the length of the bed is negligible in comparison to that transferred radially.

6) The adsorptive capacity of the charcoal for each isotope is entirely independent of all other isotopes present and as soon as an atom of a given isotope disintegrates on the surface of the charcoal a site is immediately opened to receive an undecayed atom of this isotope.

7) The time required for mass transfer of an isotope from the gas to the charcoal surface is very short relative to the time required for decay of the isotope.

The equations derived using these assumptions are:

Isotope Depletion. Depletion of isotopes along a length of adsorber bed at constant temperature is given by:

$$\ln \frac{(y_i)_2}{(y_i)_1} = - \left[\left(\frac{\lambda_i \rho_s}{G} \right) + \left(\frac{\rho_B \lambda_i}{G} \right) \left(\frac{K_i P M_i}{22,400} \right) \right] \Delta z \quad (1)$$

where $(y_i)_2$ = concentration of radioactive isotope i in the total gas stream leaving an increment of the bed with length Δz

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- $(y_i)_1$ = concentration of radioactive isotope i in the total gas stream entering the increment
- z = linear distance through the packed bed, ft.
- λ_i = decay constant for isotope i, sec^{-1}
- ρ = density of the gas, lbs/ft^3
- ϕ = fraction of voids in the bed, (about 0.8 for this charcoal).
- ρ_B = density of charcoal in the bed, lbs charcoal/ft^3 bed
- G = total mass flow rate of gas per unit cross sectional area of bed, $\text{lbs/ft}^2\text{-sec}$
- K_1 = equilibrium distribution ratio for the isotope =
$$\frac{[\text{cc (std) adsorbed i on solid}]}{(\text{g charcoal})(\text{mm Hg partial pressure i in gas})}$$

Values of K_1 are presented in Figure 5 based on data reported by Holmes⁽⁵⁾.
- P = total pressure of gas in bed, mm Hg
- M_i = molecular weight of the isotope i.

Equation 1 would be correct for any length of bed if it were at a uniform temperature or if the temperature variations in the bed had no effect at all on the equilibrium ratio K_1 . However, since the equilibrium ratios vary greatly with temperature, Equation 1 may be applied only to short lengths of the packed bed in which all portions are considered to have the same mean effective temperature. Although temperatures in the bed change rather slowly with length of the bed, their variation with changes in radial position is very large. Consequently, the mean effective temperature to be used for a given increment of the bed is a value which has been averaged with respect to temperature variations both along its length and

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at various radial positions.

Radial Temperature Distribution

2) Radial temperature distribution in a fixed cross section

$$T = \frac{1}{a} \left[\ln \frac{8k}{(r^2 + k)^2 a} - b \right] \quad (2)$$

T = temperature at any radius within the cross section at a fixed distance z through the bed, $^{\circ}\text{F}$.

r = radial distance from the center of the cross section, ft

a, b = functions of the effective conductivity of the bed, and the concentration and radioactivity of the isotopes in the gas at z .

For oxygen containing krypton and xenon isotopes flowing through 8-14 mesh charcoal

$$a = - \left(\frac{1}{122} \right) \ln \left[\frac{0.572u + 2.34v}{0.1321u + 0.442v} \right]$$

$$b = \ln [0.572u + 2.34v] - 90(a)$$

where $u = \sum_i (\epsilon_i \lambda_i p_i)_{\text{Kr}} (10)^{25}$ and $v = \sum_i (\epsilon_i \lambda_i p_i)_{\text{Xe}} (10)^{25}$

where

ϵ_i = energy of each beta particle emitted by an atom of isotope i , watt sec.

λ_i = decay constant for the isotope i , sec^{-1}

p_i = partial pressure of i in the gas at the given z value, mm Hg

The function u sums the products of ϵ_i, λ_i and p_i for all the krypton beta emitters and v does the same for the xenon beta emitters.

k = a constant which is determined by the boundary values of the maximum radius r_w and the inner pipe wall temperature T_w at this radius.

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$$k = (a - r_w^2) - \sqrt{\alpha(\alpha - 2r_w^2)}$$

$$\text{where } \left[\alpha = \frac{4}{a} e^{-(aT_w + b)} \right]$$

r_w = maximum inside radius of pipe, ft

T_w = inner pipe wall temperature, °F.

The Mean Effective Temperature. The correct mean effective temperature T_m for the small increment Δz has been shown to be approximately the arithmetic mean of the temperatures which exist at three-fourths of the maximum radius, r_w , in the cross sections at each end of the increment. This temperature is illustrated as shown below in Figure 1:

Figure 1

The correct mean temperature is thus:

$$T_m = \left[\frac{(T_1)_{r=(3/4)r_w} + (T_2)_{r=(3/4)r_w}}{2} \right] \quad (3)$$

where $(T_1)_{r=(3/4)r_w}$ = temperature at $z = z_1$, $r = (3/4)r_w$

$(T_2)_{r=(3/4)r_w}$ = temperature at $z = z_2$, $r = (3/4)r_w$

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The entire increment Δz is considered to exist at the temperature T_m when using Equation 1.

Temperature at the Wall. The cooling water surrounding the adsorber pipes was considered to be at the constant temperature of 85°F with heat transferred from the outer pipe wall by natural convection. Allowing 5°F for temperature drop through the water film and pipe wall and for variations in the cooling water temperature, a constant temperature of 90°F was assumed for the inner pipe wall along the entire adsorber length. This assumed temperature was checked by calculating the heat flux at the inner pipe wall:

$$\left[\frac{q}{A} = -k_B \left(\frac{dT}{dr} \right) \right]_{r=r_w} \quad (4)$$

where k_B = the effective conductivity of the bed, $\text{Btu/hr ft}^2 ^\circ\text{F/ft}$.

From the differentiation of Equation 2:

$$\text{at } r = r_w \quad \left(\frac{dT}{dr} \right) = - \left[\frac{4 r_w}{(r_w^2 + k)a} \right] \quad (5)$$

so that:

$$\frac{q}{A} = k_B \left[\frac{4 r_w}{(r_w^2 + k)a} \right] = \frac{\Delta T}{t/k_p + 1/h_c} \quad (6)$$

t = pipe wall thickness, ft

k_p = thermal conductivity of pipe wall, $\text{Btu/hr ft}^2 ^\circ\text{F/ft}$.

h_c = heat transfer film coefficient for natural convection, $\text{Btu/hr ft}^2 ^\circ\text{F}$.

ΔT = temperature drop across pipe wall and water film, $^\circ\text{F}$

q/A = heat flux at inner pipe wall, Btu/hr ft^2

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The film coefficient h_c was evaluated from the nomograph presented in Perry's Handbook (6).

Using the known initial gas concentration, $\left(\frac{dT}{dr}\right)$ was obtained at the wall of 1/2 inch pipe at the beginning of the bed. This value when substituted in Equation 4 should give the maximum pipe wall heat flux in the entire bed. Using this value for the heat flux, Equation 6 was solved for ΔT and the value obtained was less than 5°F. Consequently, T_w was taken as 90°F in all portions of the bed.

Calculation Procedure. The steps in calculation of the temperature and concentration distribution throughout the bed are as follows:

1) Using the known concentrations of each isotope in the gas at the entrance into the bed and assigning a constant value to the pipe wall temperature, temperatures in the cross section at the beginning of the bed are calculated at $r = 0$ and $r = (3/4) r_w$ by means of Equation 2. Only the isotopes which are beta emitters need be considered in the temperature calculations. A typical radial temperature distribution is shown in Figure 2.

2) The initial pipe diameter is sized at this point when the calculated initial temperature at the center of the bed is low enough to prevent excessive oxidation of the charcoal. Decomposition rates of charcoal in the presence of oxygen at various temperatures are shown in Figure 6 which was prepared from data reported in the HRE Design Manual (8).

3) A short length of the bed is then chosen for the first increment - lengths as large as 10 feet are often satisfactory - and a value is assumed for the mean effective temperature of this increment. An obvious choice for this temperature is a value slightly lower than the temperature

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calculated at three-fourths the maximum radius at the beginning of the increment.

4) Using the assumed mean effective temperature, the concentration of each isotope in the gas leaving the first increment is then calculated from Equation 1.

5) From these concentrations, the temperatures in the cross section at the end of the increment are calculated again at $r = 0$ and $r = (3/4)r_w$ using Equation 2. Figure 2 shows this radial temperature distribution in relation to that for the cross section at the beginning of the increment. The closer the two temperature distribution curves are to each other, the more accurate is the procedure.

6) The assumed mean effective temperature is then checked against the calculated arithmetic mean of the temperatures at three-fourths of the maximum radius at the two ends of the increment. If an appreciable error existed, the calculated value could be used as a new mean effective temperature and the process repeated. This, however, is rarely necessary since in any case the calculated bed dimensions should be oversized in the specifications. If the initially assumed temperature is high, the calculated temperatures at the center of the increment will also be high which will insure a conservative design with respect to the maximum allowable temperatures in the bed.

7) As the process continues and the concentration of the more active isotopes begins to be lowered, attempts are made to increase the pipe diameter. At various intervals the temperature which would exist at the center of a larger pipe is calculated using Equation 2. The diameter is increased as soon as this temperature is low enough to prevent an excessive oxidation rate in the charcoal.

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Figure 2

III. Sample Calculation

As an illustration of the procedure the calculations are shown below for the initial increment taken as the first 10 feet of 1/2 inch pipe in the bed.

1) Temperature distribution at beginning of increment:

The beta emitters entering each of the two beds after a delay time of 13,600 sec. between the core and charcoal pit plus an additional 2 hr delay in passing through the empty pipe before entering the charcoal are:

<u>Kr isotopes</u>	Average Beta Energy, ϵ_i watt sec	Decay Constant, λ_i sec	Partial Pressure, p_i mm Hg	$\frac{(\epsilon_i \lambda_i p_i)(10)^{25}}{(\text{watt})(\text{mm Hg})}$
85 m	34.1×10^{-15}	4.41×10^{-5}	0.004031	0.0591×10^6
87	161.5×10^{-15}	1.48×10^{-4}	0.001443	0.3448×10^6
88	* 395.5×10^{-15}	6.95×10^{-5}	0.00997	2.740×10^6

$$u = \sum_i (\epsilon_i \lambda_i p_i)(10)^{25} = 3.144 \times 10^6$$

Xe isotopes

133	18.4×10^{-15}	1.52×10^{-6}	0.00753	0.02108×10^5
135	48.3×10^{-15}	2.11×10^{-15}	0.04410	4.492×10^5

$$v = \sum_i (\epsilon_i \lambda_i p_i)(10)^{25} = 4.513 \times 10^5$$

* Includes beta energy of Rb88

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The average beta energies and decay constants for Kr and Xe isotopes were obtained from the compilation by Blomeke⁽¹⁾. Beta energy of Rb88 was supplied by Cottrell⁽²⁾.

The temperature distribution at the beginning of the increment is now obtained by using u and v above to solve Equation 2:

$$a = -\frac{1}{122} \ln \left[\frac{(0.572)(3.144 \times 10^6) + (2.34)(4.513 \times 10^5)}{(0.1321)(3.144 \times 10^6) + (0.442)(4.513 \times 10^5)} \right] = -0.01258$$

$$b = \ln \left[(0.572)(3.144 \times 10^6) + (2.34)(4.513 \times 10^5) \right] + (90)(0.01258) = 16.01$$

$$\alpha = \frac{4}{(-0.01258)} \left[e^{-[(90)(-0.01258) + 16.01]} \right] = -1.103 \times 10^{-4}$$

$$r_w = \text{for } 1/2 \text{ inch pipe, } = \left(\frac{0.311}{12} \right) = 0.0259 \text{ ft}$$

$$k = \left[-1.103 \times 10^{-4} - (0.0259)^2 \right] - \sqrt{(-1.103 \times 10^{-4}) \left[-1.103 \times 10^{-4} - (2)(0.0259)^2 \right]}$$
$$= -1.184 \times 10^3$$

Substituting in Equation 2 produces:

$$\text{at } z_1, T = \left(\frac{1}{-0.01258} \right) \left[\ln \frac{(8)(-1.184 \times 10^{-3})}{(r^2 - 1.184 \times 10^{-3})^2 (-0.01258)} - 16.01 \right]$$

which is the variation of T with r at $z_1 = 0$ ft for this first increment.

Solving gives:

$$\text{at } r = 0, T = 223^\circ\text{F}$$

This is the maximum temperature in the entire bed and it is important that it does not cause excessive charcoal oxidation. From Figure 6 the half life of the charcoal at this temperature is: 2,800 days.

$$\text{at } r = (3/4)r_w = (3/4)(0.0259) = 0.01942 \text{ ft., } T = 162^\circ\text{F}$$

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2) Trial mean effective temperature:

The mean effective temperature is initially assumed to be 150°F . This is estimated as: 162° less one-half the expected temperature drop at $r = 0.01942$ ft over the 10 ft length. The mean effective temperature is easier to establish if 5 ft lengths are chosen for the increments instead of 10 ft since the mean effective temperature is then only slightly less than 162°F . About 155°F would be nearly correct for a five foot increment.

3) Composition of gas leaving the increment:

Using the trial mean effective temperature of 150°F , Equation 1 is now solved to obtain the concentration of each isotope at $z_2 = 10$ ft for this increment.

$$\ln \frac{(y_1)_2}{(y_1)_1} = \ln \frac{(p_1)_2}{(p_1)_1} = - \left[\left(\frac{\rho \delta \lambda_1}{G} \right) + \left(\frac{K_1 P M_1}{22,400} \right) \left(\frac{\lambda_1 - \rho_B}{G} \right) \right] \Delta z \quad (7)$$

The calculation will be illustrated for Kr 85m:

at 250 std cc/min in 1/2 inch pipe, $G = 5.56 \times 10^{-3}$ lbs/ft² sec.

for Kr 85m, $\lambda_1 = 4.41 \times 10^{-5}$ sec⁻¹.

$\rho_B = 25$ lbs/ft³

$\rho = 0.0719$ lbs/ft³ for the gas at 150°F .

K at 150°F for kryptons = 0.058 cc/g mm Hg (From Figure 5)

$\delta =$ fraction voids in the bed = 0.8

$P = 760$ mm Hg

For Kr 85m Equation 1 becomes:

$$\ln \left(\frac{p_1}{p_2} \right) = - \left[\frac{(4.41 \times 10^{-5})(0.0719)(0.8)}{(5.56 \times 10^{-3})} + \frac{(0.058)(760)(85)(4.41 \times 10^{-5})(25)}{(22,400)(5.56 \times 10^{-3})} \right] \quad (10)$$

$$\ln \frac{p_2}{p_1} = - (0.00046 + 0.03318)(10) = - 0.3364$$

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$$\frac{P_2}{P_1} = (e^{-0.3364}) = 0.7143$$

$$\text{Consequently, } \left[(\epsilon_i \lambda_i P_i) (10)^{25} \text{ at } z_2 \right] = (0.7143) \left[(\epsilon_i \lambda_i P_i) (10)^{25} \text{ at } z_1 \right] \\ = (0.7143)(0.059 \times 10^6) = 0.0422 \times 10^6$$

Repeating this calculation for the other beta emitters produces the following values for $\left[(\epsilon_i \lambda_i P_i) (10)^{25} \right]$ at z_2 :

	$\frac{(\epsilon_i \lambda_i P_i) (10)^{25}}{6}$	$\frac{(\epsilon_i \lambda_i P_i) (10)^{25}}{5}$
Kr 85 m	0.0422×10^6	Xe 133 0.01927×10^5
Kr 87	0.1087×10^6	Xe 135 1.293×10^5
Kr 88	1.584×10^6	
$u = \sum_{\text{Kr}} (\epsilon_i \lambda_i P_i) (10)^{25}$	$= 1.735 \times 10^6$	$v = \sum_{\text{Xe}} (\epsilon_i \lambda_i P_i) (10)^{25} = 1.312 \times 10^5$

4) Temperature distribution at downstream end of increment:

Equation 2 is now solved as before to obtain the temperature distribution at z_2 using the u and v values calculated in step 3. The results are:

$$\text{at } z_2 = \begin{aligned} a &= -0.01237 \\ b &= 15.20 \\ k &= -1.550 \times 10^{-3} \end{aligned}$$

so that at z_2 :

$$T = \left(\frac{1}{-0.01237} \right) \left[\ln \frac{(8) (-1.550 \times 10^{-3})}{(r^2 - 1.550 \times 10^{-3})^2 (-0.01237)} - 15.20 \right]$$

Solving for T at $r = 0$ and $r = 3/4 r_w$:

$$\text{when } r = 0, T = \underline{181^\circ \text{ F.}}$$

$$\text{when } r = (3/4)r_w = 0.01942 \text{ ft, } T = \underline{137^\circ \text{ F.}}$$

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5) Check assumed mean effective temperature:

$$\text{Calculated mean effective temperature} = 1/2(162^{\circ} + 137^{\circ}) = \underline{149^{\circ} \text{ F.}}$$

This is close enough to the assumed value so that the temperatures and concentrations at z_2 are correct. Equation 1 may be solved to give the concentration of any of the other isotopes present at z_2 . This process is then repeated for the next increment.

IV. Derivation and Discussion of the Equations

Adsorption Mechanism

The calculation of the limiting design is based on the performance of the charcoal after steady-state conditions have been attained in the bed. Under steady-state conditions, if the gases passing through the bed contained only stable isotopes, no concentration changes would occur at all and each isotope in the gas would be in equilibrium with that isotope on the surface of the solid. However, if radioactive isotopes are present these isotopes continuously decay both in the gas stream and on the solid surface. But in any small section of the adsorber the atoms which are on the charcoal surface have spent, on the average, a much longer time within the adsorber bed than those in the adjacent gas. Consequently, more of the atoms on the surface have decayed and the reduction of the surface concentration of this isotope caused by this decay will be much greater than the reduction of its concentration in the adjacent gas. The equilibrium between any radioactive isotope in the gas and on the solid surface has thus been disrupted by the greater concentration decrease on the surface. Atoms of this isotope then pass from the gas to the charcoal in an attempt to restore equilibrium conditions.

This mechanism assumes that the absorptive capacity of the charcoal for each isotope is not influenced in any way by the other isotopes present and that as soon as an atom of a given isotope disintegrates on the surface a site is at

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once opened to receive an undecayed atom of this isotope. These assumptions should be good, however, for gases containing low concentrations of radioactive isotopes and for solids with a high adsorptive capacity such as activated charcoal.

It is now necessary to express the Isotope Distribution Along the Adsorber Length at Constant Radius mechanism described above in mathematical terms. This will be done by examining a differential section of a cylindrical shell within the packed bed as shown below in Figure 3:

Figure 3

It will be assumed that mixing and diffusion in the gas are sufficient to cause its composition at any given cross section to be the same at all radii. However, due to the large radial temperature variation, the equilibrium constant for the adsorption and, consequently, the concentration of the isotopes on the solid surface will vary extremely with radial position. The nomenclature used is as follows:

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z = linear distance through the packed bed, ft.

r = radial distance from the center of the bed, ft.

$\bar{G}$ = mass rate of flow per unit cross sectional area of the non-radioactive or inert components of the gas stream in the z direction, lbs./sec ft².

$d\theta$ = time required for the inert components to travers dz , sec.

y_i = mole fraction of radioactive isotope i in the gas stream at any distance z . (assumed constant at all radii within the cross section at z .)

$\bar{y}_i = \frac{y_i}{1 - \sum y_i}$ = ratio of moles of radioactive isotope i to the moles of inert gas at z .

$G = \left(\frac{\bar{G}}{1 - \sum y_i} \right) \frac{M}{\bar{M}}$ = mass rate of flow of the total gas stream per unit cross sectional area at z , lbs/sec ft².

$\bar{M}$ = molecular weight of the inert gas. M = molecular weight of total gas.

A material balance will now be made on the radioactive isotope i in the gas stream within the element dz at radius z during the time $d\theta$ required for the inert componenets to traverse dz . This balance is:

$$\left[\begin{array}{l} \text{Total rate of depletion} \\ \text{of } i \text{ from the gas in } dz \\ \text{at radius } r, \\ \text{lb/moles/sec.} \end{array} \right] = \left[\begin{array}{l} \text{Rate of decay of } i \\ \text{in the gas within} \\ dz \text{ at radius } r, \\ \text{lb moles/sec.} \end{array} \right] + \left[\begin{array}{l} \text{Net rate of transfer} \\ \text{of } i \text{ from the gas to} \\ \text{the solid surface in} \\ dz \text{ at radius } r, \\ \text{lb moles/sec.} \end{array} \right]$$

$$- \left[\left(\frac{\bar{G}}{\bar{M}} \right) (2\pi r dr) \bar{y}_i \right] = \left[\left(\frac{G}{M} \right) (2\pi r dr) \lambda_i \bar{y}_i d\theta \right] + \left[k_g a P (2\pi r dr) (y_i - y_i^*) dz \right] \quad (7)$$

where $k_g a$ = mass transfer coefficient across gas film,
lb moles/(sec)(ft³ of bed)(mmHg)

P = total gas pressure, mm Hg

y_i^* = mole fraction of radioactive isotope i which would exist if the isotope in the gas were in equilibrium with the adsorbed isotope i on the solid surface within dz at radius r .

Dividing Equation 7 through by $(2\pi r dr)$ and multiplying by $(\bar{M}/\bar{G})$ produces:

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$$- d\bar{y}_i = \bar{y}_i \lambda_i d\theta + \left(\frac{k_g^a P \bar{M}}{\bar{G}} \right) (y_i - y_i^*) dz \quad (8)$$

However,
$$\left(\frac{k_g^a P \bar{M}}{\bar{G}} \right) = \left(\frac{1}{1 - \sum_i y_i} \right) \left(\frac{k_g^a P M}{G} \right) = \left(\frac{1}{1 - \sum y_i} \right) \left(\frac{1}{HTU} \right)$$

Since the total concentration of all radioactive isotopes in the gas is on the order of 1×10^{-4} mole fraction, the $(1 - \sum y_i)$ term may be considered equal to unity with negligible error, then:

$$y_i = \bar{y}_i; \quad G = \bar{G} \quad \text{and} \quad M = \bar{M}$$

Equation 8 then becomes:

$$-dy_i = y_i \lambda_i d\theta + \frac{1}{HTU} (y_i - y_i^*) dz \quad (9)$$

where HTU = the height of a transfer unit for the bed, ft.

Equilibrium relations estimated for xenon and krypton on a typical activated charcoal are shown in Figure 5. The equilibrium ratio is expressed as:

$$K_i = \frac{(\text{std cc of } i \text{ adsorbed on the charcoal})}{(\text{g of charcoal})(\text{mm Hg partial pressure of } i \text{ in gas})}$$

The y_i^* term in Equation 9 can therefore be related to the concentration of i on the surface within dz by:

$$y_i^* = \left(\frac{22,400}{K_i P M_i} \right) x_i \quad (10)$$

where: x_i = lbs adsorbed on charcoal/ lb charcoal, M_i = molecular weight of i .

In a similar manner a material balance may be written for the isotope i on the solid surface within dz at a distance r from the center during time $d\theta$:

$$\left[\begin{array}{l} \text{Total lbs of } i \\ \text{accumulated on surface} \\ \text{per lb of charcoal in} \\ \text{dz at radius } r \text{ during } d\theta \end{array} \right] = \left[\begin{array}{l} \text{lbs. of } i \text{ depleted due} \\ \text{to decay on the surface} \\ \text{per lb of charcoal in} \\ \text{dz at radius } r \text{ during } d\theta \end{array} \right] + \left[\begin{array}{l} \text{Net lbs. of } i \text{ transferred} \\ \text{to the surface per lb} \\ \text{of charcoal in } dz \text{ at} \\ \text{radius } r \text{ during } d\theta \end{array} \right]$$

$$\left[dx_i \right] = - \left[x_i \lambda_i d\theta \right] + \left[\frac{G}{\rho_B} \frac{1}{HTU} (y_i - y_i^*) d\theta \right] \quad (11)$$

where: ρ_B = density of charcoal in bed, lbs. charcoal/ft³ bed

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In these material balance equations, θ and z are related by:

$$d\theta = \left(\frac{\rho \delta}{G} \right) dz \quad (12)$$

where: ρ = density of gas in the element dz at r , lbs/ft³

δ = fraction of voids in the bed

Substituting Equation 12 in Equation 9 causes the two material balance equations 9 and 11 to become:

$$-\left(\frac{\partial y_i}{\partial z} \right)_r = \left[y_i \lambda_i \left(\frac{\rho \delta}{G} \right) + \frac{1}{HTU} (y_i - y_i^*) \right] \quad (13)$$

$$\left(\frac{\partial x_i}{\partial \theta} \right)_{r,z} = \left[-x_i \lambda_i + \left(\frac{G}{\rho_B} \right) \frac{1}{HTU} (y_i - y_i^*) \right] \quad (14)$$

Equations 13 and 14 describe the behavior of the bed at a fixed radius r for unsteady state conditions. If y_i^* is eliminated from these equations by means of Equation 10, two partial differential equations result whose simultaneous solution relates x_i and y_i at any position in the bed at any time. With the attainment of steady state conditions, however, for which the bed is to be designed in this problem, these two equations are considerably simplified since:

$$\left(\frac{\partial x_i}{\partial \theta} \right)_{r,z} = 0$$

Equation 14 then becomes:

$$(x_i \lambda_i) = \left(\frac{G}{\rho_B} \right) \frac{1}{HTU} (y_i - y_i^*) \quad (15)$$

Equation 15 means that at steady state conditions the net rate of mass transfer of i from the gas to the solid is exactly equal to the rate of disintegration of i on the solid surface. Rearranging 15 shows that:

$$\frac{1}{HTU} (y_i - y_i^*) = \left(\frac{\rho_B \lambda_i}{G} \right) x_i \quad (16)$$

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Substituting in 13:

$$-\left(\frac{\partial y_1}{\partial z}\right)_r = \left[y_1 \left(\frac{\rho_S \lambda_1}{G} \right) + \left(\frac{\rho_B \lambda_1}{G} \right) x_1 \right] \quad (17)$$

But substituting Equation 10 into Equation 15 shows that:

$$x_1 = \left(\frac{G}{\rho_B \lambda_1} \right) \left(\frac{1}{HTU} \right) \left[y_1 - \left(\frac{22,400}{K_1 P M_1} \right) x_1 \right]$$

Solving for x_1 produces:

$$x_1 = \left[\frac{y_1}{\left(\frac{\rho_B \lambda_1}{G} \right) \left(\frac{1}{HTU} \right) + \left(\frac{22,400}{K_1 P M_1} \right)} \right] \quad (18)$$

Equation 18 relates the y_1 and x_1 adjacent to each other in the bed at steady state operation. Substituting Equation 18 into Equation 17 produces:

$$-\frac{1}{y_1} \left(\frac{\partial y_1}{\partial z} \right)_r = \left[\left(\frac{\rho_S \lambda_1}{G} \right) + \frac{1}{HTU + \left(\frac{G}{\rho_B \lambda_1} \right) \left(\frac{22,400}{K_1 P M_1} \right)} \right] \quad (19)$$

Although the concentration of any isotope in the gas is constant at all radii at a given z value, Equation 19 indicates how the amount of this isotope removed from the gas per unit linear distance through the bed varies with the radial distance from the center. The terms in the right hand side of 19 are all very nearly constant with the exception of ρ and K_1 which are both functions of temperature. However, an approximate integration of 19 may be carried out when r is held constant as illustrated below in Figure 4:

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Figure 4

Figure 4 considers a cylindrical shell of radius r and thickness dr within a short length Δz of the packed adsorber. Although the variation of temperature with radius is quite large, temperature variations along the length of the bed at a fixed radius are relatively small. The temperature within the shell may then be considered constant at:

$$T = \left[\frac{(T_1)_r + (T_2)_r}{2} \right]$$

provided that Δz is not excessively long. $(T_1)_r$ is the temperature at r in the upstream cross section at z_1 and $(T_2)_r$ is the temperature at r in the downstream cross section at z_2 . Thus when Equation 19 is applied only to this differential shell, the right side of the equation is very nearly constant.

Integration of 19 within the shell then produces:

$$\ln \frac{(y_1')_2}{(y_1)_1} = - \left[\left(\frac{\rho \delta \lambda_1}{G} \right) + \frac{1}{(\text{HTU}) + \left(\frac{1}{\rho \lambda_1} \right) \left(\frac{22,400}{K_1 P M_1} \right)} \right] \Delta z \quad (20)$$

where K_1 and ρ are evaluated at the temperature $T = 1/2 \left[(T_1)_r + (T_2)_r \right]$. The term $(y_1')_2$ is the concentration which would exist at z_2 at a distance r from the center if the gas which travelled through the differential shell did not mix at all with the remainder of the gas in the bed at z_2 . That is, $(y_1')_2$

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represents the partial concentration of i at z_2 which is produced only by the charcoal at the distance r from the center of the increment.

The actual bulk concentration in the total gas stream at z_2 is represented by $(y_i)_2$ and can be determined by summing the partial concentrations $(y_i')_2$ at all radii in the cross section at z_2 . This is done by:

$$(y_i)_2 = \left[\frac{\int_0^{r_w} (y_i')_2 \rho \delta 2 \pi r dr}{\int_0^{r_w} \rho \delta 2 \pi r dr} \right]_{z=z_2} = \left[\frac{\int_0^{r_w} (y_i')_2 \rho r dr}{\int_0^{r_w} \rho r dr} \right]_{z=z_2} \quad (21)$$

Since $\rho \propto \left(\frac{1}{T}\right)$:

$$(y_i)_2 = \left[\frac{\int_0^{r_w} (y_i')_2 \left(\frac{r}{T}\right) dr}{\int_0^{r_w} \left(\frac{r}{T}\right) dr} \right]_{z=z_2} \quad (22)$$

where $(y_i)_2$ is the overall concentration of i in the entire gas stream at z_2 .

Thus the concentration of any radioactive isotope i in the gas stream leaving a short increment of the bed may be determined by means of Equation 22 with the $(y_i')_2$ term evaluated from Equation 20. In order to solve Equation 20 for $(y_i')_2$, however, a value must be obtained for the height of a transfer unit, HTU, in the bed.

Height of the
Transfer Unit

Although no direct experimental HTU values were available for activated charcoal, it was assumed that these values were of the same order of magnitude as the HTU values for various gases absorbed on silica gel with roughly the same particle size. If it is assumed that gas film resistance is primarily controlling, the agreement should be good. Gamson, Thodos, and Hougen⁽³⁾ report for the absorption of gases on silica gel particles:

$$HTU = \left(\frac{0.55}{a_v} \right) \left(\frac{D_p G}{\mu} \right)^{0.51} \left(\frac{\mu}{\rho D_v} \right)^{0.67} \quad (23)$$

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HTU = height of a transfer unit, ft.

a_v = a function of the external surface area of the particles.
The value is approximately 300 for 0.1 inch diameter particles as used here.

G = mass flow rate of gas stream, lbs./sec ft²

D_p = average particle diameter, ft. The particles used are about 0.1 inch in diameter.

μ = gas viscosity, lbs/ft sec.

D_v = diffusivity of transferring gas through carrier gas, ft²/sec.

ρ = gas density, lbs./ft³

As a consequence of the extremely low isotope concentration in the gas, it is assumed that the diffusivities and the resulting HTU values for each isotope are unaffected by the other isotopes present. Diffusivities of xenon through oxygen and krypton through oxygen were calculated by means of the Hirschfelder, Bird, Spotz equation (4). At the extremely low flow rates in this problem values of the HTU calculated for both xenons and kryptons were from 100 to 10,000 times smaller than the values of the $\left(\frac{G}{\rho_B \lambda_i}\right) \left(\frac{22,400}{K_i P M_i}\right)$ term in the right side of Equation 19 and 20. Consequently, Equation 20 becomes very nearly:

$$\ln \frac{(y_i')_2}{(y_i')_1} = - \left[\left(\frac{\rho \delta \lambda_i}{G} \right) + \left(\frac{K_i P M_i}{22,400} \right) \left(\frac{\rho_B \lambda_i}{G} \right) \right] \Delta z \quad (24)$$

This simplification means in effect that the time required for atoms of an isotope to travel from the gas to the solid surface is extremely short in comparison with the time required for these atoms to disintegrate. This is true in this case because the more rapidly decaying isotopes have already been removed before entering the charcoal and because the extremely low gas flow favors a rapid mass transfer rate and small HTU value, as shown in Equation 23.

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Radial Temperature Distribution

Before Equation 22 can be integrated it is necessary to evaluate the temperature as a function of the radius at a fixed cross section. This is done by making an energy balance on the differential element shown in Figure 3. It is assumed first that the heat transferred along the length of the bed is negligible in comparison with the heat transferred radially. This assumption should be good as a result of the low specific heat and low flow rate of the gas stream. Secondly, it is assumed that all the beta energy released by the isotopes within the element is absorbed instantaneously at the point of its emission. This is undoubtedly incorrect, although the path of the beta particles in the bed should be short and the assumption tends to accentuate the variation of temperature with the radius, thus giving higher calculated temperatures at the center which favor a slightly conservative design. Finally, it is assumed that none of the gamma energy is absorbed in the bed. This might perhaps compensate for the assumption that all beta energy is absorbed.

With these assumptions the net rate of heat removal from the element is equal to:

$$- k_B \left(\frac{\partial T}{\partial r} \right) 2 \pi dr dz - k_B \left(\frac{\partial^2 T}{\partial r^2} \right) 2 \pi r dr dz \quad (25)$$

where k_B is the effective conductivity of the bed, Btu/hr ft² °F/ft.

At steady state conditions this rate of heat removal must equal the rate of release of beta energy within the element, which is determined as follows:

Using Equation 18, the number of adsorbed atoms of beta emitters per gram of charcoal in the bed equals:

$$\sum_i \left(\frac{N}{M_1} \right) x_i = \sum_i \frac{N}{M_1} \left[\frac{y_i}{\left(\frac{\rho_B \lambda_i}{G} \right) (HTU) + \frac{22,400}{K_1 P M_1}} \right] \approx \sum_i \frac{N y_i K_1 P}{22,400}$$

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where N = Avogadro's number and the summation is over all beta emitters.

Number of adsorbed atoms of beta emitters in the whole element =

$$\sum_i \left[\frac{N y_i K_i P (454)}{22,400} \right] \rho_B 2\pi r dr dz$$

where ρ_B = density of charcoal in the bed, lbs/ft³

Rate of beta energy release from adsorbed atoms in the element, Btu/hr =

$$(3.415) \sum_i \left[\frac{N y_i K_i P (454)}{22,400} \right] (\epsilon_i \lambda_i) \rho_B 2\pi r dr dz$$

In a similar manner, the number of atoms of beta emitters unadsorbed in the gas within the element =

$$\sum_i N \left[\frac{P \delta}{R(T+460)} \right] y_i 2\pi r dr dz$$

And the rate of beta energy release from the unadsorbed atoms in Btu/hr is:

$$(3.415) \sum_i N \left[\frac{P \delta}{R(T+460)} \right] (y_i \epsilon_i \lambda_i) 2\pi r dr dz$$

Equating the rate of heat removal to the total rate of heat generation due to beta energy release in the element produces:

$$-k_B \left(\frac{\partial T}{\partial r} \right) (2\pi r dr dz) - k_B \left(\frac{\partial^2 T}{\partial r^2} \right) (2\pi r dr dz) = (3.415) \sum_i N \left[\frac{\delta}{R(T+460)} + \frac{454 K_i \rho_B}{22,400} \right] (P y_i \lambda_i \epsilon_i) (2\pi r dr dz)$$

Dividing by $2\pi r dr dz$ and rearranging gives:

$$\left(\frac{\partial^2 T}{\partial r^2} \right) + \frac{1}{r} \left(\frac{\partial T}{\partial r} \right) + (3.415) \sum_i \frac{N}{k_B} \left[\frac{\delta}{R(T+460)} + \frac{(454) K_i \rho_B}{22,400} \right] (P y_i \lambda_i \epsilon_i) = 0 \quad (26)$$

The nomenclature and units are:

T = temperature, °F.

r = radius, ft.

N = Avogadro's number, 6.06×10^{23} atoms/ g mole

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k_B = effective bed conductivity, taken as 0.03 Btu/hr ft² °F/ft.

R = gas constant = 1.221 mm Hg ft³/ g mole ° R

δ = fraction of voids in the bed = 0.8

ρ_B = density of charcoal in bed, taken as 25 lbs/ft³

$P_i = P_{y_i}$ = partial pressure of isotope i in gas at z , mm Hg

λ_i = decay constant for isotope i , sec⁻¹

ϵ_i = energy release per beta emission from an atom of i , watt sec/atom.

K_i = adsorption equilibrium ratio for i , std cc/(g charcoal)(mm Hg.)

The summation term in Equation 26 can be represented very accurately by an exponential temperature function so that Equation 26 becomes:

$$\left(\frac{\partial^2 T}{\partial r^2} \right) + \frac{1}{r} \left(\frac{\partial T}{\partial r} \right) + e^{(aT + b)} = 0 \quad (27)$$

The solution to Equation 27 is the relation given in Equation 2:

$$T = \frac{1}{a} \left[\ln \frac{8k}{(r^2 + k)^2 a} - b \right] \quad (2)$$

where a and b are respectively the slope and intercept of the straight line which results when the logarithm of the summation term in Equation 26 is plotted against T at a given z value. The k term is an arbitrary constant whose value is determined by substituting a known temperature at a single radius into Equation 2. Analytical expressions for a , b , and k are given for this problem in the discussion following Equation 2 in Part II.

Solution of The Equations

The equations which enable the calculation of the concentration of any isotope i in the total gas stream leaving the short length of bed Δz as shown in Figure 4, are summarized below:

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- 1) Isotope depletion at a fixed radius along Δz :

$$\ln \frac{(y_1')_2}{(y_1)_1} = - \left[\left(\frac{\rho S \lambda_1}{G} \right) + \frac{K_1 P M_1}{22,400} \left(\frac{\rho_{B \lambda_1}}{G} \right) \right] \Delta z \quad (24)$$

- 2) Radial temperature distribution in the cross section at a fixed z :

$$T = \frac{1}{a} \left[\ln \frac{8k}{(r^2 + k)^2 a} - b \right] \quad (2)$$

- 3) Relation of isotope depletion at a fixed radius to its overall concentration in the total gas stream leaving Δz :

$$(y_1)_2 = \left[\frac{\int_0^{r_w} (y_1')_2 \left(\frac{r}{r_w} \right) dr}{\int_0^{r_w} \left(\frac{r}{r_w} \right) dr} \right]_{z=z_2} \quad (22)$$

The exact solution of these three equations would be to begin with a short increment at the beginning of the bed, where the gas composition entering the increment is the known gas composition at the entrance to the entire bed, and proceed as follows:

1) Assume a set of values for $(y_1)_2$ the concentration of each isotope in the total gas stream leaving the increment.

2) Using these values, the functions a , b , and k are calculated and by solving Equation 2 a plot is made of temperature vs. radius at z_2 . The known concentrations of all isotopes entering the increment are then used in the same manner to solve Equation 2 and produce a plot of temperature vs. radius at z_1 .

3) At various radii the arithmetic average of the temperatures at each end of the increment is determined from the temperature vs. radius curves at z_1 and z_2 . Using the calculated average temperatures to establish K_1 and ρ values at these various radii, a value for the partial concentration $(y_1')_2$ is determined for each isotope at different radii in the cross section at z_2 by means of Equation 24.

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4) At each radius in the cross section at z_2 this procedure has now obtained a temperature from Equation 2 and a partial concentration $(y_1')_2$ for each isotope from Equation 24. Using these values, Equation 22 is integrated graphically to determine a calculated value of the total concentration $(y_1)_2$ for each isotope in the total gas stream leaving the increment. This whole procedure is then repeated until this calculated concentration for each isotope from Equation 22 agrees with the initially assumed value.

The Mean Effective Temperature

Although the calculation procedure outlined above illustrates the performance of the equations derived, solving them in this manner would require an impossible amount of work. It is necessary to use an approximate procedure which assigns an average temperature to the entire increment. The correct mean effective temperature which may be considered to apply to the entire increment is that temperature which, when used to specify K_1 and ρ in Equation 24, produces values of $(y_1')_2$ for each isotope which are numerically equal to the correct values of the total concentrations $(y_1)_2$ for these isotopes in the total gas stream at z_2 .

Although it is not always possible to find one single temperature which will produce exact equality between $(y_1')_2$ and $(y_1)_2$ for all combinations of any isotopes, a temperature can be found which will approximate this equality if one or two isotopes make a much greater contribution to the heat load in the bed than all others. In this problem, for example, by far the greatest single contributor to the heat load is the Kr 88 as a result of the intense beta activity of its Rb 88 daughter. The temperature distribution is influenced more by the Kr 88 concentration than by any other single isotope present. Consequently, the mean effective temperature for each increment in this problem was established as follows:

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A trial mean effective temperature was assumed for the first increment at the beginning of the bed. This trial temperature can be checked since it permits the solution of the three Equations 24, 2 and 22 to be approximated by means of the following modification of the steps which were outlined for their exact solution:

- 1) Instead of assuming $(y_1)_2$ values for each isotope, a set of values for $(y_1)_2$ was calculated by replacing $(y_1')_2$ with $(y_1)_2$ in Equation 24. The equation was then solved for $(y_1)_2$ for each isotope with K_1 and ρ evaluated at the assumed mean effective temperature.
- 2) This step is unchanged and plots of T vs. r from Equation 2 were drawn for each end of the increment as before.
- 3) This step is likewise unchanged and the partial concentration $(y_1')_2$ for each isotope at various radii at z_2 were determined from Equation 24.
- 4) Equation 22 was integrated graphically in this case only for the Kr 88 isotope and the whole process was repeated with new trial mean effective temperatures until the value obtained for the total exit concentration of Kr 88 by integration of Equation 22 agreed with the exit Kr 88 concentration calculated in step 1.

In integrating Equation 22 for other isotopes present, it was observed that the mean effective temperature which proved correct for Kr 88 was also very nearly correct for Xe 135, the next largest contributor to the heat load in the bed. Thus it was possible to determine a fairly accurate mean effective temperature to fit all the krypton and xenon isotopes of this problem.

The next step was to locate the position of that temperature in the bed which is numerically equal to the mean effective temperature. It was found that the radius at which the proper mean effective temperature is the arithmetic average of the temperatures at each end of the increment at this radius, is

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very nearly three-fourths of the maximum radius, regardless of the diameter or location of the increment along the bed. Consequently, the mean effective temperature was defined as given in Equation 3:

$$T_m = 1/2 \left[(T_2)_{r=3/4 r_w} + (T_1)_{r=3/4 r_w} \right] \quad (3)$$

Using this temperature to define K_i and ρ in Equation 24 causes $(y_i')_2$ in the equation to be numerically equal to the total concentration $(y_i)_2$ at z_2 , so that Equation 24 may be written as in Equation 1:

$$\ln \frac{(y_i)_2}{(y_i)_1} = - \left[\left(\frac{\lambda_i \rho \delta}{G} \right) + \left(\frac{\rho_B \lambda_i}{G} \right) \left(\frac{K_i P M_i}{22,400} \right) \right]_{T=T_m} \Delta z \quad (1)$$

With the mean effective temperature thus located it is no longer necessary to integrate Equation 22 for each increment and the solution procedure is that outlined in part II.

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Fig. 5. Equilibrium Distribution Ratios for the Adsorption of K_r and X_e on Charcoal
DATA FROM HOLMES, J.M. (5)

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Fig. 6. Effect of Temperature on Rate of Oxidation of Charcoal

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VI. Table of Nomenclature:

English Letters

- A = surface area of cylindrical section of adsorber bed, ft^2
- a, b = functions of the effective conductivity of the bed and of the concentration and activity of the isotopes in the gas
- a_v = function of the external surface area of charcoal particles
- D_p = average particle diameter, ft.
- D_v = diffusivity of a transferring isotope through the carrier gas, ft^2/sec .
- d = differential operator
- e = base of natural logarithms
- G = mass rate of flow of the total gas stream per unit cross sectional area of the packed bed, $\text{lbs}/\text{sec ft}^2$.
- $\bar{G}$ = mass rate of flow of the inert or non-radioactive components of the gas stream per unit cross sectional area of the bed, $\text{lbs}/\text{sec ft}^2$.
- HTU = height of a transfer unit, ft.
- h_c = heat transfer coefficient for natural convection, $\text{Btu}/\text{hr ft}^2 \text{ } ^\circ\text{F}$.
- K = equilibrium distribution ratio,
$$\left[\frac{(\text{std cc isotope adsorbed on surface})}{(g \text{ charcoal}) (\text{mm Hg partial pressure of isotope})} \right]$$
- k = a constant
- k_p = thermal conductivity of the metal in the pipe wall, $\text{Btu}/\text{hr ft}^2 \text{ } ^\circ\text{F}/\text{ft}$.
- k_B = effective thermal conductivity of the packed bed under operating conditions, $\text{Btu}/\text{hr ft}^2 \text{ } ^\circ\text{F}/\text{ft}$.
- k_{ga} = mass transfer coefficient across gas film, $\text{lb moles}/\text{sec (mm Hg) (ft}^3 \text{ bed)}$
- M = molecular weight
- N = Avogadro's number
- P = total pressure of the gas in the bed, mm Hg
- p = partial pressure of a given isotope in the gas, mm Hg
- q = rate of heat transfer, Btu/hr .

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- R = gas constant, (mm Hg)(ft³)/(g mole)(°R)
- r = radial distance from the center of the packed bed, ft.
- r_w = radius of pipe at the inner wall, ft
- T = Temperature, °F
- t = pipe wall thickness, ft.
- x = concentration of an isotope adsorbed on surface of charcoal, lbs/lb charcoal.
- y = mole fraction of a given isotope in the total gas stream at a given cross section in the bed.
- y' = hypothetical mole fraction of an isotope produced in the gas only by the charcoal at a constant radial distance from the center of the bed
- $\bar{y}$ = ratio of the moles of an isotope in the total gas stream to the moles of inert carrier gas
- y^* = mole fraction of a given isotope in the gas stream which would exist if the gas were in equilibrium with the adjacent charcoal surface.
- z = linear distance through the packed bed, ft.

Greek Letters

- α = a function of the pipe wall temperature
- β = beta particle
- γ = gamma particle
- Δ = difference operator
- δ = fraction of voids in the packed bed
- ϵ = average energy of the beta particle at time of its emission, watt sec.
- λ = decay constant for a given isotope, sec⁻¹
- μ = absolute viscosity, lbs/ft sec.
- π = 3.1416

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- ρ = gas density, lbs/ft³ gas
 ρ_B = density of charcoal in the packed bed, lbs/ft³ bed
 Σ = summation operator

Subscripts

- 1 = cross section at the upstream end of an increment of the packed bed
 2 = cross section at the downstream end of an increment of the packed bed
 B = charcoal bed
 i = a particular radioactive isotope
 m = arithmetic mean value
 r = a particular radial distance from the center of the packed bed
 z = a particular linear distance from the beginning of the packed bed

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