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A Semi-Empirical Equation for Pair Production
Below 5 Mev

C. D. Zerby and H. S. Moran

It is well known that there exists a discrepancy between experimental and theoretical Born approximation calculations of the cross section for production of electron-positron pairs by gamma rays. The ratio of the cross section determined by experiment to that given by the Born approximation is greater than one at low gamma-ray energy and less than one at high gamma-ray energy. The crossover occurs at approximately 6.5 Mev as determined from the experiments of Rosenblum et al.¹

The discrepancy between experiment and theory has been reconciled in part by the work of Bethe and Maximon² who determined the differential cross section at high energy using wave functions that closely approximate the correct wave functions. The integral cross section given by Davies, Bethe, and Maximon³ compares quite well with experimental values above 88 Mev.

In the region below 88 Mev the solution of the wave equation using the correct wave functions has been done numerically for a few cases by Jaeger and Hulme.⁴⁻⁶ Their results have been compared with experiment by

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1. E. S. Rosenblum, E. F. Shroder, and R. M. Warner, Jr., Phys. Rev. 88, 612 (1952).
 2. H. A. Bethe and L. C. Maximon (Part I), Phys. Rev. 93, 768 (1954).
 3. H. Davies, H. A. Bethe, and L. C. Maximon, (Part II) Phys. Rev. 93, 788 (1954).
 4. J. C. Jaeger and H. R. Hulme, Proc. Roy. Soc. (London) A153, 443 (1936).
 5. J. C. Jaeger, Nature 137, 781 (1936).
 6. J. C. Jaeger, Nature 148, 86 (1941).

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several authors^{1,7,8} with good agreement. However, it is not easy to interpolate or extrapolate the numerical results to obtain the cross section for an arbitrary energy and element with charge Z . This has lead several authors to suggest an empirical correction to the Bethe-Heitler (Born approximation) equation⁹ based on experimental results.

The empirical equations suggested by Dayton⁷ to take care of the z dependence were

$$\sigma(\text{corrected}) = \sigma(\text{Born}) \times (1 + 1.59 \times 10^{-4} Z^2) \text{ at } 1.33 \text{ Mev} \quad (1)$$

$$\sigma(\text{corrected}) = \sigma(\text{Born}) \times (1 + 3.39 \times 10^{-5} Z^2) \text{ at } 2.62 \text{ Mev} \quad (2)$$

where σ is the cross section for pair production in the field of the nucleus and Z is the atomic charge. The empirical dependence on energy can be obtained with the use of Eqs. (1) and (2) and the additional information that Bethe-Heitler equation matches experiment at approximately 6.5 Mev.¹ The proposed empirical equation becomes

$$\sigma(\text{corrected}) = \sigma(\text{Born}) \times (1 + 7.824 \times 10^{-4} e^{-0.612 k} Z) \quad (3)$$

$$\text{for } 2 < k < 12$$

where k is the photon energy in units of m_0c^2 . The inequality limiting the upper energy bound of Eq. (3) comes from the exponential energy correction which does not go to zero at 6.5 Mev. The lower energy bound is not set equal to the threshold energy of $2 m_0c^2$ because of the uncertainty that

7. I. E. Dayton, Phys. Rev. 89, 544 (1953).

8. H. I. West, Jr., Phys. Rev. 101, 915 (1956).

9. W. Heitler, The Quantum Theory of Radiation (Oxford University Press, 1944) p. 196.

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such a simple empirical correction factor will apply. At 1.17 Mev West⁸ indicated a correction term for the Z dependence such as given in Eqs. (1) and (2) could not be made to apply to the available data. It should be pointed out, however, that the pair-production cross section is only a small percentage of the total cross section in the energy range of 1.17 Mev and below so the use of Eq. (3) down to an energy of $2 m_0 c^2$ will introduce little error to the total cross section.

A simplified approximation to the Bethe-Heitler differential cross section has been given by Hough¹⁰ which is repeated here with the empirical correction term given in Eq. (3). The limits quoted on the equations are restricted by the empirical correction terms and do not necessarily indicate the restriction of application of Hough's approximation. The simplified differential cross section is

$$\frac{d\sigma}{dE_+} = \bar{\phi} Q y \left[1 + 0.135(Q - 0.52)y(1 - y^2) \right] (k - 2)^{-1} \beta \quad \text{for } 2 < k < 12 \quad (4)$$

where the second term in the bracket is dropped when it becomes negative (below $k = 4.2$), and

$$\beta = (1 + 7.824 \times 10^{-4} e^{-0.612 k} Z^2)$$

$$\sigma = \text{cross section (cm}^2/\text{atom)}$$

$$E_+ = \text{energy of the positron on } m_0 c^2 \text{ units}$$

$$x = (E_+ - 1)/(k - 2)$$

$$y = 2 \left[x(1 - x) \right]^{1/2}$$

$$\bar{\phi} = z^2/137 (e^2/m_0 c^2)^2$$

10. P. V. C. Hough, Phys. Rev. 73, 266 (1948).

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The value of Q is given by

$$Q = (1 - \gamma) \left[\frac{1}{3} (4 - \gamma^2)(L - 1) - \gamma^2 \alpha (\alpha - 1) - \gamma^4 \alpha (L - \alpha) \right] \quad (5)$$

where

$$\gamma = \frac{2}{k}$$

$$L = \left[\frac{2}{(1 - \gamma^2)} \ln \left(\frac{k}{2} \right) \right]$$

$$\alpha = \frac{1}{(1 - \gamma)^{1/2}} \ln \left[\left(\frac{k}{2} \right) + \left(\frac{k^2}{4} - 1 \right)^{1/2} \right]$$

Integration of Eq. (4) over the range of E_+ from 1 to $k - 1$ gives the total cross section

$$\sigma (\text{corrected}) = \bar{\Phi} \left[0.776Q + 0.018Q^2 \right] \beta \quad \text{for } 4.2 \leq k < 12 \quad (6)$$

$$\sigma (\text{corrected}) = 0.785 \bar{\Phi} Q \beta \quad \text{for } 2 < k \leq 4.2 \quad (7)$$

Equations (4), (6), and (7) apply for pair production in the field of the nucleus and do not include the pair production in the field of the electrons or the effects of screening.

The pair production in the field of the nucleus as determined from Eqs. (6) and (7) plus the pair production in the field of the electrons as given by Borsellino¹¹ is presented in Table 1 for several elements. A comparison of Eqs. (6) and (7) against the experimental data of Rosenblum et al,¹ Dayton,⁷ Jaeger et al,^{4,5,6} and Colgate¹² is given in Figs. 1 through 3.

11. A. Borsellino, Nuovo cimento, 4, 112 (1947).

12. S. A. Colgate, Phys. Rev. 87, 592 (1952).

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Table 1. Pair Production Cross Sections (barns per atom)

Z	1	4	6	7	8	11	13
E (Mev)	H	Be	C	N	O	Na	Al
1.1	4.48×10^{-7}	7.19×10^{-6}	1.62×10^{-5}	2.22×10^{-5}	2.91×10^{-5}	5.56×10^{-5}	7.84×10^{-5}
1.2	4.17×10^{-6}	6.70×10^{-5}	1.51×10^{-4}	2.06×10^{-4}	2.70×10^{-4}	5.16×10^{-4}	7.27×10^{-4}
1.3	1.28×10^{-5}	2.05×10^{-4}	4.63×10^{-4}	6.31×10^{-4}	8.26×10^{-4}	1.58×10^{-3}	2.22×10^{-3}
1.4	2.63×10^{-5}	4.22×10^{-4}	9.51×10^{-4}	1.30×10^{-3}	1.70×10^{-3}	3.24×10^{-3}	4.55×10^{-3}
1.5	4.42×10^{-5}	7.08×10^{-4}	1.60×10^{-3}	2.18×10^{-3}	2.85×10^{-3}	5.43×10^{-3}	7.63×10^{-3}
1.75	1.03×10^{-4}	1.66×10^{-3}	3.73×10^{-3}	5.09×10^{-3}	6.66×10^{-3}	1.27×10^{-2}	1.78×10^{-2}
2	1.76×10^{-4}	2.82×10^{-3}	6.35×10^{-3}	8.65×10^{-3}	1.13×10^{-2}	2.15×10^{-2}	3.01×10^{-2}
3	5.60×10^{-4}	8.31×10^{-3}	1.85×10^{-2}	2.52×10^{-2}	3.28×10^{-2}	6.20×10^{-2}	8.65×10^{-2}
4	1.00×10^{-3}	1.39×10^{-2}	3.07×10^{-2}	4.16×10^{-2}	5.41×10^{-2}	1.02×10^{-1}	1.41×10^{-1}
5	1.44×10^{-3}	1.90×10^{-2}	4.18×10^{-2}	5.64×10^{-2}	7.33×10^{-2}	1.37×10^{-1}	1.90×10^{-1}

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Table 1 (cont.)

Z	14	18	20	26	29	42	50
E (MeV)	Si	A	Ca	Fe	Cu	Mo	Sn
1.1	9.14×10^{-5}	1.55×10^{-4}	1.94×10^{-4}	3.46×10^{-4}	4.43×10^{-4}	1.08×10^{-3}	1.71×10^{-3}
1.2	8.48×10^{-4}	1.43×10^{-3}	1.79×10^{-3}	3.18×10^{-3}	4.06×10^{-3}	9.77×10^{-3}	1.53×10^{-2}
1.3	2.59×10^{-3}	4.36×10^{-3}	5.45×10^{-3}	9.60×10^{-3}	1.22×10^{-2}	2.91×10^{-2}	4.51×10^{-2}
1.4	5.30×10^{-3}	8.92×10^{-3}	1.11×10^{-2}	1.95×10^{-2}	2.48×10^{-2}	5.83×10^{-2}	8.98×10^{-2}
1.5	8.88×10^{-3}	1.49×10^{-2}	1.86×10^{-2}	3.25×10^{-2}	4.12×10^{-2}	9.57×10^{-2}	1.46×10^{-1}
1.75	2.06×10^{-2}	3.45×10^{-2}	4.29×10^{-2}	7.44×10^{-2}	9.40×10^{-2}	2.13×10^{-1}	3.21×10^{-1}
2	3.49×10^{-2}	5.83×10^{-2}	7.23×10^{-2}	1.25×10^{-1}	1.57×10^{-1}	3.49×10^{-1}	5.18×10^{-1}
3	1.00×10^{-1}	1.66×10^{-1}	2.05×10^{-1}	3.47×10^{-1}	4.35×10^{-1}	9.28×10^{-1}	1.33 ---
4	1.64×10^{-1}	2.70×10^{-1}	3.34×10^{-1}	5.63×10^{-1}	7.00×10^{-1}	1.48 ---	2.10 ---
5	2.21×10^{-1}	3.63×10^{-1}	4.48×10^{-1}	7.55×10^{-1}	9.38×10^{-1}	1.96 ---	2.75 ---

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Table 1 (cont.)

Z	53	74	78	82	92
E (Mev)	I	W	Pt	Pb	U
1.1	2.00×10^{-3}	5.27×10^{-3}	6.20×10^{-3}	7.25×10^{-3}	1.05×10^{-2}
1.2	1.78×10^{-2}	4.61×10^{-2}	5.41×10^{-2}	6.31×10^{-2}	9.09×10^{-2}
1.3	5.25×10^{-2}	1.33×10^{-1}	1.56×10^{-1}	1.81×10^{-1}	2.59×10^{-1}
1.4	1.04×10^{-1}	2.59×10^{-1}	3.02×10^{-1}	3.51×10^{-1}	4.98×10^{-1}
1.5	1.69×10^{-1}	4.14×10^{-1}	4.81×10^{-1}	5.56×10^{-1}	7.84×10^{-1}
1.75	3.69×10^{-1}	8.64×10^{-1}	9.97×10^{-1}	1.14 ---	1.59 ---
2	5.93×10^{-1}	1.34 ---	1.53 ---	1.75 ---	2.39 ---
3	1.51 ---	3.10 ---	3.48 ---	3.89 ---	5.06 ---
4	2.36 ---	4.67 ---	5.21 ---	5.78 ---	7.36 ---
5	3.10 ---	6.08 ---	6.75 ---	7.49 ---	9.42 ---

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Fig. 1. Comparison of Experimental and Calculated Pair Production in the Field of the Nucleus for Copper ($Z=29$).

Fig. 2. Comparison of Experimental and Calculated Pair Production in the Field of the Nucleus for Tin ($Z = 50$).

Fig 3. Comparison of Experimental and Calculated Pair Production in the Field of the Nucleus for Lead ($Z = 82$).

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Values of the pair-production cross section above the range of application of Eq. (7) can be obtained from the tables provided by White.¹³

13. G. R. White, X-Ray Attenuation Coefficients (National Bureau of Standards report) report number NBS-1003 (1952).

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