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RDT F 9-5T

Supersedes
RDT F 9-5T, March 1974

RDT Standard

GUIDELINES AND PROCEDURES FOR
DESIGN OF NUCLEAR SYSTEM
COMPONENTS AT ELEVATED
TEMPERATURE

SEPTEMBER 1974

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FOREWARD

This standard supersedes the March 1974 issue of RDT F 9-5T and incorporates those changes to that issue of the standard that were approved for publication in this revision. These changes are identified by the following marginal notations:

C - Change approved September 1974

D - Deletion approved September 1974

Section 6.3 was deleted.

Section 6.6 was deleted.

Sections 6.4 and 6.5 were renumbered appropriately.

Editorial changes that were made during preparation of this revision are not identified.

RDT STANDARD

UNITED STATES ATOMIC ENERGY COMMISSION
DIVISION OF REACTOR RESEARCH AND DEVELOPMENT

RDT F 9-5T
DATE September 1974
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GUIDELINES AND PROCEDURES FOR DESIGN OF NUCLEAR SYSTEM COMPONENTS AT ELEVATED TEMPERATURE

1. INTRODUCTION

1.1 Scope. This standard provides guidelines and procedures which may be used by the manufacturer in satisfying the requirements given in RDT F 9-4. Guidance is given regarding planning and control of design analysis. A sequence for calculations is recommended. Methods of analysis, including procedures to account for environmental effects, are given which are acceptable in principle to the owner. A format is provided for use in documentation of design analyses.

1.2 Applicability. The guidelines and procedures of this standard are nonmandatory.

1.3 Organization. Certain subsections provide guidelines and methods which correspond to requirements stated in RDT F 9-4; however, this standard is topically organized and related considerations such as environmental effects are presented in separate subsections. Guidance on these related considerations can be found using the table of contents. Identification of specific relevant guidance is also given in the flow charts of Section 3.

2. STRUCTURAL EVALUATION PROGRAM

The design and construction of Class 1 nuclear components, parts, and appurtenances to meet the high temperature limits of RDT F 9-4 may require sophisticated and lengthy analysis and test procedures for which few precedents exist. To facilitate interaction between the manufacturer and owner, and assure efficient performance of the design analysis, the manufacturer is required to formulate a detailed structural evaluation program. The purposes of the structural program are to:

1. Assure adequate planning and effective management of the design, analysis, and structural evaluation effort by the manufacturer.
2. Identify major analysis and test programs, how they are related to the overall structural evaluation program and time-tables for completion.
3. Provide for communications with the owner, both formal and informal, to keep the owner informed of project status at all times, including a final formal report of design justification.

As specified in N-1200, RDT F 9-4, the structural evaluation program shall include two types of documentation: a Structural Evaluation Plan (SEP) to be provided as part of the proposal and updated during the course of the analysis, and preliminary and final stress reports.* Details concerning the contents of the structural evaluation program are provided in the next two sections. |C

2.1 Structural Evaluation Plan. The purpose of the SEP is to provide an overall plan for the management and performance of a structural design analysis and evaluation. The initial SEP submitted with the proposal delineates the general scope and level of effort expected by the manufacturer. The SEP should be updated and submitted as more detailed engineering and scheduling information becomes available. The amount of detail included in the SEP should be commensurate with the size and complexity of the task to be performed. A typical SEP might include the following information:

2.1.1 Identification of Major Design Milestones. The SEP should identify plans for providing the design evaluation in the detail and at the times required to meet design, procurement, and fabrication schedules. A logical sequence of events should be identified in the form of design milestones with specific requirements for each milestone. A typical set of design milestones might include the following major events.

a. Basic Design Configuration. Sufficient analysis is required at an early stage in development to establish basic shapes and sizes for interface with design of other components. Critical dimensions and methods to be used in their establishment should be identified.

b. Order of Long Lead Items. Initiation of procurement of certain long lead items, such as forgings, may be required well in advanced of completion of the detailed structural evaluation. The long lead items and analysis required in support of procurement action should be identified.

c. Release for Fabrication. The minimum level of evaluation necessary to support fabrication release should be established. The schedule for the evaluation effort and report submittal should be identified.

*Stress reports are defined as in Section III of the ASME Boiler and Pressure Vessel Code (herein referred to as the ASME Code), Appendix C, "Manufacturer's Stress Report," modified to include additional analysis which may be required in RDT F 9-4 or to define applicable loading conditions.

d. Final Design Confirmation. An outline of work items to be completed and included in the final stress report should be included. A typical format for the final stress report is given in 2.2.

2.1.2 Responsibility for the Structural Evaluation Program. The organization that will be responsible for management of structural analysis and evaluation shall be clearly identified by an organization chart or other organizational directives. Technical and organization interaction between the various support groups involved - materials, thermal and hydraulics, stress analysis, testing, etc. - should be identified, and provision made for coordinating their effort toward meeting overall objectives. An individual should be identified by name or position title to act as a contact point between the manufacturer and owner on all matters pertaining to the SEP.

2.1.3 Subcontractor Performance. The SEP should identify structural analysis tasks which may be subcontracted and should describe procedures to be used to integrate this effort into component design efforts. This might include procedures for transmitting requirements to subcontractors and keeping them updated, details concerning the type of guidance to be provided, schedules for periodic review of subcontractor progress, requirements for documentation of work, and procedures for continuing evaluation of subcontractor performance. The individual or organization responsible for subcontractor performance should be identified by component or contract.

2.1.4 Deformation Limits and Functional Requirements. The SEP should identify deformation limits established by the manufacturer for proper performance of component and the evaluation effort required to satisfy these limits.

Should the manufacturer, in the course of preliminary or screening analysis, determine that modified functional requirements are feasible and could lead to a more efficient design, the modified functional requirements should be identified to the owner.

2.1.5 Structural Evaluation. Details of how the manufacturer intends to demonstrate compliance with structural requirements should be described in the following categories.

a. Failure Modes. The manufacturer should identify the locations and failure modes (-1100(d), RDT F 9-4) which are expected to be dominant, and the load conditions (pressure, thermal seismic, etc.) associated therewith. Any failure modes not identified in RDT F 9-4 to be guarded against for specified loads should be identified. Any unspecified loads which may be anticipated should be identified.

b. Screening Analysis. The purpose of a screening analysis is to make a preliminary evaluation of the adequacy of the design, and to determine the extent and type of analysis that may be required in final structural evaluations. Where appropriate the manufacturer should

outline a screening analysis to facilitate the early stages of design, and for the purpose of ordering long-lead items such as forgings. A screening analysis is also useful in the performance of early design iterations and identification of locations in the structure requiring detailed inelastic analysis. It is often possible to establish conservative upper bounds on strains using linear elastic and simplified inelastic methods of analysis.

A typical screening analysis may consist of an initial linear elastic analysis with one or more design iterations, and comparison with design limits adjusted to account for significant fabrication and environmental effects. If design limits cannot be met by elastic calculations, simplified inelastic calculations may be made for comparison with design limits. Simplified inelastic analysis may take the form of approximate methods such as the reference stress method,[15] a one-dimensional approximation of two- and three-dimensional geometries, or simplified material models. A sample flow chart for a screening analysis is shown in Fig. 1. |C

c. Detailed Inelastic Analysis. The extent of inelastic analysis required will generally not be known until completion of the screening analysis. However, the manufacturer should identify, in the SEP, regions where initial detailed inelastic analysis may be required and the extent to which the analysis may realistically be anticipated.

The manufacturer should modify the SEP to include the extent of inelastic analysis required to satisfy design criteria. Detailed inelastic analysis should generally be performed in accordance with the procedures of Section 4. Alternate procedures proposed for use should be included in the SEP. The plan and schedule for furnishing substantiation and justification of the alternate techniques and methods should be given.

d. Support Test Program for Component Design Analysis. The manufacturer should identify structural tests required in support of design analysis in two categories: those which appear necessary from the outset, and those which may be required depending on the outcome of the screening analysis. Although test programs falling in the latter category may be highly speculative, it is important that some reasonable estimate be included in the SEP in order to scope the magnitude of the test program. Upon completion of the screening analysis, the total test program should be identified and submitted in the modified SEP.

In addition to structural tests, any basic material property tests required to obtain data in support of design analysis shall be identified. Such data will normally be provided or referenced in the design specifications or design code, but certain analysis procedures may require data not provided, or data may be required for specific heats of material to evaluate component test results. Where additional material property data are required, the manufacturer should identify the types of tests to be performed, number of samples desired, and specific data required.

Fig. 1. Sample Screening Analysis

e. Fabrication and Environmental Effects. Environmental effects which must be considered are listed in N-3214.5, RDT F 9-4. Guidance for the treatment of these effects is included in Section 7. The manufacturer should indicate how he intends to account for these environmental effects (e.g., whether in accordance with guidance provided or through alternate procedures).

For any specific application there may be environmental effects in addition to those identified in RDT F 9-4. The manufacturer should include a description of any such effects deemed significant in the operation of the component, and methods, if any, for accounting for them in the analysis to assure safe operation for the specified lifetime.

The effects of fabrication may alter the response or failure characteristics of the material. Section 7 provides guidance for the treatment of some of the effects of fabrication. The manufacturer should indicate, in the SEP, how he intends to account for fabrication effects.

f. Structural Design and Analysis Checklist. The manufacturer should provide a summary of the SEP in the form of a structural design and analysis checklist. A sample of a structural design and analysis checklist is given in Fig. 2 for guidance.

2.1.6 Test Capabilities. If a test program is proposed or anticipated in support of structural evaluation, the manufacturer should describe his testing capability relative to the proposed test program. If testing is to be subcontracted, possible contractors should be identified along with their relevant capabilities. The description of capabilities should include descriptions of test facilities, personnel qualifications, experience in high temperature testing, and tests performed on similar components.

2.1.7 Computer Programs Identification and Description. The manufacturer should identify and briefly describe the major computer programs which may be used in the various stages of analysis, and the computer on which they are to be used. This should include, but is not limited to programs to be used for thermal, elastic, simplified inelastic, detailed inelastic, dynamic, seismic, and component interaction analysis. Descriptive information may be provided by reference to generally available or published documents.

The following information should be provided for each computer program to be used:

a. Identification. Include the: 1) name and version of program and computers on which the program is currently operational, 2) availability of the program, and 3) whether the program is proprietary or non-proprietary.

[illegible]

Fig. 2. Sample Structural Design and Analysis Checklist
(continued)

[illegible]

Fig. 2. Sample Structural Design and Analysis Checklist (continued)

b. Technical Basis.* Include the basic theory, major theoretical assumptions, solution method, and class of problems to which the program is applicable.

c. Program Verification. Verification is normally the responsibility of the program developer. However, since the manufacturer is ultimately responsible for the design, he should assure the program functions properly and provide the owner with proof of verification. Guidelines for computer program verification are given in Section 3.

d. Program Qualification. Even though a program has been verified to a high level of confidence, it should be shown that the program is applicable to the particular design analysis and provides an adequate representation of component behavior for the design conditions. Guidelines for computer program qualification are given in Section 5.

e. User Experience. Capability in the use of a complex program and accuracy of results can only be attained by experience in application to actual design problems. A great deal of time and effort can be spent in learning to use a new computer program. The manufacturer should describe previous experience in use of the program including types and number of problems solved, number of people familiar with the program, and extent of current usage.

f. Graphic Display Capability. Graphic display of calculated results is a major tool for debugging input and evaluating output of computer programs, especially for complicated geometries and material behavior. The description of graphic display capability should include description of input and output display, form of display (pen on drum, cathode-ray tube, etc.) and specific hardware requirements.

2.1.8 Description of Computer Facilities. The manufacturer should describe computer facilities available for use relative to the design analysis project. The description should include:

1. Type and number of computers available.
2. Location of computers.
3. Mode of access - direct on site, remote access, commercial computer service, etc.
4. Peripheral equipment, including graphical display hardware.

2.1.9 Structural Evaluation Program Schedule and Documentation Schedule. A schedule should be included in the form of a bar chart, logic diagrams, or otherwise which identifies completion dates for each

*This information is not required for programs described in Appendix B.

level of analysis required for screening purposes and major design milestones. The schedule should include test programs with dates for first significant results and final completion identified.

Documentation should be provided at various points in the structural evaluation program for information and interaction with the owner. Points at which documentation is to be provided should be identified along with a schedule for accomplishment.

2.1.10 Structural Evaluation Program Modification and Update. The manufacturer should describe procedures for modification and update of the SEP. Points at which it is expected the SEP will be modified should be identified.

2.2 Stress Reports. The manufacturer shall prepare and submit stress reports to the owner to establish structural and functional integrity of each component based on the design specification provided by the owner. The reports should include all structural analyses and studies performed to define the system, component, subcomponent, material selection, design allowables, external and internal load distribution, load paths, and environmental effects. Conclusions concerning structural and functional integrity should be clearly stated in each report submittal. The logic and reasoning leading from the analysis and experimental results to the proposed action should also be included in each report submittal. A sample format and guidelines for the preparation of stress reports are given in the following paragraphs.

2.2.1 Informal Reports. Informal reports are unscheduled reports submitted by the manufacturer to the owner, as required, to advise the owner in a timely manner of significant findings; especially those necessary to alert the owner to possible problems. Informal reports are for information only and do not necessarily require any action on the part of the owner. Their primary purpose is to inform the owner of situations which may lead to major changes in the SEP as they become known to the manufacturer.

Informal reports should be brief, taking the form of a highlight memo, critical item report, or some similar form appropriate to the information to be conveyed. A specific format is not provided for informal reports.

2.2.2 Formal Reports. Formal reports include preliminary and final reports submitted in accordance with the SEP. Preliminary reports are submitted at times specified in the SEP to provide:

1. Results of the screening analysis, sample shown in Fig. 1
2. Proposals for modification of the SEP based on findings of the screening analysis.

3. Analyses in support of milestones such as: establishment of basic design parameters, ordering of long lead items, and drawing release for fabrication.

Final stress reports submitted to fully substantiate the structural and functional integrity of the component. Final reports are used by the owner for final review and acceptance of all components, subcomponents, parts, and appurtenances. The final report should be a compilation of the final analysis and test results, presented in volumes as a complete and organized total evaluation. The first volume should be a summary including items (a) through (g) of the final stress report format given below.

Preliminary reports should follow the same general format as final reports, but the content should be truncated to provide only that information necessary to satisfy the specific purpose of the report as defined in the SEP.

The following format and content is suggested for final stress reports:

- a. Title Page. The title page should identify the report by name, show approval signatures, and date of issue.
- b. Abstract. The abstract is a brief one or two paragraph summary of the report. It should include a statement of purpose or objective, methods and extent of investigation, and principal results or conclusions.
- c. Table of Contents, List of Tables, and List of Figures. The table of contents should contain major headings and subheadings as they appear in the body of the report listed by page number, and a summary of appendices listed by title. All tables and figures (including graphs, drawings, and photographs) should be listed following the table of contents. The list of tables, numbered by Roman numerals, and the list of figures, numbered by Arabic numerals, should include table or figure number, caption, and page number.
- d. Introduction. The purpose of the introduction is to give the reader information needed to understand results, conclusions, and recommendations. This information should include:
 1. Purpose or objective of report.
 2. Description of problem investigated.
 3. Significant design features.
 4. Description of method of analysis.
 5. Identification of design specification.
 6. Scope of report and its organization.

e. Summary of Results. The main findings of the analysis should be summarized in a concise form. The summary should include:

1. Critical modes of failure.
2. Critical stress or strain locations.
3. Calculated versus allowable values (stresses, strain, deformations, usage factors, etc.), plots or tables including part name and identification, material, load and environmental condition, and reference page number.

f. Discussion. Discussion should include interpretation of results, correlation of analysis with test data, comparison with results for similar components, adequacy of the evaluation, and discussion of anomalous results.

g. Conclusions. Conclusions should include deductions based on the evaluation relative to structural integrity.

h. Component or System Description.

1. Functional. The functions and operations of the component or assembly which affect structural integrity should be described.

2. Geometrical. A geometrical description of the part should be given and illustrated by sketches.

3. Specific Design Criteria. Identification of applicable documents such as edition of ASME Code, Code Cases and Addenda, RDT Standards, etc. The design specification should also be identified.

i. Loads. The requirements as specified in the design specification should be identified. Other loads on the component or assembly due to environment should be included. Load descriptions should include, but not be limited to the following:

1. External Loads. Any loads applied to the component or assembly from adjacent parts or supporting structure.
2. Operational Loads. Loads caused by operation of the components or assembly, such as fluid flow loads and inertia loads.
3. Dynamic Loads. Loads caused by mechanical vibration, including flow induced vibrations, seismic, etc.
4. Thermal Loads. Loads in the component or assembly caused by temperature changes or thermal gradients.

5. Internal Loads. Load interactions between various parts which make up the component or assembly should be calculated from expected combinations of the loads.

j. Thermal Analysis. The data for the various transient and operating cycles specified in the design specification and used to predict metal thermal transients for stress analysis should be described. The thermal analysis should include a description of:

1. Methods of analysis, assumptions, and boundary conditions.
2. Geometry from temperature distribution point of view.
3. Geometries from stress investigation point of view.
4. Temperature-dependent data for each thermal geometry and transient.
5. Temperature distribution for each geometry.
6. Thermal gradients.

Further guidance on above items is given in Appendix C of the ASME Code.

k. Detail Stress Analysis.

1. Methods of Analysis. This section should include a description of mathematical models and the calculation methods used in connection with the thermal and mechanical stress analysis. Explicit description of boundary conditions should be given. All computer programs used during calculations should be identified and described. The basic theories on which the calculations are based and the assumptions and their corresponding limitations should also be included.

2. Material Properties. Basic mechanical and physical properties used in the stress analysis should be given. Sources (e.g., handbook, code, or test) should be identified.

3. Sketches and Free-Body Diagrams. Loads at critical points and cross sections of the various parts should be calculated and illustrated by free-body diagrams.

4. Calculated Stresses. Stresses at various critical points and cross sections should be calculated from the most critical load conditions. This should include method of analysis, assumptions, formulae, calculations, results, and determination of critical condition.

5. Presentation of Stresses. Calculated and allowable stresses should be identified and summarized as to magnitude, location, origin, and classification consistent with the ASME Code definitions. The following are typical of stress components that should be considered:

A. Thermal stress generated by:

(1) Radial gradient:

- (a) Thermal stress.
- (b) Thermal discontinuity stress.

(2) Longitudinal gradient:

- (a) Thermal stress.
- (b) Thermal discontinuity stress.

B. Mechanical stresses generated by:

- (1) Pressure load.
- (2) Deadweight load.
- (3) Piping load.
- (4) Externally applied load.
- (5) Seismic loads.
- (6) Dynamic loads.

1. Strain, Deformation, and Service Life Evaluation.

1. Service Life. The service life requirements per the design specification should be stated along with the strain and deformation limits of the application design criteria document.

2. Histogram. The histogram provided in the design specification and any approximations used for purposes of analysis should be presented.

3. Ratcheting Evaluation. Method of analysis, calculations, and evaluations should be presented. Comparison to specified criteria limits should be given.

4. Fatigue and Creep-Fatigue Evaluation. Method of analysis, calculations and evaluations should be presented. Accumulated creep and fatigue damage should be presented in terms of usage factors for creep, fatigue, and combined creep-fatigue damage.

m. Reference. All references in the stress analysis should be listed including the title and drawing number for parts analyzed. Unique identification should be provided in the report for each computer analysis employed in the final stress report. The computer runs should be identified individually within the Reference section. Input listings,

if not an integral part of the computer output, should be separately identified.

n. Appendices. Detailed investigations, data or descriptive matter which are not essential to the body of the report, but which constitute valuable reference material, should be incorporated in the report as appendices. Typically, design drawings and detailed descriptions of computer programs are presented as appendices.

o. Miscellaneous. Presentation of information should conform to good engineering practice. Some details of format not already covered include:

1. Designer's name on first page, initial on following pages. Each page should be dated and contain the work charge number.

2. The analysis may be printed or in script; legible, sequential and uncluttered.

3. Sketches may be free-hand but should be neat and clear and not too distorted; they should contain no dimensions that are not used in the calculations and all dimensions that are used. They should be employed freely, especially when their use eliminates written text or description.

4. Graphs should conform to accepted practice. They should be fully identified by title and coordinates, and contain source references, initials of designer, page number and date.

5. Calculations should be presented in the best possible form for checking.

6. Do not use "x" to denote multiplication: use parenthesis ().

7. References should be used freely. Every item of information used for calculations on a given page of an analysis should be referenced, even if the source is in the same analysis.

8. Assumptions, when not obvious or which are reasonably challengable, should be conspicuously set forth at the place in the analysis where they are used. They should always be demonstrably conservative.

3. FLOW OF DESIGN ANALYSIS

3.1 General.

- 3.1.1 Background. In satisfying the high temperature design requirements of RDT F 9-4, many structural analyses, of varying degrees of complexity, are required. Without general guidance, each contractor

responsible for assuring the structural integrity of components must individually decide upon an analysis plan which will yield the required analytical results in a logical, timely and economical sequence. Past experience in demonstrating the acceptability of complex reactor plant components for high temperature service has indicated that wide differences exist in the degree of planning logic used. For example, detailed finite element thermal stress analysis should not in general be performed on preliminary structural configurations until it has been demonstrated that the primary stresses are within the allowable limits. This section provides guidance on the flow of analysis and covers all provisions of the high temperature design criteria given in RDT F 9-4.

Separate limits are given in RDT F 9-4 for load-controlled quantities and deformation-controlled quantities. Load-controlled quantities are either stress intensities which result from equilibrium with external loads during plant operation or deformation controlled quantities which have significant elastic follow-up. The flow of design analysis for load-controlled quantities is discussed in 3.3. Deformation controlled quantities are strains or deformations which result from load deflection, strain compatibility, or both. The flow of design analysis for deformation controlled quantities is discussed in 3.4.

3.1.2 Purpose. The purpose of this section is to provide examples of guidelines and procedures for conducting structural analyses of elevated temperature systems in accordance with the design requirements of RDT F 9-4.

Although this section defines the formal logic which may be employed in satisfying the RDT F 9-4 criteria, checks of each provision may be made with different degrees of sophistication depending upon the design phase (i.e., conceptual, preliminary and final). For example, in the concept design stage, simplified checks of each of the limits are encouraged to help prevent unforeseen violations in areas that may have initially been considered to be unimportant. These quick checks should help to eliminate major design changes from occurring in the preliminary and final design stages.

3.1.3 Organization. The overall logic which may be used for performing structural analyses and criteria checks according to RDT F 9-4 is presented in Fig. 3 in the form of a flow chart.* In using this section as a guide, it is first recommended that an in-depth understanding of the general conditions shown in Fig. 3 be obtained. This will aid the user in following the detailed logic provided within each chart.

*Terms and symbols used in the logic flow chart are in general consistent with those defined in RDT F 9-4. Unless otherwise noted all cross references to sections, figures and table numbers used in this nomenclature refer to RDT F 9-4.

Fig. 3. Overall Logic

With a few exceptions the general concept used in Fig. 3 is to check design conditions, operating load-controlled conditions, and operating deformation controlled conditions in that order. Elastic analysis methods are tried first. Inelastic analysis methods are used when elastic limits are violated. Final checks on limits are for buckling and other special provisions.

3.2 General Requirements. Before actually beginning a structural analysis for use in satisfying the requirements of RDT F 9-4, all provisions of the general requirements should be reviewed in the order shown in Fig. 4. The general requirements covered in Fig. 4 assure that:

1. The structure is part of ASME Code Class 1 system.
2. Maximum metal temperatures are within the limits of the applicable criteria.
3. Effective dates of applicable documents are known.
4. Any conflicting requirements are brought to the attention of cognizant organizations.
5. A SEP (structural evaluation plan) will be prepared.
6. Stress reports will be submitted to the owner.
7. Applicable drawings are available.
8. Materials being proposed are from the list allowed by applicable standards.
9. All environmental effects will be considered.
10. Material properties are obtained from acceptable sources.

Once all the general requirements have been satisfied, analysis effort may be initiated. In those cases where one load dominates the response of a structure it may be appropriate to use a modified sequence of analysis.

More refined analysis can often be used when the rules of RDT F 9-4 are not satisfied with the initial analysis. The conservative approximations included in most analyses can, where necessary, be removed by using refined analysis. The process of refining the analysis will often provide the relief needed to permit satisfaction of the rules of this standard.

In preparation of these flow charts it has been assumed that the design specification is correct, reasonable, and complete. Should it become apparent during the course of the evaluations that a change in design requirements is warranted or might be beneficial, it should be brought to the attention of the owner.

SHEET 1 OF CHART 1.0

Fig. 4. Logic Flow Chart 1.0 Governing Provisions
of the General Requirements

(continued)

Fig. 4. Logic Flow Chart 1.0 Governing Provisions of the General Requirements (continued)

(continued)

SHEET 3 OF CHART 1.0

Fig. 4. Logic Flow Chart 1.0 Governing Provisions of the General Requirements (continued)

(continued)

LOGIC FLOW CHART LEGEND

Fig. 4. Logic Flow Chart 1.0 Governing Provisions of the General Requirements (continued)

3.3 Load-Controlled Quantities. Load-controlled quantities are stress intensities which result from equilibrium with applied loads during plant operation. Included in this category are general primary membrane stresses, local primary membrane stresses, primary bending stresses and secondary stresses with a large amount of elastic follow-up.

There are four load-controlled conditions which must be checked. Listed in an example sequence, the criteria checks which may be made are:

1. Design Conditions.
2. Normal and Upset Operating Conditions.
3. Emergency Operating Conditions.
4. Faulted Operating Conditions.

3.3.1 Design Conditions. After primary design load stresses have been calculated, there are three basic checks which should be performed. Listed in the example sequence they are:

1. Perform a preliminary check of buckling allowables.
2. Check general primary membrane stress intensity allowable (S_0).
3. Check primary membrane (general and local) plus bending stress intensity allowables ($1.5 S_0$).

Because of the relatively simple criteria and importance to overall structural integrity, the design condition criteria can be checked first to assure that all primary stresses are within the allowable limits. The guidelines and logic for proceeding with design condition checks are presented in Fig. 5 in the form of a logic flow chart.

3.3.2 Operating Conditions. With respect to RDT F 9-4 criteria, there are four unique operating conditions which must be considered. They are Normal, Upset, Emergency, and Faulted. Each condition to which the component may be subjected is defined by the equipment specification to be within one of these categories.

3.3.2.1 Normal and Upset. Although the specification defines separate Normal and Upset Conditions, from a structural criteria viewpoint the two conditions are considered to be identical. Note that, unlike the rules in the ASME Code, the rules of RDT F 9-4 require evaluation of primary stresses for Normal and Upset Conditions in addition to the primary stress evaluation for design conditions. Thus, transient mechanical loadings in the Upset category need not be also included as design mechanical loads.

After the stresses at all critical points have been determined for Normal and Upset Conditions, they should be evaluated according to the criteria of RDT F 9-4. The suggested order for satisfying the Normal

SHEET 1 OF CHART 2.0

Fig. 5. Logic Flow Chart 2.0 for Load Controlled Design Condition Stress Limits

(continued)

SHEET 2 OF CHART 2.0

Fig. 5. Logic Flow Chart 2.0 for Load Controlled Design Condition Stress Limits (continued)

and Upset Condition rules is shown in flow chart form in Fig. 6. It should be noted from Fig. 6, that the major criteria for checking the acceptability of a structure to survive the Normal and Upset Conditions are:

1. Preliminary check to satisfy the buckling criteria (if design stresses were found to satisfy the buckling criteria it is most likely that Normal and Upset Conditions will not be critical unless life requirements are such that creep effects are dominating).
2. Demonstrate that the general primary membrane stress intensity is within the time-dependent allowable (S_{mt}).
3. Provide checks to assure that the time-fraction ratio is within the allowable limit (B).
4. Demonstrate that the combined primary (membrane plus bending) stress intensity is within the smaller of the time-dependent allowable ($1.5 S_m$) or time dependent allowable (KS_t).

In each case where the allowable limits have been violated, a check should be made to determine if either the configuration or the histogram can be modified. A possible sequence for checking the Normal and Upset Condition criteria is described in detail in Fig. 6.

3.3.2.2 Emergency. The type of criteria for Emergency Conditions are similar to those for Normal and Upset Operating Conditions except slightly higher limits are allowed. The first step in starting the criteria check for Emergency Conditions is to determine the primary (membrane and bending) stresses at all critical locations for Emergency Condition loads. These stresses are then compared with allowables from RDT F 9-4 using the sequence in Fig. 7. The basic criteria covered by this chart are:

1. Preliminary check on the buckling criteria (if not previously established by design or Normal and Upset Conditions).
2. Primary membrane stress intensity must be limited to the lesser of a time independent limit ($1.2 S_m$) and a time dependent limit (S_t).
3. The time-fraction ratio must be less than the allowable (B).
4. The primary membrane plus bending stress intensity must be limited to the lesser of a time-independent allowable ($1.8 S_m$) and a time-dependent allowable (KS_t).

SHEET 1 OF CHART 2.1

Fig. 6. Logic Flow Chart 2.1 for Load Controlled Operating (Normal and Upset) Condition Stress Limits

(continued)

SHEET 2 OF CHART 2.1

Fig. 6. Logic Flow Chart 2.1 for Load Controlled Operating (Normal and Upset) Condition Stress Limits (continued)
(continued)

SHEET 3 OF CHART 2.1

Fig. 6. Logic Flow Chart 2.1 for Load Controlled Operating (Normal and Upset) Condition Stress Limits (continued)

SHEET 1 OF CHART 2.2

Fig. 7. Logic Flow Chart 2.2 for Load Controlled Operating (Emergency) Condition Stress Limits

(continued)

Fig. 7. Logic Flow Chart 2.2 for Load Controlled Operation
(Emergency) Condition Stress Limits (continued)

SHEET 3 OF CHART 2.2

Fig. 7. Logic Flow Chart 2.2 for Load Controlled Operating
(Emergency) Condition Stress Limits (continued)

If the time-independent limits are violated with elastic analysis, inelastic analysis should be considered. If any of the Faulted Condition limits are violated, an attempt may be made to revise the configuration and histogram according to the sequence given in Fig. 8. If the load-controlled design and operating condition criteria have been satisfied, the next step is to evaluate the deformation controlled quantities.

3.4 Deformation-Controlled Quantities. Deformation-controlled quantities are strains or deformations which result from load deflection, strain compatibility or both. The strain and deformation limits described in 3.4 apply only to the combination of Normal, Upset and Emergency Conditions. Functional deformation limits may be specified for any of the operating conditions. Currently, strain limits are not imposed on design conditions.

3.4.1 Strain Limits and Ratcheting Criteria. The strain limits of RDT F 9-4 can be satisfied by comparison with the strains calculated using inelastic analysis or by using elastic stress limits to demonstrate that no ratcheting occurs. As would be expected, the stress limits imposed on elastically calculated stresses to demonstrate freedom from ratcheting are, in some cases, very restrictive. Since elastic stress intensities have been calculated for previous checks on load-controlled quantities, it is convenient to first check ratcheting adequacy using the elastic methods and criteria. Alternate inelastic analysis procedures of this standard may be used as noted in Fig. 9. If a portion of a system fails the elastic limits for ratcheting, an inelastic analysis of that portion of the system may be performed. The normal sequence of analysis is to first check with the elastic analysis methods and then use inelastic analysis if necessary. However, in some circumstances it may be desirable to go directly to the inelastic analysis rules without iterating the design to comply with the elastic analysis rules.

3.4.1.1 Elastic and Simplified Inelastic Analysis Methods. By imposing a conservative limit on the elastically calculated primary plus secondary stress intensity ranges at all points in the system, freedom from ratcheting and excessive deformation is considered to have been demonstrated. It should be noted that only Normal, Upset and Emergency Condition loadings need to be included in ratcheting checks. (This does not preclude the possible need for functional deformation limits for Faulted Condition loadings).

The procedure recommended for checking the ratcheting rules for elastically calculated stress intensity ranges is shown in the form of a logic flow diagram in Fig. 10. The major checks provided by the criteria outlines in Fig. 10 are:

1. All functional requirements must be satisfied.
2. Creep relaxation at one extreme of the cycle is negligible.

SHEET 1 OF CHART 2.3

Fig. 8. Logic Flow Chart 2.3 for Load Controlled Operating (Faulted) Condition Stress Limits

(continued)

Fig. 8. Logic Flow Chart 2.3 for Load Controlled Operating (Faulted) Condition Stress Limits (continued)

Fig. 9. Flow Chart 3.0 Response - When Elastic Checks Fail

SHEET 1 OF CHART 3.1

Fig. 10. Logic Flow Chart 3.1 Evaluation of Strain and Deformation Limits Using Elastically Calculated Stresses

(continued)

SHEET 2 OF CHART 3.1

Fig. 10. Logic Flow Chart 3.1 Evaluation of Strain Deformation Limits Using Elastically Calculated Stresses (continued)

(continued)

SHEET 3 OF CHART 3.1

Fig. 10. Logic Flow Chart 3.1 Evaluation of Strain and Deformation Limits Using Elastically Calculated Stresses (continued)

3. The maximum primary stress intensity plus the secondary stress intensity range must be less than the allowable stress (S_q).

If any parts of the criteria are violated a check can be made to determine if the configuration, histogram or both can be changed. A logical sequence for checking the possibility of modifying the configuration and histogram is outlined in the logic flow chart shown in Fig. 10. If it is found that feasible modifications can be made, this chart will provide the necessary direction for re-entering the charts and re-checking all criteria affected by the change.

3.4.1.2 Inelastic Analysis Methods. If it is found that the strain limits for elastic analysis can not be satisfied, the strain limits for inelastic analysis can not be met using simplified methods, the histogram use can not be revised and the configuration can not be changed, Fig. 10 logic directs the effort toward an inelastic analysis. At this point it is suggested that a detailed inelastic analysis be performed for all those areas which violated the elastic criteria. Only Normal, Upset and Emergency Conditions need to be considered. For inelastic analysis, the limits are applied on strains. There are three limits for the maximum positive principal inelastic strain which must be satisfied.

1. The strain averaged through the wall thickness must be less than or equal to 1% for base metal and 0.5% for welds.
2. The surface strains (based on an equivalent linear distribution of strain through the thickness) must be less than or equal to 2% for base metal and 1% for welds.
3. The local strains must be less than or equal to 5% for base metal and 2.5% for welds.

The recommended logic diagram for performing the strain limits checks is given in Fig. 11.

If any of the strain limits are violated and additional, or more refined analysis is ineffective, a change in the design configuration, histogram or both must be made as indicated in Fig. 10. After making the change the criteria check sequence leads back to either Fig. 6 (if histogram is modified) or Fig. 5 (if configuration is modified).

3.4.2 Creep-Fatigue Criteria. There are two methods which can be used to demonstrate the structural adequacy of a system for a creep-fatigue type of failure mode. One is based upon elastically calculated strains and the other is based upon inelastically calculated strains.

If an inelastic analysis has not been required thus far in the criteria checks, it is a decided advantage to first try to satisfy the creep-fatigue rules based upon elastic methods. However, if inelastic

Fig. 11. Logic Flow Chart 3.2 Evaluation of Strain and Deformation Limits Using Inelastic Analysis

analyses were required to satisfy the strain and deformation rules described in 3.4.1, there are two choices for checking creep-fatigue adequacy. One would be to go directly to the inelastic criteria and the other (the one recommended by the flow charts) would be to check creep-fatigue according to the elastic criteria. The amount of effort for these two options will depend largely upon the type of system being evaluated.

3.4.2.1 Elastic Analysis Methods. To check the creep-fatigue rules using elastic analysis methods, strain differences must be calculated from the load and deformation controlled elastic stresses. The details of this procedure are not provided. The designer should know that the extension of these rules beyond one-dimensional applications may require demonstration of the validity of the results. Fig. 12 suggests a sequence of decisions for use prior to the use of elastic analysis rules.

3.4.2.2 Inelastic Analysis Methods. Before creep-fatigue limits for inelastic analyses can be applied, a detailed inelastic analysis of the structure must be performed for Normal, Upset and Emergency loading conditions. The criteria for creep-fatigue damage using inelastically calculated strains is to assure that the total of the use fractions for time and cycles are less than or equal to a damage factor limit (D).

The guidance and logic for performing the creep-fatigue checks using inelastic analysis are outlined in logic flow chart form in Fig. 13. If any violations of the creep-fatigue criteria for inelastic methods are found, a revision to either the design configuration or the histogram as outlined in Fig. 9 may have to be made. After changing either the configuration or the histogram, the criteria check sequence is directed back to either Fig. 6 (if histogram is changed) or Fig. 5 (if configuration is changed).

3.5 Buckling and Instability Criteria. Although preliminary checks of the buckling criteria were made during the criteria checks for load-controlled stresses, all stress states must be examined to demonstrate that the complete buckling criteria is satisfied. The two basic criteria for buckling covered by RDT F 9-4 are to assure that:

1. The load which would cause instant instability if applied at time t divided by the expected load at the same time t is equal to or greater than the load factor of safety.
2. The ratio of the time to instability, if expected load history during service life is repeatedly applied, divided by specified service life is equal to or greater than the time factor of safety.

*ADVISE THE OWNER

Fig. 12. Logic Flow Chart 3.3 Creep-Fatigue Evaluation
Using Elastically Calculated Stresses

Sheet 1 of Chart 3.4

Fig. 13. Logic Flow Chart 3.4 Creep-Fatigue Damage Using Inelastic Analysis Methods

(continued)

Sheet 2 of Chart 3.4

Fig. 13. Logic Flow Chart 3.4 Creep-Fatigue Damage Using Inelastic Analysis Methods (continued)

(continued)

Sheet 3 of Chart 3.4

Fig. 13. Logic Flow Chart 3.4 Creep-Fatigue Evaluation
Using Inelastic Analysis Methods (continued)

The minimum load factors of safety depend upon the type of buckling analysis performed and whether or not the analysis is based upon theoretical calculations or experimental data.

An example procedure for proceeding with the checks of the buckling limits is presented in logic flow chart form in Fig. 14. If buckling limits can not be satisfied by any of the several alternate procedures, an alternate action is to change either the configuration or the histogram and proceed with the criteria checks according to Fig. 9.

3.6 Special Limits. There are several special limits defined in Section 6 of RDT F 9-4 which must be checked. The special limits cover the following areas:

1. Multi-Dimensional Stress Analysis.
2. Triaxial Stresses.
3. Welds.
4. Bolting.
5. Bearing Loads.
6. Elastic Follow-up.
7. Fatigue Testing.
8. Testing Conditions.
9. Minimum Wall Thickness for Valves.

A procedure for satisfying these special limits is shown in logic flow chart form in Fig. 15. This figure shows that the criteria for special limits are checked after all general provisions, load and deformation controlled criteria and buckling limits have been satisfied. Depending upon the system being evaluated this may not be the most efficient sequence. For example, if there are several bolted joints in the system, it would be more logical to check the bolting limits early, i.e., immediately after the design condition limits. For this reason, the overall logic required for special limits should be tailored to the specific system being analyzed. An efficient logic chart for one system may be a very poor one for a totally different system.

4. INELASTIC ANALYSIS METHODS

4.1 Introduction. Detailed inelastic analyses are permitted and may be required to demonstrate conformance with the requirements of RDT F 9-4. RDT F 9-4 provides criteria for determining the acceptability of stresses and strains predicted by inelastic analyses, and specifies the manner in which the predicted stresses and strains are to be combined for evaluation. If does not; however, specify the techniques and methods to be used in predicting stresses, strains, and deformations. To perform a detailed inelastic analysis, the designer must have available mathematical analogs, or constitutive equations, that describe the response of the material to time-varying load and temperature histories. The main part of this Section (4.2 through 4.4) provides acceptable constitutive equations for types 304 and 316 stainless steel. Subsequent subsections provide similar guidelines for other materials.

SHEET 1 OF CHART 4.0

Fig. 14. Logic Flow Chart 4.0 Governing Criteria For Buckling and Instability

(continued)

SHEET 2 OF CHART 4.0

Fig. 14. Logic Flow Chart 4.0 Governing Criteria for Buckling and Instability (continued)

Fig. 15. Logic Flow Chart 5.0 Governing Special Limits

Constitutive equations which consider both time-independent elastic-plastic behavior and time-dependent creep behavior are the key ingredients in an inelastic analysis. The possible formulations available to the designer in the literature are many and varied and in most the presumed material behavior bears little resemblance to the actual behavior of the materials of interest, and the predictions resulting from the various formulations can differ very markedly. Thus, to obtain realistic and meaningful predictions of high-temperature inelastic behavior, the analysis method must be based on constitutive equations that have been carefully developed and evaluated for the materials and applications of interest. The constitutive equation guidelines presented for types 304 and 316 stainless steel were arrived at in this manner. The techniques and methods that are given form a common acceptable basis for the detailed analyses that are necessary to demonstrate compliance with the requirements of this standard, and establish a minimum acceptable quality level for detailed inelastic analyses.

One of the guidelines used in arriving at the constitutive equation recommendations was that they simply describe the essential features of high-temperature material behavior and be broadly applicable to a variety of situations encountered in high-temperature design. In specific situations some of the features incorporated may not be relevant to the problem at hand and the designer may feel that alternate procedures will provide improved predictions.

Thus, alternate techniques and methods may be used by the manufacturer when it can be shown that they are more accurate, more conservative, or better suited to the specific situation being analyzed. Prior to the use of such methods the Structural Evaluation Plan (SEP) shall be revised in conformance with N-1200, RDT F 9-4 to provide the plan and schedule for furnishing substantiation and justification of the alternate techniques and methods. | C

The motivating factors that led to the selection of the currently recommended constitutive equations and procedures to be used in inelastic design analyses of types 304 and 316 stainless steel components are described in Appendix A. The basic observed features of high-temperature deformation behavior, derived largely from uniaxial tests of type 304 stainless steel, are qualitatively discussed. It can be seen in Appendix A that high-temperature material behavior, even of a single material under uniaxial conditions, is a highly complex phenomenon consisting of processes and interactions that are not completely understood. Generalizations of the uniaxial observations to multiaxial stress conditions introduce additional unknowns and raise many questions. Thus, the constitutive equations currently recommended are simplified descriptions for interim use. They are based on the current state-of-the-art and will almost certainly be updated and improved as more is learned about material behavior and as theories are developed to more properly represent the inelastic behavior that is observed. This is particularly true for general nonlinear, multiaxial behavior in response to nonradial loading conditions.

In the interim the recommended methods will provide the designer with tools for general use that are consistent with the state-of-the-art and can currently be regarded as a good engineering approach. The guidelines are formulated with sufficient generality to allow the designer some leeway in treating each situation. When carefully applied the procedures will predict the more significant and important behavioral features of the austenitic stainless steels as described in Appendix A. The procedures are also compatible with current analytical capabilities in that they can be incorporated into existing computational techniques. Finally, use of these recommended constitutive equations will provide a desirable and important consistency in inelastic design analyses and evaluations throughout a high-temperature reactor project.

It is emphasized that the constitutive relations recommended herein represent first approximations to the descriptions of inelastic behaviors to be encountered in service. Thus, although inelastic analyses can currently be used for making estimates of the basic characteristics or the essential features of inelastic response in critical situations, their use must be accompanied by a large measure of sound engineering judgment.

4.2 Preliminary Considerations. Generally, a combined time-dependent elastic-plastic analysis and time-dependent creep analysis capable of predicting stresses, total accumulated strains, and deformations as functions of time for specified load and temperature histories is required. The method of calculation should be based on the assumption that the total strain at any instant of time consists of three parts - elastic, plastic, and creep. Small, but discrete, increments of time are generally considered in which elastic-plastic strains and creep strains are computed separately and added to obtain the total strain. The strains accumulated during the expected operating life-time and loading history should be determined by summing the strains computed for the discrete time increments:

$$\epsilon_{ij} = \epsilon_{ij}^E + \sum \Delta \epsilon_{ij}^P + \sum \dot{\epsilon}_{ij}^C \Delta t, \quad (1)$$

where: ϵ_{ij} = total strain components,

ϵ_{ij}^E = elastic strain components,

$\Delta \epsilon_{ij}^P$ = increments of plastic strain components,

$\dot{\epsilon}_{ij}^C$ = creep strain-rate components,

t = time.

Although a few exploratory tests are discussed in Appendix A, available data on the effect of prior plasticity on subsequent creep and of prior creep on subsequent plasticity are not sufficient to allow formulations of constitutive equations that account for creep-plasticity interactions in a general way. In these recommendations any effects of prior plastic deformation on subsequent creep response are neglected, and except for hardening due to prior creep strains the effects of prior creep deformation on subsequent elastic-plastic behavior are neglected. Thus, the elastic-plastic and creep equations are formulated independently. Specific guidelines for the determination of the plastic strain components are given in 4.3, and guidelines for determining the creep strain-rate components are given in 4.4.

Constitutive equations for inelastic analyses that have been developed to the point of being usable in practical applications are almost invariably based on small-strain theory, and the relations recommended here are similarly based. However, this does not impose significant restraints on reactor component designs since strains in a given component should not be more than a few percent.

For compactness, the equations in the following sections are written using index notation. To assist the unfamiliar reader, Appendix C is included in this volume to explain and illustrate the notation used.

4.3 Time-Independent Elastic-Plastic Analyses.

4.3.1 Basic Model Recommendations. Three ingredients, in addition to Hooke's law, are necessary to describe material behavior for an elastic-plastic analysis. There are: (1) a yield condition specifying the states of multiaxial stress corresponding to the onset of plastic flow; (2) a flow law in the form of equations relating plastic strain increments to the stresses and stress increments subsequent to yielding; and (3) a hardening rule specifying the modification of the yield condition in the course of plastic flow. The constitutive equations for elastic-plastic behavior are determined from these ingredients.

The yield criteria currently in use are those of von Mises and of Tresca. Either is acceptable, although the paragraphs which follow assume the use of the von Mises condition which has been shown to agree reasonably well with test data for austenitic stainless steels. The von Mises flow law is recommended for use with either yield condition. The use of an associated yield criterion is recommended. The final ingredient - the hardening rule - plays an important role for cyclic elastic-plastic behavior predictions.

Examination of the available data, such as those presented and discussed in Appendix A, lead to the conclusion that of the two commonly utilized hardening rules - isotropic and kinematic - kinematic hardening more correctly describes the elevated-temperature cyclic hardening

behavior of type 304 stainless steel. During the initial cycles the measured elastic range increases, but not nearly as much as would be predicted by isotropic hardening. Subsequent to the first few plastic cycles, the extent of the elastic region is essentially unaffected by plastic flow indicating a rigid yield surface as is assumed in the kinematic hardening model. Thus, the recommendations here are based on kinematic hardening.

It is emphasized in Appendix A that cyclic stress-strain response of type 304 stainless steel is highly dependent upon its history. For example, in the uniaxial cyclic test results shown, the cyclic stress-strain curves depend strongly on strain range. Thus, in an actual structure the stress-strain relations that should be used would ideally vary from point to point. History-dependent stress-strain relations that have been experimentally confirmed do not exist. Likewise, proven multiaxial theories that can adequately predict the nonlinear, history-dependent behavior of a material cyclically loaded along arbitrary non-radial loading paths do not exist.

To avoid these unknowns, but yet to provide an approximating theory which can predict the significant features of elastic-plastic behavior and which is conceptually and mathematically consistent for general nonproportional loadings, it is recommended that bilinear representations of the uniaxial stress-strain curves be utilized. The classical kinematic hardening theory is based on a bilinear stress-strain relationship and is theoretically consistent for such a relation.

The bilinear representation used for the initial loading cycle of a component should be based on the initial loading, monotonic stress-strain curve for the virgin material. To approximately account for cyclic hardening, as well as for the effect of prior creep on subsequent plastic behavior, it is recommended that bilinear representations of cyclic stress-strain curves be used for subsequent cyclic loadings. In most cases, a single representation based on the cyclic stress-strain curve for a partially hardened material is adequate and appropriate for these subsequent loadings. The tenth-cycle stress-strain curve is recommended as an acceptable representation of the partially hardened state.* By using the initial and tenth-cycle stress-strain curves, the designer can obtain a good prediction of the initial loading response of a virgin structure and of the typical response to repetitive loadings throughout the lifetime of the component.

*As discussed in Appendix A, saturation of the cyclic hardening may not occur under some conditions until a considerably larger number of cycles than ten. On the other hand, some hardening is lost during periods of high-temperature exposure. The tenth-cycle is a compromise between additional hardening and thermal softening.

In certain design analyses, the designer may want to obtain a more realistic prediction of structural response during the first few cycles of loading of the virgin specimen than would be possible with the abrupt change in hardening that is associated with the switch from the initial to the tenth-cycle stress-strain curves. To handle these cases, optional provisions are provided for gradually changing the bilinear representation upon each loading reversal.

Although bilinear representations are recommended, a nonlinear kinematic hardening approach based on effective-stress, plastic-strain relationships that take into account the actual monotonic and cyclic stress-strain curves may be used for radial, or near-radial, loadings. Nonlinear kinematic hardening relationships are not recommended for general nonradial loadings. This approach will be advantageous in certain calculations such as instability analyses. Consequently, equations are given for the use of effective-stress, plastic-strain relations. It should be emphasized; however, that the nonlinear theory based on effective-stress, plastic-strain relations is rigorously correct only for radial monotonic loadings. For radial reversed loadings, and subsequent cyclic loadings, it generally suffers from the same shortcomings as the bilinear theory. That is, only one effective-stress, plastic-strain relationship is used despite the fact that the previous history varies from point to point in the component. Consequently, the nonlinear theory for radial reversed and cyclic loadings offers little real advantage over the bilinear theory and at the cost of a loss in simplification. The development of rigorous nonlinear, multiaxial theories which can be used for general nonradial loadings and which correctly reflect past history must await the generation of more experimental data.

In the overall sense, the use of a bilinear representation of the stress-strain curve is not much different from the use of a single representative nonlinear stress-strain relationship to describe the isothermal behavior of all materials in a given class for which the detailed compositions, prior histories, heat treatment, and grain size are unknown. An example is the use of a single correlation to represent type 304 stainless steel as a class of material. Nor is it much different from considering an unloading elastic-plastic stress-strain curve to be geometrically similar to the loading curve.

In conclusion then, the recommended bilinear approach, if carefully applied, will generally give reasonable predictions for both radial and nonradial loadings. In most cases, other approximating theories for cyclic radial loadings introduce added complexities without really significantly improving the reliability of the elastic-plastic predictions.

4.3.2 Stress-Strain Relationships. Consistent procedures to be used in constructing bilinear representations from actual nonlinear cyclic stress-strain curves are provided in this subsection. For a given material and a given temperature, the bilinear representations to be used depend on the expected range of elastic-plastic strain.

In the bilinear representations, the material is characterized by the elastic modulus E , a material constant C , which is related to the slope E_p of the elastic-plastic segment of the bilinear curve, and a quantity κ , which is related to the yield point σ_0 of the bilinear representation and which is a measure of the size of the yield surface. For a given analysis, C should be considered to be constant for a given temperature; that is, the elastic-plastic segment of each bilinear representation should have the same slope at a given temperature. However, for a given temperature κ will have a value κ_0 for the first inelastic loading, and different values for subsequent loading cycles. In most cases, the bilinear representation for the tenth-cycle cyclic stress-strain curve will be used for all loadings subsequent to the initial one; the particular κ value corresponding to the tenth-cycle is denoted by κ_1 .

4.3.2.1 Bilinear Representations of Initial Monotonic Stress-Strain Curves. For each temperature, the constants C and κ_0 to be used in a specific elastic-plastic analysis should be determined from the appropriate monotonic tensile stress-strain curves for the virgin material by the following procedures illustrated in Fig. 16.

a. The elastic segment of the bilinear curve is represented by a straight line determined by the initial response of the material.

b. For a given maximum elastic plus plastic strain, the elastic-plastic segment of the bilinear curve is determined by a straight line connecting the stress point at the maximum strain, $\epsilon_{\max}$, to the stress point at strain, $\epsilon_{\max}/2$.

c. The yield point, σ_0 , is defined as the intersection of these two straight lines.

d. The value of κ_0 is determined from the yield point σ_0 by the relation:

$$\kappa_0 = \frac{1}{3} \sigma_0^2. \quad (2)$$

e. The constant C is related to the slope E_p of the elastic-plastic line by the relation*:

$$C = \frac{2}{3} \frac{EE_p}{E - E_p}. \quad (3)$$

*The constant C is actually the slope of the elastic-plastic line in terms of deviatoric stress, σ' , vs plastic strain ϵ^p . That is, $C = d\sigma'/d\epsilon^p$, where, in the special case of a uniaxial specimen $\sigma' = 2/3\sigma$.

Fig. 16. Bilinear Representation of Monotonic, Initial-Loading Stress-Strain Curve

The choice of the maximum strain to be used in determining the appropriate bilinear representation must be left to the judgment and experience of the designer. Generally, the maximum strain should correspond to the maximum elastic-plastic strain expected in the structure. In most realistic design situations, a maximum strain of 0.2 or 0.3% would be appropriate. Values much higher than this will result in unrealistically high yield points. In multiaxial strain states, the maximum strain should be assessed on the basis of an effective strain value, $\bar{\epsilon}$, occurring at a point and defined by:

$$\bar{\epsilon} = \left[\frac{2}{3} \epsilon_{ij} \epsilon_{ij} \right]^{1/2}, \quad (4)$$

where ϵ_{ij} represents the elastic plus plastic strain tensor.

4.3.2.2 Bilinear Representations of Cyclic Stress-Strain Curves-Step Change. It is recognized that material in a structure can be hardened by prior loadings and that the use of stress-strain data from virgin material can lead to predictions of larger plastic strains than will actually occur in some applications. However, the sole use of curves obtained from material after hardening has occurred could give first loading predictions that are grossly in error. Therefore, results from initial monotonic loading tensile tests should be used for the first inelastic loading calculations. Calculations for loading cycles subsequent to the first inelastic loading should generally be based on a bilinear representation of the cyclic stress-strain curve corresponding to the curve for the tenth-cycle obtained from constant strain-range cyclic stress-strain tests as represented in Fig. 17.

The tenth-cycle stress-strain curves to be used in a specific analysis should be established on the basis of the same $\epsilon_{\max}$ as used in the initial loading analysis. That is, the curve should come from constant strain range tests in which uniaxial specimens are cycled between the fixed strain limits of $\pm\epsilon_{\max}$. Alternately, the cyclic hardening may be based on the expected amplitude of the strain cycle instead of the maximum value.

For each temperature, the constant κ_1 should be determined by the following procedures illustrated in Fig. 17.

- a. The slope of the plastic portion of the bilinear representation of the tenth-cycle (and consequently the value of C) is equal to that of the bilinear representation of the initial monotonic stress-strain curve.
- b. The intersections of the elastic and elastic-plastic lines are located so that the areas above and below the actual curve are approximately equal. This is, the two shaded areas on the tensile (positive stress) portion of the cyclic curve shown in Fig. 17 are equal, and the shaded areas on the compressive portion are equal.

Fig. 17. Bilinear Representations of Initial and Tenth-Cycle Stress-Strain Curves

c. The value κ_1 is determined from the points of intersection of the elastic lines with the elastic-plastic lines. The slope of the elastic line is E , and the slope of the elastic-plastic line is $E_p = (3/2 EC)/(E + 3/2 C)$. The method of construction is shown in Fig. 17.

d. The quantity κ_1 is related to the elastic stress range, $\Delta\sigma^E$, of the equivalent bilinear cyclic curve by the relation

$$\kappa_1 = \frac{1}{12} (\Delta\sigma^E)^2. \quad (5)$$

4.3.2.3 Bilinear Representations of Cyclic Stress-Strain Curves-Ramp Change. In using the bilinear representations for the initial monotonic curve and the tenth-cycle curve in an analysis, the size of the initial yield surface, which is used for the initial monotonic loading into the plastic region, is determined by κ_0 . The size of the yield surface for the first reversed plastic loading and for all subsequent plastic loadings is determined by κ_1 . In certain design analyses, it may be desirable to avoid the abrupt change in hardening that is associated with the switch from κ_0 to κ_1 by using one of the following alternate procedures which allow a gradual change in κ upon each loading reversal.

a. A value of κ may be obtained for each loading and unloading cycle of a constant strain-range cyclic stress-strain curve up to the n th cycle. Since the value of κ is to be changed only at each load reversal, the transition to the n th cyclic representation may be performed in $2n$ discrete steps and the flow rule given in 4.3.4 remains valid.

b. Alternatively, if sufficient data are available from cyclic stress-strain tests, the designer may be able to plot the κ values for each half cycle as a function of accumulated plastic strain (disregarding signs) and to adequately approximate κ as a function of accumulated plastic strain. The change in κ at each discrete load reversal may then be based on the actual accumulated plastic strain at that point (again without regard to direction).

In using either of the above procedures for gradually changing κ , the bilinear representation of each half-cycle stress-strain curve should be determined using the same procedures as those recommended for the tenth-cycle bilinear representations. In particular, the C value for all curves at a given temperature should be the same. Also, in either of the above procedures, consideration must be given to the effect of creep strains on cyclic hardening as discussed in 4.3.5.

4.3.3 Initial Yield Condition, Hardening Rule, and Loading Surfaces. Use is made of the six-dimensional stress space whose cartesian coordinates are defined by σ_{ij} ; that is, each axis is labeled by one of the components of the symmetric stress tensor. The von Mises flow law* is of the form:

$$d\epsilon_{ij}^P \sim \frac{\partial f}{\partial \sigma_{ij}}, \quad (6)$$

where ϵ_{ij}^P is the plastic strain. Since an associated flow law is recommended, the function f , by definition, describes the initial yield surface as well as the subsequent loading surfaces. The yield function f is thus used in the role of a plastic potential function. The flow law would be a nonassociated one if initial yielding were determined by the Tresca condition with the von Mises condition used as the plastic potential, or vice versa. In the guidelines that follow, the plastic potential function is taken to be the yield function of von Mises. The potential function describes the loading surface; the term yield surface is reserved for the designation of the initial yield surface of the virgin material.

The kinematic hardening model should be used to describe the loading surface behavior subsequent to initial yielding. The relationships for this model using the von Mises yield formulation are given below. If the Tresca condition is used to describe the yield surface, Ziegler's modification of Prager's kinematic hardening model should be used to assure invariance of the hardening rule in stress spaces of reduced dimensions. In any case, caution should be used in considering subspaces of the full nine-dimensional space.[2]

The initial yield surface and the subsequent loading surfaces for isothermal conditions are given by:

$$f = \frac{1}{2} (\sigma'_{ij} - \alpha_{ij})(\sigma'_{ij} - \alpha_{ij}) = \kappa, \quad (7)$$

$$\text{with: } d\alpha_{ij} = C d\epsilon_{ij}^P, \quad (8)$$

where: C, κ = bilinear material constants defined in 4.3.2,

α_{ij} = tensor representing the total translation in deviatoric stress space of the yield surface during plastic flow,

*In carrying out the partial differentiations, σ_{ij} and σ_{ji} ($i \neq j$) must be treated as separate quantities, that is, a nine-dimensional stress space must be considered for differentiating.

ϵ_{ij}^P = plastic strain tensor,

σ'_{ij} = deviatoric stress tensor.*

The initial yield surface and the loading surfaces for nonisothermal conditions are given by:

$$f' = f(\sigma_{ij}, \epsilon_{ij}^P) - \kappa(T) = 0, \quad (9)$$

where f is defined by Eq 7 and κ is again defined as in 4.3.2.

If the stress point is on the loading surface and a temperature change causes the surface to decrease in size, it is assumed that the surface moves in stress space (α_{ij} changes) so that the stress point remains on the surface. If the surface increases in size or if the stress point is inside the surface it is assumed that a temperature change does not cause the surface to move in stress space (σ_{ij} does not change). With these assumptions, C can be taken as a function of temperature and Eq 8 for the translation tensor becomes:

$$d\alpha_{ij} = C(T) d\epsilon_{ij}^P. \quad (10)$$

The hardening manner of the material is still characterized by a bilinear stress-strain relation at each temperature. However, the value of C must be reassigned as temperature changes. Under the assumption stated earlier and with the aid of Eq 10, it can be shown that the term $\partial f / \partial T$, which appears in the equations of the following subsection, is given by:

$$\frac{\partial f'}{\partial T} = - \frac{d\kappa}{dT}, \quad (11)$$

and the flow rule described in the following subsection is valid regardless of whether C is a function of temperature or a constant value.

*The deviatoric stress tensor is defined by:

$$\sigma'_{ij} = \sigma_{ij} - \frac{1}{3} \delta_{ij} \sigma_{kk},$$

where:

$$\delta_{ij} = \begin{cases} 0 & \text{for } i \neq j \\ 1 & \text{for } i = j \end{cases}, \quad i, j = 1, 2, 3.$$

4.3.4 Flow Rule. The basic form of the von Mises flow law is given by Eq 6. The specific equations for loading, neutral loading, and unloading under nonisothermal conditions* are given below for:

1. Loading:

$$d\epsilon_{ij}^P = \frac{1}{2C\kappa} \frac{\partial f'}{\partial \sigma'_{ij}} \left(\frac{\partial f'}{\partial \sigma'_{mn}} d\sigma'_{mn} + \frac{\partial f'}{\partial T} dT \right) \quad (12)$$

$$f' = 0 \quad (13)$$

$$\left(\frac{\partial f'}{\partial \sigma'_{ij}} d\sigma'_{ij} + \frac{\partial f'}{\partial T} dT \right) > 0 \quad (14)$$

2. Neutral Loading:

$$d\epsilon_{ij}^P = 0 \quad (15)$$

$$f' = 0 \quad (16)$$

$$\left(\frac{\partial f'}{\partial \sigma'_{ij}} d\sigma'_{ij} + \frac{\partial f'}{\partial T} dT \right) = 0 \quad (17)$$

3. Unloading:

$$d\epsilon_{ij}^P = 0 \quad (18)$$

$$f' \leq 0 \quad (19)$$

$$\left(\frac{\partial f'}{\partial \sigma'_{ij}} d\sigma'_{ij} + \frac{\partial f'}{\partial T} dT \right) < 0 \quad (20)$$

*The relations for isothermal conditions follow directly by neglecting the temperature dependence.

4.3.5 Procedures for Changing from κ_0 to κ_1 . The following procedures should be used in changing from $f = \kappa_0$ to $f = \kappa_1$ in the elastic-plastic portion of an inelastic analysis.

a. The changes should generally be made on an individual element-to-element basis, assuming finite-element techniques are used, or by an equivalent procedure if other analysis techniques are employed. Specific cases may infrequently exist for which the designer believes a more representative analysis will result if a simultaneous transition from κ_0 to κ_1 is made for all points in a structural model. This is permissible provided the designer justifies the procedure by considering the experimentally observed progression of work hardening or softening of the material, the applicable load histogram, the induced strain response, and the design requirements.

b. The change should be made immediately preceding the incurrence of reversed plastic strains after a program of initial loading in which plastic strain of any magnitude is incurred.* Reversed loading is defined as follows:

$$\alpha_{ij} \, d\epsilon_{ij}^P \leq 0, \quad (21)$$

where α_{ij} is the tensor denoting the total translation of the surface at the end of the initial plastic loading phase, and $d\epsilon_{ij}^P$ is the initial increment of reversed plastic strain.

c. Available experimental data (see Appendix A) suggest that prior creep (time-dependent) deformations have much the same hardening influence on subsequent cyclic elastic-plastic behavior as do accumulated plastic (time-independent) strains. Thus, the change from κ_0 to κ_1 should also be made when an effective creep strain of 0.2%, determined by the procedures in 4.4, has been accumulated.

4.3.6 Use of Effective Stress-Plastic Strain Relationships. In certain situations it may be desirable to have an analysis that is based on the actual monotonic, and perhaps even cyclic, stress-strain curves rather than on the bilinear equivalent. A nonlinear kinematic hardening approach based on effective-stress, plastic-strain relationships can be used for such analyses provided the loading is radial or near-radial. Equations for using effective-stress plastic-strain relationships are provided and discussed in this subsection.

*To avoid numerical difficulties in certain applications, it may be necessary to make the change before actual reverse yielding occurs. As an example, for near-radial loading situations an acceptable procedure would be to make the change when the effective stress decreases elastically by an amount $\sqrt{2} \sigma_0$ where σ_0 is the initial yield stress. The change from κ_0 to κ_1 does not cause the origin of the loading surface to translate.

When loading paths in a stress space are in radial directions (proportionally increasing or decreasing with respect to the zero-stress state), effective-stress plastic-strain relationships can be used with kinematic hardening. This was demonstrated experimentally through combined stress testing in which thin-walled tubes of type 304 stainless steel were subjected to combined tension and torsion loadings (see Appendix A). The effective stress and strain quantities under discussion here are given by:

$$\bar{\sigma} = \left[\frac{3}{2} \sigma'_{ij} \sigma'_{ij} \right]^{1/2}, \quad (22)$$

and:

$$\bar{\epsilon}^P = \left[\frac{2}{3} \epsilon^P_{ij} \epsilon^P_{ij} \right]^{1/2}. \quad (23)$$

The applicability of effective-stress, plastic-strain relationships can be deduced as follows. Under the isothermal kinematic hardening assumption, a more general expression of the form:

$$d\alpha_{ij} = g(\epsilon^P_{mn}) d\epsilon^P_{ij} \quad (24)$$

can be used in place of Eq 8. Taking the inner product of both sides and multiplying by 3/2 gives:

$$\left(\frac{3}{2} d\alpha_{ij} d\alpha_{ij} \right) = \left(\frac{3}{2} g \right)^2 \left(\frac{2}{3} d\epsilon^P_{ij} d\epsilon^P_{ij} \right). \quad (25)$$

Eq 25 can be written in the alternate form:

$$d\bar{\alpha} = \frac{3}{2} g d\bar{\epsilon}^P, \quad (26)$$

$$d\bar{\alpha} = \left(\frac{3}{2} d\alpha_{ij} d\alpha_{ij} \right)^{1/2} \quad (27)$$

$$d\bar{\epsilon}^P = \left(\frac{2}{3} d\epsilon^P_{ij} d\epsilon^P_{ij} \right)^{1/2}. \quad (28)$$

Thus, Eq 26 relates the effective translation measure ($\bar{\alpha}$) and the effective plastic strain. However, for this relationship to be usable, α must be replaced with a quantity that can be measured during test. During plastic loading along a radial path, the size of the loading surface does not change, and:

$$d\alpha_{ij} = d\alpha'_{ij} . \quad (29)$$

For the straight-line loading paths that are being considered here:

$$\begin{aligned} \int_0^{\alpha_{ij}} (d\alpha_{kl} d\alpha_{kl})^{1/2} &= \int_{\sigma'_{ij}{}^0}^{\sigma'_{ij}} (d\sigma'_{kl} d\sigma'_{kl})^{1/2} \\ &= [(\sigma'_{ij} - \sigma'_{ij}{}^0)(\sigma'_{ij} - \sigma'_{ij}{}^0)]^{1/2} . \end{aligned} \quad (30)$$

Thus, a stress quantity:

$$ds_{ij} = d(\sigma'_{ij} - \sigma'_{ij}{}^0) \quad (31)$$

can be defined, and Eq 26 can be written in terms of stress and plastic strain:

$$d\bar{s} = \frac{3}{2} g d^{-P} , \quad (32)$$

where:

$$d\bar{s} = \left(\frac{3}{2} ds_{ij} ds_{ij} \right)^{1/2} . \quad (33)$$

Here the components $\sigma'_{ij}{}^0$ correspond to the stress state at the onset of plastic flow with respect to the most current elastic stress state. The quantity s_{ij} is then the stress tensor whose components are measured from the values corresponding to the stress state when yielding first occurred along the current loading segment, and Eq 32 is an equivalent-stress plastic-strain relationship. The function g gives the instantaneous slope of the curve. In the case of uniaxial stress vs plastic strain curves:

$$g = \frac{2}{3} \frac{d\sigma}{d\epsilon^P} . \quad (34)$$

Having established the existence of the equivalent relationship, the restrictions on its use correspond to the limitations imposed for the derivation. Namely, the loadings must be radial. Although cyclic loading along radial paths can be treated using effective-stress plastic-strain relationships, this is made impractical in most situations by the fact that relationships appropriate to each segment of loading (or loading and unloading) are required. Since the influence of prior plastic deformation on subsequent deformation response is very significant, curves corresponding to the loading history of each element in a structure would be required. To use a single representative curve is no better than using a bilinear representation.

In reality, the above discussion is centered on the function g in Eq 24. This function is strongly dependent upon the loading history (ϵ_{ij}^P) and the temperature (T) at least for initial plastic deformation along any loading segment. Thus, the temperature dependence must also be factored into effective-stress plastic-strain correlations.

In summary, the rigorous use of effective-stress effective-plastic strain relationships is applicable, in a practical sense, to radial monotonic loadings only. Further, the relationships are temperature-dependent and this dependency must be accounted for in their use.

For initial radial loadings, an isotropic hardening model could be used as well as the kinematic hardening mode. However, the isotropic hardening model* should not be used for initial loading analyses when these loadings are nonradial. This is explained, in part, below. According to an isotropic hardening model the loading surface as depicted geometrically in stress space, is defined by a uniform expansion of the initial yield surface. Upon occurrence of plastic deformation, the surface expands and the center remains at the origin. Therefore, the equation describing the surface is of the form:

$$f(\alpha_{ij}) = \kappa, \quad (35)$$

where κ depends either on the plastic work or effective plastic strain.

*For the isotropic hardening model, the effective plastic strain is defined by:

$$\epsilon^P = \int_0^P \frac{1}{mn} \left(\frac{2}{3} d\epsilon_{ij}^P d\epsilon_{ij}^P \right)^{1/2}$$

rather than by Eq 23. For radial loadings the two definitions are identical.

To demonstrate that the effective-stress effective-plastic strain relationship which is inherent to isotropic hardening does not apply for the kinematic hardening case, and to demonstrate fundamental differences under nonradial loading cases, consider Fig. 18. The movement of a von Mises type loading surface in a two-dimensional stress space is shown for kinematic hardening when a monotonic uniaxial loading (σ_1) to point 2 is initially imposed. Since this loading is beyond the initial yield surface (point 1), the loading surface translates as shown. Next consider loading along the path from point 2 to point 3 which lies along a constant effective stress ($\bar{\sigma}$) surface.

$$\bar{\sigma}^2 = \frac{3}{2} \sigma'_{ij} \sigma'_{ij} \quad (36)$$

Along this surface $\bar{\sigma}$ equals the uniaxial stress value at point 2. The effective-stress plastic-strain relations associated with isotropic hardening would indicate that no change in plastic strain is experienced along this loading path. However, the loading path from point 2 to point 3 clearly extends into a region exterior (plastic region) to the kinematic hardening loading surface which was established by the loading to point 2. Therefore, additional plastic straining will occur according to the kinematic hardening model along this loading path. Obviously the effective plastic strain is not uniquely determined by the effective stress when considering kinematic hardening.

4.4 Time-Dependent Creep Analyses.

4.4.1 Basic Model Recommendations. An equation-of-state approach for creep analyses is recommended on the basis of available experimental data, computational methods, and analysis experience. Constitutive equations based on the equation-of-state approach generally require three ingredients, somewhat similar to the ingredients required to describe elastic-plastic behavior. These are:

1. A uniaxial creep law describing the experimental uniaxial, constant-stress, isothermal, creep curves.
2. A so-called "flow-rule" for multiaxial conditions.
3. A hardening law for prescribing the specific manner in which the formulation applies to variable stress conditions.

Although a specific form is generally used in the following paragraphs, the creep law can theoretically be any convenient algebraic equation that adequately describes the constant-uniaxial-stress creep curves. Existing creep data do not permit the use of a creep law for compressive stresses that differs from that for tensile stresses. The equation-of-state approaches to be used are based on the assumptions that the material is isotropic and incompressible with regard to creep

Fig. 18. Comparison Between Kinematic Hardening Loading Surfaces and Surfaces of Constant Effective Stress

strains and that the creep strains are indifferent to hydrostatic states of stress. The flow rule for multiaxial stresses is based on colinearity of creep strain rate and deviatoric stress tensors. The basic form of the constitutive equation is:

$$\dot{\epsilon}_{ij}^C = \lambda \sigma'_{ij}, \quad (37)$$

where ϵ_{ij}^C and σ'_{ij} are the components of creep strain and deviatoric stress tensors respectively, and the dot denotes creep strain rate. In general, the scalar proportionality function λ depends upon the invariants of the deviatoric stress tensor and of the creep strain tensor as well as other scalar variables such as time and temperature. Since the effective stress and strain variables ($\bar{\sigma}$ and $\bar{\epsilon}$) are proportional to the second invariant (J_2) of σ'_{ij} and the second invariant (I_2) of ϵ_{ij}^C , λ can be expressed in terms of $\bar{\sigma}$ and $\bar{\epsilon}$. Specifically:

$$\bar{\sigma}^2 = 3J_2' = \frac{3}{2} \sigma'_{ij} \sigma'_{ij} \quad (38)$$

and:

$$\bar{\epsilon}^2 = \frac{4}{3} I_2 = \frac{2}{3} \epsilon_{ij}^C \epsilon_{ij}^C. \quad (39)$$

An added condition, that the multiaxial constitutive equations must incorporate a proper description of the uniaxial stress case, leads to the form:

$$\dot{\epsilon}_{ij}^C = \frac{3}{2} \frac{\dot{\bar{\epsilon}}(\bar{\sigma}, t, T)}{\bar{\sigma}} \sigma'_{ij}. \quad (40)$$

The requirement that Eq 40 degenerate to the uniaxial-stress case requires $\dot{\bar{\epsilon}}(\bar{\sigma}, t, T)$ to be the constant-uniaxial-stress creep law with axial stress and strain variables replaced by their effective counterparts. Therefore, the constitutive equations for the basic equation-of-state approach are composed of Eq 40 and a creep law which mathematically expresses the experimentally observed axial creep strain as a function of constant-uniaxial-stress, time, and temperature.

The manner in which these equations apply to variable stress conditions depends upon the hardening law chosen. This choice is reflected by the variable used to express the effective creep strain rate $\dot{\bar{\epsilon}}$. Time-hardening results if stress, time, and temperature are used. If stress, strain, and temperature are used through the elimination of time between $\bar{\epsilon}$ and $\dot{\bar{\epsilon}}$, strain-hardening results. On the basis of

experimental observations of variable load creep tests such as those discussed in Appendix A, strain-hardening methods are recommended for creep analyses involving types 304 and 316 stainless steel. Strain hardening is considered to be applicable without modification so long as stress reversals do not occur (changes in magnitude are permitted). Auxiliary rules are recommended in 4.4.4 for use when stress reversals are encountered.

4.4.2 Uniaxial Creep Law. Equation-of-state approaches require as an input a creep law which mathematically expresses experimentally observed axial creep strain as a function of constant-uniaxial-stress, time, and temperature. The law must include both primary and secondary creep strains. The creep law considered in the following paragraphs is of the form:

$$\bar{\epsilon}^C(\bar{\sigma}, t, T) = f(\bar{\sigma}, T) [1 - e^{-r(\bar{\sigma}, T)t}] + g(\bar{\sigma}, T)t, \quad (41)$$

where: f, r, g = functions of stress and temperature

t = time

T = temperature

$\bar{\epsilon}^C$ = effective creep strain

$\bar{\sigma}$ = effective stress.

For a uniaxial specimen $\bar{\epsilon}^C$ and $\bar{\sigma}$ reduce to the axial creep strain (ϵ^C) and the applied uniaxial stress (σ) respectively, and Eq 41 represents the uniaxial creep law.

The effective creep strain given by Eq 41 may be considered to be composed of a primary, or transient, creep component, $\bar{\epsilon}^t$, and a steady-state creep component, $\bar{\epsilon}^s$, where:

$$\bar{\epsilon}^t = f(\bar{\sigma}, T) [1 - e^{-r(\bar{\sigma}, T)t}] \quad (42)$$

and:

$$\bar{\epsilon}^s = g(\bar{\sigma}, T)t. \quad (43)$$

In addition to the representation given by Eq 41, current creep laws are given in Ref 2. Representations not appearing in Ref 2 should receive prior approval by the owner.

4.4.3 Flow Rule and Hardening Law. For variable stress and temperature conditions, creep strains should be based on the strain-hardening law. Two approaches to strain-hardening computations have been used and either is acceptable. The flow laws for both are given below. These procedures may be used in a straightforward manner as long as loadings are constant or increasing. In the event of loading reversals the auxiliary rules described in 4.4.4 must be employed.

a. Strain-hardening on Total Creep Strain. The flow rule for this method is:

$$\dot{\epsilon}_{ij}^C = \frac{3}{2} \frac{\dot{\bar{\epsilon}}^C(\bar{\sigma}, \bar{\epsilon}^C, T)}{\bar{\sigma}} \sigma'_{ij}, \quad (44)$$

where: $\dot{\epsilon}_{ij}^C$ = creep strain-rate tensor

$\dot{\bar{\epsilon}}^C$ = effective creep strain rate

σ'_{ij} = deviatoric stress tensor.

The strain hardening is based on the total (primary plus secondary) effective creep strain which is calculated from the current values of the total creep strain components. The procedures are conceptually the same regardless of the creep law used since the total creep strain law is composed of at least two mathematical terms, one (or more) for the primary creep and one for the secondary creep. This precludes, except in rare cases, the elimination of time (t) between $\bar{\epsilon}$ and $\dot{\bar{\epsilon}}$ in a closed-form fashion. Therefore, analysis methods using strain hardening based on total creep strain must possess a scheme for solving numerically for time-values corresponding to given values of stress and total creep strain.

b. Strain Hardening Based on Primary Creep Strain. As an alternate, strain-hardening computations may be based on the primary creep component of the total creep strain. This method makes the elimination of time (t) between $\bar{\epsilon}^t$ and $\dot{\bar{\epsilon}}^t$ in a closed-form fashion possible for certain creep law forms, including Eq 42. When a closed-form solution is available, the need for a corresponding numerical scheme in analysis methods is eliminated. The specific flow rule depends upon the creep law used, but the general form is:

$$\dot{\epsilon}_{ij}^C = \frac{3}{2} \left[\dot{\bar{\epsilon}}^t(\bar{\sigma}, \bar{\epsilon}^t, T) + g(\bar{\sigma}, T) \right] \frac{\sigma'_{ij}}{\bar{\sigma}}, \quad (45)$$

where $\bar{\epsilon}^t$ is the effective primary, or transient, creep strain and $3/2 g(\bar{\sigma}, T) \sigma'_{ij} / \bar{\sigma}$ represents the steady-state creep rate.

The multiaxial flow rule, including the steady-state creep rate, for the creep law given by Eq 41 is:

$$\dot{\epsilon}_{ij}^C = \frac{3}{2} \{r(\bar{\sigma}, T) [f(\bar{\sigma}, T) - \frac{\bar{\sigma}^t}{\bar{\sigma}}] + g(\sigma, T)\} \frac{\sigma'_{ij}}{\bar{\sigma}}, \quad (46)$$

where the effective primary creep strain $\bar{\epsilon}^t$ is calculated from the primary creep strain components present at that time.

To avoid anomalous predictions when using Eq 46, it is necessary to equate the term $[f(\bar{\sigma}, T) - \frac{\bar{\sigma}^t}{\bar{\sigma}}]$ to zero whenever it is negative.

In principle, a numerical scheme analogous to that discussed relative to the strain-hardening procedure based on total creep strain can also be employed with the primary creep strain procedure. This would make possible the use of arbitrarily complex equations for representing primary creep. An example of a creep law which includes a two-term expression for the primary creep is:

$$\epsilon^C(\sigma, t) = f_1(\sigma, T) \left[1 - e^{-r_1(\sigma, T)t} \right] + f_2(\sigma, T) \left[1 - e^{-r_2(\sigma, T)t} \right] + g(\sigma, T)t. \quad (47)$$

A closed-form solution to this two-term primary creep expression for time cannot be easily found, and the use of a numerical scheme is considered acceptable. Obviously, the use of a multi-term expression such as that shown above for the creep law does, in general, permit a better representation of creep strain-time data than a single-term expression.

4.4.3.1 Effective Stress and Strain Definitions. The effective stress to be used in Eqs 41 through 47 is determined by the following relationship:

$$\bar{\sigma} = \left(\frac{3}{2} \sigma'_{ij} \sigma'_{ij} \right)^{1/2}, \quad (48)$$

where σ'_{ij} is the deviatoric stress. The definition of effective strain hardening depends upon the occurrence of stress reversals in the loading history. For cases where stress reversals do not occur (changes in stress magnitude are permitted), the effective total creep strain is determined by:

$$\bar{\epsilon}^C = \left(\frac{2}{3} \epsilon_{ij}^C \epsilon_{ij}^C \right)^{1/2} \quad (49)$$

Where the strain-hardening computations are based on the primary creep component, the effective primary creep strain is determined by:

$$\bar{\epsilon}^t = \left(\frac{2}{3} \epsilon_{ij}^t \epsilon_{ij}^t \right)^{1/2} \quad (50)$$

4.4.4 Recommended Auxiliary Procedures for Strain-Hardening Analysis Involving Load Reversals. The strain-hardening formulation recommended in 4.4.3 for use with the equation-of-state approach to creep analysis is considered applicable only so long as load reversals do not occur. If a change in load occurs at time t so that a load reversal does not occur, the strain-hardening procedure based on either total or primary creep strains can be applied in a straightforward manner. A specific definition of a load reversal is given later. If a stress reversal occurs at time t , auxiliary procedures should be employed along with the strain-hardening procedure previously given.

4.4.4.1 Auxiliary Procedures for Load Reversal Conditions. The general strain-hardening multiaxial creep equations are written in the form:

$$\dot{\epsilon}_{ij}^C = (\bar{\epsilon}^H, \bar{\sigma}, T) \bar{\sigma}'_{ij} = \frac{\dot{\bar{\epsilon}}(\bar{\epsilon}^H, \bar{\sigma}, T)}{\bar{\sigma}} \quad (51)$$

where: $\dot{\epsilon}_{ij}^C$ = the total strain-rate components

$\dot{\bar{\epsilon}}$ = creep rate as a function of $\bar{\epsilon}^H, \bar{\sigma}$, and temperature as determined from constant (uniaxial) stress test correlations

$\bar{\epsilon}^H$ = the current value of strain hardening on either a total or primary creep strain basis

(In the usual strain-hardening procedure this is a measure of current effective creep strain)

$\bar{\sigma}$ = the effective stress

$\bar{\sigma}'_{ij}$ = the deviatoric stress components.

The applicability of Eq 51 is extended to multiaxial load reversals by redefining the strain hardening $\bar{\epsilon}^H$ relative to reference creep strain states and by determining its value according to the steps of the following generalized procedures and as illustrated in Fig. 19.

1. Define:

Strain-hardening value, $\bar{\epsilon}^H$	=	$\begin{cases} \text{Value based on } \epsilon_{ij}^C, \text{ when strain is based on total creep strain} \\ \text{Value based on } \epsilon_{ij}^t, \text{ when strain hardening is based on primary creep strain,} \end{cases}$
Instantaneous strain value, ϵ_{ij}^I	=	$\begin{cases} \epsilon_{ij}^C, \text{ when strain hardening is based on total creep strain} \\ \epsilon_{ij}^t, \text{ when strain hardening is based on primary creep strain,} \end{cases}$
$\epsilon_{ij}^+, \epsilon_{ij}^-$	=	Two possible strain origins which exist at any time in either total creep strain space as appropriate,
Initially $\epsilon_{ij}^+, \epsilon_{ij}^-$	=	0 for fully solution treated material,
$\hat{\epsilon}$	=	An effective strain quantity which in the multiaxial case is the equivalent to the distance between origins for the uniaxial case,
G	=	A measure, on an effective strain basis, of the distance between an instantaneous strain state and the appropriate one of the two strain origins (see Eq 39),

$$G^+ = G \left(\epsilon_{ij}^I - \epsilon_{ij}^+ \right) = \left[\frac{2}{3} \left(\epsilon_{ij}^I - \epsilon_{ij}^+ \right) \left(\epsilon_{ij}^I - \epsilon_{ij}^+ \right) \right]^{1/2}, \quad (52)$$

$$G^- = G \left(\epsilon_{ij}^I - \epsilon_{ij}^- \right) = \left[\frac{2}{3} \left(\epsilon_{ij}^I - \epsilon_{ij}^- \right) \left(\epsilon_{ij}^I - \epsilon_{ij}^- \right) \right]^{1/2}. \quad (53)$$

Fig. 19. Flow Chart for Determining Creep Strain Hardening During Stress Reversals

Definition of load reversal: For multiaxial conditions a load reversal is considered to occur whenever the effective creep strain (G^+ or G^-) measured from the current origin (ϵ_{ij}^+ or ϵ_{ij}^-) begins to decrease. Since an increment in the effective creep-strain is given by a product which involves increments of the creep components (which are colinear with the deviatoric stress components), the condition for a load reversal is that the deviatoric stress be directed toward the current origin. More precisely, if the current origin is ϵ_{ij}^+ , a load reversal occurs when the product:

$$(\epsilon_{ij}^I - \epsilon_{ij}^+) \sigma'_{ij} < 0 .$$

Because no volume change occurs during creep, $\epsilon_{kk}^I = 0$, and the above inequality can be replaced by:

$$(\epsilon_{ij}^I - \epsilon_{ij}^+) \sigma_{ij} < 0 . \quad (54)$$

Similarly, if the current origin is ϵ_{ij}^- , a load reversal occurs when:

$$(\epsilon_{ij}^I - \epsilon_{ij}^-) \sigma_{ij} < 0 . \quad (55)$$

Whenever a load reversal is detected at the beginning of a time increment as described below, the origin is switched (and reset if necessary) before the incremental creep strains are calculated.

2. For the initial unloaded case (virgin material) set:

$$\epsilon_{ij}^+ = \epsilon_{ij}^- = \hat{\epsilon} = 0. \quad (56)$$

Therefore, for the initial loading of the virgin material until the first reversal occurs, the creep rate is determined from Eq 51 with ϵ^H given by:

$$\epsilon^H = G \left(\epsilon_{ij}^I \right) . \quad (57)$$

3. Analysis iterations are performed until the first reversal occurs.

$$\left(\epsilon_{ij}^I \right) \sigma_{ij} < 0. \quad (58)$$

The value of $\bar{\epsilon}^H$ is determined using Eq 57 until the first reversal occurs.

4. When the first reversal occurs, ϵ_{ij}^- and $\hat{\epsilon}$ are set equal to ϵ_{ij}^I and $G(\epsilon_{ij}^I)$ respectively, and the origin (which until now has been zero) is switched to ϵ_{ij}^- . Between the first and second reversals $\bar{\epsilon}^H$ is evaluated using Eq 53.

The next reversal is detected when Eq 55 is satisfied and if $G^- > \hat{\epsilon}$, ϵ_{ij}^+ and $\hat{\epsilon}$ are set equal to ϵ_{ij}^I and G^- respectively and the origin is switched to ϵ_{ij}^+ . Between the second and third reversals $\bar{\epsilon}^H$ is evaluated using Eq 52.

5. The procedures for determining the value of $\bar{\epsilon}^H$ for the first three reversals are provided above. They are the initial steps of the general procedures given below for detecting load reversals and determining the value of $\bar{\epsilon}^H$. The procedures given below apply generally.
6. Subsequent to each analysis iteration the response of each element should be checked using Eqs 54 and 55 to determine if a load reversal has occurred. If a reversal did occur the procedures of steps 7 and 8 should be used. Steps 8 and 9 provide the general procedure for evaluating $\bar{\epsilon}^H$.
7. In most practical cases the creep strain increments are always directed away from the current origin. However, it is possible to have a condition where both:

$$\left(\epsilon_{ij}^I - \epsilon_{ij}^+ \right) \sigma_{ij} < 0, \quad (54)$$

and:

$$\left(\epsilon_{ij}^I - \epsilon_{ij}^- \right) \sigma_{ij} < 0, \quad (55)$$

and neither origin is to be reset (see steps 8a and 8b). If the rules of step 8 and Eqs 54 and 55 are used without alteration, every increment will be interpreted as a stress reversal and thus will result in a switch of origins. To avoid this problem of possible repeated oscillations between origins when moving along a single path, the most distant origin should be used. That is, ϵ_{ij}^I should be used as the origin and the effective strain hardening determined as in step 9b if:

$$G \left(\epsilon_{ij}^I - \epsilon_{ij}^+ \right) \geq G \left(\epsilon_{ij}^I - \epsilon_{ij}^- \right), \quad (59)$$

and ϵ_{ij}^- should be used as the origin and the effective strain hardening determined as in step 9a if:

$$G \left(\epsilon_{ij}^I - \epsilon_{ij}^+ \right) < G \left(\epsilon_{ij}^I - \epsilon_{ij}^- \right). \quad (60)$$

8. a. In general, when the current origin is ϵ_{ij}^+ , a stress reversal is detected when Eq 54 is satisfied and the following steps are taken.

If:
$$G \left(\epsilon_{ij}^I - \epsilon_{ij}^+ \right) > \hat{\epsilon}, \quad (61)$$

leave ϵ_{ij}^+ unchanged and reset:

$$\epsilon_{ij}^- = \epsilon_{ij}^I, \quad (62a)$$

$$\hat{\epsilon} = G \left(\epsilon_{ij}^I - \epsilon_{ij}^+ \right). \quad (62b)$$

Otherwise:
$$G \left(\epsilon_{ij}^I - \epsilon_{ij}^+ \right) \leq \hat{\epsilon}, \quad (63)$$

and ϵ_{ij}^+ , ϵ_{ij}^- , and $\hat{\epsilon}$ are left unchanged.

In either case the origin should be reset to ϵ_{ij}^- and the value of ϵ^H is determined subsequent to the reversal by step 9a.

- b. When the current origin is ϵ_{ij}^- , a stress reversal is detected when Eq 55 is satisfied and the following steps are taken:

If:
$$G \left(\epsilon_{ij}^I - \epsilon_{ij}^- \right) > \hat{\epsilon} , \quad (64)$$

leave ϵ_{ij}^- unchanged and reset:

$$\epsilon_{ij}^+ = \epsilon_{ij}^I , \quad (65a)$$

$$\hat{\epsilon} = G \left(\epsilon_{ij}^I - \epsilon_{ij}^- \right) . \quad (65b)$$

Otherwise:

$$G \left(\epsilon_{ij}^I - \epsilon_{ij}^- \right) \leq \hat{\epsilon} , \quad (66)$$

and ϵ_{ij}^+ , ϵ_{ij}^- , and $\hat{\epsilon}$ are left unchanged.

In either case the origin should be reset to ϵ_{ij}^+ and the value of $\bar{\epsilon}^H$ is determined subsequent to the reversal by step 9b.

9. a. With the origin at ϵ_{ij}^- the effective strain hardening, $\bar{\epsilon}^H$, is defined by:

$$\bar{\epsilon}^H = G \left(\epsilon_{ij}^I - \epsilon_{ij}^- \right) . \quad (67)$$

This relation is used to determine the effective strain hardening until the next load reversal occurs.

- b. With the origin at ϵ_{ij}^+ , the effective strain hardening, $\bar{\epsilon}^H$, is defined by:

$$\bar{\epsilon}^H = G \left(\epsilon_{ij}^I - \epsilon_{ij}^+ \right) . \quad (68)$$

This relation is used to determine the effective strain hardening until the next load reversal occurs.

10. In the time-increment, initial-strain finite-element analysis procedure the above rules are to be applied on an element-by-element basis. Furthermore, these auxiliary rules for load reversal situations were formulated for use only in near-radial loading situations. Care should be taken if significantly non-radial loadings are encountered.

4.4.4.2 Interpretation of Auxiliary Procedures. The auxiliary hardening procedures are much simpler in concept than the above procedure would imply. The purpose of the following discussion is to illustrate the ideas behind the auxiliary procedures and to describe the shortcomings in the classical strain-hardening procedures when stress reversals occur.

Consider the case of a uniaxial creep specimen subjected first to a tensile stress of $+\sigma$ and then to a compressive stress of $-\sigma$. During the tensile portion of the loading strain hardening occurs, and the strain-hardening procedure would predict that upon changing to the compressive loading this accumulated strain hardening would be retained. For example, if the secondary creep portion of the creep response had been reached in tension, then the strain-hardening procedure would predict a compressive creep strain response beginning in the secondary creep region. Subsequent to the change to a compressive stress, the effective creep strain, as computed from the creep strain at any given time, decreases as the creep strain rate under the compressive stress is negative. Hence, in an incremental analysis which utilizes small time increments the effective creep strain, and hence the strain hardening, decreases from increment to increment. The net result is that the creep strain rate increases with time as the effective strain decreases toward zero. For the example in which the secondary portion of the creep response is reached in tension before reversing the stress, a time-incremental analysis using strain hardening would predict a compressive creep response starting with secondary creep and proceeding backwards into primary creep. After reaching zero the strain hardening would then start to accumulate again and the creep strain rate would start to diminish back towards the steady-state value.

The recommended auxiliary procedures are intended to overcome inconsistencies such as those described above. The procedures are relatively simple and should present no serious problems with regards to their incorporation into existing inelastic structural analysis computer programs. There are other potential inconsistencies, such as that discussed in connection with Eq 46, that may arise in connection with the use of a specific form of the creep law.

In order to assist interpretation, the recommended auxiliary procedures are described here in terms of uniaxial behavior. As a starting point consider, graphically, the application of strain-hardening procedures to situations not involving stress reversals. This is illustrated in Fig. 20 for the case of a three-step loading sequence involving the uniaxial tensile stresses $\sigma_1 < \sigma_2 < \sigma_3$. The top sketch shows the stress-

Fig. 20. Uniaxial Creep Response Predicted by Strain Hardening
When No Stress Reversals Are Involved*

*The strain hardening shown is based on total creep strains.

time history; the middle sketch, the uniaxial constant-stress creep curves and the usual strain-hardening procedure*; and the bottom sketch, the resulting predicted creep response.

It can be seen that for any stress history in which no stress reversal (change in sign) occurs, strain hardening, whether based on total creep strain or on primary creep strain, is applied in a straight-forward manner. Hardening continues so long as creep strain, either total or primary as the case may be, is accumulated.

Now consider a situation in which a stress reversal is involved such as shown in Fig. 21. During the period in which the initial tensile stress σ_3 acts, total creep strain denoted by ϵ_1 is accumulated, and on the basis of total creep strain, ϵ_1 is also a measure of the hardening accumulated. When the stress changes to $-\sigma_1$ at time t_1 , the auxiliary procedures assume that all the strain hardening (ϵ_1) accumulated in tension is lost. Thus the creep response produced by the compressive stress starts at zero strain hardening just as in the case of a virgin specimen.

When the stress is changes back to tension at time t_2 , hardening equal to $\epsilon_1 - \epsilon_2$ remains in tension if ϵ_2 , the creep strain accumulated in compression, is smaller in magnitude than ϵ_1 . The tensile creep response then starts at this value on the σ_2 creep curve as shown in Fig. 21. If ϵ_2 had been larger in magnitude than ϵ_1 , then all of the tensile strain hardening would be eliminated and the creep response on σ_2 curve would start at zero strain just as for a virgin specimen. Going one step further, if creep strain with magnitude ϵ_3 is accumulated at stress σ_2 and if ϵ_3 exceeds ϵ_2 in magnitude, the hardening accumulated in compression is lost, and a subsequent change to a compressive stress would produce an assumed creep response starting at zero strain on the appropriate compressive creep curve (assumed identical to the corresponding tensile creep curve). If a change to compression occurs before ϵ_3 exceeds ϵ_2 in magnitude, then hardening equal to $\epsilon_2 - \epsilon_3$ would remain in compression.

Consider that the equations for either of the recommended strain-hardening procedures reduce to a general uniaxial strain-hardening equation of the form:

$$\dot{\epsilon}^c = f \left(\epsilon^H, \sigma, T \right), \quad (69)$$

*Although the strain hardening is shown graphically based on total creep strain, the basis could just as well be primary creep strain.

Fig. 21. Presumed Uniaxial Creep Response When Stress Reversals are Involved*

*The strain hardening shown is based on total creep strain.

where $\dot{\epsilon}^C$ is the total creep strain (primary plus secondary) rate and ϵ^H is the current creep strain, which in the usual strain-hardening procedure is also a measure of the accumulated hardening.*

The strain-hardening law given by Eq 69 is considered applicable only when stress reversals are not considered, that is, when the stresses change in magnitude but not in direction. For cyclic uniaxial loadings involving stress reversals, the auxiliary rules extend the applicability of Eq 69 by redefining ϵ^H to be reference creep strain determined according to the following rules:

a. At any time there exists two possible creep strain origins, ϵ^+ and ϵ^- , as shown in Fig. 22. The strain ϵ^+ is a negative quantity, and ϵ^- is a positive quantity.

b. Initially for a virgin specimen, $\epsilon^+ = \epsilon^- = 0$.

c. For positive stresses, the creep rate is determined from Eq 69 with ϵ^H defined by

$$\epsilon^H = \epsilon - \epsilon^+.$$

For negative stresses, the creep rate is determined with ϵ^H defined by

$$\epsilon^H = \epsilon - \epsilon^-.$$

Here, ϵ is the current creep strain, either primary or total, depending on the strain-hardening law.

d. For arbitrary stress reversals, let $\epsilon_1, \epsilon_2, \epsilon_3, \dots, \epsilon_n$ denote the values of ϵ at the time of the first, second, etc., stress reversals, respectively, and let $\epsilon_0 = 0$ denote the initial creep strain. Then after the nth stress reversal:

$$\epsilon^+ = \min_{i=0, n} \epsilon_i,$$

$$\epsilon^- = \max_{i=0, n} \epsilon_i.$$

*The strain ϵ^H may denote either primary creep strain or total creep strain depending on the type of strain hardening adopted.

Fig. 22. Presumed Creep Response for Arbitrary Uniaxial Loading Sequences

The creep response predicted by these procedures for a case involving ten arbitrary stress reversals is depicted in Fig. 22.* Note that $\hat{\epsilon}$ corresponds here to the strain value between ϵ^+ and ϵ^- . The points indicate the creep strain origin used for the following step of the curve. In step 1, $\epsilon = 0$ is used as the origin. In step 2, which involves a stress reversal, the origin ϵ^- is reset from its initial zero value. The creep behavior for step 2 is then the virgin uniaxial-constant-stress creep curve shifted to the new origin represented by the point and reversing the direction of creep to account for the stress reversal. The shifted origins for subsequent steps are shown by the points. At each stress reversal, the origin is switched so that the strain always moves away from the current origin. The origin strain, ϵ^+ or ϵ^- , is reset only when it is exceeded. In Fig. 22 this occurs at the beginning of steps 2, 6, and 9.

4.5 Guidelines for Materials Other Than the Austenitic Stainless Steel.

(In the course of preparation)

*The response shown in Fig. 22 is based on a constant stress acting between each stress reversal. More generally the stresses in an actual case would vary in magnitude in the intervals between reversals, and the response would look something like that shown in Fig. 20.

5. VERIFICATION AND QUALIFICATION OF ANALYSIS METHODS

5.1 Introduction. This section provides guidelines for verification and qualification of analysis methods. For illustrative purposes reference is made only to structural finite element computer programs.

The use of large computer programs to perform thermo-structural design calculations introduces questions of validation and acceptance of the analysis results. These questions, in turn, involve the concepts of verification and qualification. Verification is defined as the demonstration that a computer program correctly solves the model that was programmed, independently of whether or not the model is a valid representation of any particular system. Qualification is concerned with the use of computer programs for solving the real problems encountered in design. Given that a computer program correctly solves the model programmed, does the combination of mathematical model, discretization, description of material properties, representation of the loading and temperature histories, and boundary conditions, all consistent with the program limitations, give an acceptable solution to the physical problem? RDT F 9-4 specifies that the manufacturer shall describe, in the structural evaluation plan (SEP), the extent to which the computer programs to be used have been verified, or will be verified, and the extent to which they have been qualified, or will be qualified. The intent of this section is to provide the manufacturer, as well as the owner, with background and general guidelines for determining the scope and content of a minimum verification and qualification effort.

Program verification, to a considerable degree, is performed by the program developer and program sponsors. However, the manufacturer (user) accepts the responsibility for verification since he alone has the ultimate responsibility for the validity of his design calculations. Unless the manufacturer or the owner has experience or documented evidence to the contrary, the manufacturer should assume that the program he is about to use has not been adequately verified over the combinations of parameters of interest, and he should describe in the SEP additional verification effort deemed necessary.

Qualification, as opposed to verification, is almost entirely the responsibility of the manufacturer as he attempts to apply a computer program to his specific design problems. The manufacturer should describe in the SEP the minimum qualification effort which he intends to undertake to demonstrate the validity of his computer-aided structural analyses. Full account should be taken of any previous experience and qualification efforts.

5.2 Background. The structural analysis computer programs listed in Appendix B are relatively large and complex. Consequently there are numerous sources of error in the process of program development. There may be errors in the various theories and formulations and in the numerical procedures as well as errors in the programming because of the mass of logic to be developed and translated into a machine acceptable

language. Errors can also be introduced when converting a program from one computer to another, even if they are the same model.

Most of the programming errors will hopefully have been located and corrected by the developer, the program originator or both during debugging and in the initial stages of program verification. Nonetheless, it is not realistic to believe that any large program, no matter how long it has been in use, is error free. Although good comparisons with exact solutions or with experimental data may have been obtained for a few problems covering a limited range of geometries, loading conditions, and other parameters, there is no assurance that the program is correct or will even run for another range of parameters. In fact, it seems impossible to verify a general purpose program over all ranges and combinations of parameters for all options. Thus verification of a program must, in general be approached with respect to some limited range of options and parameters determined by the intended application.

Large two- and three-dimensional finite-element programs for inelastic analyses require considerable engineering experience and ingenuity on the part of the user. There is the choice, associated with any finite-element analysis, of the finite-element model, i.e., the discretization of the actual structure into finite elements. This requires experience and judgment to obtain satisfactory results. But even more significant are the inelastic aspects of the analysis procedure. Inelastic analyses are conducted on incremental and iterative bases. The time-history of loads and temperatures must be divided into increments of loading, temperature and time. The choice of these increments is often left to the program user and depends on the expected elastic-plastic and creep behavior of the structure. A poor choice can easily lead to instabilities in the solution, erroneous results or both. Consequently, it must be recognized that, at present, performing a complex inelastic analysis of a realistic structure is very much an art and can be successfully done only by persons experienced and trained in the proper techniques. Thus, the question of validation and acceptance of inelastic design analysis results involves not only verification and qualification of the computer program but qualification of the user as well, in the sense that the manufacturer has the responsibility for assuring that his designer has the necessary training and experience to successfully apply the program to realistic inelastic design problems.

The goal of the manufacturer should be to establish and document, in an efficient manner and with as little overall duplication of effort as possible, a level of confidence in his inelastic design analyses consistent with the high level of structural integrity appropriate for high-temperature reactor structures. Although the details of how this can best be accomplished in each particular case must be decided upon by the manufacturer, and agreed to by the owner, it is suggested that use be made of a limited verification and qualification effort based on performing benchmark calculations; this is, by solving a small number of carefully chosen problems with known solutions. Collectively, the selected benchmark problems should demonstrate those aspects of the

computer program and of the user's ingenuity that are required in the design analyses to be performed. Satisfactory solution of the benchmark problems will demonstrate the user's ability to properly use the program and at the same time verify the program for the particular job at hand.

In the following discussion the benchmark problem concept is established and discussed, and typical benchmark problems are described. Guidelines for verification and qualification based on the benchmark problem concept are provided.

5.3 Benchmark Problem Calculations. The concept of benchmark problem calculations is not intended for the verification and qualification of computer programs and users per se; rather, it is intended to assist structural design analysis and to provide a basis for confidence in the results. The assessment and improvement of the correspondence between finite-element solutions and actual behavior is the real charter of a benchmark problem effort. Specifically, satisfactory solution of a carefully chosen set of benchmark problems serves to:

1. Verify the designer's skill in using the program, especially in terms of finite-element modeling and of inputting time-histories of mechanical and thermal loadings on an incremental basis.
2. Verify those specific and significant features of the program to be used in the actual design analyses of interest.
3. Verify the adequacy of the constitutive equations for the intended application.

The overall result is to qualify the program and user for the intended application and to provide a documented demonstration of the level of accuracy appropriate to the design analyses.

Candidate benchmark problems can be usefully classified into two categories; these categories and the bases for choosing each are listed below.

5.3.1 Simple Problems. Group 1 for user training and basic program verification.

1. Elementary components with simple geometries.
2. Inelastic behavior.
3. Realistic loading histories.
4. Exact solutions available for correlation.

5. Problems sufficiently varied to provide user with basic experience and to exercise basic inelastic program features.

5.3.2 Advanced Problems. Group 2 for verification and qualification of program and user.

1. Geometry relatively simple yet characteristic of typical areas of high-temperature reactor components.
2. Comparison data available from carefully conducted experiments.
3. Behavior usually inelastic, although elastic benchmark problems involving complex shell and three-dimensional geometries important.
4. Realistic loading histories.
5. Group of problems sufficiently varied to exercise each important aspect of the computer program and user's ingenuity required in the final design analyses.

A typical, but by no means inclusive, set of realistic benchmark problems is listed in Table 1 and illustrated in Figs. 23 and 24 for Groups 1 (5.3.1) and 2 (5.3.2) respectively.* Those problems in Group 1 each meet the criteria of being simple basic problems with known exact solutions. They are well suited for use in the initial stages of user training and program verification.

The problems in Group 2 are more advanced and more realistic in terms of geometry, loading, and behavior. All of the Group 2 problems represent structures and loading conditions which are currently being tested, or are definitely planned for testing, in the national base-technology programs to develop and evaluate high-temperature design methods.^[4,5] Successful completion of a small select subgroup of these, or similar problems, could verify those features of the program required for the particular design analysis at hand and, at the same time, qualify both program and user.

The selection of a potential group of benchmark problems and the choice of the specific problems to be solved for verification and qualification in connection with a given design analysis depend on many things, including the features and salient computational characteristics of the design job at hand and the availability of applicable experimental data. However, the problems listed in Table 1 should be prime candidates because of their pertinence and because carefully obtained experimental data and material properties information are being obtained.

*Additional benchmark problems are given in Reference 3.

Table 1 Typical Set of Benchmark Problems

Problem No.	Description	Comments
<u>Group 1</u>		
1.1	Uniaxial specimen subjected to reversed cyclic loading into plastic range	Analysis should produce cyclic stress-strain curve. Good for studying load incrementing choices.
1.2	Uniaxial relaxation specimen under isothermal conditions	Analysis should reproduce simple calculations or actual relaxation test results. Good for studying time increment choices in creep analyses.
1.3	Uniaxial specimen subjected to stepped and reversed creep loading.	Analysis should produce simple creep curves with hardening rules considered. Good for time-increment selection.
1.4	Thick-walled cylinder subjected to transient radial temperature gradient producing plastic behavior.	Results can be checked analytically. Good for studying choice of temperature increments for plastic loading.
1.5	Biaxially loaded plate or cylindrical shell subjected to specialized nonproportional loadings into plastic range.	Results can be checked analytically or, in the case of cylindrical shells subjected to pressure, axial, and torsional loads, by experimental results. Good for checking program treatment of nonproportional loading.
<u>Group 2</u>		
2.1	Simply-supported beams subjected to various center loading histories producing elastic-plastic-creep behavior.	For plane 2D-solid and beam analyses. Test results being obtained under ORNL High-Temperature Structural Design Methods Program.
2.2	Simply-supported circular plates subjected to various center loading histories producing elastic-plastic-creep behavior.	For axisymmetric 2D-solid and flat plate analyses. Test results being obtained under ORNL High-Temperature Structural Design Methods Program.
2.3	Capped circular cylindrical shells subjected to varying internal pressure loading producing elastic-plastic-creep behavior.	For axisymmetric 2D-solid and shell analyses. Test results being obtained under ORNL High-Temperature Structural Design Methods Program.
2.4	Finite-width flat plate with circular hole subjected to cyclic axial load producing elastic-plastic-creep behavior.	For plane 2D-solid analyses. Results being obtained under ORNL High-Temperature Structural Design Methods Program.
2.5	Stiffened shear lag panel subjected to cyclic external loading producing elastic-plastic-creep behavior.	For plane 2D-solid analyses. Results being obtained under ORNL High-Temperature Structural Design Methods Program.
2.6	Pipe thermal ratchetting specimens (straight pipe subjected to internal pressure and to periodic thermal downshocks in contained fluids).	For axisymmetric 2D-solid or shell analyses. Results being obtained under ORNL High-Temperature Structural Design Methods Program and by LMEC.
2.7	Piping elbow subjected to external loadings producing elastic-plastic-creep behavior.	For shell and special piping analyses. Test results being obtained under ORNL High-Temperature Structural Design Methods Program and Westinghouse Validation of High-Temperature Design Methods and Criteria Program.
2.8	Nozzle attachment thermal ratchetting specimen (conditions as above).	For shell analyses. Results being obtained under Westinghouse Validation of High-Temperature Design Methods and Criteria Program.
2.9	Nozzle-to-spherical shell attachments subjected to internal pressure and external loadings causing elastic-plastic-creep behavior.	For axisymmetric 2D-solid or shell analyses. Results being obtained under ORNL High-Temperature Structural Design Methods Program.
2.10	Intersecting cylindrical shells or piping tees subjected to external loadings producing elastic-plastic-creep behavior.	For shell analyses. Test results being obtained under ORNL High-Temperature Structural Design Methods Program and Westinghouse Validation of High-Temperature Design Methods and Criteria Program.

(4) THICK-WALLED CYLINDER SUBJECTED TO RADIAL TEMPERATURE GRADIENT PRODUCING PLASTIC BEHAVIOR

(5) FLAT PLATE OR CYLINDRICAL SHELL SUBJECTED TO SPECIAL NONPROPORTIONAL BIAxIAL LOADINGS PRODUCING PLASTIC BEHAVIOR

Fig. 23. Simple Benchmark Problems for User Training and Basic Program Verification

Fig. 24. Advanced Benchmark Problems for Verification and Qualification of Program and User

In selecting a specific sequence of problems, a guide similar to Table 2 might be considered. Here, the features of the design analysis to be performed are listed, and the applicable benchmark problems from Table 1 are tabulated for each specific feature. The features of interest are grouped under geometry (which at least partially dictates the choice of the particular finite element to be used), loading, and behavior. From this sort of guide, the designer should be able to select an adequate and representative sequence of benchmark problems for his application.

Rather than dwell further on guidelines for selecting an appropriate sequence of benchmark problems, a specific example based on the problems given in Table 1 will be given. Consider a manufacturer who is supplying a component with a single critical area requiring a detailed inelastic computer analysis, and assume that the manufacturer has essentially no past experience with performing inelastic analyses. The critical area is a nozzle-to-cylindrical shell attachment with cyclic external loadings on the nozzle, internal pressure, and subjected to periodic downshocks in the internal sodium temperature (conditions for thermal ratcheting). Successful solution of the sequence of benchmark problems outlined in Table 3 would sufficiently validate and qualify the user for the necessary design analysis, and the documented benchmark effort would provide a high level of confidence in the design analysis of the manufacturer's component.*

It is worthwhile emphasizing two points in connection with inelastic analyses. The first relates to the interpretation of benchmark calculation results. The correlation of analytical predictions with experimental results for inelastic problems is by no means a straightforward procedure. Unlike the elastic case where exact agreement can theoretically be expected and strived for, there will always be differences between predicted and measured inelastic structural behavior. This is because, as has been emphasized previously, high-temperature material behavior is a highly complex phenomenon consisting of processes and interactions which defy complete understanding and exact description. The constitutive equations recommended in this standard represent first approximations to the descriptions of inelastic behavior. Thus, rather than exact quantitative predictions, inelastic analyses qualitatively predict the basic characteristics or the essential features of inelastic response in critical situations; they can locate the trouble spots in a design; and hopefully they can, along with engineering judgment, bound in some manner the behavior to be expected.

*It should be emphasized that the sequence shown represents an idealized effort for a completely inexperienced manufacturer. In most cases a much smaller effort would be appropriate. The manufacturer should undertake only those problems which he feels to be absolutely necessary.

Table 2. Sample Benchmark Problem Selection Guide

<u>Features of Design Analysis</u>	<u>Possible Benchmark Problems (from Table 5.1)</u>
<u>Geometry</u>	
Beam	2.1
Plane 2D Solid	2.1, 2.4, 2.5
Axisymmetric 2D Solid	2.2, 2.3, 2.6, 2.9
Plate Bending	2.2
Shell	2.3, 2.6, 2.7, 2.8, 2.9, 2.10
Piping	2.6, 2.7, 2.10
3D Solid	2.1, 2.2, 2.3, 2.4, 2.6, 2.7, 2.8, 2.9, 2.10
<u>Loading</u>	
Mechanical Loads	1.1, 1.2, 1.3, 1.5, 2.1, 2.2, 2.4, 2.5, 2.7, 2.9, 2.10
Pressure	2.3, 2.6, 2.7, 2.8, 2.9, 2.10
Thermal	1.4, 2.6, 2.8
<u>Behavior</u>	
Monotonic Elastic- Plastic	1.1, 1.4, 1.5, 2.1, 2.2, 2.3, 2.4, 2.5, 2.7, 2.9, 2.10
Cyclic Elastic- Plastic	1.1, 1.5, 2.1, 2.2, 2.4, 2.5
Elastic-Plastic- Creep (w/o re- versed loading)	1.2, 2.1, 2.2, 2.3, 2.5, 2.7, 2.9, 2.10
Elastic-Plastic with Stepped & Reversed Load Creep	1.3, 2.1, 2.2, 2.4, 2.5
Elastic-Plastic- Creep Ratchetting	2.6, 2.8

Table 3. Sample Benchmark Calculations Set

Design Analysis Problem: Nozzle-to-Cylindrical Shell Attachment with
Cyclic External Loadings and Thermal Ratcheting Conditions.

Benchmark Problem Sequence:

Problem*	Purpose
1.1	To gain experience in load increment selection and to help verify basic treatment of plasticity in program.
1.2, 1.3	To gain experience in time-increment selection for creep analyses and to help verify basic treatment of creep in program.
1.4	To gain experience in inputting temperature loadings on an incremental basis and to help verify treatment of temperature loadings in program.
1.5	To help verify treatment of nonproportional loading in program.
2.1 (using plane 2D solid elements)	To gain further experience in load- and time-increment selection and to help verify program treatment of combined elastic-plastic-creep behavior.
2.6 (using shell elements)	To gain experience in using shell elements and performing ratcheeting analyses and to help verify program treatment of shells and ratchetting conditions.
2.8, 2.10 (using shell elements)	To verify program and qualify user in performing analyses very similar to actual design analysis.

*Refer to Table 1 and Figs. 23 and 24 for problem descriptions.

The final point is that even when using a well-verified program, the designer should always carefully review his computer results. The use of graphics to quickly assess the overall behavior is particularly helpful in inelastic analyses where the volume of output data is very large. Performing an inelastic analysis is an art and is likely to remain so for some time into the future. Thus the use of inelastic analysis design must always be accompanied by a large measure of sound engineering judgment.

6. EVALUATION

6.1 Strain Limits Using Inelastic Analysis.

6.1.1 General Guidance. The use of the inelastic strain limits of T-1300, RDT F 9-4, is conceptually simple. The inelastic strains are limited to specific maximum values. The elaboration below is intended to assist the designer in the use of these limits. The specific strain limits are:

1. 1.0% - average through the thickness.
2. 2.0% - at the surface due to an equivalent linear distribution of strain through the thickness.
3. 5.0% - at all locations.

For welds the above strain limits shall be divided by two. These strain limits apply to the maximum positive principal inelastic (plastic + creep) strains accumulated during the specified Normal, Upset, and Emergency Operating Conditions.

The 5.0% limit shall be interpreted to mean that the inelastic strain at any point in the component including gross, local, or simultaneous gross and local discontinuities shall be limited to 5.0% (2.5% for welds).

The equivalent linear distribution is that linear distribution whose average (through the wall) value is the same as the actual distribution and whose first moment is the same as the actual distribution.

Prior to employing inelastic analysis to demonstrate compliance with the inelastic strain limits the time, costs, and potential for success of all alternatives should be evaluated.

6.1.2 Specific Guidance. When the inelastic response of a (finite element) model of the component due to the repeated application of the unit histogram has been determined, there are several steps remaining before the estimated end of life inelastic strains are determined. In general the change in the response from one unit histogram to the next is extrapolated out to the end of the specified design life. The steps to do this are:

6.1.2.1 Identify the point(s) in the unit histogram which will be used to estimate the end of life inelastic strains. Note that T-1300, RDT F 9-4, indicates that the strains after periods of constant operation should be used. The intent is to limit the general accumulation of inelastic strain, not the instantaneous value in the midst of a severe transient. Consider the example given in Fig. 25.

Fig. 25 Average Accumulated Inelastic Strain vs Time

The average strain limit is first exceeded at time T_1 (during a transient). The strain limit is intended to apply to non-transient points in the histogram. Thus a non-transient point in the unit histogram should be chosen as the basis for extrapolation of the results. In this case the intent of the rules of T-1300, RDT F 9-4, is violated for design lives beyond time, T_2 .

6.1.2.2 Identify the cross-section(s) to be examined for excessive inelastic strain. In many cases several cross-sections must be examined to be sure that the points of maximum inelastic strain accumulation (average, linearized and peak) are evaluated. It is likely that the cross-section of highest average strains may not be the cross-section of highest linearized or peak strain. As experience is gained, the selection of potentially critical cross-sections will be a more direct process.

6.1.2.3 Record the data for each point on the cross-section. The components of inelastic strain ($\epsilon_r, \epsilon_\theta, \epsilon_z, \epsilon_{r\theta}, \epsilon_{\theta z}, \epsilon_{rz}$) should be recorded at each point across the section for which the strain data are available. For finite element programs these values are usually available only at the centroids of the elements. The strain

components for each point in each cross-section for each chosen point in the unit histogram should be recorded for each unit histogram analyzed.

6.1.2.4 Calculate the end of life strain components. The end of life strain components should be evaluated by extrapolating the strain components of 6.1.2.3 to the specified end of design life. For example using the data from the (N-1)th and Nth unit histogram,

$$\left(\epsilon_i^{\text{Comp}} \right)_{\text{end of life}} = \epsilon_{i,J=N}^{\text{Comp}} + \left[\epsilon_{i,j=N}^{\text{Comp}} - \epsilon_{i,J=N}^{\text{Comp}} \right] \times [M-N], \quad (70)$$

where: M = the total number of unit histograms in the design life / (j=2,3,...M)

N = the index number of the last unit histogram analyzed.

i = the element number.

j = unit histogram number.

Note that only the original (element) strains, not the strains extrapolated to the surface, are extrapolated in time. In this extrapolation the constant volume assumption may be used to adjust the $\Delta \epsilon$ components. Note that in those cases where the strain limits are not met and where the incremental strain decreases for subsequent unit histograms, it may be necessary to consider analysis of additional unit histograms in order to meet the strain limits. The extrapolation to the end-of-life should be performed only when the results of the last three unit histograms shown a consistent pattern of constant or decreasing strain accumulation per unit histogram.

6.1.2.5 Determine the end-of-life inelastic strain components at the surface. The inelastic strain components at the end-of-life should be determined for each surface. The end-of-life strains at the three or more points (element centroids) nearest the surface should be used to extrapolate the strains (component by component) to the surface. Thus, not only are the data of 6.1.2.3 required but the positions of the centroids of each element on each cross-section and the positions of the centroids of each element on each cross-section and the position of the surfaces relative to the element centroids are also needed. It is permissible to utilize the known boundary conditions ($\epsilon_{rz}, \epsilon_{r\theta} = 0$ in an internally pressurized cylinder) in calculating the surface strain components.

6.1.2.6 Determine the average strain at the component level at the end-of-life. The wall-averaged components of inelastic strain are calculated by averaging each component of strain across the section giving weight to the cross-sectional area of the element. These calculations are performed using the end-of-life values:

$$\left(\epsilon_{\text{AVE}}^{\text{Component}} \right)_{\text{EOL}} = \sum_{i=2}^{R-1} \left(\epsilon_i^{\text{Component}} \times A^C(i) \right) \div \sum_{i=2}^{N-1} A^C(i) \quad (71)$$

where: $A^C(i)$ = the cross-sectional area of element (i) for this component.

$i = 1, R$ are values of the strain component at the two surfaces.

6.1.2.7 Calculate the end-of-life wall-averaged strain. The end-of-life wall-averaged maximum positive principal inelastic strain is calculated using the data of 6.1.2.6. The strain components are resolved to obtain the principal inelastic strains. The most positive value is then tested against the 1.0% (or 0.5% for welds) limits.

$$\left[\epsilon_{\text{AVE}}^{\text{MPPI}} \right]_{\text{EOL}} = \text{most positive} \left[\left(\epsilon_i^L \right)_{\text{EOL}} \right]_{L=1,2,3} \quad (72)$$

This value is calculated for each cross-section. If all the values are less than 1.0% (or 0.5% for welds) then the average strain limit of T-1300, RDT F 9-4 is satisfied.

|c

6.1.2.8 Calculate the equivalent linear strain distribution at each section. The equivalent linear strain distribution (component by component) for each cross-section at each unit histogram is calculated as shown below.

1. Pick a reference point for moment calculations, e.g., the inside surfaces.

2. Let $\epsilon = C + Bx$ represent the equivalent linear strain distribution with $\epsilon = C$ at the inside surface ($x = 0$) and $\epsilon = C + BL$ at the outside surface ($x = L$).

3. Equate the first moment of the real strain distribution to that of the equivalent linear distribution on a component by component basis using the end-of-life values:

$$\sum_{i=2}^{R-1} \left[\left(\epsilon_i^{\text{Component}} \right) \cdot A^C(i) \cdot \ell(i) \right] = \int_0^L (C + Bx) W(x) \ell(x) dx \quad (73)$$

where: $A^C(i)$ = the cross-sectional area of element (i) for this component.

$\ell(i)$ = the distance from the inside surface to the centroid of the i'th element.

$W(x)$ = the width of the incremental cross-section [W(x) by dx]

$\ell(x)$ = moment arm of the local strain times the incremental area $(C + Bx) \cdot W(x) \cdot dx$ from the inside surface

L = distance from the inside to the outside surfaces of the section.

4. Equate the average value of the real strain distribution to that of the equivalent linear distribution. Again use the end-of-life values.

$$\sum_{i=2}^{R-1} \left[\left(\epsilon_i^{\text{Component}} \right) \cdot A^C(i) \right] = \int_0^L (C + Bx) \cdot W(x) \cdot dx \quad (74)$$

This step provides the second equation needed to evaluate the constants C and B for a given component of strain.

5. Simultaneously solve Eqs 73 and 74 to determine C and B for each strain component at each time point of interest in each unit histogram at each cross-section in the structure.

6. Calculate the strain components at each surface for each cross-section.

$$\text{Inside: } \epsilon_x = C_x$$

$$\epsilon_y = C_y$$

$$\epsilon_z = C_z$$

$$\text{Outside: } \epsilon_x = (C + B L)_x$$

$$\epsilon_y = (C + B L)_y$$

etc.

6.1.2.9 Calculate the maximum positive principal linearized inelastic strain ($\epsilon^{lin.}$) at each surface based on the components from 6.1.2.8(6).

6.1.2.10 Compare the end-of-life linearized strains at each surface to the limit (2.0% parent metal, 1.0% at welds) of T-1300, RDT F 9-4.

6.1.2.11 Calculate the end-of-life inelastic strain of each point across the section. The maximum positive principal inelastic strain should be calculated for each element and each surface on each cross-section for every point of interest in the unit histogram. Call these values (ϵ^{peak})_{EOL}, where i is the particular point on the cross-section where the strains are known. The extreme range of i ($i=1,R$) refers to the extrapolated strain at the surfaces of the section. Thus, there are ($R-2$) elements across the section.

6.1.2.12 Compare the maximum value of the maximum positive principal inelastic strain calculated in 6.1.2.11 to the peak limits 5.0% (or 2.5% for welds) of RDT F 9-4.

6.1.3 Simplified Methods for Satisfying the Inelastic Limits.
The basic limits of T-1300, RDT F 9-4 are the 1.0%, 2.0% and 5.0% inelastic strain limits. In some circumstances the inelastic strain limits can be shown to have been satisfied using simplified methods. Test No. 2 of T-1323 of RDT F 9-4 is simply a statement that when primary plus secondary stresses do not exceed yield (and creep is negligible) ratchet strains are negligible. Thus, satisfaction of Test No. 2 of T-1323 of the Code Case* automatically ensures satisfaction of the 1.0% and 2.0% limits. In accepting Test No. 2 of T-1323 as an alternate for all three strain limits, the Code Case is essentially stating that peak stresses cannot cause excessive strains by themselves. The further limitation of Test No. 2 (one end of the Q Range being below the creep region) permits extension of the rules to creep temperatures in temperature cycling cases without causing ratcheting.

T-1324 of RDT F 9-4 and Section 6.1.3.2(b) of this standard provide procedures for bounding the maximum positive principal inelastic strain using elastically calculated stresses. The technique for applying these procedures is:

a. The method of T-1324 of RDT F 9-4 (the O'Donnell-Porowski method) bounds the surface strain due to a cycle between a linear through the wall temperature gradient and a uniform wall temperature. Since the surface strain could be either a membrane or membrane plus

*The term "the Code Case" refers specifically to Code Case 1592 of the ASME Boiler and Pressure Vessel Code.

linearized bending strain, the 1.0% membrane limit is applied. Thus, the 2.0% limit is automatically satisfied. Note, however, that the effect of peak stresses are, in the derivation of Fig. 26, neglected as are the peak strains. It is judged that the peak effects can be ignored (if the 1.0% limit is satisfied) as it was in the case of test 1 of T-1322 and test 2 of T-1323 of the Code Case.

b. The Bree complete relaxation method bounds the membrane and membrane plus linearized bending strains produced by repeated thermal creep relaxation of the linear through the wall temperature gradient. Since both membrane and membrane plus bending strains are bounded, the resulting values are compared to the 1.0% and 2.0% limits. The peak stress effects and peak strains are not considered. Using the Code Case precedent the need for meeting the 5.0% strain limit is deleted when the Bree complete relaxation procedure is used to demonstrate satisfaction of the 1.0% and 2.0% limits.

It should be recognized that the use of either of these methods in nonaxisymmetric or locally discontinuous applications is subject to the judgement of the designer and the approval of the owner.

6.1.3.1 Applicability of the Simplified Procedures of This Section. The rules and procedures in this section are all based on mathematically one-dimensional inelastic analyses of thin-walled cylindrical vessels (straight pipes) under sustained internal pressure with a cyclic temperature gradient through the wall. Their conservative applicability to straight pipes with axisymmetric thermal and mechanical secondary stresses has been verified through comparisons with the results of a series of detailed inelastic analyses.^[6] Also, they have been shown to be applicable to straight beams including thin-walled pipes subjected to bending loads,^[7] and to a nozzle-to-spherical-shell geometry.^[8] Thus, on the basis of existing experience, the rules and procedures of this section can generally be applied* to axisymmetric shells subjected to axisymmetric loadings in locations where there are no local structural discontinuities (as defined in -3213.3). The elastically calculated maximum primary stress intensity and maximum secondary stress intensity range for the one-dimensional case must be equal to those for the particular point being considered in the axisymmetric structure. The rules and procedures can also be applied* to straight beams, including straight pipes subjected to net nonaxisymmetric bending. For the latter case, the maximum outer fiber bending stress must be treated as an axisymmetric axial primary stress in the one-dimensional evaluation. In any of these applications the methods should

*Provided the procedures of 6.1.3.4 are used.

Fig. 26. Effective Creep Stress, $z' = \sigma_C / S_y$, for Ratcheting

be used with extreme caution and care; the designer must keep their bases and limitations in mind and exercise sound engineering judgment in their application.*

It is expected that the rules and procedures in this section may conservatively apply to a broader class of structural situations but the experience required for more general application is not currently available. Thus, the manufacturer must justify to the owner's satisfaction any application of this section to situations other than those specified above. In those cases where the geometry or conditions do not vary significantly from the above, the designer may be able to rationalize the use of the rules on a sound engineering basis. Where the geometry or conditions do vary more significantly, the applicability can generally be demonstrated only by comparisons with detailed inelastic analyses of similar situations. As such comparisons are documented and collected, it is anticipated that support will be established for the broader application of simplified rules and procedures.

The diagram shown in Fig. 27 represents the stress regions that determine the ratcheting behavior for the one-dimensional case on which these procedures are based.^[9] The boundaries between the various regions are given in the figure. The primary stress parameter x , is $(P_L + P_b/Kt)$ nondimensionalized by dividing by a yield stress and the secondary stress parameter y , is Q^R nondimensionalized by dividing by the yield stress. The factor K_t is defined in -3223 of RDT F 9-4. |C

For the idealized material behavior considered in the development of Fig. 27 including perfect plasticity and no creep, ratcheting occurs in regions R_1 and R_2 , shakedown occurs in regions S_1 and S_2 , plastic cycling occurs in region P and the behavior is always elastic in region E . Creep can cause ratcheting to occur in all six regions. The elastic analysis strain limit rules in T-1322 and T-1323 of RDT F 9-4, are intended to confine operation to region E and to limit the creep ratcheting to insignificant levels. The elastic analysis rule of 6.2 prevents ratcheting by confining operation to regions S_1 , S_2 , and P , and stipulating that no creep can occur. Finally, the O'Donnell-Porowski method given below in 6.1.3.2a can be used for points in |C

*As an example of the importance of this point, reference is made to the nozzle-to-spherical-shell analysis described in Ref. 8. The behavior at a point in a general axisymmetric shell is dependent not just on the conditions through the wall at that location but on the behavior at other points as well. Because of this structural interaction, a one-dimensional (straight pipe) analysis overpredicted ratcheting in the junction region where the ratcheting strains were highest, but underpredicted ratcheting in some portions of the discontinuity regions. The rules and procedures of this section, were; however, adequately conservative in terms of the maximum ratcheting strain.

Fig. 27. Ratchet Limits

regions E, S_1 , S_2 , and P, and the modified Bree complete relaxation method of 6.1.3.2(b), as well as the one-dimensional inelastic analysis method of 6.1.3.3 can be used for any region.

Guidance on applying the various rules and procedures is provided in 6.1.3.4.

6.1.3.2 Simplified Procedures Using Elastically Calculated Quantities.

a. The Method of T-1324 of RDT F 9-4. This method is applicable provided that one of the extreme conditions which defines the maximum secondary stress intensity range, $(Q_R)_{\max}$, is such that negligible creep relaxation occurs at that point in the loading cycle. This requirement will be satisfied provided the temperature is below the value corresponding to the point where $S_m = S_t$ for $t = 10^5$ h.¹ | C

In this method the elastically calculated primary and secondary stresses are used to determine an effective creep stress, σ_C , which in turn is used to determine a total ratcheting strain that includes creep ratcheting (see 3213.23, RDT F 9-4). The effective creep stress, σ_C , for any combination of loading is given in Fig. 26 in dimensionless form. Note that this method is applicable only when σ_C is less than the yield stress, S_y . | C

The ratcheting strain is determined by multiplying σ_C by 1.25 and entering the isochronous stress-strain curves of the Code Case for the total operating time² and the maximum operating temperature.³ The total ratcheting strain shall be limited to 1.0% for parent material and 0.5% for weld regions.

The dimensionless expressions for the effective creep stress, $z' = \sigma_C/S_y$, in regimes S_1 , S_2 , and P of Fig. 26 are:

$$z' = x' \cdot y' \quad (75)$$

¹To use temperatures above this value, the manufacturer shall demonstrate, to the owner's satisfaction, that the hold time is sufficiently short for creep relaxation to be negligible.

²See subsection 6.1.2.4 for further guidance.

³Alternatively, uniaxial creep laws and stress-strain representations from the LMFBR Materials Handbook^[2] may be used to determine the total ratcheting strain due to σ_C . The isochronous curves of the Code Case represent average data. Since minimum properties should be used, σ_C is, in either case, multiplied by 1.25 (the approximate average to minimum yield strength ratio for types 304 and 316 stainless steel).

in region S_2 and P , and:

$$z' = (y' + 1) - 2 \sqrt{(1 - x')y'} , \quad (76)$$

in region S_1 , where:

$$x' = \left(P_L + \frac{P_b}{K_t} \right)_{\max} / S_y , \quad (77)$$

$$y' = (Q_R)_{\max} / S_y , \quad (78)$$

$$z' = \sigma_C / S_y , \quad (79)$$

and S_y is the average of the S_y values at the maximum and minimum wall-averaged temperatures during the operating conditions being evaluated, $(P_L + P_b/K_t)_{\max}$ is the maximum value of the primary stress, adjusted for bending via K_t , and $(Q_R)_{\max}$ is the maximum range of the secondary stress. P_L , P_b , and Q_R are stress intensities and are calculated on an elastic basis.

For any combination of elastically calculated primary and secondary stresses, the designer can, using the above equations or Fig. 26, and the appropriate set of isochronous stress-strain curves (or appropriate stress-strain and creep law expressions), quickly estimate the total ratcheting strains in a particular situation.

b. Bree Complete Relaxation Method. [9] Deformation and strain limits for structural integrity are considered to have been satisfied and a detailed inelastic ratcheting analysis need not be performed if the following four conditions are satisfied:

1. The approximate inelastic strain averaged through the thickness, ϵ_{avg} , is less than the limit of T-1310 of RDT F 9-4, with:

$$\epsilon_{\text{avg}} = \epsilon_c + \epsilon_r , \quad (80)$$

where: ϵ_c = cumulative creep strain based on the primary stress $(P_L + P_b/K_t)$ and the appropriate isochronous stress-strain curves,

| C

ϵ_r = cumulative ratchet strain based on Bree's complete relaxation analysis.

The values of ϵ_r are given by:

$$\epsilon_r = \sum_i N_i (\Delta\epsilon_r)_i, \quad (81)$$

where: N_i = number of type i cycles,

$(\Delta\epsilon_r)_i$ = ratchet strain per cycle for type i cycles from Bree's complete relaxation equations.

The Bree complete relaxation equations are (with x'' and y'' defined by Eqs 86 and 87):

$$(\Delta\epsilon_r)_i = \frac{S_m}{E} \left\{ 1 - x'' + y'' \left[1 - 2 \left(\frac{1 - x''}{y''} \right)^{1/2} \right] \right\}, \quad (82)$$

in region S_1 of Fig. 27*:

$$(\Delta\epsilon_r)_i = \frac{S_m}{E} \left\{ 1 - x'' + 2y'' \left[1 - 2 \left(\frac{1 - x''}{y''} \right)^{1/2} \right] \right\}, \quad (83)$$

in region R_1 of Fig. 27:

$$(\Delta\epsilon_r)_i = \frac{S_m}{E} [x''(y'' - 1)], \quad (84)$$

in regions S_2 and P of Fig. 27, and:

$$(\Delta\epsilon_r)_i = \frac{S_m}{E} [x''(2y'' - 1) - 1], \quad (85)$$

in region R_2 of Fig. 27. In the above:

*Here, $x = x''$ and $y = y''$.

$$x'' = \left(P_L + \frac{P_b}{K_t} \right)_{\max} / S_m \quad (86)$$

and:

$$y'' = (Q_R)_{\max} / S_m, \quad (87)$$

and S_m corresponds to the maximum operating metal temperature at the location being considered (see Fig. I-14.3, RDT F 9-4, for S_m values). |c

2. The maximum principal surface strain, $\epsilon_{\max}$, is less than the limit of T-1310(b), RDT F 9-4, with: |c

$$\epsilon_{\max} = \epsilon_p + \epsilon_c + \epsilon_r, \quad (88)$$

where:

$$\epsilon_p = \begin{cases} \frac{1}{E} (S_{\max} - S_m), & \text{for } S_{\max} > S_m \\ 0, & \text{for } S_{\max} < S_m \end{cases} \quad (89)$$

with $S_{\max}$ the maximum stress intensity due to primary and secondary stresses including gross but not local discontinuity stresses.

$$3. \quad y'' \leq 3. \quad (90)$$

4. To limit creep ratcheting, one of the extreme conditions which defines the maximum secondary stress intensity range, $(Q_R)_{\max}$, must be such that negligible creep relaxation occurs at this point in the loading cycle. This requirement will be satisfied provided the temperature is below the value corresponding to the point where $S_m = S_t$ for $t = 10^5 h$.*

6.1.3.3 One-Dimensional Inelastic Analysis. Both of the preceding methods are conservative approximations to detailed one-dimensional inelastic ratcheting analyses. In many cases they are

*To use temperatures above this value, the manufacturer shall demonstrate to the owner's satisfaction that the hold time is sufficiently short for creep relaxation to be negligible.

extremely conservative so that often it will be the designer's advantage to perform a detailed one-dimensional ratcheting analysis using one of the highly efficient, special-purpose pipe ratcheting computer programs such as PLACRE or Foster Wheeler Program R-1045, both of which are described in Appendix B. The detailed ratcheting behavior can be predicted using these programs, and it is possible to follow relatively complex load histories.

It should be emphasized that the applicability of these detailed one-dimensional analyses is limited in the same manner as the other methods in this section, and they are strictly rigorous only for long straight pipes.

6.1.3.4 Guidelines for Applying the Rules and Procedures for Meeting Strain Limits. Considerable care and engineering judgment must be used in applying the elastic analysis strain limit rules of T-1310, RDT F 9-4, and the simplified methods given in this section for satisfying the inelastic analysis strain limits. It must be kept in mind that the rules and procedures are based on the behavior of an infinitely long pipe subjected to a very simple repetitive loading consisting of a sustained primary stress and a cyclic radial temperature gradient with uniform hold periods during which relaxation of residual stresses and enhanced creep can occur. Actual specified loading cycles are likely to be much more complex, having events of varying severity intermixed. When using rules and procedures based on elastically calculated quantities only, there is no easy way to estimate the degree to which the residual effects of one event may remain to combine with, and possibly reinforce, the effects of a subsequent event. Thus, the designer must use very conservative approaches to allow for the possible effects of these unknown interactions. |C

The following general guidelines are to be used in applying the rules and procedures.

a. In each of the three simplified procedures given in this section for meeting the strain limits for inelastic analysis, any stresses with elastic followup (e.g., secondary stresses other than those caused by radial temperature variations) should be included as primary stresses for purposes of predicting ratcheting strains unless otherwise justified. Alternatively, the strains due to such stresses may be separately calculated and added to the predicted ratcheting strains, the sum being limited to the values of T-1310, RDT F 9-4. If the latter is done, stresses with elastic follow-up should be treated as secondary. |C

b. To apply the elastic analysis rules of T-1310, RDT F 9-4, and the simplified procedures (O'Donnell-Porowski and modified Bree complete relaxation) of 6.1.3.2, the time history of $(P_L + P_b/K_t)$ and Q , and Q_R , should be calculated elastically for all sequences of normal and upset conditions. Emergency conditions may be evaluated separately and the resulting strain added to that from the Normal and Upset conditions to satisfy the limits. |C

The maximum values of $(P_L + P_b/K_t)$ and Q_R for the entire life (even though they may not occur at the same time) should be checked against the elastic analysis strain limit rules. If they meet the rules, the deformation and strain limits of T-1310, RDT F 9-4, are considered satisfied. If they do not meet the rules, the procedures of this section should be employed.

|C

If, for some initial period of time, t_0 , prior to the occurrence of either $(P_L + P_b/K_t)_{\max}$ or the first event which describes at least one of the extremes of $(Q_R)_{\max}$, the maximum values of $(P_L + P_b/K_t)$ and Q_R within the period t_0 satisfy the elastic analysis rules (T-1322 or T-1323 of RDT F 9-4), then the events within t_0 need not be considered further and no strain increments due to these particular events need be added to the ratcheting strains. The entire operating life beginning at time t_0 must then be evaluated according to the inelastic analysis strain limits.

c. Either of the two simplified methods of 6.1.3.2 may be used to meet the inelastic strain limits. Generally, the one giving the smallest predicted strains should be employed. For large numbers of events of moderate severity, the O'Donnell-Porowski method usually provides the smaller prediction, while for a small number of events the modified Bree method usually provides the smaller prediction.

d. The O'Donnell-Porowski method in 6.1.3.2a provides an upper bound ratcheting strain by assuming the worst possible effect over a maximum period of time. Thus, if this method is used, then $(P_L + P_b/K_t)_{\max}$ and $(Q_R)_{\max}$ for Normal and Upset operating conditions must be used except as provided by guideline (f), and the times used in selecting the isochronous curves must sum to the total service life* (minus the t_0 period defined in (b)) regardless of whether all or only a part of the cycles (events) are evaluated by the O'Donnell-Porowski method. The ratcheting strains produced by intermediate events, other than those that established the $(P_L + P_b/K_t)_{\max}$ and $(Q_R)_{\max}$ values, may still require evaluation and should be looked at individually. If these events do not pass the elastic analysis strain limit rules (e.g., cycles for which Q_R is less than $(Q_R)_{\max}$ but neither extreme is such that negligible relaxation occurs as provided by 6.1.3.2a), then they may be evaluated using the O'Donnell-Porowski method, (if applicable), the modified Bree method, or a detailed inelastic analysis.

e. The modified Bree complete relaxation method in 6.1.3.2b provides a conservative ratcheting strain estimate by considering the worst possible effect regardless of any hold period times. Thus, in this method, the $(P_L + P_b/K_t)_{\max}$ and $(Q_R)_{\max}$ values should be taken as

*The total service life may be subdivided into temperature-time blocks and the strain increment for each block may be evaluated separately. The strain increments of each block are then added to obtain the total strain.

the primary and secondary stresses for all significant events (except for those accounted for in guidelines d and f, regardless of the actual stress values for the various events. Any events meeting the elastic analysis strain limit rules do not have to be considered.

f. A detailed one-dimensional inelastic analysis, as described in 6.1.3.3, may be used to meet the inelastic analysis strain limits for the entire operating life, beginning at time t_0 . It is acceptable to analyze each type cycle for only a few cycles and estimate the total accumulated strain for that type cycle by multiplying the strain accumulated in the last cycle by the remaining number of cycles of that type and adding that value to the accumulated strain for the cycles analyzed. It is also acceptable, in certain situations, to determine the total inelastic strains due to any number of selected operating cycles using a detailed one-dimensional analysis or a more general inelastic analysis. The sum of these accumulated strains plus the strains due to elastic follow-up (evaluated by either of the procedures described in guideline a above), plus the ratcheting strains for the remaining cycles as determined by either of the methods of 6.1.3.2, must not exceed the strain limits of T-1310, RDT F 9-4. |c

g. An individual cycle cannot be split into subcycles of smaller time increments in order to satisfy any of the strain limits rules or values.

6.1.4 Example of Strain Limit Evaluation. Consider a long right circular cylinder ($r_i = 4.8$, $r_o = 5.0$) of type 304 stainless steel subjected to repeated down thermal transients on its inside surface. The wall temperature returns to a uniform value between the transients. A finite element model was used with bilinear kinematic (plastic) hardening in an elastic-plastic analysis. No isotropic hardening was permitted. Thirteen constant strain quadrilateral elements were uniformly spaced in the R direction (Fig. 28). The inelastic (plastic) strain components at the center of the elements in the second and third unit histograms are given in Table 4. It is assumed that no welds exist.

The strain limit evaluation is made as described below. Following the steps given in 6.1.2:

1. Identify points in unit histogram to be used. The point where all temperatures are equal has been chosen.
2. Identify the cross section to be used. There is only one cross section in this example.
3. Record the data for each point on the cross section (See Table 4).
4. Extrapolate the inelastic strain components to the end-of-life values. The components of inelastic strain at each point across the

Fig. 28. Un-Notched Cylinder Model

Table 4. Inelastic Analysis of Uniform Cylinder

Element	Strain Component	Calculated Plastic Strain Unit Histogram		Estimated Plastic Strain at M Unit Histograms	Estimated Plastic Strain at 1003 Unit Histograms	Sum of Inelastic Strains at 1003 Unit Histograms	Sum of Inelastic Strains at 1003 Unit Histograms	Maximum Positive In-elastic Strain at 1003 Unit Histograms
		#2 (in/in)x10 ⁷	#3 (in/in)x10 ⁷	(in/in)x10 ⁷	(in/in)x10 ⁷ $\epsilon_{1,k}$	(in/in)x10 ⁷	(in/in)x10 ⁷	(in/in)x10 ⁷ MPPI, $\epsilon_{1,k}$
(Surface)	R	-	-	-	6348	-	-	-
	Z	-	-	-	-5689	-921	-12000	6348
	θ	-	-	-	-1580	-		-
1	R	-6447	-6439	-6439 + 8.(M-3)	1561	-		-
	Z	3216	3210	3210 - 6.(M-3)	-2790	Zero		1561
	θ	3231	3229	3229 - 2.(M-3)	1229	-		-
2	R	-6010	-6012	-6012 - 2.(M-3)	-8012	-		-
	Z	3008	3008	3008	3008	-1003		4003
	θ	3002	3003	3003 + 1.(M-3)	4003	-		-
3	R	-4567	-4568	-4568 - 1.(M-3)	-5568	-		-
	Z	2291	2292	2292 + 1.(M-3)	3292	+1000		3292
	θ	2275	2276	2276 + 1.(M-3)	3276	-		-
4	R	-2729	-2731	-2731 - 2.(M-3)	-4731	-		-
	Z	1375	1376	1376 + 1.(M-3)	2376	Zero		2376
	θ	1354	1355	1355 + 1.(M-3)	2355	-		-
5	R	-1006	-1008	-1008 - 2.(M-3)	-3008	-		-
	Z	510	510	510	510	-1000		1498
	θ	497	498	498 + 1.(M-3)	1498	-		-
13	R	141	141	141	141	-		-
	Z	-70	-70	-70	-70	-		141
	θ	-70	-70	-70	-70	-		-
(Surface)	R	-	-	-	212	-		-
	Z	-	-	-	-105	2	2	212
	θ	-	-	-	-105	-		-

Based on extrapolation procedure which is not recommended (extrapolated to surface and then extrapolated in time)

section were extrapolated to the end of life. The extrapolation is illustrated in Table 4. The inelastic strain components at the end of 1003 unit histograms are shown in Table 4 also. Note that the time extrapolation is performed on the strain components as originally determined by the finite element analysis.

5. Determine the inelastic strain components at the surfaces. The data of Table 4 were used to extrapolate the inelastic strain components to the inside and outside surfaces. The extrapolation procedure used a parabolic function fitted to the data of the three elements nearest the surface. The results are given in Table 4. The procedure of extrapolating in time and then to the surface is generally more accurate than the reverse procedure since compatibility is maintained during the extrapolation in time when the element strains are used. Extrapolation of strains to the surface, prior to the time extrapolation can introduce compatibility errors which are then magnified by the time extrapolation procedure. In Table 4 the resulting end of life inelastic hydrostatic strain values are given. Note that these values are within expected error bands (round off x number of extrapolated unit histograms) when the recommended extrapolation procedures are used. However, when the reverse sequence was tried, the hydrostatic strains at the inside surface were much less accurate.

6. Determine the average strain at the component level. The component inelastic strains of the thirteen elements were averaged. If the elements had substantially unequal areas (in the $r\theta$ plane) the strain averages would have been weighted by the element areas. The averaged component strains are shown in Table 5.

7. Determine the end-of-life maximum positive principal inelastic strain from the averaged components. The maximum positive principal inelastic strain, averaged across the section, is 0.00945% (See Table 5). This value is less than the 1.0% limit for parent metal and less than the 0.5% limit for weld metal. Thus the average strain limit is met for 1003 unit histograms.

8. Calculation of equivalent linear strain distribution.

Calculate ϵ_r :

$$\sum [\epsilon_i \cdot A^C(i) \cdot l(i)] = -389.87 \quad (\text{Table 5}),$$

$$\int (C + Bx)W(x)l(x)dx = \int_0^{.2} (C + Bx)(30.7876)(x)dx,$$

$$\int (A + Bx)W(x)l(x)dx = .615752C + .0821003B,$$

Table 5. Calculations to Determine the Linear Strain Distribution.

Element	Strain Component	Inelastic Strain Component at End Of Life	Product $\epsilon_i \cdot A^C(i)$ Value	Distance from Inside Surface $l(i)$	Product $\epsilon_i \times l(i)$	Product $\epsilon_i \cdot l(i) \cdot A^C(i)$
		$\text{in/in} \times 10^7$ Component ϵ_i	$\text{in}^2/\text{in} \times 10^7$	in	$\text{in}^2/\text{in} \times 10^7$	$\text{in}^3/\text{in} \times 10^7$
1	R	1561	737.4	{ 0.007692	12.007	5.687
	Z	-2790	-1321.5		-21.461	-10.165
	θ	1229	582.1		9.453	4.477
2	R	-8012	-3794.9	{ 0.023077	-184.893	-87.58
	Z	3008	1424.8		69.416	32.88
	θ	4003	1896.0		92.377	43.75
3	R	-5568	-2637.3	{ 0.038462	-214.156	-101.436
	Z	3292	1559.2		126.617	59.97
	θ	3276	1551.7		126.002	59.68
4	R	-4731	-2240.9	{ 0.053846	-254.745	-120.66
	Z	2376	1125.4		127.938	60.60
	θ	2355	1115.5		126.807	60.06
5	R	-3008	-1424.8	{ 0.069231	-208.247	-98.63
	Z	510	241.6		35.308	16.72
	θ	1498	709.5		103.708	49.12
13	R	141	668.0	{ 0.192308	27.115	12.84
	Z	-70	-332.0		-13.462	-6.376
	θ	-70	-332.0		-13.462	-6.376
Sum	R	-	-	-	-822.919	-389.78
	Z	-	-	-	324.356	153.63
	θ	-	-	-	444.885	210.72

$$\text{Average } \epsilon_r = \frac{-9291.7}{13 \times 0.47365} = -1509.0 \quad \epsilon_z = \frac{2996.3}{13 \times 0.47365} = 487. \quad \epsilon_\theta = \frac{5821.6}{13 \times 0.47365} = 945.$$

$$A^C(i) = \frac{0.2}{13} (2\pi 4.9) = 0.47365$$

where: $W(x) = 2\pi 4.9 = 30.7876 \text{ in.}$,

$$l(x) = x.$$

Therefore:

$$0.615752C + .0821003B = -389.87 ,$$

and:

$$\Sigma[\epsilon_i \cdot A^C(i)] = (C + Bx)W(x)dx ,$$

$$-9291.7 = 6.15752C + 0.615752B ,$$

solving:

$$-0.205251B = -5393 ,$$

$$B = 26,275 ,$$

$$C = 4136 ,$$

$$@ x = 0 \quad \epsilon_r = -4136 \times 10^{-7} \text{ in./in.} ,$$

$$@ x = 0.2 \quad \epsilon_r = 1119 \times 10^{-7} \text{ in./in.} .$$

Similarly for ϵ_z :

$$0.615752C + 0.0821003B = 153.63 ,$$

and

$$6.15752C + 0.615752B = 2996.3 ,$$

$$B = -7,113 ,$$

$$C = +1198 ,$$

$$@ x = 0 \quad \epsilon_z = 1198 \times 10^{-7} \text{ in./in.} ,$$

$$@ x = 0.2 \quad \epsilon_z = -225 \times 10^{-7} \text{ in./in.} .$$

and for ϵ_{θ} :

$$0.615752C + 0.0821003B = 210.72 ,$$

$$6.15752C + 0.615752B = 5821.6 ,$$

$$B = -18097 ,$$

$$C = 2,755,$$

$$x = 0 \epsilon_{\theta} = 2755 \times 10^{-7} \text{ in./in.} ,$$

$$@ x = 0.2\epsilon_{\theta} = -864 \times 10^{-7} \text{ in./in.} .$$

9. Calculate the maximum positive principal linearized inelastic strain ($\epsilon^{\text{lin.}}$) at each surface based on the components from step 8.

$$\epsilon_{\text{MPPL}}^{\text{inside}} = 2755 \times 10^{-7}$$

$$\epsilon_{\text{MPPL}}^{\text{outside}} = 1119 \times 10^{-7}$$

10. Compare these values to the limits of RDT F 9-4. These values are less than 2.0%, thus the linearized strain limits are not violated.

11. Calculate the maximum positive principal inelastic strain at each point. See Table 4.

12. Compare the value to the limit of this standard.

$$0.0006348 < 0.05$$

Thus, the peak strain limits of this standard are satisfied.

6.2 Strain Limits Using Elastic Analysis. The elastic analysis strain limit rules given in T-1323, RDT F 9-4, are considered to be generally applicable to any structure. They are, however, overly restrictive in many cases. An alternative, less restrictive elastic analysis strain limit rule is given in this section but it may be used only where creep is insignificant. Also, its applicability is limited to axisymmetric shells subjected to axisymmetric loadings in locations where there are no local structural discontinuities as defined in -3213.3 of RDT F 9-4, and to straight beams or pipes subjected to net nonaxisymmetric bending.*

*The limits of applicability of the rule given in this section are the same as those given for the rules in 6.1.3 which should be referred to for guidance.

When the maximum metal temperature at the location being considered is always below the value corresponding to the point where $S_m = S_t$ for $t = 10^5$ h (See Fig. I-14.3, RDT F 9-4, for the appropriate material) the deformation and strain limits for structural integrity of T-1323, RDT F 9-4, are considered to have been satisfied, and a ratcheting analysis need not be performed if: |C

$$x' + \frac{1}{4} y' \leq 1, \quad (91)$$

where: $x' \geq 0.5$,

$$\text{or:} \quad x' \cdot y' \leq 1, \quad (92)$$

where: $x' \leq 0.5$,

$$\text{where: } x' = \left(P_L + \frac{P_b}{K_t} \right)_{\max} / S_y, \quad (93)$$

$$y' = \frac{(Q_R)_{\max}}{S_y}, \quad (94)$$

and S_y is the average of the S_y values at the maximum and minimum wall-averaged temperatures during the operating conditions being evaluated, $(P_L + P_b/K_t)_{\max}$ is the maximum value of the primary stress adjusted for bending via K_t (see 6.5) during the operating conditions being evaluated, and $(Q_R)_{\max}$ is the maximum range of the secondary stress during the operating conditions being evaluated.*

In the above equations the quantities P_L , P_b , and $Q_R = Q_{\text{range}}$ are stress intensities calculated on an elastic basis. Further guidance on the application of the elastic analysis strain limit rules given above and in T-1323, RDT F 9-4, is provided in 6.1 |C

*The limits defined by Eqs 91 and 92 are shown in Fig. 27 with $x = x'$ and $y = y'$.

6.3 Creep Fatigue-Evaluation Using Elastically Calculated Stress Values. The procedures and limits for creep-fatigue evaluation using elastically calculated stress values are given in T-1430, RDT F 9-4. The procedures and limits are explained below.

RDT F 9-4 provides two types of design fatigue curves. The Fig. T-1420, RDT F 9-4, design fatigue curve is from rapid fully reversed, continuous cycling tests of smooth specimens with no hold periods interposed between cycles. The Fig. T-1430, RDT F 9-4, design fatigue curve has the effects of slow cycling rates and hold time effects included within its limits. Thus, part of the creep damage has already been accounted for via lower fatigue limits in Fig. T-1430 (as compared to Fig. T-1420).

In general, creep-fatigue evaluations shall be based on the elastic analysis rules unless inelastic analysis has been used. When elastic analysis is used, the creep-fatigue damage limit D , is equal to unity.

6.4 Section Properties (K_t). In -3200, a K_t factor is used in the basic stress limits on primary membrane-plus-bending stress intensities (e.g., Eq 4 for Normal and Upset Conditions) and in the strain limits for elastic analysis (T-1322, T-1323). Likewise, the K_t factor is used in the rules and procedures for satisfying strain limits given in 6.1 and 6.2. The implied definition of K_t as used in RDT F 9-4 is the ratio of the elastically calculated maximum bending stress, P_b , to the actual maximum inelastic bending stress.* The logic involved is illustrated in Fig. 29, where the bending stress distributions in a

Fig. 29. Elastically Calculated Primary Bending Stress Distribution Compared to Actual Time-Dependent Inelastic Stress Distribution

*In Article A-9000 of the ASME Code, K denotes the more traditional shape factor or section factor based on an elastic, ideally-plastic material response.

beam subjected to pure bending are depicted. The linear distribution represents the elastically calculated stresses, and the maximum value is denoted by P_b . The actual distribution, which results at a particular time from either time-dependent creep deformations or from a combination of time-independent plastic deformation and subsequent creep deformation, is depicted by the nonlinear distribution. The significant point is that the maximum stress, which is denoted by P_b/K_t , is less than the elastically calculated value, and K_t is the factor which accounts for this reduction. The subscript t is used here to denote that K_t is time, temperature, and material dependent, in addition to being dependent on the geometry of the cross section.

In the primary stress limits of RDT F 9-4, K_t is used to increase the allowable stress S_t in recognition of the fact that creep and plastic deformations will reduce the actual maximum bending stress below the elastically calculated value. In the deformation and strain limits (ratcheting rules), P_b from an elastic calculation is reduced by K_t to more realistically reflect the true maximum primary stress, which is a measure of the driving force for ratcheting.

To better understand the role of the K_t factor and the values which should be used, a number of elastic-creep and elastic-plastic-creep beam bending analyses using material properties for types 304 and 316 stainless steel have been performed, and the resulting inelastic stress distributions and maximum stresses have been examined relative to the K_t factor. The bases for the procedure for making K_t dependent on the actual cross section is given below together with guidelines for calculating alternate K_t factors for use in specific critical design situations. The validity of Eqs 7 and 8 of RDT F 9-4 will have to be established for materials other than types 304 and 316 austenitic stainless steel.

In 6.4.1, typical inelastic beam bending results are discussed relative to K_t factors and the existing equation for K_t is evaluated. Specific rules and procedures for implementing RDT F 9-4 are given in 6.4.2.

6.4.1 Inelastic Beam Bending Analysis Results. The results of typical analyses are presented in Figs. 30 through 35. Figs. 30 and 31 show the steady-state creep stress distributions in a solid rectangular beam and in a tubular beam, or pipe, of circular cross section, for various values of n in the assumed steady-state creep law $\dot{\epsilon}_c = A\sigma^n$. Although calculated for a specific bending moment and beam dimensions, these distributions are essentially independent of the stress levels and dimensions.* For the rectangular section, K_t varies from about 1.28 to 1.38 for n between 3 and 6, the usual range for the materials of

*Actually the distribution for the tubular section are independent of dimensions only as the thickness approaches zero.

Fig. 30. Steady-State Creep Stress Distributions Across A Rectangular Beam in Pure Bending*

*Having a steady-state creep law of the form $\dot{\epsilon}_c = A\sigma^n$.

Fig. 31. Steady-State Creep Stress Distributions Across a Thin Circular Tube in Pure Bending*

*Having a steady-state creep law of the form $\dot{\epsilon} = A\sigma^n$. For the particular thickness used, the value of K_t for $n = \infty$ or for elastic, ideally-plastic response is approximately 1.31. The limiting value as the thickness goes to zero is 1.27.

Fig. 32. Stress Distributions for Various Times in a Rectangular Beam of Type 304 Stainless Steel at 900°F *

*These calculations are based on the one-exponential creep law given on Data Sheet 304SS-23 in Ref. 2 and a maximum elastically calculated stress of $1.5S_m$ at 1100°F.

Fig. 33. Stress Distributions for Various Times in a Rectangular Beam of Type 304 Stainless Steel at 1100°F*

*These calculations are based on the one-exponential creep law given on Data Sheet 304SS-23 in Ref. 2 and a maximum elastically calculated stress of $1.5S_{m1100^\circ\text{F}}$.

Fig. 34. Stress Distributions for Various Times in a Rectangular Beam of Type 304 Stainless Steel at 900°F^*

*These calculations include initial elastic-plastic behavior based on the average stress-strain law given in Ref. 11. The maximum elastically calculated stress is $1.5S_{m1100^\circ\text{F}}$, and the creep behavior is based on the one-exponential creep law given on Data Sheet 304SS-23 in Ref. 2.

Fig. 35. Stress Distributions for Various Times in a Rectangular Beam of Type 304 Stainless Steel at 1100°F*

*These calculations include initial elastic-plastic behavior based on the average stress-strain law given in Ref. 11. The maximum elastically calculated stress is $1.5S_{m1100^{\circ}F}$, and the creep behavior is based on the one-exponential creep law given on Data Sheet 304SS-23 in Ref. 2.

interest at low stresses. Note that $K_t = 1.5$ for $n = \infty$ or for an elastic, ideally-plastic response (this is the traditional section factor, K , for a rectangular section).

Additional calculations based on the steady-state creep portion of the two-exponential creep law recommended in Ref. 11 were used to examine types 304 and 316 stainless steel beams in the 900 to 1200°F temperature range. For elastically calculated stresses ranging from 5,000 to 15,000 psi, it was found that the ratios of the maximum steady-state stress to the elastic stress varied from 0.70 to 0.74, giving a K_t value ranging from 1.35 to 1.43.

These results show that the value of 1.25 used in RDT F 9-4 (for $P_L = 0$) is reasonable and conservative for steady-state stress distribution in rectangular cross sections. On the same basis; however, Fig. 31 indicates that 1.25 is high for a thin circular tube. A comparable value to the 1.25 would be about 1.15 for the circular section. Thus, the K_t values used for sections other than solid rectangular should in some important cases be reduced below the 1.25 value used for rectangular cross sections.

In reality the creep response will consist of both primary and secondary creep. Also the instantaneous bending behavior can be elastic-plastic. Figs. 32 and 33 show the stress distributions for a particular rectangular beam of type 304 stainless steel at 900 and 1100°F for various times. These calculations are based on the one-exponential creep law given in Ref. 2 and a bending moment producing a maximum elastic stress of $1.5 S_m$ at 1100°F. These figures simply indicate that the appropriate K_t value is time and temperature dependent (as well as stress and material dependent). The RDT F 9-4 value of 1.25 is still conservative for long times.

Figs. 34 and 35 show results in which initial elastic-plastic response is included. These results, at 900 and 1100°F, are realistic for the conditions considered. The effect of the elastic-plastic behavior is to increase K_t and to move the 1.25 value toward conservatism.

An important question is whether or not the creep-rupture damage, $\int dt/T_d$, calculated from the elastic-plastic-creep solution will be less than that resulting from a constant stress level of $P_b/1.25$. An evaluation of the results in Figs. 34 and 35 in this respect shows that at 900°F an elastically calculated damage factor based on $P_b/1.25$ is slightly less conservative than the result of a crude evaluation of the integral using the elastic-plastic-creep analysis predictions. At 1100°F the elastically calculated damage factor based on $P_b/1.25$ is clearly conservative. These results further support the applicability of the 1.25 factor in RDT F 9-4 for use with types 304 and 316 stainless steel. The conservatism of these procedures for determining K_t for materials other than solution treated types 304 and 316 stainless steel should be demonstrated.

6.4.2 Procedures and Augmenting Eq 5, RDT F 9-4. The example calculations in the previous subsection show that K_t for a rectangular section can vary from 1.0 to at least 1.4 with time, temperature, loading, etc. Consequently use of a single value is the best interim approach for most situations, and the 1.25 value currently in RDT F 9-4 for rectangular cross sections, although not always conservative, is a reasonable compromise value for types 304 and 316 stainless steels.*

The K_t factor depends on the section shape. The simple procedure illustrated below will accomplish this.

$$K_t = 1 + k_s \left(1 - \left(\frac{P_L}{S_t} \right) \right), \quad (95)$$

where: $k_s = \alpha(K - 1)$.

Here, K is the section factor given in Table A-9221 (a)-1 of the ASME Code (reproduced below), and $\alpha = 0.5$.

**TABLE A-9221(a)-1
SECTION FACTORS**

FLANGES ONLY	—	—	—	THIN TUBE	THICK TUBE	SOLID ROUND	—

$K=1$	$K=1 \text{ TO } 1.5$	$K=1.33$	$K=1.5$	$K=1.27$	$K=1.28 \text{ TO } 1.70$	$K=1.7$	$K=2.0$

In evaluating across-the-wall bending of shell-type structures, $K = 1.5$ (for rectangular sections) shall be used. Thus, for across-the-wall shell bending, $k_s = 0.25$ in Eq 95 above. For net beam-type bending of structures, K should be determined from Table A-9221 (a)-1 or calculated. Note, however, that the classification of stresses as primary membrane or primary bending for use with these section factors must be consistent with the specific rules for the component type (see -3300 to -3600).

*Only $K_t = 1.0$ would be everywhere conservative. The validity of this value for other materials should be demonstrated by simple analyses similar to those discussed in the previous subsection.

The traditional shape factor, or section factor, as given in Table A-9221 (a)-1 is defined as the ratio of the ultimate moment that can be carried by a beam of an elastic, ideally-plastic material to the moment which just produces a maximum outer fiber stress equal to the yield stress. Thus:

$$K = \frac{M_{ult.}}{M_{y.p.}} \quad (96)$$

This ratio can be readily calculated for cross sections other than those given in Table A-9221 (a)-1 using procedures outlined in strength of materials and elementary plasticity or limit analysis textbooks (see Refs 12 and 13 for examples).

In some critical applications, it may be desirable to directly calculate K_t for given times. Recall that K_t is defined as the ratio of the maximum elastically calculated bending stress to the actual maximum inelastic bending stress for a given moment and a given time. By calculating the maximum stress using a detailed inelastic analysis, as was done for Figs. 34 and 35, K_t can be readily determined. Alternatively, the maximum inelastic stress can be determined by using a simplified strength-of-materials inelastic beam-bending approach with the appropriate isochronous stress-strain curve from Fig. I-1800, RDT F 9-4. This latter simplified procedure is briefly outlined below for a beam with a rectangular cross section. Extensions of the procedure to other cross sections are discussed in Ref. 14.

The elastic-plastic stress distribution in a beam subjected to an axial force, producing a mean stress σ_M , and a bending moment can be represented by a segment of the tension and compressive stress-strain curves as depicted in Fig. 36. Likewise, the elastic-plastic-creep stress distribution at any time, t , can be approximately represented by segments of the isochronous stress-strain curve for time, t . The outer fiber strains ϵ_1 and ϵ_2 in Fig. 36 are determined by two conditions. First, the two shaded areas must be equal, and second, the bending moment carried by the beam is related to the parameters of Fig. 36 by the equation:

$$M = \frac{bh^2}{\Delta^2} \int_{\epsilon_1}^{\epsilon_2} \sigma(\epsilon - \epsilon_0) d\epsilon \quad (97)$$

Fig. 36. Typical Stress-Strain Curve Showing Stress Distributions Across a Beam Subjected to an Axial Force and Bending Moment

Here, h is the height of the beam, b is the base, and:

$$\Delta = \epsilon_2 - \epsilon_1 .$$

For purposes of determining K_t , only bending need be considered, and if equal tension and compressive properties are assumed, $\epsilon_0 = 0$ and $\epsilon_2 = -\epsilon_1$ in Fig. 36. Eq 97 thus reduces to:

$$M = \frac{2bh^2}{\Delta^2} \int_0^{\epsilon_2} \sigma \epsilon \, d\epsilon . \quad (98)$$

For a given applied moment, M , $\sigma_{\max}$ can be determined numerically or graphically using Eq 98 and K_t can be calculated for time t .

7. FABRICATION AND ENVIRONMENTAL EFFECTS

It is important to know at all times the structural stability characteristics and degree of retention of properties of materials, with exposure at temperature.

7.1 Fabrication Effects. The influence of fabrication practices such as forming, welding, and thermal treatments on the mechanical properties of materials can be important when they result in a change of properties. Where particular temperature ranges of exposure or heat treatment, cooling rates, combinations of mechanical working and thermal treatments, fabrication practices, etc., cause significant changes in the mechanical properties, microstructure, resistance to brittle fracture, etc., it is of prime importance to consider those conditions and resulting effects.

7.1.1 Forming. Any process may be used to hot or cold form materials, including weld metal, provided the specified properties are not reduced below the minimum specified or assumed values, or they are effectively restored by heat treatment following the forming operation. The treatment of cold work effects may proceed by restricting forming operations to remain below some limiting cold work level, or by considering the particular effects associated with each level of cold work, temperature, material and property.

7.1.1.1 Forming Limits. For levels of cold work equal to or less than 5%, it may be assumed that none of the limits considered in design need be adjusted (Ref. ASME Code Case 1593, 1.4(b)).

7.1.1.2 Forming Effects. Until such time that the sources of materials data of -3200 or RDT F 9-4, contain explicit representations |C

of properties (as a function of time and temperature) versus cold work level, the following general guidelines should be considered (for austenitic stainless steels only).

a. Cold working generally increases the tensile strength (yield and ultimate) of austenitic stainless steels above the minimum expected values at least for any temperatures up to 1300°F.

b. Cold working decreases tensile ductility values at all temperatures up to 1300°F. Total tensile ductility as low as 5% can be expected for material cold formed to levels above 20%, particularly in the temperature range 600-1000°F.

c. The rupture strength (life) for austenitic stainless steels cold worked up to 20% and tested at temperatures up to 1200°F remains well above minimum expected values assumed by the ASME Code.

d. The stress for 1.0% strain for austenitic stainless steel, cold worked to levels up to 20% will exceed comparable values (all times and temperatures) assumed by the ASME Code.

e. Cold work may produce some degradation to the material creep-fatigue capability due to the strain and residual stresses imposed upon forming.

f. Cold working generally produced a beneficial affect on the fatigue crack propagation behavior of austenitic stainless steels for all temperatures up to 1200°F.

g. The observed low rupture ductility values for cold worked material suggest that strain limits lower than the values currently specified by the ASME Code may be appropriate.

7.2 Environmental Effects. The influence of the service environment such as irradiation, sodium, steam, air, etc., are important when they result in a change of properties. When properties are affected in such a way that they no longer fall within the ranges assumed by the ASME Code, then it is of prime importance to consider those conditions and resulting effects.

7.2.1 Irradiation. Neutron irradiation generally affects materials in a manner comparable to those effects produced by cold working. However, the higher levels of irradiation may also lead to a phenomenon known as irradiation induced swelling. The treatment of irradiation effects may proceed by excluding from consideration any areas that have fluences below some established limit or by considering the changes that occur in properties of materials anytime those materials are used above the limit (below which property changes are acceptable).

7.2.1.1 Fluence Limits. When the fluence is no greater than 10^{20}n/cm^2 ($E > 0.1 \text{ Mev}$) over the lifetime, the effects of neutron irradiation are considered insignificant for all mechanical properties, then no correction of the response or damage criteria is needed. This applies only to solution heat treated (annealed) types 304 and 316 stainless steels.

7.2.1.2 Fluence Effects. Until such time that the sources of materials data of -3200 or RDT F 9-4, contain all of the explicit descriptions of property changes as a function of fluence and temperature, the following general guidelines should be considered. |C

a. Tensile Properties. Tensile properties (ultimate and yield strength, total and uniform elongation), are described in Ref. 2 for type 304 stainless steel for neutrons with energies $> 0.1 \text{ Mev}$. To account for the variation of damage with different energy neutrons, the damage function approach may be used. This approach is described in Ref. 16. Until specific values are provided, type 316 stainless steel can be assumed to behave in a similar manner.

b. Swelling. Empirical relationships describing the swelling of stainless steel, as a function of fluence and temperature, are provided in Ref. 2.

7.2.2 Sodium. To be added.

7.2.3 Steam. To be added.

7.2.4 Air. To be added.

8. REFERENCED DOCUMENTS

8.1 American Society of Mechanical Engineers (ASME).

ASME Boiler and Pressure Vessel Code,

Section III Nuclear Power Plant Components.

Code Case 1592-0 Class 1 Components in Elevated Temperature Service, Section III. |C

Code Case 1593 Fabrication and Installation of Elevated Temperature Components, Section III, Class 1.

8.2 Other Referenced Documents. The following literature is referenced in the text by number or by superscripts in brackets:

1. C. E. Pugh, J. M. Corum, K. C. Liu, and W. L. Greenstreet, "Currently Recommended Constitutive Equations for Inelastic Design Analysis of FFTF Components," ORNL-TM-3602, September 1972.

2. "Liquid Metal Fast Breeder Reactor Materials Handbook," HEDL-TME 71-32.
3. Pressure Vessel and Piping /1972 Computer Programs Verification, American Society of Mechanical Engineers, New York, 1972.
4. "High-Temperature Structural Design Methods for LMFBR Components," Periodic Progress Reports from the Base Technology Programs. (Oak Ridge National Laboratory, Oak Ridge, Tennessee).
5. "Validation of High Temperature Design Methods and Criteria," Periodic Progress Reports from the Base Technology Programs. (Westinghouse Electric Corporation, Advanced Reactors Division, Madison, Pennsylvania).
6. T. W. Pickel, G. T. Yahr, W. K. Sartory, and W. L. Greenstreet, "Study of Thermal Ratcheting Behavior and Elastic Design Requirements," in "High-Temperature Structural Design Methods for LMFBR Components Quarterly Progress Report for Period Ending September 30, 1972," ORNL-TM-4058, January 1973, pp. 190-213.
7. E. O. Weiner, "A Study of the Applicability of the Simplified Inelastic Analysis Methods for Axisymmetric Shells Under Axisymmetric Loads to Pipes and Nozzles Subjected to Bending Loads and Thermal Shocks," unpublished report, Handford Engineering Development Laboratory, October 1973.
8. "Ratchet Analysis of an Axisymmetric Nozzle," Technical Report E-1414(b), Teledyne Materials Research, Waltham, MA, February 28, 1973.
9. J. Bree, "Elastic-Plastic Behaviour of Thin Tubes Subjected to Internal Pressure and Intermittent High-Heat Fluxes With Application to Fast-Nuclear Reactor Fuel Elements," J. Strain Anal. 2, pp. 226-238 (1967).
10. W. J. O'Donnell and J. Porowski, "Upper Bounds for Accumulated Strains Due to Creep Ratcheting," Welding Research Council Bulletin Number 195, June, 1974, Welding Research Council, 345 E. 47th Street, New York, New York, 10017.
11. L. D. Blackburn, "Isochronous Stress-Strain Curves for Austenitic Stainless Steels," in The Generation of Isochronous Stress-Strain Curves, pp. 15-48, American Society of Mechanical Engineers, New York, 1972.
12. S. Timoshenko, Strength of Materials, 3rd ed., Part II, pp. 346-352, D. Van Nostrand Company, Inc. Princeton, New Jersey, 1956.

13. A. Phillips, Introduction to Plasticity, Articles 2-3 and 3-3, The Ronald Press Company, New York, 1956.
14. Timoshenko, op. cit., p. 366.
15. R. K. Penny and D. L. Marriott, Design for Creep, McGraw Hill Book Company, 1971.
16. R. L. Simmons, W. N. McElroy, and L. D. Blackburn, "Damage Function Analysis of Neutron Induced Mechanical Property Change Data in Type 304 Stainless Steel," HEDL-TME 73-85, November 1973.

APPENDIX A

HIGH-TEMPERATURE DEFORMATION BEHAVIOR

A.1 INTRODUCTION

One of the most distinguishing features of design methods for high-temperature service is the requirement to include inelastic material behavior in the analysis. Both time-independent elastic-plastic behavior and time-dependent creep behavior must be considered. These individual behaviors as well as the total combined inelastic response for a given material are very complex, consisting of processes and interactions which are not completely understood and which currently defy exact description. To carry out rational design analyses and assessments, it is necessary that the essential significant features of the overall behavior be identified and understood.

The purpose of this appendix is to provide a broad-brush description of those basic observed features of high-temperature deformation behavior which influence the selection of the constitutive equations and procedures to be used in inelastic design analyses. Information is provided to form a background for understanding the inelastic behavior of the materials under consideration, and the basic behavioral trends are identified. Although these trends are discussed primarily in a qualitative sense rather than quantitatively, as would be necessary to provide detailed guidance for constitutive equation development, the motivating factors that led to the selection of the currently recommended constitutive equations are described. In addition, directions for improvements to these equations and for further development work, both experimental and theoretical, are indicated. It is intended that the information presented, although scant in some areas, will provide a better understanding of the bases, as well as the deficiencies, of those equations and procedures recommended for detailed inelastic analyses.

Most of the current knowledge of high-temperature behavior is based on observations of uniaxial test results. Consequently, this discussion is based largely on uniaxial data, although some multiaxial observations are included. The implications of generalizing from uniaxial to multiaxial conditions are briefly discussed. Also, most of the information and discussion centers largely on a single material, annealed type 304 stainless steel.*

*Throughout this appendix, the term "annealed type 304 stainless steel" refers to material that has been laboratory reannealed as opposed to material in the as-received, mill-annealed condition.

In the current state-of-the-art, plasticity and creep are treated separately and the observed interactions are accounted for in an approximate manner. In this appendix, time-independent elastic-plastic behavior and time-dependent creep behavior are treated separately and then interactions between plasticity and creep are addressed. Finally, the influence of pretest treatments, product form, and heat-to-heat variability are each briefly discussed.

It should be reemphasized that the data presented in this appendix are qualitative only. The data should not be used directly in design analyses nor for quantitatively developing constitutive relationships for use in design analyses.

A.2 TIME-INDEPENDENT ELASTIC-PLASTIC BEHAVIOR

A.2.1 Basic Features of Cyclic Behavior. In most practical design situations, plastic deformations are the result of transient loading conditions which may occur many times during the life of a component. Therefore, elastic-plastic behavior under conditions of cyclic, as well as monotonic, loading conditions must be considered. Most of the information on cyclic elastic-plastic behavior comes from uniaxial stress-strain data that are obtained by cycling uniaxial specimens between fixed limits of total strain. A typical curve for annealed type 304 stainless steel at 1100°F is shown in Fig. A.1. The curve shown was obtained by cyclically loading a specimen between fixed strain limits of $\pm 0.3\%$ (0.6% total strain range) at a strain rate of 0.005 in./in./min. Thirty cycles are plotted.

It can be seen from Fig. A.1 that considerable cyclic strain hardening occurs in the early cycles subsequent to the initial monotonic loading of the virgin specimen. The magnitude of this cyclic hardening is particularly large in a fully annealed material. By about the tenth cycle, a major portion of the cyclic hardening has occurred, and by the end of the 30 cycles shown, the hardening has begun to saturate. This saturation effect is further illustrated by the stress histogram in Fig. A.2. Here the stress range is shown against time for a cyclic test of annealed and aged type 304 stainless steel at 1050°F and a total strain range of 2.01%. It can be seen that only a very slight additional hardening occurs as the number of cycles extends into the hundreds.

Saturation of hardening is typical of cyclic elastic-plastic behavior. In fact, the saturation cyclic stress-strain curve, or stabilized loop, appears often to be about the same for a given material and strain range, regardless of the pretest history or treatment. For example, type 304 or type 316 stainless steel in the as-received, mill-annealed condition usually has some remaining cold work, and the initial loading curve might be much above the one for the fully annealed condition. The stable cyclic loop, however, would be expected to be about the same as indicated in Fig. A.1. Likewise, for a highly cold-worked material the initial loading curve might be above the stable loop, and

Fig. A.1. Typical Cyclic Stress-Strain Curve for Annealed Type 304 Stainless Steel at 1100°F

Fig. A.2. Stress Histogram (stress ranges) for Cyclic Stress-Strain
Test of Annealed and Aged Type 304 Stainless Steel at 1050°F

cyclic softening would occur until the stable loop was reached. Thus, in addition to the initial monotonic stress-strain curve, the stable cyclic curve has particular significance in elastic-plastic behavior.

To describe elastic-plastic behavior for use in inelastic analysis, the description of several aspects of material response, in addition to Hooke's law, are required. In uniaxial terms these include: (1) a yield stress* corresponding to the onset of plastic flow; (2) equations relating the plastic strain increments to stress and stress increments subsequent to yielding; and (3) a hardening rule, specifying the change in the yield stress in the course of plastic flow. The last factor, the hardening rule, plays an important role for cyclic elastic-plastic behavior. It would for example, specify the point at which yielding occurs in each of the cycles of Fig. A.1.

Two idealized hardening models are commonly used in theories of plasticity, isotropic hardening and kinematic hardening. These two models are schematically compared in Fig. A.3 for the case of a uniaxial specimen cycled between fixed strain limits. In terms of the idealized bilinear behavior shown, isotropic hardening predicts that yielding on each reversal will occur when the magnitude of the stress reaches the value attained immediately prior to the reversal. Thus, isotropic hardening predicts an ever-increasing elastic stress range, which results in an entirely elastic behavior. Kinematic hardening, on the other hand, predicts that the elastic range, on every reversal, is equal to twice the initial yield stress. Thus, in kinematic hardening the elastic range is constant.

The behavior of real material at elevated temperatures does not fit either of these hardening models exactly. A study of Fig. A.1 shows that the elastic range for each reversal increases slightly for the first few cycles. This increase, however, is not nearly as much as would be supposed at first glance. If the yield point is carefully located for each reversal, the increase in the elastic region is seen to be small. At lower temperatures, the increase is even less. A study of Fig. A.1 in terms of isotropic hardening is in considerable error even for the first few cycles. Thus, it is concluded that for annealed type 304 stainless steel, uniaxial yielding during each reversal is reasonably well predicted by the kinematic hardening theory.

A.2.2 Effects of Strain Range on Cyclic Behavior. All points in a structure are not cycled between the same strain limits, nor, in fact, is a given point usually cycled between the same strain limits throughout the structure's life. The basic effects of strain range on cyclic behavior are demonstrated in Fig. A.4 where the tenth cycle loops for strain ranges of 0.4, 0.6, 1, 2 and 3% are shown for annealed

*For constitutive equation purposes, the term yield stress or yield point is meant to be synonymous with the proportional limit.

Fig. A.3. Schematic of Uniaxial Behaviors Predicted By Isotropic and Kinematic Hardening

Fig. A.4. Monotonic Loading Stress-Strain Curve and Tenth-Cycle Stress-Strain Curves for Different Total-Strain Ranges

304 stainless steel tested at 1100°F. The cyclic hardening is increased with increasing strain range. Note that the elastic range appears to be increased with strain range for the lower strain range values. Also, it should be noted that the slope of the plastic portions of the curves, near the tips, is nearly equal for the larger strain ranges, and these do not differ too much from the slope of the monotonic curve at the corresponding strains.

It is of interest to point out that Fig. A.4 represents a nest of tenth-cycle hysteresis loops. A similar figure could be constructed for saturated loops. If the tips of such loops are joined, as shown by the dashed line in Fig. A.4, the resulting curve is sometimes called the cyclic stress-strain curve. An alternative method for developing a saturated cyclic stress-strain curve, but one which does not always produce the same results, involves incrementally increasing the strain range of each cycle until some maximum is reached. The strain range is then decreased incrementally to zero; then the process is repeated. Eventually stability develops and no further changes in the response are observed for any strain range within the maximum loop. A curve through the saturated loops forms the incremental cyclic stress-strain curve. Although quite useful in some design applications, the incremental stress-strain curve for type 304 stainless steel depends strongly on maximum strain range and does not appear to offer any advantages over cyclic stress-strain behavior based on constant strain amplitude cycling.

The effect of strain range on the saturation of cyclic hardening is shown in Fig. A.5 for 800°F and in Fig. A.6 for 1100°F. Again, these curves are for annealed type 304 stainless steel. At both temperatures cyclic hardening, as well as saturation of the hardening occurs more rapidly in terms of number of cycles as the strain range is increased. For large strain ranges like 3%, hardening may be nearly complete at ten cycles, while for a small strain range like 0.4%, two- or three-hundred cycles may be required. The relative amount of hardening that occurs for the small strain range is much less than that for the larger strain ranges.

A few elevated temperature cyclic tests have been performed in which the strain range was varied with the number of cycles. A typical example at 1200°F is shown in Fig. A.7. Here saturation is alternately reached at strain ranges of 2% and then 1%. Upon switching from 2% to 1%, cyclic softening occurs and the 1% saturation loop is approached. Upon again switching to 2%, hardening occurs and the 2% saturation loop is approached. Tests at 800°F and 1100°F are shown in a different manner in Figs. A.8 and A.9, respectively. Here, the results of incremental strain range changes are plotted on the constant strain range curves of Figs. A.5 and A.6. In the 800°F test shown in Fig. A.8 saturation was first reached at 0.6%; the strain range was then changed to 0.98% and the stress range remained initially constant (corresponding to about 15 cycles on the 0.98% strain range curve). Hardening at 0.98% then followed the base 0.98% curve until the range was increased to 1.98%; the same response was observed. The base 1.98% curve was followed until

Fig. A.5. Stress Range vs Number of Cycles for
Annealed Type 304 Stainless Steel Tested at 800°F

TYPE 304 STAINLESS STEEL
 ANNEALED 1/2 h AT 2000°F
 (HEAT 9T2796)
 TEMPERATURE: 1100°F

Fig. A.6. Stress Range vs Number of Cycles for
 Annealed Type 304 Stainless Steel Tested at 1100°F

Fig. A.7. Cyclic Response to Block Loadings Alternately at Strain Ranges of 2% and 1% for Type 304 Stainless Steel at 1200°F

Fig. A.8. Response to Incremental Changes in Cyclic Strain Range for Annealed Type 304 Stainless Steel at 800°F

Fig. A.9. Response to Incremental Changes in Cyclic Strain Range for Annealed Type 304 Stainless Steel at 1100°F

the range was reduced to 0.98%. As the strain range was reduced, softening occurred and the stress range approached the saturation values for the reduced strain ranges. The same general trends are exhibited in the several tests at 1100°F which are depicted in Fig. A.9.

The effect of strain as discussed above is seen to depend not only upon the current range but, in general, on the environmental history as well. Thus, the analytic representation of the nonlinear behavior of an arbitrarily cycled specimen requires a repository for the environmental history. This has been accomplished for some cases at room temperature, but the elevated temperature response is much more complex, and the method for accounting for the environmental history effects is incomplete. Fortunately, stainless steel at the elevated temperatures of interest is somewhat forgiving in that the cyclic behavior at a given strain range does tend toward a characteristic saturation loop dependent only on that strain range. In this sense, the material has a fading memory as it is cycled at a given strain range.

A.2.3 Effects of Temperature. In many high-temperature reactor components temperature variations are common and, in fact, much of the elastic-plastic behavior may be the result of thermal-type loadings. Thus, temperature effects must be considered.

Stress-strain curves for type 304 stainless steel show significant decreases in deformation resistance with increased temperature for initial monotonic loadings. This is illustrated in Fig. A.10 for temperatures up to 1200°F. These curves are close to parallel over the strain range from about 1.0 to 5.0%. However, the temperature influence is significant for total strains of less than about 1.0%. The differences between yield stress are small for temperatures between 1000 and 1200°F, particularly when the strains are less than about 3.0%.

From the curves shown, the effects of increasing the temperature of a loaded specimen can be reduced. Consider a uniaxial specimen which is loaded to a stress level of 20 ksi at room temperature and held at that stress level. An increase in temperature of the specimen will cause plastic flow to occur. As the temperature level is increased, the increment of plastic flow corresponding to a given temperature increment will decrease, and will be very small in the 1000 to 1200°F range. A decrease in temperature at a given stress would result in no decrease in plastic strain since plastic strain is irrecoverable. There would, however, be a small elastic strain decrease corresponding to the increase in the elastic modulus. These effects of temperature must be taken into account by the elastic-plastic constitutive equations used in inelastic design analyses. Thus, the constitutive relations recommended in this standard include nonisothermal plasticity.

The effects of temperature on cyclic elastic-plastic behavior were illustrated by some of the figures in A.2.2. These effects are partly summarized in Fig. A.11. Here, the maximum stresses (one-half the stress

Fig. A.10. Monotonic Initial Loading Stress-Strain Curves
for Annealed Type 304 Stainless Steel

Fig. A.11. Comparison of the Monotonic Initial Loading Strength and Cyclic Strengths of Annealed Type 304 Stainless Steel Tested at Different Temperatures

range) for the tenth cycle loops of 2% total strain range tests are plotted as a function of test temperature and compared to the stress at 1% strain from the corresponding monotonic initial loading curves. The variation of ultimate strength with temperature is shown for reference. It is seen that the relative hardening (tenth cycle compared to the initial loading) is lowest at room temperature, reaches a maximum at about 900°F, and then decreases with further increase in temperature. It is interesting to observe that the stress at tenth-cycle at 900°F is higher than the stress at room temperature and that the variation is relatively small from room temperature up to 1050 or 1100°F.

A.2.4 Effects of Strain Rate. In the temperature range of interest (800 to 1100°F), the relation between strain rate and flow stress* for austenitic stainless steels is quite complex. Analytical correlations do not exist. The response of the monotonic flow stress to the imposed strain rate for annealed type 304 stainless steel is illustrated in Fig. A.12. The stresses at 0.2% offset strain and 5% strain, and the ultimate strength are plotted against strain rate at room temperature, 800, and 1100°F. At room temperature and 800°F the 0.2% offset yield stress appears to increase slightly with increasing strain rate while the ultimate strength is nearly constant. At 1100°F the yield stress shows essentially no dependence on strain rate, while the ultimate strength increases with increasing strain rate.

Under cyclic straining conditions at low strain ranges (while the stress amplitudes are near the monotonic yield strength) one might expect that the strain rate dependence of the cyclic stress amplitude would parallel the trend for the monotonic yield strength. At high cyclic strain levels (when stresses build up to values near the ultimate) one might expect the strain rate dependence of the cyclic stress amplitude to parallel the trend for the monotonic ultimate strength. This seems to be the case at 1100°F. Data which were obtained by cyclic straining at a total strain range of 1% are shown in Fig. A.13. The saturation stress amplitude is plotted against the strain rate for type 304 stainless steel in two conditions: annealed and annealed plus aged for 1000 hours at 1100°F. The figure also shows tensile strength data for the same heat which are parallel to the cyclic data.

Data which were obtained at 1000 and 1200°F from incremental step tests suggest that there is an inversion in the strain rate dependency of the saturated cyclic stress amplitude with changing temperature. Data are summarized in Fig. A.14, which shows that at 1000°F the material strained at 0.0024/min is stronger than material strained at 0.24/min. At 1200°F the opposite appears to be true.

*In this appendix, "flow stress" is a term used to denote stress level in the plastic region of a stress-strain curve.

Fig. A.12. Influence of Strain Rate on the Monotonic Flow Stress of Annealed Type 304 Stainless Steel

- MONOTONIC ULTIMATE STRENGTH
- SATURATED CYCLE STRENGTH AT 1.0 % $\Delta\epsilon$:
- ▲ REANNEALED ONLY
- REANNEALED AND AGED 1000 hrs.

Fig. A.13. Influence of Strain Rate on the Saturated Cyclic Stress Amplitude of Annealed Type 304 Stainless Steel Tested at a Strain Range of 1% and a Temperature of 1100°F

Fig. A.14. Dependency of Incremental Cyclic Stress-Strain Curves on Strain Rate for Type 304 Stainless Steel Tested at 1000 and 1200°F

In summary, the amount and type of influence on the flow stress produced by varying the strain rate depends on temperature, type of straining, and the amount of strain accumulated. Data are too meager to enable a meaningful quantitative description to be developed. For general high-temperature reactor component design analyses, a strain rate of 0.005/min is appropriate for obtaining data for use in deformation analyses. Caution should be used in utilizing data obtained at significantly different rates, particularly at higher temperatures.

A.2.5 Effect of Hold Periods on Cyclic Behavior. In design applications, transient conditions producing plastic behavior rarely occur one right after the other. Rather, transient loadings are usually separated by fairly long periods of steady operation. During these periods some regions of a component may be subject to near-constant stresses, and others, to near-constant strains, or to combinations of the two. Some regions may not be loaded at all during these periods.

At sufficiently high temperatures, holding at temperature of cyclically-hardened specimens results in thermal softening. That is, if the specimen is recycled after the holding period, the initial cyclic loops will exhibit a smaller stress range than existed in the cycles just prior to the holding period. This thermal softening has been observed to occur even when the hold occurs at a constant stress (creep hold period) or at a constant strain (relaxation hold period). Thus in realistic design situations, plastic cycling, which tends to harden the material toward the saturation value, is somewhat counter-balanced by softening due to elevated temperature exposure.

In Section 4 these considerations led to the selection of the tenth-cycle stress-strain loop as the recommended stress-strain relaxation for cyclic analyses. The tenth cycle loop is a compromise to approximately account for the softening effect of high-temperature hold periods on the fully hardened material. The purpose of this subsection is to illustrate this softening effect on cyclic hardening by presenting some of the test data that are available. The thermal softening behavior discussed here is closely related to the more general topic of interactions between plasticity and creep that will be discussed in Section A.4.

The softening associated with thermal annealing at zero stress is illustrated in Fig. A.15. Data shown were obtained by interrupting continuous cycling tests in which saturation had occurred, and holding at zero stress for the indicated times. The stress ranges that were developed in the first cycle after cyclic testing was resumed are shown in the figure. The data, corresponding to two strain levels, indicate the holding periods of less than 100 hours will restore the hardening curve to one similar to that observed after ten cycles of continuous cycling.

If, rather than holding at zero stress, a relaxation-type hold at constant strain is introduced, the results are similar. The basic effect is illustrated in Fig. A.16 where the cyclic behavior of annealed

Fig. A.15. Effect of Soaking at Zero Stress on the Cyclic Stress Range for Type 304 Stainless Steel at 1100°F

Fig. A.16. Effect of Relaxation of the Continuous Cycling Behavior of Type 304 Stainless Steel at 1100°F

type 304 stainless steel at 1100°F for a 3% strain range is illustrated. The initial continuous cycle loop to 15 cycles (near-saturation) is depicted by the solid lines. After 15 cycles the strain was held constant at 1.5% for 70 hours. The dashed cyclic loops show the subsequent continuous cycling behavior. Several cycles are required to harden back to the saturation level.

The softening which occurs as a result of relaxation hold periods is further illustrated in Fig. A.17 for two heats of type 304 stainless steel at 1100°F. Here, the curves correspond to three nominal strain ranges of 0.5, 1, and 2%, and represent hold times to 66 hours. The data points at 20 and 66 hours were obtained by interrupting continuous cycling tests, whereas data for lesser times were obtained by holding for the indicated time in every cycle. The trend of these data at the 1 and 2% ranges suggests that hold times of less than 100 hours are sufficient to reduce the stress ranges to values comparable to the tenth cycle of continuous cycling. It is important to understand, however, that the softening observed both in zero stress and relaxation soak periods is thermally induced and considerably longer hold periods would be required at lower temperatures.

Fig. A.18 illustrates that even a creep hold period during continuous cycling results in some softening at 1100°F. In this figure a specimen, which is indicated by the solid lines, was continuously cycled for 20 cycles over a 0.6% strain range. After 20 cycles the stress was lowered to 15 ksi and 300 hours creep strain was accumulated. The specimen was then reloaded to an elastic plus plastic strain of 0.3% and the continuous cycling resumed. The thermal softening which would normally have occurred in 300 hours at 1100°F has, in this case, been partially counterbalanced by creep strain hardening. Some net softening has, however, occurred.

A.2.6 Multiaxial Elastic-Plastic Behavior. The previous discussions have shown that uniaxial high-temperature cyclic elastic-plastic behavior is a complex phenomenon dependent on several inter-related factors, including environmental history. The subject becomes more involved, and the available data much more when multiaxial behavior is considered. Much of our current knowledge of multiaxial behavior is based on analytical theories which are not completely verified in detail by experimental observations.

The response factors necessary to describe the uniaxial elastic-plastic behavior must be generalized to three dimensions for general multiaxial behavior. They are the yield condition, flow law, and hardening rule. In the uniaxial case it was observed that hardening is best described by the kinematic hardening model, although there is a tendency at elevated temperature for the elastic range to increase over a limited range of cyclic plastic strain accumulation from the reference or virgin state. The multiaxial implications to the kinematic, as well as to the isotropic, hardening theory are schematically shown in Fig. A.19 for a two-dimensional stress space. The initial yield

Fig. A.17. Effect of Relaxation on the Continuous Cycling Stress Range for Type 304 Stainless Steel at 1100°F

Fig. A.18. Effect of Creep on Continuous Cycling Behavior of Type 304 Stainless Steel at 1100°F

(a) ISOTROPIC HARDENING

(b) KINEMATIC HARDENING

Fig. A.19. Schematic Descriptions of Isotropic and Kinematic Hardening in Two-Dimensional Stress Space for Uniaxial Loading*

*The movement of the yield surface is denoted by α' in this figure since α denotes the movement in deviatoric stress space.

surface, shown by the solid lines, is the familiar von Mises ellipse which combined-stress experiments shows to be a good approximation. The changes in this initial yield surface as predicted by isotropic and kinematic hardening are shown for a uniaxial specimen cyclically loaded, first to points 1 and 2, and then in the reversed direction to point 3.

In both hardening theories initial yielding occurs at point 1 and plastic strains are accumulated between points 1 and 2. Upon reversed loading, the elastic range as predicted by the isotropic hardening has expanded so that yielding does not occur until point 3 is reached. Kinematic hardening, on the other hand, predicts a constant elastic range. Thus yielding upon reversed loading occurs at point 2' and plastic strains in the reversed direction are accumulated between points 2' and 3: In isotropic hardening, the yield surface expands with its center remaining at the stress origin. In kinematic hardening, the yield surface remains fixed in size, but is translated as a rigid body. When the stress is at point 2 in Fig. A.19(b) the translation of the surface is measured by α_1' , the distance the center has moved in this stress space.

More generally, the yield surface is a hypersurface in 9-dimensional stress space and its center at any time is, for the kinematic hardening theory, denoted by α_{ij} , which is the translation tensor.* Realistically, the translation tensor is a nonlinear, history-dependent function of the plastic strain. Also, it can be seen from our uniaxial discussion that both translation and growth of subsequent yield surfaces must be considered. To do this on a rigorous basis however, requires additional experimental investigation.

Because of the close tie between plastic strain increments and the description of the yield, or loading surfaces in the mathematical theory of plasticity, the behavior of the yield surface is crucial to the development of an adequate theory. Surfaces have been observed to change shape, alter in size, and translate as a result of plastic deformation. Uniaxial test results can give a bit of evidence regarding size and position only. Influences of plastic deformation on surface configurations for stainless steels are essentially unknown as are the combined effects of deformation and temperature.

Cyclic radial, or proportional, loading** tests on thin-walled, type 304 stainless steel tubes at room temperature and in combined tension-torsion space show that, on an effective stress versus effective

*Actually, in the mathematical relations for plasticity, the translation tensor is measured in deviatoric stress space.

**Radial loading is here meant to be synonymous with proportional loading and denotes loadings in which the stress components maintain the same proportionality with each other. Radial loadings are thus represented in multiaxial stress space by straight lines passing through the origin.

stress versus effective plastic strain basis, response curves from radial combined loading tests are essentially identical within experimental error to uniaxial cyclic curves. These results thus support the kinematic hardening theory. The loadings for a typical set of tests are described in Fig. A.20(a), where the von Mises yield surface appears as a circle in the special tension-torsion space shown. The loading path along the horizontal axis corresponds to a uniaxial tension-compression loading, and the one along the vertical axis corresponds to pure torsional loading. The three remaining radial loading paths are combined loading tests at three different stress ratios. Note the experimental points in Fig. A.20(a) which denote the initial yield surface. The agreement with the von Mises surface is good.

The plastic strain components were calculated from the strain measurements and the strain trajectories are shown in Fig. A.20(b). As assumed by plasticity theory, the directions of the strain trajectories closely follow the directions of the corresponding loading paths and are perpendicular to the yield surface. The test results, in terms of effective stress-plastic strain, are depicted by the plots of $\bar{\sigma}$ versus the effective plastic strain shown in Fig. A.21. The quantity $\bar{\sigma}$ represents the magnitude of the translation of the yield surface along the loading path. The response is essentially the same for each loading path, including the uniaxial case.

Although the radial loading results support a relatively simple effective-stress effective-plastic strain relationship for multiaxial behavior there can be no such relationship for general nonradial loadings as explained in Section 4. The loose coupling between stress and plastic deformation is an important factor. Consider the case in which a specimen is loaded along a radial path in combined stress space, a two-dimensional space will suffice for illustration purposes. From test results the response for reversed loading along the radial path and its extension in the opposite stress quadrant can be determined. Further it is known that the plastic response will differ significantly during reversed loading from that exhibited during the initial loading. Thus, a general observation of the relationship for the response of a prestrained material and subsequent loading path has been established. In the case of straight-line paths the direction is a determining factor; another is the accumulated plastic strain. There are data on the response associated with only two of the infinite number of possible directions - continued loading and reversed loading along a straight line. It is known that the response for other directions must be different. For continuous behavior, there must be a continuous change in response characteristics with direction over a 180-deg change in angle in both directions from that of the original path.

Another example is that of a specimen which is prestrained as before, but in this case the stress point is moved back to the center of the current loading surface. Again, the response for subsequent motion of the stress point will depend on the accumulated plastic strain

(a) Loading Paths

(b) Measured Plastic Strain Trajectories

Fig. A.20. Radial Loading Paths for Room Temperature Tension-Torsion Tests of Type 304 Stainless Steel Tubes

Fig. A.21. Comparison of Multiaxial Response to Cyclic Radial Loadings With Uniaxial Response
For Type 304 Stainless Steel Tubes Tested at Room Temperature*

*The quantity $\bar{\sigma}$ is in terms of deviatoric stress.

and the direction for straight-line loading paths. Thus, the response characteristics are much different than those associated with virgin, isotropic material.

From the illustrations given, it can be seen that improperly developed nonlinear multiaxial theories can lead to significant inconsistencies and difficulties. Presumptions based on uniaxial observations can often lead to gross inconsistencies when the multiaxial ramifications of such presumptions are considered, particularly with respect to nonradial loading paths.

In summary, the unknowns regarding elastic-plastic behavior under combined stress conditions may be classified in two basic categories: those associated with subsequent yield surface behavior, including changes in configuration, size, and location in stress space; and those associated with subsequent deformation response for prestrained specimens. These remarks apply to the isothermal case; very little information exists regarding nonisothermal multiaxial behavior.

The elastic-plastic constitutive relations for general nonradial cyclic loadings that are recommended in this standard for interim use until more rigorous theories can be developed and verified are based on simplifying assumptions that bypass some of the problems discussed above. By using bilinear representations of the uniaxial stress-strain behavior and classical kinematic hardening theory, the dilemma concerning the response as a function of direction for a prestrained material is handled by giving the same response regardless of direction. In addition, the subsequent response prediction is encumbered by a need for accurate knowledge concerning the prior deformation history.

A.3 TIME-DEPENDENT CREEP BEHAVIOR

A.3.1 Uniaxial Behavior. The design of components to operate at temperatures in the creep region requires that the fundamental features of time-dependent mechanical behavior be identified for both sustained and variable loading conditions. The basic information on creep behavior comes from uniaxial, isothermal, constant-load creep tests, and the basic input into the currently-used structural creep analysis procedures is a mathematical representation of the resulting response in terms of creep-strain versus time.

A schematic representation of a typical strain-time response curve from a creep test is shown in Fig. A.22. Here, ϵ^L represents the instantaneous elastic plus plastic strain occurring during loading, ϵ^t represents primary, or transient, creep strain, and $\dot{\epsilon}_m$ represents the minimum creep rate that occurs during the test. The linear portion of the curve represents secondary creep, and the final upturned portion of the curve represents the tertiary, or third stage, creep. Since the intent in most cases is to avoid tertiary creep, only the primary and secondary portions of the uniaxial creep response are usually considered when

Fig. A.22. Schematic Representation of Typical Uniaxial, Isothermal, Constant-load Creep Test Result

developing the mathematical representation (creep law) for use in design analyses.

A typical set of creep curves for an annealed type 304 stainless steel is shown in Fig. A.23 for stress levels ranging from 7.5 to 20.0 ksi at a temperature of 1100°F. Although not too apparent in Fig. A.23, an important point regarding these curves is that both the magnitude of the primary and secondary creep strains and the duration of the primary creep depend strongly on stress. Generally, the time required for saturation of the primary creep response increases with decreasing stress levels (in much the same manner as was seen for elastic-plastic cyclic hardening in Section A.2 to saturate less quickly with decreasing strain range). For the relatively low stresses which are of most interest in design, these saturation times are very large. Thus, it is important that accurate data be available at low stress levels and that the mathematical form chosen to describe the data be capable of incorporating stress and temperature dependencies in all parts of the creep representation.

In realistic design situations the stress is not constant but is likely to vary throughout the life of the component. Thus a procedure for using the constant-stress creep response to predict the response when the stress levels are changing is required. Procedures for coding this are referred to as hardening rules. Two are in common use - time hardening and strain hardening - and they are represented schematically in Fig. A.24. The curves labeled σ_1 and σ_2 in the creep-strain versus time plot are the constant stress creep curves. Assume that creep occurs at the σ_1 stress level until time at point 1, when the stress is changed from σ_1 to σ_2 . At this point, the time hardening rule would indicate that the creep response follows the σ_2 curve beginning at point 1'. Strain hardening would indicate that the response follows the σ_2 curve beginning at point 1". The two predicted responses are shown beginning at point 1.

To evaluate these and other hardening rules, step-load creep tests are performed. Fig. A.25 illustrates the results of one such test on annealed type 304 stainless steel at 1200°F. Here, near secondary, or steady-state creep was reached at a stress level of 15 ksi. The stress was then increased to 17 ksi, and later to 20 ksi. Note that despite the fact that the primary creep was saturated at 15 ksi, large primary creep strains occurred at 17 and 20 ksi. The predictions shown are based on two alternate forms of strain hardening. The agreement is reasonably good. Time-hardening on the other hand would not predict the occurrence of primary creep strains at the higher stress levels.

Other step-load creep tests show similar results. That is, strain-hardening predictions are superior to those of time hardening. Thus, the constitutive relations recommended for creep analyses are based on strain hardening.

Fig. A.23. Typical Creep Curves for an Annealed
Type 304 Stainless Steel Tested at 1100°F

Fig. A.24. Schematic Representation of Time-Hardening and Strain-Hardening Models of Creep Response Under a Stepwise Varying Load

Fig. A.25. Results of Step-load Creep Test Compared with Strain-Hardening Predictions for Annealed Type 304 Stainless Steel at 1200°F

In addition to changes in stress magnitude, stress reversals can occur. These reversals require special consideration to avoid anomalous behavioral predictions when the currently available response models are used in creep analyses. Thus auxiliary strain-hardening rules for cyclic creep conditions are provided in the inelastic analysis guidelines of this standard. The cyclic creep responses of two different annealed type 304 stainless steel specimens tested at 1100°F are shown in Figs. A.26 and A.27 and compared with the predictions of the auxiliary strain hardening rules recommended in this standard and with those of time hardening. In Fig. A.26 the specimen was loaded to ± 15 ksi while in Fig. A.27 the loading was ± 17.5 ksi. The predictions in both figures were made on the basis of a creep equation which was developed as a representation of a group of uniaxial, constant-load, creep tests of specimens taken from the same steel plate. The difference between the experimental and predicted creep strains during the initial loading portions of these tests is due only to the fact that the constant-stress, uniaxial creep law does not agree with the particular uniaxial test results. The measured creep response in each subsequent cycle is slightly less in each test than the response in the previous cycle. Nonetheless, the agreement with the predictions of the auxiliary strain-hardening procedure is reasonably good, particularly when compared to the predictions of time hardening.

A.3.2 Uniaxial Relaxation Behavior. In the creep analysis procedures currently used for metals the relaxation behavior is assumed to be simply a manifestation of creep; the usual creep analysis procedure readily handles relaxation as well. Thus, comparison of uniaxial relaxation test results with creep predictions offers a convenient check of this basic assumption and of the basic creep analysis procedure.

Fig. A.28 shows typical relaxation test results for an annealed type 304 stainless steel specimen at 1100°F and 0.5% strain. The predictions of the strain-hardening creep theory are presented along with the experimental data. These predictions are based on the same creep equation that was cited in connection with Figs. A.26 and A.27. The predictions were obtained in the classical manner of dividing time into small, but discrete time increments and assuming that for each increment creep occurs at a constant stress. The stress to be used for each subsequent time increment is determined by subtracting from the stress in the previous time increment the stress increment necessary to elastically restore the total axial strain to the prescribed constant value. This procedure is equivalent to the time-increment, initial-strain approach generally used in creep analyses.

Some of the disagreement between the measured and predicted behavior in Fig. A.28 is due to the fact that the uniaxial creep law does not adequately represent the very steep initial portion of the uniaxial creep response at the stress levels and very short times of interest (for example, compare the measured and predicted creep response at about 10 hours in Fig. A.26). Considering this, the agreement is reasonable.

Fig. A.26. Comparison of Measured and Predicted Cyclic Creep Response of an Annealed Type 304 Stainless Steel at 1100°F

Fig. A.27. Comparison of Measured and Predicted Cyclic Creep Response of an Annealed Type 304 Stainless Steel Specimen at 1100°F

Fig. A.28. Comparison of Measured and Predicted Uniaxial Relaxation Response of Annealed Type 304 Stainless Steel for a Strain of 0.5% at 1100°F

A.3.3 Multiaxial Behavior. In addition to a uniaxial constant-stress creep law and a hardening rule for variable loading conditions, a multiaxial flow rule, or multiaxial statement of the constitutive relations is required. Inherent in the flow rule that is commonly employed in creep analysis, and which is recommended in this standard, is the prediction that under a constant sustained multiaxial loading condition the ratio of the creep strain components will be constant. Fig. A.29 shows results for a combined tension-torsion test of a tubular specimen of annealed type 304 stainless steel at 1100°F. The stress levels for this test, which had a duration of 1000 hours, are listed in Table A.1, as test No. 1. The torsional creep strain and the axial creep strain shown in Fig. A.29 are proportional as predicted.

Table A.1. Creep Strain Ratios for Combined Tension-Torsion Loadings - Annealed Type 304 Stainless Steel, 1100°F

Test No.	Axial Stress σ (ksi)	Torsional Stress τ (ksi)	Effective Stress $\bar{\sigma}$ (ksi)	Stress Ratio τ/σ	Strain Ratio: Torsion/Axial $\epsilon_{12}/\epsilon_{11}$	
					Observed	Predicted
1	10.628	6.111	15.00	0.575	0.92	0.862
2	13.975	3.407	15.17	0.244	0.37	0.366
3	5.570	7.934	14.83	1.424	2.18	2.137

In addition to test No. 1, two other combined tension-torsion tests are listed in Table A.1. Each of these tests also exhibited proportional torsional and axial creep strains, with the experimentally observed proportionality constant shown in the next to the last column. The last column of the table shows the proportionality constants predicted by the constitutive equations recommended in this standard. The agreement between predicted and measured values is good.

The measured axial and torsional creep strains for test No. 1 are shown in Figs. A.30 and A.31, respectively, as a function of time. Also shown in these figures are the predicted creep strains which were determined using the recommended multiaxial relations and the uniaxial, constant-stress creep law that has been discussed. The agreement in each case is about the same as would be expected between the creep law predictions and a given uniaxial creep curve.

In another combined tension-torsion test at 1100°F, the sustained stress portion of loading, which had a duration of 1000 hours, was followed by a 300-hour period of complete unloading at zero stress. The creep strains that were recovered during this period of zero stress are shown in Fig. A.32 and are seen to be very small compared to the creep strains during the sustained loading portion of the similar test depicted in Figs. A.30 and A.31. This lack of significant recovery

Fig. A.29. Measured Torsional Strain vs Axial Strain for Combined Tension-Torsion Creep Test of Tubular Specimen of Annealed Type 304 Stainless Steel at 1100°F

Fig. A.30. Measured vs Predicted Axial Creep Strains as a Function of Time for Test No. 1 in Table A.1

Fig. A.31. Measured vs Predicted Torsional Creep Strain as a Function of Time for Test No. 1 in Table A.1

Fig. A.32. Measured Creep Strains Recovered at Zero Stress in a Combined Tension-Torsion Specimen at 1100°F After a 1000 hour Sustained Loading Period

of creep strains under unloaded conditions has also been observed in unloaded segments of uniaxial creep tests on at least two heats of type 304 stainless steel. In these uniaxial tests the duration of the unloaded test segments ranged up to approximately 100 hours. These results indicate that recovery strains are practically nil and the creep analysis procedure should thus predict zero recovery. The creep procedures recommended in this standard are correct in this respect.

The knowledge of creep hardening under general multiaxial, non-radial* loading conditions is, as was the case for elastic-plastic behavior, extremely limited. It is presumed that there is a now reasonable understanding of hardening along radial paths. That is, it is believed that strain hardening is adequate for strain paths along a radial line so long as stress reversals do not occur. For stress reversals resulting in creep strains along the same radial path, but in the opposite direction, strain hardening together with the auxiliary rules given in this standard can be used. Thus, the hardening behavior of a material prestrained in creep only when strains occur along either direction of a radial line is known. For any other direction it is known only that the hardening behavior must vary uniformly over the 180-deg range between the two known strain-response directions. This is an area where much additional experimental data are needed before a completely rational formulation for general nonradial multiaxial strain paths can be formulated.

A.4 PLASTICITY-CREEP INTERACTIONS

Although the current state-of-the-art requires that plasticity and creep constitutive equations be formulated largely on independent bases, elevated-temperature deformation is the result of time-dependent processes in which both plastic and creep behaviors are present simultaneously. Prior creep deformations influence subsequent plastic behavior and prior plastic deformations influence subsequent creep behavior. The extent of these interactions is discussed in this section on the basis of a very few test results for type 304 stainless steel.

A.4.1 Effect of Prior Creep on Subsequent Cyclic Plasticity.

Very little conclusive information is available regarding the influence of prior creep strain accumulation on subsequent cyclic plasticity. One reason for this is that fully annealed stainless steels often have a low yield strength relative to their creep resistance. Thus it is difficult to introduce significant creep strain without first accumulating relatively large plastic strains, especially at temperatures of 1100°F and below.

*The term "radial" is here taken to imply proportional changes in creep strains.

Results of three uniaxial cyclic tests on specimens which had previously accumulated creep strains are shown in Fig. A.33 where they are compared to continuous cycling curves for virgin specimens. The three annealed type 304 stainless steel creep specimens were first loaded to a stress of 15 ksi at 1100°F and creep strains of 0.1, 0.20, and 0.24%, respectively, were then accumulated. The loads were then removed and standard continuous cycling tests were begun with a strain range of 0.6%. For these latter tests the zero reference strain was taken to be at the unloaded condition after the creep period. The points shown for cycle 1 in Fig. A.33 are the tensile stresses reached on reloading to a strain of 0.3%. If no creep hardening had occurred, the stress levels in the first loading to 0.3% would have been about 16 ksi due to the prior plastic strain hardening.* The actual stresses reached for the specimen with 0.2 and 0.24% accumulated creep strains were higher than this, and it can be surmised that the difference is due to creep hardening, with the larger creep strain producing the larger amount of hardening. The specimens with just 0.1% accumulated creep strain showed no apparent creep hardening since the tensile stress on reloading was about 16 ksi. These test data provide some support to the standard in which it is recommended that a creep strain of 0.2% be taken as a threshold value, below which creep has no effect on the elastic-plastic behavior.

Data of a slightly different type exist to show the creep strains increase the monotonic yield stress. Studies of these data suggest that creep strains can be treated approximately as equivalent to plastic strains in regard to their hardening effect on the subsequent monotonic plastic flow behavior. Data of this type, collected at 1000, 1100, and 1200°F, are shown in Fig. A.34. These results show the measured-stress versus total-strain response from uniaxial step-load creep tests. Both the instantaneous elastic-plastic strains and the time-dependent creep strains (the horizontal lines at constant stress levels) are shown. The creep hardening effect is readily apparent in these results. Significant creep strains raise the yield stress. At 1200°F, for example, no significant plastic strains occurred in reaching a stress level of 15 ksi in the step-load creep test whereas a plastic strain of more than 1.5% would be accumulated before reaching 15 ksi in a monotonic loading test.

In summary, the available data seem to indicate that for a specimen that has not been completely hardened by cyclic plastic straining, creep strains can increase the yield strength and presumably

*The basis for the 16 ksi estimate can be seen by referring to the 1100°F monotonic stress-strain curve that was shown in Fig. A.4. If no creep hardening had occurred, the specimens would have been loaded to 15 ksi, unloaded, and upon reloading would have returned elastically to the 15 ksi point and then followed loading curve to a point corresponding to about 16 ksi.

Fig. A.33. Effect of Prior Creep on Subsequent Cyclic Elastic-Plastic Behavior of Annealed Type 304 Stainless Steel at 1100°F

Fig. A.34. Effect of Creep Strains on the Elastic-Plastic Loading Strains in Step-load Creep Tests at 1000, 1100, and 1200°F. Data Are for Annealed Typed 304 Stainless Steel

the elastic-plastic cyclic loops. However, it was seen in A.2.5 that if saturation hardening has previously occurred, it is likely that thermal softening will predominate over the creep hardening, and the net effect during the creep period will be to reduce the prior hardening.

A.4.2 Effect of Prior Plasticity on Subsequent Creep. It is generally observed that large prior plastic strains of several percent in the form of cold work can improve the creep resistance of stainless steels in the temperature range of 1000 to 1300°F.* The optimum improvement depends on the specific alloy, the creep stress, and the creep temperature. Likewise, data indicate that large plastic strain incurred at elevated temperatures can reduce subsequent creep rates. Relaxation tests performed after strain cycling over large strain ranges show this effect.

It is not necessarily true, however, that small plastic strains should produce similar creep hardening effects. In fact the data that exist suggest that perhaps the opposite is true. That is, small plastic strains slightly accelerate creep.

In conclusion, it would seem that for the relatively small plastic strains that will be induced in an actual component by the mechanical and thermal loadings, little effect on the creep rate should be anticipated. In the inelastic analysis guidelines of this standard, it is recommended that the effects of prior creep on subsequent plasticity be approximately accounted for, but that the effects of prior plasticity on subsequent creep be ignored. Needless to say, much more experimental work is needed in this area to yield quantitative relationships.

A.5 INFLUENCES OF PRODUCT FORMS AND HEAT-TO-HEAT VARIABILITY

The previous observations and discussions were based largely on test results for a single heat of one material with all specimens having had similar pretest heat treatments. In actual design applications inelastic analyses must be performed on components yet to be built and little may be known about the specific heats of material which ultimately be used. Even within a given heat properties can vary widely from product form to product form, the amount of cold work induced during manufacturing can have a large effects, and the heat treatment used for the actual part likewise has a significant effect. Thus the designer must be aware of the implications of utilizing a single set of average, or representative, stress-strain and creep behavior descriptions to predict the inelastic deformation behavior of all component parts of a given alloy.

*There is a strain limit, at each temperature, beyond which additional cold work begins to reduce the creep resistance.

In this section the magnitudes of the possible variations of properties within a given heat and from heat to heat within a given alloy are very briefly discussed. Again the discussion is restricted to type 304 stainless steel for which sufficient data are available for general observations.

A.5.1 Variation Resulting from Product Form and Heat Treatment.

The magnitude of the variation in the initial monotonic stress-strain response that can exist within a single heat of material is illustrated in Fig. A.35 where room-temperature stress-strain curves are shown for different product forms of type 304 stainless steel in the as-received, mill-annealed condition* and in the reannealed, or laboratory-annealed condition. The reannealed material from all product forms exhibits very similar behavior. However, the variations in the as-received condition are quite large and would have an obviously significant effect on inelastic analysis predictions of initial behavior. Fortunately, the relative differences exhibited in Fig. A.35 would be expected to decrease with subsequent cyclic elastic-plastic loadings.

It should be emphasized that the individual curves in Fig. A.35 are for so-called mill-annealed material. In actual reactor system components, material in some regions may receive no annealing treatment and may contain forming cold work amounting to several percent. Thus, the expected variation could be similar to that shown in Fig. A.35.

The variability in the creep behavior of reannealed specimens taken from plate, bar, pipe, and tube product forms of a single heat of type 304 stainless steel is illustrated in Fig. A.36. Here the creep strain rates and rupture times are illustrated for a stress level of 25,000 psi at 1100°F. The variation is not extremely large. However, if results for material in the as-received, mill-annealed condition were included the variation would be much more significant. Cold work, particularly at the lower stress levels, results in significant increases in creep resistance. Thus the picture for creep behavior is much the same as for stress-strain behavior. That is, the variability within a given heat is not too large provided all material is fully annealed. The variability is, however, quite large for material in the as-received condition.

A.5.2 Heat-To-Heat Variations.

The variability of the room-temperature tensile behavior of approximately 50 heats of type 304 stainless steel in the as-received, mill-annealed condition is depicted by the yield strength histogram in Fig. A.37 and the range of stress-strain behaviors at room-temperature of 16 heats in the as-received

*The as-received material was given an annealing treatment at the mill. Much of the variation in behavior exhibited in Fig. A.35 is probably due to cold work introduced in subsequent straightening operations.

Fig. A.35. Room Temperature Stress-Strain Curves Obtained for Various Product Forms From A Single Heat of Type 304 Stainless Steel*

*The individual curves are for as-received mill annealed material while the shaded band represents results from reannealed material of the same product forms.

Fig. A.36. Variation in Creep Responses of Fully Annealed Specimens From Plate, Bar, Pipe, and Tube Product Forms of a Single Heat of Type 304 Stainless Steel

Fig. 37. Histogram of 0.2%-Offset Room-Temperature Yield Strengths of Specimens From Approximately 50 Heats of Type 304 Stainless Steel in the As-received, Mill Annealed Condition

condition is shown in Fig. A.38. For comparison, stress-strain behavior at 1100°F for the same 16 heats in the as-received condition is shown in Fig. A.39. The spread in the data is not too dissimilar from that shown in Fig. A.35 for the different product forms of the one heat.

Reannealing diminishes the spread of stress-strain curves for different heats, particularly at elevated temperatures, in a manner similar to that observed for different product forms of a single heat. For example, stress-strain curves at 1100°F for the 16 heats discussed above are shown in Fig. A.40 for the reannealed conditions. The spread in yield data is smaller than shown in Fig. A.39. The differences that are exhibited in Fig. A.40 reflect mainly the effect of chemical composition and perhaps of grain size. In all of these cases the stress-strain curves are essentially parallel beyond the 0.2%-offset yield point.

The variation in the as-received stress-strain properties and the effects of reannealing are further illustrated in Figs. A.41(a) and A.41(b). In Fig. A.41(a) the 0.2%-offset yield data for the above 16 heats in the as-received condition are plotted at room-temperature, 800, 1100, and 1200°F and compared with the minimum yield strength curve given in Code Case 1592 and an average yield strength curve determined from the average stress-strain representation given by Blackburn.^[1]* This figure is a very graphic illustration of the variation in stress-strain response that can be expected by the designer. Fig. A.41(b) depicts the yield data for the same heats in the reannealed condition. Two significant observations can be made. First, the data spread has been decreased somewhat by reannealing, and second, the yield strengths of many of the heats fall below the minimum specified in Code Case 1592. This latter point should be kept in mind when analyzing components which are to be reannealed. Consideration of all the data in both Figs. A.41(a) and (b) shows that the yield strengths at a given temperature can vary by more than a factor of two.

The extent of the variability in the elastic-plastic cyclic-hardening behavior from one heat to another has not been well established. As stated earlier, it appears that fully annealed material exhibits pronounced hardening at large strain amplitudes while heavily cold worked material cyclically strain softens. Nonetheless, variability persists as evidenced by the plot of cyclic hardening data in Fig. A.42. The stress amplitude at saturation for three strain levels, 0.5, 1 and 2% are shown plotted against the testing temperature. Six heats and several different heat treatments are included. On a percentage basis, the variability in cyclic strength is smaller than the variation expected for monotonic yield and ultimate strengths.

*Superscripts in brackets refer to references in Section A.7.

Fig. A.38. Range of Room-Temperature Stress-Strain Responses for 16 Heats of Type 304 Stainless Steel in the As-received, Mill Annealed Condition*

*Data from plate (A-240) and bar (A-479) product forms are shown.

Fig. A.39. Range of Stress-Strain Responses at 1100°F for 16 Heats of Type 304 Stainless Steel in the As-received, Mill Annealed Condition*

*Data from plate (A-240) and bar (A-479) product forms are shown.

Fig. A.40. Range of Stress-Strain Response at 1100°F for 16 Heats of Type 304 Stainless Steel in the Reannealed Condition

(a) As-received, mill-annealed condition

(b) Reannealed condition

Fig. A.41. Comparison of 0.2%-Offset Yield Strength of 16 Heats of Type 304 Stainless Steel with Minimums Given in Code Case 1592-0

Fig. A.42. Stress Amplitude at Saturation for Cyclic Stress-Strain Tests on Six Heats of Type 304 Stainless Steel in Three Pretest Conditions*

*All data were obtained at a strain rate of approximately 0.24/min

The wide range of creep-rupture strengths exhibited by different heats of type 304 stainless steel is indicated by the histograms in Fig. A.43. This spread in strength reflects differences in the chemistry, grain size, and mechanical treatment. In contrast to its influence on stress-strain behavior, reannealing does not appreciably decrease the spread in creep-rupture strength. As illustrated by the creep curves for 15 reannealed heats plotted in Fig. A.44, the heat-to-heat variation in creep deformation response is also appreciable.* To gain a better appreciation of the heat-to-heat variation, two series of tests were performed to determine the variation within a single heat. The results of five identical tests on a single heat are shown as the lower scatter band in Fig. A.44. The upper scatter band represents the results for different product forms within another heat. The dashed lines in Fig. A.44 represent the predictions of the one-exponential creep law for type 304 stainless steel given in the LMFBR Materials Handbook.^[2]

Less information on heat-to-heat variation in creep deformation response is available for the lower stress levels which are meaningful in regard to long-term service. One set of curves for a stress level of 17,000 psi and a temperature of 1100°F is shown in Fig. A.45. Several of the curves shown are for material in the as-received mill annealed condition. Again the predictions of one- and two-exponential creep laws are shown.

In terms of time to produce a given deformation, the variability in Fig. A.45 is quite large. This could amount to one or two log cycles in time. On a stress basis, there is no reason to believe that the stress to produce a given deformation in a given time would be much different from that indicated for rupture in Fig. A.43. This amounts to about 0.2 log cycles in stress. In terms of temperature, a very weak heat could exhibit about the same creep response as a very strong one that is subjected to a 100°F higher service temperature.

In summary, significant variations in elastic-plastic and creep behaviors can exist in components of a given alloy, depending on such factors as composition, grain size, heat treatment, and degree of cold work.

A.6 DEFORMATION BEHAVIOR OF OTHER AUSTENITIC ALLOYS

Throughout this appendix, the basic features of high-temperature material response have been outlined using type 304 stainless steel test data to illustrate the trends. In this section the similarities and differences in the behaviors of types 304 and 316 stainless steel are discussed.

*Note that very large plastic strains would have been accumulated for some of the heats upon loading. These large strains could have an influence on the subsequent creep response.

Fig. A.43. Histograms of Creep Rupture Strengths of Several Heats of Type 304 Stainless Steel in Both the As-received Mill-Annealed Condition and the Reannealed Condition*

*The black squares correspond to a particular heat, 8043813.

Fig. A.44. Creep Curves From 22 Tests on 15 Reannealed Heats of Type 304 Stainless Steel 35,000 psi and 1100°F Compared with Predictions of HEDL One- and Two-Exponential Creep Laws

Fig. A.45. Creep Curves From Tests on Several Heats of Type 304 Stainless Steel in the As-received Mill Annealed and Reannealed Condition at 17,000 psi and 1100°F Compared with Predictions of HEDL One- and Two-Exponential Creep Laws

A.6.1 Time-Independent Behavior of Type 316 Stainless Steel. The basic features of plastic deformation in type 316 stainless steel are quite similar to type 304. Regarding the monotonic tensile behavior, type 316 stainless steel, being more highly alloyed, exhibits higher yield strengths on the average than type 304. The cyclic hardening response of laboratory reannealed type 316 stainless steel is very similar to type 304. Both exhibit considerable hardening in the first few cycles which quickly saturates at large strain amplitudes. At small cyclic strain amplitudes hardening for both materials is more gradual than at large strain amplitudes.

At intermediate temperatures, such as 800°F, where the monotonic yield strength is low and the ultimate strength is high, saturation occurs quite slowly. As in the case for type 304, the maximum difference between the initial and saturation stress amplitude occurs in the vicinity of this temperature.

The effect of strain rate on the cyclic hardening response of type 316 is also qualitatively similar to that of type 304. Both alloys show higher stress amplitudes at lower strain rates at temperatures between 800 and 1000°F. At 1200°F and above both alloys show decreased strength with decreasing strain rate.

A.6.2 Time-Dependent Creep Behavior of Type 316 Stainless Steel. Statistically, type 316 stainless steel is stronger in creep than type 304. Qualitatively it is similar in the sense that in the fully annealed condition it usually exhibits a relatively long period of primary and secondary creep.

The creep characteristics of type 316 stainless steel are influenced to some extent by the metallurgical instability which this alloy exhibits. In particular, there exists a long-time drift toward loss in creep resistance as a consequence of aging processes. The extent to which cyclic loads, temperatures, or strains accelerate the phenomena cannot be quantified at this time.

A.7 REFERENCED DOCUMENTS

1. L. D. Blackburn, "Isochronous Stress-Strain Curves for Austenitic Stainless Steels," in The Generation of Isochronous Stress-Strain Curves, pp. 15-48, The American Society of Mechanical Engineers, New York, 1972.
2. "Liquid Metal Fast Breeder Reactor Materials Handbook," HEDL-TME 71-32, Data Sheet 304SS-23, July 30, 1971.

APPENDIX B

COMPUTER PROGRAMS FOR DETAILED INELASTIC ANALYSIS

B.1 INTRODUCTION

This appendix contains brief descriptions of a number of available computer programs reported by their developers to be capable of carrying out the type of detailed inelastic design analyses specified in Section 4. In no case should this appendix be taken as an endorsement or recommendation of any of the programs described. Such could come only from a comprehensive evaluation based on first-hand experience with the programs for a variety of geometries, loading conditions, and types of material behavior. This experience is not generally available. Many of the inelastic programs need to be evaluated with regard to accuracy and efficiency before their real value can be assessed. Most programs have not been adequately verified. All must be qualified, as described in Section 5, for any particular design analysis application.

The brief descriptions of the programs and their capabilities included here were prepared in each case by the program developers or sponsors on the basis of a common outline. As can be noted, some of the programs are available at no cost, except for handling, others may be purchased, while some are available for use on a royalty or rental fee basis. For more complete information, the sources listed should be contacted.

The programs described range from large general-purpose programs to smaller special-purpose and research programs. Some of the larger codes include compatible thermal analysis capabilities. In addition to inelastic nonlinear analysis capabilities, some include provisions for performing dynamic analyses and instability (buckling) analyses. The general-purpose programs can be used on a variety of geometries since they have a large selection of finite elements. The special-purpose programs are generally limited to a certain class of geometrical shapes such as plane and axisymmetric two-dimensional solids or general three-dimensional shells. Finally, the research-type programs are specialized computer codes developed largely for in-house use but which are potentially useful to others.

Each of the programs described are presented in a uniform format. The format was established to adequately present the characteristics and limitations of the most complex program. The smaller programs do not need all of the headings. Some short standardized responses are used for brevity; clarifying statements are made when first used.

B.2 MARC SERIES

B.2.1 Brief Program Description. The programs form a general purpose capability for analysis by the finite element method. They

perform linear and non-linear stress and heat transfer analysis by the direct stiffness method for two and three-dimensional continuance.

The programs are as follows:

1. MARCHMESH2D
2. MARCHMESH3D Two and Three-Dimensional Mesh Generation and Plotting.
3. MARCSTRUCPL
4. MARCHEAT Two and Three-Dimensional Heat Transfer Analysis.
5. MARCCDC General Purpose Stress Analysis Package.
6. MARCPLOT Contour and Time History Plotting from Tapes Written by MARCHEAT or MARCCDC.

The programs were originally developed on IBM 360 series machines, but in the last two years they have been used almost exclusively on CDC 6000 series machines. All programs are written in standard Fortran IV and are operational on both CDC 6000/7000 and IBM 360/370 series machines. The versions for the IBM machines use selective double precision for stress analysis (MARCCDC: stiffness assembly and solution in double precision) but otherwise use single precision. All the programs are designed around the concept of dynamic core allocation, whereby the core is used according to individual problem size and type. There are no imposed problem (or element) size limitations other than those of the computer system used. All the programs are stand-alone, but share coding as far as possible. While this approach has some disadvantages, it has the strong advantage that the common coding becomes well checked out and reliable. The programs are widely used both in analytical applications and as research tools.

B.2.2 General Information.

B.2.2.1 Type of Structure. The element libraries cover most two and three-dimensional structures. The elements are in two main groups, (1) continuum elements and (2) reduced dimensionality elements, the latter consisting of beams (in a plane or in space, open and closed section), plates and shells. Most of the elements in the program are isoparametric in formulation.

B.2.2.2 Method of Analysis. The direct stiffness method is used, the governing equations being those obtained by a Galerkin procedure.

B.2.2.3 Restart Capability. In MARCCDC restart is provided at any load increment, at the user's option. For any part of the analysis, the user may choose the frequency of writing restart data to tape; restart is then available at any one of these points. At any restart the user may adjust his control parameters. A utility program is provided to allow the user to access the restart tape for his own use.

B.2.2.4 Computer System Requirements. The programs are all written in Fortran IV and, as such, are machine independent. The biggest program in the system is MARCCDC, and this program requires an overlaying loader. Coding takes about 26000 (decimal) words of core on CDC-6000 machines, about 230K (decimal) bytes of core on IBM 360-370 machines. Input and output (I/O) requirements vary with problem size. The programs will run totally in-core on small problems, while the biggest problems need three secondary storage allocations (2 random access, 1 sequential) plus optional restart, plot, and mesh tapes. I/O times vary for the same reason and are heavily dependent on machine configuration. For example, a typical large problem may require a wall-clock time about two to five times the Central Processor Unit (CPU) time on a CDC 6600 without extended core storage (ECS), but with a large block of ECS the same problem will take wall-clock time of about 1.2 times CPU time. An alternate loader is being developed for use on small core machines (CDC-7600).

B.2.3 Element Library.

B.2.3.1 Three-Dimensional Elements. Both MARCHEAT and MARCCDC use the eight-node, linear edge isoparametric hexahedron and the 20-node, quadratic edge isoparametric hexahedron. Wedges and tetrahedra may be generated, if desired, by degeneration, retaining or removing the gradient singularities.

B.2.3.2 Two-Dimensional Elements.

B.2.3.2.1 MARCHEAT. When used for plane and axisymmetric analysis MARCHEAT uses either:

1. Linear interpretation triangles.
2. Linear edge isoparametric quadrangles.
3. Quadratic (nine-node) isoparametric distorted quadrangles.
4. Cubic (four-node, four degrees of freedom per node) isoparametric distorted quadrangles.

B.2.3.2.2 MARCCDC. When used for plane or axisymmetric analysis MARCCDC uses either:

1. Linear interpretation triangles.
2. Linear edge isoparametric quadrangles.

B.2.3.3 Plate Element. (Information not supplied).

B.2.3.4 Shell Elements. Three doubly-curved thin shell elements and a doubly-curved thick shell element are available in

MARCCDC. The thin shell elements are based on the Dupuis formulation. This is a fully compatible, isoparametric formulation based on Koiter-Sanders shell theory. The elements are written as a triangle, a four-node quadrilateral and an eight-node quadrilateral. The thick shell element is the eight-node reduced integration element of Zienkiewicz.

For axisymmetric shells two elements are available; the Khojasteh-Bakht curved shell element, and a complete cubic isoparametric element (the one-dimensional version of the Dupuis formulation).

B.2.3.5 One-Dimensional Elements. Four one-dimensional elements are available in MARCCDC:

1. Simple straight truss.
2. Beam in a plane.
3. Closed section beam in space of arbitrary section (without warping but including torsion).
4. Open section curved beam in space with arbitrary section, including primary warping and torsion.

For all shell and beam type elements section properties are integrated numerically. For shells and plane beams eleven points are used through the thickness. For beams in three dimensions the user chooses the number and position of these integration points when he defines the section either by subroutine or on data cards. In all cases, temperature data must be provided at each of these section points. The stress-strain-temperature relations are computed at each point and then integrated numerically over the section.

NOTE: All elements include large displacement effects if flagged, and may be used with the full library of constitutive theories.

B.2.3.6 Other Special Purpose Elements. MARCCDC has special pipe-bend elements, which are generated by extension of axisymmetric shell elements. These allow for coupling of curved-beam modes along the pipe bend with shell modes (ovalization) in the section. The elements allow nonlinear analysis of pipe bends, elbows, and complete piping runs, using Karmann pipe bend theory.

B.2.4 Material Behavior.

B.2.4.1 Plasticity.

a. For the constitutive equations, flow rules are all associative; perfect plasticity and both isotropic and full-Prager kinematic hardening are available. Four yield conditions are available; the von Mises

condition, and extensions to either linear or quadratic hydrostatic stress dependence, and anisotropic yield as defined by Hill. Work hardening slopes may be input by user subroutine or by slope-breakpoint tables. The theory recommended in this standard is available as an option: initial and 10th cycle yield and work-hardening may be independently temperature dependent.

b. For the numerical approach, the tangent modulus method of analysis is used, together with mean normal and mean stiffness techniques. The load step size is user controlled, and the convergence of the solution is controlled by user-set tolerance. Residual load correction is available as a recommended option.

c. The constitutive and element libraries are independent, so that any constitutive theory will work with any element.

B.2.4.2 Creep.

a. For the constitutive equations, two creep flow rules are considered both radial to the J_2 stress surface, and volumetric (e.g., irradiation creep). The equivalent tensile creep law is input as a function of stress, temperature, time, total creep strain, and any other state variables by direct coding in a user subroutine or on data cards, so that any form of equivalent creep rate equations may be used. The theory recommended in this standard is available as an option, in combination with any equivalent rate law.

b. For the numerical approach, the initial strain is used for time dependent material behavior. The time increment selection scheme is automatic, based on user-set tolerance on stress change and creep strain increment per elastic strain. The scheme is designed to generate monotonically increasing time-step size where possible - it is described in detail in the references and in the MARC manuals. Briefly, the scheme subdivides increments which exceed either tolerance so as to generate a convergent time-step size, and increases the time-step size if all changes are below certain percentages of the tolerance. The scheme is explicit so as to avoid time-step-size oscillations often seen when implicit schemes are used with complex creep laws.

B.2.4.3 Material Property Input.

a. Any property may be temperature dependent. For thermal analysis, latent heat effects may also be included, and special procedures are invoked to handle the fusion problem. Mechanical properties are allowed to reduce the strength to zero as the material melts.

b. No properties are built-in; all are user defined.

c. Base values of properties and temperature dependence are input in tables. Work hardening slopes and creep laws may be input in tables or, more commonly, via user subroutines.

B.2.4.4 Isotropic or Orthotropic Behavior. Arbitrarily oriented orthotropy for both elastic constants and the yield function is available. The orientation and properties are defined point wise through the mesh.

B.2.5 Thermal Analysis. The program is primarily a non-linear transient thermal analyzer. However, since the time integration operator is unconditionally stable for linear problems the steady-state solution in such a case may be obtained by a single large time step or, more directly, by setting all specific heats to zero. For mildly non-linear problems (e.g., slowly temperature dependent properties or film coefficients) the steady-state solution can be stepped out with a few large time steps, but for severe nonlinearities (e.g., fusion with latent heat, or radiation) the transient solution must be followed since no artificial damping is introduced.

B.2.5.1 Method. The method of analysis is finite element, using the modified Crank-Nicolson operator for time integration.

B.2.5.2 Initial Conditions. Any general initial conditions are allowed through a user subroutine. Alternatively spatially dependent conditions may be set up from cards or tape.

B.2.5.3 Boundary Conditions.

a. Convective boundary conditions are available by programming temperature or time dependent film coefficients, or both, and sink temperatures in a user subroutine. Surface temperatures provided to this routine are obtained by linear extrapolation from the previous time step. Linear convection conditions (constant film coefficients, linear time dependent sink temperature) are available directly by data input.

b. Radiation conditions are handled by a user-supplied subroutine defining the flux in terms of linearly extrapolated surface temperature provided by the program - this is the most general form of boundary condition input.

c. Temperature and flux specification may be made node-by-node or, for flux, on a surface or unit volume basis. Both may have linear variation over a time interval (several time steps) or, by user subroutine with arbitrary time variations.

For all surface and volumetric conditions, the equivalent nodal values are obtained by consistent distribution.

For linear time-varying temperature the coding is included to avoid re-triangularization.

d. The intimate-contact boundary condition is modeled by a special impulsive equation technique.

B.2.5.4 Elements. The elements are defined in B.2.3.

B.2.5.5 Properties. The properties may be temperature dependent, including latent heat effects when the phase change occurs over a temperature range (non-eutectic alloys). Up to ten different material types with varying properties, as well as any number of constant property materials, may be used in the model. Material properties are input on cards, with their temperature variations given by slope-break-point tables identified by material type.

B.2.5.6 Thermal and Stress Analysis Interface. The two programs MARCHEAT and MARCCDC have much coding in common, but exist as separate, stand-alone packages. Output of centroidal (or integration point) temperature values is provided from MARCHEAT via a user-supplied subroutine. A corresponding input routine exists in MARCCDC. The user must supply his own spatial interpolation if necessary. For time interpolation, a subroutine is documented which takes advantage of the time-independence of elastic-plastic material behavior - this subroutine merges or sub-divides increments on the temperature tape to provide a set of increments where the maximum temperature change is always about the same, its value being chosen by the user. Thus, effectively, a non-linear time scale is adopted for the thermal stress analysis. This flexibility allows the user to restrict temperature gradient changes from one increment to the next, thus maintaining close control on solution accuracy, while creating a minimal number of increments to achieve this end.

The subroutine is not part of MARCHEAT or MARCCDC and need only be used at the designer's option.

B.2.6 Dynamics. The dynamic capability extracts frequencies and modes, beginning with the lowest frequency, by use of a power sweep technique with a shift. Transient calculations are performed by direct integration using the Newmark- β methods, $\beta = 1/4$, $\gamma = 1/2$, or by the Houbolt or central difference operators, or by mode super-position. For nonlinear problems (materials and geometric) the latter includes re-evaluation of mode shapes and frequencies at user-selected times by the method of simultaneous iteration.

The dynamic capability is available with all elements and all constitutive theories in the library.

B.2.7 Instability (Buckling) Analysis. Eigenvalue extraction is available during static analysis to provide buckling estimates. This procedure is well documented for elastic materials. For elastic-plastic analysis a generalized Shanley buckling procedure is implemented. This may also be used for creep analysis, to provide creep buckling time estimates. Alternatively the complete load-displacement history may always be stepped out.

Usually the eigenvalue predictions are improved after some part of the response is obtained in this way. Note that prestress effect may be readily included in these estimates. These options are available for all elements.

B.2.8 Kinematics. Large displacement behavior is included for all elements by a Lagrangian formulation. The coding includes finite strain effects, except that area (volume) changes are ignored in the computation of equivalent surface (body) forces and certain stress-dependent terms in the finite strain formulation of the elastic-plastic rate equations are ignored. Under these conditions, the program reduces to a finite rotation, small strain formulation.

B.2.9 Boundary Conditions for Structural Analysis. Boundary conditions include consistently distributed pressures and body forces. All appropriate nodal forces may be applied. Nodal displacements may be prescribed. The program also allows the imposition of arbitrary homogeneous, linear displacement constraints. Any or all boundary conditions may be changed, or new ones added, at any time.

B.2.10 Mesh Generation and Graphics Capability.

B.2.10.1 Mesh Generation. Mesh generation and band-width optimization are available in the separate programs MARCMESH2D and MARCMESH3D. These programs are both based on block generation, where the structure is built as a series of super-elements, each of which is automatically subdivided into elements for analysis. Various options exist to take advantage of geometric symmetry.

The mesh generators automatically output tapes suitable for input to MARCHEAT, MARCSTRUCPL and MARCCDC. Built in band-width optimization is available. MARCMESH3D also contains optional plotting output, allowing display of the super-element blocks, with a hidden line plotting algorithm.

B.2.10.2 Graphics. Mesh plotting is available in the separate program MARCSTRUCPL, using the same input data common to all the MARC programs. This allows plane or perspective viewing of all or part of the mesh (chosen by element number, type, etc.).

Contour and time history plotting is available for any element variables - for shell or beam elements the user may choose the several layers at which plot-output is required. Utility programs are provided to allow the user to access plot and restart tapes for interface with other post-processors.

B.2.11 Program Documentation.

a. Throughout the development of the computer programs documentation was prepared to provide ample source references. Among the more

recent documentation are the works of Marcal^{[1]*}; Hibbitt, Levey and Marcal^[2]; Hibbitt and Marcal^[3]; Hibbitt^[4]; and Hibbitt, Sorensen, and Marcal^[5]. A theoretical development of the large displacement capability is given by Hibbitt, Marcal, and Rice^[6]. A summary of the elastic-plastic-creep constitutive behavior provided in the program, and the theoretical development of the various features of MARCHEAT are given by Hibbitt^[4].

b. The Control Data Corporation has prepared user information manuals covering program description, input, and demonstration problems^[7].

c. The programs are being used in a wide variety of industrial applications and are available through the CDC CYBERNET system and the MARC organization. In addition various research reports in the finite element analysis and its applications, including results obtained using the programs, have been published by the group at Brown University.

B.2.12 Program Availability.

a. Availability. The programs referenced in B.2.11a and b are proprietary and are owned by the MARC Analysis Research Corp., 105 Medway Street, Providence, Rhode Island 02906. They are generally available on a royalty basis through the CDC Cybernet service. Other arrangements for the use of the programs may be negotiated with MARC and CDC.

b. User Assistance. The CDC Cybernet service provides professional structural analysis to support the package. In addition MARC provides complete support for the programs.

c. Machines on Which Operational. The program is presently installed on CDC 6000/7000 series machines, and on IBM 360-370 series machines. See introductory description above.

B.3 ANSYS

B.3.1 Brief Program Description. The ANSYS (Engineering ANalysis SYStem) computer program is a large-scale general-purpose computer program employing finite-element technology for the solution of several classes of engineering analysis problems. The program capabilities include structural analysis (static and dynamic; elastic, plastic, creep, and swelling; and small and large deflections), and heat transfer analysis (steady-state and transient; and conduction, convection, and radiation). Structural and heat transfer analyses may be made in one, two, or three dimensions, including axisymmetric and plane problems.

*Superscripts in brackets refer to references in Section B.10.

B.3.2 General Information. The ANSYS program may be applied to a large number of structures, including two- and three-dimensional frames and grids, piping systems, two-dimensional plane and axisymmetric solids, flat plates, three-dimensional solids, axisymmetric and three-dimensional shells, and nonlinear problems including interfaces and cables. Some of these are shown in Figs. B.1 through B.10. Creep and swelling options may be used in analyzing irradiated structures.

The ANSYS program uses the wave front (or frontal) direct solution method for solving the system of simultaneous linear equations developed by the matrix displacement method, and thus gives results of high accuracy in a minimum of computer time. The direct solution method does not place a band-width restriction on the problem definition. The wave front is limited by the amount of core storage required for a given problem. This tends to be restrictive only for analyses of arbitrary three-dimensional solids or in the use of ANSYS on a small computer.

The ANSYS program has restart capability for several analysis types. This feature may be used to check the solution results at intermediate points within the load history or it may be used to branch-off along different load histories. The latter method is used to analyze separate transients having a common initial portion without rerunning the common portion of the analysis. An option is also available for saving the stiffness matrix once it is calculated for the structure and using it for other loading conditions. This option, where applicable, can save a considerable amount of computer time in subsequent load steps on the same model. The computer system requirements are shown in Table B.1.

Table B.1. Computer System Requirements

CDC 6400, 6500, 6600	(120K ₈ - 350K ₈)
CDC 7600	(150K ₈ LCM, 200K ₈ ECS)
UNIVAC 1108	(55K ₁₀ minimum)
IBM 360/65 and up	(250K ₁₀ Bytes and up)
IBM 370/165 and up	(250K ₁₀ Bytes and up)

B.3.3 Element Library.

B.3.3.1 The finite elements available in ANSYS are shown in Table B.2.

Fig. B.1. Two-Dimensional Spar Element

Fig. B.2. Constant Strain Element

Fig. B.3 Three-Dimensional Spar Element

Fig. B.4. Three-Dimensional Plastic Pipe Element

Fig. B.5 Two-Dimensional Plastic Beam Element

Fig. B.6 Plastic Flat Plate Element

Fig. B.7 Plastic Flat Triangular Shell Element

Fig. B.8 Plastic Flat Triangular Shell Element

NOTE - Positive pressures act to compress the element.

Fig. B.9. Three-Dimensional Solid Elements

Fig. B.10. Three-Dimensional Plastic Curved Pipe Element

Table B.2. Summary ANSYS Finite Elements

Element Type	Name	Dimensions	Degrees of Freedom	Nodes	Plastic and Creep
1	Spar, 2-D	2	2	2	yes
2	Constant Strain Elem.	2	2	3,4	yes
3	Elastic Beam, 2-D	2	3	2	no
4	Elastic Beam, 3-D	3	6	2	no
5	Elastic Solid (CST)	3	3	4,6,8	no
6	Elas. Flat Tri. Plate	2	3	3	no
8	Spar, 3-D	3	3	2	yes
9	Elastic Straight Pipe	3	6	2	no
10	Cable	3	3	2	non-lin
11	Axisym, Conical Shell	2	3	2	no
12	Interface Elem. (2-D)	2	2	2	non-lin
13	Elas. Flat Tri. Shell	3	6	3	no
14	Spring-Damper	3	3	2	no
15	Mass with Rotary Iner.	2	3	1	no
16	Mass, 2-D	2	2	1	no
17	Mass, 3-D	3	3	1	no
18	Spring, 2-D	2	2	2	no
19	Damper, 2-D	2	2	2	no
20	Plastic Straight Pipe	3	6	2	yes
21	General Mass	3	6	1	no
22	Core Spacer and Gap	2	1	2	non-lin
23	Plastic Beam, 2-D	2	3	2	yes
24	Torsion Spring-Damper	3	3	2	no
25	Axisym, Harmonic Quad.	2	3	4	no
26	Plas. Flat Tri. Plate	2	3	3	yes
27	Input Stiffness Matrix	3	6	2	no
28	PL Flat Shell (3 Temp)	3	6	3	yes
29	Curved Pipe (Elbow)	3	6	3	no
30	Conducting Solid	3	1	4,6,8	no
31	Radiation Link	1,2,3	1	2	no
32	Conducting Bar, 2-D	2	1	2	no
33	Conducting Bar, 3-D	3	1	2	no
34	Convection Link	1,2,3	1	2	no
35	Linear Temper. El.	2	1	3,4	no

(continued)

Table B.2. Summary ANSYS Finite Elements (continued)

Element Type	Name	Dimen- sions	Degrees of Freedom	Nodes	Plastic and Creep
36	Hydraulic Conductance	1,2,3	1	2	non-lin
37	Conducting Flat Shell	3	1	3	no
39	Sliding Interface	1	1	2	non-lin
40	Dynamic Element	1	1	2	non-lin
41	Elas. Quad Memb. Shell	3	3	4	no
42	Linear Strain Isopar.	2	2	4	no
43	Elas, Flat Rect. Shell	3	6	4	no
44	Tapered Unsym. Beam	3	6	3	no
45	Elas. Solid (Isopar)	3	3	8	no
46	Elas. Flat Rect. Plate	2	3	4	no
47	Transv. Ht. Cond. Shell	3	1	5	no
48	Pl. Flat Shell (5 Temp)	3	6	3	yes
49	Plastic Solid	3	3	4,6,8	yes
50	Super-Element	3	-	-	no
52	Interface Elem. (3-D)	3	3	2	non-lin
53	Laminated Shell	3	6	3	no
54	Taper. Unsym. Beam (2-D)	2	3	2	no
55	Isopar. Quad. Temp. Elem.	2	1	4	no
57	Iso. Quad. Shell Temp.	3	1	4	no
59	Immersed Pipe Elem.	3	6	2	no
60	Plastic Elbow	3	6	3	yes
63	Elas. Flat Quad. Shell	3	6	4	no

B.3.3.2 Substructures (or Super-Element) Capabilities. ANSYS has the capability of generating substructures (or super-elements) that may be stored in a file and used in other analyses. The dynamic-matrix-condensation technique (or Guyan reduction) is used during the generation of the super-element to reduce the assembled stiffness (or conductivity), mass (or specific heat), and damping matrices of the substructure to a set of dynamic degrees of freedom. These matrices are stored in an element library and may be recalled in a subsequent analysis.

The super-element or substructure thus formed is a group of previously assembled ANSYS elements that can be treated as a single element. The super-element, once it is generated, may be included in any ANSYS finite-element model and used in any analysis type for which it is applicable. Some of the advantages of such an element are:

a. In a structure undergoing design changes, the nonchanging portion of the structure may be defined as a single super-element or substructure.

b. A compound structure may be modeled in terms of several super-elements or substructures, each super-element itself being an assembly of ANSYS elements or other super-elements.

c. In certain nonlinear analyses, the linear portion of the structure may be defined as a single super-element and iterations performed only on the nonlinear elements of the model.

B.3.4 Material Behavior.

B.3.4.1 Plasticity.

a. ANSYS kinematic hardening, isotropic hardening, and classical bilinear kinematic hardening are available. The von Mises yield criterion and associated flow rule are used. Hardening representations consistent with this standard are available.

b. For plasticity and creep problems an incremental technique is used. The loading is applied in increments, and at each loading level an elastic solution is done, with a correction applied to the next loading step to account for the plasticity and creep occurring during this loading step. In this procedure the plasticity lags the loading, and the calculated stresses are somewhat higher than the true stresses. The amount of this conservative difference can be reduced by increasing the number of load increments or by running iterations with no increase in loading to refine the solution. Unloading and reversed loading can be handled with no difficulty by this technique. The von Mises yield surface is used, along with the Prandtl-Reuss flow relations. The stress-strain curve upon reversed loading is assumed to be the same shape as the virgin stress-strain curve, but offset to account for the strain due to previous plastic deformation. Kinematic or isotropic hardening rules are also available for the treatment of cyclic plasticity.

An accelerated convergence technique is used for plasticity problems when no creep is possible (dynamic solution of 0.0 time interval) in which the plastic strain for the next load step is estimated before the load step is applied.

Another convergence criterion is available to automatically terminate the number of iterative calculations within a load step. The convergence criterion states that convergence occurs whenever the ratio of the plastic strain increment to the elastic strain, for all elements, is less than or equal to a user-specified value (1% if not specified).

c. The elements which account for plasticity and creep behavior are shown in Table B.2.

B.3.4.2 Creep.

a. A library of equation-of-state primary and secondary creep laws is available for time- and strain-hardening. Creep hardening representations consistent with Section 4 are available.

b. For the technique used for creep calculations, see B.3.4.1 on plasticity.

The program will handle creep by a similar incremental technique. Both primary and secondary creep equations are available to the user. The user has the option of selecting either a creep formulation which assumes the stresses decay due to the creep (as in thermal stresses), or a formulation in which the stresses are independent of creep (as in primary stresses).

The ANSYS program also includes irradiation induced swelling and creep for use in the analysis of nuclear reactor internals. The swelling is not stress-dependent, and is treated in a manner similar to thermal strains, while the irradiation creep is a stress and temperature-dependent phenomenon.

A creep time-step optimization procedure is available to minimize the number of iterations (sub-steps) required within a time (or load) step to give significant creep strain values. The first iteration within the specified time step is based on the user-specified parameters. If the ratio of the creep-strain increment to the elastic strain for all elements is less than a specified value, the time increment for the next iteration is increased and the number of remaining iterations required to reach the end of the time step is reduced accordingly. This optimization procedure is checked each iteration and further modifications are made to the following iterations if required. The creep-strain ratio may be specified by the user. If not specified, a default value of 0.25 (25%) is used.

c. Creep representations are available for the same elements as plasticity (except the 3-D pipe and elbow elements) as shown in Table B.2.

B.3.4.3 Material Properties Input.

a. Properties may be temperature dependent.

b. For most properties the user must supply constants or tables, which are used in conjunction with built-in equations. Linear interpolation within tabular data is also available.

c. Stress-strain data are supplied by the user in tabular form. Creep is specified by selecting an equation and supplying constants. Other properties may be supplied in the form of polynomial coefficients or as tables.

B.3.4.4 Isotropic or Orthotropic Behavior. Orthotropic material properties may be included in all plane and solid elastic structural elements and in all heat transfer elements.

B.3.5 Thermal Analysis Capability.

a. ANSYS may be used to solve for the steady-state or transient temperature distribution in a heat conducting body. Material properties may be anisotropic and temperature dependent. Output temperatures may be stored and reread for structural analyses.

b. Thermal analysis is done by the finite-element method.

c. Heat transfer analyses include internal heat generation, convection or radiation boundaries, and specified temperatures or heat flows. These inputs may be arbitrary functions of time for transient analyses. Boundary conditions may have step or ramp changes between specified load points.

d. The elements which account for thermal effects are shown in Table B.3.

Table B.3. Finite Elements Available in ANSYS for Thermal Effects

<u>Name</u>	<u>Dimension</u>	<u>Nodes</u>
Conducting Solid	3	4,6,8
Radiation Link	1,2,3	2
Conducting Bar, 2-D	2	2
Conducting Bar, 3-D	3	2
Convection Link	1,2,3	2
Linear Temper. El.	2	3,4
Conducting Flat Shell	3	3
Transv. Ht. Cond. Shell	3	5
Isopar. Quad. Temp. Elem.	2	4
Iso. Quad. Shell Temp.	3	4

e. Thermal properties are used in the form of a polynomial function of temperature. They are input either as polynomial coefficients or in tabular form.

f. The temperature output from the heat transfer analysis is in the form required for input to structural analyses. This coupling between analysis types allows a thermal-stress analysis of a structure using the same analysis model.

B.3.6 Dynamics.

B.3.6.1 Steady-State Vibration Analysis. The following types of steady-state vibration analyses are available:

a. Mode-Frequency. Used to solve for the resonant frequencies and mode shapes characterizing a structure. Full or reduced (in-core matrix condensation) analysis options are available. The structure may be subjected to a seismic loading or a force loading for a spectrum analysis option. Stresses are output in addition to the eigenvalues and eigenvectors with this option. The solution is by Jacobi iteration. Guyan reduction is available optionally.

b. Harmonic Response. Used to determine the steady-state solution of a linear elastic system under a set of harmonic loads of known amplitude and frequency. Damping may be included in the system. Complex displacements are output. The wave-front method is used for solution.

c. Linear Harmonic Response Spectrum. Used to determine the steady-state solution of a linear elastic system under a set of harmonic loads of known amplitude and frequency when several frequencies are to be analyzed. The technique of dynamic matrix condensation is used to allow a rapid and efficient response-versus-frequency solution.

A second pass through the analysis uses the previously calculated displacements and stiffness matrix to determine the stress-versus-frequency solution. Stresses may be calculated at desired points within the frequency range and the displacement solution is expended in full. The phase angle at which the stress solution is to be calculated is input by the user.

All structural elements are available for the above analyses.

B.3.6.2 Transient Time Response Calculations. The following types of transient time response calculations are available.

a. Nonlinear Transient Dynamic. Used to determine the time-history solution of the response of an arbitrary structure to a known force, displacement forcing function, or both. The mass, damping, and stiffness matrices may vary with time and may also be functions of the displacements themselves. Nonlinear effects, such as friction, plasticity, and large deflections, may be included.

b. Reduced Linear Dynamic. Used to determine the time-history solution of the response of a linear elastic structure to a known force, displacement forcing function, or both. The matrix representing the system is reduced to the degrees of freedom required to characterize the response of the system. A semilinear analysis option includes the use of interfaces (gap) between any pairs of these dynamic degrees of freedom or between any dynamic degree of freedom and ground. In this

case the mass, damping, and stiffness matrices are constants. Only one matrix inversion is required and the analysis is reduced to a series of matrix multiplications.

All structural elements are available for the preceding analysis.

B.3.7 Instability (Buckling) Analysis. If the load is applied to the structure in a single step and the rate of convergence to the large deflection is observed, an estimate of the stability of the structure can be made. In particular, if the deflection diverges the load is above the critical buckling load. This large deflection analysis then becomes a stability check.

B.3.8 Kinematics. For large deflection analysis, the geometry is modified at the end of each load increment so that the total loading is applied to the deformed structure at the next load increment. This procedure thus follows the large deflection load-deflection curve.

Large strains (greater than a few percent) are not practical.

B.3.9 Boundary Conditions for Structural Analysis. Loading inputs for structural analyses may be forces, displacements, pressures, or temperatures. These inputs may be sinusoidal, random, or an arbitrary function of time for the linear and nonlinear dynamic analyses. Mode-frequency analyses may include force spectrum or velocity spectrum loadings.

A preprocessing subroutine is available to generate boundary conditions from a minimum of input data. In addition to reading-in arbitrary tabular data for displacements, forces, pressures, and temperatures; typical functions, including sine, cosine, constant, and linear functions, can be generated. Data manipulation options include scaling, copying from one table to another or from one section of a table to another section of the same table, integration, double integration, differentiation, and double differentiation. The double integration facility is particularly suited to converting seismic accelerograms to displacement input.

B.3.10 Mesh Generation, Graphics Capability, and Post-Analysis Operations.

a. The ANSYS program includes a multiple region mesh generation capability within the main program as well as two-dimensional mesh generation and three-dimensional intersecting shell mesh generation subroutines. The multiple region generation capability is used to fill-in nodal points between two specified nodal points, to repeat specified sets of nodal points, to generate additional sets of elements from a specified set to form a group of elements, and to generate additional groups of elements.

The shell mesh generation subroutine generates geometries consisting of single or intersecting regions of shell elements. Intersecting surfaces may be planes, cylinders, cones, spheres, tori, ellipsoids, or hyperboloids. The intersection lines between the surfaces are automatically calculated.

b. Geometry plotting is available for all elements in the ANSYS library, including isometric and perspective views of three-dimensional structures. Solution of the problem is not required to obtain the geometry plots. Plotting subroutines are also available for the plotting of stresses, displacements and temperatures from two- and three-dimensional solid or shell analyses, mode shapes and amplitude-frequency plots from dynamic analyses, distorted geometries from static analyses, transient forces and displacements versus time from transient dynamic analyses, and stress-strain plots from plastic and creep analyses. Velocities and accelerations may also be plotted from dynamic analyses.

A special plotting subroutine is available for forming two-dimensional displacement and stress plots for harmonically loaded axisymmetric structures.

c. A special feature of the ANSYS program permits variables calculated from a solution run and stored on a data file to be included in a number of mathematical operations. The results may be printed, plotted, or both. The variable operations include addition, subtraction, multiplication, division, square and square-root operation, comparison and selection operations, cumulative damage integral formation, and time derivative operations.

A post-processing subroutine is available for combining the results of the mode-frequency analysis. A root-mean-square (rms) sum is performed on selected modal components, such as displacements, velocities, accelerations, forces, and stresses. The modal components may be individually scaled and added before being included in the rms sum. Modal forces and stresses may be combined with the corresponding static components. Static results may also be combined.

A post-processing subroutine is available for generating a displacement, velocity, or acceleration response spectrum based on a given (or ANSYS calculated) displacement-versus-time history. The response spectrum may provide input to the mode frequency analysis.

B.3.11 Program Documentation. The primary sources of information on the ANSYS program are the ANSYS manuals. The most important of these is the ANSYS User's Manual, which provides the instructions for the use of the program, and gives a brief description, references, or both for the theory of the methods used. The ANSYS Examples Manual is a collection of demonstration problems prepared by Swanson Analysis Systems, Inc. (SASI). The user may collect the problems which he needs for his types of analyses. The ANSYS Theoretical Manual and the ANSYS Programmer's Manual are SASI in-house concepts which are not scheduled

for release to the general user community. Portions may be released as proprietary documents to selected users on a need-to-know basis.

B.3.12 Program Availability. The ANSYS computer program is widely available in the United States, and at some locations elsewhere. The program can be used on a computer time surcharge basis, a lease basis, or in controlled circumstances, it may be purchased. Further information may be obtained from:

SWANSON ANALYSIS SYSTEMS, INC.
870 Pine View Drive
Elizabeth, Pennsylvania 15037
(412)-751-1940/(412)-872-9555

B.3.12.1 Communication between the ANSYS program developer (SASI) and the user, is available in the following forms:

a. Program Documentation. A complete user's manual (628 pages) is available. An expanding collection of example problems (Examples Manual) is also available.

b. Program Notes and Messages. New features are announced to the user on the first page of each ANSYS printout. Error messages are printed as they occur.

c. User Training Seminars. A two-day seminar may be scheduled during which the program features are explained to a group of users and example problems are individually set up.

d. SASI Consulting Services. SASI consultants are available to aid users in analyzing difficult problems at the user's location or at the SASI offices. Immediate assistance is available by telephoning SASI personnel.

B.3.12.2 The ANSYS program is currently operational on Control Data Corporation 6400, 6500, 6600, and 7600 computers, the UNIVAC 1108, and IBM 360 and 370 computers. Single precision is used on the CDC computers, double precision on the other machines.

B.4 CREEP-PLAST

B.4.1 Brief Program Description. CREEP-PLAST is a special purpose finite element program for thermal and elastic-plastic-creep analysis of two-dimensional structures. The program has been developed by General Electric under subcontract with the Oak Ridge National Laboratory (ORNL) as part of a program to develop high-temperature structural design methods for LMFBR components. The major objective for development of CREEP-PLAST is to provide LMFBR component vendors with a non-proprietary, user-oriented, inelastic-analysis program for use on their in-house computing facilities. ORNL is responsible for the verification, maintenance, modification, distribution, and partially the documentation of CREEP-PLAST.

B.4.2 General Information.

B.4.2.1 Type of Structure. The program can be applied to plane strain, plane stress, and axisymmetric problems.

B.4.2.2 Method of Analysis. An incremental matrix displacement method is used. Increments are normally controlled by the user; however, as a special option, creep increments can be controlled by the program itself to optimize creep time step selection. For transient thermal problems the temperatures and stresses can be calculated in the same run or separately. In both cases, the thermal grid is the same as the stress grid. The program is limited to 2000 elements, 1200 nodes, 100 constrained nodes, and a maximum half-band width of 102. The matrix equilibrium equations are solved by direct Gaussian elimination. Input and output (I/O) requirements during the solution of the matrix equations are determined, in part, by the user under constraint of the core storage available to him. For most practical problems solution of the matrix equations can be conducted entirely in-core, unless CREEP-PLAST is being implemented on a small computer.

B.4.2.3 Restart Capability. The program provides restarting capabilities for a virtually unlimited number of steps. By using separate input and output tapes, or by employing separate files on the same tape, restart runs can be performed using any of the previously generated restart data files. Normally, only one restart file is written for each run of the program; however, a special option is provided by which the user may choose the frequency of writing restart data files. Restart is then available from any one of these restart files.

B.4.2.4 Computer System Requirements. The program is written in Fortran-IV and is virtually machine independent. The IBM-360 version requires a minimum of 230K bytes (4 bytes per hexadecimal-8-digit word) of memory (excluding buffers) and an overlay loader. The storage requirements are determined, in part, by the size of the problem being considered; and in part, by the user based on his selection of the size of the working space allotted for the solution of the equilibrium equations. A large problem may require 500K to 600K bytes of memory on the ORNL IBM-360 system. I/O requirements vary with problem size and with user selected options. Small problems will run entirely in-core, while moderately sized problems normally require four scratch units plus optional restart, plot, temperature, and mesh tapes or data files. The ratio of I/O to CPU time ranges from about 1.5:1 to 5:1 depending upon the I/O requirements of the problem and upon such things as the hardware configuration, and the job mix, in a multi-tasking environment.

B.4.3 Element Library. The program utilizes the two-dimensional, three-node triangular solid element (plane and axisymmetric) with two degrees of freedom at each node. Uniaxial bar elements are also present.

B.4.4 Material Behavior.

B.4.4.1 Plasticity.

a. Classical non-isothermal bilinear kinematic hardening and CREEP-PLAST nonlinear kinematic hardening are included. Flow rules are associative, and the von Mises yield criterion is employed exclusively. Hardening rules for cyclic plasticity are consistent with this standard.

b. The analysis method is based on the modified stiffness approach with user selected load and temperature steps. Iteration within each step is also provided and is controlled by the user.

c. Plasticity representations are available for all elements.

B.4.4.2 Creep.

a. The program utilizes the equation-of-state creep theory with primary (closed-form) strain hardening and is consistent with this standard.

b. The time increments are normally specified by the user; however, as a special option, the increments can be controlled by the program itself to optimize time step selection. The ratio of creep strain increment to elastic strain and the relative stress change are checked on an element-to-element basis during each step, and future time steps are prescribed by the program on a previous-step to subsequent-step prediction basis to meet either a specified creep-strain ratio criteria or a stress-change criteria, if relaxation is involved. The equation-of-state creep theory is treated similarly to the plasticity theory, namely, by the modified stiffness approach. Part of the predicted creep strain increment is treated by the customary initial strain (initial stress) concept and is decomposed into pseudo-nodal loads; part of the creep strain increment modifies the element stiffness matrix. As a result, the stability of the creep analysis is less sensitive to the time step, and thus fewer steps are required. Iteration within each step is also provided and is controlled by the user.

c. Creep representations are available for all elements.

B.4.4.3 Material Properties Input. The creep and plasticity properties are temperature dependent. They are built into the program through subroutines which can be easily modified by the user. As an example, the Liquid Metal Fast Breeder Reactor Materials Handbook[8] creep (July 1971) and plasticity (April 1971) equations are coded in CREEP-PLAST. The program has isotropic capability only.

B.4.5 Thermal Analysis Capability. Transient temperature, including the steady-state condition, is considered with nonlinear boundary conditions and material properties. The thermal properties

are input through subroutines which can be modified by the user. The thermal and stress grids are the same.

B.4.6 Dynamics. The program has no dynamic analysis capability. (No capability).

B.4.7 Instability (Buckling) Analysis. The program has no instability analysis capability. (No capability).

B.4.8 Kinematics. The program is limited to small displacement and small strain theory.

B.4.9 Boundary Conditions for Structural Analysis. CREEP-PLAST considers pressure loading and concentrated forces to vary arbitrarily with time. Three independent pressure multiplier arrays are provided for mechanical loading, and one thermal multiplier array is available to vary the amplitude of a steady-state temperature distribution problem. Transient temperature field is normally specified by the element temperature tape.

Displacements vary arbitrarily with time. One displacement multiplier array is available to specify the displacement history in a constrained degree-of-freedom problem.

B.4.10 Mesh Generation and Graphics Capability. Mesh generation is included which can be used for regular cylinders, rectangular flat plates, or any large rectangular regions in an irregular structure. The mesh can be generated by the program during a data check and stored on the mesh tape, or it can be generated as part of the normal execution. CREEP-PLAST will also accept a mesh tape prepared, in a compatible format, by other external mesh generation programs available to the user.

An optional plot tape for storage of data needed for plotting is included. Complete graphical output capabilities have been added to CREEP-PLAST at ORNL (IBM-360 system). A limited plotting program has been linked directly with CREEP-PLAST. This enables the user to request immediate output plots of time and load dependent displacements, strain, and stresses at selected nodes. Nodal stresses and strains are averages of the values in the adjacent elements. Immediate output plots of stresses and strains through any cross section at selected steps in the problem history can also be obtained. A more complete stand-alone plotting program has been developed at ORNL which uses the information saved on magnetic tape from an analysis. This second plotting program will yield the same plots as the immediate-output plotting program, plus it will display stress contours, displaced mesh configurations and stress trajectories, and identify plastic and strain hardened elements with the grid. Either Cathode-ray tube or Calcomp pen-and-ink plots can be generated. Complete graphical output capabilities similar to those developed at ORNL have been independently developed at General Electric (UNIVAC 1108 system), and have been transmitted to ORNL.

B.4.11 Program Documentation.

a. The program is documented by Rashid in a theoretical report,[9] and the original program is documented by a user's manual.[10] The current version being distributed by ORNL is also documented by an ORNL User's Manual.[11] Complete verification of CREEP-PLAST is currently near completion, and the verification studies will soon be documented by ORNL.

b. A sample problem is being distributed with the program. Documented use of CREEP-PLAST has been described in many ORNL-TM reports for example, Refs 12 and 13. Use is also documented by a General Electric report.[14]

B.4.12 Program Availability.

a. The IBM-360 version of CREEP-PLAST is being maintained, updated, and distributed by ORNL. The program is available for a small service charge, and limited user assistance is available on an informal basis from ORNL. Address requests to:

Computer Librarian
Room A228, Building 4500-N
Oak Ridge National Laboratory
P. O. Box X
Oak Ridge, Tennessee 37830

b. CREEP-PLAST is operational on the IBM-360, UNIVAC-1108, and CDC-6600 computer systems. The IBM-360 version available from ORNL is partially double precision.

B.5 EPACA

B.5.1 Brief Program Description. EPACA is a special purpose finite element program designed for thermal* and elastic-plastic-creep analysis of structures that can be modeled as assemblages of flat, curved, thin, or thick shell type elements. The program has been developed by Franklin Institute under subcontract with ORNL as part of the program to develop high-temperature structural design methods for LMFBR components. As in the case of CREEP-PLAST, the major objective for development of EPACA is to provide LMFBR component vendors with a nonproprietary, inelastic analysis program for use on their in-house computing facilities. EPACA, though not a general purpose program, will handle many of the common geometries (plate and shell) for which inelastic analyses are required.

*Thermal analysis capability is not yet available.

B.5.2 General Information.

B.5.2.1 Types of Structures. EPACA, in its present form, can be used for an arbitrary structure representable by flat or curved triangular or quadrilateral plate or shell elements. Thickness variation is included and interpolation of thickness is performed from the known values at nodal points. In addition, the entire structure or parts of the surface can be supported on an elastic foundation. Accordingly heat exchangers, reactor vessels, reactor core support structures, containment building, piping tees, and elbows form but a part of structures which can be directly modeled for analysis by EPACA.

B.5.2.2 Method of Analysis. An incremental principle of virtual work is used for inelastic element formulation. Solution of the matrix equations is obtained by a direct elimination scheme developed by Jennings. An inverse power method is used for vibration eigenvalue determination. EPACA is written in such a manner that each node can have a variable number of degrees of freedom. This feature facilitates the possible incorporation of additional elements in the future.

B.5.2.3 Restart Capability. Three levels of program operation provide restart capability at the:

1. End of geometric input.
2. End of preprocessing of the input, where element files and load files are prepared.
3. Beginning of each load history interval.

Note that only the latest restart generated is available on the restart tape.

B.5.2.4 Computer System Requirements. On the IBM 360/91, an overlay structure is utilized to permit instructions to occupy about 150K bytes. On the UNIVAC 1108, EXEC-8 version, instructions, also overlaid, occupy about 18800₁₀ locations.

In addition, depending upon the problem and its size and a user-defined work array a number of I/O units are required. Excluding logical units, 5 input and 6 output, a maximum of 22 logical units may be required.

A so called STRATEGY method determines if available space is sufficient for in-core system formation and solution, otherwise external storage is utilized.

B.5.3 Element Library. The present element library of EPACA contains eight surface type elements, all suited for membrane and bending. Four of the elements are flat; the remaining four can be flat or curved.

They are summarized in Fig. B.11. The characteristics of various elements described under the code number NKOD of each element, are as follows:

a. NKOD = 3. This is a flat triangular element with three nodes, Fig. B.12(a). The interpolating functions for the in-plane displacements and the transverse rotations are linear. The interpolating function for the transverse displacements has nine cubic terms. In addition to the transverse displacement, Kirchhoff's hypothesis that lines normal to the middle surface remain straight and normal after deformation apply at the nodes. This supplies the two required additional conditions at each node.

b. NKOD = 4. This is also a flat triangular element with three nodes. It differs from NKOD = 3 in the function for the transverse displacements, which is a six term polynomial. It has the capability of representing shear more accurately.

c. NKOD = 5. This is a flat quadrilateral element, Fig. B.12(b). The interpolating functions for in-plane displacements and the transverse rotations are also linear. However, the transverse displacements are in the form of a six term incomplete cubic. It also represents shear more accurately and is recommended for thick-shell-type analysis.

d. NKOD = 6. This is also a flat quadrilateral element. In-plane displacements and transverse rotations are obtained from a bilinear interpolating function. The transverse displacements are based upon a twelve term polynomial with Kirchhoff's hypothesis applied at the four nodal points.

e. NKOD = 7 thru 10. There are four basic isoparametric curved elements incorporated in EPACA, Fig. B.13 which can also be used for flat configurations. To account for transverse shears, it is assumed that lines initially normal to the middle surface remain straight after deformation but can undergo rotation relative to the middle surface. This allows transverse shear strain of constant magnitude through the thickness of the element. These elements are invariant with rigid body motion and compatible with the adjacent elements, both in displacements and rotations.

B.5.4 Material Behavior.

B.5.4.1 Plasticity.

B.5.4.1.1 Constitutive Equations. The von Mises yield condition is used for the initial yield as well as subsequent yield surfaces. For kinematic strain hardening, Ziegler's modification of Prager's kinematic hardening is used. An associated flow rule is employed in all cases; and both isothermal and non-isothermal plasticity are included. Isotropic and kinematic hardening models are in

Shape	Flat				Curved - Isoparametric			
	Triangular		Quadrilateral		Triangular		Quadrilateral	
Code No.	3	4	5	6	7	9	8	10
Number of Nodes	3	3	4	4	6	10	8	12
Node Distribution

Fig. B.11. Summary of EPACA Element Library

Fig. B.12. Flat Plate Elements

Fig. B.13. Isoparametric Elements

agreement with this standard; bilinear material properties being used for the kinematic hardening, and bilinear or multilinear material properties for isotropic hardening.

B.5.4.1.2 Numerical Approach. The tangent stiffness (modified stiffness) method is used to solve the incremental plasticity problem. At present a user-selected load-increment scheme is incorporated. If the state-of-stress departs from the materials curve, at the end of a solution step, the unbalanced load is computed; a correction is possible either in the iteration step, within the load step, or in the next load step. Similarly, for non-isothermal cases at the beginning of the step the state-of-stress is transferred to the new temperature material properties curve before the unbalance loads are computed. If, at the end of a load step, the solution departs from the materials curve in excess of a prespecified amount (0.001 built-in into EPACA), up to four iterations (Newton-Raphson) are automatically performed to bring the stresses to equilibrium both with external loads as well as to the materials curve utilized. All elements are operational under the same numerical scheme.

B.5.4.2 Creep.

B.5.4.2.1 Constitutive Equations. The equation-of-state approach with either primary or total creep strain hardening is employed in EPACA in a manner consistent with this standard. Auxiliary hardening rules are incorporated in order to consider stress reversals in an appropriate manner. The equivalent tensile creep law is coded in EPACA as a function of stress, temperature, and time with both steady-state and primary (one-term exponential) creep terms being considered. The flow rule is of the form recommended (e.g., creep rate radial to the J_2 stress surface).

B.5.4.2.2 Numerical Approach. The initial strain approach is employed for time-dependent material behavior. The time increment for each creep step is computed internally, based on a user-specified ratio of creep-strain to elastic-strain increment. Each integration point of every element is considered in the process of selecting the subsequent creep relaxation step.

B.5.4.2.3 Creep Analysis. Creep analysis is available for all elements.

B.5.4.3 Materials Property Input. Material properties may be temperature dependent; specific built-in properties are available for types 304 and 316 stainless steel; and materials properties both for elastic-plastic and creep analysis can be input with considerable generality. For manually prepared input true-stress true-elastic strain curves for up to fourteen different temperatures at thirteen additional stress points; each on the stress-strain curve (not including the initial yield point), are allowed for each materials type. There is no limit on the number of materials types; vector storage is allocated on the

basis of input for the number of materials types. In addition, five basic material properties (E , σ_y , ν , α , ρ) can be read-in at up to fourteen different temperatures. Internally, the user provided stress-strain data is transformed into true-stress true-plastic strain curves and numerically tabulated at the same plastic strain values for all temperatures. This procedure speeds up property evaluation.

B.5.4.4 Isotropic, Orthotropic Behavior. The program assumes material behavior is isotropic throughout.

B.5.5 Thermal Analysis Capability. Thermal analysis capability is planned for the future. It will be available for steady and transient thermal analysis, for all elements of EPACA, and for temperature-dependent materials properties. Boundary conditions will be linear or bi-linear approximations with partially internal film coefficient computation. The finite element mesh used for stress analysis will also be used for thermal analysis with built-in monitoring for load history interval identification (load point identification so that maximum stress states are located). Internal transmission and selection of load history for the entire transient will be incorporated.

B.5.6 Dynamics.

B.5.6.1 Steady-State Vibration Analysis. Under $KIND = 3$ and $KIND = 4$ operations, EPACA computes eigenvalues in the specified range of frequencies with the associated normalized eigenvectors. Lumped mass is used; however, provisions for future incorporation of consistent mass are provided.* Eigenvalues are computed on the total system by use of inverse power iteration with a spectral shifts algorithm, which is a streamlined version of that used in NASTRAN. Any of the eight elements of EPACA can be used in this analysis. The structure can be preloaded with any combination of loads, load history or both prior to the evaluation of vibration frequencies. In its present version, EPACA computes eigenvalues and eigenvectors, normalizes the latter and stores them on the eigenvalue tape in increasing order. For vibration analysis, the frequencies are computed in the neighborhood of the loaded (or unloaded state (elastic or plastic)).

B.5.6.2 Transient Time Response Calculations. Transient response analysis uses lumped mass and Newmark's ($\gamma = 1/2$, $\beta = 1/4$) algorithm, which results in step by step solution of:

$$BU_2 = F_2 + \left(\frac{4}{t^2} M + \frac{2}{t} C\right) U_1 + \left(\frac{4}{t} M + C\right) \dot{U}_1 + M\ddot{U}_1 \quad (B.1)$$

*Need for consistent mass is recognized in connection with the problems involving significant reference surface shifts and generalized multi-point constraints.

where:
$$B = \frac{4}{t} M + \frac{2}{t} C + K \quad (B.2)$$

From load-history interval to load-history interval, the geometry can be updated and plasticity effects considered. Practical experience, however, indicates that system updating from dynamic plasticity analysis is rather frequently required; hence, even with an inherently stable method of solution, computer times are prohibitively large. For small deformation elastic analysis, B (Eq B.2) is decomposed only once, and the analysis time is within normal limits. All EPACA elements can be used for this analysis.

B.5.7 Instability (Buckling) Analysis. EPACA handles elastic equilibrium bifurcation buckling problems under KIND = 6. All elements can be used for this analysis. As in the case of vibration analysis, inverse power iteration with spectral shifts is used to determine the eigenvalues and mode shapes for buckling. The user can compute any number of buckling modes as desired. Results are stored in the same manner as for vibration analysis.

B.5.8 Kinematics. Large accumulated displacements can be obtained, including initial stress effects, in the absence of a snap-through condition for the incremental analysis with the geometry update option. This analysis is available in elastic, elastic-plastic and dynamic modes, if these analyses are performed incrementally with specified inclusion of initial stress effects and geometry update at each step. At this time there is no adequate experience available with EPACA to define the limit of largest strains obtainable by this analysis.

B.5.9 Boundary Conditions for Structural Analysis. A rather general specification of boundary conditions is acceptable. Surface tractions are read-in for all nodal points at the end of each load history interval. The same generality applies to the specification of concentrated loads and temperature distribution. Any number of load-history intervals can be specified.

Displacement, finite or zero, can be specified for any of the degrees of freedom on any arbitrary reference frame, also for any number of load-history intervals. Within each of the load-history intervals, the interpolation between the end points is linear. Initial velocities and displacements for KIND = 2 analysis are also read-in. In addition to physical loads and kinematic boundaries an elastic foundation, continuous over element surface and discrete elastic foundation at any of the nodes, can be specified. Contrary to other boundary conditions, the elastic foundation remains constant through the analysis over the load history.

B.5.10 Mesh Generation and Graphics Capability. Automatic mesh generation features are available at two levels. At the hand input

level, an extensive pattern propagation feature is available for element connectivity input as well as the input of joint coordinates. Completely automatic mesh generation is available for specific geometries. At present, in EPACA, automatic mesh generators are available for elbows, general shells of revolution, and pear-shaped cylinders. Elbow and general shell-of-revolution input generators operate for all elements; the pear-shaped cylinder input operates only for NKOD = 5 and NKOD = 6. A tee-mesh generator is being implemented and will be made available as soon as it is complete.

Plotting of the mesh is provided in EPACA. Here undeformed, deformed, or both mesh shapes can be plotted. Graphical output capabilities, similar to those incorporated into CREEP-PLAST (B.4.10) are to be included in the ORNL IBM 360/91 version of EPACA in the near future.

B.5.11 Program Documentation. A comprehensive user's manual for EPACA is available.^[15] The theory upon which various parts of program are based is described in detail; particularly in the areas of variational principles, constitutive relations, and description of various elements. A brief program description is given in the user's manual, and a list of program stops with the associated error messages is given. An input guide forms a major portion of the user's manual. It contains a card-by-card input flow chart and input description in two levels. First, a concise input map is given card-by-card and column-by-column. Next, detailed card-by-card instructions with graphical card field representation are described.

Several problems have been solved and documented in various ORNL-TM reports.^[12,13] Sample input and output are also available.

B.5.12 Program Availability. The IBM-360 version of EPACA is being maintained, updated, and distributed by ORNL. The program is available for a small service charge, and limited informal assistance is available from ORNL. Address requests to:

Computer Librarian
Room A228, Building 4500-N
Oak Ridge National Laboratory
P. O. Box X
Oak Ridge, Tennessee 37830

EPACA, with the available features listed above, is operational on UNIVAC 1108 and IBM 360/91 computer systems in a double precision version.

B.6 PLACRE

B.6.1 Brief Program Description. PLACRE is a finite element program for elastic-plastic-creep analysis of two-dimensional structures. It was developed as a research tool, and is essentially supplanted by CREEP-PLAST for most general usage. A special version which has been

adapted for pipe wall ratcheting calculations, with built-in thermal transient analysis, should still be of some general interest.

B.6.2 General Information.

B.6.2.1 Type of Structure. PLACRE is applicable to plane stress and axisymmetric analyses.

B.6.2.2 Method of Analysis. An incremental matrix displacement method is used. The matrix equilibrium equations are solved entirely incore, by a variation of the Choleski method.

B.6.2.3 Restart Capability. The program has no restart capability.

B.6.2.4 Computer System Requirements. About 190 K, 16-digit words of core storage are required for general two-dimensional problems. The special pipe-ratchet version requires about 33 K, 16-digit words. Two scratch disks are used, along with the standard input and output (I/O) units. I/O time is essentially limited to printed output and is small compared to C.P.U. time.

B.6.3 Element Library.

- a. No general 3-D solid elements are available.
- b. Compatible constant strain 3-node triangles are available for plane stress and axisymmetric problems. An equilibrium 3-node triangle is available for plane stress only. However, plate bending, shell, beam and bar elements are not available.
- c. Special elements are available for pipe wall ratcheting, with and without bending of the wall. Fig. B.14 illustrates the pipe wall element with bending. The solid circles are nodes and the open circle is a roller. Nodes 1 and 2 have only a radial degree of freedom. Node 3 has two degrees of freedom corresponding to rotation and axial displacement. The rigid bar attached to node 3 is an intrinsic part of the element formulation and not an added-on constraint. Note that bending refers to axisymmetric bending of the wall and not overall bending of the pipe.

B.6.4 Material Behavior.

B.6.4.1 Plasticity.

- a. Non-isothermal bilinear kinematic hardening is used with the von Mises yield criterion and associated flow rule. Requirements of this standard are included. A special cyclic nonlinear kinematic hardening law is also included.

(a) SCHEMATIC DIAGRAM OF SINGLE ELEMENT

(b) SIMULATION OF A PIPE WALL WITH BENDING

Fig. B.14. PLACRE Pipe Wall Element

b. The modified stiffness method is used for plasticity, with user-selected increments. Iteration is normally used within each increment.

c. Plasticity representations are available for all elements.

B.6.4.2 Creep.

a. Only equation-of-state creep laws are available, with time- or strain-hardening. The von Mises-associated flow rule is used. The requirements of this standard can be met.

b. The initial strain method is used. Creep increment selection is largely automatic.

c. Creep representations are available for all elements.

B.6.4.3 Material Properties Input.

a. Properties may be temperature dependent.

b. The user must supply constants for built-in equations.

c. Polynomial temperature dependence is used for most properties, and requires user-supplied coefficients. A library of creep equations which require user-supplied constants is included in subroutines. Additional creep equations can be added.

B.6.4.4 Availability. Only isotropic behavior representations are available.

B.6.5 Thermal Analysis Capability.

a. Thermal transient analysis is available for pipe walls only.

b. A finite difference method is used for thermal analysis.

c. Outer surface of pipe is insulated. Inner surface boundary condition is convection to a coolant with a time-dependent temperature. The heat transfer coefficient must be supplied by the user and can be time-dependent. Any initial temperature profile can be read in.

d. Thermal analysis representations are available for pipe wall elements only.

e. Thermal property laws with polynomial temperature dependence are used. Coefficients must be supplied by the user.

f. One finite difference thermal mesh point is used for each finite element. Transient temperature profiles are generated internally as used.

B.6.6 Dynamics. No capability.

B.6.7 Instability (Buckling) Analysis. No capability.

B.6.8 Kinematics. The program is limited to small displacement and small strain theory only.

B.6.9 Boundary Conditions for Structural Analysis. Loads, displacements, and pressures can have arbitrary time dependence.

B.6.10 Mesh Generation and Graphics Capability.

a. Automatic mesh generation is available for quadrilateral blocks of triangular elements, and for pipe walls.

b. Plots of stress, strain, and displacement history during ratcheting can be obtained.

B.6.11 Program Documentation. The theory is not documented, but a user's manual is available. Documented use of some sample problems are available. Also, program usage is documented in ORNL reports.[13]

B.6.12 Program Availability. The program is available at no charge and limited user assistance is available on an informal basis from:

W. K. Sartory
Oak Ridge National Laboratory
P. O. Box Y
Oak Ridge, Tennessee 37830

B.7 FOSTER WHEELER PROGRAM R-1045

B.7.1 Brief Program Description. R-1045 (FWC) is a special purpose FORTRAN program for an analytical solution to the elastic-plastic-creep problem of a long thick-walled cylinder of temperature-dependent material properties under cyclic loading conditions that include time varying internal or external pressure, thermal gradients, as well as axial tractions or axial strain applied in a nonproportional manner.

B.7.2 General Information.

B.7.2.1 Type of Structure. A long thick-walled cylinder under plane strain or generalized plane strain assumption.

B.7.2.2 Method of Analysis. Analytical rate formulation solved by method of successive approximation.

B.7.2.3 Restart Capability. Restart can be done from incore so that cyclic loading conditions can be changed after an arbitrary number of load cycles without using storage tapes.

Restart can also be accomplished by using storage tape so that the solution for an arbitrary number of load cycles can be studied before continuing the analysis for additional cycles.

Stress-strain history can be entered onto a user-supplied subprogram as an option so that the user may store them on tape for the purpose of further plotting and evaluation.

B.7.2.4 Computer System Requirements. It is an incore program. Two tapes are used optionally for the purposes of restart and solution storage. I/O time is negligible.

B.7.3 Element Library. No element library is required (not required).

B.7.4 Material Behavior.

B.7.4.1 Plasticity.

B.7.4.1.1 Constitutive Equations. Nonisothermal linear kinematic hardening model is used together with von Mises' yield function, its associated flow law, and a bilinear representation of uniaxial stress-strain relation, as recommended by this standard.

B.7.4.1.2 Numerical Approach. The method of successive approximation is applied to the rate formulation. In the formulation the field solution and material properties are assumed to depend on the rate of change of temperature in addition to the current temperature. Thus the time-step size need not be reduced solely due to a rapid temperature change. The user specifies the time steps at which there is a change of load pattern. The loads are then assumed to be of linear variation between two consecutive time steps. The size of any subtime steps is then determined internally by allowable plastic strain increment that is specified by the user in terms of percentage of current total strain.

B.7.4.2 Creep.

B.7.4.2.1 Constitutive Equations. At a given state of stress and strain, the effective creep strain rate is assumed to depend on the temperature, effective stress, effective total creep strain (or modified effective creep strain in case of load reversal as suggested in this standard), and as an option on effective stress rate. That part of creep strain rate dependent on effective stress rate is incorporated in the plastic strain calculation routine. This dependence will allow a larger time-step size for creep strain calculations. When stress rate dependence is disregarded customary strain hardening is recovered. The user may supply a subprogram to specify the creep relationship for his own material.

B.7.4.2.2 Numerical Approach. The incremental creep and plastic strains are calculated simultaneously. The interaction between creep and plastic behavior is linked to the stress increment. As with the numerical approach for plasticity, the subtime step size may be controlled by specifying the allowable creep strain increment in terms of total strain.

B.7.4.3 Material Properties Input.

a. The elastic modulus, elastic-plastic modulus, yield stress, thermal expansion coefficient, and creep behavior are temperature-dependent.

b. Card inputs by the user provides the material constants at base temperature. If the option of temperature-dependent material properties is chosen, the user then has to supply subroutines to specify the rate of change of material constants with respect to temperature.

B.7.4.4 Assumptions. The material is assumed to be isotropic with respect to the direction at each material point.

B.7.5 Thermal Analysis Capability. The temperature distribution histogram is input at each time step either by card or by subroutine.

B.7.6 Dynamics. No capability.

B.7.7 Instability (Buckling) Analysis. No capability.

B.7.8 Kinematics. No capability.

B.7.9 Boundary Conditions for Structural Analysis.

a. Mechanical loads that are time-varying can be applied at the inner and outer boundaries of the long cylinder, and in the axial direction.

b. The axial displacement can be specified in the generalized plane strain option.

B.7.10 Mesh Generation and Graphics Capability. Mesh generation is not required and graphics capability is not available.

B.7.11 Program Documentation. Both the theory and user's manual are documented by Foster Wheeler Corporation.[16] A revised user's manual is to be added with more complete features of post-process analyses.

B.7.12 Program Availability.

a. The source deck is listed by the Foster Wheeler Corp. document.[16]

b. User assistance may be available through special arrangement with Foster Wheeler Corporation, 110 South Orange Avenue, Livingston, New Jersey 07039.

c. Program is operational on a CDC computer.

B.8 BASIC TECHNOLOGY PROGRAM - BTI511

B.8.1 Brief Program Description. BTI511 is a special-purpose computer program for analysis of the inelastic behavior of a single slab or shell element during any combination of load, strain, pressure, or temperature transient history. The plate may be subdivided into as many as ten layers of identical material.

The object of the program is to substantially reduce the cost of computer analyses which are exceedingly expensive with general-purpose programs when the transient history is long or complicated. Beside saving cost where the single-element representation is valid, this approach lends itself to greater accuracy of the mathematical treatment.

Due to the particularly simple input structure, the applicability of the program ranges from scoping calculations at the design stage to sophisticated transients at the analysis stage.

BTI511 is growing and developing. Through its user-defined routines, the program is very adaptable. The program owner makes an effort to keep the program abreast of the latest inelastic analysis procedures.

B.8.2 General Information.

B.8.2.1 Type of Structure. The program is applicable to a single slab or cylindrical shell element.

B.8.2.2 Method of Analysis. The program owner did not supply the method analysis; see the numerical approach below under plasticity and creep.

B.8.2.3 Restart Capability. Information was not supplied.

B.8.2.4 Computer System Requirements. Information was not supplied.

B.8.3 Element Library. The single shell element of BTI511 is a thin square plate of unit edge. It may have two configurations: flat slab, or cylindrical shell, which differ in the boundary conditions. Namely, all deformations must satisfy the requirement that the edge face of the element must remain plane. Bending is allowed for the slab, but not for the shell, representation. In the cylinder option, hoop and axial loads are applied proportionately to the pressure, if any. There may be up to 11 fibers, equidistant or not, across the thickness of the element.

B.8.4 Material Behavior.

B.8.4.1 Plasticity.

B.8.4.1.1 Constitutive Equations. The plasticity option is selected by specifying a stress-strain curve (bilinear or polylinear, isothermal or not). The first point of the curve is the yield point. Isotropic and kinematic hardening are available. The von Mises criterion and the Prantl-Reuss flow rule applies. The program owner makes an effort to implement the empirical rules of this standard, and others as they become available. The user has the option of defining his own formulation of the plasticity algorithm.

B.8.4.1.2 Numerical Approach. The plastic strain increments are calculated according to either initial-strain or tangent-modulus method. The calculated increments may be increased or damped and an iterative technique applied until convergence is satisfied, in order to avoid unwarranted unloading of yielding fibers. The load and temperature incrementation schemes are user-specified.

B.8.4.1.3 Elements. Not required.

B.8.4.2 Creep.

B.8.4.2.1 Constitutive Equations. Several equations calculating the creep increment in terms of the current stress state and time are available (including 304, 316SS). The creep option is selected by specifying the coefficients of these equations. Thus, strain hardening and time hardening are available. The Prantl-Reuss flow rule applies. The program owner makes an effort to implement the empirical rules of this standard and others as they become available. The user has the option of defining his own formulation of the creep routine.

B.8.4.2.2 Numerical Approach. The creep increment, in the standard option, is actually computed for the previous time period on the average of previous and new stress states assumed to hold for the whole period. The time incrementation scheme is user-specified.

B.8.4.2.3 Elements. Not required.

B.8.4.2.4 Creep Damage. The creep damage option is selected by specifying a table of life time versus stress level. The creep damage integral is accrued for the previous time period, with the average of the previous and new stress states assumed to hold for the whole period.

B.8.4.3 Material Properties Input.

a. Material properties are either constant or temperature dependent.

b. No material property is built-in, nor are any specific set of units; a consistent set must be chosen. Property data for most commonly used materials are available.

B.8.4.4 Method of Inputting and Generality Permitted. Internally, material properties are treated as polynomials in the temperature. Input may be the polynomial coefficients, or a table of the property values versus temperature (which will be fitted to a polynomial up to the sixth degree). Plastic stress-strain relations are input by table or subroutine. Creep equations are entered through specified coefficients or by subroutine. The program has orthotropic capability.

B.8.5 Thermal Analysis Capability.

B.8.5.1 Steady-State Transient. Both steady-state and transient transverse heat conduction are optional. Thermal solutions may be turned off, or they may be calculated on a smaller time scale than the mechanical analysis (when elastic).

B.8.5.2 Method of Analysis. The solution is a one-dimensional finite-difference approximation of the heat flow.

B.8.5.3 Boundary Conditions and Initial Conditions. Initial conditions are assumed either uniform temperature or uniform gradient. Boundary conditions are either insulation or specified temperatures or convection film temperatures at either surface of the plate. The film coefficients may depend on a power of the temperature difference.

B.8.5.4 Elements. Not required.

B.8.5.5 Properties. Constant or temperature-dependent properties are input through their polynomial coefficients or through a value table.

B.8.5.6 Coupling to Structural Analysis. The coupling is direct, or immediate. The mesh is the same, there is no interpolation and the transmission of data is internal.

B.8.6 Dynamics. No capability.

B.8.7 Instability. No capability.

B.8.8 Kinematics. No capability.

B.8.9 Boundary Conditions for Structural Analysis. The program has load, pressure, and displacement capability with arbitrary time variation; however, it has either load or displacement control but they cannot be mixed.

B.8.10 Mesh Generation and Graphics Capability.

B.8.10.1 Mesh Generation. Not required.

B.8.10.2 Graphics. The program will provide output for X-Y plotting of variation of fiber quantities with time or loadings, or against each other (for the same fiber).

B.8.11 Program Documentation.

B.8.11.1 Theory. The theory is available by special arrangement with the program owner.

B.8.11.2 User's Manual. The manual consists of sixty-four pages, including examples, updates (14 complementary notes, and 9 Appendices) and documentation of use.

B.8.12 Program Availability.

B.8.12.1 Source. The program is available by special arrangements with:

Basic Technology, Inc.
201 Penn Center Blvd.
Pittsburgh, Pennsylvania 15235

B.8.12.2 Arrangements. There is a royalty charge for use of the program by the owner. Arrangements are being planned by the program owner to make the program available through a national time-sharing service.

B.8.12.3 User Assistance. User assistance can be obtained by calling phone (412) 824-9210, or through personal consultation with the program owner,

B.8.12.4 Machines on Which Operational. The program is available for use on a CDC-6600 using standard FORTRAN.

B.9 PIRAX

B.9.1 Brief Program Description. PIRAX is a special purpose computer program for the simplified inelastic analysis of moderately complex spatially three-dimensional piping systems. This inelastic behavior includes both time-independent plasticity and time-dependent creep and relaxation. The loading conditions include mechanical and thermal loadings. The program is currently under development and should be completed by the end of 1974.

B.9.2 General Information.

B.9.2.1 Type of Structure. It is for use on spatially three-dimensional piping systems.

B.9.2.2 Method of Analysis. The finite-element matrix displacement method is used with the initial strain approach utilized to solve the nonlinear problem in an incremental sense.

B.9.2.3 Restart Capability. No capability.

B.9.2.4 Computer System Requirements. Concurrently, the program is operational on only CDC 6000 series machines with extended core storage. However, at the completion of the development, any computer system capable of compiling FORTRAN should be sufficient.

B.9.3 Element Library. The program utilizes two beam elements with a circular cross section. These two elements are a straight uniform beam and a circularly curved uniform beam with an elastic, plastic creep, and relaxation flexibility factor.

B.9.4 Material Behavior.

B.9.4.1 Plasticity.

B.9.4.1.1 Constitutive Equations. The time-independent plasticity solution is based on an assumed longitudinal stress-strain relationship of the form:

$$\epsilon = \frac{\sigma}{E} + K_p \left\{ \frac{\sigma}{\sigma_{op}} \right\}^{NP},$$

where: E , K_p , σ_{op} , NP are material and temperature dependent properties.

B.9.4.1.2 Numerical Approach. The method of initial strain is applied to solve the plasticity formulation. Currently, the use must specify the incremental step.

B.9.4.1.3 Availability. The program has plasticity capability for both beam elements.

B.9.4.2 Creep.

B.9.4.2.1 Constitutive Equations. The time-dependent creep solution is based on an assumed longitudinal stress-strain rate relationship of the form:

$$\dot{\epsilon} = K \left\{ \frac{\sigma}{\sigma_o} \right\}^N, \quad (B.3)$$

where: k , σ_0 , N are material and temperature-dependent properties.

B.9.4.2.2 Numerical Approach. The method of initial strain is applied to solve the creep and relaxation formulation. Currently, the user must specify the incremental time step.

B.9.4.2.3 Availability. The program has creep capability for both beam elements.

B.9.4.3 Material Properties Input. The user must input all material properties.

B.9.4.4 Availability. Only isotropic behavior capability is available.

B.9.5 Thermal Analysis Capability. No capability.

B.9.6 Dynamics. No capability.

B.9.7 Instability (Buckling) Analysis. No capability.

B.9.8 Kinematics. No capability.

B.9.9 Boundary Conditions for Structural Analysis. Mechanical and thermal loads can be applied. Mechanical loads include forces, displacements or both.

B.9.10 Mesh Generation and Graphics Capability. No capability.

B.9.11 Program Documentation. Both a theory and user's manual will be furnished to Oak Ridge National Laboratory.

B.9.12 Program Availability. The program when completed will be transferred to Oak Ridge National Laboratory, for information contact:

W. K. Sartory
Oak Ridge National Laboratory
P. O. Box Y
Oak Ridge, Tennessee 37830

B.10 REFERENCED DOCUMENTS

B.10.1 American Society of Mechanical Engineers (ASME).

ASME Boiler and Pressure Vessel Code,

Section III Nuclear Power Plant Components.

B.10.2 Other Referenced Documents. The following literature is referenced in the text by number or by superscripts in brackets:

1. P. V. Marcal, "Finite-Element Analysis of Combined Problems of Non-Linear Material and Geometric Behavior," paper presented at the American Society of Mechanical Engineers Computer Conference, Illinois Institute of Technology, Chicago, June 19-20, pp. 133-149.
2. H. D. Hibbitt, N. J. Levy, and P. V. Marcal, "On General Purpose Programs for Non-Linear Finite Element Analysis," On General Purpose Finite Element Computer Program, pp. 98-122, The American Society of Mechanical Engineers, New York, 1970.
3. H. D. Hibbitt and P. V. Marcal, "A Numerical Thermo-Mechanical Model for the Welding and Subsequent Loading of a Fabricated Structure," Comput. Struct. 3, pp 1145-1174 (1973).
4. H. D. Hibbitt, A Numerical, Thermo-Mechanical Model for the Welding and Subsequent Loading of a Fabricated Structure, PhD. Thesis, Brown University, 1972.
5. H. D. Hibbitt, E. P. Sorensen, and P. V. Marcal, "The Elastic-Plastic and Creep Analysis of Pipelines by Finite Elements," in Pressure Vessel Technology, Part, Design and Analysis, pp. 239-251, The American Society of Mechanical Engineers, New York, 1973.
6. H. D. Hibbitt, P. V. Marcall, and J. R. Rice, "Finite Element Formulation for Problems of Large Strain and Large Displacement," Int. J. Solids Struct. 6, pp. 1069-1086 (1970).
7. "MARC-CDC User Information Manual: Volume I, Program Description, Volume II, Program Input, Volume III, Demonstration Problems," Control Data Corporation, Minneapolis, Minn.
8. "Liquid Metal Fast Breeder Reactor Materials Handbook," HEDL-TME 71-32.
9. Y. R. Rashid, "Part I. Theory Report for Creep-Plast Computer Program: Analysis of Two-Dimensional Problems Under Simultaneous Creep and Plasticity," GEAP-10546, January 1972.
10. Y. R. Rashid, "Part II. User's Manual for Creep-Plast Computer Program," GEAP-13262, March 1972.
11. J. A. Clinard and J. S. Corwell, "ORNL User's Manual for Creep-Plast Computer Program," ORNL-TM-4062, November 1973.
12. W. L. Greenstreet, J. M. Corum, and C. E. Pugh, "High-Temperature Structural Design Methods for LMFBR Components Quarterly Progress Report for Period Ending September 30, 1972," ORNL-TM-4058, January 1973.

13. W. L. Greenstreet, J. M. Corum, and C. E. Pugh, "High-Temperature Structural Design Methods for LMFBR Components. Progress Report, October 1972-March 1973," ORNL-TM-4089, July 1973.
14. F. C. Rally and W. H. Whitling, compilers, "High-Temperature Stress Analysis Program" GEAP-13944, April 1973.
15. Z. Zudans, M. M. Reddi, H. M. Fishman, and T. Y. Chow, "Theory and Users Manual for EPACA, General Purpose Elastic-Plastic-Creep Finite Element Analysis Program for Three-Dimensional Thick Shell Structures," Final Report F-C 3038, June 1972. (Franklin Institute Research Laboratories, Philadelphia).
16. J. M. Chern, "Intermediate Heat Exchanger for Fast Flux Test Facility, Elastic-Plastic-Creep Analysis of a Thick-Walled Cylinder Subjected to Time-Dependent Pressure, Temperature, and Axial Loads or Displacements," FWR-35, July 1972, (Foster-Wheeler Corporation, John Blizzard Research Center, 12 Peach Tree Hill Road, Livingston, New Jersey).

APPENDIX C

INDEX NOTATION

C.1 INTRODUCTION

In this appendix, some of the stress and strain quantities that are used frequently in Section 4 and that are written there in index notation are briefly explained in terms of the stresses and strains related to rectangular cartesian coordinates (x,y,z). It is hoped that this explanation will be useful to the reader who may be familiar with tensor quantities and index notation.

In index notation, coordinates are indicated by indexes (1,2,3) instead of letters (x,y,z). For example, the coordinates of a general point in (x,y,z) space are denoted by $x_i = (x_1, x_2, x_3)$, or more briefly by x_i , with the understanding that i takes the values 1,2,3. Similarly, axes (x,y,z) may be denoted by (x_1, x_2, x_3) or simply by x_i .

C.2 STRESS QUANTITIES

The nine components of stress taken collectively are called the stress tensor or the stress array. With the following change in notation, the stress tensor may be denoted in index notation* by σ_{ij} , where i and j can take the values 1,2,3, and where $\sigma_{ij} = \sigma_{ji}$:

$$\begin{aligned} \sigma_x &= \sigma_{11}, & \sigma_y &= \sigma_{22}, & \sigma_z &= \sigma_{33}, \\ \tau_{xy} &= \sigma_{12}, & \tau_{yz} &= \sigma_{23}, & \tau_{xz} &= \sigma_{13}. \end{aligned} \quad (C.1)$$

More generally, an array of numbers such as the stress array is called a matrix. The matrix of the stress tensor can thus be written as:

$$\sigma_{ij} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix}. \quad (C.2)$$

It should be emphasized that the notation σ_{ij} refers to the stress components collectively. However, when specific values are assigned to

*The particular letters used for the index are arbitrary.

i and j , the notation refers to a specific stress component. For example, σ_{12} refers to a specific stress component (τ_{xy} in the more familiar engineering notation).

C.2.1 Component Tensors. It is possible to resolve the stress tensor σ_{ij} into two additive component tensors:

$$\sigma_{ij} = \sigma'_{ij} + \sigma''_{ij}, \quad (C.3)$$

defined as follows:

$$\sigma''_{ij} = \begin{bmatrix} \sigma_m & 0 & 0 \\ 0 & \sigma_m & 0 \\ 0 & 0 & \sigma_m \end{bmatrix}, \quad (C.4)$$

where:

$$\sigma_m = \frac{1}{3} (\sigma_x + \sigma_y + \sigma_z),$$

and:

$$\sigma'_{ij} = \sigma_{ij} - \sigma''_{ij} = \begin{bmatrix} \frac{2\sigma_x - \sigma_y - \sigma_z}{3} & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \frac{2\sigma_y - \sigma_z - \sigma_x}{3} & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \frac{2\sigma_z - \sigma_x - \sigma_y}{3} \end{bmatrix}. \quad (C.5)$$

The quantity σ'_{ij} is called the deviatoric component of the stress tensor, and the quantity σ''_{ij} is called the spherical (or hydrostatic) component of the stress tensor.

The deviatoric component is an important, and often encountered quantity in constitutive relations. In index notation the deviatoric stress component can be written as:

$$\sigma'_{ij} = \sigma_{ij} - \frac{1}{3} \delta_{ij} \sigma_{kk} , \quad (C.6)$$

where δ_{ij} is called the Kronecker delta and has the following simple meaning:

$$\delta_{ij} = \begin{cases} 1 & \text{for } i = j \\ 0 & \text{for } i \neq j \end{cases} . \quad (C.7)$$

C.2.2 Implied Summation. To read Eq C.6 it must be recognized that a repeated index in a single term implies summation over the values 1,2,3. Accordingly, the quantity σ_{kk} becomes:

$$\sigma_{kk} = \sigma_{11} + \sigma_{22} + \sigma_{33} = \sigma_x + \sigma_y + \sigma_z . \quad (C.8)$$

Thus, as specific values are assigned to i and j in Eq C.6, the specific components of the deviatoric stress given in Eq C.5 can be readily obtained. As examples:

$$\sigma'_{11} = \sigma_{11} - \frac{1}{3} (\sigma_{11} + \sigma_{22} + \sigma_{33}) = \frac{2\sigma_x - \sigma_y - \sigma_z}{3} = \sigma'_x$$

and:

$$\sigma'_{12} = \sigma_{12} - \frac{1}{3} (0)(\sigma_{11} + \sigma_{22} + \sigma_{33}) = \tau_{xy} .$$

C.2.3 Invariants. Often, scalar quantities called invariants of the stress tensor are useful. The term "invariant" derives from the fact that the magnitudes of these quantities are independent of the particular set of coordinate axes being considered. The second invariant, J'_2 , of the deviatoric component of the stress tensor is used in this document. In terms of the usual engineering nomenclature:

$$J'_2 = - \left(\sigma'_x \sigma'_y + \sigma'_y \sigma'_z + \sigma'_z \sigma'_x - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{xz}^2 \right) . \quad (C.9)$$

In index notation, Eq C.9 can be written simply as:

$$J_2' = \frac{1}{2} \sigma'_{ij} \sigma'_{ij} . \quad (C.10)$$

Recalling the rule that repeated indexes imply summation over the values 1,2,3, Eq C.10 may be expanded as follows. Considering the index i first:

$$J_2' = \frac{1}{2} (\sigma'_{1j} \sigma'_{1j} + \sigma'_{2j} \sigma'_{2j} + \sigma'_{3j} \sigma'_{3j}) ,$$

and then considering the j index, each of the three terms in parenthesis is expanded, in turn, into three terms, so that:

$$J_2' = \frac{1}{2} [(\sigma_{11}'^2 + \sigma_{12}'^2 + \sigma_{13}'^2) + (\sigma_{21}'^2 + \sigma_{22}'^2 + \sigma_{23}'^2) + (\sigma_{31}'^2 + \sigma_{32}'^2 + \sigma_{33}'^2)] . \quad (C.11)$$

Now it is observed that:

$$\sigma'_{11} + \sigma'_{22} + \sigma'_{33} = 0 ,$$

so that:

$$(\sigma'_{11} + \sigma'_{22} + \sigma'_{33})^2 = 0 = \sigma_{11}'^2 + \sigma_{22}'^2 + \sigma_{33}'^2 + 2(\sigma'_{11}\sigma'_{22} + \sigma'_{11}\sigma'_{33} + \sigma'_{22}\sigma'_{33})$$

or:

$$\sigma_{11}'^2 + \sigma_{22}'^2 + \sigma_{33}'^2 = -2(\sigma'_{11}\sigma'_{22} + \sigma'_{11}\sigma'_{33} + \sigma'_{22}\sigma'_{33}) .$$

Substituting this last relation into Eq C.11 it is found that:

$$J_2' = -(\sigma'_{11}\sigma'_{22} + \sigma'_{11}\sigma'_{33} + \sigma'_{22}\sigma'_{33} - \sigma_{12}'^2 - \sigma_{13}'^2 - \sigma_{23}'^2) . \quad (C.12)$$

Finally, if the change in notation specified by Eq C.1 is made Eq C.12 agrees with Eq C.9.

Often a scalar quantity called the effective stress, or the von Mises effective stress, is utilized. In the usual notation, this quantity is defined as:

$$\bar{\sigma} = C_1 \sqrt{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{xz}^2 + \tau_{yz}^2)} \quad (C.13)$$

If $C_1 = 1/3$, the effective stress is equal to the octahedral shear stress; if $C_1 = 1/\sqrt{6}$, the square of the effective stress is equal to the second invariant J_2 of the deviatoric component of the stress tensor. Finally, if $C_1 = 1/\sqrt{2}$ then for a uniaxial stress, for which σ_x is the axial stress and all of the other stress components are zero, $\bar{\sigma} = \sigma_x$.

C.3 STRAIN QUANTITIES

Strain quantities are written in index notation in a manner analogous to stress quantities. The nine components of strain taken collectively are called the strain tensor. With the following change in notation the strain tensor may be denoted in index notation by ϵ_{ij} , where $\epsilon_{ij} = \epsilon_{ji}$. Then:

$$\begin{aligned} \epsilon_x &= \epsilon_{11}, & \epsilon_y &= \epsilon_{22}, & \epsilon_z &= \epsilon_{33}, \\ \frac{\gamma_{xy}}{2} &= \epsilon_{12}, & \frac{\gamma_{yz}}{2} &= \epsilon_{23}, & \frac{\gamma_{xz}}{2} &= \epsilon_{13}. \end{aligned} \quad (C.14)$$

As in the case of stress, the strain tensor ϵ_{ij} may be resolved into two additive component tensors, the deviatoric strain tensor, ϵ'_{ij} , and the spherical (hydrostatic) strain tensor, ϵ''_{ij} . The matrix of the spherical strain tensor is given by:

$$\epsilon''_{ij} = \begin{bmatrix} \epsilon_m & 0 & 0 \\ 0 & \epsilon_m & 0 \\ 0 & 0 & \epsilon_m \end{bmatrix}, \quad (C.15)$$

where:

$$\epsilon_m = \frac{1}{3} (\epsilon_x + \epsilon_y + \epsilon_z)$$

is the mean strain.

The matrix of the deviatoric strain tensor is:

$$\epsilon'_{ij} = \begin{bmatrix} \frac{2\epsilon_x - \epsilon_y - \epsilon_z}{3} & \frac{\gamma_{xy}}{2} & \frac{\gamma_{xz}}{2} \\ \frac{\gamma_{xy}}{2} & \frac{2\epsilon_y - \epsilon_z - \epsilon_x}{3} & \frac{\gamma_{yz}}{2} \\ \frac{\gamma_{xz}}{2} & \frac{\gamma_{yz}}{2} & \frac{2\epsilon_z - \epsilon_x - \epsilon_y}{3} \end{bmatrix} \quad (C.16)$$

It is important to note that in both plasticity and creep the assumption of incompressibility is made. This means that when only the inelastic portion of the total strain is considered:

$$\epsilon_m = 0. \quad (C.17)$$

Consequently:

$$\epsilon'_{ij} = 0,$$

and:

$$\epsilon'_{ij} = \epsilon_{ij}. \quad (C.18)$$

The second invariant, I'_2 , of the deviatoric component of the strain tensor is in the case of inelastic strains equal, because of Eq C.17, to the second invariant, I_2 , of the strain tensor, and is given by:

$$I'_2 = I_2 = \frac{1}{2} \epsilon_{ij} \epsilon_{ij}. \quad (C.19)$$

For inelastic strains only:

$$\begin{aligned} (\epsilon_{11} + \epsilon_{22} + \epsilon_{33})^2 = 0 &= \epsilon_{11}^2 + \epsilon_{22}^2 \\ &+ \epsilon_{33}^2 + 2(\epsilon_{11}\epsilon_{22} + \epsilon_{11}\epsilon_{33} + \epsilon_{22}\epsilon_{33}), \end{aligned}$$

so that:

$$\epsilon_{11}^2 + \epsilon_{22}^2 + \epsilon_{33}^2 = -2(\epsilon_{11}\epsilon_{22} + \epsilon_{11}\epsilon_{33} + \epsilon_{22}\epsilon_{33}) .$$

Expanding Eq C.19 and substituting this latter relation into it:

$$I_2 = -(\epsilon_{11}\epsilon_{22} + \epsilon_{11}\epsilon_{33} + \epsilon_{22}\epsilon_{33} - \epsilon_{12}^2 - \epsilon_{13}^2 - \epsilon_{23}^2) ,$$

or, in the usual engineering strain notation:

$$I_2 = -\left(\epsilon_x\epsilon_y + \epsilon_x\epsilon_z + \epsilon_y\epsilon_z - \frac{\gamma_{xy}^2}{4} - \frac{\gamma_{xz}^2}{4} - \frac{\gamma_{yz}^2}{4}\right) . \quad (C.20)$$

Eq C.20 holds only for the inelastic components of strain.

An effective strain quantity, in the usual strain notation, is defined by:

$$\bar{\epsilon} = C_2 \sqrt{(\epsilon_x - \epsilon_y)^2 + (\epsilon_y - \epsilon_z)^2 + (\epsilon_z - \epsilon_x)^2 + \frac{3}{2}(\gamma_{xy}^2 + \gamma_{xz}^2 + \gamma_{yz}^2)} . \quad (C.21)$$

If $C_2 = 2/3$, the effective strain is equal to the octahedral shear strain; if $C_2 = 1/\sqrt{6}$, the square of the effective strain is equal to the second invariant I_2 of the strain tensor, provided only inelastic strains are being considered. Finally, in the case of a uniaxial strain for the inelastic condition:

$$\epsilon_y = \epsilon_z = -\frac{1}{2} \epsilon_x ,$$

since $\epsilon_m = 0$. Therefore, $\bar{\epsilon} = \epsilon_x$ for a uniaxial test provided that $C_2 = \sqrt{2}/3$.