MASTER

Report No. BMI-APDA-638

OK
NSA

STRESS-STRAIN PROPERTIES OF IRRADIATED
URANIUM-10 w/o MOLYBDENUM

by

John E. Gates
Edward G. Bodine
J. Clarence Bell
Arthur A. Bauer
George D. Calkins

to

ATOMIC POWER DEVELOPMENT ASSOCIATES

This document is
WHOLLY RELEASABLE
Exempt from GDS
Authenticating Office
Date 4/20/04

January 6, 1958

BATTELLE MEMORIAL INSTITUTE
505 King Avenue
Columbus 1, Ohio

Encl 1^c

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

TABLE OF CONTENTS

	<u>Page</u>
ABSTRACT	1
INTRODUCTION	1
EXPERIMENTAL WORK	2
Apparatus	2
Specimens	2
Irradiation	5
Testing Procedure	5
DISCUSSION OF RESULTS	7
Unirradiated Specimens	7
Irradiated Specimens	17
Effect of Burnup	17
Effect of Irradiation Temperature	32
CONCLUSIONS	37
REFERENCES	38

APPENDIX

OBTAINING THE STRESS-STRAIN CURVE USING BEND-TEST DATA	A-1
--	-----

STRESS-STRAIN PROPERTIES OF IRRADIATED URANIUM-10 w/o MOLYBDENUM

John E. Gates, Edward G. Bodine, J. Clarence Bell,
Arthur A. Bauer, and George D. Calkins

This report describes the stress-strain properties of the APDA reference alloy of uranium-10 w/o molybdenum at various burnups up to 2.1 total a/o. Data were obtained on both irradiated and unirradiated specimens of the alloy by the use of a remotely operated bend-testing machine. Tensile properties of unirradiated material are also reported for comparison with data obtained from the bend tests. The mathematical procedure used in obtaining equivalent uniaxial stress-strain properties from the bend-test data is described in the Appendix.

The results of the bend tests indicate that the physical properties of the reference alloy were severely affected by irradiation to burnups as high as 2.1 total a/o. At a test temperature of 500 C, the modulus of elasticity, ultimate strength, and strain at fracture decreased rapidly up to a burnup of about 0.5 to 1.0 total a/o, with the rate of change decreasing thereafter. No important differences were noted between coextruded and hot-rolled material. An indication was obtained that the ultimate strength of the alloy was improved with increasing irradiation temperatures up to about 500 C. Data obtained on unirradiated material indicated that ductility of the reference alloy was lower at 500 C than at room temperature. Stress-strain curves for the irradiated specimens are included in this report. The value of these data is limited because of the large number of variables encountered in the specimen histories and the small number of tests involved in this program.

INTRODUCTION

A study of the effects of radiation on various alloys of uranium has been in progress at Battelle for the past 3 years. Preliminary studies of binary alloys of chromium, molybdenum, and zirconium with uranium indicated that an alloy of uranium-10 w/o molybdenum was relatively stable to radiation and therefore might offer good possibilities for use as a reactor-fuel alloy. During the past year, specimens of this alloy have been irradiated to burnups ranging up to 2.7 total a/o at temperatures up to 600 C. The effects of radiation upon various properties of the alloy such as density, dimensional stability, electrical resistivity, loss of fission gas, and thermal conductivity and the effects of various heat treatments on the radiation stability of the alloy have been studied. The results of these studies were included in BMI-APDA-625.⁽¹⁾ Several specimens, upon which the nondestructive tests noted above have been completed, have now been bend tested in order to obtain data on the stress-strain properties of the alloy as a function of burnup up to 2.1 total a/o. Data obtained from these tests, along with a description of the apparatus and a theoretical discussion of the testing techniques, are included in this report.

(1) References at end of report.

EXPERIMENTAL WORK

The bend test was used to obtain the mechanical properties of irradiated material because of the ease and simplicity of remote operation and because the small, simple specimens required by irradiation operations could be employed. Having selected a testing technique, it was necessary to translate the resulting data into the more familiar representations afforded by conventional test procedures. For example, the utility of a stress-strain curve in illustrating the response of a material to uniaxially applied loads is well known. However, the moment-deflection curve obtained from bend-test data is not a widely familiar representation of material properties. Therefore, a procedure was devised to yield uniaxial stress-strain information from moment-deflection data. The mathematical analysis of this problem for the case where the elementary equation $\sigma = \frac{MC}{I}$ is no longer valid is treated in the Appendix.

The above-mentioned analysis has been previously used at Battelle for finding the stress-strain relationship for rectangular bend-test specimens, and has recently been expanded to include the treatment of cylindrical specimens of the type used in this study.

Apparatus

A drawing of the testing machine which was constructed to perform bend tests on irradiated specimens is shown in Figure 1. Essentially, the device consists of a fixed upper head or platen, a screw-driven lower platen, four rods situated so as to guide the motion of the lower platen while holding the upper platen in a fixed position, and a motor-reducer arrangement operating the drive screw. The motion of the moving head can be limited in both the upward and downward direction by means of microswitches. Figure 2 shows the specimen in position during a test.

Load and deflection are measured by means of the two concentric rings shown in detail in Figure 3. The heavy outer ring is situated so as to carry the load on the specimen, while the very light gage inner ring is deformed by the amount of the specimen's deflection on its support. The strains which occur in both these rings are measured using SR-4 gages and a two-channel Sanborn recording indicator. The recorded strains are then translated into the appropriate loads and deflections.

A cylindrical furnace slides vertically on two guide rods so as to permit access to the specimen supports in the raised position, and to surround the specimen area during heating in the lowered position.

Specimens

In general, specimens of the reference alloy were produced from castings prepared from uranium and high-purity molybdenum sheet. The material was prealloyed by arc melting, remelted in a zirconia crucible, and cast at 1480 C into cold copper molds. Actual fabrication of specimens from the castings was accomplished by techniques consisting of either hot rolling and cold swaging or coextrusion.

FIGURE 1. REMOTELY OPERATED BEND-TEST APPARATUS

BATTELLE MEMORIAL INSTITUTE

FIGURE 2. DETAIL OF SPECIMEN SUPPORT

FIGURE 3. DETAIL OF LOAD- AND DEFLECTION-MEASURING RINGS

Irradiation

The specimens used for the bend tests were obtained from the APDA radiation-effects program. These specimens were irradiated in various capsules in the MTR. An irradiation history for each capsule is listed in Table 1. It was impossible to select specimens for the bend tests with either a constant burnup or a constant irradiation temperature because of the small number of available specimens. Specimens were therefore selected to provide data over a range of burnups. Both methods of fabrication were represented in the selected specimens.

Testing Procedure

The bend-test apparatus was calibrated by testing unirradiated specimens of iron and aluminum. The tensile data obtained from these tests were then compared with known data obtained by conventional means. Specimens of the uranium-10 w/o molybdenum reference alloy, fabricated by both hot-rolling and coextrusion techniques, were also bend tested for calibration purposes.

The calibration tests were performed at temperatures of 25, 250, and 500 C. At the higher temperatures, helium was passed through the furnace to reduce the amount of surface oxidation of the specimens. The electrical resistivity of some of the reference-alloy specimens was also measured in liquid nitrogen after bend testing at elevated temperatures to determine if transformation had occurred.

To perform the actual bend testing, a specimen was placed in position on the lower platen shown in Figure 1. If the test was to be performed above room temperature, the furnace was lowered to enclose the specimens, and the temperature raised to the desired point. After the testing temperature was reached, the lower platen was raised until the specimen contacted the upper platen. The moment of contact was shown on the Sanborn recorder as was the load and deflection during the test. After the test was completed, the temperature was reduced and the specimen removed from the apparatus.

Upon the completion of the tests on the unirradiated control specimens, the stress-strain properties were calculated from the moment-deflection curve. These results were then compared with stress-strain data obtained from tensile tests of specimens of the same alloy and heat treatment.

The procedure for bend testing the irradiated specimens was the same as that used for the control specimens except that fracture values were not obtained at all three temperatures due to the lack of specimens. The specimens were tested in two groups. The first group of four specimens was tested and the results analyzed before the second group was tested.

The test temperatures of 25, 250, and 500 C were selected at the suggestion of APDA for the first series of tests. Normally, the testing to fracture of one specimen at each temperature would have been desirable. However, there were not three specimens available with the same fabrication and irradiation history, without considering the number of specimens necessary to allow data to be obtained at different burnups.

TABLE 1. MTR IRRADIATION HISTORY OF VARIOUS CAPSULES

Battelle Capsule	MTR Cycle	Irradiation Time		Reactor Hole	Position from Top of Reflector, in.		MTR Quoted Flux, 10 ¹⁴ nv		Average Dosimeter Flux, 10 ¹⁴ nv
		MWD	Power Level, megawatts		Capsule Bottom	Capsule Top	Capsule Bottom	Capsule Top	
302-19	67	648	39.3	A-29			1.65	1.5	
	68	677	39.6	A-29			1.65	1.5	
	69	605	38.7	A-29			1.65	1.5	
	70	611	39.4	A-29			1.65	1.5	
9-1	68	677	39.6	A-27-NE	16-1/2	13-1/2	1.0	0.88	1.04 for total irradiation period
9-2	68	677	39.6	A-27-NE	19-3/4	16-1/2	1.1	1.0	1.1 for total irradiation period
	69	605	38.7	A-27-NE	19-3/4	16-1/2	1.0	1.0	
9-7	77	473	38.0	A-28-SE	14-3/4	7-1/2	0.51	0.20	0.8 for total irradiation period
	78	539	38.7	A-28-SE	14-3/4	7-1/2	0.51	0.20	
	79	527	38.2	A-28-SE	14-3/4	7-1/2	0.51	0.20	
	80	556	38.5	A-28-SE	14-3/4	7-1/2	0.51	0.20	
9-8	77	473	38.0	A-28-SE	22	14-3/4	0.57	0.51	
	78	539	38.7	A-28-SE	22	14-3/4	0.57	0.51	
	79	527	38.2	A-28-SF	22	14-3/4	0.57	0.51	
	80	556	38.5	A-28-SE	22	14-3/4	0.57	0.51	
9-9	78	539	38.7	A-30-NE	26	14-1/2	1.2	1.45	
	79	527	38.2	A-30-NE	26	14-1/2	1.2	1.45	
	80	556	38.5	A-30-NE	26	14-1/2	1.2	1.45	
	81	535	38.1	A-30-NE	26	14-1/2	1.2	1.45	
9-11	82	597	39.1	A-30-NE	26	14-1/2	1.2	1.45	
	83	587	39.6	A-30-NE	26	14-1/2	1.2	1.45	
	84	495	37.7	A-30-NE	26	14-1/2	1.2	1.45	
	79	527	38.2	A-36-SW	24	16	0.35	0.28	
9-14	80	556	38.5	A-36-SW	24	16	0.35	0.28	
	81	535	38.1	A-43-SW	24	16	0.73	0.63	
	82	597	39.1	A-43-SW	24	16	0.73	0.63	
	83	587	39.6	A-43-SW	24	10	0.73	0.32	
9-14	84	495	37.7	A-43-SW	24	10	0.73	0.32	
	85	516	39.6	A-43-SW	24	10	0.73	0.32	
	86	614	39.2	A-43-SW	24	10	0.73	0.32	
	87	650	38.8	A-2-NE	20	5-1/2	1.10	0.20	
302-5	60, 61, 62	743	~ 30	--	--	--	--	2.1	

It was therefore decided to test one specimen at each of the temperatures. This was accomplished by loading the specimens just enough during the bend tests at each temperature to allow a small portion of the stress-strain curve to be mapped in the elastic region. Final testing to fracture at 500 C was selected because, at the time of selection of test temperatures, it was believed that the specimens would be more ductile at the elevated temperature and greater portions of the stress-strain curves could thus be observed. The first irradiated specimen was tested in the elastic region at 500, 250, and 25 C, in the order named. The temperature was then increased to 500 C and the specimen tested to fracture. It was observed that this specimen, with a burnup of only 0.36 total a/o, fractured at a load of 40 pounds. It appeared reasonable to assume that specimens with higher burnups would break at even lower loads. It was therefore decided to not risk testing in the elastic regions at all three temperatures, and to only test the specimens to fracture at 500 C. This procedure was used on the remaining three specimens of the first group.

After the results obtained from the first series of tests were analyzed, it was decided that the tests in the elastic region at the various temperatures could be resumed using adequate care during testing. The specimens were tested in the elastic region at 25, 250, and 500 C, with testing to fracture at 500 C. This order of testing at temperature was the reverse of that used in the first series, thus avoiding the testing of a specimen at 25 C which had previously been tested at 500 C. In addition, some specimens were tested more than once at a particular temperature to evaluate the effects of time at testing temperature on the properties of the alloy.

DISCUSSION OF RESULTS

Unirradiated Specimens

The results of tensile tests performed on unirradiated uranium-10 w/o molybdenum rods are listed in Table 2. The material was cold swaged to 127 mils in diameter from 0.25-in. -diameter hot-rolled bar stock, annealed 24 hr at 900 C, and water quenched. The test section was then machined to 100 mils in diameter. The values for elongation and reduction in area from the tensile test at 500 C (Table 2) indicate that a reduction in ductility took place at that temperature. This type of behavior was probably due to decomposition of the metastable gamma phase when the specimens were heated in this temperature range, where the rate of gamma decomposition is a maximum. It is also possible that some unobserved flaw in the specimen was responsible for a premature failure. The results of bend tests on the same type of specimens are shown in Table 3. Comparing the values shown in Table 2 with those in Table 3, it can be seen that the data obtained from bend tests can be treated so as to provide acceptable uniaxial (in this case, tensile) mechanical-properties information.

Ultimate-strength values for the unirradiated material were not included in Table 3. The ductility of the irradiated specimen was sufficiently large to preclude failure in bending during the limited head travel of the bend-test apparatus. It should be noted that good correlation between tensile and bend-test data will depend on cross-sectional uniformity of the bend-test specimen. In other words, an annealed specimen would be expected to yield correlative results, while a specimen which had been cold

TABLE 2. TENSILE PROPERTIES OF UNIRRADIATED URANIUM-10 w/o MOLYBDENUM

Specimen	Test Temperature, C	Elastic Modulus, 10^6 psi	0.2 Per Cent Offset Yield Strength, 10^3 psi	Ultimate Tensile Strength, 10^3 psi	Elongation, per cent	Reduction in Area, per cent
1	25	12.8	138.6	141.8	1.5 in 2 in.	33.8
2	25	12.2	136.4	139.0	1.9 in 2 in.	36.8
Average	25	12.5	137.5	140.4	1.7 in 2 in.	35.3
1	250	12.3	86.3	91.6	10.5 in 1 in.	45.2
2	250	11.8	85.2	90.8	4.5 in 2 in.	45.7
Average	250	12.0	85.8	91.2	--	45.4
1 ^(a)	500	9.4	66.2	69.4	1.5 in 1 in.	15.1

(a) Note. Two other specimens were scheduled for testing at 500 C. These were not straight however, and the tendency for the grips to straighten the specimens during heating caused them to break before they were tested. They seemed to be very brittle at 500 C. Elevated-temperature testing was done in a helium atmosphere. Rate of straining above the yield point was 0.02 in. per min of crosshead speed.

TABLE 3. MECHANICAL PROPERTIES OF UNIRRADIATED
URANIUM-10 w/o MOLYBDENUM DETERMINED
FROM BEND-TEST DATA

Specimen	Test Temperature, C	Elastic Modulus, 10^6 psi	0.2 Per Cent Offset Yield Strength ^(a) , 10^3 psi
1	25	12.5	--
2	25	12.5	--
Average	25	12.5	--
1	250	11.4	75.6
2	250	12.1	--
3	250	12.3	86.6
Average	250	11.9	81.1
1	500	8.1	64.0
2	500	8.4	--
3	500	9.8	--
4	500	9.8	78.5
Average	500	9.0	71.2

(a) Where values are not shown, the test was stopped before plastic straining took place.

worked on the surface, or was clad with a different material than that of the core, would not necessarily yield identical results in bending and in tension. This, of course, is due to the greater dependence of bend-test results on the outer rather than the inner fibers of the specimen.

This surface sensitivity of the bend test may be significant in the testing of specimens irradiated in a thermal reactor such as the MTR since the surface burnup is normally higher than that at the center of such specimens. It should be noted, however, that such a surface effect would result in an apparent loss of strength in the whole cross section. Another factor which may serve to reduce the apparent strength of the specimens during bend testing is a "notch effect" arising from the presence of machining marks on the surface. Since these two effects would both tend to reduce the apparent strength of the test specimens, the bend tests would tend to produce lower values for the strength.

Elastic-modulus data were obtained on two additional unirradiated specimens by bend testing. These results are shown in Table 4. Specimen HRC-3 was machined from hot-rolled stock, while Specimen EC-3 was obtained from coextruded stock. Neither specimen was clad. The increase in the room-temperature modulus of elasticity after exposure to elevated temperature, as shown by the behavior of Specimen HRC-3, prompted a further study of the elevated-temperature properties of uranium-10 w/o molybdenum. An 18-in. -long by 100-mil-diameter sample of uranium-10 w/o molybdenum was obtained which had been hot rolled to 0.25 in. in diameter, cold swaged to 100 mils in diameter, heated to 1000 C, and water quenched. This material differed from the bend-test samples in heat treatment and the fact that the bend specimens had been machined from 127-mil stock. However, even though the heat treatments varied to some extent, they all had the effect of retaining the high-temperature-stable gamma phase in the reference alloy. Two sections were cut from the original 18-in. -long rod, one 8 in. and one 10 in. long. The 10-in. -long section was mounted as a torsional pendulum. Values for the natural frequency of the torsional system were obtained in vacuo at about 15 C intervals from room to 500 C and back again to room temperature. The shear modulus was then computed from these frequencies. Figure 4 shows the variation of shear modulus with temperature. The shear-modulus values indicate that a rapid decrease begins to occur at about 400 C. The rate of decrease was so great at 450 to 500 C that a small change in temperature resulted in large changes in shear modulus. The room-temperature shear modulus was also increased by the exposure of the specimen to high temperatures. Also, the shear-modulus values seem to indicate that decomposition of the gamma phase may begin to occur at about 200 C. Considerably more investigation would be needed before such a change could be definitely established. Since the shear modulus is related to the elastic modulus, a comparison of the two moduli is valuable.

Figure 5 was prepared to illustrate the effect of temperature and time on the modulus of elasticity of the reference alloy as obtained in bend testing unirradiated Specimen HRC-3. The similarity of Figures 4 and 5 is apparent, especially the rapid decrease at the higher temperature and the increase in the room-temperature modulus after exposure of the specimen to high temperature. Variation of the elastic modulus with time at 500 C was also measured. The observed increase in the modulus of elasticity with time at 500 C, and the resultant increase in the room-temperature modulus after exposure to elevated temperatures, is probably due to transformation from the gamma to the alpha and delta phases stable below about 575 C. Similar

TABLE 4. VARIATION OF MODULUS OF ELASTICITY WITH
TEST TEMPERATURE FOR TWO URANIUM-
10 w/o MOLYBDENUM BEND-TEST SPECIMENS

Test Temperature ^(a) , C	Modulus of Elasticity, 10 ⁶ psi	
	Specimen HRC-3	Specimen EC-3
25	11.8	13.3
210	--	13.4
245	12.1	--
325	--	11.1
380	8.7	--
430	--	9.6
470	6.5	--
475	--	8.9
500	6.0	6.3
500 (after 1 hr)	7.7	--
500 (after 2 hr)	8.7	--
25	14.5	--

(a) The tests are listed in the order of performance.

FIGURE 4. SHEAR MODULUS OF UNIRRADIATED URANIUM-10 w/o MOLYBDENUM AS A FUNCTION OF TEMPERATURE

FIGURE 5. ELASTIC MODULUS OF UNIRRADIATED URANIUM-10 w/o MOLYBDENUM AS A FUNCTION OF TEMPERATURE

100X

HC-1081

a. Specimen 10MoU-17 at 0.36 Total a/o Burnup

100X

HC-1083

b. Specimen 10MoU-22 at 0.50 Total a/o Burnup

FIGURE 6. MICROSTRUCTURES OF IRRADIATED SPECIMENS
AFTER BEND TESTING

Chromic-acetic electroetch, bright-field
illumination.

c. Specimen 10MoU-13 at 0.91 Total a/o Burnup

d. Specimen 10MoU-22 in the As-Polished Condition, Showing Corrosion at the Specimen Surface

FIGURE 6. (CONTINUED)

changes would probably have occurred in the shear modulus under the same conditions of time at temperature.

To further assist in the evaluation of the previous data, the elastic and dynamic moduli were also obtained at room temperature on both the 10-in. -long torsion-pendulum specimen and the remaining 8-in. -long section cut from the previously mentioned 18-in. -long rod. The elastic modulus was measured by tension tests while the dynamic modulus was measured by a sonic-vibration method. A summary of the data obtained at room temperature follows:

Specimen	Specimen Condition	Shear Modulus (25 C), G, 10^6 psi	Elastic Modulus (25 C), E, 10^6 psi	Dynamic Modulus (25 C), E _d , 10^6 psi	Poisson's Ratio ν , (25 C)
10-in. section	As fabricated and heat treated	4.55	12.05	13.08	0.324
10-in. section	After testing at temperature	4.91	13.02	13.94	0.326
HRC-B	As fabricated and heat treated	--	11.8 (by bend test)	--	--
HRC-B	After bend tests at temperature	--	14.5 (by bend test)	--	--

Poisson's ratio was calculated as a matter of general interest from the equation

$$\nu = \frac{E}{2G} - 1$$

This ratio could not be computed from bend-test data since the shear modulus cannot be evaluated from such tests.

As was mentioned earlier, the electrical resistivity of the unirradiated specimens of the reference alloy was measured before and after bend testing. This was done to determine if transformation from the gamma to the alpha-plus-delta phases had occurred during bend testing. The resistivity measurements, which were considered to be reliable indicators of phase changes, indicated that no measurable amounts of transformation had occurred in the specimens during the 30 to 60-min duration of the bend tests. This is in direct contrast to the evidences of transformation obtained from the elastic-modulus measurements mentioned above and to metallographic evidence. Apparently, electrical-resistivity measurements are relatively insensitive to small amounts of transformation. Microstructures of Specimens 10MoU-13, -17, and -22, after irradiation and bend testing, are shown in Figure 6. Specimen 10MoU-13, which had received the highest burnup, 0.91 a/o, exhibits a small amount of transformation product in the untransformed gamma matrix. The other two specimens also exhibit some degree of transformation at the grain boundary as evidenced by the dark-etching characteristics at these points. The appearance of the surface of Specimen 10MoU-22, also shown in Figure 6, indicates that some oxidation had occurred at the surface of the specimen during the heating cycle of the bend test. The amount of surface oxidation present did not appear to affect the mechanical properties of unirradiated control specimens, since the bend-test data compared well with the data obtained from well protected tensile-test specimens.

Irradiated Specimens

Mechanical-property data obtained by bend tests of both clad and unclad irradiated specimens of the reference alloy are listed in Table 5. Some of the variables which would affect the data, such as burnup, irradiation temperature, fabrication, and heat treatment are also listed. The large number of variables involved in the history of the specimens tends to obscure the effects of any particular variable on the mechanical properties. Thus, it becomes necessary to speak in terms of trends and probabilities when evaluating the data, rather than the more desirable quantitative correlations. The use of specimens with so many different variables was made necessary by the small total number of irradiated specimens available for the tests.

Since it has been decided by APDA and Battelle personnel that the most useful information would be the mechanical properties at 500 C, the stress-strain curves shown in Figures 7 through 17 were calculated from data obtained at that temperature. There is one exception. Specimen HT-1 (Figure 15) was unintentionally fractured during a bend test at room temperature, 25 C.

The data indicate that the mechanical properties of the irradiated reference alloys were affected significantly by two major variables, burnup and irradiation temperature.

Effect of Burnup

The variation of the elastic modulus with test temperature at different specimen burnups is shown in Figure 18. The values at temperatures of 25 C and 250 C were obtained from tests which did not subject the specimens to as large a value of stress as would be desirable for accurate determinations of the modulus. The stress was purposely kept low to avoid the possibility of specimen failure before data were obtained at all three temperatures. It appears from Figure 18 that a rapid reduction in the modulus is taking place in the vicinity of 500 C. This effect correlates with the elastic-modulus data obtained from unirradiated specimens. This rapid rate of change contributes to the scatter in the data obtained at this temperature.

The two uranium-9 w/o molybdenum and uranium-11 w/o molybdenum alloys were bend tested in order to obtain information on both the effects of compositional changes on the mechanical properties of the alloy and to provide a greater amount of data for analysis. The elastic modulus of both alloys did not show as great a rate change with temperature as did the uranium-10 w/o molybdenum alloy. An explanation for this behavior cannot be advanced because of the large number of variables involved in the histories of all the specimens.

The decrease in elastic modulus with temperature for any one particular irradiated specimen, as shown in Figure 18, is typical of most unirradiated metals and implies that the reference alloy would not normally be of great structural value much beyond 500 C. However, this would probably not apply during irradiation of the reference alloy, since it is known that radiation tends to maintain the alloy in the gamma phase even at 500 C where the rate of transformation is a maximum.

TABLE 5. SUMMARY OF BEND-TEST DATA ON IRRADIATED URANIUM-MOLYBDENUM ALLOYS(a)

Specimen	Capsule	Specimen History		Burnup, total a/o	Irradiation Temperature, C	Test Temperature(d), C	Elastic Modulus, 10 ⁶ psi	0.2 Per Cent		Ultimate Strength, 10 ³ psi	Strain at Fracture, per cent
		Fabrication(b)	Heat- Treatment(c)					Offset Yield Strength, 10 ³ psi			
10MoU-1	302-5	1	A	0.06	200	500	8.4	78.0		78.0	0.93
10MoU-17	9-1	1	B	0.36	500	500	7.6	--		--	--
						250	9.9	--		--	--
						25	10.2	--		--	--
						500	5.9	30.0		30.0	0.57
10CT8-4	9-14	2 (8.1-mil Zr cladding)	C	0.40	130	500	5.4	--		--	--
						25	12.2(e)	--		--	--
						500	5.4	14.6		17.3	0.97
10MoU-22	9-2	1	B	0.50	380	500	6.5	25.3		25.3	0.39
10MoU-13	302-19	1	B	0.91	550	500	7.8	18.7		18.7	0.24
10CT4-28	9-14	2 (4.7-mil Zr cladding)	C	1.0	200	25	10.0(f)	--		--	--
						250	9.6(f)	--		--	--
						500	3.2	17.0		17.1	0.76
11MoCE-4	9-11	2 (4.7-mil Zr cladding)	C	1.2	400	25	9.7(f)	--		--	--
						263	7.9	--		--	--
						25	9.5(f)	--		--	--
						500	4.7	19.9		20.1	0.76
9MoCe-9	9-11	2 (4.7-mil Zr cladding)	C	1.2	400	25	11.1	--		--	--
						250	10.2	--		--	--
						25	10.1	--		--	--
						500	7.9	29.4		29.4	0.57
HT-1	9-7	2 (unclad)	D	1.3	470	25	11.7	49.4		49.4	0.42
HT-8	9-8	2 (unclad)	E	2.0	600	25	11.7	--		--	--
						250	11.3	--		--	--
						25	14.7	--		--	--
						500	4.0	5.7		5.7	0.26

TABLE 5. (Continued)

Specimen	Capsule	Specimen History		Burnup, total a/o	Irradiation Temperature, C	Test Temperature(d), C	Elastic Modulus, 10 ⁶ psi	0.2 Per Cent Offset Yield Strength, 10 ³ psi	Ultimate Strength, 10 ³ psi	Strain at Fracture, per cent
		Fabrication(b)	Heat- Treatment(c)							
10CT4-25	9-9	2 (4.7-mil Zr cladding)	D	2.1	340	25	6.7	--	--	--
						250	6.6	--	--	--
						500	3.0	11.7	15.7	--(g)

(a) All specimens are uranium-10 w/o molybdenum except 11MoCE-4 and 9MoCE-9 which contain 11 and 9 w/o molybdenum, respectively.

(b) Type of fabrication:

1. Hot rolled to 1/4-inch diameter at 900 to 980 C, air cooled, and cold swaged to a 127-mil diameter. Irradiation specimens were then machined to 100 mils in diameter.

2. Cast billets canned in zirconium and coextruded at 790 C to a 222-mil diameter, swaged to 158 mils in diameter, and machined to size.

(c) Preirradiation heat treatments:

A. 24 hr at 900 C and 2 weeks at 500 C, furnace cooled.

B. 24 hr at 900 C, water quenched.

C. 1 hr at 800 C, water quenched.

D. 1 hr at 800 C, water quenched; 1/4 hr at 350 C, air cooled.

E. 24 hr at 900 C, water quenched; 1/4 hr at 350 C, air cooled.

(d) The test temperatures are listed in the order of performance.

(e) Average of three tests at this temperature.

(f) Average of two tests at this temperature.

(g) This value could not be computed. Refer to the stress-strain diagram, Figure 16.

FIGURE 7. STRESS-STRAIN CURVE FOR SPECIMEN 10MoU-1

FIGURE 8. STRESS-STRAIN CURVE FOR SPECIMEN 10MoU-17

FIGURE 9. STRESS-STRAIN CURVE FOR SPECIMEN 10CT8-4

FIGURE 10. STRESS-STRAIN CURVE FOR SPECIMEN 10M0U-22

FIGURE 11. STRESS-STRAIN CURVE FOR SPECIMEN 10MoU-13

FIGURE 12. STRESS-STRAIN CURVE FOR SPECIMEN 10CT4-28

FIGURE 13. STRESS-STRAIN CURVE FOR SPECIMEN 11MoCE-4

FIGURE 14. STRESS-STRAIN CURVE FOR SPECIMEN 9MoCE-9

FIGURE 15. STRESS-STRAIN CURVE FOR SPECIMEN HT-1

FIGURE 16. STRESS-STRAIN CURVE FOR SPECIMEN HT-8

FIGURE 17. STRESS-STRAIN CURVE FOR SPECIMEN 10CT4-25

FIGURE 18. EFFECT OF TEMPERATURE ON THE ELASTIC MODULUS OF BOTH CLAD AND UNCLAD URANIUM-10 w/o MOLYBDENUM AT VARIOUS BURNUPS

In general, an increase in burnup had the effect of reducing significantly the elastic modulus. The amount of the reduction appeared to depend to some extent upon the temperature at which the specimens were irradiated. The temperature effect is discussed in another section.

Figures 19 and 20 illustrate the effect of burnup on the ultimate strength and strain at fracture of the irradiated specimens at 500 C. The data indicate that a severe reduction in ultimate strength and strain at fracture occurs with increasing burnup. It is interesting to note that, initially, the ultimate strength increases with burnup and then decreases rapidly with additional irradiation. The rate of decrease is rapid up to a burnup of about 0.5 total a/o in the case of ultimate strengths and about 1.0 total a/o in the case of strain at fracture. Both thereafter decrease almost linearly to a burnup of 2.0 total a/o. Such a severe reduction in ultimate strength is not surprising, since similar reductions have been reported by other investigators⁽²⁾ studying fissionable materials. It is significant in the reference alloy that the rapid rate of decrease in ultimate strength and strain at fracture at the low burnups does not continue into the higher range of burnups.

Data for ultimate strength and strain at fracture obtained from clad specimens are also shown in Figures 19 and 20. At the lower burnups the ultimate strength for clad specimens was found to be lower than that of unclad specimens. However, the one clad specimen tested at a burnup of 2.0 total a/o exhibited an ultimate strength almost three times that of an unclad specimen with a burnup of 2.0 total a/o. The clad specimens also exhibited a greater ductility as evidenced by the data for strain at fracture. The differences between the clad and unclad specimens may be attributed to either normal scatter in the results or, as explained earlier, the particular sensitivity of bend tests to surface effects. It seems reasonable that the cladding should have a pronounced beneficial effect on both the ultimate strength and strain at fracture of the composite specimen at high burnups.

The ultimate strength and strain at fracture of the reference alloy at 25 C and a burnup of 1.3 total a/o is shown in Figures 19 and 20. These data indicate that the ultimate strength and strain at fracture are significantly higher at this temperature than at 500 C.

The data indicate in general that, when the reference alloy is irradiated to burnups of any significance, the ductility of the material is lost, and, therefore, stress concentrations, thermal or otherwise, in the alloy cannot be relieved to any extent by plastic deformation. The presence of the zirconium cladding apparently had a tendency to improve the mechanical properties of the composite specimens at burnups of 2.0 total a/o or more. It should be remembered that these conclusions were derived from tests on specimens after irradiation and that they may not necessarily apply to those same specimens during irradiation.

Effect of Irradiation Temperature

Attempts were made to evaluate the effects of irradiation temperature upon the elastic modulus and ultimate strength of the specimens. The results are shown in Figures 21 and 22. The lines on the figures represent approximately constant burnup levels of 0.4, 1.0, and 2.0 total a/o.

FIGURE 19. EFFECT OF BURNUP ON THE ULTIMATE STRENGTH OF URANIUM-10 w/o MOLYBDENUM AT 500 C

FIGURE 20. EFFECT OF BURNUP ON THE STRAIN AT FRACTURE OF URANIUM-10 w/o MOLYBDENUM AT 500 C

FIGURE 21. EFFECT OF IRRADIATION TEMPERATURE ON THE ELASTIC MODULUS OF URANIUM-10 w/o MOLYBDENUM ALLOY AT 500 C

FIGURE 22. EFFECT OF IRRADIATION TEMPERATURE ON THE ULTIMATE STRENGTH OF URANIUM-10 w/o MOLYBDENUM AT 500 C

The scarcity of specimens was a very critical factor in this evaluation. Ideally, all the specimens should have had the same irradiation and fabrication history. However, the specimens could be placed in three groups, considering only their burnups. Three specimens of the reference alloy had burnups near a 0.4 total a/o, two more had burnups near 1.0 total a/o, and two others had burnups near 2.0 total a/o. In addition, data obtained from one specimen of uranium-11 w/o molybdenum, and one of uranium-9 w/o molybdenum, both with a 1.2 total a/o burnup, were included.

The data shown in Figure 21 seem to indicate that specimens irradiated at the higher temperature exhibited higher elastic-modulus values. An explanation for the apparent improvement in the modulus value with increasing irradiation temperature is that more of the radiation damage was annealed out at the higher temperatures. A corresponding improvement would also be expected in the ultimate strength and strain at fracture.

The effect of irradiation temperature on the ultimate strength of the reference alloy is shown in Figure 22. Only two specimens, one clad and one unclad, were tested at the 2.0 total a/o burnup level. Since previous data (Figures 19 and 20) had indicated that the cladding apparently improved the ultimate strength and strain at fracture of specimens at this high burnup, the data were not included in Figure 22. The data indicate that, at a given burnup level, specimens irradiated at these temperatures exhibited higher ultimate strength.

CONCLUSIONS

The conclusions and possible trends that were derived from bend tests of specimens of the reference alloy are listed below:

- (1) The instability of the gamma phase during mechanical testing at temperatures in the vicinity of 500 C affects the physical properties of the reference alloy both in the unirradiated and irradiated condition.
- (2) No significant differences in the mechanical properties of irradiated specimens fabricated by hot-rolling or coextrusion techniques were observed.
- (3) The ductility of the reference alloy decreased with increasing amounts of irradiation.
- (4) The ultimate strength increased slightly with small amounts of irradiation, and then decreased with further irradiation.
- (5) The modulus of elasticity tended to decrease with increasing burnup.
- (6) Apparently, at any specified burnup up to 1.0 total a/o, the ultimate strength and the modulus of elasticity of the reference alloy was improved by increasing the irradiation temperature.

- (7) Zirconium cladding appeared to improve the ductility and strength of the specimens at high burnups.

It should be noted that the above conclusions have been based upon the results obtained from a relatively small number of specimens. The large number of variables also introduced difficulties in the evaluation of the data. In particular, transformation of the metastable gamma phase at the higher test temperatures is known to have occurred during testing. Thus, at the 500 C test temperature, a portion of the changes in physical properties of the alloy would be due to transformation. In addition, caution should be used in applying these data to the reference alloy while it is actually being irradiated.

REFERENCES

- (1) Gates, J. E., et al., "The Radiation Stability of Uranium-10 w/o Molybdenum as a Function of Heat Treatment and Burnup", BMI-APDA-625 (February, 1957).
- (2) Hueschen, R. E., Kemper, R. S., and Kelly, S. K., Nuclear Engineering and Science Congress Paper 57-NESC-13 (1957).
- (3) Nadai, A., Theory of Flow and Fracture of Solids, McGraw-Hill Book Company, New York (1950).

JEG:EGB:JCB:AAB:GDC/mmk:dpc:pa:dc:mk:dt

APPENDIX

OBTAINING THE STRESS-STRAIN CURVE
USING BEND-TEST DATA

LIST OF SYMBOLS

b = width of a rectangular beam

t = height of a rectangular beam

A = cross-sectional area

p = radius of curvature of the neutral axis of a beam

R = radius of a beam of circular cross section

P = applied load

M = applied moment

h = distance from the neutral axis

ϵ = unit strain

ϵ_c = unit strain on the outermost compressive fiber

ϵ_t = unit strain on the outermost tensile fiber

σ = stress

σ_c = stress on the outermost compressive fiber

σ_t = stress on the outermost tensile fiber

APPENDIX

OBTAINING THE STRESS-STRAIN CURVE USING BEND-TEST DATA

It is convenient in many instances to be able to determine the uniaxial stress-strain curve from the more easily obtained moment-deflection curve. The purpose of this Appendix is to illustrate one method of doing this in cases where the elementary relationship $\sigma = \frac{MC}{I}$ is no longer valid.

Rectangular Beams

In a beam of narrow rectangular cross section, the width of the beam offers little restraint to sidewise deformation and the corresponding stresses in this direction will be small compared with those in the longitudinal direction. Nadai⁽³⁾ has suggested that, in view of the above, each successive fiber in the beam will behave as a miniature tensile or compressive specimen and that, therefore, the stress distribution on a section for which the strains are known can be determined from a knowledge of the stress-strain properties of the material. The assumption, however, neglects the interaction effects of one fiber on another.

The converse problem, that of finding the stress-strain properties from a bend test, is solvable in the same way.

Since experiments have shown that, for beams subjected to pure bending, cross sections which were originally plane in the unbent beam will remain plane after bending, the strains on a cross section are linear and are given by:

$$\epsilon = \frac{h}{\rho} \quad (A-1)$$

The stresses on a cross section will take on values at each fiber which correspond to the stress at the same strain in an uniaxial test. A severely bent beam will, thus, have a stress distribution which has the shape of a complete stress-strain diagram up to the maximum fiber strains, as shown in Figure A-1. The effect of additional bending would be simply to extend, in both directions, the amount of the stress-strain curve enclosed by the maximum fiber strains, ϵ_t and ϵ_c .

The equilibrium considerations imposed on the section shown in Figure A-1 are such that the sum of the forces in the plane of the cross section must be zero and the internal bending moment must be equal to the externally applied bending moment. In equation form

$$\int \sigma \, dA = 0, \quad (A-2)$$

and

$$\int \sigma h \, dA = M. \quad (A-3)$$

(3) References at end of text of report.

FIGURE A-1. STRESS DISTRIBUTION IN A NARROW BEAM

B A T T E L L E M E M O R I A L I N S T I T U T E

For a slight increase in bending moment, increases will occur in ϵ_t , ϵ_c , σ_t , and σ_c in such a way as to maintain the equilibrium conditions given by Equations (A-2) and (A-3). In other words, if an increase occurs on the tensile side of Figure A-1 — represented by the shaded area $\sigma_t d\epsilon_t$ — a like increase of amount $\sigma_c d\epsilon_c$ must occur simultaneously on the compressive side. Therefore,

$$\frac{\sigma_t}{\sigma_c} = \frac{d\epsilon_c}{d\epsilon_t} \quad (A-4)$$

By adding ϵ_t and ϵ_c , and by observing that $h_1 + h_2 = t$, we obtain from Equation (A-1)

$$p = \frac{t}{\epsilon_t + \epsilon_c} \quad (A-5)$$

If in the second equilibrium equation, Equation (A-3), we make the following substitutions:

$$h = p\epsilon \quad [\text{from Equation (A-1)}],$$

$$dh = p d\epsilon$$

$$dA = bdh = b p d\epsilon$$

$$\sigma = f(\epsilon),$$

we have

$$\frac{M}{p^2} = b \int_{-\epsilon_c}^{\epsilon_t} f(\epsilon) \epsilon d\epsilon \quad (A-6)$$

The variation of Equation (A-6) with p is obtained by taking the partial

$$\frac{\partial}{\partial p} \left(\frac{M}{p^2} \right) = \frac{\partial}{\partial p} \left[b \int_{-\epsilon_c}^{\epsilon_t} f(\epsilon) \epsilon d\epsilon \right] \quad (A-7)$$

The right side of Equation (A-7) becomes

$$b \left[\int_{-\epsilon_c}^{\epsilon_t} \frac{\partial f(\epsilon)}{\partial p} \epsilon d\epsilon + f(\epsilon_t) \epsilon_t \frac{d\epsilon_t}{dp} + f(\epsilon_c) \epsilon_c \frac{d\epsilon_c}{dp} \right]$$

By substituting

$$f(\epsilon_t) = \sigma_t,$$

$$f(\epsilon_c) = \sigma_c,$$

$$\frac{d\epsilon_t}{dp} = \frac{\sigma_c}{\sigma_t} \frac{d\epsilon_c}{dp} \quad [\text{from Equation (A-4)}],$$

A-4

$$\frac{d\epsilon_c}{dp} + \frac{d\epsilon_t}{dp} = -\frac{t}{p^2} \text{ [from Equation (A-5)] ,}$$

and by noting that

$$\int_{-\epsilon_c}^{\epsilon_t} \frac{df(\epsilon)}{dp} \epsilon d\epsilon = 0 ,$$

Equation (A-7) becomes

$$\frac{\sigma_t \sigma_c}{\sigma_t + \sigma_c} = \frac{1}{bt^2} \left[2M - p \frac{\partial M}{\partial p} \right] . \quad (A-8)$$

Replacing derivatives by differences, and remembering from Equation (A-5) that

$$\frac{dp}{d(\epsilon_t + \epsilon_c)} = \frac{-t}{(\epsilon_t + \epsilon_c)^2} ,$$

we rewrite Equation (A-8) as

$$\frac{\sigma_t \sigma_c}{\sigma_t + \sigma_c} = \frac{1}{bt^2} \left[2M + \frac{(\epsilon_t + \epsilon_c) \Delta M}{\Delta \epsilon_t + \Delta \epsilon_c} \right] . \quad (A-9)$$

Equations (A-4) and (A-9) are sufficient to yield values of stress (σ_t, σ_c) for measured values of strain (ϵ_t, ϵ_c) and applied moment. If it can be assumed that the strains on the outer tensile fibers are equal to those on the outer compressive fibers, as would be the case for materials having nearly the same properties in tension as in compression, Equation (A-9) can be written as:

$$\sigma = \frac{2}{bt^2} \left[2M + \frac{\epsilon}{\Delta \epsilon} \Delta M \right] , \quad (A-10)$$

in which σ and ϵ are now the outer fiber stress and strain, respectively.

The values of ϵ can be conveniently obtained from deflection data. For example, consider Figure A-2, which illustrates a beam subjected to pure bending.

The section of the beam between the inner reactions is subjected to a constant bending moment of magnitude $\frac{Pa}{2}$, and therefore, has a uniform curvature in this section. The radius of curvature can be found from the sagitta of the arc included by ℓ , thus,

$$p = \frac{\ell^2}{8s} + \frac{s}{2} \approx \frac{\ell^2}{8s}, \quad (\text{A-11})$$

and the strain corresponding to p is given by,

$$\epsilon = \frac{4st}{\ell^2}. \quad (\text{A-12})$$

The stresses could then be found by the use of a table such as Table A-1. Note that if the assumption that $\epsilon_t = \epsilon_c$ is not made, Table A-1 would be slightly more involved and that measurements of both ϵ_t and ϵ_c would need to be made.

Round Beams

When the cross section of the beam is not rectangular, the equilibrium equations shown by Equations (A-2) and (A-3) become more complicated and a simple solution for the stresses is not possible. The expression for the incremental area, dA becomes in the case of a round beam

$$dA = 2p\sqrt{R^2 - p^2\epsilon^2} d\epsilon. \quad (\text{A-13})$$

Again, it is assumed that the properties in tension and compression are nearly equal. If the bending moment on a cylindrical rod is given by $M(R/p)$, Equation (A-3) becomes

$$\frac{M(R/p)}{4p^2} = \int_0^{R/p} f(\epsilon) \epsilon \sqrt{R^2 - p^2\epsilon^2} d\epsilon. \quad (\text{A-14})$$

Letting

$$g = \frac{p\epsilon}{R},$$

and

$$X = \frac{R}{p},$$

Equation (A-14) becomes

$$\frac{M(X)}{4R^3} = \int_0^1 f(Xg) g \sqrt{1 - g^2} dg. \quad (\text{A-15})$$

Assuming that $M(X)$ is a known empirical function of X , the function f must be obtained by a solution of Equation (A-15).

TABLE A-1. COMPUTATION OF STRESS-STRAIN CURVE FROM BEND-TEST LOAD-DEFLECTION DATA

Measured Load, P	Moment, M	Moment Differences, ΔM	Measured Deflection, s	Strain(a), ϵ	Strain Differences, $\Delta \epsilon$	Stress(b), σ
P ₀	M ₀	--	s ₀	ϵ_0	--	σ_0
P ₁	M ₁	M ₁ - M ₀	s ₁	ϵ_1	$\epsilon_1 - \epsilon_0$	σ_1
P ₂	M ₂	M ₂ - M ₁	s ₂	ϵ_2	$\epsilon_2 - \epsilon_1$	σ_2
P ₃	M ₃	M ₃ - M ₂	s ₃	ϵ_3	$\epsilon_3 - \epsilon_2$	σ_3
P ₄	M ₄	M ₄ - M ₃	s ₄	ϵ_4	$\epsilon_4 - \epsilon_3$	σ_4
...	...	...	...	...	...	...

(a) The values of strain are obtained from Equation (A-12),

$$\epsilon = \frac{4st}{l^2}$$

(b) The values of stress are obtained from Equation (A-10),

$$\sigma = \frac{2}{bt^2} \left[2M + \frac{\epsilon}{\Delta \epsilon} \Delta M \right]$$

By some curve-fitting procedure, such as least squares, $M(X)$ can be represented as closely as desired in the form of a polynomial:

$$M(X) = \sum_{n=1,3,5 \dots}^m b_n X^n \quad (A-16)$$

From this, a polynomial representation of $f(\epsilon)$ is sought in the form:

$$f(\epsilon) = \sum_{n=1,3,5 \dots}^m A_n \epsilon^n, \quad (A-17)$$

where the coefficients A_n need to be determined. Substituting these two representations, Equations (A-16) and (A-17), into Equation (A-15) shows:

$$\frac{1}{4R^3} \sum_{n=1,3,5 \dots}^m b_n X^n = \sum_{n=1,3,5 \dots}^m A_n X^n \int_0^1 g^{n+1} \sqrt{1-g^2} dg.$$

This equality can be assured by taking

$$A_n = \frac{b_n}{4R^3 \int_0^1 g^{n+1} \sqrt{1-g^2} dg} \quad (A-18)$$

Equation (A-18) can be simplified by using the following integration:

$$\begin{aligned} \int_0^1 g^{n+1} \sqrt{1-g^2} dg &= 1/2 \int_0^1 t^{n/2} \sqrt{1-t} dt, \\ &= 1/2 B(n/2 + 1, 3/2), \\ &= \frac{\Gamma(n/2 + 1) \Gamma(3/2)}{2 \Gamma(n/2 + 5/2)}, \end{aligned}$$

where B and Γ are the beta function and gamma function, respectively. Thus,

$$A_n = \frac{b_n \Gamma(n/2 + 5/2)}{2R^3 \Gamma(n/2 + 1) \Gamma(3/2)}, \text{ for } n = 1, 3, 5 \dots m. \quad (A-19)$$

Since $f(\epsilon)$ is an odd function, the constants A_n and b_n are used with only odd subscripts.

Putting $n = 2k + 1$, with $k = 0, 1, 2, 3, 4, \dots$ Equation (A-19) can be simplified by noting that,

$$\begin{aligned} \frac{\Gamma(n/2 + 5/2)}{\Gamma(n/2 + 1) \Gamma(3/2)} &= \frac{\Gamma(k + 3)}{\Gamma(k + 3/2) \Gamma(3/2)} \\ &= \frac{2^{k+1} (k + 1)!}{(2k + 2)! \pi} \end{aligned}$$

Therefore,

$$A_n = b_n \frac{2^{k+3} (k + 1)! (k + 2)!}{2R^3 (2k + 2)! \pi} \quad (\text{A-20})$$

Whenever the coefficients, b_n , and the radius R are known, the coefficients, A_n , of the stress-strain curve can be computed readily.

The solution just made can also be found in closed form by employing the Mellin transforms. Returning to Equation (A-15), two new functions, $u(X)$ and $v(X)$ are defined as follows:

$$u(X) = \begin{cases} X\sqrt{1-X^2} & \text{if } 0 \leq X \leq 1 \\ 0 & \text{if } 1 < X \end{cases}, \quad (\text{A-21})$$

and

$$v(X) = \begin{cases} \sqrt{1-X^2} & \text{if } 0 \leq X \leq 1 \\ 0 & \text{if } 1 < X \end{cases}. \quad (\text{A-22})$$

Equation (A-15) can now be written as:

$$\int_0^\infty f(Xg) u(g) dg = \frac{M(X)}{4R^3}. \quad (\text{A-23})$$

Taking the Mellin transform of the above equations:

$$\bar{f}(s) \bar{u}(1-s) = \frac{\bar{M}(s)}{4R^3}, \quad (\text{A-24})$$

or

$$\bar{f}(s) = \frac{\bar{M}(s)}{4R^3 \bar{u}(1-s)}.$$

Noting that

$$\begin{aligned} \bar{u}(s) &= \int_0^1 X^{s-1} X\sqrt{1-X^2} dX = 1/2 \int_0^1 t^{\frac{s+1}{2}-1} (1-t)^{3/2-1} dt \\ &= 1/2 B\left(\frac{s+1}{2}, \frac{3}{2}\right), \end{aligned}$$

$$\bar{v}(s) = \int_0^1 X^{s-1} \sqrt{1-X^2} dX = 1/2 B\left(\frac{s}{2}, \frac{3}{2}\right),$$

$$\frac{1}{\bar{u}(s)} = \frac{2}{B\left(\frac{s+1}{2}, 3/2\right)} = \frac{2(s+2)(s+1)s}{\pi} \bar{v}(s),$$

Equation (A-24) becomes

$$\bar{f}(s) = \frac{1}{2R^{3\pi}} \left[(3-s)(2-s)(1-s) \bar{M}(s) \right] \bar{v}(1-s). \quad (A-25)$$

Note also that:

$$\text{inverse of } (3-s)(2-s)(1-s) \bar{M}(s) = \text{inverse of } [6 + 11(-s) + 6(-s)^2 + (-s)^3] \bar{M}(s)$$

$$= \left[6 + 11 \left(X \frac{d}{dX} \right) + 6 \left(X \frac{d}{dX} \right)^2 + \left(X \frac{d}{dX} \right)^3 \right] M(X) = \frac{d^3}{dX^3} (X^3 M).$$

Therefore, Equation (A-25) can be written as

$$f(\epsilon) = \frac{1}{2R^{3\pi}} \int_0^1 N(\epsilon g) \sqrt{1-g^2} dg, \quad (A-26)$$

where

$$N(X) = \frac{d^3}{dX^3} [X^3 M(X)].$$

Equation (A-26) can be verified by assuming a power series for $M(X)$ and showing that the above equation leads to the same power series expansion for $f(\epsilon)$ as was found by the first method of solution.