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Abstract

Table 1 lists for a homogeneous reactor, and for five moderators, the maximum thermal-neutron fluxes obtainable for the case that maximum power density is limited to 100 w/cm^3 , and also for the case that total reactor power is 100 Mw.

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HOMOGENEOUS HIGH-FLUX REACTOR

In a homogeneous* thermal reactor the power density p is given by

$$p = \frac{\Sigma_f \phi}{3.1 \times 10^{10}} \text{ watt/cm}^3 \dots \quad (1)$$

where Σ_f is the macroscopic fission cross section, in cm^{-1} , and ϕ is the thermal-neutron flux, in $\text{n/cm}^2 \text{ sec}$. For a given power density, the thermal-neutron flux can be increased only by reduction of the macroscopic fission cross section, that is by diluting the fissionable material. If there were no absorption other than the absorption in the fissionable material, and if the reactor were infinite in extent, the macroscopic fission cross section could indeed be reduced without limit.

However, even in an infinite reactor, parasitic absorption of neutrons in the moderator, structures, impurities, etc., sets a limit below which the macroscopic fission cross section Σ_f cannot go. For Σ_f smaller than this limiting value $\Sigma_{f\ell}$, the reactor would not be critical.

If Σ_m is the macroscopic cross section of this parasitic absorption, then $\Sigma_{f\ell}$ is given by

$$k = \frac{\nu \Sigma_{f\ell}}{(1+\alpha) \Sigma_{f\ell} + \Sigma_m} = 1 \dots \quad (2)$$

Here ν is the number of neutrons per fission, and α the ratio of capture cross section to fission cross section. From Eq. (2), $\Sigma_{f\ell}$ can be computed and this limiting value can then be substituted into Eq. (1). If p is then

*The word "homogeneous" here refers to a reactor which can be described by one region in which the cross sections are constant. These cross sections may be space averages of varying cross sections. For instance a block of BeO, penetrated by equal and equally space fuel elements, would be regarded as homogeneous as this connection if the distance between the elements is small.

interpreted as the maximum permissible power density, the maximum obtainable flux is given by

$$\phi_{\max} = 3.1 \times 10^{10} (\gamma - 1 - \alpha) \frac{p}{\Sigma_m} \dots \quad (3)$$

With the "U.S. values" for γ and $\alpha^{(1)}$ of U^{235} ,

$$\phi_{\max} = 4 \times 10^{10} \frac{p}{\Sigma_m} \dots \quad (3a)$$

The theoretical lower limit for Σ_m would be obtained if the absorption by the moderator were the only absorption. For this theoretical case, Table 1, column 2, lists for the five most common moderators the maximum thermal flux obtainable with a power density of 100 w/cm^3 . The values of Σ_m were computed from Table 1, Ref. 2, on the basis of $\Sigma_m = \kappa^2 D$.

It may be seen that fluxes of the order of 10^{16} n/cm^2 are obtainable with 100 w/cm^3 only in D_2O and C. Beryllium and BeO would give these fluxes only at several hundred watts/cm³. H_2O could reach these fluxes only at impractical, high power densities..

If the reactor is finite, Eq. (2) has to be replaced by

$$\frac{\gamma \Sigma_f}{(1+\alpha) \Sigma_f + \Sigma_m} \frac{e^{-B^2 \tau}}{1 + L^2 B^2} = 1 \dots \quad (2a)$$

where B is the buckling, τ the age, and L the diffusion length. Since

$e^{-B^2 \tau} / (1 + L^2 B^2)$ is smaller than one, the expression $\gamma \Sigma_f / [(1+\alpha) \Sigma_f + \Sigma_m]$ and

1. D. J. Hughes and R. B. Schwartz, Neutron Cross Sections, BNL 325, Supplement No. 1, p. vii, Brookhaven National Laboratory, Upton, New York (Jan. 1957).
2. W. K. Ergen, Flux Distribution in a Reactor Consisting of a Spherical Shell of Fuel in an Infinite Moderator, ORNL CF-57-12-100 (Dec. 24, 1957).

hence Σ_f has to be increased as compared to the value given by Eq. (2). In other words, the neutron losses by leakage necessitate an improved competitive position of the fissionable material relative to the parasitic absorbers. If Σ_f is increased, and more fissionable material is used, the flux obtainable for a given power density decreases below the values given by Table 1, column 2.

We now consider the case in which the total power P is the limiting factor. For a spherical-bare reactor of radius R cm

$$P = \frac{4}{\pi} R^3 \frac{\Sigma_f \phi}{3.1 \times 10^{10}} \text{ watts ...} \quad (4)$$

and, in order to make the reactor critical, Σ_f has again to be given by Eq. (2a). Into Eq. (2a) we substitute $B = \pi/R$ and

$$L^2 = \frac{D}{(1+\alpha) \Sigma_f + \Sigma_m},$$

where the term in the denominator is the total absorption cross section, and D is the diffusion constant which is assumed to be equal to the diffusion constant of the moderator. This is a very good approximation since the amount of fissionable material is small.

Thus Eq. (2a) is transformed into

$$\frac{\nu \Sigma_f}{(1+\alpha) \Sigma_f + \Sigma_m} \frac{e^{-\pi^2 \tau / R^2}}{1 + \frac{\pi^2}{R^2} \frac{D}{(1+\alpha) \Sigma_f + \Sigma_m}} = 1. \quad (5)$$

Setting

$$\pi \sqrt{\tau} / R = x \quad \dots \quad (6)$$

$$\tau \Sigma_m / D = K^2 \quad \dots \quad (7)$$

Σ_f is calculated from Eq. (5).

$$\Sigma_f = \Sigma_m \frac{1 + x^2/K^2}{\nu e^{-x^2} - (1+\alpha)}$$

If this is substituted into Eq. (4) we obtain for the flux-to-power ratio

$$\phi/P = 7.852 \times 10^8 \frac{1}{\Sigma_m \tau^{3/2}} x^3 \frac{\nu e^{-x^2} - (1+\alpha)}{1 + x^2/K^2} \dots \quad (8)$$

This ratio has a maximum for a value of x , which depends on K , and hence on the moderator. For $\nu/(1+\alpha) = 2.06$ (the "U.S. value" for U-235⁽¹⁾) the x values are listed in Table 1, together with the corresponding value of the radius and the flux obtainable with 100 Mw.

TABLE 1. THERMAL-NEUTRON FLUXES OBTAINABLE
WITH VARIOUS MODERATORS

Moderator	Power-Density Limited	Total-Power Limited		
	Maximum Flux in n/cm ² sec for 100 w/cm ³	x	R (cm)	Maximum Flux in n/cm ² sec for 100 Mw
H ₂ O	0.201 x 10 ¹⁵	0.63	27.9	2.3 x 10 ¹⁵
D ₂ O	60.4 x 10 ¹⁵	0.46	76.4	2.8 x 10 ¹⁵
Be	3.56 x 10 ¹⁵	0.56	55.3	3.3 x 10 ¹⁵
BeO	5.36 x 10 ¹⁵	0.54	59.6	3.5 x 10 ¹⁵
C	10.2 x 10 ¹⁵	0.53	90.5	1.7 x 10 ¹⁵

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