Spontaneously Broken Abelian Gauge Invariant Supersymmetric Model*

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Abstract

We present a model that is invariant under an Abelian gauge transformation and a modified supersymmetry transformation. This model is broken spontaneously and the interplay between symmetry breaking, Goldstone particles and mass breaking is studied. In the present model, spontaneously breaking the Abelian symmetry of the vacuum restores the invariance of the vacuum under a modified supersymmetry transformation.

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We construct a Lagrangian that is invariant under an Abelian gauge transformation and a supersymmetry transformation. Supersymmetric Lagrangians were originally developed by Wess and Zumino¹ and are characterized by having a conserved spinor current. The supersymmetry requires each multiplet to have a unique mass, so it is essential to spontaneously break the symmetry to obtain a mass spectrum.

Our model is similar to that of Fayet and Iliopoulos² in which spontaneously broken supersymmetry and ordinary gauge symmetry are investigated. The notation is that of Ref.1 and the techniques for constructing the model are based on superfields originally developed by Salam and Strathdee.^{3,4}

The Lagrangian density we consider is constructed from a vector superfield $V(x, \theta, \bar{\theta})$ composed of the ordinary fields $V_\mu(x)$, $\lambda(x)$, $\chi(x)$, $D(x)$, $C(x)$, $M(x)$, $N(x)$, and a left-handed scalar superfield $S(x, \theta)$ composed of fields $\psi(x)$, $A(x)$, $B(x)$, $F(x)$, $G(x)$. $V_\mu(x)$ is a vector field, fields denoted by a Greek letter are Majorana four component spinors and fields denoted by Roman letters are either scalars or pseudoscalars. Under a generalized gauge transformation the superfields transform according to^{5,6}

$$e^{2gV} \rightarrow e^{-2ig\Lambda^\dagger} e^{2gV} e^{2ig\Lambda},$$

$$S \rightarrow e^{-2ig\Lambda} S.$$
(1)

Here $\Lambda(x, \theta)$ is a left-handed superfield composed of the fields $\psi(x)$, $Q(x)$, $\theta(x)$, $\bar{\theta}(x)$, and $\mathcal{Y}(x)$. In the superfield formalism, the superfield $\Lambda(x, \theta)$

plays a role analogous to the gauge parameter $\Lambda(x)$ in ordinary gauge invariant theories. In terms of superfields a Lagrangian density which provides an action invariant under supersymmetry transformations¹ and the generalized gauge transformation (1) is given by

$$\mathcal{L} = (S^* e^{2gV} S)_{D\text{component}} - \frac{1}{2} (V, V)_{F\text{component}} \quad (2)$$

The scalar multiplet (V, V) is constructed from V in Ref.1. As emphasized in Refs.3 and 4, under a supersymmetry transformation the D component of a vector multiplet and the F component of a scalar multiplet transform as four divergences with the result that the action $\int d^4x \mathcal{L}$ is invariant.

The infinitesimal supersymmetry transformation is tabulated in Ref.1 and with the aid of Ref.4, the infinitesimal form of the generalized gauge transformation (1) is found to be

$$\begin{aligned} \delta V_\mu &= \partial_\mu \alpha, \quad \delta \lambda = 0, \quad \delta D = 0, \quad \delta \chi = \Psi, \\ \delta C &= B, \quad \delta M = \tilde{F}, \quad \delta N = \tilde{G}, \\ \delta \Psi &= -g \{ (B + \gamma_5 \alpha) \Psi + \gamma_5 \tilde{F} A + \tilde{\Psi} B \}, \\ \delta A &= -g \{ B A + \alpha B \}, \quad \delta B = -g \{ B B - \alpha A \}, \\ \delta F &= g \{ \tilde{G} A - B F - \tilde{F} B + \alpha G + i \tilde{\Psi} \gamma_5 \Psi \}, \\ \delta G &= -g \{ \tilde{F} A + B G + \tilde{A} B + \alpha F - i \tilde{\Psi} \Psi \}. \end{aligned} \quad (3)$$

The Lagrangian (2) is non-polynomial but as shown in Ref.5 it is possible to go to a special gauge in which

$$C(X) = \chi(X) = M(X) = N(X) = 0, \quad (4)$$

and only V_μ , λ and D survive in the vector multiplet. Because a special gauge is chosen, the action is no longer invariant under the supersymmetry transformation in Ref.1 or under the generalized gauge transformation (3). The action, however, is invariant under the modified supersymmetry transformation⁵

$$\delta_S + \delta_G, \quad (5)$$

which is constructed so as to preserve the gauge condition (4). In (5), δ_S is the supersymmetry transformation with C , X , M , and N set equal to zero. The transformation δ_S violates the gauge condition (4). The transformation δ_G is the generalized gauge transformation (3) with the gauge parameters chosen to be

$$\Psi = -\lambda\alpha, \quad \tilde{F} = -i\bar{\alpha}\lambda, \quad \tilde{G} = -i\bar{\alpha}\gamma_5\lambda, \quad A=B=0, \quad (6)$$

so that the gauge condition (4) is restored. The parameter α in (6) is an infinitesimal constant spinor which anticommutes with itself, λ , and ψ , and commutes with all boson fields.

In the special gauge (4), we obtain the Lagrangian

$$\begin{aligned} \mathcal{L} = & -\frac{1}{2} (\partial_\mu A)^2 - \frac{1}{2} (\partial_\mu B)^2 - \frac{i}{2} \bar{\Psi} \gamma^\mu \partial_\mu \Psi \\ & + \frac{1}{2} F^2 + \frac{1}{2} G^2 + \frac{g}{2} (A^2 + B^2) D - i g \bar{\lambda} (B - \gamma_5 A) \Psi \\ & - \frac{1}{4} V_{\mu\nu}^2 - \frac{i}{2} \bar{\lambda} \not{D} \lambda + \frac{1}{2} D^2, \end{aligned} \quad (7)$$

where

$$\begin{aligned} D_\mu A &= \partial_\mu A + g V_\mu B, & D_\mu B &= \partial_\mu B - g V_\mu A, \\ D_\mu \Psi &= \partial_\mu \Psi + g V_\mu \gamma_5 \Psi, & V_{\mu\nu} &= \partial_\mu V_\nu - \partial_\nu V_\mu. \end{aligned} \quad (8)$$

The Lagrangian (7) is invariant under an Abelian gauge transformation but the fields A and B are not eigenstates of the charge associated with the gauge transformation. We define fields $\phi^\pm$ with definite charge by the relation

$$\phi^\pm = (A \pm iB)/\sqrt{2}, \quad (9)$$

and rewrite (7) in terms of $\phi^\pm$ as

$$\begin{aligned} \mathcal{L} &= -\frac{1}{2} |D_\mu \phi^+|^2 - \frac{1}{2} |D_\mu \phi^-|^2 - \frac{i}{2} \bar{\Psi} \gamma^\mu D_\mu \Psi + \frac{1}{2} F^2 + \frac{1}{2} G^2 \\ &+ g \phi^+ \phi^- D - \frac{g}{\sqrt{2}} \bar{\lambda} [\phi^+ (1 - i\gamma_5) - \phi^- (1 + i\gamma_5)] \Psi \\ &- \frac{1}{4} V_{\mu\nu}^2 - \frac{i}{2} \bar{\lambda} \not{V} \lambda + \frac{1}{2} D^2, \end{aligned} \quad (10)$$

where

$$\begin{aligned} D_\mu \phi^\pm &= \partial_\mu \phi^\pm \mp ig V_\mu \phi^\pm, \\ D_\mu \Psi &= \partial_\mu \Psi + g V_\mu \gamma_5 \Psi. \end{aligned} \quad (11)$$

Lagrangian (10) is manifestly invariant under the Abelian gauge transformation

$$\begin{aligned}
V_\mu(x) &\rightarrow V_\mu(x) - \partial_\mu \Lambda(x), & \lambda(x) &\rightarrow \lambda(x), \\
\psi(x) &\rightarrow e^{ig\gamma_5 \Lambda(x)} \psi(x), & \phi^\pm(x) &\rightarrow e^{\mp i g \Lambda(x)} \phi^\pm(x).
\end{aligned}
\tag{12}$$

We observe from Lagrangians (7) and (10) that it is not possible to simultaneously have a parity conserving Lagrangian and fields which are eigenstates of both parity and the charge associated with the Abelian gauge transformation. In other words, when A and B in (7) have opposite parity, the Lagrangian (7) preserves parity but A and B do not have a definite charge, whereas in (10) if A and B have the same parity, the $\phi^\pm$ fields have a definite charge and parity but the Lagrangian is parity-violating.

The Lagrangian is invariant under the Abelian gauge transformation (12) and the modified supersymmetry transformation (5), and as a result has both a conserved current

$$J_\mu = A \partial_\mu B - B \partial_\mu A - \frac{i}{2} \bar{\psi} \gamma_\mu \gamma_5 \psi - g (A^2 + B^2) V_\mu, \tag{13}$$

and a conserved spinor current j_μ

$$j_\mu \alpha = 2i \left[\frac{\partial \mathcal{L}}{\partial \partial_\mu \psi} (\delta_S + \delta_G) \psi + \frac{\partial \mathcal{L}}{\partial \partial_\mu \lambda} (\delta_S + \delta_G) \lambda \right]. \tag{14}$$

By combining Eqs.(3), (4), (5), (7), (14) and results from Ref.1, we obtain

$$j_\mu = \bar{\psi} \gamma_\mu (\not{\partial} - g \not{V} \gamma_5) (A + \gamma_5 B) + \bar{\lambda} \gamma_\mu (-\frac{1}{2} V_{\rho\sigma} \gamma^\rho \gamma^\sigma + D \gamma_5). \tag{15}$$

The potential in Lagrangian (7) is at a minimum when all the fields have zero vacuum expectation value, so it is not possible to spontaneously break either of the two symmetries. But when the Lagrangian is supplemented by terms invariant under the transformations (5) and (12), the potential is no longer at a minimum when the fields all have zero vacuum expectation value and the Lagrangian offers, because of its simplicity, a model in which to study the interplay between symmetry breaking, Goldstone bosons, Goldstone fermions, and mass breaking. We now proceed to break the symmetry of the vacuum by adding to the Lagrangian given by (7) the term

$$\mathcal{L}_B = -\xi g D, \quad (16)$$

where ξ is a constant. The $\mathcal{L}_B$ is invariant under the Abelian gauge transformation (12) and the action obtained from $\mathcal{L}_B$ is invariant under the modified supersymmetry transformation (5).

The Lagrange equations of motion for the non-dynamical fields F , G , and D are

$$D = -\frac{g}{2} (A^2 + B^2) + \xi g, \quad F = G = 0, \quad (17)$$

and may be used to eliminate the fields from the Lagrangian. The equation for D in (17) verifies that the vacuum is no longer invariant. Making the substitution (17), the Lagrangian becomes

$$\begin{aligned} \mathcal{L} + \mathcal{L}_B = & -\frac{1}{2} (\partial_\mu A)^2 - \frac{1}{2} (\partial_\mu B)^2 - \frac{i}{2} \bar{\Psi} \gamma^\mu \partial_\mu \Psi \\ & - \frac{1}{4} V_{\mu\nu}^2 - \frac{i}{2} \bar{\lambda} \not{\partial} \lambda - ig \bar{\lambda} (B - \gamma_5 A) \Psi \\ & - \frac{g^2}{8} (A^2 + B^2)^2 - \frac{1}{2} (-\xi g^2) (A^2 + B^2). \end{aligned} \quad (18)$$

Particles A and B have each acquired a mass $(-\xi g^2)^{\frac{1}{2}}$ which is real and positive provided $\xi < 0$. The supersymmetry has been broken by the degeneracy of the vacuum and all the particles in the scalar multiplet no longer have the same mass. The action is invariant under (5) and (12), so (18) still has a conserved spinor current j_μ and a conserved current J_μ . The Abelian symmetry remains unbroken and no Goldstone bosons appear. Under (5), λ transforms as²

$$\lambda \rightarrow \xi g \gamma_5 \alpha + \text{terms involving fields}, \quad (19)$$

which verifies that λ is a Goldstone spinor.⁷

Since the Lagrangian $\mathcal{L} + \mathcal{L}_B$ is invariant under rotations in the A, B plane we can, without loss of generality, spontaneously break the Abelian symmetry by making the substitution

$$A \rightarrow A + a, \quad (20)$$

where the new A field has zero vacuum expectation value and

$$a^2 = 2\xi \quad (21)$$

We are now forced to take ξ to be positive. The following broken Lagrangian is obtained after substituting (20) into (18) and diagonalizing the fermion mass terms:

$$\begin{aligned} \mathcal{L} + \mathcal{L}_B = & -\frac{1}{2} [\partial_\mu A + g V_\mu B]^2 - \frac{1}{2} m^2 A^2 - \frac{1}{2} [\partial_\mu B - V_\mu (gA + m)]^2 \\ & - \frac{i}{2} (\bar{\chi}_1 \not{\partial} \chi_1 + \bar{\chi}_2 \not{\partial} \chi_2) - \frac{i}{2} m (\bar{\chi}_1 \chi_1 + \bar{\chi}_2 \chi_2) \quad (22) \\ & - \frac{1}{4} V_\mu^2 - \frac{i}{4} (\bar{\chi}_1 \gamma_\mu \gamma_5 \chi_1 + \bar{\chi}_2 \gamma_\mu \gamma_5 \chi_2 + 2 \bar{\chi}_1 \gamma_\mu \chi_2) V^\mu \\ & - \frac{ig}{2} [A (\bar{\chi}_1 \chi_1 + \bar{\chi}_2 \chi_2) + B (\bar{\chi}_1 \gamma_5 \chi_1 + \bar{\chi}_2 \gamma_5 \chi_2)] \\ & - \frac{g}{8} [g (A^2 + B^2)^2 + 4m (A^2 + B^2) A], \end{aligned}$$

where

$$\chi_1 = (\Psi - \gamma_5 \lambda) / \sqrt{2} \quad \chi_2 = \gamma_5 (\Psi + \gamma_5 \lambda) / \sqrt{2} \quad (23)$$

and $m^2 = 2g^2\xi = a^2g^2$. The broken Lagrangian (22) conserves parity provided A is a scalar, B is a pseudoscalar, and the fermions χ_1 and χ_2 have opposite parity. This opens up the possibility of constructing more realistic parity conserving, spontaneously broken supersymmetric gauge invariant models.^{8,9} Note that with the exception of the Goldstone boson B, all the particles have acquired a mass m.

The simple model studied is invariant under a modified supersymmetry transformation and an ordinary gauge transformation. It has the following novel feature: In $\mathcal{L} + \mathcal{L}_B$, a Goldstone spinor λ appears because the vacuum is not invariant under the modified supersymmetry transformation $\delta_S + \delta_G$. When $\mathcal{L} + \mathcal{L}_B$ is spontaneously broken according to (20), the symmetry of the vacuum under the Abelian transformation is destroyed and a Goldstone boson appears. The transformation (20), however, restores the symmetry of the vacuum under $\delta_S + \delta_G$ and therefore the Goldstone spinor disappears.¹⁰ Among supersymmetric gauge invariant models involving interacting vector particles, this model possibly has the fewest fields and is therefore particularly amenable to generalizations to realistic $SU(N) \times U(1)$ models.

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References and Footnotes

1. J. Wess and B. Zumino, Nucl. Phys. B70, 39 (1974). The infinitesimal supersymmetry transformations are given by

$$\begin{aligned}\delta V_\mu &= i\bar{\alpha}\gamma_\mu\lambda + i\bar{\alpha}\partial_\mu\chi, & \delta D &= i\bar{\alpha}\gamma_5\gamma_\mu\partial_\mu\lambda, & \delta C &= i\bar{\alpha}\gamma_5\chi \\ \delta\lambda &= -\frac{i}{2}V_{\mu\nu}\gamma^\mu\gamma^\nu\alpha + D\gamma_5\alpha, & \delta N &= i\bar{\alpha}\gamma_5\lambda + i\bar{\alpha}\gamma_5\gamma^\mu\partial_\mu\chi \\ \delta\chi &= \gamma^\mu V_\mu\alpha - \partial^\mu C\gamma_5\gamma_\mu\alpha + (M+\gamma_5 N)\alpha, & \delta M &= i\bar{\alpha}\lambda + i\bar{\alpha}\gamma^\mu\partial_\mu\chi \\ \delta\Psi &= \partial_\mu(A-\gamma_5 B)\gamma^\mu\alpha + (F+\gamma_5 G)\alpha, & \delta A &= i\bar{\alpha}\Psi, & \delta B &= i\bar{\alpha}\gamma_5\Psi \\ \delta F &= i\bar{\alpha}\gamma^\mu\partial_\mu\Psi, & \delta G &= i\bar{\alpha}\gamma_5\gamma^\mu\partial_\mu\Psi\end{aligned}$$

2. P. Fayet and J. Iliopoulos, Phys. Lett. 51B, 461 (1974).
 3. A. Salam and J. Strathdee, Nucl. Phys. B76, 477 (1974).
 4. S. Ferrara and B. Zumino, Phys. Lett. 51B, 239 (1974).
 5. J. Wess and B. Zumino, Nucl. Phys. B78, 1 (1974).
 6. S. Ferrara and B. Zumino, Nucl. Phys. B79, 413 (1974).
 7. A. Salam and J. Strathdee, Phys. Lett. 49B, 465 (1974).
 8. In Ref.2, the broken Lagrangian is not parity conserving. We have recently learned that parity conserving, spontaneously broken supersymmetric models involving spin-0 and spin- $\frac{1}{2}$ particles have been constructed by L. O'Raifeartaigh, "On the Question of Global Stability for the Spontaneous Breakdown of Chiral Scalar Supersymmetry," Dublin Institute for Advanced Studies progress report.
 9. Various $SU(N) \times U(1)$ models are being studied.
 10. The reason for this curious feature is because

$$\begin{aligned}(\delta_S + \delta_G)\lambda &= D\gamma_5\alpha + \dots = \left[-\frac{1}{2}g(A^2+B^2) + \xi g\right]\gamma_5\alpha + \dots \\ &= \left(-\frac{1}{2}ga^2 + \xi g\right)\gamma_5\alpha + \text{terms involving fields} \\ A &\rightarrow A+a \\ &= 0 + \text{terms involving fields}\end{aligned}$$