SNPO-C

Richardson

M/c

Records

AGC--7730-0006R

REPORT NO. 7730-0006R

ANALYSIS OF THE ENGINE RESONANCE SURVEY
IN THE TRANSVERSE DIRECTION AT 3°-45' (SERIES I)
OF THE VIBRATION SURVEY OF XECF IN ETS-1

MAY 1968

SECRET

NUCLEAR ROCKET OPERATIONS

DISTRIBUTION

11-729

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

REPORT NO. 7730:0006R

ANALYSIS OF THE ENGINE RESONANCE SURVEY IN THE
TRANSVERSE DIRECTION AT 3°-45' (SERIES I)
OF THE VIBRATION SURVEY OF XECF IN ETS-1

MAY 1968

PREPARED BY:

J. E. JELLISON
D. TOW
R. C. ROSSER

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Atomic Energy Commission nor any of their employees, nor any of their contractors, subcontractors, or their employees makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

Approved:

I. L. Odgers

I. L. Odgers, Manager
Engineering Department
Nuclear Rocket Operations

Approved:

G. S. Kaveney

G. S. Kaveney, Manager
Structures Section
Engineering Department
Nuclear Rocket Operations

CLASSIFICATION CATEGORY	
<i>Unclassified</i>	
<i>G. S. Kaveney</i>	<i>3/13/68</i>
CLASSIFYING OFFICER	DATE

TABLE OF CONTENTS

ABSTRACT

I. INTRODUCTION

II. TESTING - SERIES I

- A. MEASUREMENT REQUIREMENTS
- B. RESONANCE SURVEY - TEST 1
- C. MODAL DWELLS - TEST 2
- D. DAMPING - TEST 3
- E. TEST OPERATIONS LOG

III. RESULTS

- A. NORMALIZED MODAL RESPONSES
- B. PRIMARY MODES
- C. MODAL STRESSES
- D. DAMPING

IV. DYNAMIC ANALYSIS

- A. METHOD
- B. MODEL
- C. EQUATIONS OF MOTION
- D. NATURAL FREQUENCIES
- E. MODE SHAPES

V. CONCLUSIONS

LIST OF FIGURES

FIGURE

- 1 Accelerometer Mounting Block Locations
- 2 Response of Control Accelerometer 29X in g's vs. Frequency in Hz
- 3 Measured Modal Response @ $f_1 = 13$ Hz
- 4 Measured Modal Response @ $f_2 = 16$ Hz
- 5 Measured Modal Response @ $f_3 = 57$ Hz
- 6 XE Engine Radial Deflection, $f = 13$ Hz
- 7 XE Engine Radial Deflection, $f = 57$ Hz
- 8 Modified Goodman Diagram for A1 6061-T6
- 9 Illustration of Critical Damping Determination
- 10 23 Block Measured Structural Damping Ratio
- 11 21 Block Measured Structural Damping Ratio
- 12 20 Block Measured Structural Damping Ratio
- 13 24 Block Measured Structural Damping Ratio
- 14 26 Block Measured Structural Damping Ratio
- 15 27 Block Measured Structural Damping Ratio
- 16 29 Block Measured Structural Damping Ratio
- 17 XE Engine
- 18 Typical Model Section
- 19 Forces Acting on a Typical Beam
- 20 Free Body of i'th Element
- 21 XE Engine Mass Points Location
- 22 Mathematical Model of XE Engine
- 23 XE Engine Mode Shape @ $f_1 = 1.6$ Hz
- 24 XE Engine Mode Shape @ $f_2 = 17.2$ Hz
- 25 XE Engine Mode Shape @ $f_3 = 47.2$ Hz
- 26 XE Engine Mode Shape @ $f_4 = 103.8$ Hz

ABSTRACT

This is a complete analysis of the engine transverse resonance survey test, test results, supporting dynamic analyses, and conclusions. This test was Part I (Series I) of the vibration survey of XECF conducted in three parts at ETS-1 in December 1967 and January 1968 by AGC and WANL.

I. INTRODUCTION

The vibration survey of XECF in ETS-1 was a three part test. Series I was an engine survey in the transverse direction at 3° - 45'. Series II was an engine survey in the longitudinal direction at 3° - 45' and Series III was a line survey of the Turbine Inlet Line at 168° - 30'. A report of the testing and analysis of the entire test is being prepared. This is the final analysis of the primary engine palpitations in Series I, engine transverse test. Detailed reactor motions, resonancies, amplifications, damping, etc. are being analyzed separately at WANL and will be combined with this series of reports for inclusion in one final joint test analysis report. A report of this series is being issued separately to expedite the dissemination of information and will constitute a part of the final XECF vibration test analysis.

The XECF vibration test was based on NRO Report No. RN-TD-0010 (revised), "Test Description for the Vibration Survey of XECF in ETS-1", Revised July 1967. Testing was performed under NTO-I-0345 (Final Issue October 17, 1967), Control Room Operating Procedure for Vibration Testing of XECF at ETS-1, including changes 1 through 5 as approved by the TRB.

The objectives of the test were:

1. To determine the principal resonant frequencies of the engine and or major engine component assemblies over a frequency range of 5 to 1500 Hz.
2. To record the shapes of the first five response modes associated with the five lowest principal resonances discovered during the survey.
3. To determine the amplification factors at these resonances.
4. To determine the mechanical impedance at these resonances, and over the engine survey frequency range, by solving;

$$Z = \frac{F}{\ddot{X}}$$

where:

Z	=	mechanical impedance
F	=	excitation force
$\ddot{X}$	=	function of $\dot{X}$
$\ddot{X}$	=	response of reference accelerometer

5. To determine the damping characteristics of the engine and its major component assemblies.

II. TESTING - SERIES I

The engine was excited by applying the driving force in the transverse direction at the pressure vessel-nozzle interface which is approximately station 267.0.

A. MEASUREMENT REQUIREMENTS

Instrumentation was judiciously positioned so that the response of all major engine components was measured. The measurement requirements list is included. There were 51 accelerometer locations. Twelve of these, in addition to the force and frequency measurements, were recorded on magnetic tape and are referred to as the control accelerometers. The responses for all 51 accelerometers as well as the frequency were recorded through a digital data acquisition system. Although actual testing time was short due to tight scheduling all possible data was acquired with this test setup. Complete photographic coverage was obtained. The MRL is presented in the following table and Figure 1 shows the various mounting block locations.

TABLE 1

MRL-TRANSVERSE EXCITATION AT 3°-45'

<u>Block</u>	<u>Description</u>	<u>Sta.</u>	<u>Rad.</u>	<u>Theta</u>	<u>Radial</u>	<u>Tan- gential</u>	<u>Longi- tudinal</u>
1	Load Cell Support Beam	-150.0	78.0	3°-45'	X		(X)
2	Load Cell Support Beam	-150.0	78.0	183°-45'	X		
3	Load Cell Support Beam	-150.0	78.0	273°-45'			
4	Test Stand Adapter	1.0	78.0	3°-45'	(X)	X	X
5	Test Stand Adapter	1.0	78.0	183°-45'			
6	Test Stand Adapter	1.0	78.0	318°-45'	X		
7	TSA-UTS Flg	42.0	51.0	3°-45'	(X)		X
8	TSA-UTS Flg	42.0	51.0	183°-45'			
9	TSA-UTS Flg	42.0	51.0	318°-45'	X		
10	TPA	89.0	6.0	3°-45'	X	X	X
11	Engine External Shield	146.0	51.0	3°-45'	(X)	X	X
12	Engine External Shield	146.0	51.0	183°-45'	X		
13	Engine External Shield	146.0	51.0	318°-45'	X		
14	Turbine Inlet Line	183.0	44.0	168°-30'	X	X	X
15	Pressure Vessel Flg	188.0	27.0	3°-45'	(X)	X	X
16	Pressure Vessel Flg	188.0	27.0	183°-45'	(X)		
17	Pressure Vessel Flg	188.0	27.0	318°-45'	X		
18	Turbine Exhaust Line	193.0	38.0	273°-45'	X	X	X
19	Pump Discharge Line	198.0	38.0	120°-	X	X	X
20	Pressure Vessel	227.0	27.0	3°-45'	(X)	X	X
21	Pressure Vessel	227.0	27.0	183°-45'	(X)		
22	Pressure Vessel	227.0	27.0	318°-45'	X		
23	PV-Noz Flg	267.0	27.0	3°-45'	(X)	X	(X)
24	PV-Noz Flg	267.0	27.0	183°-45'	(X)		
25	PV-Noz Flg	267.0	27.0	318°-45'	X		
26	Nozzle Manifold	315.0	16.0	3°-45'	(X)	X	X
27	Nozzle Manifold	315.0	16.0	183°-45'	(X)		X
28	Nozzle Manifold	315.0	16.0	318°-45'	X		
29	Reactor Core	258.0	Centerline		(X)	(X)	(X)

NOTE: X = Detail Survey (51 Total)

(X) = Resonance Survey (16 Total)

ACCELEROMETER MOUNTING BLOCK LOCATIONS

XECF RESONANT VIBRATION SURVEY
AT ETS-1 NRDS

FIGURE 1

B. RESONANCE SURVEY - TEST 1

The resonance survey was conducted over the prescribed range of 5 - 1500 Hz. The input driving force was held constant while the frequency was varied for each run. Response peaking of the control accelerometers, which were the only ones recorded during these runs, indicated system resonances. In some instances, the particular run would be terminated when it appeared that the test response limit of 1 g would be exceeded. The test was then continued using a lower force. Seven different runs were made using forces of 1 to 60 lbs. The response of the control accelerometer, 29 X located on the engine core, is presented for run 3 from 5 to 500 Hz with an input force of 15 lbs. and for run 1 from 500 to 1500 Hz with an input force of 5 lbs. in Figure 2.

C. MODAL DWELLS - TEST 2

Dwell testing was conducted at each significant resonant frequency indicated by the sweep test data. A force level sufficient to excite a measurable response without exceeding the 1 g acceleration limit on any monitored accelerometer was selected for each dwell. Thirty-nine dwells were performed in the 5 to 1500 Hz range. The last five were repeats of earlier dwells so that a check on earlier data was made. A minimum data level of .040 g was used in data reduction to account for the measurement system accuracy and noise level. Therefore, blank spaces in the data tables indicate readings below .040 g. All 51 accelerometer channels were recorded for each run. Five of these accelerometer channels never reached the .040 g minimum reading and consequently are blank for the complete frequency range. These channels are:

<u>Channel #</u>	<u>Accelerometer #</u>	<u>Location</u>
2	4R	TSA Sta. 1.0
22	4T	TSA Sta. 1.0
23	4L	TSA Sta. 1.0
24	6R	TSA Sta. 1.0
34	13R	Engine External Shield Sta. 146.0

D. DAMPING - TEST 3

Damping tests were performed at the twelve most predominant resonances. The input force was adjusted to establish a significant steady state vibration level. The power to the electrodynamic shaker was then shut off instantaneously thus leaving the structure in a state of free vibration. The vibration decay curve was recorded by the control accelerometers.

E. TEST OPERATIONS LOG

An abbreviated, edited, summary of the test operations log is included. Each individual run of the three tests in this series is documented. Major highlights and accomplishments are recorded.

FIGURE 2

RESPONSE OF CONTROL ACCELEROMETER 29X IN g's
VS.
FREQUENCY IN HZ

-7-
ENGINE TRANSVERSE VIBRATION TEST DWELL DATA
SERIES I TEST II

CHANNEL #	f (Hz) F (lbs.) RUN #	CALIB. 1-19	CALIB. 20-39	8 44 19	13 39 4	16 50 15	16 60 35	24 37 2	57 83 34	129 60 1
1	23 R	1.500	1.454		.112g	.061				.293
2	4 R	1.010	1.047							
3	7 R	1.068	1.040							
4	11 R	1.083	1.037							
5	15 R	1.062	1.020							.051
6	16 R	1.063	1.016							.052
7	20 R	1.058	1.015							.151
8	21 R	1.062	1.019		.063	.066	.078		.074	.144
10	23 L	1.041	1.012		.073	.042				
11	24 R	1.043	1.027		.080	.055	.077			.242
12	26 R	1.064	1.022		.096	.074	.060		.085	.296
13	27 R	1.070	1.026		.158	.053	.060		.122	.349
14	29 X	.633	.655							.145
15	29 Y	.701	.671							
16	29 Z	.743	.708							
17	1 L	1.023	.983	.071	.058	.065		.061		.05
18	2 R	.908	.863							

CHANNEL #	f (Hz) F (lbs) RUN #	CALIB.	CALIB.	8 44 19	13 39 4	16 50 15	16 60 35	24 37 2	57 83 34	129 60 1
19	1R	1.042	.992	.044						
20	28R	.964	.919		.098	.059	.047		.061	.151
21	27L	1.053	.999					.087		.403
22	4T	.935	.893							
23	4L	1.024	.975							
24	6R	1.021	.972							
26	7L	.955	.921	.106	.102	.108	.106	.086	.110	.103
27	9R	.922	.943				.043			
28	10R	.986	.936							
29	10T	1.018	.972							
30	10L	1.101	1.047							
31	11T	.804	.767		.048					
32	11L	.927	.880	.048						
33	12R	1.021	.969		.046					
34	13R	.756	.715							
35	14R	.931	.884		.231	.117	.169	.080	.059	.13
36	14T	.948	.902		.199	.059	.133			.041

CHANNEL#	f (Hz) F (lbs) RUN #	CALIB. 1-19	CALIB. 20-39	8 44 19	13 39 4	16 50 15	16 60 35	24 37 2	57 83 34	129 60 1
37	14L	.920	.875		.099	.065	.107	.040		
38	15T	1.043	.993							
39	15L	.939	.893		.045			.064		
40	17R	1.083	1.030							
41	18R	1.005	.951		.055	.101	.135		.086	
42	18T	1.053	1.001		.074	.197	.146	.194	.068	.054
43	18L	.980	.928		.056	.095	.041			
44	19R	1.007	.955						.095	.131
45	19T	1.043	.988		.108	.116			.156	.393
46	19L	.967	.913							.090
47	20T	1.095	1.038							
48	20L	1.112	1.051		.044	.063				.050
49	22R	1.024	.973							.087
50	23T	1.119	1.062							.056
52	26T	1.087	1.026		.047					.083
53	26L	1.088	1.017							.381
54	25R	.943	.870		.068	.045				.125

[illegible]

CHANNEL #	f (Hz)	183	228	237	248	258	274	308	323	333
	F (lbs.)	45	55	55	59	50	39	50	50	50
	RUN #	7	16	20	21	22	3	23	24	25
19	1R									
20	28R		.070	.315	.207	.071	.098	.062		
21	27L		.333	.508	.253	.225		.185		.213
22	4T									
23	4L									
24	6R									
26	7L	.084	.112	.118	.101	.102	.102	.089	.093	.117
27	9R			.041						
28	10R		.056		.054	.047				.046
29	10T			.058	.143					.066
30	10L				.123	.084				
31	11T						.048			
32	11L									
33	12R			.056	.040	.072	.046			
34	13R									
35	14R		.327	.774	.415	.781	.231	.344	.110	.572
36	14T	.041	.346	1.608	.483	.538	.199	.694	.164	.363

	f (Hz)	183	228	237	248	258	274	308	323	333
	F (lbs.)	45	55	55	59	50	39	50	50	50
CHANNEL #	RUN #	7	16	20	21	22	3	23	24	25
37	14L		.337	.835	.701	1.149	.099	.306	.171	.655
38	15T									
39	15L		.086	.080		.059	.045	.115	.082	.119
40	17R									
41	18R			.109	.058		.055		.153	
42	18T		.062	.137	.124		.074	.186		.057
43	18L					.054	.056		.042	
44	19R		.122	.178	.465	.165			.084	.106
45	19T		.205	.149	.405	.129	.108	.156	.135	
46	19L		.139	.129		.060		.051		
47	20T	.047	.044	.045		.040		.095		.044
48	20L		.105	.079	.055	.070	.044	.056		.210
49	22R	.086	.041	.065	.046	.060		.173	.070	.068
50	23T		.065							
52	26T		.354	.275	.342	.082	.047			
53	26L		.275	.353	.485	.315		.282	.136	.08
54	25R		.113	.155	.092	.156	.068	.082	.047	.102

[illegible]

CHANNEL #	f (Hz) F (lbs.) RUN #	341 42 26	381 45 8	415 45.5 5	452 29 6	452 45 36	454 36.5 12	478 65 27	519 65 28	523 50 11
19	1R									
20	28R				.180	.084	.227	.091	.106	.081
21	27L	.136	.112	.179	1.236	1.071	1.118	.446	.640	.633
22	4T									
23	4L									
24	6R									
26	7L	.116	.086	.099	.109	.099	.127	.104	.106	.101
27	9R									
28	10R						.040			
29	10T				.103	.138	.120			
30	10L				.089	.104	.092			
31	11T									
32	11L				.046	.043				
33	12R									
34	13R									
35	14R	.071	.067	.346	.370	.925	.925	.146	.087	.173
36	14T		.104	.268	.609	.711	.813	.136	.524	.607

CHANNEL #	f (Hz)	341	381	415	452	452	454	478	519	523
	F (lbs.)	42	45	45.5	29	45	36.5	65	65	50
	RUN #	26	8	5	6	36	12	27	28	11
37	14L	.049		.160	.407	.474	.550	.158	.231	.317
38	15T						.094			.043
39	15L	.457	.112	.101	.170	.254	.197	.210	.131	.110
40	17R	.052				.046	.042	.053		.029
41	18R	.078				.077			.120	.104
42	18T				.063	.074		.042	.124	.086
43	18L	.056			.043	.052	.048	.126	.118	.094
44	19R	.084		.077	.528	.527	.496	.086	.196	.103
45	19T	.174		.051	1.892	.521	1.957	.365	.139	.103
46	19L			.069	.331	.075	.316	.108	.043	
47	20T	.088		.125	.168	.157	.151	.148	.257	.202
48	20L	.582	.047	.121	.118	.260	.211	.574	.243	.025
49	22R	.144	.073	.126	.278	.201	.268	.154	.194	.158
50	23T		.068	.567	.095		.209			.109
52	26T	.065	.066		1.057	.700	.719	.553	.108	.099
53	26L	.067	.119	.397	2.517	1.457	1.854	.727	.425	.441
54	25R	.085	.052	.078	.284	.455	.396	.218	.256	.346

CHANNEL #	f (Hz)	633	633	657	658	703	816	856	876
	F (lbs.)	9	15	11.5	7	7	15	16	10
	RUN #	13	37	38	14	10	29	30	18
1	23R	.224	.328	.190	.134	.154	.051	.114	.146
2	4R								
3	7R					.050			
4	11R								
5	15R								
6	16R							.040	
7	20R	.218	.287	.143	.095	.062	.071	.165	
8	21R	.136	.144	.127		.059	.069	.129	2.342
10	23L	.142	.191	.267	.159	.142	.263	.350	.239
11	24R	.092	.172				.043	.058	
12	26R						.073	.054	.060
13	27R		.052			.147	.044		.072
14	29X	.599	.786	.669	.623	.321	.426	.568	.625
15	29Y	.108	.255	.712	.387	.883	.074	.064	.166
16	29Z					.046	.073	.081	.094
17	1L	.057			.073	.068			.055
18	2R								

CHANNEL #	f (Hz)	633	633	657	658	703	816	856	876
	F (lbs)	9	15	11.5	7	7	15	16	10
	RUN #	13	37	38	14	10	29	30	18
19	1R								
20	28R		.067	.042	.047	.096	.063	.049	.079
21	27L	.099	.185	.112		.090	.131	.566	.327
22	4T								
23	4L								
24	6R								
26	7L	.122	.105	.109	.108	.089	.104	.101	.121
27	9R								
28	10R								
29	10T								
30	10L								
31	11T								
32	11L								
33	12R								
34	13R								
35	14R					.108			
36	14T	.801	.569	.542	.914	.054		.073	

	f (Hz)	633	633	657	658	703	816	856	876
	F (lbs.)	9	15	11.5	7	7	15	16	10
CHANNEL #	RUN #	13	37	38	14	10	29	30	18
37	14L	.252	.236	.099	.171	.226		.061	
38	15T	.061							.053
39	15L		.074	.059	.051	.046	.045	.065	
40	17R								
41	18R						.061		
42	18T							.046	
43	18L			.067	.059				
44	19R	.097	.071	.078	.077		.757	.310	.081
45	19T	.048	.041	.107	.054		.764	.236	.072
46	19L	.092	.051	.097	.040		.049		
47	20T					.058			
48	20L	.088	.074	.098	.084	.071	.056	.210	
49	22R		.093	.107	.095			.042	
50	23T	.061				.142			
52	26T			.078	.044	.088	.118	.347	.114
53	26L	.078	.313	.812	.417	.300	.419	1.032	.447
54	25R	.042	.050	.059	.045			.043	

[illegible]

[illegible]

CHANNEL #	f (Hz) F (lbs) RUN #	922 11 31	931 13.5 32	988 55 33	1028 17 17	1238 5 9	1238 4 39			
37	14L		.163	.260	.151					
38	15T				.054	.054				
39	15L	.076	.096	.296		.057				
40	17R									
41	18R			.134		.076	.100			
42	18T			.085		.126	.134			
43	18L			.066						
44	19R	.168	.056	.423	.107		.045			
45	19T	.444	.346	1.111	.229					
46	19L	.174	.128	.424	.064					
47	20T	.125	.161	.112	.140		.072			
48	20L			.328	.380	.053				
49	22R	.117		.426	.232	.071	.079			
50	23T				.061	1.124				
52	26T	.740	1.247	3.626	.335					
53	26L	1.385	1.644	6.489	.509		.052			
54	25R	.056	.045	.090	.053	.099	.058			

TRANSVERSE ENGINE TEST OPERATIONS LOG SUMMARY

12/4/67 Initiated transverse engine vibration test operations at ETS-1. Completed checkout of all equipment. Channel 28 only bad channel. Drive rod, shaker, and shaker support structure positioned in place. Accelerometer mounting blocks, which were previously installed at E-MAD, are found to be too small. Additional blocks are epoxied onto original blocks.

12/5/67 Found, upon arrival, that the shaker support stand had been bumped by a truck the previous night and the drive rod had been destroyed by impact against the XECF engine. Shaker found to be undamaged. Mounted 47 accelerometers (5 remaining). Routed 34 accelerometer cables (18 remaining).

12/6/67 Accelerometer installation and wiring continued. ETS-1 facility closed at noon for rest of day due to scheduled firing of NRX-A6.

12/7/67 Completed accelerometer installation and wiring in morning. Primary test crew scheduled to work 2nd shift in order to prevent further interference with NRX-A6. Nozzle cover plate removed for first time. No material or debris fell out. Sweep testing performed.

SERIES I, TEST 1 (SWEEPS)

<u>Run</u>	<u>Frequency, Hz</u>	<u>Force Input, Lbs.</u>
1	5 - 1240	5
	1200 - 1500	1
	1500 - 1250	5
2	5 - 670	10
	1100 - 700	10
3	5 - 670	15
	1100 - 750	15
4	5 - 590	30
5	5 - 450	50

12/8/67 Reviewed sweep data from previous day. Recalibrated instrumentation. All instrumentation is recalibrated on each test day. Sweep testing is continued.

SERIES I, TEST 1 (SWEEPS)

<u>Run</u>	<u>Frequency, Hz</u>	<u>Force Input, Lbs.</u>
6	80 - 7	60
7	68 - 5	60

Dwell testing performed.

SERIES I, TEST 2 (DWELLS)

<u>Run</u>	<u>Frequency, Hz</u>	<u>Force Input, Lbs.</u>
1	129	60
2	24	37
3	274	39
4	13	39
5	415	45.5
6	452	29
7	183	45
8	381	45
9	1238	5
10	703	7
11	523	50
12	454	36.5
13	632	9
14	658	7
15	16	50
16	228	55
17	1028	17
18	876	10
19	8	44

Phase angle digital measurements may be in doubt because excessive wind (20 - 25 mph) blowing instrumentation cables causes phase meter readings to jump around. Each dwell test took from 10 to 11 min. Was unable to provide sufficient excitation force at lower frequencies. The 150 lb rated shaker system could only provide 50 to 60 lb output force at these very low frequencies.

12/11/67 Major power failure at NRDS causes two hour delay in start of testing. Dwell testing continued.

SERIES I, TEST 2 (DWELLS)

<u>Run</u>	<u>Frequency, Hz</u>	<u>Force Input, Lbs.</u>
20	237	55
21	248	59
22	258	50
23	308	50
24	323	50
25	333	50

SERIES I, TEST 2 (DWELLS) cont'd

<u>Run</u>	<u>Frequency, Hz</u>	<u>Force Input, Lbs.</u>
26	341	42
27	478	65
28	519	65
29	816	15
30	856	16
31	922	11
32	931	13.5
33	988	55
34	57	83

Damping Testing Performed.

SERIES I, TEST 3 (DAMPING)

<u>Run</u>	<u>Frequency, Hz</u>	<u>Force Input, Lbs.</u>
1	16	60
2	452	45
3	632	20
4	658	16
5	1238	4
6	13	75
7	120	75
8	228	70
9	274	70
10	523	80
11	703	85
12	876	20

Additional Dwell Testing Performed. (wind has subsided somewhat).

SERIES I, TEST 2 (DWELLS)

<u>Run</u>	<u>Frequency, Hz</u>	<u>Force Input, Lbs.</u>
35	16	60
36	452	45
37	633	15
38	657	11.5
39	1238	4

DATA CHANNEL - ACCELEROMETER NUMBER IDENTIFICATION

<u>CHANNEL</u>	<u>ACCELEROMETER</u>	<u>CHANNEL</u>	<u>ACCELEROMETER</u>	<u>CHANNEL</u>	<u>ACCELEROMETER</u>
1	23R	19	1R	37	14L
2	4R	20	28R	38	15T
3	7R	21	27L	39	15L
4	11R	22	4T	40	17R
5	15R	23	4L	41	18R
6	16R	24	6R	42	18T
7	20R	26	7L	43	18L
8	21R	27	9R	44	19R
10	23L	28	10R	45	19T
11	24R	29	10T	46	19L
12	26R	30	10L	47	20T
13	27R	31	11T	48	20L
14	29X	32	11L	49	22R
15	29Y	33	12R	50	23T
16	29Z	34	13R	52	26T
17	1L	35	14R	53	26L
18	2R	36	14T	54	25R

MAGNETIC TAPE CHANNEL IDENTIFICATION

<u>CHANNEL</u>	<u>DATA</u>
1	Frequency (sine servo)
2	Force
3	23R
4	21R
5	20R
6	23L
7	24R
8	26R
9	27R
10	29X
11	29Y
12	29Z
13	1L
14	4R

ACCELEROMETER ORIENTATION FOR THE ENGINE
TRANSVERSE VIBRATION SURVEY AT 3° - 45'

All accelerometer blocks, except special noted cases, are as viewed looking toward the centerline of the engine which is in a vertical nozzle down position.

ACCELEROMETER ORIENTATION FOR THE ENGINE
TRANSVERSE VIBRATION SURVEY AT 3° - $45'$

III. RESULTS

The data is analyzed in detail. Supporting, independent, dynamic analyses are included in a subsequent section of this report.

A. NORMALIZED MODAL RESPONSES

The various modal dwells were run with different force inputs. The input force was adjusted at each frequency to produce responses as high as possible without exceeding the 1 g test limits. All of the modal response data is now normalized to reflect an equivalent input force of 100 lbs. in order to facilitate comparison of responses. These normalized responses are presented in the following table. The maximum response in each mode is underlined.

B. PRIMARY MODES

There are three modes which it was possible to identify. These are the radial responses at the indicated resonant frequency of 13 Hz, 16 Hz, and 57 Hz. The lack of sufficient test data points with valve large enough to be recorded within the accuracy of the data acquisition system, especially in the top half of the engine assembly, have made it more difficult to determine and identify the characteristic of the engine deflected curve. The primary cause of this difficulty was due to the fact that a maximum test response valve of 1 g was imposed on any control accelerometer and therefore, any particular frequency run would be terminated whenever the test response limit of 1 g on any accelerometer would be exceeded. The driving force for all frequency runs, which was adjusted to the highest possible level without exceeding the 1 g limit, was varied from 1 to 60 lbs. These values were considered to be too low to have excited the top half of the engine assembly, which is quite remote from the exciting source. Therefore, blank spaces in the data tables do not mean that there were no data recorded in the accelerometers, but rather indicate that the response readings were below a certain prescribed test data reduction limit. In the present series of tests, a minimum data level of 0.040 g was used in data reduction to account for the measurement system accuracy and noise level. The lack of sufficient test data points, with large value, would be reflected in the subsequent frequency plots.

The modal curves presented below are plotted from the normalized measured accelerations. The following scales were used for plotting the modal curves. A one to one scale corresponds to a force input of 100 lbs.

<u>FREQUENCY, Hz</u>	<u>SCALE</u>
13	1:20 (5 lb. equivalent force)
16	1:10
57	1:10

$$\underline{f = 13 \text{ Hz}}$$

This is a pitching mode of the XECF structure (Figure 3). The motion for this frequency indicates it is the engine first transverse mode. Primary motion is confined to the pressure vessel and the nozzle. There is little bending in the structure. This mode is similar to the predicted second mode of the engine system, although the predicted frequency was slightly higher (17.2 HZ).

$$\underline{f = 16 \text{ Hz}}$$

Tangential motions are negligible at this frequency, as is in the previous frequency (Figure 4). The XECF structure is still experiencing pitching motion. Primary motion is again confined to the pressure vessel and nozzle. There is little motion in the reactor core, which implies no relative displacement between the core and the pressure vessel.

$$\underline{f = 57 \text{ Hz}}$$

This modal response indicates it is the engine second transverse mode (Figure 5). It also approaches the classical bending mode of the XECF structure. Primary deflection is still confined to the pressure vessel and nozzle with the latter having the largest transverse displacement. Most of the bending occurs in the engine lower thrust structure.

[illegible]

CHANNEL #	F (Hz) F (lbs) RUN #	8 100 19	13 100 4	16 100 15	16 100 35	24 100 2	57 100 34	129 100 1	183 100 7	228 100 16
19	1 R	.096								
20	28 R		.261	.122	.085		.080	.261		.132
21	27 L					.223		<u>.638</u>		.575
22	4 T									
23	4 L									
24	6 R									
26	7 L	<u>.252</u>	.274	.226	.192	.243	<u>.144</u>	.180	.195	.213
27	9 R				.076					
28	10 R									.103
29	10 T									
30	10 L									
31	11 T		.153							
32	11 L	.118								
33	12 R		.116							
34	13 R									
35	14 R		<u>.636</u>	.251	<u>.319</u>	.232	.080	.243		.639
36	14 T		.538	.124	.246			.072	.096	.664

CHANNEL #	F (Hz) F. (lbs.) RUN #	8 100 19	13 100 4	16 100 15	16 100 35	24 100 2	57 100 34	129 100 1	183 100 7	228 100 16
37	14 L		.276	.141	.204	.118				.666
38	15 T									
39	15 L		.123			.184				.167
40	17 R									
41	18 R		.140	.201	.237		.110			
42	18 T		.180	<u>.374</u>	.243	<u>.498</u>	.082	.085		.107
43	18 L		.147	.194	.074					
44	19 R						.120	.217		.220
45	19 T		.266	.222			.190	.628		.357
46	19 L							.155		.261
47	20 T								.095	.073
48	20 L		.101	.113				.075		.172
49	22 R							.142	.187	.073
50	23 T							.083		.106
52	26 T		.111					.127		.592
53	26 L							.584		.46
54	25 R		.185	.095				.221		.218

CHANNEL#	f (Hz) F (lbs) RUN #	237 100 20	248 100 21	258 100 22	274 100 3	308 100 23	323 100 24	333 100 25	341 100 26	381 100 8
1	23R	.254	.127	.298	.096	.186	.212	.253	.273	.382
2	4R									
3	7R						.088	.088		
4	11R									
5	15R									
6	16R			.108				.152		
7	20R	.118	.107	.303	.596	1.143	<u>.646</u>	.432	.411	<u>.431</u>
8	21R	.175	.228	.304	<u>.916</u>	.972	.343	.398	.593	.199
10	23L	.131	.236	.093		.383	.182	1.063	.656	.107
11	24R	.489	.325	.339	.233	.370	.230	.450	.158	.241
12	26R	.662	.708	.476	.147	.115			.119	
13	27R	.800	.628	.556			.088	.125		.253
14	29X	.605	.318	.412	.332	.180	.211	.223		
15	29Y	.387		.161				.247	.365	.244
16	29Z									
17	1L				.156					.125
18	2R					.095				

CHANNEL #	f (Hz) F (lbs.) RUN #	237 100 20	248 100 21	258 100 22	274 100 3	308 100 23	323 100 24	333 100 25	341 100 26	381 100 8
19	1R									
20	28R	.623	.382	.155	.261	.135				
21	27R	.925	.429	.450		.370		.426	.324	.236
22	4T									
23	4L									
24	6R									
26	7L	.233	.186	.221	.274	.193	.202	.254	.300	.200
27	9R	.079								
28	10R		.098	.100				.098		
29	10T	.108	.249					.136		
30	10L		.199	.160						
31	11T				.153					
32	11L									
33	12R	.105	.070	.149	.116					
34	13R									
35	14R	1.592	.796	1.767	.637	.778	.249	.887	.191	.160
36	14T	<u>3.241</u>	.908	1.193	.538	<u>1.539</u>	.364	.805		.244

CHANNEL #	f (Hz) F (lbs.) RUN #	237 100 20	248 100 21	258 100 22	274 100 3	308 100 23	323 100 24	333 100 25	341 100 26	381 100 8
37	14L	1.735	<u>1.358</u>	<u>2.626</u>	.276	.699	.391	<u>1.497</u>	.134	
38	15T									
39	15L	.163		.132	.123	.258	.184	.267	1.218	.265
40	17R								.120	
41	18R	.208	.103		.140		.322		.195	
42	18T	.249	.210		.180	.372		.114		
43	18L			.116	.147		.091		.144	
44	19R	.339	.825	.346			.176	.222	.209	
45	19T	.274	.695	.261	.266	.316	.273		.419	
46	19L	.257		.131		.112				
47	20T	.079		.077		.183		.085	.202	
48	20L	.137	.089	.133	.101	.107		.400	<u>1.318</u>	.094
49	22R	.121	.080	.123		.356	.144	.140	.352	.158
50	23T									.135
52	26T	.487	.565	.160	.111				.151	.135
53	26L	.631	.808	.619		.555	.267	.157	.157	.24
54	25R	.324	.179	.359	.185	.189	.108	.234	.233	.123

[illegible]

CHANNEL#	f (Hz) F (lbs.) RUN #	415 100 5	452 100 6	452 100 36	454 100 12	478 100 27	519 100 28	523 100 11	633 100 13	633 100 37
19	1 R									
20	28 R		.644	.203	.645	.152	.177	.168		.486
21	27 L	.374	4.048	2.382	2.909	.687	<u>.986</u>	1.202	1.045	1.235
22	4 T									
23	4 L									
24	6 R									
26	7 L	.228	.394	.239	.364	.174	.177	.212	1.419	.760
27	9 R									
28	10 R				.111					
29	10 T		.349	.316	.323					
30	10 L		.279	.221	.250					
31	11 T									
32	11 L		.171	.109						
33	12 R									
34	13 R									
35	14 R	.860	1.370	2.325	2.722	.254	.151	.372		
36	14 T	.653	2.215	1.752	2.350	.232	.894	<u>1.281</u>	9.388	4.205

CHANNEL	f (Hz) F (lbs.) RUN #	415 100 5	452 100 6	452 100 36	454 100 12	478 100 27	519 100 28	523 100 11	633 100 13	633 100 37
37	14L	.382	1.525	1.204	1.638	.278	.406	.689	3.043	1.798
38	15 T				.247			.082	.650	
39	15 L	.236	.624	.632	.575	.362	.226	.234		.552
40	17 R			.099	.106	.079		.054		
41	18 R			.180			.194	.207		
42	18 T		.206	.164		.065	.191	.163		
43	18 L		.151	.125	.134	.209	.196	.192		
44	19 R	.168	1.808	1.226	1.349	.139	.316	.205	1.070	.496
45	19 T	.107	6.255	1.172	<u>5.141</u>	.568	.216	.198	.511	.277
46	19 L	.157	1.180	.183	.895	.182	.072		1.057	.372
47	20 T	.251	.529	.336	.378	.219	.381	.369		
48	20 L	.239	.366	.550	.520	.840	.356	.153	.879	.469
49	22 R	.270	.936	.459	.717	.243	.307	.309		.637
50	23 T	<u>1.114</u>	.293		.512			.195	.606	
52	26 T		3.353	1.516	1.812	.829	.162	.182		
53	26 L	.802	<u>7.977</u>	<u>3.184</u>	4.669	1.100	.643	.811	.797	2.01
54	25 R	.182	1.039	1.162	1.151	.385	.453	.734	.495	.383

[illegible]

CHANNEL #	f (Hz) F (lbs.) RUN #	657 100 38	658 100 14	703 100 10	816 100 29	856 100 30	876 100 18	922 100 31	931 100 32	988 100 33
19	1R									
20	28R	.397	.697	1.423	.457	.333	.820			.716
21	27R	.975		1.221	.874	3.541	3.105	5.897	6.177	8.885
22	4T									
23	4L									
24	6R									
26	7L	1.029	1.616	1.331	.753	.685	11.267	.948	.780	.209
27	9R									
28	10R									
29	10T									
30	10L									
31	11T									
32	11L									.087
33	12R								.344	
34	13R									
35	14R			1.657					.360	.21
36	14T	5.225	13.773	.814		.506				

CHANNEL #	f (Hz) F (lbs) RUN #	657 100 38	658 100 14	703 100 10	816 100 29	856 100 30	876 100 18	922 100 31	931 100 32	988 100 33
37	14L	.984	2.655	3.509		.436			1.380	.54
38	15T						.508			
39	15L	.575	.776	.700	.336	.455		.774	.796	.603
40	17R									
41	18R				.428					.256
42	18T					.287				.154
43	18L	.628	.860							.129
44	19R	.710	1.092		<u>5.284</u>	2.029	.804	1.599	.434	.805
45	19T	.942	.740		5.155	1.493	.690	4.085	2.594	2.045
46	19L	.924	.591		.358			1.733	1.038	.844
47	20T			.757				1.095	1.149	.196
48	20L	.811	1.079	.912	.355	1.249				.567
49	22R	.956	1.325			.270		1.093		.796
50	23T			1.813						
52	26T	.661	.578	1.157	.767	2.114	1.049	6.557	9.003	6.426
53	26L	6.943	5.475	3.939	2.747	<u>6.342</u>	4.108	<u>12.380</u>	<u>11.974</u>	<u>11.6</u>
54	25R	.590	.682			.309		.585	.383	.188

CHANNEL #	f (Hz)	1028	1238	1238					
	F (lbs)	100	100	100					
	RUN #	17	9	39					
1	23R	1.086	1.267	1.358					
2	4R								
3	7R								
4	11R		.813						
5	15R								
6	16R	.238							
7	20R	1.006	2.722	3.670					
8	21R	1.014	1.695	2.061					
10	23L	5.447	18.578	<u>21.072</u>					
11	24R	1.404		1.217					
12	26R	.531	1.447	1.468					
13	27R	2.254							
14	29X								
15	29Y	1.443							
16	29Z	.364							
17	1L	.385	1.173						
18	2R								

CHANNEL #	f (Hz) F (lbs) RUN #	1028 100 17	1238 100 9	1238 100 39					
19	1 R								
20	28 R	.311							
21	27 L	<u>11.044</u>	6.040	2.903					
22	4 T								
23	4 L								
24	6 R								
26	7 L	.782	2.031	2.796					
27	9 R								
28	10 R								
29	10 T								
30	10 L								
31	11 T								
32	11 L								
33	12 R								
34	13 R								
35	14 R	.594							
36	14 T								

CHANNEL #	f (Hz) F (lbs) RUN #	1028 100 17	1238 100 9	1238 100 39						
37	14L	.965								
38	15T	.305	1.035							
39	15L		1.214							
40	17R									
41	18R		1.512	2.629						
42	18T		2.393	3.347						
43	18L									
44	19R	.625		1.178						
45	19T	1.292								
46	19L	.389								
47	20T	.752		1.734						
48	20L	2.010	.953							
49	22R	1.333	1.387	2.030						
50	23T	.321	<u>20.089</u>							
52	26T	1.813								
53	26L	2.752		1.278						
54	25R	.331	2.100	1.667						

XE ENGINE

FIGURE 3
MEASURED MODAL RESPONSE
@ $f_1 = 13$ Hz

XE ENGINE

FIGURE 4
MEASURED MODAL RESPONSE
@ $f_2 = 16$ Hz

ϵ
Y

XE ENGINE

FIGURE 5
MEASURED MODAL RESPONSE
@ $f_3 = 57$ Hz

C. MODAL STRESSES

The primary modes have been determined and the measured modal deflections presented. Only two modes; namely, 13 Hz and 57 Hz, are presented for modal stresses calculation as these are the most significant modes. The measured elastic deflection curves can be represented by mathematical expressions. The modal curves were actually plotted from acceleration responses and then the deflection curves are determined from the relationship for sinusoidal vibration of:

$$a = (2 \pi f)^2 \frac{y}{g}$$

The elastic deflection is determined as:

$$y = \text{function } (x) = \text{fct } (x)$$

$$\text{and } \frac{M}{EI} = \text{fct } '' (x)$$

$$\overline{\sigma}_B = \frac{Mc}{I} = \frac{cEI \text{ fct } '' (x)}{I}$$

$$\therefore \overline{\sigma}_B = cE \text{ fct } '' (x) = cE \frac{d^2 y}{dx^2}$$

The dynamic bending stresses are determined and compared with the allowable vibratory stresses after mechanical and thermal stresses are considered. The final result is the determination of an allowable engine acceleration for each frequency. This can then be compared to the predicted vibration levels during nuclear testing.

$$\underline{f = 13 \text{ Hz}}$$

The deflection curve is shown in Figure 6. The solid line is the measured test data; the dash line is the mathematical curve fit; the zero ordinate is the XE engine center line.

$$y = 0.434 \times 10^{-9} [2L^3 - 3L^2 x + x^3] \quad \text{where } x \text{ is measured from the nozzle end.}$$

$$\frac{dy}{dx} = 0.434 \times 10^{-9} [-3L^2 + 3x^2]$$

$$\frac{d^2 y}{dx^2} = 0.434 \times 10^{-9} (6x)$$

$$\overline{\sigma}_{\max} \text{ occurs at } x = 273 \text{ in.}$$

$$\therefore \overline{\sigma}_B = \pm 48.8 \times 10^7 \times 0.434 \times 10^{-9} \times 6 \times 273 = \pm 347 \text{ psi/100 lb force}$$

$$\overline{\sigma} \text{ at the pressure vessel-lower thrust structure interface, } x = 127$$

$$\overline{\sigma} = \pm 25.66 \times 10^7 \times 0.434 \times 10^{-9} \times 6 \times 127 = \pm 85 \text{ psi/100 lb force.}$$

FIGURE 6
XE ENGINE RADIAL DEFLECTION
 $f = 13 \text{ Hz}$
 $y = 0.434 \times 10^{-9} [2L^3 - 3L^2x + x^3]$
For $0 \leq x \leq 273$ and $L = 273$

ZERO ORDINATE IS THE XE ENGINE CENTER LINE

$$\underline{f = 57 \text{ Hz}}$$

The deflection curve is shown in Figure 7.

$$\text{for } 0 \leq x \leq 215$$

$$y = 2.123 \times 10^{-11} [2L^3 - 3L^2 x + x^3] \quad \text{where } x \text{ is measured from the nozzle end}$$

$$\frac{dy}{dx} = 2.123 \times 10^{-11} [-3L^2 + 3x^2]$$

$$\frac{d^2y}{dx^2} = 2.123 \times 10^{-11} (6x)$$

$$\sigma_{\text{MAX}} \text{ occurs at } x = 215''$$

$$\therefore \sigma_B = \pm 48.8 \times 10^7 \times 2.123 \times 10^{-11} \times 6 \times 215 = \pm \underline{13 \text{ psi}} / 100 \text{ lb. force}$$

$$\sigma \text{ at the pressure vessel-lower thrust structure interface, } X = 127$$

$$\sigma = \pm 25.66 \times 10^7 \times 2.123 \times 10^{-11} \times 6 \times 127 = \pm \underline{4 \text{ psi}} / 100 \text{ lb. force}$$

FIGURE 7
XE ENGINE RADIAL DEFLECTION
 $f = 57 \text{ Hz}$
 $y = 2.123 \times 10^{-11} [2L^3 - 3L^2x + x^3]$
For $0 \leq x \leq 215$ and $L = 215$

$\delta \sim 10^{-4}$

SUMMARY

A summary of stresses for the most critical structural components of the XE engine is presented in the table below. The maximum allowable dynamic stress is found for each frequency using the modified Goodman Diagram given in Figure 8. The maximum allowable vibrational accelerations are then found, and margins of safety are shown. The XE engine, in its present configuration, is structurally adequate to withstand the vibratory deflections expected during nuclear firing.

XE ENGINE PRESSURE VESSEL STRESS SUMMARY

<u>f</u> (Hz)	<u>n¹</u> (cycles)	<u>$\sqrt{\phi_{MAX}^2}$</u> (psi)	<u>$\sqrt{\sigma_{DYN-ALL}^3}$</u> (psi)	<u>$\sqrt{\sigma_{DYN-MAX}}$</u> 100 LB F. (psi)	<u>ϵ_{MAX}</u> 100 LB F.	<u>g</u> ALLOW	<u>g</u> PREDICT	<u>$\sqrt{\sigma_{DYN-PREDICT}}$</u> (psi)	<u>M.S.</u>
13	46,800	28,000	11,500	85	.19	25.7	.50	224	Large
57	205,200	28,000	11,500	4	.04	115.0	1.00	100	Large

1. Based on 60 Minutes Nuclear Firing
2. Reference: RN-TM-0338, Structural Analysis of the XE-1 Pressure Vessel Components and Assembly, Feb. 1967
3. From Modified Goodman Diagram. Figure 8

XE ENGINE UPPER THRUST STRUCTURE STRESS SUMMARY

<u>f</u> (Hz)	<u>n¹</u> (cycles)	<u>$\sigma_{\phi \text{ MAX}}^2$</u> (psi)	<u>$\sigma_{\text{DYN-ALL}}^3$</u> (psi)	<u>$\sigma_{\text{DYN-MAX}}/$ 100 LB F. (psi)</u>	<u>$\epsilon_{\text{MAX}}/$ 100 LB F.</u>	<u>g</u> ALLOW	<u>g</u> PREDICT	<u>$\sigma_{\text{DYN-PREDICT}}$</u> (psi)	<u>M.S.</u>
13	46,800	21,750	17,000	347	.058	2.8	.15	897	18.0
57	205,200	21,750	17,000	13	.006	7.8	.15	325	Large

1. Based on 60 Minutes Nuclear Firing
2. Reference: RN-TM-0331, Stress Analysis of Upper Thrust Structure XE-1
3. From Modified Goodman Diagram, Figure 8

FIGURE 8
MODIFIED GOODMAN DIAGRAM
FOR AL 6061-T6

D. DAMPING

The differential equation of motion for a single degree-of-freedom system in free vibration is:

$$m \ddot{x} + c \dot{x} + k x = 0.$$

The same equation holds true for a multiple degree-of-freedom system if matrix notation is used. The minimum value of c , the viscous damping coefficient, which will allow the displaced system to return to its initial position without oscillation is called the coefficient of critical damping C_c . The ratio $\zeta = c/C_c$ is defined as the fraction of critical damping. The degree of damping in a system having $\zeta < 1$ may be defined in terms of successive peak values in a record of a free oscillation. The expression for free vibration of a damped system is:

$$X = A e^{-\zeta \omega_n t} \sin(\omega_d t + \theta)$$

Consider any two maxima (i.e., value of X when $dx/dt = 0$) separated by n cycles of oscillation as shown below:

The ratio of these maxima is:

$$\frac{X_n}{X_0} = e^{\frac{-2\pi n \zeta}{(1-\zeta^2)^{\frac{1}{2}}}}$$

Values of X_n/X_0 for $n = 1, 2$, and 3 are plotted in Figure 9 which illustrates the method. The data are for 27R at 13 Hz from Series I, Test 3, Run 6. Every readable maxima for 27R was taken from the oscillogram playback of the test data. Then all possible ratios of X_n/X_0 were calculated for $n = 1, 2$, and 3 . These ratios were then plotted on the appropriate curves. If conditions had been perfect for performing the test, recording and reducing the data, the various data points in Figure 9 would all lie on one horizontal line. Since they do not, a certain amount of engineering judgment is used to determine the values of ζ from the plots.

ILLUSTRATION OF CRITICAL DAMPING DETERMINATION

FIGURE 9

A summary of all values of $\mathcal{J}$ determined is given in the following table. Not all responses were large enough to allow reduction. These damping fractions are also plotted versus frequency in Figures 10 through 16. The damping values are seen to be generally less than 4% except at the low frequencies of 13 and 16 Hz and for the core accelerometers between 500 and 600 Hz range.

FRACTION OF CRITICAL DAMPING
XECF ENGINE RESONANCE SURVEY IN THE
TRANSVERSE DIRECTION

f (HZ)	13	16	120	228	274	452	523	632
F (lbs)	75	60	75	70	70	45	80	20
Run #	6	1	7	8	9	2	10	3
Accel. #								
23R	.09	.04	.02	.017	.015	.04	.01	.015
21R		.02	.03	.016				.035
20R			.012	.04	.022		.02	
23L	.10	.04	.025	.014	.007	.025	.012	.05
24R	.06	.03	.027	.035	.035	.01	.013	
26R	.05	.02	.013	.023	.013		.01	
27R	.055	.045	.035	.015	.018	.015	.04	.04
29X	.06	.10	.04	.015	.011	.018	.074	.06
29Y	.095	.10		.011			.10	.05
29Z	.08			.037		.018	.05	

f (HZ)	658	703	876	1238
F (lbs)	16	85	20	4
Run #	4	11	12	5
Accel. #				
23R	.025	.006	.011	.03
21R	.022	.011		.017
20R	.035	.012	.018	
23L	.012	.01	.04	.055
24R		.022		
26R	.008	.01		
27R	.018	.02	.02	.04
29X	.05	.007	.05	
29Y	.02	.01		
29Z		.02

FIGURE 10

FIGURE 11

FIGURE 12

FIGURE 13

FIGURE 14

FIGURE 15

FIGURE 16

IV. DYNAMIC ANALYSIS

A dynamic modal analysis of the XE engine system was performed. The primary objective of this analysis was to predict the natural modes of vibration of the engine system. This analytical solution provides information on the structural vibration characteristics of the XE engine system. The secondary objective was to provide analytical information for the vibration test program of XECF. The analytical results were then used as a basis to correlate test data in order to accurately determine the engine free vibration modes and viscous-damping characteristics.

A. METHOD OF ANALYSIS

The XE engine system as shown in Figure 17 is dynamically simulated by dividing it into a series of concentrated masses connected by short massless beams and/or idealized springs. Calculation of the free-vibration modes requires sectional representation of the engine system mass and stiffness characteristics. The mass representation includes linear mass, mass unbalance, and mass moment of inertia of each section. Mass points are assumed at the ends of each section with half the mass properties of each section lumped at each end of the section. The stiffness representation of the system is in the form of a set of influence coefficients at each station, as though that station was cantilevered from the adjacent station. The influence coefficients as well as the equations of motion are formulated as follows:

A typical section of the model is shown in Figure 18, where $i - 1$, i , and $i + 1$ are mass points at $i - 1$, i , and $i + 1$ elements.

Figure 18 - Typical Model Section

Figure 19 - Forces Acting on a Typical Beam

XE ENGINE

FIGURE 17

For a typical beam (Figure 19) the following matrix equation can be written to relate the forces and moments to the deformations:

$$\begin{bmatrix} \Delta_i \\ \theta_i \end{bmatrix} = \begin{bmatrix} a_{vi} & a_{mi} \\ b_{vi} & b_{mi} \end{bmatrix} \begin{bmatrix} F_{i+1} \\ M_{i+1} \end{bmatrix}$$

This matrix equation can be inverted to give:

$$\begin{bmatrix} F_{i+1} \\ M_{i+1} \end{bmatrix} = \begin{bmatrix} K_{Fi} & K_{FTi} \\ K_{TFi} & K_{Ti} \end{bmatrix} \begin{bmatrix} \Delta_i \\ \theta_i \end{bmatrix}$$

where

- a_{vi} = Deflection due to unit force
- a_{mi} = Deflection due to unit moment
- b_{vi} = Rotation due to unit force
- b_{mi} = Rotation due to unit moment
- K_{Fi} = Force required to produce a unit deflection
- K_{FTi} = Force required to produce a unit rotation
- K_{TFi} = Moment required to produce a unit deflection
- K_{Ti} = Moment required to produce a unit rotation

For a uniform beam, these influence coefficients become:

$$\begin{aligned} a_{vi} &= \frac{L_i^3}{3EI_i} & K_{Fi} &= \frac{12EI_i}{L_i^3} \\ a_{mi} &= \frac{L_i^2}{2EI_i} & K_{FTi} &= \frac{-6EI_i}{L_i^2} \\ b_{vi} &= \frac{L_i^2}{2EI_i} & K_{TFi} &= \frac{-6EI_i}{L_i^2} \\ b_{mi} &= \frac{L_i}{EI_i} & K_{Ti} &= \frac{4EI_i}{L_i} \end{aligned}$$

By applying the force and moment equations (derived above) to the typical section of the model, the forces and moments acting on the i th element are found to be those shown in Figure 20. It follows that the equations of motion are:

$$M_i \ddot{y}_i + y_{i-1} (-K_{Fi-1}) + y_i (K_{Fi-1} + K_{Fi}) + y_{i+1} (-K_{Fi}) + \theta_{i-1} (-K_{FTi-1} - K_{Fi-1} L_{i-1}) + \theta_i (K_{FTi-1} + K_{FTi} + K_{Fi} L_i) - \theta_{i+1} (K_{FTi}) = 0 \quad (1)$$

$$I_i \ddot{\theta}_i + y_{i-1} (-K_{TFi-1}) + y_i (K_{TFi-1} + K_{TFi} + K_{Fi} L_i) + y_{i+1} (-K_{TFi} - K_{Fi} L_i) + \theta_{i-1} (-K_{Ti-1} - K_{TFi-1} L_{i-1}) + \theta_i (K_{Ti-1} + K_{Ti} + K_{FTi} L_i + K_{TFi} L_i + K_i L_i^2) + \theta_{i+1} (-K_{Ti} - K_{FTi} L_i) = 0 \quad (2)$$

For a uniform beam, the following relationships hold:

$$K_{FTi} = K_{TFi}$$

$$K_{FTi} + K_{Fi} L_i = -K_{FTi}$$

$$K_{Ti} + K_{FTi} L_i = -1/2 K_{Ti}$$

$$2K_{FTi} L_i + K_{Fi} L_i^2 = 0$$

Therefore, Equations (1) and (2) may be simplified to:

$$M_i \ddot{y}_i + y_{i-1} (-K_{Fi-1}) + y_i (K_{Fi-1} + K_{Fi}) + y_{i+1} (-K_{Fi}) + \theta_{i-1} (K_{FTi-1}) + \theta_i (K_{FTi-1} - K_{FTi}) + \theta_{i+1} (-K_{FTi}) = 0 \quad (3)$$

$$I_i \ddot{\theta}_i + y_{i-1} (-K_{TFi-1}) + y_i (K_{TFi-1} - K_{TFi}) + y_{i+1} (K_{FTi}) + \theta_{i-1} (1/2 K_{Ti-1}) + \theta_i (K_{Ti-1} + K_{Ti}) + \theta_{i+1} (1/2 K_{Ti}) = 0 \quad (4)$$

Figure 20- Free Body of i 'th Element

Equations (3) and (4) are the general equations of motion for the i th element of a continuous elastic beam assuming each element to be uniform in itself.

There are as many equations of motion as there are number of degrees of freedom for the system. Using the classical, normal mode theory in conjunction with matrix formulation of the governing coupled differential equation (the type shown by Equations (3) and (4)), frequency and vibratory mode solution can be obtained using the Givens method on AGC Computer Program 998.

B. MODEL

The critical task of formulating a mathematical model to dynamically and structurally simulate the XE engine assembly is to understand the basic characteristics of the system and the corresponding component load-transfer paths. By knowing these different characteristics, a mathematical model (which is in every sense dynamically similar to the actual structure under consideration) can be derived with sufficient accuracy.

The mathematical model selected for the XE engine assembly consists of a system of 16 masses (see Figure 21). Because of the variation in stiffness in the downward section, the nozzle assembly was resolved into an equivalent dynamic system of three masses. Mass points 7 and 8 represent mass concentration at the critical reactor components: only axial motion is allowed in these two mass points.

The turbopump assembly is mounted on the propellant inlet line and has a tendency to vibrate in the axial as well as transverse directions. Two vibratory motions were assigned to specify its motion. Mass point 16 (which consists primarily of half the mass of the engine test stand adapter) can be treated as a boundary-mass concentration whose force and moment spring rates can be evaluated. All other mass points will have three vibratory motions: one rotational (8) and two translational (x and y) motions.

The complete mathematical model of the engine assembly, therefore, consists of a system of 16 masses interconnected by linear and/or rotational springs for a total of 43 degrees of freedom (see Figure 22).

Figure 21 - XE Engine Mass Points Location

Note: Number within the circle refers to beam number (See Table 2)

Figure 22- Mathematical Model of XE Engine

WEIGHT, MASS, AND MASS MOMENT OF INERTIA

<u>POINT</u>	<u>WEIGHT</u> <u>LB</u>	<u>MASS</u> <u>LB-SEC²/IN</u>	<u>I_θ</u> <u>LB-IN²</u>
1	380	0.983	146.8
2	352	0.911	89.0
3	1114	2.883	731.4
4	383	0.991	374.6
5	383	0.991	374.6
6	2766	7.158	4110.0
7	4800	12.422	----
8	2800	7.246	----
9	11251	29.117	936.8
10	11310	29.270	1229.3
11	465	1.203	1485.7
12	465	1.203	1485.7
13	769	1.990	----
14	465	1.203	1485.7
15	2322	6.009	10502.0
16	2260	5.849	16428.0

BEAM PROPERTIES

<u>BEAM</u>	<u>EA</u> <u>LB</u>	<u>EI</u> <u>LB-IN²</u>
1	10.88×10^8	6.4×10^{10}
2	30.00×10^8	40.0×10^{10}
3	10.50×10^8	30.0×10^{10}
4	10.50×10^8	30.0×10^{10}
5	10.50×10^8	30.0×10^{10}
6	2.60×10^8	16.0×10^{10}
7	3.04×10^8	48.0×10^{10}
8	3.00×10^8	54.0×10^{10}
9	3.00×10^8	54.0×10^{10}
10	3.00×10^8	54.0×10^{10}
11	3.00×10^8	54.0×10^{10}
12	6.80×10^8	140.0×10^{10}

C. EQUATIONS OF MOTION

The equations of motion for the stiffness matrix form a 43×43 matrix. These equations can easily be written knowing the model, method, and properties given in the previous section. The elements and their coupled degrees are listed in the following table.

COUPLED DEGREES

X_1	•	X_2				
Y_1	•	θ_1	Y_2	θ_2		
θ_1	Y_1	•	Y_2	θ_2		
X_2	X_1	•	X_3			
Y_2	Y_1	θ_1	•	θ_2	Y_3	θ_3
θ_2	Y_1	θ_1	Y_2	•	Y_3	θ_3
X_3	X_2	•	X_4	X_7		
Y_3	Y_2	θ_2	•	θ_3	Y_4	θ_4
θ_3	Y_2	θ_2	Y_3	•	Y_4	θ_4
X_4	X_3	•	X_5			
Y_4	Y_3	θ_3	•	θ_4	Y_5	θ_5
θ_4	Y_3	θ_3	Y_4	•	Y_5	θ_5
X_5	X_4	•	X_6			
Y_5	Y_4	θ_4	•	θ_5	Y_6	θ_6
θ_5	Y_4	θ_4	Y_5	•	Y_6	θ_6
X_6	X_5	•	X_8	X_9		
Y_6	Y_5	θ_5	•	θ_6	Y_9	θ_9
θ_6	Y_5	θ_5	Y_6	•	Y_9	θ_9

COUPLED DEGREES

X_7	X_3	.	X_8			
X_8	X_6	X_7	.			
X_9	X_6	.	X_{10}			
Y_9	Y_6	θ_6	.	θ_9	Y_{10}	θ_{10}
θ_9	Y_6	θ_6	Y_9	.	Y_{10}	θ_{10}
X_{10}	X_9	.	X_{11}			
Y_{10}	Y_9	θ_9	.	θ_{10}	Y_{11}	θ_{11}
θ_{10}	Y_9	θ_9	Y_{10}	.	Y_{11}	θ_{11}
X_{11}	X_{10}	.	X_{12}			
Y_{11}	Y_{10}	θ_{10}	.	θ_{11}	Y_{12}	θ_{12}
θ_{11}	Y_{10}	θ_{10}	Y_{11}	.	Y_{12}	θ_{12}
X_{12}	X_{11}	.	X_{14}			
Y_{12}	Y_{11}	θ_{11}	.	θ_{12}	Y_{14}	θ_{14}
θ_{12}	Y_{11}	θ_{11}	Y_{12}	.	Y_{14}	θ_{14}
X_{13}	.	X_{15}				
Y_{13}	.	Y_{15}	θ_{15}			
X_{14}	X_{12}	.	X_{15}			
Y_{14}	Y_{12}	θ_{12}	.	θ_{14}	Y_{15}	θ_{15}

COUPLED DEGREES

θ_{14}	Y_{12}	θ_{12}	Y_{14}	•	Y_{15}	θ_{15}	
X_{15}	X_{13}	X_{14}	•	X_{16}			
Y_{15}	Y_{13}	Y_{14}	θ_{14}	•	θ_{15}	Y_{16}	θ_{16}
θ_{15}	Y_{13}	Y_{14}	θ_{14}	Y_{15}	•	Y_{16}	θ_{16}
X_{16}	X_{15}	•					
Y_{16}	Y_{15}	θ_{15}	•	θ_{16}			
θ_{16}	Y_{15}	θ_{15}	Y_{16}	•			

D. NATURAL FREQUENCIES

The first four transverse natural frequencies of the XE engine system obtained from the computer program are 1.6, 17.2, 47.2, and 103.8 Hz, respectively. The next few higher modes are 148.4, 202.5, 309.5, 399.6, 429.5, 632.1, 715.0, and 816.1 Hz.

E. MODE SHAPES

The modal data corresponding to each of the first four transverse frequencies are shown in the following tables (a dash in these tables indicates a non-data point). The mode shapes for these four frequencies are plotted in Figures 23 through 26.

XE Engine Transverse Vibration Modal Response No. 1 - $f_1 = 1.6$ Hz

This modal response is primarily an engine translation mode. There is no bending in the entire system and the motion is purely a rigid body translational mode. This mode is made possible by the fact that the four long, slender thrust measuring columns, which mated the engine to the ETS-1, combine to produce a very weak lateral spring (K13y). This lateral spring thus presents little or no restraint to the lateral motion of the engine system.

XE Engine Transverse Vibration Modal Response No. 2 - $f_2 = 17.2$ Hz

This is a pitching mode of the XE engine system with little or no bending produced in the entire system. It is also the first transverse mode of the engine. The largest transverse displacements occur at both ends with the mode occurring at approximately the midpoint of the engine external shield. Again, this mode is made possible by the weak lateral spring at top of the engine, which provided very little restraint in the engine transverse direction.

XE Engine Transverse Vibration Modal Response No. 3 - $f_3 = 47.2$ Hz

This is the second transverse mode of the engine system. It is also the classical bending response of the structure in its deflected shape. Most of the bending occurs in the engine lower thrust structure. The largest transverse displacement again occurs at the nozzle end.

XE Engine Transverse Vibration Modal Response No. 4 - $f_4 = 103.8$ Hz

This is the engine third transverse mode. It is also one of the higher bending modes. Most of the bending occurs in the engine lower and upper thrust structures. The most significant response was in the turbopump assembly which exhibits large transverse displacement perpendicular to the engine center line. However, this large magnitude of displacement is localized, and is not a representative of the entire engine system.

X E ENGINE

FIGURE 23
XE ENGINE MODE SHAPE
@ $f_1 = 1.6$ Hz

X E ENGINE

$$f_1 = 1.6 \text{ Hz}$$

LOCATION	MODAL DATA	
	Y	θ
1	1112	0
2	1101	0
3	1095	0
4	1086	0
5	1078	0
6	1069	0
7	—	—
8	—	—
9	1059	0
10	1054	0
11	1047	0
12	1040	0
13	1037	—
14	1034	0
15	1029	0
16	1021	0

X E ENGINE

FIGURE 24
XE ENGINE MODE SHAPE
@ $f_2 = 17.2$ Hz

X E ENGINE

$$f_2 = 17.2 \text{ Hz}$$

LOCATION	MODAL DATA	
	Y	θ
1	3955	- 26
2	3167	- 26
3	2646	- 26
4	2002	- 26
5	1374	- 25
6	772	- 23
7	—	—
8	—	—
9	141	- 18
10	- 145	- 17
11	- 543	- 16
12	- 899	- 14
13	- 1114	—
14	- 1207	- 12
15	- 1459	- 9
16	- 1796	- 8

X E ENGINE

FIGURE 25
XE ENGINE MODE SHAPE
@ $f_3 = 47.2$ Hz

X E ENGINE

$$f_3 = 47.2 \text{ Hz}$$

LOCATION	MODAL DATA	
	Y	θ
1	3871	- 41
2	2672	- 38
3	1915	- 37
4	1013	- 34
5	218	- 29
6	- 411	- 21
7	—	—
8	—	—
9	- 624	8
10	- 463	12
11	- 106	17
12	339	20
13	664	—
14	820	20
15	1288	18
16	1980	16

X E ENGINE

FIGURE 26
XE ENGINE MODE SHAPE
@ $f_4 = 103.8$ Hz

X E ENGINE

$$f_4 = 103.8 \text{ Hz}$$

LOCATION:	MODAL DATA	
	Y	θ
1	875	- 17
2	398	- 14
3	127	- 13
4	- 165	- 10
5	- 365	- 6
6	- 438	0
7	—	—
8	—	—
9	- 226	13
10	- 9	14
11	289	11
12	460	3
13	6185	—
14	408	- 8
15	47	- 23
16	- 872	- 23

V. CONCLUSIONS

It has been concluded from the results of this test that:

1. Data acquisition was complete.
2. All predominant engine resonant frequencies were determined.
3. Engine mode shapes for the three lowest resonances were identified.
4. Mechanical impedances could not be determined due to the present lack of a computer program necessary for this operation.
5. Damping characteristics for the complete engine were determined.
6. The vibratory deflections of the engine, due to nuclear testing, were predicted and the stresses resulting from these deflections were calculated. A high margin of safety exists at all times.
7. A 1 g acceleration limit precluded the finding of additional data. A more realistic limit would have been a deflection limit which should be used during any future testing.