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OSCILLATIONS IN THE ANGULAR DISTRIBUTIONS OF HEAVY-ION TRANSFER REACTIONS*

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Oscillations in the angular distributions are established as a general feature of heavy-ion induced, few-nucleon transfer reactions at incident energies sufficiently above the Coulomb barrier. For proper Q-matching with incident and exiting energies well above the Coulomb barrier, the angular position of the most forward maxima in the differential cross section is characteristic of the transferred angular momentum. The threshold for such oscillations is particularly sensitive to the absorptive potential near the nuclear surface requiring potentials that are strongly absorbing in the nuclear interior but nearly transparent at the surface. Evidence also is presented for an energy dependent surface absorption. Certain anomalous cases where the oscillating cross sections are predicted to be out of phase with the data are discussed.

1. Introduction

Recent experimental studies (c.f. refs. ¹⁻¹⁰) have indicated that large forward angle cross sections which oscillate as a function of angle are a general feature of heavy-ion induced, few-nucleon transfer reactions at incident energies sufficiently above the Coulomb barrier. This development has stimulated renewed experimental and theoretical activity in such measurements that previously had been considered by many as uninteresting. I should like to discuss some of the features of these angular distributions concentrating on the heavy-ion physics that can be learned from such studies. Since most of the audience is familiar with the subject, I intend to outline only briefly the general features of such angular shapes, and then to discuss some of the recent data and what can be learned from it. Finally, perhaps I will be able to speak to some questions that were raised at our Brookhaven informal study on this subject last summer. I will use Brookhaven data to illustrate my points, but I wish to stress that similar data has been measured at many other European and American laboratories.

It must be remarked that large, oscillating forward angle cross sections previously had been anticipated by several theorist (c.f. refs. ¹¹⁻¹³) and in fact studied in detail for light targets here at the Max-Planck-Institute ¹⁴) as well as

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elsewhere. As early as 1961 large cross sections were observed forward of the grazing angle in radiochemical total cross section data obtained at Yale¹⁵). The new element revealed by the recent detailed measurements is the low threshold for the large forward angle cross sections in certain cases. Furthermore, measurements at higher energies suggest³) that such structure is a general feature of few-nucleon transfer reactions sufficiently above the Coulomb barrier. Oscillating forward angle cross sections have been observed¹⁰) for few nucleon transfer induced by light heavy ions on intermediate mass targets having Sommerfeld parameters, $\eta = Z_1 Z_2 e^2 / \hbar v \sim 20$.

2. General features of oscillating angular distributions

Typical angular shapes observed in such transfer reactions are shown in fig. 1

Fig. 1. Angular distributions of the $^{40}\text{Ca}(^{13}\text{C}, ^{12}\text{C})$ transition to the ^{41}Ca ground state at 40, 60, and 68 MeV incident energies (refs. 3 and 16). The DWBA predictions (curves) reproduce the shape change between 40 and 68 MeV.

for the $^{40}\text{Ca}(^{13}\text{C}, ^{12}\text{C})$ transition populating the ^{41}Ca ground state^{3,16}) measured at incident energies of 40, 60, and 68 MeV. At 60 and 68 MeV pronounced oscillations are observed in the data. At 40 MeV incident energy, however, a well developed, broad peak is observed near the angle corresponding to a grazing trajectory and at more forward angles a smaller peak is observed. The detailed change in shape and magnitude is reproduced in terms of DWBA predictions.

The shape change from a relatively smooth grazing peak at 40 MeV to rapid oscillations at 68 MeV also has been explained³) in terms of a diffractive model based on a partial wave expansion of the cross section^{1,11}). Such a parameterization approximates the l space localization of the transfer amplitude by a Gaussian distribution and assumes a linear variation with l of the transfer phase. Until recently such a semiclassical description

had been applied only to $L=0$ or to the $M=0$ magnetic substates for $L \neq 0$ transfer. Even though such a model is qualitatively successful in describing the existence of oscillations, it fails to describe the richness of heavy-ion induced transfer phenomena. One improvement would be to extend such an approach to include $M \neq 0$ components and to incorporate an expansion which is valid at small angles. Such a description has been developed by Kahana, Bond, and Chasman^{17,18}). I do not wish to discuss the details of such a description, since it has been covered explicitly in a recent publication¹⁷) and in the talk of Kahana at the Nashville Conference on Reactions Between Complex Nuclei¹⁸). Such a description, however, shows that the partial cross section corresponding to the $|M|=L$ magnetic substate will dominate over the $|M| < L$ substates when: i) the reaction is well matched, i.e. when the localization in r space is the same for entrance and exit channels, and ii) the incident and exiting energies are well above the Coulomb barrier. For such cases, the angular position of the forward most peak is given by the first maximum in the Bessel function $J_M[(\epsilon_f^0 + 1/2)\theta_f]$ and satisfies the inequality

$$(M(M+2))^{1/2} < (\epsilon_f^0 + 1/2)\theta_f < (2M(M+1))^{1/2}.$$

Such "L" or really M dependence is observed in experimental measurements and reproduced in the DWBA analysis^{1,6}). Figure 2 demonstrates such dependence for the $^{48}\text{Ca}(^{16}\text{O}, ^{14}\text{C})$ transitions⁶) to the lowest 0^+ , 2^+ , 4^+ , and 6^+ states of ^{50}Ti . The solid curves in the figure are DWBA cross sections calculated using the finite range, recoil code LOLA¹⁹) with a two-nucleon cluster form factor as described in ref. ⁶). This figure indicates the necessary precision in the extreme forward angle data to distinguish transferred L values using such reactions. It is apparent that it is easier to extract transferred L values from light-ion induced transfer if the equivalent transfer is possible with available projectiles.

3. DWBA analysis

Although the simple models are successful in describing the general features of heavy-ion induced transfer reactions, and are an aid in understanding the physics involved, the DWBA formalism still remains the main prescription in the analysis of such data. The remainder of this talk will be based on such an analysis. It already has been indicated that the energy dependence (see fig. 1) of the shape and magnitude of the $^{40}\text{Ca}(^{13}\text{C}, ^{12}\text{C})^{41}\text{Ca}_{\text{g.s.}}$ transition and the observed L dependence (see fig. 2) is reproduced in DWBA predictions.

The DWBA differential cross sections for single-nucleon transfer studies discussed were calculated using the finite-range DWBA code SRC of Baltz²⁰). This code includes the effects of recoil in a proper Taylor series expansion²¹) keeping terms to second order. For single-nucleon transfer the predicted cross section for the reaction $A(a,b)B$ is related to the stripping cross section through spectroscopic

Fig. 2. Angular distributions for $^{48}\text{Ca}(^{16}\text{O}, ^{14}\text{C})$ transitions at an incident energy of 56 MeV populating the lowest 0^+ , 2^+ , 4^+ , and 6^+ states in ^{50}Ti (ref. 6). The solid lines are DWBA predictions calculated using the code LOLA. The inset shows an energy spectrum.

ing that the details of the imaginary potential near the nuclear surface are important for transfer reactions.

Angular distribution shapes with very large forward peaks and a well formed "bell-shaped" peak at the "grazing" angle are particularly sensitive to the details

factors.

$$C_{\text{exp}}(\theta) = N C^2 s_{ab} C^2 s_{AB} C_{\text{SRC}}(\theta)$$

A value of the normalization constant N near one, therefore, indicates agreement between measured and predicted absolute cross sections. The two-nucleon transfer version²²⁾ of the finite range code RSDC²³⁾ was used to predict the $^{60}\text{Ni}(^{18}\text{O}, ^{16}\text{O})^{62}\text{Ni}$ and $^{26}\text{Mg}(^{16}\text{O}, ^{14}\text{C})^{28}\text{Si}$ cross sections.

This code uses a microscopic form factor but does not include the effects of recoil. The parameters, spectroscopic factors and two-nucleon spectroscopic amplitudes used in these calculations are tabulated in the appendix.

4. Sensitivity to optical potentials

The r -space localization of the heavy-ion induced transfer amplitude is a consequence of the decay of the radial form factor affecting the fall off toward higher r and the absorptive effects of the ion-ion optical potential governing the decay toward lower r 's. The phase in the region of the r window also is quite sensitive to the nuclear absorption. For larger r 's the phases will be identical to Coulomb phases. Near the r window the phases will deviate from the Coulomb phase (for energies above the Coulomb barrier) as the effects of the real nuclear potential become important. However as the absorption increases the effects of the nuclear potential will decrease (see e.g. figs. 19 and 21). It, therefore, is not surpris-

of the imaginary potential. Such shapes represent the onset of oscillations. A typical example of such an angular shape is the $^{40}\text{Ca}(^{13}\text{C}, ^{14}\text{N})$ transition to the ^{39}K ground state, $J^\pi = 3/2^+$ (fig. 3), measured^{16,24}) at an incident energy of 40 MeV.

Fig. 3. Angular distribution of the $^{40}\text{Ca}(^{13}\text{C}, ^{14}\text{N})$ transition to the ground state of ^{39}K measured at an incident energy of 40 MeV (refs. 16,24). The curves are DWBA predictions using optical-model parameter set 2, table A1. Only the magnitude of the absorption has been varied for the two different calculations shown.

A large, rapidly varying differential cross section is observed at the most forward angles while smaller amplitude oscillations, superimposed on a broad peak, are observed near angles corresponding to the classical grazing trajectory. Shown with the data are DWBA calculations based on optical-model parameters having identical geometry for both the real and imaginary potential wells. The dashed curve represents calculations based on parameters (set 2 of Table A1) which reproduce ^{13}C and ^{14}N elastic scattering on ^{40}Ca over a range of incident energies^{3,24}). The calculation reproduces neither the large forward angle cross section nor the variation of the cross section as a function of angle. If the imaginary potentials in both the entrance and exit channels are reduced to $W = 5.5$ MeV, while leaving the other parameters

unchanged, the forward angle cross sections are strongly enhanced and strong oscillations are predicted (solid curve in fig. 3). However, the oscillations of the curves based on these weakly absorbing potentials are out of phase with the most forward angle data points. The angular

positions of the maxima at the forward angles also have changed relative to their predicted positions when the more strongly absorbing potentials are used.

The change in the angular positions of the forward angle maxima is the result of contributions from amplitudes corresponding to large nuclear overlaps. The predicted angular distributions for the $^{40}\text{Ca}(^{13}\text{C}, ^{14}\text{N})^{39}\text{K}_{\text{g.s.}}$ reaction as a function of a lower radial cutoff radius, R_{co} , are shown in fig. 4. These calculations indicate that for such weakly absorbing potentials contributions are obtained well inside the channel radii. In fact, at forward angles large contributions are observed for less than a 5 fm separation of the nuclear centers. The angle of the most forward maxima

Fig. 4. Predicted angular distribution of the $^{40}\text{Ca}(^{13}\text{C}, ^{14}\text{N})^{39}\text{K}_{\text{g.s.}}$ cross section at an incident energy of 40 MeV as a function of a lower cutoff on the radial integration, R_{co} . Optical-model parameters set 2 of table A1 with an absorption of 5.5 MeV, (which correspond to the solid curve in fig. 3) were used in all the calculations.

also changes when the contributions from the nuclear interior are removed.

With this extremely weak absorptive potential the lower partial waves have a significant effect upon the cross section. In fig. 5 the transition amplitudes B_L^{LM} as a function of the outgoing partial wave l are shown for the $M=1$ substate. The change in the position of the calculated maxima in the weakly absorbing case arises from a beating of the frequencies of oscillation corresponding to the two peaks in B_L^{LM} of fig. 5. When the first 15 partial waves are removed from this calculation the position of the maxima returns to that calculated using the strong absorptive potential (see figs. 3 and 6).

The contributions to calculated heavy-ion transfer cross sections from deep within the nuclear interior that arise as a result of weakly absorbing ion-ion potentials are unphysical. The absorption in the nuclear interior must be increased to reduce such contributions, and yet it is necessary to maintain a weak absorption in the nuclear surface region to obtain the large forward-angle

Fig. 5. Calculated $M=1$ transition amplitude as a function of outgoing l corresponding to the angular distribution shown as a solid curve in fig. 3. Large values are predicted for low partial waves.

Fig. 6. Comparison of predicted angular distribution shown as a solid curve in fig. 3 with a calculation having the contributions for all partial waves < 15 removed (see fig. 5).

cross sections that are observed, e.g., in the $^{40}\text{Ca}(^{13}\text{C}, ^{14}\text{N})^{39}\text{K}$ ground-state transition (see fig. 3). It is particularly important that potentials which do not have unphysical contributions from the nuclear interior should be used when spectroscopic information is to be obtained from the analysis. As seen in figs. 4 and 6 such contributions can shift the position of the most forward maxima which is sensitive to the L dependence of the transfer.

At Brookhaven we have been using an imaginary potential shape which combines a strong absorption in the nuclear interior and weak absorption at the nuclear surface with an imaginary diffusivity at the nuclear surface similar to that of nuclear matter. The sum of a Woods-Saxon shape with a small diffusivity and a surface derivative term with a larger diffusivity is used for the imaginary potential. Strong absorption from the Woods-Saxon shape reduces the contributions from the nuclear interior while the weaker imaginary potential at the nuclear surface with a "reasonable" diffusivity gives the absorption at the radii where transfer takes place. Predictions based on such surface transparent potentials are shown in fig. 7 for the 40 MeV $^{40}\text{Ca}(^{13}\text{C}, ^{14}\text{N})$ and $^{40}\text{Ca}(^{13}\text{C}, ^{12}\text{C})$ transitions to the ground states of ^{39}K and ^{41}Ca . The shape and magnitude of the $(^{13}\text{C}, ^{12}\text{C})$ angular distribution is well reproduced by these calculations. The general shape and the magnitude of the more structured $(^{13}\text{C}, ^{14}\text{N})$ angular distribution is given, but the details of the most forward angle data still are not completely reproduced. However, predictions based on these potentials give a better description of the forward angle $^{40}\text{Ca}(^{13}\text{C}, ^{14}\text{N})^{39}\text{K}$ g.s. data than calculations using potentials with the same real and imaginary Woods-Saxon geometry (see, e.g., fig. 3).

The $^{40}\text{Ca}(^{13}\text{C}, ^{14}\text{N})^{39}\text{K}$ g.s. transition amplitudes, B_{ℓ}^{LM} , calculated for the $M=1$ magnetic substate using these potentials are shown as a function of ℓ in fig. 8. As a result of the stronger absorption in the nuclear interior, the large contributions that were observed for the small partial waves using the weakly absorbing Woods-Saxon potentials (fig. 5) are greatly reduced in these calculations. In fig. 9 the 40 MeV ^{13}C elastic scattering from ^{40}Ca predicted using these surface transparent potentials is compared with experimental data¹⁶). The experimental cross sections are reproduced.

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Fig. 7. Comparison of measured and predicted angular distributions for the 40 MeV ^{40}Ca ($^{13}\text{C}, ^{14}\text{N}$) ^{39}K and ^{40}Ca ($^{13}\text{C}, ^{12}\text{C}$) $^{41}\text{Ca.s.}$ transitions. The calculated angular shapes used surface transparent potentials as discussed in the text and given as set 1 in table A1.

Fig. 9. Comparison of measured and predicted 40 MeV ^{13}C elastic scattering from ^{40}Ca using the surface transparent potentials (set 1 table A1) that reproduce the transfer cross sections (fig. 7).

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Fig. 8. Calculated $M=1$ transition amplitude as a function of outgoing l using surface transparent potentials. These potentials were used to calculate the ($^{13}\text{C}, ^{14}\text{N}$) angular distribution shown in fig. 7. The large values predicted for low partial waves using weak absorptive potentials of Woods-Saxon geometry (fig. 5) have been reduced.

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In figs. 10 and 11 the differential and total calculated cross sections for the $^{40}\text{Ca}(^{13}\text{C}, ^{14}\text{N})^{39}\text{K}$ ground-state transition at an incident energy of 40 MeV is shown

Fig. 10. Calculated 40 MeV $^{40}\text{Ca}(^{13}\text{C}, ^{14}\text{N})^{39}\text{K}_{\text{gs}}$ differential cross sections as a function of a lower cutoff on the radial integration, R_{CO} . These calculations used the surface transparent potentials (set 1 of Table A1) discussed in the text. Predictions based on these potentials are shown with the experimental data in fig. 7. The radial shape of the Coulomb plus real nuclear and imaginary nuclear potentials for $^{13}\text{C} + ^{40}\text{Ca}$ also are shown.

Fig. 11. Predicted total cross section for the 40 MeV $^{40}\text{Ca}(^{13}\text{C}, ^{14}\text{N})^{39}\text{K}_{\text{gs}}$ transition as a function of a lower cutoff on the radial integration. See fig. 10 and text for potentials used. R_{R} and R_{R} indicate the real nuclear radius and the Rutherford radius in the entrance channel.

as a function of lower cutoffs on the radial integration. These calculations are based on the surface transparent potentials. For comparison the real plus Coulomb and imaginary nuclear potentials for the $^{13}\text{C} + ^{40}\text{Ca}$ channel also are shown in fig. 10. It must be emphasized that the use of a radial integration cutoff is a computational technique

and that it does not completely remove the effects of the nuclear interior. It does indicate, however, at what radii large contribution to the cross section are obtained.

The predicted cross sections corresponding to radial cutoffs of 6-7 fm were nearly identical both in magnitude and shape to the calculation with no radial cutoff. Such was not the case for the weak Woods-Saxon imaginary potential which had significant contributions from the nuclear interior (see fig. 4). The rise in the total cross section for cutoffs between 7 and 8 fm result from removing certain contributions which cause cancellations in the cross section. The main contributions to the total cross section still are obtained near the Rutherford radius (i.e. the radius at which the Coulomb plus real nuclear potential is maximum--see fig. 10). The forward-angle cross section, however, starts to decrease relative to the "grazing" peak cross section for radial cutoffs between 7.5 and 8.0 fm and has nearly disappeared after $R_{co} = 8.5$ fm. Therefore the large forward peak seems to have its origin at radii inside the Rutherford radius, suggesting that this peak is connected to contributions from the opposite side of the nucleus that is either diffracted or refracted around the nucleus.

By using surface transparent potentials (set 1 of table A1) it also has been possible to reproduce the details of the oscillations observed in the $^{60}\text{Ni}(^{18}\text{O}, ^{16}\text{O})$ transition to the ^{62}Ni ground state¹⁰) (see fig. 12). Prediction using surface

Fig. 12. Angular distribution corresponding to the $^{60}\text{Ni}(^{18}\text{O}, ^{16}\text{O})$ transition to the ground state of ^{62}Ni measured at an incident energy of 65 MeV (ref. 10). Data indicated by solid points were recorded using the BNL QDDD spectrometer. The DWBA curves were calculated using surface transparent potentials (set 1, table A1).

transparent potentials which reproduce the experimental data¹⁰) are compared in fig. 13 with predictions based on weakly absorbing imaginary potentials of Woods-Saxon geometry (set 2 of table A1) which have been used in a previous analysis of this reaction⁴). The strong absorption in the nuclear interior has decreased the magnitude of the oscillation at the more backward angles. The different period of the oscillations at the forward angles in the calculations based on weakly absorbing imaginary potentials of Woods Saxon geometry probably results from a beating between contributions from the interior and the surface of the nucleus. Furthermore, nearly all the angular dependence on the single-particle configurations which is predicted using the potentials that are too weakly absorbing in the nuclear interior is removed in the calculation based on the surface transparent potentials fig. 14.

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Fig. 13. Predictions based on surface transparent potentials (set 1 table A1) which reproduce the experimental data (fig. 12) are compared with predictions based on weakly absorbing potentials of Woods-Saxon geometry (set 2 of table A1) which were used in a previous analysis of this reaction (ref. 4). We have made progress in a year, haven't we!

The important features of the potentials which we have been using are: i) strong absorption in the nuclear interior, ii) weak absorption near the nuclear surface, and iii) a diffusivity near the nuclear surface that approximates the nuclear matter distribution. The suggested parameterization is only one that incorporates these features. In fact, imaginary potentials with a small diffusivity

and a radius somewhat smaller than the real nuclear radius are presently being used by several groups, c.f. refs. ^{6,8}). Such an imaginary potential incorporates the first two of the proposed features.

Fig. 14. The configuration dependence observed in $^{60}\text{Ni}(^{18}\text{O},^{16}\text{O})^{62}\text{Ni}_{g.s.}$ transition at 65 MeV using weakly absorbing potentials (set 2, table A1) is almost completely removed when surface transparent potentials are used (set 1, table A1).

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5. Total quasielastic cross sections

(12)

Such imaginary potential shapes may have a physical basis. The interior and surface absorption almost certainly arise from different processes--compound processes dominating the interior and quasielastic processes important in the surface region. A sharp increase in the absorption might then be expected at the radius where the more violent compound processes become important. In order to determine if quasielastic processes contain sufficient total cross section to account for the partial cross sections of the surface partial waves, we have measured²⁵⁾ the total few-nucleon transfer reaction and inelastic scattering cross sections induced by ^{16}O on ^{208}Pb at bombarding energies of 82, 88, and 94 MeV. Heavy targets were chosen to allow the transfer cross sections to be measured above the Coulomb barrier without competition with reaction products from light target impurities. For the cases measured the reaction kinematics completely separate the groups of interest and products resulting from light target impurities in the angular region which contribute significantly to the total cross section. Angular distributions of the total yield of inelastically scattered ^{16}O and O, N, and C resulting from transfer is shown for 82 MeV ^{16}O incident on $^{206,208}\text{Pb}$ in fig. 15. Much smaller yields also were observed for Be and B. Total reaction cross sections were determined from the optical model using parameter giving a best fit to the

Fig. 15. Differential cross section as a function of angle for the total yields of O, N, and C resulting from transfer and inelastically scattered ^{16}O induced by 82 MeV ^{16}O incident on ^{206}Pb and ^{208}Pb (see ref.²⁵). The summed cross section for all channels also is shown.

elastic scattering at each energy. The total reaction cross section is the sum over l 's of the partial cross section $\sigma_l(\theta)$ for each l is given by

$$\sigma_j = \pi \lambda^2 (2j + 1) T_j$$

where T_j is the transmission coefficient as calculated using the optical model code ABACUS²⁶). The results of a preliminary analysis for ^{208}Pb are given in table 1.

Table 1. Summary of $^{16}\text{O} + ^{208}\text{Pb}$ Total Cross Section Data.

E_{lab} (MeV)	$E_{\text{c.m.}}$ (MeV)	σ_T^a (mb)	σ_{QE}^b (mb)	$\sigma_{\text{QE}}/\sigma_T$	l_{crit}^c
82	76.14	219	94	0.43	16
88	81.71	553	257	0.46	23
94	87.29	876	331	0.38	32

a) From optical model fit of elastic scattering.

b) Integrated experimental cross sections.

c) See fig. 16--assumes sharp cutoff.

Fig. 16. Sum of partial cross section σ_j for $l < l_i$ as a function of l_i . For comparison transmission coefficients based on a fit to the $^{16}\text{O} + ^{208}\text{Pb}$ elastic scattering data at each energy is shown. The values of l_{crit} separating contributions from quasielastic and other processes in a sharp cutoff approximation are indicated with the transmission coefficients. These values are determined from the measured quasielastic and the calculated total reaction cross sections given in table 1.

A surprisingly large fraction of the total reaction cross section $\sim 40\%$ is accounted for by the quasielastic processes even as high as 15 MeV above the Coulomb barrier. The sum of the partial cross sections σ_l for $l < l_i$ is shown as a function of l_i in fig. 16. The values of l dividing the quasielastic cross section from other processes are shown as l_{crit} on the plot of transmission coefficients in fig. 16. The l_{crit} are obtained assuming that the partial cross sections for large l 's are all quasielastic and a sharp division in l space between quasielastic and other processes. It is seen for the system studied in this energy region that the quasielastic cross sections are sufficiently large to account for the cross sections of the surface partial waves. Such a division between quasielastic and other more violent processes in l -space is consistent with a difference in the radial dependence of the absorption near the nuclear surface and in the nuclear interior.

6. Energy dependence of the surface absorption from the $^{26}\text{Mg}(^{16}\text{O}, ^{14}\text{C})^{28}\text{Si}_{\text{g.s.}}$ (14) transition

I would now like to turn my attention to the $^{26}\text{Mg}(^{16}\text{O}, ^{14}\text{C})$ transition to the ^{28}Si ground state. Experimental data now exists for this transition at 33 and 40 MeV (ref.²⁷), 45 MeV (refs.^{2,7}), 60 MeV (ref.²⁸) and 128 MeV (ref.²⁹) (see fig. 17).

Fig. 17. Differential cross sections for the $^{26}\text{Mg}(^{16}\text{O}, ^{14}\text{C})^{28}\text{Si}$ ground-state transition at 33 (ref.²⁷), 40 (ref.²⁷), 45 (refs.^{2,7}), 60 (ref.²⁸), and 128 (ref.²⁹) MeV. DWBA predictions based on identical optical-model parameters (set 1, table A1) are shown with the 33-60 MeV data. At 128 MeV it is necessary to increase the absorption in the region of the nuclear surface to $W_{\text{SD}} = 10$ MeV to reproduce the shape of the angular distribution. Note the change in angular scale between the left and right portions of the figure.

The weaker absorption for the surface region in these potentials presumably allows the transfer amplitude to build up faster, but then the rapid increase in absorption at smaller radii causes a sharp drop in the amplitude on the low l side of the l window. Similarly for the phase derivative a weaker absorption at large radii permits the real nuclear potential to have a greater effect and $\psi(l)$ deviates more from the

Oscillating cross sections are observed at all energies. Once again the lowest energy data is particularly sensitive to the imaginary potential shape. Calculations based on three different sets of optical model parameters are shown in fig. 18. The two dashed curves which have relatively weak absorption in the nuclear surface but are of Woods Saxon geometry with a diffusivity $\sim 0.5-0.6$ do not reproduce the oscillating experimental cross sections. Attempts to "fit" the data by reducing the imaginary potential keeping a fixed geometry has been unsuccessful. The solid curve in fig. 18 is based on the same real geometry as the long dashed curve, but with surface transparent imaginary potentials given as set 1 in table A1. The amplitudes, B_l^{LM} and phase derivatives $\psi(l)$ are compared in fig. 19 for the calculations using weak

Woods-Saxon imaginary potentials (long dashed curve of fig. 18) and surface transparent potential (solid curve of fig. 18). The surface transparent potential which produces oscillating cross sections is more localized in l space and has a more pronounced dip in the phase derivative, $\psi(l)$, in the " l -window".

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Fig. 18. Comparison of DWBA predictions for the $^{26}\text{Mg}(^{16}\text{O}, ^{14}\text{C})^{28}\text{Si}$ g.s. transition at an incident energy of 33 MeV using three different optical potentials. The solid curve corresponds to surface transparent potentials (set 1, table A1) and is shown with data in fig. 17. The long and short dashed curves correspond to sets 2 and 3, respectively in table A1.

Coulomb phase change for surface transparent potentials. The added localization and a smaller $\psi(l)$ both enhance the probability of oscillations^{1,3}.

The 33 MeV $^{26}\text{Mg}(^{16}\text{O}, ^{14}\text{C})^{28}\text{Si}$ g.s. data are sensitive to the magnitude of the surface absorption in the surface transparent potentials (see fig. 20). An adequate

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Fig. 20. Effect of increasing the surface absorption from $W_{SD} = 1$ to 5 for 33 MeV $^{26}\text{Mg}(^{16}\text{O}, ^{14}\text{C})^{28}\text{Si}$ g.s. angular distribution.

description of the experimental data can be obtained for $W_{SD} \leq 2$ MeV. Using identical parameters with a $W_{SD} = 1$ MeV absorption in the surface region, the shapes of the oscillating cross sections can be reproduced for 33, 40, 45, and 60 MeV. The ratio of the experimental to calculated cross section, however, decreases a factor of 2 between 33 and 60 MeV incident energy (see table 2). The

Fig. 19. Amplitudes, B_l^{LM} , and phase derivatives, $\psi'(l)$, as a function of l for the solid and long dashed calculations shown in fig. 18. The $1/e$ half widths, Γ , are indicated for the B_l^{LM} 's.

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Table 2. Surface Absorption and Normalizations for $^{26}\text{Mg}(^{16}\text{O}, ^{14}\text{C})^{28}\text{Si}_{\text{g.s.}}$ Transition analysis.

E_{lab} (MeV)	$E_{\text{c.m.}}$ (MeV)	W_{SD} (MeV) to fit shape ^{a)}	$\sigma(\theta)_{\text{exp}} / c(\theta)_{\text{DRC}}$	
			$W_{\text{SD}} = 1 \text{ MeV}$	Energy Dep W_{SD} ^{b)}
33	20.43	≤ 2	130	130
40	24.76	≤ 4	100	130
45	27.86	≤ 8	90	130
60	37.14	≤ 10	65	130
128	79.24	$\geq 5^{\text{d)}$	c)	37

a) Range of surface absorption with parameter set 1, table A1 which fits angular shape.

b) $W_{\text{SD}} = 0.54$ ($E_{\text{c.m.}} = 18.58$) MeV.

c) Shape of 128 MeV angular distribution not reproduced for $W_{\text{SD}} = 1.0$. $\sigma_{\text{exp}}(\theta) / \sigma_{\text{DRC}}(\theta) = 20$ for $W_{\text{SD}} = 10$ MeV.

d) Values as large as 100 MeV still reproduce the angular shape.

calculated cross section for the 128 MeV data using parameters identical to those used at the lower incident energies do not reproduce the shape of the experimental angular distribution. This problem is remedied by increasing the surface absorption to $W_{\text{SD}} = 10$ MeV. A calculation based on such parameters is shown with the experimental data in fig. 17. Therefore, an energy dependent surface absorption is necessary to reproduce simultaneously the shape of the angular distribution at 33 and 128 MeV incident energy. An energy dependent surface absorption decreases the variation with incident energy of the normalization between calculated and experimental cross sections. In fact, for a surface absorption of

$$W_{\text{SD}} = 0.54 \text{ (E. c.m. = 18.58) MeV}$$

the variation of the normalization with energy is removed between 33 and 60 MeV. Such an energy dependence also is consistent with the surface absorption needed to reproduce the shape of the single-nucleon transfer angular distributions for 60 MeV ^{16}O incident on ^{26}Mg . The normalization between experimental and calculated cross sections for the 128-MeV data is increased by the energy dependent absorption; however, it still is a factor of ~ 3.5 less than that of the lower energy data. This is not surprising, since the recoil corrections, which are not included in the present two-nucleon transfer calculations, are expected to vary considerably between 60 and 128 MeV.

Using such potentials the absolute experimental $^{26}\text{Mg}(^{16}\text{O}, ^{14}\text{C})^{28}\text{Si}$ ground state cross section is under predicted by a factor of ~ 130 (see table 2). It is known³⁰⁾ that a proper treatment of recoil increases the calculated cross section for this transition at 45 MeV incident energy. This is in contrast to a predicted³¹⁾ decrease in the $^{62}\text{Ni}(^{18}\text{O}, ^{16}\text{O})^{64}\text{Ni}$ ground state cross section at an incident energy of 65 MeV. Recent calculations³²⁾ for $(^{16}\text{O}, ^{14}\text{C})$ reactions on ^{208}Pb and ^{48}Ca targets suggest that

increasing the basis of single particle states over what are used in the present calculations might increase the predicted cross sections by as much as an order of magnitude. It seems imperative that large-basis full-recoil calculations be applied to this and similar cases where data exists over a large range of energies.

The transfer amplitudes, and change of phases, are shown in fig. 21 for the DWBA

Fig. 21. Transfer amplitudes, B_L^{LM} , and phase derivatives, $\psi'(L)$, for the DWBA calculations shown with the $^{26}\text{Mg}(^{16}\text{O}, ^{14}\text{C})^{28}\text{Si}$ g.s. data in fig. 17. Values both for $W_{SD}=1$ (solid curve) and 10 (dashed curve) are shown for 128 MeV.

calculations shown with the data in fig. 17. Several interesting features are noted. Between 33 and 60 MeV the L space localization remains nearly constant. The increased oscillations for higher incident energies probably are the result of a decreased ψ . The amplitudes for small L 's increase in magnitude with increasing incident energy. Contributions from such interior partial waves probably causes the anomalous angular shape predicted for 128 MeV incident energy and too weak of a surface absorption.

An anomalous behavior is observed in the phase derivative, $\psi'(L)$, for L values just below the L window (see fig. 21). A shoulder is observed in the L dependence of ψ for the lower energy data, but a sharp peak develops at higher energies. The peak is correlated in L with a minimum observed in the amplitudes B_L^{LM} ; therefore it has been associated^{33,34} with a zero in the transfer amplitude at a nonphysical value of the angular momentum not far from a physical angular momentum. Similar behavior also has been observed in calculations based on weakly absorbing potentials for several other reactions, e.g., $^{40}\text{Ca}(^{13}\text{C}, ^{14}\text{N})^{39}\text{K}$ g.s. and $^{48}\text{Ca}(^{14}\text{N}, ^{13}\text{C})^{49}\text{Sc}$ g.s. transition¹ at 50 MeV incident energy, do in some cases show similar rapid changes in phase. The effect of such an anomaly on the cross section is not large. In fig. 22 the calculated angular distribution that was shown with the 60 MeV $^{26}\text{Mg}(^{16}\text{O}, ^{14}\text{C})^{28}\text{Si}$ g.s.

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Fig. 22. Comparison of calculated angular distribution for the 60 MeV $^{26}\text{Mg}(^{16}\text{O}, ^{14}\text{C})^{28}\text{Si}$ transition shown with the data in fig. 17 with a calculated distribution having the contributions of the partial waves below $l = 25$ removed.

data in fig. 17 is compared with a calculation based on identical parameters in which the contributions from the partial waves below $l = 25$ were removed. Such a comparison indicates that the anomalous phase change near partial waves 23 and 24 and the other contributions from lower partial waves do not significantly change the character of the forward angle cross sections. Such contributions do, however, alter the pattern of amplitude modulations which Friedman, McVoy, and Shuy have shown^{34,35}) can result from the dip in $\psi(l)$ in the l window. The phases in the l

window are identical for the two curves shown in fig. 22, and indeed amplitude modulations are observed in both. Contributions from the lower partial waves (see fig. 21), which are removed in the case of the dashed curve, do change the modulation. Such amplitude modulations may be observed in our 68 MeV $^{40}\text{Ca}(^{13}\text{C}, ^{14}\text{N})$ data in fig. 1.

7. Difference in angular distribution shapes for single- and two-nucleon transfer

Significant differences between the angular shapes of single-nucleon and two-nucleon transfer are well known (c.f. ref.^{2,36,37}). Smoother single-nucleon transfer angular distributions have been explained³⁷) as less l -space localization resulting from a less rapidly decaying form factor in single-nucleon transfer and the incoherent contributions from several magnetic substates $|M| \leq L$. A dramatic example of the difference is shown in fig. 23 where experimental angular distributions for the $^{26}\text{Mg}(^{16}\text{O}, ^{14}\text{C})$ and $^{26}\text{Mg}(^{16}\text{O}, ^{15}\text{N})$ transitions to the ground states of ^{28}Si and ^{27}Al are compared. These two transitions were measured²⁸) simultaneously at an incident energy of 60 MeV using the MIT-Brookhaven time-of-flight system. In fig. 24 calculated angular shapes are shown for a progression of changes between correct two-proton and single-proton transfer. The amplitudes and phase derivatives are shown in fig. 25 as a function of l corresponding to the various curves shown in fig. 24. The change in the angular shape produced by changing the form factor in two-nucleon transfer to that of a single proton having the same binding as one of the protons in the two

Fig. 23. Comparison of the experimental angular distributions (ref. ²⁸) for the 60 MeV $^{26}\text{Mg}(^{16}\text{O}, ^{14}\text{C})^{28}\text{Si}$ and $^{26}\text{Mg}(^{16}\text{O}, ^{15}\text{N})^{27}\text{Al}$ g.s. transitions.

Fig. 24. Comparison of predicted angular shapes showing a progression of changes between the correct two-proton transfer $^{26}\text{Mg}(^{16}\text{O}, ^{14}\text{C})^{28}\text{Si}$ and one-proton transfer $^{26}\text{Mg}(^{16}\text{O}, ^{15}\text{N})^{27}\text{Al}$ g.s. The calculations were for ^{60}MeV incident energy and used optical-model parameter set 1, table A1 with $W_{SD}=10$ MeV. The solid and long dashed curves are for identical $^{26}\text{Mg}(^{16}\text{O}, ^{14}\text{C})^{28}\text{Si}$ g.s. calculations except the former has a correct microscopic two-proton form factor, whereas the latter has a single-proton form factor. The short dashed and dot-dash curves are for $^{26}\text{Mg}(^{16}\text{O}, ^{15}\text{N})^{27}\text{Al}$ transitions with spins changed to give $L=0$ and the correct $L=3$ transfer respectively.

Fig. 25. Transfer amplitudes, B_L^{LM} , and phase derivatives, $\psi(L)$, corresponding to the calculated angular shapes shown in fig. 24. The various curves correspond to calculations as described in the caption of fig. 24.

proton form factor is not as large as might be expected³⁷⁾, even though the l window is now about two units wider at the $1/e$ width. Changing the kinematic from that of $^{26}\text{Mg}(^{16}\text{O}, ^{14}\text{C})^{28}\text{Si}_{\text{g.s.}}$ to that of $^{26}\text{Mg}(^{16}\text{O}, ^{15}\text{N})^{27}\text{Al}$ makes a more dramatic change in the angular shapes (compare the long dashed and short dashed curves in fig. 24) than a change between two-nucleon and single-nucleon form factors. The correct $(^{16}\text{O}, ^{15}\text{N})$ kinematics gives an even wider l -window and makes a large difference for the phase derivative, $\psi(L)$, in the l window. Finally, additional smoothing is obtained from contributions of the various magnetic substates and the angles of the peaks are moved when the correct angular momentum is included in the $L = 3$ $^{26}\text{Mg}(^{16}\text{O}, ^{15}\text{N})^{27}\text{Al}$ calculation. Thus for the case studied the kinematics and the different angular momentum are important effects in the observed shape change between single-proton and two-proton transfer. The role of the phase in producing more or less oscillatory angular distributions also must be stressed. Similar conclusions have been made³⁸⁾ for comparison of two- and single-nucleon transfer using other targets and projectiles.

8. Difficulties in reproducing the correct phase of oscillations for $\text{Ca}(^{13}\text{C}, ^{14}\text{N})\text{K}$ transitions

Finally, I would like to discuss an anomalous behavior of the forward angle oscillations. The DWBA cross sections predicted for the $^{40,42,44}\text{Ca}(^{13}\text{C}, ^{14}\text{N})$ transitions^{9,24,39)} to the $^{39,41,43}\text{K}$ ground states shown in figs. 26 and 27 correctly reproduce the magnitude and the general shape of the angular distributions as well as

Fig. 26. Angular distributions of the $^{40}\text{Ca}(^{13}\text{C}, ^{14}\text{N})^{39}\text{K}$ transition at incident energies of 60 and 68 MeV (refs. 9,24). The data represented by open circles were obtained using the QDDD spectrometer. The curves correspond to DWBA calculations as described in the text.

the period of the oscillations; however, the oscillations predicted are out of phase with the 68 MeV data for all targets and with both 60 and 68 MeV data for the ^{40}Ca target. This is in contrast to the case of the $^{40,42,44}\text{Ca}(^{13}\text{C}, ^{12}\text{C})^{41,43,45}\text{Ca}_{\text{g.s.}}$ transitions at the same incident energies as the $\text{Ca}(^{13}\text{C}, ^{14}\text{N})\text{K}$ transitions

figs. 1 and 28 where the details of the oscillations are reproduced. The shape of the $^{40}\text{Ca}(^{13}\text{C}, ^{14}\text{N})^{39}\text{K}_{\text{g.s.}}$ transition at an incident energy of 40 MeV also is reproduced using surface transparent potentials (see fig. 3). A wide variety of optical parameters have been tried; however, we have not been able to reproduce this angular shape. It

Fig. 27. Angular distributions of the $^{40,42,44}\text{Ca}(^{13}\text{C}, ^{12}\text{C})^{41,43,45}\text{Ca}$ and $^{40,42,44}\text{Ca}(^{13}\text{C}, ^{14}\text{N})^{39,41,43}\text{K}$ transitions at an incident energy of 68 MeV (refs. 9, 39). The data on $^{42,44}\text{Ca}$ were obtained using the QDDD spectrometer. The DWBA calculations shown as curves reproduce the shape of the $(^{13}\text{C}, ^{12}\text{C})$ angular distributions, but the predicted oscillations are out of phase with the $(^{13}\text{C}, ^{14}\text{N})$ data for all three targets. The optical model parameters used were set 1, table A1. For the $^{42,44}\text{Ca}$ targets the radii were scaled to account for the increased A.

is easy to change the period of the oscillations, but that does not allow a "fit" over a large angular range. We originally thought that the explanation might be a large non-normal contribution from recoil. The $L = 2$ non-normal cross section is indeed out of phase with the $L = 1$ normal predicted cross section (fig. 28); however, the normal term is predicted to dominate.

Furthermore, the oscillations also are predicted to be out of phase for the $^{40}\text{Ca}(^{13}\text{C}, ^{14}\text{N})$ transition to the $J^\pi = 1/2^+$, 2.53 MeV excited state of ^{39}K (fig. 29). Since a $s_{1/2}$ proton is picked up from ^{40}Ca , there are no non-normal contributions for this case. Therefore, recoil effects apparently are not the source of the discrepancy.

Similar difficulties in reproducing the phase of the oscillations were experienced by deVries, et al.⁴⁰) for the $^{12}\text{C}(^{14}\text{N}, ^{13}\text{N})$ transition to the $J^\pi = 1/2^+$ state at an excitation of 3.09 MeV in ^{13}C . Therefore, similar problems exist in fitting the detailed oscillation of five different transitions--all $L = 1$! In the case of $^{40}\text{Ca}(^{13}\text{C}, ^{14}\text{N})^{39}\text{K}_{g.s.}$ the Q matching is reasonably good (see fig. 30) and the differential cross sections are several mb/sr. It, therefore, would require an unusually large second order process to explain this anomaly. Some type of a resonance phenomena seems equally unlikely, since similar effects are observed at different incident energies and at different Q values. It is imperative that such discrepancies be understood before heavy-ion transfer reactions are used as a spectroscopic tool.

9. Acknowledgements

I must acknowledge the contributions of several collaborators (listed in table 3) among the Brookhaven theory and experimental groups as well as several summer visitors and a collaboration with several experimentalist from MIT. I also wish

to thank D. Sinclair for allowing the use of his 128 MeV $^{26}\text{Mg}(^{16}\text{O}, ^{14}\text{C})^{28}\text{Si}_{\text{g.s.}}$ data before it appeared in publication.

No. Neg. #

Fig. 28. DWBA predicted angular distribution for the $^{40}\text{Ca}(^{13}\text{C}, ^{14}\text{N})^{39}\text{K}_{\text{g.s.}}$ transition at 68 MeV incident energy showing the normal $L = 1$ and non-normal $L = 2$ contributions. This predicted angular shape is shown with the data in figs. 26 and 27.

Fig. 29. Angular distributions of the $^{40}\text{Ca}(^{13}\text{C}, ^{14}\text{N})$ transition to $1/2^+$ state at 2.53 MeV excitation in ^{39}K . The open circles show data measured using the QDDD spectrometer. The curves correspond to DWBA calculations described in the text.

Fig. 30. Plot of the total cross section as a function of varying Q-value for the reaction $^{40}\text{Ca}(^{13}\text{C}, ^{14}\text{N})^{39}\text{K}$ at an incident energy of 68 MeV and assuming the configuration of the ^{39}K ground state. The proper Q-value for the transition to the ground state of ^{39}K is indicated by Q_0 .

Table 3 Collaborators

BNL			MIT
Experimentalist	Theorist	Visitors	
P. D. Bond	E. H. Auerbach	C. K. Gelbke	T. M. Cormier
C. Chasman	A. J. Baltz	O. Hansen	E. R. Cosman
M. J. LeVine	R. Fuller	W. von Oertzen	A. Sperduto
A. Z. Schwarzschild	S. Kahana		K. van Bibber
C. E. Thorn			A. J. Lazzarini
H. E. Wegner			A. Graue

Appendix

Table A1. Optical-model parameters used in calculation.

$$U(r) = V_c - V \frac{1}{1 + e^x} - i W_{\text{WS}} \frac{1}{1 + e^{x'}} + i W_{\text{SD}} 4a'_{\text{SD}} \frac{d}{dr} \left(\frac{1}{1 + e^{x''}} \right)$$

where

$$x = \frac{r - R}{a} \quad x' = \frac{r - R'_{\text{WS}}}{a'_{\text{WS}}} \quad x'' = \frac{r - R'_{\text{SD}}}{a'_{\text{SD}}}$$

Reaction/Channel	V (MeV)	R (fm)	$a = a'_{\text{SD}}$ (fm)	W_{WS} (MeV)	W_{SD} (MeV)	R'_{WS} (fm)	R'_{SD} (fm)	a'_{WS} (fm)
$^{40}\text{Ca}(^{13}\text{C}, ^{14}\text{N})^{39}\text{K}$ and $^{40}\text{Ca}(^{13}\text{C}, ^{12}\text{C})^{41}\text{Ca}$								
$^{13}\text{C} + ^{40}\text{Ca}$								

40 MeV	Set 1	33.4	7.33	0.55	18.	4.5	6.75	6.75	0.05
	Set 2	33.4	7.33	0.55	18.	-	7.33	-	0.55
60 MeV		33.4	7.33	0.55	18.	9.0	6.90	6.90	0.05
68 MeV		33.4	7.33	0.55	18.	9.0	7.00	7.00	0.05
$^{14}\text{N} + ^{39}\text{K}$									
40 MeV	Set 1	34.2	7.37	0.55	18.	0.3	7.10	7.10	0.05
	Set 2	34.2	7.37	0.55	12.	-	7.37	-	0.55
60 and 68 MeV		34.2	7.37	0.55	18.	4.5	7.00	7.00	0.05
$^{12}\text{C} + ^{41}\text{Ca}$									
40 MeV		33.4	7.29	0.55	18.	4.5	6.75	6.75	0.05
60 MeV		33.4	7.29	0.55	18.	9.0	6.90	6.90	0.05
68 MeV		33.4	7.29	0.55	18.	9.0	7.00	7.00	0.05
$^{60}\text{Ni}(^{18}\text{O}, ^{16}\text{O})^{62}\text{Ni}$									
	$^{18}\text{O} + ^{60}\text{Ni}$	70.0	8.68	0.40	18.	1.	8.38	8.68	0.05
	$^{16}\text{O} + ^{62}\text{Ni}$	70.0	8.31	0.40	18.	1.	8.01	8.31	0.05
$^{26}\text{Mg}(^{16}\text{O}, ^{14}\text{C})^{28}\text{Si}$									
	$^{16}\text{O} + ^{26}\text{Mg}$								
	Set 1	100.	6.69	0.50	65	1. ^{a)}	6.40	5.40	0.05
	Set 2	100.	6.69	0.50	20	-	6.69	-	0.50
	Set 3	35	6.25	0.69	35	-	5.70	-	0.61
$^{14}\text{C} + ^{28}\text{Si}$									
	Set 1	100	6.64	0.50	65	1. ^{a)}	6.35	6.35	0.05
	Set 2	100	6.64	0.50	20	-	6.64	-	0.50
	Set 3	35	6.21	0.69	35.	-	5.66	-	0.61
Bound State									
$^{40}\text{Ca}(^{13}\text{C}, ^{14}\text{N})$	b		$1.2A^{1/3}$	0.65					
$^{40}\text{Ca}(^{13}\text{C}, ^{12}\text{C})$	b)		$1.2A^{1/3}$	0.65					
Other Reactions	b)		$1.25A^{1/3}$	0.65					

- a) For 128 MeV it was necessary to increase this value. The calculation in fig. 17 used $W_{SD} = 10$ for 128 MeV. See also text for discussion of energy dependence of W_{SD} for this reaction. The comparison of one-proton and two-proton transfer in figs. 24 and 25 used $W_{SD} = 10$ as single-proton data seemed to prefer such a value.
- b) The bound state well depths were adjusted to give the transferred nucleon the proper binding energy.

Table A2. Summary of transitions analyzed.

Reaction	Incident Energy (MeV)	J^π	Transferred L		$C^2S_{ab}^a)$	$C^2S_{AB}^a)$	N^a Figure ^{b)}	
			Nor.	NN				
$^{40}\text{Ca}(^{13}\text{C}, ^{14}\text{N})^{39}\text{K}_{g.s.}$								
	40	$3/2^+$	1	2	0.69	4.0	1.25	7
	60	$3/2^+$	1	2	0.69	4.0	1.00	26
	68	$3/2^+$	1	2	0.69	4.0	1.25	26
$^{40}\text{Ca}(^{13}\text{C}, ^{14}\text{N})^{39}\text{K}_{2.53}$								
	60	$1/2^+$	1	-	0.69	1.53	1.60	29
	68	$1/2^+$	1	-	0.69	1.53	2.00	29

$^{40}\text{Ca} (^{13}\text{C}, ^{12}\text{C}) ^{41}\text{Ca}$		25						
40	$7/2^-$	4	3	0.77	0.80	1.25	1.67	
60	$7/2^-$	4	3	0.77	0.80	1.25	1	
68	$7/2^-$	4	3	0.77	0.80	1.50	1	

a) Normalization and spectroscopic factors as described in text. For sources of spectroscopic factors see ref. 41.

b) Fig. in which calculated cross section shown with data.

Table A3. Two-nucleon spectroscopic amplitudes used in calculations.

$^{18}\text{O} \rightarrow ^{16}\text{O} \text{ a)}$	$^{60}\text{Ni} \rightarrow ^{62}\text{Ni} \text{ b)}$	$^{16}\text{O} \rightarrow ^{14}\text{C} \text{ c)}$	$^{26}\text{Mg} \rightarrow ^{28}\text{Si} \text{ c)}$
$2s_{1/2}$ 0.450	$2p_{3/2}$ 0.795	$1p_{1/2}$ 0.88	$1d_{5/2}$ 1.03
$1d_{5/2}$ 0.893	$1f_{5/2}$ 0.991	$1d_{5/2}$ 0.08	$2s_{1/2}$ 0.51
	$2p_{1/2}$ 0.395	$2s_{1/2}$ 0.11	$1d_{3/2}$ 0.25
		$1d_{3/2}$ 0.06	

a) Ref. 22).

b) Determined from pairing wave functions of ref. 42.

c) Ref. 37).

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