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CF-56-7-49

OAK RIDGE NATIONAL LABORATORY

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CHEMISTRY--SEPARATION PROCESSES

FOR PLUTONIUM AND URANIUM

UIC

POST OFFICE BOX P
OAK RIDGE, TENNESSEE

DATE: July 16, 1956

SUBJECT: HRP-BP: A Mathematical Analysis of the
HRT Blanket Processing Plant

TO: W. E. Unger

FROM: W. L. Carter

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0.0 SUMMARY

A mathematical analysis of the solid-liquid separation step for the HRT blanket processing facility has been made. In this study equations were derived which relate the variables x , y and e where

x = solids concentration in underflow receiver at any time
 y = solids concentration in blanket circulating loop at any time
 e = time

These equations were derived for the specific condition of starting up both the HRT blanket system on UO_2SO_4 solution and the blanket processing plant (hydroclone system) at the same time and allowing the two processes to operate concurrently. It was assumed that the contents of the underflow receiver were well mixed at all times. Three equations were developed which are

$$y = \left[R - \frac{k_2}{k_1} \right] \frac{e}{V} - \frac{k_2}{k_1} P \left[e^{\frac{k_1 e}{PV}} - 1 \right]$$

$$x = \frac{k_2 e}{k_1 P} + \frac{k_2}{k_1} V \left[e^{\frac{k_1 e}{PV}} - 1 \right]$$

$$fL(1-b)x - ay = \frac{k_2}{k_1} \left[e^{\frac{k_1}{k_2 V} x} + \frac{k_1}{k_2} \{ ay - fL(1-b)x \} \right] - 1$$

The significance of the terms of these equations is explained in Paragraph 2.0.

Calculated elsewhere* but recorded here because of its pertinence to this study is the expected composition of the solids produced in the blanket system.

<u>Component</u>	<u>% weight</u>
Fe_2O_3	70
Cr_2O_3	18.1
SiO_2	9.7
ZrO_2	1.2
SnO_2	0.02
PuO_2	0.86

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1.0 INTRODUCTION

The proposed process for separating plutonium and fission and corrosion product solids from the uranyl sulfate blanket of the HRT is to use a hydroclone on the let-down stream from the high to low pressure system. In a manner similar to that of the HRT core processing plant, the solids will be concentrated in an underflow receiver as a slurry in blanket solution. Further steps in processing the contents of the underflow receiver are manifest in heavy water recovery from the slurry and plutonium recovery from the fission and corrosion products.

The low production rate of plutonium, 3.5 gm per day, suggests that the solid-liquid separation step be operated continuously while the UO_2SO_4 blanket is in the HRT. Since there is an affinity for plutonium to plate out on surfaces, it is desirable to collect this substance in one place to minimize the chance of having it distributed over the entire blanket. However, even under the most favorable conditions there is still much more opportunity for plutonium to deposit in the reactor system. The rather severe corrosion rate of the system components will contribute about 400 gm of corrosion product oxides each day, and it is possible that the plutonium will be scavenged by the much larger quantity of corrosion products. Even if the plutonium deposits on the surfaces of the system, there is a possibility that when corrosion particles flake off the walls, PuO_2 will be tied up in the particle and also flake off.

Presented in this memorandum is an analysis of the blanket system and an associated hydroclone-induced underflow receiver unit for solids removal. The study has been made from the view point of starting up the blanket and hydroclone at the same time and operating both concurrently. The desired information from such an analysis is a prediction of the solids concentration in the blanket and in the underflow receiver at any time during the operation. This type of data will allow one to choose intelligently the proper size to make the underflow receiver for the proposed operating period. This vessel should be of such capacity that the total solids concentration does not exceed ca. 200 grams per kilogram.

By way of introduction to the problem it is well to describe the action of a hydroclone and induced underflow receiver. Such a unit is diagramed in Figure 1. The feed stream containing entrained solids is introduced into the hydroclone tangentially, and the centrifugal field thus created throws the particles to the periphery. The hydroclone has two exit ports--an underflow discharge emptying into the underflow receiver and an overflow discharge carrying clarified fluid away from the system. The ports are designed so that about 2% of the feed stream reports to the underflow. The solid particles flow with the underflow stream along and down the wall of the hydroclone into the underflow receiver. The underflow receiver is much greater in cross section than the underflow port, so the down flowing stream diffuses into the contents of the receiver and loses some of its turbulence.

The underflow receiver has no open drain, and the only exit from the unit is through the hydroclone underflow port. Therefore downflowing slurry from the hydroclone must displace an equal volume of fluid from the underflow receiver through a vortex at the center of the vessel. This is called the "induced underflow". This vortex flows into the hydroclone through the center of the underflow port and outward with the hydroclone overflow stream. There will be a loss of particles from the underflow receiver via the vortex.

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FIGURE NO. 1
HYDROCLONE AND
UNDERFLOW RECEIVER
SHOWING INDUCED
UNDERFLOW PATTERN

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FIGURE 2
SCHEMATIC FLOW DIAGRAM FOR
PROCESSING UO_2SO_4 BLANKET IN HRT

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In speaking of the separating ability or efficiency of a hydroclone, two efficiencies have to be considered. One of these is the efficiency of the unit to separate a particle from the feed stream and deposit it in the underflow stream; the other is the efficiency of the unit for removing a particle from the induced underflow stream, which is leaving the receiver, and returning that particle to the receiver. Hydroclone experiments have indicated that these two efficiencies are in almost all cases equal. Of course these efficiencies will vary with particle size, but values of 60% have been obtained consistently for submicron size solids; for 2-5 micron solids, values of greater than 90% are easily obtained.

It might be expected that solids in the underflow receiver will settle and create a region of higher concentration at the bottom of the vessel. The relatively small flow from the hydroclone underflow probably will not create much turbulence. This being the case the induced underflow will not have a solids concentration representative of a well mixed underflow receiver but instead will be somewhat leaner. An assumption in the accompanying derivations is that the concentration of the induced underflow is equivalent to the concentration of a well agitated underflow receiver. This assumption was made because there was no means of knowing the degree of mixing in the receiver. The net result of such a supposition is that the underflow receiver is more concentrated at any given time than the derived equations indicate.

2.0 NOMENCLATURE

The following definitions of terms are offered:

- V = volume of blanket high pressure system, liters
- P = volume of underflow receiver, liters
- R = solids production rate in blanket (fission and corrosion products and plutonium), gm/day
- S = total solids in underflow receiver at any time, gms
- W = total solids in blanket high pressure system at any time, gms.
- L = liquid draw-off rate from the blanket to feed hydroclone, liters/day
- t = time, days
- x = solids concentration in underflow receiver at any time, gm/liter
- y = solids concentration in high pressure blanket system at any time, gm/liter
- a = efficiency of hydroclone to separate particle from feed and deposit in underflow
- b = efficiency of hydroclone to return particle leaving underflow receiver in induced underflow stream back to the underflow receiver
- f = fraction of hydroclone feed reporting to underflow

In addition to the basic quantities given above, the derivations were simplified at appropriate points by introducing the following terms:

- A = $fL(1-b)$
- B = aL
- z = $By - Ax = aLy - fL(1-b)x$
- $k_1 = PB + AV = aLP + fLV(1-b)$
- $k_2 = PBR = aLPR$
- C_1, C_2, C_3 and C_4 = integration constants

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Figure 2 has been prepared to show the flow pattern in the system and to introduce the above properties at the appropriate points.

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3.0 CALCULATIONS

There have been derived in the Appendix equations relating the concentration, x , of solids in the underflow receiver and the concentration, y , of solids in the blanket circulating system with time, e . These equations have been plotted in Fig. 3 & 4 as x and y versus e using several values of the underflow receiver volume, P , as parameters. The time cycle includes a 60-day period, which is probably representative of the time that the BRT will operate with a UO_2SO_4 blanket.

Of the several properties of the system enumerated in Paragraph 2.0, only the value of P has been varied. This is the parameter about whose influence on the system the least is known and for which a design value needed to be established. Other properties, such as the two hydroclone efficiencies a and b and the underflow fraction f , may not be equal in actual plant operation to the values used in these calculations; however, it is believed that their values are known accurately enough from laboratory work that the figures used here are approximately correct. Therefore a set of curves in which these terms were used as parameters has not been drawn. The numerical values of V , R and L are fixed by the system, and advantage has been taken of the latest information to establish their values.

The equations for x and y as functions of e are listed below. These equations are solved for the several cases following.

$$x = \frac{k_2 e}{k_1 P} + \frac{k_2}{k_1^2} V \left[e^{\frac{k_1 e}{PV}} - 1 \right]$$

$$y = \left[R - \frac{k_2}{k_1} \right] \frac{e}{V} - \frac{k_2}{k_1^2} P \left[e^{\frac{k_1 e}{PV}} - 1 \right]$$

Case I: $P = 10 \text{ gal} = 37.85 \text{ liters}$

$V = 1785 \text{ liters}$

$R = 407 \text{ gm/day}$

$L = 1 \text{ gal/min} = 5,450 \text{ liters/day}$

$a = 0.6$

$b = 0.6$

$f = 0.02$

$k_2 = aLPR = (0.6)(5450)(37.85)(407) = 50.4 \times 10^6$

$k_1 = aLP = fLV(1-b) = (0.6)(5450)(37.85) + (0.02)(5450)(1785)(0.4)$
 $= 12.4 \times 10^4 + 7.8 \times 10^4$
 $= 20.2 \times 10^4$

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$$\frac{k_2}{k_1 P} = \frac{50.4 \times 10^6}{20.2 \times 10^4 \times 37.85} = 6.6$$

$$\frac{k_2 V}{k_1^2} = \frac{50.4 \times 10^6 \times 1785}{(20.2 \times 10^4)^2} = 2.2$$

$$\frac{k_1}{PV} = \frac{20.2 \times 10^4}{37.85 \times 1785} = 2.99$$

$$\left[R - \frac{k_2}{k_1} \right] \cdot \frac{1}{V} = \left[407 - \frac{50.4 \times 10^6}{20.2 \times 10^4} \right] \frac{1}{1785} = \frac{(407-250)}{1785} = 0.088$$

$$\frac{k_2 P}{k_1^2} = \frac{50.4 \times 10^6 \times 37.85}{(20.2 \times 10^4)^2} = 0.047$$

When these values are substituted into the above equations, there results

$$x = 6.6e + 2.2 \left[e^{-2.99e} - 1 \right]$$

$$y = 0.088e - 0.047 \left[e^{-2.99e} - 1 \right]$$

These equations are plotted on Figures 3 and 4.

Case II: P = 20 gal = 75.70 liters

V = 1785 liters
R = 407 gm/day
L = 1 gal/min = 5450 liters/day
a = 0.6
b = 0.6
f = 0.02

When the constants are evaluated, there obtains

$$x = 4.1e + 1.7 \left[e^{-2.4e} - 1 \right]$$

$$y = 0.054e - 0.072 \left[e^{-2.4e} - 1 \right]$$

These equations are plotted on Figures 3 and 4.

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Case III: P = 30 gal = 113.6 liters

V = 1785 liters
R = 407 gm/day
L = 1 gal/min = 5450 liters/day
a = 0.6
b = 0.6
f = 0.02

When these constants are evaluated, there obtains

$$x = 2.96e + 1.33 \left[e^{-2.21e} - 1 \right]$$
$$y = 0.04e - 0.085 \left[e^{-2.21e} - 1 \right]$$

Case IV: P = 40 gal = 151.4 liters

V = 1785 liters
R = 407 gm/day
L = 1 gal/min = 5450 liters/day
a = 0.6
b = 0.6
f = 0.02

When these constants are evaluated, there obtains

$$x = 2.32e + 1.09 \left[e^{-2.12e} - 1 \right]$$
$$y = 0.031e - 0.093 \left[e^{-2.12e} - 1 \right]$$

These equations are plotted in Figures 3 and 4.

An examination of the above equations and their graphs shows that after about four days operation both x and y are essentially linear functions of time. This happens because the exponential term, $e^{-\frac{k_1}{PV}}$, becomes negligibly small. The blanket let-down rate L, appearing in the constant k_1 , is large with the result that solids are removed quite rapidly from the system. The rate is such that slightly more than three blanket volumes are processed each day. As would be expected, the effect of using a lower let-down rate would be to cause a slower build-up of solids in the underflow receiver and lead to higher solids concentration in the blanket at any given time.

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FIGURE 3

HRT OPERATION WITH UO_2SO_4 BLANKET:
SOLIDS CONCENTRATION IN UNDERFLOW
RECEIVER AS A FUNCTION OF OPERATING
TIME

OPERATING CONDITIONS

FLOW RATE TO HYDROCLONE	= 1.0 GPM
HYDROCLONE EFFICIENCY FEED TO UNDERFLOW	= 60%
HYDROCLONE EFFICIENCY IN INDUCED UNDERFLOW	= 60%
SOLIDS PRODUCTION RATE	= 407 gm/day
FRACTION OF FEED STREAM TO UNDERFLOW	= 2.5%
VOLUME OF BLANKET SYSTEM	= 1785 liters

FIGURE 4

HRT OPERATION WITH UO_2SO_4 BLANKET:
SOLIDS CONCENTRATION IN BLANKET
AS A FUNCTION OF OPERATING TIME

OPERATING CONDITIONS

FLOW RATE TO HYDROCLONE	= 1 GPM
HYDROCLONE EFFICIENCY, FEED TO UNDERFLOW	= 40 %
HYDROCLONE EFFICIENCY IN INDUCED UNDERFLOW	= 60 %
SOLIDS PRODUCTION RATE	= 401 gm/day
FRACTION OF FEED STREAM TO UNDERFLOW	= 2 %
VOLUME OF BLANKET SYSTEM	= 1785 liters

underflow receiver volume

10 gal
20 gal
30 gal
40 gal

SOLIDS CONCENTRATION, gm/liter

TIME, days

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4.0 RECOMMENDATIONS

On the basis of this study as well as previous knowledge on the performance of hydroclones, the following recommendations are made:

1. It is desirable to maintain the following conditions in the hydroclone system.
 - a. Total solids concentration less than 120 gm/liter in the underflow receiver to enhance hydroclone operation.
 - b. Plutonium concentrations greater than 150 mg/liter in underflow receiver to facilitate detection and measurement.
 - c. Plutonium concentrations less than 10 mg/liter in the blanket system.
 - d. An operating cycle (i.e. dump cycle) on the underflow receiver of less than 15 days to permit acquisition of a convenient number of data points during the anticipated short operating period of the HRT with a UO_2SO_4 blanket. However, because of the low production rate (3.5 gm/day) of plutonium the operating cycle should be sufficiently long to enable plutonium to build-up in the system.

An examination of Figures 3 and 4 shows that there are several combinations of operating time and underflow receiver volume that will fulfill these criteria. However, it is recommended that a 10-gallon underflow receiver be employed and operated on a 10-day dump cycle.

2. It is desirable to route the entire blanket let-down stream through the hydroclone on the way to the dump tank. This will permit the first opportunity for separation of solids into the underflow receiver before the possibility of settling in the dump tanks.

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5.0 APPENDIX

A material balance can be made around the underflow receiver to determine the build up of solids in this vessel by equating the accumulation in the vessel to difference between the quantity of solids entering and the quantity leaving. If at any time, e , the quantity of solids in the underflow receiver is represented by S , over a differential period, de , the accumulation is dS . The solids entering the system during this differential time interval are given by the product of flow rate, concentration and time, that is,

$$\text{solids entering hydroclone system} = yLde.$$

The solids leaving the system have to be represented schematically by two streams: One is the portion of the feed solids which reports to the overflow, and the second is the portion of the solids which originally reported to the hydroclone underflow but are lost from the underflow receiver via the induced underflow stream. For the first case,

$$\text{solids reporting to overflow} = yL(1-a)de.$$

The terms are defined in the preceding table of nomenclature.

For the second case,

$$\text{solids leaving via induced underflow} = fLx(1-b)de.$$

" fL " is the flow rate of the induced underflow; " x " is the concentration of solids in the underflow receiver, and " b " is the fraction of the particles leaving with the induced underflow which are returned to the underflow receiver by the hydroclone. In other words " b " is an efficiency for separation of particles from the induced underflow. This separation occurs principally in the hydroclone since the induced underflow makes its way out of the system as a vortex in the hydroclone.

The material balance equation becomes

$$dS = yLde - [yL(1-a)de + fLx(1-b)de] \quad (1)$$

The variables are S , y , x , and e . The other terms of the equation are parameters which can be considered as constants for any particular design. There is a simple relationship between x and S ; at any time

$$x = \frac{S}{P} \quad (2)$$

$$dS = Pdx \quad (3)$$

Substituting Eqn. 3 in Eqn. 1

$$Pdx = yLde - yL(1-a)de - fLx(1-b)de \quad (4)$$

$$Pdx = ayLde - fLx(1-b)de \quad (5)$$

$$\frac{dx}{de} = \frac{aLy}{P} - \frac{fL(1-b)x}{P} \quad (6)$$

The rate of change of the underflow receiver concentration is therefore proportional to the difference between the incoming concentration and the concentration of the underflow receiver.

Equation 6 contains three variables, x , y and e , one of which needs to be eliminated before the equation can be integrated. By making a material balance around the blanket system another relationship can be found. In a manner similar to that above it is seen that

$$\text{accumulation in system} = dW.$$

Solids entering the system are the sum of those produced within the system plus those brought in with the hydroclone overflow. That is

$$\text{solids entering system} = Rde + [yL(1-a)de + fLx(1-b)de].$$

Solids leaving the system are those carried away with the feed stream to the hydroclone.

$$\text{solids leaving system} = yLde.$$

By equating the accumulation to the difference between the entering and leaving solids, the material balance becomes

$$dW = Rde + [yL(1-a)de + fLx(1-b)de] - yLde \quad (7)$$

$$\frac{dW}{de} = R - aLy + fL(1-b)x \quad (8)$$

At any time, e , the concentration in the blanket system is

$$y = \frac{W}{V} \quad (9)$$

$$dW = Vdy \quad (10)$$

"V" is the volume of the blanket high pressure system. When Eqn. 10 is substituted into Eqn. 8, there results

$$\frac{dy}{de} = \frac{R}{V} - \frac{aL}{V} y + \frac{fL(1-b)x}{V} \quad (11)$$

In order to simplify let

$$A = fL(1-b) \quad (12)$$

$$B = aL \quad (13)$$

Equations 6 and 11 will therefore become

$$\frac{dx}{de} = \frac{1}{P} (By - Ax) \quad (14)$$

$$\frac{dy}{de} = \frac{1}{V} [R - (By - Ax)] \quad (15)$$

The prominence of the term (By-Ax) suggests a substitution of the form

$$z = By - Ax; \quad (16)$$

whereby,

$$\frac{dx}{ds} = \frac{z}{P}; \quad (17)$$

$$\frac{dy}{ds} = \frac{R-z}{V}. \quad (18)$$

Division of Eqn. 18 by Eqn. 17 to eliminate ds gives

$$\frac{dy}{dx} = \frac{\frac{R-z}{V}}{\frac{z}{P}} = \frac{P}{V} \frac{(R-z)}{z}. \quad (19)$$

Differentiation of Eqn. 16 and solving for dy, gives

$$dy = \frac{dz + A dx}{B}. \quad (20)$$

This equation is substituted into Eqn. 19.

$$\frac{dz}{dx} + A = \frac{PB}{V} \frac{(R-z)}{z} \quad (21)$$

$$dz = \left[\frac{PBR}{Vz} - \frac{PB}{V} - A \right] dx \quad (22)$$

Perform the indicated algebraic operations, transpose terms and there obtains

$$\frac{Vz dz}{-(PB + AV)z + PBR} = dx, \quad (23)$$

$$\text{For simplicity let } k_1 = PB + AV; \quad (24)$$

$$k_2 = PBR. \quad (25)$$

$$\text{Therefore } \frac{Vz dz}{-k_1 z + k_2} = dx. \quad (26)$$

If the indicated division is performed, the equation is in an integrable form.

$$V \int \left[\frac{1}{k_1} + \frac{k_2}{k_1 k_2 - k_1^2 z} \right] dz = \int dx \quad (27)$$

$$V \left[-\frac{z}{k_1} - \frac{k_2}{k_1^2} \ln(k_1 k_2 - k_1^2 z) \right] = x + C_1 \quad (28)$$

The boundary conditions are such that at start-up the blanket circulating system contains no solids and the underflow receiver contains no solids. That is at

$$e = 0 \quad (29)$$

$$x = y = 0 \quad (30)$$

Since

$$z = By - Ax, \quad (16)$$

$$\text{also } z = 0. \quad (31)$$

Substituting the values of Eqs. 30 and 31 into Eqn. 28, the constant, C_1 , can be evaluated.

$$C_1 = \frac{-Vk_2}{k_1^2} \ln k_1 k_2 \quad (32)$$

Substitute Eqn. 32 into Eqn. 28 and rearrange terms.

$$\frac{Vk_2}{k_1^2} \ln \frac{k_1 k_2}{k_1 k_2 - k_1^2 z} = x + \frac{V}{k_1} z \quad (33)$$

$$\text{Remembering that } z = By - Ax, \quad (16)$$

$$B = aL, \quad (13)$$

$$\text{and } A = fL(1-b), \quad (12)$$

these substitutions can be made into Eqn. 33.

$$\frac{Vk_2}{k_1^2} \ln \frac{k_1 k_2}{k_1 k_2 - k_1^2 [aLy - fL(1-b)x]} = x + \frac{V}{k_1} [aLy - fL(1-b)x] \quad (34)$$

When written in the exponential form Equation 34 becomes

$$fL(1-b)x - aLy = \frac{k_2}{k_1} \left[e^{\left[-\frac{k_1^2}{k_2 V} x + \frac{k_1}{k_2} \{aLy - fL(1-b)x\} \right]} - 1 \right] \quad (34a)$$

Equation 34a expresses a correlation between the concentration, x , of solids in the underflow receiver and the concentration, y , of solids in the blanket circulating system at any time. It should be remembered that the boundary conditions are for starting the solids removal system and the blanket circulating system at the same time and allowing the two plants to operate simultaneously. The equation would be different if solids were allowed to build up in the blanket before the hydroclone was put into service. Equation 34 is most simply expressed in this form; the values of the constants k_1 and k_2 in terms of system parameters are

$$k_1 = PB + AV = aLP + fLV(1-b)$$

$$k_2 = PBR = aLPR.$$

It remains that there be developed an expression correlating the concentration, x , in the underflow receiver with time, e . Equations 14 and 34 provide the necessary relationships for obtaining an expression in terms of x and e ; unfortunately though Eqn. 34 is not readily solvable for y in terms of x . Nevertheless if Eqn. 28 is differentiated with respect to e , there obtains

$$\frac{dx}{de} = v \left[\frac{k_2}{k_1 k_2 - k_1^2 z} - \frac{1}{k_1} \right] \frac{dz}{de} \quad (35)$$

From Eqn. 17 it is known that

$$\frac{dx}{de} = \frac{z}{P} \quad (17)$$

Equating Eqns. 17 and 35 eliminates $\frac{dx}{de}$ to give an expression in terms of z and e .

$$z = PV \left[\frac{k_2}{k_1 k_2 - k_1^2 z} - \frac{1}{k_1} \right] \frac{dz}{de} \quad (36)$$

The variables can be separated giving

$$\frac{k_2}{z(k_1 k_2 - k_1^2 z)} dz - \frac{1}{k_1} \frac{dz}{z} = \frac{1}{PV} de \quad (37)$$

For convenience in integrating simplify the expression.

$$\frac{1}{k_1 z} \left[\frac{k_2}{k_2 - k_1 z} - 1 \right] dz = \frac{1}{PV} de \quad (38)$$

$$\int \frac{dz}{k_2 - k_1 z} = \frac{1}{PV} \int de \quad (39)$$

$$-\frac{1}{k_1} \ln(k_2 - k_1 z) = \frac{e}{PV} + C_2 \quad (40)$$

The integration constant, C_2 , of Eqn. 40 may be evaluated from the boundary conditions of

$$e = 0 \quad (20)$$

$$\text{at } z = 0 \quad (31)$$

Therefore

$$C_2 = -\frac{1}{k_1} \ln k_2 \quad (41)$$

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Substituting this value of C_2 into Eqn. 40 and solving for z , there obtains

$$z = \frac{k_2}{k_1} \left[1 - e^{-\frac{k_1 \theta}{PV}} \right] \quad (42)$$

Substituting for z its value as given in Eqn. 16, and the values of the constants A and B as given by Eqns. 12 and 13,

$$aLy - fL(1-b)x = \frac{k_2}{k_1} \left[1 - e^{-\frac{k_1 \theta}{PV}} \right] \quad (43)$$

$$\text{Where } k_1 = PB + AV = aLP + fL(1-b)V \quad (44)$$

$$k_2 = PBR = aLPR \quad (45)$$

Equation 43 defines a relationship between time, θ , the underflow receiver, solids concentration, x , and the blanket circulating system solids concentration, y . Since there are two dependent variables, this expression is not too useful in determining the value of either concentration at a particular time; however, Eqn. 42 or 43 can be used to eliminate one of the concentration variables from the original differential equations--Eqns. 17 and 18. If the value of z in terms of θ from Eqn. 42 is substituted into Eqn. 17, there is obtained

$$\frac{dx}{d\theta} = \frac{k_2}{Pk_1} \left[1 - e^{-\frac{k_1 \theta}{PV}} \right] \quad (46)$$

$$\int dx = \frac{k_2}{Pk_1} \int \left[1 - e^{-\frac{k_1 \theta}{PV}} \right] d\theta \quad (47)$$

$$x = \frac{k_2}{Pk_1} \left[\theta + \frac{PV}{k_1} e^{-\frac{k_1 \theta}{PV}} \right] + C_3 \quad (48)$$

It is known that $x = 0$ at $\theta = 0$; therefore

$$C_3 = -\frac{k_2 V}{k_1^2} \quad (49)$$

Therefore

$$x = \frac{k_2 \theta}{k_1 P} + \frac{k_2 V}{k_1^2} \left[e^{-\frac{k_1 \theta}{PV}} - 1 \right] \quad (50)$$

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Where

$$k_1 = aLP + fL(1-b)V \quad (44)$$

$$k_2 = aLPR \quad (45)$$

Equation 50 is an expression for the concentration of solids in the underflow receiver as a function of time only. The equation might be plotted as x versus e with any of several parameters; however, the parameter which influences the design the most is P , the volume of the underflow receiver, since this is the factor over which there is the most control. The other quantities such as R , L , V , a , b and f are essentially fixed by the design of the reactor and the hydroclone and are therefore essentially constant.

In a manner similar to the derivation of an equation of $x = F(e)$, an expression of $y = F'(e)$ can also be found by using Eqs. 18 and 42. Substituting Eqn. 42 into Eqn. 18

$$\frac{dy}{de} = \frac{R}{V} - \frac{k_2}{k_1 V} \left[1 - e^{-\frac{k_1 e}{PV}} \right] \quad (51)$$

$$\int dy = \frac{R}{V} de - \frac{k_2}{k_1 V} \int \left[1 - e^{-\frac{k_1 e}{PV}} \right] de \quad (52)$$

$$y = \frac{R}{V} e - \frac{k_2}{k_1 V} \left[e + \frac{PV}{k_1} e^{-\frac{k_1 e}{PV}} \right] + C_4 \quad (53)$$

Since $y = 0$ at $e = 0$,

$$C_4 = \frac{k_2 P}{k_1^2} \quad (54)$$

Inserting this value of C_4 into Eqn. 53 and rearranging terms,

$$y = \left[R - \frac{k_2}{k_1} \right] \frac{e}{V} - \frac{k_2 P}{k_1^2} \left[e^{-\frac{k_1 e}{PV}} - 1 \right] \quad (55)$$

Where $k_1 = aLP + fL(1-b)V$

$$k_2 = aLPR$$

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Equation 55 is the desired expression in y and e , and using it the solids concentration in the blanket circulating system at any time e can be evaluated. It is pointed out again that these derivations are based on the fact that at $e = 0$ both $x = 0$ and $y = 0$, i.e., both the blanket circulating system and the blanket processing plant are started up at the same time. If the blanket is allowed to accumulate solids before the blanket processing plant is put on stream, then $y \neq 0$ at $e = 0$ and $x = 0$. In this case the constant of integration would take on a different value.

Any of the terms R , P , L , V , a , b and f may be used as parameters when plotting Eqn. 55; however, it is probably more significant to use P , the volume of the underflow receiver, since the other terms are more closely fixed by reactor and hydroclone design.

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